Mathematical investigations: A primary teacher educator's narrative journey of professional awareness

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Abstract

Over a period of twenty months a mathematics teacher educator uses narrative inquiry, a form of story-telling, to investigate her professional practice in working alongside pre-service primary teachers. Two main themes emerge in this research.

The first of these centres around the use of mathematical investigations as a vehicle for supporting pre-service primary teachers to consider what the learning and teaching of mathematics may entail. As part of this process the author personally undertook several mathematical investigations. This resulted in significant learning about previously unrecognised personal beliefs about the nature and learning of mathematics. These beliefs were discovered to include ideas that 'real' mathematicians solve problems quickly, do so on their own and do not get stuck. Surprisingly, all of these subconscious assumptions were contrary to what the author espoused in the classroom.

A consequence of this learning included some changed beliefs and teaching practices. One such change has been moving from a conception of mathematics as a separate body of 'correct' mathematical ideas, and where the emphasis when doing mathematics was on attaining the correct answer, to now viewing mathematics as a sense-making activity involving discovering, doing and communicating in situations involving numbers, patterns, shape and space. Thus, mathematics is now perceived to primarily be found in the 'doing' rather than existing as a predetermined body of knowledge. As such one's interpretations of a mathematical problem are important to consider.

Changes in teaching include using mathematical investigations as a teaching approach with the belief that students can effectively learn mathematical ideas using this approach; an acceptance that this may involve periods of being 'stuck' and that this does not mean that the teacher needs to immediately

support the students in becoming 'unstuck'; more in-depth interactions, including questioning, to support this mathematical learning; and an acceptance that mathematics can be learned by people working in a collaborative manner.

The second theme encountered in this narrative inquiry involves the exploration of narrative as a powerful means with which to pursue professional development. Narrative inquiry, including a mention of differing forms of narrative writing, is described. Issues also considered include the place of reflection in narrative; the notion of multiple perspectives that are encountered in qualitative research such as this; issues of validity and authenticity; a consideration of what the products of narrative research might be and who may benefit from such research; a brief mention of collaboration; and the place of emotion in qualitative research.

The concept of change occurring within a narrative inquiry is not seen to imply an initial deficit position. Rather the research process is regarded as the building of a narrative layer that supports and grows alongside the writer's life as it occurs (Brown & Jones, 2001). Thus there is not a seeking of perfection or an ideal, but a greater awareness of one's professional practice. The results of narrative research therefore, are not definitive statements or generalisations about an aspect of that which is being researched (e.g., Winkler, 2003). As such, a definitive statement about how to be a teacher of pre-service students learning mathematics is not offered. Rather, a story is shared that may connect with the stories of the reader.

Acknowledgements

Firstly, I would like to thank my colleagues for their support in many and varied ways throughout the last two years. I value our informal collegial discussions - these have been and continue to be an important source of ongoing learning for me. In addition I have appreciated the mathematical wisdom and computer guidance of these same colleagues!

My supervisor, Tony Brown, has consistently provided me with encouragement, both informally and through various written reports. Introducing me to narrative inquiry as a research methodology is something I have particularly valued, and completing this thesis has become a learning experience that I treasure. Thank you.

I would also like to acknowledge and thank the students who were prepared to be observed and taped during our classes together. I am also grateful for the four students who were willing to offer their thoughts in more depth. This sharing provided me with valuable insights as we discussed our experiences with respect to mathematical investigations and learning. It has been a particular pleasure to have you continue communicating with me, as you share your ongoing thinking and experiences.

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Chapter 1:

Introduction

Some students have interpreted the plantation problem differently to how I have. I'm not sure about how this will evolve. Part of me is quite happy to let them explore their idea - I would feel better though if I could keep an eye on their writing. It feels scary not to be able to pre-prepare for wherever it is they are heading. Will I be able to ask appropriate questions on the spot? And what will their learning be? I think the crunch time for me will be next week. 'Crunch' in terms of, will the students re-engage? Will those who have just started to be frustrated, be able to be encouraged to continue? Will I ask questions that will support their movement towards a solution and will they have learned something? In the past this is the point we usually don't get to, engaging more fully in the mathematics, because of the pressure to move on. So this is new. Scary. Exciting.

Thus read one of my journal entries written during this narrative research inquiry in which I, a mathematics educator of pre-service primary teachers, use narrative inquiry to investigate my professional practice. Undertaking this inquiry was motivated by a desire to more closely examine my professional practice whilst simultaneously meeting the requirements for a Masters of Education. Reflection, albeit informally done, has always been an integral part of my teaching practice. Narrative inquiry however, has provided a more structured vehicle for the ongoing critical reflection of my professional role, and has resulted in significant personal learning.

Two main themes emerge in this research. The first of these centres around the use of mathematical investigations as a vehicle for supporting pre-service primary teachers to consider what the learning and teaching of mathematics may entail. As part of this process I undertook several mathematical investigations

myself as a learner. My conception of a mathematical investigation is one of an open-ended problem or statement that lends itself to the possibility of multiple pathways being explored in the process of undertaking the investigation. Such investigations tend to take more time than usually encountered in more traditional mathematics problems frequently used in schools, and can lead to a variety of mathematical ideas and/or solutions.

The second main theme in this research involves the exploration of narrative inquiry as a means with which to pursue professional development. Narrative inquiry, which in essence is a form of story-telling, has become a valued form of educational research. It is regarded as a powerful means by which learners (in this case the learner is also a teacher) can reflect on and develop their professional practice. A goal of narrative inquiry is for the participant to learn and possibly change their thinking as a result of this learning. One example of such learning that has occurred for me in this research is the discovery of changing beliefs about the nature of mathematics.

This story begins with the 2003 academic year. I began to record my reflections about my professional practice. I initially wrote in response to questions posed by my supervisor, such as; what is effectiveness?'; what criteria do I judge effectiveness by in terms of teaching?; in terms of mathematics?; how do my assumptions about being a teacher and a learner shape my beliefs?'; what influences affect my questions?'; and who am I trying to please?'. I also wrote as I reflected on my teaching of two classes of second-year student teachers. It was during this initial period of writing that I first began to think more deeply about the nature of mathematics. Considering the nature of mathematics became an ongoing consideration in my writing.

A pivotal point in this research period was teaching using a mathematical investigative approach. This was an approach that I had previously not encountered, either as a learner or teacher. During this semester I kept a journal in which I recorded my thoughts and reflections of my teaching and learning experiences, with a particular emphasis on the investigative experiences.

Towards the latter part of the semester I personally engaged in doing two mathematical investigations myself. Each investigation took some hours to work on, each over a period of 4-5 days (between teaching and family commitments). Personally undertaking these two investigations was one of two highlights that occurred during this research process. Whilst working on these two investigations I discovered previously unknown beliefs that I held about learning mathematics. It was somewhat uncomfortable to discover that these unknown beliefs were contrary to what I espoused in the classroom.

As a consequence of my teaching and learning experiences with mathematical investigations, I decided to use this teaching approach again. Having gathered some experience with investigations as both a learner and teacher, this second experience was an opportunity to further develop and reflect on this approach as a vehicle to support my pre-service primary teachers in their learning about becoming teachers of mathematics. My reflections at this time indicate some changes in my beliefs and teaching practice.

Approximately a year after this research began, I collated my writing of the previous year's reflections. I gave this piece of writing the title, 'A journey of professional development'. I was unaware at that time how my choice of title was closely aligned with what I subsequently learned about narrative inquiry. 'A journey of professional development', later renamed to 'Mathematical investigations: A journey of professional awareness' chapter 2, thus begins this story and tells the journey of my first year of research, as briefly outlined above.

Towards the end of the second investigative experience I interviewed four students who had now participated in two series of investigations. I was keen to hear how they had experienced participating in mathematical investigations. The interviews were informally conducted and all four students appeared to appreciate, although not uncritically, an investigative approach to learning mathematics and learning about being teachers of mathematics. A description of these interviews and my analysis of the student's thoughts and reflections is found in chapter 3.

A second pivotal point in this research process occurred when I began to read literature about narrative inquiry. I had not initially understood that narrative (writing) was recognised as a legitimate form of action research and also regarded as an effective means with which to develop one's own professional practice. My educational background in completing a Bachelor of Science had predisposed me to expect and understand research models that were empirical and underpinned with criteria such as being objective, value-free, replicable and therefore, valid. Learning about narrative inquiry therefore helped me to make sense of what I had been doing over the past three semesters.

Themes encountered in literature about narrative inquiry, and included in this research, are presented in chapter 4. They include the beginnings of some description of narrative and differing forms of narrative writing; the place of reflection in narrative; the notion of multiple perspectives that are encountered in qualitative research such as this; issues of validity and authenticity; a consideration of what the products of narrative research might be and who may benefit from such research; a brief mention of collaboration; and finally the place of emotion in qualitative research.

Reading literature about narrative inquiry provided me with a framework with which to then further reflect upon and analyse my earlier writing. This section of the report (chapters 5 and 6) ties together the themes encountered within the literature about narrative research, and the themes that emerged in my writing contained in chapter 2. Although somewhat intertwined, this analysis is presented in two chapters. Chapter 5 primarily focuses on some aspects of my practice as a mathematics educator of pre-service primary teachers, which have emerged during this narrative inquiry. Chapter 6 looks more closely at various aspects of narrative in terms of my research experience.

In chapter 5 I begin by aligning my research with narrative inquiry and reflect that I have found this to be a very powerful process for learning about and developing my professional practice as a mathematics educator. I also outline

some more deliberations about the nature of mathematics, and ponder the implications for my teaching practice. Change is an integral aspect of narrative inquiry and I describe the changes that I perceive have occurred with respect to my teaching practice. I also look at changes that have occurred in my beliefs about the notion of seeking an ideal. This chapter finishes with a consideration of who may benefit from this narrative research.

In chapter 6 I firstly further discuss the issue of multiple perspectives and describe how a contrasting perspective became a pivotal point for reflection over the period of twenty months. In response to statements encountered in the literature regarding the importance of closely examining and stating the context of such research, including becoming aware of those things we take for granted, I then describe in more detail the professional context within which I work. This includes an examination of past influences on the current research context, assumptions and judgments that have come to the fore, collegial influences and the influence of literature itself upon the research process. I move onto outline how reading literature acknowledging the role of emotion in research has had an impact on my subsequent writing. My writing is also analysed in terms of the different narrative and reflection techniques described in the literature. Lastly, I re-examine issues of validity and authenticity, linking to the writing of various authors.

The thesis finishes with a conclusion (chapter 7) summarising the main points of this story, of one mathematics educator of pre-service primary teachers who has been seeking to examine her professional practice more closely. In line with the literature that suggests narrative inquiry is an ongoing process, although this section is presented as a conclusion, it simply marks yet another moment in time, as this story nudges towards the future.

Questions at the end of this research:

More about narrative inquiry, and particularly the place of reflection

Reading about current trends re: mxl investigations

Rdg about the nature of mx, linking this to post-modernist trends in education, and critically analysing this move.

Chapter 2:

Mathematical investigations: A journey of professional awareness

(a)   An overview of the journey

As explained in the introduction, during the academic year of 2003 I reflected on and recorded aspects of my professional work as a mathematics educator of pre-service primary teachers. In section 2(b) my writing initially reveals the taking of too much personal responsibility for students' learning, and debating whether or not a body of mathematical correctness or truth exists. Other considerations in this initial writing included my thoughts about what makes an effective mathematics educator, and the values, assumptions and influences that I perceived to affect my beliefs.

During this same period of time the team of mathematics educators with whom I work decided to use a mathematical investigative approach with our first year pre-service primary teachers. Following my writing in section 2(b), I go on to explain the process of planning for and teaching using these mathematical investigations (section 2(c)). This writing is a mixture of original writing and summaries of my writing that I prepared at various points throughout the semester. These were analysed with the emergence of six themes.

The writing describing these six themes is then followed by an account of one of several mathematics investigations that I personally undertook as part of this research (section 2(d)). Personally undertaking these mathematical investigations became a pivotal point in this journey of professional development leading to the discovery of previously unrecognised beliefs pertaining to my own mathematical learning, and more generally beliefs about the nature of learning mathematics. A shift also occurred in my beliefs about the

nature of mathematics. The experience of working on these investigations affected my subsequent teaching practice and this is also described.

This chapter then, is a description of my 'journey' that unfolded over the first twelve months of this research period.

(b) Initial steps on the journey

(i) Teacher responsibility

Two themes were apparent in the early stages of my writing. The first was that I was taking too much responsibility for student learning and was aware of some of the effects this was having on my teaching. Efforts at changing this were trialled and noted:

> When opportunities arise within the teaching time (and many, many do) I feel almost a desperation to seize upon <u>all</u> of the opportunities. Otherwise that moment is lost, and we have so little time and so much to cover….. . So sometimes I crowd too much into the available time. I think this also links to my tendency to take on too much responsibility for their [students'] learning. With these thoughts in mind… I approached my lesson with the other class this morning in a different frame of mind. I did not feel an urge to solve all their mathematical problems in one swoop and was prepared to let them make choices about their use of the learning time/activity that I created. As a result I felt much more relaxed, positive and also wonder if I was more in tune with the needs of the class (an interesting paradox). (05/03/03)

(ii) Mathematical correctness

The second theme to emerge at this time evolved from a collegial debate as to whether or not there is a 'body of mathematical correctness or truth'? This debate occurred in response to my written reflections about supporting students on a one-to-one basis outside of class time. Evident in my writing is considerable tension about the existence or otherwise of mathematical correctness or truth, and some discomfort at the thought that maybe I was being 'exclusive' in my teaching practice.

It seems that implicit in my thinking is the notion that the student needs to come to my understanding. [A colleague] however suggests that perhaps we need to move towards a shared understanding and/or not see this process as moving towards the right answer. A question that arises for me is 'does this compromise mathematical truth?' though I hesitate to use that word 'truth' because what is 'truth'? So I think I would rather use the word 'mathematical correctness'! If I simply want to 'share' understandings does that mean that understanding does not develop or improve? Or do I need to share her understanding in order to know what to say/ask/do to help her move towards mathematical correctness? This feels like a circle. Is there a mathematical correctness or not? I guess I have an underlying assumption that there is.

I do agree that I need to move to her understanding/share a perspective, but I think this so I can more effectively help her move where? To my understanding? So is that exclusive? By wanting the student to move to my position/mathematical correctness am I being exclusive? [A colleague] also used the

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phrase "soften" with respect to communication. Well, I think I'm

as soft as marshmallow and as 'inclusive as' so it is rather a shock

to consider that MAYBE I am being exclusive or needing to

soften. BUT does that compromise mathematical understandings?

(17/03/03)

Further reflection at that point in time appears to concede that maybe aspects of my thinking about mathematics and thus my teaching practice were indeed exclusive:

I mainly perceive mathematics as stuff rather than doing, and that

stuff does not include the student's stuff - now that is exclusive

(17/03/03)

The idea of mathematics being 'stuff' or 'doing' emerges again when teaching first-year student teachers using mathematical investigations and personally working on several mathematical investigations later in the year. A major shift in thinking appears to have occurred during this time (see 'Considerations about the nature and learning of mathematics', pp. 18-20).

Some of the tension apparent in the above writing appears to be resolved in an extract written approximately a week later. There also seems to be more consideration of the student's perspective. Further analysis also reveals a contradiction within this section of writing - namely, that I initially state that "yes, there is a body of mathematical correctness" but later do not want the students to regard me as "a source of rightness". I now see this as a contradiction because if there is a body of mathematical correctness surely the teacher would know that 'body' and thus be a source of 'rightness'.

I'm now thinking that 'yes, there is a body of mathematical

correctness', but that is not to say that a student's current

understandings are not correct – they are understanding something

else and even the same thing can be understood in a variety of

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ways. So, moving towards a shared understanding is likely to be a useful process that enables me to be more helpful in enabling the student to move to new understandings, and indeed myself too. So I don't think there is compromise happening – rather it is all learning. An ideal for me would be to have students feel able to be confident to state their understandings and work through a process of learning together rather than regarding me as a source of 'rightness'. So all perspectives matter in this process – there is not an emphasis on being correct although I do want there to be an emphasis on thinking/engagement/making sense of what is happening. (24/03/03)

(iii) Effective mathematics educators

Towards the end of the first month the focus of my writing was considering my effectiveness as a mathematics educator of pre-service pre-service primary teachers. In my writing I reflect on the question 'what is effectiveness?':

I perceive an effective pre-service mathematics educator to be firstly a person who can motivate their students to learn about being a mathematics teacher. This is very important, particularly in this subject where so many students have negative experiences in their mathematical past. They are likely to be motivated to 'hide' their insecurities rather than be motivated to learn about being a mathematics teacher. It is interesting to note however that many students are determined to create better mathematical experiences than they themselves have experienced. I also believe that unless these students engage in reflecting on these past experiences they are more likely to recreate their own past rather

than something new.   I think motivation is not only the educator's responsibility but also the student's.

A safe environment needs to be created where students feel able to engage in "doing" mathematics which probably involves taking risks.   I think this is a big ask. I would like to ponder if I still have moments of insecurity how much greater might this be for my students, and what a risk it may be for them to even walk into my classroom.   To what extent is this the case, and how can I respond accordingly?

Following on from this, an effective pre-service mathematics educator is one who provides opportunities for 'doing mathematics' so that students come to understand some mathematical ideas and make sense of and connections with these. I do not believe it is possible nor desirable to cover all mathematical topics in our courses.   I would rather create the skills/attitudes whereby a student is willing to engage in doing and learning about the mathematics at hand, wants to make sense of it, and connect it with other known 'stuff'.

I think an effective mathematics educator probably initially creates opportunities or an environment of success. (31/03/03)

I believe my writing on 'success' at this point was likely to have been underpinned by an assumption that success occurs when a student reaches a 'right' answer. I now (one year later) think that mathematical success could be

measured in terms of process rather than product - that success is apparent when the student is engaged in 'doing' mathematics, i.e., engaged in the process of inquiry, seeking a variety of solution paths and critical thinking rather than reaching one "correct" answer.

Critical examination by students was also seen to be a mark of effectiveness:

> The effectiveness continues if a student can be persuaded to critically examine their own experiences and beliefs in light of new alternatives that may be presented. This critical examination will hopefully enable the students to make sound pedagogical decisions about their teaching in light of their experiences including their 'mathematical knowledge'. (31/03/03)

Also revealed within this period of writing is the existence of a personal insecurity regarding my own level of mathematical knowledge. Here my definition of 'success' is the finding of a solution to a mathematical task or problem rather than my 'newer' ideas of success as described above.

> I still take care to protect what I perceive to be a 'weak' mathematical background, but I have also become more willing to engage and take risks as I explore mathematical problems. I was interested to note at a recent mathematics day (for mathematics educators) that my anxiety rose when questions were put to the audience [including me]. My belief appears to be that I should know all, particularly as a pre-service mathematics educator! However, I also noted that I was willing to give something a go and work through it rather than actually pretend I did know it all. Success and a newly developed belief that it is okay to not know and recognise these moments as opportunities for learning also

helps. (31/03/03)

At the end of this section of writing I summarised my thoughts as follows:

> In summary, I believe an effective pre-service mathematics educator is
>
> a person who:
>
> motivates;
>
> engages students in 'doing' mathematics in order for students to
>
> understand some mathematical ideas;
>
> creates a 'safe' environment;
>
> creates an environment of success; and
>
> encourages critical reflection. (31/03/03)

I go on to state that the criteria against which I would judge my effectiveness
would be the extent to which I would achieve the above list of bullet points. In a
later analysis of my writing I notice the dualism in my thinking and writing:

> Looking back on my writing… it seems I have an either/or,
>
> black/white way of thinking. ie. that if these things are achieved
>
> then I will be considered 'effective'. I still believe these factors are
>
> important yet now have an awareness of this either/or mentality.
>
> This is reflected in my personal writing, 'I'm really challenged by
>
> observing another way and question myself as to how much do I
>
> subconsciously act from a belief of 'this is the right way to do
>
> things?' ' (22/08/03)

(iv) Values, assumptions and influences on this research process

There appears to be an awareness in my writing throughout this initial period
that my beliefs, values and assumptions 'colour' what I perceive both with

respect to my thoughts on 'what is effectiveness?' and my 'noticing' and interpretation of data:

> I'm quite clear that my assumptions (known and unknown) about being a teacher shape my beliefs. How could it be otherwise?… . I think that my assumptions not only shape my beliefs but also influence how I 'see' situations. For example, a person with different assumptions is likely to perceive a situation quite differently to me. (31/03/03)

This awareness appears in my writing again some months later where I reflect that my interpretations of incidents within my class are only my interpretations of any particular situation.

> Six themes appear to have emerged in my writing to date. I'm very conscious however, that other themes would be apparent to a different reader and indeed myself on a different day and when in a different mood. Also affecting the themes that I perceive is the notion that I'm more likely to notice issues of interest to me at the moment. So, this is all subjective. (15/08/03)

I initially pinpoint my colleagues' ideas and the reading of research as major influences on my reflecting, writing and beliefs about the nature of learning and teaching mathematics. For example, through discussions with colleagues and reading I have come to believe that:

> A 'good' mathematics teacher is one who enables children to make sense of situations in a mathematical way – probably in a way where new knowledge fits in with/builds-on/refines their existing knowledge. The teacher is likely to do this by providing experiences likely to support the development of particular

mathematical ideas; asking questions; clarifying ideas; and

pointing to connections. (31/03/03)

What appears to be missing in this earlier writing about what influences my reflecting, writing and beliefs about the nature of learning and teaching mathematics, is an awareness of the impact of my own learning and teaching experience. During the last ten years there has been a significant shift in how I teach and my beliefs about the learning and teaching of mathematics. For example, I used to believe that a good mathematics teacher was one who could explain things carefully and break things down into manageable steps that could be understood. More recent experience has led me to now believe that 'good' mathematics teachers teach as described above.

There would thus appear to be a change in my awareness of what influences my questions and beliefs. An initial emphasis was placed on others (i.e., colleagues and reading). Now I am also conscious of the influence of my own personal experience (i.e., self).

(c) Journeying

(i) Moving towards mathematical investigations

During the first five months of my research time the pre-service mathematics educators with whom I work met weekly with a goal of revising what and how we teach in our two mathematics education papers that our pre-service students take in their teacher education. During these discussions we decided to try to engage our students with an investigative approach to some mathematical problems over a sustained period of time in our first-year paper. The focus was for the students to engage in doing mathematics, hopefully make sense of some mathematical ideas and to reflect on their own personal learning of mathematics. This was a new approach for me (both in terms of teaching mathematics using an investigative approach and as a vehicle for supporting pre-service primary teachers in their learning about the learning and teaching of

mathematics) and has become a major focus in subsequent writing. It has also led me to personally reflect more deeply about my own thinking about the nature of mathematics and what might be involved in the learning of this subject. This became one of six themes that I later analysed from this period of writing.

When the first year paper began I kept two journals. I wrote in my 'student journal' during classes whilst the students were also writing in their journals (which were an integral part of their assessment for this paper). Our instructions for the students were that, "the journal will consist of ongoing, regular entries which focus on you [i.e., the student] making sense of some mathematical ideas, and your insights about this process" (University of Waikato, 2003, p. 6). My 'student journal' was an attempt to model this process to the students and was available for the students to read if they wished. An interesting development of both this journal, the process of planning for these investigations and collegial discussions was my delight in 'doing' mathematics. I have sensed a growing personal understanding of links between mathematical topics and an increase in confidence in being able to work on mathematical investigations.

The second journal (which I refer to as my personal journal) was used for reflective writing focussing on my experience of planning, teaching and evaluating this investigative approach as a vehicle for supporting pre-service primary teachers in their learning about the learning and teaching of mathematics.

Audio-tapes and transcripts of student conversations were collected as they worked in pairs on mathematical investigations during class time in the second week of the paper at the beginning of undertaking a two-hour investigation. Two pairs were taped in two classes and one pair was taped in a third class. An observer organised and instructed each student pair in the use of the tape-recorder and then withdrew to a corner of the classroom where she observed the student pairs and the class as a whole. These observations were recorded alongside each transcript of the audio-tapes.

An informal discussion with a student (student A) who was struggling with this investigative approach to learning mathematics was also audio-taped and transcribed. This discussion took place in my office after I had become aware of this student's discomfort both during class and as written about by the student in a concurrently occurring online-discussion. Participation in this online discussion was an essential requirement of the paper and provided an opportunity for students to discuss a set topic. This topic centred on how the learning of mathematics may occur, and it was hoped that students would link their past experiences with their current learning and relevant literature about the learning of mathematics. Student A, during this online discussion, wrote about his discomfort and willingly agreed to discuss it on a one-to-one basis.

I also participated in collegial observation. Colleague N observed my teaching of one class during this investigative approach, and I observed colleagues N and T. Notes were recorded during each observation and informal discussions took place after each observation. Reflections on these were recorded in my personal journal.

These journals, transcripts of student conversations, observations and the transcript of the tutor-student discussion were written and collected during the first five teaching weeks of the first year paper. They were analysed shortly after this five week period with six themes appearing to emerge. I described the six themes as:

- Considerations about the nature and learning of mathematics;
- Thoughts/issues to do with mathematical investigations;
- Student beliefs about mathematics learning;
- Student behaviours during a mathematical investigation;
- My role as a pre-service mathematics teacher; and
- Collegial liaison.

My writing on these six themes at this 'point' in the journey follows.

(ii) A middle point: Six themes

- Considerations about the nature and learning of mathematics

It is evident in my writing that my views about the nature of mathematics have been undergoing a change for some time, not just during the time-frame of this study.

> As already alluded to my views of the nature of mathematics and mathematics learning have changed dramatically since I began teaching in 1984. At the beginning of my teaching career I taught in a very behaviourist fashion and viewed mathematics as a body of rules to be taught to students. It was always my preference for students to understand but did not really expect that - more a regurgitation of facts and procedures at the right time and place.
>
> Now it seems I believe something quite different. In the informal discussion with student A I describe my idea of mathematics as a subject that can be made sense of, "… something I'm keen about… is people being able to make sense of mathematics rather than viewing it as a set of rules that somehow fell out of the sky" (Judy in discussion with student A, 12/08/03). This expectation of sense-making is also evident in my 'student journal' where I take delight in personally making sense of previously learned procedures such as trigonometry. (21/08/03)

In contrast to earlier writing (see Mathematical correctness, pp. 8-10) about the nature of mathematics, where I refered to mathematics as 'stuff' and debate the existence of a body of mathematical correctness, in the following writing I now refer to mathematics being about discovering and doing, with what seems to be an increasing openness to the idea of multiple interpretations.

"What you have just described to me … that you had a problem that involved space and involved numbers… and you were thinking through how to solve it. So in my way of thinking, what you were doing was maths". (Judy in discussion with student A, 12/08/03)

"I believe multiple interpretations and understandings are always present in any mathematical learning situation". (02/08/03)

I also ponder whether mathematics can be likened to a 'language'.

"Is mathematics a language for number and pattern and shapes and space ideas?" (02/08/03)

This transition in belief is not occurring without some discomfort and tension, as is evident in the following extracts of writing.

Some tensions appear to be evident between my former beliefs about the nature of mathematics and my current vision of mathematics as sense-making, discovering, doing, and a 'language'. As already mentioned I noted a contradiction in my early thinking and writing regarding the nature of mathematics - namely, that I initially state that "yes, there is a body of mathematical correctness", but later do not want the students to regard me as "a source of rightness". (02/08/03)

In contrast to this espoused belief I wonder if the language I use when conversing with students reveals a different belief tying back to the idea of 'correct answers'. The words "that is perfect" (24/07/03) may be evidence of a persisting underlying belief of

'this is the right way'. Alternatively this may be a phrase to try and encourage the students. (02/08/03)

Also evident in summaries of my writing from this stage is confusion about how mathematics is learned. This contrasts with my later experience of teaching using an investigative approach with second year students where I seem to have clearer ideas about how mathematics is learned (see Teaching using an extended mathematical investigation, pp. 40-42).

> Considerable confusion is also evident in my writing about what the learning of mathematics entails. Cognitive, social and environmental factors are mentioned but a more precise response to the question 'how might mathematics be learned?' is "I don't know" (02/08/03). Questions are asked as to whether it is possible for all mathematical ideas to be discovered by learners or whether some transmission of information needs to occur. An example is offered where I consider whether or not it is possible for a learner to discover 'trigonometry'. I do recognise however that this may "merely reflect my lack of knowledge of where trigonometry comes from and thus I can not imagine how a teacher could have students discover trigonometry". (13/08/03)

- Thinking about mathematical investigations

I obviously initially had considerable concern about whether or not an investigative approach for learning mathematics was effective. Whilst the concern is obvious I also appear to have the belief that this teaching approach definitely has potential. Also included in this summary are some suggestions that I acted upon in the next semester.

A major concern that also appears considers 'what' and 'if' the students are learning. The question "will they have learned something?" appears early in my journal and is repeated throughout ongoing reflections on my teaching in these classes. For example, "I have questions though about what is being learned" (22/07/03). A persisting hunch that this approach can result in learning, if structured carefully, is also apparent in my writing. I also suggest in the future it might be better to have one extended investigation over five weeks rather than the three separate investigations that we trialled this year. I write that if given one extended investigation "students may be more likely to engage more deeply and learn some mathematics" (13/08/03). My concerns about whether or not learning has occurred appear to be 'justified' when reading the transcripts of the students working on the investigations. (21/07/03)

Further evidence of my belief that this approach has potential is found when I refer to openness and flexibility. "I feel that this investigative approach has good potential for meeting individuals' learning needs because of its open-ended nature. A positive that I perceive is the flexibility of this process…". (24/07/03)

Tensions caused by my changing ideas about the nature of mathematics and mathematics learning appear in several places in my writing and continue to cause some discomfort.

Multiple interpretations have been a feature of the first investigations. This would appear to fit with my developing ideas about the nature of mathematics and mathematics learning. My

writing reveals at times a willingness to accept this. "Part of me is quite happy to let them explore their idea" (22/07/03). Tensions are also apparent however, as I consider what learning this may lead to. (24/07/03)

Using a 'context' as a background for the investigations appears to have mixed effects. One transcript of two students working on the popcorn problem reveals a prolonged struggle to understand the context (21/07/03) with no significant learning appearing to have taken place. It is necessary to note however, that this audio-tape was recorded at the beginning of the investigation. The issue of making sense of the context is also apparent in the plantation investigation (24/07/03) although as I write I wonder if this is part of 'real' mathematising. (21/08/03)

Again I refer to feeling anxious or insecure about my own mathematical knowledge. This underwent a process of change when teaching using an investigative approach for the second time (see Teaching using an extended mathematical investigation, pp. 40-42).

Another feature of this approach is the creation of some anxiety regarding the extent of my own mathematical knowledge. "It feels scary not to be able to pre-prepare for wherever it is they are heading" (22/07/03) and "I do admit… that I have definite 'safety' boundaries - the thought of coming across several questions… that I did not know how to tackle would be embarrassing and I would be scared of my credibility being undermined" (02/08/03). I recognise another paradox here. I now believe that being a

mathematician is about exploring ideas/doing and yet if I am in a position where I need to 'explore' I feel that my credibility will be undermined! On the other hand when questions have arisen lately in individual discussions in class I sometimes feel quite content to say 'I don't know, I'll find out'. (21/08/03)

There is an awareness in the following writing of the different experiences that occur within any one situation. At this point I still have doubts about this approach and wonder if students' beliefs and past experiences further impact on the outcomes of using such an approach.

> A question also arises re: even if I consider this approach to be effective (by which I presume I mean it ties in with my current beliefs about mathematics and mathematics learning) is it appropriate in light of our student's past experiences? Can our students access the possible mathematical learning when they do not even have the expectation that mathematics makes sense? (21/08/03)

- Pre-service teacher beliefs about mathematics learning

I go on to write:

> As alluded to above I think it likely that my students have a different picture of mathematics to mine. I believe our students probably think of mathematics as a subject that does not make sense, is not relevant to their lives and is a matter of finding the correct answer by remembering rules and formulae that are found in textbooks. It may even involve being tricked!

"Several times in class yesterday students asked me about the right answer - 'what's the right answer?', 'what's the formula?'" (22/07/03)

"We need a textbook…". (Transcript, 24/07/03)

"I don't know. I have forgotten mathematical concepts". (Transcript, 21/07/03)

"Maybe that is the whole point, trying to trick us…". (Transcript, 24/07/03)

These quotes certainly suggest that these students do not perceive mathematics to be about sense-making and solving problems from a place of understanding.

This dichotomy of views about the nature of mathematics is discussed by Bradford (2002) who wrote:

> … commonly held perceptions of mathematics lie between two extremes. On the one hand, it can be regarded as an abstract subject about pure mathematical facts and truths that hold true wherever you are in the world or it can be regarded as a social activity bound in our environment (p. 21).

She goes on to ponder the learning outcomes when students and teacher hold opposing views about mathematics. I also wonder if there will be difficulties bridging this 'gap' between student beliefs about mathematics and this teaching approach (i.e., investigative approach).

This is also a particularly prominent theme in the discussion with student A. This student clearly perceives mathematics to be a body of ideas that I can give him and that he needs to learn so he can pull out the appropriate formula at the appropriate time to find the correct answer. As a consequence he is feeling very uncomfortable about an investigative approach where the learner is expected to find and make sense of the mathematics they encounter. (21/08/03)

- Pre-service teacher behaviours during mathematical investigations

In the following summary of my writing I ponder observed student behaviours during these mathematical investigations:

A fourth theme apparent in the data concerns comments regarding student behaviours whilst participating in a mathematical investigation. Overall I think there has been an increase in the degree to which my first year students are engaged and involved in these initial 'Learning and Teaching Mathematics' classes. I comment in my personal journal that the "majority of class [are] thoroughly engaged" (22/07/03).

I am also aware however that off-task behaviour is still apparent with some students. This was noted in the transcript of 21/07/03 where the observer noted "one group was not on-task preferring to discuss social issues for a significant period of class time". I also noticed this same group, "one table of 6... I suspect [were] dissecting their weekend adventures" (22/07/03). Off-task

behaviour was also noted for a short period of time during the recording of the transcript on 22/07/03.

Mathematical behaviours that I perceive to be useful when solving mathematical investigations appeared to be notably lacking in all 3 of my groups. Very few students (if any) showed any attempt to be systematic, to search for patterns and be accurate. Also of note was the lack of persistence in their search for an answer. In an investigation students were required to search for a maximum area (in this context the maximum was 6 1/2 square units). Most students were content to stop when they had found an area greater than 4 square units as was proposed in the original question and subsequent searching for about 10 minutes found nothing greater than their first findings (eg. 4 1/2 or 5 square units).

There was some evidence in one transcript of a student making a conjecture, although this did follow a clear statement from me about making conjectures. In this instance however it appeared to be quite spontaneous and was some time after they had been asked to write their first conjectures. The conjecture made was "Maybe you can only make shapes of equal numbers" (Transcript, 24/07/03). (21/08/03)

With the benefit of hindsight I now wonder if the apparent lack of mathematical behaviours was partly a reflection of the fact that this investigation took place at the beginning of the paper when the students were probably still feeling their way into the course. It may also of course be a reflection of a lack of experience with this approach to learning mathematics.

Another student behaviour that I noticed during an observation of colleague N's class was the difficulty one student had putting her, what I initially termed as, 'understanding', into words. There appeared to be a 'gap' between what she 'knew' or had 'glimpsed' (phrase coined by colleague N, 14/08/03) and being able to articulate it. This led me to ask "when students are still struggling with their own understanding is it reasonable to ask them to listen to alternative explanations let alone follow them?" (13/08/03).

- My role as a pre-service mathematics education teacher

As mentioned in my earlier writing (see 'Teacher responsibility, p. 7) I recognise that I have a tendency to take too much responsibility for student learning, and have endeavoured to change this with what seems to be limited success:

This ability to 'be' different however appears to be transitory and my writing five months later still reveals images of tension and a desire for change. "An image of a 'tightly wound clock' comes to my mind compared with a 'softly set jelly'… . Desire change". (08/08/03)

Linked to these feelings of responsibility is a recognition of wanting to please students although it appears this is not as strong as it once was. "I'm less dependent on needing the approval of my students, although I definitely still want to respond to their needs" (21/03/03). I recognise that sometimes I will 'push' some students to engage in situations and this "does not please some of them some of the time but nevertheless I will do it - operating from my

belief that they need to learn to engage in problems in a mathematical way". (21/03/03)

Congruent with my beliefs about mathematics learning involving discovery and sense-making is my inclination not to 'tell' students the answer but ask questions to support their thinking and discovery of mathematical ideas. This behaviour is evident in places in the transcripts. I wonder what the student's experience of this is, particularly if their beliefs around mathematics learning are not congruent with my philosophy of learning. I know in the past that some students become highly frustrated and just want to be told the answer. I have also wondered if asking questions does in fact support learning. It would appear that it can do, for example, when exploring the understanding of the formula for finding the area of a triangle with student A it would appear that questioning did support A's learning. (12/08/03)

- Collegial liaison

Once again, the importance and influence of my colleagues with respect to my teaching is highlighted. It is interesting to note that I value and am quite at ease when liaising with colleagues to support my professional development. In contrast, later on, whilst personally working on mathematical investigations I refer to collegial liaison as possibly "cheating".

A final theme that appears in my writing during this period has been my valuing the process of liaising frequently with my colleagues as we plan, prepare and evaluate our teaching experiences using an investigative approach and participate in

online discussions with our students. This theme occurs both in my student and personal journal. The fact that it appears in my student journal would suggest that I value this process and wish to model it for my students. My colleagues have indeed been a valuable source not only in pedagogical discussions but also of aiding my mathematical understanding as I work on the mathematical investigations we have set the students. It is also apparent that observing colleagues has been a useful stimulus for further personal reflection (21/08/03).

(d) A journey highlight: Gaining personal experience of mathematical investigations

(i) The billiard table investigation

Following the completion of investigative work with our students I embarked on gaining further personal experience in working on mathematical investigations. Apart from participating in collegial planning with the investigations we presented to our first year classes this was my first experience with an open-ended mathematical investigation, and certainly the first investigation that I initially worked on, on my own. I will call the investigation the 'Billiard Table' investigation. A diagram was presented (see figure 1) with the instructions 'a billiard ball enters the table (pockets only in each corner) at the bottom left-hand corner. It bounces around the table reflecting off the walls at 90 degree angles. Experiment with different sized tables'.

Figure 1. Diagram showing the path of a billiard ball for the 'billiard table investigation'.

I recorded all of my work on each of the investigations, with my concurrent reflections, in my journal (see appendix A for a copy of my original journal entries for the billiard table investigation). My initial entry for the billiard table investigation read:

> Look for patterns - alter one variable at a time - first alter the
>
> length of the table. (08/09/03)

This comment, together with subsequent experiences in the second investigation, has revealed my assumption that all number sequences follow a pattern and can be defined by an algebraic expression. I have since learned that this is not necessarily the case.

Using grid paper I then drew billiard tables all with a width of 3 squares, and of varying lengths from 1 square through to 11 squares. I drew up a corresponding table (though not in the order of lengths at this stage) noting the number of times the ball touched the side of the table and where the ball exited. I also initially paid attention to whether or not a symmetrical pattern was created by the ball's trajectory, but did not pursue this as once I had drawn several tables of different lengths I recognised that the patterns were similar and predictable.

Once I had drawn tables of length 4 - 8 squares I noted a pattern and began to predict the number of touches (except for lengths that were multiples of 3 the number of touches was increasing 1 each time) and where the ball would exit (if the length was an even number the exit would be the bottom right corner, if the length was an odd number the exit would be the top right corner). I also briefly

explored what would happen if the length of the table was not a whole number but a fractional number (eg. 1 1/2units long). I did not pursue this further at this stage beyond looking at lengths of 1 1/2 and 2 1/2 units, deciding to concentrate on whole numbers.

At this point I wrote in my journal:

> There is obviously a pattern - I'm rapt that I spotted it after looking
>
> at table lengths 4 - 8 squares. I'm going to do an 'orderly' table
>
> [see table 1] to see if I can find an algebraic rule for this pattern - I
>
> have doubts because it's obviously not linear but will try!
>
> (08/09/03)

Table 1. Relationship between the length of billiard table (width 3 squares); the number of times the ball touches the side and the point of exit.

Length of billiard table (width 3 squares)	No. of times ball touches a side	Point of exit
1	2	TR
2	3	BR
3	0	TR
4	5	BR
5	6	TR
6	1	BR
7	8	TR
8	9	BR
9	2	TR
10	11	BR
11	12	TR

(TR= top right corner BR= bottom right corner)

It is interesting to note my willingness to 'try'. Until recently I would not have had the courage to 'try' . There has certainly been progress over the past five years (corresponding to my time as a pre-service mathematics educator) both in

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my willingness to 'try' and persist.

Having created the table the algebraic rule $y = x + 1$ was immediately apparent for all lengths except when 'x' was a multiple of 3 (where y equals the number of times ball touches a side and x equals the length of the billiard table). There then followed a period of exploring exponential functions, e.g., $y = 2^x - 2^x$, $y = 2^x - (2^{(x)} - 1)$,
$y = 2^x - (2^{(x)} - 2)$ and so on wondering if an exponential function would define the x's that were a multiple of 3. I was finding it frustrating at this stage not being able to find a single algebraic expression that would define all components of this pattern, and my memories of more formal algebraic mathematics were too distant to remember. My journal entry at this point read,

> Have I got something here or not? Frustration. Probably need to ask for some help but do not want to! Want to be able to solve this on my own! (08/09/03)

This was the first occasion where it becomes evident that I seem to believe that 'mathematics is a solitary endeavour'.

Despite my frustration I immediately persisted in more exploring, pondering whether a graph (see figure 2) would help my investigating. It is evident from my writing at this stage that I was unclear as to whether the relationship could be defined by one algebraic expression that would fit all values. Interestingly, this was despite the fact that I had already been exploring separate expressions. I had not yet remembered that it is possible for a graph to have more than one algebraic expression depending on the value of 'x'. Drawing the graph appeared to help my thinking with regard to making sense of this context. I began to question whether it was appropriate to link the values of 3, 6, 9 and so on.

Figure 2. Relationship between length of billiard table and number of times ball touches a side.

I comment at this stage, in my writing, that I would normally be feeling very frustrated but was instead content to continue to explore and consider alternative options. My next thought was to explore varying the width of the table and thus drew a series of diagrams, tables and a graph to explore the pattern when the table was 4 squares wide and of varying lengths. However, having taken some time to do this, I appeared to be no further ahead with my question, 'is there a single algebraic expression to fit these relationships?'. I was obviously very focussed on finding a single algebraic expression and this appeared to block my thinking with respect to looking for other relationships that might exist within the data. I was unaware of this at the time.

At this stage I decided to enter the number sequences I was exploring into Google, a search engine on the world-wide net, but this did not yield any useful results. My writing once again reveals that I appear to view mathematics as a solitary endeavour.

> I did wonder whether searching on the net was a legitimate
> mathematical thing to do - I'm not sure. (09/09/03)

I also ponder that there appears to be links between geometry and algebra, something I had not previously understood.

At this point having worked on the investigation over a few days my writing

reveals considerable frustration and the labelling of seeking support as 'cheating'. I write:

> [I] need support to continue but that feels like cheating - why can't I work this out on my own? (09/09/03)

I then decided to do some more exploring on the web and found a site that explores the same problem (http://www.k12science.org/IMATTT/billiards_t.html). I was immediately interested to note the interpretation of the data on this web-site was slightly different to my own. They had also counted the entry and exit points as 'touches'. The site also referred to 'relative primes' which at this stage merely added to my feelings of anxiety not knowing what 'relative primes' were.

Despite my frustration and anxiety that were apparent at this stage I kept going and explored another web-site to find out what relative primes were. Having done this I wrote:

> This feels better. That horrible feeling in my stomach is receding. My other thought at the moment is the importance of interpretation. Do different interpretations get in the way? Or can I still explore my interpretation? I think so. (09/09/03)

Even though I had previously written about (see Thinking about mathematical investigations, pp. 20-23) and begun to accept the idea of multiple interpretations here I was experiencing doubts about the validity of my interpretation.

I had also now become aware of what I called 'subconscious beliefs' and appeared to be ready to experiment with changing some of these:

> This is a very interesting exercise - some subconscious beliefs appear to include:
>
> 1. I should be able to get this by now (after a couple of hours) - do I not genuinely believe that learning takes time?
>
> 2. 'Real' learning does not include others. So how does that fit

with my professed constructivist/enactivist beliefs?

3. Still struggling with the 'OK-ness' of being stuck.

So I will now consider this website more carefully - thinking it is

OK to collaborate with others. (09/09/03)

My exploring in conjunction with the web-site then leads to the understanding and conclusion that:

A relationship exists to predict the number of touches: number of

touches = width + length if the dimensions are relatively prime.

For example, if the width of the table is 3 and the length 4 the

number of touches will be 7 (counting both the exit and entry

points as one touch each). If the width and length are not relatively

prime reducing them to the smallest rectangle of similar

dimensions will reveal the number of touches (eg. If the width is 3

and the length 6, reduce to 1 and 2 which gives a total of 3 touches

counting the entry and exit points as one touch each). [Note that in

table one], I list a table that has a width of 3 and a length of 6 to

have one touch - this is because my interpretation did not include

the entry and exit points as 'touches'. (09/09/03)

I then went onto explore whether I could predict where the ball exits for any width and length of table. My conjectures at this point were:

If the width and length are both odd and relatively prime the ball

will exit the top right corner.

If the width is even and length odd and are relatively prime the ball

will exit the top left corner.

If the width is odd and the length even and are relatively prime the

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ball will exit the bottom right corner.

If the width and length are not relatively prime find the smallest

rectangle with the same ratio and apply the above conjectures.

(09/09/03)

Testing these against my data found them to be true.

I went onto write, once again revealing my discomfort about not working on my own.

This is satisfying but I didn't find it on my own - I had not

explored this interpretation - would I have found it? I still have this

niggle about not finding it on my own. (09/09/03)

Not content to leave my question regarding a single algebraic relationship I consulted a colleague about my findings as shown in table 1 and figure 2 and learned that 'piece-wise' functions do exist where different equations can be linked to different values of 'x'. Also in consultation with a colleague I examined the multiples of 3 - looking at just these on their own (see table 2) clearly revealed the algebraic expression y = 1/3 x - 3.

So, in summary, when x belongs to natural numbers but not the

multiples of 3, y = x + 1;

When x is a multiple of 3, y = 1/3 x – 3. (12/09/03)

Table 2. Relationship between the length of billiard table (width 3 squares) and the number of times the ball touches a side when the length is a multiple of 3.

Length of billiard table (width 3 squares)	Number of times ball touches a side
3	0
6	1
9	2
12	3

Now knowing about the existence of piece-wise functions, I returned to looking at my data for when the width of the table was 4 squares. I found 3 relationships within this data (see table 3).

The relationships were:

When x belongs to an odd number, y = x + 2;

When x is a multiple of 4, y = 1/4 x - 1; and

When x is an even number but not a multiple of 4, y = 1/2x. In all relationships x belongs to the set of natural numbers.

Table 3. Relationship between the length of billiard table (width 4 squares) and the number of times the ball touches the side.

Length of billiard table (width 4 squares)	Number of times ball touches a side
1	3
2	1
3	5
4	0
5	7
6	3
7	9
8	1
9	11
10	5
11	13
12	2

I also returned to my earlier explorations when the length was a fractional number, eg. When the width was 3 squares and the length 2 1/2 squares.

I then thought I would find the simplest fractional ratio of these two numbers, ie. 6 and 5. I then applied the conjecture for an even width and odd length to predict the point of exit to be top left and

the number of bounces should be 11. I tested these conjectures and found them to be true. Ah, very satisfying, so the rules even apply to fractional numbers. (12/09/03)

At the conclusion of this investigation I wrote:

Overall this has been very satisfying. I have learned a lot both about maths and my attitudes towards learning. Thoughts at the moment include:

I'm now prepared to search further than I used to.

I recognise that for me searching and trialling new ideas is a valid and important part of the process. Is it in this 'searching and trialling' that the possibilities for learning (which is different for different individuals) exist? I think so.

I have some unhelpful behavioural and belief patterns that can block my progress.

I appear to have had or still have the belief that [mathematical] learning is a solitary business and only really 'real' if done/solved by myself. I need to ponder/explore this further because it is contrary to my espoused belief! Interesting!

I would like to develop the 'habit' of thinking in multiple interpretations because this could help solve problems. (12/09/03)

This last point would appear to represent a shift to thinking more positively about multiple interpretations, compared with an earlier stage within the investigation.

(ii) Another investigation and reflections

After completing the billiard table investigation I immediately began

investigating a number investigation. As the investigation progressed I noted a positive change in how I felt when stuck - rather than the negative feelings previously experienced I simply felt interested and stimulated. I write that this is a significant change:

> No 'ucky' feelings on this one yet… A significant change!
>
> (12/09/03)

I also noted that I recognised when I needed to look at the investigation in a new way. This would seem to link to beginning to develop the 'habit' of thinking in multiple interpretations as desired at the end of the billiard table investigation. Also evident was enjoyment and persistence:

> When did I last spend about 6 hours doing maths in one day?!
>
> (12/09/03)

Some change also appears to have occurred in my thinking about whether mathematics is a solitary endeavour or not.

> It is interesting to note that the forum "Ask Dr Math" (a web site)
>
> is about people sharing information and seeking answers - so does
>
> that mean math does not always need to be a solitary
>
> endeavour?(12/09/03)

Reflecting back on the experiences of working on these two investigations led me to further identify another discrepancy between what I believe and what I say.

> I believe part of a teacher's role is about supporting learners at
>
> appropriate moments with a useful question and/or piece of
>
> information, and yet I have been unwilling to accept that for
>
> myself as a learner. This appears yet again to point to a
>
> discrepancy in my professed beliefs and actual beliefs. I expect
>
> myself to be able to 'do' these investigations on my own, i.e.,

know everything! (12/09/03)

In terms of my own mathematical achievement I found I was satisfied or felt that I had 'achieved' something when (1) I had found the answers to the questions I posed in the course of the investigation; and (2) that I had learned something new (even in the form of applying 'old' knowledge in a new context). I have become aware that the possibilities within any one investigation of this nature are numerous. Previously I would have had a tendency to want to answer 'all' of the possible questions. I did not feel that need in the billiards table and number investigations.

A crucial question that arose from working on these investigations was 'in which ways does this shed light on how I set targets in my own teaching, and how this results in working with students in particular ways?'. An initial response to this question was to return to an idea formed during the teaching of the first five weeks of the first year paper where our students participated in a number of smaller investigations (approximately 2 - 3 hours spent on each). That idea being that I believed it may be valuable for students to participate in an extended (more than 5 hours) investigation rather than several smaller (2 - 3 hours) ones, thus creating more opportunity to pursue and learn some mathematical ideas. I moved onto to develop this idea in the next semester, as described below.

(iii) Teaching using an extended mathematical investigation

The way in which I approached the first teaching unit in the second year mathematics education paper at the beginning of the next semester (semester A, 2004), some 5 - 6 months after working on the billiard table and number investigations, was directly influenced by my investigative experiences, and different to other teaching approaches I have previously tried. I had personally experienced that it is possible to learn mathematics through this approach and was therefore willing to try an extended investigation with my students. I also believed it was important to reinforce the investigative approach that they had first encountered in the first year paper.

To begin I created a number of 'stations' where students were asked to engage in an activity and identify questions that they had about geometry and/or measurement as stimulated by the activity. I then created or found five different open-ended investigations based on the students' questions. Students were asked to choose one investigation to pursue over a period of approximately six hours in class time. They were free to work on their own or with others. With the exception of two students (out of 44) all chose to work with one other or within a small group. I have no hesitation about students working collaboratively whilst learning mathematics - this is interesting considering the dilemma I encountered when reflecting on my own learning in mathematics.

I was delighted by the students' engagement with the investigations. Indeed they chose to present what they had learned to the class (writing the mathematical ideas they had learned onto an overhead transparency and presenting this to the class with demonstrations and models as appropriate) at the end of the six hours. Whilst their teaching/presenting skills in such situations are still developing it was evident that they certainly had understood various mathematical ideas. This was also apparent when working alongside the students during the six hours. For some students some of the mathematics ideas had been encountered for the first time whilst others found they developed an understanding of a particular procedure for the first time. For example, one group of students developed an understanding of why π is equal to approximately 3. Student A, whom I had an informal discussion with during the first year paper (as described on page 17), showed particular pleasure at coming to understand the meaning of π, and appears to have a greater appreciation of mathematics as a sense-making experience rather than an arbitrary set of rules.

I also noted that my own mathematical knowledge continued to grow using this teaching technique. Questions from the students were an important prompt to re-examine my knowledge about tessellations in particular, and refine my understanding of various definitions pertaining to tessellations. In contrast with previous writing which revealed the taking of too much responsibility for students' learning, this approach lessened my anxiety about answering students' mathematical questions. I believe this was in response to planning for a more in-

depth coverage of one particular topic creating more time to pursue the topic in greater depth.

Another change that I believe occurred involved the nature of interactions between students and myself. When a student asked a question I tended to respond with a question, listen to the reply and continue to work through the particular issue being discussed. I believe this resulted in the student having more space to articulate their own thoughts and thinking. Whilst this is a teaching technique that I have used over the past five years I believe I was more effective at following through on the reply to my question, ie. persuing the topic to an appropriate point to help support the learning.

At this point in my writing I went onto summarise the process that I had experienced during teaching and learning using mathematical investigations. This writing follows below.

(e) Continuing the journey

As already described there have been a number of changes in both my beliefs and teaching practice over this period of time. Personally working on two mathematical investigations was an important time in the journey, leading to the discovery of previously unrecognised beliefs pertaining to my own learning in mathematics, and more generally linking to the nature of learning mathematics. There appears to have been a shift in my thinking from a point where mathematics was perceived as a separate body of correct mathematical ideas to now viewing mathematics as a sense-making process (involving discovering and doing) to do with numbers and pattern and shape and space. Similarly 'success' has been redefined from the obtaining of one answer to:

> Success is apparent when the student is engaged in 'doing'
>
> mathematics, i.e., engaged in the process of inquiry, seeking a
>
> variety of solution paths and critical thinking rather than reaching
>
> one 'correct' answer. (23/04/04)

Earlier uncertainty about how mathematics is learned and whether investigations are a useful approach to support the learning of mathematics ideas have at least been partially resolved with the positive experiences encountered whilst using mathematical investigations as a teaching approach, and as a vehicle for personal mathematical learning. Questions remain about how pre-service primary teachers experience the use of investigations as a learning tool, although working with the second-year student teachers suggests that for some at least, that this approach can lead to the learning of some mathematical ideas.

I am comfortable with the notion that mathematics learning can be aided by the collaboration between students and between teacher and student. I suspect however, that I would still prefer to be able to independently solve mathematical problems that I encounter.

In summary, at this moment in time, I believe the experience of personally working on two extended mathematical investigations has led to significant change in my teaching practice when using an investigative approach. The changes include a belief that students can effectively learn mathematical ideas using this approach; more in-depth interactions to support this mathematical learning; less teacher anxiety and an acceptance that mathematics can be learned by people working in a collaborative manner.

During this research period I developed an awareness that aspects of my thinking were dualistic. Not wishing to continue this and in line with the theme of multiple interpretations I know that teaching is not a profession that can be characterised by doing it this way or that way. Whilst I have undergone valuable personal learning I do not wish to become a crusader advocating that using mathematical investigations will solve all challenges involved in supporting our pre-service teachers to become more skilled at teaching and learning mathematics. Rather it has been a personal journey that at this point has found mathematical investigations to be useful for the learning and teaching of mathematics.

Chapter 3:

Mathematical Investigations: Students' experiences

At the end of chapter two I raised the question about how the pre-service primary teachers in my classes experienced the use of investigations as a tool for learning mathematics, and learning about the teaching of mathematics. Whilst I was aware that I had had a mainly positive experience I believed it was important to more formally gauge how some of the students had experienced this process. I had already informally noted the student's responses and some of these general comments are included in chapter two. In order to gain a more detailed insight into their experiences I decided to ask the students in one of my second-year classes if they would be willing to be informally interviewed. I had taught this class of students in the second semester of their first year and the first semester of their second year, and therefore had been teaching them for a total period of 28 weeks over the preceding 12 months.

(a) Interview procedure

To set up the interviews I reminded the students of the research I was undertaking. This had previously been explained to them at the beginning of their first year mathematics education paper. This procedure was an oral explanation of the research during their first mathematics class with me, with each student being provided with a letter outlining the research and seeking their written consent if they wished to be involved (see appendix B). As explained in the letter, participation was entirely voluntary, and would in no way affect the outcome of their achievement in the paper.

Following reminding the students of the research, I asked if some individuals would be willing to be informally interviewed about their experiences and thoughts regarding participating in mathematical investigations. Four students

volunteered and I subsequently made arrangements to individually interview them at a time suitable to both parties. At the beginning of each interview I showed the students the letter of consent they had signed the previous year. I went over the letter with them to remind them of their rights, including being able to withdraw from the interview at any time, and asked their permission to audio-tape the interview. All signed the letter of consent for a second time indicating their continued consent at this latter time within the research period. All students agreed to be audio-taped, although in one interview the student became uncomfortable with the audio-taping. At this point the tape recorder was turned off. For this interview, with the student's permission, I then took informal, handwritten notes. These were available to the student for reading, as were the transcripts of each audio-tape (each transcript only being available for that respective student's interview).

It is interesting to reflect that all of the four students who volunteered are considered 'mature' students, (i.e., they are not recent school-leavers). Maybe mature students feel comfortable sharing their experiences, or perhaps an investigative approach is more successful with an 'older' student. I also think it more likely that students who appreciated this approach would volunteer to be interviewed. Thus, whilst these four student's responses contain a wealth of information regarding their experiences and thoughts, I believe that they cannot be construed to be a representative sample.

The students were provided with a copy of the questions I wanted to ask (see appendix C). Discussion during each interview was informal, and the questions only loosely followed. As is evident in the interview questions I also included questions regarding the student's thoughts about the nature of mathematics. Later however, whilst analysing the interviews I decided to concentrate on their comments and thoughts with respect to mathematical investigations and collaboration whilst learning mathematics.

Student A, with whom I had had an informal conversation in the first year of the research, agreed to be interviewed again. It was therefore his second interview with me. It was the first interview for the other three students whom I shall refer

to as students B, C and D.

(b) Student experiences of mathematical investigations.

(i) Initial student discomfort

All four students appeared to find participating in mathematical investigations to be a positive experience. For the student I had previously interviewed however, the experience only became positive after some time. This student initially struggled with the investigative approach and was resistant to being involved. This seemed to change during his second paper in mathematics education. In his second interview he stated, "The way I am [now] acting during an investigation is that I am not putting as many barriers in the way of it". A second student also indicated that it took a while to understand what was happening during the investigative process. She stated, "I suppose last year I didn't get the gist of what was going on…".

It would appear, for these two students at least, that there was an initial period where the process of participating in an investigation was an unfamiliar experience, and created some feelings of discomfort. This was particularly evident for the student who now recognises that he was actually initially creating 'barriers' to the process. These findings link with my previously stated concerns regarding whether mathematical investigations are appropriate to use, given some of our student's past mathematical experiences. I previously asked the question, can our students access the possible mathematical learning (during an investigation) when they do not even have the expectation that mathematics makes sense? It certainly seemed that at least initially an investigative approach posed problems for some of the students. I also wonder what part I may have played in these initial feelings of discomfort? It is certainly evident in my writing that, early on, I also had concerns regarding using investigations. I believe it is possible that my early attempts at using mathematical investigations may have influenced the students' experiences.

Because I now have more experience with this approach as a learner and teacher I believe that I can take this awareness of possible feelings of discomfort into my teaching, and as a beginning point, be able to empathise with students who experience this. Having also experienced the learning that can result from this approach I believe it to be pertinent to also point to the possibilities of learning that can occur if the student can be encouraged to persevere through these initial feelings of discomfort. Student beliefs about the learning and/or nature of mathematics could also be openly acknowledged and discussed within a supportive environment. It would also appear that using this approach for a second time (for example, in the second year paper) is productive and enables the students to make deeper connections with the issues that arise.

(ii) Positive experiences

It also appears from the interviews that, for these four students at least, following some initial discomfort they had positive experiences. One of the four students that I interviewed described how she found investigations to be less threatening and experienced them as being less pressured. Another student described how coming to understand why π is equal to '3 and a bit more' was an 'a-ha' moment. This appeared to be quite a pivotal experience for him in developing a new enthusiasm for the investigative process. He said, "… realising, like that activity, when we went outside. My thinking was 'if you want us to go outside, I'll enjoy some sunshine and that's about it'. Little did I know that I was going to have an a-ha moment and that was great". As a mathematics educator receiving such positive feedback is certainly encouraging when considering whether or not to continue using this approach with future cohorts of pre-service primary teachers.

(iii) Learning mathematics using an investigative approach

All four students stated that they learned some mathematical ideas, or understood a previously learned concept for the first time, through participating in the investigations. This is congruent with my own personal experience. The

four interviewed students also alluded to a deeper level of learning using this approach. This level of learning could be contrasted with a more traditional approach where a teacher might impart some knowledge (eg. telling students a piece of information, e.g., the value of π; or showing a particular procedure) followed by students practicing numerous examples. One student described her experience of the deeper learning saying:

> With the traditional method, I can sometimes see there is [sic] good points to it, but then when we did that ' one', obviously I would've been told what π was (referring to her past), but I never remembered it. So when we started, I thought 'really, what is it?' and when I found out, it'll be in my head for the rest of my life. I found out for myself.

This student seems to link this deeper learning with 'doing it herself' rather than being told something. Another student described the difference in learning as follows, "it is learned today, but it was taught in the old days". This same student stated that the investigations had "reignited the flame" with respect to her enjoyment of mathematics, and also referred to the importance of being able to relate previously learned mathematical ideas to a context. She referred to the "privilege… of putting that formula to life's learning experiences and into a context". It would certainly seem that for these four students that an investigative mathematical approach has been worthwhile.

Three of the four students also said that traditional methods of teaching mathematics still have their place, either in addition to doing investigations, or as part of an investigation. One of the students said that within investigations a teacher may still need to provide some specific teaching, and gave an hypothetical example, suggesting a teacher may need to teach a class about graphing skills in order to support the progress of an investigation. Student A referred to feeling 'torn' in his thinking when responding to a question as to whether or not he still wanted an 'A to Z poster' of maths ideas (this was mentioned in his first interview). He stated, "I am torn in my thinking. I have

really embraced investigations but I can see that I would still like a poster. I can see how that contradicts my excitement about investigations".

The suggestion that traditional teaching or 'skills' teaching continue to have a place in the primary mathematics classroom, alongside or as part of an investigative approach, is congruent with suggestions proposed by Lovitt (2004). This certainly occurred, in an informal-as-needed manner, within the context of the investigations I used, but is something that I did not raise as a discussion point with the class. Perhaps this could be explored in more depth in future teaching, and make a point of discussing with the students as part of their analysing of the investigative process.

One of the students whilst not referring to including traditional methods of mathematics teaching in her forthcoming teaching career, expressed concern about the time taken to learn mathematical ideas via an investigative approach. She stated that for her, "it is more time consuming" and asked the question, "have we got more hours in the day to spend on maths…?".

I too have had that concern. However, based on my experiences both as a learner and teacher using this investigative approach, I believe that the level of learning is deeper and more meaningful and thus warrants the required time. Also, when considering my new ideas about what the learning of mathematics may entail, I now believe that this approach more closely captures the essence of what mathematics is actually about, ie. a process of making sense of situations involving number, patterns, shape and space, rather than the finding of a particular answer using a set procedure that someone else has previously discovered.

Whilst the four students said they had learned some mathematical ideas whilst participating in an investigation, one student described how she believed that some of the mathematical ideas encountered within the investigations had not been completely understood by some students. This was a concern that I also had, particularly during the first time I used an investigative approach. Because of this concern, in the second year paper where we were using an investigative approach for the second time, I encouraged the students to collate their findings

onto an overhead transparency which was subsequently presented to the class. I believe that this helped to consolidate students' learning, although from this student's comments it would seem that some students were still not understanding some of the mathematical ideas being encountered.

> I remember having outline presentations for tessellations and getting down logically what we had learned and then having to summarise it and putting it on an OHT was not actually an easy task. It put it in a neat little box for us. As a group, we still churned through this and still argued and sorted it out. Some of us were able to sought it for ourselves. But I thought it was a little bit dangerous leaving it up in the air….

I wonder if the phrase 'leaving it up in the air' points to a need for a more rigorous checking of individuals' learning? Perhaps a more formal assessment of the mathematics being learned would reveal whether or not this is a widespread problem. However, numerous issues arise from this suggestion. Firstly, is something being 'left up in the air' problematic? If mathematics is viewed as a process rather than product is it helpful to assess the product of learning as is traditionally done? If one wishes to assess learning in terms of 'process' what indicators of learning might one look for? And, because this is a mathematics education paper rather than a mathematics paper, and because of university assessment requirements the suggestion of more formally assessing the mathematics being learned is problematic. Nevertheless, I think it is important that I do not consistently and inadvertently leave things 'up in the air'. While I believe that some degree of uncertainty is helpful for learning, I believe that too much does not support learning. I need therefore, to continue to carefully ponder the issue of mathematical learning when teaching using this approach.

(iv) Students' mathematical behaviours

The student's mathematical behaviours appeared to undergo some change over

the two semesters whilst participating in mathematical investigations. One student stated that she is now more thorough in her approach, and referred to thinking about the investigations in an ongoing manner which lead to learning. Another student also indicated that mathematical discussions and thinking occurred beyond the class environment. She said, "we didn't just walk away from your lessons and forget about it". The first student (mentioned in this paragraph) also described that wanting or needing to work on an investigation was an important part of the process. She was clear that she did not want to choose some of the investigations because she already knew the mathematical ideas involved in those, and wanted to explore something new to her. This student also believes that being able to identify what she already knew, and what she then needed to know were important parts of the investigative process.

Two students both described how they are now more open to the possibilities of alternative approaches to solving problems and the multiple interpretations that might occur within any one investigation. One also stated that she was more 'on-task' than she usually is because of the hands-on nature of the investigations she chose, and also because a mathematics text book was provided as a resource to consult if needed. This student also appeared to have a sense of ownership over her learning. She described how "it was better because I was actually doing it all myself and then learning at the same time". Some of these comments would appear to support the previously made suggestion that it is worthwhile pursuing this approach for a second time in the second year paper.

(v) Writing in mathematical investigations

In line with the belief that it is beneficial for students to write whilst learning mathematics (Flores & Brittain, 2003; 2004), during some of the investigations that the students participated in there was a requirement to provide written evidence of their mathematical thinking. This could include diagrams, tables, graphs as well as written descriptions. This appeared to have mixed results. One student said that this was sometimes difficult to do and had the effect of interrupting her stream of thinking, which tended to slow the process down. She also simultaneously acknowledged however, that writing was important as a

way in which ideas could be clarified. Writing during mathematical investigations certainly appears to be complex with both positive and negative influences on the process. While I value and enjoy writing it is good to be reminded that writing may not always be a helpful activity, and that for some students it may interfere or be perceived to interfere with their mathematical thinking. Being open to different forms of writing, for example, accepting summative and/or formative writing, may be an important point to consider in the future.

(vi) Students teaching using a mathematical investigative approach

Some of the students' beliefs and/or ideas about teaching mathematics also appeared to change. Three of the students interviewed indicated they would try using an investigative approach when they begin to teach. One student also indicated feeling "frightened" that she would be unable to deal with the possibilities that the children might raise within the course of an investigation. She stated however, that, "I believe I am now preparing myself to work through whatever their ideas are, which I think is really positive". Once again, I can empathise with this student's experience. I have already described in chapter two how I too, found this approach to be initially somewhat unsettling with respect to possibly not knowing the mathematics that might be encountered during the course of the investigation. Perhaps this is part of a process of moving from viewing mathematics as a discipline where it is important to know the answer, to an alternative view of seeing mathematics as a process of doing and discovery. In this alternative view not knowing the answer would be seen as an exciting and natural part of doing mathematics.

A student also stated that her learning of mathematics,

> … has shifted from being formula based mathematics [to] social
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> constructivism… you are interacting with others, you are using
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> your previous knowledge and ideas and you are experimenting
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> with it. I had never been allowed to do that with maths before and I
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> enjoyed it.

There also appears to have been some shifting in the beliefs of the student whom I interviewed twice. He stated "maths is the ideas and the curriculum" which still seems to indicate a belief of mathematics as a separate body of facts. However, he also indicated that maths is everywhere saying, "I would have said maths is about those subjects at school that I had to be taught. Now, I don't know where to begin…. I am seeing maths in a whole lot more things". This student's beliefs regarding collaboration also changed. In response to the question, "does collaboration help your learning in mathematics?" he stated, "I am a believer, yes. At first I wasn't but with time, I know that my thinking has changed".

Three of the interviewed students referred to the teacher's role when using mathematical investigations to support learning in mathematics. Two students in particular, seemed to suggest that the teacher had a pivotal role. One described how the collaborative aspect; the way the teacher "ran the classroom"; and the teacher's use of questions were all important and positive aspects of the process. Another thought that a teacher's own knowledge is important, stating that "the teacher's [sic] having that knowledge also helps".

Whilst one student felt positive about the teacher's use of questions another student appeared to have sometimes had a negative experience when a teacher asked questions. She said, "sometimes I get frustrated when the teachers keep asking me questions and I don't get it and I don't want to play guessing games". This is interesting to reflect on given my previous statements regarding my questioning. I have written:

> When a student asked a question I tended to respond with a
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> question, listen to the reply and continue to work through the
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> particular issue being discussed. I believe this resulted in the
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> student having more space to articulate their thoughts and thinking.
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> Whilst this is a teaching technique that I have used over the past
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> five years I believe I was more effective at following through on

the reply to my question, ie. persuing the topic to an appropriate

point to help support the learning (chapter 2, p. 42).

I now wonder if my questioning was helpful for this student, or perhaps was appearing to be a 'guessing game'. Perhaps these differences point to differing perceptions of what it is to 'do' mathematics. Embedded in this student's response may be the belief that mathematics is about getting answers and that the teachers have the answers. In contrast it is now my belief that mathematics is about doing and being engaged in thinking about and solving mathematical problems, and thus my questions are designed to encourage thinking and making connections rather than providing an answer.

Another student is quite clear that mathematical investigations on their own will not support children's learning in mathematics. As previously described this student believed that investigative work needs to be supported by some more traditional methods of teaching. She stated, "I don't think kids can just learn/discover things by themselves".

(vii) Collaboration in learning mathematics

The four students had varying thoughts regarding the place of collaboration in the learning of mathematics. Two of the students referred to needing a 'safe' class environment for working collaboratively. Knowing one's fellow students was particularly important for one student. Negative aspects of collaboration that were mentioned included being time-consuming; the possibility of individuals being distracted; and, a 'slowing down' because of working in a group. Positive aspects of collaboration included the sharing of ideas that might not have otherwise been considered. This was seen to stimulate further ideas and sharpen one's thinking. It was stated that "the new collaborative learning type thing is more exciting, more stimulating and there is more of a desire to be there…".

(c) Other perspectives

It has been valuable to gain an insight into the experiences of these four students. Whilst some of their experiences resonate with mine (e.g., being concerned about not knowing enough about mathematics given the open-ended nature of these investigations when in a teaching role), others were different and I was able to gain new insights into this learning and teaching approach.

Of particular note are several possibilities to consider when using this approach with future cohorts of students. The first of these is the possibility of more deliberately discussing student beliefs about the learning and nature of mathematics in an effort to acknowledge these in a more specific manner, and to simultaneously provide encouragement to continue with the investigations. The second possibility involves discussing the place of more traditional skills teaching that may occur within this approach. The third is to continue to carefully observe and consider the learning of mathematics using an investigative approach. Being open to differing forms of writing may also be necessary to cater for those students who find writing interferes with their mathematical thinking. For example, accepting writing at the end of an investigation as an option to the current requirement of writing throughout the investigative process.

It was encouraging to also note that, in the main, the students seemed to have had positive experiences with this approach. Of particular note was the deeper learning that appeared to occur. The students spoke of truly coming to understand particular mathematical concepts for the first time. They also perceived their mathematical behaviours to have changed in a positive manner. For example, ongoing reflection and a more thorough approach were described. An openness to the idea of multiple solution strategies was also developed, and all four students expressed an interest in trialling this approach once teaching in their own classrooms.

I think it important to remind the reader at this stage that while I value the experiences and insights that these interviewees have shared, I am also aware that there will be other experiences and interpretations of the mathematical investigative approach that are not represented by these four students. It is not

my intention in this study to gather quantitative data that is representative of all students but I believe it is nevertheless important to remain open to the ideas and insights that others may hold.

Chapter 4:

A theoretical perspective of narrative inquiry

At this point in the research process I began to read about narrative inquiry in more depth. This was a rewarding experience as I began to understand more about this method of qualitative research from a theoretical point of view. In this chapter I write about the themes I encountered in the literature about narrative inquiry. I also begin to make links with the writing I had already done although these are developed much more fully in chapter 5. The themes I encountered and have chosen to explore here include: a consideration of what narrative inquiry is, including a brief description of various narrative approaches; an examination of the role of reflection in narrative inquiry; looking at the notion of multiple perspectives, and the influences of assumptions and judgments on perceptions and writing; a consideration of research issues such as validity, authenticity, the products and dissemination of research such as this; a brief look at the role of collaboration and lastly, considering the affective dimension of the research process.

(a) Narrative inquiry and narrative approaches

Over the past two decades the practice of reflection has been recognised as a legitimate aspect of action research in education (Adler, 1993; Francis, 1995; Schon, 1983). More recently, narrative inquiry has also become a valued form of research (Chambers, 2003; Luwisch, 2001; O'Connell Rust, 1999). With the development of action research there has been an associated move away from empirical analytical models of research in education towards that of teacher practitioner research (a form of action research) where the practitioner (whether also the researcher or not) is central to the research (Brown, 2001; Winkler, 2003). The place of theory in narrative research is also different when compared with empirical analytical models of research. "Work done in the field of

narrative studies is concerned with … the development of theory in terms of practice rather than in the analysis of practice in terms of theory" (Beattie, 1995, p. 63).

As implied above, narrative inquiry can be seen as a form of action research. Winter (2002) and Beattie (1995) state that the central purpose of action research is to create some form of change in practice. There is a similar emphasis on change with narrative inquiry. Clandinin and Connelly (2000, p. 2) state that a "goal of narrative inquiry is mutual learning among the participants that changes their thinking and their lives". Other authors also referring to change include Brown and Jones (2001) who state that writing is "an important marker of time" (p. 55) in monitoring the process of change; and Winter (2003) who in an article that draws parallels between the basic principles of action research and some key Buddhist doctrines, also points to the central importance of change. Winter regards this to be a source of understanding, and also refers to the human tendency of wishing to avoid change. This research process has certainly resulted in significant changes both in my beliefs and teaching practice. I further examine this concept of change with respect to this narrative inquiry in more depth in chapter 5.

Narrative inquiry is perceived to be a powerful means with which learners can reflect on and develop their own professional practice (Chambers, 2003; McCormack, 2002; Rushton, 2001), and perhaps even recast their lives (Winkler, 2003). Beattie (1995) makes a particularly strong case for the use of narrative inquiry within educational research, writing, "at the heart of meaningful educational reform and change, lie the narratives" (p. 66). Narrative is a practice that has wide applications ranging from being a form of professional development/training in sectors such as health and social care (Chambers, 2003; McCormack, 2002) through to the development of teacher practice (Beattie, 1995; Brown, 2001; Brown & England, 2004; Clough, 2002; Doecke, Brown & Loughran, 2000; Eick, 2002; Johnson, 2002; Martin, 2000; O'Connell Rust, 1999; Olson & Craig, 2001), and more specifically for the purposes of this paper, being an invaluable means of professional development for teacher educators (Brown, 2001; O'Connell Rust, 1999; Tzur, 2001) .

Narrative in essence is a form of story-telling. Stories are regarded as a means through which teachers are able to make sense of their work of teaching (O'Connell Rust, 1999). Beattie (1995) refers to narrative as a way in which teachers "find voices to tell their own stories" (p. 59) and gain new understandings of their lives and the communities within which they live. Clandinin and Connelly (2000) write in a similar vein stating, "the study of narrative is the study of the ways humans experience the world" (p. 2). It is also thought that by describing our narrative identity we come to know who we are (Ricoeur, 1986, as cited in McCormack, 2002). Narrative research would thus appear to be a journey during which the researcher comes to know more deeply about their life and who they are as a person. Once again this resonates with my experience, and is evidenced in the preceding chapters. For example, I have become aware of previously unknown subconscious beliefs about my own learning in mathematics.

Using narrative as a research methodology necessarily locates the researcher within the research (Adler, 1993; Brown, 2001). Chambers (2003) describes how the narrative authors are not merely "component parts" (p. 413) but are an integral part of the research. Brown (2001) writes in a similar vein stating, "the trend towards 'practitioner research' increasingly accommodates an understanding of how researchers are practically related to the situations they investigate, where their actions, as teacher-researchers, are seen as an essential part of situation [sic] being described" (p. 211).

Given that narrative is "a story of events, experiences or the like, or; a written or spoken work containing such a story" (Stein, 1975, p. 885) language is necessarily an integral and inseparable feature of the work (Brown & England, 2004; Brown & Jones, 2001). However the language individuals use is not value-free nor does it exist as an objective entity on its own (Wilber, 1998). Brown and England in referring to Habermas' work write, "we must adopt a critical attitude to the language that we use in describing our professional practice" (p. 69). Similarly, Brown and Jones suggest that their view of the emancipatory aspect of research may be seen as the seeking "to break free of the ideological distortions intrinsic to the language itself" (p. 34).

Brown and England (2004) also refer to the work of Lacan, with a similar theme of the paramount importance of language. They write, "Lacan, we believe, assists us in examining our own language with view to locating how our desires, our fears, our hidden motivations govern our professional practice" (Brown & England, 2004, p. 72). Language used is also thought to reflect the society from which it comes including that societies power relations and inequalities (Brown & England, 2004; Brown & Jones, 2001).

Unlike more traditional research, in narrative research, "the subject is never given at the beginning, but it unfolds as the story is told" (Ricoeur, 1986, as cited in McCormack, 2002, p. 337). Beattie (1995) also points to the lack of predetermined goals. Similarly, McCormack (2002, p. 338) writes, "predicting the outcome is less important than understanding the journey". This aspect of narrative research is something that I actually initially felt considerable discomfort about. As explained later, my scientific background predisposed me to understanding and accepting more scientific models of research, and as such, the notion of an 'unfolding' subject caused considerable unease. I discuss this more fully in the next chapter.

Whilst most narrative research to date has primarily been in written form, other possibilities such as visual narrative (picture book genre, collage and photographs) exist (Johnson, 2002). A number of differing forms of narrative writing are described in the literature (Chambers, 2003; McCormack, 2002). McCormack (2002) suggests readers use a knowledge of narrative types as " 'listening devices', i.e., devices to help the listener understand the focus of the narrative and thus engage in active listening" (p. 337).

McCormack (2002) refers to three types of narrative including restitution narrative, chaos narrative and quest narrative. Restitution narratives have a focus on the future whilst chaos narratives relate a story where life is thought to never get better. Quest narratives in contrast, "meet suffering head on. Such narratives accept what is happening, and seek to use such happenings positively on a journey of growth and change" (McCormack, 2002, p. 337).

Chambers (2003) also presents a range of narrative approaches. This author describes three techniques which he calls 'spontaneous', 'realistic' and 'anecdotal'. In the first technique the writer recreates a particular setting or context through spontaneous writing. There is an attempt to recreate and begin to understand the context including the role the writer is playing within the context. Parallels are seen by Chambers with Schon's (1983) 'reflection-in-action'. Ethical questions are raised about this approach however, including the question, "is there, for example, any voice other than that of the writer?" (Chambers, 2003, p. 406) and thus, may be perceived to be intrusive in nature. Attention to detail, imagery and the use of metaphor are seen to be important aspects of this narrative technique and render it personalised and subjective. Chambers concludes that this type of narrative whilst ethically problematic "at least allows for a glimpse of light between the idea and the reality" (p. 407).

The second narrative technique described by Chambers (2003) involves an attempt by the writer to 'replicate the scene' reproducing a realistic version of events as they occur. The aim of the observation is authenticity and invites reflection-on-action (Schon, 1983). This includes the use of direct quotes and is much less creative than the spontaneous technique. The voice of the writer is not of any particular significance in this style of writing although it is noticed that the value system of the writer is still evident within the writing.

Chamber's (2003) third technique, the anecdotal technique, produces an objective view of a scene where the writer is in the role of a non-participant observer, observing carefully in a detached mode. This contrasts markedly with the spontaneous technique where there is complete engagement. The purpose of this third type of narrative is seen to be the creation of a visual picture, written at a distance, in order to bring reflection to the fore (Chambers).

It was something of a surprise to read about these differing narrative techniques. It had not occurred to me, once again, probably because of my scientific background, that one could legitimately write in a spontaneous manner. Indeed, I now perceive a struggle in my writing to acknowledge 'myself'. Although it now seems somewhat surprising this is initially evident in the delayed awareness of the impact of my own past experiences on my current situation, as

described in chapter 2. I pursue this and link my writing to differing techniques more fully in chapter 5.

(b) The role of reflection in narrative inquiry

Reflection is an integral part of narrative inquiry and is linked to the gaining of new understandings. Brown (2001, p. 211) states that, "self-reflection is integral in the teacher's self-positioning in the teaching act and in assessing its affect on the student". Chambers (2003) points out that "both the narrative itself and reflections upon the narrative appear to facilitate understanding and to generate new knowledge" (pp. 404-5). Similarly, Johnson (2002) indicates that the process of (re)reading one's narrative can offer new and alternative learning. As such narrative inquiry has many levels of potential for reflection (Brown & Jones, 2001; Chambers), and such reflection is regarded as important in facilitating teachers' personal and professional growth (Johnson).

Korthagen (2004) also refers to the possibilities of multifaceted reflection. In an article he poses the question of 'what are the essential qualities of a good teacher?' and proposes a more holistic approach in teacher education. He also presents an "umbrella model of levels of change that could serve as a framework for reflection and development" (p. 77). Korthagen suggests that there has been a considerable emphasis on promoting reflection in teachers but also points to a lack of direction about what teachers might reflect upon. Therefore, he proposes an 'onion' model where a series of layers provide a possible structure for the content of reflection. These layers include a consideration of the teaching environment, teacher and student behaviours, teacher competencies, beliefs, professional identity and mission, with mission being the innermost layer or core.

Korthagen (2004) also refers to developments in teacher education where more recently the trend is for researchers to use a narrative approach. He writes that a narrative approach is based on the premise that teachers' thinking is embedded in the stories they tell each other and themselves. Korthagen also refers to what he perceives to be a shift of accent within the narrative approach with an

increasing emphasis placed upon the beliefs people hold about themselves. These beliefs are linked in the 'onion' model to one's professional identity. As I have already mentioned, an examination and growing awareness of some of my beliefs has been a feature of this research.

The innermost level within Korthagen's (2004) model is referred to as the level of mission and includes a consideration of professional identity. 'Mission' is what is "deep inside us that moves us to do what we do" (Korthagen, 2004, p. 85), and is about what gives meaning to one's existence. Korthagen believes that this level may have a very concrete significance in teacher's professional development, but also recognises that little theoretical research has yet focussed on professional identity and mission.

Brown and England (2004) also refer to the notion of identity, questioning models where there is a supposition of a deficit position from which the researcher seeks to free him/herself in efforts to attain an ideal. Instead an alternative model of emancipatory practitioner research, based on the work of Lacan, is offered whereby the identity of the practitioner researcher is seen to be more fluid. Rather than seeking resolution or an end point (implied within the work of Habermas, 1972, 1976, 1984, 1987 and Foucault, 1997, 1998 as cited in Brown & England, 2004) the research process is regarded as the building of a narrative layer that supports and grows alongside the writer's life as it occurs (Brown & Jones, 2001). This is a model of research that has personal appeal, and is alluded to again in chapter 5.

(c) Interpretations and influences on perception

When writing narrative, different perspectives or interpretations are always possible (Brown & England, 2004; Mason, 2002; Wilber, 1998; Winkler, 2003; Winter, 2002, 2003). Not only are different perspectives of events or contexts a possibility but alternative 'personas' are referred to in the writing of Brown and England. They suggest the task of the researcher is a re-examination of one's life with an aim to become aware of alternative persona that may be adopted, and the subsequent relationships to the world that these alternatives create

(Brown & England). I see this in my experiences regarding viewing my role as a pre-service mathematics educator in slightly different ways which had a subsequent impact on my teaching practice. For example, as described in chapter 2, in taking less responsibility for the students' learning (an alternative persona) I experienced feeling more in tune with the student's needs (subsequent relationships are changed).

The concept of multiple interpretations is seen to be a positive aspect of reflection. Chambers (2003) writes, "different perspectives further open up possibilities for engaging in the process of reflection in that they offer specific and sometimes comparable or contrasting points of view" (p. 412). The notion of differing perspectives offering a contrasting point of view was a pivotal point in the reconsideration of my beliefs about the nature of mathematics (see chapter 5 for more detail).

Wilber (1998) however, warns against the extremes of postmodernism which take the position that all interpretations are equally valid. While Wilber agrees that, "the world is in part a construction, an interpretation" (p. 34), he clearly states that "all interpretations are not equally valid: there are better and worse interpretations of every text" (p. 34). It seems therefore, that whilst the proposition of multiple interpretations is valid, it should not be taken to the extremes of accepting all interpretations as equally valid.

It seems to follow from the notion of multiple perspectives or interpretations that no piece of writing "has an absolute meaning" (Brown, 2001, p. 218). Brown suggests that another story can always be placed alongside any particular piece of writing. However various authors caution against accepting all stories as being equally valid (Brown, 2001; Wilber, 1998). It is also suggested that the stories teachers write can not necessarily be resolved with one another. "There are multiple stories of what it is to be a teacher to be negotiated - stories that do not lend themselves to final resolution in relation to each other" (Brown & England, 2004, p. 71). Not only can multiple perspectives of a situation be written about but understandings of any writing are also always temporary and subject to reformulations (Brown & Jones, 2001).

Becoming aware of those things we take for granted is an integral part of narrative research (Francis, 1995; Mason, 2002; Winkler, 2003). McLaughlin (2003) explicitly states that, "we are often unconscious of our assumptions and judgements" (p. 68). Brown and Jones (2001) recognise that layers of assumptions may result in constraining individuals and their actions. I would understand this to suggest that given the deeply embedded nature of social values (Brown, 2003; McLaughlin) that it can be difficult to identify social norms that the researcher may be unconsciously operating under, and thus because of these assumptions, choices and/or actions might be limited.

Nevertheless Adler (1993) and Brown (2001) state that the context of the research must be made clear. Brown sees that "teachers working on building a picture of their practice face a necessary task of developing a sense of the context in which they see themselves" (p. 218). Another author, Winkler (2003), believes that whilst the researcher's own personal assumptions and preoccupations are a part of collaborative narrative research, that the work must also be "deeply empirical, grounded in systematically collected data, sceptical questioning and rigorous examination of meanings that are conveyed" (p. 400).

With the possibility of multiple interpretations and the existence of unconscious assumptions and judgments, writing and analysing narrative is obviously not a straightforward task. Winkler (2003) writes that "reality is seen as a multiple complex construct", and as such "reality is not fixed out there, but is an intricate, collaborative experience informed by purposes and intentions of those who live it" (p. 390). Winkler also recognises how her own personal stories strongly influenced her intuition and determined the kind of knowledge that she constructed within her research. In a similar vein, Brown and England (2004) refer to the work of Lacan writing "I notice what I do in so far as my actions inhabit my fantasy frame of who I am" (p. 73). They go onto state however that observations are also "haunted by the aspects I choose not to see" (p. 73). This point is further explored in chapter 5.

Such complexity is a recurrent theme encountered in the literature located for

this review on narrative methodology (e.g., Luwisch, 2001; McLaughlin, 2003; O'Connell Rust, 1999). Winter (2003) continues this theme, writing, "our analysis of data must trace the links between physical events, social relationships, organisational structures, psychological states of mind, and moral values in order to formulate wise and compassionate action" (p. 148). Winter suggests that in reflecting on data it is important to seek out contradictions and recognise the reflexivity that most of our statements/judgments contain. Learning to distinguish between an experience and our "conceptual response to that experience" (Winkler, p. 392) is also perceived to be an integral part of narrative research. Seeking out contradictions was a notion that appealed having already noticed one such contradiction in my writing (refer chapter 2, p. 19).

(d) Issues of validity, authenticity, who benefits from this research, and dissemination.

A premise of positivist scientific and empirical analytical models of research is that research must meet criteria such as being objective, value-free, scientific and therefore valid (Sikes, 2002, in Clough, 2002). Narrative research, because of it's inherent subjectivity, would therefore appear to be problematic. The literature encountered for this review however does not perceive a loss of objectivity to be a problem (e.g., Mason, 2002). Brown (2001) acknowledges the seeming loss of supposed objectivity inherent in this form of research but suggests that this is replaced with an account of what might be seen and how best to see it. Winkler (2003) also supports the notion of subjectivity going so far as to suggest that a rational and objective framework for research could compromise "the generative force of each teacher's story" (p. 393).

Wilber (1998) however, cautions against narcissistic possibilities when there is no demand for evidence at all. He states that the idea, "there is nothing but interpretation, and thus we can dispense with the objective component of truth altogether" (p. 119) as absurd and self-defeating. Thus, while subjectivity appears to be acceptable, we are warned against totally dispensing with searching for an objective component of truth.

It has been suggested that the validity of narrative research reports resides in their authenticity (Winter, 2002) or trustworthiness (Winter, 2003). Winter goes on to write however, of the ambiguities in the concept of 'authenticity' as used in relation to the narratives of action research. Differences between other forms of research and action research, which has an underlying principle of providing 'culturally silenced' people to find a voice, are pointed to, with regard to a report being considered authentic if it provides this voice for the culturally silenced (Winter, 2002). However, not only are there multiple understandings of the term authenticity, but analytical problems and questions regarding 'truth' emerge. Winter (2002) suggests that rather than asking the question, " 'is this narrative true?' " (p. 145), it may be more helpful to ask the question,

> … is this narrative shaped and moulded in such a way that we feel
>
> it is trustworthy, i.e. does it persuade us that we might helpfully
>
> rely on the insights it presents about that particular situation to
>
> guide our thinking about other situations? (p.145).

Winter (2002) also suggests that "an understanding of the complexity of one's 'existential condition'" (p. 149) and emphasising the dialectical reflexivity of narrative are ways of resolving the dilemmas posed when considering authenticity, i.e., being aware that there is not a single, correct perspective and that any analysis is tentative and cannot be regarded as "'accurate' but merely as trustworthy'" (p. 148). As such, Winter proposes a modernist aesthetic for narratives to avoid "reproducing the authoritarian texts of realist fiction and of hierarchically organised research" (p. 143). I further ponder these issues in relation to my own work in chapter 5.

Winkler (2003) also refers to the notion of validity and links this to the theme of multiple realities (already discussed above). She writes, "the validity and ethical defensibility of collaborative research ultimately depends on the critical acknowledgement of multiple realities, and on self-aware, discriminating and informed judgements about these realities" (p. 400). Like Brown (2001), she too points to the importance of extending the research to others. She also suggests that readers of the research will make links with the research in terms of their own lives. "Readers… invariably busy themselves questioning and

reconstructing the original story in terms of their own" (Winkler, 2003, p. 393). She goes further to state that the credibility, transferability and validity of a study is ultimately dependent upon the quality of the final text. This point is also further explored in chapter 5.

Winkler (2003) also points to other issues to be aware of with narrative research including the "seductive power of authoring lives the way we want them to be" (p. 393); creating an illusion of a coherent purpose when in fact none exists; and incorporating the complex nature of 'authoring' into the writing of the final text.

Personal reflection and narrative alone would not be viewed as research unless there is a communication of the ideas being explored, with others (e.g., Winkler, 2003). Thus the literature appears to point to a requirement for some form of dissemination to occur. However, Brown (2001) and Winkler (2003) raise some issues regarding the dissemination of this form of practitioner research. Brown states that,

> … the product of practitioner research does not result in statements
>
> of practical implications common to all. Rather it gives an account
>
> of a practitioner examining specific issues within their practice and
>
> how these were addressed as problems within the research process
>
> (p. 248).

Links with subsequent readers of the research are nevertheless important however but in a different way to traditional research, as described in the following paragraph.

As implied above, the result of narrative research is not a definitive statement or generalisation about an aspect of that which is being researched (e.g., Brown & Jones, 2001). Brown (2001) uses an analogy of the research resulting in a 'traveller's guide' rather than a map or encyclopedia entry. McCormack (2002) also refers to this research not providing a 'map' but allowing "the reader to witness the process of the story's construction and its meaning for the storyteller" (p. 337). Brown also states that the reader of such research has a right, having read the research, to tell stories about how it may connect with

their own practice. "As such the task of research is not to provide a mapping of 'how things are' but rather is about production that triggers renewal" (Brown, 2001, p. 249). Winkler writes in a similar way suggesting "the narrative nature of the study … allows insights to be transferred from one context to the next" (p. 392) . These ideas are congruent with my closing statements in chapter 2 where I do not necessarily suggest that using mathematical investigations will be appropriate in all scenarios.

Some authors (e.g., Winter, 2003) suggest that narrative research is a way in which one can contribute to humankind. Chambers (2003, p. 413) writes, "the essence of reflective practice is that it can make a difference to individuals". Brown and England (2004) state that much practitioner research in education is based on an emancipatory model taken from the work of Habermas, in which the teacher researcher is understood as being an agent of change for the better. As already alluded to, I believe that this narrative research has made a difference to my practice, and I explore this in more detail in chapter 5.

Brown and Jones (2001), and Winter (2003) write however, that emancipation within action research is a contested concept, and that agreements negotiated within collaborative research may be merely "temporary pragmatic political or interpersonal compromises" (Winter, 2003, p. 151). Francis (1995) believes that the high profile of reflection in teacher education is only warranted if it impacts on more equitable and just outcomes for the preservice teachers with whom she works. She also raises the important question as to whether teachers being more reflective will positively impact on children's learning.

Brown and Jones (2001) write that emancipatory views of practitioner research supposedly allows the practitioner to organise the complexity of the teaching situation with a view to controlling the change, for the better. These authors propose however, that the desire for control can obscure the complexities of a situation. Instead they suggest that postmodernist analysis "offers opportunities to conceptualise the world in different ways" (Brown & Jones, 2001, p. 6). A part of this process is seen to be developing the facility "to recognise the ways in which dominant ideologies and social structures work at coercing and oppressing" (Brown & Jones, 2001, p. 18).

One might suppose that narrative research would involve the seeking of an ideal, for example, becoming the 'ideal' teacher. Such possible striving for an ideal, including an endpoint, where there is a supposition of a deficit position from which the researcher seeks to free him/herself, is one particular stance described in the literature (Brown & England, 2004). An alternative to this is rejecting the notion of achieving an ideal and having a perception of research that creates "stories that help us for the present, as we make sense of the past, as we nudge to the future" (Brown & England, 2004, p. 77). This is an idea that appeals to me, and I refer to it again in latter writing. Although there is a notion here of rejecting an ideal or resolution, there is still a belief in the possibility of social change. This belief is tempered however by the recognition that such social change would always occur through a filter of one's own fantasy frame (Zizek, 1989, as cited in Brown & England, 2004).

Avoiding the seeking of idealism is also supported by Brown and Jones (2001) and Brown (2001) who in describing the work of Elliott (1987, 1993 as cited in Brown, 2001, p. 214) state that, "in addressing the changes in practice the central task is not to learn new techniques but rather to locate oneself in one's own current practice and build a notion of a way forward". O'Connell Rust (1999) also writes that "newness is not the point" (p. 370).

> As such, research becomes the instrument through which we build
>
> and understand our practice, not to reach some higher plane of
>
> perfection, nor to be more in touch with where we are in life, but
>
> rather to make explicit a reflective/constructive narrative layer that
>
> feeds, while growing alongside, the life it seeks to portray (Brown
>
> & Jones, 2001, p. 69).

This desire for research to make a positive contribution in the lives of all participants, not only that of the researcher is evident within recent research literature (e.g., Winter 2003). Winkler (2003) however, raises an issue regarding "the ethical implications of managing the fluid relational boundaries that characterise narrative research" (p. 388), where the possibility of oppressive relations and exploitation occurring when working in a collaborative narrative

setting is raised. The question then of 'who benefits?' might be perceived as problematic.

(e) Collaboration

In much of the literature on narrative research there has been an associated collaborative dimension (e.g., Olson & Craig, 2001). Collaboration appears to be extremely complex (Olson & Craig, 2001; Winkler, 2003) and may have a variety of potential effects. McLaughlin (2003) writes that, "through listening and discussion we can raise our awareness of our unconscious modus operandi" (p. 68). However, we are also "asked to see things differently and this involves a great risk and challenge to … feelings of professional and intellectual security" (McLaughlin, 2003, p. 69). Whilst there are inherent risks in being asked to see things differently there is also the possibility of appropriate supports being in place in order "to protect the teacher's professional identify [sic] and sense of competence, but at the same time opens them to challenge and the possibility of learning" (McLaughlin, 2003, p. 74). Tension and 'power issues' may also form part of collaborative relationships albeit in an obscured form because of a desire to be 'nice' (Winkler, 2003).

(f) The affective dimension in research

Research literature acknowledges that emotion is an integral aspect of teaching that is worthy of consideration (Cobb & Mayer, 2000; Confrey, 1995; Hargreaves, 2000; Pool, 1997; Zembylas, 2004). Some recent research is now also acknowledging that emotions have a role in the research process (eg. McLaughlin, 2003). McLaughlin makes a strong case for acknowledging and working with emotion in the research process stating that "emotional blindness will not enhance the research process: it will only drive underground the examination of assumptions" (p.76). As previously alluded to research work is perceived to be values-driven (Chambers, 2003; McLaughlin) including data analysis which is also "a deeply emotional process" (McLaughlin, 2003, p. 72).

Chambers (2003), in his article describing a range of approaches to narrative, also acknowledges the role of 'feelings' in narrative writing. Whilst Chamber's second two approaches (realistic and anecdotal) appear to be more objective, he writes that all three narrative approaches reveal beliefs, values and feelings of the writer. He suggests that "a creative approach to the writing of narratives can promote learning from practice which is affective as well as cognitive" (p. 412). McLaughlin (2003), referring to Abercrombie's work, goes further to suggest that even the 'state' of the perceiver (writer) at any particular moment will influence what is perceived at that moment in time. Winter (2003) also refers to how our psychological states of mind can impact upon data analysis.

McLaughlin (2003) cites authors who have challenged the rational and cognitive models of reflection, and the notion of reason and emotion being opposites. She suggests that reason and emotion are linked and that, "more attention needs to be given to the importance of the role of emotion in understanding and developing the capacities for reflection which facilitate personal, professional and ultimately systems change" (McLaughlin, 2003, p. 66). Whilst acknowledging that emotion is central to reasoning and decision making, McLaughlin also states that too much or too little can hinder the process. McLaughlin also cites Claxton's (2000) writing about the role of intuition as being a part of most knowledge generation. McLaughlin further develops Claxton's idea, writing that intuition has been wrongly associated with "the untrammelled forces of repressed emotion" (p. 66).

McLaughlin (2003) also describes how researchers who are involved in examining their own practice may experience defensive or threatened feelings when their professional and intellectual security is challenged. The ability to endure these feelings is seen to be crucial to the process of appraising one's practice. Elbaz-Luwisch (1997, as cited in Winkler, 2003, p. 399) states that our narratives, "are most instructive and revealing when they are most personal, and often when the owners of the stories are most vulnerable". McLaughlin suggests the practitioner researcher may also encounter confusion, anxiety, exhaustion, frustration, doubt, feelings of inadequacy and a desire for clarity. McLaughlin (2003) goes on to write that one needs to be able to "live with the ambiguity and lack of clarity long enough to formulate a specific focus to research" (p. 70).

Beattie (1995) also refers to the triumphs and setbacks of professional growth and further suggests that narrative ways of knowing teaching and learning can be a difficult process that requires introspection and the reformation of held beliefs.

Reading that emotion is now an acknowledged aspect of research work was encouraging, and I believe was a pivotal part of becoming aware of my previously unrecognised assumptions. I explore this in more depth in chapter 5.

Chapter 5:

Sense-making: linking my writing with theoretical perspectives of narrative inquiry

As previously described, and pointed to in chapter 4, in the process of reading literature about narrative inquiry I began to make links between my writing and the themes that emerged in the literature. In the following two chapters I endeavour to make sense of this research experience by linking the themes that I encountered within the literature with the themes that emerged in my writing in chapters 2 and 3 and in my continued journal reflections. In this chapter I firstly align my work with narrative inquiry and then primarily focus on some aspects of my practice, as a mathematics educator of pre-service teachers, which have emerged during this research process.

(a) Narrative inquiry – a powerful methodology

My work is closely aligned to the process undertaken by students completing masters degrees at the Manchester Metropolitan University (Brown, 2001). These students create "pieces of writing reporting on practice [that] become data within practitioner research inquiry" (p. 226); and through "successive accounts in writing the practitioner can become aware of the changes taking place in himself, in the situation and in his way of describing it" (p. 227). In the third year of the Manchester course students identify a specific theme upon which they then focus their inquiry and eventually produce a dissertation. Parallels with my own work include the production of successive pieces of writing initially culminating in a piece of writing titled 'Mathematical investigations: A journey of professional awareness'. Another parallel can be found in the personally significant changes that have occurred for this writer throughout this process. These are described later. Also, similarly to the Manchester students, I chose a specific theme upon which I focused in the latter stages of my research. While several themes were present in the initial piece of writing a predominant theme was the exploring of mathematical investigations within the context of

pre-service teacher education and within the context of my own personal mathematical learning.

Given that my pieces of writing tell a story about aspects of my professional practice my methodology can simultaneously be aligned with narrative inquiry. Narrative inquiry can be an effective means with which to reflect on and learn about one's practice. This is alluded to by a number of authors (e.g., Rushton, 2001). Together with reading literature written about this way of researching, narrative inquiry has been a very powerful process for my learning. As suggested in the literature review, narrative research can be a journey during which the researcher comes to know more deeply about their life and who they are as a person. This has been the case for me. My learning has been multi-faceted, encompassing not only learning about narrative inquiry as a research methodology, but also thinking about the nature of mathematics, learning about mathematics teaching and learning in general, and more specifically, learning about my own professional practice as a pre-service mathematics educator with particular reference to the use of mathematical investigations.

> So much thinking and learning has taken place whilst completing
> the literature review, not only about narrative inquiry but further
> reflection about mathematics learning in general, and my teaching
> as a mathematics educator. I have formed new 'flavours' about
> what I believe 'learning' in mathematics to entail. (28/06/04)

My deliberations about the nature of mathematics have been ongoing. Eighteen months after beginning to think about this issue I wrote:

> I still struggle with the notion of 'mathematical correctness or
> truth'. How do my newer beliefs that mathematics is about 'doing'
> fit with the existence of mathematical rules and proofs? Is it, that a
> rule or proof only exists in the 'doing' or 'discovering'. That is, it
> does not exist without or outside the mathematician, and thus must
> only be found in the doing? (13/08/04)

Recently, following discussions in class sessions regarding whether or not there could be more than one answer to a mathematical problem, I had a discussion with a colleague regarding the issue of the validity of multiple answers. During this discussion I was able to clarify and refine my ideas. I later wrote:

> My thoughts are that the answers were all correct (referring to a problem in class) given the differing sets of assumptions or interpretations that each person/group made. Usually these interpretations have to be the teacher's and thus teachers (and the children who think in the same way as the teacher) have been the ones who hold the power. Thus, mathematics has not been accessible to many people. Learners justifying their answers with their own reasoning relocates the power to the learner (this does not allow for 'shoddy' thinking however).
>
> I propose that always defining problems so tightly as to create only one correct answer does not lead to useful life or problem-solving skills. Nor does it lead to 'real' learning, rather the 'game' of 'let's guess what the teacher wants us to do/say now', i.e., it is the teacher's interpretation that matters. Thus accepting multiple interpretations supports the learner to 'really' learn, and creates an expectation of learners making sense of contradictions and a range of perspective. (13/08/04)

I now believe that there is not an absolute body of mathematical truth that exists somewhere as a separate body of knowledge. Rather, that one's interpretation and understanding of the context of a mathematical problem will determine the 'truth' that may or may not exist within any given context. Thus, mathematics

primarily lies in the 'doing' rather than existing as a predetermined body of knowledge. In light of Wilber's (1998) challenge, although he was not referring at this point in his book to mathematics in particular, that "all interpretations are not equally valid" (p. 34), I think it important to point out that although I have indicated an acceptance of multiple interpretations of a mathematical problem, I also believe that the learner needs to be able to justify his/her thinking given their particular interpretation of a problem. Thus, accepting multiple interpretations does not become an excuse for an acceptance of any and all ideas. For example, if a problem '9 and 4 more' is given the answer might well be 13, or even 1. The learner may justify that 9 and 4 more is 1 because 4 more hours after 9 o'clock is 1 o'clock. '9 and 4 more' can simultaneously be justified as 13 if we are referring to our base ten number system. If a learner had clarified that they were working in a base ten number system and then tried to explain that 9 and 4 more was 14, I would have difficulty accepting that answer. It would be a valuable starting point however for more investigation and learning.

I think this shift in belief is likely to result in a 'softening' in my communication when teaching mathematics. For example, I can immediately recall a recent incident when talking with a student who had come to a different answer to the one recorded in my planning. Rather than immediately thinking 'this is wrong' or 'you have misinterpreted the question' I asked the student to explain his understanding of the problem, and thus was able to share his understanding and see the 'right-ness' of his answer given his interpretation of the problem. This would support recent writing where I state "learning in mathematics is about being engaged in thinking, experiencing and communicating (although this does not have to be present in all learning) without fear or the pressure to conform to social norms of right and wrong" (15/06/04). This is congruent with Brown's (2003) suggestions that perhaps mathematics needs to become more inclusive where the student's mathematical ideas do not have to be the same as the teacher's.

So, I now wonder, to what degree does my teaching reflect this? Do I leave enough space for thinking to occur or are my questions, time movements within

the lesson, and other ways of being still influenced with my 'old beliefs'? And once again, the questions are raised, that even if this is my new conception of mathematics learning, how does it intersect with my students' conceptions about mathematics learning, and how does it impact on their learning about becoming mathematics teachers?

I think these changes in belief will create more space for my students. Previously I wanted to convince and 'convert' my students to the newer (constructivist/enactivist) possibilities that exist for mathematics teaching and learning. Now, while I still believe in these new possibilities, probably with more passion than before, I have less need to be 'evangelical' in my teaching. I think this may be linked to my ideas about engagement, space and the student's own personal thinking (rather than parroting) being required for learning. Thus, I now perceive my role to be, not one of conversion, but to provide situations where the students can experience, engage, and think for themselves about what is happening for them. As such multiple possibilities exist for what may be learned.

(b) Personal change

The concept of change occurring within narrative inquiry is referred to by various authors (e.g., Clandinin & Connelly, 2000). There is a congruency here with an underlying principle of action research, concerning the issue of 'change'. As Winter (2003, p. 146) writes, "action research actively seeks change as its main resource for learning". This also parallels with the Buddhist concept of "impermanence (annicata) [as] the first and most fundamental characteristic of existence" (Winter, 2003, p. 416).

O'Connell Rust (1999) suggests however that newness is not the point of narrative research, and yet some of my thinking and discoveries (e.g., of personal unknown assumptions) are new, to me, at the very least. While I do not perceive my task to be to create a 'new' me I believe there has been significant change in both my beliefs and my practice, albeit tentative and transitory in some cases, as is evident in the following extract.

As I reflect on some recent teaching I have been doing, in a one-to-one or small group situation, I'm surprised by how entrenched old tendencies and behaviours are. I'm aware of a desire for a student to say the answer I want to hear, be it in agreement with my philosophy of teaching and learning mathematics, or a particular answer that I want to hear in response to a closed mathematical question. I ask the question, "am I endeavouring to have her construct her own ideas or am I imposing my own?" I'm also aware of some change in my practice. I'm less wanting students to conform to 'my way', and am looking for reflective, thoughtful engagement. It seems I am hovering between a 'new' and an 'old'. (30/06/04)

There have been several changes that have occurred for me during the process of narrative inquiry. For example, having previously had concerns regarding whether or not mathematical investigations would result in learning, I have now, for the moment, embraced the use of mathematical investigations as one means with which to hopefully initiate and encourage mathematical learning with our pre-service teachers. There also appears to be change in what I 'expect' within a mathematics lesson. Whereas I previously would have wished for a definitive statement of learning about some mathematical idea, there is more room now for students to explore, conjecture and think, and for these processes to be valued, rather than the sole focus being on attaining a definitive statement of learning about some mathematical idea.

Initial changes occurred as a result of discovering my unconscious assumptions regarding my own personal learning in mathematics (see chapter 2). My beliefs that 'real' mathematicians solve problems quickly, do so on their own and do not get stuck have experienced a shift. Whilst I do not yet quite experience being stuck "as an honourable state" (Collier, 1999, p. 500) there has certainly been change, and because I have personally struggled with these issues

I believe my practice in the classroom, with respect to this issue, is now more congruent with what I have espoused for a number of years. An example of this occurred this year whilst working alongside my first year students working on an algebra investigation. When students became stuck, rather than rushing in to 'relieve' them from being stuck (because I now believe being stuck to be a part of mathematical learning) I was able to stand back if I judged that to be most helpful, or ask questions and/or provide hints if I judged that to be more helpful.

Another belief that has undergone considerable change, that I have already partially described, concerns my thinking about the nature of mathematics. The following writing describes this change:

> Whilst all this learning was going on both with mathematics, and learning about my learning in mathematics I was simultaneously thinking about what mathematics is. In the past when we have asked the question in our meetings "what is mathematics?" I have always wanted to avoid the topic, and was relieved when the conversation turned! I was not able, back then, to articulate what I thought mathematics was. If I had been, I probably would have spoken about mathematics as a body of rules and procedures that existed as a separate entity 'out there'.

> I now can describe what mathematics is for me, at this moment in time. I also believe that this is not a definitive statement of what mathematics is, but rather just my current thinking about what mathematics is. I expect this to continue to change, and this may even do so today in response to questions or comments from you.

> So, there has been a shift in my thinking from a point where mathematics was perceived as a separate body of 'correct'

> mathematical ideas to now viewing mathematics as a sense-making
>
> activity (involving discovering and doing) to do with numbers and
>
> pattern and shape and space. Also, that one's interpretation and
>
> understanding of the context of a mathematical problem will
>
> determine the 'truth' that may or may not exist within any given
>
> context. (22/08/04)

My own personal behaviours, and thoughts about where learning can occur, whilst engaged in a mathematical investigation have also undergone some change. These include being more persistent and being prepared to search much further than I used to; recognising that the 'searching and trialling' part of investigations creates possibilities for learning and thus valuing this aspect of the process much more and giving it more time; and being less resistant to being stuck. I also recognise that I would like to develop the 'habit' of thinking in multiple interpretations because I now perceive this to be helpful in solving problems. Interestingly whilst I am comfortable for others to learn mathematics in a collaborative setting, I still appear, with respect to my own mathematical learning, to retain my past belief that 'real' mathematics learning should occur independently:

> I am comfortable with the notion that mathematics learning can be
>
> aided by the collaboration between students and between teacher
>
> and student. I suspect however that I would still prefer to be able to
>
> independently solve mathematical problems that I encounter.
>
> (22/08/04)

As previously described, another change that I believe has occurred involves the nature of interactions between students and myself.

> When a student asked a question I tended to respond with a
>
> question, listen to the reply and continue to work through the
>
> particular issue being discussed. I believe this resulted in the
>
> student having more space to articulate their thoughts and thinking.

Whilst this is a teaching technique that I have used over the past five years I believe I was more effective at following through on the reply to my question, ie. persuing the topic to an appropriate point to help support the learning (chapter 2).

Another example of change has been the noticing of dualism in my writing, and the beginnings of a move away from this way of viewing the world. My writing suggests, on several occasions, that I believe dualism exists in a situation. For example, "I obviously had considerable concern at this point about whether or not an investigative approach for learning mathematics was effective" (chapter 2). My writing suggests that I perceive an approach to be effective, or not. I now ponder that, in looking at things or moments in time in this dualistic way, that much 'richness' is potentially lost. Mason (2002) suggests that cultivating an inner witness that is mindful and notices what is happening in any one moment without passing judgment (e.g., the dualism, 'effective or not') is a valuable practice to cultivate. Similarly, Wilber (1998) suggests that in adding up all perspectives one can begin to grasp the integral or whole.

It also appears to be easy to adopt the position whereby the 'old' (ie. 'before' change, if viewed in a dualistic way!) is perceived to be a deficit position from which the researcher seeks to free him/herself (Brown & England, 2004). I prefer the perception of research described by Brown and England which suggests narrative research creates stories which help the researcher make sense of the past in the moving towards the future. As such, rather than interpreting my practice as either belonging to the 'old' or 'new' it may be more helpful to just 'notice' (Mason, 2002) what is happening at any moment in time.

(c) Seeking an ideal

My ideas of change itself have also changed. Rather than searching for the 'end' and becoming 'the effective mathematics educator' I now regard change as constant and ongoing. It is a relief to no longer be endeavouring to reach the unattainable position of perfection! Avoiding the seeking of idealism is

supported by Brown (2001) who in describing the work of Elliott (1987, 1993 as cited in Brown, 2001, p. 214) states that "in addressing the changes in practice the central task is … to locate oneself in one's own current practice and build a notion of a way forward". Having rejected idealism one must also be aware however, of avoiding the possibility of accepting mediocre practice under the guise of there being no ideal to strive for.

The process of letting go of the idea of an'ideal' has not been a smooth one. Interestingly, when I was first asked to write about 'what makes an effective mathematics educator?', I had no problem with the idea of setting out what I thought. It certainly seems evident that I had a fixed notion of what constitutes an effective mathematics educator, and what was needed to reach such an ideal (see chapter 2). If I was asked to write in response to the same question now, I'm not sure I could. Whilst I still agree with what I have written and still hold certain beliefs (and no doubt other unknown assumptions) about what makes an effective mathematics educator, I feel much more attuned to the unlimited, complex and changing range of influences and factors operating in a classroom at any one ever-changing moment in time. Such thoughts can be aligned with the writing of Korthagen (2004) who refers to the difficulties of considering what makes a good teacher; to Winter (2003) who refers to impermanence; and to Brown (2001) who refers to the complexities of multiple interpretations.

As well as considering ideals with respect to becoming 'the effective mathematics educator' I have also discovered I hold the notion of 'ideal' with respect to the research process. My writing reveals my struggles to reconcile my usual tendency of thinking in 'rights (ideals) and wrongs' with the idea of simply locating myself in my own current practice and building a notion of a way forward.

> As I set out to write this morning I saw myself thinking, "I will re-read my literature review - I want to do this properly". Also, last week, I was thinking I will re-read Mason's points about 'see-ing' one's writing - again, I want to do this properly". But what is properly? Is intuition not proper? That writing is already in the

> past, and today is fresh with possibilities. So, today I set out to re-
>
> read my narrative, and to see what I see. Not that this is the new,
>
> right way, but merely this is the way I set out to do it in the
>
> moment. There is such freedom in letting go of rightness, proper-
>
> ness, perfectionism. (30/06/04)

And later I wrote:

> As I write I still have and see the idea, that this is not complete,
>
> right, finished, perfect. There is tension between new beliefs and
>
> the recognition of impermanence, and the unlimited nature of
>
> interpretation vs. producing the 'correct' version. (22/08/04)

Looking for an 'ideal' is also apparent in my writing when pondering how mathematics might be learned. In my earlier writing I stated that "I seem to have clearer ideas about how mathematics is learned" (see chapter 2). I now notice that embedded in this writing is the belief that there is a definitive answer or ideal about how mathematics might be learned. I can certainly now articulate my personal ideas about this topic more clearly than in the past, but also see my thinking that there is one 'right' or 'ideal' way to learn mathematics. I do not believe that to be the case, and yet this writing once again points to the tensions embedded in change.

As indicated above I also no longer seek an endpoint, but regard this process as ongoing, with my writing merely marking moments in time, with words approximating the meanings I wish to convey (Brown & Jones, 2001). I have thus adopted the position taken by Brown and England (2004) who have a perception of narrative research as creating "stories that help us for the present, as we make sense of the past, as we nudge to the future" (p. 77). As already referred to, Brown and England also describe the possibility of a supposition of a deficit position from which the researcher seeks to free him/herself. I reject such a notion and regard my beginnings as not being a deficit position. Rather,

this is a journey with the narrative supporting the journey. This belief is evident in how I labelled the headings and sub-headings of my initial narrative (see chapter 2).

Another characteristic of narrative that is described in the literature review is, "the subject is never given at the beginning, but it unfolds as the story is told" (Ricoeur, 1986, as cited in McCormack, 2002, p. 337). I initially found this aspect of narrative research to be very unsettling. I was sure I should have some predetermined goal or 'thing' to be investigating. However, the story has and continues to unfold, despite my worst fears and enduring resistance that it would not. I now trust the process, and perceive it to be a powerful and liberating one. It has certainly been in the ongoing reflection and writing that I have come to more fully understand the journey, with the prediction of an outcome being less important - an idea proposed by McCormack (2002). This also links with the writing of McLaughlin (2003) who suggests that the practitioner researcher needs to be able to, "live with the ambiguity and lack of clarity long enough to formulate a specific focus to research" (p. 70). Having done this I can concur with McLaughlin's suggested feelings of confusion, anxiety, frustration, doubt, feelings of inadequacy and a desire for clarity as the research process unfolds.

(d) Some products of this narrative research, and who benefits?

The results of narrative research are not definitive statements or generalisations about an aspect of that which is being researched (e.g., Brown & Jones, 2001). This statement is congruent with sentiments expressed in my writing regarding the place of mathematical investigations in pre-service teacher training. Towards the end of chapter 2 I wrote:

> Whilst I have undergone valuable personal learning I do not wish to become a crusader advocating that using mathematical investigations will solve all challenges involved in supporting our pre-service teachers to become more skilled at teaching and

learning mathematics. Rather it has been a personal journey that at this point has found mathematical investigations to be a useful learning and teaching tool (chapter 2).

Although definitive statements or generalisations are not made, links with the subsequent readers of the research are nevertheless important. Perhaps readers of this report will be able to witness the process of this story of professional development, and also make connections with it, maybe in terms of their own practice/stories. I have certainly found I have been able to make connections with other stories of professional development that I have read for this research (Jaberg, Lubinski & Yazujian, 2002; Tzur, 2001).

As indicated in chapter 4 the question of who benefits from narrative research is complex. It is my belief that I have benefited from this process in many ways. These include learning more about some aspects of my professional life; namely, learning about narrative inquiry as a research methodology, considering more deeply the nature of mathematics, what learning mathematics can involve, and about mathematical investigations. However, as Francis (1995) suggests, reflection of one's teaching is only worthwhile if it has a positive impact on one's students. Like Francis, I would ultimately wish for the pre-service student teachers to benefit from this research, and in turn for the children these pre-service teachers will eventually teach to also benefit.

It would seem from the interviews that I held with four students (see chapter 3) that this may be the case for at least these four students. It also gave me great pleasure when recently one of the four student teachers whom I interviewed returned to see me after her second year practicum to show me the results of the children's work with whom she had worked. This work was the culmination of the children working on an extended algebra investigation. The student teacher was very enthusiastic about the learning the children had achieved, and the enthusiasm with which they had approached their mathematics lessons.

Chapter 6:

More sense-making: linking my writing with theoretical perspectives of narrative inquiry

In this chapter I continue to endeavour to make sense of this research experience by linking the themes that I encountered within the literature with the themes that emerged in my writing in chapters 2 and 3, and in my continued journal reflections. I now look more closely at various aspects of narrative inquiry in terms of my research experience. These aspects include a further discussion of the challenges of attempting to capture the essence of a complex situation; the impact of emotion in this research; examining my writing in terms of narrative approaches; and lastly revisiting the issues of validity and authenticity.

(a) Catching complexity:

(i) Multiple interpretations

As described in the literature review, multiple perspectives or interpretations of narrative are always possible (e.g., Brown & England, 2004). This is so not only when writing narrative but firstly in one's experience and interpretation of any event, and also in recalling an event. The possibility of multiple interpretations is an awareness I had very early on in my research period (see chapter 2). My awareness that interpretations might also be affected by one's mood at the time of writing is congruent with the suggestions made by McLaughlin (2003).

The idea of multiple interpretations appears in Wilber's (1998) description of three core assumptions of postmodernism. These are that "reality is not in all ways pregiven, but in some significant ways is a construction, an

interpretation…; meaning is context-dependent…; cognition must therefore privilege no single perspective (this is called 'intergral-aperspectival')" (p. 121). Like Winter (2002) who suggests that there is no one single perspective, Wilber writes, "any single perspective is likely to be partial, limited, perhaps even distorted, and only by taking multiple perspectives and multiple contexts can the knowledge quest be fruitfully advanced" (p. 131). Thus there is a need to consider many perspectives thereby attempting to grasp the integral or whole (Wilber). Different perspectives then, open up possibilities for engaging in the process of reflection because of the likelihood of sometimes comparable or contrasting points of view (Chambers, 2003). One such example I have experienced within my research is when a colleague,

> … used the phrase "soften" with respect to communication. Well, I
>
> think I'm as soft as marshmallow and as 'inclusive as' so it is
>
> rather a shock to consider that MAYBE I am being exclusive or
>
> needing to soften. BUT does that compromise mathematical
>
> understandings? (17/03/03)

My writing clearly reveals the fact that this was a contrasting point of view to my own. As already described, this became a pivotal point for further reflection as I pondered at length how one might or whether one should 'soften' one's communication when teaching mathematics. My early struggles with this, linked to my debate about the notion of mathematical correctness, are initially recorded in chapter 2, and were still present in my writing 18 months later.

Winter (2003) suggests that seeking out contradictions when reflecting on one's writing is also part of the process of narrative inquiry. I have experienced discovering such a contradiction. When I initially pondered the existence or otherwise of a body of mathematical correctness I stated that "yes, there is a body of mathematical correctness" but later do not want the students to regard me as "a source of rightness" (see chapter 2). It seems that on the one hand I believed in 'correctness' or 'rightness' and on the other hand I was wanting students to explore their own understandings or interpretations of a particular mathematical context rather than looking for a 'right' answer. This 'contradiction' has been an important 'point of reflection' resulting in a changed

belief regarding this aspect of mathematics, and perhaps a greater congruence in what I believe, do and say in my teaching. For example, because I am now more fully aware that multiple interpretations can be made (within a mathematical investigation or problem), I am more open to listening carefully to my student's interpretations when discussing a mathematical problem.

The existence of multiple interpretations (speaking generally, rather than referring to multiple mathematical interpretations) is linked to the inherent subjectivity of narrative research. I raised this issue in my writing midway through last year (see chapter 2). What may or may not be apparent in my writing at this time were feelings of discomfort about the inherent subjectivity linked with the notion of multiple perspectives and/or interpretations. I wrote, "so, this is all subjective" (15/08/03). Underlying this writing was an unarticulated concern regarding the subsequent validity of such work. This is likely to stem from my educational background (completing a Bachelor of Science degree, and later teaching science and mathematics) which was embedded in scientific research. As described in the literature review, a premise of positivist scientific and empirical analytical models of research is that research must meet criteria such as being objective, value-free, scientific and therefore valid (Sikes, 2002, in Clough, 2002). Hence my discomfort at that time with subjective research.

However, having pondered that subjectivity is inescapable in research such as this, and also having read that such subjectivity is not necessarily perceived to be a problem (e.g., Winkler, 2003) I now feel willing to honour my interpretations as valid, authentic and trustworthy, with the provisos that this is but one account of my professional situation, and the simultaneous acknowledgement of the possibility of multiple interpretations. Such provisos begin to counter the narcissistic possibility of one's ego claiming a particular view of reality for which there is no supporting evidence (Wilber, 1998). For example, it would be possible to claim a single particular interpretation of what the students I interviewed said. I could have claimed that questions that encourage students to explore, connect and think are supportive and effective. However, alternative interpretations need to also be considered, particularly given another student's perspective of questions as the 'playing of guessing

games'. That interpretation might be that questions need to be more judiciously used, along with sensitive observations being made of the learner, particularly when the learner is new to participating in mathematical investigations.

(ii) More on the context of this research

A second aspect regarding narrative that I will now consider concerns the 'context' of narrative inquiry. Various authors point to the necessity of clearly stating the context of narrative research (e.g., Adler, 1993). Simultaneously there is recognition that often we are unaware of assumptions and judgments operating within any particular context (e.g., Wilber, 1998).

Within this research I have endeavoured to describe the context within which I work as a pre-service educator, both in terms of describing my current professional situation and also referring to past experiences and influences. Some of the detail referring more precisely to the current pre-service context within which I work is found in chapter 2. I think it pertinent at this stage to also include a 'position' statement that outlines the philosophy of the team of mathematics educators with whom I work. The team at the time of these discussions consisted of six full-time mathematics educators and myself (a part-time mathematics educator).

This statement was written by one member of our team and whilst never formally adopted or ratified I believe it summarises the main points of discussions we shared whilst meeting weekly to begin to articulate our philosophy and revise what and how we teach in our two papers that our pre-service undergraduate students take in their teacher training in mathematics education. The statement begins to give an indication of the context within which our team operates.

Key issues with core plan:

The course is designed to enable students to engage in a broad

range of issues relating to the teaching of mathematics in schools.

A particular focus entails building an environment in which the

students can become confident with their own content knowledge

of mathematics and develop a positive attitude to the subject. This

however is not a mathematics received intact from the gods. Rather

mathematics is seen as alive and still in the process of its own

creation. So viewed mathematics is a subject available to all where

everyone's perspective on what it might be is taken in to account.

In this way mathematics will be built as a highly inclusive activity

with responsibilities to a diverse range of participants. Thus the

course will be attending to pedagogical issues relating to this

evolving subject and will enable students to build effective

approaches to facilitating learning with children across the primary

and intermediate age range. More broadly issues relating to public

understandings of mathematics will be tackled and attention will

be given to how students develop the capacity to be critical

participants in curriculum initiatives. Moreover, these objectives

will be achieved within a framework designed to enable students to

become professional in a school context equipped to take

responsibility for their own professional development needs. The

particular approach to be taken here will focus on the student

developing a reflective attitude to their teaching studies to be built

through a research led engagement with everyday teaching issues.

A core feature of this reflective attitude will be a specific attention

to how mathematics is understood and further developed in the

context of their professional life and in the lives of the children they teach. Through this route students will develop a critical capability necessary to make possible the ongoing adjustments crucial to contemporary life.

Method:

The initial stages of the course are designed to immerse new students in mathematical activity designed to develop the student's own understanding and to foster a positive attitude to the subject. A core issue will be to share alternative perspectives and show how these perspectives combine to create a rich conception of mathematics valid to everyone's needs. Centred around issues relating to the effective learning of numeracy as a foundation stone of mathematics a broad range of generic issues will be tackled, such as:

1. Mathematics - the nature of mathematics, mathematical experiences, affective responses to mathematics, primary teachers as mathematicians.

2. Children as mathematicians - exploring children's understandings and knowledge about mathematics.

3. Critical analysis of learning theories and frameworks.

4. Investigating mathematical ideas - one or two significant investigations into a mathematical problem, concept or idea.

5. Mathematical pedagogy - preparing, teaching and evaluating mathematics.

The second year of the course offers a broader perspective whilst showing how core issues tackled in the first year support the learning of mathematics across a broad range of mathematical curriculum concerns. Particular attention will be given to geometry, algebra and probability and statistics. Meanwhile first year work in numeracy will be linked to topics such as fractions, decimals and measurement. The overall intention will be to develop core issues relating to the learning and assessing of mathematics by children whilst ensuring that these concerns are understood across a range of mathematical areas. This work will be supported by guided introduction to research literature to assist the student in becoming aware of how their own teaching concerns are addressed in the broader arena. The course however cannot hope to be comprehensive in the limited time available. Rather it is designed on the premise that students can be equipped to become autonomous learners able to take responsibility for their own professional development in mathematics.

The two papers, referred to throughout this research that our pre-service undergraduate students take in their teacher education in mathematics education comprise a total of 75 hours of contact teaching time with an expected further 200 hours of independent study and research. This time is divided between two papers, the first of which is held in the second semester (two semesters per year) of the first year (75 hours), and the first semester in the second year (200 hours).

(iii) Past influences on context

As previously described my scientific background had a bearing on what I initially regarded as valid within this research context. I believe such past

experiences influence my current research in a myriad of definable and unknown ways. This is an idea supported by Winkler (2003), and Brown and England (2004). Another definable example that I perceive to influence my research is my involvement over the past eight years with Buddhist teachings and practice. Had this not been the case I think it would be less likely that I would have 'noticed' and connected with the writing of Winter (2003) and Mason (2002). Thus, I believe my links with Buddhist teachings and practice have influenced my noticing and sense-making of these ideas. For example, Winter (2003) refers to the central importance of change within action research. Given my awareness of the Buddhist principle of impermanence this concept is one that I have some understanding of and connection with, and it subsequently appears in my research. This raises a question similar to one asked by Brown and England raising the issue of what ideas (embedded within the literature and my writing) have not been noticed?

(iv) Influences on perception

As previously indicated we are often unaware of our assumptions and judgements (McLaughlin, 2003). Indeed a major benefit that I have experienced during the course of this research project is the uncovering of at least three such unconscious assumptions and judgments. These are described in chapter 2 and include beliefs that 'real' mathematicians solve problems quickly, do so on their own and do not get stuck. As previously discussed in chapter 5 all of these unconscious assumptions were contrary to what I espoused in the classroom.

I continue to become more aware of some of the assumptions under which I operate, including identifying the powerful nature and influence of the context within which I work. For example, my interpretations of collegial discussions regarding learning theory over the past five and a half years have left me with an impression that behaviourist learning theories (Barker, 2001) where concepts to be taught are broken down into manageable parts are less than satisfactory when we consider learning in mathematics. Rather, there has been a preference for social constructivist and/or enactivist theories of learning to underpin our mode of teaching. I was unaware how much I had taken this position for granted

when a colleague stated during a recent meeting that all theories have their good points. This challenged me to realise how much I had uncritically adopted the position of promoting the more recent theories of learning.

(v) Collegiality and collaboration

As alluded to above, collegial liaison is a feature of my professional work scene, and has been so throughout the entire period of the research. Whilst my research was primarily individually conducted, collegial liaison certainly influences my thinking and reflections. Interactions with my supervisor have also been influential. Collegial and supervisory liaison thus form part of the context of my research.

Most collegial discussion occurs informally or at planning meetings where ideas are shared for teaching our pre-service students. One such incident occurred recently, where during an informal discussion about an issue that had surfaced while teaching, ideas were exchanged and clarified. Such discussion allowed me to articulate some loosely-formed ideas and questions, and continue my deliberations about the issue of mathematical truth (see chapter 5).

While there has not been formal collaboration in this research, the collegial and supervisory relationships that are an important and integral aspect of my thinking, might be likened to aspects of collaboration. Winkler (2003) indicates such collaboration can have a variety of potential effects. My experience re: 'softness in mathematical communication' (see chapter 2) supports McLaughlin's (2003) statement that, "through listening and discussion we can raise our awareness of our unconscious modus operandi" (p. 68). Through hearing my colleague talk about 'softness in mathematical communication' I was led into sustained reflection where some of my fundamental beliefs about the nature of mathematics were challenged. McLaughlin also refers to the risks and challenges to one's sense of professional and intellectual security that occur when "asked to see things differently" (p. 69). As previously described the degree of 'challenge' I experienced is evident in my writing.

Another aspect of collaboration includes tension and power issues (Winkler, 2003). I wonder to what extent such issues form a part of my research context, both in collegial and supervisory relationships and also in the relationships between my students and I. Approximately fifteen months after I began my research I wrote:

> I hear [my supervisor] in my writing, and even in my speech in
> matters other than academic. Once again, I am aware of the
> powerful impact of the social context within which I operate. Do I
> need to think and question outside what may be a 'new square'?
> (14/05/04)

I believe that any team develops ways of communicating that will contain unconscious (or not) assumptions and judgments, and are likely to share or develop similar ideas. These are at least partially evident in the statement of philosophy (see chapter 6). Beyond this of course also exist institutional beliefs and ways of operating, and the influences of current trends in mathematics education discourse. Thus, my professional development does not occur in isolation.

Power relations are bound to be evident within my interactions with students. When I have the power to assess their work and award a pass or fail, I think it is likely that communication between the student and I might be less than full and honest. As such, this may change what students feel able to share within the interviews I conducted for this research.

(vi) The influence of literature in this writing

Another influence on the context of this research is the literature I read during this narrative inquiry. I found reading about narrative inquiry as a research methodology to be a pivotal and rewarding experience. Learning that narrative is regarded as a respected form of research was affirming.

Realising that the role of emotion in the research process is beginning to be acknowledged (e.g., McLaughlin, 2003) also influenced the 'context' of the research. For example, the nature of my writing changed following my discovery that the role of emotion in research is acknowledged. This is discussed in more detail below.

Reading for and writing chapter 4 has also given me the confidence to begin believing in the validity of my own experience. Not only am I more "conscious of the influence of my own personal experience" (chapter 2), I am more willing to acknowledge and honour the validity of my experience. In the past I believe I have had a tendency to negate my own personal experience and defer to others. In contrast I now recognise myself as one of the "humans [that] experience the world (Clandinin & Connelly, 2000, p. 2). As suggested in chapter 4, narrative research has, for me, been a journey during which I have come to know more deeply about my life and who I am as a person.

(b) The affective dimension in this research

My initial ideas about what was appropriate to write about in this narrative inquiry influenced the content of my earlier writing. McLaughlin (2003) suggests the ignoring of emotion can have a negative impact on the examination of assumptions, and I wonder if this occurred in the early stages of my writing. Having read literature that acknowledges the role of emotion in research (e.g., McLaughlin, 2003) I now write more fully, and with reference to the emotions encountered in this research process. Soon after reading this literature I wrote the following after re-reading my writing about 'softness in mathematics communication':

> I also remember however, the feelings that I did not write about. These included feeling affronted and defensive ("you mean, I haven't got this right"). At that time I was certainly unwilling to record the depth and/or true nature of my experience. I believe this was to protect myself, and also stemming from a belief that surely,

> it is unhelpful, or at the very least, inappropriate, to record such
>
> matters or feelings (30/06/04).

I also wrote, "this process is 'thick' with emotion, ranging from despair to elation" (30/06/04).

So, I currently feel willing to reveal more of the 'true' nature of what is happening in this narrative study. In doing so, I recognise that I now operate from a belief that this is likely to be a helpful part of my research, and most importantly, my learning. Such a notion is supported by McLaughlin (2003) who suggests that there is a link between emotion, reflection and thus "personal, professional and ultimately systems change" (p. 66).

It is also recognised that too much emotion can hinder reasoning and decision making (McLaughlin, 2003). I have certainly experienced moments during this research of feeling overwhelmed (too much emotion), and this had the effect of temporarily impeding progress. One such moment occurred while reading literature. I found reading and encountering new ideas to be both exciting, but also challenging and when struggling to understand, immensely frustrating.

My work on mathematical investigations was also an emotional process. Words such as rapt, courage, frustration, anxiety and discomfort appear throughout my writing (see chapter 2). The emotional aspect of what I was experiencing was a vital and integral part of my learning process. Without this I think it unlikely that I would have uncovered some of my previously unrecognised assumptions. For example, without the feeling of frustration I would have been unlikely to have searched on the internet, which led to discovering and pondering about the place of collaboration in mathematical learning. This also links to Elbaz-Luwisch's (cited in Winkler, 2003) suggestion that our narratives are most instructive and revealing when personal and revealing vulnerabilities. There have certainly been times during this research when I have experienced this vulnerability.

(c) Narrative approaches in this research

I perceive a number of narrative techniques within my writing. There has been an element of 'realistic' narrative (Chambers, 2003) where I have attempted to provide a replication of a scene including the use of direct quotes from students. The description of the scene is found in my original journal whilst some of the student's quotes are to be found in chapter 2. I have made little use of the anecdotal technique (Chambers, 2003), although when a colleague visited me and wrote notes these could be perceived to be an anecdotal account of what was happening, upon which I was later able to reflect.

Much of my writing could be seen to be reflection-on-action (Schon, 1983). Interestingly my earlier writing suggested I did not perceive my voice to belong within the research process even though the research was centred upon my professional practice. For example, there is little spontaneous writing (where the role of the writer is included) although I simultaneously realised and was aware that my beliefs and values affected my writing which is consistent with Chamber's (2003) realistic narrative technique.

Concerns similar to those raised by Chambers (2003) regarding the ethical implications of spontaneous narrative excluding other participants, are reflected in my attempts to hear from my students via audio-taping during classes and semi-formal interviewing at two different points within the research process. Thus, while I now recognise myself and my experiences as valid, I am also aware of the inherent dangers of not including or considering the perspectives of others central to the research, most notably my students.

As described in chapter 4, reflection is an integral part of narrative inquiry. I would certainly concur with Brown (2001), Chambers (2003) and Johnson (2002) who suggest that both the process of writing narrative and reflecting back on one's narrative creates new understandings and knowledge. For example, whilst writing in the early stages I learned that I believed in mathematics as truths that existed separately to those engaged in the mathematics. An example of some new understandings that have developed

whilst re-reading was noticing the contradiction within my writing regarding the nature of mathematics. Through re-reading I was led into reflection and new understandings about the nature of mathematics.

Another example that highlights the value of reflecting on one's writing is my discovery of the possibility of a persisting belief (at that point in my research) in a behaviourist theory of learning, although I had been espousing the merits of social constructivism for five years and enactivism more recently. This is evident in my concerns regarding whether or not using an investigative approach, which might be seen to be more aligned with social constructivist or enactivist learning theories, would 'really' result in mathematical learning. Unknowingly, I appeared to have more faith in the more behaviourist theories of learning.

Korthagen (2004) proposes an 'onion' model whereby a series of layers provide a possible structure for the content of reflection. These layers include a consideration of the teaching environment, teacher and student behaviours, teacher competencies, beliefs, professional identity and mission, with mission being the innermost layer or core. Several of these layers are evident within my writing. The first layer that I can identify links to 'teacher and student behaviours'. For example, I was initially concerned with my own (teacher) behaviour regarding taking too much responsibility for student learning. Not much later in my writing, the fourth theme that I identified within my initial data, was my concern regarding student behaviours whilst participating in mathematical investigations. The next layer regarding teacher competencies also appears in my writing. I initially write of my own personal insecurities regarding my own level of mathematical knowledge, and later note a growing confidence as a result of undertaking the two mathematical investigations.

Writing and reflecting upon my beliefs, Korthagen's next layer after teacher competencies, has occurred throughout this narrative. Some of these beliefs have been explicit whilst others have been discovered, and some have changed. For example, I explicitly identify my earlier beliefs about what makes an effective mathematics educator; and in the process of undertaking mathematical investigations discover previously unknown assumptions/beliefs. Changes in

beliefs have also taken place. For example, I now believe that mathematical investigations can result in mathematical learning.

As described in the literature review, Korthagen (2004) refers to a shift within narrative research, one that places an increasing emphasis being placed upon beliefs that people hold. This has certainly been true, and useful, within my narrative experience. Discovering my beliefs about my own personal learning in mathematics, and my changing beliefs regarding the nature of mathematics have been integral to my subsequent considerations about learning and teaching mathematics. For example, finding out that I subconsciously believed that 'real' mathematicians work alone, together with initially thinking of mathematics as a separate body of knowledge, have been pivotal to my changing thoughts about the roles of collaborative work and multiple interpretations in mathematical learning.

Korthagen (2004) writes that beliefs are linked to one's professional identity, which is a consideration of the innermost level (referred to as the mission level) of the 'onion' reflection model. Whilst little theoretical research has yet focussed on professional identity and mission (Korthagen), I think it is likely from my experience that one's beliefs and sense of mission are intimately linked. Whilst not stated explicitly in my writing, part of my 'mission' is care and concern for my student's and subsequently children's learning; and my beliefs form an inextricable and integral, if not always congruent, part of how I carry out this 'mission'.

Reflection is a complex process, and one that I believe that I have only just begun to know more about. The current scope of this piece of research precludes a more in-depth examination of reflection and it's role in narrative inquiry. However, I am aware that this is an area that could be explored much more fully. Mason (2002) (see also Mason, 2003; Schoenfeld, 2003), who offers a variety of techniques one might employ in endeavours to become more skilled at noticing, would be a valuable starting point for further considerations.

(d) Issues of validity and authenticity again

As mentioned in chapter 5 the products of practitioner research are not statements of practical implication that can be generalised to other situations. Nevertheless there is a simultaneous belief that such research can be judged as credible, transferable and valid, and that this is ultimately dependent upon the quality of the final text (Winkler, 2003). Whilst I would agree that the quality of the final text is important I wonder how much credibility, transferability and validity can be suggested when considered in conjunction with Brown's (2001) idea that "any accounts… reflect the society from which they [writers] come and have, built within the language itself, layers of assumptions endemic in that society's view of the world" (p. 217). Beattie (1995) and McLaughlin (2003) also write in a similar vein. McLaughlin states that "how we see what we see is learned from personal and cultural experiences" (p. 67).   Because of this I wonder if it is possible to be certain of credibility, transferability and validity even if a text is well-written given the reader and writer may well come from different 'personal and cultural experiences'.

As already described in chapter 5, it is a premise of narrative research that readers may make connections with the research in terms of their own practice and/or stories. Such a connection may depend on the 'trustworthiness' of the narrative (Winter, 2002). Winter goes onto suggest that emphasising the dialectical reflexivity of narrative is a way of resolving the dilemmas posed when considering authenticity; i.e., being aware that there is not a single, correct perspective and that any analysis is tentative and cannot be regarded as "'accurate' but merely as trustworthy'" (p. 148). As such, I would once again point to the multiple interpretations that exist at every level of such research, and that this is merely one account written at this moment in time.

Another issue concerning the validity and authenticity of narrative research is the "seductive power of authoring lives the way we want them to be" (Winkler, 2003, p. 393), and creating an illusion of a coherent purpose when in fact none exists (Winkler). And yet, narrative writers are also challenged with the importance of the quality of the final text, as described above. These

demands/issues might appear to be contradictory. Brown and England (in press) describe how, "life resists being depicted un-problematically in research constructions as a singular or tangible entity". I have certainly struggled with telling a story that appears to be linear in it's unfolding, and yet this has not been the case. Attempting to capture the complexities of this snapshot of my professional life has, at the very least, been problematic. These difficulties are revealed in at least two different stages in my writing:

> My mind absolutely races when writing, and has also done so whilst writing the literature review. So many ideas pop up, and multiple links wait to be made. I develop an aura of desperation wanting to catch it all! Shall I tell this here or here?'. How do I begin to capture the 'full' picture, and which interpretation of the picture?
>
> During the process of writing I once again became aware of my tendency to want to 'wrap ideas up into a neat package that can subsequently be labelled, and regarded as complete'. (30/06/04)

Questions remain as to whether I have authored my life the way I want it to be. I partially query this however, as I certainly had no intention of 'laying bare' some of the issues that have surfaced during the process of this narrative inquiry (e.g., my feelings of insecurity regarding the depth of my own mathematical knowledge). With regard to an illusion of a coherent purpose, I do think that some coherent purpose has been reached (e.g., experiencing and reflecting on issues pertaining to mathematical investigations both as a learner and pre-service teacher educator). Perhaps the challenge is to not become focussed on what may appear to be a 'purpose' with the inherent risk of failing to notice other emerging 'purposes'.

Chapter 7:
Conclusion

There have been two main foci of learning in this research process. Narrative inquiry as a qualitative research methodology has been explored, and a range of issues pertinent to this pre-service teacher's teaching and learning of mathematics have emerged and been reflected upon. The primary focus of these reflections has centred on the use of mathematical investigations for teaching pre-service primary teachers; and participating in mathematical investigations as a learner.

Many authors (e.g., Clandinin & Connelly, 2000) cite narrative inquiry as a powerful means with which learners can reflect on and develop their professional practice, and this has certainly been the case for this narrative inquirer. Narrative is a form of story-telling. Beattie (1995) refers to narrative as a way in which teachers "find voices to tell their own stories" (p. 59). This research then is one telling of my own professional story that has taken place over a period of twenty months.

I began to read more deeply about narrative as a research methodology part way through this research process. Doing so was empowering as I came to understand that writing and reflecting on one's writing is a recognised means by which to examine one's professional life more carefully. This growing understanding of narrative inquiry subsequently provided me with a framework with which to make sense of the personal journey that I was undertaking with respect to my professional awareness.

Various characteristics about narrative inquiry were particularly important to

me. One of these was coming to understand that in this research methodology the focus of research can emerge as the process unfolds (McCormack, 2002). Initially this was an unsettling aspect, and is probably linked to my previous experiences with more traditional scientific models of research. Another significant learning was coming to accept my place within the research process. In narrative inquiry the researcher is necessarily and inextricably located within the research (Brown, 2001), and there is never only one story to be told (Wilber, 1998). Indeed the idea of multiple perspectives is embraced and seen to add richness to the process. Once again, these aspects did not initially 'sit' well with me. However, in learning more about narrative inquiry together with reading about the role of emotion within research (e.g., McLaughlin, 2003), I moved to a place where I believed my experience was valid and worthy of consideration. As a consequence in writing the later chapters (4-8) I found it easier to write from the position of 'I', and was also more willing to embrace and reflect on the 'whole' experience rather than only those parts I initially regarded as 'appropriate'.

As mentioned above, some recent research is now acknowledging that emotions have a role in the research process (e.g., McLaughlin, 2003). Rather than viewing reason and emotion as opposites McLaughlin (2003) suggests that they are linked, and that, "more attention needs to be given to the importance of the role of emotion in understanding and developing the capacities for reflection which facilitate personal, professional and ultimately systems change" (p. 66). I believe that my initial writing was influenced by my perception that emotion did not belong in research. Since reading literature to the contrary my writing has changed to include the emotional aspects encountered. McLaughlin also states that, "emotional blindness will not enhance the research process: it will only drive underground the examination of assumptions" (p.76). This is congruent with my experience of participating in mathematical investigations as a learner. I believe that the emotional aspect of participating in a mathematical investigation

led to learning about previously unrecognised assumptions.

A central tenet of narrative inquiry, and of action research in general, is one of 'change' (Clandinin & Connelly, 2000). This action research principle (i.e., research resulting in change) also parallels the Buddhist concept of impermanence (Winter, 2003). As already mentioned, a number of issues pertaining to the learning and teaching of mathematics emerged during this narrative inquiry. Many of these illustrate this concept of change that is regarded as an integral part of narrative research.

One such change came about through prolonged reflection about the nature of mathematics. The noticing of contradictions (Winter, 2003) within my writing, and the offering of a contrasting interpretation (Chambers, 2003) led to this reflection, including a consideration of what it means to 'do' mathematics. There has been a shift in my thinking away from a point where mathematics was subconsciously perceived as a separate body of 'correct' mathematical ideas, and where the emphasis when doing mathematics was on attaining the 'correct' answer. I now view mathematics to be more of a sense-making activity involving discovering, doing and maybe communicating, to do with numbers, patterns, shape and space. Thus, I now perceive mathematics to primarily be found in the 'doing' rather than existing as a predetermined body of knowledge. I also believe that one's interpretation and understanding of the context of a mathematical problem will determine the 'truth' that may or may not exist within any given context. Therefore, it is important to explore these interpretations within the learning process. However, Wilber's (1998) challenge, that not all interpretations are equal, perhaps suggests that accepting multiple interpretations of a mathematical problem should not become an excuse for an acceptance of any and all ideas.

Discovering those things taken for granted is also an integral part of narrative

research (e.g., Mason, 2002). This has occurred for me not only with my beliefs about the nature of mathematics, but also with regard to the learning of mathematics. I discovered that these beliefs included ideas that 'real' mathematicians solve problems quickly, do so on their own and do not get stuck. Surprisingly, all of these subconscious assumptions were contrary to what I espoused in the classroom. Because I have now personally experienced struggling with these issues I believe this aspect of my teaching practice is now more congruent with what I have espoused for a number of years. These changes in teaching include using mathematical investigations for one of our teaching approaches with the belief that students can effectively learn mathematical ideas using this approach; an acceptance that this may involve periods of being 'stuck' and that this does not mean that I need to immediately support the students in becoming 'unstuck'; more in-depth interactions, including questioning, to support this mathematical learning; and an acceptance that mathematics can be learned by people working in a collaborative manner. Interestingly, I still perceive a personal resistance to working collaboratively in a mathematical context. My long-held belief, which can probably be explained by my own school experiences, that a 'real' mathematician solves problems alone appears to be resistant to change.

My own personal behaviours, and thoughts about where learning can occur, whilst engaged in a mathematical investigation have also undergone some change. These include personally being more persistent and being prepared to search much further than I used to; recognising that the 'searching and trialling' part of investigations creates possibilities for learning and thus valuing this aspect of the process much more and giving it more time; and personally being less resistant to being stuck. I also recognise that I would like to develop the 'habit' of thinking in multiple interpretations because I now perceive this to be helpful in solving problems. There also appears to be change in what I 'expect' within a mathematics lesson. Whereas I previously would have wished for a definitive

statement of learning about some mathematical idea, I perceive there is more room now for students to explore, conjecture and think, and for these processes to be valued, rather than the sole focus being on a definitive statement of learning about some mathematical idea.

Whilst I now recognise myself and my experiences as valid, I am also aware of the inherent dangers of not including or considering the perspectives of others central to the research, most notably my students. I therefore interviewed four students as part of this inquiry and gained valuable insights in doing so. These students' experiences of an investigative approach appeared to be mainly, but not uncritically, positive. Whilst some of their insights resonated with my experiences, others were different and I was able to gain new perspectives about this learning and teaching approach.

The first of these is the possibility of more deliberately discussing student beliefs about the learning and nature of mathematics in an effort to acknowledge these in a more specific and open manner. Hopefully this would simultaneously provide encouragement to continue with their efforts in mathematical investigations. The second possibility involves discussing the place of more traditional skills teaching that may occur within this approach. The third, arising from a concern from one interviewee that some students did not grasp the mathematical concepts being explored, is to continue to carefully observe and consider the learning of mathematics using an investigative approach. Being open to differing forms of writing (part of the investigative process) may also be necessary to cater for those students who find writing interferes with their mathematical thinking. For example, accepting writing at the end of an investigation as an option to the current requirement of writing throughout the investigative process.

It was encouraging to also note that, in the main, the students had positive

experiences with this approach. Of particular note was the deeper learning that appeared to occur. The students spoke of truly coming to understand particular mathematical concepts for the first time. They also perceived their mathematical behaviours to have changed in a positive manner. For example, ongoing reflection and a more thorough approach were described. An openness to the idea of multiple solution strategies also developed; and all four students expressed an interest in trialling this approach once teaching in their own classrooms. At least one of these students has gone on to do so during a teaching practicum, experiencing a positive response from the children with whom she worked.

Narrative inquiry by its very nature is subjective. However, such subjectivity is not perceived to be a problem (e.g., Brown, 2001). Winter (2002) suggests that rather than asking the question, " 'is this narrative true?' " (p. 145), it may be more helpful to ask the question,

> … 'is this narrative shaped and moulded in such a way that we feel it is trustworthy, i.e. does it persuade us that we might helpfully rely on the insights it presents about that particular situation to guide our thinking about other situations?' (p.145).

It is my hope that this narrative is indeed shaped and moulded in a 'trustworthy' way.

Whilst the concept of change might be seen to imply an initial deficit position, Brown and England (2004) reject such a notion. Instead an alternative model of emancipatory practitioner research is offered. Rather than seeking resolution or an end point the research process is regarded as the building of a narrative layer that supports and grows alongside the writer's life as it occurs (Brown & Jones, 2001). Thus there is not a seeking of perfection or an ideal, but a greater awareness of one's professional practice with the likelihood of change occurring.

This has been my experience, and this philosophy is one that appeals to me, and is evident within my writing in later chapters.

Therefore the results of narrative research are not definitive statements or generalisations about an aspect of that which is being researched (e.g., Winkler, 2003). McCormack (2002), like Brown (2001), refers to research not providing a 'map' but allowing "the reader to witness the process of the story's construction and its meaning for the storyteller" (p. 337). The reader of such research has a right, having read the research, to tell stories about how it may connect with their own practice. As such, I do not have a definitive statement about how to be a teacher of pre-service students learning mathematics. Rather I share my story with you, and what it means to me, and then you can, if you wish, tell stories about how it may connect with your own story/practice.

As alluded to above and stated in chapter 2 I have undergone valuable personal learning but do not wish to become a crusader advocating that using mathematical investigations will solve all challenges involved in supporting our pre-service teachers in becoming more skilled at teaching and learning mathematics. Rather it has been a personal journey that at this point has found the use of mathematical investigations to be personally rewarding, and a means with which our pre-service teachers (and the researcher herself) can be encouraged to participate in mathematical learning and reflect upon teaching and learning mathematics.

Questions that remain for me at the end of this research fall into three main categories. Firstly, I am left with a desire to learn more about narrative inquiry, with a focus on the place and nature of reflection within this process. It would also be interesting to read literature about the use, including other's experiences, of mathematical investigations, particularly in the pre-service teacher education setting. And lastly, it would be valuable to learn more about other's perceptions

about the nature of mathematics, looking for links to post-modernist trends in education, and critically analysing this move.

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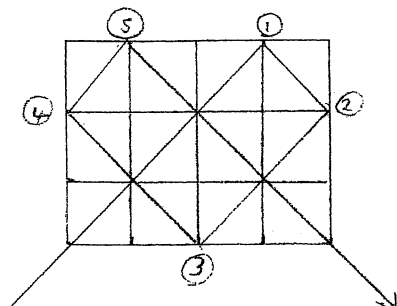
Appendices

Appendix A:

18/09

Experiment with diff. sized tables.

– look for patterns – alter 1 variable @ a time – first alter length of table.

	No. of times it touches a side	point of exit	symm.
3 × 4	5	BR	S
3 × 5	6	TR	S(R)
3 × 6	1	BR	
3 × 7	8	TR	

Notice that 3 × 4 gives a symmetrical pattern
1st line always same length.

3 × 8	9	BR
3 × 9	2	TR
3 × 10	11	BR

Creates a pattern that is repeated (see above).

Can I predict 3 × 9? Predict top right with 2 touches. I'm right. I now predict 3 × 10 will have 10 touches + BR, and 3 × 11 will have 11 touches. No. 3 × 10 → 11 touches + BR   Yes – I'm right!
3 × 11 → 12 touches + TR.   "   "   ".

3 × 11	12	TR

So now I would predict that

3 × 3	0	– oh dear but no. that will be OK

it will just go straight thru' with no touching

3 × 1	2 ✓	TR ✓
3 × 2	3 ✓	BR ✓

There is obviously a pattern – I'm rapt that I spotted it after 3×8. I'm going to do an 'orderly' table to see if I can find an algebraic rule for this pattern – I have doubts bn its obviously not linear but will try!

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3 × 5

3 × 6

3 × 7

3 × 8

3 × 9

3 × 10

3 × 11

3 × 3

3 × 2

3 × 2½

3 × 1

3 × 1½
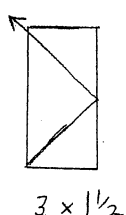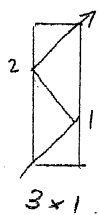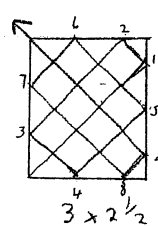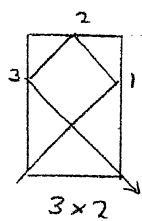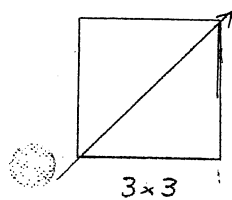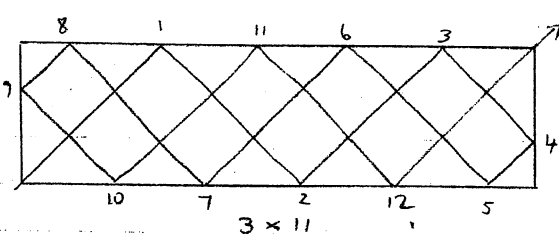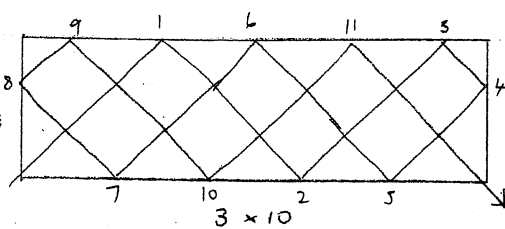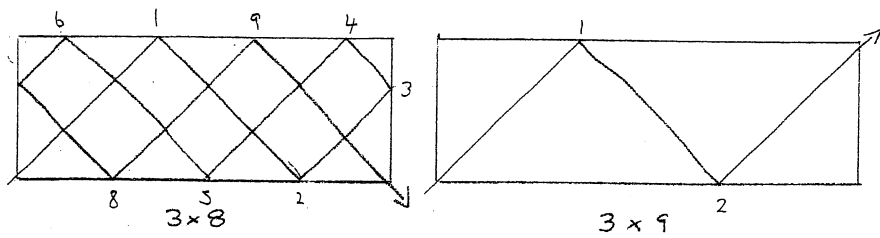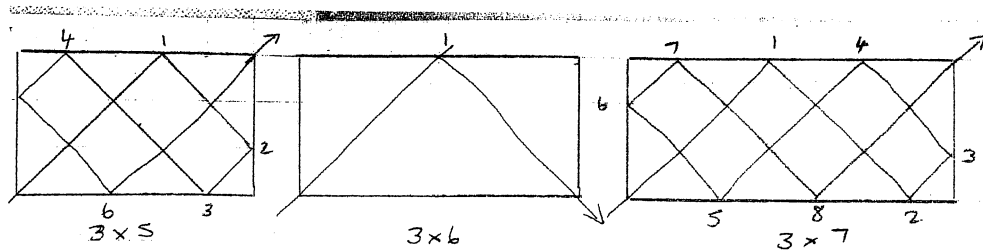
		no. of boxes	no. of lines
4	$\frac{1}{3}$	1 , 2	3
5	$\frac{2}{3}$	1½, 1	2
6	$\frac{3}{3}$	2 , 3	4
		2½, 9	10
	$\frac{3}{3}$	3 , 0	1
7	$\frac{4}{3}$	4 , 5	
8	$\frac{5}{3}$	5 , 6	
9	$\frac{6}{3}$	6 , 1	
		7 , 8	
		8 , 9	
		9 , 2	
		10 , 11	
		11 , 12	

Size of billiard tab.	No. of times touches a side	Pt of exit
3 × 1	2	TR
3 × 2	3	BR
3 × 3	0 (4)	TR
3 × 4	5	BR
3 × 5	6	TR
3 × 6	1 (5)	BR
3 × 7	8	TR
3 × 8	9	BR
3 × 9	2 (6)	TR
3 × 10	11	BR
3 × 11	12	TR

$x \quad y$

1 , 2
2 , 3
3 , 0
4 , 5
5 , 6
6 , 1
7 , 8
8 , 9
9 , 2
10 , 11
11 , 12

x y	diff.
3 , 0	3
6 , 1	5 ⟍ 2
9 , 2	7 ⟍ 2
12, 3	9 ⟍ 2

$y = x + 1$ except for 3 , 6 , 9 ie, multiples of 3.
If x = multiple of 3 ?

Some dim memory is ringing re: '2nd order difference'
Is it a quadratic or ?

x^2 9   36   81
        ↓   ↓   ↓
        0   1   2

(circled note) 12/9 What was I doing? — looking @ diff btw x & y not helpful in this context!

Doesn't look likely.
What if x^{12} ?
I'm familiar with exponential function $y = 2^{x-1}$
What if 2^x ?
It can't be $y = 2^{x-1}$ biz that yields 1, 2 , 4 , 8, 16. etc where x is 1, 2, 3. But here my x are not linear.
So ?

** So a question I have @ this pt is to find out if it is possible to have one expression for this pattern.

2^x $2^3 = 8 - 8 = 0$. $(2^3 - 0)$ $\Big\} y = 2^x - 2^{(x - \underline{?})}$
            $2^6 = 64 - 63 = 1$ $(2^6 - 1)$
            $2^9 = 512 - 510 = 2$ $(2^9 - 2)$

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Have I got something here or not? Frustration. Probably need to ask for some help but do not want to! Want to be able to solve this on my own!

Will a graph help?

No. of times touches the side

Size of billiard table
(dimension of length).
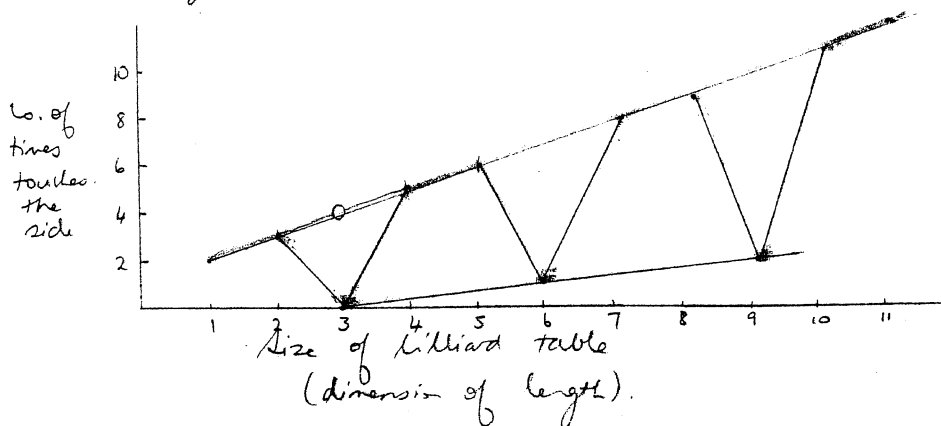
So are there two relationships? - I don't think so biz it's not appropriate to link 3 directly to 6 (eg. biz 4 is not 4, 0.25 - 4, 5).
Is it appropriate to link ___ ? I guess this is possible biz the length of a billiard can be any amount btw 2 & 3 etc.

I think I will have to ask someone the Q @ ▇ on the last pg

Looking @ table again now that I consider that there is probably 1 relationship. Thinking 2^{x} etc, 3^{x} ..., 4^{x} ... but in my experience that sort of R yields an increasing y whereas in my relationship it goes up and down.

It would be my usual tendency to be berating my stupidity by now! And then rush off for help. But as I'm doing neither - happy to explore, think, dwell, consider options.
eg. I wonder if it would be helpful to explore diff. width of table eg. 4 ×1, 4 × 2 etc.

Another bell is ringing of equations for set values / parabolas? Will try then call it a day for today.
Mmm, tired brain?

/9 A new question - would these points be connected by straight lines or a curve?

Looking @ 4 x 1 etc there appears to be a pattern for the odd need length eg

$$x \quad y$$

x	y	
1	3	
3	5	$y = x + 2$
5	7	
7	9	
9	11	

Even nos

x	y		x	y
2	1		12	2
4	0		14	7
6	3	Pattern ?		
8	1			
10	5			

What does this graph look like?

No. of times it touches the side — length of billiard table
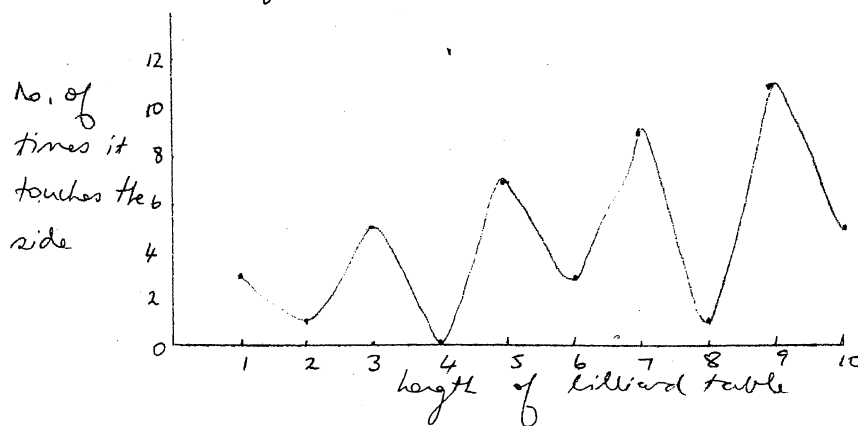
Feel stuck.

My question remains "is there a single algebraic expression to fit these relationships?"

What else could I look at?

For billiard tables width 3 — if length is odd no billiard ball exits from top right (TR)
— if length is even no. billiard ball exits from bottom right (BR)

4×1

4×8

4×9

4×10

4×12
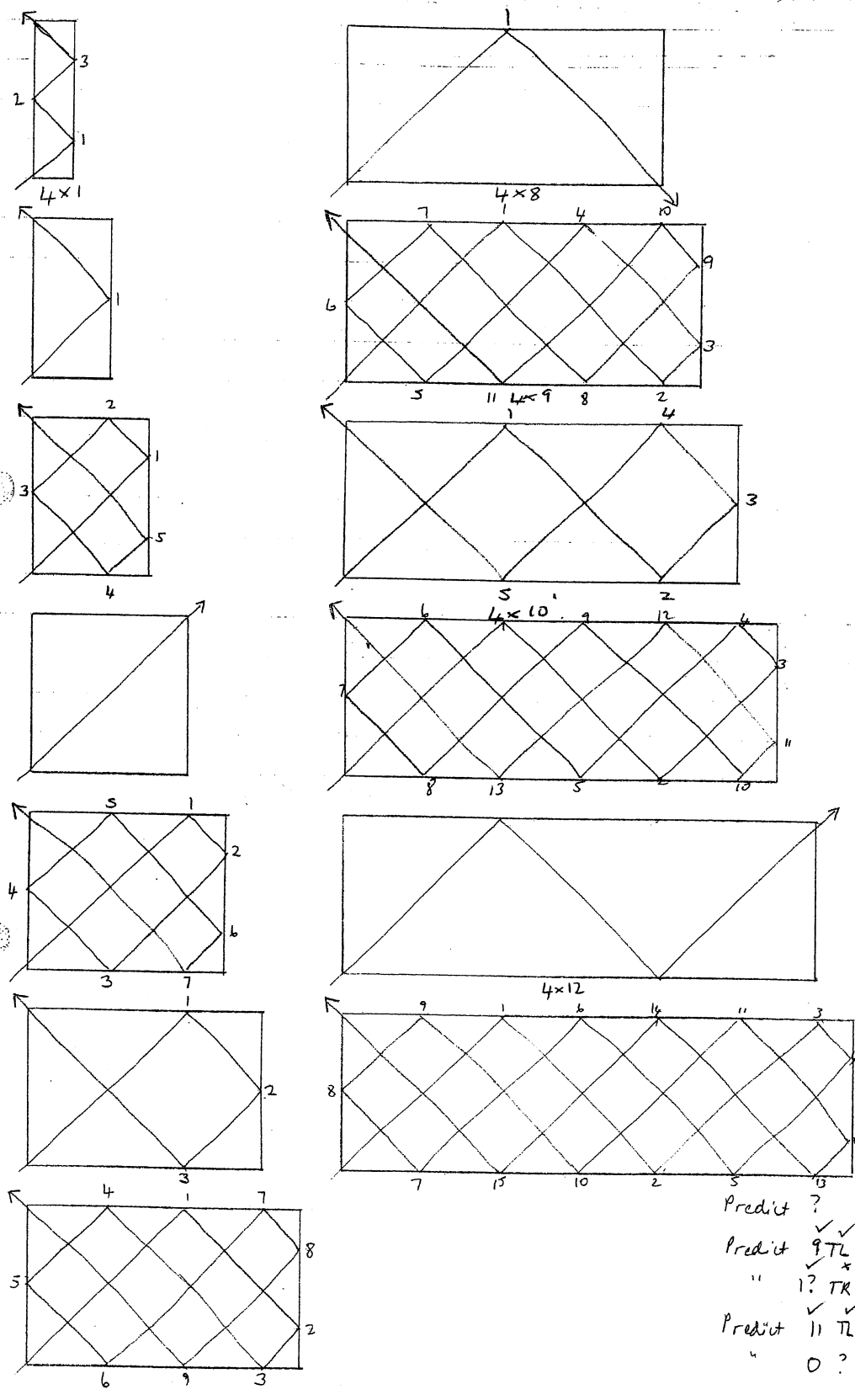
1 , 3		TL
2 , 1		TL
3 , 5		TL
4 , 0		(TR)
5 , 7		TL
Predict ?	6 , 3	TL
Predict 9 TL	7 , 9	TL
" 1? TR	8 , 1	(BR) !
Predict 11 TL	9 , 11	TL
" 0 ?	10 , 5	TL

For billiard table width 4 if length is a multiple of 4 does not exit TL but either TR or BR. Will explore a bit more.

Predict 13 TL 11 13 TL

2 TR 12 2 TR

15 TL 13 15 TL

7 TL 14 7 TL

I wonder if I put these no seq. into Google whether that will help. No it doesn't (after a quick try).

Yes - if multiple of 4 it exits TR or BR which makes sense when you consider the symmetry of the table.

I was thinking last night that there seems to be a link btw geometry and algebra. Interesting.

I feel like I'm almost clutching @ straws here although such exploring does feel OK. I did wonder whether searching on the net was a legitimate mxl thing to do - I'm not sure.

What if I look @ no of lines - patterns?

"   "   "   "   "   ratio of sides - like trig.?

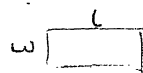|	No. of lines
3 × 1	3
3 × 2	4
3 × 3	1
3 × 4	6
3 × 5	7
3 × 6	2
3 × 7	9
3 × 8	10
3 × 9	3
3 × 10	12
3 × 11	13
3 × 12	4

Of course - it is 1 more than no. of corners.

OK, so what about the ratio of sides?
Look @ length and width.

$\frac{length}{width}$ and $\frac{width}{length}$

	$\frac{length}{width}$			$\frac{width}{length}$	
3 × 1	$\frac{1}{3}$	$\frac{0}{0}$ TR		$\frac{0}{0}$	$\frac{3}{1}$
3 × 2	$\frac{2}{3}$	$\frac{E}{0}$ BR		$\frac{0}{E}$	$\frac{3}{2}$
3 × 3	$\frac{3}{3}$	$\frac{0}{0}$ TR		$\frac{0}{6}$	$\frac{3}{3}$
3 × 4	$\frac{4}{3}$	$\frac{E}{0}$ BR			$\frac{3}{4}$
3 × 5	$\frac{5}{3}$	$\frac{0}{0}$ TR			$\frac{3}{5}$
3 × 6	$\frac{6}{3}$	$\frac{E}{0}$ BR			$\frac{3}{6}$

	$\frac{L}{w}$			$\frac{w}{L}$
4 × 1	$\frac{1}{4}$	$\frac{0}{E}$ TL	$\frac{E}{0}$	$\frac{4}{1}$
4 × 2	$\frac{2}{4}$	$\frac{E}{E}$ TL	$\frac{E}{E}$	$\frac{4}{2}$
4 × 3	$\frac{3}{4}$	$\frac{0}{E}$ TL		$\frac{4}{3}$
4 × 4	$\frac{4}{4}$	$\frac{E}{E}$ TK		$\frac{4}{4}$
4 × 5	$\frac{5}{4}$	$\frac{0}{E}$ TL		$\frac{4}{5}$
4 × 6	$\frac{6}{4}$	$\frac{E}{E}$ TL		$\frac{4}{6}$

So if the ratio is an $\frac{odd\ no}{odd\ no}$ it exits via TR when width is 3

" " " " " $\frac{odd\ no}{even\ no}$ or $\frac{even\ no}{odd\ no}$ it exits via BR when width is 3.

Pattern not clear-cut for 4.
I wonder if I need to concentrate on just 1 width @ a time.
g. 3 then do 4 later.
Is there an 'answer' here? I certainly want an answer re
algebraic expression but then would one go on exploring w/o ending?
Is looking on 'google' for a similar investigation allowed or not?

I'm now feeling annoyed!

So I've looked at the internet site — they
speak of relative primes — what are they?! So haven't got any
further ahead. http://www.k12science.org/IMATTT/billiards_t.html.

I've reached that spot I referred to a few pages back — I can't
do this which feels very annoying. Need support to continue but
that feels like 'cheating' — why can't I work this out on my own?

Going back to the web-site their interp^n of no. of touches is diff
to mine — I don't include where it enters, they do. or exits.

Even so, my data doesn't fit their pattern.

width	length	touches (net)	mine (with net interp⁻)	
3	1	4	4	} does fit
3	2	5	5	} but
3	3	6	2	} doesn't !

but could reduce 3-3 to 1-1 then 2 would fit.

So what is rel. prime?

Used http://mathforum.org/library/drmath/view/57121.html.

"2 or more nos are said to be relatively prime if their greatest common factor is 1."

for 14, 15

2	14	1	2	7	14
	15	1	3	5	15

GCF = 1.

Use 'relatively prime' nos every time you reduce a fr to its lowest terms. The fr in its simplest form are 2 RP numbers.

This feels better. That horrible feeling in my stomach is receding. My other thought @ the moment is the importance of interpretation. Does 'other' interpretation get in the way? Or can I still explore my interpretn. I think so. The website takes you to rel. prime nos biz of what was said in the explanations which was diff. to T's expln. So when do you know the sum can be used or need to find relatively prime no?

This is a very interesting exercise - some subconscious beliefs appear to include:

1. I should be able to get this by now (after a cycle of Lono) - do I not genuinely think lg takes time?
2. 'Real' lng does not include using others. So how does that fit with my professed constructivist / enactivist beliefs?
3. Still struggling with the 'OK-ness' of being stuck.

Another thought links back to the approach - I end up lg what I don't know.

So I will now consider more carefully this website - thinking it is OK to collaborate with others.

"If the dim. are rel prime the path (no of touches) will be the sum of the length + width".

"If not find the smallest rect. that will have the same path".

eg.

W	L	Touches	
5	7	12	
8	3	11	
5	4	9	
10	13	23	
6	9	5	← not relatively prime Their GCF is 3!

So for 6-9 find the smallest rect. which would be 2-3. and hence 5.

I also appear to think that 'I should be able to 'instantly' understand and berate myself / which puts up blocks if I don't. Scary realisations.

Let's see if this fits with my work on width of 4.

	W	L	touches (not interpret ie add 2 to mine)	
TL	4	1	(3) 5	✓
TL	4	2	(1) 3	← not liz not RP. Reduce to 2-1
TL	4	3	(5) 7	✓
TR	4	4	(0) 2	← not liz not RP. Reduce to 1-1

Yes it does.

So a relationship to predict the no. of touches exists; it being ∞ no. of touches = width + length if dimensions are rel. prime.

Let's now explore where the ball ends up:

W	L		RP	Conj.	
5	7	TR	✓	✓	
8	3	TL	✓	✓	
5	4	BR	✓	✓	
10	13	TL	✓	✓	
5	9	TR	✓	✓	
6	9	TL	✗		← not RP 2-3 E O so TL
7	10	BR	✓	✓	

So some conjectures:
- if W + L are both odd and RP — TR
- if W is even + length odd " " TL
- " " " odd + " " even " " BR
- if not RP find smallest rectangle with same ratio and apply above.

Test conjectures with my data.

4	1	TL ✓		
4	2	TL ✓	2	1
4	3	TL ✓		
4	4	TR ✓	1	1
4	5	TL ✓		
4	6	TL ✓	2	3
4	7	TL ✓		
4	8	BR ✓	1	2

Yes the conjectures appear to hold.
Test with 3.

3	1	TR ✓
3	2	BR ✓
3	5	TR ✓
3	8	BR ✓

Yes, again they hold.

This is satisfying but I didn't find it on my own – I had not explored this interpretation – would I have then found it? Probably not biz I didn't know about relatively prime numbers. So I now know how to predict the no. of bounces and place of exit:
No of bounces = sum of width + length if RP. If not finding smallest rectangle in same ratio and sum that with Pt of exit – as described above.
So the understanding is satisfying and I have learned about relatively prime numbers. I still have this niggle about not finding it on my own! So I now will return to my interpretation and the pattern there.
Why did I not think to look @ ratios in their simplest form?

I have just asked S— about the table on p.17. I have learned 2 things: 1) "piece-wise" functions do exist where diff eqⁿ can be linked to diff values of x.

2) I have made an assumption regarding values of x btw 2 and 3 - ie, do √ exist? I need to check this out.

So for x ∈ natural numbers y = x + 1
for x ∈ multiple of 3 y = x - ?

I had the glimmer of thinking re you can't connect 3 - 6 - 9 etc which is correct but an eqⁿ may exist just for the multiples of 3.

What if length (x) = 1.5?

2/09 I trialled a length of 1.5 and found it linked to 1 bounce - 2½ had 9 bounces!

At this stage I do not wish to explore fractional nos in this problem any further.

I asked P— about

x	y
3	0
6	1
9	2
12	3

I have learned that if the gaps between the y-values are even then the R must be linear. Of course - that does make sense - I was exploring exponential functions which in light of the new info. doesn't make much sense.

So the relationship here is $y = \frac{1}{3}x - 3$ or $y = \frac{x-3}{3}$.

Of course!

Summary ~~x ∈ natural nos, but not the multiples of 3 ⇒ y = x + 1~~

~~x ∈ multiple of 3 ⇒ y = x - 3~~

Questions that could be further explored:

- for my interpretation, what is the algebraic relationship for fractional nos. when width is four?
- same question when width is 4 and the length is an even no?

NB, thinking back to $3 \times 2\frac{1}{2}$ I was wondering if fractional nos might be considered O or E. I then thought I would find the simplest final form/ratio of these 2 nos ie $6 \times 5 \rightarrow$ even width, odd length so pt of exit should be TL and it is! No. of bounces should then be 11 (using net interpretation) and it is! Ah, very satisfying so this (the rules!) even applies to fractional numbers!

And now ———— ideas also make sense!

Overall this has been very satisfying. I have learned a lot both about maths and my attitudes towards learning. Thoughts at the moment include: (also see back to p. 22 and 23).

- I'm now prepared to search further than I used to.
- I recognise that searching and trialling new ideas is a valid and important part of the process. Is it in this 'searching and trialling' that the possibilities for learning (which is different for diff. individuals) exist? I think so.
- I have some unhelpful behavioural + belief patterns that can block my progress. or still have?
- I appear to have had the belief that lrg is a solitary business and only really 'real' if done/solved by myself. I need to ponder/explore this further because it is contrary to my espoused belief! Interesting!
- I would like to develop the 'habit' of thinking in multiple interpretations! biz this could help solve problems.

On to the next investigation and yes, I do have butterflies! Will I get stuck? Yes, so I guess the real question is will I get unstuck? If so, by myself or not? and does this matter?

When width = 4.

x	y
2	1
4	0
6	3
8	1
10	5
12	2
14	7
16	3
18	9

$x \in$ odd no. $y = x + 2$.

$x \in$ multiple of 4 $y = \frac{1}{4}x - 1$

$x \in$ even no, not mult of 4 $y = \frac{1}{2}x$.

Went back to this —
I found myself say
"I can't predict 0=..., you!
Can, I predicted 16 would
be 3 and voila! It
all fell into place!

4   0
8   1
12   2
16   3

The
University
of Waikato
Te Whare Wānanga
o Waikato

14 July 2003

To students enrolled in TEMS120-03B Learning and Teaching Mathematics.

Kia ora,

Staff involved in teaching Mathematics Education at Waikato University are doing some research that looks at ways to improve pre-service teacher education in mathematics. It has been decided that this Mathematics Education paper (TEMS120-03B) will be one vehicle for this research.

This letter is to invite you to participate. Participation may include the following aspects:

- Your lecturer observing and keeping notes about what happens during on-campus classes and online discussions. Another observer may also be invited into classrooms (on-campus and online) from time to time. You will be informed if and when that happens.

- Your lecturer observing you and keeping notes about what happens as you work with a young child in a normal school (working with a young child is part of your second assignment). Note well, the child will not be observed.

- Making your journal, which is an essential requirement for all students taking this paper, available to form part of the research data. Using the journal as part of the research data would not conflict with or prejudice your grades in any way.

- Your lecturer possibly interviewing you (informally) about what you are learning. A time for this would be arranged that is mutually decided upon.

  If you choose to participate in the research:

  - Your permission is sought for making copies of your journal including the assignments and online contributions.

  - If you are interviewed, you can choose not to answer a question, or to stop the interview at any time.

  - Any interview or class observation will be audio-taped only with your consent. A transcript of the interview and/or observation will be made available for you to check if you wish. If you are interviewed, the only people to have access to the tape will be staff in Mathematics Education.

  - Your name will not be used in the final research report and all communication will remain confidential to the staff in Mathematics Education.

- When the research report is complete copies of it will be put in the School of Education library so that you can read it. Information in the report is likely to be included in academic outputs (eg. theses); an article that will be sent to a journal for publication; and may be presented at a conference.

- Participation in this research is entirely voluntary. A decision not to participate will **not** disadvantage you with respect to any Mathematics Education paper.

- You may withdraw from the research without explanation at any time, even if you have previously given consent.

This project has been approved by the School of Education Ethics Committee. If you are willing to be involved, please fill out the consent form on the next page. If you have any questions or require more information, please feel free to call us, the researchers, on (07) 838 4500 (Merilyn x7727, Tony x4955, Peter x7846, , Sashi x6298, Ngarewa x7848, Nigel x5308, or Judy x7742).

If you have any concerns about this project at any time please feel free to contact your lecturer or Ian Taylor ((07)8384500 x 7872) who has agreed to be an independent contact person.

Yours sincerely,

for Tony Brown, Peter Grootenboer, Merilyn Taylor, Sashi Sharma, Ngarewa Hawera, Nigel Calder, Judy Bailey.

Informed Consent

I have read and understand the attached letter.

I am willing to participate in the research project that is taking part within my Mathematics Education paper TEMS120-03B.

I consent to my journal including both assignments, and online contributions being copied for the research project.

If I am interviewed I can choose not to answer any question and/or stop the interview at any time.

I give consent to be audio-taped during class discussions and interviews, knowing that I can veto the use of any excerpt when checking the transcript.

I realise that in any report my name will be changed to ensure my anonymity.

I understand that I may withdraw from the research at any time, even if I have previously given consent.

I understand that my asessments in this paper will be prejudiced in any way as a result of my participation in this research.

Name: _____

Appendix C:

Interview questions

1. What do you think mathematics is about?

2. Why do you think this? What experiences have led to these thoughts?

3. Remembering the investigations we did last year, and this year; have these influenced your thoughts about:
   - the nature of mathematics?
   - teaching mathematics?
   - learning mathematics?

4. How would you describe your mathematical behaviour when doing an investigation?
   Have you noticed any changes over the two semesters?
   What would these be?
   What do you attribute these to?

5. What do you think about mathematical investigations with respect to learning mathematics?

6. What do you think about collaboration with respect to learning mathematics?

7. How do you think mathematics is learned?