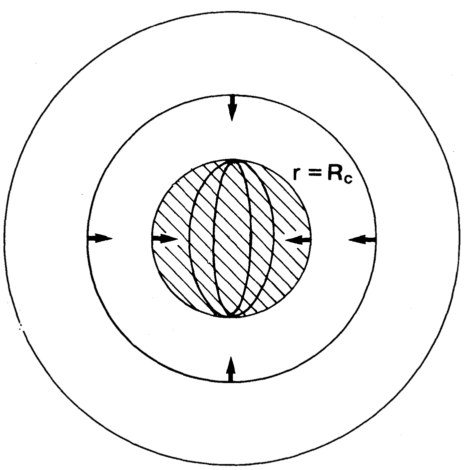
$$\pi R_c^4 \approx \delta E ,$$



the problem is that small disturbances in the outer field drive wave motions which strongly localize near the origin, generating large currents: thus incoming waves are resistively attenuated as they traverse the diffusion region prior to reflection from the stagnation point. In this way, advective motions of the outer field drive fast oscillatory reconnection with a fundamental frequency $\omega = -\pi/\ln \eta$ and asymptotic decay rate $\omega^2/2$.

Perhaps of greater physical significance is the fact that the bulk of the perturbation energy is lost as a result of the initial implosion prior to the asymptotic decay. This can be understood in terms of the very fast decay of higher nodal ($n > 0$) components in the eigenfunction representation of the initial conditions. Yet fast linear reconnection is not restricted to cylindrical geometries: Cartesian formulations maintain all the characteristics of fast reconnection, in particular the scalings of current density and current-sheet area (as discussed in § 4). The effects of finite gas pressure, however, wash out the oscillatory reconnection phase and, if sufficiently large, can stall—at least in the present geometry—the initial implosive energy release. This result also holds good for large-amplitude disturbances, but its physical significance is compromised by the fact that more complex magnetic flow topologies may allow fast implosive reconnection to persist even for incompressible plasmas (DeLuca & Craig 1992).

In § 5 we considered the breakdown of the linear theory. In general, a low-amplitude topological disturbance is manifested as an inward-propagating cylindrical wave that gradually steepens and eventually becomes nonlinear at some radius R_c determined by the energy of the perturbation. We can think of the neutral point focusing the perturbation energy toward the origin. The outer field is left current-free, but inside R_c the perturbation energy becomes increasingly localized into a quasi-one-dimensional current sheet whose thickness is limited by resistive diffusion. The current structure now resembles a rectangular "tombstone" rather than the cylindrical "spike" of the linear theory. This change in morphology means that the magnetic flux now has to pile up at the edge of the sheet in order to maintain fast reconnection: in this case the length of the sheet, though ultimately limited by the global geometry, depends mainly on the energy of the perturbation (via eq. [4.4]), whereas the sheet thickness scales as $\eta^{1/2}$ or faster. In such cases the perturbation energy is mainly converted to heat via Ohmic dissipation rather than to the kinetic energy of mass motion. DeLuca & Craig (1992) provide concrete examples of dynamic flux pileup solutions.

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APPENDIX

GLOBAL ENERGY CALCULATION

The fact that the zeroth-order field is a magnetic equilibrium implies that the first-order energy variations must vanish. This means that the excess energy of the linearized system is given by the second-order variations of magnetic and kinetic energy, $\delta^2 M$ and $\delta^2 K$, respectively. The kinetic energy variation is given by

$$\delta^2 K = \langle \tfrac{1}{2} v^2 \rangle ,$$

where $\langle \ldots \rangle = \int \ldots dV$, with $dV = r\, dr\, d\theta$ in cylindrical polars. The magnetic energy variation initially appears more complicated:

$$\delta^2 M = \tfrac{1}{2}\langle \delta \boldsymbol{B} \cdot \delta \boldsymbol{B} \rangle + \langle \delta^2 \boldsymbol{B} \cdot \boldsymbol{B}_E \rangle ,$$

but recasting this equation in the form

$$\delta^2 M = \tfrac{1}{2}\langle \delta \boldsymbol{B} \cdot \delta \boldsymbol{B} \rangle - \tfrac{1}{2}\langle \delta \boldsymbol{B} \cdot (\boldsymbol{\xi} \times \boldsymbol{J}_E) \rangle$$

by introducing the fluid displacement $\boldsymbol{\xi}$ shows that the second-order contribution $\langle \delta^2 \boldsymbol{B} \cdot \boldsymbol{B}_E \rangle$ vanishes, since the equilibrium current $\boldsymbol{J}_E$ is zero. Hence we can write the change in global energy in the simple form

$$\delta^2 U = \tfrac{1}{2}\langle \delta \boldsymbol{B} \cdot \delta \boldsymbol{B} \rangle + \tfrac{1}{2}\langle v^2 \rangle .$$

To calculate the fluid energy, we first note, from the momentum equation (3.2), that any velocity increment $d\boldsymbol{v}$ is in the direction of $\nabla \psi_E$, which is perpendicular to the magnetic field lines. This allows us to write $\boldsymbol{v}(r, \theta, t) = v_m(r, t) \cos m\theta\, (\nabla \psi / |\nabla \psi|)$, and we can compute the velocity magnitude from the induction equation via

$$v_m(r, t) = \frac{1}{r}\left(\eta \nabla^2 \psi_m - \frac{\partial \psi_m}{\partial t}\right).$$

Thus a simple space quadrature over the flux function and its derivatives suffices to compute the global energy variation

$$\delta^2 U = \tfrac{1}{2}\int [(\nabla \psi_m)^2 + v_m^2] dV .$$

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