Abstract—The Cuk Converter offers simultaneous buck-boost operation, but requires careful design owing to its having a fourth-order transfer function and numerous practical design constraints. We exploit a serendipitous overlap between the converter circuit and the equivalent circuit of a dc motor to design a motor controller that can operate with supply voltage that is lower than the motor full-speed requirement. We examine the transfer function when such a topology is used to control the speed of a small motor. We conclude that the approach is relatively straightforward owing to the impact of the motor's inductance. Measurements agree with theory.

Index Terms—motor speed control, Cuk converter, switchmode power conversion.

I. INTRODUCTION

A. Cuk Converter

When Middlebrook and his graduate student Slobodan Ćuk presented in 1983 the detail of their 1977 design for a switchmode converter, the design grabbed the imagination of researchers because it approximated the “ideal dc transformer” [1]. The topology offered conversion between input and output dc voltages, with the voltage magnitude ratio set purely by the duty cycle of drive to a single switch, one side of which could be at ground potential. The outline circuit is shown in figure 1.

For a number of reasons the design was slower to take off than its promise might have lead the reader to expect. The transfer function is fourth order; the position of poles varies not only with component values, but also the controlling duty cycle; parasitic resistance in the two inductors and the so-called “Ćuk capacitor” all tend to significantly displace the poles of the characteristic equation; and the circuit was relatively complicated compared to other topologies [2]. Only now in the 21st century is the converter used with any confidence. For example, modern switching methods are just being applied [3], and a search of the Xplore Digital Library yields a total of 48 papers with “Ćuk” in the title published between 1990 and 1999, but 137 papers in the decade and a half since January 2000 [4]. Along with its variant the Single-Ended Primary-Inductance Converter (SEPIC converter) that swaps the position of output switch and inductor, and is thus able to generate an output voltage of the same polarity as the input, the Ćuk converter is the most versatile, but hardest to design, of switchmode converters.

B. Brushed DC Motor

A brushed dc (BDC) motor has an equivalent circuit that consists of the series combination of a voltage source, a resistance and an inductance. The voltage source represents the energy sink that is the mechanical output of the motor, or the source of electrical energy coming from the mechanical components when the motor operates in generator mode. The I-V characteristic of this voltage source embodies the pole that arises from the mass of the mechanical components as well as the loss inherent in doing mechanical work. Previous work has shown that feedback control of the speed of small BDC motors presents greater difficulty that control of larger motors, as the mechanical pole tends not to be dominant [5].

The equivalent circuit of a BDC motor, shown in figure 2, presents 2 poles. It also bears a strong resemblance to some components of the Ćuk topology. Consider in the circuit of figure 1 that the parallel combination of $C_o$ and $R$ will resemble a voltage source if $C_o$ is large, and $L_2$ with its inescapable parasitic resistance map exactly to $L_m$ and $R_m$ in figure 2. Overlaying these two circuit diagrams with the motor replacing the equivalent parts of the Ćuk circuit leads to the circuit of figure 3.

II. TRANSFER FUNCTION

We will now consider the transfer function of the circuit of figure 3 driving a typical, physically-small BDC motor. The mathematical derivation of the state-space equations is given in the appendix. Predictions will be made using these equations, evaluated in Matlab.
The series inductance of a small motor is typically a few millihenries. Our example motor has $L_m = 16\text{mH}$. Motors with rotors of about 1 cubic centimetre typically have this or even smaller an inductance.

Given the ability to switch at around 100kHz, we expect values for $L_{in}$ in the range up to a hundred $\mu$H; we will start with $10\mu$H which is quite practical a value for small converter circuits. Practical values of the Čuk capacitor are around a few $\mu$F; we will start with $2.2\mu$F.

We next consider the variation of duty cycle, $D$. Motor inductance is held at $16\text{mH}$, corresponding to our test motor, $L_{in}$ is kept at $10\mu$H and the Čuk capacitor stays at $2.2\mu$F. Figure 7 plots the interesting (close-in) situation. The two close poles, 3 and 4, actually separate as the duty cycle is increased, so that stability is likely to be better at higher loads. Crucially, this figure suggests that the converter-motor system will be no more difficult to control once feedback is applied than was the motor alone, that is with the motor powered with something close to an ideal voltage amplifier (or PWM equivalent).
Reassured by the above analysis of the transfer function, we construct a converter with the default values above, a 5V input supply, and a 12V BDC motor with $L_m = 16\, \text{mH}$ series inductance and $12\, \Omega$ resistance, $L_{in} = 10\, \mu\text{H}$, $R_{in} = 1.2\, \Omega$, $R_C = 0.1\, \Omega$, $R_t = 0.1\, \Omega$, $R_d = 0.1\, \Omega$, $J = 0.05$, $b = 0.8$, and $K_b = 0.01$.

Figure 8 is a screen capture from measurements made on the prototype. To a remarkable degree to time-domain waveforms have the instantaneous shape that is to be expected, except for finite risetimes and small “wiggles” that are attributed to measurement artefacts and extraneous parasitic impedances. The upper part of the figure shows the evolution of waveforms on a longer time scale. Figure 9 shows the variation of shaft speed on yet longer a time scale. Matlab simulation agrees with measured data where available. Figure 10 shows the input and output currents in response to the same step input. While the input inductor current may spike, the output current describes, on average, a smooth response reminiscent of a single-pole exponential change. Figures 11 and 12 show the evolution of predicted and measured mean input and output current. There is a discrepancy about 1 second that is attributed to mechanical imperfections in the apparatus and errors in our values for electrical parasitics.
IV. CONCLUSION

We have shown that the Čuk topology is readily applied to drive a small motor. The overlap of circuit topologies results in poles from the motor replacing poles in the Čuk transfer function to yield only the same number of poles (order of the system) as existed without the motor. Component values can be chosen so as to leave a dominant pole, and a system around which feedback can be applied with no more complexity than existed in the case of the motor driven by a perfect analog source.

APPENDIX

There are various ways to model a switchmode circuit [6]. Here we develop the state-space equations for the circuit of figure 3 [7], [8]. Including the mechanical pole of the motor and load, the system will be of fourth order. Let the state be

\[ x = \begin{bmatrix} I_{in} \\ I_m \\ V_C \\ \omega \end{bmatrix} \tag{1} \]

where \( I_{in} \) is the (input) current drawn from the source through inductor \( L_{in} \), \( I_m \) is the motor (output) current, \( V_C \) is the voltage across the Čuk capacitor, and \( \omega \) is the (desired) motor shaft output rotational speed. Next we define the input variables

\[ u = \begin{bmatrix} V_{in} \\ T_L \end{bmatrix} \tag{2} \]

where \( V_{in} \) is the supply (input) voltage and \( T_L \) is the torque (load) encountered on the motor output shaft. Then

\[ \dot{x} = Ax + Bu \tag{3} \]

\[ y = Cx \tag{4} \]

where \( y \) is the output of the system. The state-space equations are perturbed with

\[ d = D + \dot{d} \tag{5} \]

\[ x = X + \dot{x} \tag{6} \]

\[ y = Y + \dot{y} \tag{7} \]

\[ u = U + \dot{u} \tag{8} \]

In steady state

\[ \dot{x} = AX + BU = 0 \tag{9} \]

\[ X = -A^{-1}BU \tag{10} \]

\[ Y = CX \tag{11} \]

We have a continuous, time-varying system, as the switch has two states. The state matrix \( A \) is represented by two matrices, \( A_1 \) and \( A_2 \) representing the switch-closed and switch-open conditions. The variable \( D \) weights the two condition state matrices. Next we write

\[ \dot{x} = A\dot{x} + B\dot{u} + [(A_1 - A_2)X + (B_1 - B_2)U]\dot{d} \tag{12} \]

\[ \dot{y} = C\dot{x} + (C_1 - C_2)Xd \tag{13} \]

\[ A = DA_1 + (1 - D)A_2 \tag{14} \]

\[ B = DB_1 + (1 - D)B_2 \tag{15} \]

\[ C = DC_1 + (1 - D)C_2 \tag{16} \]

A. On state

When the switch is closed the circuit becomes that shown in figure 13. Application of Kirchoff’s laws yields:

\[ \frac{dI_{in}}{dt} = \frac{1}{L_{in}}V_{in} - \frac{R_{in} + R_d}{L_{in}}I_{in} - \frac{R_b}{L_{in}}I_m \tag{17} \]

\[ \frac{dI_m}{dt} = \frac{1}{L_m}V_C - \frac{k_b}{L_m}\omega - \frac{R_m + R_C + R_d}{L_m}I_m - \frac{R_b}{L_m}I_{in} \tag{18} \]

\[ \frac{dV_C}{dt} = -\frac{1}{C}I_m \tag{19} \]

\[ \frac{d\omega}{dt} = \frac{k_t}{J}I_m - \frac{b}{J}\omega - \frac{T_L}{J} \tag{20} \]

B. Off state

When the switch is closed the circuit becomes that shown in figure 14. Again the application of Kirchoff’s laws yields:

\[ \frac{dI_{in}}{dt} = \frac{1}{L_{in}}V_{in} - \frac{R_{in} + R_C + R_d}{L_{in}}I_{in} - \frac{R_d}{L_{in}}I_m - \frac{1}{L_{in}}V_C \tag{21} \]

\[ \frac{dI_m}{dt} = \frac{k_b}{L_m}\omega - \frac{R_m + R_d}{L_m}I_m - \frac{R_b}{L_m}I_{in} \tag{22} \]

\[ \frac{dV_C}{dt} = \frac{1}{C}I_{in} \tag{23} \]

\[ \frac{d\omega}{dt} = \frac{k_t}{J}I_m - \frac{b}{J}\omega - \frac{T_b}{J} \tag{24} \]
C. State Space Matrices

We are finally able to derive the two matrices

\[
A_1 = \begin{bmatrix}
-\frac{R_{in} + R_t}{L_{in}} & -\frac{R_t}{L_{in}} & 0 & 0 \\
-\frac{R_t}{L_{in}} & -\frac{R_{in} + R_t + R_s}{L_{m}} & \frac{1}{L_{m}} & -\frac{k_s}{L_{m}} \\
0 & -\frac{1}{C} & 0 & 0 \\
0 & \frac{k_s}{J} & 0 & -\frac{b}{J}
\end{bmatrix}
\]  
(25)

\[
A_2 = \begin{bmatrix}
-\frac{R_{in} + R_s}{L_{in}} & -\frac{R_s}{L_{in}} & -\frac{1}{L_{in}} & 0 \\
-\frac{R_s}{L_{in}} & -\frac{R_{in} + R_s}{L_{m}} & 0 & -\frac{k_s}{L_{m}} \\
\frac{1}{C} & 0 & 0 & 0 \\
0 & \frac{k_s}{J} & 0 & -\frac{b}{J}
\end{bmatrix}
\]  
(26)

and

\[
B = B_1 = B_2 = \begin{bmatrix}
\frac{1}{L_{in}} & 0 \\
0 & 0 \\
0 & 0 \\
0 & \frac{J}{T_s}
\end{bmatrix}
\]  
(27)

\[
C = C_1 = C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}
\]  
(28)

which will permit calculation of the performance of the circuit using a tool such as Matlab.

REFERENCES