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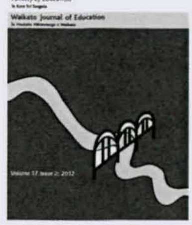

Lessons from children in Māori medium for teachers: Encouraging greater efficiency when learning to multiply

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"He nui maunga e kore e taea te whakaneke,
he nui ngaru moana mā te ihu o te waka e wāhi"

Abstract

This research explores the responses of 44 Year 7–8 students from four Māori medium schools who were asked to solve a multiplication word problem. The findings show that there was a range of mental strategies displayed by the children, 29 of whom were able to solve the problem. However, data also indicates that 15 children were not able to either access the problem or utilise an appropriate strategy to solve it. This paper discusses the strategies shared by all of these children and suggests avenues to further support learners to become multiplicative thinkers.

Keywords

Māori, children, multiplication, mental, strategies.

Background

In 2002 in New Zealand, Te Poutama Tau (Ministry of Education, 2002) was produced to place focus on the teaching and learning of numeracy in Māori medium schools. The intent was to enhance children's learning of number ideas through teachers' professional development. The resource was introduced as a result of increasing expectation that mathematics education emphasise the solving of problems in a variety of ways. Learning and remembering set procedures for solving number problems was not strongly advocated (Ministry of Education, 1992, 1996).

The mathematics learning area in Te Marautanga o Aotearoa (Ministry of Education, 2008a) also emphasises that children should be making connections between

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number ideas. The ability to make such connections is perceived to be critical for the development of number sense (Anghileri, 2006). In addition, Whanaketanga Pāngarau: He Aratohu mā te Pouako (Ministry of Education, 2010) was produced to support teachers in their assessment and teaching of children's mathematics learning. By the end of Year 8, children are expected to have developed a wide range of strategies to support their multiplicative thinking.

The ability to mentally compute is part of being able to think multiplicatively. It requires the use of cognitive processes to solve mathematics problems and is the most common form of computation used by adults (Northcote & McIntosh, 1999). Mental computation involves estimation and offers an opportunity to be creative when working with number. Children who can compute mentally with ease have developed a rich sense of number. They display flexibility of thinking and an awareness of the links and relationships between number ideas. These children are able to make generalisations about number and choose the most efficient and appropriate operations to solve problems in ways that make sense to them. Such processes can aid problem solving in alternative situations (Angliheri, 2006; Dowker, 2005).

In recent years there has been a concentration on the development of robust methods of mental computation in mathematics education in the western world. In Britain for example, the National Numeracy Strategy advocates exposing children to a variety of mental strategies. From such a range of strategies, children are encouraged to explore and choose the most appropriate for the situation or problem they are engaged in. This process assists them to develop confidence in their ability to problem solve (Suggate, Davis, & Goulding, 2006). This precept is also embedded in Te Marautanga o Aotearoa (Ministry of Education, 2008a).

Being able to articulate a mental computation strategy is deemed beneficial for children's learning in mathematics (Ittigson, 2002; Zevenbergen, Dole, & Wright, 2004). Encouraging appropriate articulation of mathematics ideas has proved challenging in some Māori medium settings (Christensen, 2004). Many children are not only learning mathematical concepts but are also doing so as second language learners of the Māori language (Maangi, Smith, Melbourne, & Meaney, 2010). Being articulate means children must develop the ability to participate in meaningful mathematical discourse (R. Hunter, 2006; Moschkovich, 2002). The skill to engage in mathematical discourse must be learned and interwoven with the conceptual development of mathematics ideas (Anthony & Walshaw, 2007; Barton, 2008; Christensen, 2004). Recent curriculum documents and support materials reflect the importance of learners developing proficiency in the articulation of their mathematical thinking.

Mathematical discourse involves the acquisition of terminology that needs to be used appropriately for any ensuing discussion to be meaningful. However, the development of appropriate vocabulary for exploring mathematics ideas in Māori medium is a recent phenomenon. The continuing evolution of new terms in Māori presents a challenge for teachers who need to be able to support the development of discourse with learners (Hāwera, 2011; Hāwera & Taylor, 2011; Murphy, Bright, McKinley, & Collins, 2009).

An important part of being proficient in mathematics is being a multiplicative thinker. Siemon (2005) suggests that multiplicative thinking is about an individual's "capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in various contexts" (p. 1). The ability to reason multiplicatively is essential for learning mathematical concepts such as ratio and proportion, area and volume, probability and data analysis (Mulligan & Watson, 1998). Society also presents situations on a regular basis that require multiplicative reasoning. The development of such thinking demands an appreciation of the underlying pattern and structure of numbers so that these can be manipulated in a variety of ways (Mulligan & Mitchelmore, 2009). It is this proficiency that impacts on the extent to which children are able to develop strategies for thinking multiplicatively.

The simplest form of a multiplicative situation that children will meet is one where there is a one-to-many correspondence between two sets e.g. one car, four wheels. The most basic strategy children use to solve this type of problem is repeated addition. This is when they add the multiplier the number of times indicated by the multiplicand: 4x17 becomes 4+4=8+4=12+4 … (Nunes & Bryant, 1996).

Children who can only use a simple additive strategy are unlikely to access and solve a range of multiplication problems efficiently. A crucial idea for children learning to move beyond relying on repeated addition is to understand the commutative property, which involves recognising and accepting that "3x4" gives the same result as "4x3". Those children who develop this understanding will have more flexibility to consider and employ the most efficient calculation strategy to solve any given problem (Anghileri, 2006; Haylock, 2010; Mulligan & Watson, 1998).

Another strategy children might use is to double numbers to create "clumps" that are added together to find a total. These can include a double double strategy where 17×4 becomes (17+17)+(17+17)=34+34=68. When 17×4 becomes (17×2)×2=34×2=68, this times doubling strategy demonstrates the use of the associative property where the product is always the same regardless of the way the factors are grouped. A doubling strategy may be less efficient when larger numbers are involved because of cognitive overload and the length of time taken to arrive at a solution (Ambrose, Baek, & Carpenter, 2003; Baek, 2005/2006).

The standard place value partitioning strategy for 17×4 means that 17 is partitioned into 10 and 7, each of which is then multiplied by 4 and the products added together (i.e. 10×4=40 and 7×4=28, 40+28=68) (Anghileri, 2006; Haylock, 2010; Young-Loveridge & Mills, 2010). Children who utilise this strategy understand the distributive property where the multiplication process can be "distributed" over the sum of two parts.

An alternative strategy that can be employed for solving multiplication problems is compensation. This strategy requires flexible thinking and a fluent understanding of both the numbers and the process of multiplication (Baek, 2005/2006). For example, 17×4 can become (17+3)×4 then calculated as 20×4=80. Following this, 80–(3×4) (the extra amount added on to the 17 at the beginning) is the same as 80–12=68.

In order to develop effective strategies for multiplication, children need to learn and readily access a wide base of number facts. Knowledge of number facts can provide a platform to develop further number ideas. The more known facts (and the connections between them) that children can access mentally, the greater their potential for constructing strategies to solve number problems (Dowker, 2005; Thompson, 1999). Utilising known facts can help children to derive others (Young-Loveridge & Mills, 2010). However, if children do not appreciate the connections between individual number facts, this can result in cognitive overload that will impact on a learner's capability to reason through situations requiring multiplicative thinking (Anghileri, 2006).

Children need to have exposure to a variety of problems to support their development of multiplicative thinking. Greer (1992) cited in Anghileri (2006, p. 84) identifies four main categories of problems. These are

- equivalent groups (e.g. 3 rows of 4 children);
- multiplicative comparison (e.g. 3 times as many boys as girls);
- rectangular arrays (e.g. 3 rows of 4 children); and
- cartesian product (e.g. the number of different possibilities for girl-boy pairs from 3 girls and 4 boys).

It is important when solving problems that children have opportunities to record their mathematical thinking in ways they decide. These recordings can provide valuable insights into children's thinking and provide catalysts for initiating discussion (Anghileri, 2006; Mulligan & Watson, 1998). When recording, children might attempt to use traditionally taught written procedures. Difficulties can arise if they do not have a clear understanding of place value and try to perform calculations by following poorly understood rules (Baek, 2005/2006; Lawton, 2005).

The traditional recording of algorithmic procedures has not often closely reflected mental computation processes. This occurrence has contributed instead to "cognitive passivity" for many children (Thompson, 1999, p. 173). It is argued that mathematics instruction for multiplication should therefore include ways of helping all children to integrate their mathematical thinking and recording so that they are not using procedures by rote in meaningless ways (Gilmore & Bryant, 2008). The recording of children's strategies should reflect their mathematical thinking and can include the use of empty number lines, arrays, equipment, children's diagrams and jottings (Anghileri, 2006; Ministry of Education, 2008b, 2012).

Method

Participants

This study focuses on the responses of 44 Year 7–8 children in four Māori medium schools. All schools were level 1 Māori medium where 81–100 percent of the instruction was in te reo Māori. Three of the schools had participated in Te Poutama Tau, the Māori medium equivalent of the Numeracy Development Project, for some years prior to the study. Learning multiplicative strategies is a component of the Mahere Tau (Number Framework). The stages of the framework describe the expected progression of multiplicative strategies that children should develop. Eleven of the children were from Decile 1 schools, 21 from a Decile 2 school and 12 from a Decile 5 one.

Table 1. Composition of the Children by School and Year Level:

School/Kura	Year 7	Year 8	Total
1*	0	6	6
2*	3	2	5
3*	8	4	12
4	12	9	21
Number of children	23	21	44

* Poutama Tau participants

Procedure

Schools were asked to nominate children from across a range of mathematics levels. Children were interviewed individually for about 30 minutes in Māori or English (their choice) in a quiet place away from the classroom. They were told that the interviewer was interested in finding out about their thoughts regarding their learning of mathematics.

The question this paper focuses on is part of a larger collection that the children were asked to respond to. Other questions have been previously analysed and discussed elsewhere (Hāwera & Taylor, 2008, 2009, 2010, 2011; Hāwera, Taylor, Young-Loveridge, & Sharma, 2007). The multiplication question was selected on the basis that the mathematics involved should be accessible to the children. The question was designed to elicit the use of important and relevant strategies and knowledge that this age group could be expected to employ. The question analysed here is:

- E hanga motokā ana te kamupene o Hera. E 4 ngā wīra mo ia motokā. E hia katoa ngā wīra mo ngā motokā 17?
  (Hera has a car manufacturing company. She needs 4 wheels for each car. How many wheels does she need for 17 cars?)

Audiotapes of the interviews were transcribed by a person fluent in the Māori language. Transcripts were subjected to a content analysis to identify common strategies in the students' responses. The children's responses have been coded to maintain confidentiality and to be consistent with the reporting of other data from the larger study.

Results

Multiplication strategies

Thirty-six out of 44 children attempted to solve the multiplication problem. Of these, 29 were able to do so correctly. The children used a range of strategies to solve this problem. These responses have been categorised into the following:

- (SPVP) is the standard place value partitioning strategy e.g. $4\times17=(4\times10)+(4\times7)=40+28=68$;
- (DF) is the derived fact strategy e.g. $4\times17=(4\times10)+(4\times5)+(4\times2)=40+20+8=68$;

- (TD) is the times doubling strategy e.g. 4×17=(2×17)+(2×17)=34+34=68;
- (TT) is the times twice strategy e.g. 4×17=(2×17)×2=34×2=68;
- (DD) is the double double strategy e.g. 4x17=(17+17)+(17+17)=34+34=68;
- (C4) is the counting up in fours strategy e.g. (4,8,12,16 … 68);
- (ALG) is a traditionally taught written procedure; and
- (NA) is when No attempt made and no strategy offered.

Table 2. Strategies used for the multiplication task:

Kura	Number of Year 7–8 children	SPVP	DF	TT	DD	TD	C4	ALG	No attempt made or strategy offered
1*	6	0	0	0	0	1	0	1 (1W)	4
2*	5	1	1	1	1	1	0	0	0
3*	12	6	0	0	0	1	0	2	3
4	21	4 (1W)	4 (1W)	0	0	1	5 (1W)	6 (3W)	1
Total	**44**	**11**	**5**	**1**	**1**	**4**	**5**	**9**	**8**

* Poutama Tau participants (nW) indicates the number of incorrect solutions

Eleven of the children used the standard place value partitioning strategy. An example of this is:

> I took the 7 away and I just did 10 times 4 equals 40, then I done 4 times 7 which equals 28. Then I added those 2 answers together and got 68. (K68m8)

The derived fact strategy was used by five of the children to reach a solution. Further analysis indicates that this group of children was able to make use of known facts that they were instantly able to recall. For example:

> Whā whakarau tekau ka puta whā tekau, a, whā whakarau rima ka puta rua tekau, a, whā whakarau rua, ka puta waru, a, ka tāpiri ērā mea kia ono tekau mā waru. (K25f7)
>
> (4 times 10 makes 40, and 4 times 5 makes 20, and 4 times 2 makes 8, and you add those to make 68)

Another student who worked from a fact that she knew said:

> … um, I went 4 times 12 which is 48 and then I just went 4 times 5 is 20 and then added the 20 to the 48. (K65f7)

Variations of the doubling strategy were used by six of the children.

> I rounded the 17 down to um 15
>
> I timesed the 15 times 4, timesed 15 times 4 which equals 60

[How'd you know that equals 60?]

Two 15's equals 30 then two 30's equal 60

[60 and the two that you took off?]

You times it by 4 which equals 8 and 60 plus 8 equals 68. (K51m7)

Counting up in fours was used by five of the children. The traditionally taught written procedure (ALG) was used by nine of the 44 children, a slightly greater proportion of whom were from the non Poutama Tau school. Half of the children who used the algorithmic strategy were able to provide the correct solution.

An example of an incorrect solution from an attempt to use an algorithm is:

... um, it will be 428

I stuck the 4 up the top and 17 down bottom

... and then I went 4 times 7 equals 28

and then 4 times 1 equals 4

and then you put the 28 behind the 4 which equals 428. (K69f7)

Eight of the children indicated that they did not know how to do the multiplication task and made no attempt to do so. Of the 36 children who did attempt the problem, seven of these provided an incorrect solution.

For the multiplication task, nine of the children shared more than one strategy for finding the solution. All of these solutions were correct.

Table 3. Multiplication-2 strategies (as stated by the children)

Name	Kura	Strategy 1	Strategy 2
K27f8	2	2×17=34+34=68	4×10=40,4×7=28, 40+28=68
K25f7	2	4×10=40, 4×5=20, 4×2=8, 40+20+8=68	4×20=80, 4×3=12, 80−12=68
K36m7	3	17×2=34+34=68	4+1=5, 5+5=10, 10×17=170/2 = 85−17=68
K37f7	3	10×4=40, 7×4=28, 40+28=68	17×2=34×2=68
K46m7	3	10×4=40, 7×4=28, 40+28=68	* 4×7 =28, 4×1=4+2=68
K38f7	3	4×10=40,4×7=28, 40+28=68	17+17+17+17=68
K610f7	4	4,8,12, 13, 14, 15, ... 68	* 4 times 7 is 28 and then you stick the 2 there and then ... 4 times 1 is 4 plus the 2 is 6.
K64f8	4	4×7=28, 4×10=40 28+40=68	17×2=34×2=68

K57f7	4	4,8,12 … 68	* 4 times 7 is 28. Put the 8 down here and the 2 up there. 2 and 4 is 6

* asked for paper to record on.

An explanation from one child about his two strategies is:

- First strategy: Um, tekau mā whitu whakarau rua, ka toru tekau mā whā, toru tekau mā whā tāpiri toru tekau mā whā, ka ono tekau mā waru.
  (17 times 2 equals 34, 34 plus 34 equals 68)
- Second strategy: Ka taea ki te tāpiri te kotahi i runga i te whā ka rima, kātahi huri te rima ki te tekau, tekau whakarau tekau mā whitu ka tahi rau whitu tekau, kātahi me hāwhe te tahi rau whitu tekau

  Ka waru tekau mā rima, kātahi tango tekau mā whitu Maangi, Ka ono tekau māwaru
  ("You can add 1 to the 4 to make 5, then change the 5 to 10; 10 times 17 makes 170, then halve the 170 makes 85, then take away 17 equals 68") (K36m7)

Discussion

Numeracy in the 21st century demands that learners become numerate and develop positive dispositions towards learning mathematics. They should display a willingness and confidence to engage with mathematical tasks (Goos, Dole, & Geiger, 2012). It is heartening to see that two-thirds of these children seemed to readily engage with and make sense of the multiplication problem.

Many children were able to demonstrate the ability to reason from what they knew and manipulate numbers to derive a solution. They had enough basic fact knowledge to support them to solve the multiplication problem. They very quickly understood the meaning of the problem, recognised the operation that was required and were able to take action. They showed flexibility with number that is an important aspect of developing a rich number sense (Anghileri, 2006; Dowker, 2005).

It is significant that some children were able to construct more than one pathway to their solution, indicating an awareness of being able to use number knowledge in different ways (Young-Loveridge, 2006). The ability to choose critically from a range of strategies is an important principle embedded in the Poutama Tau (Ministry of Education, 2002). However, it is difficult to have a full appreciation of the range of strategies these children may have had at their disposal when they were only asked to solve one mathematics problem. Asking a variety of questions may have encouraged children to divulge the greater array of multiplicative strategies that would be expected at Year 7–8 level.

While most of the children could solve the problem, some used less efficient strategies that proved cumbersome and time consuming. For example, some children displayed use of the counting up in fours strategy when multiplying 17×4 or 4×17. If children are to become proficient multiplicative thinkers, they may need extra support and time to understand that 4+4+4+4+4+4+4+4+4+4+4+4+4+4+4+4+4 gives the same total as 17+17+17+17.

Children need support to recognise that greater efficiency is afforded when adding or multiplying fewer numbers. Numerous opportunities to understand and articulate this idea may be required for learners to accept this concept. While each side of the equation actually looks different to the other, they both equate to the same number. Such thinking links to a basic precept of equality in algebra which children need to understand (Lamon, 2007). Five of the 44 children in this study who utilised skip counting in fours to solve the problem showed that they had yet to appreciate this notion.

Children need to also understand that an addition equation involving equal groups can be transformed into a multiplication one. This means understanding that 17+17 is the same as 17×2 in Māori medium (Ministry of Education, 2008c, 2012; Young-Loveridge & Mills, 2010). For children to derive the final solution of the word problem efficiently about the number of wheels required, it is essential that they know some facts; that is, 17+17=34 or 2×17=34 or 17×2=34.

It is noted that 11 of the 44 children in this study were able to draw on the more efficient standard place value partitioning strategy to solve this problem (Dowker, 2005). Having the facility to use this strategy requires students to understand the pattern and structure of whole numbers. While children have spent time in earlier years at school to understand that numbers like 17 are made up of one group of 10 and a group of 7, they now need to transfer that idea to multiplicative situations. Part whole thinking is essential if children are to make sense of and utilise the standard place value partitioning strategy effectively. It is interesting that four of the 11 had developed this strategy even though their school had not participated Poutama Tau.

Analysis of findings indicates that of the 15 children who did not solve the problem, seven did make an attempt but were unable to reach the appropriate solution. Six out of these seven had no formal experience with the teaching of multiplicative strategies in Te Poutama Tau. From this group of seven, four attempted to use the procedure of a standard written algorithm. These children did not appear to fully understand the process of manipulating numbers when using an algorithmic procedure for multiplication (Gilmore & Bryant, 2008; Thompson, 1999). While their thinking indicated some basic fact knowledge, there was a lack of clarity regarding the structure and numerical value of each digit represented in their solutions. These children seemed to be attempting to recall a procedure they had learned by rote and merely accepted the total they arrived at.

Current practice in mathematics education promotes the notion of encouraging children to develop efficient mental strategies for single and double-digit multiplication in ways that make sense to them (Dowker, 2005; Young-Loveridge, 2010). Making sense when learning mathematics is an important principle for young learners to understand and apply. Some children may require more opportunities than others to explore multiplication in ways that they understand. For children who struggle to make sense of situations that require multiplicative thinking, working with arrays and other materials can be fruitful (Young-Loveridge, 2010). Early teaching of an algorithmic procedure may be unnecessary and merely result in confusion for some.

Another strategy for supporting children learning to think multiplicatively is that of encouraging them to make "jottings" when they are attempting to solve problems. These jottings can help teachers to understand children's mental processes. This information can provide insights for assessment and support teachers to make decisions about ways they might scaffold learners to multiply more efficiently (Anghileri, 2006).

Eight Year 7 and 8 children did not seem to have a strategy to solve or even begin the multiplication problem. This is significant given that seven of this group would have had experience with the multiplicative strategies in Te Poutama Tau. Given that these children are likely to have been exposed to the process of multiplication for at least three years (Ministry of Education, 2007, 2008b), it is a concern that they appeared unable to access and use any mathematical strategy to solve the problem. While they were offered paper and pencil at the time, no child in this group took advantage of these materials to support their thinking.

It may be that these children were not able to make the necessary connections between their mathematical thinking and the language required to express it, even though the choice of language for the interview was theirs. When helping children in Māori medium learn about multiplication, attention has to be paid not only to the mathematical concepts but also to the acquisition of the language required to understand and express ideas. Language acquisition may have to include explicit discussion of new terminology (Maangi et al., 2010). Displays of appropriate terminology according to the current learning context can also be a feature on a classroom wall (Meaney, Trinick, & Fairhall, 2009). Learning about multiplication is a complex process and research indicates that the development of multiplicative thinking is a challenge for many children (Lamon, 2007). Multiple factors need to be considered when planning for children's learning.

An emphasis on communication is reflected in recent curriculum documents and support resources in New Zealand (Ministry of Education, 1992, 1996, 2002, 2007, 2008). Barton (2008) argues that mathematical development is affected by language development and vice versa. Parallel advancement in both of these aspects has implications for children's ability to learn and share mathematical ideas. Zevenbergen, Dole and Wright (2004) and J. Hunter (2009) maintain that expecting children to explain, listen to and reflect on a range of ideas helps them make better sense of the mathematics they engage with. Twenty-nine children that solved the problem correctly were able to express their mathematical reasoning clearly and succinctly. This indicates a confidence in their knowledge and use of appropriate mathematical vocabulary and discourse. There is a concern that the other 15 children had not yet developed this proficiency by Years 7 and 8.

Although data has been collected from four Māori medium schools, a limitation of this paper is that these findings pertain to just 44 children in total. Also, while the mathematics problem presented to them was set in a context that was considered familiar to children, it did not allow for any variation in the context. For example, some children may have wondered why there was no spare tyre allowed for in the cars. This knowledge may have impeded their access to the problem.

The fact that there was only one problem presented may not have given all children an opportunity to show what they could do or how they might think regarding their solving of a multiplication problem. More cognisance could have been taken of the suggestion by Greer (1992, cited in Anghileri, 2006) that different types of multiplication problems should be presented to children. It might have been more illuminating as well to have a problem that could have been illustrated by the numbers 18×4 instead of 17×4. 18×4 would easily lend itself to being solved by utilising a doubling and halving strategy that children at this level could be expected to include in their repertoire. The limitations of the sample size and the investigation of only one question make it difficult to generalise about children in Māori medium settings who are learning to be multiplicative thinkers. This study, however, does provide some insights into how these children approached the given task.

Conclusion

This research shows that many of these children were able to demonstrate a variety of strategies between them to solve a multiplication problem. Some of these strategies were more efficient than others. Overt teaching that promotes the development of efficient strategies and appropriate number knowledge seems necessary for increasing all children's facility with number. Even then, some children require more focused scaffolding to ensure that they can access and solve mathematical problems. Any teaching of mathematical concepts such as multiplication needs to also incorporate relevant language acquisition and encourage appropriate discourse. These aspects require explicit planning and overt attention.

If children in Māori medium are to gain equitable access to higher levels of schooling and enjoy subsequent wider opportunities offered in the New Zealand and global community, it is essential that they develop greater proficiency in multiplicative thinking. The children in this study have provided messages for teachers to consider when endeavouring to support learners in this mathematical domain.

Hei Mihi

Hei kapi ake, ka haere tonu ngā mihi ki ngā tamariki me ō rātou whānau i whakaae kia uru mai ki te rangahau nei. Mei kore rātou, e kore e pēnei rawa te puta o ngā māramatanga me ngā momo kōrero hei tautoko i te kaupapa. Mauriora ki a tātou!

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