Entrepreneurship and Debt:

Growth Aversion, Debt Aversion, Overconfidence, and the Effects on Small Business Credit Decisions

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Abstract

Widespread empirical consistency with the pecking order theory of capital structure (Myers & Majluf, 1984) has led researchers to conclude that small and medium sized enterprises conform to this theory's predictions. In chapter 2, a formal model is presented that allows for plausible and empirically supported psychological owner/manager objectives in addition to the profit motive. This chapter provides an alternative explanation of preferences for low leverage that does not rely on informational asymmetry, as well as predicting limits on firm sizes, and the existence of collateral.

In chapter 3, a formal model of the entrepreneurship decision with credit is presented for firms with managers who overestimate their probability of success. Explanations for credit rationing, predatory lending, and the existence of collateral are produced and the welfare implications of overconfidence are investigated in an equilibrium model. It is found that overconfidence can increase overall welfare but harms entrepreneurs who are able to engage in poor projects.

Acknowledgements

Friends and family were immensely supportive during the writing of this thesis. Particularly, in keeping with theme, I have accumulated a large debt with my mother, mostly due to the countless dinners she provided during this time. I expect to be repaying this obligation, in one way or another, for many decades to come.

John Tressler, my supervisor, provided invaluable insight into many aspects of this work. I’m especially appreciative of his patience during busy times and his friendliness.

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1. Introduction

Small and medium sized enterprises (SMEs) make up more than 99% of businesses in New Zealand and provide around 60% of employment. Around the world, SMEs make up similar proportions of the business population. It is clear that the contribution of SMEs to society is substantial and the need to understand SMEs is an important issue for economists and policy makers.

The particular issues of SME financing, entry, and growth are important ones. Economic growth is an important public goal and a popularly cited path to this growth is through SME investment. This thesis provides two formal models of the borrowing and investment/entry decision of the firm. The first models non-financial motivations and explains limits on firm size and debt amongst SMEs, as well as the presence of collateral. The second models firm managers as overconfident in their business prospects, providing explanations for predatory lending, credit rationing, and collateral.

The two main chapters in this thesis are written in a paper-like format and can be read independently. Each sub-topic is introduced in more detail within the chapter itself. Conclusions for each model are also provided within the chapters.

2. Non-financial motivations and the entrepreneurship decision with debt

> What can be added to the happiness of a man who is in health, out of debt, and has a clear conscience?
>
> - Adam Smith (1790, p. I.III.7)

2.1 Introduction

The well-known classical argument that justifies profit maximisation as the firm's objective relies on the assumption that the firm has many individual owners with heterogeneous preferences. It would be impractical and costly for managers to directly arrange firm activities to meet the diverse individual preferences of the firm owners. Maximising firm profit allows owners to satisfy their preferences through the product markets. However, since privately held small and medium sized enterprises (SMEs) *by definition* have few shareholders, the classical argument is not appropriate and, in reality, the firm goal of gaining profits is combined with other personal objectives of the SME shareholders. These objectives may conflict with wealth maximisation.

The dominant financing theories, based on the simplifying assumption that firms maximise the financial wealth of shareholders, have questionable applicability to small and medium enterprises. The pecking order theory assumes informational asymmetry between firms and financiers and shows that wealth maximising firms should first finance investment internally, given available funds (Myers & Majluf, 1984). Theories predicting an optimal target capital structure balance the wealth enhancing aspects of debt, such as tax advantages (Kraus & Litzenberger, 1973;

Modigliani & Miller, 1958, 1963), and control of agency costs to shareholders (Jensen & Meckling, 1976), against the wealth reducing aspects of debt, such as financial distress costs and agency costs to debt holders.

The reluctance to relinquish control and the desire for independence are oft-cited examples of real life attitudes small firm owner/managers hold (Ang, 1992; Bolton Committee, 1971). Lifestyle factors are also considered important in explaining SME behaviour, such as the trade-off between leisure and work (Kirkwood & Tootell, 2008). The concept of 'positive atmosphere' in the workplace, contributing to job satisfaction, is another important issue for SMEs (Wiklund, Davidsson, & Delmar, 2003). All of these objectives can conflict with the goal of profit maximisation and all are important to consider when developing policy and theory.

Previous empirical studies of SME capital structure have used the conventional large-firm theories in their motivations and implicitly assume SMEs maximise profits. As a corollary to this assumption, these studies have also assumed that SMEs, in general, desire growth and seek external finance (Beck, Demirgüç-Kunt, & Maksimovic, 2008; Cassar & Holmes, 2003; Chittenden, Hall, & Hutchinson, 1996; Hall, Hutchinson, & Michaelas, 2004; Michaelas, Chittenden, & Poutziouris, 1999; Ramalho & da Silva, 2009; Sogorb-Mira, 2005). Exceptions include Lucey and Mac an Bhaird (2006) and, to a lesser extent, Degryse, de Goeij and Kappert (2009), and Psillaki and Daskalakis (2009). Lucey and Mac an Bhaird examine 299 Irish SMEs and conclude that the desire for independence and control is important in SME capital structure decisions, while Degryse, de Goeij and Kappert, and Psillaki and Daskalakis cite independence and control as a possible explanation of their finding related to profitability. The general conclusion drawn by these studies is that SMEs conform to the pecking order theory and that it is informational asymmetry which explains the observed preference for internal funds.

Few direct tests have been performed to determine the severity of the informational asymmetry problem. Hyytinen and Pajarinen (2008) report credit rating disagreements (a measure of informational opacity) in Finland SMEs to be around 50% suggesting this problem does indeed exist; however, the extent to which this affects credit markets was not investigated. Interestingly, Hyytinen and Pajarinen find the frequency of rating disagreement is unrelated to SME size when controlling for firm age. It is, however, conventionally assumed that firm size and informational opacity are inversely related. Until there exists substantial direct empirical evidence demonstrating the effect of informational asymmetry on SME credit markets, a convincing argument for the dominance of the traditional pecking order theory in SME credit markets cannot be made.

It has been shown empirically that SME managers have preferences for independence, control, (Bolton Committee, 1971; Cressy, 1995), employee well-being (TD Bank, 2010; Wiklund et al., 2003), and the leisure/labour balance (Kirkwood & Tootell, 2008). Vos, Yeh, Carter, and Tagg (2007) also paint the small firm as one which is unconcerned with the excesses of wealth and document that only around 8% of UK SMEs have the objective to grow rapidly. Growth aversion in SMEs has also been documented in several other works (Davidsson, 1989; Kolvereid, 1992; Storey, 1994). However, many recent papers, continue to represent small firms as wealth oriented, and even at some stage on the path to an IPO (e.g. Beck & Demirgüç-Kunt, 2006).

Wiklund et al. (2003) attempt to find the relationship between the expected consequences of firm growth and actual growth motivation using Swedish SMEs. In this important paper, a linear regression model is estimated with standardised coefficients and which shows that concerns for independence, control, and positive atmosphere as well as personal income have the strongest relationships with stated growth motivation. This allows the inference that small firm owners do indeed perceive their firms as having a better atmosphere than hypothetical larger firms and that this preference is important for growth decisions.

In the economics of entrepreneurship literature, there are few microeconomic theory papers which deviate from the profit maximising assumption. Levesque, Shepherd, and Douglas (2002) develop a dynamic extension to Douglas and Shepherd (2000) who model the small firm entry decision to be a utility-maximising response that depends on independence as well as other self-employment perquisites. Generally, amongst the entrepreneurship literature, the complex motivations of entrepreneurs are acknowledged.

Some psychologists have also participated in this line of research. Schindehutte, Morris, and Allen (2006) directly show, using in-depth psychological interviews, that entrepreneurs are more concerned with psychological aspects of entrepreneurship such as peak performance, peak experience, and flow, compared with extrinsic rewards such as money. Their results suggest we should place less emphasis on small business as simply a mode of wealth generation and economic growth and more emphasis on entrepreneurship as a mode of increasing well-being and happiness directly.

A previous paper that attempts to theoretically model non-financial motivations in the entrepreneurship decision with debt is Cressy (1995). This paper models debt as a variable which contributes negatively to an entrepreneur's decision to proceed with a variable sized project that requires external financing. The term for entrepreneurs' dislike of external debt in Cressy is [external] "control aversion" (pp. 261). In Cressy, control aversion represents an assumed negative and concave relationship between debt and decision utility. The current chapter extends Cressy by modelling collateral as a variable which reduces the disutility of debt and project size as a variable which contributes negatively to the decision to go ahead with the project.

In Cressy (1995), the justification for including debt as a negative decision factor in his model was that lenders impose monitoring and controls on the firm, presumably either in the form of restrictive covenants or by collecting information from the firm for valuation, which is disliked by the firm. Debt impacts decision utility independently of the level of profits, so this is interpreted as a purely psychological aversion to this monitoring. Because of the empirical observation of an aversion to outside control amongst small business owners, only monitoring which impacts on control in the form of restrictive covenants is justified here. The common foundational justification for monitoring in the form of restrictive covenants is to mitigate the problems of moral hazard (e.g. Holmström & Tirole, 1997). This can be the interpretation of the reason why the lender engages in this monitoring. Following from this, any other device which removes moral hazard risk for the lender will reduce monitoring intensity and, consequently, the disutility to the entrepreneur. Collateral is one of these devices. Collateral reduces the risky portion of repayment to the lender and therefore reduces the incentive to monitor. If collateral is included in a model of this sort, it should reduce the disutility to debt. Because this important effect is omitted in Cressy, the optimal level of collateral is found to be zero when it can potentially be positive.

Preferences for independence, positive atmosphere, and the leisure/labour balance are able to be incorporated into the model through the project size variable. Independence (that is unrelated to profits) can be broadly interpreted but typically is related to self-reliance. A larger project requires more input from outside sources, such as higher external capital and extra managerial staff, increasing the degree to which the entrepreneur relies on others. An entrepreneur who prefers to be self-reliant will choose a smaller project than an otherwise similar entrepreneur who is indifferent to their level of self-reliance.

The leisure/labour balance motivation interacts with the preference for independence/control in the in the context of entrepreneurship. Entrepreneurs looking to increase income must take on larger projects. Since larger projects are more complex, entrepreneurs must forgo either control to another manager or leisure in order to spend more time managing the firm. Either is a potential source of disutility to the manager that is related to project size.

In summary, the aforementioned non-financial motivations are able to be modelled most parsimoniously using two variables in the decision utility function in addition to profit. External control arising from monitoring intensity can be measured by the risky portion of a loan, which is reduced by collateral. Independence, the leisure/labour balance and positive atmosphere can be measured by the size of the project itself (which has an identity relationship with the size of the loan if no internal funds are used).

2.2 The model

An entrepreneur is to decide whether or not to engage in a risky project of variable size that is dependent on external capital investment. Firms are financially risk averse and exhibit risky-debt aversion (resulting from, for example, aversion to outside control or monitoring) and aversion to firm size (resulting from, for example, preferences for firm manageability, positive atmosphere, and the non-risky portion of the debt itself).

The firm is assumed to have no endowment of investable wealth so the entire cost of the project, I, is to be borrowed from the credit market. Extending this model to allow for partial self-financing is straightforward and does not substantially alter the results of the model. The firm is endowed with $\bar{C}$ of non-liquid assets that are unable to be invested directly in the project. The firm is able to offer any portion of this endowment as collateral. The amount offered is denoted as C and these assets have value βC to the lender with $0 < \beta \leq 1$. βC is interpreted as the liquidated value of the collateral and takes account of direct liquidation costs as well as any difference in the value of use to the entrepreneur and the market value of the assets. The project is assumed to return $R(I)$ on success, with probability p, and zero on failure. It is assumed that the project has positive net present value (NPV) or $pR(I) - I > 0$ for all values of I. Both the opportunity cost of lending (the market interest rate) and the opportunity cost of engaging in the project are normalised to 0.

The value of the loan which will be subject to monitoring (the risky portion of the loan) is $L \equiv I - \beta C$ and the variable that represents firm size is the investment required, I. Monitoring from the lender is assumed to be costless and occurs exogenously.

The lender offers a set of acceptable lending contracts to the entrepreneur that depend on the offered collateral and the size of the project. The share of the project return on success that goes to the lender is $R_l(I, C)$ and the share which goes to the entrepreneur is $R_b(I, C) \equiv R(I) - R_l(I, C)$.

Lenders are assumed to be risk neutral and competitive so the expected profit for the lender must be zero:

$$pR_l + (1 - p)\beta C - I = 0. \tag{1}$$

This yields the offer set in (I, C) space which is presented to the entrepreneur:

$$R_l = \frac{I - (1 - p)\beta C}{p}. \tag{2}$$

The entrepreneur then maximises expected utility over I and C. The final wealth to the entrepreneur upon success is denoted by

$$F(I, C) \equiv R(I) - R_l(I, C) = \frac{pR(I) - I + (1 - p)\beta C}{p}, \tag{3}$$

and the entrepreneur pays C to the lender upon failure. The utility function depends on three factors, the final wealth of the entrepreneur Y, the risky or monitored portion of the loan $L \equiv I - \beta C$, and the size of the project I. The program the entrepreneur faces is

$$\max_{I,C} \mathrm{E}[U(Y(I,C), L(I,C), I)]$$

$$\text{s.t. } 0 \leq C \leq \bar{C}, \qquad I - \beta C \geq 0,$$

where $U_Y > 0, U_{YY} < 0, U_L < 0$, and $U_I < 0$. Note that the first and last constraint together imply that $I \geq 0$. The final constraint is not a physical feature of the model but it is included so that the objective function is smooth across the feasible region and it only excludes a suboptimal region. Without this constraint the negative contribution to utility that comes from the risky loan portion would fall to zero and remain at zero if collateral were rise above I/β. Because collateral costs the entrepreneur financially ($\beta \leq 1$) and the entrepreneur gains no psychological benefit from collateral above $C = I/\beta$, any acceptable contract with $C > I/\beta$ will be weakly dominated by one with $C \leq I/\beta$.

Expanding the expectation and replacing Y with the final wealth to the entrepreneur in the respective states, the program becomes

$$\max_{I,C} \left[pU\left(\frac{pR(I) - I + (1-p)\beta C}{p}, I - \beta C, I \right) + (1-p)U(-C, I - \beta C, I) \right] \tag{4}$$

$$\text{s.t. } 0 \leq C \leq \bar{C}, \qquad I - \beta C \geq 0.$$

At this stage it is convenient to assume additive separability in the three decision factors in order to simplify the analysis. With this assumption, the marginal utility of one decision factor is restricted so that it does not depend on the other decision factors. That is, it does not allow for any interaction effects between the decision factors. For example, an increase in the risky portion of the loan is equally harmful (in certain dollars) to an entrepreneur who engages in a highly profitable project as it is to the same entrepreneur who engages in a less profitable project. The utility function then becomes

$$U(Y(I,C), L(I,C), I) = u_1\big(Y(I,C)\big) - u_2\big(L(I,C)\big) - u_3(I), \tag{5}$$

and the program (4) becomes

$$\begin{gathered}\max_{I,C}\left[pu_1\big(F(I,C)\big) + (1-p)u_1(-C) - u_2\big(L(I,C)\big) - u_3(I)\right] \\ \text{s.t. } 0 \leq C \leq \bar{C}, \qquad I - \beta C \geq 0.\end{gathered} \tag{6}$$

Assuming an interior solution, the first first-order-condition is

$$\frac{\partial \mathrm{E}[U]}{\partial I} = pu_1'\big(F(I,C)\big) * \left(R'(I) - \frac{1}{p}\right) - u_2'\big(L(I,C)\big) - u_3'(I) = 0,$$

$$\Leftrightarrow p\left(R'(I) - \frac{1}{p}\right) = \frac{u_2'\big(L(I,C)\big) + u_3'(I)}{u_1'\big(F(I,C)\big)},$$

$$\Leftrightarrow pF_I(I) = MRS_{IF}, \tag{7}$$

where MRS_{IF} denotes the number of dollars in the good state that the firm would require in order to induce them to take on an extra unit of risky debt/firm size. So this condition says that, given C, the firm will choose a value of I such that the expected marginal dollar gained by increasing size I (left hand side) equals the sum of the marginal rate of substitution of income for risky loans and the marginal rate of substitution of income for firm size, when evaluated at the entrepreneurs good-state income (right hand side). The expected extra amount earned as a result of the last unit of I must just compensate the entrepreneur for taking on the larger loan and the larger firm size.

The second first order condition when assuming an interior solution is

$$\frac{\partial \mathrm{E}[U]}{\partial C} = pu_1'\big(F(I,C)\big)\left(\frac{1-p}{p}\beta\right) - (1-p)u_1'(-C) + \beta p u_2'\big(L(I,C)\big) = 0,$$

$$\Leftrightarrow (1-p)\left(\beta u_1'\big(F(I,C)\big) - u_1'(-C)\right) + \beta u_2'\big(L(I,C)\big) = 0. \qquad (8)$$

Note that

$$\beta u_1'\big(F(I,C)\big) - u_1'(-C) < 0,$$

as $u_1'' < 0, \beta \leq 1$. Collateral reduces the negative effects of gaining a loan through reduced monitoring but also costs the risk averse entrepreneur through both the loss of value of collateral when transferred to the lender and the increase in the dispersion of returns. This condition explicitly balances these benefits and costs; for a given I, the marginal psychological benefit of collateral through reduced risky loan size, $\beta u_2'\big(L(I,C)\big)$, must equal the expected marginal financial cost of collateral, $-(1-p)\left(\beta u_1'\big(F(I,C)\big) - u_1'(-C)\right)$.

Under certain collections of assumptions on u_1'', u_2'', u_3'', and R'', the second order conditions for a maximum can be met. A special case for these quantities is presented in the following section which allows the model solution to be tractable.

2.3 The simplified model

In this section, the model is fully solved in the special case when the entrepreneur has constant absolute risk aversion a, the risky loan and firm size decision factors have constant marginal utilities b and c respectively, and the project exhibits constant financial returns to scale so $R(I) = dI$.[1] The assumption of constant absolute risk aversion removes the income effect of changes in wealth on risk preferences. For example, a very wealthy individual would view a $500 bet as just as risky as a very poor individual under this assumption. Thus, changes in the parameters that affect average income will not take into account this income effect.

The assumptions that the risky loan and firm size decision factors have constant marginal utilities are made for simplicity and they remove any curvature effects in these variables. The utility function being globally concave is all that is required to guarantee a solution to the utility maximisation problem. Risk aversion in final wealth with constant marginal utilities in the risky loan and firm size decision factors yields a globally concave utility function.

Constant financial returns to scale occurs when the firm's price and marginal cost are constant[2]. This is a reasonable assumption for small firms who have little impact on their product or input prices. Constant returns to scale in the production

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[1] The second order conditions and comparative statics were also computed assuming the utility function was concave in the negative decision factors. The expressions are rather unwieldy and the only interesting result noticed which is not captured by the presented model is that R'' must be bounded at a positive value to ensure a solution exists.

[2] The profit function upon success can be represented as $R = \big(p(Q) - c(Q)\big)Q$. Investment is $I = c(Q)Q$ yielding $R = \left(\frac{p(Q)}{c(Q)} - 1\right)I$. Thus, if $p(Q)$ and $c(Q)$ are constant, the profit function will exhibit constant financial returns to scale.

function can be justified by the replicability argument. The utility function in (5) can now be expressed as

$$U(Y, L, I) = 1 - \frac{e^{-aY}}{a} - bL - cI, \tag{9}$$

and the expected utility maximisation problem is

$$\max_{I,C} \left[1 - \frac{p}{a} e^{-a\left(\frac{pdI - I + (1-p)\beta C}{p}\right)} - \frac{(1-p)}{a} e^{aC} - bL - cI \right]$$

$$\text{s.t. } 0 \leq C \leq \bar{C}, \qquad I - \beta C \geq 0. \tag{10}$$

The Lagrange function for this problem is

$$\mathcal{L} = 1 - \frac{p}{a} e^{-a\left(\frac{pdI - I + (1-p)\beta C}{p}\right)} - \frac{(1-p)}{a} e^{aC} - b(I - \beta C) - cI + \lambda_1(-C)$$
$$+ \lambda_2(C - \bar{C}) + \lambda_3(-I + \beta C).$$

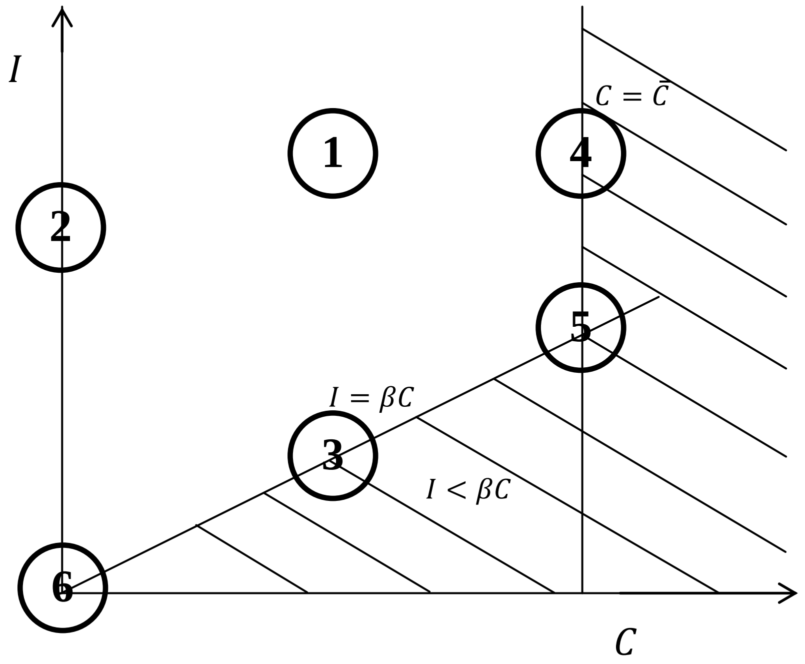
The necessary conditions for a maximisation when the constraints are all less than or equal to zero are

$$\frac{\partial \mathcal{L}}{\partial I} = \frac{\partial \mathcal{L}}{\partial C} = 0, \qquad \lambda_i \leq 0, \qquad 0 \leq C \leq \bar{C}, \qquad I - \beta C \geq 0,$$

$$\lambda_1(-C) = \lambda_2(C - \bar{C}) = \lambda_3(-I + \beta C) = 0.$$

The full solution to this programme was solved using Maple. The Maple output used is presented in Appendix 1. The solutions for the λ's are used to find conditions on the parameters which will yield the different solutions. The presentation approach is to number the six different classes of solution from 1 to 6. The KKT multipliers are denoted as λ_{ij} where i is the KKT multiplier number and j is the solution number. Figure 1 graphically depicts in (I, C) space the feasible region along with the locations of the various numbered solutions. The solutions themselves are listed below.

Figure 1 – Feasible solution region with locations of various numbered solutions.



The interior solution $(I_1, C_1, \lambda_{11}, \lambda_{21}, \lambda_{31})$ is $\lambda_{11} = \lambda_{21} = \lambda_{31} = 0$ and

$$I_1 = \frac{\ln\left(\left(\frac{dp-1}{b+c}\right)^p \left(\frac{(dp-1)(1-p)}{\beta\left(c(1-p)+bp(d-1)\right)}\right)^{\beta(1-p)}\right)}{a(dp-1)}, \tag{11}$$

$$C_1 = \frac{1}{a}\ln\left(\frac{\beta\left(c(1-p)+bp(d-1)\right)}{(dp-1)(1-p)}\right).$$

Conditions which will yield this solution are $\lambda_{12} > 0,\ \lambda_{33} > 0,\ \lambda_{24} > 0$:

$$(dp-1)(1-p) < \beta\left((1-p)c + pb(d-1)\right) < e^{a\bar{C}}(dp-1)(1-p), \tag{12}$$

$$(dp-1)e^{-\beta a(d-1)C_3} > b + c,$$

where C_3 is the unique solution to

$$-a(1-p)e^{C_3} + \beta a(d-1)pe^{-\beta(d-1)C_3} - \beta c = 0.$$

The solution $(I_2, C_2, \lambda_{12}, \lambda_{22}, \lambda_{32})$ with $I_2 > 0$ is $C_2 = \lambda_{22} = \lambda_{32} = 0$ and

$$I_2 = \frac{p}{a(dp-1)} \ln\left(\frac{dp-1}{b+c}\right),$$

$$\lambda_{12} = \frac{\beta\big((1-p)c + pb(d-1)\big) - (dp-1)(1-p)}{dp-1}. \qquad (13)$$

Conditions which will yield this solution are $\lambda_{12} < 0$, $\lambda_{36} > 0$:

$$\beta\big((1-p)c + pb(d-1)\big) < (dp-1)(1-p),$$

$$dp - 1 > b + c. \qquad (14)$$

The solution $(I_3, C_3, \lambda_{13}, \lambda_{23}, \lambda_{33})$ with $I_3 > 0, C_3 = \frac{I_3}{\beta} < \bar{C}$ is $\lambda_{13} = \lambda_{23} = 0$ and

$$I_3 = \beta C_3,$$

$$\lambda_{33} = (dp-1)e^{-\beta a(d-1)C^3} - b - c, \qquad (15)$$

where C_3 is the unique solution to

$$-a(1-p)e^{C_3} + \beta a(d-1)pe^{-\beta(d-1)C_3} - \beta c = 0. \qquad (16)$$

Conditions which will yield this solution are $\lambda_{33} < 0, \lambda_{16} > 0, \lambda_{25} > 0$:

$$(dp-1)e^{-\beta a(d-1)C_3} < b+c,$$

$$\beta p(d-1) > \beta c + 1 - p, \tag{17}$$

$$(1-p)e^{a\bar{C}} + \beta c > \beta(d-1)pe^{-\beta a(d-1)\bar{C}}.$$

The solution $(I_4, C_4, \lambda_{14}, \lambda_{24}, \lambda_{34})$ with $0 < \beta\bar{C} < I_4$, $C_4 = \bar{C}$ is $\lambda_{14} = \lambda_{34} = 0$ and

$$I_4 = \frac{p\ln\left(\frac{dp-1}{b+c}\right) - a\beta(1-p)\bar{C}}{a(dp-1)},$$

$$C_4 = \bar{C}, \tag{18}$$

$$\lambda_{24} = -\frac{\beta\big((1-p)c + pb(d-1)\big) - e^{a\bar{C}}(dp-1)(1-p)}{dp-1}.$$

Conditions which will yield this solution are $\lambda_{24} < 0, \lambda_{35} > 0$:

$$\beta\big((1-p)c + pb(d-1)\big) < e^{a\bar{C}}(dp-1)(1-p),$$

$$(19)$$

$$(dp-1)e^{-\beta a(d-1)\bar{C}} > b+c.$$

The solution $(I_5, C_5, \lambda_{15}, \lambda_{25}, \lambda_{35})$ with $I_5 = \beta\bar{C}$, $C_5 = \bar{C}, \lambda_{15} = 0$ is

$$\lambda_{25} = (1-p)e^{a\bar{C}} - \beta(d-1)pe^{-\beta a(d-1)\bar{C}} + \beta c,$$

$$\lambda_{35} = (dp-1)e^{-\beta a(d-1)\bar{C}} - b - c.$$

Conditions which will yield this solution are $\lambda_{25} < 0, \lambda_{35} < 0$:

$$(1-p)e^{a\bar{C}} + \beta c < \beta(d-1)pe^{-\beta a(d-1)\bar{C}},$$
$$(dp-1)e^{-\beta a(d-1)\bar{C}} < b + c. \tag{20}$$

The solution $(I_6, C_6, \lambda_{16}, \lambda_{26}, \lambda_{36})$ with $I_6 = C_6 = \lambda_{26} = 0$ is

$$\lambda_{16} = \beta p(d-1) - \beta c - (1-p),$$

$$\lambda_{36} = dp - 1 - b - c.$$

Conditions which will yield this solution are $\lambda_{16} < 0, \lambda_{36} < 0$:

$$\beta p(d-1) < \beta c + 1 - p,$$
$$dp - 1 < b + c. \tag{21}$$

It can be easily verified that the objective function is globally concave[3], so the first order conditions are sufficient for a maximum. Because the solution to this problem depends on six parameters, the various solution regions are not able to be visualised graphically.

Solution 6 represents those individuals who do not engage in projects they have access to. This group is of particular social interest as it represents people who choose not to become entrepreneurs due to the negative aspects of risky loan monitoring and firm size. The second condition in (21) shows that for investment to occur, the expected marginal financial return to the first unit of investment must

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[3] This is shown in

Appendix 2.

exceed the marginal psychological cost of that first unit. Thus, increasing project returns, the probability of success, and the liquid ratio of collateral will all increase investment for marginal firms. Decreasing the marginal disutilities of risky loans and firm size will also increase investment for marginal firms. Importantly, there are individuals in this group who are non-marginal who will not respond to policies intended to increase investment that do not push them over this threshold.

Note that, in this formulation, ignoring the unconventional psychological considerations $(b = c = 0)$ will result in investment for all firms with positive NPV projects.

2.4 Comparative statics

The following comparative statics analysis will investigate the effects on firm size I and collateral C of the six various parameters: coefficient of absolute risk aversion a, coefficient of risky debt aversion b, coefficient of firm size aversion c, revenue-investment ratio (upon success) d, probability of success p, and liquidity ratio of collateral β. These are ordered by parameter and are obtained using the interior solution.

$$\frac{\partial I}{\partial a} = -\frac{\ln\left(\left(\frac{dp-1}{b+c}\right)^{p}\left(\frac{(dp-1)(1-p)}{\beta\big(c(1-p)+bp(d-1)\big)}\right)^{\beta(1-p)}\right)}{a^2(dp-1)},$$

$$= -\frac{I_1}{a} < 0,$$

$$\frac{\partial C}{\partial a} = -\frac{\ln\left(\frac{\beta\big(c(1-p)+bp(d-1)\big)}{(dp-1)(1-p)}\right)}{a^2} = -\frac{C_1}{a} < 0.$$

Increasing risk aversion will decrease both investment size and collateral. Increasing each of the variables increases the dispersion of returns to the entrepreneur (regardless of the liquidity ratio β), so a more risk averse entrepreneur will use less of each.[4]

$$\frac{\partial I}{\partial b}=\frac{-p\big(c(1-p)+bp(d-1)+\beta(d-1)(b+c)(1-p)\big)}{(b+c)\big(c(1-p)+bp(d-1)\big)a(dp-1)}<0,$$

$$\frac{\partial C}{\partial b}=\frac{p(d-1)}{a\big(c(1-p)+bp(d-1)\big)}>0.$$

Increasing risky loan aversion will decrease firm size and increase collateral. The result for collateral is intuitive as collateral directly reduces the negative effects of the risky loan portion. In the presence of collateral, the result for firm size is non-trivial, as the firm could intuitively only make more use of their available collateral and leave investment unchanged in response to a change in risky loan aversion. This result shows that a combination of increasing collateral and reducing firm size is optimal for the firm. Recall result (7) that stated the marginal rate of substitution of income for firm size/risky loan must equal the expected marginal dollar earned from increased investment. Increasing risky loan aversion and collateral both contribute to increasing the marginal rate of substitution of income for firm size/risky loan. The risky loan aversion directly increases the amount of income required to substitute for a unit of firm size/risky loan and the increase in collateral increases the income the lender will offer in the good state, causing the marginal utility of good-state income to decrease. Firm size must decrease in order to compensate for both of these changes.

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[4] There is a problem with the setup of the utility function in that the marginal rate of substitution between a certain dollar and one of the negative decision factors depends on risk aversion. Intuitively, if there is no variability in returns ($p=1$), a change in risk aversion should not affect the MRS between certain wealth and one of the other factors.

$$\frac{\partial I}{\partial c} = \frac{-p(c(1-p) + bp(d-1) + \beta d(b+c)(1-p)^2)}{(b+c)\big(c(1-p) + bp(d-1)\big)a(dp-1)} < 0,$$

$$\frac{\partial C}{\partial c} = \frac{1-p}{a\big(c(1-p) + bp(d-1)\big)} > 0.$$

Increasing firm size aversion decreases firm size and increases the use of available collateral. The reduction in firm size is intuitive as firm size itself is made more undesirable. The increase in collateral arises as the financial cost of collateral from (8) must be held constant. Condition (8) is repeated below in the simplified context:

$$(1-p)\left(\beta e^{-a\left(\frac{pdI - I + (1-p)\beta C}{p}\right)} - e^{aC}\right) + \beta b = 0.$$

The investment size I decreases, causing the good state profit to decrease and, consequently, the marginal utility of good state profit (the first exponential) to increase. Collateral increases so that both the good state profit increases and the bad state loss increases in order to keep the total marginal financial cost of collateral constant.

It is also interesting to note that the magnitudes of both derivatives with respect to c are smaller than the respective magnitudes of both derivatives with respect to b. This is due to the entrepreneur being able to increase collateral in direct response to the increase in risky debt aversion, which then causes an even larger decrease in firm size. This result shows that for firms that use collateral, a small increase in risky debt aversion will more severely impact on the size of firms than an equivalent increase in firm size aversion.

$$\frac{\partial I}{\partial d}=\frac{p}{a(dp-1)^2\big(c(1-p)+bp(d-1)\big)}\Bigg(p\big(c(1-p)+bp(d-1)\big)$$

$$+\beta(b+c)(1-p)^2$$

$$+\beta(1-p)\big(c(1-p)$$

$$+bp(d-1)\big)\ln\left(\frac{\beta\big(c(1-p)+bp(d-1)\big)}{(dp-1)(1-p)}\right)$$

$$-p(c(1-p)+bp(d-1))\ln\left(\frac{dp-1}{b+c}\right)\Bigg),$$

$$=\frac{p}{a(dp-1)^2\big(c(1-p)+bp(d-1)\big)}\Big(\big(c(1-p)+bp(d-1)\big)(p$$

$$-a(dp-1)I_1)+\beta(b+c)(1-p)^2\Big),$$

$$=\frac{p^2}{a(dp-1)^2}-\frac{pI_1}{dp-1}+\frac{p\beta(b+c)(1-p)^2}{a(dp-1)^2\big(c(1-p)+bp(d-1)\big)}\lesseqgtr 0,$$

$$\frac{\partial C}{\partial d}=\frac{-(1-p)(b+c)p}{a(dp-1)\big(c(1-p)+bp(d-1)\big)}<0.$$

I either increases or decreases as a result of profitability increasing. Firstly, from condition (7), the expected marginal return to increasing size I (left hand side) increases. Thus, the marginal rate of substitution of income for risky loan/firm size (right hand side) must also increase, meaning income $F=\big((dp-1)I+(1-p)\beta C\big)/p$ must increase. For high values of d, the increase required is small enough so that I must decrease in order to counter the initial increase in d. Intuitively, the marginal utility of income is so small that the entrepreneur is better off taking a small increase in income by reducing firm size and gaining the benefit of reducing the risky loan size and firm size. This captures the idea of satiation.

Note that, even if investment falls as a result of the increase in profitability, the overall effect on good state profit F will be positive. This is what causes collateral to decrease when d increases. Because good-state profit increases as a result of d increasing, collateral must decrease to maintain the constant financial cost of collateral in condition (8).

$$\frac{\partial I}{\partial \beta} = -\frac{(1-p)\left(1+\ln\left(\frac{\beta\big(c(1-p)+bp(d-1)\big)}{(dp-1)(1-p)}\right)\right)}{a(dp-1)} < 0,$$

$$\frac{\partial C}{\partial \beta} = \frac{1}{a\beta} > 0.$$

Both results relating to the liquidity of collateral are intuitive. Increasing liquidity increases the use of collateral as there is less inefficiency created by its use. For the same reason, the lender demands less repayment in the good state, increasing income for the borrower. This decreases the marginal utility of income which, by condition (7), must be held constant. Thus, investment I must decrease.

$$\frac{\partial I}{\partial p} = \frac{b(d-1)(p^2d(1-\beta)+\beta)+dc(1-p)\big(p+\beta(1-p)\big)}{a(dp-1)^2\big(c(1-p)+bp(d-1)\big)}$$

$$+\frac{\big(c(1-p)+bp(d-1)\big)\left((d-1)\beta\ln\left(\frac{\beta\big(c(1-p)+bp(d-1)\big)}{(dp-1)(1-p)}\right)-\ln\left(\frac{dp-1}{b+c}\right)\right)}{b(d-1)(p^2d(1-\beta)+\beta)}$$

$$\lesseqgtr 0,$$

$$\frac{\partial C}{\partial p} = \frac{b(d-1)(dp^2-1)-cd(1-p)^2}{a(dp-1)(1-p)\big(c(1-p)+bp(d-1)\big)} \lesseqgtr 0.$$

Increasing the probability of success can either increase or decrease both investment size and collateral. From both (7) and (8), it is clear there is a complicated relationship between the variables and p making the intuition for these effects difficult. Numerical trials show up to two turning points in the p-I relationship. Two turning points occur if profitability d is high. If d is high, for low values of p there is a sharp increase in I up to a relative plateau with a slight decrease in I until some middle value of p and then I continuing to increase slightly for higher values of p. For low values of profitability d the p-I relationship is increasing. The p-C relationship is U-shaped for low values of d and increasing for high values of d.

The general increasing relationship between p and I is intuitive. The average profit gained from an extra unit of I increases with p, making increases in I more attractive.

The signs of the above comparative statics are summarised in Table 1.

Table 1 – Comparative statics for solution 1 in non-financial motivations model.

Parameter	$\frac{\partial I}{\partial \alpha_i}$	$\frac{\partial C}{\partial \alpha_i}$
a – Coefficient of absolute risk aversion	$\frac{\partial I}{\partial a} < 0$	$\frac{\partial C}{\partial a} < 0$
b – Coefficient of risky loan aversion	$\frac{\partial I}{\partial b} < 0$	$\frac{\partial C}{\partial b} > 0$
c – Coefficient of firm size aversion	$\frac{\partial I}{\partial c} < 0$	$\frac{\partial C}{\partial c} < 0$
d – Revenue-investment ratio	$\frac{\partial I}{\partial d} \lesseqgtr 0$	$\frac{\partial C}{\partial d} < 0$
β – Liquidity ratio of collateral	$\frac{\partial I}{\partial \beta} < 0$	$\frac{\partial C}{\partial \beta} > 0$
p – Probability of success	$\frac{\partial I}{\partial p} \lesseqgtr 0$	$\frac{\partial C}{\partial p} \lesseqgtr 0$

2.5 Discussion/Conclusion

Given that small businesses provide such a large proportion of employment, the understanding of how these entities operate is paramount for policy making in this area. The model presented in this chapter attempts to incorporate the most important decision factors that influence small business owner/managers in their borrowing and investment decisions. In the most recent Survey of Small Business Finance (SSBF), around 50% of small firms in the USA reported that they had not recently applied for financing nor were they discouraged from applying for fear of denial (Cole, 2010). It can be inferred that these firms either chose to forgo any new investment beyond what their ongoing profit allowed them to make (due to nonfinancial considerations) or these firms had no profitable growth opportunities. The model in this chapter provides a plausible explanation of small firms' lack of growth motivation, their low observed leverage, and their use of collateral. The entrepreneur trades off the negative aspects of increasing firm size and debt against generated profit, yielding limits on firm size and debt. In a dynamic setting, it's easy to see that a firm can potentially choose to reduce debt using retained profits in order to gain non-financial benefits, at the expense of either immediate consumption or firm investment.

In order for the profit-only model to have empirical traction, one must invoke decreasing returns to scale as the main driver behind the observed finite firm sizes. Financial constraints alone cannot explain the observed variation in firm size as such a large proportion of firms do not seek financing. The latter explanation is still often assumed or implied in the literature.

This model can provide insight into Rice and Strahan (2010) which found a measure of increase in credit supply in the USA did not bring about an increase in borrowing amongst SMEs. As in previous SME papers, the explanation in Rice and Strahan is that lenders restrict quantity due to asymmetric information and thus credit supply increases do not necessarily translate into increased observed leverage. However, if the population contains many firms whose managers have high aversion to the negative aspects of engaging in further projects, an empirical increase in credit supply can potentially show a negligible impact on borrowing rates.

There does exist current evidence that small business owners value independence, control, positive atmosphere, and leisure and the model presented provides a simple framework for incorporating these nonfinancial motivations into the credit and growth decision of a firm. The challenge for future research is to devise experimental testing that can isolate the relative importance of each of these decision factors amongst small business owners. Policy makers can also take note that there are negative welfare effects of growth at the firm level. While firm growth can increase employment, firm growth can negatively affect the welfare of the owner, as well as the job satisfaction of the employees. In addition, artificially increasing incentives to invest through, for example, reducing the cost of borrowing for small firms won't increase investment from non-marginal non-borrowers in a model of this type. Thus, the predictions of increased investment and increased employment, as a result of a stimulus policy such as subsidising loans, may be overstated.

3. Overconfidence and the entrepreneurship decision with debt

No. I see it as an asset. Absolutely. I am not joking. I am not overconfident.

\- Romano Prodi, former Italian prime minister
and economics professor speaking on the euro.
(Quoted in Dickey, 2005)

3.1 Introduction

Eighty percent of people believe they are above average drivers; seventy percent of people believe they are above average leaders; seventy percent of teachers believe they are above average in terms of teaching ability. We've all heard statistics such as these quoted that demonstrate that humans are somewhat prideful beings. We discount the abilities of others and we overestimate our own. Referred to by economists (usually) as 'overconfidence'[5], or 'optimism', this effect has important implications for entry into entrepreneurship as well as entry into projects by existing firms. Due to this overconfidence, potential entrepreneurs with fundamentally poor projects can engage in investment despite being expected to be left worse off in reality.

Entrepreneurs with sufficient personal assets are able to invest in any project which they believe will be beneficial. Despite harming the entrepreneur in expectation, a poor project going ahead still has social value as it increases total production. The expected loss in wealth to the entrepreneur is effectively transferred to the resource holders, such as employees, who benefit from the

[5] Psychologists have referred to this effect as "illusory superiority', the 'above average effect', 'superiority bias', 'leniency error', 'sense of relative superiority', and 'the Lake Wobegon effect'.

investment. To the extent that resources are not diverted from more productive uses, the overall effect of overconfidence on welfare can be positive. For example, in an economy with high unemployment, poor projects going ahead can put unemployed resources to use and create value.

Credit markets increase the incidence of this investment as lenders can profit from enabling partially funded entrepreneurs to invest. Overconfidence in the credit market setting was modelled in de Meza and Southey (1996) (with a note in Hillier (1998))[6] which provides a plausible explanation for the existence of collateral, as well as credit rationing. Predatory lending is also a result implied by their model. The driver of these results is asymmetry in the assessments of the probability distribution of outcomes, potentially as a result of overconfidence.

The current chapter adds to de Meza and Southey (1996) by setting up a similar entrepreneurship situation and investigating the implications of overconfidence for aggregate welfare. In addition, the directional effects of the various parameters on both welfare and the prevalence of credit rationing are provided, it is demonstrated that overconfidence is not analytically equivalent to situations where the entrepreneur gains some private benefit upon success of the project, and the explanation for collateral is explicitly provided.

3.2 Literature review

Credit rationing was defined in Stiglitz and Weiss (1981, pp. 394-395) as a situation in which either: “among loan applicants who appear to be identical some receive a loan and others do not, and the rejected applicants would not receive a

[6] Note to marker: The current model was developed independently prior to coming across these similar papers.

loan even if they offered to pay a higher interest rate; or, identifiable groups of individuals in the population who, with a given supply of credit, are unable to obtain loans at any interest rate, even though with a larger supply of credit, they would". Unfortunately, no model of credit rationing has yet been widely accepted. The Stiglitz and Weiss paper has been criticised by Arnold and Riley (2009) and Hart (1985) (referenced in Tirole (2006)) who show that the assumed funds supply curve is not possible and the structure of their moral hazard problem can be solved by using a profit sharing contract.

In the context of moral hazard, Tirole (2006) shows credit rationing can occur if the entrepreneur does not have sufficient funds available for partial self-financing. This model assumes entrepreneurs have access to private benefits which are realized if they choose to misbehave. These private benefits are interpreted as lower cost of effort or diverted resources. Firms are considered credit rationed in this model if they could benefit from a loan but no loan contract exists that both induces good behaviour and is acceptable to the lender[7].

Overconfidence as an explanation for credit rationing was first considered in the context of credit markets in de Meza and Southey (1996). In this paper, a firm is considered credit rationed if a firm has access to a project they see as profitable but cannot gain the financing required to invest at its perceived optimal size. In this model, as in Tirole (2006), firms with insufficient personal assets for investment are not able to gain financing.

[7] This doesn't quite fit the definition provided in Stiglitz and Weiss.

Overconfidence has been used in other instances in the economics literature. Landier and Thesmar (2009) construct a model to show that overconfident entrepreneurs will choose short term finance over long term finance. However, this model may have questionable validity as it assumes realistic entrepreneurs possess projects with no potential for loss. Nonetheless, this paper also shows empirically, using French data, that overconfident managers do tend to prefer short term finance. In an interesting experimental paper, Camerer and Lovallo (1999) show that individuals enter skill-based competition when it is against their best interest and demonstrate the concept of 'reference group neglect'. This concept refers to situations in which individuals excessively enter skill-based competition even when they know the group has self-selected with the knowledge that the competition would be skill-based. This model provides empirical justification for using overconfidence in a formal model.

3.3 The model

A population of entrepreneurs are assumed to have access to an indivisible project requiring investment I. Entrepreneurs' initial investable wealth, A, is assumed to be small enough such that the firm requires external financing to proceed with the project $(I - A > 0)$. Both the return to their next best employment alternative and the alternative return to the investable assets A are normalised to zero. The alternative return on funds for the lender is also normalised to zero.

The project returns R upon success, with $R > I$, and 0 upon failure. The return R is then split into a repayment R_l which is returned to the lender and profit R_b which is returned to the firm. The central feature of this model is that entrepreneurs have systematically higher estimates of their probabilities of success than the estimates made by lenders. The borrower's assessment of the probability of success is denoted by p_b and the lender's assessment of the probability of success is denoted by p, with $p_b > p$. In the welfare analysis, it is assumed that lenders know the true probability of success.

Entrepreneurs first have the choice between applying and not applying to the lender for financing. If the firm applies, the lender either offers a contract (R_l, R_b), which is either accepted or rejected by the entrepreneur, or the lender declines to offer a lending contract. Situations in which the firm applies for financing and then finds the terms of the contract unacceptable, or the lender declines to offer a contract to the firm, are interpreted as credit rationing. If the firm applies for financing, they do so with the expectation that the lender will come to the same assessment of the likelihood of success. This causes the firm to have a different expectation of the structure of the lending contract prior to applying, compared to the one actually offered by the lender. The amounts which the firm expects to go to the lender and themself respectively, prior to applying, are denoted as R_l^* and R_b^*. Lenders are competitive, both firms and lenders are risk neutral, and there are no transaction costs.

An important assumption is that firms do not update their probability of success upon viewing the lending contract. This is interpreted as the firm manager viewing the bank's assessment as inferior, consistent with the idea of overconfidence. Partial updating would retain the main results of this model but full updating would result in all overconfident firms with poor projects failing to invest following application.

The variables in the model are summarised in Table 2.

Table 2 – Summary of variables in overconfidence model.

Variable	**Description**
I	The required investment in order to proceed with the project.
A	The entrepreneur's initial assets able to be invested in the project.
R	Total return to the project upon success.
R_l	The share of R specified in the lending contract that goes to the lender.
R_b	The share of R specified in the lending contract that goes to the borrower.
R_l^*	The share of R the entrepreneur expects to go to the lender prior to applying.
R_b^*	The share of R the entrepreneur expects to go to the borrower prior to applying.
p	The lender-assessed probability of success.
p_b	The borrower-assessed probability of success.

Proposition 1

Firms will apply for financing if they believe they have a positive net present value (NPV) project.

The firm applies for financing using their own probability assessment to determine what they expect the lending contract to be. Since firms must expect to gain a payoff larger than consuming their assets, the rationality constraint for the firm to apply is

$$p_b R_b^* \geq A. \tag{22}$$

Since lenders are competitive, the expected profit the entrepreneur anticipates the lender to gain must be zero:

$$p_b R_l^* - (I - A) = 0. \tag{23}$$

Combining conditions (22) and (23) yields the result

$$p_b \left(R - \frac{I - A}{p_b} \right) \geq A,$$

$$\Leftrightarrow p_b R - I \geq 0. \tag{24}$$

So the firm will apply for financing if it believes the net present value to the project is positive.

Proposition 2

Lending always occurs if both firms and lenders believe the firm has a positive net present value (NPV) project.

The previous result shows that the firm will apply if it believes it has a positive NPV project. The lender offers a contract based on the zero profit condition:

$$pR_l - (I - A) = 0. \tag{25}$$

R_l is obtained from (25) and $R_b \equiv R - R_l$. Thus, the contract that the lender presents to the firm is

$$(R_l, R_b) = \left(\frac{I - A}{p}, \frac{pR - (I - A)}{p} \right). \tag{26}$$

Since the contract is acceptable to the lender by construction, to show that lending occurs we simply need to show that this contract is acceptable to the borrower. This is true if

$$p_b R_b \geq A,$$

$$\Leftrightarrow p_b \frac{pR - (I - A)}{p} \geq A.$$

And, replacing p_b with its least value from (24) (note that this includes both pessimistic and overconfident borrowers), the previous statement becomes

$$\Leftarrow \frac{I}{R} \frac{pR - (I - A)}{p} \geq A,$$

$$\Leftrightarrow I\big(pR - (I - A)\big) \geq ApR,$$

$$\Leftrightarrow (I - A)pR - I(I - A) \geq 0,$$

$$\Leftrightarrow (I - A)(pR - I) \geq 0.$$

So, when $p_b R - I > 0$ and $pR - I > 0$, this is true. Despite overconfident borrowers obtaining less profit than they expected to upon application, the project nonetheless goes ahead if both parties agree that it is profitable.

Interesting outcomes occur when the firm believes it has a positive NPV project and the lender believes the firm has a negative NPV project. In what follows, it is assumed that $p_b R - I > 0$ and $pR - I < 0$.

Proposition 3

The lender declines to offer financing if the firm's initial investable wealth is less than the expected loss to the project divided by the probability of failure or $A < \frac{I - pR}{1 - p}$.

From (26), the best contract return the borrower is able to obtain is

$$R_b = \frac{pR - (I - A)}{p}.$$

The lender will decline to offer financing if it is sure the contract will be unacceptable to the borrower[8], no matter what their assessed probability of success. The contract is unacceptable to the borrower if

$$p_b R_b < A.$$

The lender does not know the firm's degree of overconfidence; however, the bank can place the upper bound of certainty on the borrower's probability assessment. Assuming a certain borrower with $p_b = 1$, the contract will be unacceptable if

$$\frac{pR - (I - A)}{p} < A,$$

$$\Leftrightarrow A < \frac{I - pR}{(1 - p)}. \qquad (27)$$

Thus, an overconfident firm requires some level of self-financing in order for the lender to offer them a contract.

Proposition 4

The firm will decline the lenders offer for financing if the level of overconfidence is such that: $p_b < \frac{pA}{A-(I-pR)}$.

Given a contract is offered by the lender, the contract will be unacceptable to the borrower if

$$p_b R_b < A,$$

[8] In this formulation, the lender is actually indifferent between offering a contract and not. Introducing an infinitesimal cost of offering the contract is enough to guarantee the lender will decline the application.

$$\Leftrightarrow p_b\left(\frac{pR-(I-A)}{p}\right)<A. \qquad (28)$$

In order to solve for p_b, it must be confirmed that the left hand side of (28) is positive. Since a contract is offered, condition (27) must be false:

$$A \geq \frac{I-pR}{1-p},$$

$$\Leftrightarrow A \geq I-pR+pA,$$

$$\Leftrightarrow pR \geq I-A+pA,$$

$$\Rightarrow pR \geq I-A.$$

Thus, the left hand side of (28) is non-negative, immediately yielding the result

$$p_b<\frac{pA}{A-(I-pR)}. \qquad (29)$$

The interpretation of this situation is that the borrower views the contract on offer as both unfair and unacceptable and the entrepreneur is left better off (in belief) by consuming the assets A. It is necessary to confirm that there exists a borrower's probability assessment that is both high enough to induce the initial application for financing and low enough to satisfy condition (29). There exists a p_b that satisfies both (24) and (29) if

$$\frac{I}{R}<\frac{pA}{A-(I-pR)}.$$

This holds true if

$$I>A, \qquad I-pR>0,$$

both of which are assumptions of the model. The proof is given in Appendix 3.

Corollary 1

The firm will accept the lenders offer for financing if they have sufficient investable wealth and optimism so that $A \geq \frac{I-pR}{(1-p)}$ *and* $p_b \geq \frac{pA}{A-(I-pR)}$.

This follows directly from propositions 3 and 4. If condition (27) is violated the lender will offer a lending contract and the lending contract will be accepted so long as condition (29) also does not hold. Note that the violation of condition (27) also implies the right hand side of condition (29) is less than or equal to one so there does exist a level of overconfidence that allows lending to occur.

3.4 Welfare and overall investment

Overall welfare can be calculated as the sum of consumer surplus, producer profit, and payments to resource holders. In what follows, it is assumed for simplicity that consumer surplus is zero, that is, firms are able to extract all surplus from consumers.

Firstly note that, if an investment I would otherwise not be put to more productive use, a firm investing causes an increase in overall welfare, even if the NPV of the project is negative. To see this, note that the expected profit for a project is $pR - I$ and the payments to resource holders is I, making the overall change in welfare $pR > 0$. Thus, the value of the total expected production will be the measure of welfare used.

Consider an economy in which only genuinely positive NPV projects exist and there is no overconfidence. When overconfidence is introduced, three effects occur. Firstly, firms with genuinely negative NPV projects begin to invest, causing an increase in investment and production. Secondly, as more projects are invested in, demand for resources (such as labour) increases, increasing costs for all firms and causing marginal realistic firms to not invest. And thirdly, as more individuals become entrepreneurs, more individuals withdraw from the labour workforce, resulting in a decrease in labour supply. This again increases costs for all firms and causes marginal realistic firms to not invest.

If the production gained from the new investment outweighs the production lost from other projects no longer going ahead, an overall increase in welfare will result. If an economy's resources are fully employed in positive NPV projects, introducing overconfident firms with negative NPV projects will divert resources to less productive uses. Thus, in the full-employment scenario, overconfidence reduces welfare. However, in a more realistic economy where there is unemployment, introducing overconfident firms can potentially increase the total use of resources and result in higher production.

An attempt was made to formally demonstrate the overall equilibrium effect of introducing overconfidence in the presence of unemployment. In order to capture the important effects from the previous section, firms varied by initial wealth A and self-assessed probability of success p_b. In addition, in order to include both firms with positive and negative NPV projects, firms varied by p. The variables $p_b, p,$ and A were uniform distributed and independent, labour was the only resource, and the cost of a unit of labour was assumed fixed. An expression for welfare was obtained using Maple which ran 6 screens long and is thus omitted. At the end of this section is a summary of the method employed. All sample numerical increases in the level of overconfidence, holding other parameters constant, resulted in an *increase* in overall welfare. This result is driven by the assumption that the cost of labour is fixed, regardless of the level of employment. More realistic modelling of the labour market is required to show that overconfidence can still decrease welfare in the presence of unemployment.

In order to investigate the effects of other parameters on the level of investment within a group of overconfident firms, a simpler economy is set up where attention is limited to only overconfident firms with negative NPV projects.

Suppose there is a density of individuals with access to identical projects who vary by overconfidence and initial wealth. Individuals are able to become entrepreneurs or workers; entrepreneurs invest and employ all non-entrepreneurs. Labour is the only resource and the required investment I is interpreted as a fixed amount of wages that is distributed evenly to a firm's share of workers. The total external labour (excluding the entrepreneur) required to complete the firm's project is assumed to be fixed and provided collectively by the firm's share of workers (to capture unemployment, labour capacity constraints are assumed to be non-binding). The wage rate per unit of labour is also assumed to be fixed. This implies that more firms operating will increase the amount of labour each worker provides and, subsequently, their total wages. It is assumed there is no opportunity cost of labour other than entrepreneurship itself.

Let the proportion of entrepreneurs in the population be λ. The wages per worker is then[9] $I\lambda/(1-\lambda)$. It is assumed that A is uniform distributed on $[0,\bar{A}]$ and p_b is

[9] Let the number of entrepreneurs be n_e and the number of workers n_w. The wages paid by each firm is I so the rate paid per worker is $n_e I/n_w$. $\frac{\lambda}{1-\lambda}=\frac{\frac{n_e}{n_e+n_w}}{1-\frac{n_e}{n_e+n_w}}=\frac{n_e}{n_w}$.

uniform distributed on $[\underline{p}_b, 1]$ and both variables are independent. It is assumed there are some individuals who do not apply for financing in equilibrium $\left(\underline{p}_b R - I - \frac{\lambda}{1-\lambda} I < 0\right)$ and there are no realists or pessimists $(\underline{p}_b > p)$. The opportunity cost of entrepreneurship was assumed to be zero in the previous section. Because the opportunity cost of entrepreneurship is now the value of wages per worker, each of the above propositions must be modified.

Proposition 1a

Firms will apply for financing if they believe they have a positive net present value (NPV) project, net of their opportunity cost of investment.

The condition for application is modified to

$$p_b R_b^* \geq A + \frac{\lambda}{1-\lambda} I. \tag{30}$$

Again, the firm will expect their share of the repayment to be based on the bank's zero profit condition but using the firm's probability assessment. The above condition then becomes

$$p_b \left(R - \frac{I-A}{p_b}\right) \geq A + \frac{\lambda}{1-\lambda} I,$$

$$\Leftrightarrow p_b R - I - \frac{\lambda}{1-\lambda} I \geq 0. \tag{31}$$

This has the same interpretation as earlier. The firm will apply if it believes the net present value to the project, net of their opportunity cost, is positive. The difference with the previous proposition simply exists due to being unable to normalise the opportunity cost to zero.

Proposition 2a

Lending occurs if both lenders and firms believe the firm has a positive net present value (NPV) project net of the firm's opportunity cost.

Lenders believe the firm has a positive NPV project net of their opportunity cost if $pR - I - \frac{\lambda}{1-\lambda}I > 0$. As previously shown, the firm applies for financing if they believe they have a positive NPV project net of the opportunity cost of entrepreneurship. They will accept the contract if the actual contract offered obeys their rationality constraint:

$$p_b R_b \geq A + \frac{\lambda}{1-\lambda}I,$$

$$\Leftrightarrow p_b \frac{pR - (I - A)}{p} \geq A + \frac{\lambda}{1-\lambda}I.$$

And, replacing p_b with its least value from (31), the previous statement becomes

$$\Leftrightarrow \frac{I}{R}\frac{1}{1-\lambda}\frac{pR - (I - A)}{p} \geq A + \frac{\lambda}{1-\lambda}I,$$

$$\Leftrightarrow I\big(pR - (I - A)\big) \geq pR(1-\lambda)A + pR\lambda I,$$

$$\Leftrightarrow pR(1-\lambda)(I - A) - I(I - A) \geq 0,$$

$$\Leftrightarrow (pR(1-\lambda) - I)(I - A) \geq 0,$$

$$\Leftrightarrow \left(pR - \frac{I}{1-\lambda}\right)(I - A) \geq 0,$$

$$\Leftrightarrow \left(pR - I - \frac{\lambda}{1-\lambda}I\right)(I - A) \geq 0.$$

This is true for firms for which the lender believes the firm has a positive NPV project, net of the firm's opportunity cost, showing the proposition is true. Note that this result also extends to pessimistic individuals who still believe they have a positive NPV project net of their opportunity cost. As was the case earlier, interesting cases occur when the lender believes the firm has a negative NPV project net of the firm's opportunity cost and the firm believes they have a positive NPV project net of their opportunity cost. Thus, it is assumed that $pR - I - \frac{\lambda}{1-\lambda} I < 0$ and $p_b R - I - \frac{\lambda}{1-\lambda} > 0$.

Proposition 3a

The lender declines to offer financing if the firm's initial investable wealth is such that $A < \frac{p\frac{\lambda}{1-\lambda}I + I - pR}{(1-p)}$.

By the zero profit constraint, the best contract return the borrower is able to obtain is

$$R_b = \frac{pR - (I - A)}{p}.$$

And, as before, the lender won't offer financing if it knows the contract will be unacceptable to the borrower. The contract is unacceptable to the borrower if

$$p_b \frac{pR - (I - A)}{p} < A + \frac{\lambda}{1-\lambda} I.$$

And, assuming a certain (infinitely overconfident) borrower, the contract will be unacceptable if

$$pR - (I - A) < pA + p\frac{\lambda}{1-\lambda} I,$$

$$\Leftrightarrow pR - I + A < pA + p\frac{\lambda}{1-\lambda} I,$$

$$\Leftrightarrow A(1-p) < p\frac{\lambda}{1-\lambda}I + I - pR,$$

$$A < \frac{p\frac{\lambda}{1-\lambda}I + I - pR}{(1-p)}. \tag{32}$$

Proposition 4a

The firm will decline the lenders offer for financing if the level of overconfidence is such that: $p_b < \frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}$.

Given a contract is offered by the lender, the contract will be unacceptable to the borrower if

$$p_b R_b < A + \frac{\lambda}{1-\lambda}I,$$

$$\Leftrightarrow p_b\left(\frac{pR-(I-A)}{p}\right) < A + \frac{\lambda}{1-\lambda}I.$$

Since a contract is offered, condition (32) is false implying the left hand side of the above inequality is non-negative, immediately yielding the result

$$\Leftrightarrow p_b < \frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}. \tag{33}$$

And again, to confirm there exists a level of overconfidence where the firm both applies and finds the offered contract unacceptable the following inequality must hold:

$$\frac{I\left(1+\frac{\lambda}{1-\lambda}\right)}{R} < \frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}.$$

This holds true if

$$I > A, \qquad I - pR + \frac{\lambda}{1-\lambda} I > 0,$$

both of which are assumptions of the model. The proof is similar to that given in Appendix 3.

A formula for the proportion of entrepreneurs in the population λ will now be derived. From proposition 4a, the marginal entrepreneur satisfies

$$p_b = \frac{p\left(A + \frac{\lambda}{1-\lambda} I\right)}{A + pR - I} > p. \qquad (34)$$

This is a negative relationship[10] between p_b and A. The lower initial assets are, the higher the overconfidence threshold for accepting the lending contract is. The range for the A variable for which individuals become entrepreneurs is from A^*, such that $p_b(A^*) = 1$, to $\bar{A}$. Rearranging (34) with $p_b = 1$, A^* is found to be

$$A^* = \frac{p\frac{\lambda}{1-\lambda} I + I - pR}{(1-p)}.$$

When $A = \bar{A}$, p_b is

$$p_b = \frac{p\left(\bar{A} + \frac{\lambda}{1-\lambda} I\right)}{\bar{A} + pR - I}.$$

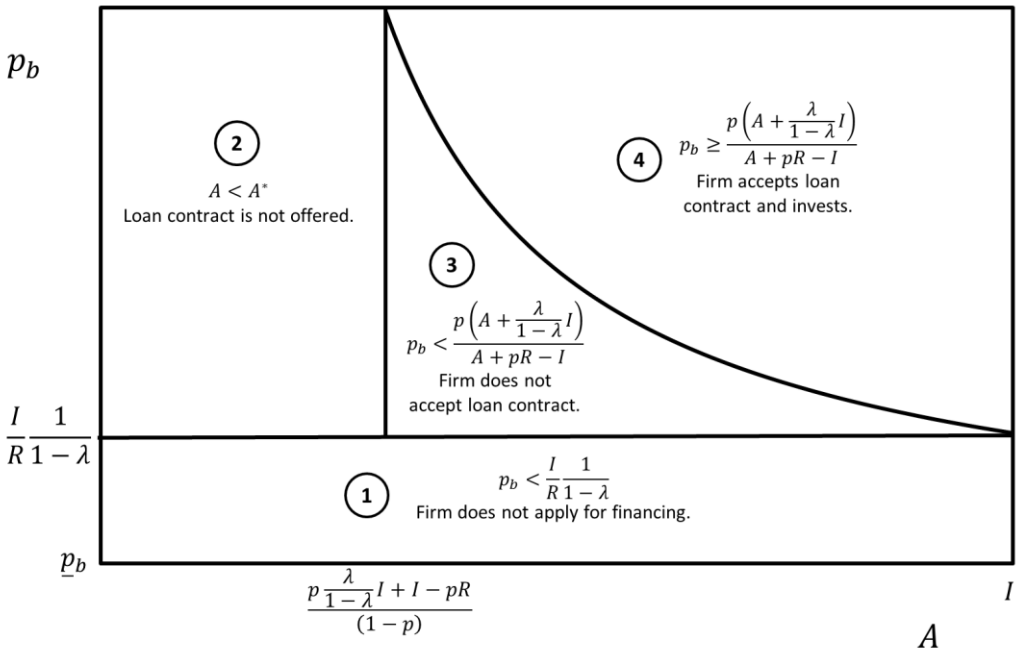
When $A = \bar{A} = I$, p_b is

$$p_b = \frac{I}{R(1-\lambda)}.$$

[10] The numerator of dp_b/dA is: $p(A + pR - I) - p\left(A + \frac{\lambda}{1-\lambda} I\right) = p\left(pR - I - \frac{\lambda}{1-\lambda} I\right) < 0$

These relationships are graphed in Figure 2 below with $\bar{A} = I$.

Figure 2 – Solution regions for economy of individuals with negative NPV projects with $\bar{A} = I$.



Area 1 in Figure 2 represents those firms who do not apply for financing as they have negative self-assessed NPV. Area 2 represents those firms who are immediately denied financing as there is no level of overconfidence which would make an offered loan contract acceptable. Area 3 represents those firms who are offered a contract but do not accept. Area 4 represents those firms who do accept the offered contract and invest. Area 4 divided by the total area of the plot yields the proportion of entrepreneurs λ. For $\bar{A} \leq I$, this can be calculated as the solution to

$$\frac{1}{\bar{A}(1-\underline{p}_b)} \int_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+I-pR}{(1-p)}}^{\bar{A}} \int_{\frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}}^{1} dp_b\, dA = \lambda. \qquad (35)$$

Calculating the inner integral yields

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\int_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+I-pR}{(1-p)}}^{\bar{A}}\left[1-\frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}\right]dA=\lambda.$$

Using the substitution $u = A + pR - I$ and simplifying yields

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\int_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+p(I-pR)}{(1-p)}}^{pR+\bar{A}-I}\left[1-\frac{p\left(u-pR+I+\frac{\lambda}{1-\lambda}I\right)}{u}\right]du=\lambda,$$

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\int_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+p(I-pR)}{(1-p)}}^{pR+\bar{A}-I}\left[1-p-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)u^{-1}\right]du=\lambda,$$

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\left[(1-p)u-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln(u)\right]_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+p(I-pR)}{(1-p)}}^{pR+\bar{A}-I}=\lambda,$$

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\left[\left[(1-p)(pR+\bar{A}-I)-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln(pR+\bar{A}-I)\right]\right.$$

$$-\left[pI\left(\frac{\lambda}{1-\lambda}\right)+p(I-pR)\right.$$

$$\left.\left.-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln\left(\frac{p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)}{(1-p)}\right)\right]\right]=\lambda,$$

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\left[\left[(pR+\ \bar{A}-I)-p\bar{A}-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln(pR+\ \bar{A}-I)\right]\right.$$
$$-\left[pI\left(\frac{\lambda}{1-\lambda}\right)\right.$$
$$\left.\left.-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln\left(\frac{p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)}{(1-p)}\right)\right]\right]=\lambda,$$

$$\frac{1}{\bar{A}(1-\underline{p}_b)}\left[(pR+\ \bar{A}-I-p\bar{A})\right.$$
$$-p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln\left(\frac{(pR+\bar{A}-I)(1-p)}{p\left(I-pR+\frac{\lambda}{1-\lambda}I\right)}\right) \tag{36}$$
$$\left.-pI\left(\frac{\lambda}{1-\lambda}\right)\right]-\lambda=0.$$

And when $\bar{A}=I,\lambda$ satisfies

$$\frac{1}{I(1-\underline{p}_b)}\left[R-I-I\left(\frac{\lambda}{1-\lambda}\right)\right.$$
$$\left.+\left(I-pR+\frac{\lambda}{1-\lambda}I\right)\ln\left(\frac{(1-p)R}{I-pR+\frac{\lambda}{1-\lambda}I}\right)\right]-\lambda=0. \tag{37}$$

There is no closed form solution for λ for the above equation. Solutions can easily be found numerically for given p,I,R using a numerical solver. Appendix 4 justifies why there is a unique real solution to this equation that satisfies the assumptions of the model.

To sum up, the most wealthy and overconfident individuals are those that end up becoming entrepreneurs. Those that enter entrepreneurship effectively transfer part of their initial wealth to workers with the process enabled by predatory lending (lending which is, in reality, harmful to the borrower). Because the firms do have positive production, an overall welfare gain results from overconfidence when there are no realists. The expected final wealth for an entrepreneur with initial wealth A_e is

$$pR_b,$$

and substituting from (26),

$$= p\frac{pR - (I - A_e)}{p},$$

$$= A_e - (I - pR).$$

So, in reality, entrepreneurs can expect to lose the expected loss to the project. The final wealth for a worker with initial wealth A_w is

$$A_w + \frac{\lambda}{1-\lambda}I.$$

Thus, workers gain the wages per worker $\frac{\lambda}{1-\lambda}I$. The average gain in welfare is the same as the average production per person:

$$W = \lambda(pR - I) + (1-\lambda)\frac{\lambda}{1-\lambda}I = \lambda(I, p, R)pR. \qquad (38)$$

Thus, despite the projects not being economic for the entrepreneurs, the projects have social value as there is still a positive probability of success.

3.5 Comparative statics

In order to investigate how the parameters alter total welfare in an economy of overconfident firms, the boundaries of the region in p_b, A space that represents the population must be fixed at $[\underline{p}_b, 1] \times [0, \bar{A}]$ so the λ derivatives must be calculated using (36). The expressions for several of the derivatives are quite long and are thus omitted. For those derivatives which are omitted, the sign of the expression was determined numerically. The left hand side of (36) is denoted as $F(\lambda, I, p, R)$.

The effect on average welfare of an increase in the probability of success of all projects is

$$\frac{\partial W}{\partial p} = \frac{\partial \lambda}{\partial p} pR + \lambda(I, p, R)R,$$

$$= -\frac{\frac{\partial F}{\partial p}}{\frac{\partial F}{\partial \lambda}} pR + \lambda(I, p, R)R > 0.$$

The effect of the true probability of success on welfare comes from two sources. Firstly, the lender offers contracts with lower repayments and to more potential borrowers, causing more borrowers to find the contract acceptable. However, as more firms enter, the opportunity cost of entrepreneurship increases, somewhat mitigating the increase. Note that this doesn't affect the firm's own assessments of their probability of success, only that of the lender, and thus the proportion of firms applying for financing is only affected by the change in the opportunity cost.

The effect of increasing the cost of investment (wages) on welfare is

$$\frac{\partial W}{\partial I} = -\frac{\frac{\partial F}{\partial I}}{\frac{\partial F}{\partial \lambda}} pR \Bigg|_{\bar{A}=I,\ \underline{p}_b=p},$$

$$= -\frac{(1-\lambda)p\left(R(1-\lambda) + pR\ln\left(\frac{(1-p)R}{I - pR + \frac{\lambda}{1-\lambda}I}\right) - I\right)}{I\left(\ln\left(\frac{(1-p)R}{I - pR + \frac{\lambda}{1-\lambda}I}\right) + (1-\lambda)^2(1-p)\right)} < 0.$$

The result for the cost of investment also comes from two sources. Firstly, increasing the cost of investment directly increases costs for firms so fewer firms will apply for financing and fewer firms will find the offered contract acceptable. Also, because wages increase, the opportunity cost of investment increases, further decreasing entry into entrepreneurship.

The effect of increasing the return to the project upon success on welfare is

$$\frac{\partial W}{\partial R} = \frac{\partial \lambda}{\partial R} pR + \lambda(I,p,R)p,$$

$$= -\frac{\frac{\partial F}{\partial R}}{\frac{\partial F}{\partial \lambda}} pR \Bigg|_{\bar{A}=I,\ \underline{p}_b=p} + \lambda(I,p,R)p,$$

$$= \frac{p}{I\left(p\ln\left(\frac{(1-p)R}{I - pR + \frac{\lambda}{1-\lambda}I}\right) + (1-p)(1-\lambda)^2\right)}\left(p\ln\left(\frac{(1-p)R}{I - pR + \frac{\lambda}{1-\lambda}I}\right)(pR(1\right.$$

$$\left. -\lambda)^2 + \lambda I) + (1-\lambda)^2\left(p\left(R - \frac{1}{1-\lambda}I\right) + I\lambda(1-p)\right)\right) > 0.$$

The project return upon success R affects overall welfare in a similar way to the probability of success. Again, the lender offers contracts with lower repayments and to more potential borrowers, causing more borrowers to find the contract acceptable. However, increasing R also directly increases the number of individuals who apply for financing, further increasing investment. And again, as more firms enter, the opportunity cost of entrepreneurship increases, mitigating the increase in investment as with the probability of success.

The degree of credit rationing is another important empirical statistic so it is interesting to investigate how the parameters affect this. Credit rationing can be affected both by the proportion of applicants being credit rationed and the change in the proportion of individuals applying for financing. The proportion of firms applying is gained by taking the total of areas 2, 3, and 4 and dividing by the total area. The proportion of firms credit rationed, denoted by α, is then gained by subtracting the proportion of entrepreneurs λ:

$$\alpha = \frac{\bar{A}\left(1 - \frac{I}{R(1-\lambda)}\right)}{\bar{A}\left(1 - \underline{p}_b\right)} - \lambda,$$

$$\alpha = \frac{1}{1 - \underline{p}_b} - \frac{1}{1 - \underline{p}_b} * \frac{I}{R(1-\lambda)} - \lambda,$$

The various derivatives can be calculated again using (36).

$$\frac{\partial \alpha}{\partial I} = -\frac{1}{1 - \underline{p}_b} * \frac{1}{R(1-\lambda)} - \frac{1}{1 - \underline{p}_b} * \frac{I \frac{\partial \lambda}{\partial I}}{R(1-\lambda)^2} - \frac{\partial \lambda}{\partial I} > 0.$$

The effect of I on the equilibrium level of credit rationing works in several ways. Given a firm has applied for financing, an increase in I increases the repayment required by the lender, causing the borrower to be less likely to find the loan contract acceptable, increasing credit rationing. However, the increase in I makes the application decision less attractive so fewer borrowers apply for financing in the first place, decreasing credit rationing. The reduction in the total proportion of individuals being entrepreneurs also reduces the opportunity cost to entrepreneurship, making entrepreneurship more attractive and making it less likely for firms to reject the lender's offer for financing as well as increasing the number of individuals applying. The positive result is interesting here as it shows that, using the uniform distribution, all these competing effects net out to be overall positive.

$$\frac{\partial\alpha}{\partial R} = \frac{I}{(1-\underline{p}_b)R^2(1-\lambda)} - \frac{I\frac{\partial\lambda}{\partial R}}{(1-\underline{p}_b)R(1-\lambda)^2} - \frac{\partial\lambda}{\partial R} \lesseqgtr 0.$$

The effect of increasing the project return on equilibrium credit rationing is positive for low values of pR and negative for high values of pR. Firstly, when the project return increases, the proportion of firms applying for financing increases, increasing the proportion of firms being credit rationed. Since the required repayment decreases, the proportion of applicants finding the lending contract acceptable will increase, decreasing the proportion of firms being credit rationed.

When pR is low, the proportion of firms applying for financing is low and increasing R will cause a large increase in the proportion of applicants and a smaller increase in the number of firms finding the lending contract acceptable, thus increasing credit rationing. Once R is high enough, the increase in the proportion of firms applying for financing slows and the increase in the number of firms accepting the contract begins to dominate, causing the proportion of rationed firms to decrease.

$$\frac{\partial\alpha}{\partial p} = -\frac{I\frac{\partial\lambda}{\partial p}}{(1-\underline{p}_b)R(1-\lambda)^2} - \frac{\partial\lambda}{\partial p} < 0.$$

As would be intuitively expected, increasing the objective probability of success will reduce the incidence of credit rationing due to overconfidence as the lending contract becomes more attractive to the lender. The proportion of firms applying remains constant so all of the effect is due to more lending contracts being accepted.

In order to capture the effect of overconfidence on diverting resources from more productive projects, the economy must be set up so that firms with good projects can potentially be displaced by firms with poor projects. Thus, firms are varied by probability of success in order to include individuals who have both positive and negative NPV projects. As more firms with poor projects invest due to overconfidence, more resources can potentially be diverted from genuinely good projects.

Now, p_b, p, and A vary uniformly across $[\underline{p}_b, \bar{p}_b] \times [\underline{p}, \bar{p}] \times [0, \bar{A}]$. The parameters are chosen such that both optimistic and pessimistic firms exist in the economy. The proportion of entrepreneurs λ can then be determined as the solution to

$$\frac{1}{\bar{A}(\bar{p}-\underline{p})(\bar{p}_b-\underline{p}_b)}\left(\int_{\underline{p}}^{\frac{I}{R}\frac{1}{1-\lambda}}\int_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+I-pR}{(1-p)}}^{\bar{A}}\int_{\frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}}^{\bar{p}_b}dp_b\,dA\,dp\right.$$

$$\left.+\bar{A}\left(\bar{p}-\frac{I}{R}\frac{1}{1-\lambda}\right)\left(\bar{p}_b-\frac{I}{R}\frac{1}{1-\lambda}\right)\right)=\lambda,$$

where the integral represents all firms with negative NPV projects that are able to obtain financing and the product represents all firms sufficiently optimistic to apply for financing who have positive NPV projects. The total represents all firms who engage in their projects.

Total welfare is then represented by the integral of pR across all entering entrepreneurs with respect to p:

$$W=\frac{1}{\bar{A}(\bar{p}-\underline{p})(\bar{p}_b-\underline{p}_b)}\int_{\underline{p}}^{\frac{I}{R(1-\lambda)}}pR\left(\int_{\frac{pI\left(\frac{\lambda}{1-\lambda}\right)+I-pR}{(1-p)}}^{\bar{A}}\int_{\frac{p\left(A+\frac{\lambda}{1-\lambda}I\right)}{A+pR-I}}^{\bar{p}_b}dp_b\,dA\right.$$

$$\left.+\bar{A}\left(\bar{p}_b-\frac{I}{R(1-\lambda)}\right)\right)dp.$$

As aforementioned, sample numerical derivatives of this expression with respect to $\bar{p}_b$ and $\underline{p}_b$ showed that increasing overconfidence (an increase in $\bar{p}_b$ or $\underline{p}_b$) increases welfare in this economy.

This result is driven by the assumption that the cost of labour is fixed, regardless of the level of employment. To see this, consider an individual who becomes an entrepreneur due to a small increase in overconfidence. This individual is removed from the labour force and also employs a share of workers. To compensate for the loss of workers, the amount of labour provided by each individual increases. However, the total cost of labour per firm does not increase as wages per unit of labour are assumed to be fixed.

The only effect that can drive down investment following the entry of this new firm is the increase in wages per worker as this is the opportunity cost of entrepreneurship. However, if a single entrepreneur with a genuinely good project now finds it better to cease entrepreneurship and enter the workforce, the opportunity cost would return to the original value. Thus, the number of firms that enter must outnumber the number of firms that exit as a result of an increase in overconfidence. The trialled numerical derivatives also suggest that the total production gained also outnumbers the total production lost when using the uniformly distributed population as in this section. Incorporating an increasing labour supply curve and general population distributions into this analysis is a task for future work that will yield conditions for increases in overconfidence causing either increases or decreases in total welfare.

3.6 Collateral

In this section overconfidence is shown to be a justification for collateral. Firms have the option to offer collateral C, with maximum value $\bar{C}$, on their loan contract which is interpreted as illiquid assets which are worth more to the entrepreneur than to the lender. The firm's value of collateral is C and the lender's value of collateral is βC with $0 < \beta < 1$. Firms apply expecting a continuum of contracts (R_l^*, C^*) to be offered by the lender. Following application, the lender either offers a continuum of acceptable contracts (R_l, C) or declines to offer a contract. The entrepreneur then chooses their optimal lending contract, or declines the lender's offer for financing. All other notation is unchanged.

Proposition 1 still holds if collateral is available. The rationality constraint for the firm's application decision is now

$$p_b R_b^* + (1 - p_b)(-C^*) \geq A.$$

The firm anticipates the lender to require zero profit so the contract satisfies the zero profit constraint:

$$p_b R_l^* + (1 - p_b)\beta C^* = I - A.$$

Therefore R_b^* satisfies

$$R_b^* = R - R_l^* = \frac{p_b R - (I - A - (1 - p_b)\,\beta C^*)}{p_b}.$$

Substituting this into the firm's rationality constraint yields

$$p_b R - I - (1 - p_b)(1 - \beta)C^* \geq 0.$$

The firm will also anticipate that they will choose the optimal contract which has $C^* = 0$ so their application constraint is unchanged from that in proposition 1:

$$p_b R - I \geq 0.$$

Proposition 2 also holds if collateral is available. The offered contract set obeys the zero profit condition:

$$p_b R_l + (1 - p_b)\beta C = I - A. \tag{39}$$

Thus, the relationship between R_l and C in the contract set is

$$R_l = \frac{(I - A - (1-p)\beta C)}{p}, \tag{40}$$

with $C \in [0, \bar{C}]$. The firm will accept if any contract satisfies their rationality constraint for acceptance. So setting $C = 0$[11] immediately yields the result from proposition 2.

Proposition 3b

The lender declines to offer financing if the firm's initial investable wealth and available collateral are such that $A + \beta\bar{C} < \frac{I-pR}{(1-p)}$.

The lender will decline to offer financing if there is no level of overconfidence which will make the most attractive available contract acceptable to the borrower. The firm's rationality constraint for the acceptance of the lending contract (R_l, C) is

$$p_b R_b + (1 - p_b)(-C) \geq A. \tag{41}$$

Substituting in (40), (41) becomes

$$\Leftrightarrow p_b\left(R - \frac{I - A - (1-p)\beta C}{p}\right) + (1 - p_b)(-C) \geq A,$$

$$\Leftrightarrow p_b\big(pR - (I - A - (1-p)\beta C)\big) + p(1 - p_b)(-C) \geq pA,$$

$$\Leftrightarrow p_b pR - p_b I + p_b A + p_b(1-p)\beta C + p(1 - p_b)(-C) \geq pA,$$

[11] Note that this is not necessarily the actual contract chosen.

$$\Leftrightarrow (p_b - p)A + \big(p_b(1-p)\beta - p(1-p_b)\big)C \geq p_b(I - pR). \qquad (42)$$

The firm chooses C to maximise the left hand side of this expression. Thus, for collateral to appear in the most attractive contract, the coefficient on C here must be positive. This is true for firms whose liquidation value of the collateral satisfies

$$\beta \geq \frac{p(1-p_b)}{p_b(1-p)}. \qquad (43)$$

Given condition (43), the entrepreneur will choose the maximum collateral $\bar{C}$. Under this model, collateral can be arbitrarily high as long as R_l is allowed to be negative. That is, a nonstandard lending contract is possible where the entrepreneur is first financed by the bank and the bank gives the entrepreneur further funds upon success while the entrepreneur hands over a large amount of collateral (larger than the size of the loan) upon failure. Note that (43) also implies that the most overconfident firms (with $p_b \sim 1$) will always make use of collateral as long as it has some positive value to the lender ($\beta > 0$) and the least overconfident firms (with $p_b \sim p$) will not make use of collateral as $\beta < 1$.

Since those with the highest level of overconfidence will find it optimal to use collateral, the most overconfident firms will reject all contracts if condition (42) fails for $p_b = 1$:

$$(1-p)A + (1-p)\beta\bar{C} < (I - pR),$$

$$\Leftrightarrow A + \beta\bar{C} < \frac{I - pR}{1-p}. \qquad (44)$$

Proposition 4b

The firm will decline the lenders offer for financing if the level of overconfidence is such that: $p_b < \min\left\{\frac{pA}{A+pR-I}, \frac{p(A+\bar{C})}{\left(pR-I+A+\left(\beta+p(1-\beta)\right)\bar{C}\right)}\right\}$.

The condition that determines which element of the minimum is active is shown below:

$$\frac{pA}{A+pR-I} < \frac{p(A+\bar{C})}{\left(pR-I+A+\left(\beta+p(1-\beta)\right)\bar{C}\right)},$$

$$\Leftrightarrow \frac{pA}{(A-I+pR)} < \frac{pA+p\bar{C}}{(A-I+pR)+\left(\beta+p(1-\beta)\right)\bar{C}},$$

$$\Leftrightarrow pA\left(\beta+p(1-\beta)\right)\bar{C} < p\bar{C}(A-I+pR),$$

$$\Leftrightarrow A(\beta+p(1-\beta)-1) < -I+pR,$$

$$\Leftrightarrow I-pR < A(1-\beta)(1-p),$$

$$\Leftrightarrow A > \frac{I-pR}{(1-\beta)(1-p)}. \qquad (45)$$

For firms which fail condition (43), proposition 4 holds. It must be shown that this is consistent with the inequality in the proposition. The first element of the minimum is active for this firm if condition (45) holds. If condition (45) fails, condition (43) failing implies that p_b is less than the second element of the minimum. This is shown below:

$$\beta < \frac{p(1-p_b)}{p_b(1-p)},$$

$$\Leftrightarrow p_b < \frac{p}{(1-p)\beta+p}.$$

The right hand side of the above inequality is less than the second element of the minimum if condition (45) fails:

$$\frac{p}{(1-p)\beta+p} < \frac{pA+p\bar{C}}{(A-I+pR)+\big(\beta+p(1-\beta)\big)\bar{C}},$$

$$\Leftrightarrow p\Big((A-I+pR)+\big(\beta+p(1-\beta)\big)\bar{C}\Big) < (pA+p\bar{C})\big((1-p)\beta+p\big),$$

$$\Leftrightarrow p\big((A-I+pR)\big) < (pA)\big((1-p)\beta+p\big),$$

$$\Leftrightarrow A(1-(1-p)\beta-p) < I-pR,$$

$$\Leftrightarrow A(1-p)(1-\beta) < I-pR,$$

$$\Leftrightarrow A < \frac{I-pR}{(1-p)(1-\beta)}.$$

Thus, p_b is less than the second element of the minimum if both conditions (43) and (45) fail.

If condition (43) holds, the firm will find the offered contract set unacceptable if

$$(p_b-p)A+\big(p_b\beta-p+p_bp(1-\beta)\big)\bar{C} < p_b(I-pR),$$

$$\Leftrightarrow p_b\big(A+\big(\beta+p(1-\beta)\big)\bar{C}-I+pR\big) < p(A+\bar{C}),$$

$$\Leftrightarrow p_b < \frac{p(A+\bar{C})}{\big(A+\big(\beta+p(1-\beta)\big)\bar{C}-I+pR\big)}.$$

Which is the smallest in the minimum when condition (45) fails. There are no firms which will decline the lenders offer of financing if both conditions (43) and (45) hold. As shown above, if condition (43) holds the firm will reject the offer of financing if

$$p_b < \frac{p(A+\bar{C})}{\left(A+\left(\beta+p(1-\beta)\right)\bar{C}-I+pR\right)}.$$

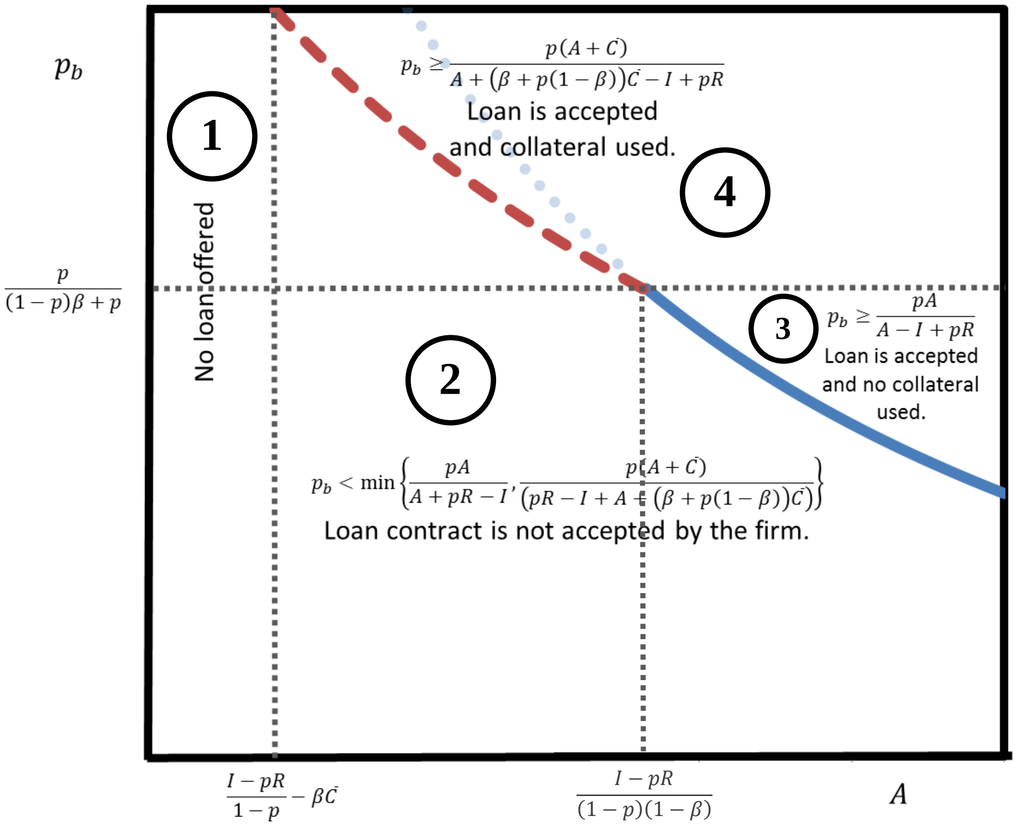
And, as shown above,

$$\frac{p(A+\bar{C})}{\left(A+\left(\beta+p(1-\beta)\right)\bar{C}-I+pR\right)} < \frac{p}{(1-p)\beta+p},$$

if condition (45) holds. Since $p_b > p/\left((1-p)\beta+p\right)$ when condition (43) holds, no firms will reject the offer of financing.

For values of p_b which satisfy (43), the firm's rationality constraint can be satisfied by offering collateral. Intuitively, the firm has such confidence in its project that the reduction in interest rate in the good state gained by pledging collateral outweighs their value of the expected loss of collateral in the bad state. The results from above are shown in the A, p_b plane in Figure 3. The effect of the opportunity cost of entrepreneurship and the application constraint are suppressed.

Figure 3 – Graph illustrating various solution regions across the A, p_b plane.



Area 1 in Figure 3 represents those firms with insufficient investable assets and collateral for the lender to offer a contract. Area 2 represents those firms who find the lender's offer of financing unacceptable. Area 3 represents those firms who accept the lending contract but do not offer collateral as the liquid value is too low. Area 4 represents those firms who accept the lending contract and do find the liquid value high enough to offer collateral.

Note that the availability of collateral increases the number of projects that are funded. Prior to the introduction of collateral area 4 would have stopped at the light dotted line. With collateral, those firms between the light dotted line and the heavy dotted line are now able to invest.

Proposition 5

For given values of other parameters, the availability of collateral reduces or does not change the level of credit rationing.

Since the contract $C = 0$ is always available and the application condition is unchanged by the introduction of collateral, no extra firms will be credit rationed under collateral. It is clear from Figure 3 that some firms who previously would have been credit rationed can make use of collateral and gain funding for their project. These are the firms in area 4 between the heavy and light dotted lines with p_b, A satisfying

$$\frac{p(A+\bar{C})}{\left(pR - I + A + \left(\beta + p(1-\beta)\right)\bar{C}\right)} \leq p_b < \min\left\{\frac{pA}{A + pR - I}, 1\right\},$$

$$\frac{I - pR}{1 - p} - \beta\bar{C} \leq A < \frac{I - pR}{(1-p)(1-\beta)}.$$

The solution to the model when considering collateral is summarised in Table 3.

Table 3 – Summary of solution for overconfidence model with collateral.

Conditions on p_b, A for given $\beta, \bar{C}, p, R, I$	**Solution**
$p_b \leq \frac{I}{R}$	Firm-assessed NPV is negative so firm does not apply.
$p_b \geq \frac{I}{R}, \qquad A + \beta\bar{C} < \frac{I - pR}{1 - p}$	Firm-assessed NPV is positive so the firm applies for financing. Investable assets plus collateral are too low for the lender to justify making an offer of financing to the firm as the lender knows even the most overconfident entrepreneur

	would reject any offer of financing that is acceptable to the lender.
$\frac{I}{R} \leq p_b < \min\left\{\frac{pA}{A+pR-I}, \frac{p}{(1-p)\beta+p}\right\}$, $\frac{I-pR}{1-p} - \beta\bar{C} \leq A \leq I$	Firm-assessed NPV is positive but collateral is not valuable enough to the lender to make it worth the loss of efficiency for the firm. Investable assets plus collateral are high enough for the lender to offer a contract but credit rationing occurs as the borrower finds all offered contracts unacceptable.
$\frac{I}{R} \leq \frac{p}{(1-p)\beta+p} \leq p_b$, $p_b < \frac{p(A+\bar{C})}{\left(pR-I+A+\left(\beta+p(1-\beta)\right)\bar{C}\right)}$, $\frac{I-pR}{1-p} - \beta\bar{C} \leq A \leq \frac{I-pR}{(1-p)(1-\beta)}$	Firm-assessed NPV is positive and collateral is valuable enough to the lender to make it worth the loss of efficiency for the firm. The firm is, however, not overconfident enough to accept the contract so credit is rationed.
$\frac{pA}{A+pR-I} \leq p_b \leq \frac{p}{(1-p)\beta+p}$, $\frac{I-pR}{(1-p)(1-\beta)} \leq A \leq I$	Firm-assessed NPV is positive, assets are high enough for the lender to offer a contract and the firm is overconfident enough to accept the lender's offer of financing. Collateral is not valuable enough to the lender to make it worth the loss of efficiency to the firm so no collateral is offered.

$\max\left\{\frac{p(A+\bar{C})}{\left(A+\left(\beta+p(1-\beta)\right)\bar{C}-I+pR\right)},\frac{p}{(1-p)\beta+p}\right\} \leq p_b,$ $\frac{I-pR}{1-p}-\beta\bar{C} \leq A \leq I$	Firm-assessed NPV is positive and collateral is valuable enough to the lender to make it worth the loss of efficiency for the firm. Lending occurs as the borrower is overconfident enough to gain a high enough return from the offered financing deal.

A testable prediction is implied by this model that is not made explicit in the paper by de Meza and Southey (1996) is that, amongst overconfident entrepreneurs, potentially only a fraction will make use of illiquid assets for collateral[12]. This fraction of firms using collateral is positively related to the liquidation fraction β and negatively related to the probability of success p.

A similar equilibrium model as in the previous section can be set up which explicitly models the firm's opportunity cost of entrepreneurship when taking explicit account of collateral. Since part of the value of collateral is lost if this is transferred to the lender, there is a dead weight loss associated with the use of collateral in this context. To briefly analyse this, assume that the liquidation value of collateral is such that all entrepreneurs make full use of collateral. If the proportion of entrepreneurs in the economy is again λ, the overall average change in welfare, assuming the lender has the correct probability assessment, can be expressed as

$$\Delta W = \lambda(pR_b - (1-p)\bar{C} - A) + (1-\lambda)\left(\frac{\lambda}{1-\lambda}\right)I,$$

[12] All entering firms will use collateral if $\frac{p}{(1-p)\beta+p} \leq \frac{I}{R}$.

$$\Delta W = \lambda(pR - I - (1-p)(1-\beta)\bar{C}) + (1-\lambda)\left(\frac{\lambda}{1-\lambda}\right)I,$$

$$\Delta W = \lambda(p, R, I, \bar{C}) * (pR - (1-p)(1-\beta)\bar{C}).$$

Thus, the average change in welfare is the average production per firm less the average expected loss in value of collateral. Since $\bar{C}$ is able to be made arbitrarily large, this expression can be potentially negative. Because there is a loss in value of the collateral as a result of liquidation, there is potentially a loss in overall welfare as a result of overconfidence, even amongst a population of all overconfident firms.

3.7 Relation to private benefits of successful entrepreneurship

Intuitively, one would suspect similar seemingly high entry into entrepreneurship if firms were to gain some private benefit upon the success of their firm. This could be interpreted as psychological satisfaction of success, preferences for independence, or preferences for control, for example. This can explain excess entry into entrepreneurship, compared to what would be expected if income was the only consideration (see, for example, Levesque et al. (2002)), but cannot explain credit rationing or collateral. This section demonstrates why.

The firm now applies for financing using the true probability assessment but they gain an extra private payoff in the good state. Thus, their application constraint is

$$p(R_b + B) - (1-p)C \geq A.$$

And since the firm knows the lending contracts which will be offered the inequality becomes

$$\Leftrightarrow p\left(\frac{pR-(I-A-(1-p)\beta C)}{p}+B\right)-(1-p)C \geq A,$$

$$\Leftrightarrow pR-I-(1-p)(1-\beta)C+pB \geq 0.$$

Thus, the firm will choose the minimum collateral $C = 0$, making private benefits unable to explain the existence of collateral. In addition, the firm must be able to fully repay the lender upon success and so the firm has the additional application constraint: $R \geq (I-A)/p$. So the firm will apply for financing if they believe they have a positive private NPV project. This is the same as the original application constraint in proposition 1 if $B = (p_b - p)R/p$.

However, since the firm has anticipated the actual lending contract which will be offered by the lender, the application constraints guarantee the acceptance constraints will be satisfied. Thus, no immediate denials will occur and no unacceptable contracts will be offered. Private benefits of successful entrepreneurship are unable to explain credit rationing.

3.8 Discussion/Conclusion

Similar to de Meza and Southey (1996), this chapter has shown that overconfidence in the probability of success of a project can explain credit rationing, both in the sense of firms receiving a denial of financing when they expected an acceptance and the sense of firms being offered lending contracts which they found to be unacceptable.

In addition to adding some extra precision to the process of application and acceptance of lending contracts, this chapter shows that overconfidence can increase the employment of resources, resulting in a gain in overall welfare. In all cases, overconfident firms unwittingly transfer wealth from themselves to resource holders (such as their employees) and the predatory lending markets facilitate this transfer.

This chapter also makes explicit why collateral is able to be explained by overconfidence. The most overconfident firms are able to gain what they see as value from collateral as they can gain lower interest rates. These firms weight the probability of losing the collateral lower than they should and subsequently lose value on average in reality.

An interesting practical question is: should government encourage or discourage overconfidence? If employment is low, extra overconfident firms are unlikely to divert significant resources from more productive firms. Thus, in times when employment is low, it may be in society's interest to encourage wealthy individuals to be overconfident in their business prospects. However, there are some clear ethical problems with encouraging individuals to engage in behaviour which is harmful to them and there is no obvious mechanism by which this could be achieved. It may be ethically justifiable in the same way that progressive taxation is justifiable as overconfidence results in potentially desirable distributional outcomes.

More realistic modelling of the labour market is required to enhance the welfare analysis in this chapter. The amount of labour provided by an entrepreneur is larger than that provided by a single worker when there is unemployment. Since cost of effort was assumed to be zero, the implications of this were suppressed. Since entrepreneurship has a higher cost of effort than working, the opportunity cost of entrepreneurship is understated in the current version of the model. Modelling cost of effort in a way which yields an upward sloping labour supply curve will make it possible to capture the effect of resources being diverted from more productive uses.

Potential extensions to this model include introducing more general distributions of the parameters to better investigate the effects of potential changes in parameters. Adding reasonable distributional judgements to the social welfare function is another possible avenue for future models. In the current model, an extra dollar for a low wealth individual is worth just as much as an extra dollar for a high wealth individual. Thus, introducing risk aversion over final wealth could provide some more insight into making recommendations for policy here.

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5. Appendices

Appendix 1 - Maple output for debt and firm size aversion model

$u(Y, L, i) := 1 - \frac{\exp(-a \cdot Y)}{a} - b \cdot L - c \cdot i$

$$(Y, L, i) \rightarrow 1 - \frac{e^{-a Y}}{a} - b L - c i \quad \textbf{(1)}$$

$U(i, C, p, \beta) := p \cdot u\left(R(i) - \frac{i - \beta \cdot C}{p} - \beta \cdot C, i - \beta \cdot C, i\right) + (1 - p) \cdot u(-C, i - \beta \cdot C, i)$

$$(i, C, p, \beta) \rightarrow p\, u\left(R(i) - \frac{i - \beta C}{p} - \beta C, i - \beta C, i\right) + (1 - p)\, u(-C, i - \beta C, i) \quad \textbf{(2)}$$

$R(i) := d \cdot i$

$$i \rightarrow d\, i \quad \textbf{(3)}$$

$L(i, C, p, \beta, \lambda 1, \lambda 2, \lambda 3) := U(i, C, p, \beta) + \lambda 1 \cdot (-C) + \lambda 2 \cdot (C - Cbar) + \lambda 3 \cdot (\beta \cdot C - i)$

$$(i, C, p, \beta, \lambda 1, \lambda 2, \lambda 3) \rightarrow U(i, C, p, \beta) - \lambda 1\, C + \lambda 2\, (C - Cbar) + \lambda 3\, (\beta\, C - i) \quad \textbf{(4)}$$

$(solve(\{diff(L(i, C, p, \beta, \lambda 1, \lambda 2, \lambda 3), C) = 0, diff(L(i, C, p, \beta, \lambda 1, \lambda 2, \lambda 3), i) = 0, \lambda 1 \cdot C = 0, \lambda 2 \cdot (C - Cbar), \lambda 3 \cdot (\beta \cdot C - i) = 0\}, \{i, C, \lambda 1, \lambda 2, \lambda 3\}))$

$$\left\{C = \frac{\ln\left(-\frac{\beta\,(c - b\,p - c\,p + b\,d\,p)}{-d\,p + 1 + p^2\,d - p}\right)}{a}, i = -\frac{\ln\left(\frac{b + c}{d\,p - 1}\right) p + \ln\left(-\frac{\beta\,(c - b\,p - c\,p + b\,d\,p)}{-d\,p + 1 + p^2\,d - p}\right)\beta - \ln\left(-\frac{\beta\,(c - b\,p - c\,p + b\,d\,p)}{-d\,p + 1 + p^2\,d - p}\right)\beta\,p}{a\,(d\,p - 1)}, \lambda 1 = 0, \lambda 2 = 0, \lambda 3 = 0\right\}, \left\{C = \frac{RootOf\left(e^{-\_Z} - e^{-\_Z}\,p + \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,p - \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,d\,p + \beta\,c\right)}{a}, i = \frac{\beta\,RootOf\left(e^{-\_Z} - e^{-\_Z}\,p + \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,p - \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,d\,p + \beta\,c\right)}{a}, \lambda 1 = 0, \lambda 2 = 0, \lambda 3 = e^{-\beta\,RootOf\left(e^{-\_Z} - e^{-\_Z}\,p + \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,p - \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,d\,p + \beta\,c\right)(d - 1)}\,d\,p - e^{-\beta\,RootOf\left(e^{-\_Z} - e^{-\_Z}\,p + \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,p - \beta\,e^{-\beta\_Z\,d + \beta\_Z}\,d\,p + \beta\,c\right)(d - 1)} - b - c\right\}, \left\{C = Cbar, i = -\frac{a\,\beta\,Cbar - a\,\beta\,Cbar\,p + \ln\left(\frac{b + c}{d\,p - 1}\right)p}{a\,(d\,p - 1)}, \lambda 1 = 0, \lambda 2 = -\frac{\beta\,c - b\,\beta\,p - \beta\,p\,c - e^{a\,Cbar}\,d\,p + e^{a\,Cbar} + b\,\beta\,d\,p + p^2\,e^{a\,Cbar}\,d - p\,e^{a\,Cbar}}{d\,p - 1}, \lambda 3 = 0\right\}, \left\{C = Cbar, i = \beta\,Cbar, \lambda 1 = 0, \lambda 2 = \beta\,e^{-a\,\beta\,Cbar\,(d - 1)}\,p + e^{a\,Cbar} - p\,e^{a\,Cbar} - \beta\,e^{-a\,\beta\,Cbar\,(d - 1)}\,d\,p + \beta\,c, \lambda 3 = e^{-a\,\beta\,Cbar\,(d - 1)}\,d\,p - e^{-a\,\beta\,Cbar\,(d - 1)} - b - c\right\}, \left\{C = 0, i = -\frac{\ln\left(\frac{b + c}{d\,p - 1}\right)p}{a\,(d\,p - 1)}, \lambda 1 = \frac{\beta\,c - b\,\beta\,p - \beta\,p\,c + 1 - d\,p + b\,\beta\,d\,p + p^2\,d - p}{d\,p - 1}, \lambda 2 = 0, \lambda 3 = 0\right\}, \{C = 0, i = 0, \lambda 1 = -\beta\,p - 1 + p + \beta\,d\,p - \beta\,c, \lambda 2 = 0, \lambda 3 = d\,p - 1 - b - c\} \quad \textbf{(5)}$$

Appendix 2

The second order conditions for the unconstrained optimisation problem are shown below:

$$\frac{\partial^2 \mathrm{E}[U]}{\partial I^2} = -pa^2\left(d-\frac{1}{p}\right)^2 e^{-a\left(dI-\frac{I-\beta C}{p}-\beta C\right)} < 0$$

$$\frac{\partial^2 \mathrm{E}[U]}{\partial C^2} = -\frac{(1-p)^2}{p}a^2\beta^2 e^{-a\left(dI-\frac{I-\beta C}{p}-\beta C\right)} - (1-p)a^2 e^{aC} < 0$$

$$\left(\frac{\partial^2 \mathrm{E}[U]}{\partial I^2}\right)\left(\frac{\partial^2 \mathrm{E}[U]}{\partial C^2}\right) - \left(\frac{\partial^2 \mathrm{E}[U]}{\partial I \partial C}\right)^2 = \frac{a^4(dp-1)^2(1-p)e^{-a\left(dI-\frac{I-\beta C}{p}-\beta C-C\right)}}{p} > 0$$

Appendix 3

Proposition:

$$\frac{I}{R} < \frac{pA}{A+pR-I}.$$

$A + pR - I > 0$, by (27):

$$\Leftrightarrow I(A+pR-I) < pRA,$$

$$\Leftrightarrow I(A+pR) - I^2 < pRA,$$

$$\Leftrightarrow I^2 - (A+pR)I + pRA > 0,$$

$$\Leftrightarrow (I-A)(I-pR) > 0.$$

This will be positive if both terms are positive. So, $I - A > 0$ and $I - pR > 0$ guarantee there is value of p_b which satisfies both (24) and (29).

Appendix 4

Maple shows the solution for λ depends on the roots of the following equation:

$$(pR)^2xe^x + xpR^2(1-p) - e^{2x}((I-pR)^2 + IpR) + e^x(2(pR)^2 - IpR - pR^2 + IR) - (pR)^2 + pR^2 = 0.$$

The left hand side $\to -\infty$ as $\to \infty$. Dominated by the term

$$-e^{2x}((I-pR)^2 + IpR).$$

The left hand side goes to minus infinity as $x \to -\infty$. This is dominated by

$$xpR^2(1-p).$$

If $x = 0$, the left hand side is

$$I(R-I) > 0.$$

Thus equation (37) has at least 2 solution and has an even number of real solutions.

All numerical examples trialled yielded two solutions for $\lambda \in [0,1]$ with only the least solution satisfying $R - I - \frac{\lambda}{1-\lambda}I > 0$, which is the expected return on the project to the most optimistic borrower. If λ were at this higher solution, there would be no firms applying for financing.