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Students' Perceptions of the Secondary Numeracy Project

A thesis submitted in partial fulfilment of the requirements for the
degree of

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by

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Abstract

This thesis explores the perceptions and experiences of twenty four Year 9 students from two schools in New Zealand who had participated in the Secondary Numeracy Project. The two schools were in their first year of SNP at the time of data collection. The main focus was on four areas of mathematics learning: group work, equipment, communication and teachers. Data was collected mainly by using semi-structured and clinical interviews. Findings revealed that equipment was particularly important and were used more frequently by students in Low ability group. Students liked using equipment, working in groups and sharing multiple solutions. However, communication was not used much as a means of making sense of mathematics in these classrooms. Furthermore the students' responses depended on the ability groups. These findings complemented the work of other researchers who have explored students' perspectives at the primary level. These findings suggest that the SNP would be more successful if students were explicitly taught good communication and cooperative learning skills.

DEDICATION

This thesis is dedicated to my loving mum
Without whose unconditional love and constant prayers
I wouldn't have made this far
Thank you Mum for always being there for me

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“In the name of Allah, the most Merciful, the most Beneficent”

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Chapter 1: INTRODUCTION

The Numeracy Development Project was initiated by the Government of New Zealand in the year 2001 to improve students' achievement in mathematics. The focus of the Numeracy Project was to give students a better understanding of number properties. The three key features of this project are the research-based approach, number framework and ongoing professional development for teachers. Initially, emphasis was given to young children. However, the project was introduced to intermediate schools and started in secondary schools in 2006 as the Secondary Numeracy Project (SNP).

This research investigated the perceptions and experiences of Year 9 students of mathematics, from two different schools in their first year of the SNP. The study examined students' views about; the nature of mathematics, students' perceptions and experiences on group work, use of equipment, sharing multiple solutions or making sense of mathematics and their teachers' role. In addition, the students were also given some mathematics tasks. The data was collected using semi and clinical interviews. The analysis also took into account the mathematical ability of the group of students.

Rationale

My past experience as a secondary mathematics teacher in the Maldives led to my present curiosity and eagerness in investigating students' low achievement in mathematics. It has always been my desire to improve the students' mathematics score even if it was limited to the student's in my classroom. My theory was that, if students had a good grasp of the content, then they would not have to memorize long tedious mathematical formulae and or procedures. When I started my studies here at the University of Waikato I realized that this theory was necessary but not sufficient to raise the achievement level of students. It occurred to me that I had not been doing any better, it was just another way of teaching mathematics in a

rote manner. During my studies here, I came across the Numeracy Development Project (NDP) which interested me. My main interest in the project was that the project, it was about reform mathematics. I wanted to gain a better understanding of the project so that I could take the knowledge and experience back with me to the Maldives. I want to be able to make a difference to the way mathematics is taught in the Maldives, after all big changes happen with small steps.

Purpose and research question

A lot of research had been done to find the perceptions and experiences of mathematics learning in young children. However, secondary students' perceptions haven't been investigated much in the past. In addition, Secondary Numeracy Project officially started in 2006. Hence, it is important to find out the students' perceptions, attitudes, achievement and experiences about the project.

The main research question addressed in this study is:

What are the perceptions and experiences of Year 9 students of mathematics from two different schools in the first year of the Secondary Numeracy Project in New Zealand?

Overview of the thesis

This chapter looks gives an overview of this thesis, rationale and presents the research question. Chapter two reviews the literature on mathematics educational reform focusing mainly on mathematics education in New Zealand Education. The chapter concludes with the research methodology. Chapter three gives an overview of the method undertaken in collecting data and analysis of the methods. The findings of this research are presented in Chapter four. The chapter looks at students' perceptions about mathematics, their experience and views towards group work, using equipment, communication and students' responses to the mathematical tasks they were provided. The findings are discussed in Chapter five. Limitations of the study, implication and recommendations for further research are discussed in Chapter six.

Chapter 2: LITERATURE REVIEW

2.1 Introduction

This chapter reviews the literature starting from the students' voice and their perceptions of mathematics. This is followed by international mathematics educational reforms and the Numeracy Development Project (NDP) as the reform movement of mathematics education in the New Zealand. However, before going on to NDP a small discussion of the relationship between Numeracy and Mathematics is provided. Initially the theoretical framework of the reform structure will be explored, moving on to discussing the literature on group work, equipment, communication, followed by the methodology of research methods undertaken. The chapter concludes with a summary.

2.2 Students' voice

United Nations Convention on the Rights of the Child in 1989 ensured the children got their rights to actively participate in all matters concerning them (Noyes, 2005). Noyes admits that this was a starting point for using student perspectives to develop educational processes internationally. Pupil voice is considered as an significant element in furthering our understanding of teaching and learning as they are seen as an important part of their own learning (McCallum, Hargreaves, & Gipps, 2000). Duffield, Allan, Turner and Morris (2000) demonstrated the importance of listening to the pupil's voice in order to focus on learning and achievement rather than on their performance and 'standards' of achievement. Moreover, at times McCallum et al (2000) found that the student's thinking echoed formal theories of learning. For example the students talked about listening and discussion fostered connecting ideas and helped problem solving. Furthermore, McCallum et al also found that children as young as seven years were mirroring some of the thinking of secondary school

students. Learning what students know and what they don't know would help the teachers and educators create a more student friendly environment for them (Guillaume & Kirtman, 2005).

An important example of usefulness of students' voice in New Zealand could be *Te Kotahitanga* project by Russell Bishop (Kane & Maw, 2005). This project is one of the most influential researches funded by the Ministry of Education. This project explores the experiences of young Maori students in secondary school, why the achievement of Maori students are so low and drop out rates are high. One of the most important findings of the research was the discovery that young Maori students valued teachers who would enable them to bring their cultural experiences to the learning conversation. This research helps Maori students achieving better. Also, students need be aware of their rights. For example, Taylor, Hawera and Young-Loveridge (2005) found that the students they interviewed weren't aware of the opportunities offered in mathematics learning, like communication.

2.3 Students' perceptions of mathematics

Students' perception of mathematics is linked with teachers' beliefs and curriculum. Beaton, Mullis, Martin, Gonzalez, Kelly and Smith (1996) found a clearly positive relationship between a stronger liking of mathematics and higher achievement. So it is important for students to have a positive attitude towards mathematics. According to Grootenboer (2002) students' view of mathematics is structured by their school experiences, teachers. However, teachers' philosophy of teaching is deeply rooted in their own beliefs (Dindyal, 2005). Grootenboer (2002) found that teachers who perceived mathematics as unchanging, fixed subject made up of abstract concepts and rules would give importance for drill and practice in his or her teaching. It would be difficult for such a teacher to teach mathematics in a constructive way. In addition, Franke and Carey (1997) and Aldridge, Fraser and Huang (1999) had the view that the nature of curriculum influences the learning environment. For example, exam-driven curriculum leads to teacher-centered learning approaches in the classroom. However, teachers are

pressured to prepare students for the exams, and at the same time they are pressured to implement constructive learning techniques (Aldridge et al., 1999).

Students need to have a positive attitude towards the subject. This would be the first step towards learning mathematics. Teachers need to ensure that the students enjoy mathematics and they want to engage in mathematics activities (Steward & Nardi, 2002). This would mean teachers need to make the class an interactive environment where students' voice is valued and interests met. However, Dindyal (2005) believes that as long as the traditional teachers don't change their core beliefs, there is little hope for successfully implementing the reform mathematics.

2.4 Theoretical Framework

The four major learning theories in mathematics are the Behaviorist, Developmental, Humanistic and Social Constructivist Learning Theories (Biddulph & Carr, 1999). Behaviourism focused on observable behaviours where attention was given to teach step-by-step computational procedures (Battista, 1994). On the other extreme, constructivism emphasize on learning rather than teaching which is the latest form of theoretical personality of mathematics education (Gadanidis, 1994). Confrey and Kazak (2006) states that the evolution of constructivism was centred around researchers' interests in the child's reasoning to understand the richness of students strategy and approach. Constructivists believes that knowledge is constructed by the individual learners (Barton, 1993; Confrey & Kazak, 2006) were the students' previous knowledge are the basis of construction (Barker, 2001). Constructivists believe that the teacher cannot transmit mathematical knowledge directly to students, but students construct it by resolving situations they find problematic (Alsup, 2005; Anthony, 1995). Harlow, Cummings and Aberasturi (2006) had the idea that external reality can be observed and critically evaluated by the students. Harlow et al. believe that 'without such evaluation, concepts cannot be accepted, rejected, integrated, or refined' (p.42). In fact, Confrey and Kazak (2006) located misconceptions as one of the roots of constructivism.

Misconceptions established clearly that learning was not

a simple and direct accumulation of ideas and beliefs, with simple correction and replacement, but that its course would be circuitous, demanding revisiting and revising ideas as they gained intellectual breadth and power, and requiring careful attention to learners' thoughts and perceptions. It further signalled that one's view of epistemology and one's philosophy of intellectual development in mathematics could be seen as relevant to an understanding of learning (p.308)

The purpose of learning theories is to improve student achievement and understanding of mathematics. Findings show that there is a strong relationship between the learning environment and student outcomes (Aldridge et al., 1999). Teachers should aim to make the classroom a community of inquiry, a problem solving and problem- posing environment and teacher playing the key part as an advisor, supporter and guide (Biddulph & Carr, 1999). In a community of inquiry the students are encouraged to think about the wider issues to understand the concept rather than remembering the algorithms. Emphasis is given to social interaction and to developing a collaborative learning environment (Barker, 2001; McClintock, O'Brien, & Jiang, 2005). Critics would say that students discovering concepts is too Time consuming (Gadanidis, 1994). However, the end product would be fruitful. Also Clements (1997) points out that constructivism is neither a switch to put on or off nor a type of learning where we can say today we learn constructively and tomorrow we will use another type of learning theory. According to Clements it is a learning philosophy where all people learn all the time. Constructivism is a way of thinking, a philosophy of learning, not a methodology.

2.5 Mathematics Reform - International

The qualitative methods could be adapted as a means of exploring multiple realities of individuals and obtains detailed in-depth information about the individuals (Mertens, 1998). Interviews are the most common method used to collect qualitative data. Semi structured Interview provides greater flexibility and

permits a more valid response from the informant's perception of reality (Burns, 2000). Interview is a social and interpersonal encounter so the interviewer would need to make the participant feel secure to talk freely (Cohen, Manion, & Morrison, 2000). One very recent method for collecting data particularly used in mathematics education is clinical interviews.

A major turning point for the reform of mathematics education came from the United States. The release of Curriculum and Evaluation Standards for School Mathematics in 1989 (United States), the National Council of Teachers of Mathematics (NCTM) increased a major mathematics reform movement in the schools (Battista, 1994). The National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000) encourages teaching for understanding with the students actively building new knowledge from experience and prior knowledge. NCTM advocates moving away from the traditional focus on listening to the teacher for acquisition of facts, memorization of isolated information, computational skills, drill and practice and textbook-based instruction (McClintock et al., 2005). Instead, NCTM advocates moving towards a curriculum of hands-on activities and intellectually challenging problems. Where the focus is on conceptual understanding, making connections to real-life, problem solving, reasoning, sense making and communication as a fundamental goals of instruction (Battista, 1994; Ollerton, 2007; Sylvia & Lynn, 1999; Wood, 2001). This complex and more sophisticated forms of interaction where students are involved in mathematical arguments, reasoning and justifying themselves places students rather than teachers, at the centre of instruction (Wood, 2001). Hence the focus in reform is on learning instead of teaching (Drake, n.d). Generation of knowledge could be fostered by developing a classroom environment where students valued individual voices, promoted risk taking, supported the active participations of all students, and allowed them to be reflective about their thinking (Jones & Underwood, 2007; NCTM, 2000; Whitin & Cox, 2003). In such an environment the teacher acts as an organizer, challenger, and a facilitator of student achievement (McClintock et al., 2005; Wood, 2001).

The importance of an effective classroom learning environment has been

increasingly recognized over the past twenty years (Aldridge, Fraser & Huang, 2001). In addition, the poor performance of most western countries in the Third International Mathematics and Science Study (TIMSS) reinforced the need for a different approach in mathematics teaching (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, & Gould, 2005). The Third International Mathematics and Science Study (TIMSS) is the largest of the international comparative studies undertaken by International Association for the Evaluation of Educational Achievement (IEA) in the past thirty seven years (Beaton et al., 1996). The main purpose of the TIMSS was to improve mathematics and science learning by focusing on educational policies, practices, and outcomes. According to Beaton et al. (1996) more than half a million students in 15,000 schools from more than 40 countries around the world took part in the study and the data was collected in more than 30 languages. Singapore was the top performing country in the TIMSS followed by other Asian countries like Korea, Japan and Hong Kong (Beaton et al., 1996; Holden, 2000; Prystay, 2004; Schemo, 2000; Weisman, 2005).

Hence, United States turned to Asia, mainly Singapore for answers on how to improve the standard of mathematics in their students (Prystay, 2004; Vogel, 1996). Prystay acknowledged that the Singapore curriculum was developed over the past few decades by experts hired by Ministry of Education in Singapore. The experts worked in conjunction with the maths teachers. The elementary textbooks in Singapore covered only one third of that of USA but it goes into much more detail, and on the other extreme Singapore students are taught high school algebra at grade seven and eight (Prystay, 2004). This contrasts with the USA curriculum, where the topics are not covered thoroughly and the problems not challenging (Vogel, 1996). Vogel (1996) also revealed that the countries at the top in TIMSS are not ideal either. For example, Korea had the largest class size in the study but the highly motivated students and the need to prepare for the major exam in the ninth grade more than compensate for the class size. Similarly Ramakrishnan (2000) argues that the success of Singapore students in the test was not due to the superior conceptual understanding, problem solving capabilities, or genetic makeup of Singapore students but their rigorous curriculum, attitude towards education, preparation for paper and pencil examinations. Based on the TIMSS

1999 Video Study, Anthony and Walshaw (in press) concluded that the student achievements cannot be linked with teaching styles.

Teachers try to incorporate reform methods in their teaching. For example, Bucholz (2004) discovered that the two months she spent in teaching number sense, by developing the students' communication skills was a considerable amount to spend on today's crowded curriculum. However, her students emerged from this study as amazing thinkers, ready to take on any challenge that came their way and having a solid understanding of number sense. In another study, Williamson (2007) was teaching children multiplication tables. Williamson found that it was vital to help children develop a range of calculating skill as well as emphasising on process instead of focusing on the end result. She believed that the emphasis on process had important implications for children's willingness to 'give tasks a go' and become more active learners (Williamson, 2007). Jensen, Whitehouse and Coulehan (2000) investigated the process (observing, making an abstraction, recording and then communicating using the language of mathematics) followed by today's mathematicians to solving problems. They then used the process as an effective framework for students' learning of mathematics in their classrooms. All these teachers had to turn to another teaching approach as they figured out that the traditional approach was effective for teaching mathematics for their students.

Whitin and Cox (2003) made up a Bill of Rights for mathematical thinkers using her children's reflections. This bill captures many of the children's comments and reflects the mathematical communities advocated by NCTM. The Bill is a good summary of the expectations for the reform mathematics in today's classrooms.

A Mathematician's Bill of Rights

As a mathematician I have the right to:

Pose my own questions

Create, revise, and abandon hypotheses

Hear and reflect on the thinking of my peers

Initiate my own investigations

Share my rough-draft thinking

Solving problems in ways that make sense to me
Question the reasons behind the procedures
investigate the unexpected
Be sceptical of numerical information
Describe and define mathematical ideas in my own language
Build on the ideas of my peers
Represent my ideas in my different ways
Capitalize on mistakes as sites for learning
Challenge ideas in a respectful way
Learn about the history of mathematical ideas and language
Solve a problem in more than one way (p. 139)

Problem solving

Problem solving is an important component of reform mathematics (Celia & Angiline, 2005; Collier, 1999; NCTM, 2000; Ollerton, 2007; Sylvia & Lynn, 1999; Wood, 2001). Collier (1999) defined problem solving as ‘the art of knowing what to do when you do not know what to do’ (p. 28). An essential feature of the problems is the presence of an element of challenge and puzzlement, otherwise pupils are faced with triviality and a potential lack of stimulation (Ollerton, 2007). Problem solving provides students with opportunities to develop knowledge and at the same time consolidate this knowledge (Ollerton, 2007). The example given by Ollerton was the ‘Palindromes’ problem where students were required to choose two digit number, reverse the number and add this to the original number. If the answer was not a palindrome, they have had to repeat the procedure until they got one. For example, take 57, reverse is 75, add them to get 132, not palindrome. So continue, reverse of 132 is 231, add them to get 363 which is a palindrome. When students are engaged in this activity they are using addition and at the same time working systematically, classifying results, recognising patterns and seeking generality.

Problem solving tasks are a challenge to many students, but there are things teachers could do help them. Clarke (2003) investigated students’ belief about the nature of mathematics. They found that students believed that if they could not solve a problem almost immediately, then it was impossible to solve it. In

addition, students had a negative attitude towards getting stuck or struggling with a problem for long (Collier, 1999). Teachers could help change their attitude by building confidence and overcoming the anxieties. This could be achieved by finding a starting point that all students could initially engage in, then extending the task for the students who needed a challenge (Ollerton, 2007). Moreover, Confrey and Kazak (2006) claims that the problems existed independent of the solver. Hence another way of helping the students become efficient in problem solving was by teaching a fruitful set of techniques (Confrey & Kazak, 2006). For example Singapore students visual tools to understand concepts called “bar modelling” which when used for a number of years help them to breakdown complex problems and do rapid calculations in their head (Prystay, 2004). Finally, an important way of building confidence in students is by showing them the vulnerability of the teachers (Whitin & Cox, 2003). The more risks teachers take by opening up to the unexpected and unanswerable situations, the more confidence students get. Thus teaching students the attitude that not all mathematics problems could be solved immediately.

2.6 Numeracy versus Mathematics

People need basic literacy and numeracy skills to survive. Hence students need essential skill including being numerate, able to read, write, spell and communicate (Young-Loveridge, 2002a). Numeracy is not just about basic numbers or arithmetic skills (Hogan, 2000; Stoessiger, 2002). To be numerate in New Zealand, has been defined by the Ministry of Education (2001) as ‘the ability and inclination to use mathematics effectively in our lives – at home, at work, and in the community’ (p.1). Moreover, to be numerate a person needs a combination of mathematical, contextual and strategic knowledge (Hogan, 2000). Hogan explained the three type of knowledge in the following manner. Mathematical knowledge is the use and understanding of the mathematical ideas and techniques in Number, Space, Chance and Data, Algebra and Measurement. Contextual knowledge is the ability to link mathematics to life experiences. Finally, Strategic knowledge is the ability to identify and use the appropriate knowledge of

mathematics in solving a problem and interpret the outcomes. Therefore, numeracy is a subset of mathematics (Stoessiger, 2002).

2.7 New Zealand Reform – Numeracy Development Project

New Zealand developed Numeracy Development Project to raise the mathematical achievement of the students. Like many other western countries, New Zealand students performed poorly on the Third International Mathematics and Science Study (TIMSS) in 1995 (Beaton et al., 1996). As a result, New Zealand put greater emphasis on teaching and learning of literacy and numeracy particularly in the early years (Bobis et al., 2005; Young-Loveridge, 2002a). The definition for being numerate in New Zealand was (and is) ‘the ability and inclination to use mathematics effectively in our lives – at home, at work, and in the community’ (Ministry of Education, 2001, p.1). The consequence of the focus in numeracy was shown in the launching of Numeracy Development Project (NDP) initiated by the government in 2001 after a pilot project in 2000 (Bobis et al., 2005; Higgins, Bonne, & Fraser, 2004; Holton, 2005).

The NDP has developed rapidly. New Zealand NDP initially started with the Early Numeracy Project [ENP] for children in years 0 to 3. This was followed by the Advanced Numeracy Project [ANP] for children in years 4 to 6. The Intermediate Numeracy Project [INP] for students in years 7 to 8 followed. The Secondary Numeracy Project [SNP] for students in years 9 and 10 was launched in 2005. Also Te Poutama Tau a programme for years 1 to 8 students in Maori medium settings (Young-Loveridge, Taylor, & Hawera, 2005). The Numeracy Development Project has a dynamic approach which includes development of a research-based framework to describe mathematics learning, individual task-based interviews to assess children’s mathematical thinking and ongoing professional development for teachers (Bobis et al., 2005; Higgins et al., 2004; Ministry of Education, 2004; Young-Loveridge, 2005c).

Teachers play a very important part in the NDP to raise the students’ achievement. Increasing levels of abstraction in children’s level of understanding mathematical

concepts and representing them are the core ideas of NDP (Holton, 2005). Teachers' confidence, subject matter and pedagogical content knowledge are seen as critical factors in advocating these core ideas (Bobis et al., 2005; Ministry of Education, 2006a). Hence, the project focuses on improving student achievement by improving the professional capability of teachers (Bobis et al., 2005). Young-Loveridge (2002b) points out the importance of providing learning opportunities for the teachers. If not, the teachers would just stick with the traditional approach or add a few features like group work while retaining their same goals and lesson designs (Hiebert, 1999; Young-Loveridge, 2002b).

The research based Number Framework consists of two main sections: strategy and knowledge which are interdependent on each other (Ministry of Education, 2006a). Bobis et al. (2005) define strategy as a 'sequence of global stages describing the mental processes students use to solve problems with numbers' (p.44), and knowledge as the key pieces of information that students need to learn in order to be able to use strategies effectively. Strong knowledge is essential to broaden the strategies used with numbers as knowledge is usually a pre-requisite for the development of advanced and sophisticated strategies (Ministry of Education, 2006a). For example, new strategies learnt would become knowledge through appropriate repetition.

The strategy section of the framework (*see* Appendix A) is a model consisting of nine stages. New strategies are built on existing ones, and if faced with unfamiliar problems or when mental load gets high, students frequently revert to previous strategies (Ministry of Education, 2006a). The first five stages (Stage 0 to 4) show the increasing sophistication of the counting strategies. The transition from stage four to five is a major step for students as this is the stage where they stop counting and start using part-whole strategies for addition and subtraction problems (Young-Loveridge, 2005b). The remaining four stages (stages 5 to 8) require students to use part-whole strategies. These strategies are based on the knowledge of using number properties to 'break numbers' apart (partitioning) and recombine them in ways that make the problem easier to solve without counting (Young-Loveridge, 2001) (*see* Appendix A for a copy of the framework detail of

the characteristics of each stage). For example, students could use bridging through ten or compensation to solve addition/subtraction. Three operational domains of the number framework consists of addition/subtraction (goes up to stage seven), multiplication/division and proportion/ratio (go to stage 8) (Bobis et al., 2005).

Effective development of number sense can enhance students in learning algebra. Being able to use part-whole strategies marks the beginning of additive thinking which leads to multiplicative thinking which in turn helps to develop proportional reasoning, prepares the students for algebra (Irwin & Britt, 2005a; Ministry of Education, 2006d; Young-Loveridge, 2005b). The potential benefit of the Numeracy project is the methods students acquire in the project, particularly the use of multiplicative part-whole thinking may provide a foundation for algebraic thinking (Irwin & Britt, 2005a; Ministry of Education, 2006c; Young-Loveridge, 2005b). The students in NDP are encouraged to adopt a variety of mental strategies to solve arithmetic problems such as compensation, factorization, and maintaining equivalence which were traditionally first learnt in algebra (Irwin & Britt, 2005a). These students have the opportunity to apply them to numerical problems before they are exposed to literal symbols of algebra (Irwin & Britt, 2005a). Furthermore, present researchers' advice is to introduce algebraic thinking from an early stage (Irwin & Britt, 2005a). This could be accomplished in young children through activities that encourage students to move beyond such stages as numerical reasoning to more general reasoning about relationships, ways of notating and symbolizing (Ruopp, Cuoco, Rasala, & Kelemanik, 1997).

2.8 Group Work

There is a difference between students *in* groups and students *working in* groups. Being in a group doesn't mean that students are learning together as a group (Johnson & Johnson, 1999; Leikin & Zaslavsky, 1999). Some kinds of learning groups facilitate student learning while other types of learning groups hinder student learning and create disharmony and dissatisfaction (Johnson & Johnson, 1999). Hence the structure of a group can determine success or failure (Lindaucer

& Petrie, 1997). Lindauer and Petrie distinguished three different structures for learning: individual learning, competitive learning and cooperative learning. Most commonly used was individual learning, where each student is responsible for his/her own learning. Second most widely used was competitive learning, which forced students to become winners or losers. The least used method was cooperative learning, where students were expected to learn to work in mixed ability groups to achieve common goals. In cooperative learning, students were structured so there was interdependency, individual accountability, face-to-face interaction, and processing of social skills among the students. Ideally, the latter structure is what teachers would want to achieve when forming group, provide less threatening environment for students to work successfully together to achieve their goals (Dossey, Giordano, McCrone, & Weir, 2002; Thomson & Brown, 2000). This type of grouping is known as cooperative learning group.

Cooperative learning has some distinguishing features. The five distinguishing characteristics of cooperative learning outlined by Johnson and Johnson (1999) which Thomson and Brown (2000) found to form the acronym PIGSF (Pigs Fly) are:

- P** Positive interdependence
- I** Individual accountability
- G** Group reflection
- S** Small group skills
- F** Face to face interaction

Thomson and Brown (2000) describe PISGF as follows: Positive interdependence is the 'heart of co-operative learning' where students develop 'sink or swim together' attitude to working in their groups. A job is not done if even one student hasn't finished. The members perceive that if one fails then all fail. Individual accountability requires students to take responsibility in his or her own learning and for contributing to the group. The goal of co-operative learning is, what students do in group today, they can do individually tomorrow. Group reflection is the key to continuous improvement as the students reflect on the functioning of the group. Small group skills like strategies for learning and interpersonal skills should be taught for effective group work. Finally, a face to face interaction

encourages talking and discussions.

Leikin and Zaslavsky (1999) proposed the above conditions apart from group reflection and face to face interaction for the characteristics of cooperative learning. They described cooperative learning as ‘exchange-of-knowledge’ where students were given opportunities to play the role of a teacher to offer explanations to their peers and students were allowed to work individually when appropriate. In fact for quality teaching Anthony and Walshaw (in press) highlighted the importance of using both individual and group processes to enhance and engage the students in creating mathematical knowledge. They acknowledged that all students need time alone to think and work quietly.

Cooperative learning offers great deal of positive contribution to mathematics education. Cooperative learning has many positive effects if properly implemented in the mathematics classrooms (Thomson & Brown, 2000; Walmsley & Muniz, 2003). First, cooperative learning enhances students’ achievement (Qin, Johnson, & Johnson, 1995; Whicker, Bol, & Nunnery, 1997). However, it is important for the group to be heterogeneous in ability, motivation, age and race (Lindaucer & Petrie, 1997). Providing rewards for the group with provisions for individual accountability was successful (Lindaucer & Petrie, 1997). It was also found that students liked working in groups, especially to learn difficult concepts (Whicker et al., 1997). However, the positive effects of cooperative learning results in successful implementation (Walmsley & Muniz, 2003). This is a skill students had to learn explicitly from their teachers (Thomson & Brown, 2000).

2.9 Ability Grouping

Ability grouping is defined as the practice where “students of perceived similar achievement levels are placed in the same classroom or group” (Zevenbergen, 2003; 2). According to Linchevski and Kutscher (2002) this is the most commonly used technique for grouping students in the same grade. Ability grouping is frequently described in terms of ‘Tracking’ where students are grouped on a subject-by-subject basis or ‘Streaming’ where students are grouped

for all subject at once or ‘setting’ where students are placed in ability classes for certain subjects only (Boaler, 1997; Cahan & Linchevski, 1996; Linchevski & Kutscher, 1998; Zevenbergen, 2003).

Teachers mostly prefer ability grouping. Linchevski and Kutscher (1998) states that a high percentage of mathematics teachers advocate ability grouping claiming that it is the best way to improve the scholastic achievements of all students. One reason for this view is their belief, that mathematics is a hierarchical discipline where concepts build on previous concepts (Zevenbergen, 2003). Another reason for the preference of ability grouping is to reduce the heterogeneity of the learning group by keeping students with similar level of prior knowledge together (Cahan & Linchevski, 1996). Linchevski and Kutscher (2002) have highlighted that the proponents of Tracking and Streaming justified themselves for grouping students saying that it is easier to adapt class content, pace and teaching methods to students who are functioning on same level.

Teachers find ability grouping favourable but the researchers think it is not the best way for the students. According to researchers such as Boaler (1997), Cahan and Linchevski (1996), Zevenbergen (2003), there is no evidence that ability grouping improved scholastic achievements of students in homogeneous classes. In fact the scholastic achievements of students in low and average ability group decreased considerably, whereas, there was no significant increase in achievement among the students of high ability group (Linchevski & Kutscher, 2002). Young-Loveridge (2005d) acknowledged that students who gets most from ability grouping is the higher achieving students. This kind of pattern she referred to as the “Matthew Effect” because the higher achievers’ gets better while the low achievers’ achievement decreases.

There are a variety of reasons for low achievers to decrease their achievements. According to Perso (2006) one reason for this lack of success by low achieving group might be because explicitly lowering their expectations by labelling the classes as ‘basic’ or ‘fundamental’ maths classes leads students to perform more poorly. Other disadvantages faced by the low achieving students evident in the

research literature include tendency of the teachers to under-estimate the low ability students, reduced pace and the likelihood of providing less qualified and less experienced teachers resulting in valuable class time spent on classroom management by mostly giving emphasis on ‘seatwork’ to manage students (Boaler, 1997; Gamoran, 2002; Linchevski & Kutscher, 1998).

In fact, ability groups perpetuate inequality. Dividing students into ability groups segregates students by race, ethnicity, and social class (Gamoran, 2002). Boaler (1997) argues that ability grouping has also created and maintained inequality. This, in turn, widens the gap between the low achieving students and those in high achieving groups (Linchevski & Kutscher, 2002). Research has shown that the achievement gap has widened with students on the NDP as well (Bobis et al., 2005). It has been found by Cahan and Linchevski (1996) that ability grouping has cumulative effect on students’ achievement. According to Cahan and Linchevski the longer students stay in these homogeneous groups, the wider the gaps between the students, unlike students in mixed ability groups. A study done by Linchevski and Kutscher (2002) explored the gap between the students in the two extremes learning together was different from the gap that would be expected on the basis of initial differences between the two groups.

Mixed ability whole class teaching or within class ability grouping is becoming more acceptable now (MacIntyre & Ireson, 2002; Mevarech & Kramarski, 1997; Pratt, 2006). Linchevski and Kutscher (2002) found that mixed ability grouping prevented the widening of the gap between the students in the two extreme groups (low and high ability groups). In fact Linchevski and Kutscher’s research showed that the students in average and low achieving groups significantly gained in their achievement while the achievement gain of students in the high ability group was negligible. In addition, the study highlights that the gap did not increase in mixed ability classrooms, concluding that students in all ability groups can learn in mixed ability classes. Also Mevarech and Kramarski (1997) used an innovative instructional method called IMPROVE¹ (Introducing the new concepts, Metacognitive questioning, Practicing, Reviewing and reducing difficulties, Obtaining

¹ The acronyms of IMPROVE represents all the teaching steps used in the method.

mastery, Verification, and Enrichment) for teaching mathematics in heterogeneous classrooms and investigated the effects of using IMPROVE on students' achievement. They found that IMPROVE students significantly outperformed the control group. Findings like these show that mixed ability groups could be used for successful teaching. Thus, Boaler (1997) points out that now psychological and educational researchers widely acknowledge that students do not have a fixed ability which can be determined at an early stage of their lives. Hence, placing them in a fixed ability group can fix their potential achievement in that level (Boaler, 1997).

2.10 Equipment

Students can involve in hands-on activities by using Equipment. Equipment represents concrete materials which symbolize mathematical situations and ideas (Dossey et al., 2002). English (1998) believed that effective equipment or mathematical tasks would generally be “those engaging students’ intellect, capturing their interest and curiosity, developing their mathematical understanding and reasoning processes, and allowing for different solution strategies, solutions, and representational forms” (p. 67). Choosing a ‘transparent²’ equipment to represent the mathematical concept is very important (Clements & McMillen, 1996), so as to inevitably lead the students to the correct mathematical construction (Moyer, 2001). Moreover, the students should be given opportunities to explore equipment through social activity (Pape & Tchoshanov, 2001). In addition, Wheatley (1991) emphasized the need to present the equipment or activities in a familiar setting so the students will have a greater opportunity to use their prior experience in giving meaning to the task. Furthermore, Piagetian theory suggest students should be given a chance to reflect on the physical *or mental* actions as well (Baroody, 1989). This is crucial as students’ knowledge is strongest if they can make the connection between real world situation, equipment and written symbols (Lesh, 1990).

² *Transparent* in this context means that the equipment present abstract relationship in an easily apprehensive form so that the teacher can draw students’ attention to them (Cobb, 1988)

Equipment is very helpful in learning mathematical concepts. Equipment is useful for learning and doing mathematics, communicating and making connections (Clements & McMillen, 1996; Dossey et al., 2002; Meira, 1998; Ministry of Education, 2006c; Moyer, 2001; National Council of Teachers of Mathematics, 2000). Equipment helps students to organize their thinking about a concept and to shed light on an idea not fully understood in another form (Dossey et al., 2002; Fennell & Rowan, 2001). According to Cobb et al. (1992) students experience mathematical understanding when they create and manipulate equipment in ways that they can explain and justify when necessary. One very important use of equipment acknowledged by Wheatley (1991) was to encourage students' use of imagination. He explains that the use of imagery could be developed by encouraging students to build mental pictures and communicating their images by designing activities that promote the use of visual imagery. Most importantly, teachers should make the value of imagery explicit to the students (Higgins, 1999; Wheatley, 1991). Imagery is an essential step towards learning abstract concepts in mathematics as described by the teaching model developed for the NDP (Ministry of Education, 2006c). The model advocates the students' use of concrete materials to develop mathematical ideas and then start using images to solve problems and finally use number properties to represent number ideas (Higgins et al., 2004). An essential characteristic of good representations is that the students should grow out of equipment; the purpose of them is to enable students to think through the problem (Fennell & Rowan, 2001).

Teachers play a major role by observing and guiding the students to make the connection between the equipment and the mathematical concept. This enables students to see the relationship between their informal understanding of the concept and new knowledge (Ministry of Education, 2006c; Moyer, 2001), translate the mathematical idea into a form that they can mentally or physically manipulate to gain understanding (Fennell & Rowan, 2001), and finally help them to build 'integrated-concrete ideas' (Clements & McMillen, 1996). English's (1998) study revealed that students liked using equipment when learning mathematics too. The fifth and seventh graders in his study requested for more representational material. English's finding reminds the educators and teacher that use of equipments and hands-on activities should not decline with increasing

grade level. Moreover, there is evidence that the effectiveness of equipment is seen most clearly in long-term use of equipment with symbolic instruction, for specific topics and entire mathematics curriculum, across different grade levels (Sowell, 1989).

Baroody (1989) and Moyer (2001) agree that using equipment does not guarantee acquisition of conceptual understanding. The process of using equipment is more complex than it might appear. Equipment has to be used judiciously and carefully for the best results (Baroody, 1989), otherwise students might get more confused or use equipment remotely. Students could complete mathematical tasks by applying procedural instructions to manipulate symbols and use this procedure as a strategy to solve problems (Baroody, 1989; English, 1998; Fennell & Rowan, 2001). For example, if students were given equipment and asked to follow a set of instructions in order to get the answers, they would get the correct answer. However, this does not require any level of mathematical thinking on the students' part, equipment does not have meaning on its own.

Humans interpret and gives meaning to the equipment and symbols in terms of his or her own mathematical ways of knowing (Cobb et al., 1992; Moyer, 2001). However, Moyer (2001) has acknowledged three problems teachers face when trying to incorporate students' exploration and discussion of equipment in the classroom. The first is the huge curriculum to complete and prepare students for the end-of-year exam. The second is the pressure on teachers to keep their classroom quiet. The third is the teachers' limited knowledge of mathematics to guide the students' discussions to make the connection between the equipment and relevant mathematical concepts. Meira (1998) pointed out the meaning of equipment attached by the manufacturer is not always transparent to the students or even the teachers. Hence it is important to choose meaningful equipment.

Teachers need to be knowledgeable about the equipment as well as have strong mathematical content knowledge to be able to help the students. Moyer's (2001) study showed that teachers use manipulatives for all sorts of reasons ranging from problem solving, reinforcement and enrichment of concepts to have a change of pace in a mathematics classroom, as a reward and privilege, a visual model, like another strategy and just to make maths fun. It is easy to misuse equipment (Thompson,

1994). For example, Pearson (1996) discovered that it was important to check whether the equipment was appropriate for the students, had the prerequisite knowledge needed for the activity and adapted the activity if needed to cater for the targeted students.

2.11 Communication

Mathematics is a body of knowledge, evolved gradually through a process of argument and proof (Brosnan, 2002). Hence Brosnan believes that learning to argue about mathematical ideas should be fundamental to understanding mathematics. Therefore, the most significant change in recent mathematical education is the emphasis on the thinking process students go through to get the answer, not the answer (NCTM, 1989, 2000). The thinking process needs to be communicated in order to understand and make sense of mathematical concepts (Leikin & Zaslavsky, 1999; Ministry of Education, 2006c; NCTM, 2000). Hence students need to communicate mathematically.

Cobb, Wood, Yackel & McNeal (1992) defined communication in mathematics classrooms as a process in which the teacher and students come to a mutual agreement on mathematical meanings and practices. However, communication is not just being able to identify and use correct mathematical vocabulary, notation, and structure to express ideas but also to think and reason mathematically (Cai & Kenney, 2000). Students need to communicate if they want to validate his/her thinking, justify an answer or just want to clarify their thinking to others (Boaler, 1997; Pratt, 2006; Whitenack & Yackel, 2002; Yackel & Cobb, 1996). Moreover, students' reasoning is viewed as acts of participation in communal practices that they and the teacher establish in the course of their ongoing interaction (Cobb, Stephan, McClain, & Gravemeijer, 2001). Hence, carefully listening to others' ideas is just as important (Cai & Kenney, 2000; Pratt, 2006). These situations can create learning opportunities for students and help them to understand and make sense of the particular concepts they learn (Boaler, 1997; Whitenack & Yackel, 2002). In fact, Jones and Tanner (2002) and Wood (2001) found that students learn most quickly when they get an opportunity to identify and resolve

discrepancies between then current understanding and new information. Therefore, the clarity and completeness of students' explanation, argument or justification determine their level of understanding (Cai & Kenney, 2000).

Fostering an environment where students feel comfortable in sharing and discussing ideas, asking questions and taking risks is a critical role for the teachers (Kazemi, 1998; Reinhart, 2000; Whitenack & Yackel, 2002; Yackel & Cobb, 1996). Teachers could 'set the stage' providing students with opportunities and expectations where the students need to use reasoned arguments to critique, examine and validate their mathematical knowledge (Anthony & Hunter, 2005; Cobb et al., 1992). The normative aspects of mathematical discussion specific to students' mathematical activities are known as Sociomathematical (Yackel & Cobb, 1996). Hence, sociomathematical norms govern mathematical discourse in a classroom (Kazemi, 1998; Yackel & Cobb, 1996). Kazemi (1998) observed four sociomathematical norms taking place in his study; explanations were supported by mathematical reasons, each student was accountable for the work of the group, students drew mathematical connections between strategies and mistakes created opportunities to engage further with mathematical ideas. Therefore, to create these situations for mathematical learning in classrooms, teachers must resist their temptation to give students information, make the task simpler, or step in and do part of the task (Wood, 2001).

Teachers need to encourage students to seek, formulate and critique explanations, creating a community of inquiry (Bobis, Mulligan, Lowrie, & Taplin, 1999; NCTM, 2000). Such an environment make use of open-ended investigations where students are encouraged to find multiple solutions and in the process becomes autonomous learners (Bobis et al., 1999). One misconception about autonomous learners is that learning would result in noisy and chaotic classrooms and the teacher end up *teaching* nothing (Moyer, 2001). But in reality, such an environment must be highly organised to ensure learning occurs with minimal distraction (Bobis et al., 1999).

One way of fostering communication is by encouraging students to find multiple

solutions to problems (Cai & Kenney, 2000). Mathematical communication will help students to let go of ‘the fixed mathematics’ taught in the traditional maths (using formula/algorithms) and start trying to make sense of the problems and answers they get. Anthony and Hunter (2005) emphasize that students should not only identify what procedures they have used but also should know *how* they work in order to make sense of mathematical concepts. Once again teachers play an important role here by directing the students to the desired path by asking questions which make them think and using strategies to require all students to participate (Ministry of Education, 2005; Reinhart, 2000).

Reinhart (2000) lists a few key points to help teachers to encourage students thinking. Mostly it is learning of questioning techniques, teachers should ask good questions which require more than just recalling a fact or reproducing skills. Reinhart discovered changing his lesson plan to a set of good questions to be a very useful strategy for improving his teaching. Ollerton (2007) defines this type of questions as open-ended questions. According to Ollerton, an open-ended task has several variables that the students can choose to change. Instead of asking ‘What is 4×6 ?’ This question can be opened up by asking, ‘I have multiplied two numbers together and the answer is 24. What could the two numbers be?’

The current New Zealand Curriculum highlights the use of communication in learning mathematics for understanding (Ministry of Education, 1992). A New Zealand study involving a content analysis of School Certificate examination papers in mathematics from 1992 to 1997 revealed that an increased emphasis had been placed on communication; writing explanations and justifications which the students did not respond to well (Bicknell, 1999). A recent research done by Young-Loveridge, Taylor and Hawera (2005a) on students’ ideas about communicating mathematical thinking and strategies with their peers showed that students are still not used to the idea of engaging in mathematical arguments in their classrooms. A number of students who had participated in the NDP did not support the idea of sharing their answers or solution strategies. Students could not distinguish between cheating and being helpful or co-operative.

A number of studies had been done in New Zealand on reforming communication. A study done by Hunter (2006a) on students involved in an inquiry classroom for one year revealed that communication affects their construction of mathematical understandings. Students in the research saw inquiry and debate as a tool for analysing and reconstruct reasoning. Hunter's (2005) study on one teacher's journey to communication reform showed the students shift from context of strategy reporting communities to inquiry and argument communities. The study also revealed that students constructed increasingly sophisticated discourse with the changes in the classroom norms and increased student autonomy. However, the teacher need to take multiple roles; as a facilitator of the discourse, participant in the discourse and a commentator about the discourse (Hunter, 2006b) . These findings shows implementing communication reform takes time for the students and teacher (Anthony & Hunter, 2005). It is clearly important to stress and expose students to mathematical arguments and communication from kindergarten to college (Walter, 2003).

2.12 Methodology

Piaget's belief that a child is not a miniature adult led him to the invention of clinical method for exploring children's ideas (Vygotskii, 1986). Clinical interviews are now used commonly by mathematics educators to explore children's ideas/understandings, misunderstanding or strategies they used to solve mathematical problems (Biddulph, Carr, Gerhrke, Taylor, Hawera, & Bailey, 2003; Ginsburg, 1997; Heirdsfield, 2002; Hunting, 1997; Storey, 2001; Vygotskii, 1986). Ginsburg defines clinical interview methods as involving 'intensive interaction with the individual child, an extended dialog between adult and child, careful observation of the child's work with "concrete" intellectual objects, and flexible questioning tailored to the individual child's distinctive characteristics' (p. ix). The goal is to keep an open mind to learn how the *child* thinks and how the *child* constructs a personal world (Ginsburg, 1997). This is particularly helpful as Sylvia and Lynn (1999) found that students' oral explanations were superior to their written work as the verbal explanations were clearly expressed and included

supportive details omitted in writing. Hence, avoid exclusive reliance on counts of correct answers associated with pencil-and-paper test (Goldin, 1997).

There are some important points to keep in mind when doing a clinical interview. Biddulph et al. (2003) and Ginsburg (1997) highlight a few key points to keep in mind during a clinical interview session:

- Ask questions in a non-threatening way
- Adopt an unhurried relaxed approach by using understanding voice tone and body language
- Listen a lot
- Avoid telling/letting the child know the answer is right/wrong
- Seek clarification when necessary
- Being flexible with the questions (eg. Abandon or make up questions as appropriate) and avoid using mathematical terms like ‘plus’, ‘minus’ in the questions.
- Use short prompts like “How did you solve that problem?” would be handy
- Most importantly, interviewers should be prepared for the interview.

Experience and preparedness of the interviewer is crucial in clinical interview. Students might respond differently to teachers with different levels of experience with the Number Framework as they might ask more specific questions to get a better responses (Irwin & Niederer, 2002). Language also plays an important role in the method and clarification of the meaning as researchers ask questions and children talk about their mathematics (Hunting, 1997). The interviewers should have a good understanding of the concepts being explored and know the norm of the type of study so that they would be successful in interpreting, theorizing, trying to make sense of what the child does and says (Ginsburg, 1997, p. 120). Hence, Hunting (1997) advices practitioners to be both humble and wise to be successful; humble in the sense of being prepared to gain insights into the mathematical learning process of students, and wise in understanding strengths and limitations of the method.

2.13 Gap in the Literature

There are not many researches done on teaching and learning in secondary schools internationally and in New Zealand as well. There are researches about young children's perceptions of mathematics learning in New Zealand. However, Secondary Numeracy Project (SNP) had started in 2006 and hardly any research had been done on students' perceptions of SNP.

Chapter 3: METHOD

3.1 Introduction

This research investigated the perceptions and experiences of Year 9 students of mathematics, participating in the Secondary Numeracy Project at two schools in their first year of the project. This chapter outlines how the participants were selected, the procedure used for gathering and analysing the data.

3.2 Participants

Two coeducational secondary schools in New Zealand, catering for students in Year 9 to 13 were chosen. Both schools had started Numeracy Project at the beginning of the year 2006. Twelve students from each school were selected; four students (two girls and two boys) each from a low, average and high ability class. An attempt was made to get an equal number of boys and girls in each ability group to try to ensure comparability of the groups. However, in School A there were three boys and one girl in the low ability group (see Appendix C for student profile). In addition, an attempt was made to involve only three classrooms from each school so that each set of students would be in the same classroom so that the difference in teaching won't be a factor.

All the students participated in the study were in Year 9 at the beginning of 2006. Hence they had been in the school for about ten months when the interview took place. Some students had participated in the Intermediate Numeracy Project (INP) and/or the Early and Advanced Numeracy Project (ENP and ANP). However, this was only a small proportion of the students. Most of the primary and intermediate schools the students came from had not been involved with any of the Numeracy Project. Nevertheless these schools were using 'Figure it out' books and a variety

of equipments to enhance learning of mathematics. A few more details about the two schools; School A and school B are given below.

Background of the Schools

School A

School A was a decile 5 school from a suburban area of the city. This school has a roll of over 1800 students and about 110 teachers. The ratio of boys to girls was 53 : 47 respectively. And the ethnic composition of the school was 63: 22: 15 for Pakeha: Maori: Others. Their mathematics department had developed differentiated learning programmes. In the mathematics department, students were assigned into low, average and high levels. They were initially streamed into the three ability groups, based on a combination of mathematics (PAT - Progressive Achievement Test) and English (AsTTLE - Assess To Learn) marks.

School B

School B was a large urban school of decile 9 with a roll of 1500 (50% males and 50% females). This school provided numerous opportunities for students to take part in academic, sporting and cultural pursuits where many students achieved highly at regional and national levels. Parents and staff had high expectations for students' learning and behaviour. This school reflected the increasing multicultural society of New Zealand, representing 28 different national groups. Their roll consisted of 68% Pakeha/European, 15% Asian, 12% Maori and 5% other nationality groups. The school's extra curricular programme encouraged all students to become involved in the full life of the school, irrespective of expertise or experience. The students in this school were also streamed but this was based only on their mathematics results on CEM (Canterbury Educational Management System). In this school students were given the same test at the end of the year, regardless of their levels. This is done to maintain the standards of the schools.

3.3 Procedure

University of Waikato's ethical procedures were followed (*see* Appendix B). The Numeracy Advisor selected three volunteer teachers from Year 9 classes who wanted to participate in the study. That is, one teaching a Low, one Average and one High ability class. Each teacher was asked to select four students from their class, preferably two girls and two boys. They were advised to do 'Purposeful selection' concentrating on students who were able to express their ideas effectively. The twelve students from each school were interviewed in their school. The students were taken out from their classrooms with the approval of their teacher. Students were interviewed in a quiet place in the schools.

The interview protocol used in this research was initially developed by Young-Loveridge, Taylor, Hawera and Sharma (in press) for primary children over a number of years. This interview protocol was adapted by including questions targeted for secondary students involved in the SNP (*see* Appendix D). Students were asked to sign a consent form (*see* Appendix E) stating the main purpose of the study, before the start of the interview. The length of the interviews ranged from 15 to 50 minutes. They were told that the interviewer was interested in their opinion about mathematics learning. The interviews were recorded using a digital mp3 recorder at the time of the interview and later transcribed. The interview was mainly semi-structured which allowed for probing whenever necessary and reiteration of detail to clear the doubts and make sure we understand each other. Students had to perform three tasks which took in the form of clinical interview. After a few interviews, the interview protocol was revised as it was proving difficult to get clear answers from the students. Most of the questions were opinion questions, asking them for their attitudes, beliefs, views and experiences about mathematics learning and their classroom practices. It also included the presentation and discussion of three mathematical tasks; a subtraction, division and a fraction task. They were all written as word problems. Following are the three tasks and the backup task for students who had difficulties with the first three tasks.

1. You have 47 dollars in your piggy bank. You take 9 dollars to buy a toy. Now how many dollars do you have in your piggy bank?
2. Teacher has 98 marbles which she wants to distribute among 14 students. How much will each student get?
3. Harry and Sally bought two pizzas. Harry ate $\frac{3}{4}$ of a pizza while his friend Sally ate $\frac{7}{8}$ of a pizza. How much pizza did they eat?
4. You had six lollies and you got eight lollies from your brother, how many lollies do you have altogether?

The first task ($47 - 9$) was a subtraction word problem with the need to use regrouping. The basis for using this was as a ‘warm up’ and to boost the confidence of the students, keeping in mind the different ability groups so that it wouldn’t be too easy for the high achievers and not too difficult for the low achievers. The second ($98 \div 14$) was a two-digit division problem which could not be solved by the written algorithm for division using times table knowledge. This task required students to analyze and make use of their knowledge of number properties. The third task ($\frac{3}{4} + \frac{7}{8}$) involved addition of fractions with different denominators. The fourth ($6 + 8$) was used only with students who couldn’t do other tasks. This task required addition of two single digit quantities. Students who answered each of the tasks were asked to think of another way to solve it.

3.4 Data Analysis

The data was analyzed by using the method proposed by Miles & Huberman (1984). First the interviews were transcribed on the computer. While transcribing, notes were made about important similarities and differences between the interviews for future use. The transcripts were read and re-read to identify and highlight common themes, ideas, trends, patterns and words. Each transcript was divided into chunks based on the themes. Then the transcripts were coded to make it easy to locate data. Decisions were made on which parts of the interviews to incorporate in the study. The responses of the twelve students for each of the selected questions were cut and pasted under the appropriate question. The data

fell into three main categories: students' perceptions and awareness of mathematics learning, students' perceptions and experiences of the four areas of mathematics learning and students' mathematical expectations and abilities.

To maintain the confidentiality of the students, their names were coded. The codes given to the students were as follows:

There were two schools: School A and School B

There are three levels³: L = Low, A = Average and H = High

Gender; G = Girls and B = Boys

Two students of each gender from each class; 1, 2

So each student is named in the order: School, Level, Gender and number.

For example, the girl two from High Ability Group in School A was represented by AHG2 and so forth.

³ From now onwards, students from Low/Average/High Ability Group would be addressed as Low/Average/High students or Low/Average/High ability students.

Chapter 4: FINDINGS

4.1 Introduction

This research investigated the perceptions of students on aspects of the Secondary Numeracy Project (SNP). The main research question explored in this study was:

What are the perceptions and experiences of Year 9 students of mathematics from two different schools in the first year of the Secondary Numeracy Project?

The focus was on students' perceptions of group work, equipment, communication and their teachers. The analysis also took into account the mathematics ability groups of the students.

The key findings of the study are organized and presented in this chapter. The three main categories are students' perceptions of mathematics (4.2 to 4.3), students' perceptions and experiences about their mathematics classes (4.4 to 4.9) and students' responses to the mathematical tasks (4.10 – 4.11). The first section, 4.2 presents the students' views and perceptions on the nature of mathematics while 4.3 discussed its importance. The next section, 4.4 explores the students' awareness of the Secondary Numeracy Project. Sections 4.5, 4.6, 4.7, 4.8 and 4.9 reports on students' experience and their views: on group work, social activity, communication, use of equipment in mathematics classroom and view of their teachers. Final sections, 4.10 and 4.11 presents students' views on how good they thought they were at mathematics and how they went about doing the three (or four) tasks.

4.2 Students' Perceptions of the Nature of Mathematics

Students were asked: "What do you think maths is all about?" Students' responses varied. Their answers were classified into three different categories; numbers and basic operations in mathematics (e.g. addition and subtraction), other mathematics

topics they have studied in mathematics, and problem solving. The following are some of the comments made by students.

Number and operations

Fifteen out of twenty-four (five from each ability group) students felt that mathematics is all about numbers and mathematical operations or time tables.

Numbers... to confuse people [BAG2]

Numbers... it is adding and finding different ways to add, subtract, divide and stuff [BHB1]

One student justified his reason for feeling that mathematics is numbers.

Because when we are doing maths we are always using numbers [ALB1]

Other Mathematics domains

Two students from each of the Low and High group mentioned topics they had covered in mathematics. Interestingly none of the students in Average group came up with this reason as a definition of mathematics.

Measurements, angles and numbers [ALB1]

Like algebra, number, geometry... to be able to times things and subtract things... doing shapes and reflection and equations and that sort of things [BHG2]

Problem Solving

A small group of students talked about mathematics as problem solving. That is, three students from Average group and two each from Low and High group had a broader perspective of the nature of mathematics.

Working out things, how to solve things, how to figure out things [ALB2]

Maths is all about having fun.. problems, working out problems... challenges [AHB1]

Problem solving... most maths problems you need solving problems... yeah all maths problems needs to solve problems [AAB1]

If you don't have maths, then if you went to a shop and says you want this much lollies and they could give you the wrong amount and you wouldn't know [AAB2]

It was interesting to note that no one among this twenty four students mentioned

logic, reasoning or thinking as the nature of mathematics. Students from the Low and High groups included all four categories in their responses. However, five of the Average students mentioned numbers and the rest of the students talked about as problem solving.

Link to other subjects

One student viewed mathematics as a basic foundation for all other science subjects.

Maths is the heart of all learning because if you don't know maths you can't do science. Science and maths are like the two most important branches of learning [BHB1]

4.3 Students' Views on the Importance of Mathematics

Students were asked, "Do you think maths is important? Why?" All students agreed that they felt that maths is important (unanimous agreement). Students had a variety of reasons for holding this view. Their responses were categorized into students who considered using mathematics for the immediate use and those who were concerned about their futures in terms of mathematics utility.

Everyday life here and now

Students thought mathematics was important for their everyday life and talked about needing mathematics when they go shopping or the need for mathematics when studying other subjects or when handling money.

Everyday and subjects

Students were aware of the use of mathematics to do some basic operations in their everyday life.

We use in it everyday life [BHB2]

Because if you buy something.. and like taking 60% of something.. [ALG1]

Because it is used in everyday life... like the supermarket say.. you try to add price and you need maths for that [BLB2]

There were a few students who acknowledged the use of mathematics in other subject areas.

Well for me maths is in every subject, like science, economics, even information technology for the formula and English [BLG2]

Handling money now

A lot of students talked about the need for mathematics when dealing with money.

'Cause a lot of it deals with money and maths and stuff like that [AHG1]

'Cause in everyday life you need maths for.. like if you are buying stuff from the canteen you need to know how much change you get otherwise you might get ripped off or something like that.. hehe.. That is not good [BHB1]

Life in the Future

More students thought that they would need mathematics in the future, when they find a job or to handle money in the future.

Job

A lot of students knew that they had to use mathematics in the future as many jobs require the use of mathematics.

Basically in any job you got to know how to do maths [BHG2]

Most jobs now need maths...Architectural designing [AAB1]

As you grow older it becomes important, when you are applying for jobs.. like NCEA credits and stuff like.. if you passed your exam [inaudible] and it's on your CV they know you are not dumb or anything [BAB2]

Money handling jobs

Some students believe that they need mathematics for jobs that deals with money.

Because they help you with the job.. like you don't want to give them like more money than what they suppose to or not give them enough [ALB3]

For some jobs you need to know how to calculate problems even little things.. umm any job really.. especially you are behind a counter and hand you over money of something and you need to be able to think how much change... and things like that [BHG1]

Handling money in future

Students mentioned the need for mathematics knowledge when they use it to pay bills and deal with money.

Because everyday, whatever you do, you always come across maths... like when you grow up when you have bills and you don't know how to add them up, you won't know how much to pay [AHG2]

In summary, the Low and Average students predominantly talked about mathematics needed for future jobs (five students from each of these groups) whereas, five students from High achieving group mentioned utility in everyday life and when dealing with money.

4.4 Students' Awareness of the Numeracy Project

In response to the question whether they were aware of the Numeracy Project, the majority of the students had heard about Numeracy or the Numeracy Project. They had an idea that it has something to do with numbers. Few students were aware that they were on the Numeracy Project at some stage of this year. Some thought that it was something to do with mathematical operations, using equipment or just a different way of teaching mathematics. Some of the responses are given below;

It's about numbers.. [AAG2]

Is it using counters and stuff?[AHB1]

Adding and subtracting and stuff... doing easy maths [BLG1]

Yeah that's the fun one where we got to play lots of games [BAG2]

I don't know what's it about but I heard it from my sister... is Numeracy numbers or something? ... and different ways of showing kids how to do the same questions to try and make it easier? [AAB1]

Overall, no one was sure what Numeracy Project was really about. However, most of them had the idea that it has something to do with numbers and mathematics.

Areas of focus in the Secondary Numeracy Project

It was clear that most of the students did not have a clear understanding of the project. The Secondary Numeracy Project (SNP) emphasises on improving students' understanding and achievement in mathematics by advocating group work, using equipment, sharing multiple solutions with the help of the teacher. Hence, it was found that students' views and experiences about the SNP could be explored indirectly by getting information from students about these components.

The following sections explore these areas in depth. Section 4.5 and 4.6 present students' awareness and perceptions of group work respectively. Section 4.7 explores students' views on sharing multiple solutions via communication. The following section, 4.8 presents the students' perceptions on equipment. Finally, section 4.9 presents data on the students' views of their teachers.

4.5 Students' experiences of Group work in mathematics

This section explores the "Group Formation" which includes how the students are grouped, whether they had a choice when forming groups and how they worked at mathematics in the classrooms. There are differences between the ability groups so each ability group is discussed separately.

Low Ability Students

Table 1 (below) reveals that the Low ability students in both schools sat in groups. However, data from School A revealed that students do not have direct control over the group formation.

Teacher put us into groups [ALB1]

He kinda waits till we pick [the groups] and then if he thinks it is a good idea then he will put us in groups [ALB3]

Students from School A were encouraged to work in groups as they revealed that the teachers had set a 'point system' to ensure student involvement.

Look out for people who help each other and gives points to them [ALB1]

They were then rewarded every week.

By the end of the week the group which gets most points [are rewarded with] lollies and stuff like that [ALG1]

Students believed that this system of grouping avoided unwanted behaviours. One student said that;

They [teacher] give us points if we don't talk [ALB1]

Table 1

Seating and Working arrangements of the students in Low Ability Group

Ability Groups	Schools	Current Seating Arrangements	Students' Choice Of Seating ⁴	Working Arrangement ⁵	Comments About Groups
LOW	A	Groups	No	Groups always	Points system
	B	Groups	Sometimes	Mostly individual	'Desk groups'

Students from School B sat in groups called 'desk groups of four [4 students per group]' [BLG1]. Sometimes the students had a choice in forming the groups and sometimes they don't.

Just choose where we want to sit at the start of the lesson and that is our group [BLB2]

Sometimes teachers do it [put them into groups] ... sometimes we can sit in our own group [BLG2]

However, there is a condition.

If you talk too much with people then you will have to go to another group [BLB1]

They all acknowledged that they sit in groups but work individually unless they were playing a game or activity.

Most of the time we work like individually... and sometimes if we play like maths games we do it in groups [BLG1]

⁴ Whether or not students have a choice to sit anywhere or in any group they want

⁵ How the students work at mathematics in their classrooms

Average Ability Students

Average students from both schools said that they usually did not have group work during mathematics classes (see Table 2 below). Students in School A were encouraged to do individual work as they “are not allowed to talk” [AAG2]. Students mentioned that the only time they do group work was when the teacher gives work to the whole class and teaches to a selected group of students in a corner (small group teaching).

She [the teacher] would pick a few [students] and explain [AAG1]

Table 2

Seating and Working arrangements of the students in Average Ability Group

ABILITY GROUPS	SCHOOLS	CURRENT SEATING ARRANGEMENTS	STUDENTS' CHOICE OF SEATING	WORKING ARRANGEMENT	COMMENTS ABOUT GROUPS
AVERAGE	A	Rows	-	Individual	Small group Teaching
	B	Pairs	Sometimes	Groups sometimes	Groups for games

Table 2 shows that the students of Average group in School B sat in pairs. Sometimes the students were allowed to select their groups but not always.

Sometimes we do sometimes we don't [have a choice in forming groups] [BAB1]

The teacher sort of sets us in groups [BAG2]

One student believed that they were grouped according to their ability levels.

Depend on the level of smartness [BAG1]

It is important to note that half of the students thought that they didn't have a choice when forming groups while the other half believed that they were allowed to select sometimes⁶. All students from this school revealed that they form groups only when they played games or activities.

⁶ This is one drawback of getting the perceptions of people; different students felt differently.

High Ability Students

Table 3

Seating and Working arrangements of the students in High Ability Group

ABILITY GROUPS	SCHOOLS	CURRENT SEATING ARRANGEMENTS	STUDENTS' CHOICE OF SEATING	WORKING ARRANGEMENT	COMMENTS ABOUT GROUP
HIGH	A	pairs	Yes	Groups sometimes	Groups on every Wednesday
	B	pairs	Yes	Individual / Pair	Groups for research

The Table 3 above shows that High students in School A sat in pairs. Students revealed that they were allowed to choose their group.

Just make our own group [AHB2]

They formed groups once a week and other times work individually.

Have a special period where we get into groups and play games and stuff [AHB1]

For things like problem solving.. we get into groups of three [AHG2]

Students of School B in the High group also sat in pairs. Like the High students in School A, School B students were allowed to choose where they want to sit.

Can move around but generally we stay where we are [BHB2]

Students usually worked individually but occasionally they worked with partners.

Work with partners for long term assignments [BHB1]

Sometimes we are grouped for researching like how to learn algebra, or sometimes before we learn a topic" [BHG1]

In summary, Low students in both schools were grouped while the Average and High students were not. On the other hand, the Low and Average students did not have much of a choice when forming groups. Only the Low students in School A worked at mathematics in groups. The Average and High students worked in groups only to play games or activities.

4.6 Students' Perceptions of Mathematics as a Social Activity

This section discusses the students' perceptions of work in mathematics classroom. They were asked, "Would you like to work in maths on your own, in groups or both"? Their answers to this question were classified into three categories; individual, group and both (sometimes individually and sometimes in groups). The most striking finding shown in the Table 4 below was that none of the girls liked to work individually⁷ at any time. Also it can be seen that mostly girls in Low and High preferred to work both individually and in groups. Table 4 below summarizes the information. Details of the responses for each ability group are discussed below.

Table 4

Students' preferences to work at maths

LEVEL	SCHOOL	INDIVIDUAL		GROUP		BOTH	
		BOYS	GIRLS	BOYS	GIRLS	BOYS	GIRLS
Low	A	2		1			1
	B	1		1			2
Average	A	1			2	1	
	B				2	2	
High	A			1	1	1	1
	B	1		1			2

Low Ability Group

Table 4 shows that students in Low Ability Group preferred to working under all three conditions. Students wanted to work individually because if their concentration was disturbed, cheating or fighting could occur.

[Sharing] disrupts me [ALB2]

No fighting or anything [BLB2]

⁷ It is in the scope of this study to explore gender difference. Hence this issue will not be discussed further.

Sometimes people cheat [when in groups] [ALG1]

I concentrate more when I am working individually [BLG1]

Students wanted to work in groups to bring some fun and enjoyment into their mathematics lesson, or to get help from other students.

It's easier... cause we all work out sums and pass them [BLB1]

If you are by yourself then it is no like ah.. fun... it's boring [ALB1]

If they or other students needed help it was useful to be in a group but if the students did not need help there was a preference to work individually.

If they can do it they can do individually, if they are having problems then they should be in a group [BLG2]

Well, in maths I have to be ahead [when working out maths problems] cause some people... like in my group needs serious help and things.. [BLG2]

Average Ability Group

The Table 4 above reveals that Average students mostly preferred either group work or a combination of group and individual work. These students wanted to work in groups so that they could provide and get help whenever needed.

You can discuss with your mates and if somebody doesn't get it then you can help them out and stuff [BAG1]

One student wanted to work in groups because she wasn't confident to work alone.

Because I don't like working on my own.. it is hard [AAG1]

One student wanted to work in groups to get help but preferred the students who did not take their work seriously to work alone, individually.

it's good to work in groups because then if you don't know something, you don't have to wait for the teacher to come around but can just ask other people, in your group... some people, like when they are with other people all they do is muck around so it is better for them to work by themselves [BAG2]

Once again, the majority of the students preferred both – they only wanted to be in

groups if the work was difficult - otherwise they preferred working individually.

depends on what [they] are doing, sometimes it is hard and other people knows things and sometimes its real easy then you just do by yourself [AAG1]

High Ability Group

Like the Average students, the High students mostly preferred to work in groups or both (see Table 4 above). Only one student from High Ability Group wanted to work alone. He believed learning was an individual process.

I think you will be able to learn more from it [working individually] than relying on other people's knowledge [because] if someone else solves it you still don't know [BHB2]

Students preferred to work in groups so learning would be fun and it would be easier to work as help is available.

I think it is fun, 'cause we can talk about it [AHB2]

I think it is easier in groups but if you work on your own you show how good you are [to the teacher and others], like how much you can do [AHG1]

The comments of the High level students who preferred both (individual and group) showed that their liking for group work was based on the help they could get.

If you don't understand then you need to have other people to work with [BHG2]

If we don't understand something they can ask another person in the group and that person can explain... like they learn together [BHB1]

Sometimes it's nice to work with other people, explaining to me.. sometimes its quite hard to try and answer questions individually [AHB1]

I like to do problem solving in a group cause sometimes you really get stuck and other people can help you. But normal work.. it's better to do it by yourself otherwise you don't really learn cause other people will be telling you the answer and stuff [AHG2]

Overall, the Low students preferred working at mathematics individually, in groups and both (individually and in groups) whilst the Average and High students preferred working in groups or both. The High students preferred

working in groups mostly for the help they could get from others.

4.7 Communication

This section presents the students' perceptions and experience of communication or making sense of mathematics by interacting with peers and teachers. The three main questions asked were, "Do you think it is important for you to know how others got their answers? Do you think it is important for you to be able to explain your thinking to your peers? Do you think it is important for you to explain your thinking to your teacher?" Students' responses for each of the questions are given below.

Importance to know how others got their answers

There were four students from Low group who were not interested in how others got their answers.

I am not interested in how others do it [ALB2]

Because the way I do it is usually the way I do it. I don't really take note of what other people.. how other people do it [solve problem] [BLB2]

In contrast, one student from Average and one from High were not interested in others answers if they got their answers correct.

If I get it correct then usually just carry on doing like how I do it [BHG2].

If I feel I am confident in the way I am doing then I usually don't want other people to tell me how they did it because it will confuse me [BAB1]

There was one student who thought that if another student had a better way of solving a problem then that meant his/her own way was wrong. This came from a student who said that they played starter games where all students share their strategies to solve a problem.

BLB2: I would be wrong isn't it, if they have a better way.. so if my one is wrong then I would listen to them, see what's right

I: So do you get explain your..

BLB2: Yup.. yup

I: How does that...

BLB2: Usually everyday we do this thing called Starter and what we do is we get to fill out questions on the board and teacher asks us how we did it and there's lots of different ways of how we do a question.

Another student shared his experience of 'Sharing' solutions they practice in their classrooms.

Sometimes we do a little sharing.. umm Sharing time where.. sometimes the teacher wants to know answers from different people.. so we throw our answers and we work it out and see who got it correct [AHB1]

Students from all ability groups were aware that there are a variety of ways to solve a problem and students particularly from School B, practiced answer sharing sessions.

Different strategies [AHG1]

Always more than one way to get an answer [AAB2]

Some other ways...to get the same number [solution] [ALB1]

Every time when we do something we always figure out how many different ways we can do figure the thing out [BAB2]

The main reason why most of the students wanted to listen to other's solution strategies was to find easier or quicker ways to solve problems or just explore the different strategies so that one of the methods would come in handy one day.

Quicker ways [AHB2]

Easier ways [AHB1, AHG1]

Learn different strategies [AAG2]

Because they might work out an easier or quicker way [ALB3]

If you couldn't remember the way that the teacher said [explained] then you could just think back to the way someone else does it [someone else explained in the classroom discussion] [BAG2]

Importance of explaining their answers to peers

Overall, students were more willing to explain their thinking to others than listening to other students' solution strategies. However, there was one student from Low group who did not feel that it was that important to explain his thinking

to others.

No, not really... If they ask me to explain then I will explain it but if they just gonna not listen to what I'm saying, why waste my breath [BLB2]

The rest of the students felt strongly that it was important to explain their thinking to their peers to help their friends in need. Students also shared their successful strategies with their classmates.

It helps if they haven't got the answer [BAB2]

Help them out of something [help them out to solve a problem] [ALG1]

Explain the answer to the rest of the class if got a quicker way to do it [ALB2]

Some students took the opportunity to explain their thinking to other students as a time to check their understanding of mathematical concepts. There was a student who thought that if you could not explain what you have done then it meant a lack of understanding.

If you can't explain then it's practically as if you don't know it [BHB1]

If you don't know how to explain it you are not helping anybody else [BAG2]

If you can't explain to others then you haven't really worked it out because you just got an answer.. you haven't understood it [BAB1]

There was one student who was not confident with her answers to share it.

I think that my answer is wrong [AAG1]

Importance of explaining their thinking to their teachers

Students wanted to explain their thinking to the teacher so that the teacher could provide help.

She understands what I think about the topic [BLG2]

If you do something wrong, he or she can help you work it out [ALB3]

Then if you are not doing it properly the teacher would say 'oh if you do this way it would be much easier [AHG2]

Some students use the teacher as a way to communicate to the whole class. They let the teacher know if they had a better way so that she could tell the whole class.

You can tell more like ideas to the teacher and he can just tell the whole class [ALB1]

If your thinking is a lot simpler than her way.. you probably should tell her that her way is complicated and you've got a better way [BAB1]

Students felt that it was necessary to explain their thinking to their teachers only during a test to show the teacher their level of understanding which is required in tests.

In the test and stuff she like ask us to answer them and put your working out and it helps the teacher because she knows what you are talking about [BAG1]

Students felt explaining their answers to the teacher were for self assurance and letting the teacher know that they didn't cheat.

Then she knows that we haven't cheated [AHB2]

Then she knows that you haven't cheated [BLB2]

So that she knows that I can do that and I know that I can do it [BHB2]

One student was aware that maths was not just about getting the correct answer but understanding the procedure was important as well.

In maths it's not just about the answer, you've got to explain how you got the answer [AHB1]

Overall, Low ability group students were not interested in understanding how others got their answers. However, Average and High ability students value listening to others to learn effective strategies. This might be due to the 'Sharing time' experienced by High ability group in School A and all the students in School B. Most of the students wanted to explain their thinking to others to help their friends. Explaining to the teacher was done solely so that the teachers could help them or know that their work was not copied.

4.8 Equipment

This section explores the students' experience and perceptions of the usefulness of equipment in the two schools. It was found that Low group students in both schools used a variety of equipment compared to the High students.

Equipment used by the students

All students mentioned using ‘Alpha mathematics’ textbook quite a lot. Apart from textbooks, students from the two schools used a variety of equipment including games and activities.

- Counters and block things [ALG1]
- Dice when playing games [BLB1]
- Compass, rulers, protractors [ALB3]
- Shapes for fraction [AAB1]
- Ten Pin⁸ [BAB2]
- Logic Puzzle... Sudoku [BAB1]
- Sudoku, bingo... and Wai maths [AHB1]
- Matches, polygon things [for learning algebraic patterns] [BHB1]
- She’s got this white board things that we have and we do some of our equations on them and she gives out number lines [BAB1]

Most of the students get a chance to use equipment during their mathematics lessons. However, the type of equipment used by students from Low ability group is quite different from that used by students from Average or High ability group.

When they use equipment

Students of School A in the Low Ability Group play games and activities after they have finished their class work. One student responded to the question “Do you play games and activities a lot?” as:

- Sometimes when we have finished all the stuff [doing the class work] we do it [play games and activities] [ALB2]

High ability students of School A had a dedicated period once a week to play games and activities.

- Every Wednesday we have a special period where we get into groups and play games and stuff [AHB1]

Low ability students of School B played games and activities when the students got stuck with a problem.

- In my maths class some people get confused with the question so what my teacher

⁸ Students were given three or four numbers between 11-20. The students had to multiply & divide or add & subtract using these numbers.

does is she will take all these games so we can play and understand better [BLG2]

Calculators

Calculators play an important part in mathematics education at the secondary level. Each student is expected to have a calculator when he/she goes to the secondary school. However, they are not allowed to use them in all areas of mathematics.

Like integers and stuff [AHG1]

Allowed to use it [calculators] for some topics [AAG1]

Some subjects [topics] like numbers we have to do it in our head, but like Algebra is really hard so we need to use the calculators and you are allowed to [BHG2]

Students acknowledged that the teacher in School B discouraged them from using calculators.

If we are allowed to use the calculators, most of the time the calculators wouldn't help that much [BLG1]

We are not allowed to use them... and if you use the word 'calculate' you must calculate in your head, not on the calculator [BLB1]

Used to use it... but now I use my brain... when using calculators you would be like 'where did I get this number from'? [BLG2]

Students in High ability group were given reasons by their teachers, discouraging the use of calculators.

Because the teacher says it's to develop our numbers skills, for example, multiplying, dividing [BHB2]

Her [teacher's] Year 11 classes are all using calculators and now don't know how to do it in their head [mental mathematics] [AHG2]

Mental Calculations

Students had some interesting thoughts about mental mathematics. Most students compared mental mathematics with either calculators or pen and paper. A reason why they might have compared mental mathematics with calculators might be that the preceding question was about using calculators.

Because it stays in there but calculators just turn off [AAB1]

Sometimes you need calculators and sometimes you need your head [AAG2]

Yeah, she is trying to make us do that [promote mental mathematics] [BLB1]

One student thought mental mathematics was important as his alternative was fingers and he felt that it was a primary thing.

It's better in head because finger is more of a primary school thing and when you go to high school you should learn to do stuff in your head [BLB2]

One student explained what using calculator ultimately meant.

Cause if you are just using the calculators you are not learning anything cause you are just putting $5 + 6$ and automatically comes the answer, without you having to think... [if doing mentally] you figure it out yourself [AAB2]

Students, particularly from High ability group felt the need to use pen and paper instead of just doing mathematics mentally. A few students thought that mental mathematics is the first step of learning mathematics.

Yes [good to learn how to do mental mathematics] but I find it easier to write it down when working out... cause then you could see visually [AHG2]

It's good to have the skills in your head, then you know what to do, and then you can just put on a piece of paper and work out the question. Cause if you don't have the skills in your mind.. well then you might write on a piece of paper and lose the piece of paper but if in your mind, it's there [AHB1]

I think in some ways it is good [mental mathematics] but in some ways it isn't because I find it more easy to see things on paper than be able to work out from my head. If it is like a longer one then I would be like, yeah, yeah, yeah and then I will be like 'where am I again?' [BAG2]

Mental maths is important if you don't have pencil and paper but if you have it's better to do it on paper... so if you get them wrong then you know where you went wrong [BHB1]

Usefulness of equipment

All students in the Average Ability Group had the perceptions that equipment help them when they do mathematics. Some students found equipment not useful at all while others thought they are helpful to learn concepts, visualize or as a faster way to get the answer without having to think.

Ineffective

Not all the students were positive about using equipment. However, it was only students from Low and High ability group who thought that equipment were not helpful and could learn without using them.

Doesn't help me at all... I just like playing them [BLB2]

Yeah we kind of learn...not directly [ALB2]

Yes it's easy.. you can just as easily do it in your head [AHB2]

It [usefulness] depends on the games as well, sometimes we do some really helpful games but sometimes.. no use..

Some students in the High ability group felt that they did not need equipment because they belong to the top group or they seemed like expectations from them were different.

This year it's just to listen to the teacher and work [BHG2]

Cause we are in the enrichment class so we don't really need those stuff. We use calculators and stuff like that [AHG2]

One student thought the equipment was not helpful because they were learning things she knew.

Well the stuff that we do in maths.. is really easy and I know these stuff also. So it doesn't help [BAG1]

One student thought that there should be a better way than using equipment as when they grow up they can't keep on using equipment.

Should try some other ways... better way [without using equipments] [ALG1]

Sometimes students did not have a clear understanding of the actual use of the games utilised. One student responded to the question of using equipment being helpful by explaining the procedure to play Sudoku.

You learn like what's the numbers, and where they should go like take them from one place and putting them in another place [ALB3]

Faster in calculating basic operations or exercise for brain

Some students felt that the use of playing some games like Bingo or Ten Pin, would make them faster in calculations or help them exercise their brains.

Ten pin... is good for your brains... it helps you remember [BAB2]

More like get faster at your times tables... just like get faster in add and subtract and all that [BAG1]

When asked of a student what she learnt from Bingo, she responded;

Learn times table.. maybe [ALG1].

Substitute for fingers or using head

Students felt that use of equipment helped them avoid using their fingers or head.

Learn like how to put things in groups and you don't need to use fingers [BLB2]

Yes.. it gives you the answer... instead of thinking about it [BLB1]

Hands on and fun

There were a number of students from Average and High ability group who thought that using equipment was fun and they could figure out what was happening.

They are way more fun than any other stuff [BAG1]

It's fun way of learning and more interesting [BHG1]

Cubes are helpful ... we are doing rules and its is a lot easier [BHB2]

It's better than working from a book cause you can move shapes around [AAB2]

I find it more helpful to use equipment than seeing it up in numbers because it's like hands on [BAG2]

Two students reasoned out why they need to have fun in the class.

When we are doing a lot of activities we didn't really realize we are learning things but we were [BHG1]

Like when you are bored you can't really learn.. like you learn but it's not really going into your head.. like you don't really want to remember it [AHG1]

Visualize

Once again students from High and especially Average group thought that equipment helped them to visualize what was going on. This made it easier to understand the concepts.

It's easier to understand like you can see it visually... not just read it [BAB2]

Fun way of learning... you can like see what's happening... visualize in your head [AAG2]

You can like find a pattern that is more practical to form a pattern. If you draw you won't understand how many... won't understand directly where they are coming... [3D shapes] [BHB1]

Yes... in a way... it gives you more of a sort of visual so you could sort of see how it's getting worked out [BAB1]

Helpful in learning new concepts

There were a few students from Low and High group who thought using equipment made it easier for them to learn concepts.

They make it easier to understand and to work it out [BLG2]

Helpful getting used to it [concept] ... now I've got over it, got the skills [AHB1]

The fraction magnets were really helpful for me cause it's [Fractions] quite difficult for me... I can do some problems without the magnet [BLG1]

In summary, students from Average and High ability groups talked about equipment helping them to visualizing mathematical concepts while Low ability students seemed more attached to them or took the meaning of usefulness at face value. This became evident from their explanations of the use of particular games.

4.9 Students' view of their maths teachers

Students were asked about the things they liked about the mathematics lessons that year and the things they did not like. The majority of the students mentioned teachers making a big difference to their experiences of maths learning. Below is a discussion of students' perceptions of their teachers in learning mathematics.

Helpful

Mostly Low ability students appreciate the teachers' help and their approachability. Also the way some teachers invited them to ask questions which made them feel not dumb to be asking questions.

Teachers, they are pretty helpful [ALB1]

If you understand you put your thumbs up and if you don't understand you put your thumbs down [BAG2]

The teachers will come around and help us if we are in trouble or show us what we need to do. And if we do real good sometimes we get achievement certificates [ALB3]

She don't make you feel dumb if you don't know something.. you can go and ask her... she is really nice and smiling.. like she feels approachable [BHG1]

Availability

Low ability students from School B mentioned the availability of a teacher or teacher aide which made it easier to get help most of the time.

The students who don't know the question or if they get confused they have a teacher to go to... a teacher aid and our teacher [BLG2]

Interactive and makes the lesson fun (Personality)

Low and Average ability students liked their teachers because they make mathematics lessons more interactive and enjoyable.

She [teacher] makes maths fun [BLG2]

The teacher is a lot more interactive of the class this year [BLB2]

She is probably the best maths teacher I've had since primary [BAB2]

I like the fact that its more social this year and easy going.. it's not like really strict [BAB1]

Explanation or Teaching

Average and High ability students were very concerned particularly with the teaching and contents covered in their mathematics lessons. Another thing High ability students liked was the fact that their teachers (in secondary) are thorough with the topics.

She teaches easy [AAG1]

The teacher explains well [AAG2]

They look at different topics every three weeks [AHB1]

The teacher more focused on maths than other things [BHB2]

He explains everything first so we are not just sitting there puzzled [BHG2]

The teacher is more straight forward.. like teaching, teaching and teaching [AHG1]

She [the teacher] gets you to do the work done quickly so that we can move on ...
this year we have covered 6 or 7 topics [AHB2]

One student when asked for any final suggestion about teaching and learning of mathematics advised “if you are teaching at a High School, be helpful and explain it properly” [ALB1]. Overall, it seems that students from Low ability like their teachers to help them and be more approachable. However, the students in High ability group wanted their teachers to have a good understanding of the content knowledge, explain well and cover as many topics as possible.

4.10 Students’ Self Assessments

Students were shown the following scale and asked “How good do you think you are at maths? Where will you be in the scale?”

Very poor	Poor	Average	Good	Very Good
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Table 5

Students ranking of their own mathematics ability

School	Levels	Students	Students’ Responses				
			V. Poor	Poor	Average	Good	V. Good
A	Low	Boys				3	
		Girls			1		
	Average	Boys				2	
		Girls			2		
	High	Boys					2
		Girls				1	1
B	Low	Boys			2		
		Girls			2		
	Average	Boys				1	1
		Girls			2		
	High	Boys				2	
		Girls				2	

Table 5 highlights the fact that the ability of the students has an impact on their self esteem. In general students had a positive attitude towards their mathematics learning. While Low students ranked themselves from ‘average’ to ‘good’, the High students rated themselves from ‘good’ to ‘very good’. The Average students assessed themselves from ‘average’ to ‘very good’.

It is interesting to note that in School A, all of the girls with the exception of one girl, ranked themselves one level lower than the boys in the respective ability groups⁹. In contrast, as seen in Table 5, there was no difference in self ranking between the genders among the students in School B for Low and High students. Furthermore, the girls and boys in the Average group from both schools ranked themselves almost exactly the same.

Students were further asked, “How do you know you are [Very Good/ Good/ Average/ Poor/ Very Poor] on the scale?” Students based their judgment of their self evaluation on their Mathematics Knowledge, their Assessment Results or their Ability group.

Mathematics Knowledge

Most of the students in the Low group based their assessment of themselves on their knowledge of mathematics and compared their knowledge with other students in either their classroom or other students in the Year level.

I don’t know everything about maths like Year 9 should... you can always learn new stuff [BHB1]

I know that I am not very good, otherwise those questions [the three word problems given in the interview] would have gone easy peazy[BLB2]

Assessment Result

Some students believe that assessment results provide them with a good judgment of how good they are. This is particularly the case for students in School A High

⁹ It was not within the scope of this research to explore the gender differences, hence this matter is not addressed further in this study.

achievers.

In test I always get over a half [ALB3]

Ability Group

Students in Schools A were streamed by their English and Mathematics ability while the students in School B were streamed solely on their Mathematics ability. Some students were conscious of their ability groups. Students compared with other students in the class or other year 9 students.

Got into the top class [AHG1]

Not in the intelligent class, just in a normal class [AAB2]

Am one of the top students in the class but not in the top class [BAB1]

There was one student who was comparing her ability with those of Achievement Group.

I'm average because I can't do good maths and I can do easy maths... [What do you mean good maths?] well you know, the hard stuff like all the technical stuff [problems] that good people do [BAG1]

Based on all three reasons

Only one student assessed himself based on all three reasons.

Problems like that [the three assessment tasks given to them]... they just come easy for me ...I do ok in tests.... I get quite good marks [BAB2].

Overall, most students in School B assessed themselves based on their knowledge of mathematics and ability while the students in School A based their judgment mostly on their assessment results and their mathematical knowledge. Majority of the students who reasoned out 'ability grouping' to base their judgment of self assessment were from the Average group. Table 6 below, is a summary of the students' responses in the two schools.

Table 6

Reasons for students' self assessment

School	Mathematics Knowledge	Assessment Results	Ability Group
A	4	6	2
B	8	1	3

4.11 Students' Responses to the Tasks

This section presents the students' responses to the tasks. Following are the three tasks given to the students.

Task one: $47 - 9$

Task two: $98 \div 14$

Task three: $\frac{3}{4} + \frac{7}{8}$

Task four (backup task): $6 + 8$

The techniques used by students also helped to determine and validate students' responses and their understanding of the tasks. For example, a student who said, "Nah, I just know the answer in my head" [ALB2], when asked whether he used fingers in mathematics was found to be dependent on his fingers to solve the majority of tasks given. Differentiating the techniques used by the students from the two schools was not an intention of the researcher. However, the data revealed a striking difference in the techniques used by students of the two schools. The following table gives a summary of the responses given by the students to the above tasks.

In the Table 7 below, the tick (✓) means that the student did the problem correctly, the cross (✗) means their answer was wrong and dash (-) means basically the student did not know. A cross followed by a tick (✗✓) means the student got the right answer with a little help.

Table 7

Summary of student's responses to the three tasks

Level	Students	TASK 1 (Subtraction)		TASK 2 (Division)		TASK 3 (fraction)	
		Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
LOW A & B	ALB1	✓	–	✗	✓	✗	–
	ALB2	✓	–	✗	–	✗	–
	ALB3	✓	–	–	–	✗	–
	ALG1	–	–	–	–	–	–
	BLB1	✓	✓	✗	–	✗	–
	BLB2	✓	✓	✗	✗	✗	–
	BLG1	✓	–	–	–	✗	–
	BLG2	✓	✓	✗	–	✗	–
AVERAGE A & B	AAB1	✓	–	✗✓	✓	✗	✗
	AAB2	✓	✓	✓	✓	✗	–
	AAG1	✓	✓	–	–	–	–
	AAG2	✓	–	–	–	–	–
	BAB1	✓	–	✓	–	✗✓	–
	BAB2	✓	✓	✓	✓	✗✓	✓
	BAG1	✓	✓	✗✓	✓	✓	–
	BAG2	✓	✓	✓	✓	✓	–
HIGH A & B	AHB1	✓	✓	✓	✓	✓	✓
	AHB2	✓	✓	✓	–	✓	✓
	AHG1	✓	–	✓	–	✓	–
	AHG2	✓	✓	–	✓	✓	–
	BHB1	✓	✓	–	✓	✓	–
	BHB2	✓	✓	✓	✓	✓	–
	BHG1	✓	✓	✓	✓	✓	✓
	BHG2	✓	✓	✓	✓	✓	–

The Table 7 above shows, students of High ability group were more successful in solving the tasks than students from low or Average ability groups. The techniques used by students of High ability group were becoming more sophisticated. The Table 7 also reveals that the students of School B were more successful in their attempts than students of School A. The following subsections address each ability group separately. A table showing the techniques used by the

students in each ability group and their success in solving the tasks is shown below under Low, Average and High Ability Groups.

Low Ability Group

The following table shows the strategies students used to solve the tasks and their success in first attempt and alternative method/idea.

Table 8

Strategies used by low ability students to solve the tasks

Levels	Students	TASK 1 (Subtraction)		TASK 2 (Division)		TASK 3 (Fractions)		
		1st method	Alter. Method-idea	1st method	Alter. Method-idea	1st method	Alter. Method-idea	
LOW A & B	ALB1	✓ Finger	-	✗ Halve 100..	✓ tally	✗ $7+3=10$	-	
	ALB2	✓ Brdg thr 10	-	✗ Subt.	-	✗ Diagram	-	
	ALB3	✓ Brdg thr 10	-	-	-	✗ Diagram $\frac{13}{16}$	-	
	ALG1	Couldn't do any task so she was given the 'back up' task and she got it right.						
	BLB1	✓ Brdg thr 10	✓ Fingers	✗ check	-	✗ Diagram	-	
	BLB2	✓ Brdg thr 10	✓ Compn	✗ Guess	✗	✗ Guess $1\frac{1}{2}$	-	
	BLG1	✓ count down	✓ G & Ch	-	-	✗ Diagram	-	
	BLG2	✓ Std Algrthm	✓ Number line	✗ tally	-	✗ Diagram $\frac{13}{16}$	-	

G & Ch = Guess & Check

Compn = compensation

Brdg thr 10 = bridging through ten

Std Algrm = Standard algorithm

- = couldn't do or did not know how to attempt

Subt = subtraction

Task one (47 – 9)

As evident from the Table 8, half of the students of the Low ability group solved the first task (47 – 9) using 'Bridging through ten' (Part-whole) strategy.

Well I went 47 dollars and take 9 dollars away, 9 minus 7 is 2 and then you got minus 2 from that which is 38 [BLB2]

38... because 7 plus 2 is 9, so $47 - 7$ is 40 and 40 take away 2 [ALB3]

BLG2 used a pen and paper to do the ‘Standard Algorithm’ to solve the first task, as shown below.

$$\begin{array}{r} 3 \\ \cancel{4}7 \\ - 9 \\ \hline 38 \end{array}$$

There were students who found it difficult to explain their thinking. It was a little difficult to know what the students had in mind. Also this student was counting on to solve the task.

I: ok.. How did you get the answer for the first one?

BLG1: umm. I just took away 9 from 47.. 47 – 9.

I: How did you work it out?

BLG1: I just counted down.. I don’t know.. hehe

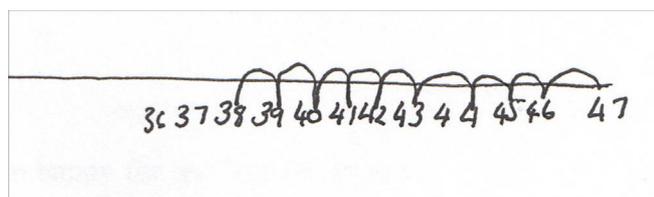
I: ok.. was it like first you took 7 from 47 and another 2?

BLG1: no, I just did 47 – 9 in my Head and work it out..

I: do you think you can like.. get the same answer from another way?

BLG1: yeah I could Have counted like... could Have counted like I could Have started with 45 and 46, 47, ... 52, 53, 54 which would be too much.. and so could Have started and just keep on working my way.. until I get 38..

This student mentioned using Number line for an alternative approach as shown below.



It was very difficult for some students to explain what they did.

BLG1: umm.. I just took 9 from 47.. 47 minus 9

I: how did you work it out?

BLG1: I just counted down... I don’t know.. hehehe... in my head

For an alternative method this student mentioned guess and check

I could have counted like I could have started with 45 and 46, 47, ... 52, 53, 54 [add 9] which would be too much and so could have started and just kept on working my way.. until I get 38 [BLG1]

Task two (98 ÷ 14)

Apart from one student, none of the other students in this ability group were able

to do the second task. The student who got a correct answer was initially unsuccessful trying to divide 100 by twos. He succeeded in his second attempt when he was trying for an alternate approach for the task. He used counting by ones with the help of a diagram, drawing 14 lines of tallying (see Figure 1 below).



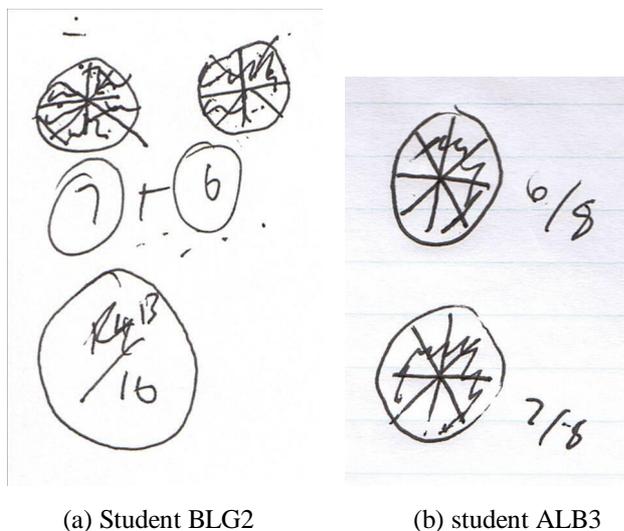
Figure 1: Student ALB1 used grouping to solve task two (98/14)

Another student started tallying but was unsuccessful as she drew only 13 boxes instead of 14. Some other methods used by the students were 'Guess & Check', Subtract instead of divide.

Task three ($\frac{3}{4} + \frac{7}{8}$)

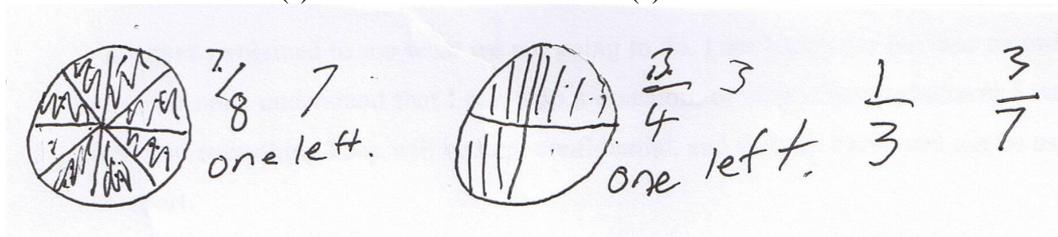
As it was observed from the Table 8, majority of the Low ability students used diagrams to attempt Task three. However, none of the students were able to solve this task. From the five students who used the diagram, only three students were able to draw it correctly (see Figure 2). Two students got $\frac{13}{16}$ as they saw a total of

16 pieces in the two pizzas.



(a) Student BLG2

(b) student ALB3



(c) Student BLB1

Figure 2: Diagrams used by the students to solve Task 3

Student in (a) and (b) got 13/16, Student in (c) was unsuccessful

Average Ability Group

Task one (47 – 9)

Students in the Average ability group were more successful in solving the tasks than the students in the Low Ability Group (see Table 9 below). Three students in this ability group used ‘Bridging through ten’ strategy. Two out of the three students who used ‘Standard Algorithm’ imagined the algorithm in their head.

BAB2: Like you start with the base.. like the main number you are looking at is 47 cause it's how much you have and it's basically just subtracting 9 off that base number and then it's asking how much you've got left... so just a basic subtraction.

I: so did you imagine 47 and 9 below.. like in algorithmic form?

BAB2: yeah

Table 9

Strategies used by average ability students to solve the word task

Level	Student	TASK 1 (Subtraction)		TASK 2 (Division)		TASK 3 (Fraction)	
		1st method	Alter. Method-idea	1st method	Alter. Method-idea	1st method	Alter. Method-idea
AVERAGE A & B	AAB1	✓Brdg thr 10	-	* tally	✓Std PVP	* add num	* Eq. frac.
	AAB2	✓Brdg thr 10	✓Compn	✓R. additn	✓ $14 \overline{)98}$	* Dia. & Ef	-
	AAG1	✓G & Ch	✓Calculator	-	-	-	-
	AAG2	✓Std Algrm	-	-	-	-	-
	BAB1	✓Brdg thr 10	-	✓R. additn	-	* Eq. frac✓	-
	BAB2	✓Std Algrm	✓Brdg thr 10	✓G & Ch	✓ $14 \overline{)98}$	* Eq. frac ✓	✓+ leftovers
	BAG1	✓Std Algrm	✓Compn	* R. additn✓	✓Sim. ⁹⁸ ₁₄	✓ Eq. frac	-
	BAG2	✓Finger	✓ $9 + ? = 47$	✓Finger	✓Sim. ⁹⁸ ₁₄	✓ Eq. frac	-

G & Ch = Guess & Check

R. additn = repeated addition

Std Algrm = Standard algorithm

+ numerator = adding the numerators

Dia & Ef = Diagram and Equivalent fraction

Compn = Compensation

Sim. = Simplify

Eq. frac. = Equivalent Fraction

Add num = add numerators

Only one student used fingers in Average group:

I just went $47 - 9$.. counting my fingers [BAG2]

There was one student in this ability group who mentally tried Guess and check to get the answer for this first task. At first she was hesitant to explain her procedure thinking her answer was wrong but once assured by the researcher, she was happy to give her reasoning:

AAG1: it would be [the answer for the first task]... Thirty... eight

I: 38, why did you say 38? How did you work it out?

AAG1: I don't know

I: you don't know? You got the correct answer... so how did you get the answer?

AAG1: I just write a number then add, like 38 and then add 9 and see if it is correct

I: so you randomly chose a number... 38 and you just check whether the answer is correct?

AAG1: yes...

Task two ($98 \div 14$)

Once again the success rate of the students in this ability group for the second task was higher than those of the Low Ability Group. These students had a better understanding of how to get the answer for this task. They used techniques such as ‘Guess & Check’, ‘Repeated Addition’, and ‘Standard Place Value Partitioning’ to attempt this task. Five out of the six students who got the correct answer were able to provide an alternative idea for solving this task.

AAB1 first tried to solve this task by drawing 14 boxes and tallying the boxes. However, instead of 14, mistakenly he drew 13 and didn’t realize it until at the end. So didn’t want to repeat the whole procedure again. So when asked him whether he can think of another way to do the problem, he attempted Place Value Partitioning, mentally as follows.

AAB1: It would be probably 7

I: Why did you say that?

AAB1: Because 7 times 10 is 70 and 7 times 4 is 28 and you got 28 and 70 is 98.

There was one student who did ‘Guess and check’ with place value partitioning to solve $98 \div 14$, however, the way he did it was different from that of what the Low group students did.

BAB2: Well... first you got the base number again... 98 marbles and you got to divide it between 14 students ... so you got $98 \div 14$ but I’m not good in dividing so I went guess and check in my head... like 14 times 8, it would be like 8.. 4 times 8 and then it would be too much for 98 so I just went down, then I calculated 7...

I: so you just calculated the last number? for 14 times 8 did you calculated 4 times 8?

BAB2: 10 times 8 and then 4 times 8...

All the tasks were word problems. Hence it was interesting to note that none of the students wrote $98 \div 14$ as a fraction. So some students were asked if the question was written as $\frac{98}{14}$ how will they solve it like $\frac{98}{14} = \frac{49}{7} = \frac{7}{1} = 7$. All six students who were shown this fraction, solved the task successfully by reducing fraction. That was two from Average group and three from High ability group.

Task three ($\frac{7}{8} + \frac{3}{4}$)

This task was a challenge for half of the students in this group as well. Out of six students who attempted, only four students got the correct answer. From them, two students were able to do it successfully on their own. The two students who performed this task successfully did so mentally, using the concept of equivalent fractions which is a strategy used by students at stage 7 on Numeracy Development Project.

I: how did you do the last one (task three)? .. why did you write $\frac{6}{8}$?

BAG1: because it's $\frac{3}{4}$ and if you make the bottom numbers the same then you like double that or times it by two (double the denominator, 4) to get 8. so you got to times the top number by 2 as well.. so that's like $\frac{13}{8}$.. so it's like $1\frac{5}{8}$

They also showed some understanding of when the denominators need to be the same.

I: why did you double $\frac{3}{4}$?

BAG2: so that you will get the same as that one (showing the denominator of $\frac{7}{8}$)?

I: why do you need to make the denominators same?

BAG2: umm.. I don't know...

I: so when do you need to make the denominators same?

BAG2: if you are plus-ing or times-ing or just.. I don't know! You are confusing me now!

High Ability Group

As the table reveals (*see Table 10 below*) all students from High Ability Group were able to answer all the tasks correctly. In addition a lot of them were able to give an alternative idea to solve the tasks. All students used more systematic and sophisticated techniques such as 'Bridging through ten', 'Compensation', 'Standard Place Value Partitioning', 'Cross Multiplication' and 'Equivalent Fractions' to tackle the tasks. However, some of the students who used 'Cross Multiplication' did so in a rote manner.

Task one (47 – 9)

All the students used either ‘Compensation’ or ‘Bridging through ten’ to solve $47 - 9$ in a similar way except for one student. When asked BHG1 for an alternative approach she responded in the following manner.

You could take 9 away from 40 dollars so you get 31 dollars and add 7 dollars back on so you get 37 back on [BHG1]

Task two ($98 \div 14$)

Only one student used a trick to divide 98 by 14 using short division. He used the fact that $7 \times 2 = 14$. He divided 98 by 7 instead of 14 and then divided the answer by 2 to get the final answer. He did write the steps down.

$$\begin{array}{r} 14 \\ 7 \overline{)98} \end{array} \text{ and } 14 \div 2 = 7$$

AHB2: Probably divide 98 by 7 and you get 14 then divide that by 2 to get 7

I: Why will you choose 7 [to divide 98 instead of 14]?

AHB2: Times 2 is 14

Table 10
Strategies used by high ability students to solve the tasks

Levels	Students	TASK 1 (Subtraction)		TASK 2 (Division)		TASK 3 (Fractions)	
		1st method	Alter. Method-idea	1st method	Alter. Method-idea	1st method	Alter. Method-idea
HIGH A & B	AHB1	✓Compn	✓Count from 9	✓Std PVP	✓ Std Algm	✓ Cross. Mult'n	✓ Eq. frac
	AHB2	✓Brdg thr 10	✓Compn	✓ $7\overline{)98}$	-	✓ Eq. frac	✓ Cross. Mult'n
	AHG1	✓Compn	-	✓Std PVP	-	✓ Eq. frac	-
	AHG2	✓Brdg thr 10	✓Compn	-	✓Simplify $\frac{98}{14}$	✓ Eq. frac	-
	BHB1	✓Compn	✓Brdg thr 10	✓Std PVP	✓Std Algm	✓ Eq. frac	-
	BHB2	✓Compn	✓Brdg thr 10	✓ $5 \times 14 ..$	✓Simplify $\frac{98}{14}$	✓ Eq. frac	-
	BHG1	✓Compn	✓40- $9=31+7=38$	✓ $5 \times 14 ..$	✓Simplify $\frac{98}{14}$	✓ Eq. frac	✓2-leftover
	BHG2	✓Compn	✓Std Algorithm	✓G & Ch	✓Simplify $\frac{98}{14}$	✓ Eq. frac	-

PVP = Place Value partitioning

G & Ch = Guess and Check

Cross. Mult'n = cross multiplication

Std. Algm = Standard Algorithm

Brdg thr 10 = bridging through ten

Eq. frac = Equivalent Fractions

Compn = compensation

Two students solved the task by first finding 5 times 14 and then adding till they got 98 in the following manner:

BHB2: I went up in 14 to 98. So 5 times 14 is 70, next is 84

I: How did you know 5 times 14?

BHB2: Because 5 times 10 is 50 and 4 times 5 is 20 then you add them up

The student who did use 'Guess and Check' to solve the task did it very fast mentally.

Well, I was just working out in my head like cause I don't actually know and I was

like 14 times 3, 14 times 5 and 14 times 7.. like guess and check [BHG2]

An example of a student who used ‘Place Value Partitioning’ (mentally) is given below. He did use his fingers to “count how many 14s there were”.

What I did was if you have 10 it would be 140 (10×14), a half is 70. So you got 28 left, take another 14 and that is 6 and take another 14 which is 7 [AHB1]

Task three ($\frac{7}{8} + \frac{3}{4}$)

As visible from the Table 10, students of this ability group used two techniques to solve the last task. They used Cross multiplication and halving Equivalent fractions. Three students used Cross multiplication method and they are all from School A. All three students used the same steps to derive at the same answer. Following is how AHB1 solved the task using Cross multiplication method.

$$\frac{7}{8} \times \frac{3}{4} = \frac{28+24}{32} = \frac{52}{32} = \frac{26}{16} = \frac{13}{8} = 1\frac{5}{8}$$

It could be observed that the students who used cross multiplication were unable to provide a valid reason for cross multiplying, and the fact that they did not recognise the relationship between eighths and quarters, implying they have learnt the procedure in a rote manner. When students were asked why they had to multiply 7 by 4 or 8 by 3 students’ responses included:

I am not too sure why [AHB1]

That’s the way we used to do it [when they did that topic] [AHB2]

You have to multiply those two and then multiply these two to make it equal...[reason is] the same thing you do to the denominator you do to the numerator [AHG2].

None of these students mentioned any terms like ‘making the denominator the same’ or ‘LCM’ (Lowest Common Multiple). On the other hand, all the students in School B used Halving of Equivalent fractions. Following is an example of a student who used ‘Equivalent Fractions’ to solve the task.

$$\begin{aligned}
 & \frac{3}{4} + \frac{7}{8} \\
 \Rightarrow & \frac{3 \times 2}{4 \times 2} + \frac{7}{8} \\
 \Rightarrow & \frac{6}{8} + \frac{7}{8} \\
 \Rightarrow & \frac{13}{8} \\
 \Rightarrow & 1\frac{5}{8}
 \end{aligned}$$

I: why did you multiply these [$\frac{3 \times 2}{4 \times 2}$] by 2?

BHB1: because you find that the lowest common denominator is 8

Another student who used equivalent fractions justified himself as:

BHB2: Change $\frac{3}{4}$ to $\frac{6}{8}$ and $\frac{7}{8}$, add them together

I: why did you change $\frac{3}{4}$ to $\frac{6}{8}$?

BHB2: Because you can't add fraction without same denominator

I: why do you think that is?

BHB2: Because it wouldn't really make too much sense

Another student also agreed with that idea:

Because to multiply you need to have the same denominator [BHG1]

Overall, the students in this ability group showed much better understanding of the concepts and used less algorithmic procedures to solve the tasks compared with the other two ability groups. The two schools approached task three very differently. School A used mainly Cross multiplication in a rote manner while School B used halving quarters of equivalent fractions. School B students knew the reason for making the two denominators same.

Chapter 5: DISCUSSION

5.1 Introduction

This chapter presents the discussion of the findings. First students' perception of the nature of mathematics were discussed, followed by the discussion of students' perception and experiences of group work, equipment, communication, teachers and finally students' responses to the tasks.

5.2 Students' Perceptions of the Nature of Mathematics

The majority of the students viewed mathematics as numbers or operations. This finding is consistent with Young-Loveridge et al. (2006) where they found students' view of mathematics revolved around numbers. Similar with Young-Loveridge et al.'s (2006) finding, there were a few students in the present study who defined the nature of mathematics as problem solving. However, it was interesting to note that only one student linked mathematics with other subjects and none of the students mentioned logic, thinking or reasoning as the nature of mathematics. Students' view of the mathematics could be broadened by linking mathematics to real life, involving students in problem solving where the problems are taken from areas of interest for the students.

All students agreed that mathematics was helpful. Some students thought it was helpful in issues related with handling of money and for jobs. This finding is also consistent with Young-Loveridge et al. (2006) where she found students' view of the use of mathematics for the future jobs. The more students know about the use of mathematics the more importance students would give to the subject. This is true as teachers' beliefs impact on their students (Grootenboer, 2002). If teachers focus on procedures rather than understanding then students would believe maths is all about following procedures. On the other hand, if teachers use an inquiry

base teaching, students view about mathematics would be different.

5.3 Group Work

Students with Low ability were grouped for mathematics lessons. Low ability students from School B sat in group but worked individually. According to Thomson and Brown (2000) this is not an effective way to get the most out of group work. However, Low ability students from School A engaged in some sort of cooperative learning. These students were given reward points if they helped each other or contributed to the group work. Lindaucer and Petrie (1997) suggests giving reward points for the whole group with individual accountability as an effective way to make the students work in groups. The findings revealed that majority of the Average and High ability students formed groups only when they needed to play games or activities. Thomson and Brown (2000) believed that cooperative learning can not be implemented just by grouping students. Teachers would need to learn about how to teach the students the skills they need to successfully work in cooperative learning groups (Johnson & Johnson, 1999). It was clear that the teachers lacked the knowledge of how to make the students work *in* groups.

Most of the students in the present study preferred working either in groups or both in groups and individually. This result is consistent with cooperative learning method proposed by Leikin and Zaslavsky (1999) as exchange-of-knowledge method where the students were allowed to work individually when appropriate. The main purpose of learning in groups is so that the students could learn from their peers. ‘What they learn today in groups, they do alone tomorrow’ is the main idea. Hence, the attitude of High ability students in the present study, working in groups when they encountered with challenging problems and working alone otherwise, serves the purpose.

5.4 Equipment

Students particularly in the Low Ability Groups used a variety of equipment including games and activities to enhance learning. Some students from Low group used them as a strategy to get answers, for others it was ‘fun’, or as a substitute when they had nothing else to do after an exam, or as a reward when they finish their work. This finding was consistent with Moyer (2001) who outlined a variety of reasons teachers used equipment in mathematics classroom apart from conceptual understanding. Researchers have emphasized the importance of giving a chance for the students to explore the ‘concrete’ materials, communicate with peers to link with their existing knowledge to form new mathematical concepts (Baroody, 1989; Ministry of Education, 2006d). The discussions, justifications and reasoning are the key components needed to trigger their thinking and leads to conceptual understanding (Cobb et al., 1992; English, 1998; Fennell & Rowan, 2001; Wheatley, 1991).

The majority of students in the Average ability group and a few from High ability group talked about the use of equipment as an aid in visualizing the mathematics problem. This shows that students use equipment to visualize either the mathematical concepts or the problem so they don’t always had to depend on the equipment. Ministry of Education (2006c) calls this as using equipment to provide a concrete foundation to build their mathematical concepts. Teachers are encouraged to use equipment to teach any new concept. The ultimate goal of using equipment effectively leads the students to let go of the equipment when they grasp the concept.

The finding revealed that students’ chances and expectations of using equipment to learn mathematics decreased with their ability group. Most of the students from the High ability group had a negative attitude towards using equipment. They felt that because they were in the top class they did not need equipment. These students felt that they had learnt all the concepts they needed to know using the equipment in primary and intermediate school and now they were able to visualize the concepts. Hence there was no need for using it anymore - they only need the

teachers to ‘teach, teach and teach’. This contradicts the findings of English (1998) who found that students’ perceptions of the usefulness of equipment did not decline with grade levels. In fact, English suggested that students in all grades be presented with concrete materials when learning new concepts. The perceptions of the High ability students might have been negative because they did not get a chance to use appropriate equipment to learn the concepts. Students in secondary learn new concepts all the time and it is essential to use appropriate material for the students to effectively grasp these complex concepts.

The study also revealed that the students in both schools, especially the high achievers were discouraged from using calculators. This was done to encourage the students’ use of mental strategies to develop number sense. Students also believed that the use of calculators did not enhance understanding mathematical concepts as they could get an answer by using the calculator but sometimes the answer does not make sense. This finding contradicts Grant (1996) and Huinker’s (2002) research about how calculators could be used to extend students’ mathematical thinking. However, the reason for not using calculators could be due to Ministry of Education (2004) discouraging the use of written algorithms before the students get a good understanding of part-whole strategies. In this modern world, it is important to teach students to use technology to enhance learning though effort is need to develop guidelines on how to use new technology in constructive ways (Lawrenz, Gravely, & Ooms, 2006).

5.5 Communication

Students in the present study wanted to share their answers with their peers for a variety of reasons but no one mentioned sharing for understanding. Students felt it was important to explain their thinking to their teachers so that the teacher knew they were not cheating or the teacher could help them or reassure them. This finding is consistent with Young-Loveridge et al. (2005) where they found that students felt sharing answers were cheating. This might be because the students were not used to sharing answers, or when they share they just want to get the answer correct, not to understand the mathematical concepts. If they got a chance

to practice mathematical arguments from a very young age then they would have been more confident with the process (Walter, 2003).

The students' exchange of multiple solution strategies revealed that what they exchanged or discussed was only the procedures and their answers. Read out the steps they have followed does not need any level of thinking. This type of communication is referred by Monaghan (2006) as exploratory talk. The students in present study are at context of strategy reporting stage as mentioned by Hunter (2005). She found her students initially at this stage in a mathematics class undergoing reform in communication. The classroom norm was set for communication and by the end of the year students had started using inquiry and explanation in the classroom building up to argumentative discourse.

The ultimate reason for sharing solution strategies for the students in the study seemed to be to find a variety of procedures to remember when they forget one. This finding is consistent with that of Pratt (2006) who found the students in the study giving priority for listening. As Pritt points out, this could be due to the student's conceptions of memorizing the 'best' result. Knowing how to solve a problem is important as they would have to sit in exams. However, if they really understand then they do not have to memorize the procedures!

Some students in the study showed their understanding of some mathematical terms by using them correctly. However, just using terms like that is not enough to communication in mathematics although it is a necessary starting point to build confidence to engage in a mathematical conversation (Cai & Kenney, 2000). One important thing to keep in mind to encourage students sharing their thinking would be to let the students know if their answer is correct before asking him/her to explain the procedure (Ministry of Education, 2006b). There were a few students, especially from the Low Ability Group who were concerned about their answer. They were reluctant to share their answers, thinking their answer was wrong. Yackel and Cobb (1996) and Kazemi (Kazemi, 1998) talked about the importance of setting a sociomathematical norm in mathematical communication. This could be achieved by learning the appropriate skills and getting opportunities

to practice these skills in a favourable environment where students are confident to take risks and mistakes are viewed as an opportunity to learn.

5.6 Teachers

Students in this study identified teachers as the most important element in their mathematics classrooms. This finding is consistent with that of Aldridge et al. (1999) where the students valued teachers as the most important factor in a positive classroom learning environment. Teachers can make all the difference in a classroom. They can make the subject interesting, easier or even have a positive attitude towards the subject.

All students expected their teacher to be able to explaining the contents well. This finding is consistent with Kyriacou's (1986) findings where most students wanted their teacher to be able to explain content clearly and at their level. This shows that students are mostly doing the 'listening' instead of talking. As Taylor et al.'s (2005) suggested, these students' perceptions about the role of their mathematics teachers need to be changed to that of a mentor in order for them to learn mathematics effectively. Teachers teach more than contents, they teach attitudes (Whitin & Cox, 2003). It is important to let the students know that teachers do not know all the answers and takes risk and are vulnerable too. Whitin and Cox found that this knowledge gives students the confidence to the students to do the same.

5.7 Students' responses to the tasks –develop this section

Students' strategies for solving the tasks ranged from counting on (stage 4) to advanced proportional part-whole thinking (stage 8) on the number framework (see Appendix F). The students from the Low Ability Group struggled with the tasks using techniques such as counting on using fingers, guessing and standard algorithms. At the other extreme students from the High Ability Group solved the tasks with ease and using sophisticated techniques like place-value partitioning or compensation (Yackel, 2001; Young-Loveridge, 2002b). Almost all students were

able to solve the addition task, but there were a few students who used counting-on to solve it. Young- Loveridge (2002a) has suggested some activities to help students move from counting-on to part-whole thinking.

Only half of the students were able to solve the multiplication task and less than half the students were able to do the fraction task. From the students who were able to solve the task, some were not able to give reasons for their actions, showing that they have learnt rote procedures. Students who were successful in solving the fraction task used cross multiplication and equivalent fractions. Some of the students who used cross multiplication were not able to give reasons for making the denominator same or why they had to multiply the numerators by the denominator of the other fraction. This is consistent with the findings of Young-Loveridge, Taylor, Hawera and Sharma (in preparation) where they found only a few students had a deeper understanding of fractions. Young-loveridge et al. (in preparation) found that only 13% of the students of Year 7 and 8 could do $\frac{3}{4} + \frac{7}{8}$ correctly. Understanding of fraction is vital for algebra (Irwin & Britt, 2005b). Hence the lack of success in late Year 9 is of concern in relation to helping students think algebraically and the teaching of formal algebra.

Chapter 6: CONCLUSIONS AND IMPLICATIONS

6.1 Conclusions

This research investigated the perceptions and experiences of Year 9 students in mathematics, participating in Secondary Numeracy Project at two schools in their first year of the project. The findings revealed that a majority of students believed the nature of mathematics to be numbers and used mostly when handling money and for future jobs. They also perceive mathematics learning as following different procedures. If they share multiple solutions, they share the procedures and answer. The group work is not structured to advocate for cooperative learning. If students were in groups, they were mostly working individually. Students enjoyed using equipment, playing games and activities in mathematics classroom because it was fun. However, they had the idea that the materials used in their classroom did not help them to learn any of the concepts they were being taught. Communication is a basic requirement for learning mathematics for understanding, however, this area is not developed much among the students involved in this study. Teachers were mainly considered good if they could explain the lesson well. The study reveals that students form groups, use equipment and sometimes share solution strategies. However, these areas need to be improved in order to effectively implement SNP. Hence improve students' attitude towards mathematics and raise mathematics achievement.

6.2 Limitations

These findings add to our knowledge about students involved in the Secondary Numeracy Project. The study explored students' perspectives on the nature of mathematics, and experiences and attitudes towards mathematics learning. However, there are some limitations to be acknowledged. First of all, this study

was done on only twenty four students so the findings might not be generalized to all Year 9 students involved in the SNP in New Zealand. Second limitation is that the investigation relied on the children's oral responses and explanations, frequently to questions not previously considered. Some students frequently found it difficult to express their ideas as clearly as other children saying 'I don't know'. This could have masked their actual perceptions and sentiments (Garner & Alexander, 1989). The third limitation is what Cohen, Manion and Morrison (2000) have talked about as the students' will to please adults – what the children said was what they really felt or what they thought the researcher wanted to hear.

Fourth limitation is my origin. I am from another country and English is not my native language. Hence, there may have been cases of misunderstanding (Cohen et al., 2000). Fifth limitation was my skills as a researcher. Ginsburg (1997) acknowledged that conducting clinical interviews requires a long process of preparation and training. I am new to interviewing. In addition, I had been teaching mathematics for a few years so sometimes it was difficult for me to separate the teacher and the researcher in me. I provided hints to solve the mathematical tasks to get the correct answer. Also clinical interviewers should not let the students know whether the answer was right or wrong (Biddulph et al., 2003). However, some students thought their answer was wrong and hesitated to explain how they arrived at the answer. So I had to let them know whether the answer was right or wrong.

6.3 Implications

Implication of this study is divided into two parts; implications for further research and implications for the schools and teacher, as teachers are the key to the reform movement (Battista, 1994; Ministry of Education, 2006a).

Implications for further research

It might be interesting to do a further study on these students by involving the students in an inquiry base classroom where they get to use equipment to explore

in small groups and involve in explorative talk to make sense and understand the mathematical concepts. Then do the same interview to find their experiences and perceptions of mathematics. This study could be developed by observing some lesson, or collecting data on teachers' perceptions towards the project and towards teaching mathematics and their philosophy. This could be developed to a national study by collecting data from all the secondary schools involved in the SNP to get a better view of the students' perception in general.

Implications for the Schools and Teachers

Teachers need to have a good knowledge of contents to initiate an open-ended communication on a mathematics topic. However, not all teachers are confident in their mathematical knowledge. Hence it might be useful for the schools to conduct short courses or classes to upgrade their mathematical content knowledge. The content knowledge could provide the teachers with the confidence they need in advocating communication and relating equipment or activities used in the classroom to develop mathematical understanding. Also, most secondary school teachers are pressured to cover the curriculum but if the students come to secondary schools, well equipped with communication and cooperative learning skills, the teachers might find group work speeds up the learning process.

To effectively implement the project and raise student achievement, there is as much learning for teachers as for students (Hiebert, 1999). Ongoing professional development for teachers is one of the key features of the NDP. The teachers need extensive in-service training programs providing both curriculum materials as well as instruction on mathematics and mathematics learning (Battista, 1994).

Supporting teachers by developing appropriate teaching materials, games and activities especially targeted for secondary school students (Higgins, 1999). This will help teachers and students to connect the equipment with the appropriate mathematical concepts. Another issue is teachers might worry that if students engage in communication the class might get noisy and out of control. There will be some noise but the key is to structure. The lesson should be structured in a way

to minimize the distractions.

Overall, the study explored students' perception of SNP indirectly via their involvement in group work, communication and use of equipment. Their attitude was mostly positive towards the change in their learning style. However, teachers need to learn and teach the students some skills necessary to successfully implement this reform project which will enhance students' mathematical learning for understanding and raise their achievement. However, we have to keep in mind that changing a teaching method is a process that takes time. When the change involves implementation as well as learning the procedures to be implemented, it could be very fragile (Lawson, 1997). The process will take time, effort and positive attitude. Hence teachers need to be supported by the encouragement of those who have experienced the process, fellow teachers and school as a whole.

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Appendix A

STAGES IN NUMBER FRAMEWORK

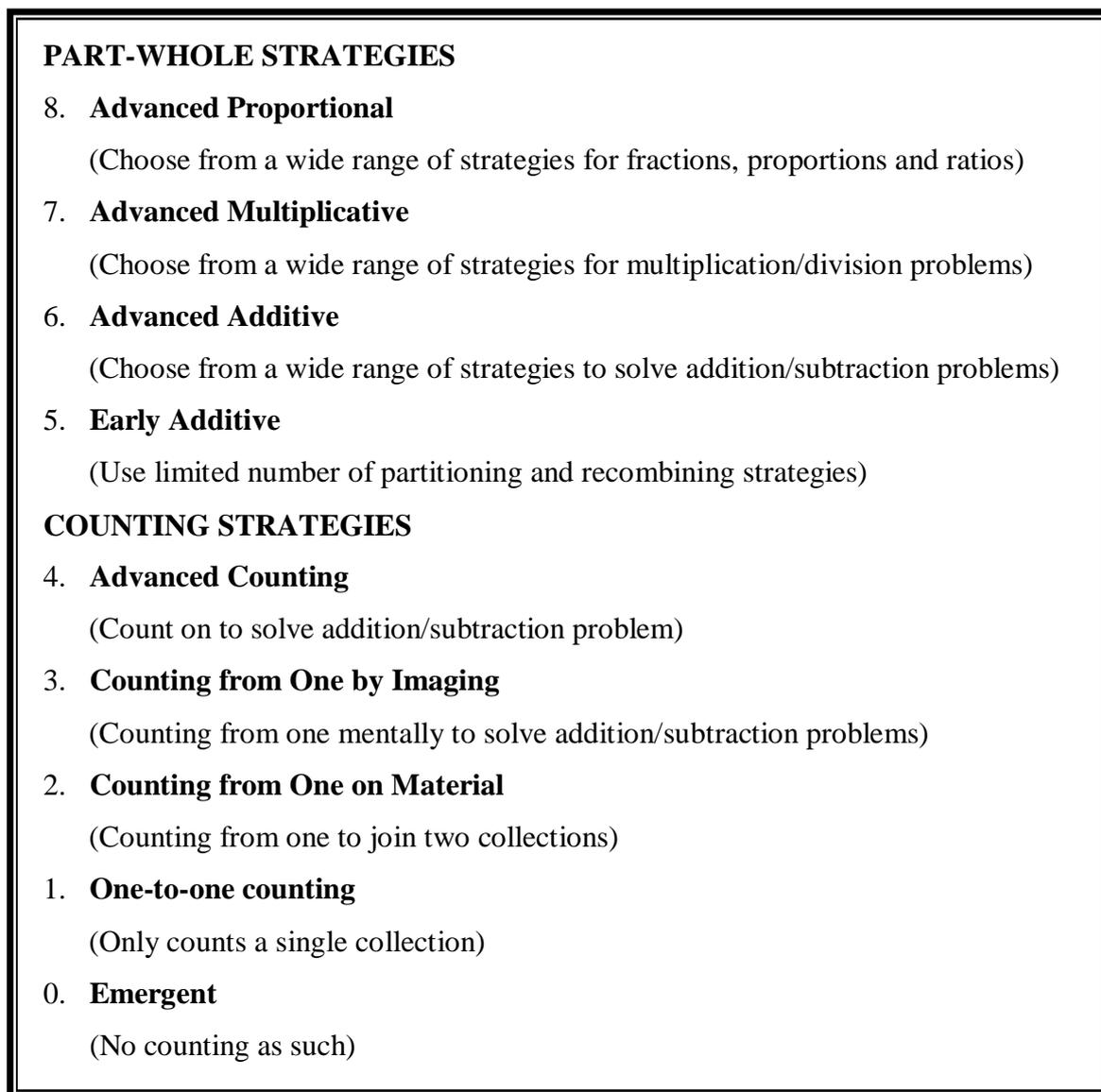


Figure 3: Stages of the Number Framework (Ministry of Education, 2006a)

Appendix B

Introductory letter to the Principal

264 Clyde Street,
Hillcrest,
Hamilton

Date: _____

Principal,
X School

Dear _____,

I am a Postgraduate student at the University of Waikato, enrolled in a Master of Education degree in the School of Education. I would like your help in fulfilling the research requirements for a three-paper thesis, which forms a significant part of this degree. This thesis aims to explore the impact of the Secondary Numeracy Project on students and teachers.

For this research I would like to interview 15 participants from your school. That is 12 students and three teachers. Preferably I would like to choose students from three classes; high, average and low ability. Then ask the class teachers to fill a questionnaire. I would like to give some mathematics problems in order to gather information about the strategies students use to solve subtraction, multiplication and proportional problems. The interview for the students is semi-structured, which will allow for a more informal discussion between us. In addition to that there will be a brief 2 page long questionnaire for teachers. It is to find teacher's view of mathematics and their practice in the classrooms. It will take around 10-15 minutes to complete.

I would like to talk to children individually for about 30 minutes; at a time the teacher indicates will be least disruptive to their school-work. Participation will be entirely voluntary. The children may choose not to answer a question, or stop the interview at any time. The interview will be audio-taped with the child's consent.

The child's name will not be used in the final research report and everything s/he tells us will remain confidential. The only people to have access to the tape will be my supervisors (Young-Loveridge & Sashi Sharma), University of Waikato staff transcribing tapes and myself. When the research report is complete, I will forward a summary of the thesis to the school for staff and parents.

I would value your help in arranging for children to interview, if possible in mid-July. I enclose letters of information and consent forms for parents or caregivers of the children to be interviewed. A separate letter of information for teachers is also enclosed. If you need more clarification on the topic or more information on this research study, please contact me on (07) 858 2063 (evenings), (021) 0336771 (mobile) or email me at ft18@waikato.ac.nz.

Yours Sincerely,

Fathimath Thereesha.

Letter to the Mathematics Teacher

264 Clyde Street,
Hillcrest,
Hamilton.

Date:

Dear Mathematics Teacher,

Kia ora,

I am a Postgraduate student at the University of Waikato, enrolled in a Master of Education degree in the School of Education. I would like your help in fulfilling the requirements of research for a three-paper thesis, which forms a significant part of this degree. This thesis aims to explore the impact of the Secondary Numeracy Project on students and teachers.

For this research I would like to interview 15 participants from the school. That is 12 students and three teachers. Preferably I would like to choose students from three classes; high, average and low ability. And you will be asked (maths teachers from these classes) to fill a questionnaire. I would like to give the students some mathematics problems in order to gather information about the strategies they use to solve subtraction, multiplication and proportional problems. The interview for the students is semi-structured, which will allow for a more informal discussion between us. A brief questionnaire on teaching and learning of mathematics is prepared for the teachers which will take around 10 minutes to complete.

I would like to talk to children individually for about 30 minutes; at a time the teacher indicates will be least disruptive to their school-work. Participation will be entirely voluntary. The children may choose not to answer a question, or stop the interview at any time. The interview will be audio-taped with the child's consent. The child's name will not be used in the final research report and everything s/he says will remain confidential. The only people to have access to the tape will be my supervisors (Jenny Young-Loveridge & Sashi Sharma), University of Waikato

staff transcribing tapes and myself. When the research report is complete, I will forward a summary to the staff and parents.

I would value your help in arranging for children to interview, if possible in mid-August. I enclose letters of information and consent forms for parents or caregivers of the children to be interviewed. A separate letter of information for Principal is also enclosed. If you need more clarification on the topic or more information on this research study, please contact me on (07) 858 2063 (evenings), (021) 0336771 (mobile) or email me at ft18@waikato.ac.nz.

Yours Sincerely,

Fathimath Thereesha.

Letter to the parent/caregiver

95B Aurora Terrace,
Hillcrest, Hamilton.

To the parent/caregiver of 13th
September '06

Kia ora,

I am a post graduate student at the University of Waikato, enrolled in a Master of Education degree in the School of Education. I am doing some research to find out about students' perspectives towards learning mathematics as part of the Secondary Numeracy Project. I would like to give your child the opportunity to talk about what s/he thinks about mathematics learning this year.

I would like to have (or I had) a chat with your child. S/he have/had the choice not to answer a question, or stop the interview at any time. The interview was/will be audio-taped with your child's agreement. Your child's name won't be used in the final report and any other publications. I will forward a summary to the school for staff and parents.

It would really be appreciated you could authorize me to use the data for my thesis. If you are happy about this, please fill in the consent form below and return it to school with your child by 15th September '06. If you have any questions or require further information, please feel free to call me on (07) 858 3424 or (021) 0336771 or email on ft18@waikato.ac.nz.

Yours sincerely,
Fathimath Thereesha.

Parent/Caregiver Consent

I agree (child's name)’s data to use in . I understand that the interview was/will be audio-taped with my child's agreement, and that all information will be kept private. I realize that my child's name will not be used in the report or any other publications so that s/he cannot be identified. I understand that my child had the choice to skip any question, or stop the interview at any time.

Signed:

Name:

School:

Date:

Appendix C

Student Profile

School	level	Student	Date of Birth	Gender	Ethnicity	Last school	form	Previous school	Years	No. of other schools	
A	LOW	ALB1	? - 92	M	FI	Ma	F1-2	India			
		ALB2	12-92	M	M	Ma	F2	LW	Y5-F1		
		ALG1	02-93	F	P	C	Y1-F2				
		ALB3	04-93	M	P	WH	Y4-F2	Ha	Y1-2	2	
	AVERAG E	AAB1	03-93	M	P	StA		P		1	
		AAB2	02-93	M	P	Ma	F1-2	F	Y4-6	2	
		AAG1	08-92	F	P	Ma	F1-2	Me	Y2-4	1	
		AAG2	11-92	F	FI	StA	F1-2	Va	Y5-6		
	HIGH	AHB1	11-92	M	P	Ma	F1-2	F	Y1-6		
		AHB2	08-93	M	P	Ma	F1-2	SC	Y1-6		
		AHG1	09-92	F	P	Ma	F1-2	A	Y1-6		
		AHG2	01-93	F	P	Ma	F1-2	F	Y4-6		
	B	LOW	BLB1	12-92	M	P	PI	F1-2	N	Y1-6	
			BLB2	08-92	M	P	PI	F1-2	R	Y1-6	
			BLG1	10-92	F	P	PI	F1-2	K	Y1-6	
			BLG2	07-92	F	FI	PI	F1-2	K	Y6	
AVERAG E		BAB1	10-92	M	P	PP	Y1-F2				
		BAB2	10-92	M	P	B	F1-2	S	Y1-6		
		BAG1	07-92	F	P	B	F1-2	N	Y4-6	1	
		BAG2	10-92	F	P	B	F1-2	S	Y1-6		
HIGH		BHB1	06-93	M	A	B	F1-2	H	Y6	2	
		BHB2	12-92	M	A	B	F1-2	H	Y5-6		
		BHG1	01-93	F	P	B	F1-2	T	Y4-6	1	
		BHG2	12-92	F	P	B	F1-2	N	Y1-6		

FI – Fiji Indian

M – Maori

P – Pakeha

A – Asian

Appendix D

STUDENT INTERVIEW PROTOCOL

(Adapted from Young-Loveridge et al., in press)

INTRODUCTION

I am trying to find out more about how kids learn maths and how teachers can help them. I am especially interested in what kids themselves think about mathematics. Would you be willing to talk to me about what happens when you're learning maths?

I can't write very fast, so it would help me if I could turn on the tape recorder. Then I can concentrate on listening to you without having to write. Is that OK with you?

If there are any questions you want to skip, just let me know.

If you want to stop talking to me, that's fine too.

Everything you say today will be kept completely confidential.

This piece of paper [consent form] says:

I've explained what we are going to do.

You are happy about the tape recorder being on.

You know you can skip a question, or stop talking at any time.

Everything you say will be kept confidential.

Your name will be changed in the report so no one will know it's you.

Is all that OK with you?

Could you please sign your name here to show that you are happy about this?

Possible follow-up probes

Can you tell me (or explain) why you think that?

Can you give me an example?

Tell me about that?

Can you tell me how you would start to do that question?

Start Tape & say: can you say your name into the tape to start?
So, [NAME], can u tell me what you've been studying in mathematics recently?
Do you like that topic? Why? Why not?

GROUPS

Do you have groups for maths? Which maths group are you in?
Why are you in that group?
Do you contribute during group work? How much?
How does the teacher know whether all students are contributing?
Do you think that people should work at maths on their own, or should they work in groups? [Probe: small or big?] why?
What do you prefer?

EQUIPMENT

Do you ever use equipment (like counters) when you're doing maths? How?
Is it helpful? Would you like to use equipment in maths?
Do you think equipment helps people learn maths? How?
Do you ever use your fingers in maths? When do you use them?
Do you use calculators? When did you start using them?
Is it important to work out maths problem in your head? Why?
When you are doing maths, do you ever use;
Work sheets? [Probe: how often? How do you feel?]
Textbooks? [Name]
Figure It Out books?
Computers?
Or play games or do activities? [How often? Do you like it?]

SOCIAL DIMENSIONS

How do you check your answer?
Is it important for you to know how other people get their answers? Why?
Is it important for you to be able to explain to other people how you worked out

your answer? Why?

What about being able to explain your thinking to your teacher? Is that important?

Why?

Do you get a chance to explain your answers to anyone?

[teachers/students/parents]

Do you talk about/discuss maths problems at home?

Do you get help from your family members when doing your homework?

Are they good in maths and do they like maths?

WRITTEN PROBLEMS

I'm going to show you a problem and ask you to work out an answer. Then I'm going to get you to explain how you worked out your answer and make some notes about it here. Is that OK?

You have 47 dollars in your piggy bank. You take 9 dollars to buy a toy. How many dollars you have in your piggy bank now?

Teacher has 98 marbles which she wants to distribute among 14 students. How much will each student get?

Harry and Sally buy two pizzas. Harry eats $\frac{3}{4}$ of a pizza while his friend Sally eats $\frac{7}{8}$ of a pizza. How much pizza did they eat?

For each question ask:

Can you think of another way to do that question?

What's the best way for you of working out an answer?

Backup Problem for students who cannot answer more than 2 questions:

You have 6 lollies a you got another 8 lollies from your brother. How many lollies do you have now?

IMPORTANCE OF MATHS

Do you think maths is important? Why?

Do you see maths anywhere around you? Anywhere else?

Do you do things at home or in other places that might involve maths?

What does your family feel about maths (important or not)? How do you know?

VIEW AND ATTITUDE

What do you think mathematics is all about? (Probe: If you were going to tell someone about what maths is, what would you say to them?)

If maths were a food what food would it be? why did you choose that food?

Have you always felt like this about maths?

When did it change?

Can someone who is really bad at maths – could they get to be really good at it?

How?

SELF ASSESSMENT

Very poor	Poor	Average	Good	Very Good

How good are you at maths? Which one will you choose from the above box?

How do you know that?

Have you always been that good at maths?

Does the maths you do usually make sense to you?

What happens when the maths you are doing doesn't make sense to you? What do you do then?

NUMERACY PROJECT

Have you heard about the Numeracy Project or SNP? What do you know about SNP?

Did you have ENP or ANP in your primary school(s)? Or INP at your intermediate school?

Do you feel or see a difference in the way you were taught this year from last year(s)?

How is it different?

Do you like the way your teachers teach this year?

Is the way s/he teaches easier or more confusing?

What are the things you like about your maths class this year? [and don't like]

What are the things you liked about maths class last year or previous years?

CONCLUSION

Is there anything more you want to tell me about learning maths?

Thanks for helping me understand what kids think about learning maths.

Appendix E

Student's Consent

It has been explained to me what we are going to do. I am happy for the tape recorder to be turned on. I understand that I can skip a question, or stop talking whenever I want. I know that everything I say will be kept confidential, and that my name will not be used in the report.

Signed:

Name:

School:

Room: Year:

Date of Birth: Age:

Date of Interview:

Gender: Male or Female

Ethnicity: NZ Maori

Pasifika: Island group

Pakeha New Zealander

How long have you been in this school?

What is your last school?

Previous School(s):

Intermediate School From to

Primary School From to

THANK YOU

Appendix F

Stages of the students in the study

Ability groups	I	F	I	F	I	F	I	F	I	F	I	F
	add	add	mult	mult	prop	prop	frac	frac	pv	pv	bf	bf
ALB1	4	5	5	4	5	5	6	6	5	5	5	5
ALB2	5	5	5	6	5	5	4	4	5	4	6	7
ALB3	5	5	6	7	5	7	5	5	5	5	5	6
ALG1	5	4	4	4	4	5	4	5	5	5	5	5
BLB1	5	6	5	5	5	5	4	4	4	5	5	4
BLB2	6	6	6	7	6	7	5	ab	5	ab	6	ab
BLG1	4	4	4	5	2_4	5	5	6	5	5	6	6
BLG2	5	5	ab	6	5	5	5	5	5	5	5	6
AAB1	ab	5	ab	5	ab	7	ab	5	ab	5	ab	6
AAB2	ab	6	ab	6	ab	6	ab	6	ab	7	ab	6
AAG1	5	5	4	4	5	5	5	5	5	5	5	6
AAG2	5	5	5	5	6	7	5	6	6	7	6	6
BAB1	5	6	7	7	7	6	6	6	7	7	6	7
BAB2	6	7	6	6	7	7	7	7	6	6	6	6
BAG1	7	7	6	7	7	6	6	ab	5	ab	7	ab
BAG2	4	6	6	7	6	7	6	6	5	8	5	6
AHB1	7	7	7	8	7	8	8	8	7	8	7	7
AHB2	7	7	8	8	7	8	7	8	8	8	7	7
AHG1	5	6	6	6	7	7	8	7	8	8	7	7
AHG2	5	5	6	6	7	7	7	7	6	7	7	7
BHB1	7	7	8	8	8	8	7	8	6	7	6	7
BHB2	7	7	8	8	7	8	8	7	8	8	6	6
BHG1	7	6	6	8	7	7	6	8	7	7	6	6
BHG2	6	7	6	7	7	7	5	6	7	8	6	6