

CRICKET BOWLING: A TWO-SEGMENT LAGRANGIAN MODEL

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In this study, a Lagrangian forward solution of the bowling arm in cricket is made using a two-segment rigid body model, coupled with projectile equations for the free flight of the ball. For given initial arm positions and constant joint torques, the equations are solved numerically to determine the ball speed and arm angle at release so that the ball can land on a predetermined position on the pitch. The model was driven with kinematic data from video obtained from an elite bowler. The model can be analysed in order to study the biomechanics of the bowling arm as well as to quantify the effects of changing input parameters on the trajectory and speed of the ball.

KEY WORDS: cricket, bowling biomechanics, Lagrangian model

INTRODUCTION: The function of bowling in cricket is to deliver the ball to the batter. In the majority of deliveries, the ball bounces once off the ground. The laws of cricket specify that any straightening of the bowling arm must occur well before the time of ball release. The basic bowling action can be described as a series of sequential steps (Figure 1).

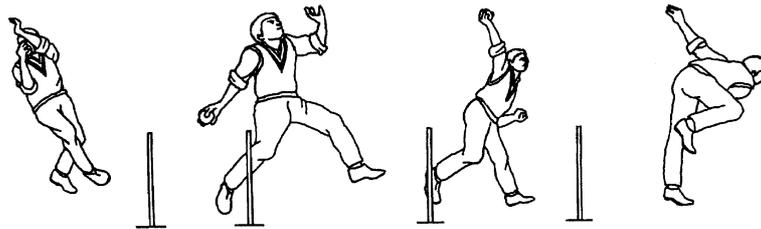


Figure 1 - The basic bowling technique (right-hand bowler): the run-up, leap, right foot contact, left arm motion, bowling arm rotation, left foot contact, ball release, and follow-through. A good technique allows the bowler to deliver a ball with speed at a chosen point on the pitch while maintaining a straight bowling arm [Picture from Biddle, S., Morris, P., de Looy, A. & Thomas, P. 1991. *Get ready for cricket. A complete training programme.* Courtesy of the The Crowood Press Ltd., Wiltshire.].

To produce successful bowlers, it is imperative that correct bowling technique is taught at a young age. However, much of the available instruction in this area is subjective, based largely on the experiences of successful past bowlers. Previous bio-mechanical research in bowling has been largely confined to the analysis of kinematic data. In this paper, a forward solution model of the bowling arm is developed which allows a range of joint torques and initial arm positions to be specified to determine which combination of ball speed and arm angle at release will land a ball on a predetermined position on the pitch.

MATERIALS AND METHODS:

Link segment model. Using the approach of Winter (1990), a two segment link model of the bowling arm was constructed with respect to a global reference system and two local reference systems. This was combined with three generalised coordinates [q_1 (linear shoulder displacement), q_2 (upper arm angular displacement), and q_3 (forearm angular displacement)] (Figure 2b). Segment lengths are represented by l_j , the distances to the centres of mass (measured from the proximal end) represented by r_j , the segment masses by

m_j , and the joint torques T_1 , and T_2 chosen to represent the shoulder and elbow torques, respectively (Figure 2a, b). Note that m_2 includes the mass of the forearm, hand and ball. The origins of the local reference systems are at the proximal ends of the segments. Reaction forces at the joints are not required, so the bowling arm model can be completely specified using two local reference systems, two segments, three output displacement variables and two torques.

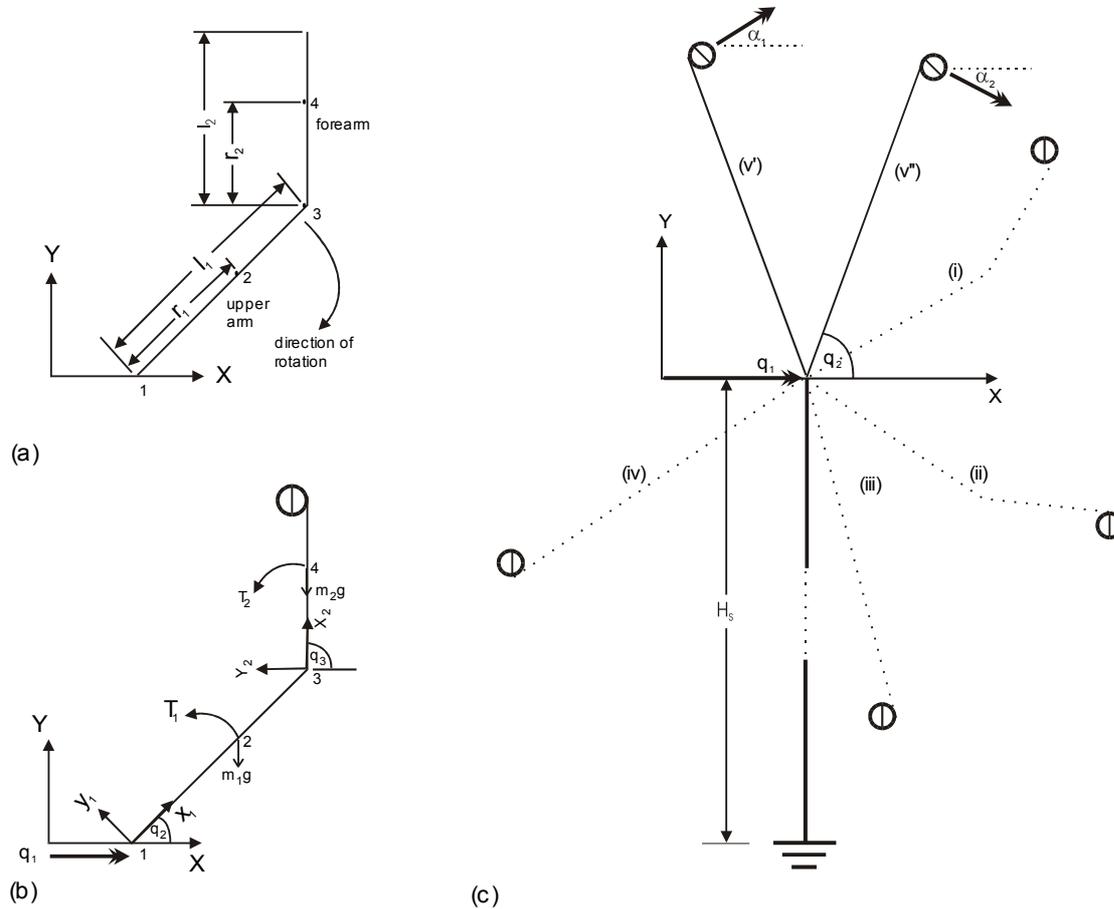


Figure 2 - (a) Two segment link model of the bowling arm prior to straightening (i.e. before locking). (b) The link segment model showing the global reference system, two local reference systems, two joint torques (T_1 , T_2), and three output displacement variables (q_1 , q_2 , q_3). (c) The dotted stick figure sequence (i) to (v) shows how the rotation of the link segment model can represent the rotation of the bowling arm in cricket. Stages (i) and (ii) depict the bowling arm before locking, which occurs at (iii). Release positions (v') and (v'') represent two possible solutions at α_1 and α_2 , respectively.

Lagrangian equations of motion. The Lagrangian mechanics approach requires the formulation of the Lagrangian L , given by

$$L = T - V \tag{1}$$

where T and V are the kinetic and potential energies of the system, respectively, written in terms of the independent variables q_i . The resulting equations of motion are generated by using the following:

$$\frac{\partial (\partial L / \partial \dot{q}_i)}{\partial t} - \frac{\partial L}{\partial q_i} = Q_i, \quad (i=1, \dots, 3) \quad (2)$$

where the Q_i are the virtual work expressions which involve the applied joint torques.

Projectile study. Projectile equations mimic the goal-oriented approach of the bowler. In this case, the objective is to hit a target on the pitch at a given distance from the bowler's end, given a specified speed and trajectory. Ball release in cricket is performed when the arm is straight (i.e. locked), and therefore the projectile equations are developed for the case of a straight segment, $l_1 + l_2$ in length, with the release height H_G given by

$$H_G = H_s + (l_1 + l_2) \sin(q_2) \quad (3)$$

where H_s is the height of the shoulder (Figure 2c).

Manipulation of the standard projectile equation of motion for which the projection point is at a different level from the impact point ($Rg = u \cos \alpha \pm \sqrt{u^2 \sin^2 \alpha - 2gh}$) (de Mestre, 1990), yields the following equation for *ball release speed* u :

$$u = \sqrt{\frac{R^2 g}{2 \cos \alpha (H_G \cos \alpha + R \sin \alpha)}} \quad (4)$$

where R is the range of the ball, and α is the *ball release angle* which can be related to q_2 through simple geometry.

Numerical solution. The system is considered an initial value problem, where the choice of initial conditions and input torques was based on actual 2D kinematic data captured and analysed by the Peak5 motion analysis system at the Centre for Sport Studies at the Waikato Polytechnic, Hamilton.

The system response $\{q_1(t), q_2(t), q_3(t), \dot{q}_1(t), \dot{q}_2(t), \dot{q}_3(t)\}$ can then be found by numerically solving the second order differential equations generated by (2). This was performed by Senac (Broughan, 1992), a symbolic computer language developed by the University of Waikato. The projectile equations were linked to the Lagrangian solution by

$$\dot{q}_2(l_1 + l_2) = u \quad (5)$$

RESULTS: In many of the modelled bowling trajectories, it was found that two release points satisfied the solution criteria (Figure 3). For instance, consider the particular case where T_1 and T_2 are equal to 10 and 12 Nm, respectively, for $q_1(0) = 0$ m, $q_2(0) = -45^\circ$, $q_3(0) = 60^\circ$, $\dot{q}_1(0) = 5$ ms⁻¹, $\dot{q}_2(0) = 0$, and $\dot{q}_3(0) = 0$ in order to deliver a ball at a target on the pitch 20 m away.

The first release point (0.241s) achieved a speed of 88.6 km/h at an arm position 61° behind the vertical, whereas the second point (0.272s) achieved a speed of 138 km/h for an arm position 5° behind the vertical. The first release point is unlikely to be used in match situations and can be disregarded. However, the second release point correlates reasonably with the actual release point of a bowler delivering a yorker (i.e., a ball at a batter's feet) as observed from video footage.

It was also found that increasing the initial angular displacement of the upper arm from $q_2(0) = -40^\circ$ to $q_2(0) = 0^\circ$ produced an 11.7% increase in ball release velocity.

Interestingly, increasing the initial position of the upper arm also had a significant effect on the locking angle. At $q_2(0) = -40^\circ$, the arm locked at -79.2° , whereas when $q_2(0)$ was increased to 0° , the arm locked at -25.4° .

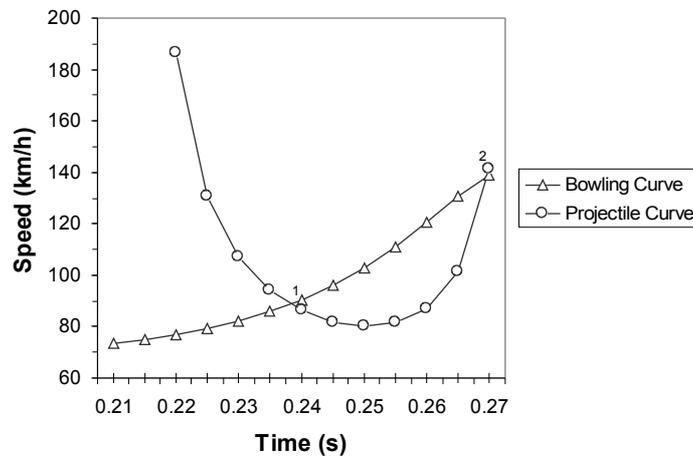


Figure 3 - Projectile equations determine the range of validity for the solution of $\dot{q}_2(t)$. In this case, there are viable solutions at $t = 0.241$ s (1), and $t = 0.272$ s (2).

DISCUSSION: In this study, a general approach was developed, which allows a range of joint torques and initial arm configurations to be chosen, in order to investigate their effect on the required ball release speed and arm angle, required to land a ball on a predetermined position on the pitch. Certain characteristics of the bowling action, such as initial arm position and locking angle, were found to have a bearing on ball release speed.

There were a few simplifying assumptions of the model, such as constant torques, and the small number of segments (two). Also, standard tables were used to select the anthropometric constants for the model. Despite its limitations, even the simplified model used here, could predict that a typical yorker length can be achieved by releasing the ball at about 5° before the arm is vertical. This is reasonably consistent with the corresponding video footage.

CONCLUSION: At present, there is no consensus on which elements of bowling technique contribute most to the generation of ball release speed (Bartlett et al., 1996). To achieve this would require the further development and validation of forward solution models such as the one described here. For example, incorporation of variable joint torque patterns and optimisation over input parameter ranges would facilitate future research into bowling technique.

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