Supporting lower-achieving seven- and eight-year-old children with place value understandings

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Children can sometimes appear to understand a concept such as place value without really having a deep understanding. Judy Bailey stresses the importance of listening carefully to children to identify their current understandings and then building on them systematically, using a range of materials, to promote a deep conceptual understanding.

Introduction

A reality for classroom teachers is that they have responsibility for supporting all children in their class even when the children represent a wide range of mathematical understandings and competence. The spread of number understandings in any one class can be broad (Gervasoni, 2011). Catering for lower-achieving children in such settings can be quite a challenge, particularly for new teachers.

This paper reports on the collaboration of the author (a university mathematics educator) and a relatively new classroom teacher. The teacher sought to address the needs of a group of lower-achieving children within the regular class mathematics programme. This small study was part of a two-year project focused on raising school-wide achievement in mathematics in the school.

The setting and people involved

The class was a coeducational composite class of seven- and eight-year-old children in a relatively large urban primary school (catering for children aged five–eleven years) within a middle socio-economic suburb. The teacher, whom I shall call Linda, was in her second year of teaching, and it was her first year working with children this age. As a collaborative teacher/investigator, I brought to the classroom my own teaching experience together with knowledge about mathematics teacher education.

Linda had previously gathered and examined test data (Progressive Achievement Tests) and through working with these children for the previous two terms had identified nine of the class of 27 children as not achieving at the expected levels in mathematics. During the study the author conducted individual interviews, covering a range of number ideas, with the nine children. These confirmed Linda's prior observations and concerns about their level of understanding.

The focus

Linda wanted to support these children with their mathematics development, but was uncertain about how to do this within the normal mathematics programme. The syndicate, including this class, was scheduled to undertake a unit on number, with an expected focus on place value and addition and subtraction. The intended unit focus aligned with Linda's knowledge of the children's learning needs and the author's observations of the children's understanding of whole number place value. This investigation took place during a series of lessons over a period of two weeks.

Since place value is so fundamental to developing deeper number sense, we hoped that an initial focus on this topic would enable the children to

make progress with their learning in mathematics. We were especially interested in beginning to explore what it takes to help lower-achieving children develop understandings of place value.

Place value

Place value involves the learning of a number of ideas. Children need to learn to integrate their experience of counting by ones with grouping numbers in tens, and linking these groupings to the corresponding written numeral and how numbers are said (Van de Walle, Karp, Lovin and Bay-Williams, 2014). It is noted that some children, even if successful at reading, writing and ordering numbers, do not necessarily fully understand two and three digit numbers (Gervasoni, 2011). A number of approaches are described in texts supporting teachers of primary mathematics, with some advocating for a gradual teaching of place value through the development of mental strategies for adding and subtracting numbers, with more formal approaches using base ten materials being another possibility (Jorgensen and Dole, 2011).

Nature of the collaboration

It would be fair to say that in the course of this investigation I acted as Linda's assistant. We discussed our ideas for the lessons, shared planning, and we both interacted with and observed the children during the lessons and collected samples of the lower-achieving children's written work. We both also recorded notes after the lessons. Analyses of these data enabled us to consider (a) what was happening with the children's learning, (b) other issues as they arose during the lessons, and (c) ongoing adjustments that we needed to make to better support the children's learning.

For convenience, we called the nine lower-achieving children Group A, and the rest of the class Group B.

Our joint planning of the unit

To help with planning we first looked to a number of sources including:

- the school's syndicate unit plan for place value and associated resources;
- the *New Zealand Curriculum* document (Ministry of Education, 2007);
- a *Ministry of Education* website (www.nzmaths.co.nz); and the
- *New Zealand Mathematics Education Handbook for Primary and Intermediate School Years* (Biddulph, 2011).

We found that thinking about and researching the range of ideas children need to encounter when learning place value was initially time-consuming but essential and valuable. We were reminded of the need for teachers to not only know about children's current understandings, but also have a full understanding about place value themselves including the likely progression of children's ideas. This planning process can be challenging for new teachers in terms of the many demands made on their time. Research has shown that teachers also need to customise learning experiences to meet children's varying learning needs (Gervasoni, 2011; Young-Loveridge, 1999).

Having consulted the resources and considered the prior assessment data, we decided that a key focus for development in this short series of lessons would be for the children in group A to learn how many tens are in numbers up to and including 100, and Group B children would look at how many tens there are in numbers up to 1000. We decided to start with two-digit numbers for Group A because as Young-Loveridge (1999) explains, children need to understand that tens are a different kind of unit compared to ones, before moving onto working with numbers in the hundreds and thousands. While it is important not to place a ceiling on children's learning, the challenges they are set also need to be achievable.

Our planning also extended to a consideration of suitable equipment to use with the children given it is thought the use of equipment is vital for helping children to develop their understandings (Van de Walle, Karp, Lovin and Bay-Williams, 2014; Young-Loveridge, 1999).

Equipment and discussion

It is widely recognised that using equipment can help children learn mathematics. The use of equipment alone, however, does not ensure children will make connections between the physical representations and abstract mathematical ideas (Chandler and Kamii, 2009). Students need an opportunity to discuss the ideas being modelled and explored, and need to mentally construct ideas for themselves (Higgins, Wakefield and Isaacson, 2006). In this respect, questions asked by the teacher are critical (Reinhart, 2000). A wide range of materials are available for teachers to use in their teaching of place-value, for example, small objects that can be grouped into tens, tens frames, Slavonic abacuses and so on (see Young-Loveridge, 1999). These considerations formed a back-drop to our planning. We settled on using four pieces of equipment: a Slavonic abacus (an abacus of 100 individual beads arranged in rows of ten), place value blocks, numeral cards and calculators.

What we did, and what happened

Our initial efforts

As an introductory activity we gave the children an opportunity to practise skip-counting in tens. We used the abacus to support this process with the intention of physically demonstrating, particularly to the children in Group A, that each time a new number was said (10, 20, 30…), another group of 10 was being added to the previous number. After orally chanting the sequence from 10 to 100 a few times with the aid of the abacus, we asked the children what number comes after 100 if we are counting in tens. Not surprisingly, at least one-third of the class, including children from both Groups A and B, seemed uncertain. With the benefit of hindsight we realised it would have been helpful to have another abacus available to show the children groups of 10 over 100. If two abacuses were not available, then multiple tens frames, an easily available and cheap resource (see http://www.nzmaths.co.nz/sites/default/files/Numeracy/2007matmas/Bk4/MM%204_6.pdf for a master copy) or another quantity picture (see Young-Loveridge, 1999) could be used for the same purpose.

Following the introductory task, the numerals one to nine were set out on the floor in front of the circle of children and questions were asked to establish how many small cubes should be placed to match each numeral. (Dienes blocks representing one were used—see figure 2.)

Figure 1. Tens frames.

The children could do this easily. We then asked them how many cubes there would be if one more was added to the nine. The children agreed there would be 10, and the numeral 10 was duly placed by the pile of 10 cubes.

To provide opportunities for discussion we asked the children (who were sitting in a large circle) to work with the person they were sitting next to. Each pair of children was then given a 'long' (a Dienes block representing ten, with the ten units marked), and asked what they noticed about the long in comparison to the pile of ten cubes. During this activity we observed a couple of interesting things, namely:

- a few of the children in group A in particular did not appear to make the connection that one ten had the same numerical value as ten ones; and

- all children were excited and engaged by the use of the equipment, and were very eager to participate and be involved in handling it.

A similar process was then followed for numerals between 10 and 20 starting with the number 11. Questioning was used again to support the children to make connections between the equipment and numerals.

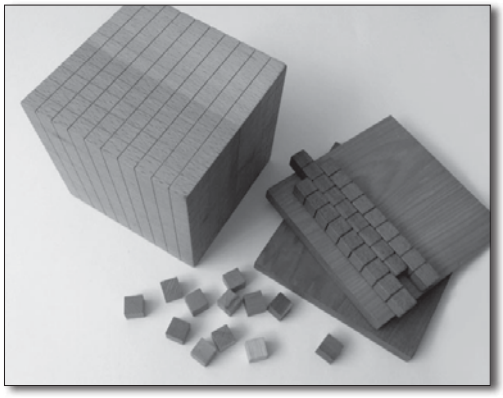

Figure 2. Dienes blocks showing thousands, hundreds, tens and ones.

For example, "Now, I'm going to add one more to the 10. How many do I have now? How do I write that? 11—what does this 'one' mean? (Pointing to the one in the tens position) What does this other 'one' mean? (Pointing to the one in the ones position)". At this stage the answers of children in Group B showed they understood the links between the numerals, the position of each digit in the number and the place-value equipment.

When one more block was added to the 19 (making 20) some children could see the pattern of two (in the numeral 20) meaning two tens. More 'longs' were then added (showing the numbers 20–90) and children were asked more questions to highlight how many groups of ten were in each of the numbers 20–90. Most children appeared to grasp this well, maybe because of the link to the introductory activity that had focused on skip-counting in tens.

While we acknowledge the rather procedural nature of this introductory task, the main intent was to support the children to begin working with the equipment and have the opportunity to make links between the equipment and their existing or developing place-value ideas. We have also realised we could have used more open questions at this early stage to encourage more thinking amongst the children. Linda also made suggestions about using money for future lessons, and we acknowledge incorporating real-life contexts may have also further enhanced our initial efforts.

Consolidating learning

The class was then divided into the two groups. The children in Group A were asked to choose three decade numbers (10, 20, 30, 40 …), write how many groups of 10 were in their chosen numbers, and draw a picture of each number, showing the groups of 10. For reasons of confidentiality, the names of children that appear below are pseudonyms.

The pictures drawn by four of the nine children clearly showed an appropriate number of groups of 10 (most drew a 'skinny' rectangle or a straight line representing the 'long' Dienes block representing 10). Another three of the group appeared to have the beginnings of some understandings. For example, Daniel drew an appropriate number of skinny rectangles (representing the longs) for two out of three of his chosen numbers. Daniel's remaining picture showed eight skinny rectangles for 80, and also 18 single small squares, suggesting some confusion. The work of the remaining two children showed no understandings of groups of 10, their drawings showing a collection of discrete squares corresponding to the chosen number but no groupings of 10. For example, Alice drew a long line of 20 individual squares for the numeral 20. We were not surprised by these findings, knowing that learning takes time, and often many experiences, before a concept is learned. It also seems possible that for at least these two children more experiences were needed with discrete materials that could be gathered into groups of 10 before moving to pre-grouped place-value blocks. This would be congruent with the suggestion in literature that for children at the beginning stages of learning about place value it is important for materials to be proportional and groupable, that is, single units such as beans or small cubes that can be gathered into groups of ten (Van de Walle, Karp, Lovin and Bay-Williams, 2014; Young-Loveridge, 1999).

We noticed that eight of the nine children drew their pictures showing the ten ones within the groups of ten. We wonder if this suggests that

these children do not yet 'trust' the count of 10 within one group of 10. Again, this would not be surprising. As Van de Walle, Karp, Lovin and Bay-Williams (2014) suggest, children need to establish for themselves that counts by ones and an equivalent count by tens and ones will result in the same number.

Catering for diversity: Same key idea, different numbers

In the meantime, Group B children explored how many tens were in numbers up to 1000. Once we had recapped the number of hundreds in a number such as 200 (which was easily achieved) the key question being explored was, "How many groups of ten are there in 200 (300…)?" The children were given opportunities to share their thinking with another child whom they were sitting next to, and were also asked to justify their thinking. For example, the question, "How do you know?" was asked. The children were also asked if they could see a pattern or a 'clue' in the numeral that told them how many tens were in that number, that is, there are 30 tens in 300. Most of the children were able to justify the number of tens in a number such as 300, and some made reference to the place value equipment to support their explanations. Interestingly, however, when the numeral 300 was written in a place value chart, not all of the children could make the connection between 30 tens and 300. When asked in this setting, how many tens were in 300, their answers ranged from zero to three to 30 and 300. This idea clearly needed more exploring and time to develop a solid understanding. We were also left wondering whether it had been wise to introduce the place value chart at this point in the lesson.

An interesting happening

Towards the end of this lesson, one child who had appeared to not be paying much attention to the final discussion suddenly asked a question. He had heard the statement, "there are 10 groups of 10 in 100" and thoughtfully asked, "How do we know that?". It was delightful to see his interest and efforts at making sense of this particular idea for himself.

Next lessons

During the following lessons, Linda was keen to continue planning for and teaching two groups for mathematics. She provided opportunities for the Group A children to consolidate and then extend the learning that had begun, for example, by including numbers with a digit in the ones column (e.g., 23). In the first follow-up lesson, she asked the children to again choose a number (from the range 10–90) and draw a picture to show how many groups of ten. Interestingly, one child who had previously been drawing groups of 10 reverted to drawing single squares in the first follow-up lesson. She noticed the other children continued to initially count the 10 ones in the tens bar, before eventually appearing to make a connection with groups of ten. More detailed observations and interviews would have been needed to verify this suggestion. Once working with numbers such as 23, it was encouraging to note that just three children needed some initial assistance to engage in the set task.

Meanwhile, the children in Group B worked with three digit numbers exploring different ways of representing a number of their choice. For example, 234 has two 100s, three 10s and four 1s; it also has 23 10s and 4 1s, and so on.

Introducing the calculator

In the final lesson that was recorded for this small study, the children were introduced to using calculators to help support their counting in groups of 10 (or a different self-selected number for children in Group B) and making connections to the number of tens in a number. This proved to be useful consolidation.

The children were initially given an opportunity to 'play' with their calculator, and were then

asked to key in + 10 = = =. After exploring the pattern created by repeatedly pushing the equals button, the children were asked to 'predict' the next number before pressing the 'equals' button. This was particularly useful when counting across the hundreds, that is, from 90 to 100 and 100 to 110. Place value equipment was used in conjunction with the calculators to illustrate the groups of ten in each number.

We noticed that the children were absolutely delighted with their explorations, and went on to 'discover' all sorts of things for themselves. For example, one of the children in Group B was very excited that she had created and correctly named a large number in the tens of thousands.

To encourage the children, all were given a long strip of paper and asked to record their number sequences. Children in Group A focused on counting in tens. We were fascinated to notice that by the end of the lesson some of the children in Group A were successfully able to predict the 'next number' including when crossing the hundreds—something they had previously been unable to do. For example, Elisabeth was able to correctly predict that 110 comes after 100 and, when asked what comes after 230, said 240. This contrasted markedly with her incorrect answers the previous day when she had been asked what comes after 40. Some of the children in Group A were also beginning to make connections with the digit in the tens column indicating how many groups of ten were in that number. However, not all children made these links. For instance, we noticed two children in particular who seemed to have difficulty making connections between the varying contexts, that is, how numbers on the calculator related to the place value equipment.

Other teachers' efforts

We found out subsequently that several other teachers in the school were working with their children on place value at about the same time as us. Two of their experiences are relevant for this paper. A teacher of middle and higher achieving six-year-olds used ice block sticks as equipment. She fastened 10 sticks into one lot of 10 with a rubber band but when she asked her children how many groups of ten they had, they said 10 and wanted to write 10 in the tens column. Clearly they could see 10 things and they were not going to be persuaded otherwise.

Meanwhile a teacher of five-year-olds introduced her children to place value through the use of Cuisenaire rods. The children soon came to see that if the small white cube was one, then the orange rod was 10. They could see it as one 10, and happily went on to show their teacher that 12 was one 10 (an orange rod) and one 2 (a red rod), and eventually got to showing her that 23 must be two 10s (two orange rods) and three more (a light green rod). This finding is worth further investigation, particularly in light of various authors' suggestions that an extra effort must be made when working with pre-grouped materials such as a Cuisenaire rod to highlight the 'ten-ness' of the pre-grouped piece of equipment that represents ten ones. A critical point to explore is whether the children understood that two orange rods and the one light green rod did indeed represent 23 ones. Is it possible that some equipment is more helpful in supporting children to make connections to ten-ness than others?

What can be learnt from these experiences?

As most primary teachers know, helping all children understand the abstract mathematical idea of place value is quite a challenge. Although the teaching efforts reported in this paper were not all successful, it is nevertheless possible to gain some insights from them.

- Being very clear about the key idea(s) we hope the children will learn is a critical first step in the planning/teaching process, and one that is not necessarily easy for very busy, new teachers to establish.

- To provide direction for a series of lessons it is important to (a) listen to children to identify their current thinking and understanding, and (b) refer to multiple teacher resources about likely progressions in understanding.

- Getting some children beyond focusing on ones to making connections between a group of 10 ones and a group of 10 can be quite tricky. The choice of equipment (from the

experiences reported in this paper it would appear that Cuisenaire rods may have potential for supporting children's place-value learning) and/or providing alternative activities such as children inventing their own ways of solving addition and subtraction problems (Chandler and Kamii, 2009) are matters to further explore.

- Just because children appear to understand, and can say how many groups of tens there are in a hundreds number (for example that there are 30 tens in 300), we found this does not necessarily denote full understanding; they may become confused for instance when the same number is written on a place value chart. The timing and value of introducing place-value charts is another issue needing further exploring.

- Lower-achieving children's drawings may assist them in their learning, and can also reveal their understandings. This can be useful for formative assessment.

- Giving all children opportunities to talk with each other and justify their thinking, seems to support engagement and learning. This includes children asking their own questions, and making an effort to make sense of the ideas for themselves.

- The introduction of calculators generated considerable child involvement and also appeared to lift the ceiling on all children's developing mathematical understanding. Expect to be surprised by some children's depth of understanding, as revealed through their use of the calculator.

- It is possible to cater for lower-achieving children in a regular classroom to enable them to make progress, but they need unhurried time and multiple experiences to make the necessary connections. As a relatively new teacher, Linda found working with two groups one way of catering for the range of mathematical understandings in her class.

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Acknowledgements

My thanks to "Linda" for welcoming me into her classroom, and her focus on supporting all learners in her care; and to my colleague, Fred Biddulph, for his ongoing support during the research and writing of this article.