Understanding the beliefs and behaviours of low-skilled adults as they re-engage with mathematics

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Abstract

This thesis explored low-skilled adult learners' beliefs about mathematics and how they engaged with mathematical content delivered as part of foundation-level vocational programmes in New Zealand. It also examined how low-skilled learners holding procedurally-oriented beliefs responded to a classroom environment that emphasised conceptual understanding, collaboration and discourse. An 'insider research' approach was adopted to capitalise on the researcher's prior experience in the sector. Analysis of data utilised an interpretive approach drawing on Bandura's triadic reciprocal determinant model that posited beliefs, behaviour and the environment as interacting factors. Methods of data collection included surveys, observations, interviews and an intervention in which the researcher took a dual tutor/researcher role. Observation and intervention data were collected through multiple audio-recording devices that recorded private and public utterances as learners participated in lessons.

The survey and interview findings showed that most low-skilled adults believed mathematics to be procedural, performance-oriented, and learned by adopting passive strategies. Many held non-mathematical identities, described school and foundation-level mathematics lessons as potentially shame making, and reported censoring their behaviours to avoid shame.

The use of multiple-recording devices was effective for capturing the complexity of classroom interactions. Learners' behaviours within lessons were consistent with procedural/calculational beliefs and reflected performance-oriented goals such as completing tasks quickly and accurately. Some learners displayed negative affective responses when expected to engage with mathematical content, and adopted behaviours designed to reduce, or eliminate, public exposure to failure. Group problem-solving was characterised by an unequal division of labour between 'solvers' and 'supporters'. Lower-skilled learners tended to adopt peripheral support roles, while procedurally proficient learners either took, or were assigned, responsibility for all mathematical thinking. These behaviours enabled groups to complete tasks and achieve pseudo-success yet constrained the lower-skilled learners' engagement in mathematical thinking, while also presenting little mathematical challenge to higher-skilled learners. This pattern appeared to contribute to the maintenance of learners' low skills and non-mathematical identities by routinising their deferral of agency to more proficient learners.

The intervention indicated that low-skilled procedurally-oriented learners tended to resist conceptually-oriented activities that emphasised mathematical discourse, collaboration, and inquiry. This appeared to be because these activities increased the threat of a shameful episode and were perceived as superfluous to their procedurally-oriented goals. Learners reported that the intervention was more effective than their traditional classes, but attributed this to better tutor explanations, rather than their own active engagement in mathematical thinking. The overall findings indicated that low-skilled procedurally-oriented adults preferred and expected a traditional approach, and many used its familiar routines to reduce their level of engagement and exposure to shame. Implications for educators include the need to differentiate authentic from pseudo-engagement, to understand the impact of shame on behaviour, and establish positive patterns of engagement.

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Chapter 1: Introduction

1.1 Background to the study

This thesis explores the beliefs, behaviours, and environments of low-skilled adult learners as they re-engage with mathematics in foundation-level vocational programmes. These programmes offer lower-skilled adults an opportunity to develop valuable mathematical skills within the context of vocational training. However, they are typically the learners' first experience with mathematics education since leaving school. In many cases, their school mathematics experiences were negative, and may have contributed to lasting beliefs about what mathematics is, how it is learned, or whether they can be mathematically successful. These beliefs, associated behaviours and learning environments, may perpetuate the difficulties learners have with mathematical content, negating the opportunity to learn from mathematical provision.

Despite the best intentions to support low-skilled adult learners, little is known about how such learners with negative school experiences of mathematics re-engage with provision in foundational-level vocational contexts. There is little, or no, research that explores the mathematical beliefs held by low-skilled adult learners, and even less that explores how these might influence their behaviours within vocational learning environments. However, there is a large body of research conducted with different cohorts, that raise concerns about the influence of negative beliefs on such learners.

Research conducted with school and university students, and pre- and in-service teachers, suggests that over the course of an individual's educational experience some learners develop negative patterns of beliefs about the nature of mathematics and how it is learned (Goldin, Rösken, & Törner, 2009; Hannula et al., 2016). These beliefs, in turn, go on to shape how mathematics is engaged with and learned. Research also shows that negative mathematical experiences have a profound impact on individuals' affective responses to mathematics, leading to anxiety, shame, and avoidance strategies (Brown, Brown & Bibby, 2008; Evans, 2000; Mumcu & Aktas, 2015). Negative beliefs appear to influence behaviours, emotions and attitudes and undermine learner success when engaging in mathematical tasks (DeBellis & Goldin, 2006; Schoenfeld, 1989). Furthermore, personal factors, such as beliefs, not only affect behaviour and the environment, they reciprocally interact, giving rise to a variety of potential behaviours and environments (Bandura, 2006). It

is possible that these dynamic combinations may give rise to adult classroom environments that constrain, not enhance, learning.

Given the potential impact of negative beliefs to interfere with learners' engagement patterns, it is important to learn whether low-skilled adults, who have had poor mathematical experiences, hold beliefs about themselves and mathematics that have implications for how they engage with mathematical content delivered within their vocational programmes. Additionally, it is important to understand the patterns of behaviour that emerge as these learners interact in mathematics lessons, as these may have implications for learner engagement.

Situating the research

The research in this study encompasses a broad range of research from several domains: Adult numeracy and literacy, mathematics education, lifelong learning, and adult mathematics literature. These domains are diverse, overlapping, and contested Coben (2006). Wedege, Benn and Maaß (1999) have described the research domain of adults learning mathematics as a moorland, less of a bounded field and rather more wild and uncultivated. In 2006 Coben (p. 18) described the domain as "coming to be recognised as worthy of serious research but… still beset by conceptual difficulties". In 2016 conceptual challenges remain (Safford-Ramus et al., 2016). While a strong body of research is growing and with it a community of researchers, studies of low-skilled adults remain rare, suggesting links from research to practice should be prioritised. FitzSimons, Coben and O'Donoghue (2003, p. 117) suggest that "research must be closely linked with practice in a field where development and improvement in practice have priority status". Likewise, Wedege (2001) argues that a research aim ought to be to 'empower' adults learning mathematics. To maximise the effectiveness of this study I have situated it within the domain of adults learning mathematics, aware of the tensions, debates and contested domains. This, I believe, will better cope with the array of research and findings. The term 'mathematics' is used broadly and inclusively in a way consistent with FitzSimons et al.'s (2003) description of mathematics as inclusive of: specialised mathematics and service mathematics, school mathematics, vocational mathematics, street mathematics, mathematics for everyday living, and adult numeracy.

Adult numeracy levels

Understanding the mathematical beliefs and behaviours of low-skilled adults is essential for three reasons. First, the current level of numeracy skills amongst New Zealand's working age population is alarmingly low and has changed little since

measurements began. As early as 1996 the International Adult Literacy Survey (IALS) identified that half of the adult population had poor quantitative literacy skills, defined as the ability to apply arithmetic operations to information embedded within written text (Walker, Udy & Pole, 1996). However, the two most reliable and often-cited statistics, the Adult Literacy and Life Skills Survey (ALL) (OECD & Statistics Canada, 2011) and the Programme for the International Assessment of Adult Competencies (PIAAC) (OECD, 2016a), provided explicit information on the distribution of numeracy skills within the adult population aged 16-65 and how these related to various outcomes.

The ALL and PIAAC surveys share a common, yet evolving, framework that measures numeracy proficiency against five levels distributed across a 500-point scale[1]. Level 5, the highest, indicates "expert" level, while level 2 or below indicate low skills (see Appendix A for descriptions) (Gal, van Groenestijn, Manly, Schmitt, & Tout, 2005; Grotlüschen, Mallows, Reder, & Sabatini, 2016; PIAAC Numeracy Expert Group, 2009). Both surveys found that New Zealand adults with no qualifications had lower skills than most of their counterparts in other participating countries. For example, the average score of non-qualified New Zealand youth, aged 16 to 24, was level 1 (Ministry of Education & Ministry of Business, Innovation and Employment [MOE & MBIE], 2016a; Satherley & Lawes, 2009). The ALL results found that 87% of adults who left school before Year 11 had scored at level 2 or below, and 51% scored within level 1, indicating very poor skills (Lane & Smyth, 2009; Satherley & Lawes, 2009). Ethnicity was also a factor, with Māori and Pasifika over-represented among adults with no qualifications and low numeracy scores (MOE & MBIE, 2016c). It is such adults, from here on referred to as *low-skilled adults*, who are the target population of the mathematical provision delivered within foundation-level tertiary education vocational programmes (Tertiary Education Commission, 2015).

A concern is that low-skilled adults are at a disadvantage in an increasingly mathematical society. The ALL and PIAAC literature make the case that mathematical skills are, and increasingly will be, an imperative for full participation in the workplace and community (Gal et al., 2005; PIAAC Expert Numeracy Group, 2009). Studies of workplace mathematical practices reveal that even within so-called low-skilled jobs, employees are required to engage with complex mathematics (FitzSimons & Coben, 2009; FitzSimons, Mlcek, Hull, & Wright, 2005; Marr & Hagston, 2007). Furthermore, even relatively non-complex mathematical tasks are

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[1] The PIAAC analysis also included adults scoring below level 1.

undertaken within complex and highly variable circumstances (Keogh, Maguire & O'Donoghue, 2014). Low numeracy skills are considered inhibitory to full participation in the civic, recreational and cultural activities of adults (Antoni & Heineck, 2012; Bynner & Parsons, 2006; Marcenaro Gutierrez, Vignoles, & de Coulon, 2007; OECD, 2013; Steen, 2001). Those with low skills are at considerable risk of being unable to meet the demands of life and work, and of being able to engage in the practices likely to improve their skills (Grotlüschen et al., 2016).

The need for strong numeracy skills
The second reason it is important to explore the beliefs and behaviours of low-skilled adults is because low numeracy skills are associated with negative life outcomes for individuals and negative outcomes for businesses and the wider economy. Low-skilled adults are more likely to leave full-time education earlier than those with stronger skills and participate less in further training opportunities (Carpentieri, Litster, & Frumkin, 2010; Grotlüschen et al., 2016; Ministry of Education, 2017a; MOE & MBIE, 2016a; Parsons & Bynner, 2005, 2007). They are more likely to experience frequent periods of unemployment and casual work, occupy low paying, labour-intensive jobs, have fewer opportunities for promotion (Bynner & Parsons, 2006; Carpentieri, et al., 2010; de Coulon, Meschi & Yates, 2010; Grotlüschen et al., 2016; Hanushek, Schwerdt, Wiederhold & Woessmann, 2015). Additionally, low-skilled adults are more likely to live in substandard and/or overcrowded housing, to suffer depression, and are more likely to have reported poor physical health in the last year (Carpentieri et al., 2010; de Coulon et al., 2010; OECD, 2013; Parsons & Bynner, 2005).

Employees with low numeracy skills negatively impact businesses, employers and the wider economy (Coulombe, Tremblay, & Merchand, 2004; Mallows, Carpentieri & Litster, 2016; Tu et al., 2016). Employer surveys have shown that low numeracy skills are associated with the following: increased workplace accidents; loss of customers due to errors; the requirement to recruit from external sources because of insufficient internally skilled staff to fill new roles; and lower overall productivity (Mallows et al., 2016; Tu et al., 2016). Upskilling the adult working population is considered a competitive imperative for many OECD countries (Boaler et al., 2017; OECD, 2013). In summary, improving the mathematical provision provided to low-skilled adults could benefit individuals, families, businesses, and society.

Increase in societal and workplace numeracy demands

The third reason why understanding the beliefs and behaviours of low-skilled adults is important is that the mathematical demands of society are expected to increase, meaning the current disadvantages will be exacerbated. This demand for higher numeracy skills is thought to be driven by evolving technology, globalisation, the shift to knowledge-based industries, and an increasing business focus on quality and efficiency (Confederation of British Industry [CBI], 2015; Expert Group on Future Skills Needs [EGFSN], 2015; Tu et al., 2016). The workplace is expected to become more automated, data-driven, outsourced and lean, increasing the demand for individuals with multi-disciplinary technical, mathematical, management, and design skills (Störmer et al., 2014). The demand for lower-skilled workers is expected to decrease, along with reduced training opportunities, creating a potential social and economic divide (Marr & Hagston, 2007; Störmer et al., 2014).

There is also evidence that the mathematical demands of societal contexts are increasing. Technological advances have been cited as increasing the amount of quantitative data individuals are exposed to (PIAAC Numeracy Expert Group, 2009). This is evident within the domains of finance, health, education, and official statistics (Störmer et al., 2014). Additionally, individuals are increasingly required to interact with quantitative data, rather than simply receive it. Such interaction requires more critical mathematical skills to understand, interpret, and analyse complex information (Mevarech & Kramarski, 2014; Perso, 2006). The constant increase in mathematical demands is used to support the proposition that working adults will be required to learn mathematics as a lifelong endeavour, engaged across working life, rather than simply within the school sector (CBI, 2015; FitzSimons, 2010; Siivonen, 2013).

New Zealand embedded numeracy provision

The TEC responded to the above issues by implementing several strategies, one of which was to embed numeracy provision within foundation-level vocational education programmes that cater for low-skilled learners (TEC, 2008b, 2015). A supporting TEC resource articulated the expectation that "Links between literacy and numeracy learning and vocational learning are clearly and explicitly identified" and that "Explicit literacy and numeracy instruction is provided for the vocational task at hand" (TEC, 2009, p. 9). The strategy was supported by the Adult Literacy and Numeracy Learning Progression Framework (TEC, 2008a), the Literacy and Numeracy for Adults Assessment Tool (LNAAT), and other infrastructural elements[2] (TEC, 2008b).

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[2] See http://www.literacyandnumeracyforadults.com

The strategy to embed numeracy into vocational programmes drew on international research showing that embedding literacy and numeracy improves retention and programme achievement when numeracy is: deliberately connected to the vocation, or to real-life contexts; supported by both content *and* numeracy tutors; and supported with a whole organisational approach (Alkema & Rean, 2013; Leach, Zepke, Haworth, Isaacs, & Nepia, 2009; Office for Standards in Education [Ofsted], 2011). However, studies present a mixed view about whether the learners in such programmes improved their numeracy skills, how the mathematical content was delivered, or how the learners engaged with the mathematical content. Dalby and Noyes (2015) compared academic with embedded vocational lessons and reported that the vocational learning environment supported learner engagement by delivering mathematics in the context of vocational values. Yet others (e.g., Casey et al., 2006), found that when a single tutor held dual responsibility for vocational content and numeracy delivery, the results were worse than when numeracy was not embedded within provision. What is known is that low-skilled adults learn less from educational programmes, their skills tend to remain weak or deteriorate over time, and this inhibits their participation in further learning activities (MOE, 2017; OECD, 2013). This raises concerns about the effectiveness of numeracy provision with low-skilled adults, and the potential for a vicious cycle of low skills and poor engagement.

Difficulties with numeracy are related to beliefs
Definitions of numeracy have increasingly extended beyond mathematical skills and knowledge, to also include dispositional and behavioural aspects (Coben, 2003; Gal et al., 2005; Maguire & O'Donoghue, 2003). The theoretical work conducted by the PIAAC Numeracy Expert Group (2009) refined a conception of numeracy based on the premise that numeracy competence is reflected in *numerate behaviour*. Numerate behaviour requires skills, but also importantly, enabling factors and processes that support the individual in attempting to engage with a mathematical task, rather than delegate it to others or simply ignore the mathematical content. For example, the PIAAC Numeracy Expert Group describes "numerate behaviour" as follows:

> Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors, and processes (Numeracy Expert Group, 2009, p. 20).

The enabling factors for numerate behaviour include positive beliefs, attitudes, habits of mind, and prior experiences. The authors argue that these are essential for autonomous engagement with numeracy and suggest that negative beliefs about the nature of mathematics, how it is learned, and beliefs about personal performance may undermine productive engagement. Negative beliefs, attitudes and prior experiences appear to be intertwined with poor skills and may contribute to adults avoiding engagement with mathematical tasks and upskilling opportunities. If so, this may diminish the effectiveness of embedding mathematical provision in foundation-level vocational training. Yet, there little research that explores the beliefs and behaviours of low-skilled adults who may not want to participate in mathematical provision, but who must attend as part of their vocational training. Neither is there research that explores patterns of engagement within the foundation-level vocational classrooms in which negative beliefs might be the norm, not the exception.

1.2 Rationale

Few, if any, studies have explored the mathematical beliefs held by low-skilled New Zealand adult learners. Neither have studies observed low-skilled New Zealand adults as they participate in embedded numeracy lessons in foundation-level vocational programmes. Secondly, there are few observational studies of low-skilled adult learners as they interact in vocational mathematics lessons, and those that do so are outside of New Zealand (Dalby & Noyes, 2015). Furthermore, the studies that observed adults participating in numeracy lessons utilised observational methods from a single perspective, with the researcher taking notes while observing, or using a single video, or audio recorder. While these methods are highly informative, it is possible that the single view may miss important, and perhaps covert, behaviours occurring within lessons. In addition there is little 'insider research' or use of dual tutor/researcher roles to shed light on the nuances of classroom interactions in these environments. In short, little information is available on what behaviours learners adopt in such lessons, and how these behaviours enhance or constrain learning. This is a pertinent point because, despite a substantial investment of public resources, and the substantial number of adult learners involved, little is known about the kinds of mathematics with which learners engage, how they engage with it, and the extent to which they benefit from the experience.

This research explored a pivotal, yet under researched, aspect of adult numeracy; the influence of low-skilled adult learners' beliefs on their engagement with mathematics in foundation-level vocational settings. The study explored low-skilled

adults' beliefs about what mathematics is, how it is learned, their relationship with it, their behaviours as they took part in embedded mathematics lessons, and the broader classroom patterns these led to. It raises questions about how learners who may have disengaged from mathematics in school re-engage in ways that develop their understanding and use of mathematics, and to what extent learners' negative beliefs contribute to, and possibly perpetuate, continued difficulties with mathematics.

1.3 Researcher positioning

This study has its origins in two influential professional experiences. The first was an eight-year teaching experience working with low-skilled adults who had been identified as in need of further numeracy skills to prepare for training or employment. The role involved wrap-around support and tuition which included delivering mathematics lessons to up to 30 learners at any one time, supplemented by one-to-one tuition. Additionally, during learners' transition into employment, the role included observing the transferability and efficacy of their numeracy skills and providing additional support where necessary.

I had observed many adult learners with negative emotional and attitudinal responses to mathematics resist engagement in group contexts, preferring instead to engage with mathematics in private one-on-one situations. Their accounts of school often included negative references to mathematics and many engaged tentatively in classroom sessions, reluctant to contribute their own ideas or solutions. Many preferred to work in isolation or confide in a single friend on whom they relied for support. Others became frustrated or upset when they made errors. Some vehemently declared the futility of mathematics, while others rejected it completely.

For some reason, unclear to me, the learners themselves resisted engaging in the practices I believed were likely to contribute to improved mathematical understanding and performance. The issue was rarely addressed within the professional development opportunities offered to tutors which tended to draw on school-based research. The focus of the professional development was on *preventing* the onset of negative attitudes, rather than remediating existing negative attitudes or beliefs, or understanding how these might influence behaviour.

Another influential experience was a ten-year role providing professional development to tutors working within tertiary organisations as part of a coordinated strategy to develop adults' numeracy capability. This role provided insight into the concerns and needs of tutors, businesses and tertiary providers. Tutors reported that

learners either struggled or resisted engagement, and real mathematical improvement was rare. Careful not to blame the learner, many tutors felt that learners had already been 'turned-off' mathematics before their tertiary sector provision. The concern was that despite being present within mathematical lessons, many learners were not engaging with content in a meaningful way.

These experiences with low-skilled adult learners and numeracy tutors have contributed to a knowledge of much of the culture, norms, traditions, challenges, and power struggles that exist within the New Zealand foundation-level embedded numeracy environment. This prior experience situates me as an 'insider researcher' because the study is situated within a context with which I am deeply familiar and therefore know about "how the system really works" (Teusner, 2016, p. 85). This knowledge is an asset as can be used to inform the design, implementation and analysis of the study and contribute what I hope is a more nuanced analysis of the data (Coghlan, 2007).

My experience as a practitioner also contributed to my decision to adopt a dual tutor/researcher role for the final data collection component of this study, the intervention. This approach enabled the inclusion of my experiences within the tutor role, while continuing to uphold valid forms of data analysis. Another reason to adopt the dual tutor/researcher role was that observations of adult numeracy lessons, (e.g., Coben et al., 2007), typically reveal a narrow repertoire of pedagogical approaches used by tutors. Adopting the role of tutor as well as researcher created the opportunity to introduce a wider array of pedagogical approaches and explore learners' responses to these.

In summary, my experiences in the sector have raised questions about the impact of learners' prior experiences and beliefs, their behaviours as they participate in mathematics lessons, and what might be done to improve these. This thesis was undertaken to explore these questions and develop practitioner and researcher understanding to improve adult learner engagement and outcomes.

1.4 Research questions

The research considered three questions:

1. What beliefs do low-skilled adults hold about mathematics?
2. How do low-skilled adults engage with mathematics within vocational lessons?
3. How do low-skilled adults respond to a classroom environment that emphasises conceptual understanding?

1.5 Research design

This study comprised four elements used to explore the three research questions. The first, second and third elements utilised surveys, observations, and interviews, respectively, to explore the first two research questions. The fourth element, an intervention, explored the behaviours of low-skilled learners as they participated in an innovative classroom environment that emphasised conceptual understanding. An interpretive approach to analysis was used drawing on Bandura's (2006) triadic reciprocal determinant model that posited beliefs, behaviour and the environment as interacting factors. Data were thematically analysed within this framework. The data collection methods used within the observations and intervention included the use of multiple audio-recording devices designed to record otherwise private interactions, self-talk, group and whole-class discussions.

The thesis is structured into eight chapters. Following this introductory chapter, Chapter 2 reviews the relevant literature pertaining to mathematical beliefs and their influence on learners' engagement with mathematics. Chapter 3 outlines the methodological approach, including the epistemological and theoretical underpinnings, the methods, and data analysis. Chapters 4 through 7 present the findings of the research and make links to relevant literature reviewed in Chapter 2. Finally, Chapter 8 draws together the key findings of the research, discusses the study's limitations, draws out implications for practice (in teaching and professional development) and makes suggestions for further research.

Chapter 2: Literature Review

This chapter reviews research pertaining to mathematical beliefs and the impact they have on learners' engagement with mathematics. I begin by examining literature on beliefs about the nature of mathematics and how these beliefs shape learners' views about how mathematics ought to be learned. I then look at literature on beliefs about how mathematics is learned and how these beliefs influence goals, strategy use and achievement. Following this I focus on mathematical beliefs related to motivation before reviewing the beliefs that learners hold about themselves as doers of mathematics. The next section examines affective factors such as identity, shame, emotion and attitude and how these relate to engagement and avoidance. The literature review ends with ways in which beliefs might be developed within classroom environments, and interventions designed to influence belief formation. I conclude with a description of the aims of the study.

2.1 Mathematical beliefs

Mathematical beliefs have been described as an individual's "mathematical world view", which influences their perspective of mathematics, themselves, and various contexts (Schoenfeld, 1985, 2011). Beliefs are similarly described as the *lens* through which people view and interpret the mathematical world, and through which they make their mathematical decisions (Philipp, 2007). More precisely, they have been defined as cognitive and affective *configurations*, to which the holder attributes a truth value of some kind; for example, empirical truth, validity, or applicability (Goldin, Rösken & Törner, 2009). Other researchers have emphasised the subjective and often unconscious nature of beliefs (Cross Francis, 2015; Furinghetti & Pehkonen, 2002). For example, Op't Eynde, De Corte and Verschaffel (2007) have described beliefs as implicitly or explicitly held subjective conceptions that people hold to be true. It is the individual attribution of truth that transforms conjectures, propositions, stories or hypotheses into beliefs (Goldin, 2002).

Secondly, there is wide agreement that individuals do not hold beliefs in isolation, but rather that beliefs exist within clusters or networks of other similarly connected beliefs (Goldin et al., 2009; Green, 1971). Green termed these networks "belief systems", consisting of relatively stable primary beliefs that support less stable derivative beliefs. Goldin and colleagues (2002, 2009, 2011) similarly described the set of beliefs held by an individual as a *belief structure*, reserving the term *belief system* to mean an elaborated set of interconnected beliefs that are shared socially or

culturally. Despite these terminology differences, most conceptualisations of beliefs posit that the beliefs within the system, or structure, are mutually reinforcing, although not always logically so (Mason, 2003).

Beliefs are not only cognitive in nature but are also integrated into the affective domain, in many cases included as an aspect of the affective domain with values, attitudes and emotions (Grootenboer & Marshman, 2016; Hannula et al., 2016; McLeod & McLeod, 2002; Philipp, 2007). The theoretical work of McLeod (1992), and its elaborations (Grootenboer & Marshman, 2016), situates affective aspects on a continuum with beliefs situated as more cognitively stable and less affectively intense, while the other aspects are situated as less stable and more intense (See Figure 1) (Leder & Grootenboer, 2005; McLeod, 1992). Beliefs were thought to create the conditions for learners to make positive or negative evaluations of their performance, leading to a rise in more transient emotions, that over time, would contribute to the formation of more stable attitudes (McLeod, 1992; Philipp, 2007).
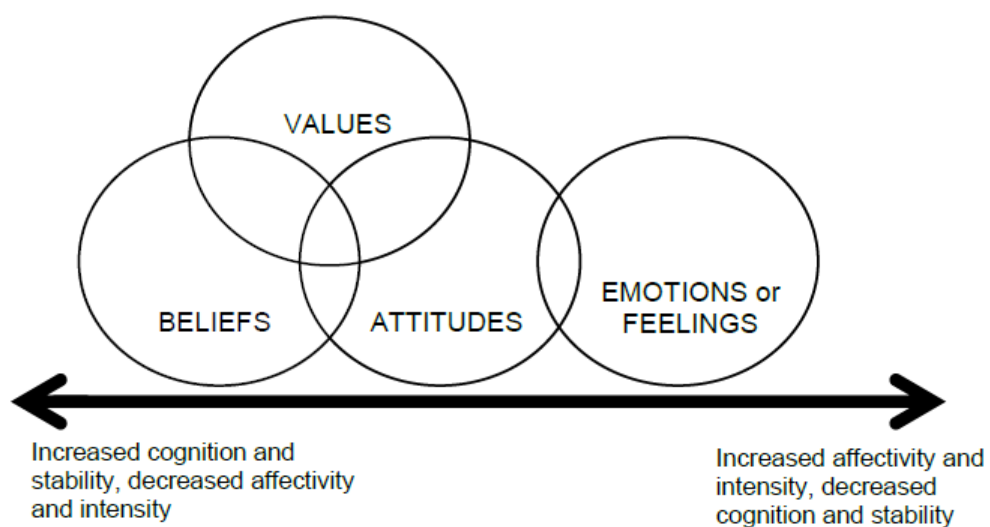
Figure 1: Conception of the affective domain
(Source: Leder & Gootenboer, 2005, p. 2)

Other conceptions posited belief systems as constructs shaped by experiences that encode into affective and cognitive domains (DeBellis & Goldin, 2006; Goldin, Epstein, Schorr & Warner, 2011; Haser & Doğan, 2012). The influence of beliefs on other affective responses was found to be reciprocal, as not only did beliefs set the conditions for affective responses, but affective responses also contributed to the development of beliefs (Goldin et al., 2011; Op't Eynde, Corte, & Verschaffel, 2006). For example, Goldin et al. hypothesised that a learner might experience pride or

pleasure when they answer arithmetic problems quickly and accurately, and develop beliefs that speed and accuracy are good measures of ability. The repetition of such events is argued to lay down affective pathways that become interwoven with cognition, in which beliefs about ability, speed and accuracy serve to support positive affective responses.

Beliefs have also been explored within several other constructs that combine beliefs with affective, cognitive and conative domains. They have been explored within the context of learners' image of mathematics (Lane, Stynes & O'Donoghue, 2014, 2016). Drawing on the conceptual work of Lim (1999) and Wilson (2011), Lane et al. (2016) explored beliefs as a cognitive aspect of a learner's image of mathematics. Wilson (2011) explored beliefs within the broader context of 'dispositions' comprising beliefs/values/identities, affect/emotion, behavioural intent/motivation, and needs. Finally, self-efficacy beliefs have also been explored across affective, cognitive and conative domains (Tait-McCutcheon, 2008).

To summarise, there is broad agreement that beliefs shape the way a person perceives the nature and purpose of mathematics; how it is learned and why; their affective responses to it; and their own relationship with it. Beliefs are considered to reciprocally influence affective, cognitive and conative domains. Because of this, many researchers have suggested that beliefs underpin *all* mathematical thinking and cognitive activity. Beliefs may be even more important in cases where the beliefs that have developed to *constrain* positive mathematical engagement. Despite widespread awareness of the importance of beliefs for adult education, there is no research that explores the beliefs of low-skilled adult learners as they re-engage with mathematics in vocational settings.

Antecedents of beliefs research
Mathematical beliefs have been investigated via two streams of research. The first emerged from educational psychology and investigated mathematical beliefs in the context of epistemic beliefs: beliefs about the nature of knowledge, how it is acquired, and how it is evaluated. Early epistemological work conceptualised the focus of beliefs slightly differently. For example, Perry (1968) explored the development of epistemic beliefs as students progressed through college. Belenky, Clinchy, Goldburger and Tarule (1986) explored the development of epistemic beliefs held by women within diverse intellectual environments. King and Kitchener (2004) explored the development of epistemic beliefs by analysing how individuals justify their knowledge. The emergent models broadly represented beliefs as developing along a

single continuum from "naïve", believing knowledge is absolute, simple and handed down by authority, to "sophisticated", believing knowledge is complex, subjective, tentative and derived through reason (Belenky et al., 1986; Hofer, 2002; King & Kitchener, 2004; Perry, 1968). However, the unidimensional view of epistemic beliefs that developed in fixed stages of progression was found to be inadequate in the face of growing research and epistemological beliefs were conceptualised as a system of interrelated but independent beliefs (Schommer, Crouse & Rhodes, 1992). Finding that a single continuum did not represent the various dimensions of epistemic beliefs, Schommer (1990) introduced a multi-dimensional model by synthesising the previous research into a system of five independent beliefs. These were beliefs about: the structure, stability, and source of knowledge; the speed of learning; and the ability to learn. Interest turned to determining whether these beliefs were held regarding knowledge generally, or whether they differed between specific domains, mathematics being a prime area of investigation (De Corte, Op't Eynde & Verschaffel, 2002). Both general and domain-specific beliefs were thought to interact, but specific mathematical beliefs were found to have a greater effect on learner behaviours (Choi & Kwon, 2012; Schommer-Aikins, Duell & Hutter, 2005). Beliefs about the structure, source and stability of mathematics, how long it took to learn, and whether mathematical ability is innate or not were found to relate to learners' strategy use, time spent on tasks, mathematical achievement and motivation (Liu, 2010; Mason, 2003; Muis, 2004; Op't Eynde, et al., 2006).

The second stream of mathematical belief research emerged from mathematics education research and focused on three main areas: beliefs about mathematics, mathematics learning and problem-solving; beliefs about the self in relation to mathematics; and beliefs about the social context of learning. However, there is much overlap, and widespread agreement that cognitive, affective and social processes are reciprocally interrelated (DeBellis & Goldin, 2006; Goldin, et al., 2011; Middleton, Jansen & Goldin, 2017; Schoenfeld, 2011). For example, mathematical beliefs have been studied in relation to mathematical anxiety, mathematical identities, emotions, values, and attitudes (DeBellis & Goldin, 2006; Evans, 2000; Goldin, Roskin & Törner, 2009; Grootenboer, & Jorgensen, 2009; Hannula et al., 2016; Maaß & Schlöglmann, 2009; McLeod, 1992; Mendick, 2005; Perry, 2004). They have also been studied in relation to learners' problem-solving behaviours, including beliefs that promote good dispositions (Berkaliev & Kloosterman, 2009; Francisco, 2013; Schoenfeld, 1985, 2011), and in relation to problem-solving heuristics and strategies (Lester, Garofalo & Lambdin Kroll, 1989; Liu, 2010; Mason & Scrivani, 2004; Schoenfeld, 1992, 1989; Stylianides & Stylianides, 2014). Research has explored the

impact of beliefs on the social context of learning, including discourse patterns, classroom behaviours, shame, image-management, and avoidance strategies (Bibby, 2002; Goldin et al., 2011; Lampert, 1990; Yackel & Cobb, 1996). In summary, beliefs have been the focus of investigation for decades, yet little research has explored the beliefs of low-achieving adult learners.

The early epistemological research terms for the quality of beliefs, "naïve" and "sophisticated", have come under criticism for their inherent epistemological judgement (Muis, 2004; Perry, 1968). Muis adopted the terms "availing" and "nonavailing" to distinguish beliefs on the degree to which they avail learning. Availing beliefs were associated with better outcomes, while nonavailing had no effect or a negative effect on learning. In a similar fashion, the terms "adaptive" and "maladaptive" have been used to indicate how beliefs about mathematics might develop in ways inconsistent with positive behaviours (De Corte, Mason, Depaepe & Verschaffel, 2011). Further terms used in the research include "positive" or "negative" beliefs, and others have been used to indicate specific types of beliefs; "instrumental", "traditional", and "procedural". However, Goldin et al. (2009) made the point that regardless of definitions and terms, researchers ought to make the *object* of the beliefs explicit to avoid "slippery" terms. Therefore, I begin with beliefs *about* the nature of mathematics, then progress to beliefs *about* how mathematics is learned, beliefs *about* oneself as a mathematician, and beliefs *about* the social contexts of mathematics.

2.2 The nature of mathematics

Learner beliefs about the nature of mathematics were found to influence a range of behaviours and outcomes. These included: how learners engaged with mathematics (FitzSimons et al., 2003; Op't Eynde, De Corte & Verschaffel, 2002; Schoenfeld, 1985, 1992; Schommer-Aikins et al., 2005); the types of strategies learners used to solve problems (Francisco, 2013; Jäder, Sidenvall, & Sumpter, 2017; Liu, 2010; Schoenfeld, 2011); adult learner achievement, particularly in university settings (Briley et al., 2009; Stage & Kloosterman, 1995); the values associated with mathematics (Ernest, 1989); and how learners linked what was learned in their classrooms to their everyday practices (Presmeg, 2002). There is evidence to indicate that beliefs about the nature of mathematics have serious ramifications for how learners engage with mathematics.

Beliefs about the nature of mathematics were found to be represented by two sets of what Coben et al., (2003, p.26) called "competing and incompatible epistemologies of

mathematics". The first of these competing epistemologies is represented by absolutist and fallibilist views of mathematics. These present a dichotomy of mathematical knowledge as *either* neutral, certain, unchanging and independent of consciousness, *or* corrigible, subject to constant revision in light of new arguments or evidence, respectively (Ernest, 1991; FitzSimons, 2010; FitzSimons et al., 2003). The second set of competing epistemologies involves constructivist and socio-cultural epistemologies of mathematics, which represent mathematical knowledge as *either* the individual construction of knowledge, *or* as the enculturation of social practices (Cobb, 1994; Coben et al., 2003; Fosnot & Dolk, 2005).

Divergent views about the nature of mathematics were found to have important implications for teaching and learning because they represent incompatible beliefs about the structure, certainty, and source of mathematics (Ernest, 1991; Viholainen, Asikainen & Hirvonen, 2014). The *structure* of mathematics was broadly conceptualised as *either* a corpus of unconnected "pieces" of knowledge, or in contrast, an interrelated and connected system (Ernest, 1989; Schommer-Aikins et al., 2005). The *certainty* of mathematics was broadly conceptualised as *either* static and unchanging, or in contrast, as a dynamic and continually expanding field of human invention and innovation (Ernest, 1989, 1991). The *source* of mathematical knowledge was dichotomised between the Platonist view that posits mathematical knowledge as independent of human existence, or in contrast, entirely socially constructed (Brown, 2005; Viholainen et al., 2014). Thus, the Platonist view posits mathematical knowledge as discovered, while the sociocultural view considers it created (Ernest, 1989; Schommer-Aikins et al., 2005; Viholainen et al., 2014). While the terminology varies, the former perspectives are broadly consistent with the terms "absolutist" (Ernest, 1991; FitzSimons et al., 2003), "traditional" (Šapkova, 2014; Stipek, Givven, Salmon & MacGyvers, 2001), "computational" (Philipp, 2007; Thompson, Philipp, Thompson & Boyd, 1994), and "instrumental" (Skemp, 1978; Thompson et al., 1994). The latter is broadly consistent with the terms "fallibilist", "relational", "conceptual" and "constructivist" (FitzSimons et al., 2003; Philipp, 2007; Šapkova, 2014; Viholainen et al., 2014; von Glasersfeld, 2005).

Absolutist and traditional views have come under criticism in mathematics education for potentially disempowering learners by supporting a product view and situating mathematics as something external to the learner (Benn & Burton, 1996; Bibby, 2002; Ernest, 1989; FitzSimons, 2002). Ernest et al. (2016) noted how various metaphors used in language captured this hidden assumption. Phrases such as "getting it" signal a view of mathematical knowledge as a thing one possesses or

controls, and therefore presuppose a static banking model. Stipek et al. (2001) argued that this view conveyed to learners that mistakes are to be avoided, which in turn contributes to a high-risk environment, particularly if learners view proficiency as an indication of ability. This view of mathematics is argued to contribute to a climate that is competitive and judgemental, because successes and failures are highly salient and obvious, further inhibiting engagement by lower-skilled learners (Bibby, 2002; Boaler & Greeno, 2000). Ernest (2014) argues that this approach sanctions a philosophic view of mathematics as rigid, fixed, logical, absolute, inhuman, cold, objective, pure, abstract, remote and ultra-rational. These characteristics are argued to be *particularly* adverse for adults who have had difficulties with mathematics because they further promote exclusion and alienation from the subject (Benn & Burton, 1996; FitzSimons, 2002).

Absolutist and fallibilist views have ramifications for the type of understanding learners are expected to develop within educational environments (Ernest et al., 2016). Skemp (1978) described two competing perspectives regarding the notion of understanding. The first, termed "instrumental", he described as knowing *how* to do something and related this to procedural knowledge. The second, "relational", he described as knowing *how* and *why*. Recently a call has been made to justify Skemp's position that relational understanding is superior to instrumental understanding (Ernest et al., 2016; Hossain et al., 2013). However, Skemp posited four advantages. First, relational understanding is more adaptable to new tasks, and second, easier to remember due to its emphasis on sense-making. Third, it is effective as a goal because it enhances intrinsic motivation, and fourth, it promotes self-discovery through exploration. He argued that instrumental approaches, in contrast, situated learners as less autonomous and creative, limiting the type of mathematics users they were to become.

Many mathematicians, philosophers of mathematics, and mathematics educators also tend to espouse a more fallibilist and relational orientation toward mathematics (Burton, 1999, 2004; Ernest, 1991; Lakatos, 1976; Pólya, 1954). Burton found that practices of mathematicians included being collaborative not individualistic, intuitive not procedural, and seeking connections between ideas, rather than merely seeking answers. They were also found to be comfortable with uncertainty, and possessed moral courage to make repeated conjectures and critiques, knowing and accepting that proof of their inaccuracy is a positive step forward (Burton, 1999; Lakatos, 1976; Lampert, 1990). This is quite different to the view held by many learners and teachers who are frequently found to believe that incorrect answers are errors to be

avoided (Bibby, 2002; Stipek, et al., 2001). A pedagogical approach that treats *learners as researchers,* rather than reproducers, is consistent with the views and practices of mathematicians (Burton, 2004). This view aligns with reform-oriented approaches such as those espoused by the U.S. National Council of Teachers of Mathematics (NCTM) (2000) which has emphasised the teaching of mathematics as a dynamic tool of thought, not a set of procedures to be learned, and learned through discourse and problem-solving, with a focus on understanding rather than correct answers (Reid, Wood, Smith & Petocz, 2005). Absolutist beliefs were found to promote a use of mathematics inconsistent with notions of numerate behaviour argued to be required for the modern economy (Boaler, 2008; NCTM, 2000; PIAAC Numeracy Expert Group, 2009). Yet, despite the broad rejection of absolutist views and traditional approaches by the mathematics education research community, these views have remained a persistent characteristic of the field (Barlow & Reddish, 2006; Liljedahl, 2009; Lomas, Grootenboer & Attard, 2012; Maasepp & Bobis, 2014).

Interest in teacher beliefs and their relationship with pedagogical approaches is influenced by research suggesting that beliefs about the nature of mathematics are transmitted to learners, explicitly and implicitly, through classroom practice (Mewborn, & Cross, 2007; Thompson, 1984; White, Perry, Way & Southwell, 2006). While there are caveats, teachers *tend* to use instructional strategies that are consistent with their beliefs about the nature of mathematics (Beswick, 2012; Cross, 2009; Pajares, 1992; Philipp, 2007; Polly et al., 2013; Stipek et al., 2001; Swan, 2006; Thompson, 1992; Viholainen et al., 2014). There is evidence that teacher beliefs about what it is to *be* numerate, and how one *becomes* numerate, influence their teaching decisions (Askew, et al., 1997; Philipp, 2007; Swan, 2007).

A distinction has been made between three philosophical perspectives as they pertain to teachers' beliefs and instructional approaches: transmissional, discovery-oriented, and connectionist (Askew et al., 1997; Ernest, 1989; Swan, 2006). Transmission-oriented teachers were found to tend to *instruct,* exert greater control over their learners, and prefer to teach procedural mastery, rules, and competence with basic skills (Cross, 2009; Stipek et al., 2001). This was consistent with their belief that mathematics consists of a body of rules, procedures and facts. Discovery-oriented teachers tended to *explain,* to guide the learner toward an individual discovery of various methods. This was consistent with their Platonist belief that mathematics exists independently of human existence and therefore is discovered, not created. Finally, connectionist-oriented teachers tended to *facilitate*, emphasising the links between ideas and encouraging learners to draw on their understandings to

solve non-routine problems, make conjectures, conduct critiques and present justifications (Askew et al. 1997; Cross, 2009; Swan, 2006). These teaching decisions, in turn, communicate messages to learners about what mathematics is, and how it is learned (Goldin et al., 2011; Philipp, 2007; Reusser, 2000; Schoenfeld, 1988).

Learner beliefs about the nature of mathematics

While there is a paucity of research exploring the mathematical beliefs of low-skilled adults, research over the last three decades has identified common mathematical beliefs held by school and university students, and pre-and in-service teachers. Several studies have found that substantial numbers of learners hold absolutist, instrumental beliefs about the nature of mathematics (Berkaliev & Kloosterman, 2009; Francisco, 2013; Kloosterman & Stage, 1992; Lampert, 1990; Muis, 2004; Schoenfeld, 1988; Schommer, Crouse, & Rhodes, 1992; Spangler, 1992). Younger learners are frequently found to describe mathematics primarily as arithmetic and computation (DÍaz-Obando, Plasencia-Cruz & Solano-Alvarado, 2003; Frank, 1988; Fuson, Kalchman & Bransford, 2005; Young-Loveridge, Taylor, Sharma, & Hawera, 2006). Many older students also believe that mathematics is a collection of rules, facts, skills, and algorithms that must be memorised and followed (Crawford, Gordon, Nicholas & Prosser, 1994; Hekimoglu & Kitrell, 2010; Mtetwa & Garofalo, 1989). For example, Petocz et al. (2007) found that half of first year undergraduate mathematics students conceptualised mathematics very narrowly, describing it simply as functions that involved numbers, or as a set of procedures that could be used to solve equations. Broader conceptions, such as seeing mathematics as a way of thinking, were more likely to be found in later year students.

Younger students are regularly found to believe that mathematics problems *always* have a rule to follow and that problems can be solved using facts, procedures, algorithms or formulas that the teacher has taught (Crawford et al., 1994; DÍaz-Obando et al., 2003; Garofalo, 1989; Kloosterman, 2002; Silver, 1985). For example, Silver found that over 80% of seventh and eighth grade students agree with the statement "There is always a rule to follow in mathematics", and Lampert (1990) found that learners believed knowing mathematics meant knowing the rule. Many studies have found that learners believe that the objective of mathematics is to find the one correct answer, and to do so quickly (Anku, 1996; Frank, 1990; Garofalo, 1989; Reid, Wood, Smith, & Petocz, 2005; Schoenfeld, 1989).

In addition to believing there is only one answer, students have been found to believe that there is only one way to *solve* a problem and this is the method shown in the text book or endorsed by the teacher (Garofalo, 1989; Lampert, 1990; Silver, 1985). Many believe that the difficulty of a problem is dictated not by the complexity of the problem but by the size and quantity of the numbers (Garofalo, 1989; Lucangeli, Coi & Bosco, 1997). This may be related to the belief that mathematicians work with abstract symbols, not ideas (Agac & Masal, 2015; Lampert, 1990). Finally, many students believe that mathematics topics and procedures are fragmented and not related (Garolfalo, 1989; Muis, 2004; Schommer-Aikins et al., 2005). It is argued that these beliefs lead learners to orient toward memorisation and repetition, ultimately inhibiting their performance and enjoyment of mathematics (Bibby, 2002; Crawford et al., 1994; Jonsson, Kulaksiz & Lithner, 2016). Learners with these beliefs have been found to apply algorithmic solutions to non-routine problems, and to fail more than students with broader views toward mathematics (Jäder et al., 2017). Finally, there is some evidence to suggest that they exert less effortful struggle than conceptually-oriented learners, which inhibits their ability to develop conceptual understanding (Jonsson et al., 2016).

Learners' beliefs about the source of mathematical knowledge have been explored. These studies found that many learners believe that mathematical knowledge is passed down through sources of authority, such as the teacher or text books (Muis, 2004; Schoenfeld, 1989; Spangler, 1992). Several studies found that learners believed only very prodigious and creative people can create mathematics, and that the others must learn this information from an authority figure (Garofalo, 1989; Schoenfeld, 1988). This belief in authority as a source of mathematical knowledge is also linked to a non-critical and shallow approach to learning mathematics (Schommer-Aikins et al., 2005). Learners with such beliefs were found to accept information without question, and perform to the letter the tasks given by the teacher (Lampert, 1990; Schoenfeld, 1988, 1991). Frank (1988) found that learners dichotomised between correct and incorrect answers and believed the teacher was the only reliable source for determining which was which. A slightly different but telling example of this was found in a study by Spangler (1992), in which learners faced with two potential answers did not critically re-evaluate them, but rather accepted the "smarter" student's answer. The notion that both could be correct was rarely considered. Finally, research found that many learners believed that when they could *mimic* a procedure, and solve a problem, they had understood mathematics (DÍaz-Obando et al., 2003; Skemp, 1978). Thus, beliefs about the

nature of mathematics influenced learners' perceptions of what it means to do and understand mathematics.

In summary, the research indicates that many teachers hold absolutist and instrumental beliefs about the nature of mathematics. They tend to adopt concomitant instructional approaches that reflect their perspective about what a numerate student is, and how they develop. Many learners appeared to develop similar absolutist, transmissional views of mathematics. These beliefs about mathematics run counter to those endorsed by reform-oriented researchers, and what we know of the practices of many professional mathematicians. How these beliefs impact on learning behaviours is reviewed below.

2.3 How mathematics is learned

A key issue raised in the mathematical beliefs research is that beliefs about the nature of mathematics, and beliefs about what learning is, led to behaviours that act either as affordances to learning, or constrain learning (Crawford, Gordon, Nicholas & Prosser, 1994; Goldin, et al., 2011; Muis, 2004). Beliefs about learning in general can be crudely divided into whether the focus is on reproducing knowledge transferred from an external source, or whether the focus is on the construction of meaning. The second approach is widely considered to be a more sophisticated conception (Marton, Watkins & Tang, 1997; Petcoz et al., 2007; Schommer-Aikins, et al., 2005). For example, Hadar (2011) reviewed several of the key conceptualisations of learning, including the surface/deep distinction, and found that surface learning focused on outcomes, materialistic absorption from the teacher, and repetition. Deep conceptions of learning focused on learning processes that were more active, self-driven, and emphasised the self-structuring of knowledge and understanding. Although a synthesis between understanding and memorisation is considered effective, a *reliance* on memorisation is widely considered ineffective (Echazarra, Salinas, Méndez, Denis & Rech, 2016; Ranellucci, et al., 2013). This may explain why conceptions of learning are found to be predictors of achievement (Cano & Cardelle-Elawar, 2004). In fact, the development of deep conceptions of learning, with a focus on the active construction of meaning, is considered a goal of higher education in and of itself (Ritchhart, Turner, & Hadar, 2009). The concern for adults learning mathematics is that the evidence suggests that learners who believe that mathematics is certain, fixed and unchanging, that it consists of discrete isolated pieces rather than being interconnected, is handed down by authority rather than discovered, learned quickly or not at all, and that their own ability is fixed rather than improvable, approach mathematics in ways consistent with surface conceptions of

learning (Crawford et al., 1994; Muis, 2004; Schommer-Aikins et al., 2005). Such approaches are ineffective and potentially constraining to learner achievement (Jäder et al., 2017; Ranellucci, Muis, Duffy, Wang, Sampasivam & Franco, 2013). Several reasons for this have been identified in the research. I begin below with learner goals.

Learners' beliefs about what ought to be learned, how learning occurs, and what standards signal success, in conjunction with contextual factors, have been found to shape learners' longer-term goals and shorter in-the-moment goals, and how they prioritised them (Goldin et al., 2011; Hadar, 2011; Midgley, Kaplan & Middleton, 2001; Muis & Franco, 2009; Schoenfeld, 2011). The goals that learners set have been argued to be *pivotal* to their subsequent use of strategies, their evaluation of their own performance, and the behaviours they adopted when they judged that they were not moving suitably toward them (Hofer & Pintrich, 2002; Labuhn, Zimmerman & Hasselhorn, 2010; Muis, 2008; Schunk, 2001).

The longer-term learning goals of learners have been described as oriented toward either mastery or performance goals and are thought to be developed in conjunction with beliefs (Linnenbrink-Garcia, Middleton, Ciani, Easter, O'Keefe, & Zusho, 2012; Midgley, 2002). These orientations are linked to mathematical engagement, whether deep or shallow learning strategies are used, and the behaviours learners adopt when learning (Park, 2005; Ranellucci et al., 2013; Senko, Hulleman, & Harackiewicz, 2011; Skaalvik, & Federici, 2016). Mastery goals emphasise the *development* of competence, while performance goals emphasise the *demonstration* of competence. Mastery goals are thought to be underpinned by the belief that learning, understanding, and solving problems are ends in themselves, and that learning is the result of effort. In contrast, performance goals are characterised by a belief that what counts are displays of performance such as test scores, good grades and public comparisons between oneself and others (Skaalvik & Federici, 2016). When combined with fixed beliefs about intelligence, poorer performing learners often adopt performance-avoidance goals (Blackwell et al., 2007; Pintrich, 2000; Rattan, Good & Dweck, 2016). This is widely considered maladaptive because learners avoid activities that might expose their performance, such as seeking help (Linnenbrink, 2007; Meece, Anderman, & Anderman, 2006; Ryan, Pintrich Midgley, 2001).

Beliefs were also theorised to influence the shorter-term goals of learners (Goldin et al., 2011; Schoenfeld, 2011). The relationship between beliefs and goals was noted

as particularly problematic for learners holding absolutist and procedural beliefs about the nature of mathematics (Briley et al., 2009; Hadar, 2011; Schoenfeld, 1985, 2011; Schommer-Aikins et al., 2005). It is argued that a learner who believes mathematics is a static and unconnected corpus of rules, facts and procedures, is likely to set goals that relate primarily to *recalling* those facts, rules and procedures when needed (Schoenfeld, 2011; Schommer-Aikins et al. 2005). A range of studies have found that many learners, particularly lower performing, believe that school mathematics is about memorisation of fixed procedures (Crawford et al., 1994; DÍaz-Obando et al. 2003; Hadar, 2011; Schoenfeld, 1988).

Some research has indicated a relationship between negative beliefs and learners setting *no* goals, but rather simply attempting to meet the demands of the immediate situation (Hadar, 2011; Schommer-Aikins et al., 2005). In such cases, many learners were found to adopt a minimal compliance orientation to classroom activities, in which they focused on *completing* immediate teacher-assigned tasks to the letter, and participating minimally, rather than focusing on learning (Hadar, 2011; Schoenfeld, 1989). Similarly, Schommer-Aikins et al (2005) identified that some learners studied mathematics *aimlessly*, and did not believe learning was strategic but rather saw it as a chance event. This aligned with studies associated with attribution theory, in which it was found that some learners attributed success to luck, not effort (Dweck, 2006; Weiner, 2010). Schommer-Aikins et al. noted that learners felt that learning was out of their control, certainly not strategic and effortful. In a similar fashion, Meyer and Parsons (1996) also found high instances of learners self-reporting disorganised study methods in relation to mathematics.

A range of researchers agreed that once learners set goals, they also select what they believe to be appropriate learning strategies to reach the goal (Hofer & Sinatra, 2010; Schoenfeld, 2011). Strategies are often categorised regarding their objective; thus, rehearsal strategies are used to facilitate recall, while elaboration strategies emphasise expansion, connectedness, integration and application (Muis & Duffy, 2013). Learners who set mathematical goals related to recalling rules, facts or procedures are likely to adopt rehearsal and repetition strategies designed to facilitate the memorisation of content (Muis, 2004). This approach was summed up by the comments of an adult learner of mathematics in Meyer and Parsons' (1996, p.749) study, "I copy it directly (the example in the text book) and then I'll go through it, read through it again, close the book and now attempt it without looking at it, and it usually works". Garofalo (1989) found that some learners' strategies were directed entirely toward the memorisation of single-solution strategies. In contrast, learners

with more sophisticated beliefs regarding doing, validating and learning mathematics were found to use more elaborated and effective strategies, and achieved more highly (Briley et al., 2009; Schommer-Aikins et al., 2005). Their strategies included exploring multiple solutions, making connections between ideas and prior knowledge, sharing solution-strategies and explaining to others (Briley et al, 2009; Hofer, 1999; Meyer & Parsons, 1996; Muis, 2004).

Poor strategy use, such as the use of memorisation as a primary learning strategy, is associated with lower mathematical achievement (Areepattamannil, 2014; Echazarra et al., 2016; Kilic, Cene & Demir, 2012). In fact, a review by Kilic et al. (2012) of the practices and performance of mathematical school students in eight countries found that the use of rote memorisation as a strategy had a *negative* effect on achievement. Likewise, a large-scale OECD review found similar findings (Echazarra et al., 2016). Countries in which *fewer* 15-year old students self-reported the use of memorisation strategies performed higher in PISA results. Interestingly, New Zealand students reported the second highest use of memorisation strategies across OECD participants. Passive rehearsal strategies were widely considered to be insufficient to facilitate the level of cognitive processing necessary to result in meaningful learning (Echazarra et al, 2016; Weinstein, Acee & Jung, 2011). While there was some evidence that active, as opposed to passive, rehearsal strategies were effective, they only appeared to be so when used to facilitate meaningful learning (Simpson, Olejnik, Tan, & Supattathum, 1994). Briley et al. (2009) found that adult learners in remedial college mathematics programmes with negative beliefs performed substantially worse than those with more positive beliefs because of poor self-regulation skills and poor learning strategies such as rehearsal. In fact, the authors conjectured that this may be why up to 70% of US students fail university remedial mathematics programmes at their first attempt.

In conclusion, these beliefs have serious implications for low-skilled adult learners re-engaging with mathematics because it suggests that even motivated adult learners might set goals and strategies that are ineffective for the development of mathematical understanding. Yet, little is known about how such beliefs might impact such learners re-engaging with mathematics in vocational environments.

2.4 Motivation

A range of beliefs related to mathematics have been found to influence learner motivation. These include beliefs about: how long mathematical tasks should take to

complete; the role of effort in learning; the role of the teacher and learner; the usefulness of mathematics; and how mathematical problems are solved.

Beliefs about how long a mathematics task should take to solve were found to have negative ramifications for learner engagement, persistence, and the amount of time spent working on a problem (Mason & Scrivani, 2004; Schoenfeld, 1988; Schommer-Aikins et al., 2005). Several studies found that learners believed that mathematical problems should be solved quickly, often in less than five minutes (Garofalo, 1989; Kloosterman & Stage, 1992; Lampert, 1990; Schoenfeld, 1988; Spangler, 1992). Various classroom practices were thought to impart messages about producing quick answers. These included deriving answers from algorithms as quickly as possible; completing myriad routine problems in a single session; and providing little or no time for exploration (Carter & Yackel, 1989; Frank, 1988; Garofalo, 1989; Schoenfeld, 1989, 1992).

Mathematical word-problems in which the context serves merely as a medium for routine calculations is also linked to learners disregarding contextual features in favour of surface level evaluations and rapid calculations (Reusser, 2000; Schoenfeld, 1991). This phenomenon, the "suspension of sense-making" is used to describe learners who have come to believe that common-sense or realistic considerations about the context of a problem is of no concern to the successful completion of the problem. Learners attempt, often unsuccessfully, to identify key words to rapidly apply procedural solutions (Schommer, 1990). Additionally, when learners are conflicted between using real-world knowledge or producing the 'expected' answer, they tend to disregard their own judgement (Alacaci & Pasztor, 2002). Such problems are common in mathematical resources (Verschaffel, Greer, de Corte, 2000), and have also been identified within assessment tasks (Drake, Wake & Noyes, 2012).

Other beliefs about solving tasks quickly were also found. Learners saw quick responses from other learners as a sign of intelligence (Spangler, 1992). Several studies found that some learners who were unable to solve a problem quickly concluded that they personally were unable to solve the task at all, while others viewed the task itself as unsolvable (Frank, 1988; Schoenfeld, 1988). Learners were also found to believe that problems should be solvable in only a few steps (Frank, 1988; Lampert, 1990; Spangler, 1992). Finally, learners who believed that problems should be solved quickly were found to spend less time assessing whether their answers made sense (Lampert, 1990; Muis, 2004).

A body of research has explored whether intelligence and mathematical achievement are believed to be the product of effort or ability. The research indicated that individuals tend to hold implicit theories of intelligence, often dichotomised between two distinct beliefs (Blackwell, Trzesniewski & Dweck, 2007; Burnette, O'Boyle, VanEpps, Pollack, & Finkel, 2013; Dweck, 2016). The first, termed "fixed" or "entity theory" is characterised by the belief that intelligence is an innate and fixed attribute. The contrasting belief, termed "incremental", or "growth" is characterised by the belief that intelligence is a malleable quality able to be developed through effort, good strategies and instruction. In the mathematics education context, researchers have argued that belief in innate intelligence supports the conclusion that learners either possess mathematical ability or not, and there is nothing that can be done to change this (Burnette, et al, 2013; Schommer & Walker, 1997; Yeager & Dweck, 2012). This belief shares a common thread with a widely held "math myth", that some people are born with a mind for mathematics, and some are not (Barlow & Reddish, 2006; Frank, 1990). Studies show that large portions of the general population hold this belief (Barlow & Reddish, 2006; Berkaliev & Kloosterman, 2009; Dweck, 2006).

Fixed and growth beliefs have been postulated to be linked to motivation, goal setting, self-regulatory processes, strategy use, social comparison, learning, and achievement (Blackwell, Trzesniewski & Dweck, 2007; Burnette et al., 2013; Dahl, Bals & Turi, 2005; Dweck, 2017; Haimovitz & Dweck, 2016). The contrasting beliefs have been shown to influence an individual's interpretation of their own, or others', successes or failures. Those holding fixed beliefs were found to interpret failure or setbacks as indicative of their own limited ability, and therefore debilitating and unchangeable, and consequently reduced effort, or stopped trying altogether (Grant & Dweck, 2003; Haimovitz & Dweck, 2016; Haimovitz, Wormington & Corpus, 2011). These learners were also found to compare themselves to others, and in the case of a negative discrepancy, make negative judgements about their own ability to learn. Furthermore, learners holding fixed beliefs were found to orient toward performance-avoidance goals to avoid judgement by others, or to avoid confirming to themselves that they are 'unable' (Haimovitz & Dweck, 2016; Rattan et al., 2012).

In contrast, learners who believed that intellect is malleable, and that effort helps overcome failures, were found to enact more active strategies in the face of difficulty, set better learning goals, and make less "helpless" attributions than learners with fixed beliefs (Blackwell et al., 2007; Dweck & Leggett, 1988). Blackwell et al. argued that these beliefs influence a divergence in learner performance, particularly when the learners are in a failure-prone environment such as mathematics instruction.

Differing interpretations of failure, attributions and responses were argued to crystallise on these occasions, resulting in divergent performance patterns. The crystallisation of entity beliefs was found to link to a wider framework of negative beliefs and goals (Blackwell et al., 2007; Dweck, 2017; Haimovitz & Dweck, 2016).

The findings bode ill for adults who consider themselves as having failed mathematics in school. Effort versus ability measures have been incorporated into both epistemic and mathematical belief surveys and questionnaires. International use of Kloosterman and Stage's (1992) belief scale "Effort can increase mathematical ability" has found that large numbers of learners do not believe effort is a key factor in mathematical performance (Berkaliev, & Kloosterman, 2009). Similarly, the Schommer's Epistemic Beliefs Questionnaire (Schommer, Crouse & Rhodes, 1992) fixed ability scale included such statements as "Some people are born smarter than others and you can't do anything to change that". International use finds high numbers of learners agreeing (Mason & Scrivani, 2004; Schommer-Aikins et al., 2005). However, little is known about the beliefs of low-skilled adult learners despite the clear connection between these beliefs and low performance.

Beliefs about the role of the teacher
Learner beliefs about the role of the teacher have also been an area of interest because of concomitant perceptions for the role of the learner. Studies show that many school students develop a view of the teacher as a manager and transmitter of information and themselves as responsible for complying with instructions and reproducing information (Campbell et al, 2001; Jones, Jones & Vermette, 2013; Taylor, Hawera & Young-Loveridge, 2005). The concern with such views is that they shape the learner's expectations for their own behaviours and those of the teacher (Brousseau & Warfield, 1999; Heyd-Metzuyanim, 2013; Kinchin, 2004). Learners tend to situate the responsibility for their learning with their teacher, adopting passive roles in which they receive knowledge, rather than construct it (Boaler & Greeno, 2000; Huak, 2005; Schoenfeld, 1988; Sutherland & Singh, 2004; Taylor et al., 2005). Taylor et al., (2005) found that only 16% of learners viewed their teachers as mentors and themselves as responsible for actively seeking out and constructing knowledge. Maladaptive views about teacher and learner roles were found to have a negative impact on learners' development of skills and the development of identity, autonomy and agency (Amit & Fried, 2005; Brousseau & Warfield, 1999; Heyd-Metzuyanim, 2013; Sutherland & Singh, 2004).

Studies indicated that learners continued to hold beliefs about the role of teachers and themselves into adulthood, and continued to adopt passive, reception-based behaviours in tertiary mathematics education settings (Briley et al, 2009; Huak, 2005; Yoon et al, 2011). For example, Huak found that adult learners had difficulty taking ownership of mathematical knowledge because they had an external locus of control regarding mathematical authority. The learners believed that the text book and the teacher were the ultimate arbiters of truth. This belief that there is a *certain* source of knowledge, and that it is externally situated, is consistent with absolutist views of mathematics (Ernest, 1991). Although there is limited research on low-skilled adults, adult learner beliefs about what makes a good tutor provide insight into their beliefs about the teachers' role. Several studies found that adult learners value tutors able to explain and break concepts into small steps (Coben et al., 2007; Whatman et al., 2010). Interestingly, a number of learners in Coben et al.'s (2007) study also mentioned the need for the teacher not to talk *too* much, in order to keep the lessons interesting, suggesting they see their role during teacher expositions as passive.

Beliefs about the usefulness of mathematics

A belief in the usefulness of mathematics is thought to relate to learners' intrinsic motivation to engage with the topic, particularly with adult students (Berkaliev & Kloosterman, 2009; Briley et al., 2009; Fennema & Sherman, 1976; Schommer-Aikins et al., 2005). A range of studies show that learners who express a greater belief in the usefulness of mathematics tend to have higher achievement rates, both in school and tertiary study (Berkaliev & Kloosterman, 2009; Briley et al, 2009; Kloosterman, Raymond & Emenaker, 1996; Lane, Stynes & O'Donoghue, 2016; Quilter & Harper, 1988; Schommer-Aikins et al., 2005). Adult learners were typically found to believe that mathematics is useful (Berkaliev & Kloosterman, 2009; Brown et al, 2008; Casey et al., 2006; Coben et al., 2007). Swain and Swan (2007) found that many adult learners were motivated to do mathematics to get good jobs, suggesting that these adults made a link between work-skills and mathematics. Furthermore, learners' beliefs about the usefulness of mathematics were found to increase as learners progressed through courses, showing that instruction can have a positive impact on learners' attitudes toward mathematics (Casey et al., 2006; Coben et al., 2007).

Several studies have found that a small but consistent proportion of individuals believe that mathematics is *not* useful, and when it is, only basic skills are necessary (APU, 1988; Brown et al., 2008; Mason, 2003). These findings were echoed in learner statements such as "Who needs to know trigonometry in everyday life?"

(Brown et al., p.11). These findings suggest that a belief in the usefulness of mathematics may not translate into the motivation to learn trigonometry in a vocational programme. Moreover, some studies found that while some students stated that mathematics was not useful for their career, a far smaller number stated that it was not useful for everyday life (Brown et al., 2008; Matthews & Pepper, 2006). However, the evidence is clear that those who state that mathematics is not useful are at greater risk of failing or leaving mathematical study (Brown et al., 2008; Mason, 2003; Quilter & Harper, 1988).

Adult learners were motivated to engage with numeracy for reasons other than careers or usefulness. These included the desire to prove to themselves they could succeed in a high-status subject, the desire to gain qualifications, the desire to help their children, or simply the desire to pursue an intellectually stimulating subject (Coben et al., 2007; Swain, Baker, Holder, Newmarch & Coben, 2005; Wedege, 2002). Swain et al. (2005) found that numeracy education became meaningful when it related to an adult's purposes for learning. The authors stated, "Meaningfulness is a feature of the quality of an individual's engagement with learning rather than of the utility of the numeracy content learned" (p.34). Their findings show that the applicability of mathematics to the "real world" was only one piece of a larger motivational jigsaw.

The impact of beliefs on problem-solving behaviours
A large body of research has explored the influence of mathematical beliefs on learners' problem-solving behaviours (Francisco, 2013; Goldin et al, 2011; Kloosterman & Stage, 1992; Mason, 2004; Schoenfeld, 1988; Stylianides & Stylianides, 2014). Mathematical problem-solving is frequently described as being at the heart of mathematical activity, and distinguished from procedural, computational or instrumental types of mathematics because of its use of non-routine tasks and solution strategies (Goldin et al, 2011; NCTM, 2000; Op t' Eynde et al., 2007; Schoenfeld, 1985; Stylianides & Stylianides, 2014; Thompson et al, 1994). Callejo and Villa (2009, p. 112) described a problem-solving situation as "a situation that proposes a mathematical question whose solution is not immediately accessible to the solver, because he or she does not have an algorithm for relating the data with the unknown or a process that automatically relates the data with the conclusion". Thus, problem-solving requires *non-routine* thinking, an emphasis on exploring a range of potentially unproductive problem solutions, collaboration, trial and error, and the construction of new representations (Goldin et al, 2011; Schoenfeld, 1985). This is argued to be time consuming, to entail higher levels of risk, and be cognitively

demanding (Stylianides & Stylianides, 2014). Thus, positive dispositions for problem-solving include persistence, resilience, and courage (Lakatos, 1976; Lampert, 1990; Pólya, 1954). Negative beliefs were found to erode both the behaviours and the dispositions associated with successful problem solving (Goldin et al., 2011; Muis, 2004; Schoenfeld, 1992).

Learners' beliefs have been linked to their behaviours during problem-solving activities (Goldin et al., 2011; Muis, 2004; Op't Eynde et al., 2007). Goldin et al. (2011) identified nine distinct engagement structures based on classroom observations that were categorised by the learner's behaviours, motivating desires, and in-the-moment goals. The authors posited that these were the result of learners' beliefs and while they were careful to avoid labelling some as "good" and others "bad", in my view, there were clear positive and negative patterns. Some of these engagement patterns were positive because they reflected increased learner engagement with the mathematics at hand, and usually resulted from their pursuit of something deemed of value, such as status, pride, or some other "pay off". Other patterns reflected a shift away from engagement with mathematics toward a greater emphasis on mitigating potentially negative social factors. These appeared to be initiated by a perceived threat to a learner's dignity, status, or sense of self-respect, and led to an orientation toward self-protective behaviours. These affective factors will be reviewed in depth in the next section; however, it is worthwhile briefly outlining several of the pertinent engagement patterns here.

Adult learners are often described as autonomous consumers of education, able to draw on their rich life experience to learn new content, in pursuit of their own self-determined goals (Compton, Cox & Laanan, 2006; Knowles et al., 2015). From this perspective four of the engagement structures identified by Goldin and colleagues are particularly concerning. The first engagement structure is "get the job done", which describes a pattern whereby learners adopt an attitude of *deference* and seek primarily to complete assigned tasks. Satisfaction is gained from completing the task in any way necessary, including enlisting others to achieve the goal, rather than understanding the mathematics. Also negative is the "Don't disrespect me" structure. This describes a pattern in which learners are motivated to avoid conditions that lead to belittlement. The attempt to "save face" overrides the desire to understand the mathematics at hand. Next, the structure "Stay out of trouble" describes a pattern in which learners seek to avoid conflict, embarrassment, humiliation or anger that might occur between themselves and another learner or teacher. Learners adopt avoidance behaviours, including striving not to be noticed, which supersedes their engagement

with mathematics. Finally, "pseudo-engagement" describes a pattern in which learners are motivated by the desire to look good to the teacher and/or peers, yet who are driven by the need to avoid blame or rejection. While they wish to overtly disengage, they fear that doing so might evoke disapproval. The authors readily admitted that these patterns overlapped and required more conceptual work. However, these patterns, identified within school mathematics classrooms, raise questions regarding the types of behaviours that might be present in foundation-level vocational programmes populated by adults likely to have experienced problematic histories with mathematics during their schooling. Surprisingly, little research could be found that explored the relationship between the beliefs of low-skilled adult learners and their behaviours in foundation-level vocational mathematics education. While mathematical belief research continues to focus on teachers, school-aged and university students, this continues to be an important but under-researched field.

2.5 Self-concept and identity

This section reviews the relationship between beliefs and the affective domain, and how these relate to engagement with mathematics. Broadly speaking these are beliefs about the "self" as a doer of mathematics, and beliefs about the social meaning of mathematics. These have been investigated in terms of various concepts, including identity, shame, anxiety, and emotional and attitudinal factors. I begin with research on identity and shame because they provide a theoretical basis for understanding why adults experience mathematics emotionally and develop various attitudes toward it.

Beliefs and mathematical identity

Identity research has stemmed primarily from socio-cultural theories of learning. Proponents assert that individuals occupy diverse social worlds and construct unique roles and expectations for themselves within each of these (Hannula, 2012; Hannula et al., 2016; Turner, 2011). A distinct *mathematical* identity is argued based on the networks of relationships inherent in mathematical instructional environments and the roles and expectations learners assume within these environments. Mathematical identities are not seen as static but under constant revision and construction (Latterell & Wilson, 2017; Sfard & Prusak, 2005).

Mathematical identities are argued to act as constraints or affordances to an individual's *participation* in mathematics (Boaler & Greeno, 2000; Darragh, 2013; Sfard, 2012). This is because learning mathematics is viewed by many as the *actualising* of one's identity through participation (Hannula et al., 2016). Therefore, a

learner who identifies as 'non-mathematical', is likely to adopt and perpetuate patterns of behaviour that conform with a non-mathematical role, rather than developing positive patterns of participation within the mathematics community (Hannula, 2012; Hannula et al., 2016). Negative identities have ramifications for participation and motivation (Brown et al., 2008); the development of agency (Boaler, 2003; Grootenboer & Jorgensen, 2009); and emotional responses (Black, Mendick, & Solomon, 2009; Evans, 2000).

Identities, much like beliefs, appear to develop based on the broader societal representations of mathematics and mathematicians, and from cumulative experiences during mathematics instruction (Mendick, 2005). Researchers argue that societal representations often present mathematics as cold, theoretical, and ultra-rational, and doers of mathematics are presented similarly (Ernest et al., 2016; Evans, Tsatsaroni & Staub, 2007; FitzSimons, 2002; Lim & Ernest, 2000). Several researchers have identified clusters of stories held by adults about mathematicians and mathematics that reflected clichéd images from media. They note that popular films and television shows present mathematicians as socially awkward "nerds", "geeks" or "hackers", or as unstable geniuses who have difficulty developing meaningful relationships with others (Moreau, Mendick & Epstein, 2010; Wilson & Latterell, 2001). Such stories link mathematical expertise with negative characteristics such as a lack of social skills. Also, societal representations of mathematicians were found to associate notions of logic, reason and independence with gender, class and race. This is thought to support the belief that mathematicians are *born*, not made, and are typically middle to upper class European males (Good, Rattan & Dweck, 2012; Martin, 2007; Mendick, 2005; Siivonen, 2013).

The societal representations described above are argued to constrain the development of positive mathematical identities because learners see them as incompatible with their own. Mathematical expertise is associated with both *unattainable* characteristics, in the case of gender, class and race, and with *undesirable* characteristics, such as poor social and relational skills (Mendick, 2005; Mendick & Moreau, 2014). This is also reflected in a binary view of mathematical identity that posits individuals as *either* a 'maths person' or not (Black et al., 2009; Mendick, 2005; Siivonen, 2013). A number of studies find that adults hold either a positive or negative mathematical identity (Coben, 2002; Coben & Thumpston, 1996; Latterell & Wilson, 2017; Mendick, 2005).

The research also shows that in addition to social representations of mathematicians, mathematics instruction is the crucible in which mathematical identities are developed and consolidated. The experiences, interactions, and interpretations of events that occur within the social context of mathematical instruction are thought to contribute to the formation of one's mathematical identity (Grootenboer, Smith & Lowrie, 2006; Hossain et al., 2013; Jonker, 2006; Turner, 2011). Wenger described this process as "a layering of events of participation and reification by which our experience and its social interpretation inform each other" (1998, p. 151). Sfard and Prusak (2005) argue that experiences and interpretations become adopted narratives, or in other words, that identities *are* the stories that learners hear and tell about themselves. These narratives can be authored by others, such as other learners or teachers, or may be institutional narratives that might include diagnoses, qualifications or ability groups. They are also theorised to be authored, and therefore reified, by themselves.

The formation of mathematical identities has been found to begin at least as early as primary school (Black, 2004). Some studies indicate that secondary school is the prime location of identity formation (Brown et al., 2008; Darragh, 2013). Researchers found that learners' participatory roles in classrooms varies from that of *active* participant (Black, 2004), to passive receivers of knowledge or complete non-participants (Boaler, 2003; Brown et al, 2008; Siivonen, 2013; Solomon, 2007; Winbourne, 2009). Many students struggle to develop a sense of belonging in the secondary school mathematics environment during the transition from primary school, and the transition is argued to lead to diffracted learning trajectories. Noyes (2006) found that high-achieving learners enjoyed even higher participation in high school than in primary school, while disadvantaged learners participated less and became even more disadvantaged. Adults tend to express greater enjoyment of mathematics in primary school than high school, which many cite as the beginning of mathematical difficulties (Evans, 2000).

Adult mathematical identities

Research indicates that mathematical identities developed during school persist into adulthood (Coben, 2002; Coben & Thumpston, 1996; Evans, 2000). While exploring adults' mathematical life histories, Coben (2002) found that many adults recollected the experience of a "brick wall", described as a mathematical procedure or concept they were unable to make sense of. These events were often traumatic for the individual and long-lasting. References were also made to a significant other, often a teacher or parent, who had a negative or positive long-lasting influence on them.

Many of the adults continued to view mathematics as "what they could not do", and tended to see the mathematics they did in their current practice not as mathematics but rather as "common sense". Because the mathematics they engaged in was invisible to them, they were never able to see themselves as mathematically successful. Other adult recollections of school mathematics experiences were found to include accounts of intimidating teachers, feelings of powerlessness, the inevitability of failure, and shame and embarrassment (Carroll, 1994; Grootenboer, 2001; Huak, 2005). These findings, and a host of others, suggest that once negative identities are developed, learners readily describe themselves as "non-maths" people, and often hold negative attitudes and feelings toward mathematics (Brown et al., 2008; Coben, 2002; Siivonen, 2013; Swain et al., 2005, 2007; Wedege, 2002).

In summary, non-mathematics identities were often packaged with traumatic experiences and memories, poor patterns of participation, and perhaps most difficult to overcome, the belief that they were just *not the right type* of person to be mathematically successful. These identities persisted into adulthood and set expectations for their participatory roles and performance. What remains unknown is what identities are held by low-skilled adult learners in foundation-level vocational programmes, given many may have already been through potentially negative identity-forming experiences.

Beliefs and shame

The construct of shame has been used to describe *why* adults experience negative affective responses within mathematics classes and why they adopt avoidance strategies (Bibby, 2002). Strong theoretical support can be found for the notion that individuals require a sense of self-worth and personal significance, and this is supported by a positive sense of location relative to others (Dunn & Creek, 2015; Habermas, 1984; Scheff, 1988; Schwalb & Mason-Schrock, 1996; Turner, 2011). Several theorists posit that individuals seek respect and esteem from others, and spend much time and effort evaluating how others perceive them (Goffman, 1967; Scheff, 1988; Turner 2011; Wertsch, 1991). Sabini, Siempann and Stein (2001) presented compelling evidence that that our perceptions of others' thoughts about us are so influential, that our desire to avoid negative evaluation may override our moral code and personal values. Adults may dramatically modify their behaviours in adult classrooms to avoid shameful episodes brought on by displays of incompetence (Bibby, 2002; Tennant, 2012).

The notion that individuals attend to how they are perceived by others, and act accordingly, forms the basis for several social theories, including Habermas' (1984) theory of communicative action, Scheff's (1994) notion of the "social bond" and Symbolic Interactionism (Mead, 1934; Turner, 2011). The evaluation of others' perceptions of ourselves, whether positive or negative, is argued to influence our sense of location within our social network, and consequently induce shame or pride (Scheff, 1994; Turner, 2010, 2011). Scheff posited shame as a reaction to a threat, or actual harm, to one's place within the social bond, just as fear might signal a threat to one's physical being. These experiences were argued to be deeply emotional and identity-forming (Dunn & Creek, 2015; Scheff, 1988, 1994).

Mathematical environments are thought to facilitate conditions that allow for frequent judgements to be made about oneself and others, and are therefore problematic for lower-skilled participants (Bibby, 2002; Boaler & Greeno, 2000; Malmivuori, 2006; Tennant, 2012). Tennant noted that adult learners feared appearing ignorant to others and this influenced their engagement. Similarly, Bibby explored how mathematical environments might threaten pre-service teachers' sense of social connectedness by exploring several themes, including: exposure to judgement; the need to be right; the vulnerability of performing mathematics; the vulnerability of making a written record; and the turmoil of being "thrown", a moment in which the individual was unable to make sense of content. She found that the participants were highly sensitive to the perceived judgement of others, and enacted strategies to mitigate this. Her findings are consistent with Habermas' (1984) notion of image-management, in which individuals were theorised to manage their public-image to invoke a stylised impression to others. The participants in Bibby's study attempted to manage their social image by avoiding damaging events such as being asked a question they were unable to answer. Such behaviours are discussed below under "avoidance strategies".

Beliefs and emotion

It is well established that mathematics evokes strong and often negative emotions in many people (Evans, 2000; Hannula et al., 2016; Tobias, 1993). A range of studies show that responses to mathematics instruction can range from panic to anxiety, to humiliation and shame, and are shown to be typically aroused when an individual is expected to engage in mathematical thinking in social situations (Buxton, 1981; Evans, 2000; Evans, Morgan & Tsatsaroni, 2006; Goldin, 2014; Hannula, 2012). Moreover, evidence suggests that when these responses become patterned they have an equal, if not greater, negative effect on achievement than mathematical

disabilities (Ashcraft, Krause & Hopko, 2007; Ralston, Benner, Tsai, Riccomini, & Nelson, 2014). Several studies have found that students with emotional and behavioural disorders perform *worse* over time than learners with mathematical disabilities (Anderson, Kutasch & Duchnowski, 2001; Epstein, Nelson, Trout, & Mooney, 2005; Ralston et al., 2014). There is consensus that negative emotional responses to mathematics are widespread and destructive to mathematical engagement and achievement (Ashcraft & Moore, 2009; Burns, 1998; Evans, 2002).

Despite agreement on the importance of emotion, the topic has suffered from theoretical and methodological difficulties (Grootenboer & Marshman, 2016; Hannula et al., 2016; Leder & Grootenboer, 2005; Lewis, 2013). However, McLeod's (1992) conception of emotion is widely cited, in which emotions are described as 'hot', unstable, and short lived. Emotions differ from beliefs and attitudes which are conceptualised as more stable and cognitive in nature. The notion that emotions are intense and quickly activated is widely shared; for example, Malmivuori (2001) describes emotions as instinctive, highly intense, weakly controllable, and short-term. Hannula's (2012) affective metatheory incorporates the psychological notion of trait and state to better describe stable and unstable affective features. The framework attributes both traits and states to emotion by continuing to posit them as having state-like tendencies (unstable and short lived), yet also having trait-like characteristics. Individuals with underlying tendencies to experience various emotions, such as anxiety, are more likely to experience these emotions in the moment and have less control over them.

Emotions have been typically explored as they are felt in-the-moment, often in the context of problem solving (Middleton, Jansen & Goldin, 2017; Schoenfeld, 1985). While descriptions vary, emotions studied include anxiety, fear, panic, anger, apprehension, sullenness, humiliation, shame, embarrassment, resentment, guilt and boredom (Bibby, 2002; Buxton, 1981; Goldin et al., 2011; Lewis, 2013). The feelings vary in intensity, ranging from mild and manageable to more severe emotional and physiological responses that appear to overwhelm the learner and disrupt engagement (Ashcraft & Moore, 2009: Evans, 2000). A key difference between adults and children is that adults are found to report the emotion of anger in response to mathematics difficulties more than school students (Carroll, 1994; Evans, 2000; Lewis, 2013). Lewis found that feelings of anxiety, in addition to a growing frustration with the mathematics at hand, develop into anger at a certain point, disrupting mathematical engagement. Likewise, Evans (2000) found that many adults experience extreme anxiety in response to doing mathematics, and many of these

described the emotion as having a debilitating effect on their progress. Examples can be seen in Ashcraft (2002), who described an adult crying in response to an arithmetic task, and Carroll (1994), in which a pre-service teacher recalled becoming so angry and frustrated at her own performance that she wanted to walk out of class.

Experiencing negative emotions during mathematical problem-solving has also been reported by higher-skilled mathematics learners, yet these learners were found to demonstrate greater control and management of their emotions, while less experienced learners were often overwhelmed and abandoned the task (Allen & Carifio, 2007). Several researchers argued that in response to the arousal of negative emotion, lower-skilled learners began to transition from engagement in mathematical thinking toward appraising the wider situation (Goldin et al, 2011; Malmivuori, 2001). It has also been suggested that learners possess different thresholds for how much frustration they can tolerate before they disengage and abandon the task (Malmivuori, 2001; Sutherland & Singh, 2004). Research on learned helplessness also indicates that when negative emotion is combined with helpless beliefs about one's ability to be successful, the threshold for disengagement is extremely low. In many cases learners adopt avoidance strategies, or simply give up trying and opt out altogether (Agaç & Masal, 2017; Sutherland & Singh, 2004; Yates, 2009).

Mathematical anxiety

The cognitive consequences of emotion are evident from the large body of research on mathematical anxiety. Mathematical anxiety was found to cause an "affective drop", or decline in performance, that was independent of the learner's competence, IQ, or mathematical achievement (Ashcraft & Moore, 2009; Hembree, 1990; Ma, 1999). Furthermore, the decline in performance increased as emotional responses increased (Ashcraft & Moore, 2009). Three reasons for the affective drop were postulated. In the first, "local avoidance", in which learners sought to end their uncomfortable and potentially embarrassing and shameful experience quickly. Several studies found that learners provided quick and irrational responses to difficult problems in what seemed an abandonment of effort to solve the problem in favour of ending the task (Chinn, 2012; Wagner, Rachlin & Jensen, 1984). Furthermore, anxious learners were found to trade accuracy for speed to minimise the time spent doing maths (Ashcraft et al., 2007).

Secondly, anxiety disrupted working memory (Ashcraft et al., 2007; Ashcraft & Moore, 2009). Mathematically anxious individuals are thought to devote some portion

of finite working memory resources to thoughts about their own anxiety reaction (Ashcraft & Moore, 2009). As the individual increasingly attends to their own emotional reaction, their performance decreases, further increasing anxiety. In addition, working memory processes are thought to maintain task focus, which permits the individual to resist interferences and distractions. An increase in anxiety was thought to limit the allocation of resources to maintain attention (Engle & Kane, 2003). As such, learners experiencing anxiety were thought to experience increased difficulty maintaining the focus necessary to engage deeply with mathematics.

Thirdly, given that beliefs, identities and attitudes are intertwined, mathematically anxious learners were found to globally avoid mathematics (Ashcraft et al., 2007). Typically, individuals with higher mathematical anxiety reported lower enjoyment and confidence with mathematics and reported an avoidance of mathematics courses in school and lower intention to enrol in college mathematics (Brown et al., 2008). Ashcraft et al. (2007) note that while causation is not certain, a pattern emerges in which anxious learners take fewer mathematics courses, and when they do, they learn less than others. Thus, a highly mathematically anxious learner, once an adult, has a high chance of having low mathematical skills, poor attitudes toward mathematics, an orientation toward avoiding mathematical situations, and a potential trait for experiencing anxiety in future classes. Given that many adults are compelled to take part in mathematical provision as part of foundation-level vocational programmes, these findings are concerning.

Beliefs and attitude, disaffection and disengagement
Attitudes have been described as positive or negative *inclinations* or relatively stable *feelings* toward mathematics, or more complexly, as a three-dimensional construct comprising dispositions, beliefs and behaviours (Di Martino & Zan, 2010; Hannula et al., 2016; McLeod, 1992). However, despite ongoing theoretical difficulties with the construct of attitude, it is well established that if you ask people how they feel about mathematics, many report holding strong negative attitudes toward mathematics generally (Burns, 1998; Mumcu & Aktas, 2015; Williams & Williams, 2010). In addition, many people state that mathematics is inherently difficult and that they personally cannot do it, while others simply express the view that mathematics is boring or unenjoyable (Brown et al., 2008; Kislenko, 2009; Lane et al., 2016; Williams & Ivey, 2001). In addition to general negative attitudes, the perceived difficulty of mathematics, its value, perceptions of ability, and general enjoyment of mathematics, worsened more than for any other subject as students aged (Midgley, Feldlaufer & Eccles, 1989; Noyes, 2006). Other studies found that as achievement decreased,

negative attitudes increased, influencing students' decisions to discontinue elective mathematics (Brown et al., 2008). Many negative attitudes are severe , for example, "I hate mathematics and I would rather die" and "because it sucks and I wouldn't want to spend any more of my time looking at algebra and other crap" (Brown et al., 2008, p. 10). Vocational students, typically lower-skilled, were found to hold particularly negative attitudes toward mathematics compared to regular school students (Mumcu & Aktas, 2015). Consistent and coherent negative experiences with mathematics were widely thought to develop more stable attitudes, dispositions and identities (McLeod, 1992; Middleton, Jansen & Goldin, 2016).

Negative attitudes were found to influence individuals' long-term and short-term decision-making (Hannula., et al., 2016). Learners holding negative attitudes enrolled in fewer elective mathematics programmes, avoided courses and careers such as science that included mathematics, and tended to avoid formal and informal training (Brown et al., 2008; Parsons & Bynner, 2007). When learners were enrolled in mathematical programmes their negative attitudes translated into poorer engagement and higher drop-out rates (Mayes, Chase, & Walker, 2008; Xin & Willms, 1999). Ashcraft et al. (2007) warned of an emergent cycle in which poor engagement led to further difficulties resulting in an ever-decreasing self-concept. This might be particularly problematic for adults, many of whom have had such experiences and continue to do so (Evans, 2000). Perhaps more problematic, however, are findings that suggest that a "group attitude" can develop in classrooms, which influences learners' behaviours *beyond* individual attitudes (Webel, 2013). Worryingly, Webel found that even though individual learners might be motivated, they were still influenced by the norms of the classroom, their relationships with other students, and general class affect. This is pertinent to this study if low-skilled adults' share negative or positive attitudes, this may influence the general class attitude.

The impact of shame, emotion and attitude on engagement
The research suggests that learners adopt strategies designed to prevent harm by controlling the risk of embarrassing or shameful episodes (Bibby, 2002; Chinn, 2012: Turner et al., 2002). These could be broadly divided into strategies designed to avoid engagement altogether and strategies designed to mitigate potentially embarrassing situations. Both strategies erode engagement and are particularly damaging when viewed from Lakatos' (1976) perspective of mathematical engagement. Lakatos argued that mathematical discovery was the result of a zig-zag path facilitated by making conjectures or "conscious guesses" followed by attempts to disprove the conjectures through counter-examples or refutations. Because learners will invariably

be proved incorrect, several researchers have argued that doing so requires intellectual *courage* and intellectual *honesty* (Gómez-Chacón, 2016; Lampert, 1990; Pólya, 1954;). Making a public conjecture required the admission that one's assumptions, insights and conclusions may be incorrect. This increases personal risk and vulnerability, particularly in classrooms where many learners believe that mathematics is a static body of knowledge that is learned by memorising content from an expert source. These environments may lead learners to interpret incorrect answers as mistakes, and something to be avoided (Bibby, 2002; Turner et al, 2002). Additionally, this view posits discourse as an *essential* element to doing mathematics and suggests that environments that lacks the process of making naïve guesses and refutations fails to meet the criteria for mathematical engagement. This suggests that strategies adopted by learners to avoid contributing to discourse, or public engagement, inhibit mathematical learning.

Avoidance strategies

Avoidance strategies are covert or overt behaviours adopted by learners to reduce or eliminate interaction with mathematics. Examples found in the literature include actively avoiding any participation, disrupting lessons or teacher, cheating, self-denigrating oneself, or subtler responses, such as self-censoring participation in discourse, feigning understanding or copying others (Bibby, 2002; Huak, 2005; Turner et al., 2002). Others have found similar strategies such as avoiding novel academic work and self-handicapping (Chinn, 2012; Turner et al., 2002). Covington (1992, p. 85) described these as strategies as "ruses and artful dodges" designed to avoid being labelled stupid.

Learners not only adopted strategies to avoid engaging with mathematics, but where this was impossible, adopted strategies to mitigate, or cope with, an experience they felt to be shameful. Bibby (2002) noted that because teachers could not abscond physically from training courses they did so mentally, mimicking engagement while "mentally absconding" or "shutting off", and "passing and disguising" in an effort to create an impression of engagement. Furthermore, studies found that learners avoided seeking help when they did not understand mathematical content, and that some learners did not try at all when failure might occur, preferring to do nothing rather than risk failure (Chinn, 2012; Turner et al., 2002). Turner and colleagues theorised that by refusing to try, these learners removed the opportunity for others to make negative judgements about their ability.

Other strategies to avoid or mitigate shame were also identified, including adults publicly self-denigrating themselves before their peers, advertising their difficulties with mathematics to the group before engaging (Bibby, 2002). Although this strategy seemed contradictory, Bibby suggested that the confession itself appeared to have no stigma attached, but had the effect of lowering the social expectations for the learner's performance. This protected the learner from the moment of shock or awkwardness when others became aware of their failing. Bibby theorised that some individuals needed to create emotional distance from mathematics and associated feelings of confusion or frustration. Methods to achieve this include sitting as far from the teacher as possible, copying others' work, responding to a teacher's question by raising their hand and "praying" they would not be asked or noticed by the teacher, and generally disguising a lack of understanding (Bibby, 2002; Hauk, 2005). Beliefs that posit mathematics as potentially stigmatising appear to lead to protectionist behaviours that inhibit engagement.

Observations of behaviours within adult classrooms

Studies of adult classrooms show that a portion of adult learners do not engage fully with lesson content. First, adult classroom-based studies find an unequal distribution of interaction between the learners themselves, and between the learners and the tutor (Fritschner, 2000; Howard & Baird, 2000; Howard, James & Taylor, 2002; Tennant, 2012; Weaver & Qi, 2005). Karp and Yoel (1976) coined the phrase "the consolidation of responsibility" to describe the recurring phenomenon of only a few learners in adult classes interacting consistently with the tutor, as few as five in most cases (Howard et al., 2002; Weaver & Qi, 2005). Furthermore, Weaver and Qi revealed that while these few learners dominate most interactions, the other learners maintain "civil attention", the practice of appearing attentive without risking involvement. These roles were so distinctive that Howard et al. (2002) argue that it is meaningless to speak of the average adult learners' patterns of interaction, and better to characterise learners as "talkers" or "non-talkers". While some of this research was conducted in non-mathematical environments, the same patterns appear to be evident in mathematics classes (Ashcraft & Moore, 2009; Bibby, 2002; Tennant, 2012).

Secondly, a body of research has explored, and recommended, developing communities of mathematical discourse within school classrooms as a tool to develop mathematical skills (Hufferd-Ackles, Fuson, Sherin, 2004; Mendez, Sherin & Louis, 2007). Although there is little comparative research with low-skilled adults, the studies of adult classes show that the mathematical discourse that does occur often

lacks the features of effective discourse such as conjecture, justification, or proving one's ideas. Several studies found that adult classrooms lacked sustained discussion or debate, and that tutors were the primary initiators, and maintainers, of discourse (Benseman et al., 2005; Mesa, 2010; Ofsted, 2011; Scogins & Knell, 2001; Tennant, 2012). For example, Mesa found that despite learners answering multitudes of questions, the mathematical discourse lacked sufficient complexity to develop meaningful skills. Traditional 'chalk and talk' approaches appear to be the dominant pedagogies in adult numeracy classes (Coben et al., 2007; Swain & Swan, 2007).

Third, although several studies found that tutors frequently initiate group work, the discourse that occurs between them may be of low value (Coben et al., 2007; Swain & Swan, 2007). Swain and Swan noticed that tutors spent little time setting-up or organising group work and noted a distinction between working *in* a group and working *as* a group. Despite being asked to work collaboratively, learners often did not do so, and when group work did occur, one learner would often tell the others how to think. Similarly, Coben et al. (2007) found that despite ample opportunities for learners to engage with each other, few observations were made of learners actually learning from each other. Interestingly, interviews conducted by Coben et al. (2007) found that in some cases, despite the tutors' efforts to cultivate group discussions, the learners continued to work independently, leaving the tutor somewhat resigned to individual work and transmissional approaches. Johnson et al. (2009) suggest that the reason for poor interaction between learners in groups is that the learners themselves are resistant to group work. There are very few studies, or transcripts, to inform how low-skilled learners engage with each other, and mathematical tasks, while group problem-solving.

The above classroom behaviours are concerning considering recommendations from several research projects on what constitutes a productive learning environment. The following is not an exhaustive list of recommended practices but does demonstrate the tensions between recommended environments and the behaviours described above. It is recommended that mathematical tutors of adult learners will:

- determine, and build on, what learners already know about a topic (Black & Wiliam, 1998; Gal, Ginsburg, Stoudt, Rethemeyer & Ebby, 1994; Ofsted, 2011; Swan, 2005; Swain & Swan, 2007);

- develop a community of discourse engaged in activity, reflection and conversation (Gal et al, 1994; Glass & Wallace, 2001; Mercer, 2000);

- use rich collaborative tasks and provide opportunities for group work (Askew & Wiliam, 1995; Gal et al., 1994; Swan, 2005; Swain & Swan, 2007);

- expose and discuss common misconceptions (Askew & Wiliam, 1995; Condelli et al., 2006; Swain & Swan, 2007; Swan, 2005);

- encourage reasoning, sense-making and the demonstrate the interconnected nature of mathematics rather than emphasising rote learning and getting the answer (Swain & Swan, 2007; Swan, 2005);

- use effective questioning to generate deep thinking (Askew & Wiliam, 1995; Hodgen, Coben, & Rhodes, 2010; Swain & Swan, 2007; Swan, 2005);

- address and evaluate attitudes and beliefs regarding both learning mathematics and using mathematics (Gal et al, 1994);

- situate problem-solving tasks within familiar, meaningful, realistic contexts (Gal et al., 1994; Ofsted, 2011; Swain & Swan, 2007);

- develop understanding by providing opportunities to explore mathematical ideas with concrete manipulatives or visual representations and hands-on activities (Gal et al, 1994; Glass & Wallace, 2001; Hodgen et al., 2010; Ofsted, 2011).

It is important to note that Coben et al. (2007) observed numerous examples of good practice in adult numeracy classes, yet were unable to recommend specific practices based on the correlative data they collected. However, drawing on qualitative aspects of their data, they noted that an important aspect of a tutor's practice was his or her flexibility in deploying a well-grounded pedagogy which included adapting to the diversity of adult numeracy learners and organisational contexts.

2.6 Changing beliefs

There is strong support in the literature for the claim that learner beliefs can be developed in an appropriate classroom environment (Higgins, 1997; Mason, 2004; Stylianides & Stylianides, 2014; Verschaffel, De Corte & Lasure, 1999; Verschaffel, Greer & De Corte, 2000). The catalyst for change is thought to be a result of

cognitive dissonance or disequilibrium that arises between existing beliefs and new experiences (Hekimoglu & Kittrell, 2010; Kienhues, Bromme, & Stahl, 2008; Muis & Duffy, 2013). Exposure to different beliefs can be a catalyst for change (Bendixen, 2002). Hence, changes in teacher practice often feature in belief interventions. New beliefs are thought to be adopted when an individual's current beliefs are judged to be inadequate to meet the demands of a situation (Liljedahl, 2010, 2015; Vosniadou, 2006). Liljedahl (2010) suggested that a "conceptual change" occurs when the current beliefs are the result of lived experiences, such as might occur in a traditional mathematics instructional environment, and are found to be no longer plausible in a new situation. In such a case, the beliefs were already being rejected, and able to be displaced by more meaningful beliefs. Many belief interventions have focused on making participants aware of their current beliefs through reflexivity and self-consciousness so that contradictions between these and new understandings are made explicit (Nespor, 1987). Grootenboer and Marshman (2016) note that supporting learners to revisit and revise episodes that gave rise to the held beliefs is useful, in conjunction with creating encounters where new and desirable beliefs can be experienced. Stylianides and Stylianides (2009) used the notion of two pillars, one at either end of an intervention, to describe this. The first was to make participants aware of their current beliefs, and the second to provide meaningful alternatives. Finally, because beliefs are thought to exist within systems, in which central beliefs provide the stability for connected yet peripheral beliefs, challenging and changing the 'leading beliefs' is thought to result in a change to the configuration and composition of the system of beliefs (Green, 1971; Liljedahl, 2010, 2015).

Despite the recognition that various beliefs influence behaviours there are few classroom-based studies that seek to improve learner beliefs through innovative classroom practices. Mason (2004) investigated the impact of a unique classroom environment on Italian fifth-graders and compared this to a traditional mathematics class. The unique aspects of the class included the following: placing an emphasis on increasing the students' role within the classroom by encouraging them to understand mathematics rather than obtain correct answers; generating multiple solutions to problems; and evaluating other learners' solutions. Learners were also encouraged to use Verschaffel et al.'s (2000) problem-solving heuristic in small-group work and whole class discussions, and to consider the various questions drawn from Schoenfeld (1992) such as "What did I want to find?", "Should I try another strategy?", and "Does my answer make sense?" The class was also exposed to non-routine problems, some of which were unsolvable or indeterminate, in addition to a range of word problems. The results indicated that after the intervention learners'

beliefs, overall mathematical performance, and self-evaluations were higher than those of the traditional class.

Similar approaches were used by Muis and Duffy (2016) who explored the impact of teacher modelling of critical thinking, evaluating multiple approaches to problem-solving, and making connections to prior knowledge, followed by opportunities for learners to practise the skills in small groups. Learners self-reported an increase in the use of elaboration strategies and critical thinking, and they achieved higher scores than the control group. Stylianides and Stylianides (2009) used a single problem to challenge teachers' beliefs about mathematics and problem-solving behaviours. The problems differed from traditional problems and required a collaborative, problem-solving approach. Teachers were asked to reflect on how the problems differed from their usual experiences and changed the way they thought about mathematics. Several other studies have sought to change learner beliefs by engaging them in problem-solving situations, emphasising multiple solution strategies, and evaluating them (Francisco, 2013; Higgins, 1997).

Several studies have provided information to learners about the nature of mathematics and how it is learned. Hekimoglu and Kittrell (2010) used a documentary about how mathematicians do mathematics to develop college students' mathematical beliefs. They found that learners broadened their stereotypical view of mathematicians and began to see them as intellectual explorers who pursued lives of challenge, adventure and excitement. Similar studies used the history of mathematics to successfully improve pre-service teacher beliefs about mathematics (Charalambous, Panaoura, & Philippou, 2009), and learners' beliefs about problem-solving (Philippou & Chistou, 1998). Blackwell et al. (2007) successfully developed 7th grade learners' beliefs about incremental theory through a series of discussion-based lessons that included how the brain worked, incremental theory, anti-stereotypes, study skills and discussions about how learning makes you smarter.

The majority of belief interventions conducted with learners have used classroom practices to create cognitive dissonance between beliefs about what mathematics is and how it ought to be learned and their current practice. They have tended to use problem-solving tasks to model effective approaches to solving the tasks, followed by group work, discussions, and the use of heuristics. Additionally, they have directly delivered information about beliefs and generated discussions.

2.7 Summary and aim of study

The research suggests that beliefs influence almost every aspect of a learner's engagement with mathematics. They influence learners' motivation to learn, their goals, standards for success, and the types of strategies they use to learn. Negative beliefs have been found to undermine effective engagement with taught content, and with problem solving, an essential component of mathematical instruction. Beliefs also influence learners' mathematical identities, which set expectations for participation and lead to passive non-participatory roles and avoidance of mathematics. Additionally, beliefs influence how learners view themselves in comparison to their peers, and may contribute to their orientation toward avoiding social harm. This in turn may lead to a range of avoidance strategies designed to mitigate potential shameful episodes. These findings suggest that negative beliefs held by low-skilled adults might lead to negative patterns of behaviour in embedded vocational programmes.

Little is known about the beliefs of low-skilled adults who are expected to re-engage with mathematical provision in vocational programmes, yet improving their mathematical skills is likely to have a considerable positive impact on their life outcomes. This study explores the beliefs held by low-skilled adults, and their behaviours while participating in mathematics education in the context of foundation-level vocational programmes. It also explores how low-skilled learners engage with mathematics in an innovative classroom environment that aligns with more positive beliefs about mathematics, and how it is learned.

Chapter 3: Methodology

This chapter outlines the methodological approach of the study. I begin with the underpinning epistemological position of the study, followed by the theoretical framework, details of the methodology, and the methods employed to collect and analyse the data.

3.1 Epistemology

A social constructionist approach is adopted as the underpinning epistemology because the phenomena under investigation, beliefs about mathematics and self, and engagement with mathematics, are taken to be contingent on human activity and take place within a socially constructed educational environment. This research tradition posits that individuals construct realities based on their interaction with society, embedded within culture and history (Gergen, 2015; Lock & Strong, 2010). Hjelm (2014) notes that whether a thing is true or not, people act based on whether they believe it to be true, regardless of a so-called objective reality. Therefore, this research adopts the view that understanding the meaning ascribed to situations and events is of upmost importance. This necessitates an interpretive paradigm in which the participants are considered to act with meaning and their behaviours are intentional.

Given the adoption of an interpretivist paradigm the notion of the objective researcher is rejected. The criteria for reliability and validity is transparency, reporting the findings in such a way that readers have enough information to 'get inside' the context, understand the researcher's interpretations, and make their own judgements (Lincoln & Guba, 1985; Walford, 2001). This can be done by including within the findings low-inference empirical evidence to illustrate concepts, supporting contextual information, and coherent explanations of interpretive judgements (Eisenhart, 2006). Therefore, the criteria for validity are emphasised, including: fidelity, rich descriptions (Geertz, 1973), and triangulation (Lincoln & Guba, 1985).

3.2 Theoretical perspective

The triadic reciprocal determinism model (TRD) (Bandura, 1999, 2001) is used as an organising framework to explore beliefs, behaviour and the environment (see Figure 2). The model describes human functioning as a product of continuous reciprocal interaction between intrapersonal, behavioural and environmental determinants. These three factors are useful to explore intrapersonal factors such as beliefs, the

behaviours learners engage in, and the classroom environments in which they occur. They are used to construct a framework for discussing agency, shame, and identity used to explore and describe the research questions.
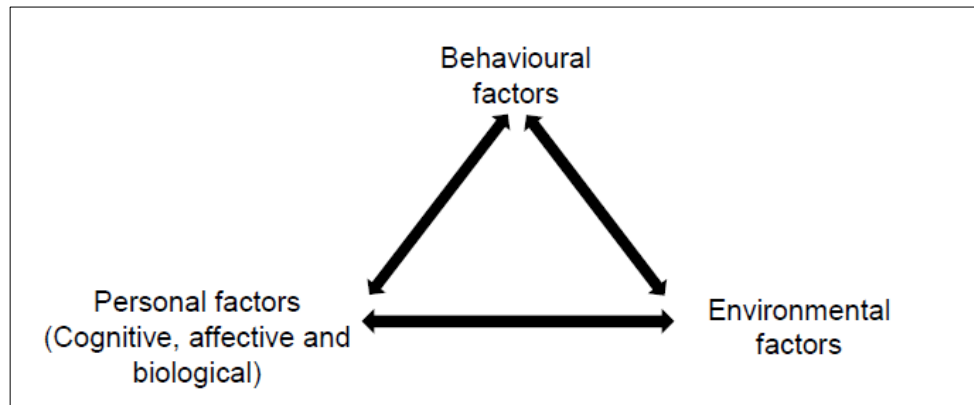
Figure 2: Bandura's (1999) model of triadic reciprocal determinism

Personal factors

Personal factors are inclusive of biological, cognitive and affective events. A large body of research has explored expectancy and self-beliefs as a domain of personal factors. Specific areas of research include beliefs about self-efficacy and their relationship with aspirations and achievement, self-appraisal of capability, motivation and resilience to difficulties (Bandura, 2012; Pajares & Usher, 2008). Beliefs about mathematical self-efficacy, perceptions of the classroom environment, interest in mathematics, and achievement have also been explored (Fadlelmula, 2010; Tosto, Asbury, Mazzocco, Petrill, & Kovas, 2016). There is substantial research support that beliefs influence: the meanings ascribed to events and environments and the affective responses to these; desired outcomes, goals, and actions; anticipated outcomes of various behaviours; how outcome states are internally represented or visualised; and how information is organised for future use (Bandura, 2006, 2012; Pajares & Usher, 2008). Given that beliefs are an important determinant of human functioning, below I describe the theoretical perspective of mathematical beliefs used in this study.

Mathematical beliefs

Three propositions about the nature of beliefs are adopted for this study. The first is that beliefs are ideas, understandings, premises or propositions thought by the holder to be true (Philipp, 2007; Richardson, 1996), and are the lens through which people view and interpret the world (Green, 1971; Pajares, 1992; Philipp, 2007). The distinction between beliefs and knowledge has been the topic of much discussion,

and conceptual disagreements remain unresolved (Kislenko, 2011; Liu, 2010; Österholm, 2010). However, I tentatively make the following differentiations, aware of the inherent tensions. This study adopts the view that beliefs differ from knowledge with respect to the degree of certainty an individual has regarding the truthfulness of a statement or proposition. Beliefs are less able than knowledge to be validated by a means other than external sources of authority, and therefore more subjective (Viholainen et al., 2014). Therefore, beliefs are more likely organised in a quasi-logical structure, in contrast to knowledge, which is more likely to be built up using logical principles (Furinghetti & Pehkonen, 2002; Green, 1971).

The second proposition is that beliefs relate to, and influence, both the cognitive and affective domains. I draw on Goldin and colleagues' (2002, 2011) description of beliefs which emphasises the interrelated nature of the two domains.

> Beliefs are defined to be multiply-encoded cognitive/affective configurations, to which the holder attributes some kind of truth value (e.g., empirical truth, validity, or applicability) (2002, p.59).

This description indicates the cognitive nature of beliefs while giving equal consideration to affective aspects. The framework developed by McLeod (1992) and elaborated by others (Grootenboer et al., 2008), situates beliefs within the affective domain and differentiates between beliefs, values, attitudes and emotions regarding their cognitive stability, and affective intensity. Beliefs are situated as increasingly cognitive and stable, and decreasingly affective and intense. However, the cognitive and affective domains are intertwined, as the onset of affective responses or the success or failure of various cognitive strategies will influence the other (Goldin et al., 2011; Grootenboer & Marshman, 2016).

The third proposition concerns the organisation of beliefs which many researchers agree have three features. The first is that beliefs are not held in isolation but rather exist as part of a belief system organised around a key idea or object (Cooney, 2001; Goldin et al., 2009; Green, 1971; Philipp, 2007). Many agree that beliefs are best categorised by the specific idea or object to which they belong (Goldin et al., 2009; Muis, 2004). Beliefs about mathematics, beliefs about self, and beliefs about the social context of mathematical learning are persistent categorisations used across the domain (Goldin et al., 2009). Secondly, the stability of an individual belief can be thought of regarding its centrality within the system (Green, 1971; Op't Eynde, et al., 2002; Philipp, 2007). Peripheral beliefs are considered less stable and possibly more

easily changed, while primary beliefs are considered more strongly held. Third, the relationships between beliefs within a system may be held in a 'quasi-logical structure' in which primary beliefs serve as foundational to derivative beliefs (Goldin et al., 2009; Philipp, 2007; Thompson, 1992).

Behaviour

A wide range of behaviours have been investigated in the context of TRD including class participation, persistence, and speech. However, a notable portion of research has explored the self-regulated learning strategies (SRL) used by learners within learning situation (Pajares & Usher, 2008; Zimmerman & Schunk, 2011). SRL is an important aspect of mathematical behaviour because the ability to manage resources, employ effective learning strategies, meaningfully organising information, monitor progress and respond appropriately is consistently associated with higher achievement (Fadlelmula, 2010; Pintrich, 2000; Zimmerman & Schunk, 2011). Examples of active learning behaviours include elaborating, mathematizing, planning and implementing problem-solving strategies (Stylianides & Stylianides, 2014). In contrast, passive, or aimless, strategies include listening, copying, and not seeking help when needed (Schoenfeld, 1988; Schommer-Aikins et al., 2005; Turner et al., 2002).

Environment

Environmental factors include cultural, contextual, social and physical features. For example, lesson structures, grouping practices, teacher beliefs and expectations, social and verbal persuasions (Pajares & Usher, 2008). Importantly for this study, the people involved in interpersonal transactions are also considered part of the environment. These individuals bring with them their own unique personal factors, beliefs, agentic dispositions and behavioural tendencies. Additionally, the interactions and relationships between people may settle into specific roles, identities, that they adopt in specific environments. The social systems generated from social transactions organise, guide, and regulate societal prescriptions and sanctions, which in turn influence further behaviour. Despite social prescriptions and sanctions there is considerable personal variation in how individuals interpret and respond to social rules (Bandura, 2006).

One way that the environment influences individuals and groups is through reinforcement or punishment, and through feedback (Akers & Sellers, 2004). For example, grade scores act as an external signal of success or failure, whereas feelings of shame or pride are internal reinforcements. Individuals adopt successful

actions and discard those that are perceived negative (Bandura, 2012). Beliefs play a role in this because they influence an individual's evaluation of feedback signals, whether it is positive or negative. For example, an adult learner who volunteers an incorrect mathematical answer in a plenary exchange may evaluate the tutor's public correction as either a learning opportunity, or a shameful lesson on not volunteering in the future. Pertinent to this study, experiences of shame or pride, are reinforcements, providing feedback that various behaviours should be adopted or abandoned.

Three types of environmental structure are distinguished based on graduations of control; imposed, selected and constructed (Bandura, 1999). The first is an environment in which the physical and social environment is *imposed*, whether people like it or not, although people do have choice in how they respond to it. The reinforcing potential of the environment comes into effect when individuals are able to select and activate courses of action. These actions constitute a *selected* environment. The combined generative efforts of individuals produce a *constructed* environment. A classroom environment is constructed as learners select and undertake actions based on their desired outcomes and their perspective on their likelihood to succeed. These actions and interactions in turn influence others' behaviours, personal factors and the environment in a reciprocal manner.

Personal, proxy and collective agency

A key part of the framework is that adults possess agency, the ability to intentionally influence their functioning and outcomes. Bandura (2006) describes agency as an outcome of human consciousness, an individual's ability to visualise a future state and construct, evaluate and modify courses of action to achieve desired outcomes. Agency features heavily in the mathematical literature, particularly relating to learners adopting agentic problem-solving behaviours, and being able to "dance" between personal agency and the agency of the discipline (Boaler, 2003; Pickering, 1995). Agency has a relationship with mathematical beliefs (Boaler & Greeno, 2000; Grootenboer & Jorgensen, 2009; Schoenfeld, 1985, 2012), and self-efficacy beliefs (Bandura, 2012).

Agency comprises four core properties; intentionality, forethought, self-reactiveness and self-reflectiveness (Bandura, 2006). Regarding intentionality, individuals form intentions and plan strategies to achieve them. Because the social world inevitably requires working with others who may have intersecting or complementary desired outcomes, individuals can accommodate each other's self-interests and work

together. This may play out in a classroom situation when learners' intersecting intentions and desired outcomes provide a basis for various behaviours.

The second property, forethought, involves the cognitive representation, or visualisation, of future states that act as guides or motivators for behaviour (Bandura, 2006). Regarding future states, Sfard and Prusak (2005) posit that an individual's representation of a potential future state is bounded by their designated identity, underpinned by beliefs about what is and is not possible. They distinguish between an *actual* identity described as an individual's personal narrative about their current identity, and a *designated* identity that describes what the individual expects to become in the future. An individual's designated mathematical identity may shape, and perhaps constrain, their representations of future states, thereby influencing their actions.

Self-reactiveness refers to the self-regulation skills applied during actions undertaken to achieve desired states. Individuals regulate their motivation, compare performance against goals and standards and take adaptive action where necessary. A limited ability to self-regulate is associated with poorer performing mathematical learners (Briley et al., 2009). Finally, individuals self-reflect on their own functioning, their personal efficacy, the meaning of their pursuits, the soundness of their thoughts, and can make corrections if necessary.

Agency can be exercised individually, through a proxy, or as part of a collective (Bandura, 1999; 2006). These are often blended because rarely are individuals able to operate with complete autonomy. Individuals can influence, or use, others with more effective resources, knowledge and means, to achieve mutually desirable outcomes. Individuals can also surrender their agency to intermediaries to achieve goals, while avoid burdensome aspects of the task. Low-skilled adult learners working in groups may adopt a blend of individual, proxy or collective modes of agency to complete mutually desired, or partially aligning, outcomes in vocational classrooms.

Identity
Identities are widely thought to be context or role-specific (Sfard & Prusak, 2005; Turner, 2010). Some researchers argue that individuals develop as many identities as they have distinct networks of relationships (Hannula, 2012; Stryker & Burke 2000), and these are 'activated' as individuals become aware of the situational ecology (Goffman, 1981). For example, this might occur when an adult enters a

vocational classroom and becomes aware of mathematical content. Embedded within activated identities are expectations for the role the individual will enact within those situations. Sfard (2009a, p.10) described identities as the "…stories about who we are, with whom we belong, and what positions we occupy among those who constitute our human environment". In this view identity is defined as a set of reifying, endorsable and significant stories about a person (Sfard & Prusak, 2005). Adult learners have such stories about themselves regarding mathematics (Coben, 2002; Evans, 2000; Heyd-Metzuyanim, 2013).

A feature of identity adopted within the framework is that of the previously mentioned *actual* identity, and *designated* identity (Sfard & Prusak, 2005). Actual identity is used to describe identity 'as it is' in the current state, while designated identity is an expectation of what one may become in the future. Dunn and Creek (2015) argue that when an individual discerns a growing discrepancy between their current and expected identity they experience negative emotion and anxiety. These negative responses are increase when accompanied by the self-perception that they are largely helpless to prevent the circumstances contributing to it (Turner & Stets, 2005). This elaborates on McLeod's (1992) assertion that repeated negative emotions lead to increasingly stable negative attitudes. Individuals who are aware of the growing gap between actual and designated identities may attend more closely to perceived episodes of failure, such as answering a mathematical question incorrectly, leading to more stable negative attitudes and beliefs about self (Brown et al., 2009).

Finally, individuals are motivated to protect themselves from social harm and may use their agency to avoid potentially shameful events. Individuals require a sense of belonging, feelings of personal significance, a sense of location relative to others, a sense of continuity and coherence, and feelings of worth (Dunn & Creek, 2015; Schwalb & Mason-Schrock, 1996). These are issues of identity because a persons' identity is related their social location within intersecting systems of stratification, and therefore relate directly to significance and esteem or stigma (Dunn & Creek, 2015). Scheff argues that 'shame' signals harm to the social self in the same way pain signals harm to the physical self. Perceived damage to identity may manifest as anger, fear, anxiety or shame (Scheff, 1988; Schwalbe & Mason-Schrock, 1996; Turner, 2010). Given this, learners may engage in *image-management,* the practice of modifying their behaviours to present a tailored image of themselves to others consistent with their perceptions of social safety (Bibby, 2002; Goffman, 1959).

In summary, a triadic reciprocal determinant model is used to explore learners' intrapersonal factors, specifically beliefs, their behaviours, and the environments in which they operate. Belief systems are considered to influence learners desired outcomes, expectations for future states, goals, strategic plans, actions, perception of success, and their evaluation and interpretation of feedback. These are not bidirectional but function as part of a dynamic reciprocal relationship comprising personal factors, behaviours and the environment. Learners possess individual agency but are not autonomous, they may exercise proxy, or collective agency to achieve outcomes and adopt collective behaviours to achieve mutually desirable outcomes. Learners may be motivated to protect their social self, and engage in image-management to do so, perhaps by avoiding events leading to negative judgement by others.

3.3 Methodology

The methodological approach comprises a range of data collection methods; surveys, observations, interviews, and an intervention. These were organised to explore three aspects of learner functioning. First, the learners' beliefs as a component of personal features. Second, the learners' behaviours and the environment in which they occur. Third, the learners' behavioural responses to a change to the mathematical environment. These were organised as three research questions.

1. What beliefs do low-skilled adults hold about mathematics?
2. How do low-skilled adults engage with mathematics within vocational lessons?
3. How do low-skilled adults respond to a classroom environment that emphasises conceptual understanding?

A key decision in exploring the research questions was to utilise my insider knowledge within the foundation-level sector to explore the research questions. Insider research can be described broadly as research that is directly concerned with the setting in which the researcher works (Robson, 2002). An 'insider' is someone who has worked in the setting in which they are researching, providing an intimate knowledge of the culture, norms, and traditions of the environments they are researching (Coghlan, 2007; Teusner, 2016). This knowledge facilitates insights into the formal and informal aspects of research domains, but particularly the less tangible aspects such as the people dynamics, pressures, power struggles and

nuances that exist within the environment. Coghlan (2007) notes that the insider can research something they are close to precisely because they know it well, and through reflection, reframe their tacit knowledge as theoretical knowledge. Insider research also benefits the research process because deeper knowledge informs the decision-making regarding the design, implementation and analysis of data (Fuller & Petch, 1995). Hence, although I have not worked directly with the organisations or people within this study, my insider knowledge as a practitioner working with low-skilled adults and as a professional 'consultant' working with foundation-level organisations and tutors, presents an opportunity to add value to the study.

A key area that my insider knowledge may contribute to is that of learners' behaviours within numeracy lessons. The current adult numeracy research presents a somewhat opaque view of the subtle behaviours of adult learners within numeracy lessons. Although there are observational studies that have explored adult numeracy lessons, these have primarily been conducted by a single observer using observation schedules, and/or single recording devices, and have emphasised the tutors' behaviours rather than those of the learners (e.g. Benseman et al., 2005; Coben et al., 2007). Yet much has been made of 'hidden dimensions' of behaviours within school mathematics classrooms brought to light by Bauersfeld (1980). In the less explored adult foundation-level numeracy domain there are likely to be hidden dimensions of learner behaviours that have remained opaque to traditional researcher approaches and could be illuminated. Drawing on my experience and knowledge of foundation-level classrooms and learner dynamics may provide insight into these areas of interest.

Insider research raises questions of validity regarding researcher bias and the ways in which it may emerge during the research process (Drake, 2010; Ravitch & Writh, 2007). Coles (2015) noted bias is not something that can simply be identified early in the research process, because the effects unfold during the process. Reflexivity is a process of ongoing self-examination of ones' active role in shaping the research process and the knowledge produced from it (King, 2004). Bias cannot be eliminated, but it can be identified and examined during the research process by questioning assumptions, interpretations and prior expectations for findings. Rooney (2005) has provided several questions to explore threats to validity designed to be used throughout the research process. Validity is discussed in depth in the data analysis section below and reflections recorded in Chapter 8.

The methodological requirements to answer the three research questions were varied. The first was to gather broad data regarding the beliefs about mathematics and learning held by low-skilled adult learners. The second was to explore the learners' own perspectives of their beliefs, historical and current experiences, feeling and thoughts about mathematics in-depth. The third was to explore learner behaviour *within* the context in which it took place, and with as little intervention or manipulation as possible (Hammersley & Atkinson, 1983). Additionally, because participant utterances are behaviours and interactions environmental factors, verbal interactions needed to be collected that were ecologically valid while being of high fidelity (Geertz, 1973). Fourth, the use of several methods would ensure high reliability by providing a measure of triangulation. An overview of how the data collection methods met these requirements is described below.

Surveys are often not used within an interpretive design, as they tend to align with positivistic approaches, however, one was included on pragmatic grounds. Very little relevant research has been conducted with adult learners in foundation-level programmes in New Zealand, and surveys are an effective way to obtain detailed descriptions of existing conditions (Cohen et al., 2007). Additionally, surveys are a well-established method of exploring beliefs (Hannula et al., 2016), making it an appropriate method for gaining initial insights into the mathematical beliefs held by low-skilled adults. The details of the survey, its design, piloting, and administration are covered in the method section below.

Observations are a powerful method for gaining insight into situations, events and behaviours in authentic contexts (Cohen et al., 2007). The ability to observe first hand and gather 'live' data ensures ecological validity (Moyles, 2007; Robson, 2002). Additionally, the use of observational data addresses a weakness of some studies on learner' beliefs, that is, the reliance on self-report data and the risk that participant responses reflect appropriated 'rhetoric' about mathematical learning rather than substantive insights (Francisco, 2013; Schoenfeld, 1989). However, while observations of behaviours in authentic mathematical sessions are essential, there are challenges associated with observing affective responses within this environment.

Observing and understanding participants' affective responses to mathematics requires discriminating between behaviours that may or may not be influenced by affective factors. The distinction in McLeod's (1992) framework between affective factors based on their intensity and stability provides a basis for making judgements

of such phenomena. Turner and Stets' (2005) list of observable manifestations of emotion is a complementary tool for making observations. The list includes speech (specific words, interjections or exclamations), para-linguistic moves (tone, pace, pitch), eye-contact or lack thereof, facial expressions and body language. These can be differentiated into either *molecular* units, small actions such as gestures, or *molar* units, larger units such as short phrases or conversations (Wilkinson, 2000). Selecting both presents difficulties largely related to the observers' singular view, and a tendency to focus on high frequency events (Cohen et al., 2007). However, capturing and analysing the wider body of 'live' data can be facilitated by using recording technology that allows further analysis of the data to occur after its collection rather than relying on note-taking methodologies that require the researcher to make in-the-moment interpretations (Erickson, 1992).

Even though the use of recording devices overcomes the partiality of the researcher's singular view, even movable recording devices are vulnerable to the researcher's selections (Morrison, 1993). However, the use of *multiple* recording devices placed throughout an environment avoids these limitations; collecting all audio data simultaneously facilitates the capacity for "completeness of analysis and comprehensiveness of the material" (Cohen et al., 2007, p. 407). It also reduces dependence on interpretations made during the observation and eliminates the need to distinguish between structured and unstructured observations, as pre-ordinate categories are not necessary (Lofland, Snow, Anderson, & Lofland, 2006). The use of multiple recording devices enables all utterances to be analysed in the context in which they occur (Alton-Lee, Nuthall & Patrick, 1993), thus enhancing the likelihood that the antecedent is identified. Additionally, as the observations progress it is important to respond to situations as they emerge and utilise 'progressive focussing' (Parlett & Hamilton, 1976) to identify, and focus on, the salient aspects of the various phenomena.

The process of collecting observational data must also be able to adapt to changes within a dynamic and unpredictable environment (Lincoln & Guba, 1985; Newby, 2010). The target environments (mathematical lessons) may have been situated in workplace environments or within rooms not designed to be classrooms, potentially constraining the boundaries of what was observable, and/or, compromising the collection of auditory data because of ambient noise. The use of multiple recording devices, in conjunction with field notes and video, ought to mitigate much of that risk.

Interviews are an effective method of gaining access to an individual's understandings, interpretations and motivations for their own, and others', actions (Cohen et al., 2007; Kvale, 1996). They have a history of being used successfully to explore participants' thoughts, knowledge, values, beliefs and attitudes in mathematical contexts, particularly when used in conjunction with surveys and/or observations (Beswick, 2007; Mason, 2003). This is because interviews allow access to rich data regarding participants' interpretations of experiences and situations, the identification of variables and relationships, and can function as a tool to validate the other methods used within a study (Cohen et al., 2007). However, gaining access to 'what is inside a person's head' has methodological challenges (Tuckman, 1972).

An interview is widely considered not a data transfer event but rather a social event and the data generated thought to be a co-construction between the interviewer and interviewee (Cicourel, 1964; Walford, 2001). Cicourel identified several factors that influenced the way in which an interview proceeds. These were the level of trust between individuals, the social distance, the interviewer control, and how each of these might influence the interviewee's responses, whether in attempts to enact avoidance strategies or to hold back information, perhaps by becoming nervous or anxious, or because of the possibility of either party misinterpreting the other. Drake (2010) noted that interviewers themselves are unable to be neutral and are always positioned in some way by the participants and/or organisation. These critiques are accepted as an unavoidable aspect of the interviews in this study, particularly given that mathematical ability is linked to an individual's perception of social worth (Räty, Kasanen, & Kärkkäinen, 2006; Siivonen, 2013), and that the participants were likely to have had difficulty with mathematics, making them potentially self-conscious.

Consequently, the notion of 'reflexivity', described as a circular relationship between cause and effect, was built into the design and approach to the interviews. A semi-structured interview is a better "fit for purpose" for this because of the latitude it provides in freedom and flexibility while maintaining purpose and direction (Cohen et al., 2007). This format enables sequential questions and probes to pursue leads provided by the participants, enabling them to elaborate on aspects of their personal experience (Lofland et al., 2006). It also leaves room for rapport building throughout the interview, rather than only at the initial stages.

The third research question explored the behavioural patterns of low-skilled adult learners as they participated in a conceptually-oriented mathematical programme. Given that the environment was highly fluid, contained a multitude of inter-related

and complex variables, and was highly unpredictable, a flexible intervention design was appropriate (Hitchcock & Hughes, 1995). The intervention was designed to collect a rich and vivid description of events that focused on individual actors and groups, environmental factors and behaviours. It enabled specific events relevant to the research question to be highlighted and explored by including the participants' perception of events, while also enabling the researcher to be integrally involved (Cohen et al., 2007).

I considered, too, that adopting the roles of both researcher and tutor would provide greater insight into the social experiences of the participants and ensure a measure of control over the features of the intervention. It has been suggested that this approach allows one to "get under the skin" of behaviours, particularly over time, as in this case (Cohen et al., 2007). The dangers inherent in 'observer as participant' roles, such as adopting the values, norms, and behaviours of the participants (Gold, 1958) are minimised by the fact that my role as tutor/researcher would differ from that of the participants and would also be known by the participants. Newby (2010) referred to this as 'active and known', in other words, the researcher is actively involved with the community, yet known not to be a member. Foundation-level classrooms are unique dynamic social environments with a myriad of human interactions, each interacting in a reflexive communicative dialogic. Therefore, the intervention design was adopted to generate the 'thick descriptions' (Geertz, 1973) necessary to illuminate the research question.

3.4 Method

There are four elements to this study used to explore the three research questions (see Figure 3). The first is a survey used to explore learner beliefs. The second comprises classroom observations to explore learner behaviours as they engage with mathematics. The third uses interviews to explore both behaviours and learner beliefs in greater detail. The fourth element is an intervention that explores the responses of low-skilled learners to conceptually-oriented mathematics. For ease of reference, these elements are referred throughout the thesis as 'The survey', 'The observations', 'The interviews', and 'The intervention'.

The four data collection methods were organised to facilitate a 'funnelling' process by which the data collection transitioned from the general to the more specific (Cohen et al., 2007). The survey was used as an initial method to collect broad data about learners' beliefs about mathematics and provide an indication of the type of behaviours that might manifest in the classroom observations. Learners were then

observed as they participated in mathematical lessons. Note, because of absentees not all the learners who completed the surveys were observed. Twelve learners who participated in both the survey and the observations were interviewed to explore their responses to the survey and perspectives on their observed behaviours. Finally, a new group of learners who had not yet taken part in the study were observed over 10 weeks as they took part in an innovative intervention tutored by myself. The data were collected and analysed in sequence, beginning with the survey, then the observations, the interviews, and finally the intervention.

Question one	Question two	Question one and two	Question three
Survey (n=119)	Observations (8 sessions observed)	Interviews (n=12)	Intervention (19 sessions)
Methods • Likert scale items • Open response questions	**Methods** • Audio-recordings • Video footage • Field notes • Direct observations	**Methods** • Semi-structured interviews	**Methods** • Audio-recordings • Field notes • Direct observations • Interviews • Surveys
Participants	Participants		Participants
Partial overlap		No overlap	

Figure 3: Data collection process

Belief survey

Participants

The survey was distributed to participants enrolled across 11 foundation-level vocational programmes or in pre-vocational numeracy and literacy programmes. The vocations included those developing general employment and work skills, sport and recreation, hairdressing and beauty, agriculture (farming and forestry), hospitality, retail, and mechanical and automotive engineering. A total of 119 participants across 11 programmes completed the survey (see Table 1).

Table 1. Survey participants by programme area and gender

		Gender	
Programme area	Total	Male	Female
Employment skills	6	3	3
Confidence for work	8	3	5
Mechanical engineering	13	12	1
Hairdressing	20	3	17
Agriculture 1	12	9	3
Service for work	17	9	8
Entry business skills	10	3	7
Retail	6	1	5
Automotive engineering	10	8	2
Sport and recreation	7	4	3
Agriculture 2	10	8	2
Total	119	62	57

Overall, the participants were approximately equally balanced in terms of gender (62 men, 57 women), predominantly Māori (55%), followed by European (21%) and Pasifika (17%) (See Table 2). Learners' ages ranged from 16 to 57 years ($M = 24.6$, $SD = 9.1$). Those 36 years and older (n=13) were combined with those aged 25-35 (n=21) to create a larger group.

Table 2. Survey participants by age, gender and ethnicity

Age	Gender		Ethnicity*				
	Male	Female	Māori	European	Pasifika	Other	Total
16-19	20	21	21	10	7	3	41
20-24	21	23	25	8	8	3	44
25 +	21	13	20	7	6	1	34
Total	**62**	**57**	**66**	**25**	**21**	**7**	**119**
Percent	**52**	**48**	**55**	**21**	**18**	**6**	**100**

Process

The survey was given out as a paper-based survey through tertiary providers of foundation-level programmes within Waikato/Auckland regions. Ten tertiary education providers were invited to participate and seven agreed. I visited 11 separate classrooms across seven locations.

To mitigate potential literacy issues, I entered each class in person and explained the purpose of the study and survey in a consistent way, avoiding information that might influence learner responses. The presentation was standard across all classrooms; the accompanying information and consent forms can be found in Appendix A. Once

learners consented to participate, the survey was handed out and read aloud to the class while they completed it.

The instrument

The survey instrument was constructed using four existing surveys, Kloosterman and Stage's (1992) Indiana Mathematical Belief Scales, the Fennema-Sherman Usefulness Scale (1976), Schommer's Epistemological Questionnaire, and two open-ended questions adapted from Young-Loveridge and Mills (2010) (see Appendix E).

The first, Kloosterman and Stage's (1992), Indiana Mathematical Belief Scales measures the strength of five mathematical beliefs in relation to their responses to a series of statements across five sections. Section one, 'I can complete time-consuming mathematical problems' measures learners' beliefs about their ability, and willingness, to complete problems that cannot be solved quickly. Section two, 'There are word problems that cannot be solved with simple step-by-step procedures' explores learners' beliefs regarding the utility of rules and procedures. Section three, 'Understanding concepts is important in mathematics', measures learner beliefs regarding whether procedures, rules and algorithms should be accepted without understanding how they work. Section four, 'Word problems are important in mathematics', measures the extent to which learners believe mathematics is a problem-solving activity related to real world applications, or a computational activity carried out with little context. Finally, the fifth section, 'Effort can increase mathematical ability' measures the role that learners believe effort, rather than innate ability, plays in increasing mathematical ability.

The second part of the instrument is the Fennema-Sherman Usefulness Scale (1976) 'Mathematics is useful in daily life'. This belief is used as an indicator of motivation to learn mathematics (Fennema & Sherman, 1976; Malmivuori & Pehkonen, 1996; Mason, 2003).

The third part of the questionnaire was adapted from Schommer's Epistemological Questionnaire (SEQ) (Schommer, 1998). The survey consists of nine subscales; 'learning is quick', 'learn the first time', 'success is unrelated to hard work', 'ability to learn is innate', 'knowledge is certain', 'avoids integration', 'avoids ambiguity', 'depends on authority' 'don't criticise authority'. The SEQ has been used with high school students (Schommer-Aikins et al., 2005), college students (Schommer & Walker, 1997) and adults from various educational backgrounds (Schommer, 1998). These items provide complementary information regarding: learner beliefs about

whether learning happens quickly or gradually, whether intellect is fixed or fluid, beliefs about the effectiveness of effort on ability and whether knowledge comes from external sources or can be discovered.

A four-point Likert scale ranging from 1 = "strongly agree" to 4 = "strongly disagree" was used (scoring reversed for negatively-worded items). The decision to remove the 'not sure' option was based on research that showed participants held widely varying interpretations of the mid-point option which reduced reliability (Nadler, Weston & Voyles, 2015), and that many participants who would have selected a definite position shift to a neutral position if it is available (Schuman & Presser, 1996).

Two open-ended questions were added, adapted from Young-Loveridge and Mills (2010). These were "What is mathematics?" and, "If a new student started your course and wanted to learn numeracy, what advice would you give them?"

The survey was piloted before use, with an emphasis on readability of the statements. A full description of the piloting process and changes can be found in Appendix G.

Any surveys that did not have matching signed consent forms, or completed demographic details (gender, age and ethnicity) were removed from the data set (n=6). The results of the survey can be found in Chapter Four.

Classroom observations

As noted in the methodology the use of multiple audio-recording devices addressed several practical challenges to observing adult classrooms. These included the need to cope with dynamic and unpredictable environments, such as mathematical lessons situated in workplace environments or within rooms not designed to be classrooms. Such elements had the potential to limit access altogether, constrain the boundaries of what was observable, or due to ambient noise, limit the capture of auditory data. Additionally, lessons might include whole-class, group or individual activities, a combination of the two (meaning seat changes), or require learners to leave the environment entirely. The use of multiple audio-recorders placed throughout rooms provided a solution and was able to capture all the conversations occurring simultaneously within a classroom, and yet situate them all within the broader classroom context.

Participants

The organisations that had taken part in the survey were approached and asked to participate in the observations. I met first with managers, then tutors, and upon the agreement of both, approached the learners in whole-class situations. Four programmes were observed from the following vocations: hairdressing, agriculture, employment skills and health and fitness (see Table 3). Some of these classes contained learners from other programmes. For example, the employment skills classes included some learners from a retail programme seeking to improve complementary skills. The programmes were selected because they catered for learners who had left school with no qualifications and reflected a range of vocational settings. Discussions with the organisations confirmed that except for a few unique cases, all learners had difficulties with numeracy (see Chapter section 5.1 for a detailed description). The four programmes were delivering foundation-level qualifications full-time and each reported embedding mathematics provision into its delivery. The observations took place mid-year, by which time learners were familiar with each other and the tutor.

Table 3. Observation participants by programme area and gender

Programme area	Number of learners	Male	Female
Hairdressing	11	2	9
Agriculture	12	9	3
Employment skills	23	6	17
Health and fitness	7	6	1
Total	**53**	**23**	**30**

Procedure

In preparation for the observations, I met with the learners, explained the purpose and procedure of the study, and gave all information regarding ethical boundaries regarding safety of the data, confidentiality and anonymity. The learners were provided with information sheets and I returned one week later to answer further questions and re-clarify information (see Appendix A). All learners gave full informed consent to be audio recorded, and each completed a consent form agreeing to participate.

For each observed session, I entered the classroom with the learners, waited until they were seated and then briefly informed the class of my presence and that the session would be recorded. I then placed the recording devices in their various locations, in full view of the learners, and informed them that from that point on all

their comments would be recorded. The audio-recorders recorded learners' private reactions to tutor instructions, their private-talk while solving problems, and their general speech during sessions. I sat at the rear of the class and let the tutor introduce me and then proceed with the session.

The observation data were collected in three ways: with multiple audio-recording devices; direct observation with field-notes; and video-recording. Up to six recording devices were used in each session, recording up to 12 hours of conversation within an average two-hour mathematics session.

Field-notes

During each of the observations field notes were taken regarding: the organisation of the class; events that occurred; content that was used (the collection of worksheets, photos of whiteboards, projections and equipment); thoughts regarding learner responses and behaviours with time signatures to the recordings; and critical events that were particularly pertinent to the research question.

Video Recording

A video recorder was used for two reasons. Firstly, the layout of each class was video recorded at the initial stages of each session to provide information on organisational and learner layout when it came time to transcribe the recordings. This allowed voices to be matched with the correct individuals. Second, the video recordings were used to record aspects of body language to complement and add fidelity to the audio recordings. However, the presence of the recorder was found to visibly change the class dynamics in several cases, and it was removed when this was so.

Learner interviews

The interviews were conducted with 12 learners, generally following the structure of the interview schedule (see Appendix I). The content included:

- Experiences of school mathematics
- Beliefs about mathematics
- Beliefs about tutors
- Reflections on current numeracy learning
- Goals and strategies
- Epistemic beliefs

The mathematical and epistemic belief sections were informed by the learners' survey responses. The learners' response from one of each of the belief scales was used to explore the learners' reasons. However, as described in the methodology, the interviews were conducted in a reflexive way, so this order was not always adhered to. The interview results are presented in Chapter 6. The interviews took approximately one hour to complete. The interviews were audio-recorded and transcribed. Data analysis is presented in the final section.

Participants

Twelve interview participants were selected from the classroom observations. Ten were selected due to behaviours that indicated some trepidation with the content. These behaviours included: showing some evidence of constrained participation (such as asking a partner for advice covertly rather than the tutor): and engaging to some degree with content. It should be noted that these behaviours were typical of those observed. To provide a contrast, two learners, Troy and Hahona, were selected due to behaviours that indicated higher confidence with mathematics. These behaviours included; teaching others, offering alternative strategies to the tutor and class and interacting with the tutor more frequently. These behaviours and interview results are elaborated in Chapters 5 and 6. Finally, the participants' numeracy scores on the LNAAT were also obtained where possible.

Table 4. Interview participants' age, gender, ethnicity, programme and LNAAT numeracy scores

Name	Age	Gender	Ethnicity	Programme	LNAAT Step
Troy	20	Male	European	Employment skills	5
Hahona	19	Male	Māori	Hairdressing	6
Trudy	26	Female	Māori	Beauty	4
Pita	19	Male	Māori	Employment skills	1
David	23	Male	European	Employment skills	3
Abbie	26	Female	Māori	Hairdressing	4
Niki	21	Female	European	Hairdressing	4
Sonja	26	Female	Māori	Hairdressing	NC
Kelly	24	Female	European	Agriculture	3
Mary	57	Female	Cook Island	Employment skills	NC
Tina	25	Female	Māori	Retail	NC
Anna	24	Female	Māori	Retail	2

*NC - Not complete

Procedure

The participants were approached following the observations and asked to take part in an interview at any time of their choosing. Interview times were arranged and took place in a private room at the learner's organisation. Seats were arranged in a neutral format and every attempt was made to reduce potential power imbalances and make participants relaxed. Participants were informed of the interview topics, process and timeframe. Interviews were recorded and transcribed.

Given the need for reflexivity, the interviews were conducted with a commitment to 'naturalness' (Gillham, 2000), in that they resembled discussions more than standardised interviews. Oppenheim's (1992) notion of 'stimulus equivalence' was adopted, in that the emphasis was on interviewees' equal understanding of the questions rather than relying on the replication of exact wording of each question. Additionally, in line with the recommendations of Kvale (1996) all questions were open, often rephrased in language suited to the participant and designed to engage the learner in sharing and interpreting their own experiences, thoughts, opinions and feelings.

The intervention

The intervention was designed to address the third research question:

3. How do low-skilled adults respond to a classroom environment that emphasises conceptual understanding?

The intervention was used to explore the behaviours of low-skilled learners who held negative beliefs about mathematics, as they participated in lessons that presented mathematics as an interconnected domain learned through inquiry, collaboration, discourse, exploration and personal meaning-making.

Data collection

The methodological requirements and challenges of the intervention were similar to that of the observations and therefore similar data collection methods were used including: the use of multiple audio-recorders to record individual self-talk, quiet interactions between learners, group and whole-class discussions; direct observations and field-notes during and after each lesson: surveys: semi-structured interviews that were completed at the end of the intervention. It was planned to video each class however the learners directly requested the video not be used.

Recorded learner conversations

The process of recording the classroom interactions was the same as the observations. Audio-recorders were distributed throughout the class once the learners were present and seated. They were informed formally that the recorders were on, at which time the session began. A wide range of dialogue was recorded, including conversations related to the mathematical content of the lesson, learners' memories of school, their feelings toward school and their current programme, family life, weekend activities and general casual conversations. The recorders were left on during a 15-minute break with the full knowledge of the learners. During this period, many learners stayed in the class and engaged in discussions with myself or asked for specific help regarding content areas. These interactions are included in the analysis. A sample of these transcripts was presented to the learners following the intervention, and all agreed to their use.

Learner interviews

Seven interviews were conducted with learners following the intervention. This number reflected the large attrition rate in terms of attendance and difficulties arranging meeting times and schedules. The same interview schedule was used as with the learners who took part in the observations, but also included additional questions about their reflections regarding the intervention and any differences they felt had been made. The interviews took approximately one hour in duration, were audio-recorded and transcribed.

Observations

The observations made during the sessions were written as field-notes during and immediately following the class. The notes taken during the class were opportunistic and took place in the moments when the learners were occupied or during breaks. These were few, as opportunities to disengage from the class were rare. Following each session, I immediately completed field-notes regarding pivotal events, learner responses, and thoughts, queries and conjectures regarding learner behaviours, responses and occurrences. These notes provided a level of triangulation with the audio-recordings and the interviews.

Design of intervention

An inductive approach to the design was adopted because of the vast range of variables. These included, but were not limited to: the mathematical skills of the

learners; their attitudes toward mathematics; the content required by their programme; the age and maturity of learners; the participating organisation and site location; the equipment available; and the unpredictable nature of the learners' responses to the content and pedagogical approach. The programme needed to be reflexive to learner needs yet oriented toward establishing a learning environment in which learners were presented with the opportunity to engage in relational mathematics and exposed to practices that promoted positive mathematical beliefs. As such, the design below was a tentative outline designed prior to the programme.

The programme adopted an approach in which learning is viewed as an interpretive, recursive, non-linear process through which learners actively build their understandings by interacting socially with a view to being enculturated into a culture of mathematising. This approach is consistent with the effective practice recommendations from a range of mathematics education research projects (reviewed on page 41). An important aspect of a tutor's numeracy education practice is their flexibility in deploying a grounded pedagogy which includes adapting to the diversity of adult learners and organisational contexts (Coben et al., 2007). The intervention adopted a similar flexible approach enabling me to adapt to various constraints or opportunities as they became apparent.

Participants
Several organisations were approached and asked to participate in the programme. They were informed that the programme was specifically for learners struggling with mathematical content in a foundation-level vocational course. A Private Training Establishment in the North Island was selected due to availability and characteristics of the learners.

The organisation specialised in working with disenfranchised adult learners, most of whom had left school without qualifications. Two mechanical engineering programmes were selected from which to draw participants. The first programme included youth learners, aged 16 to 19 years old, and the other adults aged 20 to 46. The programmes were selected because the engineering programmes had reasonably high mathematics demands which required the completion of two Unit Standards: 'Demonstrate knowledge of trade calculations and units for mechanical engineering'; and 'Demonstrate knowledge of basic mechanics for mechanical engineering trades'. These Level 2 Units required the application of a range of mathematical skills, such as the use of Pythagoras' theorem and trigonometric calculations. Additionally, many of the learners had been identified as 'at risk' of not

meeting the mathematical criteria, and on their current trajectory were likely to fail the programme. The organisation was enthusiastic about the intervention and it was offered as an elective for those wishing to participate.

Twenty learners from a group of 42 volunteered to participate in the intervention (see Table 5). The class was primarily male (n=18), with only two females attending. Most learners in the programme had left school before the age of 16 and many had histories of disruptive behaviours. Most of the learners had completed the Literacy and Numeracy for Adults Assessment Tool (LNAAT) with the average result of Step 3 indicating substantial numeracy needs (see Table 5). Others were yet to complete the assessment, and several had declined. This was most likely due to difficulty with assessment tasks and the onset of negative affective responses (as was confirmed within the intervention). For this reason, I did not compel learners to undertake the LNAAT, so complete initial assessment data was not collected. I assessed the learners using the Number Knowledge assessment (Ministry of Education, 2008). However, when completing the assessment, the recordings revealed that many learners exchanged answers. Consequently, the learners' numeracy scores were likely inflated. One learner, James, scored Step 6 on the LNAAT, indicating reasonable mathematics skills, yet was requested by the organisation to attend. Finally, most learners completed the mathematical belief survey , the results of which indicated all learners had negative beliefs toward mathematics. More information regarding assessments, mathematical skills and beliefs is available in Chapter 7. All names are pseudonyms.

Procedure

I met with the learners who had been identified by the organisation and explained the purpose of the intervention (see Appendix B for information and consent forms), the data collection methods, and the ethical procedures. Following an open forum, during which the learners asked about the content and confidentiality regarding the data audio-recordings, they were given the information and consent forms. I returned one week later to answer further questions and collected the signed consent forms. The first lessons began the following week.

The intervention was run over a ten-week period and consisted of two two-hour sessions per week.

Table 5: Intervention participants' gender, age, ethnicity and LNAAT numeracy results

Name	Gender	Age	Ethnicity	LNAAT	Number knowledge
Denzel	M	16	NZ Māori	3	4
Malcolm	M	32	Pākehā/European	2	2
Nathan	M	25	NZ Māori	NC	3
Rawiri	M	16	NZ Māori	NC	NC
Kerri	F	18	NZ Māori	3	4
Tyrone	M	16	NZ Māori	4	3
Sam	M	17	Not stated	2	NC
James	M	17	Pākehā /European	6	NC
Kevin	M	35	NZ Māori	3	NC
Tania	F	16	NZ Māori	NC	3
Clint	M	16	Pasifika/NZ Māori	NC	4
Jarred	M	18	Pākehā /European	4	4
Fisa	M	17	African	4	2
Efren	M	37	Filipino	NC	2
Mulia	M	41	Cook Island Māori	4	3
Jamie	M	20	NZ Māori	NC	NC
Hamish	M	16	Pākehā /European	2	NC
Matius	M	49	Filipeno	3	NC
Terry	M	46	Pākehā /European	3	NC
Josh	M	24	Pākehā /European	NC	NC

NC = Not complete

Ethical considerations

Ethical considerations were built into the research design and adhered to the regulations and guidelines detailed in the Ethical Conduct in Human Research and Related Activities regulations (University of Waikato, 2012). A comprehensive ethical proposal was submitted and accepted by The Waikato University Ethics committee in October 2012 (see Appendix D).

Three aspects of the research presented potential harm for those involved. The first was the potential harm for the participants if the findings reflected poorly on aspects of their performance. This included not only the learners but also organisations and tutors. Secondly, learners were being asked to share potentially sensitive information regarding their feelings, experiences and thoughts about potentially unpleasant events. Thirdly, the content of the audio recordings had potential to collect harmful data. These areas were mitigated through adherence to clear ethical guidelines and a commitment to reflexivity. As such, ethical considerations were built into the

research design and were an *ongoing* consideration throughout the research process. Guiding principles included: full disclosure; beneficence; ensuring confidentiality; and gaining fully informed consent. Additionally, these were underpinned by a commitment to easy and open communication.

All participants were informed in person and in writing of their right to decline to participate in the research, their right to withdraw any information they had provided until the analysis had commenced, a process for doing so, and a complaints procedure. Participants were also informed about the form in which the findings would be published, how the data would be secured, the duration it would be kept, and their ability to access and correct personal information. All participants were given the opportunity to feedback on the process and this was met with a positive response in every case (see Appendix B).

Data analysis

The codes and themes were developed inductively. The process of familiarising myself with the data took the form of repeatedly listening to the recorded classroom observations and the interviews. This included listening to each session from beginning to end, and, in the case of the observations and intervention, the same event from different audio-recorders, facilitated through NVivo's audio coding system. Initial codes were generated and refined, with new themes and sub-themes added. Once an initial level of coding had occurred I could listen to coded events across the data set. For example, 'disguising a lack of understanding' (DLU) was an early code, and utilising NVivo I could repeatedly listen to each occurrence of this. Items for coding were selected for a range of reasons including frequency, importance, emphasis and with a relationship with the research findings identified in Chapter 2. Units of analysis included phrases, and larger social interactions, defined and critical events. This process was iterative and included constant comparison and re-organisation until saturation occurred.

The process continued into the second stage which was to begin the act of transcribing the recordings. The process of transcribing is arguably a key phase of data analysis (Bird, 2005), and was so particularly in this case. Transcribing the classroom observations was a difficult process due to the use of multiple recording devices, yet it led to increased analysis. The devices recorded utterances from all directions simultaneously. This meant that in many cases peripheral utterances were unclear, on the edge of the volume range, or submerged under ambient noise. Additionally, the number of utterances was substantial, for example, in one 34-minute

episode, 19 distinct conversations were recorded. The NVivo software allowed me to align recordings sequentially and move between them to hear the same utterance from alternate positions in the lessons. This meant that each utterance recorded was heard and verified several times in its full context, including the conversation, the event preceding it, and the current event. It also meant that the analysis was conducted at the micro-word level and at a macro-conversation level. Although time consuming, the analysis that resulted was thorough. Transcriptions were made with a commitment to an orthographic and verbatim account of speech; this was considered practically suited to the analysis (Edwards, 1993) as it facilitated ease of understanding for readers, enabling them to gain insight into the situation.

Once events, episodes and patterns of interaction had been coded, the codes were organised into themes, and, through many iterations of writing, were reorganised, reviewed and refined. This continued until the 'principle of decreasing returns' indicated that the data had yielded accurate and substantive findings.

Validity

An accurate account of the events described in this study has been achieved in two ways. Firstly, I have taken great care to include participants' utterances, interpretations and accounts of events throughout the findings. This is to enable readers to 'step inside' the environments and draw their own conclusions. The method used facilitated this to some degree. The use of recording devices used within classes in conjunction with field-notes and video meant that the data had high fidelity and were authentic. Presentations of the data and transcripts were also provided to participating classes, and. where learners could be located, they were given classroom transcripts and asked to comment. Finally, the learner interviews validated the events of the observation. The learners provided interpretations of the events that occurred during the observations. The interviews purposely included a section that asked the learner to describe their experiences in the class.

To ensure interpretive validity I spent much time immersed in the data and in the environments from which the data were taken. This included visiting eleven training organisations, meeting and talking with groups of tutors and learners (few of which visits provided usable data) and frequent observations of, and discussions with, classes and learners. The decision to be present during the distribution of the survey acted as an opportunity for me to develop the necessary rapport to observe lessons.

Secondly, I have represented the learner's 'voice' as much as possible. Findings are illustrated with transcript samples, supported with narrative commentary, enabling the reader to 'enter the world' of the learner and form their own evaluation of events. Finally, evaluative validity relates to the judgement statements that I, the researcher, have made in relation to learner behaviours. Interpretations of the data are included throughout each of the findings chapters providing an understanding of my approach.

In summary, the research adopted an interpretivist approach designed to explore learners' beliefs and behaviours. The theoretical perspectives of beliefs as relatively stable systems and the triadic reciprocal causation model provide a framework with which to explore and describe individual and group behaviours, while making links to specific beliefs. A thematic analysis approach to the methodology enabled all three of the dominant belief methods to be utilised: surveys, observations in authentic environments, and comprehensive interview data. This reduced the weaknesses inherent in any one approach. In addition, the intervention provided a fourth approach, observing learners with negative beliefs, in an environment closer to that recommended by leading adult numeracy researchers. The data were collected using a range of methods yet drawing heavily on audio recordings that enabled high fidelity to the events, and situations in which they occurred. Finally, the data was analysed thematically, providing findings able to be presented in rich, detailed, and complex ways so that readers are able to understand the researcher's interpretation and form their own. The following four chapters present the findings of the survey, observations, interviews and the intervention.

Chapter 4: Survey: Results and Discussion

"To me mathematics is numbers, numbers and more numbers.
Numbers that come together to make more numbers."
(*Tane, 18-year-old employment skills learner*)

This chapter investigates the first research question: What beliefs do low-skilled adults hold about mathematics? It reports on the results of the belief survey used with adult learners attending foundation-level vocational programmes that embed mathematics into provision. An overview of the purpose, instrument, process, and the participants is reported in Section 3.4. A brief overview of the beliefs measured is included below, followed by the results.

4.1 Piloting process and changes made to beliefs survey

The belief scales had never been used in a New Zealand context or with adult learners who may have literacy difficulties. Therefore, the survey was piloted before use. A focus group was used to test for vocabulary and comprehension and slight modifications were made to contextualise the survey questions where necessary. The key areas of focus for the test were:

- Is the reading level appropriate to maintain comprehension?
- Is the vocabulary interpreted consistently?

I met with 10 adult learners enrolled in a level one and two employment skills programme which catered to learners with low levels of literacy and numeracy. Following the consent process the survey was distributed and participants were instructed to begin working through the survey making notes of which questions were difficult to understand, difficult to read, or that contained words that were difficult to understand. Once completed, the group as a whole was asked to give general feedback. Prompt questions included: 'Tell me what you thought of the survey?', 'Which questions were the hardest to understand?' and 'Which questions stood out as being a bit strange?' I made notes of all feedback and identified questions that were referred to more than once.

Following this the participants were asked to explain their interpretations of questions that they had marked while reading. Finally, the participants were asked to select questions that they thought adults who struggle to read may have difficulties with. This produced similar responses as the initial questions.

The questions that four or more participants indicated as confusing and the reasons were:

Question 10: 'Computational' (vocabulary)

Question 14: 'Predetermine' (vocabulary)

Question 33: 'Emphasise' (vocabulary)

Question 51: 'Truth is unchanging' (comprehension)

Therefore, the survey question containing these terms were modified.

Question 10: 'Computational' became 'calculation'.

Question 14: 'Predetermined' became 'fixed'.

Question 33: 'Math classes should not emphasise word problems' became 'Maths classes should not make word problems so important'

The decision was made to leave question 51 as it was, as the difficulty in understanding may have related to beliefs, not because of literacy challenges. The changes that were made lowered the Flesch-Kincaid reading level of each of the questions.

4.2 Survey item results

Table 6 shows the percentage responses to each of the Indiana Mathematics Belief items (IMBS) and Table seven the results of Schommer's Epistemological Questionnaire (SEQ). There were occasional non-responses to items, these were removed from the total percentages provided. Percentages have been rounded to the nearest whole number.

Table 6: Percentage of participant responses to IMBS survey items

		SD	D	A	SA
Belief 1: I can solve time consuming maths problems					
Q. 3	If I can't solve a maths problem quickly, I quit trying.	24	45	25	7
Q. 7	If I can't do maths problems in a few minutes, I probably can't do it at all.	29	49	16	6
Q. 13	Maths problems that take a long time don't bother me.	11	36	49	4
Q. 19	I feel I can do maths problems if I just hang in there.	2	7	61	30
Q. 26	I find I can do maths problems that take a long time to complete.	4	25	62	9
Q. 31	I'm not very good at solving maths problems that take a while to figure out.	4	30	50	15
Belief 2: There are word problems that cannot be solved with step by step procedures					
Q. 1	Any word problem can be solved if you know the right steps to follow.	2	4	57	37
Q. 8	Memorising steps is not that useful for learning to solve word problems.	20	50	23	7
Q. 14	There are word problems that just can't be solved by following a predetermined sequence of steps.	9	43	44	4
Q. 20	Word problems can be solved without remembering formulas.	9	47	39	5
Q. 27	Most word problems can be solved using the correct step-by-step procedure.	1	8	64	27
Q. 36	Learning to do word problems is mostly a matter of memorising the right steps to follow.	1	15	66	18
Belief 3: Understanding concepts is important					
Q. 2	A person who doesn't understand why an answer to a maths problem is correct hasn't really solved the problem.	5	15	66	15
Q. 9	Time used to investigate why a solution to a math problem works is time well spent.	1	12	60	27
Q. 15	It doesn't really matter if you understand a maths problem if you can get the answer.	10	48	36	6
Q. 21	Getting a right answer in maths is more important than understanding why the answer works.	10	53	27	10
Q. 25	It is not important to understand why a mathematical procedure works as long as it gives a correct answer.	10	58	26	6
Q. 35	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	0	3	45	52

		SD	D	A	SA
Belief 4: Word problems are important in mathematics					
Q. 4	Word problems are not a very important part of mathematics.	21	50	25	4
Q. 10	Calculation skills are of little value if you can't use them to solve word problems.	11	48	34	8
Q. 16	A person who can't solve word problems really can't do maths.	28	61	11	1
Q. 22	Mathematical skills are useless if you can't apply them to real life situations.	12	44	30	15
Q. 29	Learning mathematical skills is more important than learning to solve word problems.	4	47	42	8
Q. 33	Maths classes should not make word problems so important.	7	54	32	8
Belief 5: Effort can increase mathematical ability					
Q. 5	Ability in maths increases when one studies hard.	4	16	51	30
Q. 11	By trying hard one can become smarter in maths.	3	11	46	40
Q. 17	I can get smarter in maths by working hard.	4	7	50	40
Q. 23	Working hard can improve one's ability in maths.	2	8	45	46
Q. 28	I can get smarter in maths if I try hard.	2	8	48	43
Q. 34	Hard work can increase one's ability to do maths.	1	13	50	36
Belief 6: Mathematics will be useful in daily life					
Q. 6	Maths is a worthwhile and necessary subject.	0	3	47	50
Q. 12	Maths will not be important to me in my life's work.	50	42	6	2
Q. 18	Maths is of no relevance to my life.	39	51	8	3
Q. 24	I study mathematics because I know how useful it is.	2	15	57	27
Q. 30	Studying maths is a waste of time.	51	39	7	3
Q. 32	Knowing mathematics will help me earn a living.	1	10	56	33

Table 7: Percentage of participant responses to SEQ survey items

		SD	D	A	SA
	Belief: Learning is quick				
Q. 37	If you are going to be able to understand something, it'll make sense to you the first time you hear it.	7	42	38	13
Q. 42	Smart students understand things quickly, usually the first time.	6	27	48	19
Q. 53	If a person can't understand something in a short amount of time, they should keep on trying.	7	5	58	32
	Belief: Learn the first time				
Q. 46	Going over and over a difficult textbook chapter usually won't help you understand it.	9	41	41	10
Q. 57	Almost all the information you can learn from a textbook you will get during the first reading.	13	58	28	2
Q. 65	If I find the time to re-read a textbook chapter, I get a lot more out of it the second time.	4	12	58	26
	Belief: Success is unrelated to hard work				
Q. 38	Being good at mathematics is 90% ability and 10% hard work.	10	37	43	10
Q. 40	Getting ahead takes a lot of work.	1	13	48	38
Q. 52	The really smart students don't have to work hard to do well in school.	13	58	20	1
	Belief: Ability to learn is innate				
Q. 49	Some people are born smarter than others and you can't do anything to change that.	28	32	27	13
Q. 55	You can learn new things but you can't really change your basic intelligence.	12	40	40	9
Q. 61	An expert is someone who has a special, natural gift or talent in some area.	7	31	46	9
Q. 43	You have a certain amount of intelligence and you really can't do much to change it.	12	53	26	9
	Belief: Knowledge is certain				
Q. 39	The only thing that is certain is certainty itself.	3	19	60	18
Q. 44	If scientists try hard enough, they can find the truth to almost everything.	8	33	43	17
Q. 47	Scientists can ultimately get to the truth.	13	33	46	9
Q. 51	If something in maths is true it never changes.	6	28	56	10
Q. 58	Things that are believed to be 'facts' today may be viewed as just opinions in the future.	3	21	64	13

		SD	D	A	SA
	Belief: Simple knowledge/Avoids integration				
Q. 41	I try my best to understand the connections between the subjects I learn.	3	11	58	28
Q. 45	To me, studying means getting the big ideas from a text, rather than the details.	11	53	31	5
	Belief: Simple knowledge/Avoids ambiguity				
Q. 50	I enjoy thinking about issues that experts are uncertain about.	1	34	55	10
Q. 56	It's a waste of time to work on problems that have no possibility of coming out with a definitive answer.	7	52	33	8
	Belief: Depends on authority				
Q. 59	When you first encounter a difficult concept in a textbook, it's best to work it out on your own.	3	44	47	6
Q. 66	How much a person gets out of school depends on the quality of the teacher.	5	29	36	31
Q. 69	Sometimes you just have to accept answers from a teacher even though you don't understand them.	15	31	48	6
	Belief: Don't criticise authority				
Q. 63	Often, even advice from experts should be questioned.	2	9	71	19
Q. 64	People who challenge authority are over-confident.	7	39	43	11
Q. 67	You can believe most things you read.	15	48	36	1

Individual questions were analysed to ascertain learners' beliefs toward aspects of mathematics. The ratings from each item were aggregated between learners who agreed and disagreed ("Strongly agree" with "agree" and "strongly disagree" with "disagree").

Belief 1: I can solve time consuming problems

Responses to individual statements presented a nuanced view of learners' beliefs about their motivation and behaviour when engaged with time consuming and difficult problems. There were substantial discrepancies in the learners' responses to scale items that may be explained by the items differing emphasis on either self-efficacy or behaviour. Overall, almost two-thirds (65%) of the learners agreed with the statement "I'm not very good at solving maths problems that take a while to figure out". This suggested a general lack of confidence and poor self-efficacy amongst many participants. A further one-third (32%), agreed with the statement "If I can't solve a maths question quickly, I quit trying", indicating the act of disengaging from the task, rather than an evaluation of their ability. There was an apparent contrast however, in

that close to one quarter (22%) agreed with the statement, "If I can't do a maths problem in a few minutes, I probably can't do it at all" suggesting that most of the learners believed that if they did persist with a problem they would be likely to have success.

The discrepancy may be a result of the items slight shift in focus. The three questions can be distinguished somewhat by their transition from the learner's perception of ability, (I am not very good…), to behaviours (…I quit trying), to a final judgement (…I probably can't do it at all). If this is the case, a pattern emerges with most learners (65%) agreeing they are "not very good", fewer agreeing to the behaviour of quitting (32%), and fewer again agreeing that they probably could not solve it even if they tried (22%). The responses overall, suggest that many learners do not feel confident solving a problem if it isn't solved quickly, are not motivated to persist, and subsequently tend to disengage from problems before solving them.

Belief 2: There are word problems that cannot be solved with step by step procedures

The responses to statements that measured the learners' beliefs about the utility of step-by-step procedures indicated that almost all learners held procedural beliefs about mathematics. Ninety-four percent of learners agreed, or strongly agreed, that *any* word problem could be solved if the right steps were known. Moreover, 56% of learners disagreed that "word problems can be solved without remembering formulas" indicating a belief that word problems can *only* be solved with a formula. The responses also indicated an orientation toward memorisation with over four-fifths (84%) of learners agreed that learning to do word problems is mostly a matter of memorisation. These responses indicate a strong belief in the complete utility of step-by-step procedures, and a strong orientation toward memorisation as a key strategy.

This result was consistent with the learners' responses to the open question "What is mathematics?" in which almost all described mathematics in instrumental terms. For example, the view of what mathematics is, was summed up by the following written response:

> *A subject, a concept, made up of formulars* (sic) *to get an answer to almost anything not just numbers, hard!*

Additionally, the results of the SEQ items indicated that the learners had an expectation that mathematical content, such as mathematical procedures, ought to

be transmitted from an expert, rather than taking responsibility for constructing their own understanding. A strong orientation toward teacher as the expert was evident in the "depends on authority" scale in the EBQ (see Appendix F). Approximately two-thirds (67%) of learners agreed that "How much a person gets out of school depends on the quality of the teacher" and over a half (54%) agreeing with "Sometimes you just have to accept answers from a teacher even though you don't understand them".

Belief 3: Understanding concepts is important

Learners had a more positive orientation toward "understanding" than might be expected from their strong procedural orientation, yet the results indicated that learners prioritised performance over understanding. Forty-two percent of learners agreed with the statement "It doesn't really matter if you understand a maths problem if you can get the answer" and one-third (37%) agreed with the statement, "Getting a right answer in maths is more important than understanding why the answer is correct", both of which suggest an orientation away from understanding.

Responses to other statements appeared to contradict this. Most learners (87%) agreed with the statement "Time used to investigate why a solution works is time well spent". Likewise, 81% agreed that "A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem". This discrepancy may be a result of competing priorities between performance and understanding. While understanding is viewed as important, it is perhaps secondary to performance outcomes. The discrepancy may also be a consequence of learners interpreting the term "understanding" as knowing "how" to apply a procedure, rather than knowing "how and why".

Belief 4: Word problems are important in mathematics

The learners' responses showed a strong preference toward equations, rather than word problems. Only 12% of the learners agreed with the statement "A person who can't solve word problems really can't do maths", suggesting that most learners do not see problem solving as the primary goal of mathematics, but rather the ability to compute number problems. This was supported by a surprisingly low response to the statement "Mathematical skills are useless if you can't apply them to real life", which approximately half (45%) agreed with. This suggests that slightly over half (55%) of learners do not believe that proficiency with mathematics requires being able to apply the skills to real world situations. These views were also evident in the learners' open descriptions of mathematics, described frequently as "equations", and rarely as a skill used outside of an educational context (reviewed below). The responses suggested

that over half of learners view mathematics as primarily computational skills and not real world problem-solving skills.

Belief 5: Effort can increase mathematical ability

Almost all the learners indicated positive beliefs about the role of effort, with 90% agreeing with the statement "I can get smarter in maths by trying hard" and four-fifths (80%) agreeing with the statement "Ability in maths increases when one studies hard".

Although not as positive, the responses to the epistemic beliefs scale "Success is unrelated to hard work" were also favourable. Three-fourths (76%) agreed that "Getting ahead takes a lot of work" and close to three-fourths (71%) disagreed that "The really smart students don't have to work hard to do well in school". However, this may represent a belief that working hard is necessary for mathematical success, but is not the only factor. The responses to the SEQ scale "Ability to learn is innate" were less positive than those of the IMBS scores. Approximately half (48%) of the learners agreed with the item "You can learn new things, but you can't really change your basic intelligence", and two-fifths (40%) agreed that "Some people are born smarter than others and you can't do anything to change that".

Belief 6: Mathematics will be useful in daily life

Almost all the learners (97%) either agreed, or strongly agreed, with the statement, "Maths is a worthwhile and necessary subject". However, the individual item with the lowest agreement, although still high at 83%, was "I study mathematics because I know how useful it is". The 14% discrepancy between the item scores suggested that there were learners who believe that mathematics is useful, but did not study it because it was useful, i.e., its utility was not their primary purpose in studying mathematics. The distinction may be found in the reasons learners are studying mathematics, for example, perhaps some feel coerced in some way, given that numeracy instruction is embedded into their programmes as a mandatory component.

There were a small number (11%) of learners who felt mathematics had no relevance to their lives, suggesting little motivation to engage with embedded mathematics. An analysis was done to explore the beliefs of these learners. All held either negative, or very negative, beliefs about their ability to solve difficult problems, about the influence of effort on ability, and the utility of step-by-step procedures. Hence, negative

responses to this statement appeared to relate to overall negative beliefs about mathematics and their own mathematical ability.

4.2 Responses to open-ended questions

The learners' responses to the two questions, "What is mathematics?" and "If a new student started your course and wanted to learn numeracy, what advice would you give them?" were organised according to emergent themes.

There were only 74 completed written responses from the 119 completed surveys, suggesting respondents may have avoided writing responses due to literacy issues. Alternatively, they may have avoided writing responses because the open-ended questions were at the end of a very large survey. The percentages below are taken from the 74 responses. The learners' responses to "What is mathematics?" indicated that many learners held a view of mathematics as a non-contextualised subject consisting of numbers and equations. A thematic analysis identified three categories of response, each reflecting more purpose and meaning than the last. The first category comprised descriptions of mathematics that contained no purpose or context. The second consisted of responses in which mathematics was described as the act of using numbers, yet with no real-world purpose. The third category comprised responses in which mathematics was described as a tool useful for solving problems related to life. The learners' responses are given below as the original comments.

Lack of context/meaning

Close to one-third (35%) of the responses used the term "number/s" with no context or purpose. In these cases, mathematics was described as a subject that lacked any context or meaning.

> What is mathematics?
> *Numbers and equations*
>
> *It's a subject that you learn numbers*
>
> *To me mathematics is numbers, numbers and more numbers. Numbers that come together to make more numbers*

Limited context or purpose

Mathematics was described by almost two-fifths (39%) of the learners as a subject in which the purpose was limited to solving non-contextualised number problems. Hence, mathematics had a purpose, but this purpose had no practical application and was limited to the domain itself. However, these responses suggested a more active and engaged view of mathematics than the previous responses.

> What is mathematics?
>
> *A study of numbers and equations.*
>
> *Know how to calculate with numbers.*
>
> *Putting numbers together in such a way you can get your answer.*

Mathematics described as a tool to solve real problems

One-quarter (26%) of the responses described mathematics with some reference to utility, or being *used* as a problem-solving tool to achieve a purpose, either in life, or at some stage in the future.

> What is mathematics?
>
> *Figures, numbers; that you use through your life.*
>
> *Useful tactics when using numbers in the future.*
>
> *It is a subject that you learn from, and may get you somewhere in life.*

There were only four examples given for the utility of mathematics. These were all in the context of money:

> What is mathematics?
>
> *You need to learn how to count money or calculate things to add up.*
>
> *How to understand income.*
>
> *[You] use it everyday like paying bills and everything.*

Learner advice to new students

Two themes emerged from responses to the question, "If a new student started your course and wanted to learn numeracy what advice would you give them?"; Ask the teacher for help, and work hard.

Asking the teacher for help

A theme that emerged was the directive by learners to seek help from the tutor. These directives included advice to "listen" to the tutor, "ask questions", and when unsure of content to ask the tutor for help:

> *Listening to the teacher.*
>
> *Go to the teacher and ask him/her about it.*
>
> *Just try and tell them what the teacher told us to do.*

These responses suggested a belief in the role of the tutor as the primary source of information, and that learning numeracy is a matter of assimilating the teachers' knowledge.

The imperative to work hard

The imperative "work hard" was used frequently in relation to successfully learning numeracy. This supported the findings of the scale "Effort can increase mathematical ability" that indicated effort was believed to be positively related to mathematical proficiency:

> *Work hard and keep at it.*
>
> *To be committed and work hard.*
>
> *Keep at it and try hard.*

The responses suggested that the learners held a view of numeracy in which mathematical proficiency is not a 'norm' but rather an *exceptional* achievement. This was supported by an analysis of high-frequency words that showed that knowing mathematics was viewed as a result of exceptional effort. For example, when giving advice to other learners, the analysis identified that the word "hard" almost always related to an imperative to "work hard" in order to learn mathematics:

> *Go hard.*
>
> *Just to try hard and give it your best.*
>
> *Keep trying and NEVER give up. Because anything is possible.*

A word frequency analysis using the word "try" as a root word identified similar themes:

Give it a go and try your best.

Try hard and good luck.

Nothing is too hard if you try your best.

Try, try, try. If you still need help I can teach you the skills but not the

answers.

It is worth noting that these responses provide some explanation for the positive beliefs learners indicated toward effort rather than ability. While learners believed effort was a key determinant of mathematical success, the level of effort required may have been perceived as beyond the ability of most learners.

In summary, the responses to the open survey questions indicated that most learners held a perception of mathematics as a procedural, non-contextualised discipline rather than a contextual problem-solving discipline. The majority of descriptions were narrow, emphasising numbers and equations. However, others described mathematics as something useful for their lives. Learning numeracy was largely viewed as a process of obtaining help from an expert, such as the teacher, and working hard. Finally, becoming proficient with numeracy appeared to be perceived as an exceptional achievement, rather than a norm.

4.3 Discussion

The results indicate that low-skilled adults possessed largely positive beliefs regarding the role of effort and the usefulness of mathematics but held very negative beliefs regarding the nature of mathematics and how it is learned.

The results of the "effort can increase mathematical ability" items indicated that almost all learners believed that effort had some positive effect on their mathematics performance. The Epistemic Belief Scale statement "success is unrelated to hard work" also showed a positive orientation toward effort, and further evidence was found in the written responses in which "hard work" was frequently recommended as integral to success. These results are more positive than many other studies that indicate lower-achieving learners tend to attribute poorer performance to innate and fixed abilities (Blackwell et al., 2007; Dweck, 2006;). Given that a belief in the

productive effect of effort, rather than in an inherent ability, has been associated with a range of positive behaviours, such as focusing on learning rather than proving one's ability, seeking challenges, being highly strategic in the face of setbacks, and showing higher levels of motivation (Dweck, 2006; Rattan et al., 2012), these results differ positively from what might have been expected. However, there was a persistent 10% of learners who indicated that they believed that effort had either no or little effect on their mathematical performance. This is a concern considering the "helpless" and aimless associated learning behaviours (Schommer-Aikins et al., 2005; Yates, 2009).

Beliefs about effort have been linked to behaviours that contribute to the time a learner spends working on a problem and the types of strategies they use (Muis, 2004; Schoenfeld, 1989; Schommer-Aikins et al., 2005). Given this, it may have been expected that responses to "I can solve time consuming problems" would be similar. However, the discrepancies of percentage responses to the item statements raise questions. The results appear to indicate that approximately one-third (Q3) of learners do not persist with time consuming tasks, but a portion of these believe that if they did persist, they would be likely to solve the problem. This is interesting because one interpretation of a learner's decreased time on task is that their negative beliefs about effort leads them to rapidly diagnose themselves as "unable" to solve a problem, resulting in a complete reduction of effort (Blackwell et al., 2007; Schoenfeld, 1989). However, the results suggest that these learners do not diagnose themselves as unable to solve a time consuming problem, but disengage for other reasons.

The learners' written responses to the open questions offer a possible explanation. The responses suggest that learners believe that mathematical success is possible, but is an "exceptional achievement" and, as such, atypical. Mathematical success was posited as the result of extraordinary effort and somewhat out of reach for the average learner. This provides some explanation for why mathematical success, although viewed as an outcome of effort, is something that belongs to "other people". The learners may see mathematical success as requiring more effort than is within *their* personal means. While effort may be perceived as an essential factor for *others*' success, for those in this study, whether effort is a factor or not may be moot, as success has not been achieved. Given that the learners in this study had very low mathematical skills, they may have already disengaged and possess poor mathematical-identities (see Chapter 6). The present study suggests while low-skilled

learners believe effort is a factor, they may also believe that the effort required is beyond their personal means.

Usefulness of mathematics

Results on the usefulness of mathematics items in previous studies are typically high, and thus while the number of learners in this study agreeing that mathematics is useful is positive, the small number of learners expressing negative beliefs is of concern because the belief is linked to a reduction in participation of further mathematics study, and low motivation to engage (Brown et al., 2008; Kloosterman & Stage, 1992; Quilter & Harper, 1988). Previous research has suggested that the high rates of agreement that mathematics is useful may reflect an automatic response rather than a belief shaped by experience (Leitze, 1996). If this is the case, further investigation into *how* mathematics is useful may reveal less positive results, perhaps limited to money, as in the learners' responses to the open questions. This is discussed below in relation to the learners' lack of reference to any mathematical application in the open questions.

A further point needs to be made regarding the assumption that belief in the usefulness of mathematics is an indicator of a learner's motivation to engage in mathematical provision. Swain (2005b) posited motivation not as a product, but as a process, synonymous with the word "reasons". Thus, a belief in the usefulness of mathematics in daily life is a reasonable reason for adults to learn. Yet Swain's and his colleagues' findings regarding the reasons learners engage in numeracy were generally not related to daily life application but rather to factors such as enjoyment, to prove to oneself that one could learn mathematics, or to help one's children. This is explored further in Chapter 6.

Beliefs about the discipline of mathematics

The most negative results were on the items that measured learners' beliefs about the utility of step-by-step procedures, indicating that participants believed that rules, methods and procedures are the primary content of mathematics. For example, over four-fifths of learners agreed that a memorised procedure could be used to solve *any* word problem. A similar number agreed that learning to do word problems is mostly a matter of memorising. The learners in this study showed stronger procedural beliefs than participants of other studies, such engineering, college, and high school students (Berkaliev & Kloosterman, 2009; Kloosterman & Stage, 1992; Mason, 2003). For example, Brown et al. (1988) found only slightly over half of students

between the 7th and 11th grades agreed that there is always a rule to follow and that learning mathematics meant mostly memorising.

Further support for the learners' strong orientation toward instrumental beliefs was given in their responses to items regarding the distinction between computational skills and word problems. Surprisingly, over half of the learners (59%) agreed that mathematical skills did *not* have to be applicable to real-world contexts and 89% agreed that the being unable to solve word problems does not mean you cannot do mathematics. In other words, learners appeared to endorse the view that being able to solve equations without being able to solve situational or contextual mathematics problems reflected mathematical skill. This seems counter-intuitive, given that all the participants were attending contextualised mathematical provision that ought to have made vocational applications explicit.

The belief that rules, methods and procedures are completely effective for solving problems is found to relate to ineffective approaches to learning and poorer performance (Englebrecht, et al., 2009; Goldin et al., 2009; Jäder, et al., 2017; Mason, 2003; Muis, 2004). For example, learners holding such beliefs have been found to follow the methods of the teacher to the letter (Schoenfeld 1988, 1989); rely on memorisation rather than sense making (Briley et al., 2009; Garofalo, 1989); neglect metacognitive analysis of problems and instead simply apply arithmetic operations until an answer seems right (Lester & Garofalo, 1987; Schoenfeld, 1983, 1985); and use the teacher, not reasoning, to determine between correct or incorrect answers (Frank, 1988). Procedural beliefs lead to a reliance on memorisation, imitative reasoning, or mimicking the teacher, and success is dependent on the learner's ability to recall a solution method (Sidenvall et al., 2015; Sumpter, 2013). Learners with these beliefs have been found to unsuccessfully apply algorithmic solutions to non-routine problems (Jäder, et al., 2017).

Finally, and perhaps most concerning for the learners in this study, is that learners limited to procedures/algorithms were found not to engage in sufficient 'effortful struggle' to develop conceptual knowledge (Jonsson et al., 2016). Given that the learners in this study came from backgrounds of mathematical difficulty, such limitations may ultimately set them up for further failure. Moreover, reproducing taught procedures is insufficient to prepare adults for the changing, and growing, mathematical demands predicted to be a characteristic of the modern workforce (EGFSN, 2015). Such learning behaviours do not lend themselves to engaging in the

deep processing necessary to be mathematically successful (Echazarra et al., 2016; Hadar, 2011).

Open question: What is mathematics?
The responses to the open questions were consistent with the procedural beliefs evident in survey items. One-third of those who responded to the open question "What is mathematics?" described mathematics simply *as* numbers and equations, and made no reference to purpose or utility. Other than the blanket terms "numbers" and "equations", there were only four references to specific mathematical content, three of which were arithmetic operations ("adding, subtracting, dividing and timesing") and one which was "ratios and fractions". A further third of the responses posited mathematics as an activity, but limited the descriptions to solving number problems such as equations. Although this second-group was more positive, both described mathematics as an activity done for its own sake, with no mention of meaningful application, or to mathematics as a way of thinking. Such responses are consistent with the 'narrow conceptions' of mathematics described by Petcoz et al. (2007). Only 19 responses suggested any real-world application, and these were all in the context of money. It might be expected that adult learners would have had more sophisticated views regarding the application of mathematics than younger learners. Yet the responses lacked the references to utility frequently given by nine to 11-year olds, and six to 13-year olds who, unlike the adults in this study, frequently referred to mathematics as something useful both now and in the future (Young-Loveridge et al., 2006; Young-Loveridge & Mills, 2010). Hence, the low-skilled adults appeared to prize utility less than did normally-progressing school children.

Secondly, the learners' responses about what mathematics is did not reflect the nature of the vocational mathematics provision they were receiving. A three-year longitudinal study of students across grades one to six found that students' descriptions of the usefulness of mathematics moved from abstract to more personal examples of usefulness over the course of three years (Kloosterman et al., 1996). Yet, there were no references to vocational applications in the learners' responses in this part of the study. This is a concern considering the learners are immersed in vocational training that, according to research, is dense with mathematical demands (McCloskey, 2007).

No learner indicated that they enjoyed mathematics, and a number included negative evaluations. For example, one learner responded to "What is mathematics?" with "It's boring". This contrasts with the Young-Loveridge and Mills (2010) study in which

children described mathematics as fun and enjoyable in response to the same questions as those used in this study. It also contrasts with two studies conducted with adults learning mathematics who expressed positive views toward the mathematics they were learning, including some adult learners with poor histories of learning mathematics in school (Coben et al., 2007; Swain, 2005a). The adults in both studies reported that they enjoyed the experience, and found it challenging, fun, and engaging. The difference between those cohorts and this present one is that the learners in those studies volunteered to participate in the numeracy provision and could cite a range of reasons for attending, suggesting they sought and valued the experience. Some responses in this current study were more like 16-year-old school students attending mathematics classes only because they had to (Brown et al., 2008). The learners' attitudes toward their current numeracy study is addressed in the following two chapters.

Beliefs about how mathematics is learned

The learners' positive orientations toward effort and the usefulness of mathematics suggest a positive level of motivation toward learning mathematics, although this requires more data to verify. However, into what behaviours this motivation might be directed is important to discover, given that beliefs about how mathematics is learned influence goals, learning strategies, monitoring and the evaluation feedback, all of which have a substantial impact of performance (Echazarra et al., 2016; Hadar, 2011; Schoenfeld, 2011). The responses to the open question, "If a new student started your course and wanted to learn numeracy what advice would you give them?" revealed a passive approach to learning, such as listening, and a strong reliance on the teacher.

The advice given to learners wanting to learn numeracy were frequently admonitions to simply seek help from, and listen to, the tutor, suggesting that the tutors are viewed as the valid and primary source of knowledge. A strong orientation toward expert authority was also evident in the "depends on authority" responses. The view that the teacher is the sole source and explainer of information has been linked to passive classroom behaviours (Muis, 2004; Schoenfeld, 1985; Young-Loveridge et al., 2006). The concern is that even learners who are motivated to work hard may put their energies into behaviours that do not develop conceptual understanding, resulting in further difficulties. Furthermore, passive learner roles have negative impact, not only on skills, but also the development of identity, autonomy and agency (Amit & Fried, 2005; Brousseau & Warfield, 1999; Heyd-Metzuyanim, 2013; Sutherland & Singh, 2004).

In conclusion, these findings present a potentially worrying picture of learner beliefs. They suggest that the learners' perspective of learning mathematics in an adult context is less about developing mathematical skills and agency to further their career or enhance their lives in meaningful ways but rather to acquire generic information in the same manner they adopted during school, putting them at substantial risk of repeating their poor school performance. It may be that the learners do not draw on their greatest assets: personal agency, life experience, and meaningful contexts for mathematics. In the following two chapters, interview and observation data are used to explore these questions in more detail, and to ascertain how beliefs relate to the learners' engagement patterns within actual lessons.

Chapter 5. Observations: Results and Discussion

"Getting the right answer is more important… Like, you're a big failure if you don't get the right answer."
(*Tina, 25-year-old retail learner*)

This chapter investigates the second research question: How do low-skilled adults engage with mathematics within vocational lessons? It reports on observations of adult learners participating in mathematical lessons in vocational contexts. The findings are organised broadly into four sections. These sections are in no way exclusive of each other, but present the findings in a manner that attempts to describe the full mosaic of classroom activity. The first is an overview of the influence the tutor appeared to have on learner behaviours. This provides a context for the second section, which reports on the learners' affective responses to the lessons. The third section reports on learners' behaviours that appear to reflect their beliefs about what mathematics is, how mathematical problems should be solved, and the goals of mathematical tasks and activities. The final section reports on the learners' engagement with mathematical concepts within the lessons and how this may be related to efforts to protect status and avoid shame. The chapter concludes with a discussion.

5.1 Tutor influence on engagement patterns

The phenomena under investigation are the behaviours learners engage in while participating in mathematical provision. These behaviours are theorised to be underpinned by beliefs about the nature of mathematics, how mathematics is learned, personal mathematical ability, and how performance influences social status. Therefore, the influence of the tutor needed to be described to differentiate between the learners' voluntary behaviours and those undertaken in response to the tutor's instruction. Accordingly, before presenting the findings regarding the learners' behaviours, a brief account is given of the context, lesson structure, design of numeracy problems and standards for success, and the general classroom discourse.

Content delivery

Mathematical content was typically delivered in a transmissional mode with a focus on a single procedural solution strategy applied to a small number of compatible

problems. In most cases, the tutors adopted an authoritarian approach and began by explaining how the content related to the vocation. Learners were provided opportunities to contribute, and to work together in groups, yet aspects of the tutors' approaches conformed to a transmission approach. For example, the following transcript is taken from an introduction to a lesson delivered to a class of eight learners within a sport and fitness class:

> Tutor: *This is the definition we got from the unit standard. So, numeracy is the bridge between mathematics and daily life. It includes knowledge and skills needed to apply mathematics to everyday family and financial matters, learning, work and community tasks, social and leisure activities. So what I want you to take away, if anything, is that first sentence, the bridge between mathematics and daily life. Can everyone repeat that?*
> Class: *The bridge between mathematics and daily life.*
> Tutor: *Again.*
> Class: *The bridge between mathematics and daily life.*
> Tutor: *One more time.*
> Class: *The bridge between mathematics and daily life.*
> Tutor: *Okay, so the key words there is it's a bridge between mathematics. So mathematics is what you learned at school, all the algorithms and Pythagoras' theorem and all that sort of stuff.*

Following such introductions, the tutors tended to demonstrate a method using the whiteboard while the learners mostly remained quiet and observant. Following the demonstration, the tutors handed out a set of problems and learners worked together in pairs or small groups to solve them. In only two cases were learners asked to discuss how they might solve a problem before being shown a procedural method. For example:

> Agriculture tutor*: So here's our petrol tank, this big bit here [points to container on bench]. And they… left us some oil but there's no instructions on how much, do we tip the whole lot in or just put a teaspoon in, or whatever? Have I got enough oil for that five litres of petrol?*

These occasions prompted discussions, yet these were brief and often dominated by only a few learners. The tutors took control of the discourse within a few minutes and returned to explanations, followed by problems to be worked through in groups. Once the groups finished the problems, the tutors again took the lead role, asking groups

for answers, demonstrating how the problems could be solved and marking the groups' work. Subsequently, the structure of most lessons conformed to the following sequence of phases:

1. Tutor linking the mathematics being taught to a contextualised task
2. Tutor demonstrating several worked examples of problems and solution strategies
3. Groups, or pairs, of learners working through a range of numeracy problems set by the tutor
4. Tutor reviewing and marking the problems as a whole class, during which some learners asked questions and shared ideas.

The effect of the reviewing and marking phases was twofold. The tutors often transitioned to the marking phase when most, but not all, groups were finished. This may have contributed to the rapid pace at which the groups worked, in what appeared a rush to complete all problems before time ran out. During the marking phase, the groups' answers and the number of problems they had solved were made public to the wider class. This appeared to raise the pressure on learners to complete tasks quickly. Second, the answers were verified by the tutor and learners typically only celebrated once the tutor endorsed the answer. This had the effect of reducing personal agency, while further establishing the tutor as expert.

Numeracy problems
The numeracy tasks/problems that were used in the numeracy sessions conformed to traditional mathematical curriculum topics such as finding percentages of whole numbers, adding fractions, mixing ratios, or adding and subtracting whole numbers. Each of these skills was situated in a task context that resembled those in the target vocation but were not authentic, in the sense that the problems were unlikely to occur or be solved in the way suggested in an authentic context. The operational aspects of the problems were almost always made explicit to the learners. For example, worksheets often used a bolded font to indicate the relevant information and equation structure (see Figure 4).

> Amy has been saving up for a camera to start her photography business.
>
> She has budgeted that she can save **15%** a week of her weekly pay check.
>
> She currently earns **$450 a week.**
>
> How much can she save a week?

Figure 4: Numeracy problem taken from an employment skills programme

In cases where the equation structure was not made explicit through text features the problems still emphasised specific mathematical operations. For example, in the following example, the hairdressing tutor described the task she had drawn on the whiteboard (Figure 5) during which the operation (addition of fractions) was made explicit:

> *I've given you an equation though, so what I've actually done is worked out, is added a couple of fractions together so all I want you to do first is do this column here* [Column 2] *and that is creating the fraction of this equation added together. Okay so just concentrating on that first column please.*

Fraction	Fraction of tube	Grams
$\frac{1}{2} + \frac{1}{2}$	$\frac{1}{1}$	60g
$\frac{1}{2} + \frac{1}{4}$		
$\frac{1}{8} + \frac{3}{8}$		

Figure 5: Numeracy problem in hairdressing programme

Step-by-step solution strategies were demonstrated in almost every lesson on a sample problem while the learners sat quietly and took notes. For example: The sport and fitness tutor played a video clip explaining how to find the percentage of a runner's race time:

> *Now if we have 10% of a number which is ten parts out of a hundred it's really easy because it's the same as dividing by ten. Um now for example, I've got three numbers here so I'm gonna use these for my example. If I want to work out ten percent of fifteen it's a matter of just moving the decimal place, the decimal place is usually here and I'd move it back left so it would come here. So yeah, ten percent of fifteen would be one-point-five.*

There was little emphasis on developing conceptual understanding in any of the lessons. However, the hairdressing tutor differed from the other tutors by introducing a paper fraction strip that learners continued to halve until the strip was in eighths. It was designed to resemble a tube of colour and promote a conceptual understanding of fractions.

> Tutor: *Okay so all the same sort of rules are going to apply but it's really important that you know that you can divide your tube of colour in half, in quarters, and into eighths. Alrighty?* [The tutor hands out strips of paper the same size as a tube of colour] *This is your tube of colour. The same sort of thing what we did yesterday. We're going to fold this, fold it in half, and again.*

At the end of this activity a procedure was given for the addition of fractions, but the key difference was the focus on why and how it worked.

Standards for success - Timeframes and public accountability
Tutors set the standards for success for learners both explicitly and implicitly. This was done by setting time-frames for the completion of numeracy tasks and suggesting that learners' successes or failures might be made public to the class following the activity. This appeared to be done to help give learners a sense of how long they had to complete the task. For example:

> Employment skills tutor: *If you can take one* [worksheet] *and put it between your pair that would be great and as soon as you get it you can start working on it.* **I'll give you about ten or so minutes to do that, and then we'll come back in a bit and have a test.**
> [Emphasis mine]

The messages inherent in the words "*ten minutes or so*" and "*have a test*" may have contributed to the learners' orientation toward speed and accuracy, which, as can be seen in the third section, was considerable. However, further analysis indicated that learners were already oriented toward speed and accuracy, suggesting a reciprocal relationship between the tutor's behaviour and the learners' behaviour. The learners' behaviours are reviewed below.

Tutors' use of questions

In each of the sessions observed, the tutor initiated more questions than the learners. These were coded between questions intended to verify understanding (e.g., "Do you understand?") and questions intended to elicit or engage learners in mathematical thinking (e.g., "What will a 10% wage increase be on one hundred dollars?"), both types were typically delivered as closed questions.

The verification questions were routinely asked during lessons, but their high frequency suggested they were a habitual aspect of tutor discourse and designed to maintain learner attention, rather than to genuinely check learner understanding. For example, no learners responded to the questions below, and the tutors continued as though the learners understood:

> Hairdressing tutor: *Questions on that? Just what you're looking at the moment. Is that clicking into place, is that sort of making sense?*

> Employment skills tutor: *See how people kind of got that answer as well, does that make sense? Yip? Okay cool.*

There were no occasions of learners responding to verification questions in the negative or asking the tutor to repeat or clarify concepts in response to the tutors' questions. The tutors appeared to interpret all, or no, responses to these questions as an affirmation that learners were understanding the content, despite the observations showing that large numbers did not understand. In the following illustrative example, the tutor demonstrated a procedural method to the class, and the class, not understanding the process, remained completely silent, despite a verification question:

> Employment skills tutor: *So if you want to find the percentage off something you take the amount, you divide it by a hundred and you come up with the decimal and you times the decimal into the amount. Does that make sense? Fairly straightforward for people?*
> [No response from the class at all. The class appear to be confused.]
> Tutor: *Yeah? I know it can be a little confusing.*
> Lisa: *Oh gosh* [whispered quietly to herself].

> Tutor: *So can we try this one together and then give you a sheet?*
>
> [No response again from class. Tutor is waiting for response]
>
> [Five second silence]
>
> Dan: *Yes, yes!* [Shouting to the tutor].
>
> Tutor: *So, we'll do this one first. We have to figure out what 30%*
> *off forty dollars is.*

An analysis of the learner-to-learner conversations and learner self-talk identified that most of the learners did not understand the strategy, yet none responded to the tutor's question, "*Does that make sense?*" I interpreted Dan's eventual responses ("*Yes, yes*") as an effort to relieve the growing tension. The response appeared to be interpreted as an affirmation of understanding by the tutor, who, rather than return to the content and address the potential confusion, continued with the problem.

Learners tended to wait until the opportunity for a private conversation with the tutor presented itself before asking about their misunderstandings or errors. However, these occasions were rare, and never in the context of the tutor asking whether they understood during a whole-class interaction. As will be shown in the following section, many learners feigned understanding.

Tutors also asked questions intended to engage learners in mathematical thinking. These questions were almost always delivered as closed questions. For example:

> Employment skills tutor: *So, let's say someone works for 30 hours at KFC or Carls Junior, and they earn 14* [dollars] *an hour… at the end of the week they get their pay slip, how much before tax will they have earned?*

The almost exclusive use of closed questions by the tutor contributed to a whole class discourse pattern that conformed to the traditional "Initiate, response, evaluation" classroom discourse pattern. Learners rarely engaged in verbal reasoning, and instead responded to tutor questions with unelaborated answers. The discourse patterns within groups of learners differed. These are described in sections 5.3 and 5.4.

5.2 Affective factors

Affective responses were common within the lessons and clustered around key events. These included: manifestations of apprehension when learners were informed of their pending participation in a mathematics lesson or activity; frustration

or despondency when encountering challenging tasks; and responding with elation or dismay to success or failure.

Feelings of apprehension

Many learners responded apprehensively to the proposition that they would be taking part in a mathematics lesson or activity. They expressed their apprehension verbally and in several cases other learners reciprocated, resulting in shared expressions. For example, the statements below were uttered in a highly negative and dismayed tone when learners entered a classroom, noticed a fraction written on the whiteboard, and became aware of an impending lesson:

1. Sonja: *Oh, are we converting like into percentages or something?* [Expressed in a defeated tone].
2. Trudy: *Ohhh* [loud], *fractions ooohh* [Also expressed in a defeated tone].
3. Abbie: *Fractions, I hate fractions.* [Angry tone]
4. Sonja: *I hate fractions.* [Angry tone]
5. Trudy: *I don't even know how to do fractions.*
6. Hahona: *Ohh, fractions, ohhh rat s**** [Mimicking and teasing the others]
7. Trudy: *I really didn't like these at school.*
8. Abbie: *I hate fractions.*

Note also that no prior lessons on fractions had taken place within the programme, indicating the responses may be related to the learners' school experiences. The negative utterances appeared to stimulate similar misgivings in other learners. The interview data suggest that learners' responses to the lesson were linked to the expectation of re-experiencing unpleasant episodes of the past, underpinned by an anticipation of failure. The purpose of the utterances appeared to be to advertise, share, and perhaps dissipate, feelings of trepidation with others in the class. I also interpreted the broadcasting of their historical difficulties with fractions as a strategy to lower the tutors' and other learners' expectations for performance.

The tutors in all classes responded to negative comments from learners by quieting their talk and initiating a discussion about the purpose of the lesson. For example:

> *Hairdressing tutor: When you're ready.* [Pauses while class talk reduces].
> *Fractions. What do you think we are going to need that for?*

This approach had the effect of orienting learners toward the tutor but had little effect on reducing learner apprehension. In no case did a tutor address the learners' emotions, historical experiences, or attitudes toward mathematics. Yet the interview data showed that during these initial stages of the lessons some learners were making risk evaluations about whether to engage with content (Section 6.2). The learners noted that they made immediate judgements about their ability to be successful with the lesson content:

> Kelly: *Yeah and I make the decision pretty fast as soon as I see what's going on, I'm like 'oh yip' or 'Oh nah'.* [Deciding whether to participate]
> Damon: *When you say what's going on there, you mean when you see what's going up on the whiteboard?*
> Kelly: *Yeah on the whiteboard, problems or something, and I'm just like 'oh no'.*

The tendency to make immediate judgements about whether they would be successful appeared to be informed by their histories with mathematics, which in many cases were negative. The audio-recordings also revealed a subtle, and private, layer of negative utterances communicated either as self-talk or to an immediate colleague. For example, the following comments were typical of learners' private self-talk while working on tasks:

> Flora: *I hate maths* [Whispered quietly to partner in response to a challenging problem]
> Harley*: I'm s**t at numbers.*
> Christine*: F*** this…Nasty little guys* [In reference to a percentage problem]
>
> Lora*: Oh my gosh why am I so bored?* [In response to doing a problem]

Other behaviours also indicated that learners felt a level of apprehension throughout the lessons, much of which appeared to relate to the desire to avoid shame. There were many occasions of learners disguising their lack of understanding, in what appeared to be attempts to avoid this becoming public. For example, in the following episode, Sonja was observed to mislead the tutor into believing she understood a concept despite her deep concern that she did not:

> Tutor: *How many quarters go into that?*
> Several learners: *Two.*
> Tutor: *Two? Can everyone see that?*

> Hahona: *Yep.*
>
> Tutor: *Sonja?*
>
> Sonja: ***Yip*** [Sonja nods, yet does not appear to understand, despite her enthusiastic reply; my emphasis].
>
> Tutor: *You with me girl?* [Sonja nods that she does understand] *And then how many eighths is that?*
>
> Hahona and Trudy*: Four.* [The tutor's attention leaves Sonja and shifts to other class members]

Despite Sonja's verbal and physical response that she understood the concept, in a later interview she confirmed that she did not understand and continued to experience negative emotions throughout the lesson. She described her behaviour as an attempt to avoid embarrassment (see Sonja in Chapter 6).

Learners were also found to protect, or 'save' others from making errors in public. In these cases, when a learner who did not understand was called on to provide an answer to a problem, surrounding learners would whisper the answer to them, allowing them to answer and avoid further attention. These behaviours suggested a shared understanding that making a mistake or not understanding content was to be avoided as it was a "shameful" experience. The interview data supported this, finding that learners were sensitive to appearing "dumb":

> Interviewer: *Why does that matter?* [Whether you answer a question correctly or not]
>
> Niki: *Um, I suppose 'cause at school you don't want to look stupid. Yeah, you don't want to actually look dumb.*

Giving answers to others was perceived as beneficial in an environment in which avoiding failure was an ongoing concern.

Ongoing affective tensions

Various situations continued to evoke emotional responses from learners throughout the lessons. The first situation was the need to persist with challenging or time-consuming tasks. In most cases when learners worked on difficult tasks a more proficient learner solved it for them. However, when they did work on difficult tasks themselves they frequently disengaged while expressing frustration or discouragement. To illustrate, Lisa began to read a problem aloud from a worksheet,

and became increasingly aware of and attentive to its complexity. Finally, appearing overwhelmed by emotion she disengaged:

> Lisa: *Fifteen percent of four-hundred and fifty. 'As she was looking at the camera she noticed a sign saying twenty-five percent off until the end of the year, now the camera is reduced, how many weeks will it take her to save for it now?'* **Oh my gosh, I hate maths…** [dejected tone; emphasis mine]

Lisa disengaged at this point, and did not re-engage during the lesson; rather, she began subtly using her phone in what appeared to be texting, while occasionally feigning interest in the lesson. These occurrences suggested many learners had a low threshold for complexity and persisting with challenging problems.

The presence, and behaviours, of some high-skilled learners increased feelings of inadequacy amongst some low-skilled learners, further reducing their participation. Several learners were observed advertising their high proficiency with mathematics to the other learners and the tutor with what appeared to be a status-raising objective. These learners' behaviours included: answering questions aloud before others and elaborating their answers more than necessary; explaining content to others without being asked; asking the tutor challenging questions; using and broadcasting alternative methods of solving problems. Although these behaviours suggest positive engagement, they appeared designed to advertise superiority rather than be genuine efforts to increase their understanding. For example, when interviewed and asked about his classroom goals, one such learner responded:

> Hahona: *That's easy. To win, to be the best.*

The behaviours of these 'mathematical experts' appeared to raise the perceived expectations for mathematical performance to unachievable levels. This appeared to increase some lower-skilled learners' attention to their performance which further increased apprehension. These learners subsequently reduced public participation to avoid shame. There is a hint of this in Trudy's response to Hahona after he asks the tutor an elaborated question, with the intent of displaying his proficiency:

> Hahona: *Do we have to turn the whole into fractions as well?* [Does one and a quarter need to be expressed as the improper fraction ten-eighths]
> [Class goes quiet and tutor rolls her eyes indicating frustration]

Trudy: *Don't be difficult. You're so brainy.* [Shakes head at Hahona]

Trudy's "*Don't be difficult*" is suggestive of her wish for Hahona to not raise the standards any higher by adding unnecessary complexity. The content was already challenging for the remainder of the class. Secondly, I interpreted her utterance "*You're so brainy*" as a way of acknowledging his skill, but also as a subtle message to him to stop. The comment appeared self-deprecating, in that it also expressed the corollary, "**we** *are not so brainy*" as a subtle message as well.

During the interviews, repeated comments were made regarding so-called mathematical experts. The following comments were made regarding to the impact of "experts":

> Abbie*: ... and it makes people feel like "oh okay, you don't know what you're doing, so be quiet".*

> Damon: *Does his vocalness hinder you or help you?*
> Trudy: *Not helping, because he's just like making us feel like we're dumb.*

These types of responses were common, and indicated that many utterances made by 'experts' made other learners feel that their own contributions were unintelligent by comparison, leading to a reluctance to share their thinking for fear of embarrassment.

Responses to success or failure

Many learners responded emotionally to discovering their answers were incorrect as tutors worked through problems on the board. The responses by learners were expressed privately either as self-talk or to close peers:

> Tutor [speaking to class]: *Nine hours, okay so he starts work at seven a.m. and finishes work at five p.m. and but we're talking about hours that he has actually worked, eh. So, seven, eight, nine, ten, eleven, twelve …*
> Christine**: *Oh f**** [dismayed tone as she realises her answer is wrong].

Learners responded negatively when their errors were made public to the class. In the following example, Sonja, who until now had only tentatively engaged with the

content and feigned understanding, made her first contribution to the class. Believing she had discovered that fractions can be added directly across, she excitedly volunteered her understanding in response to a tutor question:

Sonja [has her hand up, waving enthusiastically]: *Can we go just go one plus one, and then two plus four and comes out with a number, oh two sixes* [pauses and reads tutor's face]. *Na wrong, eh?* [Said in a defeated tone. She appears to lose all confidence and looks down].

Hahona: *Yeah* [said sarcastically]

Sonja: *Sorry 'bout it* [expressed in a defeated tone. Sonja pushes her chair back and looks down]

Tutor: *That's a bit too past me* [said softly to console Sonja]

It is difficult to convey the tone and context of Sonja's comments in a transcript, but her tone reflected deep disappointment at believing she had been successful only to discover she was incorrect. Sonja confirmed this was a painful episode for her in her interview, discussed in Chapter 6.

A further example of negative emotional responses is illustrated below. The hairdressing class had been asked to colour in seven-eighths of a paper strip. The tutor noticed Trudy's negative facial expression at her response and probed:

Tutor: *So what have you got left Trud?* [Should have one-eighth left]

Trudy: *None.* [Trudy has coloured in the entire strip]

Tutor: *None?* [Sounds surprised. The class becomes quiet and all attention turned to Trudy].   *Where'd you go wrong, Honey?*

Trudy: *Um, I thought one-eighth over all of it.*

Tutor: *Why? Why did you think that?* [Accusatory tone, although perhaps unintentionally so]

Trudy: *'Cause there were eight squares.*

Hahona [Whispering to Trudy]:   *So that will be eight-eighths and that'll make a whole.*

Trudy: *Ohh, I don't really get fractions* [Frustrated, sits back away from her book and looks down].

The public exposure of an error often led to temporary disengagement from the lesson. However, in many cases the learners did re-engage in further activities later

in the lesson. For example, both Sonja and Trudy reengaged later in the lesson and experienced success with the fractions concept.

Positive emotion

Not all emotion was negative or resulted in disengagement. While many learners became frustrated when tasks could not be resolved or solved quickly, some continued to engage with tasks and demonstrated considerable tenacity while working on problems. The lack of a satisfactory solution or resolution resulted in frustration, yet did not result in total disengagement, but rather a determination to complete the task. For example, in this episode James and his partner had been attempting to solve a task for four minutes and had tried a variety of approaches, so far unsuccessfully:

> James: *Five times one, he has only paid off four thousand five hundred. Oh raaaaahh! [angry yelling followed by 20 seconds of silence] That's wrong! I can see that's wrong! [sounds angry].*

The frustration expressed by James was not accompanied by negative emotional expressions that indicated apprehension or despondency. Rather, the frustration appeared to be the result of an unmet expectation of success. It did not cause him to disengage, but rather to persist, which he did until solving the problem. As a 'persister' his attention was focussed on the task. This contrasted with Sonja, whose focus appeared to be on evaluations of her performance (Sonja's interview confirmed this).

Many learners invested time and effort into completing mathematical tasks or problems and experienced positive emotions when successful. These learners acted as though their success was a personal victory for them, and expressed their feelings as self-talk or private utterances to other learners. For example, in the following episode, Dan had spent several minutes working on a challenging problem by himself. Although difficult to appreciate in the transcript, his success at solving the problem correctly produced a subtle emotional response that would have been missed by a casual observer, and his general attitude may have been perceived, incorrectly, as indifferent:

> 1. Tutor to class: *After at the end of the week they get their pay slip, how much before tax will they have earned?*

2. Dan [Shouts to the tutor]: *Four-twenty.*

3. Tutor: *Four-twenty. Does that look right to people? Four twenty. So, we went fourteen times thirty. Does that look right to people? Four-twenty, well done.*

4. Dan [Quietly to himself]: *Chur chur.*

Dan's utterance "*Chur chur*" (4) was a private affirmation and celebration of his success captured only due to the proximity of an audio-recorder. When listened to from other recorders, such as a colleague or tutor might hear, Dan's answer appears to lack emotion [2]. No one else seemed to have heard Dan's self-affirmation, yet it suggested that Dan cared about his ability to solve the problem and felt a sense of accomplishment at solving it.

A further example of positive emotions was the response of Troy and Curtis to solving a task. Troy, supported by Curtis, had been attempting to solve a mathematical problem that had proved difficult for him. Troy's response at solving it indicated his investment in the process:

Curtis: *What's going on?*

Troy: *Nine-six-five divided by a hundred.* ***Oh yeah boom!*** [Loud and very excited].

Curtis: ***Boom!*** [Sharing in the excitement; my emphasis] *Straight up?*

Troy and Curtis' celebratory "boom" indicated a positive emotional response after having invested time and effort to solve a task. Interestingly, Curtis celebrated despite having contributed little to finding a solution (discussed further below). Positive emotional responses were also observed when learners completed worksheets or group tasks within set time-frames:

Carol: *How many of the newer larger boxes… thirty-six.*

Lisa: *Yay!*

Marea: *I think that's it. Did you boys get it? [Asking another group if they got it correct]*

Tutor: *So you guys are done?*

Lisa: ***Yay!***

Tutor: *You guys done as well? Awesome.*

Lisa: ***Yip, awesome. Yay.***

Attitude changes

There was evidence of learners' improved attitudes toward mathematics because of lessons. The following private conversation provided critical insight into the thoughts and experiences of learners observed in these programmes. It took place in response to the tutor telling two learners, David and Tane, that they were out of time. David revealed that he disliked mathematics and had struggled with it in the past, yet was now starting to like it:

> 1. Tane: *Hard out, there's nothing* [annoyed at the tutor]. *Good thing I like maths…*
> 2. David: *Now that I'm getting maths, I'm starting to like it.*
> 3. Tane: *Yeah yeah.*

David's utterances suggested several things. In the utterance *"Now that I'm getting maths…"* the temporal word "*now"* indicated that there was a point in David's past when he was aware he was not doing as well as he wanted to. Second, "*…I'm getting maths…"* suggested David was aware of his increased success with mathematics. Third, *"I'm starting to like it…"* suggested he attributed his improved attitude toward mathematics to his increased ability to "get" mathematics.

The following comments supported this further:

> 4. David: *I can feel it. F*** primary man [Primary school], I sucked at maths, I used to always ditch maths.*
> 5. Tane: *Oh yeah did you? I used to ditch everything [laughs].*
> 6. David: *I didn't like school.*
> 7. Tane: *Na, me nether.*

David's previous negative attitudes related directly to his school experiences. His utterance in line 4, "*I can feel it*", was expressed in a tone that suggests "*feel*" is a term that is related to "understand". Perhaps, "I can understand it" is an appropriate translation. It was certainly positive and suggested a substantial shift from his previous attitude.

These types of statements may not represent permanent attitudinal changes; in fact, participants' rapid shift from negative utterances to positive are indicative of unstable rapidly changing emotional shifts, yet these, and other examples, suggest that

positive changes did occur, at least in the short term. Unfortunately, these changes may be temporary as they were based on *perceptions* of success which in the next section will be shown to be of arguable quality.

5.3 Orientation toward mathematics

The category "orientations toward mathematics" emerged in response to learner behaviours that reflected beliefs about what mathematics is, how mathematical problems are solved, and the goals of mathematical tasks and activities. An analysis identified seven features and behaviours that occurred while learners engaged in group problem-solving. These were:

1. the use of procedural solutions to solve problems;
2. a lack of conceptually-oriented discussions;
3. the uncritical acceptance of proposed procedural solutions;
4. learners teaching only procedures to other learners;
5. non-elaborated answers to problems;
6. prioritisation of quick accurate answers before understanding;
7. the division of roles for higher and lower-skilled learners.

The first behaviour was the almost exclusive use of procedural step-by-step methods to solve all problems. The learners within groups quickly scanned word problems to identify the problem situation and quantities, and immediately began a calculation. Secondly, despite most problems being situated in a work context, there were almost no discussions of any factors other than the procedural solution method. As problems were scanned for the problem situation and quantities, learners either ignored contextual features, or the pace at which calculations were initiated was too rapid to allow wider discussions. Consequently, there were no learner discussions regarding the context.

For example, Lisa, Carol and Marea were given a worksheet containing several problems related to the retail sector. They began to work on the following problem: "Amy saves $67.50 per week for a $945 camera. The camera is reduced by 25%. How many weeks will it take Amy to save the money?" The group by-passed any discussion regarding the situation, and immediately began to calculate an answer:

> 1. Lisa [reading the problem aloud to the group]: *'As she was looking at the camera, she noticed a sign saying all cameras 25% off 'till the end of the*

year. Now the camera is reduced. How many weeks will it take for her to save for it now?'

2. Carol: *So then it's 25…*

3. Lisa: *So if it's 25% off that…*

4. Carol [interrupting]: *…nine-four-five divided by a hundred equals that and times zero point two five?*

5. Lisa: *Yip?* [Expressed as though a question]

6. Carol: *Equals three times … two, two-hundred and forty-six* [incorrect].

7. Lisa: *For the camera?*

8. Carol: *Yeah.*

Line 3 appeared to be the beginning of Lisa attempting to conceptualise the problem by rearticulating it in her own words. However, Carol immediately introduced a procedural solution strategy to the group, cutting off any discussion of the situation and moving the group immediately to the calculation stage (line 4). Despite it being incorrect, the other members of the group uncritically accepted the procedural solution. No evaluative discussion took place and Marea did not comment on the problem at all. The pattern of one learner moving directly into calculating and subsequently skipping any discussion of the context or problem situation was common within group work.

The third behaviour was the uncritical acceptance by group members of the first solution strategy proposed by any individual within the group. Often, the most skilled learner asserted the procedural solution, took a lead role, and set the pace for the group. The monopolisation by the 'dominant solver' is evident in the following example. Troy and Curtis were working through a series of problems. Troy, adopted the lead role, read the problem aloud, and then immediately told Curtis what to input into the calculator.

1. Troy [reading aloud]: *'Tony works at a factory that imports and exports exotic plants. They receive a huge order to send out on Monday 6000 plants. Each box they send can hold 40 plants each. How many boxes will they need?'*

2. Curtis: *Six thousand* [Spoken slowly as he enters '6000' into calculator].

3. Troy: *Six thousand divided 40 equals, yep divided by 40* [checking Curtis has entered the correct operation]*, equals hundred fifty boxes.*

4. Curtis: *How many boxes?*

5. Troy: *Right there bro* [points to answer on calculator], *them total boxes but he only has 72, so minus 72 and that's what we need.*
6. Curtis [using calculator]: *78.*

Learners were frequently observed to cede their agency to others they perceived as more capable, adopting supporting roles in the process, such as using the calculator or recording answers on worksheets. In most cases, this occurred at a rapid pace caused by a 'dominant solver' answering problems procedurally and quickly moving from problem to problem while the others recorded the previous answers. This meant that groups moved at the rate of the fastest learner, severely limiting the others' engagement in tasks.

The fourth behaviour that indicated a procedural/calculation orientation was the approach used by learners during the frequent occasion that they taught other learners. In these private exchanges, the interactions almost always conformed to descriptions of procedures, typically delivered in a transmissional approach. The "learner-teacher" approach dominated the exchanges and typically took the learner through a series of steps to answer the problem.

For example, Holly, a hairdressing learner, had been struggling to understand how fractions could be used to determine the required amount of colouring product. The learners were using a paper fractions strip that represented a 60ml tube of colour and had divided it into eighths. The tutor asked the class to use the paper strip to find three-eighths of sixty. Holly had remained attentive, despite not participating in any discourse except to quietly ask the person beside her, Hahona, for the answers to various questions. While the tutor was pre-occupied with another conversation, Hahona attempted to teach Holly.

Hahona: *Three eighths is half of. So you go seven and a half times three, so you go seven times three is twenty-one, one five equals so twenty-two.*
Holly: *Can you do that again because that's just confusing.*
Hahona: *'Cause three-eighths. So you go so its three-eighths, you know a quarter is fifteen, and then you plus on another seven-point-five 'cause you know one eighth is seven-point-five.* [Pauses to see if Holly understands. She shakes her head slightly]. *So, seven-point-five is a quarter, and that's your answer 'cause you're adding three of them up so you go a quarter plus another eight. Twenty-two-point-five. You know what I mean?*
Holly: *I'll just go …*

> Hahona: *So two-eighths is the same as a one-fourth* [interrupted by the tutor - ends].

The approach adopted by Hahona was to tell Holly the steps that would result in a correct answer ("So you go…"). The assumption appeared to be that Holly would understand the process as it was described. She did not make sense of the fraction content, yet managed to feign understanding throughout the lesson, and avoid revealing this to the rest of the class. The discourse patterns used by learners to teach peers conformed in *every* case to a transmissional approach of typically procedural steps.

A fifth behaviour indicating a calculation orientation was revealed in the learners' patterns of interaction with the tutor and themselves. Almost all the learners' responses to tutors' mathematical questions were correct, suggesting that only those learners who knew the answer responded. Additionally, answers were frequently expressed as strings of digits, as though they did not represent entities in the real world, which obscured the relationship between the problem and the meaning of the answer. For example, the responses from two separate groups who had procedurally calculated 45% of $10500 were expressed as digits:

> Tutor: *So our answer was, Bernie?* [Points to Bernie]
> Bernie: *Four, seven, two, five.*
> Vernon: *Four, seven, two, five.* [Also responding from another group]
> Tutor: *Four, seven, two, five.*

Answers of this nature suggested that despite the tutor's attempt to link mathematical tasks to meaningful contexts, learners ignored meaning in favour of fast, accurate answers.

Very rarely did answers include explanations, justifications or caveats even when the tutor explicitly invited these responses. The episodes below were typical of all classroom interactions between tutors and learners:

> Employment skills tutor: *Who would like to go through this question for us?*
> Jenny: *Nine hours* [Shouting the answer out quickly]
> Nick: *Nine hours* [Also shouting quickly]

Even in cases when the question was a request for a method or explanation, the learners often failed to recognise this and continued to provide a non-elaborated numerical answer.

> Tutor: *Cool, does that make sense, see how we go there, in dollars how much does Henry have left to pay off?* **How do we do that?** [Emphasis mine]
>
> Marea: *Five-thousand, two-hundred and seventy-five* [Shouting the answer quickly].

Additionally, once the answer was made known to the class, they immediately moved to the following problem and never returned to it.

The final behaviour relates to the learners prioritising completing all the problems as quickly and as accurately as possible rather than developing understanding. This was evidenced by the way groups typically progressed from one problem to the next without discussing, evaluating, proving or reviewing their answers. This constrained their engagement in mathematical thinking, reducing the role of most members to support positions.

The emphasis on how many questions groups managed to answer correctly is evident in the following example taken from the employment skills programme. Following the whole-class marking session, each group of learners counted how many tasks they had correctly solved. For example, James and Marea counted the number of problems they had solved correctly:

> James: *One, two, three, four, five, six, seven, eight.*
> Marea: *There's eight questions.*
> James: *One, two… Oh there is seven questions. How many did we get? Seven? Six.* [Marea indicates six only]. *Better not cheat, it's numeracy.*

James was careful to distinguish between whether they had solved six or seven correctly. His tone indicated that he might have been offering to inflate their score to seven had Marea agreed. His comment, "*Better not cheat, it's numeracy*" suggested that it matters 'in numeracy' that one make the distinction. The reason why, in this case, is made clear by the tutor's next comment, in which he implicitly reinforced correct answers as the performance criterion.

Tutor [to whole class]: *Cool, okay who got over three right?* [Lots of hands raised] *Cool. Who got over four right or four and over right? Yip. Who got five or over right? Six or over right? And who got all seven right?* [Only a few hands left up. These learners are looking proud] *Oh well done. Give all of yourselves a hand.*

This activity facilitated the public display of each group's score, and valued those scoring higher. The learners who got all seven problems correct were identified (hands up), and congratulated by the tutor. There was no mechanism by which understanding was affirmed or rewarded. Moreover, learners frequently responded negatively to incorrectly solving a problem, even if gaining understanding from the marking and review process. In the following statement, Christine realised one of her answers was incorrect, but was unable to correct it before the tutor answered it publicly:

Christine: *F**k, I think I know how to do that last one.*
[Tutor states the answer]
Christine: *So that's thirty-one hours, that's thirty-one hours, oh f**k I had that wrong* [sounding angry].

Despite having realised her error, the fact that Christine did not have the answer written on the sheet before the marking session was upsetting to her. It is worth noting that the sheets were not handed back to the tutor but rather were solely the possession of the learner. The only person who would have known was her.

Role-taking within groups
A key feature of group problem solving was the diverse roles learners adopted resulting in an unequal engagement with mathematical content and thinking. Most learners adopted support roles such as using the calculator or recording answers on a worksheet. In contrast, the procedurally proficient learners took responsibility for solving the problems, worked quickly through the tasks, and either dictated the procedures and numbers to be entered into the calculator, or answers to be written down. An illustrative example is evident in the approach adopted by Christine and Jaz as they were working through a worksheet together. Both appeared to tacitly agree that Jaz would adopt a support role while Christine took responsibility for solving the tasks. The problem as written on worksheet stated:

Tane is an apprentice builder. Each day he starts work at 7 a.m. and finishes at 5 p.m. He has half an hour smoko break in the morning and half an hour for lunch.

1. How many hours does Tane actually work for (not including breaks)?

2. Tane earns $15 an hour, and his employer pays him for his breaks. How much does Tane earn a day (before tax)?

3. Unfortunately Tane falls off a roof while clearing the gutter. He is off work Monday, Tuesday and Wednesday. How much will he be paid TOTAL this week (before tax)?

Christine: *… seven, eight, nine, ten, so he works ten hours.*

Jaz: *Er, good* [Laughs, indicating she is unsure].

Christine: *Ten hours* [Indicating to Jaz to write it down].

Jaz: *So he works for ten hours?* [Jaz writes this down on worksheet]

Christine: *Yeah, how many but not including breaks. Ah yeah, yeah ten, eerrrr.*

Jaz: *Yeah?*

Christine: *How much does Tane earn an hour? So fifteen times…* [pause]. *One hundred and fifty. Yes, one-fifty.* [Indicates to Jaz to write down one-hundred and fifty]

Jaz: *So will Tane..?* [Jaz appears unsure what to write, seeks to clarify]

Christine: *Unfortunately on Monday…* [Christine continues reading question]

Jaz: *He works fifteen dollars an hour?* [Jaz appears unsure what to write down]

Christine: *One-fifty* [repeats answer to Jaz]. *Three hundred d.m.c.* [The answer to the last question].

Jaz: *Perfection. So that times …* [Jaz doesn't get to finish as Christine begins reading the next question aloud].

The pattern of Christine taking the role of 'solver' and Jaz taking the role of 'supporter' continued throughout the activity*:*

Christine: *Destiny is a hairdresser. Twenty percent pay rise. How do you do, ah what's that? Twenty, that's zero-point-two so she currently, what's that? So, two dollars, oh not, that's currently for 18.*

Jaz: *Eleven? Oh, eighteen plus* [Apparently not understanding, but attempting to follow Christine's working]

Christine: *Yeah, eighteen plus, twenty-one dollars sixty. Yeah… So, three-sixty. Can you just write that three-sixty* [Tells Jaz to write it down]. *I hope that's right* [laughs].

Jaz: *You're good at math, eh.*

Episodes of this nature revealed the tendency of some learners within groups to assume sole responsibility for solving tasks, and others supporting roles. This distribution of task between 'supporters' and 'solvers' proved to be an efficient method for completing tasks quickly and accurately. However, it inhibited the participation of 'supporters' and eliminated almost all their engagement in mathematical thinking. In addition to removing most learners from the activity of problem-solving, it also constrained their ability to reflect on problems once they had been solved. This occurred because the 'solver' set a pace too fast to enable slower learners to make sense of content.

The following example illustrates how a procedurally proficient learner, in pursuit of completing the tasks quickly, dominated the problem-solving process and constrained another learner's engagement. Pete and Tom were provided with a worksheet and had been explicitly asked to work *together* to solve several problems concerning an apprentice builder in which hours of work and pay were to be calculated:

1. Tutor: *See how you go.*

2. Pete [reads problem]: *Ah seven a.m. Ten hours, nine hours, nine hours?* [This is not directed at Tom but is self-talk]

3. Tom: *Shall we say…*

4. Pete: *Eight, nine, ten, eleven, twelve, one, two, three, four, five, so yeah that's ten hours, and then take half an hour that will be nine hours and a half they got and now nine hours.*

5. Tom: *Is that nine hours, is that that?* [Points to question one on the worksheet]

6. Pete: *So how many hours a day is Tane actually working a day not including breaks? Nine hours.* [Indicates to Tom to write it down].

In this example, Pete's immediate engagement in calculation (2) constrained an opportunity to discuss the problem situation and potential solution strategies. Tom's comments were interrupted repeatedly, limiting his opportunity to engage in the calculation because Pete had already calculated and provided an answer. Pete's

utterances in lines 2 and 4 are simply his own verbalisation rather than an attempt to include Tom in the solution strategy. Tom appeared to attempt in line 5 to connect Pete's answer to the worksheet, yet this effectively reduced his role to one of pursuing Pete to ascertain which of his answers aligned with which questions. Pete responded by telling Tom the answer, which Tom accepted without further consideration. The conversation continued with the following question:

> Tane earns $15 an hour, and his employer pays him for his breaks. How much does Tane earn (before tax) in a day?
>
> 7. Tom: *Nine hours [writes it down]. So that's fifteen hour…*
> 8. Pete: *That's fifteen bucks an hour. So that's fifteen times nah. Oh, the employer pays him fifteen bucks an hour so that's fifteen times ten before tax. That'd be one hundred and fifty before tax. Three hundred, there's three days that he's off sick.*
> 9. Tom: *So six, seven-fifty?* [Tom is working out the weekly earnings unaware that the problem includes sick days]
> 10. Pete: *So how much will he now be paid? Well he's only got two days.*
> 11. Tom: *Oh yeah sure. Four…*
> 12. Pete: *So three hundred.*
> [Tom writes down three hundred, but does not appear sure how the answer was obtained]

Again, Pete dominated the process by immediately beginning to calculate, eliminating any discussion regarding the context, solution strategy, or collaboration with Tom. Tom eventually yielding to Pete by accepting his statement that the answer was three hundred dollars without understanding why. Tom, who appeared capable of solving the tasks, was unable to participate in the task because of the pace set by Pete. At no point during the task did Tom or Pete present a supporting argument for any solution.

Two patterns are described above. In the first Christine took the responsibility for solving all problems while Jaz passively accepted a support role. In the second, Pete monopolised the 'solver' role and Tom was pressured into a support role. The implications were that proficient learners like Pete and Christine were able to answer all problems quickly and easily, yet possibly without sufficient challenge to develop their mathematical skills, while lower-skilled learners were unable to engage with content to a degree necessary to develop their mathematical skills.

Finally, the following example illustrates how this somewhat dysfunctional pattern facilitated a form of pseudo-success, answering problems, yet prevented learners engaging in the thinking required to develop mathematical skills. Curtis and Troy were working together at a table situated at the back of the classroom, and had completed a series of problems. Troy, the 'solver' quickly answered several problems by himself, until Curtis took a support role using the calculator and writing the answers on the sheet. Curtis missed the first few questions, and returned to the first problem to learn the procedure and obtain the answer. The problem was: "*Amy can save 15% of her weekly $450 pay check. How much will she save each week?*". Curtis' strategy was to immediately ask Troy for the procedure to solve the problem. Troy's response is illustrative of memorising a procedure without an expectation of meaning:

1. Curtis to Troy: *Did you do this one?* [Points to problem on worksheet]
2. Troy: *Four-fifty, um four-fifty divided by fifteen times a hundred. Oh nah nah it was four-fifty divided by a hundred.*
3. Curtis: *Yeah* [Curtis sounds confused yet begins entering numbers into a calculator]
4. Troy: *Yeah times…*
5. Curtis: *Times four…* [begins entering numbers]
6. Troy: *Nah times fifteen.*
7. Curtis: [Clears calculator] *Oh. I better write that down so…*

Curtis's implicit request (line 1) was met with a procedural solution strategy (line 2) that Troy supplied with no hesitation, suggesting this was a familiar pattern. Despite Curtis' positive response (3), he was in fact still attempting to enter the initial string of digits into the calculator when Troy realised his mistake and changed the formula ("*Oh nah nah it was four-fifty divided by a hundred*"). Curtis's statement in line 5 reveals his lack of connection with the solution to the problem. He appears to have begun to say, "*times four-fifty*" despite having previously entered 450 divided by 100. Finally, having lost the formula, he attempted to write it down to accurately recall it. The solution strategy had been reduced to a series of digits that, when entered in the correct order, produce an answer. Curtis wrote the formula in his book as directed by Troy (450 / 100 x 15 = 67.50).

Curtis's lack of understanding of the connection between solution and answer was further revealed in the review and plenary marking phase of the class. The tutor

asked learners to volunteer their answers to each of the questions. These were written on the board and marked. Curtis voluntarily responded to the tutor's request and read his strategy aloud while the class listened attentively. Curtis read his method aloud comfortably (450 / 100 x 15 = 67.50) until the tutor asked a question that required additional understanding of the process (line 6 below).

> 1. Curtis [to the whole class and tutor]: *Ah, four-fifty divided by a hundred, time, times fifteen* [Reading directly from his notes].
> 2. Tutor: *Okay, so four-fifty, anyone else got a calculator want to do this equation. So four-fifty.*
> 3. Curtis: *Divided by a hundred…*
> 4. Tutor: *Divided by a hundred.*
> 5. Curtis: *Times…*
> 6. Tutor: *Which equals what percent, ah sorry what decimal?*
> 7. Curtis [pauses for an awkward 2 seconds]: *Fifteen, oh…* [nervous and unsure]

Curtis' silence contributed to an increased focus on his performance by the other learners. Classroom talk decreased, the tutor stopped writing on the board, and the class became attentive toward him. However, before Curtis' lack of understanding was revealed, in what appeared would be an embarrassing situation, another learner, Carol, a member of another group interjected.

> 8: Carol: *Four-point-five and then we've…*

While Carol began to explain, Troy took the moment to support Curtis by quietly whispering the answer to 450 divided by 100.

> 9. Troy [Whispers quietly to Curtis]: *Four-point-five.*
> 10. Tutor [responding to Carol]: *Oh, four-point five yip, and then we've…*
> 11. Curtis [calling out to the tutor]: *Timesed it by fifteen.*
> 12. Tutor: *Timesed it by fifteen. And that gives us…*
> 13. Curtis: *Sixty-seven fifty.*
> 14. Tutor: *Sixty-seven fifty. Alright!*

This episode requires some unpacking as it holds insights into class pressures and subsequent behaviours. Curtis did not know the answer to the tutor's question in line 6 due to a lack of understanding of the procedure. Moreover, he appeared nervous in

line 7 as the class's attention was on him during this moment. However, before his lack of knowledge was made evident to the class, Carol [8] spoke from another group and answered the tutor's question. Curtis was then told the answer privately by Troy and able to complete the formula publicly (line 11 and 13). This episode highlights several important findings that speak to the classroom context.

First, Curtis appeared to believe that the procedure and answer were what the tutor and learners wanted. Second, despite not possessing an understanding of how the procedure worked, he was able to answer the problem and therefore meet the demands of the lesson. Third, the tutor and other learners perceived Curtis to be engaging successfully. In fact, the class celebrated the production of the answer ("*Alright!*"). Finally, Troy appeared to believe that telling Curtis the answer was appropriate, perhaps to save Curtis from embarrassment (discussed in a later section) or because it was consistent with his transmissional procedural beliefs.

5.4 Learner engagement with mathematics

The focus of this section is on individual learners' engagement with mathematical concepts, and how this might be impacted by attempts to preserve their status and avoid shame. The findings continue to expand on the learners' behaviours discussed in the previous section.

The engagement patterns of learners were categorised into three levels. The first, "non-engagement", applied to learners who did not engage in any mathematical activity at all, either in whole-class or group activities. The second, "surface engagement", categorised learners who resisted engaging deeply with content, preferring routine or unchallenging thinking. The third, "deep engagement", described learners who engaged deeply with mathematical tasks and thinking. The learners' behaviours and potential motivations are also discussed.

Non-engagement

The observations revealed that some learners avoided engaging with all activities related to mathematics during lessons. These learners tended to position themselves away from the tutor, sit alone, make little or no eye contact with others, and often spent time appearing immersed in a task such as doodling or drawing. Because no transcript is available due to lack of discourse by these learners, the following example is taken from my field-notes while observing a learner in the agriculture class. The class was learning about petrol and oil mixes and the tutor had introduced the concept of ratios. The tutor was attempting to engage the learners in an open

discussion about the need for the correct ratio of petrol to oil in the context of a chainsaw seizing.

> [In-lesson notes] *Darrell does not appear interested. His hood is up, eyes focussed on his drawing, and sitting adjacent to the whiteboard. Has not looked at the whiteboard or the tutor once yet. When asked to talk about ratios, he didn't move. Has yet to talk to another student. The tutor has not approached him directly… Darrell has not participated in the class at all. He shows little expression and barely moves.*

> [Later in lesson]: *Darrell has not participated in the class at all. He did laugh when another student joked about leaving early but otherwise has spent his time drawing. He shows little expression and barely moves.*

> [Post-lesson notes] *Talked to Darrell after the class about how the course was going, how he found the session, and if he would be interested in taking part in an interview. He said yes to being interviewed* [was later removed from course and unable to be located]*, had little to say about the numeracy in the course, but did inform me that he left school at 12 and worked on a farm. He never learned to read and doesn't like maths… Him and two other learners reiterate that they do not like mathematics and would rather be on the quad bikes (four-wheeled motorbikes).*

Darrell's behaviours suggested a routine designed to create distance between himself and the learners and tutor, to avoid engagement with the mathematical content. Given Darrell's particularly low skill level, I interpreted his behaviour as a strategy designed to prevent a situation that might lead to a shameful experience. The tutor did not attempt to engage Darrell, but rather appeared to collude in what resembled a co-constructed "didactic contract" (Brousseau 1997, p.141) designed to avoid a mutually uncomfortable situation. Despite being present throughout a mathematics lesson with a focus on an important workplace skill, Darrell did not engage at any point in the lesson.

Other learners used more subtle methods to avoid engagement, such as relying on others and feigning engagement. In the following example, Flora was asked to work

with Teresa to solve several problems. Flora did not engage with her group or any mathematics yet feigned having done so to the tutor. Preceding the interaction below, both Teresa and Flora were exposed to the same instructions, yet their engagement patterns were substantially different in that Flora ignored the worksheet while Teresa attempted to collaborate with her.

Teresa: *Do you want to work with me on this?*

Flora: [Sighs in an uninterested way] *Are we up?*

Teresa: *You can, you have to answer the questions of the people.* [Hands Flora the sheet for her to read] *Murray, tell her what it was. Murray …*

[Two minutes pass, during which there is no dialogue. Teresa assumes that Flora is solving problem one]

Tutor: *So have you figured out the first answer yet?* [Spoken directly to Flora and Teresa]

Flora: *Hhmm* [In a tone that suggests she is working on it – the tutor moves on]

Teresa [1 minute of silence later]: *Have you done the first answer?* [Stares at Flora and appears annoyed]. *I've got it there. Okay, I've got the first answer, do you want to figure out the second answer?* [sounding annoyed]

Flora: *You do it.*

Teresa: *No? Can you write it down?* [meaning, can you at least write the answer down]

Despite not engaging with any of the problems the impression Flora gave to the tutor and class during the marking session was that she had worked with her group successfully to solve the problems. She did this by feigning interest in the answers, attending to the front of the class (at other times Flora faced away from the front of the class), and looking at the answers on the worksheet in an interested way when the tutor asked for groups to provide answers.

Finally, there were occasions where learners relied completely on their peers, 'solvers' who willingly completed the work for them. This tacit agreement was evident in the Sport and Fitness class in which Becky and Elisa sat quietly together while Elisa completed all the tasks. As with Flora, Becky projected a look of having completed the tasks by watching the tutor and appearing interested, yet did not engaged in any mathematical thinking.

Ironically, the class finished with the tutor describing to the learners the criteria for a one-month self-management project which was to replace the mathematics lessons. The task for the learners was to source and study different strategies to arithmetic and percentage problems using the internet or available resources. This was to be completed independently from the course, in their own time. The ability of the learners to self-regulate and manage this process was assumed. Given the avoidance strategies observed, the success of such a strategy seemed highly unlikely.

Surface level engagement

Unlike the learners in the previous section, many of the learners did engage with mathematical content, yet moderated their engagement by avoiding situations in which their thinking might be made public, in what I interpret as attempts to protect their self-image.

The following critical event gives insight into the nature and impact of learners moderating, and even resisting engagement, and of attempts by tutors to compel learners to engage. The agriculture tutor attempted to engage his class in group discussions regarding what a ratio is. Following an introduction of the content and workplace context, the tutor wrote 25:1 on the board and asked the learners to discuss what it meant:

> Tutor: *What does the ratio part of what I'm saying here mean?* […] *And just have a discussion for five minutes on what a ratio sort of means.*
> Harley: *F***, I don't know what it means*. [Pushes his chair away from the desk, indicating non-participation]

Harley's response set the tone for other members of his group, each of whom also made no attempt to discuss the topic. The tutor, aware of this response, approached and spoke directly to Harley and the group in what appeared to be an effort to encourage them to engage. In response, Harley articulated his current understanding of the ratio, yet did so in an emphatic tone suggesting certainty, and an unwillingness to consider an alternative view:

> Tutor: *Maybe how would you work that ratio out, if you worked out what a ratio means?*
> Harley: *Twenty-five mls to litre* [aggressive tone; meaning 25ml of oil to 1 litre of petrol].

> Kelly: *Go ask a man at the petrol station.*

Harley's statement was indicative of an incorrect non-commensurate understanding of the ratio, in that, for him, 25:1 meant 25 millilitres of oil to 1 litre of petrol, rather than 25 parts of petrol for every one part of oil. This did not change throughout the session, despite the tutor and another learner from another group providing the correct explanation. The tutor did not correct Harley's assertion, but instead introduced the prospect of public exposure to, and judgment by, others:

> Tutor: *'Cause in a minute you're gonna have to explain that. Explain to the class what your little group came up with, eh?* [Tutor moves to another group]
> Harley: *That's what we came up with.* [Aggressive tone]
> Tane: *That's our petrol station.*
> Harley: *Twenty-five mls to a litre. If it's wrong, then God bless my soul.*

Harley's awareness that he was going to have to explain the ratio to the class publicly in a few minutes appeared to contribute to what I interpreted as his strategies to mitigate damage in an embarrassing situation. In the meantime, Harley and his group sat in silence. During this time, they listened to a discussion occurring between other learners, two of whom revealed that 25:1 was not as Harley had described:

> Andrea [loudly to whole class]: *Oh my God! Twenty-five to one is actually twenty-five mls of gas [petrol] with one ml of oil! And then you times that by fifty to make it one litre!*

The threat of having to publicly reveal his lack of skill appeared to influence Harley to orient his efforts toward mitigating any damage from this event. He did so by using self-denigrating language to lower others' expectations for his performance.

> Harley: *F*** I'm s*** as at numbers. It's twenty-five mls to one litre. That's what I just said. Everyone's doing two-point-fives, doubling everythink [sic].*

And again, several minutes later:

> Harley: *I'm s**t with numbers! But not when it comes to money, mate.*

These comments suggested that Harley's strategy was two-fold. The first aim was to lower others' expectations of his impending public explanation of the 25:1 ratio, to

minimise potential criticism. The second was to reassert his status, by emphasising that he is not deficient in his understanding of what really matters: money. Over the remainder of the session, Harley repeated similar phrases throughout the session: "*F**k I don't know. Money's what I'm good at*". "*I'm not good with numbers. [But] when it comes to my money*".

When the moment came for Harley to explain his thinking, the class became quiet. Instead of asserting his previous understanding, he offered a superficial response that he may have perceived as less risky than expressing his non-commensurate view.

> Tutor: *So did you come up with? An explanation of what a ratio actually is, or means, or anything like that?*
> Harley: *Numbers put together.*
> Tutor: *Numbers put together? Yip.*
> Tom: *A maths genius.* [Whispered sarcastically to Harley]

The tutor did not ask for an elaboration of his superficial answer. From this point in the lesson, Harley's behaviour shifted toward bringing the class to an end, while continuing to avoid potentially embarrassing experiences. He noted that 25:1 was better interpreted as time for lunch ("*Twenty-five to one. Lunch time*"). His final statement followed extensive explanations by the tutor, during which Harley made few comments. Finally, the tutor looked directly at Harley and asked if the lesson had been successful:

> Tutor: *So is everyone happy with that?* [Looking at Harley]
> Harley: *Yip, thumbs up. Thumbs up all round.*
> Tutor: *Thumbs up all round for that one?* [The class nods as though they understand]

'"Thumbs up" was a signal that meant the lesson was successful. The lesson ended, and Harley and his group members left with what appeared to be the same understanding of the ratio of 25:1 as when they began. During a practical section of the class Harley was unable to calculate independently the oil required for one litre of petrol. The episode above was one of many that indicated many learners engaged only at a superficial level with content and resisted engaging to the degree that their existing understandings were developed. However, there were some encouraging counter-examples that demonstrated positive engagement.

Positive engagement with mathematics

There were episodes of learners engaging positively, and deeply, with mathematical content. However, these were the exception to the types of interactions shown above. Positive engagement included learners exploring ideas, expressing understanding, and engaging in positive self-talk.

The following episode is an example of positive engagement and provides a counter-example to the patterns described previously. Despite the learners' tendency to use authority rather than argument to support ideas, in the beginning of the Agriculture lesson, two learners, John and Andrea, engaged deeply in a mathematical discussion. Both had worked to make sense of a 25:1 petrol to oil fuel mix. At the time both held an incorrect non-commensurate understanding of the ratio in which 25:1 was interpreted as 25ml of oil to 1 litre of petrol, rather than the correct commensurate understanding that 25:1 represents 25 parts of petrol to one part of oil. Their ongoing conjectures and counter arguments eventually led to a correct understanding:

> Tutor: *What does the ratio part of what I'm saying here mean?* [Points to 25:1 on the board] *What's that mean? So I'll get you guys to pair up. You might even want to get a bit paper out and put some notes down on what you think that sort of word ratio sort of means?*

> 1. John: *Parts, part to it.*
> 2. Andrea: *Two parts. And that one there is like twenty-five mls to one litre* [demonstrating incorrect non-commensurate understanding]
> 3. John: *So that's 25%?*
> 4. Andrea: *No, 25mls, be 25mls to 1 litre so it's more like two-point-five, two-point-five percent? **Is that right? Let's look at that, get a calculator.*** [Emphasis mine]

John's opening statement initiated a discussion that went on to include conjectures and attempts to prove or disprove them. Andrea's response in line 4 indicated an aspect of personal agency not observed in many other cases. Not only were conjectures made, but actions taken to test them.

> Andrea: *Oh my goodness!* [Made in response to the complexity of the task]

John: *The point is twenty-five percent, so that's twenty-five percent isn't it? That's a quarter.*

Andrea: *No, it's not twenty-five percent though.*

John: *No, it's two-point-five like you said… but… twenty-five is a quarter of a whole isn't it? So you've got twenty-five per litre.* **That doesn't sound right.** [Emphasis mine]

I interpreted the phrase, "*That doesn't sound right*", as an indication of John's willingness to evaluate and refute his conjecture. Throughout the discussion both learners continued to introduce ideas and to test them. For example, John tested whether their approach would work with a 50:1 fuel mix.

John: *Fifty to one, fifty mls?... So that's, fifty to ah fifty, fifty to one, oh fifty ml per litre… A five litre container is twenty-five* [he means 250ml]. *Ah, that makes more sense. And, and, right, and, and the twenty-five to one is a hundred and twenty-five mls per five litre can.*

Although continuing with the incorrect non-commensurate line of reasoning, John continued to question and explore the relationship between 25:1 and 50:1 ratios. Later Andrea introduced the concept of fractions to help them make sense of the ratio.

Andrea: *So, the way to think of it is like a fraction, ah you know like, so you've got twenty-five over one thousand like that. So, that would bring it down to…*

The conversation reflected engagement, high levels of agency, and persistence not observed in other classes. Even though they were uncertain of what the ratio represented they postulated and explored important ideas and had an orientation toward sense-making. While there was much misunderstanding in their conversation, the discussion centred on what the numbers represented such as whether the "25" represented a decimal or a percentage. They generated, explored, clarified or rejected ideas with little guidance from the tutor.

Later in the session, Andrea made a substantial breakthrough. Up to this point she and the others had misinterpreted 25:1 as 25 millilitres of oil to one litre of petrol.

Andrea: *Oh my God. 25 to 1 is actually 25mls of gas with 1ml of oil* [Correct]. *And then you times that by 50 to make it 1 litre* [Incorrect].

Tom: *You just go um.*

Andrea: *He just changed it now, look* [points to the tutor who is writing on the board]

Taynesha: *What?*

Tom: *Whatever one is, divided by 25.*

Andrea: *No, I understand that, but what I'm saying is. Is that, hang on, what I'm saying is that instead of looking at the mls and the litre, every, you change it round, every 20, so 25mls for gas, 25mls of petrol and then the dot with the 1 is the 1ml.*

Tom: *Oh the 1. Oh, that's easier.*

In this brief episode, Andrea had begun to understand the commensurate nature of the ratio, in that the same unit of measure is used to represent a relationship between quantities. She was the first learner in this class who had this insight without the tutor's direct instruction. She quickly revealed this to the others in the class, some of whom rearticulated her idea. What was unique about Andrea and the group she was a part of, was the positive and exploratory discourse.

Some learners were observed to engage deeply with mathematical thinking as the tutor worked through problems with the class. These were most effective when the dialogue with the tutor was open. In the following example, the hairdressing class were discussing equivalent fractions, specifically how quarters could also be expressed as eighths. Up until this point there had been much misunderstanding about how one quarter could be expressed as two-eighths. The learners each had a strip of paper folded into eighths and were working together.

1. Tutor: *How are we feeling about that? So, on here as well it's also telling us it's broken into the quarters there and the eighths. So, remember when I was saying that you could add your eighths on your fractions table, you could make them into the actual specific fractions, that's what I was meaning. So, you can see on the last column it says one-eighth and then two-eighths.*

2. Niki: *Yep*

3. Tutor: *Does that make sense Cara?* [Cara has not spoken in the lesson and continues to remain quiet]

4. Niki: *Is that because it can be two one-eighths? Two-eighths?*

5. Tutor: *Yeah, exactly right, that's two one-eighths, together. Yeah definitely, well done, and you can see the quarters are the same there as well.*

6. Niki: *Yeah.*

7. Trudy: *Oh, so that's two lots of um eighths, like two lots, of sets of four?* [signalling the entire strip]

8. Niki [To Trudy]: *So like two one-eighths, but you can have two-eights* [sic].

This episode represented a substantial conceptual leap for Niki and for Trudy. In line 7, Trudy began to make sense of the concept of equivalent fractions. She made a connection between two-eighths and their equivalence to one-quarter, and the equivalence of four-quarters to a whole ("*two lots, of sets of four*"). Although, this dialogue may not have the appearance of valuable mathematical discourse it revealed learners using new vocabulary to express new understandings. As Trudy explained regarding her new understanding of equivalent fractions:

Trudy: *It's in my head but it's hard to explain what I know.*

The willingness to engage in discussions, to generate and explore ideas was linked to learners developing new understandings in the sessions. Unfortunately, these occasions were rare.

Self-talk

A third finding that related to deep engagement in tasks was the occurrence of learners engaging in self-talk as they worked through problems. While there were few occasions of evidence of reasoning through self-talk, those that did occur revealed meaningful engagement with numeracy problems. In the following example, James engaged in self-talk while reading and solving a numeracy problem:

James: *Convert the above number to the number of years including any* [reading the problem]. *What do ya mean convert it? Convert the above number to the number of years, convert that into the number of years. Oh okay, so how would you do that? Oh divided by. Maybe it's multiplied, maybe it's multiplied. That don't make sense either. Convert the following number to a number of years convert that into… Convert it to a number of years. I've got two-point-five but I don't think that's right.*

This episode demonstrated a dialogic interaction between James and the content. James asked the text a question ("*What do ya mean convert it?*") and then proceeded to answer the question ("*Convert **that** into the number of years…*" *my emphasis*). This level of engagement contrasted with most learners who did not

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engage with new concepts or change their thinking, or who simply accepted answers based on who said them, or who requested answers from other learners.

As shown, learners engaged with content along a spectrum; however, three broad patterns arose. Some avoided engaging in any mathematical content at all. They isolated themselves and avoided participation by using body language and space to signal non-interest to those around them. Most learners engaged only superficially with the content. While they often expressed their own existing ideas or opinions, they failed to engage with, explore, or generate new concepts or ideas. They also tended to cede responsibility for problem solving to more proficient learners rather than persisting with tasks. There were also occurrences, albeit rare, of learners engaging to a greater degree with mathematical content by exploring and generating ideas and communicating these to others, or as self-talk. In many cases these episodes resulted in new understandings.

5.5 Discussion:

Tutor influence on engagement patterns

The pedagogical approaches adopted by tutors can be thought of as environmental factors within Bandura's (2006) triadic model, creating constraints and opportunities for various behaviours. Unfortunately, the approaches typically lacked characteristics of good practice recommended for adult numeracy provision, such as effective formative assessment (Hodgen et al., 2010), the use of authentic numeracy problems (Coben et al., 2003; Gal et al., 1994), and the development of conceptual understanding (Swain & Swan, 2007). The content delivery, lesson structure, design of problems, and implicit standards for success all reflected a transmissional pedagogy, as did the high quantity of tutor talk, the predominant use of closed questions, and the limited amount of sustained discussion or debate between learners. These are consistent with other adult numeracy classrooms (Benseman et al., 2005; Mesa, 2010; Swain, 2006). While there were opportunities for learners to solve problems in groups, solution procedures were usually demonstrated beforehand in a 'watch, remember, repeat' format. The problem-solving sessions were always followed by a marking and review phase that concluded the lesson. These findings echo other observational studies of adult numeracy lessons that have expressed some concern about the quality of provision (Benseman et al., 2005; Benseman & Sutton, 2007; Coben et al., 2007; Swan, 2006).

The tutors' behaviours shed light on why there is some evidence of a divergence between the positive performance of learners in classes that employ vocational *and*

numeracy specialists, and the neutral or negative performance of learners in programmes in which the mathematics is taught by non-specialists (Casey et al., 2006). The pedagogical approach adopted by the tutors, suggests limited mathematical pedagogical and content knowledge, and reflects those of some pre-service teachers, who before training, appear to adopt the transmissional practices they experienced in their own schooling (Viholainen et al., 2014). Although the problems the learners worked on were in a vocational context, the tutors appeared to mimic traditional mathematics classroom values, rather than connecting the mathematics to the vocational context (Dalby & Noyes, 2015). The findings support the argument that tutor's low pedagogical and content knowledge, due to a lack of training, may contribute to poor mathematical skills in adult learners (Young-Loveridge, 2012).

The practices also supported behaviours associated with negative beliefs about how mathematics is learned, what behaviours are appropriate, and what constitutes success (Goldin et al., 2009; Muis, 2004; Schoenfeld, 1985, 1991). For example, the problems the learners worked on were designed to facilitate the practice of routine procedures but were packaged within pseudo 'real-world' contexts. These had similar structures to various assessment tasks identified by Drake et al. (2012) that although attempting to reflect real world situations were cover stories for calculations. Reusser (2000) argues that the repeated use of such problems leads to learners suspending sense-making as they realise that the contexts are simply covers for routine calculations. The instruction in step-by-step procedures and the use of multiple routine problems are linked to the belief that all problems will yield quickly to the correctly applied procedure (Schoenfeld, 1985, 1991). Transmission approaches are linked to beliefs that the teacher's role is to tell learners the mathematical content (Taylor et al., 2005; Turner et al., 2002; Yoon et al., 2011). The use of external sources of authority, such as the tutor, to verify answers has been linked to the belief that the teacher or the textbook are the only valid source of all such knowledge (Schommer-Aikins et al., 2005). The conclusion is that the lessons observed are consistent with the absolutist/procedural beliefs evident in the survey results. However, the observations showed that it was not only the tutors' contribution to the environment, but also the learners' reciprocal behaviours that contributed to this. The following sections review the learners' own choices, behaviours and responses within the lessons, beginning with affective factors and responses.

Affective factors and responses

Many learners expressed apprehensions about either taking part in a lesson or engaging in specific activities or tasks. The initial negative responses to participating in either a mathematics lesson or an activity were consistent with research that identifies extremely negative attitudes held by adults toward mathematics (Burns, 1998; Coben et al., 2003; Evans, 2000; Grootenboer & Marshman, 2016). It also supports the findings of Carpentieri et al. (2010) who reported on a body of adult numeracy research showing significant numbers of adults not only dislike mathematics, but *fear* it, experiencing anxiety when asked to engage. Interestingly many of the verbal expressions related to previous experiences with mathematics and apprehensions of failure, shame or embarrassment. For example, Trudy's response to learning that she was about to participate in a fractions lesson, following a groan, was a heartfelt "*I really didn't like these at school*". This is consistent with Coben and Thumpson's (1996) findings that a common school experience was the "brick wall", a point at which mathematics stopped making sense, which was described, for many, as *traumatic* and long lasting. As Evans (2000) found these experiences overflow into adult learning environments, raising anxiety and even spreading it to others. Interviews with Trudy confirmed this was true of her school experience (reported in Chapter 6), and the observations show that her and others' apprehension and anxiety were present throughout the lesson.

The learners responded in two contrasting ways to challenging problems. The first group consisted of learners who experienced frustration yet persisted until they achieved success. This was exemplified by James, whose self-talk revealed strong emotion while working on tasks ("*Raahhh. I can see that's wrong!*"), and yet he persisted and achieved success. The second, and more common, pattern was exemplified by Flora, who after reading almost to the end of a word problem, became frustrated at the complexity, expressed her hatred of mathematics and disengaged ("*Oh my gosh! I hate maths*"). These two broad patterns are consistent with the findings of Allen and Carifio (2007) who found that higher-skilled mathematics learners were able to manage their emotions, while lesser-skilled learners were often overwhelmed by them. This seems the case with Flora, whose behaviours support the hypothesis that at the onset of negative emotion, learners transition from attending to the task to appraising its difficulty, to disengaging altogether to make appraisals of the wider situation (Goldin et al, 2011; Malmivuori, 2001). The findings support the notion that learners' appraisals are underpinned by self-concepts of ability and therefore, learners with lower self-concept have lower thresholds for negative engagement patterns (Goldin et al., 2011).

Learners' feelings of inadequacy were aggravated by the behaviours of 'mathematical experts', higher-skilled learners who corrected and taught others or asked and answered questions with excessively complexity. The behaviours appeared designed to advertise their own proficiency, and in doing so may have contributed to the re-establishment of a classroom hierarchy. The ability to make comparisons with others, or be judged by others, is cited as a danger of absolutist/product-oriented classrooms (Bibby, 2002). These learners appeared to establish such an environment by causing others to become more attentive to their perceived inadequacy, leading to feelings of doubt, shame and reinforcing non-mathematical identities. This may have inhibited participation as learners sought to reduce opportunities for judgements to be made about their skills. The behaviour of the experts also appeared to erode the shared sense of non-judgement between lower skilled learners. In Chapter 6 the learners revealed that knowing that other learners have, and continue, to struggle with mathematics reduced feelings of inadequacy, and reduced inhibitions around participation. However, the behaviours of highly verbal 'numeracy experts' interfered with this dynamic. Consequently, the impact of the numeracy experts' behaviours was to exacerbate the existing negative engagement patterns already occurring in numeracy classrooms. These findings show that concern with how others perceive them is debilitating. These responses are consistent with Tennant (2012), who found the fear of looking ignorant to peers in adult classes reduced participation, while the ability to disregard the perceptions of others increased it.

Finally, the observations showed that learners who did engage with mathematical tasks often used self-talk to express emotion, both positive and negative. Learners talked to themselves, and to the problem, in ways that suggested they had invested emotionally into solving a problem. They celebrated success ("*Chur chur*") or berated themselves for failing ("*Oh f****"). These findings are consistent with those of Goldin et al. (2011) who posited that negative patterns of affect can also be associated with positive engagement, which may include frustration, impasse and disappointment, on the way to success. If the journey to success is difficult, it is likely to result in greater positive affect once success is achieved. It is encouraging that the learners cared deeply and experienced a range of positive emotions while engaging in what is known by practitioners to be a 'hot' emotional topic (Hekimoglu & Kittrell, 2010).

Orientations toward mathematics

In general, the learners' behaviours were consistent with beliefs that mathematics is a fixed body of procedures and rules, that problems are solved by extracting

quantities and applying routine procedures, and that the purpose of problem-solving is to demonstrate performance, not develop mastery (Boaler & Greeno, 2000; DÍaz-Obando et al, 2003; Frank, 1988; Jäder et al., 2017; Lampert, 1990; Thompson et al., 1994). The propensity to solve problems by 'recalling' the correct procedure from memory rather than developing a solution strategy is consistent with studies that found learners holding procedural beliefs tend to apply procedures to all problems, even those intended to promote conceptual thinking (Engelbrecht et al., 2009; Jäder et al., 2017; Sumpter, 2013). The behaviours were also consistent with studies that found learners believed that the ability to *mimic* a procedure to solve a problem is the purpose of problem-solving (DÍaz-Obando et al., 2003; Jäder et al., 2017).

Almost all the mathematical problems were word problems written with a context in mind, for example, finding discounted prices or calculating income. However, consistent with the suspension of sense-making learners acted as though they knew that the word problems were cover stories for mathematical equations (Reusser, 2000). Although it is positive that various members of the groups could map an abstract problem into a mathematical equation, the practice of ignoring the context of the problems and extracting only the quantities was consistent with the behaviours of learners holding procedural beliefs (Engelbrecht et al., 2009; Jäder et al., 2017; Schoenfeld, 1991). The rapid and single reading of the problems resembled Garofalo's (1989) findings, in which learners believed reading the whole text of the problem was extraneous because the operation could be identified from key words, indicating that contextual features are of little consequence in the minds of some students. The behaviours also had parallels to the practices of learners who believed that knowledge was simple and consequently exerted less time and effort comprehending, and solving, word problems (Schommer et al., 1992).

The lack of a checking or reviewing process was consistent with learners holding procedural beliefs, due to their orientation toward external authority and validation (Garofalo, 1989; Lampert, 1990; Muis, 2004). Lampert noted that answers became "true" for many learners when they are endorsed by an external source of authority, rather than self-evident. Almost all the learners uncritically accepted other learners' procedural solution methods, as though their very act of proposing an idea indicated expertise. Additionally, many learners in this study only celebrated or bemoaned their answers when the tutor wrote them on the board, showing that this was the moment of verification, not the moment of producing the answer. The notion of checking or reviewing their original work was absent, consistent with learners who believe that

mathematical knowledge resides with experts (Schoenfeld, 1985; Stylianou & Blanton, 2011).

The learners' behaviours reflected a strong emphasis on speed and performance at the expense of understanding. Wood and Kalinec (2012) similarly found that much of the talk that took place in a problem-solving group was devoted to getting *through* the task, not mathematizing. The adults in this study acted consistently with studies that find children and teachers act as though speed, accuracy and using the "right" method are the goals of mathematical activities (Kotsopoulos, 2007; Muis, 2004). This orientation toward procedures, speed and accuracy is consistent with research indicating that many learners believe speed is a sign of ability and therefore highly valued (Blackwell et al., 2007; Garofalo, 1989; Solomon, 2007; Stodolsky, 1985). Unfortunately, the speed at which learners progressed through the problems left most group members with little opportunity to engage with the task or take part in productive discourse. In several cases, learners were unsure where on a worksheet to write an answer because of the speed at which the "solver" progressed. Supporters were effectively reduced to scribes, entering digits with no sense of meaning.

More concerning was the practice of group members to taking separate roles, with the most skilled learner taking 'solver' roles while others took 'supporter' roles. These asymmetric arrangements influenced the methods and speed at which the groups progressed. Higher skilled learners either took, or were encouraged to take, responsibility for solving tasks while others took support roles. This constrained the role of supporters because it removed their participation in mathematising, and the solvers did not scaffold or support other' learners' understanding. Although not as prevalent as in this study, these roles were similar to those identified by Evans et al. (2006) who identified learners adopting *leader/follower* and *evaluator/evaluated* roles, and with Kotsopoulos (2007) who noticed some 'expert' learners in groups dominated the problem-solving process. Swain and Swan (2007) also observed group members occasionally "telling" others how to think. They also noted the distinction between working *in* a group and *as* a group, an apt distinction in this study. The findings are consistent with, and illuminate, previous studies in which whole class discussions were dominated by a few vocal members, because many adults were content to let others assume responsibility for various tasks (Benseman et al., 2005; Howard et al., 2002; Weaver & Qi, 2005).

The divergent roles acted to constrain learner engagement more than they advanced it. This differs somewhat with the findings of Marr (2001) who noted that the multiple roles adopted within group work sessions allowed for diversity of participation. Drawing on Rogoff's (1995) notion of "peripheral participant", Marr rightly noted that learning as a group participant, even when limited, is more productive than ineffective individual work. However, it is questionable whether the *quality* of participation of the supporters in this study was likely to result in either mathematical or dispositional development. In the example provided of Jaz and Christine, Jaz abstained from *any* mathematical thinking in favour of writing in answers for Christine, and this was not unusual. There is the possibility that the supporters' practice of ceding responsibility to a 'solver' consolidated non-agentic behaviours and subsequent non-mathematical identities. It may be that these learners were familiar enough with classroom practices to 'use' higher skilled learners as resources to meet classroom demands.

The impact of the different roles within groups on the quality of mathematical engagement raises questions regarding the preparedness of learners to engage in productive ways in groups, particularly given the frequent recommendations that numeracy tutors incorporate group problem-solving sessions into their practice (Condelli et al., 2006; Marr, 2001; Swan, 2005). Marr's argument that group problem-solving develops a range of skills, including time keeping, maintaining relationships, bringing in outside knowledge and including learners with lower mathematical skills, may be true with learners with more conceptually oriented beliefs, such as found in Francisco's (2013) study, but less so with learners holding such procedural beliefs. The behaviours in this study suggest that shared beliefs that problems ought to be solved quickly and in the right way contributed to unequal distribution of engagement within groups. Given Schoenfeld's (2011) contention that goals are a primary driver of human activity, the approach adopted is entirely rational. Furthermore, the arrangements were *effective* in meeting the goals and therefore would be unlikely to change without new goals. Role allocation may be a direct result of learners' mathematical belief systems, reflecting goals, priorities and the purpose of mathematical problems.

A second overlapping reason that may also account for learners adopting different roles is that those who perceive themselves as less "able" in mathematics may have been attempting to avoid a potentially socially damaging situation, that of being made to look ignorant in front of the group or an individual (Tenant, 2012). Certainly, the learners' affective responses support this. Bibby (2002) argued that absolutist/product-oriented environments are connected to experiencing shame and

lead to strategies to avoid this. Thus, some learners may have tacitly agreed to the allocation of roles as an avoidance behaviour. A further possibility is that learners avoided engaging in tasks to avoid failure from an intrapersonal perspective. It has been suggested that some learners avoid trying their best, because failure would confirm their suspected inability (Dweck, 2017; Midgley et al, 2001). Learners cannot blame themselves for failure if they do not try, and hence avoid doing so. It is likely that the learners' adoption of various roles was driven by a mixture of these motivations. Some, driven by performance goals, may have viewed the arrangement as an efficient configuration for solving multiple tasks quickly and accurately. Others, driven by performance avoidance goals, may have seen it as a way to avoid the embarrassment that might occur if they were unable to perform individually, and others, as a method for avoiding confrontational evidence of their ability. If correct, this is consistent with Webel's (2013) findings that learners working in groups hold multiple goals across varying dimensions.

There was a minimal amount of discourse in the classes studied that could be considered mathematically constructive, such as making conjectures, clarifying, questioning, constructing arguments that are explained and justified, or responding constructively to others' ideas (Hufferd-Ackles et al., 2004; Tsay et al., 2011; Yackel & Cobb, 1996). The interactions that took place between the tutor and the learners during whole-class discussions were similar to patterns identified in traditional mathematics learning environments, such as the initiation, response, and evaluation (IRE) pattern identified by Mehan (1979). However, unlike studies with younger students (Hufferd-Ackles et al., 2004; Mendez et al., 2007; Yackel & Cobb, 1996) this did not appear to be entirely a result of a traditionally teacher-directed classroom, but driven by the learners' own expectations about what was appropriate, partly evidenced by their practice of not complying with requests from the tutor to elaborate their answers. Further evidence that these patterns stem equally from the learners is the tendency of tutor questions to be answered by only a few "dominant solvers", also consistent with studies of adult classrooms (Karp & Yoels, 1976; Tennant, 2012; Tsay, Judd, Hauk & Davis, 2011). The practice of adult learners choosing to assign responsibility to other more capable learners is consistent with adult studies (Howard et al., 2002; Weaver & Qi, 2005). The patterns also support Turner et al. (2002) who found that learners in performance-oriented classrooms avoided engaging in discourse to a greater degree than in mastery-oriented classrooms in an effort to avoid embarrassment.

A final finding is that many learners appeared to believe that they were succeeding within the classes. Learners celebrated their successes even in cases when they did not personally solve a problem but were simply part of a group that did. This supports the notion that the learners' goals are prioritised toward completing tasks and avoiding shame. For example, Muis and Franco (2009) argued that learners' goals set up standards for success, against which learners evaluate their progress. According to Butler and Winne (1995) learners do this by comparing their progress against their standards to determine whether the "products" being produced meet the expectations. The products, in this case, appear to have been correct answers completed on time, but not self-produced answers nor increased mathematical understanding. Hadar (2011) similarly found that learners were able to meet short-term school performance criteria without actually learning. The learners in the lessons observed were able to achieve a pseudo-success, such as completing tasks, without developing new understanding.

Engagement with mathematics

Finally, three broad levels of learner engagement patterns were observed. The first reflected an attempt by learners to not engage at all, achieved through overt or surreptitious methods. The second pattern was partial, or shallow engagement, in which learners avoided engaging in mathematical reasoning. The third pattern was rarer, but reflected deep engagement by learners, characterised by rich mathematical discourse and persistence with difficult concepts or problems.

Non-engaged learners

There were learners in each of the classes who adopted strategies to avoid any engagement with mathematics during the lessons. Some of these opted for overt non-engagement while others adopted more inconspicuous approaches designed to give an *impression* of engagement. The learners were successful in doing so, with or without the knowledge of the tutor. They withheld effort, disguised and passed themselves off as competent, and did not seek help when needed. They also created distance between themselves and the tutor, using physical barriers such as sitting at the back of rooms, or in positions blocked by desks and other students. Some pulled hoods over their heads and acted as though preoccupied in an alternative task such as drawing, texting or reading and made no perceptible eye contact with the tutor or other learners. Other learners, unable to create distance due to class size, pretended to contribute to group work or discourse without doing so, and feigned understanding and engagement when interacting with the tutor. These behaviours are consistent with studies that have found learners sitting as far from the teacher as possible,

copying others' work, self-censoring their participation in discourse, and generally disguising their lack of understanding (Bibby, 2002; Hauk, 2005).

The behaviours suggested a history of failure with mathematics and an expectation of repeated failure. The avoidance patterns resembled those found in studies in which the authors propose that learners adopt such behaviours as an attempt to stave off public judgement and, importantly, to ensure that the causes of failure remain unclear to others (Chinn, 2012; Turner et al., 2002). Moreover, the results support the notion that learners engage in mental "what if's" during lessons, ruminating on possible embarrassing eventualities (Yoon et al., 2011). The interviews in the following chapter find evidence that learners evaluated engagement in various activities against the risk of "looking dumb" to others. It appears some choose to accept judgement for not engaging, rather than face the shame that might occur if their peers knew the true state of their ability (Bibby, 2002; Chinn, 2012).

Surface level engagement

Most of the learners did engage with mathematical content, but only to a limited degree. Learners rarely engaged in discourse that could be said to develop their ability to "think like a mathematician" (Burton, 2004; Lakatos, 1976). There was a distinct lack of mathematical 'talk' such as making conjectures or proofs, explaining thinking, or investigating other learners' ideas. Much of the discourse related to known mathematics, such as procedures, with almost no engagement in non-routine problems or reasoning. This appeared to be due to learners moderating their engagement as a form of self-protection.

Harley's engagement pattern shared similar features to the structure 'don't disrespect me' theorised to be motivated in part by an attempt to avoid stigma and maintain status (Goldin et al., 2011). This behaviour is thought to be initiated when a learner perceives a threat to their dignity, status, or sense of self-respect. The tutor's insistence that Harley '*Explain to the class what your little group came up with,*" although presumably intended to overcome Harley's lack of engagement, probably contributed further to Harley's awareness of a risk. The heightened risk probably contributed to his transition from the desire to learn towards avoiding a situation which may have led to belittlement, a common human motivation, according to Murray (2008).

The strategies Harley adopted to avoid shame, such as the statements "*I'm s**t with numbers! But not when it comes to money, mate*", and frequent iterations of the

same point, were consistent with findings of learners seeking to protect themselves by using self-denigrating utterances (Bibby, 2002). As Bibby noted, stating that one is not good at mathematics carries little stigma, yet still works to lower the social expectations for the learner's performance, thus protecting them from the awkward moment of shock or surprise when others became aware of their failing. That this was a strategy designed to protect Harley's status was evident by his *immediate* declaration of inability. His initial utterance *"F***, I don't know"* appeared reactive, uttered before he had even investigated the problem.

A second strategy that appeared designed to maintain status was Harley's attempt to distinguish between his ability in the classroom context ("*I'm s**t with numbers!*"), and his ability in the context of money ("*but not when it comes to money, mate*"). One might argue that being good with money is of greater value than merely 'numbers'. Hence, Harley lowered expectations for his performance in the classroom, yet bolstered his status in the high value area of money. In some respects, this hints at a further concerning issue, that Harley *purposely* distinguished between the 'numbers' within the classroom and *meaningful* mathematics. Moreover, given that Webel (2013) found that 'group attitudes' developed when learners worked together, it is of concern whether Harley promulgates this opinion to others also, particularly if he continues to engage in such a manner. Finally, Harley's beliefs about his poor ability may have been confirmed by his experience in the lesson. He entered the lesson thinking he was "*s*** with numbers*" and his subsequent failure to make sense of the content, may have further embedded this belief.

Deep engagement by learners was rare but encouraging because in almost all cases they appeared to develop genuine new understanding. Andrea and John's engagement was consistent with the argument that learning mathematics requires possessing an inductive attitude, the need to make conjectures and refutations, and the courage to take a personal risk by making one's conscious guesses public (Lakatos, 1976; Lampert, 1990; Pólya, 1954). The discourse between Andrea and John demonstrated Lakatos' argument that coming to know mathematics is the result of a 'zig-zag path' made by consistently positing decreasingly naïve conjectures and refutations. The *way* that Andrea and John engaged with each other can be contrasted with those who did not do so, such as Jaz and Christine. Andrea and John's discussion resulted in an increased conceptual understanding of ratios and a partial understanding of their relationship with percentages and decimals.

A key feature of Andrea and John's engagement pattern was their apparent lack of concern with preventing episodes of shame. They both shared similarities with the engagement structure "check this out", which Goldin et al. (2011) speculate may be motivated by the desire to obtain something of value, in this case, a useful and relevant workplace skill. Andrea's behaviour also resembled the structure "I'm really into this" which describes a learner whose self-concept is as an effective problem-solver and engaged thinker. This is important because (although this is conjecture) Andrea's self-concept *did not* appear to include being proficient at mathematics. At various times through the class she commented on her difficulties with mathematics. She was as aware of her lack of knowledge and potential for failure as the other learners, yet persisted regardless. Unfortunately, across all the classes only several episodes of such engagement were observed. The findings support Tenant (2012) who found that a disregard for the perceptions of other learners is a key factor in engagement.

In summary, the behaviours observed were illustrative of many of the warnings articulated about the impact of negative beliefs on learners (Boaler & Greeno, 2000; Goldin et al., 2009; Muis, 2004;). This study indicates that these warnings are valid. Learners rarely engaged to a degree sufficient to construct new mathematical understandings. Worse, the behaviours seemed likely to entrench negative patterns of behaviour that *reinforce* negative beliefs, potentially leaving the learner worse than when they began. The following chapter explores low-skilled learners' own accounts of their mathematical histories, current classroom experiences, identities and beliefs.

Chapter 6. Interviews: Results and Discussion

Damon: "What do you think the others are thinking?"

Niki: "When the tutor's like trying to help me?

That I'm dumb"

(*Niki, 21-year-old hairdressing learner*)

This chapter presents the results of twelve interviews conducted with learners who were participating in embedded mathematical instruction in foundation-level vocational programmes. It addresses the first and second research questions by exploring learners' own perspectives and interpretations of their historical and current mathematical experiences, their beliefs about mathematics, how it is learned, and their relationship with it. Ten were selected due to behaviours that indicated some trepidation with content. Two additional learners, Troy and Hahona, were selected because of behaviours that indicated higher confidence (see Methodology section 3.4 for the details of the selection criteria).

The interview findings are presented in three sections that correspond with the structure of the interview (see Appendix I):

- Challenges faced by learners re-engaging with mathematics
- Mathematical identities
- Beliefs about mathematics

Presentation of the findings is followed with a discussion.

6.1 Challenges faced by learners re-engaging with mathematics

This section reports on the learners' perspectives of their school experiences and how these influenced their thoughts and beliefs about mathematics. It then reports on the relationships between these experiences and their current engagement with mathematics.

School experiences

The lower-skilled learners described their school experiences almost entirely in negative terms. Their comments reflected perceptions of being alienated from school, of being judged, and having social and academic difficulties. While some positive

comments were made regarding having friends and enjoying various subjects, these were only mentioned by five of the learners: Kelly; Trudy; Sonja; Troy; and Hahona. Nine of the 12 learners had attended multiple schools. The earliest school leaver was Pita, who reported leaving school at age 11, while the others typically reported dropping out early. Frequent references were made to distressing aspects of school, such as struggling with judgemental peers, having difficulties with teachers and students, and disorganised classrooms and instruction.

> Damon: *Tell me about school, what was school like for you?*
> David: *Just letting you know, I probably didn't enjoy it, probably hated it. I didn't wake up wanting to go to school in the morning.*

> Kelly: *Well, I hated the learning part, but I used to like to go to play with my friends and stuff.*

The learners often indicated that school was a judgemental environment, particularly in the domain of academic competence.

> Niki: *Whereas, like, everyone judges you at school.*
> Damon: *Yeah?*
> Niki: *Yeah, I sort of think there's not a lot of judging in here* [in the current course].
> Damon: *Tell me more about that judging thing, what do you mean by that? Like at school they judge you?*
> Niki: *Um, judging like, like they're higher than me, they're like, 'I'm better than you'.*

In contrast, the two higher-skilled learners, Troy and Hahona, reported positive experiences of learning at high school both socially and academically.

> Hahona: *Yeah, it was actually cool. Like, I mostly went to hang out with my mates really. Yeah, just do all the fun stuff, just passed on the normal stuff… Oh 'cause, I'm always mucking around, but when it comes down to doing work, I'd just do it. My teacher trusted me for doing that. Like, he'd let me go to PE* [physical education] *and stuff from his class* [mathematics] *'cause he knew I'd get the job done at the end of the day, or something.*

The experience of school mathematics

The learners' experiences with mathematics were similar to their general school experience but increasingly negative. Several themes emerged, including animosity toward mathematics, perceptions of being judged, having feelings of inferiority compared to others, and disengagement from mathematics.

Each of the lower-skilled learners' attitudes toward school mathematics was negative, while Troy's and Hahona's were positive. For example, the learners' responses to the question, "Did you enjoy maths in school?" indicated negative attitudes toward mathematics.

> Trudy: *Not really. I didn't like maths.*
>
> David: *Nah* [aggressive tone].
>
> Tina: *Not really. I hated numbers, that's why.*
>
> Abbie: *Yeah, so maths wasn't a good time in third form.*
> Damon: *Ah, so was it at the point where you dreaded going to maths classes? Was it that bad?* [Previously mentioned by Abbie]
> Abbie: *Yeah. I hated maths. To this day I think I still dislike it.*

In contrast, Troy and Hahona described their mathematical experience in positive terms:

> Troy: *I really only liked practical things really.*
> Damon: *Although maths, is maths practical or not?*
> Troy: *Oh, fun!*
> Damon: *Good.*
> Troy: *'Cause I was a smart student in my school. Always in the top classes, top grades.*
>
> Damon: *Tell me about maths, what was the maths experience like?*
> Hahona: *Oh, that was one of my favourite classes, it was just like another PE class.*

Further questioning revealed that those who perceived they were good at mathematics had positive attitudes, while those who perceived themselves as poor at mathematics had negative attitudes toward mathematics.

Observation and judgement

A recurring theme from the lower-skilled learners was their perception that their behaviours in the classes were observed and judged by other learners. They reported unfavourable value judgements being made about their intellect based on their performance. Behaviours that were judged included answering a question incorrectly; asking a question that revealed a lack of understanding; and not solving a problem correctly. The consequences of these public events were to appear and feel inadequate, resulting in a loss of status and self-worth. For example, Abbie described how answering a question incorrectly in class resulted in laughter from other learners:

> Damon: *You'd answer the question and you'd got it wrong, what would've happened?*
> Abbie: *The girls would laugh, pretty much. And that's what they would do to us, to our group. And, 'cause it was, when you're in your group in class, it was like that outside of class too so....*
> Damon: *Yeah, I see. So, it'd be like exposing a weakness, is that...?*
> Abbie: *Pretty much. Make you feel pretty s***.*

Being observed and then negatively judged was described in terms of having a social cost. The learners' accounts revealed the potential of an incorrect answer to make them appear inadequate to others in the class. This occurrence was described with derogatory language such as "dumb" or "stupid".

> Damon: *Why does that matter?* [Whether you answer a question correctly or not]
> Niki: *Um, I suppose 'cause at school you don't want to look stupid. Yeah, you don't want to actually look dumb.*

> Sonja: *And I just think, at that time, probably, maybe it had to do with my self-confidence and doing maths and stuff was just like, got a few things wrong and I thought ah, just can't do it. I'm dumb.*

> Kelly: *Yep, sometimes, like I used to* [not answer questions in maths classes]. *Not so much now, but when I was younger and stuff I did. Like, 'cause you know, you were always worried that they were brainier than you and they'd think you're dumb or... you know?*

Learners were concerned with the perception of others toward themselves. Pita described having a support worker sitting with him in class. However, as he was the only student to do so, the effect was to isolate and draw attention to his difficulties.

> Pita: *I remember having a lady there with me, helping me learn.*
> Damon: *Was that good?*
> Pita: *Oh, not really, 'cause I was the only one in the class who had an adult there and a teacher. Felt stink. Yeah bro.*

Feelings of being judged were compounded by perceptions of inferiority compared to peers. The learners often felt that the other learners in the class were competent, and that they alone lacked understanding.

> Damon: *Would it bother you if everybody thought or realised that you didn't know how to do something?*
> Niki: *Yeah, yeah.*
> Damon: *How come?*
> Niki: *Shame. 'Cause everyone's just staring at you like, 'Oh you don't even know the answer', **but everyone else probably does** [Emphasis mine].*

The perception that "*everyone else*" understands while they did not was common. For example, Kelly believed that the other learners would 'get it' while she would not:

> Kelly: *Just like, I don't know, I just never really understood where the class was, like what the teachers were going on about all the time, where everyone else would get it, you know? Or put their hand up 'cause they knew the answer and stuff, but I was just like* [makes a confused expression], *and I didn't really know.*

The third theme emerging from school experiences was that of disengagement from the mathematics class and content, and subsequent membership of groups of other similarly disconnected learners. Disengagement was related to two inter-related factors. First, learners attributed their disengagement to a lack of support from teachers, poor learning environments, and their own lack of focus. Second, learners also reported hitting 'blocks'; that is, areas of mathematics that they could not overcome. These blocks often related to specific content areas, such as algebra, fractions or percentages.

For example, David attributed his disengagement from mathematics to a poor learning environment and his own behaviours:

>David: *Like, the classroom was just chaos, like, like people just turned up whenever they wanted, walked out, people yelling, people listening to music, doing whatever they wanted to do. She's* [the teacher] *constantly yelling at people, people just doing whatever they wanted. Couldn't understand her.*
>Damon: *Oh no.*
>David: *You know, was like, I was just sort of, there's no learning happening here. You know? So probably just checked out, probably, for most of maths then, in that environment.*

He also attributed his disengagement to his own lack of focus:

>David: *I just probably wasn't really, my head wasn't in learning, and that was the biggest thing, that the head wasn't there to learn, it was, you had to go to school so you went. And I was probably there, you know, I didn't understand how important it was, and how it will affect you and how fun, and how you want to be smart, how you wanna learn and you wanna, you know?*

Abbie's comment below revealed that she perceived her teacher's lack of support as intentional, due in part to her inability. Her comments indicated a social and physical distance between herself and the teacher that would be unlikely to lead to re-engagement:

>Damon: *Did the teacher know or not?* [that you didn't understand the content]
>Abbie: *She didn't come down often* [to the back of the class]. *So, I s'pose, 'cause she knew we weren't getting it, she didn't help. So it was a waste of her time.*

Learners also made references to mathematical content that acted as blocks to further advancement in mathematics:

>Tina: *Just like, oh if you have to divide it and stuff, I just don't bother.*

Anna: *I didn't understand what a lot of the numbers and the letters meant. Algebra was, errr* [groan]. *I hate algebra. I still hate it… Algebra came in and I was like 'oh no'.*

As learners disengaged from the formal classroom activities, they described themselves as connecting with similarly disengaged or struggling learners. These groups were often described in terms of behaviour, such as "naughty", and seemed to act as a support system in the face of potentially difficult environments.

Niki: *Um, well I suppose the class I was in, it was kind of like a… the naughty class, if you call it that. Yeah, so we kinda, I suppose we all understood each other.*

Learners also described geographically where the groups were positioned within the class. Sitting at the front was associated with good behaviour, while sitting at the back was associated with bad behaviour.

Abbie: *That's where I kinda gave up on maths.*
Damon: *Do you think there were other people in the group the same as you or not?*
Abbie: *Ummm, there was three of us that were moved. So, we kinda ended up sitting at the back of the class....*
Damon: *Together?*
Abbie: *With each other, yeah…*
Damon: *And so tell me about that class, what happens?*
Abbie: *And just doodle. And just doodle on paper.*

The increasing distance between teacher and learner was evident as the learners described strategies to either avoid the teacher through attempts at invisibility or in contrast, by misbehaving or challenging the authority of the classroom and teacher. Many learners rejected the authority of the classroom and actively rebelled against it:

Trudy: *I didn't like maths. There used to be a group of us and we had a teacher, we always used to make fun of her, this was in my, start of fourth form, yeah and we always used to be like little rebels and…*

David: *I didn't not enjoy it then* [high school]. *But, you know, as the level goes up and up and up, I felt the teaching quality disappeared, classroom*

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> *environment became worse, and I probably, you know, culturally, you know,*
> *became more interested in social things instead of education.*

In summary, other than Troy and Hahona, the learners reported disengaging from mathematics during their school years. While they were compelled to attend classes, they situated themselves in ways to create and maintain distance from the teacher. Similar behaviours were evident in the classroom observations (Chapter 5).

The social cost of low mathematical proficiency

The social cost of being viewed as having poor mathematical skills featured strongly in learner accounts. Learners felt their lack of understanding invited contempt from peers and was related to personal intellectual deficiencies. Learners described being positioned by others as "lesser", which led to avoidance behaviours to minimise further judgement. The positioning was described in derogatory language, including "cabbage maths", and being thought of as "dumb":

> Damon: *So, tell me about Waikato maths. What do they do differently?*
> Abbie: *I think Waikato maths they simplified everything for you. Whereas in the normal maths, well, we got called cabbage maths.*

Learners referred to being positioned as 'dumb' for the simple action of asking the tutor for help:

> Mary: *'Cause if you don't know you just ask for help and or sometimes you just get nervous asking for help. Yeah, I'm dumb.* [Meaning that she is letting everyone know she is dumb]
> Damon: *What makes you think that people will think that you're dumb? Like where is that coming from?*
> Mary: *Just me I s'pose. Yeah. I'm sure people are saying it, well not saying it out, but probably saying it in their head.*

> Damon: *What do you think the others are thinking?*
> Niki: *When the tutor's like trying to help me?*
> Damon: *Yeah.*
> Niki: *That I'm dumb* [laughs].

Finally, there was evidence that the learners themselves made judgements about others' abilities. Although not necessarily meant in a derogatory manner, they were clear:

> Hahona: *Sonja's loud but she's useless* [at mathematics].

In sum, learners' accounts of mathematics included not only perceptions of being observed and judged by peers, but also being positioned by others as lesser. These experiences contrasted with Troy's and Hahona's accounts, who described themselves as being engaged, having had good relationships with teachers, and enjoyable experiences of school mathematics.

Re-engaging with mathematics as an adult

The challenges experienced re-engaging with mathematics instruction were related to their school experiences. Themes included: the expectation that as an adult they should be proficient with mathematics; an ongoing tension related to avoiding negative judgements; the impact of learners having different skill levels within a numeracy class; and the impact of numeracy experts on engagement.

Social expectation to be proficient at mathematics

Despite disengaging from mathematics while at school, the learners felt pressure to be proficient with mathematics because they were adults. For example, when discussing her recent classroom experience learning about fractions, Sonja, aged 26, lamented her poor performance and expressed regret at her perceived failure during her school years. Moreover, she believed that she should "*know this stuff*" because of her status as an older person and cited this as a factor related to her negative feelings:

> Sonja: *Yeah. And for me I was just like, bloody, man!* [Frustrated]. *I wish I was just like* [pauses], *on board with all of this at school.*
> Damon: *You got that though, didn't you, on that day?* [Referring to her fraction lesson]
> Sonja: *Yip, yeah, later on. After a* [pauses]. *Yeah, but to me it just felt like, I just felt like a … 'scuse my language, but I felt like just a dumb c***.*
> Damon: *Why?*
> Sonja: *'Cause I'm one of the oldest in the class, the others are all round like bloody nineteen, twenty, and stuff like that, and it's just like sh**, man. I should know this stuff.*

The negative affective responses experienced in school were reported as being present in adult numeracy classes, but not as strongly felt. For example, Abbie was asked to compare her feelings in her current adult numeracy class to her unpleasant third-form class:

> Damon: *And so, for you, are the feelings the same for you if you get it wrong, are the feelings the same as in third form or..?*
> Abbie: *Not, not the same. Nope. I think I'd get a little bit bummed and then I'd be like, oh well. There's other people that got it wrong too.*

Note that Abbie still felt "*a little bit bummed*", but the knowledge that others were also incorrect reduced the negative impact. However, when asked directly about her classroom behaviour during the observation, Abbie revealed that the reluctance to be considered "dumb" still had some role in inhibiting her from answering questions.

> Damon: *What about answering questions and calling out questions and things like that? You weren't doing much of that in the observation. Why not?*
> Abbie: *'Cause I didn't want to get it wrong!*
> Damon: *What would've happened if you'd got it wrong?*
> Abbie: *I dunno, would've been a little bit bummed, I guess.*
> Damon: *Yeah, how come? Like what?*
> Abbie: *I think just because of my experience at school, I just kept quiet anyway. And just did it.*

Mary also spoke of her reluctance to speak up in school classes for fear of looking dumb. The fear of looking dumb or suffering some social damage continued to inhibit her participation in her current class:

> Mary: *Asking the teacher takes me a while.*
> Damon: *How come?*
> Mary: *Nervous, and...* [Pauses]
> Damon: *Are you nervous of the teacher, or are you nervous of making it public to everyone else?*
> Mary: *Acting dumb.*
> Damon: *So are you worried that the tutor will think that, or the other students?*
> Mary: *Other students – oh both.*

Other learners expressed the idea that specific situations elicited anxiety. In most cases, the anxiety was related to answering questions correctly or incorrectly under observation:

> *David: If I got called up in front of that classroom, you know, here's a maths question, I want you to come up and do it, it, I'd probably start to freeze a little bit, mentally, you know.*

Learners stated that they disengaged quickly in numeracy classes if they judged the content as not immediately achievable:

> *Kelly: I guess, if I get it or not, once they start the lesson, if they, if I get it then I get it, and if I don't, then I don't.*
> *Damon: I suppose that makes their start of the lesson pretty important, to make sure they're drawing everybody in and starting at an appropriate level.*
> *Kelly: Yeah and I make the decision pretty fast. As soon as I see what's going on I'm like, 'oh yip' or 'Oh nah'.*

In summary, learners recalled unpleasant affective responses during school mathematics classes and described experiencing similar behaviours and feelings in their current programmes. These included avoiding behaviours that may have led to others judging them negatively, feeling as though they were being judged by others, experiencing anxiety, and feeling shame because they were not as proficient as the other learners. Learners also described not engaging if the content was deemed not immediately achievable, or looked as though it might be challenging.

Differences in skill level as a contributor to feelings of inadequacy
A contributing factor to negative responses and disengagement from mathematics during school was the learners' perception of a skill difference between themselves and other learners. However, several of the learners cited the fact that because the learners in their adult class also struggled with content, it made them feel better about interacting in ways that revealed their own lack of understanding, such as asking for help from the tutor or other learners:

> Pita: *No, it's good 'cause we're all on the same boat.*

The perception that other learners in the class were experiencing the same difficulties with content reduced the feelings that contributed to inhibiting behaviours, such as perceptions of being judged.

The impact of 'mathematical experts'

The learners' perception that all learners shared a difficulty with mathematics was disrupted by the inclusion of a 'mathematical expert' in the classes. Mathematical experts were learners in classes who were not only proficient, but advertised their proficiency to the class. The observations revealed that these learners achieved this by frequently answering questions, teaching others, completing work quickly and asking the tutors overly complex questions. The behaviours of these learners were cited as contributing to feelings of inadequacy in their fellow-learners. When asked whether the behaviour of these learners was helpful or not, the response was that they were not:

> Trudy: *He's 'like this is easy, you just do it this way and that way', and it's just like, he gets real irritating sometimes… Not helping, 'cause he's just like making us feel like we're dumb.*

The behaviours of the experts were interpreted by the other learners as highlighting their inadequacies, evoking negative emotional responses. The behaviours also undermined the perception that all learners shared the experience of struggling with mathematics, which lowered the shared feeling of inclusiveness the learners said they had when they felt equal within a class.

6.2 Mathematical identities

The lower-skilled learners' descriptions of their experiences, relationships and ability with mathematics indicated identities of mathematical inadequacy. Reifying utterances were used to describe their relationship with mathematics and their perception of their ability. These utterances described their ability with mathematics as fixed:

> Kelly: *I don't know, I just don't get it, I just don't get, I just don't get maths very well.*

> Sonja: *I'm dumb.*

> Pita*: I'm bad at my maths too bro*

David: *Uh, coming into high school, I just couldn't hack it.*

Tina: *Like, I could say I'm smart, but I'm not.*

Moreover, the learners' relationship with mathematics was expressed as a stable relationship. For example, many learners when asked to discuss mathematics simply expressed their feelings toward it:

Kelly: *It just isn't for me. I hate it.*

The learners also clearly differentiated between mathematically successful groups and unsuccessful groups and firmly positioned themselves within the later:

Abbie: *And there were the overachievers and then us down the back.*

Trudy: *No, I didn't really succeed in maths. I tried it, but I just thought, Nah, it's not me, so, stuck to the basics of what I know.* **Yeah. I'm one of those ones** [Emphasis mine].

Trudy's statement "*I'm one of those ones*" was a common utterance type that indicated membership of non-mathematical groups. Furthermore, the learners' responses indicated a belief in a distance between themselves and the type of people they viewed as mathematically successful. For example, in response to the question "what does a successful maths class look like to you?" Niki stated:

Niki: *It would feel nerdy* [laughs].
Damon: *Okay, so you're feeling nerdy, right.*
Niki: *You'd feel nerdy, yeah. A big class of nerds, like brainy people. Yeah, um, I'd probably just hide in the corner somewhere.*

Niki distinguished between herself and the type of people that experience success in a mathematics class. Her use of the word "nerdy" did not appear to be derogatory but rather a term for "brainy" people, a group to which she felt she did not belong.

The learners' responses also indicated the belief that a lack of mathematical proficiency represents a personal deficiency. Not understanding mathematics was associated with notions of deficit, described in terms of a dichotomy between the

"smart" and the "dumb". This notion of being intellectually less able ran consistently throughout the interviews. A word analysis using the word "dumb" revealed that learners related mathematical proficiency with being "smart" and lack of proficiency as "dumb".

Table 8: Frequency and use of the term "dumb" by interview participants
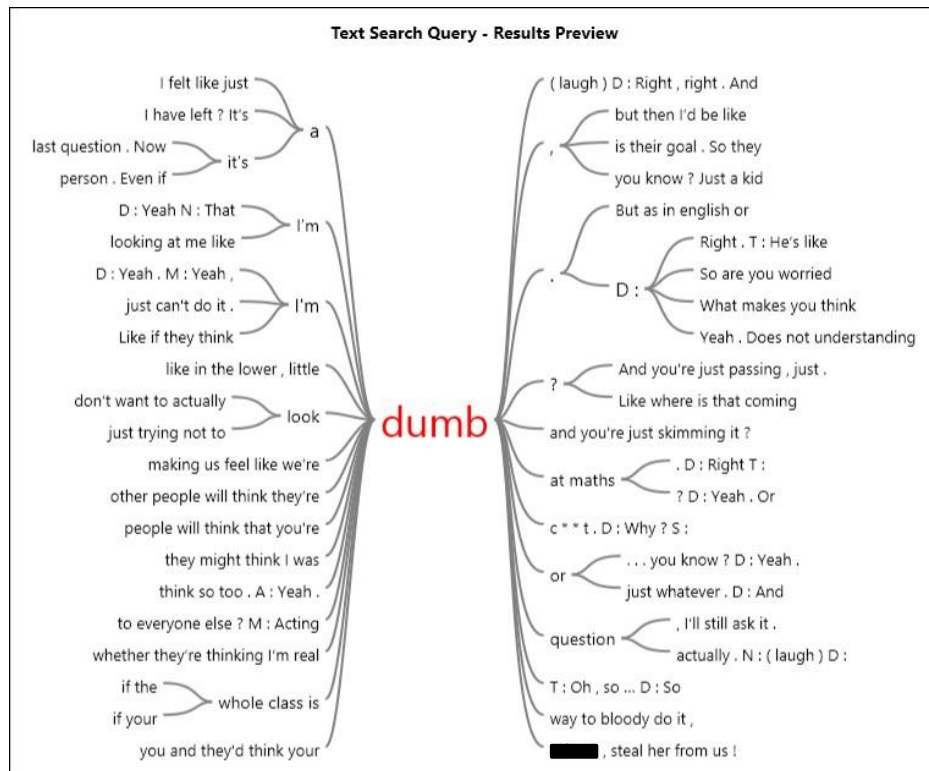
The term "dumb", above, was used in three distinct ways that aligned with the findings of this study regarding low-skilled learners' school experiences. Firstly, the learners felt dumb when exposed as not understanding or misunderstanding mathematics, particularly when the perception was that other learners did understand the material, for example, *"I feel like just a dumb c***."*

Secondly, learners perceived that others viewed them as dumb if they revealed their lack of understanding or misunderstanding in some way, for example, *"… looking at me like I'm dumb."*

Third, there was a tendency to avoid potentially exposing behaviours for fear of looking dumb, for example, *"Just trying not to look dumb".*

This was related to a recurring theme of feeling judged by others as inadequate and a subsequent allocation of attention toward avoiding actions that may be used by others to form further judgements.

Furthermore, learners' views of themselves were supported by negative experiences in mathematics classes. For example, Kelly provided evidence that she is a person in a mathematics class who simply does not understand the concepts.

> Kelly: *Just like, I don't know, I just never really understood where the class was, like what the teachers were going on about all the time, where everyone else would get it, you know?*
> Damon: *Yeah.*
> Kelly: *Or put their hand up 'cause they knew the answer and stuff, but I was just like* [makes a confused expression], *and I didn't really know.*

All the learners, except Troy and Hahona, positioned themselves as people who do not succeed at mathematics. Rather than being in a process of forming, or negotiating this identity, they had resolved that mathematics was not for them.

> Abbie: *That's where I kinda gave up on maths.*

> Mary: *So probably just checked out, probably, for most of maths then, in that environment.*

In contrast to the others, Troy and Hahona held positive math-identities. Both described themselves as "smart" and as high-achievers in school. They reported enjoying mathematics at school, and described their school position as within the "top" classes and as being able to help other learners:

> Troy: *'Cause I was a smart student in my school. Always in the top classes, top grades.* […] *I was always the first one finishing exams.*

In the same way, Hahona describes himself as being good at mathematics and known as being "smart" by his friends in school, and capable of helping them. This response was given to the question "Why do some people struggle with mathematics?":

Hahona: *Oh, here's the answer! They all struggled heaps, 'cause all my mates, they knew I was pretty smart, and I'd just be like, here bro, I want to go to lunch too, I know you want to go to lunch, copy my answers.*

He positioned himself as a helper to his friends, using his ability to ensure they were all finished in time to go to lunch. Both learners were observed engaging in similar behaviours in the observations. Their responses indicated that they were seeking to progress the lesson because of boredom and the desire for more challenging work. Both learners answered tutor questions quickly seeking to move the lesson forward and showed little sympathy for other learners:

Damon: *Are you trying to speed this process up*? [speaking of the lesson]
Hahona: *Yeah! Just so we can get onto harder questions, and we lost a whole page of work eh? 'Cause of people being slow. Whenever I can speed things up, I will I guess.*

Troy and Hahona also tended to attribute poor mathematics performance to controllable factors such as effort or motivation:

Damon: *Why do you think some people fail with maths or really struggle with it?*
Hahona: *Lazy I guess. Just don't want to try and find the solution, try and find the easy way I guess.*

Damon: *Why do you think some people fail with maths?*
Troy: *They don't find it interesting or it's just real hard for them.*
Damon: *How come do you think? Is there a reason?*
Troy: *Ah like with me, I always wanted to learn maths 'cause my Dad told me that maths would really be good for your future.*

Learner goals

The learners' identities had relationships with the types of goals they set for themselves. Both Troy and Hahona viewed numeracy and mathematics competitively. Their goals were oriented toward achieving at a high level and to some extent, out-performing others. Hahona described his goals in the following discussion:

Damon: *When you hear that you're going to have a numeracy class, what do you plan to get out of the lesson?*

Hahona: ***Kick some ass!*** *That's my answer for everything like, when I started this school, like my goal for the month was, kick some ass.*

Damon: *What does kicking ass look like?*

Hahona: *It's like me being... the best I can. Like, yeah.*

Damon: *So, it means winning?*

Hahona: Oh yeah, all the time, always gotta win.

Damon: *And what does winning look like in a maths or a numeracy class?*

Hahona: *A plus plus [A++].*

However, where the two differed was in respect to the horizon of their goals. While Hahona's goals were somewhat future oriented, Troy's goals were oriented toward the present:

Damon: *Describe your goals, when you come into a numeracy class, what are you trying to achieve?*

Troy: *um…to get all the answers right.*

The goals of the learners with poor mathematical identities were almost always framed in terms of meeting the present demands of the current lesson, rather than the development, or the application, of skills. These goals were expressed in a variety of ways but most often oriented toward meeting immediate performance goals, described by one learner as '*getting through*'. Secondly, the utterances indicated a somewhat fatalistic view of meeting the goal or not, as though being able to complete the work was dependant on the material, rather than the learner:

Kelly: *I just hope that every time I go and do a maths lesson, I just hope that I'll be able to do it or, that I'm going to be able to understand it 'cause I never know what's coming, you never know what maths you're going to learn that day so you never know what's coming.*

Abbie: *My goals would be to achieve what there is to do. If I needed to have a certain amount right, then I'd aim for that.*

Niki: *Um, I don't really set any goals. Nah, I just come in and do the work and carry on.*

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> Trudy: *'Cause sometimes it goes in my ear and out the other. So trying to focus, trying to complete what I'm asked to do, um, yeah, complete.*
>
> Damon: *Talk more about being able to complete what you've been asked to do.*
>
> Trudy: *Like try to complete whatever they throw at me.*

These utterances also reiterated the reified nature of the learners' mathematical identities. The comments indicated that the learners did not see their skills as improving, but rather hoped that they would simply possess the skills required to complete immediate work. Subsequently, the learners' goals appeared to be oriented toward meeting classroom criteria, not becoming users of mathematics.

6.3 Mathematical beliefs

Learners were asked to elaborate on their responses to the mathematical and epistemic belief survey reported in Chapter 4. A statement read from each of the item scales was used to initiate the discussions. A thematic approach resulted in six themes, each presented below.

Fixed and incremental beliefs

When asked directly whether some people are born smarter than others, most learners rejected the idea of innate intelligence and indicated incremental views of intelligence:

> Abbie: *I don't think people are born smarter, I think they learn differently, so you're not inferior to them, really. If you learn, yeah, I think if you learn what you need to learn, you are smart.*

> Troy: *I think everyone's born the same, no one's born brainy-er than anyone else, I think. 'Cause when you're little you don't know how to do anything. I reckon it's the way the parents bring you up.*

Mathematical proficiency was typically attributed to factors other than innate intelligence, such as effort, the quality of teachers, learner attitudes, or upbringing.

> Sonja: *Like, say from a young age, if you're just going to keep practicing and practicing your maths, then I reckon you can get better but starting from a young age, if you don't practise your maths, then you don't practise, you won't get better.*

Damon: *Why do some people struggle?*

Niki: *I think a lot of it's to do with family, just your surroundings at home, you know sometimes you don't even want to go to school because people at home are keeping you at home.*

The learners believed that learning mathematics was able to make people smarter, rather than one needing to be smart to learn mathematics, although they did not believe they personally had profited from this effect:

Abbie: *I suppose if you learn, and work harder towards something, you will gain more knowledge. And then become smarter.*

Many learners expressed disappointment at not having worked harder to learn mathematics because of its perceived ability to raise intelligence.

The nature of mathematics

The responses to questions about the nature of mathematics aligned with those from the open survey question "What is mathematics?" emphasising numbers, arithmetic functions and equations. They also indicated the perception that mathematics was a static body of disconnected knowledge learned by memorising individual methods and processes. For example, learners often evaluated their own, and others' proficiency by describing which arithmetic methods they were able to perform.

Mary: *I was all right with adding and subtracting and multiplication, but division. That was my only main problem was the division.*

The learners framed mathematics in terms of being able to 'do' addition, subtraction, multiplication and division rather than 'use' these functions to complete day-to-day tasks or complete problems. Being able to 'do' the procedure or method was considered the whole of the mathematical knowledge, with little thought given to underpinning conceptual understanding.

Damon: *How do you know if someone is good at maths?*

Troy: *Oh…if they know how to plus and minus.*

Damon: *Right*

Troy: *Subtract and add.*

All the learners stated that most word problems could be solved by using step-by-step procedures and that people who were good at mathematics knew the procedures. This aligned with the learners' beliefs about the nature of mathematics as a static body of knowledge.

Question: Most word problems can be solved by using the correct step-by-step procedure?

Abbie: *Yep. If you follow it.*

Niki: *That's the only way I can work it out.*
Damon: *Do you think **any** problem can be worked out if you know the right step-by-step process?*
Niki: *Yeah* [Assertive confident tone].

Beliefs about mathematics as a set of procedures related to notions of having to have each step correct in order to solve problems:

Kelly: *Yeah well in maths it just takes that one little thing and then it makes your whole answer wrong, you know? So, you've got to get it right, every single time, to work it out.*

The idea that procedures were the sum of mathematical content was also evident in attitudes toward calculator use held by many of the learners. The knowledge of the procedure used to solve any given problem was viewed as mathematics; therefore, using a calculator was viewed as subverting this. For example, Tina was asked why she did not use a calculator:

Tina: *Just to use my brains.*
Damon: *Why don't you like calculators?*
Tina: *Because it's like cheating.*
Damon: *How come?*
Tina: *Like the numbers are like right there on the screen.*
Damon: *So, using the calculator stops people thinking through it properly, is that the* [idea]*?*
Tina: *Yeah.*

Damon: *How do you know if someone's bad at maths?*

> Trudy: *Someone who doesn't listen. That doesn't try. Who just sits there and just can't be bothered. Just maybe cheat with a calculator, write the answer in.*

This view that calculators inhibited thinking did not relate to algorithms. Knowing how to use an algorithmic procedure to solve a mathematical or numeracy problem was highly valued.

In summary, the learners' beliefs reflected a limited experience with mathematical content outside of arithmetic and viewed mathematics as a system of procedures that could be selected and used to solve problems. There was no mention from any learner that mathematics could be used as an investigative or exploratory tool.

Conceptual understanding

Most learners indicated that understanding mathematics was something they valued and believed to be essential for further mathematical proficiency. However, the learners' notions of the term "understanding" were synonymous with "knowing a method" for solving a problem, rather than developing conceptual understanding. They were unable to distinguish between procedural knowledge and conceptual understanding, described as "knowing why and how" versus "knowing a method". Hence, questions that contrasted "understanding" with getting answers correct such as, "Which is more important in maths, understanding the question or getting the right answer?" were often difficult for learners to answer. For example:

> Damon: *If you're answering the questions correct, is that a sign that you understand what's going on?*
> Sonja: *Yip. Well obviously, eh, if I'm getting the answers correct.*

Another learner stated, "*I don't know how you could answer a question without understanding it*". Further probing revealed the belief that one could either solve an addition problem or not by using a procedure, there was no mention of reasoning through to an answer. Therefore, while learners often stated that it was more important to understand mathematics than "just" answer question correctly, "understanding" had no deeper meaning than "apply a method correctly". For example, when I described to David that a person might have the understanding to solve a problem without knowing the arithmetic procedure he stated:

> David: *It's like, I don't know, maybe if it was a small problem, you could get away with it, but you need to know how to do it,* **you need to know the**

method, so you can do it. I don't know how, you know, without the method there'll be things you can't work out without knowing how to do that, I think. [Emphasis mine]

In the episode below Niki described that understanding is important to get the right answer:

1. What's more important in maths, understanding the question or getting the right answer?
2. Niki: *Both*
3. Damon: *How come?*
4. Niki: *'Cause then if you get the same question again, you'll want to understand it the next time. But I think that getting the answer's more what everyone thinks about. You know, getting the answer.*

Further discussions with Niki confirmed that her utterance in 4 ("*you'll want to understand*") related to "knowing how" to solve the problem, not understanding "why" it works. Moreover, her desire to understand appeared to be motivated by the need to get her answers correct, to satisfy her need to progress through her workbook:

4. Niki: *But I think that getting the answer's more what everyone thinks about. You know, getting the answer.*
5. Damon: *Right, how come?*
6. Niki: *Um, 'cause you want to get it right. Yeah, 'cause you want to get the right answer so you can get that tick in your book, you're like yeah!*

Tina had no doubt about her preference for correct answers over understanding. When asked whether 'understanding' was more important than correct answers responded:

Tina: *Getting the right answer is more important.*
Damon: *How come? Tell me why.*
Tina: *Like, if you don't get the right answer you'll fail… Like, you're a big failure if you don't get the right answer.*
Damon: *So, getting the answers right means you're succeeding in maths?*
Tina: *Yeah.*

While many of the learners professed to value understanding above correct answers, all the learners in the classroom environment acted as though obtaining correct answers was of most value.

How mathematics is learned

Learners were asked how they went about learning mathematics and the types of strategies they used to do so. The responses revealed that the learners had a limited repertoire of learning strategies. Several were unable to provide any strategies used in their current learning situation or any historic learning situation. Others described some strategies; however, these were almost always limited to rehearsal or passive listening strategies:

> Damon: *When you're learning maths or numeracy what kind of strategies do you use, or have you used, to try and learn the material?*
> Kelly: *Dunno. I try not to try and learn maths* [laughs]

> Damon: *How often do you study maths outside of class time?*
> David: *Never.*
> Damon: *Okay. Do you plan your study before you begin?*
> David: *No.*

> Damon: *How do you go about learning numeracy or maths?*
> Tina: *I have no idea.*

Learners who did provide accounts of how they went about learning mathematics mentioned strategies that focused on listening to content or repeating methods covered in class. The primary strategy was that of listening to the tutor:

> Damon: *How do you go about learning mathematics or numeracy? What strategies do you use?*
> Mary: *What strategies? Listening.*
> Damon: *Great. Listening. Listening to who?*
> Mary: *Tutors, to the tutors.*
> Damon: *And you also mentioned earlier that you sometimes asked other people?*
> Mary: *Yeah, listening to friends, asking the other students. Listening to how they work it out.*

Moreover, as with the open survey question, in which learners were asked to give advice to a friend regarding how to learn numeracy, the answers centred on listening and concentrating on the tutor:

Damon: *If you were to give a friend advice on how best to learn maths, what would you say?*
Trudy: *Um, I would tell them to listen, listen to the tutor or the teacher and don't get distracted.*

Other learners tended to focus on repeating problems to learn or memorise how to do a specific type of problem:

Sonja: *Yeah try to memorise it and stuff like that, memory isn't the greatest, but I try.*

Damon: *How do you go about learning mathematics? What strategies do you use?*
Abbie: *What strategies?* [whispered to herself and thinking]. *I like to repeat, like I'll do it once, then do it again, to make sure I know how I got that answer.*

Abbie: *So I think every time I sat a test at school, I'd cram the night before, and the morning of. 'Cause then I'd be like: alright I did that this morning, I know what I'm gonna do now.*
Damon: *When you're cramming, what does that look like? What are you actually doing?*
Abbie: *Oh, when I used to cram, I'd just have a piece of refill and I'd just write.*
Damon: *So, it's like blank.*
Abbie: *Blank, and then I'd put equations on it that I know I'm gonna do, and then I'd do them more than once.*

Most responses from the learners indicated little orientation toward unpacking problems or seeking to understand how they could be solved. Rather, they emphasised being able to reproduce a method that could be applied directly to a specific problem. The strategies were limited to the types of behaviours likely to be inculcated in classrooms that use a transmissional pedagogy. The responses reflected a belief in a 'watch and repeat' model of learning in which the goal is to reproduce the teacher's method to solve specific types of problems.

The learners were also asked whether they planned their study in advance and if so how they plan for mathematical content. No learners reported making any plans for learning content. Moreover, none of the learners planned to review material covered in class in their own time:

> Damon: *Do you plan your study before you begin? Ever made a study plan?*
> Niki: *Mm, no.*

> Damon: *When you know that you have to learn a certain thing, do you ever plan out your study, how you're going to learn?*
> Mary: *Not that I can remember!*

The responses indicated a belief that the educational institution, tutor and classroom activities would provide the structure and conditions through which the learning would happen. While learners did take responsibility for learning, their responsibility only extended to being present both mentally and physically. A statement made by Trudy summed up the learners' approaches to learning:

> Trudy: *Like, write out ways to solve it the best way that suits you, ask them for help* [the tutor], *and don't hang out with the naughty people that don't listen.*

Learners were asked what actions should be taken if they did not understand information on first hearing. Most learners stated that "help" should be sought from the teacher, typically in the form of repeated explanations. The only alternative strategy mentioned by one learner was to re-read the content. Otherwise, all the responses indicated beliefs that knowledge is transferred via clear explanations from an expert to a learner rather than from self-engaged meaning-making. For example, what should somebody do if they don't understand information the first time they hear it?

> Trudy: *Ask the tutor to repeat it so that they understand it again and if they don't maybe get the tutor to come over and, one on one time. To repeat it until you get it.*
> Mary: *Ask for help.*
> Damon: *Right. From who?*
> Mary: *From whatever class they're in. The tutor.*

This aligned with learners' statements regarding perceptions of good teachers. Good teachers were described as those who explained content repeatedly until the information was understood.

> Damon: *What makes a good tutor?*
> Abbie: *Somebody who helps you, really clear on the explanation of the problem that you're doing. And explains how you get your answer as well. Yeah, somebody who takes the time to teach you properly.*

When the learners were asked what they did when they did not understand content even after the tutor has explained it repeatedly, tended to accept this rather than seek a solution:

> *Trudy: I'll probably be like, I don't get this and then see what they say and I'll tell the tutor that I don't get it and then I'll move on and then I'll come back, and **if I don't get it I'll just leave it*** [Emphasis mine].

These responses contrasted with Hahona's who indicated that he believed understanding occurs in response to making connections, and/or recognising relationships between ideas. For example:

> Hahona: *So hard* [speaking of a not understanding a geometry table]. *But then I got half way through the year and then started noticing the pictures, like the pictures in here, the same like on our equation sheets and stuff and I was like nah, yeah, that angle is the same as that angle and stuff like that. I'd get it like that and I'd be like yeah, that's cool.*

This attribution of understanding to personally recognising relationships and making connections was in stark contrast to the other learners' dependence on repeated teacher explanations. Moreover, the statements suggest that Hahona viewed understanding as taking place over an extended amount of time, rather than during any single exposure to the content.

The notion that learning ought to result from hearing information presented challenges for the learners. The pace of the current delivery was a recurring theme during discussions, with many of the learners stating that the pace of their current course was too fast to learn the content in the time given. This extended to school also as a key attribution to a lack of learning, regarding teachers moving too quickly

through content. This pattern of being reliant on tutor explanations for understanding was evident as a source of frustration in current programmes. For example, learners described their frustrations with the pace of course content:

> Niki: *Like we get it explained to about the colour, it's just like fast and you've got to take it in all at once and you've only got a little amount of time and you just gotta get it done, sort of thing.*
> Damon: *Is that how it comes across?*
> Niki: *I find this course is very rushed. I think a lot of things is quite rushed.*

> David: *It's just, that we um, like how he does it* [the numeracy tutor], *is like he writes it down and explains it and then boom, that's it. And then we're supposed to learn another thing.*

In summary, the learners' belief that a failure to understand mathematical content quickly combined with the belief that understanding happens by listening to explanations created a dependence on the tutor. The beliefs also appeared to contribute to negative affective responses if learners failed to make sense of the content the first time. The learners' sense of frustration was evident and related to feelings of being overwhelmed by content and convinced of their own mathematical inadequacies.

Time to solve mathematical tasks

Learners were asked how long mathematical tasks should take to solve and how long they themselves worked on tasks. Two primary findings emerged. Firstly, learners believed mathematical tasks should be completed within a short timeframe rather than being worked on over longer periods of time. Secondly, while the learners believed that *others* ought to spend a reasonable amount of time on tasks, they themselves did not.

The learners' responses to how long they believed a task should take to solve ranged from three to 20 minutes. However, any time-frame beyond a few minutes was considered atypical. There was no reference to working on a problem over the length of a lesson, or over several sessions, days or weeks. The responses also suggested that the phrase "maths problem" was likely interpreted as single answer problems, possibly within the domain of arithmetic:

> Damon: *What's the longest you've ever worked on a maths problem?*

Abbie: *Quite a long time… It depends if it was, um, adding, it probably wouldn't of taken me that long, if it was dividing, divided by, it would of took me a while, 'cause I'd still be trying to work it out… Um…probably three or four minutes, maybe longer.*

Trudy: *Mmm, not that long, maybe a few minutes.*

Damon: *How long should a maths problem take to solve?*
Anna: *Not long* [laughing]

How long do you think a maths problem should take to solve?
Kelly: *Probably pretty fast, like some people can do it just like that* [clicks fingers]. *Or like five minutes, ten minutes, they've got all their working down.*

Tina mentioned that tasks that took 'too long' would cause her to switch off.

Damon: *How long will you work on a maths problem, before you decide to switch off?*
Tina: *Maybe say five minutes. I look at it, and if I can't do it, I'll push it to the side.*

Learners were also asked about their persistence when completing tasks in their lessons. Most indicated that they quickly became frustrated by tasks that did not yield to their efforts and subsequently disengaged from the task and the wider lesson. Additionally, the learners expressed a sense of futility in persisting independently, and, as with the previous section, sought support from the tutor:

1. Damon: *Maths problems that take a long time, don't bother me. Agree or disagree?*
2. Abbie: *Oh, I get frustrated if they take me ages.*
3. Damon: *Okay, how long is ages?*
4. Abbie: *Fifteen, twenty minutes. If you're stuck on a problem for that long, then you need help.*
5. Damon: *Tell me about the frustration.*
6. Abbie: *I think you just sit there and you're like, I can't get this, you don't know what you're doing, so why bother.*
7. Damon: *Yeah. So you're either going to get it in what, a certain amount of time or* [Abbie cuts in]

8. Abbie: *I just think you're sitting there for such a long time and you're thinking, thinking, thinking and you just get p****d off. You're like, I can't do it.*

This series of utterances from Abbie revealed a pattern in which she evaluated her performance, made a judgement on her ability to be successful, followed by disengagement. Abbie's comment in line 4 revealed her negative belief about her ability to solve problems through exploration and persistence; "*If you are stuck on a problem for that long, then you need help*". She articulated her conclusion in line 6; "*… you're like, I can't get this, you don't know what you're doing, so why bother.*" Abbie's final utterance in line 8 revealed something of her internal dialogue, in which she concluded "*I can't do it*".

The learners reported a rise in negative affect when unable to solve tasks quickly:

Anna: *I'm gonna sit here and get shitty.*
Damon: *And then once you're frustrated?*
Anna: *Yeah, just push it aside.*

Damon: *How long do you think you'll sit there and work on it?*
Kelly: *Probably not very long, like I get frustrated:*

Sonja noted that if she was interested in the topic she would seek help from the tutor. However, if she was not interested she would likely disengage.

Sonja: *Maths problems that take a long time don't bother me. Depending on if I'm getting it or not. If I don't get it then I just get really bothered with it, frustrated.*
Damon: *And how long does that take you, roughly?*
Sonja: *To get frustrated? Ah, if I'm not getting it, it'll be very, very short. At the time I just don't want to know about it, if I've got heaps of people around me, I just don't want to know about it, and then if I'm really into it, and interested, and wanting to know what it is, then I'll try to have a one to one with the person that's teaching it, so I can get it.*

These responses had connections with the beliefs that understanding should happen quickly and that it occurs primarily in response to verbal explanations of content. The learners appeared unaware that mathematical knowledge could be generated from their own exploratory efforts or that they could act as their own source of

mathematical knowledge. Thus, problems that could not be solved quickly were regarded as unsolvable without external support:

> Trudy: *They do bother me* […] *'Cause it's like, solving it takes for ages to solve, and you gotta do this and this and this to get this answer and you got to solve it and I prefer just to use a calculator.*

Tina noted that when she became aware of the amount of work required she tended to switch off because the content was demanding and uninteresting:

> Tina: *I switch off when it's written on the board, I switch off.*
> Damon: *Do you? How come?*
> Tina: *'Cause I know they're gonna do a lot of work and I just, switch off, nah.*
> Damon: *Tell me about times when you don't switch off, what makes that different?*
> Tina: *Oh when, when maths is like interesting I don't switch off, like when it's something easy and I need to learn it then I don't switch off.*

The learners saw little value in spending time on tasks that were difficult to solve; instead they described ways to minimise the work or disengaged from the tasks. However, a contrast was evident in the responses of Troy and Hahona. Hahona at first appeared to describe a similar pattern of frustration and disengagement when confronted with a task he is unable to solve quickly. However, his comments did not include notions of failure, the need to seek support from a tutor, or of disengaging from the task. If anything, he seemed determined to return and solve a difficult task at any cost:

> 1. Damon: *Maths problems that take a long time don't bother me – you disagreed. Tell me about that.*
> 2. Hahona: *'Cause they do bother me! They bother everyone. Isn't it frustrating how you know you can get an answer but you can't get it at that time? You're just like thinking, man, I know this, but I can't get it. If it takes too long them ah man! Sometimes I just skip to the next one, and then go back to it later on.*

The key difference may be found in his use of the phrase "*I know this…*". Hahona exhibited confidence that he possessed the skills that would allow him to solve the problem given enough time. His frustration stemmed less from the difficulty of the

task and more from his temporary inability to bring his skills to bear on it. His response reflected high levels of agency in the face of difficulty. Similarly, Troy clarified a key difference between himself and the others when working with time consuming or difficult tasks:

> Damon: *Maths problems that take a long time don't bother me?*
>
> Troy: *No, they don't bother me, it's like, a challenge. Like I wanna get you done.*
>
> Damon: *That's cool, a great attitude. What's the longest you've ever worked on a problem?*
>
> Troy: *Maybe twenty minutes. Fifteen minutes [laugh], trying to figure out ways, and how to do it and stuff.*

Troy's utterance "*trying to figure out ways, and how to do it and stuff*" is in stark contrast to Anna's "*I'm gonna sit here and get shitty… Yeah, just push it aside."* One suggested a sense of self-agency while the other resembled behaviours associated with learned helplessness.

The usefulness of mathematics

The learners' beliefs about the usefulness of mathematics were nuanced. They all stated that mathematics is and will be useful in their life's work. Most examples related to day-to-day activities such as counting money or doing the shopping:

> Abbie: *Maths is involved in everything.*
>
> Damon: *Give me an example.*
>
> Abbie: *When I was working at the butchery I needed to know how much stuff weighed, you couldn't go over, worked at Pak'n'Save, so, when you're on the tills you need to know how much change to give back. Working at McDonald's you couldn't short-change people you had to know how much stuff was.*

> Anna: *Maths will be important in my life's work. Maths, I just reckon maths is important there, you know, like money and everything.*
>
> Damon: *How useful is the numeracy that you're learning on your course?*
>
> Anna: *Um, useful. Yeah, very useful especially if you want to do childhood education.*

However, several learners stated that the usefulness applied primarily to 'basic' maths and less so to more complex domains. For example, algebraic concepts were raised repeatedly as never having been useful in their lives.

> Troy: *I've never used it ever [*algebra*]. I've always tried to like, test my little brother and sister, only you know, little simple ones that I could remember, like simplifying A plus B.*

> Kelly: *Like I kind of think that too, like to me maths has never been important, never really needed it, apart from the basic maths, which I sort of know, with money adding money you know? That's all I really use maths for. Like, I haven't really needed it yet, as yet.*

> David: *Nah, I think maths is important to my, to life's work.*
> Damon: *Would you say for your life, it's been important, or not?*
> David: *Uh, basic maths has been. I don't, I'm not, I don't do algebra ever, so far.*

In summary, all the learners believed mathematics was important to their lives. However, learners limited this to "basic" arithmetic and disassociated these skills from other aspects of mathematics such as algebra.

6.4 Discussion

The results reveal two divergent experiences of school mathematics and the current mathematics provision. For Troy and Hahona the experience was, and is, socially and academically positive. They felt connected, valued and that they were achieving well. There was little in their interviews that indicated concerns regarding how others perceived them, and there was no mention of poor teaching, poor attitudes or disengagement. In contrast, all the other learners described the experience as unpleasant, fraught with social tensions, and culminating with disengagement and exclusion from successful groups and pathways. Their accounts were consistent with a collection of negative experiences identified in previous research that illustrate the ways in which the school experience impacts learners' beliefs about themselves, their ability to do mathematics and their inclusion or exclusion from educational institutions in general (Brown et al., 2008; Darragh, 2013; Lane et al., 2014; Noyes, 2006; Siivonen, 2013).

Learner challenges re-engaging with mathematics

The learners' accounts of school aligned with research that shows that the transition through high-school mathematics perpetuates existing disadvantage or advantage, resulting in diffracted learning trajectories by the time learners exit (Noyes, 2006). Consistent with other studies, the quality of the teachers featured strongly in the learners' accounts of school (Coben, 2002; Lane et al., 2014; Singh, 1993). Unfortunately, most described their relationships with teachers as combative or estranged. Troy and Hahona's accounts indicated positive relationships with the institution of school and with their teachers, who they described as supportive, secure and trusting. For example, Hahona's account of his relationship with the teacher "*My teacher trusted me for doing that. Like, he'd let me go to PE and stuff from his* [mathematics] *class 'cause he knew I'd get the job done…"* was in stark contrast to Kelly's view that her teacher had decided not to '*waste her time*' with her. This is problematic because the learner's perception of their teacher's expectations for them, and their sense of trust, are keys factor related to positive classroom interaction (Wentzel, 2002).

The decreased engagement with mathematics expressed by the learners indicated a growing gap between mathematical practitioners and non-practitioners. Considering a body of research suggests that the role of mathematics instruction is to induct learners into a practice of mathematising (Boaler, 2008; Grootenboer & Jorgensen, 2009; Lerman, 2009), the exclusionary nature of the learners' accounts is concerning. Their comments reflect their perception of being excluded from this process of induction, and alternatively inducted into a community of non-practitioners, a process described in several studies (Siivonen, 2013; Solomon, 2007). The learners talked of being part of disengaged groups such as the "*naughty ones*", "*the group sitting at the back*", and "*the worst class*". Given that adolescence is a time when "belonging" is so essential to identity development (Turner et al., 2002), these learners were experiencing exclusion from successful pathways and subsequent submersion into lower-achieving classes, both of which have been identified as devastating to learners' identities and outcomes (Solomon, 2007).

The learners were acutely aware of and sensitive to the thoughts and judgements of others, both in school and in their current classes. This included the perception of being judged as 'dumb' for failing to meet classroom expectations for proficiency. These fears appeared justified because the learners recalled being called derogatory names, such as "dumb", or being stigmatised as "cabbages", a derogatory slang term that positioned the intended audience as intellectually deficient. Unfortunately, the

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perception of being judged and positioned in derogatory ways by other learners is not unique in the mathematics education literature (Brown et al., 2008; Darragh, 2013; Evans, 2000; Zevenbergen, 2003). The results expand on Bibby's (2002) findings by showing that it is not only pre-service teachers who are extremely concerned with the shame that accompanies negative judgement, but also lower-skilled learners in vocational settings. They also support Tennant's (2012) findings in which the fear of being perceived as ignorant reduced adult learners' participation, and consistent with Darragh (2013), who found high school students were keenly aware of others' thoughts toward them and were often intimidated by the proficiency of other students.

The belief that because they were adults they ought to be proficient in mathematics, or at least more proficient than younger learners, added to the participants' feelings of inadequacy. Sonja's rationale for her statement "*I should know this stuff*" was that she was older than the others and as such had a social obligation to know more than them. Coben (2002) found similar attitudes among adults with lower skills, who felt that knowing mathematics was an expectation for adults. The findings contrast somewhat with earlier reports that there is less stigmatisation associated with poor mathematical skills (White, 1974). Perhaps this reflects changing social expectations. Recent evidence suggests that there are social disadvantages to being perceived as having poor mathematics skills, due in part to their being taken as a proxy for intelligence (Blackwell et al., 2007). Intelligence has social value because it is thought to improve future status (Räty et al., 2006). To quote Siivonen (2013, p. 516) "Learning mathematics is related to individual conceptions of ability in an intrusive way that has consequences far beyond ability to learn the subject". A consequence for the learners in this study was increased apprehension based on unrealised social expectations for performance.

The notion that learners "*should know this stuff*" was also evident in their hesitancy to be negatively judged by peers and their subsequent avoidance of active roles in class. The learners reported resisting making comments in class or solving problems on the board, sitting at the back of the class disengaged, or doodling to pass the time. These findings support observational research in adult classrooms that finds adults do reduce participation to avoid revealing their lack of knowledge (Howard & Baird, 2000; Howard et al., 2002; Tennant, 2012).

The learners attributed a reduction in feelings of inadequacy to the class's shared skill levels and problematic histories with mathematics. They noted that at school the skill differences between themselves and other learners was potentially

embarrassing, but that this was less in adult classes. Lewis (2013) found that learners felt nervous and frustrated when they thought all the other class members could complete tasks, yet they could not. While little direct research could be found that explored lowered feelings of shame due to feelings of shared skill levels, Tennant interviewed an adult learner who felt less anxiety because "*everyone is on the same page*" in his college mathematics class (2012, p. 30). This is similar to Pita's statement, "*We're all on the same boat*", and raises questions about why this is so. One reason may be that the shared history of difficulties lowers the expectations for performance. Research on ability grouping in schools has identified that the high expectations for learners in higher ability groups creates pressure that inhibits performance (Boaler, 1997; Solomon, 2007; Zevenbergen, 2003). One learner noted that she felt "dumb" when asking questions because the teacher expected her to know the answer (Zevenbergen, 2003). It appears that the learners' perception of the expectations of others for their performance contributes to experiences of shame.

The shared understanding of the difficulties of mathematics, and subsequent lowered expectations, was eroded by the actions of a few higher-skilled learners. The feelings of inadequacy that learners had described from their school experiences, were again described as occurring in their current classes, for example "'*cause he's just like making us feel like we're dumb*". The behaviours of these learners were cited as creating an environment in which learners again felt inferior and inadequate. There is little research on the effect of the behaviours of higher-skilled adult learners on the engagement of other learners, yet the impact appeared substantial. While there were no occasions in the observations of learners explicitly putting others down, there were comments that could be taken as condescending or simply embarrassing. For example, Hahona's helpful public explanation to Trudy about how to add fractions following her unsolicited public exposure of an error appeared to extend the moment for Trudy, drawing further attention to her failure. It also established her role as "inferior learner" below Hahona, a "superior knower".

The learners reported feeling negative emotions in their classes that were similar to those experienced at school, albeit to a lesser extent. While several of the learners stated that they were less concerned with the thoughts and opinions of others as adults, they also stressed that they felt negative emotions when asked to demonstrate knowledge in public, and were concerned with how their performance would be perceived by the others. The feelings described, such as nervousness or concern with looking dumb, were almost identical to those described in school. For example, there was symmetry between the feelings Mary described from her school

experience and her numeracy class. Speaking of school, Mary said "*And sometimes they'd [the teachers] pick the kids out to go and work it out, that was a bit of a nerve rush for me*". Regarding her current class Mary stated that she was hesitant to answer questions because she was nervous of both the tutor and the students thinking that she is "dumb". The persistence of negative feelings experienced in secondary school into adult environments is well established (Evans, 2000; Wedege & Evans, 2006). This research suggests that despite some adults stating they care less about the opinions of others as adults (Tennant, 2012), the concern with others' opinions appears to persist into adult environments, and constrains participation.

Mathematical identities

The learners' identities were consistent with those who have had particularly negative experiences with mathematics (Brown et al., 2008; Coben, 2002; Coben & Thumpston, 1996; Evans, 2000). Their accounts reflected an either/or view of mathematical performance dichotomised between inadequacy and ability. They situated "smart", "brainy" or "nerdy" people on the one hand, while others were described as "dumb" or "stupid" on the other. This is consistent with previous findings that have included lower-skilled learners (Bishop, 2012; Brown et al., 2008; Mendick, 2005; Mendick & Moreau, 2014). Unfortunately, apart from Troy and Hahona, the learners identified themselves firmly as members of inadequate groups. Abbie illustrated the distinction in her comment "*there were the over-achievers, and then us down the back*". Reifying utterances that demonstrated identifying language were used repeatedly. The utterance by Sonja*, "I'm dumb"* described a crystallised state, suggesting she viewed this as an unchanging state and therefore an "actual" identity (Sfard & Prusak, 2005). The utterances also meet Sfard and Prusak's criteria of 'identifying' language, in that the learners endorsed their identities through their narratives by providing evidence of their inability. Additionally, Kelly's account of how her teacher gave up on her, or Abbie's account of other students laughing at her, indicated the emotional impact and significance of such events.

The learners appeared to have constructed their identities, in part, through their interpretations of those around them. That the learners were sensitive to the opinions of others was evident from the frequent references to the thoughts and perceptions of others, such as "*They think…*", "*They're like, I'm better than you*", "*The girls would laugh...*". Sfard and Prusak (2005) noted that designated identities are created from the narratives "floating around us" (p.18), and discuss the influence of "significant narrators". That is, the voices of those we view as significant to our identities. The pronoun "*they*" was used repeatedly to refer to other class members. Coben and

Thumpston (1996) found that "significant others" played a considerable role in the formation of a mathematical identity, in their case an individual of personal standing. It seems that "they" are a very significant other. The opinion of peers featured strongly in learner discourse in this study, suggesting these voices carried weight with the learners and contributed powerfully to their views of themselves.

The learners' positioned themselves as not only poor at mathematics, but likely to remain poor at mathematics. Consistent with the beliefs of school students in lower-streamed classes, there was a distinct lack of positive expectations for their mathematics outcomes (Huak, 2005; Zevenbergen, 2003). Instead, the emphasis was on merely meeting the demands of individual lessons with no emphasis on 'becoming' or 'developing' into a mathematics user. Designated identities differ from actual identities, in that they reflect our *expectations* for what we might become (Sfard & Prusak, 2005). The experience of school for the learners in this study appeared to have damaged their expectations for mathematical success. Such identities have been summed up with the phrase "Maths – that's what I can't do" (Wedege, 2002, p. 63). The belief that they cannot learn can succinctly be described as 'identities of inability'. This may be compounded by Coben's (2000) findings that because the mathematics adults use is often perceived not as mathematics, but as common sense, mathematical success becomes almost unattainable. Learners believe they are unable to learn mathematics and pointing to the mathematics they do use as evidence that they can, is not accepted as evidence.

One of the concerning findings was the learners' classroom goals. The goal of developing their mathematical skills was not mentioned; rather learners' goals reflected the theme of "getting through" lessons with their current skills. Learner comments had a fatalistic nature, for example, "*…I just hope that I'll be able to do it…*" or "*trying to complete whatever they throw at me*". Mathematics classes then, had more in common with an exam environment, in which a learner's skills are tested, rather than developed. Again, this is likely a further implication of the learners' identities. If a learner believes they cannot be mathematically successful, then a mathematics class is likely to be viewed as a series of hurdles to overcome. Hadar (2011) found 43% of secondary students named 'completing classwork' as a key goal and conception of learning. Hadar summed up their goals as "*To work in class, do homework, and what the teacher tells you*" (p. 201). Similarly, the primary goal for learners in this study was not to develop mathematical skills to use in a future context, but to meet minimal compliance standards. These goals have been linked to

passive learning approaches (Biggs, 1987; Boaler, 2003), which are also apparent in this study.

Beliefs about mathematics

The learners' confirmed that mathematics was viewed as a fixed body of unrelated methods and procedures. They had difficulty with questions that suggested a difference between conceptual understanding and procedural knowledge. The learners' interpretation of the word "understand" as "knowing what to do" was consistent with Skemp's (1978) argument that a student who has learned a formula may argue that they *do* understand, despite not understanding "why". This was reinforced by the fact that the procedure allowed them to answer problems correctly, the primary goal of many of those in this study. As Skemp argues, the interpretation reflects the learners' inculcation into a particular "type" of mathematics. Given that instrumental approaches were a key feature of the learners' behaviours in the observations, and the overwhelming view in the survey data, it is evident that instrumental mathematics is the prevailing "type", and as such the learners' difficulties with notions of conceptual understanding are not unexpected.

Other than Troy and Hahona, mathematics was viewed as a series of unrelated procedures and operations. For example, the arithmetic procedures were described as separate pieces of knowledge that were learned individually. The belief that mathematics consists of isolated "bits" of knowledge rather than a system of integrated concepts is related to poorer mathematical performance (Paulsen & Feldman, 2005; Schommer-Aikins et al., 2005). Muis (2004) found this belief was common across all levels of school mathematics, but particularly in lower-performing students. This may be because beliefs regarding the structure of knowledge have been found to become more sophisticated (from isolated bits to integrated concepts) the longer learners are in education (Perry, 1968; Schommer, 1998; Schommer-Aikins et al., 2005). Given that foundation-level programmes cater to early school leavers, it seems likely that their limited experience with mathematics reduced their exposure to content that could lead to more sophisticated beliefs. Without an understanding of the related nature of mathematics there is little hope learners within imbedded provision will have the motivation to engage in conceptual development, particularly when discrete knowledge of methods continues to meet classroom demands.

The learners had a limited array of learning strategies that could be employed independently. The few examples of learning strategies that were given related to

complying with classroom norms such as listening when the tutor spoke, completing work, or repeating procedures to memorise them. The sole use of rehearsal strategies is consistent with the behaviours of learners who set goals related to recalling rules, facts or procedures (Briley et al., 2009; Dahl et al., 2005; Meyer & Parsons, 1996). Unfortunately, these strategies are found to be educationally unproductive compared to more effective deep processing strategies (Echazarra et al., 2016; Kilic et al., 2012). Other than Hahona, there were no references to other strategies such as exploring multiple solutions, making connections between ideas and prior knowledge, sharing solution-strategies or explaining to others.

Learners lacked any self-directed organising processes, such as planning, monitoring or regulating their activities and approaches. Learning was attributed to complying with traditional classroom norms and tutor instructions, abdicating the responsibility for organising learning to the institution and tutor. For example, Trudy's instruction to those wishing to learn was *"I would tell them to listen, listen to the tutor or the teacher and don't get distracted."* The act of listening was frequently referred to as the key mechanism by which learners learnt mathematics. This explains why learners mentioned avoiding associating with "naughty" students, talking, or engaging in off-task activities, as was evident in the survey open question "If a new student started your course and wanted to learn numeracy what advice would you give them?". The learners' survey responses indicated a complete dependence on the tutor's input, and so did the interview responses. These passive learning approaches were also evident when learners were asked what actions they took when having difficulty with a problem. Learners typically responded that they sought help from the tutor, and suggested that the tutor should repeat the content multiple times until understanding occurred. These behaviours resemble helpless behaviours, in which learners are completely dependent on external support (Agaç & Masal, 2017; Hadar, 2011; Yates, 2009). This may also explain why adult learners tend to value tutors who can patiently, and repeatedly, explain mathematical concepts (Coben et al., 2007). The reliance on understanding content as the tutor talked explains the frequent frustration expressed by learners at the fast pace of teaching. Most appeared to lack self-learning skills that could be applied outside of the classroom.

Unfortunately, all the lower-skilled learners indicated that they responded poorly to not understanding quickly and tended to disengage in frustration. The belief that they *ought* to "get it", in a moment, and their subsequent inability to do so, resulted in negative affective responses and disengagement. This included being unable to solve tasks quickly. For example, Kelly indicated her tendency to disengage from

problems in the face of difficulty "*Yeah, I just left them. 'Cause I just didn't get it… "'*. The belief that problems should be solved quickly, followed by disengagement when tasks could not be solved in a few minutes, is well established (Mason, 2003; Schoenfeld, 1988; Turner et al., 2002). In fact, this belief was the strongest predictor for low mathematics achievement in Mason's (2003) study of Italian high school students. The pattern suggests the negative reinforcement cycle in which learners experiencing failure come to doubt their intellectual ability, leading to beliefs that they are unable to overcome their difficulties (Sutherland & Singh, 2004). Trudy's comments "…*and if I don't get it I'll just leave it"* indicates downward cycle of engagement. Her disengagement eliminates opportunities to ever learn the content, while also consolidating negative beliefs.

The learners' emotional responses to such events were often described as "frustration" but reflected what might better be described as 'anger'. Abbie, for example, paraphrased her thinking when working on a task for an extended period "… *you're like, I can't get this, you don't know what you're doing, so why bother… You just get p****d off. You're like, I can't do it".* Emotional responses to mathematics such as this are a common research finding (Carroll, 1994; Evans, 2000). Yet the emotion of anger seems to arise more in adult contexts than compulsory education. Consistent with previous research (Evans, 2000), these responses also led to disengagement.

The negative responses described by the lower-skilled learners contrasted with Hahona and Troy who expressed an aggressively determined attitude toward solving challenging problems. Their approach resembled James, described in Chapter Five (p. 102), who when struggling to solve a problem became aggressive, yet remained focused on solving the problem. This supports research that finds learners with contrasting beliefs, such as that problems can be solved through persistent work, will act as affordances to working on challenging non-routine tasks (Francisco, 2013; Stylianides, & Stylianides, 2014). For example, both Troy and Hahona enjoyed difficult problems because they represented a challenge to be overcome. Troy even personalised the problem, "'*Cause I wanna get you done*". This reflects both learners' positive experiences working through challenging problems. Overcoming impasse and frustration leads to positive emotion once success occurs (Goldin et al., 2011). The experience of impasse develops greater control of emotion, thus more experienced and successful students, while feeling frustration, can control it (Allen & Carifio, 2007). As McLeod and McLeod (2002) note, learners can take a meta-affective perspective, in which they are aware of their frustration, but situate it in the

context of "doing maths" and as such do not become submerged in the emotion. They use the emotion to fuel attempts to solve the task. These findings help explain why James, Troy and Hahona, experience frustration, yet in a way that leads to greater engagement. Their frustration appeared linked to an expectation of success, rather than expectations of failure.

The learners strongly endorsed the usefulness of mathematics and gave examples of how it was used in their lives. The examples were generally in the context of working with money, and the uses given limited to basic mathematics, and in most cases, little more than counting, suggesting adults' beliefs about the usefulness of mathematics reflect their own conceptions of it. For example, the learners' descriptions did not include the broad range of uses, considered societal demands, described by Gal et al. (2005), which included aspects of numeracy such as dimension and shape, data and change, and patterns, functions and relationships. Much has been made of the need for these skills in daily life (McCloskey, 2007), yet the learners' comments suggested they were unaware of such uses. This may be related to Coben's (2000) findings that the mathematics adult use in life is often invisible to them, so they see these uses as "common sense" rather than "mathematics". It may also reflect the learners' limited experiences and opportunities to link school mathematics to daily tasks. For example, as other studies have found, algebra was repeatedly identified as a non-relevant component of mathematics (Brown et al., 2008), suggesting an inadequate intersection between the classroom context and the learners' actual activities. A range of research shows that what is learned in the classroom often fails to be transferred to a real-world application for normally achieving students (Reusser, 2000), let alone learners experiencing difficulties (Allsop et al., 2007).

The learners disagreed with the items that posited mathematical ability as innate and fixed. Dweck (2006) has suggested that attributing failure to innate factors may be a strategy to relieve the individual from personal responsibility. However, the learners in this study typically held themselves responsible for a lack of mathematical proficiency. While several learners were scathing of their school mathematics teachers, holding them partially responsible for the failure, they still referred to their own behaviours as a reason for difficulties. Therefore, the learners' responses to the innate/incremental items did not fit with much of the research that finds lower-skilled learners hold fixed beliefs about intellect (Blackwell, et al., 2007; Rattan et al., 2012). However, the learners' indirect comments did at times imply the notion of a fixed inability to learn mathematics. For example, appeals to '*just not getting*

mathematics', or '*I never understood it*', or even, '*brainier people are faster*' suggest that the distinction between the innate and incremental beliefs is not as clear as might be suggested from some research.

Conclusion

This chapter set out to understand learners' interpretations of their mathematical experiences, their beliefs and how these are related. The learners' configuration of beliefs appears to create a "catch 22" type situation for them. If they behave in ways consistent with their beliefs, such as listening to the tutor, rehearsing and memorising procedural methods, asking the tutor for help before developing their own understanding, and judging their performance by the number of answers they get correct; then they fail to develop agency and remain dependant on others. These behaviours were evident in the observations. Learners navigated the demands of their lessons, yet rarely exercised personal agency by attempting to solve a problem independently if it challenged their capabilities. While meeting classroom demands might satisfice classroom expectations, the practice may also be simultaneously consolidating maladaptive, non-agentic behaviours.

At the beginning of this thesis, it was suggested that learners who had experienced failure with mathematics and are now re-engaging as adults may be fundamentally different from those who haven't. The results of the previous three chapters support this. This is important, because almost all the 'beliefs' research has been conducted with learners in a voluntary educational pathway, suggesting they are self-motivated, and possessing longer-term learning goals. The results of this study indicate that the learners' configuration of beliefs contributes to negative behaviours, such as approaching learning of mathematics ineffectively, evaluating themselves using self-defeating measures, while also experiencing negative affective responses that disrupt engagement. The following chapter explores how low-skilled adults respond to a classroom environment that emphasises conceptual understanding, promotes positive beliefs, and more interactive approaches to learning.

Chapter 7: Intervention: Results and Discussion

Damon: Any other weird words that have been popping up?

Terry: Yip. Py, thag, or, us. I hate that bastard.

Damon: Do you? I've got a plan for Pythagoras…

Terry: I'm not interested in buying what you're selling. You're trying to sell me something I don't want.

(*Terry, 46-year-old engineering learner*)

This chapter investigates the third research question: How do low-skilled adults respond to a classroom environment that emphasises conceptual understanding? It reports on the findings of a ten-week teaching intervention through which I, as tutor and researcher, explored participant engagement patterns when mathematics was presented as an interconnected domain learned through inquiry, collaboration, discourse, exploration and personal meaning-making. The participants were adults who had problematic histories with mathematics and low mathematical skills, and who had re-engaged with mathematics as part of a foundation-level engineering programme. The mathematical content was a barrier, and, as will be shown, their existing beliefs about their ability to learn mathematics, and how it ought to be learned, perhaps constituted an even greater barrier to their success.

Data were collected through multiple audio-recording devices, direct observation, field notes, limited video footage, surveys and interviews (see Section 3.4). These learners did not participate in the previous components of the study.

The findings are organised into four sections. While they overlap, each presents a unique aspect of learner engagement. The first section provides the context for the intervention by describing the learners' backgrounds, goals and beliefs. The second reports on the learners' engagement with mathematical provision that emphasised enquiry-oriented problems, the construction of conceptual understanding, shifting the responsibility to learners for their own self-management, and encouraged group work and mathematical discourse. The third section reports on the learners' affective responses during the programme, and the fourth on learners' post-intervention reflections.

7.1 Backgrounds and beliefs

This section describes the wider context of the programme, including the learners' educational backgrounds, the nature and influence of a mathematics programme the learners were concurrently attending, and their mathematical skills and beliefs.

The learners shared similar backgrounds to those reported in Chapter 6. Their mathematical education experiences were typically negative and included absenteeism, trouble with teachers and authority figures, low mathematical achievement and early school departures. The comments below are representative:

> Tyrone: *And I was in s**t in Year 9, and then, um 'cause before I went to the school it'd already started. Got to take a test and I just went through it quickly. To try and hurry up. And got in s**t, and then, yeah, our class was just so f**ken, we'd* [we had] *all the naughty kids. And no one used to listen to him, no one* [the teacher]. *But I felt sorry for him, I used to listen to him… Then he ended up leaving, then we got this other teacher, and she ended up leaving as well.*

> Terry: *I've probably had more schools than I've had f**ken shoes. Maths, I didn't, I had no time for maths. Straight up, I had no time for maths. Yeah 'cause I got expelled one time because I swore at a teacher, 'cause she tried telling me that a gurnard was a flying fish… Got kicked out for the day, for swearing at this lady teacher… I left school at 13.*

This extended to behavioural issues in the current programme also. The organisation catered to younger learners who had been excluded from attending public school, hence, many participants had similar histories. Several were on probation and were released from restrictions to attend the programme from 8:30am until 3:30pm. The learners had a reputation for being challenging and the lessons often included confrontational episodes between learners. Before the intervention was complete eight learners had left the programme: five for breaching programme rules, two because they had moved to other towns, and one who gained employment. Attendance also decreased as the intervention progressed, due to events occurring in the lives of the learners.

Mathematical identities

Consistent with the learners interviewed in Chapter 6 almost all the learners held low perceptions of their numeracy skills and these were reflected in poor mathematical

identities. Utterances that reflected these were made throughout the programme, particularly in the earlier stages. These were often in response to affirming comments from me, as self-talk, or during conversations with other learners.

For example, several lessons into the programme Rawiri, who had remained very quiet up to this point, answered his first mathematical problem in the programme. It was evident that his volunteering the answer took courage and I sensed his confidence increased at this success. I sought to build on his success by asking him to explain how he solved the problem:

> Rawiri: *Just draw them.* [Has solved the problem by drawing it]
> Damon: *Nice. How do you work out the area? 'Cause you're quite sharp with maths right?*
> Rawiri: *No, no, no.* [Assertive]
> Several learners [shouting]: *No he's not!*

My comment was sincere. I had observed and talked with Rawiri privately regarding his reasoned approach to solving problems. However, advertising his success was perceived as an overstatement by the other learners and they were quick to correct the situation. Learners often situated others as mathematically unable. For example, Denzel made a point to inform the class that Kerri had incorrectly answered 8 + 8 as 12. This was a simple error on her part, not indicative of an inability to add:

> Denzel [Speaking loudly to the whole class]: *Not like 8 plus 8 equals 12. This one here,* [pointing to Kerri], *you might need to teach her,* [teach] *this one adding. She said 8 plus 8 equals 12.*
> Kerri: *Hey I was tryna…*
> Damon: *What does it equal?*
> Kerri: *16!*
> Denzel: *You were way off!*
> Kerri: *'Course I'm way off, I can't even do it, f**kin' hell.*

In other cases, learner identities were evident from their own self-talk during the lessons:

> Terry [whispering to himself under his breath]: *Maths, not my strong point …*
> *Yeah, I'd put myself lower too. Number one* [referring to the lowest possible score].

Kerri [speaking quietly to herself]: *Really? Can't even count in my head let alone do that.*

Or, from comments to other learners about themselves:

Rawiri [speaking quietly to another learner]: *Back in the intermediate and primary days and maths and s**t and um, I was dumb.*

Damon: *Tell me about how you feel about the maths content on this course.*
Denzel: *Yeah, um … I'm lost. Yeah, nah I'm lost as.*
Damon*: Did that happen slowly or was that like the second you arrived, it was like full on?*
Denzel: *Um, I's never good at maths. I never spent any time at all in my math classes... I'm a complete idiot if you want me to explain in an easier way.*

These identities reflected the negative beliefs learners held about themselves and the lenses through which they evaluated themselves. Additionally, they dichotomised between "types" of people, and situated themselves as non-mathematical.

Kerri: *…and I was more of a hands-on person, and I didn't like anything that had to do with maths and science, but I loved sport.*

Some, utterances also expressed the notion of giving up, and disowning mathematics:

Terry: And here's me getting dragged up from town to town to town, and every different school has a different way of teaching. Well, I've been trying to get my head around all these different ways of teaching, and I just gave up. I just totally gave up on everything. Maths, I didn't, I had no time for maths. Straight up, I had no time for maths.

In sum, the learners' sense of not being mathematically able permeated the intervention and seemed to lead to a growing frustration with the mathematical content.

The influence of the concurrent mathematics programme

The learners attended their regular mathematics programme (RMP) concurrently with the intervention programme and had begun four weeks prior to the intervention. The pedagogical approach of the RMP class was transmissional, typified by tutor demonstrations of the applications of formulas, which the learners were expected to memorise through rehearsal. Learners sat in rows, listened and took notes. Additionally, the tutor took an authoritarian approach:

> Kerri: *Like, you can't talk over him if he's talking. And that's...*
> Damon: *There's only one person talking at a time?*
> Kerri: *Yeah, one person talking at a time, and if you talk, you get out.*

> Rawiri: *I ask for help, and still don't get it.*
> Damon: *Yeah? Do you ask, if you are in that zone?*
> Rawiri: *Yeah, sometimes. He just gets a bit angry when he has to repeat himself.*

Many of the learners expressed concern with their learning in the RMP, that they were not able to keep up with, or apply, the content. The learners' attitudes toward the programme were largely negative, with many expressing animosities toward it:

> Rawiri: *It's just hard, he like, 'cause he rushes it, he's like "write this down" and while you're writing it down, he just would've said everything when you are writing it down, and when you've finished he gives the next page… And all that, and then yeah, he doesn't explain it properly.*

Consistent with the findings of the observations and interviews, the speed of delivery was a recurring theme as a cause of difficulty. Additionally, learners were concerned that their lack of mathematical knowledge was impairing their ability to learn from the course:

> Jarred: *Nah, it's his teaching bro. He just goes way too fast. Way too ahead of us.*

> Kerri: *And I was just like, looking at all the um worksheets and stuff, that they had on trigonometry, Pythagoras and all of that. And then, 'cause they started teaching us, at first it was too fast. Because he'll be teaching us a subject*

every day. Like Pythagoras one day, ratios the next day, trigonometry the next day and it's just like, you can't just hold that all in your head.

However, other learners enjoyed the authoritarian approach utilised in the class, partly because it facilitated the completion of work:

Tyrone: *'Cause, work gets done. Like all the time Les* [tutor of another class], *will be f**king, like 'cause he doesn't, he's not that strict. And then a lot of others just f**k around, and then when we went to Vernon's class, 'cause he's strict, there… Yeah, and we all got the work done.*

Learner reports indicated that classroom work was more instrumental than in the observations and largely consisted of the learners watching demonstrations of formulas, then copying notes directly from the board into their books. Despite the learners' conceptual difficulties with content (see next section), the strategy employed by the tutor was to teach the required formulas, so that the learners could memorise and then apply them in any future context. This approach was thought to circumvent the learners' low conceptual knowledge. However, as is described below, the learners were unable to draw on procedural knowledge to solve routine problems in the intervention.

Mathematical skills of learners

The majority of the learners scored Step 3 or below on the LNAAT Numeracy assessment. The results of the Number Knowledge Assessment (Tertiary Education Commission, 2008c) were largely consistent with this, with the mode score, step 3. However, the audio recordings revealed that the scores on both assessments may have been inflated as the learners discussed copying answers.

The learners' knowledge and skills were considerably lower than expected. For example, an activity in Session One required learners to work in groups to estimate and cut a piece of tape one-metre-long. The class was then to discuss and vote which strip of tape was the closest to one metre and confirm their estimate by measuring it with a measuring tape. A group of three older adults placed a 48cm strip of tape on the floor. When I questioned them about how close they thought they were, they stated that the length was "about right". They then attempted to find one metre on a measuring tape. However, they were unable to do so because they did not understand the tape format. The inclusion of the tape's total length, printed at the

beginning of the tape in both imperial and metric systems, caused confusion (see Figure 6 for tape format):

Dean: *So, is that right or not? How much is a metre? It says feet, where's the metre?* [Looking at the red text that shows how long the tape is]

Malcolm: *It isn't here* [studying the tape].

Dean: *3 metres?* [Looking at red 3m]

Dean [looking at the 1-foot mark]: *Ah, so that's a metre right there. Nah.*

Malcolm: *Nah. It doesn't even say metres. It goes 3 metres over here* [pointing to the '3m'].

Dean: *It doesn't say metres.*

Damon: *Ah, so you go down here, you've got 10, so that's centimetres, you want 100cm.* [Pulling the tape out to one metre]. *That's a metre right there.*
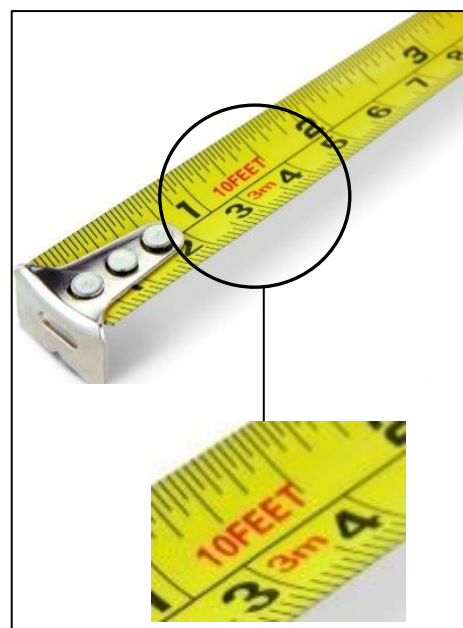
Malcolm and Dean: *Oh!*

Figure 6: Measuring tape

Only two groups were within 10cm to one metre, and most were unaware of the features of the tape such as dual metric and imperial measures, colour coding, or the conventions of reading a measurement. Additionally, many learners were unable to identify objects in the classroom close to 1cm or 1mm.

Learner beliefs about mathematics and how it is learned

The learners completed the Belief Survey (Appendix E) at the beginning of the programme. An analysis of the intervention learners' answers to specific survey questions indicated that they held predominantly absolutist, procedural oriented beliefs, consistent with learners surveyed and reported in Chapter 4.

All respondents agreed that *all* mathematics problems could be solved by using step-by-step procedures. Half of the learners believed that getting an answer correct was more important than understanding why the answer is correct, and three-fifths of the learners felt that computational skills were useful even if they could not be applied to real world scenarios. Eighty-three percent indicated that they continued to try and solve problems even when it took a long time and three-fifths indicated that they believed they were unable to solve a problem if it took longer than a few minutes. Almost all learners (92%) believed mathematical ability could be improved with hard work.

The learners viewed mathematics as primarily calculation. In response to the open survey question 'What is maths?', learners' responses indicated perceptions of mathematics as simply numbers and equations:

> Tyrone: *Maths is working with numbers, learning how to subtract*
> *divide multiply and add to get one answer*
> Vincent: *Numbers*
> Terry: *Don't know*

The learners' beliefs about how mathematics ought to be learned were also consistent with the survey and interview findings. They reflected passive learning approaches such as being in attendance, listening, and not going off-task. In response to the survey question, 'If a new student started your course and wanted to learn numeracy, what advice would you give them?' learners' recommended seeking out and listening to an expert:

> Dean: *Listen and learn*
> James: *Listen*
> Vincent: *Ask someone else*
> Terry: *go c the teacher* [sic]
> Matius: *I want to step-by-step procedure*

These views were strongly evident in both the learners' dialogue and their behaviour. For example, when asking learners about how mathematics should be taught:

> Tyrone: *You have an equation and get the teacher to teach you*
> *the formula to that. If you just know the formulas it's way easier.*

A further example was evident from discussion with the class during the second session about the need to be pro-active when learning mathematics, to engage with mathematical tasks, and to participate to a greater degree in mathematical discourse. I informed the class that we would be getting "hands on". This proposition was largely rejected by the class, and the prevailing attitude was summed up concisely by one learner who shouted the preferred behavioural expectations to the other learners:

> Kyal: *Let's just look, listen, and learn.*

This comment appeared to be accepted as the common-sense approach to learning mathematics, despite my exhortations that this was *not* sufficient, and that pro-active engagement was more likely to lead to success. That the comment was indicative of the learners' beliefs about how mathematics was best learned was confirmed over the duration of the intervention.

Beliefs about the usefulness of mathematics

Despite the survey results indicating a strong belief in the usefulness of mathematics comments made within the lessons indicated that the learners had defined the survey's usefulness items as relating to basic numeracy, counting money, and arithmetic, rather than the mathematics covered in the programme. The learners did not believe the mathematics they were learning on the engineering programme was useful. For example:

> Rawiri [pointing to an algebraic equation on the board]: *Yeah and it's like, what does that, like have to do with mechanical engineering? Like, once you get a job do you really need to learn that? I'm sure you don't* [laughs]. *Mmm. I'm sure like you only need your basics. 'Cause I'm pretty sure, 90% of the people that have jobs like this, oh like not teaching, but mechanical engineering, they won't pass a test or anything.*

> Jarred: *Yeah, I asked my Uncle, my uncle's an engineer, he works down at Forsters* [Pseudonym]. *I asked him if you needed to know all this maths, and he just looked at me goes, 'Oh, so long as you know how to read a tape measure'. 'Cause he's like pretty thick eh. He just reckons if you know how to read a tape measure. Millimetres.* [i.e., millimetres are important].

> Damon: *Have you learnt stuff here that you could use out in the workshop, in terms of maths?*
> Terry: *Nah. 'Cause I know how to read a ruler. I know how to subtract and divide and stuff like that. It's just common sense really, if you kind of think about it, but I don't even see what all these digits and decimals and s**t's got to do with it.*

Therefore, although the learners agreed that basic mathematics was important, they did not believe the mathematical programme content was important. As such, the learners' high survey results in 'usefulness of mathematics' was a poor indicator of motivation to engage in the more specialised domain of engineering.

In summary, the learners had negative experiences of learning mathematics, low mathematical skills, and negative beliefs about what mathematics is and how it is learned. The RMP they were attending was instrumental and authoritative, and emphasised rehearsal and memorisation. The learners' attitudes toward the programme and the tutor were generally negative. Accordingly, the conditions I set for the intervention: to explore how adult learners with negative beliefs would engage with a constructivist learning environment, were met.

7.2 Engagement with a constructivist-oriented pedagogy

This section reports on the learners' engagement patterns with mathematical provision that emphasised enquiry-oriented problems, the construction of conceptual understanding, shifting the responsibility to learners for their own self-management, and encouraging group work and mathematical discourse. To do this, I attempted to establish my role as that of a facilitator rather than transmitter of information. I emphasised whole-class, group and peer discussions, and sought to generate an environment in which all thinking was valued and in which explanations of thinking were a valuable source of mathematical content for review and discussion.

Resistance to the establishment of constructivist approaches

The establishment of a constructivist-oriented pedagogy was challenged by three learner behaviours. First, the learners were reluctant to engage each other in mathematical discourse and instead attempted to maintain a tutor-centred discourse. Second, learners engaged increasingly in off-task talk during activities in which they were required to take more responsibility for structuring and managing tasks. Third, learners preferred to shift the responsibility to others when asked to collaborate and engage in mathematical discourse. These three behaviours limited the effectiveness of the planned pedagogy and created conflict as learners sought to shift responsibility back toward myself.

Tutor-centred versus learner-centred discourse

The learners were reluctant to engage in mathematical discussions amongst themselves, preferring the discourse to be directed by the tutor from the front of the class. In the initial stages of the intervention, learners typically addressed all mathematically related utterances toward me, even when *directly* asked to work in groups and discuss topics amongst themselves. Compounding this was the tendency of all the learners to remain silent while I responded to an individual learner. This

suggested that group discussions of mathematics, without the presence of a tutor or teacher, was an unusual phenomenon for the class. The following is taken from my field notes at the end of Session Two:

Field notes:

The learners are very reluctant to have discussions between themselves when I ask them. When asked to discuss a topic, they remain facing me and most remain silent as though waiting for the activity to end so they can continue with the lesson. Others continue to talk directly to me, even though I have clearly and repeatedly asked them to talk to each other. Perhaps, in their other class they are not allowed to have discussions? Perplexing, given that they engage readily in off-task talk and the topics I have given are non-threatening.

In the two examples below, I indicated to learners that the *method* was to be discussed in groups, yet the tendency of learners to immediately proclaim the answer was typical of their response to being asked to engage in a group discussion.

Damon: *This is to discuss in groups… Have a discussion…*
Rawiri [Yelling]: *Three pallets, 4 boxes and 7 ones!*
Damon: *Don't talk to me about it though, talk to the people in your group.*
Dean [yelling out]: *Three pallets, 4 boxes and 7 ones…* [Trying to get my attention]

At first glance, this episode may appear to be the result of content that was too easily solved by learners. However, in most cases the learners immediately ventured wild guesses, which I interpreted as attempts to keep me at the front of the class talking, to avoid having to engage each other in dialogue. Learners resisted discussions even when the topic of discussion did not require mathematical thinking:

Damon: *Discuss in groups* [third time I have asked this], *how was maths for you during school?*
Denzel [Speaking directly to me loudly]: *I never spent time in maths. I never spent time in maths. Do I not have to discuss this?*
Kerri [Speaking directly to me, at the same time as Denzel]: *I liked my primary school maths. I liked my primary school maths. 'Cause it got me right up to high school. Then at high school I hated it.*

While these learners continued to engage with me the remainder of the class did not engage with each other but rather listened to the interaction. By responding to individuals, I re-established a teacher-centred pattern, as all learners oriented toward me. Whole-class discourse, with the tutor as the central and primary node, was the pattern the learners sought. My attempts to change this were resisted throughout the intervention.

Off-task talk during group work

A second way the learners resisted a constructivist approach was by increasing their off-task talk when engaging in unstructured tasks. While off-task utterances were expected within a class in which collaborative discourse was emphasised, off-task *conversations* (consisting of four or more interactive utterances) were viewed as indicative of momentary disengagement. Off-task talk was highly prevalent, and most of it occurred below my awareness while tutoring. The recordings revealed on-going conversations continuing throughout each of the sessions.

The nature and quantity of off-task talk was summed up by the field notes written directly after the first class.

> **Field notes: Day one**
>
> *Classroom management may be the primary concern and potential derailment of the intervention. It takes time to engage learners in tasks and the slightest potential for off-task talk is seized at every opportunity. Moments of on-task attention were only a few minutes at best. The interaction I observed between the regular tutor and the class was extremely authoritarian, with the tutor threatening punishments such as expulsion several times throughout… When asked directly to discuss with their group various topics or questions, most learners will take the time to engage in off-task talk or attempt to engage me in conversation, the others then listen to this.*

Off-task conversations were more frequent during group work, when I was not part of the conversation. For example, in session 3, there was a 34-minute episode in which learners engaged in a group problem-solving activity. During this period 19 episodes of distinct off-task conversations occurred between various learners. One of these conversations ran almost throughout the entire session, indicating the complete disengagement of the two learners involved. In contrast, other conversations continued for less than one minute before the learners returned to the mathematical

task. In the example below learners were asked to work together to solve several engineering-based problems based on powers of ten. The group did work on the task but were prone to transition into off-task talk:

Damon [to whole class]: *How many tens in one box?*

Malcolm [quietly to his group]: *How many boxes to getting pissed?* [Intoxicated]

Jamie: *A hundred thousand.*

Jarred: *I'm rocking tens of thousands.*

Rawiri: *eh?*

Jarred: *Those whole ship crates.*

Malcolm: *Yeah, what does the brother sip on?* [Question to Rawiri about his preferred beverage]

Rawiri: *Everything. What about you cuz?*

Malcolm: *On everything too, brother.*

Macus: *Everything bro, home brew, that s**t gangsta. Tasted some mean one of those.*

Terry: *How much for one of those?*

Malcolm: *I get it for free.*

Rawiri: *Eh!*

Malcolm: *My brother makes it.*

Rawiri: *Eh!*

Jarred: *Can he make all that moonshine and that?*

Malcolm: *Yip, and he makes some… based on pure f**kin' vodka…*

Jarred: *60% Eh.*

Malcolm: *Na he makes,* [unintelligible]. *But I love my top shelf aye. Those are the best.*

As this was occurring I was moving between groups. The group returned to the task when I approached and asked them to explain their thinking. They reengaged and despite regularly moving into off-task talk completed the problems and made advances in their understanding of the concepts. The tendency to engage in off-task talk appeared related to the learners' beliefs about who was responsible for managing the learning process, themselves or the tutor. The learners expected, and wanted, to be managed in an authoritarian manner. For example, when asked about the role of the tutor, the learners stated that a good tutor not only managed *others* behaviour, but also their *own*.

> Kerri: *And that's what kind of tutor I would like. Someone that will growl you,*
> *for mucking around.* [Meaning reprimand her if she "mucks around"]

> Damon: *What could be improved?*
> Rawiri: *Um, making sure is that… um making sure that everybody is like*
> *listening.*
> Damon: *Yeah?*
> Rawiri: *Instead of like drawing, like what I sometimes do.*
> Damon: *Do you mean you, when you say that?*
> Rawiri: *Yeah.*
> Damon: *Or do you mean the others?*
> Rawiri: *Yeah, me. Me as well.*

The desire to be managed indicated an abrogation of personal responsibility for controlling their attention. It manifested as a lack of effort to stay on-task during activities in which the task structure was consigned to the learners. The result was off-task talk and in several cases frustration toward me, for not explaining the content to a sufficient degree.

Ceding responsibility to others for reasoning

The third challenge to establishing a constructivist pedagogy was the tendency of learners to cede responsibility to 'solvers'. In contrast to the learners in the Chapter 5 observations, the groups would often not interact, but remain silent while one group member solved the problem silently. The arrangements were not tacit, but purposeful and explicit. For example, when provided with a problem to solve, Denzel stated to Tania what appeared to be an established practice:

> Denzel: *We'll just let the brainy two in our group do it, eh Tania?*

Furthermore, the practice was successful, as other learners were content to assume the role of problem solvers. For example, Tyrone, Denzel and Kerri were asked to work together to discuss and generate solution methods to a problem, yet Tyrone and Denzel yielded all authority to Kerri:

> 1. Kerri: [Engages immediately in self-talk oriented toward solving the
>   problem not discussing it] *Three, two, two, one, one, three, four, one.*
>   *Done! Finished!* [Directed at the tutor loudly]

2. Damon: *Give it some more time Kerri, double check it with your group.*
3. Kerri: *Yeah, we did.* [Not true]
4. Tyrone: *It's double checked, its triple checked, quadruply checked. We're right, we're the hard ones.*
5. Damon: *Come on.*
6. Denzel: *You know we got it.* [The answer is incorrect]

Kerri's utterance *"Done! Finished!"* was typical of learner behaviours early in the intervention and again, indicative of beliefs that solving problems quickly and getting them verified is the purpose of mathematical activities. The episode also revealed the passive learners' endorsement of allowing others to take responsibility for solving tasks. The comments by Kerri and Tyrone above, "*Yeah, we did* [discuss it as a group]" and *"It's double checked, its triple checked, quadruply checked"* were a fabrication, yet indicated the shared agreement within the group to allow a single member to take responsibility for the solution. At this point, despite Tyrone's and Denzel's assurance, "*You know we got it*" (6), neither Tyrone nor Denzel had engaged with the problem at all or even looked at Kerri's solutions. I interpreted this as an attempt to shift the responsibility for the management of the session back to myself to re-establish a less-demanding and threatening environment.

Despite the resistance described above, the learners did engage increasingly with various aspects of the intervention including mathematical discourse. A key strategy used throughout was to ask learners to explore and describe *solution strategies* rather than answers. The explanations could then be used as the objects for further investigation. This was modelled to learners throughout the intervention. However, as identified in the observations, learners rarely explained their thinking even when directly asked. They appeared to be unfamiliar with this practice, having difficulty understanding the objective of the task:

Damon: *In your groups, could you come up with three different ways of solving this problem.*
Tyrone: *So we've got to find out what? What do you mean?*

Yet following demonstrations and discussions on explaining thinking, learners continued to resist doing so:

Damon: *Tyrone could you explain this bottom one for us?*

Tyrone: *Three hundred and twenty* [correct].

Damon: *Could you explain it? Here's the marker, could you talk us through it?*

Tyrone: *Oh. I don't know how to do it* [Recordings confirmed that Tyrone did know how to do it].

Tyrone's hesitancy to provide an explanation appeared to be related to the potential for embarrassment, rather than an inability to explain his thinking. I also interpreted this behaviour as a reluctance to engage in more demanding thinking, particularly in public. The learners were also highly oriented toward producing answers quickly and did not value explanations despite my exhortations to the contrary. For example:

Damon [Talking to Marcus]: *Explain to us how you worked it out.*

Marcus: *Three hundred and thirty.*

Kerri: *Three hundred and twenty!*

Tyrone: *Three-twenty!*

Kerri: *Two boxes…*

Marcus: *Yeah, yeah, three-twenty.*

I continued to demonstrate how a method was explained and continued to prompt. A learner volunteered in response and came to the front of the class to use the whiteboard. Once there however, the learner (Jamie) wrote only the answer:

Damon: *Explain it man, explain it. You're right. You're absolutely right. But, can you tell us how you worked it out?*

Jamie: *Oh. With my brothers* [class laughs – meaning the class told him the answer]. *The same, that's where I got to*. [Meaning he has the same answer as the class]

Jamie appeared willing, yet unable to provide an explanation. The lack of learner explanations made it difficult to make the *explanation* the object of discussion, rather than the answer. For example, during a session that focused on partitioning strategies I asked learners to explore three methods of solving various addition problems. The first, designed to be easy enough that the focus would be on the strategy, not the answer, received less than enthusiastic responses.

[25 – 9 written on board]

Damon: *In your groups, could you come up with three different ways of solving this problem.*

Terry: *Get a hammer.*

Once explained, and modelled, the learners still interpreted this task as requiring an answer and shouted them aloud. As I continued to model the practice of identifying and evaluating solution strategies one learner began to perceive the shift from an answer orientation to a solution orientation.

[32 + 13 written on board]
Damon: *Where did you get 45 from?*
Kerri: *Because I got 30. I added the 1 to the 3 and the 2 to [get] the 5.*
Damon: *Hey check this out. This is what Kerri is saying. Kerri is saying…*
[Points to Kerri to share]
Kerri: *Add the 1 to the first 3 that equals…* [40]
Damon: *You added that to that?*
Kerri: *Yip, and add the second. Four.* [I demonstrate it on the board as she speaks]. *And 2 to the second 3 which equals 5.*

Yet the other learners were uninterested with the solution strategies and did not engage in evaluative discussions. Rather, they appeared bored at discussing how others solved the problem:

Tania [while yawning loudly]: *I could dream about maths.* [Meaning she is bored]

The learners acted as though unfamiliar with the notion of learning from other learners' explanations or approaches and avoided sharing their thinking.

Engagement with conceptually-oriented mathematics
The learners were reserved in their responses to messages and discussions that mathematics was an interconnected system, and that by developing a deep understanding of the connections they would be able to better apply mathematics and solve problems. The learners listened to these messages quietly, occasionally making comments, yet did not participate deeply in questions designed to instigate discussions:

Damon: *Some people call it a "profound understanding of fundamental mathematics". That means understanding how and why things work. Not just trying to remember stuff, but understanding how and why it works.*

Aaron: *Interesting.*

Damon: *… The reason we used the paper yesterday* [manipulatives] *was so we could see and understand what was happening.*
Jarred: *Right*
Terry: *Okay.*

Other than these general utterances of agreement, the learners added little to the conversations about the need for meaning and understanding: despite many prompts. Furthermore, they engaged to a greater degree with instrumental rather than relational content. This was consistent with the survey results in which 100% of the respondents agreed that *any* word problem could be solved if you knew the right steps to follow and 92% agreed that learning to do word problems was mostly a matter of memorising the right steps. These views persisted despite the reiterated messages that understanding how and why formulas worked would lead to an improved ability to learn and apply content related mathematics. These beliefs also persisted *despite* the learners' growing awareness that they were unable to successfully apply 'learned' formulas to the mathematical problems presented within the intervention.

The following episode illustrates how the learners' off-task talk, reliance on formulaic approaches, preference for transmissional approaches, and authoritative classroom management combined to erode the effectiveness of a conceptually-oriented lesson. A task was designed to support learners to recognise the connection between the formula for finding the area of a rectangle and that of a circle (see Figure 7). Learners were provided with an overview of the outcomes of the session which included demonstrating the direct relevance of the topic to both the vocational area and assessment. I followed this discussion by handing out cardboard circles and scissors to each learner and directed learners to discuss and explore in groups whether a circle could be restructured into a rectangular shape. For the following five-minutes off-task talk continued to disrupt the learners' engagement in productive mathematical talk and little progress was made. The utterances made by the learners, while directed to their immediate colleagues, were spoken loud enough for the entire class to hear. The class had previously discussed area, and I at this point attempted to link finding the area of a rectangle to the concept of the area of a circle.
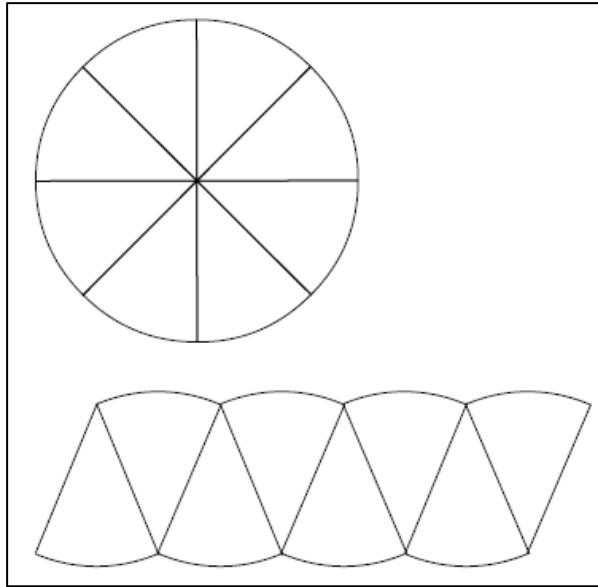
Figure 7: Manipulative to support finding the area of a circle

Damon [to class]: *You know how the area of a rectangle is found…*

Tyrone [Quietly to the people around him]: *I want to get my abs up* [abdominal muscles]. *I just want to do a little pump up.*

Denzel: *Why?*

Damon [to class]: *The same principles that apply to …*

Tyrone [loudly]: *Kerri can your boyfriend lift weights?*

Kerri: *What?*

Tyrone: *Can your boyfriend lift weights? Is he big?*

Kerri: *Na, he lost all his muscle.*

At this point the class oriented toward Tyrone and Kerri. The discussion being cultivated moments before had ceased.

1. Damon [Attempting to address and redirect the talking in the room]: *Are you guys into getting strong?*
2. Tyrone: *Yeah.*
3. Tania: *You should see them at home. Tyrone is, he's like the mega f**kin'. You should watch Tyrone do his weights, he cracks me up.*
4. Tyrone: *Do you lift weights?* [I nod in the affirmative] *Man eh.*
5. Damon: *Yip, I did used to get into weights. So, so bear with me, let's do this circle thing and then I'll tell you about the weights.*
6. Kerri: *'Bout what weight?*

7. Damon [Attempting to return to the concept of area*]: Okay. Here it is, so you can work out the area of a rectangle, a nice shape like this right* [working from a diagram on the board a 5 x 7 rectangle*], by working out how long it is. 1, 2, 3, 4, 5 across the bottom. How high it is. 1, 2, 3, 4, 5, 6, 7.*

8. Kerri [beginning to talk with Tania and laughing]

9. Tania [talking about boys and distracting class]

It is at this point that I began to ask closed questions to bring learners back to the task.

10. Damon [An attempt to re-engage learners]: *Hey, what are 5 sevens?*

11. Tyrone: *35*

12. Denzel: *35*

13. Damon: *Okay, but here's the question about area. What if it's a dirty rotten circle? How do you measure a circle when you only measure area in squares?*

14. Kerri: *You measure the whole square.*

15. Tyrone [Derailing utterance]: *Oh can I just say something?* [Entire class pauses to listen]. *Oh. No, no, I'll wait until you finish this.*

16. Damon: *Yeah.*

17. Kerri: *Don't you measure the whole square, half the square, and quarter of a square?*

18. Damon: *That's an interesting idea, we'll come back to that. It turns out, that it is horrible, that in maths you measure everything in squares and then you get this* [points to the circle]. *So, we are going to explore this issue. Okay? So, take your plate* [circle] *and fold it perfectly in half.*

19. Tyrone [Derailing utterance]: *Do you know. Do you's think there is someone faster than a calculator?*

20. James: *Depends on the calculator.*

21. Tyrone: *No* [aggressive].

22. Damon: *Take a seat.* [An attempt to continue the activity] *Okay you have it in half… Open it and draw a line down there with a pen.* [Indicating the diameter]

23. Tyrone [Derailing utterance]: *Damon, do you know that there is someone faster than a calculator.*

24. James: *He's not faster than the actual calculator.*

25. Tyrone: *Yes.*

26. James: *He's faster than the people putting in the numbers.*

As can be seen, the frequency of off-task talk prompted me to act to bring learners back to the task, by either asking an open or closed question, or giving directives to learners to move to the next stage. My question, "*What are 5 sevens?*" (Line 10) was an attempt to redirect the learners' attention toward the task. This strategy (used infrequently) was generally successful in gaining learner attention for a short period of time. Yet the practice re-established the 'initiate, respond, evaluate' discourse pattern, and re-established my role as responsible for learning and classroom management. Secondly, my final utterance (line 22) was authoritative as I sensed the lesson outcomes were at risk and took a more controlling role.

An outcome of my taking an increasingly instructive/directive approach in response to off-task talk was that the activities took on the properties of a step-by-step task. As learners completed part of the task and then moved off task, I again prompted them back to complete the next part. This had the effect of "proceduralising" an activity designed to be enquiry-oriented, removing the linking threads by segmenting the activity into micro-tasks. Yet, by taking a more controlling role, such as asking direct closed questions, I was able to reduce off-task talk by gaining learner attention. Subsequently, a tension existed between using traditional approaches to maintain learner engagement and allowing learners to disengage with the hope that they would develop more agentic roles and engage in their own construction of understanding.

The persistence of procedural approaches
The learners' first, and often only, strategy to solve problems was to apply a formula. Even when the problem could be solved through relatively low-level mathematical reasoning, the learners persisted, unsuccessfully, with formulaic approaches that often lacked any coherent relationship with the problem, indicating a complete absence of meaning-making or understanding of the formula. However, the difficulties with the formulaic approaches did not lead the learners to value or engage with conceptually oriented understanding.

For example, session eight included a whole-class discussion regarding the concept of area, its use, and how the area of rectangular shapes could be found. A 5x7 rectangle was drawn on the board with the columns and rows visible, as a prompt to a discussion. Tyrone, responding to my open question to the class regarding the area of the object, stated a formula aloud to the class. However, he incorrectly referenced the formula for finding the area of a triangle:

Tyrone: *Half base times height.*

Although Tyrone self-corrected the formula later, this somewhat simple mistake reflected the learners' first response to match a formula to a problem, while neglecting any attempt to make sense of the problem.

Secondly, the formulaic approaches were typically applied incorrectly because the learners did not connect the memorised content to real applications. For example, the learners had spent considerable time in their regular mathematics programme finding the area of circles and had memorised Pi to the third decimal place. Despite this they remained unaware of the relationship between Pi's numerical representation and its empirical application. For example, the learners lack of relational understanding was evident within a session focusing on the relationship between Pi, the circumference, and diameter. A string was used as a measuring device, and held across the diameter of a plate in full view of the class:

Damon: *How many of those* [pointing to the diameter] *would it take, that's exactly the diameter, to go right around the outside* [of the circle]*?*
Tania: *Four.*
Clint: *Four.*
Tyrone: *Yeah, it is four.*

The learners consistently articulated incorrect, or hybrid formulas throughout the lessons. This led to their awareness, and growing frustration with difficulties applying formulas to actual problems. The growing frustration of the learners was evident in the following example.

During week eight the learners spent an intensive week within the regular mathematics programme reviewing and learning mathematical formulas in preparation for a mathematics assessment. The approach adopted was to memorise specific formulas and record them in their books, so they could be referred to later, in what was to be an open book assessment. Despite the increased frequency of mathematics instruction, the learners remained aware of their inability to apply their learning. I asked them to discuss the problems they had been working on all week and to draw an example of a particularly difficult problem on the board (see Figure 8). The following comments made by Jarred and Terry were typical of the class response:
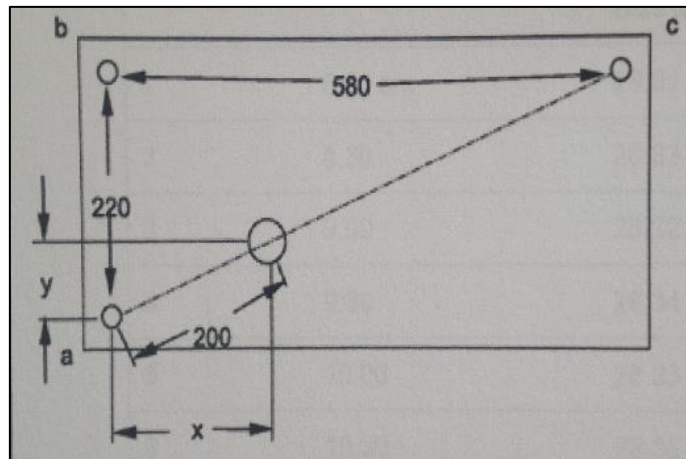
Figure 8: 'Hard problem' diagram

Jarred: *Yeah these ones are f**kin' hard.*

Terry: *Find x. I've written all that one down.*

Jarred: *I don't even know where to start with this s**t eh, basically.*

Terry: *I don't even know where to start with it. This is all new to me, this wasn't in my class.* [Meaning he hadn't covered it during his school years]

Jarred: *Yeah, yeah, yeah, f**k I've never seen any of this.* [Meaning he too, didn't do these at school]

Following this, I again informed the learners that we would be continuing to work toward *understanding* the concepts represented by the various formulas. I referred to this as "back-filling" and described this as building an understanding of "how" to solve problems and "why" the formulas work. The learners agreed with this approach:

Jarred: *Yeah, back-fill will be good, 'cause we're not learning the formulas right through until we can remember them.*

However, despite their misgivings about their current approach to learning mathematics, the learners continued to hold the view that memorising formulas was how mathematics should be learned and expressed naive beliefs regarding the efficacy of formulas. For example, in a conversation between two learners in which Jarred showed Terry a formula, he made the following comment:

Jarred: *He* [the RMP tutor] *reckons this formula will let you find any area.* [Said in a way that suggested complete belief in what the tutor has told them]

Such comments indicated the way learners believed procedures and formulas to be the substance of mathematics and reliable in all circumstances, in contrast to understanding the connections within the system. Despite being unable to apply any formula to situations presented in the intervention, the learners still held to the view that it was the superior approach:

Damon: *What's the best way to learn maths?*
Tyrone: *Best way to learn maths, bring the formulas. And that's the best way. Oh what do you, is that what you, how you mean it?*
Damon: *Yeah, that's how I mean it… Do you reckon you've got a good handle on those formulas here or not?*
Tyrone: *Yeah* [sounding confident]

Again, at no time was Tyrone able to apply a formula to the situations presented in the intervention. However, the learners acted as though they were very proud of the work they had completed, particularly the work they had copied into their books. The books were tidy and had pages of formulas written into them. Part of the appeal may have related to the impression the books gave to the learner. For example:

Tyrone: *Yeah when I get stoned man, I'll go up to my room and I like to sit there and work, 'cause it's mean. Eh, yeah, and I just sit there for ages, just,* **write out the formula**, *how Vernon* [the tutor] *did it.* **I write out everything** *of my information that's there, that's how I like to work it out. '***Cause it looks, it looks flash**. [Emphasis mine]

Tyrone's approach was to copy the formula, and the result, a page of mathematical formulas, gave him pleasure. The books themselves, rather than internalised skills and understanding, appeared to be perceived by the learners as collections of their knowledge. For example, Kerri demonstrated her knowledge to me by presenting her written formulas as evidence, even though the content had been copied:

Kerri: *Have a look at this book and I'll show you how much I know.*

Kerri meant this literally, indicating her belief that completing work, even copying a formula onto a page, was considered a productive learning activity, despite the lack of understanding. Others clearly related work completed in books as indicative of learning:

> Tyrone: *Yeah, and we all got the work done. Oh not all of us, but most of us. We learnt heaps. I filled up a whole book when I was in his class for a week. A whole book!*

Subsequent work with the class revealed the learners had not understood the formula, but primarily copied it from demonstrations tutors had written on the whiteboard. Finally, the resilience of instrumental beliefs was revealed by Terry who felt that although *he* was unable to apply formulas to problems, *others* would be able to use them to solve problems:

> Damon: *Can math problems be solved by using the right formula?*
> [reading a survey statement]
> Terry: *Mate, I don't even know which is the right formula to start with… But ah with them* [other learners], *knowing how to use the right formula to start with, I reckon they'll crack it.*

This suggested that Terry attributed his inability to apply instrumental understanding to a personal failing, not a failing of the approach itself. This view was held by other learners who felt that if they only managed to memorise the content better, then they would be able to solve any mathematical problem.

Engagement with procedural mathematical tasks
The learners engaged strongly with procedural instruction that was delivered in a transmissional approach from the front of class, followed by time to practice the procedures, and a final marking session. This was similar to the lesson structure identified in the observations. This approach resulted in a dramatic reduction in off-task talk and learners ceding responsibility to others.

For example, in response to a request from a learner to clarify a multiplication technique, the first ten minutes of session four inadvertently conformed to a transmissional "chalk and talk" approach. The learners had been highly distracted and had continued to engage in off-task talk despite the lesson having begun. A learner asked if I had seen a technique used to multiply a double-digit number by

eleven and if I could demonstrate it. The technique was a formulaic method of acquiring the answer to multiplication equation that incorporated the number 11 (11 x 33 = 3|3+3|3=363). I demonstrated the procedure:

> Damon: *For example, say it was 11 times 33. You try and work out what 11 times 33 is?*
> Tyrone: *363 eh?*
> Damon: *Oh man, you are sharp. 363 is the answer.*
> Tyrone: *That's how I do it.*
> Damon: *So, for now it's a secret.* [Asking the learners to solve how it is done]. *Give me an 11 times table and see if you can get it.*

The moment "11 x 33 = 363' was written on the board the class immediately became quiet and all learners turned toward the equation. Several learners noticed the pattern and explained it to the other learners:

> Kerri: *Add the 3 together and put it in the middle.*
> Damon [repeating]: *Add the 3s together and put it in the middle?*
> Kerri: *Yeah, put in the middle of the 3s.*

At this point all the learners ceased talking again and attended to the explanation. Kerri even asked me to move so she could see the diagram on the board:

> Kerri: *I can't see!*

After giving a brief explanation of the method, I wrote five equations on the board which the learners immediately began to solve. The self-talk revealed that all the learners were engaged in the process:

> Kerri: *That one goes here and that one goes here.*

> Malcolm: *Oh good eh. Got a reason to do my times tables now.*

> Nathan: *Yeah, it's easy eh.*

> Efren: *Good technique eh.*

> Tyrone: *Is that it? Yeah!*

During the whole-class marking discussion I purposely gave an incorrect answer, 453, instead of 473.

> Damon: *Ah, would that be four-hundred and fifty-three?*
> Matius: *No! 473*! [Yelling]
> Class: *473!* [Yelling]
> Terry: *I got it right, you got it wrong.*

The class continued to ask for more information and we continued for fifteen minutes. During this time, none of the learners engaged in off-task talk, they wrote answers in their books and continued to ask for more such methods. The recordings revealed that the learners continued to talk about, and experiment with, the method during the remainder of the lesson. At the end of the session I was literally patted on the back:

> Matius: *Thank you. That was great.*
> Efren: *Really good.*

Both were referring to the formulaic approach I adopted. It is interesting to note that Matius requested this type of instruction in his survey comments (*"I want to step by step procedure").* No such thanks were forthcoming following the enquiry-oriented lessons. Additionally, the method was referred to in the interviews as something learners had shown members of their family, suggesting it was highly valued.

Similar patterns of engagement were also present when using worksheets. Initially, the decision was made not to use individual worksheets that contained multiple problems as it reinforced a "task completion" approach. However, worksheets were used on three occasions and each in different ways. The worksheets were engaged with enthusiastically. Learners who usually avoided engagement, choosing to remain separate from much of the class, engaged immediately with them. Likewise, learners who regularly engaged in off-task talk became silent when they were handed out. The recordings reveal these were among only a few occasions of silence in any of the lessons.

During session 14, the learners had been in the class for 40 minutes and had engaged in a range of activities designed to develop their knowledge of angles. Often at the forty-minute mark learners became more susceptible to distraction, and this was the case in this session with off-task talk on the increase. The moment the sheet

was handed out, the class immediately stopped all conversation and began to work on the sheet in a way like that during the episode with the 11 times-tables. This continued for fifteen minutes. The worksheet had the effect of eliminating both positive mathematical discourse and off-task talk. It also enabled a formative assessment, in which I could communicate one-to-one with learners as I navigated the class. For example, in classroom discussions Rawiri was quiet and would often disengage from the content if unable to make immediate sense of the material:

> Damon: *How's it going Rawiri?*
> Rawiri: *I don't get it.*

In previous lessons, Rawiri would disengage at this point, however, because the class was engaged in their work, we were able to address the difficulties.

> Damon: *So, start from there, 27 degrees, go on the inside.*
> Rawiri: *Oh yeah! … Oh, so you go there* [continues to solve the problem independently].

Rawiri continued to engage with the worksheet for 15 minutes, completing it successfully. Additionally, the learners' work on the sheet was used for a further activity lasting a further five minutes. At the end of the session several of the learners requested extra copies of the worksheet to repeat and others to take home to their families. This pattern was consistent with the other two uses of worksheets.

In summary, the learners continued to apply formulaic approaches to mathematical problems despite frequent overt messages about the benefits of conceptual understanding, demonstrations of how problems could be solved using an understanding of connections between concepts, and their growing sense of difficulty applying the formulas to problems. This appeared related to the learners' belief that formulas are *the* content of mathematics. It may also have related to the *tangible* nature of formulas, and the fact that they can be written down in workbooks, and therefore represent completed work, rather than the less tangible notion of "understanding". That learners valued tangible work appeared to reflect a belief that it is how much is *done*, not how much is *learned,* that is important. Finally, the learners' approach suggested that they believed that their inability to apply formulas to problems was not a failing of the approach but rather a limitation of their own ability. A troubling conclusion is that the learners continued to emphasise the memorisation

of formulas at the expense of developing conceptual understanding, even when aware that memorisation was not working for them.

Positive engagement with problem-solving tasks

The findings presented thus far describe learner engagement with problem-solving tasks somewhat negatively. However, there were many occasions in which learners engaged and persisted with problems that took time to resolve. This led to a range of behaviours not seen in the observations.

Strategies to end engagement

The learners frequently attempted to end their engagement with time-consuming tasks as quickly as possible, preferring the tutor to transfer understanding via explanations, rather than the learners personally constructing it. They did this in three primary ways. The first was simply to resign from the task without solving it. The second was to attempt to guess the answer, assuming a correct guess would end the activity, as was likely to be the case in their experience. Third, learners sought to circumvent the parameters of the task by proposing a non-mathematical solution. These behaviours peppered the learners' engagement with enquiry-oriented tasks.

Each of these behaviours is evident in the following episode in which the learners engaged with a problem despite struggling to solve it. It is not a typical episode, because of the extended length of time that the learners engaged. However, it is useful to illustrate the learners' strategies which were typical of their behaviours when they experienced difficulties with tasks.

The water bottle problem was selected as an engaging warm-up exercise with the outcomes being to link one litre to one kilogram, develop persistence with trial and error, and to promote mathematical discourse (see Figure 9). A feature of the problem is that it has several solutions, each of which requires learners to explain their thinking.
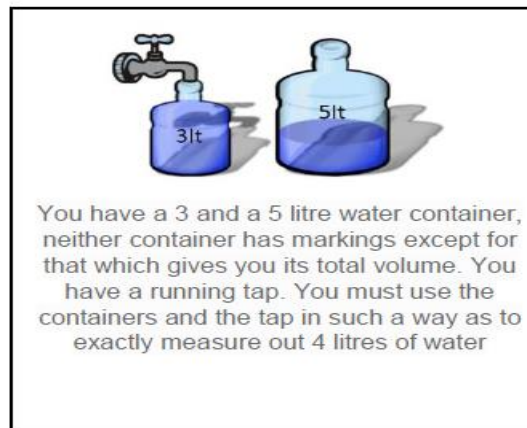
Figure 9: The water bottle problem

The problem did not include actual bottles, rather learners were encouraged to work in groups to draw diagrams on paper or on the whiteboard to discover a method. For 15-minutes learners engaged in whole-class discussions and individual work, and had to cope with, and overcome, repeated errors and frustration. The key problem centred on how to measure and add the final one litre to three litres already in the 5-litre container. This difficulty resulted in learners seeking to resign from the task, to guess answers, or circumvent the parameters of the task. For example, after much vociferous whole-class discussion the learners agreed that three litres should be tipped into the five-litre bottle, leaving one litre to be added. Tyrone offered the following:

Tyrone [speaking to the class]: *No, but you tipped in the 3 litres, and you know what's left, and you just like measure that, and get half, 'cause half of it would be a litre.* [Guessing the fourth litre]
Damon: Good thinking, keep going with that idea. Come on man, Tyrone you are so close. Don't give up, you can do it.
Kerri: *I die.* [Resignation to failure]
Damon: *You don't need to die.*
Jarred: *I'd quickly run to the shop and buy a four-litre bottle.* [Circumventing the problem]

When the learners appeared ready to disengage a prompt was used to encourage them to continue:

Damon: *Yeah. What if you tipped that in here and tipped it back…*
Kerri: *There you go, here's how you do it!*

Tyrone: *Oh, I know how you do it hoa* ['hoa' is Māori for 'friend']. *You'd chuck half in the 3 and half in the 5.*

Kerri: *But how do you know how much half is?*

Tyrone: You just have to look, look down the bottle [guessing].

The learners continued to struggle to find a method by which one litre might be found and continued to either choose to guess or circumvent the problem. However, they also continued to generate further courses of action.

Damon: *How do you know exactly where that half-way mark is?*

Tyrone: *Ruler* [Circumventing]

Damon: *Oh you've got a ruler in your pocket?*

Tyrone: *Come on man, fire-fighters always have rulers.*

Kerri: *Use the other bottle,* [new action]

Damon: *How?*

Kerri: *Well, how much more is that other bottle than compared to the other one? Two litres smaller?*

Tyrone: *No, 5 litres smaller* [teasing]

Kerri: *Then put that next door and then you measure where 3 litres is on the 5-litre bottle.*

At this point many learners in the class were working quietly, some in pairs, but most individually to solve the problem while also listening to the class-discussion. I made the decision to move to the back of the class behind the learners and reduced all prompts unless directly asked:

Damon: *Show us on the board if you want.* [Speaking to Kerri and moving to the back of the class]

Working together the class was unable to work out a way to add the remaining one litre. At this point frustration began and learners reverted to guessing in what appeared to be an effort to end the task.

Rawiri: *How about just filling it up with f**kin' four litres* [guessing].

However, this statement, designed to end all engagement with the problem, resulted in an explanation by another learner of why the fourth litre could not be estimated,

resulting in further engagement by the class. For a further two minutes the class engaged in conjectures regarding finding the fourth litre.

> Tyrone [Arguing against an idea by Kerri]: *Just listen, just listen. And 3 will be about, say 3 was there, and then you add a cap, that that will make it like that. It won't be a litre.*
> Kerri: *I'm saying a cap about this high.*
> Tyrone: *Yeah but that the, how's that going to be a litre, ah half a litre. If a litre's like that big, then then you just add like that little much to it.*
> Kerri: *Oh I don't know.* [Disengaging]
> Jarred: *Oh we all over* [Declaring that the class cannot solve the problem and disengaging].

The final two comments signalled the third time the learners made statements that signalled resignation to failure. At this point I offered an encouragement prompt:

> Damon: *You are totally on to it and your like about this far away from solving it* [holding figure and thumb close to each other]
> Jarred: *Then why can't we solve it?*

In earlier sessions, the most frequent behaviour was for learners to become discouraged and negatively responsive in the face of problems that could not be easily resolved (presented in the next section). However, apparently angry at the unresolved nature of the problem, Jarred returned to the notion of guessing the answer and aggressively stated the following:

> Jarred: *Why can't we, why can't we just put four litres in that five-litre bottle? Just fill, fill that 3 litre up, chuck him in, and then just fill about 1 litre up just a little bit* [guessing the fourth litre].
> Tyrone: *How you gonna know it's a litre though my man?*
> Jarred: *Just f**kin', go buy a …*
> Nathan: *Unless you just burn it.*
> Jarred: *He said we got scales. Aye? He said we got scales* [referring to me in the third person]

The discussion went on for some time regarding various aspects of the problems. The points raised were a combination of guesses, circumventions and one step

reasoning. The lack of resolution led to further frustration and negative affective responses:

> Tyrone: *Unless. F**k!* [Frustrated. Room goes quiet for 10 seconds]. *Nah I don't know.* [4 second pause] *Unless you fill that up, chuck 3 in there?*
> Damon: *Yeah…* [Encouraging prompt]
> Tyrone: *And then…*
> Damon: *What does that leave in your 5?*
> Several members of the class: *Two.*
> Kerri: *So you fill that up and chuck 3 in there and you got 2 litres*
> Tyrone: *Nah! You'll fill that up, chuck 3 in there. Tip that out, which will have two in there, and then* [Another pause]. *F**k!*

A whole class discussion, which excluded me, continued for several minutes, the discussion again being largely made up of conjectures and circumventions. The learners continued to engage with the problem for over 15 minutes. Finally, the lack of resolution began to erode motivation:

> *Kerri: I give up, that's too confusing. Just show us how to do it. Show us how to do it.*

The danger at this point was the risk of reinforcing the belief that problems cannot be solved through reasoning and that an expert is required. Therefore, I rearticulated the learners thinking by clearly stating the steps they had taken. This was enough for some learners to see a solution.

> Tyrone: *Oh! I can see.*
> Kerri: *So you fill that up.. yeah!...*
> Damon: *Explain it from the beginning…*
> Kerri: *You fill that up, and you tip it in here. You're gonna have 3 litres in here, and 2 in left in here. And you tip that out. Still got 2 litres in here. That's empty, you tip them 2 litres in here, and you only got a litre left. You fill that up again, you tip into here. You got three litres in here and four litres in there!*

The learners were elated with the solution, many commenting that they had shared it with friends and family. Their behaviours in this problem revealed a change in learner engagement from that observed in the earlier sessions. Although it required many prompts, they remained engaged despite the length of time it took to resolve the

problem. Additionally, the learners discussed amongst themselves possible solutions to the problem without demanding that the tutor intervene.

Engagement and increased potential for negative peer judgements

The greater emphasis on participation created a vulnerable situation for some learners, which intertwined with their strategies for completing tasks, such as resigning, guessing and circumventing. Positively, learners began to increase their participation, yet were often subject to discouraging situations as a result. These were difficult to manage, and had a substantial impact on learners' engagement.

The following critical incident illustrates the complex behaviours a learner adopted as she attempted to engage in a task while also protecting her social image by avoiding a public display of her skills. Until Session 5, two learners, Denzel and Tania, had engaged reluctantly in tasks, generally ceding responsibility to others in their group. Additionally, both typically engaged in more off-task talk than other learners and returned to on-task talk less frequently.
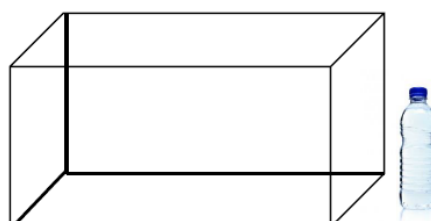
Figure 10: Volume problem

I introduced a volume problem that utilised equipment and a video to engage learners in thinking about the connection between units of measure, volume, and general problem-solving skills (See Figure 10 and 11 for lesson plan). A rectangular fish tank and a one-litre bottle were brought into the class. Learners were provided with measuring tapes and were able to work in groups or individually. A pre-prepared video was played in which I was shown filling the tank with water using the one-litre bottle. The video was paused after two litres had been added and learners were asked to work out how many more litres it would take until the tank was full. Once learners had submitted their answers, in groups or individually, the video was watched to the end with a counter keeping track of the litres added, and with regular pausing for learners to re-evaluate their initial responses.

	Content/Tutor activity	Learner activity	Resources
Objectives	Make reasonable guesses based on evidence Develop and evaluate methods to find volume Collaborate to use measuring tools (measuring tapes) Explore L x H x D		
Introduction and recap	1. Set up groups to discuss previous lesson on volume specifically recalling the 'blocks' concept. 2. Pose questions: What is volume? What was discussed in the previous lesson? 3. Show Tyrone's statement that volume is "how many blocks can fit in a space" and discuss 4. Recall volume measured in cubic centimetres (10cm³)	Recalling and describing the 'blocks' concept in groups Giving feedback to others Learners to provide example from current project work (area of small tub and trailer) Whole class discussion while handing around the dm³ and cm³ blocks	Tyrone's statement on PP Volume blocks
Making guesses	1.Show class model of one litre cubed and 1cm³ and draw attention to the fish tank and measuring tapes. 2. Ask: How many litres will it take to fill the fish tank? 3. Set up challenge: Groups to estimate number of litres it will take to fill the fish tank. 4. Show approximately 20 seconds of video footage of the tank being filled and then pause.	Learners discuss and make guesses in groups, explore the dimensions of the tank. Watch video of fish tank being filled up with water with a one litre container.	Fish tank and video
Discuss and explore ways to solve problems	1. How might working out the volume provide an answer? How might you accurately find the volume? 2. Allow appropriate time for group work 3. Write estimates on the board and continue the video.	Learners explore and discuss range of options including: • Estimating total litres from the current water height shown on video • Using the square litre block to 'count out' the quantity (physically or imaging) • Measuring the length, height and depth and calculating the volume and convert to dm³	Blocks, measuring tapes
Discuss methods with class and increase accuracy	1. Organise groups to share and explain methods Expand on and clarify where needed 2. Watch video to the end and evaluate 3. Discuss why some were closer than others (key point, the smaller the unit the more accurate the result)	Groups explain their methods, why, how, and the arising issues (can re-state their estimate) Whole class discussion of each method Comparing accuracy of each method	
Explore and describe LxHxD formula	1. Explore the formula behind the blocks method – (demonstrate with blocks) 2. Groups to identify L x H x D and make link to smaller unit of measure used by the measuring tape approach (identical approach, smaller unit of measure) The mathematics behind the measuring tape method (centimetres or millimetres as units and converting to dm³) Question: What are the challenging parts of this lesson. Prompt for questions, clarifications.	Open discussion/sharing around either counting all or multiplying LxHxD. Discuss the same approach but with smaller 'blocks' 1cm³ Watch video model demonstration (blocks filling a space and then being divided into smaller blocks).	Blocks video
	Ask groups to summarise their findings and write them on the board. Overview of next lesson	Groups to summarise in their own words the learning from the day.	

Figure 11: Volume 'fish tank' lesson plan

Learners initiated strategies such as measuring the dimensions of the tank or extrapolating the height of the water shown on the video screen. To my surprise, Tania, who had contributed little to any mathematical discussion in any session, immediately began to guess the answer, based on seeing how full the tank was after two litres had been added:

Tania: *Are you only filling it to that white line?*
Damon: *I'm filling it to overflowing.*
Tania: *Should be about 8 containers. Is it?*

As with many other learners, Tania sought to circumvent the problem-solving process by estimating the answer:

> Damon: *You're in the ball park.*
> Tania: *What does that mean?*
> Damon: *It means you are close.*
> Tania: *So, am I too high or too low?*

Tania continued to attempt to elicit an answer, yet already her interest was higher than at any preceding point in the sessions. Upon not receiving a straight answer to her guess, and encouragement from me to discuss her guess with other learners, she attempted to 'read' my response to find the answer:

> Tania: *Ten? Nine? And a half?*
> Damon: *Okay, I'll put it* up [writing nine and a half on the board but giving no indication of whether it is correct or not]
> Tania: *Maybe eleven? Maybe 12? Maybe 13. Nah, I don't know.*

Although attempting to circumvent mathematical reasoning, Tania was invested in discovering an answer. My evasive response was designed to encourage her to investigate the fish tank and join in the dialogue with other learners:

> Tania: *I reckon it's fourteen.*
> Damon: *Okay. We have our first guess. Fourteen. How did you work that out?*
> Tania: *I just guessed. Oh, plus I just looked at that and I seen that as like that, 1, 2, 3, 4, 5.*

Tania's guess was based on a method. She had looked at the tank on the screen, and taking the water level as 'two' had counted how many of these it would take to reach the top. Yet, she continued to tell others she had guessed.

> [Private conversation spoken quietly]
> Kerri: *So how did you work it out?*
> Tania: *I didn't, I just guessed.*

The primary strategy used by the learners was to find the volume of the tank by measuring and then multiplying the length, width and height. As with the other cases, various learners took the lead roles in doing this, while others observed. Although engaged and interested in finding an answer Tania struggled to understand the growing complexity of the classroom discourse as other learners began to describe their solution strategies. Unfortunately, her participation, which was her first meaningful participation at this point, placed her in a vulnerable position. She asked a question aloud within the class which was designed to help her maintain comprehension of the conversation:

1. Damon [repeating a learners' measurement]: *29 point 5 centimetres. Do we all kind of agree? You got 30.* [Acknowledging a learners' answer]
2. Kerri: *I got 30 point 5.*
3. Efren [self-talk]: *30 point 5.*
4. Tania: ***Isn't it the same?***
5. Tyrone: ***No!*** [A strong tone of ridicule]. *Isn't what the same?*
6. Tania: [appears to consider rephrasing the question]. *Oh, oh, never mind.*

It is difficult to convey in words the tone of Tyrone's utterance or its effect on Tania. At this point she completely reduced her engagement making few utterances related to the topic. She returned to her desk and continued to talk quietly off-task despite my encouragements to her during and after the class. Additionally, I valued her guess, noting how close it was to the final result. My interpretation of this episode was that Tania made herself vulnerable by asking what she reflected was a foolish question in line 4, despite it being a genuine question about the nature of decimals, or perhaps simply an attempt to maintain connection to the conversation. Her genuine engagement in the task appeared to reduce her awareness of the need to censor her participation in case she embarrassed herself. Tyrone's demeaning tone instantly returned her attention to protecting her social image. Her continued limited engagement as the programme progressed suggested that the experience made her vulnerable and led to her decision not to participate.

In summary, learners did engage, albeit to a limited degree, with tasks that were both challenging and that required greater self-management. Successful tasks were those that were accessible and deemed interesting. While challenges still existed around learners' poor self-management, their tendency to cede responsibility to others, and attempts to maintain tutor-dominated discourse, much of the engagement was positive and resulted in increased discourse and positive feedback from learners (see

section 7.4). However, when tasks became challenging, learners adopted strategies to end engagement, either by resigning completely or attempting to meet, or circumvent, the minimum requirements. Furthermore, for some learners, increased engagement led to an increase in vulnerability, bringing with it the potential for loss of status and negative affect. These factors are discussed further below.

7.3 Affective engagement

The learners' affective responses in the intervention differed from those in the observations because they were exposed, to a greater degree, to content that required active meaning-making and problem-solving, rather than passive reception. This placed the learners in situations in which they were required to persist with unresolved understandings of mathematics. These periods led to affective responses unlikely to have occurred within transmissional procedurally-oriented classes because learners could disengage or obtain answers from the tutor or other learners.

The findings showed that despite frequent messages about the benefits of working through confusion to make sense of mathematics, the learners' threshold for working with ambiguity was low. Learners preferred direct explanations and became frustrated at my emphasis on personal discovery.

Below I draw on several critical events to illustrate affective responses that occurred throughout the programme. These events illustrate the learners' growing apprehension about failing the engineering programme and their attribution of failure to themselves, their tutor and the complex nature of mathematics. They also illustrate the learners' persistent interpretation of mistakes as failures, rather than a natural and essential part of the learning process.

Learner apprehension of failure and frustration

Almost all learners held reservations about their mathematical ability and expressed concerns about their ability to learn the content. Consequently, they were anxious about completing the mathematical assessments that were a part of the engineering programme. The mathematical difficulties experienced on the programme, and a lack of progress, appeared to bring non-mathematical identities to the foreground, evoking self-doubt, anxiety, and frustration. These affects were palpable throughout each of the lessons and are all evident in the following spontaneous conversation that occurred early in the programme. The episode revealed negative emotion, "distance" between themselves and the mathematically successful, and, finally, a growing animosity toward the domain of mathematics and how it is taught.

The conversation took place during a regular 15-minute class break, within which six learners remained in class with the sole purpose of discussing their concerns regarding their progress within the RMP:

1. Terry: *Damian* [referring to me] *can you sort of help me out with suppoetry* [trigonometry] *… I have no friggin idea about how it's working or operating* [emotional and angry].
2. Damon: *Talk me through it.*
3. Terry: *I do not even know where to start* [angry]. *Yeah, no, I'm in his class, but my maths was rats**t from days of school. And I have no idea of what he is doing or trying to explain to me to try and get these friggin' answers.*
4. Damon: *Just talk me through it a little bit…*
5. Terry: *I just told you. But nothing is f**king corresponding.*
6. Damon: *I'm getting that frustration man.*
7. Kerri: *The formula, he's trying to teach us the formula.*
8. Jarred: *Yeah.*
9. Malcolm*: Yeah, the way he taught us. Like it's sort of there and it's sort of not.*
10. Terry: *And then it goes away. Pretty hard for 50-year olds to understand what he's talking about. I left school in the '80s. There was no numbers, or letters as numbers.*
11. Matius: *No, no.*
12. Terry: *There was no calculator back in the day with the didgeridoos and what they've got now. You know?*
13. Matius: *Yeah there was nothing.*
14. Terry: *So how are you going to teach a old dog new tricks?*
15. Malcolm: *I f**kin' didn't learn it at school. I was a bad fella, I wish I never was now.*
16. Rawiri: *Bro you got the same as me. You see all those pretty numbers and think, oh man, I can't do that.*
17. Kerri: *It's 'cause he's trynna do the formula.*
18. Rawiri: *And he goes way too fast eh?*
19. Malcolm: *He's just giving us all the answers, all the basic info to get us through.*
20. Jarred: *Just get us through.*

> 21. Terry: *Well, he's just getting us to, well explaining to us, but it's not corresponding in our heads.*

The conversation continued in a similar vein throughout the break and contained several recurring themes. The first was the emotional language and tone of the learners' utterances. For example, the tone of Terry's initial utterances signalled anger and frustration *("I have no frggin' idea", "…nothing is f**king corresponding")*. This was in response to a lack of progress, and a deeper sense of helplessness ("*I do not even know where to start"*) in response to the failure to learn content from the RMP ("*Yeah, the way he taught us. Like it's sort of there and it's sort of not")*. These types of comments were made frequently throughout the programme.

The second theme was the learners' tendency to exclude themselves from mathematically successful groups. This was evident in the above episode as learners expressed their unsuitability for success. For example:

> Terry: *Pretty hard for 50-year olds to understand what he's talking about… I was rats**t from days of school. There was no numbers, or letters as numbers.*

> Malcolm: *I f**kin' didn't learn it at school. I was a bad fella, I wish I never was now.*

The reasons cited for having difficulty included age, the time span since attending school, their own behaviours, a change in technology or having not been taught the required content. Learners used these to illustrate a divide between their actual educational experience and what was required to be successful.

A third theme was the growing animosity between the learners and the domain of mathematics. The animosity tended to orient toward the agents of mathematical instruction such as tutors or teachers. Terry's question hinted at his frustration with his perception of his own limitations, and that of the current learning approach.

> Terry: *So how are you going to teach a old dog new tricks?*

The learners' responses reflected a growing discontent with the approach the tutor was adopting.

> Kerri: *It's 'cause he's trynna do the formula.*

Rawiri: *And he goes way too fast eh?*

Malcolm: *He's just giving us all the answers, all the basic info to get us through.*
Jarred: *Just get us through.*
Terry: *Well, he's just getting us to, well explaining to us, but it's not corresponding in our heads.*

The notion that content was delivered too rapidly was consistent with the learners' responses in Chapter 6, who were dependant on teacher input and had few alternative strategies for learning content other than to listen and rehearse procedures. Again, while these themes were evident in this one episode, they were manifest throughout the programme, frequently arising in conversations. This apprehension about, and discontent with, the RMP provision provided the backdrop to the negative affective responses that occurred within the intervention sessions. In the remainder of this section I illustrate learners' affective responses within the programme when attempting to establish a collaborative, discourse-oriented, problem-solving approach.

Emotional responses while engaged
Expressions of negative affect were less frequent during the intervention lessons than in the observations. This likely related to the numeracy content beginning at the learners' level and thus minimising, although not eliminating, the learners being in a position where they felt overwhelmed by the complexity of the content being worked on. Secondly, the socio-mathematical norms being promoted, such as open constructive dialogue, helped defuse frustration by enabling me to recognise and respond to learners' affective responses as they occurred, reducing the potential for accumulated frustrations.

However, negative affect was an on-going issue for many learners and one which I needed to constantly be aware of and navigate. The negative affective responses that did occur were often in response to frustration at not gaining understanding quickly. This created a tension between managing learners' affective states and engaging them in productive tasks. The tension became difficult to manage on several occasions. The challenges are illustrated in the incident below.

The ability to use the Pythagoras theorem to derive lengths was a requirement of the programme, and the learners had been exposed to instruction in their regular

mathematics class. However, their exposure had been limited to the formula $a^2 + b^2 = c^2$ and the related formulations. High emotion surrounded the subject as *all* learners were confused, and many concerned that a failure to grasp the concept would result in their failure to complete the engineering certificate. I had informed the class that we would address the theorem in-depth in future classes, and told them that the learning we were currently doing provided a foundation that would ensure they were all successful.

Given that many learners felt overwhelmed when content became complex, it was beneficial to approach the theorem in a sequenced methodical manner while ensuring it was engaging. For example, the learners were unfamiliar with the conventions of square numbers and roots and hence a learning trajectory was planned and in place (for example, many learners interpreted 3^2 as 6). Yet, when the subject arose, the learners asked persuasively for an immediate explanation of the theory. This resulted in a tension between presenting a mathematical concept that may leave many learners with ambiguity regarding the concept, and taking advantage of the high motivation and full attention of the class. However, their request reflected increased agency and self-management and I decided to do as they asked.

I initially asked the learners to discuss what they already knew, but this did not result in productive talk, with almost all learners remaining silent (see Section One for an overview of learner resistance to discussions). I then described the theorem without referring to numbers, explaining (while drawing) that the theorem states that if the sides of a right-angled triangle become the basis for squares, they will both equal the square of the hypotenuse. I then constructed a 3:4 triangle on the board, adding the squares as we progressed (Figure 12), and we discussed the concept of squaring the

Figure 12: Diagram used to discuss Pythagoras theorem

legs, squaring the hypotenuse, and identifying the root. The class followed this process, were highly engaged, and asked for more. I judged at this point that the class would benefit from seeing the usefulness of the theorem and proceeded to present a problem which we worked through together. The problem contextualised a
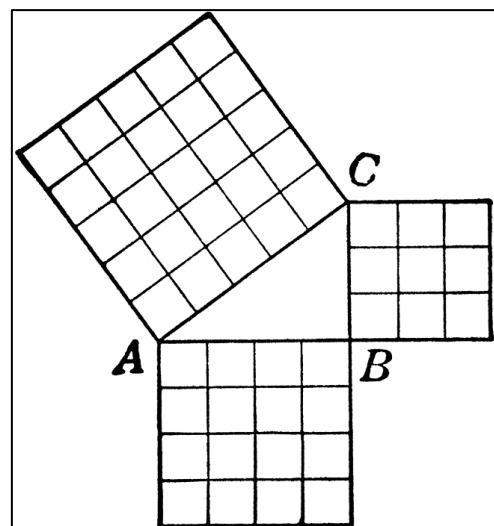
3,4,5 triangle, the purpose of which was to establish the concept as a useful tool to solve problems.

The recordings revealed that most of the learners were engaged and making sense of the concept of squaring the known sides (3, 4 and 5). However, several learners (Terry, Nathan, and Jarred) were having difficulty with the process and continued to *add* three and four, rather than first squaring them. Despite watching the process and answering 3 squared ("9") and 4 squared ("16"), Terry continued to add three and four and became increasingly frustrated at the discrepancy between his and the class's answers. Note that his comments were very quiet and only heard once the recordings were listened to:

> Damon [addressing whole class]: *4 times 4 is…* [Continuing to dialogue with class]
> Terry and Nathan [mimicking the class]: *16.*
> Terry [Self-talk]: *3 plus 4 is 7.*
> Damon [Responding to the other learners' correct responses]: *That's right, 5.* [i.e., the root of 25]
> Terry [self-talk]: *Well, how did I get 7?* [pauses to listen to my explanation] *F**k this s**t! I'm going home.*

Terry's awareness that other learners were understanding the content appeared to aggravate him further and may have consolidated his belief in the futility of the provision he expressed earlier. The decision to explore the Pythagorean theorem at the request of class members was made based on high learner motivation. Yet, the episode produced a negative affective response for Terry who missed an essential concept and began to disengage. The source of Terry's frustration was revealed in the following utterances:

> Damon: *Shall we move on from this? If you're in the deep end, like if you're doing stuff with Vernon* [RMP tutor], *and you're finding it hard to understand, don't worry about it. It's totally natural. We **are** going to get this.*
> Terry: *I put my hand up when he asks* [referring to RMP tutor], *and he talks about something totally different. "Put your hand up if you do not understand"* [Said in a mock voice]. *I put my hand up, then he goes over to him* [points to another learner] *and talks about something else.*

Terry's frustration at both his inability to grasp the concept and the perceived lack of tutor support contributed to his affective response. At this point, Terry pushed his chair back, folded his arms and disengaged. He and Nathan re-engaged several minutes later when the lesson content level was lowered, enabling them both to experience success. The task was for learners to discuss multiple solution strategies to double-digit additive problems (32+13) in the context of estimating measurements. Yet his, and Nathan's, frustration about the Pythagoras theorem were evident within this quiet conversation:

> Terry: *45. Is it half of 90?* [Speaking to me]. *Half of 90, 45.*
> Terry [quietly to Nathan]: *Look, look* [referring to me writing '45' on the board].
> Nathan: *This is easy, I can do this.*
> Terry: *Yeah, I can do this s**t.* [Said in a positive way]
> Nathan: *I can do this s**t.*
> Terry: *But when you talk about Pythag-ath-oras.*
> Nathan: *And get all f**king b, c and f**k.*

The threshold for persisting with difficult content remained low in successive sessions. When the theorem was discussed again several sessions later as scheduled, Terry continued to hold a very negative attitude toward it. For example, he made this comment to himself when the topic was introduced.

> Terry [self-talk]: *F**k, I'm over this s**t.*

Terry did not re-engage but rather became visibly agitated. The accumulation of negative experiences with the theorem appeared to have lowered his tolerance for the complexity that accompanied it to the point that he became angry almost immediately. Thus, reviewing the theorem before Terry's underpinning knowledge was consolidated appeared to entrench his negative attitude toward it and constrain his ability to engage. However, the event had been a positive experience for most of the other learners. The tension between responding to learner requests for explanations of more complex mathematics while simultaneously avoiding reinforcing learners' beliefs that mathematics was too complex to learn and activating negative emotions remained a constant challenge throughout the programme. Although the above episode focuses on Terry, most learners were found to have strong affective responses when confronted with difficult content which led to disengagement. For example, discussions with learners revealed their emotional responses and strategies of avoidance rather than engagement:

Damon: *Describe your feelings when you first saw the numeracy that was going to be in this course.*

Rawiri: *With you, or with Vernon?* [In the regular mathematics programme]

Damon: *With Vernon.*

Rawiri: *Um speechless. Like how to build a rocket wif no instructions. Like, I's like seeing all these numbers, and I's like, okay. I didn't know any of this, and then yeah* [stops talking].

Damon: *Do you get nervous about that? Or do you just kind of…*

Rawiri: *Yeah, I get nervous, and like try stay away from it. Like, nah I can't do it. What's next? Yeah, try to go for the easiest stuff.*

These responses were similar to those found in the observations and interviews in which learners made rapid evaluations of their ability to be successful with the content, and consequently disengaged immediately based on their evaluation.

Mathematics viewed as an imposition

Some learners perceived mathematical provision as an imposition, something done 'to" them by educators. For example, in the first lesson the class discussed what content would be useful and what areas learners were struggling with. The conversation turned to a review of mathematical terminology, such as hypotenuse, cosine and the Pythagoras theorem:

Damon: *Any other weird words that have been popping up?*

Terry: *Yip, Py, thag, or, us.*

Damon: *Pythagoras, yeah.*

Terry: *I hate that bastard.*

Damon: *Do you? I've got a plan for Pythagoras…*

Terry: *I'm not interested in buying what you're selling. You're trying to sell me something I don't want.*

I interpreted Terry's comments above as more indicative of his apprehension about mathematics than an authentic desire to never learn the Pythagorean theorem. However, comments like his suggested that the learners felt the mathematics they were expected to learn was someone else's knowledge which they were to some extent compelled to acquire. The following event illustrates how beliefs that mathematics belongs to the "other", that it is conspiratorially complex, and that errors

are failures rather than important steps toward understanding, manifested as frustration and anger, leading to disengagement.

The learners were working on the concept of area. I was at the front of the class, and in the context of a whole-class discussion drew a rectangle on the board marked 12m by 8m (see Figure 13).

Damon: *What would the area of that rectangle be?*
Terry: *Two 12's are 24. Two 8's are 16.*
Jarred: *24 plus 16, 24 plus 16.*
Damon [speaking to Terry]*: Very good, keep going with that*. [Mistakenly interpreting this as a mental strategy to multiply the numbers, not realising Terry was finding the perimeter]
Terry [spoken quietly to Jarred]: *Yeah, I'm just actually going to forget it.*
Jarred [quietly]: *24 plus 16.*
Terry: *40 cubic metres* [he has found the perimeter]
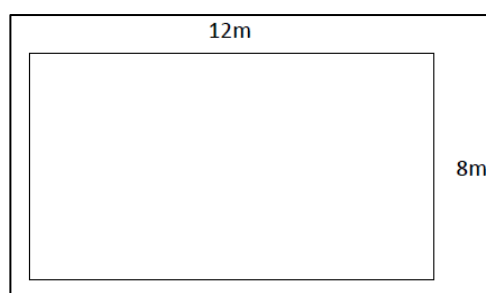
Figure 13: Dimensions of area problem

Terry had interpreted the task as a directive to find the perimeter, but had answered it in the format of cubic metres suggesting a conceptual misunderstanding. The majority of learners in the class correctly identified the area.

Damon [speaking to class]: *So what do we do for times?*
Terry [answering the question aloud]: *I just went 12 plus 12. 'Cause of that top line at the 12. Just that 12. Your 12 at the bottom is the same distance as the top one. So, I made that one there, as a 12 too, and add those two twelves together.*
Damon: *Oh I see* [said encouragingly].
Terry: *And then the 8 on this side.*
Damon: *And then you've added those all together?*
Terry: *Yip.*
Damon: *So what you've worked out is the perimeter*. [This is said in an accepting manner, validating the answer]. *You've worked out how far it would be if you were to start here, and were to start walking around that. And what was your answer?* [Validating the answer]
Terry: *40.*

Damon: *So that's perimeter...* [Writing "Perimeter = 40m" on board]

Terry [speaking quietly to himself]: *I thought you had to* [inaudible]

Damon: *… equals 40. Which is different to area. The area is what again?* [Addressing the class]

It became evident that Terry, several other learners, and I, were operating under different assumptions about what the purpose of activities were and what was at stake. I believed I had established a risk-free environment, in which learners were free to make conjectures, which whether correct or incorrect, where valuable contributions to the discussion. Terry however, responded as though he believed his response was a failure, and a further reminder that mathematics is a domain that excludes him.

The discussion continued with an interaction in which area was described as "how many squares fit inside", and the perimeter as the "distance around the outside". At this point a learner (James) shared his method of mentally solving eight times 12 to identify the area:

James: *96 is the correct answer. 10 times 8 is 80. 2 times 8 is 16. 80 plus 16 equals 96.*

It is possible that this explanation from James was further evidence to Terry and Jarred of the over-complicated nature of the question. Terry cut in, and taking an aggressive interrogative tone, began to argue that the task was designed to be misleading, and that I had instigated it. His utterances positioned the content and delivery of mathematics as belonging to an 'other'. This was evident by his frequent use of the second person pronoun, referring to both myself and the mathematical content:

Terry [speaking loudly and angrily]: *But **you** didn't ask how much, to calculate the area on the inside of **your** square.* [Emphasis mine]

Damon: *What did I ask?*

Terry: ***You** didn't ask us to calculate what was inside it.   **You** only asked to...* [Does not finish].

Jarred: *Yip true that, yip* [I had asked the class to find the area]

The comments suggested that Terry viewed the content as *my* content, and viewed himself as a compelled participant rather than an agentic self-directed learner. He then raised his voice further:

> Terry: *Well I still, I still don't even know how **you** got that 96 then too anyway.* [Accusatory tone]
> Damon: *Okay, let's look at this.*
> Jarred: *How would we usually do this anyway bro?* [Spoken to Terry and suggesting they have their own method of solving the problem]
> Terry: *With a pencil and throw this s**t away* [yelling in an angry tone]. *I'm still learning.*

At this point I attempted to rearticulate the concept of area, but also included the perimeter, to demonstrate that Terry's previous answer had merit:

> Damon: *So, there's our rectangle. There's two, you might get two questions. You might get one looking for the perimeter. And the perimeter is: how far is it all the way around? Which is exactly what you did.*
> Terry: *Yeah.*
> Damon: *Add them up* [adding them on board]. *40. The other one is the area, which is how many squares fit into that. We know that there are 8 up* [drawing rows within the rectangle]. *You know, just like with that bit of paper* [reference to a manipulative used earlier in the lesson]. *And 12 across…* [Drawing 12 columns]

The explanation was made using the diagram. At this point I asked Jarred to explain the concept:

> Damon: *…8 times 12. Like back here when we did this, and we went 9 times 12. Does that make sense? Explain it back to me.*
> Terry [Cutting in]: ***Yous guys** always trying to confuse people.* [Accusatory tone. Emphasis mine]

The shift from the known (perimeter), to the unknown (area), appeared to be interpreted by Terry as a deliberate attempt to make the content complex. I interpreted Terry's use of "*Yous guys*" as referring to mathematical educators, demarcating himself from them. Terry continued to use pronouns that positioned mathematics as an imposition actioned by me against himself:

> Terry: **You've** confused me. **You've** put me in a circle and asked me to find
> four corners [Angry tone].
> Damon: Okay, let's go back and jump out of that circle.

At this point I led the class in a discussion of area beginning with a rectangle and asked learners to share their methods and ideas. Terry began to experience success with an 8 x 10 rectangle and began again to engage in mathematical discourse. The misunderstandings that he and others held about the concept were recognised and discussed between them. Despite the positive ending to the lesson, Terry made a final comment to another learner that illustrated his sense that mathematics was made more complex than necessary by the teachers of mathematics:

> Terry: **They** confuse you man.
> Jarred: Yeah hard out.

The episode illustrated several challenges that existed when working with adults with established non-mathematical identities who were experiencing frustration with content. First, some learners responded as though they believed they were being compelled to learn externally dictated content rather than their own choice of content that would assist them to achieve personal goals. Second, despite my attempts to establish an environment in which errors were normal, and in fact encouraged, learners continued to interpret them as failures. Third, when learners perceived themselves to be failing, negative emotions quickly surfaced, this limited the presentation of complex content.

7.4 Reflections on the programme

The learners' feedback about the programme was highly positive. They stated that they enjoyed the programme more than their regular mathematics programme, that they understood and remembered more of the mathematics, and that the tasks were easier to complete and more interesting. Learners also tended to contrast the intervention and the RMP. For example, regarding the RMP, Kerri emphasised the pedagogical approach of tutors and the need to write and record content in her book. In contrast, the mathematics in the intervention was referred to as easier to remember, to interpret, understand and apply:

> Kerri: Even though Vernon went about it, and I had it all in my book, I could
> remember our teaching ways. Like our zombie ways, stuff like that. [A

reference to an activity that required trigonometric functions]. *Because, how it was like, he taught us like five different ways to do transformations and stuff like that. But in like our subjects it was so much easier the way that we did it. Because it was easier to interpret.*

She also noted that the approach was more effective:

Like Vernon had to teach us like three or four days. Just to get one subject. But when you taught us one or two days, for the session, we got it… And we'd have a couple of sessions on that subject. So it would actually, jump into my brain like Pythagoras and circles and all of that. I have that all now.
Damon: *So did that help, that stuff we did with Pythagoras?*
Kerri: *It really helped. Like our last session, I didn't know how to use a protractor.*

Others also noted that the departure from procedural teaching resulted in an understanding of otherwise confusing concepts. For example, Rawiri had experienced persistent difficulties with square and root numbers:

Rawiri [discussing squared numbers]: *I'd like to say, that that paddocks, that really made me think about it um simpler.* [We had discussed squared numbers in the context of paddocks on a square farm]
Kerri: *Yip, me too.*
Rawiri: *Just the way he* [referring to me] *explains it makes it easier to relate than Vernon's way.*

Others expressed satisfaction that they were beginning to enjoy and learn mathematics:

Denzel: *And there's a couple of times, or a few times that actually happened eh, and I was like "Oh yeah, sweet as"* [Because he understood]. *Come the lunch break, lunchtime everyone's got to go, I don't want to, I want to stay here [and] do this. I want to carry on* [working with the intervention mathematics].

Jarred*: Finally seeing it. Seen the light.*

Even Terry valued the programme:

Damon: *Some students wanted to stay the full two hours, others wanted to leave as quick as they could, why do you think that is?*

Terry: *I don't know, I'd prefer to stay here a bit longer* [laughs].

Such comments were typical of learner reflections. However, many framed the primary benefit of the programme as having provided better *explanations* of content, rather than attributing progress to the increased emphasis on meaning-making and shared discourse. Additionally, the learners did not appear to change their beliefs, nor did they reference any of the content regarding the explicit messages about beliefs:

Terry: *You gave us a bit more info.*

Damon: *Right. So that was better or not?*

Terry: *Yeah, **you explained it a bit more** than what Vernon did. It's not a language barrier or anything like that. You **just gave us a bit more explanation**.* [Emphasis mine]

Rawiri: ***I liked them*** [the lessons] ***because you know how to explain stuff***, *just like that, and like with my, our brains you know how to explain it right.* [Emphasis mine]

Denzel: *'I's like sweet as. I finally get it', **'cause someone actually helped me out and explained it a lot easier…** Um, but yeah, I was like "sweet as" and I started pumping it out, getting it all done, and I was getting it, I was finished before any of the other people were done.* [Emphasis mine]

These comments were telling, given that while tutoring I made efforts not to be drawn into the role of an explainer of information, although, as was evident, I was not always successful. However, the sessions were not characterised by improved explanations from myself, but rather by increased learner participation coupled with an emphasis on conceptual understanding. The comments suggested that the learners' view of the programme was framed by their beliefs about what makes a good class, and a good tutor.

It was difficult to determine whether the programme improved the learners' views of themselves as doers of mathematics. In most cases learners expressed positive

changes. For example, Kerri indicated a transition from a non-mathematical to mathematical identity.

> Kerri: *Because when I first came here, I was step three, in my numeracy and literacy. And so far, **I reckon, I've come up more**. So I reckon I'll be about a Step 4, Step 5. Yeah, '**cause I'm the first one to finish all my assessments in my class**. You know, 'cause I'm here every day… **I ended up passing all my assessments, I passed every single one**… Like, I didn't even know what the hell Pi was. **But now**, I just can't get the number out of my head.* [Emphasis mine]

Her final sentence expressed the contrast between her skills at the beginning and at the end. Unfortunately, such positive changes were rare, and interestingly, Kerri recognised this challenge for the other learners.

> Kerri: *For kids that, you know dropped out of school, hated school, for them to actually learn that? I reckon that's a big step.*

Other learners acknowledged that they had improved their mathematics skills and felt more positive about mathematics, yet did not appear to change their beliefs about their ability to learn mathematics. For example, Rawiri indicated more positive attitudes toward mathematics, yet qualified his statements with a critique of the tutor and approach to learning. This suggested that his improved attitude was likely dependent on external factors, rather than genuine improvement in his own ability to learn mathematics.

> Damon: *How has your attitude towards maths changed since you started this course?*
> Rawiri: *Yeah, it's getting better. Oh like, what do you mean by attitude, like my behaviour? How I feel?*
> Damon: *Just the way you think about maths, do you like it?*
> Rawiri: *Oh yeah, I like it, but then again, it's hard to learn. But nah, yeah.*
> Damon: *So you are constantly trying to learn new stuff?*
> Rawiri: *Yeah. It's just, that we um, like how he does it, is like he writes it down and explains it and then boom, that's it. And then we're supposed to learn another thing.*

Finally, despite unanimous positive attitudes toward the intervention programme and the learning it provided, the success of the programme was enveloped within the wider educational context of the engineering programme. For some learners, the overall experience was negative, negating much of the success of the intervention. An example of this was evident in comments by Tyrone in which he stated that his lack of skills, attributed to poor teaching, would likely result in a loss of employment:

> Tyrone: *He* [Denzel] *reckons that "Oh yeah, we're going to pass this, get jobs". Like it's not going to be that easy. We're going to get the boot pretty much straight away.* [Going to get fired]
> Damon: *You reckon?*
> Tyrone: *Yeah when they* [the employers] *find out we don't know nothing.*

These statements highlight the potential damage such an experience can have on not only an adult's mathematical identity, but also core identity. It also demonstrates that a repeated failure to learn may consolidate negative beliefs, further distancing the learner from a successful outcome. Fortunately, such comments were rare, and the typical experience was positive.

7.5 Discussion

The purpose of the intervention was to explore how low-skilled adults respond to a classroom environment that emphasises conceptual understanding. The attempt was made to establish an environment that valued participation, encouraged sense-making, and emphasised the interconnectedness of mathematics using rich collaborative tasks that built on the learners' prior knowledge. This was supported by a pedagogical emphasis on valuing and responding positively to all learner conjectures, answers, or suggestions, as positive additions to the mathematical discussion, with a view to mitigating a belief in the right/wrong dichotomy associated with an absolutist conception of mathematics. Learner beliefs and attitudes were openly discussed throughout the programme, as were negative school experiences and how these might influence behaviours. Discussion was had as to why pursuing understanding rather than memorisation was effective and how mathematical discourse facilitated this process. In short, to the extent that I was able, I implemented the adult-based recommendations made within a range of studies (Condelli et al., 2006; Swan, 2005; Swain & Swan, 2007). Discussed below are the learners' beliefs, their engagement with a constructivist pedagogy, their affective responses, and, finally, their reflections on the programme.

Backgrounds and beliefs

The learners' preference for a classroom environment in which the tutor was the classroom manager, authority figure, and expert, and their passive expectations for their own roles was consistent with the perspectives of urban school children who prefer authority figures to use their authority (Delpit, 1995; Jones et al., 2013). The findings support the link between these beliefs and passive learning roles (Campbell et al., 2001; Taylor et al., 2005). This may also reflect the learners' histories of behavioural difficulties during school, and the authoritarian approach adopted by the tutor of the RMP. Interestingly, both the observation and intervention interviews found that the learners were critical of teachers unable to maintain class discipline which suggested that the learners had little experience of autonomy within a classroom because of participating, perhaps exclusively, in classes in which management emphasises the need for control, obedience and compliance. The findings support the argument that once beliefs about learner roles are developed they continue to persist into adulthood (Briley et al., 2009; Huak, 2005; Yoon et al., 2011).

The learners' beliefs about the usefulness of mathematics revealed a divide between valuing daily, or basic, mathematics, and more complex mathematics. Learners described key content areas, such as the Pythagoras theorem and trigonometry, as *not* useful, and this appeared to contribute to the sense that these were an imposition, particularly given the learners' growing frustration with the concepts. Given that a belief in the usefulness of what one is learning is related to motivation (Berkaliev & Kloosterman, 2009), and predicts performance (Briley et al., 2009) this is a concern. It also explains the emerging anger several of the learners expressed during the sessions. The inability to master what was perceived as superfluous content threatened not only their sense of self, but also their future employment. This also explains why some learners viewed aspects of mathematics as a contrived impediment to their goals, and attributed it and its agents with dubious motives, such as actively making mathematics complex ("***Yous guys** always trying to confuse people"). The comments made by Terry and others may have been hyperbolic, and made in the heat of the moment, yet they still reflected anger and led to disengagement. The situations may be related to the increased reference by adults to feelings of anger in relation to mathematics, in contrast to anxiety and worry expressed by children (Carroll, 1994; Evans, 2000; Lewis, 2013).

The study also revealed the limited extent of the learners' mathematical skills and the way this constrained their success on the programme. The learners' lack of place value knowledge, number skills and familiarity with conventions of measurement put

them at a serious disadvantage when they were expected to learn more complex content. For example, some learners had memorised Pi, yet lacked the understanding of what the decimal system represented. The reader might be surprised at the notion of placing such learners into a relatively demanding mathematics programme (as was I). The suitability of these learners at their current levels for the programme raises questions. However, while the skill difference in this case was large, the observations also found that very low skilled learners were being asked to learn comparatively complex content in other programmes also. Additionally, it was evident that the strategy of circumventing the learners' low skills by focusing entirely on memorisation of procedures was not only ineffective but eroded positive attitudes toward the content. This supports other studies that document the weaknesses of this approach, despite it still being professed by some educators (Echazarra et al, 2016; Kilic et al., 2012; Pesek & Kirshner, 2002).

Engagement with a constructivist-oriented pedagogy

The learners' ongoing attempts to apply memorised formulas to conceptually-oriented problems were consistent with studies that indicate that once exposed to procedural instruction learners continue to apply them at the expense of developing conceptual understanding (Engelbrecht et al., 2009; Jäder et al., 2017; Pesek & Kirshner, 2002). The learners' dependence on using formulas *during* conceptual work parallels Pesek and Kirshner's (2002) findings in which students exposed to instrumental instruction prior to conceptual instruction continued to apply formulas incorrectly despite being able to use conceptual approaches. Interestingly, the authors found that even when correctly applied, students' justifications for their use of formulas were incoherent, indicating a disconnect between the mathematical situation and the procedure. The findings of this study support the contention that initial exposure to instrumental instruction contributes to learners depending on procedures to the detriment of their engagement with conceptual content, and lacking understanding of how the formulas relate to problems (Engelbrecht et al., 2009; Goldin et al., 2009; Thompson, 1992).

Despite the above, learners did engage to a varying degree with conceptually oriented problems that were unable to be solved procedurally. However, an interesting behaviour of learners was the attempt to end their engagement prematurely, by either resigning completely, guessing the answer, or attempting to solve the task in a non-mathematical way. These behaviours did not appear to be shame avoidance strategies, but rather a result of unmet expectations, underpinned by procedural beliefs. As shown in other studies, the belief in the utility of procedures,

and in the 'one right way', can result in learners continuing to apply the same strategy repeatedly, yet unsuccessfully (Jäder et al., 2017; Lerch, 2004; Sumpter, 2013). This behaviour was highly evident in the water bottle problem, in which learners not only repeated the same strategy, but repeated whole sentences almost verbatim multiple times. Neither were these strategies reasoned conjectures or refutations, rather they were typically composed of assertions and rejections with no explicit 'warrant' given to support either (*I think this since…*), or backing, indicating a lack of mathematical reasoning (Toulmin, 1969). This supports findings that procedural beliefs orient learners toward imitative reasoning to the extent that they do not engage at all with creative reasoning (Sumpter, 2013).

There is also some evidence to suggest that the inability to apply a procedure to the problems created a sense of insecurity, eroding task tenacity and contributing to attempts to prematurely end engagement. Sumpter (2014) found that memorised algorithms provided learners with a measure of safety. When algorithms proved unsuccessful, some learners felt insecure, rather than trust their reasoning they relied on the teacher. In the context of this study, the learners' disengagement strategies may have been attempts, not only to end feelings of insecurity and growing frustration, but also attempts to re-assert the role of the tutor as the 'expert explainer'. The behaviours are similar, albeit not as extreme, as those identified in the learned helplessness literature (Yates, 2009). This is interesting because most of problems used were within the mathematical ability of learners to solve, yet they only succeeded with prompts from myself, adding support to the contention that beliefs dominate problem-solving behaviours more than mathematical knowledge (Goldin et al., 2009; Sumpter, 2013).

The learners' reluctance to engage in discourse appeared to be motivated, in part, by a desire to avoid a threat to their status, dignity or sense of self-respect. This persisted despite my attempts to establish a safe environment in which participation, not performance, was valued. This is consistent with other studies in which adults reduced interaction when unsure of content, partly due to perceptions of being judged ignorant, and an attempt to avoid looking foolish or ignorant in front of classmates (Bibby, 2002; Tenant, 2012; Yoon et al., 2011). The reluctance also aligned with Goldin et al.'s (2011) engagement pattern 'Don't disrespect me' in which the learner's motivating desire is to avoid conditions that lead to belittlement. Learners struggled to accept the premise that making errors was a natural and essential aspect of learning mathematics, or that conjectures were an essential component of problem solving. It may be true that while *some* did accept the

premise, because they believed that the other learners would still judge incorrect utterances as evidence of inability on the part of the speaker, the behaviours continued to have a social cost.

Secondly, the resistance to engage in collaborative discussions was consistent with the learners' view of learning as acquisition, rather than participation (Muis, 2004; Sfard, 2012). This orientation was evident in their preference for me to explain and demonstrate, and their conformity to procedural 'chalk and talk' instruction. Moreover, learners were under pressure to meet external assessment criteria which they believed was best prepared for by memorising formulas. Their belief that learning was a product of "*look[ing], listen[ing], and learn[ing]"* contributed to seeing constructivist activities as interesting, yet superfluous to their needs. The learners' responses in follow up interviews shared elements with those reported in several studies in which learners reported that while problem-solving and discussions were enjoyable, they were not the 'serious' work necessary to develop the required skills (Cooney, 1985; Johnson et al., 2009). The learners' belief that they needed to focus on *serious work* increased with the approaching assessment deadlines and the learners' evaluation that they were not making the necessary progress. Engaging in discussions was not viewed as a learning mechanism, and therefore not serious work.

These findings are consistent with studies of school classrooms that find that learners who adopt a procedural approach are concerned with getting answers, not reasoning (Cobb, Yackel & Wood, 1989; Jäder et al., 2017; Nichols et al., 1990), and that learners with a calculation orientation see explaining their reasoning as irrelevant (Johnson et al., 2009; Thompson et al., 1994). However, the difference between those studies and this, is that the former studies tend to posit these behaviours as outcomes of teacher beliefs and the associated pedagogical approach. Many school-based studies show that a teacher can draw out student reasoning, and make the reasoning itself the object of discussion, by continuing to ask more probing questions and managing the conversation (Hufferd-Ackles et al., 2004). However, the findings of this study suggest that in the adult domain, the learners' patterns of behaviours are more resilient and difficult to direct. For example, the contrasting episodes below give some indication of the differences. The following transcripts are taken from Thompson et al. (1994, p. 81). A seventh-grade classroom is reviewing a mathematical problem, and the teacher has asked a student to elaborate on their thinking:

> Teacher: *How is it that you thought about the information in it?*
> S1: *Well, you gotta start by dividing 38 by 3. Then you take away…*
> Teacher: [Interrupting] *Wait! Before going on to tell us about the calculations you did, **explain to us why you did what you did?** [Pause] What were you trying to find? [Emphasis mine]*
> S1: *Well, you know that John is 3 times as old as Sally, so you divide 38 by 3 to find out how old Sally is.*

Note that S1 responded confidently and did elaborate on her thinking. Following this, the teacher built on her response to cultivate a conceptually-oriented discussion. However, this was not the pattern of discourse observed in this study. For example, in the episode below, the learner does not respond as S1 did above, but rather disengaged when asked to further elaborate on an answer (see p. 115).

> Tyrone: *Three hundred and twenty* [correct].
> Damon: *Could you explain it? Here's the marker, could you talk us through it?*
> Tyrone: *Oh. I don't know how to do it.*

Despite having solved the problem, and knowing how he did so, Tyrone professed ignorance and refused to contribute until later in the lesson. His reasons for doing so were likely related to an evaluation of social risk related to an image-management orientation as described in detail by Bibby (2002).

A further reason why public reasoning was perhaps avoided was that it invited critique from other students, not only for the quality of the reasoning, but also for assuming an undeserved authority. In other words, learners avoided situations in which they may have been perceived as having the audacity to presume to be able to teach others. For example, in the episode in which Tyrone asked James to explain his thinking (p. 195), he also made sure to undermine James' authority in what I interpreted as an attempt to maintain his status. The episode suggested that learning from a peer in a public setting created tensions regarding status. James received criticism for sharing more advanced mathematical skills, and it appeared to set him apart from his peers. The reluctance by many learners to assume the perceived role of 'expert' may have contributed their reluctance to demonstrate their thinking. Similar motives have been suggested for learners' performance-avoidance behaviours (Linnenbrink-Garcia et al., 2012), including Goldin et al.'s (2007) archetypal affective structure 'Stay out of trouble', in which learners seek to avoid conflict and strive not to be noticed.

A notable feature was the quantity of off-task talk in the intervention. Although mathematical discourse was limited, the learners engaged readily in off-task talk when not being directed or instructed by myself, such as during group discussions. The striking difference between the findings of this study and others in the literature was the extent of off-task talk that occurred. For example, Howard's (2002) observation of ten interactive university classrooms found that while older adults (25+) interacted verbally more than younger students, the mean number of verbal interactions was only 42 per hour, with a mean of 21 learners in attendance. This is considerably less than occurred in this study. For example, in session three 19 off-task *conversations* took place over 32 minutes with only 13 learners present. The number of individual utterances was in the hundreds. No account in the literature on adult mathematics could be found in which the off-task talk was as prolific. It is not my belief that this was due to uninteresting content or boredom but rather to three factors.

The first, but lesser, factor was that the multiple recording devices captured utterances designed to be covert, and therefore recorded more than the frequently used single-point observational methods. Alton-Lee et al. (1987) also recorded a substantial number of utterances using a similar method suggesting that this sub-dialogue exists but is typically unable to be detected by a single, and perhaps more visible, observer. The observation findings also showed that private dialogue between learners was a consistent feature of the observed classrooms.

Second, the organisation in which the programme took place specialised in working with learners with problematic school histories and behavioural difficulties. School-based behaviour difficulties include a range of disruptive and escape behaviours (Sutherland & Singh, 2004), and there is some evidence to suggest that these become somewhat entrenched (Turner, 2002), and perhaps become a normative part of classroom culture. The learners' beliefs and preference for an external authority to control their own personal behaviours also indicated undeveloped ability to self-regulate behaviour and motivation (Zimmerman, 2002).

Third, the off-task talk may have been used as a passive avoidance strategy to manage the potential harm that might occur from an embarrassing exposure of mathematical thinking. Simply put, if learners are talking about anything other than mathematics there is less chance of saying or doing something embarrassing. The behaviour was a characteristic pattern of groups and produced a consistent outcome;

less time spent engaging in mathematical discourse. Although not as individually agentic as some strategies, it is a possibility that the learners chose not to exert effort to prevent the on-set, or continuation, of off-task talk because it acted as an effective buffer against potentially shameful situations. A further benefit of the behaviour is that it distributes the responsibility for non-participation among group members, rather than the culpability resting on a single individual. Off-task talk may have provided a group shield from the tutors' corrective attention. If this is the case, off-task talk can be added to a growing list of performance avoidance strategies (Bibby, 2002; Linnenbrink-Garcia et al., 2012).

Attempts to generate whole-class discourse were more successful than group discussions due to my ability to facilitate, yet most learners remained silent while other more vocal class members dominated the exchanges. The use of non-routine problems meant that learners were unable to 'lock in' a correct answer that they could then use to avoid exposing their thinking through explanation. Thus, the shift away from the safer routine of the closed question/single answer pattern, may have led to the more confident learners taking even more dominant roles, because others reduced public participation to lower the risk of expressing seemingly ignorant thinking. Despite my attempts to draw all members of the class into a dialogue the patterns were consistent with the 'consolidation of responsibility' in which most adult learners remain silent, trusting others to engage in pertinent discourse (Fritschner, 2000; Howard, James & Taylor, 2002; Karp & Yoels, 1976; Weaver & Qi, 2005). The practice of consigning the role of answering and asking questions to others was clearly an established practice that persisted despite my efforts to change it.

The findings of both the observations and the intervention indicated that beliefs that posit mathematical performance as indicative of social and personal worth are likely to underpin the desire to avoid shameful and embarrassing episodes (Bibby, 2002; Siivonen, 2013). Established traditional classroom norms offered greater security from shameful moments while increased collaboration and participation in enquiry-oriented approaches increased the risk. Moreover, learners were familiar with the conventions of a traditional approach, evidenced by their ready conformity to traditional roles, which means that self-protection strategies are routine. The traditional classroom environment afforded protections against unvolunteered engagement, it reduced the pressure to share their thinking, allowed answers to be verified by others before having to state their own to the group, and enabled learners to cede responsibility for engaging with the tutor to others. A traditional environment

offered greater security to learners seeking to avoid harm, and hence most were hesitant to contribute to an alternative environment perceived as high risk.

Affective responses to the programme

A key difference between the learners' affective responses in the intervention than in the observations was the greater public expression of negative affect. In the observations expressions of negative affect were typically expressed privately, either as self-talk, or to a peer, and below the awareness of the tutor. Furthermore, the private expressions often preceded a subtle, (or not so subtle), disengagement from content. As discussed in Chapter 6, this was in large part because the discourse patterns within each of the classrooms limited opportunities for learners to express their ideas or feelings. Positively, the emphasis on open discourse in the intervention was different enough that learners felt comfortable to publicly express these emotions and trepidations which allowed the opportunity to diffuse these by addressing the learners' concerns and adapting the content and approach to suit. Even though the expressions of affect appeared dramatic in many cases, the fact that learners were able to express them was a positive step forward, as the lines of communication were maintained between myself and the learner. The isolating nature of mathematical difficulties, in which learners seek to avoid participating because of perceptions of being judged (Bibby, 2002; Turner et al., 2002) was somewhat mitigated by the open discourse-oriented environment. I conjecture that while the establishment of a discourse-oriented classroom facilitated more expressions of negative affect to the tutor, it enabled the learners and myself to negotiate a way forward. Cultivating greater learner discourse in adult classrooms has been recommended as a method of facilitating formative assessment (Hodgen et al., 2010; Nonesuch, 2006), however, the approach may also facilitate a 'release valve' for learners, while enabling the tutor to respond and adapt to signals of negative affect.

Negative affect peaked at certain moments, the primary one being the perception that an individual was either alone, or part of a small group, that did not understand content. Learners appeared to attribute substantial negative importance to their errors or misunderstandings, despite my attempt to mitigate this. For example, Terry's reaction to learning that he had identified the perimeter rather than the area suggested he may have been evaluating his ability to learn mathematics, not simply his ability to solve a problem. His response was consistent with students with 'so called' behavioural disorders, who were found to hold such low thresholds for self-evaluations of failure that they simply stopped working on problems as soon as they

experienced any difficulties and adopted helpless roles (Sutherland & Singh, 2004). Terry rejected my assertion that his answer was valid and began what Op't Eynde et al. (2009) describe as a shift from a focus on the mathematics to a focus on the situation. The cause of this was Terry's belief that an incorrect answer signified failure, rather than engagement in a process leading toward understanding. This belief was resistant to my efforts to change them, at least in the moment. This highlights a key finding of this thesis. Once beliefs are consolidated and intertwined with emotions, they are highly resilient. Key events are interpreted differently and act as watersheds for personal evaluation and subsequent affective responses. Terry and I ascribed different meanings to the same event, yet it was Terry's that determined his engagement. I will discuss possible reasons why the learners did not change their beliefs in the next chapter. However, for now, it is important to note that despite my efforts to treat Terry's response in line with constructivist interpretations, Terry did not, and began to orient toward 'appraising' the situation rather than continuing to engage in mathematical thinking.

Terry's emotional response to learning that he had identified the perimeter rather than the area might seem disproportional to the event, until considering the wider set of circumstances. Firstly, Terry's negative response began when he perceived that his answer was not accepted as the correct answer (despite my attempt to validate his response). Secondly, he appeared aggravated by the fact that James, and other learners, *had* successfully solved the problem under the same circumstances as himself, and, that he was unable to make sense of their solutions. His awareness of their success, and his perceived failure, is likely to have contributed to his sense of exclusion from the mathematically 'able', of which he clearly distinguished himself with comments such as "…*my maths was rats**t from days of school… I just totally gave up on everything… … I have no friggin' idea about how it's working or operating"*. Immediately following his realisation that others had solved the problem, Terry abandoned further attempts to engage with the mathematics and instead transitioned to making judgements on the immediate situation, before turning to the wider situation. By the time James shared his solution to the problem, Terry had disengaged and was proceeding to make appraisals of the situation ("*Well I still, I still don't even know how **you** got that 96 then too anyway"*, and "…*throw this s**t away"*). He then transitioned to appraising the nature of mathematics teachers ("***Yous guys** always trying to confuse people")*. I interpreted his comments as a transition from frustration at his own failure, to anger which became focussed on the 'injustice' of the wider situation. His attention turned from the mathematical content,

to his own performance, to appraising my role as tutor and the possibility that I was being deceptive, and finally that mathematics is purposely designed to be complex.

Terry's rapid shift to anger suggests it was initiated not only by the denial of an in-the-moment goal, but also his interpretation of the event as indicative of his likelihood to be mathematically successful. Sfard and Prusak's (2005) terms *actual* and *designated* identities are useful to distinguish between Terry's actual mathematical identity and the designated identity that he expected to become. Terry's experience within the engineering programme might be thought of as a process of working out, through participation, which identity would be realised, either his historical identity reflected by his narratives of school and current mathematical failure, or the potential positive identity of an 'able' doer, and perhaps overcomer, of mathematics. By publicly sharing his initial answer with the class he was actualising an identity of an 'able' doer of mathematics (Heyd-Metzuyanim et al., 2016). Given that he acted consistently with this positive identity, the realisation that he was incorrect, and the rising emotional reaction, may have been interpreted as a signal that he was mistaken and that in fact, he was, and would remain, 'unable'. Given the common belief that one either is or is not a maths person (Black et al., 2009), failure may have been interpreted as the *inability* to become mathematically able and with it the inability to complete the entire engineering programme and gain meaningful employment beyond this. If this interpretation is correct, a task as mundane as finding the area of a 12 x 8 rectangle became a touch-stone for a deeper internal conflict between what was, and was not, possible in Terry's life. The ability to solve a single problem may be a fulcrum between an 'unable identity' fused with perceptions of historical failure or a new 'able identity' fused with overcoming and achieving mathematical success. Terry's possible evaluation that he would never reach his goal explains the anger, the blame, and the lack of motivation to continue.

In conclusion, some low-skilled learners may be so attentive to evaluating their in-the-moment performance as a proxy for wider success, that errors are rapidly interpreted as signals of permanent inability. Rather than perceiving incorrect answers as an essential component in developing understanding, the ability to solve a problem becomes a fulcrum for judgements about their actual and designated identities. These judgements appear intertwined with affective responses resulting in the learner shifting attention from mathematics to the causes of this uncomfortable situation.

Learner reflections

Despite resistance during the intervention, the learners interviewed considered the intervention a success. It was recognised as substantially different from their regular mathematics instruction and learners specifically stated that the tasks were more interesting than their normal activities, and that they remembered and understood more of the mathematics than within their regular programme. References were made to the coherency of the delivery and content, in contrast to the disconnected delivery of content and rapid pace within the regular programme. These results are positive given research that finds learners with lower skills and attitudes have more negative responses to constructivist approaches (Johnson et al., 2009; Sonnert, et al., 2015). It is also positive in light of Pesek and Kirshner's (2002) research which found that learners exposed to instrumental before relational instruction found relational lessons enjoyable yet continued to state that they learned more from instrumental formula-based instruction.

Despite this positive result, a key finding is that the learners attributed their improved learning to better explanations of content rather than their own increased participatory role in constructing understanding. In other words, they attributed their learning to my ability to 'explain'. This suggests that their beliefs about how mathematics is learned shaped, and perhaps reinforced, their perceptions that mathematics is learned by acquiring knowledge from experts who transmit information. It also indicates the persistence of beliefs that mathematics is learned by remaining quiet, listening to the expert, and persisting with rehearsal strategies.

The learners' attribution of their learning to factors that aligned with their mathematical beliefs may have undermined one of the key conditions theorised to modify learner' beliefs: cognitive dissonance. It was expected that the learners would attribute their improved mathematical understanding to their engagement with the unique delivery approach, and that this would result in a disequilibrium, or dissonance, between their beliefs about how mathematics is learned and how it was actually learned. People are theorised to be motivated to maintain consistency between their beliefs and as such a dissonance or disequilibrium between existing beliefs and contradictory information motivates a recalibration (Hekimoglu & Kittrell, 2010). Unfortunately, the attribution of learning to better 'explanations' undermined the development of a dissonance between how learners believed mathematics was learned and how they actually came to learn it.

Secondly, learners tended to attribute their poor performance in applying their memorised formulaic knowledge to a lack of mastery, not to any limitations in the

approach itself. Terry's statement that learning formulas was the best approach, even though aware that he struggled to do so, implied that he would continue to attempt to memorise formulas. In fact, some of the learners' responses suggest that no matter how poor their performance, they would continue to pursue ineffective memorisation strategies. This is a concern considering rehearsal strategies correlate with poor performance (Echazarra, et al., 2016). Given this belief, it is unlikely that the use of memorisation as the primary strategy will ever prove unsatisfactory.

In conclusion, because memorising procedures was only considered ineffective because of a lack of effort, and because the learners' attributed their learning to better explanations, they did not come to see their procedural mathematical beliefs as neither plausible or satisfactory, events considered pivotal for belief change to occur (Liljedahl, 2010; Pajares, 1992). The findings then, support theorists who have stated that belief networks are internally robust and very difficult to change in the face of argument or reason, but rather require a 'conversion' experience (Green, 1971; Nespor, 1987). The intervention was unable to provide the conditions necessary to prompt such an experience. The implications of the findings of the study for educational practice are drawn out in the following chapter.

Chapter 8: General Discussion and Conclusion

"So how you gonna teach a old dog new tricks?"
(*Terry, 46-year-old engineering learner*)

This study began with questions regarding how adult learners' beliefs about mathematics are related to their engagement with mathematical provision. The impetus for this was the body of research indicating that as students learn mathematics in school, they also develop beliefs about what mathematics is, how it is learned, and what their relationship to it is. Given that developing the mathematical skills of low-skilled adults is a priority of the New Zealand Government, and that the low-skilled adults typically have poorer mathematical experiences than those represented in the current beliefs literature, it was important to ascertain the beliefs they held about mathematics, and how these related to their re-engagement with mathematics in foundation-level vocational programmes.

8.1 The beliefs and behaviours of low-skilled adult learners

The first two research questions were:

1. What beliefs do low-skilled adults hold about mathematics?
2. How do low-skilled adults engage with mathematics within vocational lessons?

In response to the first question, the survey and interview results showed that most learners held negative beliefs about the nature of mathematics and how it is learned, and many believed that mathematical failure was shameful and had developed non-mathematical identities. In response to the second question, the results showed that most learners oriented strongly toward calculational/procedural mathematics and focused primarily on performance outcomes at the expense of developing mathematical understanding. Additionally, the results showed that many learners moderated their participation to avoid potentially shameful episodes. These behaviours combined to establish classroom norms that facilitated an unequal distribution of roles, allowing learners to disengage, or engage superficially, while still meeting classroom demands. These norms could be interpreted as maladaptive, in the sense that they did not result in the development of mathematical understanding and may have contributed to the consolidation of non-mathematical learner identities.

Beliefs about the nature of mathematics

The survey results identified that virtually all learners (99%) believed mathematics was primarily a procedural process and that most problems could be solved by implementing a step-by-step procedure. This was first evident in the responses to the survey statement, "All mathematical problems can be solved by step-by-step procedures" which received the highest level of agreement. It was also evident in the responses to the open question, "What is mathematics?". Mathematics was described very narrowly, with most learners describing it as "numbers", and the most elaborated responses describing mathematics in terms of arithmetic, in which problems could be solved by using various operations. The number of low-skilled adult learners holding procedural beliefs was greater than those identified in comparable studies that found procedural beliefs held by many, but certainly not all, students (Berkaliev & Kloosterman, 2009; Garofalo, 1989; Mason, 2003).

The responses to the open survey questions indicated that most learners' conceptions of mathematics were consistent with the narrowest described within the literature (Ernest, 1991; Petocz et al., 2007; Viholainen et al., 2014). The interview findings revealed learners held absolutist beliefs that situated mathematics as a static and disconnected body of knowledge rather than an interconnected system. Mathematics was typically framed as the four arithmetic operations (addition, subtraction, multiplication and division), and proficiency was described as the ability to 'do' all four procedures. Consistent with Ernest et al. (2016) these related to the notion that mathematics is external, something to be "banked". The learners' responses were consistent with other studies that found learners equate mimicking teachers' procedures with mathematical understanding (DÍaz-Obando et al., 2003; Garofalo, 1989; Mason, 2003). The learners did state that they valued understanding yet described "understanding" in terms consistent with instrumental, not relational knowledge (Skemp, 1978).

Beliefs about how mathematics is learned

The majority of learners believed that knowledge, in the form of procedures, was transferred to them from an expert authority, rather than self-constructed. The responses to the open survey question, "If a new student started your course and wanted to learn numeracy, what advice would you give them?" indicated a reliance on receiving information directly from the tutor and rehearsing what had been demonstrated. Almost every response indicated the need to 'listen' to the tutor, and nearly three-quarters (73%) of learners agreed that being a good student meant memorising facts. These results are consistent with research regarding negative

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beliefs about how mathematics is learned (Hadar, 2011; Kloosterman, 2002; Muis, 2004; Schoenfeld, 1988) and supports the research that links procedural beliefs to passive learning approaches (Briley et al., 2009; Crawford et al., 1994; Frank, 1988; Goldin et al., 2011; Hofer, 1999).

The learners' strategies for resolving a lack of understanding were limited to seeking tutor support, generally a request to repeat explanations. These strategies were similar, although not quite as dependent on the tutor, as those found within the learned helplessness literature (Sutherland & Singh, 2004; Yates, 2009). The learners' comments indicated little behavioural agency and instead reflected dependence on external support. Their goals were also consistent with passive approaches, oriented toward completing immediate classroom tasks assigned by the tutor, or avoiding embarrassing episodes, rather than more purposeful goals such as developing new skills. The results support previous studies that found that the less sophisticated a learner's conception of mathematics the less goal oriented they were, and the less likely they were to use active and dynamic learning strategies (Briley et al., 2009; Schoenfeld, 1985, 2011).

Beliefs and motivation to learn mathematics
The surveys identified positive and negative beliefs related to the motivation to learn mathematics. Almost all learners (92%) held positive beliefs about the usefulness of mathematics, and understandably, those that did not, held negative beliefs across all the scales. However, the interview data, and the responses to the open survey questions, indicated that learners had limited notions of *how* mathematics was useful, and examples were limited to money rather than the mathematical demands embedded within their target vocations. This is an interesting finding because the programmes the learners were enrolled in were funded to explicitly describe and teach the mathematics necessary for their vocations (TEC, 2014). One might have expected agriculture learners to reference ratios for fuel mixes, hairdressers to mention fractions for mixing hair colours, or sport and fitness learners the percentages they use on a regular basis, yet there was no mention of any vocational application. Swain et al. (2005) also identified that although adult learners reported positive experiences learning numeracy, they did not relate what they learned to their lives, but rather continued to reference the use of money as their primary numeracy practice. It is worth noting that a portion of the lesson content in the observed lessons was in the context of money and budgeting. However, there was no evidence that the learners' awareness of mathematical vocational demands contributed to a positive belief in the usefulness of mathematics. This is a concern because the learners'

perception of the usefulness of mathematics is widely considered a motivational factor, particularly in embedded vocational training (Berkaliev & Kloosterman, 2009; Casey et al., 2006; Cooper, 2001; Lizzio & Wilson, 2004).

Three-fourths of survey respondents agreed that effort had a positive impact on performance and the interview data indicated that learners typically believed in an equal distribution of intellect and cited social and environmental factors as those that contributed to mathematical proficiency. These results are considered positive indicators of personal motivation and persistence (Berkaliev & Kloosterman, 2009; Blackwell, et al., 2007; Dweck, 2016). However, further analysis indicated that learners held a more nuanced and limited view regarding the efficacy of effort. While mathematical proficiency was considered an outcome of effort and favourable social and environmental factors, it was also considered an atypical achievement rather than an expectation. Thus, while learners believed that effort related to achievement for *others*, the effort required for them personally to be successful exceeded their willingness. Therefore, the hypothesis that this belief leads to higher levels of effort, is questionable in this situation, given the self-beliefs that surround it. What the data suggested is that despite a positive belief in effort, non-mathematical identities had become reified. Therefore, while learners may hold a positive belief in the general sense of 'it is true for others', they did not apply it to themselves.

The learners' mathematical identities were found to be consistent with 'non-mathematical' identities found in previous research (Brown et al., 2008; Coben, 2002; Solomon, 2007). Their school experiences of mathematics shaped their view of themselves as learners of mathematics, and their ability to be mathematically successful. The interview participants' school histories were almost completely negative and their comments indicative of being alienated from those they identified as mathematically successful. They also revealed an increasing identification with other disengaged learners. Learners recalled being positioned by their peers as mathematically unable and most adopted non-mathematical identities. Learners believed that their own performance was poor, that they often did not understand content while others did, that they were judged as inferior by their peers, and that they were unable to learn mathematical content. These themes were couched in emotional narratives such as being teased or ostracised by other learners, or the realisation that their life choices had become limited. The learners' experiences, and subsequent mathematical identities, were consistent with the most serious cases in the literature (Axelsson, 2009; Brown et al., 2008; Evans, 2000; Wedege, 2002).

The learners' school experiences related to their behaviours in adult learning contexts. Learners indicated that they avoided behaviours in their current classes, such as answering questions or asking for help, because of the potential for judgement from other learners. For example, Niki's response to the question: *What do you think the others are thinking* [when the tutor is helping you]*? "That I'm dumb",* revealed a possible reason for why she remained quiet in class when she did not understand a fractions concept in her hairdressing programme. The presence of more competent learners in classes also appeared to inhibit public participation. Learners became more aware of their comparatively lower skills and this contributed to feelings of inferiority. For example, speaking of the presence of more proficient members of the class, Niki stated, "*And it makes people feel like 'oh, okay you don't know what you're doing, so be quiet'".* These results add to studies that find negative school experiences may contribute to anxiety and fear when adults return to study, and that they tend to compare themselves against their peers (Swain, et al., 2005; Tennant, 2012). While some learners made references to being more resilient to judgement in an adult context, generally they still reported limiting their interactions due to feelings of judgement. This contrasts with United Kingdom findings in which learners reported feeling freer to interact in adult classes than in their school classes, citing a more relaxed atmosphere (Coben et al., 2007). This may be due to an environment more conducive to learning cultivated by the more highly-qualified tutors in the Coben study, which suggests possibilities for improvement in New Zealand. The lack of New Zealand research that explores early school leavers' experiences, backgrounds and beliefs is a concern. The lack may have resulted in policy-makers generalising blanket themes such as 'math anxiety', or 'poor numeracy', yet lacking an understanding of the deep emotional and identity-forming experiences that occurred in learners' school lives.

Classroom discourse patterns

The second research question explored how low-skilled adults behaved within vocational mathematical lessons. A key finding was the impoverished level of mathematical discourse occurring at both the whole-class level and between learners. Other than the high frequency of off-task talk, the discourse patterns resembled those of other adult classrooms in which tutors dominate the discourse, adopting transmission approaches as the primary pedagogical approach (Coben et al., 2007). Arguably, one advantage of being an adult learner is the ability to draw on rich experiences and prior knowledge to engage in argumentation (Knowles, et al., 2015). Yet, the discourse observed aligned with the lowest description of mathematical discourse in Hufferd-Ackles et al.'s (2004) framework. Learners rarely

engaged in mathematical arguments, and therefore were unable to engage in a 'negotiation of meaning' (Voigt, 1994). There was a distinct pattern of deferring to the authority of more "able" learners whether an answer was correct or not. This extended to ceding responsibility for answering tutor questions to 'solvers', a practice consistent with the "consolidation of responsibility" (Howard et al., 2002). Solvers monopolised the class discourse by replying immediately to almost all tutor questions with short unelaborated responses that required little effort. As such, the whole classroom discourse lacked mathematical complexity, and was arguably insufficient to develop mathematical understanding (Mesa, 2010; Yackel & Cobb, 1996).

The pattern of discourse described above was limited to mathematical discourse, not discourse in general. A key difference from other observation studies was that learners engaged, to a far greater degree, in off-task talk than is generally found in adult contexts (Mesa, 2010; Tenant, 2012; Weaver & Qi, 2005). The off-task talk resembled Wood and Kalinec's (2012) study in which only 10% of the talk within a group of school students was devoted to mathematizing, the rest was oriented toward people, events, and what learners were doing, or should be doing. This behaviour may have been part of a passive avoidance strategy designed to reduce mathematical discourse and with it the risk of a potentially embarrassing episode. Yet, the stark contrast between mathematical and non-mathematical discourse patterns was indicative of beliefs about appropriate behaviours and objectives within a mathematics context. The learners' general discourse, both private and whole-class, was oriented toward sharing information. In contrast, their mathematical discourse was oriented toward *receiving* information, and only reluctantly sharing it.

Group problem-solving and engagement
A second key finding was the poor engagement levels observed during group problem-solving sessions. Many learners abdicated responsibility for problem-solving to more procedurally proficient learners which resulted in an unequal distribution of mathematical engagement. The procedurally proficient learners took the lead roles, quickly applied procedural solutions, and elaborated answers and methods. They set the pace, rapidly working through problems on behalf of the group, and dominated the discourse. Because these learners posited a procedural solution before the others, and without group discourse, the group transitioned immediately to calculating the proposed procedural strategy rather than discussing context, the validity of the proposed method or alternative approaches. No opportunity was made for learners to draw on prior experience or make real-world connections. The speed of procedural solutions also inhibited critical discussions of the solution strategy

because other members were yet to make meaning of the problem, and were therefore, not able to critique whether the proposed calculation was, or was not, valid.

The learners who left mathematical thinking to the procedurally proficient either adopted, or were consigned to, support roles within the group, such as recording the answers, reading the problem aloud to the group, or manually using the calculator while another learner dictated. These support roles enabled them to be active participants yet engaged only in activities peripheral to mathematical reasoning. The deferment to higher skilled learners has been identified in adult contexts (Weaver & Qi, 2005), and in mathematical contexts (Goldin et al., 2011). The behaviour of the learners adopting 'supporter' roles is consistent with the engagement pattern labelled "get the job done" which Goldin et al. theorised was driven by the desire to complete a task out of a sense of obligation. They argued that emotional satisfaction resulted from completing the task, and to do so learners in group settings would enlist the help of others to fulfil the goal. Similarly, the behaviour of the learners in the vocational classes appeared directed toward *using* higher performing learners as tools to achieve desired outcomes. They demonstrated satisfaction when the group solved the tasks, such as clapping or shouting, despite not having contributed to the process. However, the desire to "get the job done" out of obligation to the tutor explains only an aspect of the learners' motivation. Learners also appeared motivated for their group to complete all problems quickly and accurately so that they could safely answer the questions during the whole-class marking session.

This unequal division of labour identified within groups as learners adopted distinct roles, such as 'solvers' and 'supporters', was highly efficient in producing rapid and accurate solutions to multiple problems and avoiding shame. The decision to organise in this way is rational when considering the learners' probable underlying beliefs and goals. A concern is that the patterns above contribute to the establishment, or maintenance, of learners' non-mathematical identities by routinising their practice of ceding mathematical agency to other more skilled learners. This practice of lower-skilled learners merely supporting the higher-skilled learners, and higher-skilled learners accepting and encouraging this behaviour, is similar to that observed between some teachers and their students with learning difficulties (Brosseau, & Warfeild, 1999; Heyd-Metzuyanim, 2013). In these cases, the interaction between teacher and learner settles into a pattern in which the teacher becomes complicit in the learners' non-participation. A pertinent reason why the teacher does this is to avoid placing the learner in a position of potential

embarrassment. It may be that higher-skilled learners give answers to their peers to save them from embarrassment, as appeared the case with Troy and Curtis. According to Heyd-Metzuyanim (2013) such arrangements contribute to the co-construction of a disabled-mathematical identity. Their established roles allow higher skilled learners to participate and succeed, and allow lower-skilled learners to fulfil the classroom demands of solving all problems, while avoiding shame. Unfortunately, as with Heyd-Metzuyanim's findings, the result is a pattern that constrains the learning opportunities of the lower skilled, and, possibly worse, consolidates non-participatory identities, beliefs about non-ability, and instils dependence on others.

The number of learners abdicating responsibility to higher skilled learners raises concerns about the development of an adult's agency within the lessons. The adoption of passive mathematical roles by less proficient learners appears non-agentic from an adult numeracy perspective. For example, the PIAAC Numeracy Expert Group (2009) suggested that avoiding, delegating, or only completing a portion of a task, due to negative self-concept, falls short of autonomous engagement and may lead to a failure to achieve learning goals. Yet, it is beneficial to examine the learners' goals in this situation and whether the learners achieved them or not. The goals appeared to be an amalgam of shame avoidance, group acceptance, and task completion goals. Viewed from the perspective that these were the desired goals, the learners' behaviours do reflect agentic autonomous behaviours, by meeting four criteria of agency: intentionality, forethought, self-reactiveness, and self-reflectiveness (Bandura, 2006).

Lower-skilled learners demonstrated *intentionality* by taking advantage of converging self-interests of the individuals within the group. 'Solvers' sought to solve all problems quickly and accurately while 'supporters' may have sought security in having all problems completed accurately. The supporters' awareness that the action of relinquishing all problem-solving to the solvers would successfully achieve these shared goals suggests *forethought*. It also suggests that the pattern is sufficiently established that learners are familiar and skilled navigators of it. Furthermore, supporters were not typically passive onlookers, (although some were) but better described as 'active supporters'. They demonstrated s*elf-reactiveness* by pro-actively implementing strategies to facilitate the problem-solving process. While not actively involved in mathematical thinking, they took responsibility for important peripheral tasks such as recording the answers to the problems for later reference. One purpose of these behaviours may have been to add efficiency to the process by removing time-consuming tasks from high-skilled learners (other possible reasons

are discussed below). This suggests that to some extent learners *self-reflected* on their choices, fully aware that these actions would achieve their goals. In some sense, the lower-skilled learners were using the environmental resources, in the form of procedurally-proficient learners, as tools to achieve their goals.

This suggests that low-skilled adults do exercise agency but have developed an alternative 'dance of agency' than that described by educators (Boaler, 2003; Grootenboer & Jorgensen, 2009). The 'dance' is described as the interplay between the personal agency adults use to make decisions, including which mathematical tools, processes, or machines to use to accomplish their goals, and the so-called discipline of mathematics, the fixed processes that produce answers (Pickering, 1995). Yet, from a mathematical education perspective, rather than engage in the dance between their agency and that of the discipline, low-skilled learners engaged in a dysfunctional dance. They appear to exploit the routine practice of mathematical group work, specifically leveraging the performance goals of proficient procedural learners by conscripting them as willing tools to achieve their goals. Or, in Bandura's (1999) terms, exercising their own agency through the proxy agency of intermediaries. Unfortunately, while this dance was successful in achieving their goals, it removed them from engaging at a level sufficient to develop conceptual understanding, and is likely to reinforce learners' non-mathematical identities and their expectations for future identities.

The diverse participatory roles may also have reinforced learners' perceptions that procedural proficiency is the mathematical goal, because the application of procedural approaches by the 'solvers' was nearly always successful. Lower skilled learners observed all problems being solved with a pre-learned procedure, without the need to investigate the problem, consider the context, draw on their prior knowledge, establish relationships, or to manage affective responses. A common characteristic of a mathematical problem in the literature is that the solution is not immediately accessible to the learner via an algorithm or procedure (Callejo & Vila, 2009). This may have been the case for most learners, however the practice of the 'solvers' to rapidly produce procedural solutions prevented any opportunity for others to work on such problems. This also perpetuated the higher/lower procedural proficiency status of group members by maintaining the distinction between the mathematically able and the less able.

It is also worth noting that the patterns identified also undermined the advantages attributed to adult learners. Adult learners are thought to be more self-directed than

younger learners (Knowles et al., 2015; Ross-Gordon, 2003), more goal-oriented (Kasworm, 2008), and possess valuable life experience that can be used to process and assimilate new information (Compton et al., 2006). Given these assets, group problem-solving ought to facilitate the sharing and relating of life experiences with other learners regarding mathematical scenarios. It is largely for these reasons that group problem-solving is cited as a key recommendation for developing adult learners' mathematical skills (Condelli et al., 2006; Nonesuch, 2006). Group work is argued to facilitate the *inclusion* of learners with lower mathematical skills, because these learners can contribute by drawing on their prior knowledge (Marr, 2001). Yet the belief systems, behaviours, and environment, led learners to apply procedures so quickly that there was no opportunity for them to discuss the context, meaning, or make links to personal experience. As Schoenfeld (2011) has argued, learners' beliefs influence what goals are prioritised in the moment, and in this case, these related to the rapid and accurate completion of problems, and the avoidance of shame. The fact that learners who did not engage in mathematical thinking displayed positive emotions when their group's answers were validated by the tutor as correct, suggests their belief that they were successful, regardless of their non-engagement in personal meaning-making.

The patterns observed in this study cannot necessarily be generalised beyond the situations in which they occurred. Yet they raise concerning questions about the ability of foundation-level embedded mathematical provision to improve a low-skilled learner's relationship with mathematics and develop meaningful skills. Adults require mathematical agency *and* mathematical skills to be successful (Gal et al., 2005). Higher-skilled learners (the 'solvers') appeared to remain largely unchallenged by the content and used their agency to exercise their procedural knowledge, despite evidence that procedural learning does not lead to effective problem-solving skills and may even interfere with it (Echazarra et al., 2016; Jäder et al., 2017). Lower skilled learners used their agency to avoid engaging in mathematical thinking while continuing to meet classroom demands. The idea that a mathematical task can be completed by using a higher-skilled person as a proxy for the learner's own engagement is contrary to the findings of research in adult numeracy and the wider mathematics education research. Ongoing participation in the practice may be worse than no provision at all.

Beliefs and affective engagement
The lessons triggered positive and negative affective responses in the learners and these appeared to have a direct impact on engagement. Lessons were interspersed

with negative utterances and gestures that revealed emotional and attitudinal responses to content. Key moments included: learners becoming either aggressive or despondent when struggling to make sense of mathematical content, and reducing their engagement accordingly; 'Evaders' refusing to engage at all, choosing to engage in alternative tasks such as texting, reading or doodling; learners passing off others' work as their own; learners becoming overwhelmed by a moment of perceived failure, and disengaging. However, few of these responses were overt. Rather they were expressed privately to a select few, or quietly to themselves as self-talk. Without the judiciously placed recording devices, the research would have failed to capture this 'hidden dimension' of classroom interaction (Bauersfeld, 1980). The affective responses typically occurred outside of the awareness of the tutor and other learners. The results confirm other research that finds negative emotions suppress and constrain engagement and that learners hide these responses from others (Bibby, 2002; Goldin et al., 2011; Weaver & Qi, 2005).

Avoidance strategies

Learners used a variety of strategies to exert control over the degree to which their mathematical performance was made public to the class. Strategies were both active and passive. The passive strategies of lower-skilled learners appeared intended to avoid public displays of mathematical performance by becoming inconspicuous, or in other words, hiding in plain sight. Strategies included those identified in previous studies, such as "civil attention", "consolidation of responsibility", "passing and disguising", "shutting off", and avoiding verbal interaction (Bibby, 2002; Howard & Baird, 2000; Karp & Yoels, 1976; Turner et al., 2002). Learners adopted 'civil attention', the act of attending to the tutor while feigning understanding. This behaviour appeared intended to project an impression to the tutor that there was no need to change the pedagogical approach or implement a formative assessment, such as asking the learner a question (Weaver & Qi, 2005). Likewise, the 'consolidation of responsibility' reduced the risk for the lower-skilled to be addressed by the tutor because other learners were well established as speakers.

A range of more active disengagement strategies were also evident, such as projecting their emotional distress as a signal for others to leave them alone, in contrast to those designed to subtly avoid attention. These were observed when learners appeared unable to maintain control of the access others had to their thinking - for example, when a tutor directly asked the learner a question. This was evident with Sarah who expressed a desire to control the degree of public access to her mathematical thinking *("I just don't want them to think I'm a dumb c**t")*. In

response to answering a question incorrectly, Sarah uttered a loud sound that expressed disillusionment with her own ability, pushed her seat back, physically moved away from her work, and adopted a posture that clearly projected her unwillingness to participate any further. These actions appeared to be intended as a signal to others of her discomfort and her need for space, providing her with protection against a repeat of the episode. The purposeful display of emotional reactions has been used to achieve similar goals (Murray, 2008).

However, some learners signalled their disengagement from the beginning of the lesson. For example, Darrell, during a ratio lesson, sat apart from other learners with his hood up covering his face and engaged in drawing pictures (p. 117). Rather than this strategy being a *response* to a shameful event within the lesson, the behaviour was likely due to events having already occurred in his history. Darrell's complete disengagement is consistent with Turner and colleagues' (2002) contention that some students do not try at *all* to stave off public judgement, and importantly, aim to ensure that the causes of failure remain uncertain to others. Perhaps, by making no effort, Darrell believed that no one was able to make a confident judgement on his degree of inability. It is also consistent with Chinn's (2012) suggestion that some learners will make no attempt, rather than risk failure. This again suggests the persuasiveness of beliefs about the relationship between mathematics, intellect and social value reported in the literature (Boylan & Povey, 2014; Siivonen, 2013).

Finally, one more pattern of engagement requires discussion. There were occasions when learners were unable to abdicate reasoning to others and had to engage mathematically with problems. These occasions resulted in two broad patterns, positive and negative. The negative responses related to situations when learners appeared to evaluate a problem as too difficult to solve. The behaviour patterns reflected the idea that learners make conscious or unconscious evaluations about their performance in relation to their target goals, and if negative, shift into an appraisal orientation and enact strategies to mitigate harm (Bibby, 2002; Malmivuori, 2001). For example, Tane, a learner in the agriculture class, overwhelmed by the complexity of a 25:1 petrol to oil ratio, stated somewhat aggressively, "*Just ask the man at the petrol station*". This response suggested that he appraised the context and concluded that it was unnecessary to do the calculation given that service station personnel would complete the task for him. Such disharmony between a learner's actual experience and taught mathematical content is cited as contributing to learner resistance (Jarvis, 2001; Wedege & Evans, 2006). Yet, the utterance may have also had a strategic intention, that of convincing class members that the mathematics was

irrelevant, mitigating potential social damage from failure. In short, the evaluation that a problem was unable to be solved, led to affective responses, appraisals of the context, and strategies to mitigate harm. The learners' actions were consistent with strategies intended to mitigate the social harm often associated with failure (Bibby, 2002).

In contrast, some learners, perhaps characterised as 'persisters', continued to engage with challenging tasks, and solved them despite frequent errors, difficulties and frustration. Their behaviours were consistent with Allen and Carifio (2007) finding that mathematically sophisticated learners were able to manage their affective responses, specifically making ongoing self-evaluations of their progress. This differs from those who appeared to make a single evaluation of the difficulty of the content and an immediate decision to disengage. Allen and Carifio recommended pairing these diverse learners together in order for affective management to be modelled to the lower skilled learner. However, in this study, the learners who did demonstrate higher affective management showed little interest in collaborating, almost actively reducing the role of the other. Improving collaborative models in such settings is an important area of future research.

Finally, it must be noted, that positive emotions were also evident within the lessons. Learners celebrated successes. They clapped hands, high-fived each other, whooped and cheered. Amongst a range of negative findings, what is clear is that the learners *cared* about their performance with mathematics. Many invested emotionally into problems and responded positively when successful.

Conclusion

What is evident from the results of this study is that the elements that lead to the development of mathematical understanding, such as engaging in mathematical discourse and meaning-making, are almost entirely absent from the learners' current classroom experiences. Instead what is observed is a 'dance of dysfunction', a pattern of behaviours that reduce engagement in mathematical thinking to the degree that the development of conceptual understanding is unlikely, yet simultaneously enabling the learners to experience a form of pseudo-success within the lessons.

The patterns of behaviour adopted by the learners were consistent with their beliefs. Overwhelmingly, learners believed that mathematics was a fixed body of procedures, rules and methods. True to the warnings of researchers (Goldin et al., 2009; Schoenfeld, 1988), these beliefs led to an orientation toward completing routine tasks

quickly and efficiently rather than developing conceptual understanding. What was not as evident in the literature, is that groups of learners would arrange themselves in ways that would efficiently achieve the completion of tasks, such as assigning responsibility for mathematical thinking to higher skilled learners (the "solvers"), while the less skilled took support roles (the "supporters"). The arrangement provided safety for those concerned with avoiding shame and enabled the procedurally proficient learners to solve problems without any conceptual challenge at all. Not only did this dilute any mathematical learning that may have occurred for either high or low skilled learners, it also seemed likely to consolidate lower skilled learners' non-mathematical identities by routinising the assignment of mathematical thinking to others.

8.2 Learner responses to the intervention

Given that the findings of the survey, observations and interviews, indicated that learner beliefs negate much of the positive benefits of embedded mathematics instruction in vocational programmes, the third research question explored how low-skilled learners would engage with a conceptually-oriented mathematics environment. The third research question was:

> 3. How do low-skilled adults respond to a classroom environment that emphasises conceptual understanding?

The results of the programme revealed a mixture of positive and negative responses, each of which offers insights into how educators might improve learner engagement. The learners generally resisted engaging in conceptually-oriented content. Their strong beliefs about what mathematics is and how it ought to be learned contributed to the view of conceptually-oriented activities as superfluous to their learning goals. Additionally, the learners were wary of the potential social harm inherent in activities that emphasised sharing and negotiating mathematical-meaning.

Learner background and beliefs

The learners' beliefs, school experiences, mathematical identities and mathematical skills were similar, if not worse, than those reported in Chapter 6. Without the intervention it was unlikely the participants would develop the mathematical skills to complete the engineering programme and gain related employment. This contributed to learner frustration, a vivid example of which was evident with Terry, aged 46, who had reported being able to "*pull a car apart and put it back together*", but unable to do

"*Pythagoras and all this s**t*". He resented the system which he perceived compelled him to get his 'ticket'" to do a job he felt he was already able to do. Furthermore, the learners' beliefs were strongly procedural, and based on their utterances in interviews and during the intervention most had non-mathematical identities. The findings illuminated how poor numeracy skills may contribute to negative life outcomes (Bynner & Parsons, 2006; Marcenaro Gutierrez et al., 2007). A lack of mathematical skills meant that passing the programme required substantial upskilling, and their configuration of beliefs made this almost impossible. Their beliefs about what mathematics is and how it is learned, led them to adopt ineffective approaches and beliefs about themselves and the social meaning of mathematics contributed to negative affective responses that further disrupted learning. The findings reaffirm the deep personal and social costs adults incur regarding self-identity, civic inclusion, and their limited employment opportunities (PIAAC Numeracy Expert Group, 2009).

Pedagogy

Learners resisted efforts to participate in active learner roles associated with a constructivist pedagogy and instead sought to maintain a pattern that conformed to traditional passive learner roles. Three compounding factors contributed to difficulties implementing the change: the learners' beliefs regarding the roles of the tutor and the learner; beliefs about how 'good' learners learn mathematics; and the risk to their social status within the class.

Firstly, the learners' beliefs about my role as the classroom manager, and theirs as passive participants, constrained the establishment of a constructivist pedagogy. The learners' expectations, and preferences, were for an authoritative tutor who set and policed boundaries for behaviour. Tyrone's justification for a strict tutor summed up most learners' attitudes, "*'Cause, work gets done*". In contrast, in a class without a strict tutor he states "[we] *just f**k around*". These preferences were consistent with research findings undertaken with urban school students, who preferred the teacher to set and enforce rules because of disruptive behaviours from other leaners (Delpit, 1995; Jones, et al., 2013). Surprisingly however, the learners in this study wanted not only the behaviour of *other* learners controlled, but their own as well. Harkin (2010) identified a similar tension between the expectations for discipline of 14-year old students who entered an adult training environment. The teachers expected the learners to "behave" in an adult environment, yet some students preferred to be strictly managed, and misbehaved when not. My decision to step back from an authoritative classroom manager role created a tension as the learners struggled with

less direct control. As I reduced control, the learners tended to disengage with content and increased off-task talk, creating a pedagogical conundrum. Increased control led to engaged yet passive learners, while less control resulted in an increase in off-task talk and non-engaged learners.

Compounding this issue, the learners in both the observations and intervention appeared to attribute learning mathematics to passive receptive behaviours. This can be summed up by Kyal's utterance "*Let's just look, listen, and learn*", which he requested the class to do following my discussion on the benefits of active learning behaviours. The learners' behaviours reflected passive approaches associated with the belief that teachers are responsible for student learning (Taylor et al., 2005) consistent with the belief that mathematics is learned by listening to and mimicking the teacher (Muis, 2004). A challenge for establishing greater learner participation was that although learners frequently breached behavioural boundaries (such as off-task talk) whenever they wanted to 'learn', they conformed to 'civil attenders' adopting passive roles, which included refraining from discourse with others.

These behaviours are consistent with two beliefs. First, the belief that the tutor ought to strictly manage the class; this was associated with poor self-management skills. Second, the learners' belief that learning mathematics is accomplished by sitting quietly and watching and listening to the tutor. This explains the learners' shift to off-task behaviours during activities not directly managed by myself and their ready conformity to a traditional student role during transmissional teaching. This leads to an important insight: there might be a temptation for tutors to take advantage of the learners' beliefs as a classroom management tool. Tutors themselves, struggling to manage adults who breach classroom norms, may adopt traditional approaches as a management tool. Given that most tutors have little, or no, mathematics education training, it seems unlikely that they will persist with constructivist approaches if they appear to contribute to disruptive behaviours, particularly if the benefits are not clear.

In addition to a reduction in off-task behaviours, the learners themselves provided positive feedback about procedural approaches. On the few occasions that I delivered procedural content in a traditional manner, the class worked quietly and completed problems. Considering the almost continual classroom talk at other times, this was striking. These episodes finished with learners approaching me and thanking me for delivering the content. I received a literal pat on the back for delivering one procedure in a traditional manner. I received no such thanks for preparing manipulatives, sequencing activities and organising more contextualised

content to develop conceptual understanding. Learners requested procedural content and resisted constructivist approaches. Unfortunately, as was evident in the observations, the pseudo-achievement gained from attention to procedural content did not necessarily translate into mathematical understanding.

The establishment of mathematical discourse *between* learners was also difficult to cultivate and this also interfered with my ability to establish a constructivist pedagogy. For example, the learners' typical response to being asked to discuss content with each other was to avoid engaging each other *at all* and remain oriented toward the front of the class. The majority remained silently attentive, while the 'talkers' attempted to re-engage myself, the tutor, in further conversation. This behaviour improved slightly throughout the programme, yet the learners consistently avoided mathematical discussions that did not include me as the central coordinator of discourse. Studies have observed that adults in numeracy classrooms often engage in simple rather than complex mathematical discourse (Coben et al., 2007; Mesa, 2010). Yet the limited discourse between learners in this study was considerably more constrained than previous studies have identified, suggesting that low-skilled procedurally-oriented learners are particularly resistant to inter-learner dialogue.

One explanation for the lack of learner-to-learner discourse was the learners' desire to avoid damaging the status balances that existed within the classroom. This included avoiding losing status by appearing ignorant which is consistent with other findings (Bibby, 2002; Tenant, 2012). It also included avoiding displays of proficiency that might breach others' expectations of their ability, which would also interfere with existing status balances within the group. One strategy to avoid either situation was to avoid any public displays of performance.

Maintaining, not just improving, one's status is thought to be of essential importance to individuals because it signifies their social location within intersecting systems of stratification and therefore relates directly to the learners' sense of stigma or esteem (Dunn & Creek, 2015; Scheff, 1994). Learners' identities are deeply connected to their roles and their positions among others within a community, and shame signals a threat to the social bond (Bibby, 2002). The learners' risk of damaging their status through poor performance appeared to inhibit their engagement in public activities such as sharing ideas with others. An example of the discomfort of lowering one's status was evident in the interaction between Tyrone and James, in which Tyrone grudgingly had to take a lower status role than James to elicit from James how he had solved a problem. Both Tyrone and James struggled with their shifting

hierarchies, with James uncomfortable with the role of a teacher and Tyrone uncomfortable with being positioned as a learner with respect to James. The result was an uncomfortable exchange that included Tyrone using sarcasm and insults to maintain his status, all the while clearly wanting to learn how James solved the problem.

The risk of *raising* status by performing better than anticipated also appeared to constrain learners' participation in mathematical discourse. The impact of this was evident when I asked learners to share and elaborate on their ideas. The act of sharing a mathematical argument drew evaluative and critical attention from other learners, not on the ideas, but on the learner's status. This included my attempts to use learners' contributions as objects of discussion, or even encouragements. For example, my response to a participant's tentative answer to a question, "… *you're quite sharp with maths right?*", drew a negative response from others, who exclaimed immediately "*No he's not!".* The learner did not contribute again in the lesson. Although not a result of poor performance, it fits the notion that shame is caused by losing or gaining status, because it damages the existing social bond (Bibby, 2002; Scheff, 1994). The safest way to avoid damaging the social bond may have been to avoid active behaviours altogether and adopt passive traditional learner roles.

Traditional pedagogies are the norm in most adult numeracy classrooms (Benseman et al., 2005; Coben et al., 2007; Ofsted, 2011). They are also typically viewed as teacher driven (Mewborn & Cross, 2007; Rozelle & Wilson, 2012; Swan, 2006). However, the results of this study suggest that these pedagogical environments are at-least partially constructed by the learners themselves, rather than imposed. The learners' ready conformity to traditional roles, hesitancy to discuss ideas, preference for individual worksheets and other similar behaviours, all exert pressure on the tutor to assent to their preferences.

This may explain the dominance of transmission approaches in adult numeracy classrooms. For example, the following quote from Coben et al. (2007, p. 38) reveals a tutor attempting to make adult numeracy lessons more collaborative, yet encountering resistance from the learners who appear to prefer individualistic approaches:

> I have tried to make it as student-centred as possible, trying to get them to work together. But they are such a hard-working group, they just want to work independently. Or they want to get through the work really fast […] I mean, some people just naturally work

together, and some people you have got to make them. I mean, I am trying, but you saw, they just didn't speak once, just working away.

(Coben et al., 2007, p 38)

In light of the findings of this study, the learners' desire to work independently, the "really fast" pace at which they work, and their lack of mathematical discourse might be better explained by their beliefs about the role of the tutor, the nature of mathematics and how it is learned and attempts to avoid damaging the social bond. For example, the class referred to above may have resisted collaborating because they believed that mathematics is learned by listening to experts and completing tasks independently. They may have believed that collaborating with non-experts is superfluous, and some may have viewed sharing their thinking as socially and personally harmful. The tutor noted that she had tried, and continued to try, to implement a 'student-centred' model in which learners discussed problems, yet they insisted on working quietly and individually. Interestingly, the tutor attributed the learners' behaviour to an orientation toward 'hard work' and her comments suggests a resignation to the fact that the learners were 'just this way'. Learner resistance to learner-centred pedagogies is not new (Johnson et al., 2009), but the findings of this present study suggest the pressure on tutors to conform to traditional pedagogies is considerable.

Engagement with conceptual mathematics
The low-skilled learners' beliefs about the absolute utility of procedural approaches did not appear to change despite messages about the benefits of conceptual understanding and activities demonstrating how conceptual understanding could support problem-solving. This was surprising considering the learners were experiencing substantial difficulties applying the procedures covered in their regular mathematics programme and were increasingly aware, as time went on, that they were failing to meet the mathematical demands of the programme. Being dissatisfied with the current mathematical beliefs, and being exposed to new conceptions, are considered catalysts for learners to change beliefs (Bendixen, 2002; Hekimoglu & Kittrell, 2010). However, the learners were not dissatisfied with their mathematical beliefs because they did not attribute their difficulties to a failure of the procedural approach or a lack of conceptual understanding. Instead they cited factors like those reported in Chapter 6, such as their own inability to learn, the speed of delivery, or the fault of the tutor. The learners continued to reference the memorisation of

formulas as their favoured learning approach in the post intervention interviews. This result differs from several school-situated studies that show some evidence of a belief change in response to the introduction of a conceptual approach (Lampert, 1990; Mason & Scrivani, 2004; Muis, 2004; Verschaffel et al., 1999).

There are several factors that may have contributed to the persistence of procedural beliefs. First, consistent with other studies, the learners appeared to perceive conceptually-oriented activities to be *superfluous* to their mathematical goals (Cooney, 1985; Johnson et al., 2009). Conceptual understanding, and the activities that were designed to develop it, were not considered by the learners as the 'serious' work necessary to acquire essential skills. The learners' belief that they needed to focus on serious work was driven in part by their apprehension of failing critical mathematical assessments and the belief that procedural knowledge was essential to pass. This is consistent with the notion of the 'backwash' effect of examinations, in which learners' goals are increasingly dominated by considerations of assessment (Skemp, 1978). The mathematical assessments the learners were required to complete were 'open book', which, given that the book contained the formulas required, implied to learners that the objective was to identify and apply the formula to the corresponding problem. This was a message expressed by the regular mathematics tutor as often happens in a vocational context (Engelbrecht et al., 2009; Swain & Swan, 2007). Many conversations, two of which are included within Chapter 7, revealed the rising tension fuelled by an increasing awareness of the discrepancy between their own skills and the assessment criteria. Research shows that adult learners engage with mathematics primarily to get a qualification or a job (Coben et al., 2007; Swain et al., 2005). Thus, many are motivated to learn the mathematics required to pass an assessment, rather than investing time to develop conceptual understanding that may be fun, but non-essential (Johnson et al., 2009).

A further interesting insight on the learners' preference for procedural instruction was their notion of 'work' and how this was related to progress. A typical transmissional classroom experience for the learners included completing multiple problems and copying formulas into personal workbooks. The learners considered that work of this nature resulted in *tangible* evidence of progress. The learners appeared to equate their recorded book work with their actual knowledge. For example, Kerri was proud of her copied book work stating, "*Have a look at this book and I'll show you how much I know",* despite not being able to apply the formulas within it during the intervention. Adults are often motivated to re-engage in mathematics to prove to themselves that they can learn a high-status subject (Swan, 2005). It appears that

the nature of a procedural approach, such as recording formulas, satisfied the learners' judgement that they were 'doing maths', more so than the less tangible outcomes of conceptual approaches. The procedural approach had tangible outcomes that the learners took as evidence of progress, such as completing a certain number of problems in a lesson, working in mathematics book that looks 'flash' due to the copied formulas, or passing an assessment. Conceptual understanding might not provide the concrete feedback required by such learners.

Despite the difficulties I encountered in attempting to engage learners in conceptually-oriented instruction, they did nevertheless engage with inquiry-oriented problems on occasions and improved over time. This resulted in a range of engagement patterns, a better distribution of mathematical discourse and higher quality mathematical thinking. The most successful problems were those that allowed learners to contribute by making safe estimates of an answer, which avoided risking the social harm that might have come from an overt failure. Once engaged, many learners were motivated to find a solution. However, they were uncomfortable with 'effortful struggle' and attempted to disengage with problems that they were unable to solve quickly. These behaviours are consistent with studies that linked beliefs about the length of time a problem ought to take to solve to early disengagement, because learners evaluated them as personally unsolvable (Berkaliev & Kloosterman, 2009; Kloosterman & Stage, 1992; Schoenfeld, 1989). However, what is not evident in the adult numeracy literature is the specific strategies learners used to disengage from the problem which were found in this study; namely: resigning themselves to failure, making guesses, or attempting solve the problem in a non-mathematical way. The second two behaviours are consistent with the notion of learners attempting to passively satisfy the minimum demands of a lesson (Hadar, 2013). All three signalled the learners' preference for shifting the responsibility for solving the problem back to the tutor. The implication of 'passing back' responsibility to me was that I as the tutor was expected to explain, and/or provide the solution.

Furthermore, learners' attempts to disengage shed light on Jonsson et al.'s (2016) finding that procedurally-oriented learners incorrectly applied procedures to problems rather than engaging in creative mathematical reasoning. They noted that these learners engaged less in "effortful struggle" and this was thought to be a key contributor to poor performance. The low-skilled learners' disengagement strategies observed in this present study, such as guessing, raise the possibility that the learners incorrectly applying procedures in Jonsson et al. were also using guessing as a strategy to end the task rather than persist with effortful mathematical thinking.

The findings of this study support the notion that procedurally-oriented learners avoid creative mathematical reasoning because it does not align with their beliefs and it requires effort that may be considered uncomfortable (Jonsson et al., 2016; Stylianides, & Stylianides, 2014; Sumpter, 2013).

Despite, being uncomfortable with effortful struggle, the learners could be engaged longer in mathematical thinking through use of verbal prompts and encouragement such as "*You've worked out so much already. Keep going.*" Such behaviours are recommended as key adult instructional approaches (Hodgen et al., 2010). However, for some learners, the extended engagement with a single problem without an apparent solution resulted in negative affective responses.

As discussed above, there was a growing apprehension amongst learners in the intervention about not meeting the mathematical demands of the programme. This growing frustration appeared to aggravate the in-the-moment, negative responses to difficult problems. The expectation of failing the programme reduced learners' tolerance for tasks that did not yield to procedural approaches and became part of the appraisal content when learners became overwhelmed by emotion. For example, learners reacted negatively when unable to solve a task, and attributed blame either to themselves, or to the discipline of mathematics, or its agents, the tutors. This is concerning because it suggests that the continuation of difficulties may drive negative beliefs about mathematical identity and the nature of mathematics deeper into the learners' self-perceptions. In a similar fashion to McLeod's (1992) conjecture that continued negative experiences embed negative attitudes, continued 'failure' appears to embed negative conceptions of mathematical ability. If this is the case, then some learners may exit mathematical programmes with more negative beliefs than those with which they entered.

The final finding was that the learners' beliefs about mathematics changed little over the course of the intervention. The learners reflected that they enjoyed the class, that they learned more than in other classes, and they had developed a better understanding of mathematics because of it. However, they did not attribute this to their own engagement with mathematics, rather to my ability to explain content. Beliefs are thought to be self-confirming, a lens through which people interpret and make sense of the world (Cross Francis, 2015; Green, 1971). The fact that the learners attributed their learning to better tutor explanations, rather than their own engagement, suggests that beliefs shaped how they judged the intervention. This raises the issue of whether beliefs can be changed and the conditions necessary to

do so. The learners received overt messages based on relational views of mathematics and they were engaged in activities designed to modify their beliefs. Bluntly, despite viewing me as the mathematics authority, the learners did not accept my perspective on how mathematics was best learned, nor did they adopt the behaviours I suggested. The findings support the notion that beliefs, once developed are very difficult to change (Liljedahl, 2010; Pajares, 1992). This result contrasts with many studies that identified a change in beliefs following interventions (Higgins, 1997; Mason & Scrivani, 2004; Verschaffel, et al., 1999). However, it should be noted that these latter studies had longer timeframes, more highly skilled learners with relatively positive views of their ability and did not have participants concurrently attending a procedurally-oriented class.

8.3 Contribution to the international literature

The relationship between mathematical beliefs, learner behaviours, and authentic learning environments has been under-researched in general, specifically within the learning environments of low-skilled adult learners. This study makes several important contributions to the international literature regarding the beliefs and engagement patterns of these learners within foundation-level mathematics lessons. First, the study found that low-skilled adult learners had particularly narrow views of mathematics, held strong procedural beliefs, and had limited learning repertoires. While research has explored adults' affective relationship with mathematics (Safford-Ramus et al., 2016), beliefs about the nature of mathematics and how it is learned has received less attention. The findings suggest that these beliefs orient learners towards procedural proficiency and passive learning behaviours at the expense of conceptual understanding. These behaviours are contrary to those recommended for lower-skilled learners (Allsopp et al., 2007), and suggest that learners may continue to experience difficulties.

Second, the study showed that learners are impacted by the roles that other learners adopt. These roles can be considered social environmental determinants as they created conditions that exerted influence on other learners' behaviours. While they do require deeper analysis, examples from this study include: 'mathematical experts', the talkative experts who advertise their high skills; 'dominant solvers', the procedurally proficient group members who dominate group problem-solving sessions; 'protectors', learners seeking to protect the lesser skilled from shame by providing them with answers; 'evaders', those learners seeking to establish and maintain non-participatory patterns; 'supporters', learners seeking to meet classroom

goals by using experts as proxy agents to solve problems. These roles, and others, are shown to be substantial environmental features of the embedded lessons.

Third, the study showed that mathematical group problem-solving, so often used in adult settings (Swain & Swan, 2007), resulted in an unequal division of labour, yet presented an impression to the external observer that all members contributed. Although, some research has identified unequal participatory roles in general group discussions (Coben et al., 2007; Howard et al., 2002), it was the degree of the discrepancy between 'solvers' and 'supporters' engagement with mathematical thinking that was unknown. Many learners did not engage in mathematical thinking at all, and these appeared to be well established practices. Importantly for the international literature is that the practice was designed to be covert in most cases. The practice was only evident in this study because of the use of multiple audio-recording devices, the use of a single video recorder, direct observation and note-taking failed to identify the practice. Although caution must be applied in generalising these results, they do raise questions about the effectiveness of observational studies that utilise only a single view to observe interactions amongst low-skilled adult learners, or perhaps any group or learners.

Fourth, the intervention revealed limited learner engagement with conceptually-oriented approaches and the pressures on the tutor to conform to the learners' pedagogical preferences. Again, with the benefit of the multiple audio-recorder methodology the learners' experiences and behaviours during the lessons were revealed. Although tutor difficulties implementing constructivist, or problem-solving pedagogies, in school and university environments have been discussed (Johnson et al., 2009), this study revealed the tensions between delivering conceptually challenging content to low-skilled adults and managing their sensitivity to ambiguity and frustration. Learners' engagement patterns, and affective responses, in such environments had not been observed and this study revealed a network of forces combining to influence behaviour and affect.

The research might be advanced by further identifying key determinants within each of the triadic domains (personal features, behaviours and environment). For example, learner roles as aspects of the environment, beliefs as aspects of the personal features, and levels of engagement with mathematical thinking as behaviours. Once these aspects are clarified, the framework has potential to explore the reciprocal interplay between the domains in authentic learning environments, particularly given the use of multiple-recording devices and analysis tools such as those provided by

NVivo. The framework and methodology may be able to expand investigations into beliefs, behaviour, and environmental factors within the foundation-level environment

8.4 Limitations

Key limitations of this study relate to methodological and logistical issues. Firstly, beliefs as a concept for investigation proved problematic. While the survey and interview data provided an indication of learner beliefs, attempting to link these to classroom behaviours was difficult. This was because learners were not only engaging with mathematics, but rather engaging with others in a dynamic social situation. A detailed view of how specific beliefs related to specific mathematical behaviours was difficult. The reliance on inferring mathematical beliefs based on observed behaviours is a methodological weakness of this study, even though exploring beliefs in action is recommended, in most studies mathematical behaviours are observed in controlled problem-solving environments (Francisco, 2013).

The open question 'What is mathematics?' used in the survey has been critiqued for its limited ability to evoke elaborated responses (Latterell & Wilson, 2017). This may explain the narrow responses elicited in this study, although the interviews did provide a measure of triangulation. Latterell and Wilson (2013) found that requesting learners to generate metaphors for mathematics has proved useful and may provide richer data in the future. These may have to be generated orally due to literacy concerns with low-skilled adult learners.

The use of multiple-recording devices was highly effective for collecting ecologically valid classroom discourse. Yet, the copious amounts of recorded dialogue at various levels of interaction led to analysis difficulties. The boundaries of recorded conversations proved to be highly fluid when learners engaged in dialogue. For example, a group of learners might discuss a problem while two of the group members conducted their own private "sub" conversation, all while the tutor was speaking to the whole class. Such data was difficult to analyse, and this is reflected in transcription which either risked redundancy or becoming so complex as to be incoherent. This required limiting the focus of analysis to various aspects of the dialogue and therefore imposing my judgement on the boundaries. These decisions were informed by the research questions and theoretical framework yet added a further layer of interpretive decision-making. The rich data collected is evidence that the method is effective, yet its future use will require a clear theoretical approach, possibly drawing on discourse analysis.

Finally, caution needs to be applied in generalising the intervention results for several reasons. First, the programme was highly contextual due to my role as researcher/tutor and the negotiated environment produced from this. Secondly, the learners were attending a traditional regular mathematics class while participating in the intervention: this may have reinforced their procedural beliefs by being the primary mathematics programme. For example, the assessment criteria and tasks originated from their regular programme tutor. This is likely to have given extra weight to the tutor's content selection and possibly the pedagogy employed in the programme.

8.5 Reflections

The paucity of observational research in foundation-level programmes is surprising because it is such a rich environment for exploring. However, a reason for this may be the difficulty collecting data. There were several challenges that arose during accessing learners and classrooms. Firstly, access to the learners required multiple layers of permission; the organisation, area manager, the tutor, and the learner. While most organisations and managers agreed to participate, there was hesitation from some tutors at the prospect of a researcher entering their classes with a potentially critical perspective. I addressed this with tutors, and within ethics, being sure tutors understood the parameters of the study, and spent considerable time talking with staff, answering questions and feeding back to them.

An example of these difficulties was evident in the number of tutors who declined to take part in the observation aspect of the study. Consent from managers was obtained by four organisations to participate in the observations and yet were subsequently declined by the tutors. There were also several occasions I was invited to speak to a class only to find the class was not present on that day. I suspect that the tutors felt vulnerable about being observed and anxious that my findings might reflect poorly upon them. Their apparent reluctance perhaps reflected a lack of confidence among many tutors expected to deliver numeracy and, as discussed below, the potential for negative learner feedback.

The learners themselves were often suspicious or reserved, and rarely impartial. My insider understanding of classroom and learner contexts informed my approach. Aware I was perceived as an outsider by the learners, I understood the need to develop rapport and invest time into 'a getting to know you process'. I spent time in classrooms cultivating conversations and navigating a diversity of attitudes. Initial sarcastic or aggressive comments from learners were opportunities to laugh and

discuss mathematics, and self-denigrating comments were opportunities to share thoughts on what might have led to this. Understanding the challenges regarding specific numeracy Unit Standards, assessment practices, and timeframes was valuable in finding points of connection. The ability to navigate the dynamics of a specific context was a strength of this insider research study, involving work with learners who were potentially unwilling participants (see Coghlan, 2007; Teusner, 2016). The time and approach spent developing rapport contributed to learners completing the surveys, agreeing to being recorded, and participating fully when observed. This supported the view that developing relationships with participants minimises their potential to censor their behaviours because of self-consciousness and enables them to share potentially sensitive information rather than the researcher adopting a non-relational clinical approach (Newby, 2010), It also strengthens support for insider research as a valuable way to access and enter specific communities otherwise difficult to access (Robson, 2002).

In addition to gaining rapport and access to participants, the decision to draw on insider knowledge and adopt a dual tutor/researcher role within the intervention produced unique findings. The ability to implement various pedagogical approaches, to actively interact with learners and navigate learners' shifting affective responses contributed to insights that possibly would not have been identified otherwise. The dual role also enabled me to reflect and record the pressures felt when tutoring, such as tensions between classroom management and learner-centred discourse, or emotionally charged responses from learners frustrated with their progress. These perspectives, although common among practitioners, are rarely mentioned in the literature. It is also worth noting that the environments were at times uncomfortable, and possessing prior experiences of these environments reduced the 'shock' that may otherwise have impacted researchers unfamiliar with the environment. For example, the classroom environments were at times aggressive, there were occasional bouts of violence, and occasionally learners were under the influence of drugs. My experience supports the view that an insider's knowledge informs the planning, implementation and analysis aspects of the research process and improves the ability to navigate people dynamics, pressures, and power struggles that exist within the environment (Cohen et al. 2007; Robson, 2002; Teusner, 2016).

Although my insider knowledge was an asset to the research, it presented challenges that required examining the connection between myself and the context within which the study took place. An example can be found in the tensions that arose when learners reflected negatively on their current tutors, specifically how my role as a

researcher/practitioner may have contributed to this, and what content to include in the thesis. A common recommendation for insider researchers is to be actively alert for potential areas of tension arising from their activities (Fuller & Petch, 1995; Ravitch & Wirth, 2007). Criticisms of tutors emerging from the research was one source of tension. When learners were discussing their mathematical experiences, they frequently gave unprompted personal critiques of their current tutors, rather than more objective reflections of their tutors' pedagogical approach. Adults critiquing their school teachers is not unusual (Evans, 2000). However, I was surprised by the nature of the comments about current tutors. It was important to include these accounts because they revealed the learners' perspectives of their classroom experiences, attributions of difficulty, and their affective responses to the situations. Yet, these personal criticisms toward tutors raised questions as an insider researcher about how my role may have contributed to these and how to frame these critiques.

I reflected on a key question posed to insider researchers by Rooney (2005) concerning my own role on the participants' behaviours. That was, whether the learners' critiques of other tutors emerging from the intervention arose as the result of my active involvement delivering the intervention programme. Had I created conditions that encouraged comparisons between the intervention and the learners' regular mathematics programme, and between myself and other tutors. It is possible that it contributed to comparisons but to what degree was difficult to determine.

One solution was to include as transparent a view of my own teaching practice as practical to demonstrate the extent that my programme was comparative. Transparency is arguably of utmost importance in presenting interpretive research (Kvale, 2002). As is clear in Chapter 7, my tutoring performance in the intervention was not a paragon of good teaching practice. I too was critiqued by learners during lessons. For example, Terry accused me of making the content difficult to confuse him, and he and others made frequent outbursts expressing negative views at various times. These outbursts were born of frustration but indicate that I was not perceived as the 'better' tutor, but part of the 'mathematical teaching cohort' perceived by some learners as part of the problem, at least in that moment. Initial discussions with tutors about the potential for this were undertaken in addition to follow-up conversations about the findings. However, ongoing discussions with tutors would have been useful as a practice of reflexivity, by providing exposure to multiple perspectives, and examining my own my interpretations of events and situations.

Finally, although insider research and interpretive designs are limited in the conclusions that can be drawn from the data (Cohen et al., 2007), the strength of this study is that it penetrated some of the "hidden dimensions" of the learners' classroom experiences. This may provide the impetus for others to conduct further research using an array of approaches and methodologies.

8.6 Implications for practice

A key finding was that many learners were able to participate only peripherally in class activities and yet have the appearance of fully engaging. The ability of tutors to distinguish pseudo from authentic engagement is important, as is the ability to identify the dynamics that facilitate the process. To do this, tutors require an understanding of how their pedagogical approach contributes to classroom routines, and how learners navigate these routines to avoid shame while meeting classroom demands. It also requires being aware of the dynamics of classroom interaction, role taking, and practices such as assigning responsibility to other learners for problem-solving and civil attention.

As shown in the intervention, merely changing the pedagogy to a more constructivist model is unlikely to result in full engagement in classroom activities because of ongoing learner resistance. The challenge tutors face is that once a learner is identified as only partially engaging, their re-engagement seems likely to require the tutor to balance on the one hand, the learner's sensitivity to shame and setback, and on the other hand, setting expectations for the learner's engagement and providing a level of accountability. To do so effectively would require understanding the unique needs and concerns of each learner.

Adult learners reference the quality of the relationship with their tutor as an important factor in their enjoyment and success with numeracy, particularly trust and respect (Coben et al., 2007). This may reflect a sense of safety learners have in a tutor who understands what exposure to pressure they are comfortable with, such as knowing when and what to ask a learner in a public forum. This requires an informed knowledge of a learner's affective resilience and their capacity to reason in front of peers. Knowing a learner's prior experiences, apprehensions, sensitivity to shame, and skill level, is a starting point to engaging them in challenging mathematics. The interview process used in this study facilitated this, and although time consuming, some form of the interview schedule, albeit streamlined, could be appropriate for tutors to use with learners. Additionally, an emphasis on formative assessment as described by Hodgen et al. (2010) in which tutors utilise discussion, use open

questions, to gain in-the-moment insights into not only the learners' understanding, but their affective state as well.

Tutors did not address affective responses in the observations, even when aware of them, and this often led to disengagement. Addressing adult learners' prior experiences and negative affect is an established recommendation (Condelli et al., 2006; Tobias,1993). The intervention showed that a more open dialogue created space where negative responses could be made public, allowing them to be addressed. Learners could benefit from tutors addressing affective factors at initial stages of their programmes and lessons, opening a classroom dialogue about experiences, anxiety, shame, and strategies to manage negative emotions. Understanding and attending to signals of negative affect exhibited by learners would also be useful.

A further implication for practice is the challenge for tutors to actively avoid co-constructing unhealthy 'didactic contracts' with learners who attempt to avoid engagement (Brousseau & Warfield, 1999). Darrell in Chapter 5 presents an example, in which the tutor appears to collude with Darrell's complete disengagement. The challenge for educators is that some learners are highly motivated to establish these relationships and given that they are adults, it is neither wise nor effective to force them because to do so may actualise their fears, driving the negative affect deeper. Rather than allowing these behavioural patterns to develop out of classroom interactions, it may be beneficial to negotiate expectations for interaction privately. Interaction goals could be set, emphasised, and monitored over time(Heyd–Metzuyanim, 2013).

The dysfunctional group dynamics that emerged from so-called group problem-solving might be improved by addressing environmental factors. The seemingly ubiquitous practice of giving groups a worksheet with multiple problems to be solved in a short time frame, followed by a marking session, although convenient, provided the conditions to cultivate poor engagement. This practice has been shown to work well (Marr, 2001), but with low-skilled learners the use of several options may be more prudent. Providing ample time, so that groups do not feel compelled to hurry may reduce the orientation toward speed (Schoenfeld, 1988) and improve discussions. Groups finishing early might benefit from activities regarding checking their working or engaging in discussion of context and meaning.

Finally, a clear case can be made for increased professional development opportunities for tutors to ensure sufficient *quality* of provision, not merely quantity of mathematical provision. This study, like others, indicates that educational institutions are struggling with the practice of embedding mathematical provision into vocational contexts (Alkema & Rean, 2013; Casey et al., 2006). Much has been written about good instructional practice with adults and the need for skilled tutors (Swain et al., 2005; Swain & Swan, 2007). Given the learner needs identified in this study, any expectation of success requires an equal expectation for high quality provision.

8.7 Implications for further research

The use of multiple audio-recorders made it possible to capture the submerged learner experience. It is unlikely that traditional observation methods such as researcher observations with field notes, video footage, or checklists would have captured the private experiences of learners in the busy, unstructured and fluid environment of a foundation-level class. This use of multi-recording devices, and the issues raised by this study, present rich directions and considerations for exploring the authentic experience of learners as they re-engage with mathematics in foundation-level courses. These areas of exploration might include:

1. Further exploration of the strategies used by learners to meet classroom criteria without engaging deeply with mathematical thinking, across a broader range of contexts.
2. The impact of shame on learners' engagement patterns, its precursors, and how various classroom strategies might reduce its influence.
3. The construction of learner roles within groups, how equal participatory roles might be developed, and how introducing structures and tools, or changing group members, might influence the group work dynamics.
4. The impact of an intervention that starts at the beginning of the year with new learners, thus pre-empting the establishment of negative patterns of behaviour.
5. The role of mathematical discourse in developing conceptual understanding.
6. How negative mathematical beliefs and behaviours, once established, can be effectively broadened, and how this relates to quality mathematical engagement.

8.9 Conclusions

The notion that low-skilled adult learners hold negative beliefs is not altogether surprising. As reported, many have had negative experiences with mathematics that have contributed to negative beliefs about their ability. The extent of the procedural beliefs was surprising but explainable given many had incomplete school experiences, perhaps having left before being exposed to more advanced mathematics or reform-oriented approaches. However, what was unexpected was the impact the learners had on their learning environments, due to the various behaviours they did, or did not engage in. Belief studies typically find that the instructional environment contributes to learner beliefs (Ernest, 1989; Schoenfeld, 1988). The intervention element in this study differs precisely because the instructional environment to a large extent was shaped by the *learners*' behaviour. In addition, the learners resisted my attempts to change this. Problematically, the instructional environment preferred by learners conforms with widely identified models of 'bad practice' (Carpentieri et al., 2010). The observations revealed that for many learners, engagement was so superficial that it called into question whether any meaningful mathematical development took place. Within the intervention the learners experienced extreme mathematical difficulties despite being provided with a structured conceptually-oriented approach. The combined impact of the learners' beliefs on their pedagogical practices, discourse patterns, emotional responses and attempts to manage their self-image neutralised the learning potential of mathematical provision.

Furthermore, the beliefs and behaviours impacted not only learners seeking to avoid engagement, but also on those motivated to learn. Motivated learners in this study appeared to embrace the opportunity to learn important mathematical skills and apply themselves fully to the task. Yet their efforts were oriented toward the rapid application of procedures to routine problems, and their standards for success were speed and accuracy, not understanding. Completing tasks quickly and accurately was achieved at the *cost* of understanding, not *because of* understanding. Those who did study in their own time reported adopting rehearsal strategies by copying formulas into their workbooks. In fact, they saw this as tangible evidence of work and learning, despite being unable to draw on the knowledge in the classroom. Their efforts are likely to have led to a form of pseudo-success, such as being able to answer tutor questions, solve routine problems, and complete worksheets quickly.

A problem with such success is that although the learner may benefit from a growing mathematical confidence and an improved attitude toward mathematics, the

evidence suggests that such procedural skills do not lead to improved non-routine problems solving (Jäder et al., 2017), nor necessarily contribute to further learning (Pesek & Kirshner, 2000). Moreover, while procedural fluency is considered useful in some industries (Engelbrecht et al., 2017), conceptual understanding is considered vital to meet the sophisticated and changing mathematical demands of the workplace (Keogh et al., 2014; McCloskey, 2007). In time, the skills are likely to prove insufficient to meet the demands of a changing workplace or personal life, and several experiences of difficulty, or failure, may erode the confidence developed in class, causing the adult to question the value of what they learned.

Learners with negative beliefs about themselves and their ability to learn mathematics are at a greater risk than learners with procedural beliefs yet who continue to engage. As was evident in this study, being compelled to be present in a mathematics lesson induced feelings of apprehension that reminded them of their unpleasant school experiences. Low-skilled learners appeared to exploit the classroom routines in order to meet classroom demands. Many adopted a mode of 'civil attention' during classes, let other members of the class take responsibility for asking and answering questions, and feigned understanding when seeking to avoid awkward moments during which their limited skills might be exposed. The interviews showed that even if a learner did act to address their lack of understanding, their strategic repertoire was limited to asking the tutor for help, and then asking for repeated explanations. If the tutor's explanations did not result in understanding, these learners tended to attribute their lack of understanding to their own inability, consolidating negative beliefs, leaving them to retreat into strategies designed to mitigate social harm and avoid further engagement.

Embedding mathematical provision into foundation-level vocational training is a policy intended to improve the outcomes of low-skilled adult learners (TEC, 2014). This study shows that a proportion of learners within such programmes experienced some level of success, but many did not. Attempts to meet their learning needs by providing *more* provision, but not necessarily higher quality provision, ignores the influence of learner beliefs on learner engagement. Likewise, the notion that contextualised learning, or the self-determining nature of adults, will overcome embedded beliefs and behavioural patterns, ignores the realities of the classroom experience seen within this study.

However, as I reflect on the process I am deeply hopeful about the future of adults with negative beliefs re-engaging with mathematics. The observations, discussions

with learners, and interviews made it clear that despite painful school experiences, setbacks and challenges, most seek desperately to improve their skills and strive to overcome difficulties with mathematics. Mathematics loomed large in the lives of the learners and in some respects, reflected their battles with self-worth generally. In short, learners cared about their mathematical performance and this is very encouraging. Despite all the negative associations many learners hold toward it, success with mathematics has the potential to rebuild self-belief and repair damaged identities. Mathematics can isolate and demoralise, but conversely it has the potential to encourage and build up adult learners. However, drastic action need to be taken to channel learners' desire and energy to improve their skills into effective, meaningful and successful provision.

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Appendices

Appendix A: PIAAC Numeracy Levels

Numeracy levels and their meaning

Numeracy level	Type of tasks someone can perform
Below level 1	At this level, people can carry out single tasks, such as counting, sorting and performing basic arithmetic with whole numbers and money. They can also recognise common spatial dimensions.
Level 1	At this level, people can carry out basic mathematical processes where the mathematical content is made explicit and there are few text distractions. They are also able to understand simple percentages such as 50%.
Level 2	At this level, people can perform mathematical tasks with two or more steps where the mathematical content is explicit. These operations may include common decimals, percentages and fractions. They are also able to interpret relatively simple graphs and spatial representations.
Level 3	At this level, people can understand mathematical contexts that are subtly embedded in text. They can make choices between different problem solving strategies. They can also perform basic analyses of statistics in texts, tables and graphs.
Level 4	At this level, people can understand mathematical information that may be complex or abstract and embedded in unfamiliar contexts. They can analyse complex reasoning about quantities, data, statistics, chance, spatial relationships, change, proportions and formulas.
Level 5	At this level, people can integrate and interpret several types of mathematical information. They can understand complex representations and abstract mathematical ideas embedded in complex text.

Figure 14: PIAAC Numeracy Levels

(MOE & MBIE, 2016c)

Appendix B: Survey, observation and interview information and consent forms

Damon Whitten

Hamilton

Email: damon@waikato.ac.nz

Mobile: 021 863 279

Date

ABC Training LTD

Auckland

Dear Sir/Madam

This letter is a request for your participation in a research project.

As you will be aware for many young adults in the tertiary sector the school years were difficult and often negative. In some cases, the experience of school and learning has been so negative that learners may have developed patterns of negative beliefs about themselves, their ability to learn and the nature of learning. These negative beliefs may continue to undermine their success in their current fields of study. The aim of this research project is to prevent the same patterns of failure experienced in school being repeated in tertiary programmes.

I am initiating a doctoral research project exploring the beliefs of young adults and the impact these beliefs have on their learning, in particular their engagement in, and achievement of, numeracy. I am requesting that ABC Training be a part of this research.

As a participating organisation ABC Training would have access to all findings and recommendations and a professional development session for tutors. In turn, your organisation would provide access to learners aged 18 to 24 years old enrolled in level one to three NZQF programmes. Participation is completely voluntary and would require fully informed consent from each learner. This study falls under The University of Waikato's Ethical Conduct in Human Research and therefore requires that stringent confidentiality and full disclosure principles apply.

The study

The study has three components to it. Your organisation may take part in one or all of the components.

The first part of the study involves collecting information about learner beliefs through the use of a survey. The survey will take approximately ten minutes for each leaner to complete and can be completed on-line or as a paper-based version. I will be available to discuss the survey with groups of learners and distribute and collect the survey at a time convenient to your organisation.

The second part of the study is a classroom observation. The researcher will observe two or three classroom sessions that include numeracy content. The focus of the observation is the learners' reaction and response to mathematical content, not the tutor. Participating students will be fully informed and must give consent before the observation takes place. With the participating learners' permission, the observations will be recorded on a digital video recorder. This will allow for visual aspects of learners' responses to be included in the study.

The third component of the study involves individual interviews with several learners. The learners will be fully informed of their rights to abstain from participating or to leave the research at any time without giving a reason. The interviews will take up to one hour.

Privacy and confidentiality

The study complies with The University of Waikato's Ethical Conduct in Human Research Regulations. As such the study operates under the principles of full informed consent and confidentiality. All collected information is confidential and kept in a secure location. All place names, organisational names and personal names will be replaced with pseudonyms. Additionally, any identifying characteristics will be removed from all published material. All recordings (video recordings and notes) will be kept private and in a secure location. Access is limited to the researcher, and the researcher's two supervisors. All video footage will be erased upon completion of the thesis.
Once the results of the findings have been compiled the researcher will present the findings to your organisation if requested. This presentation can be made to managers, tutors and learners.

Your rights during the project

During the data collection procedure your organisation has the right to withdraw at any time without giving a reason. Once the surveys and observations are complete you have two weeks to withdraw from the project without giving a reason.

Complaints procedure

Concerns can be addressed to the researcher's supervisors, Associate Professor Jenny Young-Loveridge or Professor Diana Coben.

If ABC Training would like to be a part of the research or would appreciate more information I am more than happy to meet in person and discuss the research details with you and your staff.

You can contact me via the contact details provided on the letterhead. Please call at any time. Additionally, if you have any concerns regarding this project please contact:

Associate Professor Jenny Young-Loveridge
Waikato University
Department of Mathematics, Science & Technology Education
Email: educ2233@waikato.ac.nz

Yours sincerely

Damon Whitten

Exploring the mathematics beliefs of adult learners

Consent Form for Participating Organisations

ABC Training has understood the terms of the study and agrees to take part. The training centre will allow the researcher to initiate contact with learners in order to:

- complete a survey
- observe learners in a classroom setting
- interview selected learners

ABC Training understands that:

- learners will not be coerced in any way
- learners must give free and fully informed consent to the study
- recordings will be kept secure and be destroyed when the study ends
- confidentiality is guaranteed
- withdrawal can occur at any time during the data collection process

ABC Training knows that this research is being carried out by Damon Whitten at The University of Waikato and that his chief supervisor is Associate Professor Jenny Young-Loveridge.

Finally, the training centre has been informed of the complaints procedure and understands their rights as a participating organisation.

Organisation: ……………………………………………………..………………………..

Name: …………………………………………… Position: ………………………….

Signature: …..…………………………………………… Date: ………………………..

Contact details:
Damon Whitten
Email: damon@waikato.ac.nz
Mobile: 021 863 279

Chief supervisor
Associate Professor Jenny Young-Loveridge
Email: educ2233@waikato.ac.nz

Exploring the mathematics beliefs of adult learners

Information sheet

My name is Damon Whitten. I am a PhD student at The University of Waikato. I am conducting a survey as part of a study into the beliefs that young adults hold about mathematics. If you are aged between 18 and 24 years old, you are eligible to participate in the survey.

The survey involves answering questions about your beliefs about mathematics and learning. The survey takes about 15 minutes to complete. The purpose of the survey is to help me understand the beliefs that adults hold about mathematics and how these beliefs influence learning. The results will be used to help students to overcome negative beliefs and improve their numeracy skills.

Your participation is completely voluntary, and your responses are confidential. You may decline to answer any questions in the survey. If you choose not to participate you will not be disadvantaged in anyway.

If you agree to complete the survey, please write your name and the name of the course you are on directly on the survey. After you finish filling the survey out, please put the survey in the envelope provided.

If you do not want to complete the survey, simply return the blank form and envelope to your tutor.

This study is supervised by Dr Young-Loveridge. If you have any questions about the study you can email Dr. Young-Loveridge at educ2233@waikato.ac.nz. This project has been approved by The University of Waikato's Ethical Conduct in Human Research Regulations.

The results of this project will be available in late 2015. If you would like a copy of the results of the project or have any questions, please contact me via the email address below.

Damon Whitten
damon@waikato.ac.nz.
Mobile: 021 863 279

Please keep this letter for your records. Thank you for your participation.

Exploring the mathematics beliefs of adult learners

Consent Form for Participants

I have read the Information Sheet for this study and have had the details of the study explained to me. My questions about the study have been answered to my satisfaction.

I also understand that:

- I may ask questions about the study at any time.
- I am free to withdraw from the study.
- I may decline to answer any particular questions in the survey.
- My survey results will remain confidential.
- Survey results will be kept in a secure storage area.

I agree to participate in this study.

Signed: _____

Name: _____

Date: _____

Researcher's name and contact information:

Damon Whitten
Email: damon@waikato.ac.nz
Mobile: 021 863 279

Supervisor's name and contact information:

Associate Professor Jenny Young-Loveridge
Email: educ2233@waikato.ac.nz

Exploring the mathematics beliefs of adult learners

Information Sheet for Participants

My name is Damon Whitten. I am a PhD student at The University of Waikato. I am conducting a study into the beliefs that young adults hold about mathematics, and how they influence an adult's engagement with numeracy. As part of this research I am observing learners as they participate in their normal classes. I am asking your consent to observe several learning sessions that include numeracy.

If you agree to take part in the project you will allow me to sit in your class, watch, record, and take notes of how you interact with the numeracy being taught. I will be watching how all learners engage with the numeracy content, with each other, the tutor and the workbooks. The class will be audio recorded and may also be video recorded. The observation will not exceed two hours.

By agreeing to take part in the classroom observation, you may be asked to take part in an interview. You do not have to take part in the interview.

Privacy and confidentiality

The results of the study will be included in a doctoral thesis. However, your confidentiality is guaranteed absolutely. No individual participating in the research will be identified. All personal names, place names and organisational names will be replaced with pseudonyms. Additionally, any identifying characteristics will be removed from all published material. All collected information is confidential and kept in a secure location and will be destroyed following the completion of the study. Access is limited to the researcher, and the researcher's two supervisors Associate Professor Jenny Young-Loveridge and Professor Diana Coben.

The results of the research will be used to tell us what adults believe about learning and mathematics and how these beliefs influence how learners respond to numeracy content in courses.

Further information on this study can be gained by emailing:

Damon Whitten
Email: damon@waikato.ac.nz
Mobile: 021 863 279

If you have any concerns regarding this project please contact:

Associate Professor Jenny Young-Loveridge

Waikato University

Department of Mathematics, Science & Technology Education

Email: educ2233@waikato.ac.nz

Thank you for your participation. Please complete the attached form to agree.

Exploring the mathematics beliefs of adult learners

Consent Form for Learners

I have read the Information Sheet and fully understand what is required for my participation in this section of the research project. I will be asked to do the following as part of the research:

Take part in several scheduled lessons that include numeracy and be observed by a researcher.

It has been explained to me what we are going to do. I agree to allow myself to be observed while taking part in several classes. I understand that:

- All recordings will be kept secure and private.
- My identity will remain confidential.
- My name will not be used in the report.
- This research is being carried out by Damon Whitten at The University of Waikato as part of doctoral study.
- The chief supervisor is Associate Professor Jenny Young-Loveridge.

Finally, I have been informed of the complaints procedure and understand my rights as a participant.

I agree to participate in this study under the conditions set out in the Information Sheet.

Name: ………………………………………………………………………………

Age: ……………………………………… Date: ....

Signed: ……………………………………………………………………………….

Researcher's name and contact information:
Damon Whitten
Email: damon@waikato.ac.nz
Mobile: 021 863 279

Supervisor's name and contact information:
Associate Professor Jenny Young-Loveridge
Email: educ2233@waikato.ac.nz

Exploring the Mathematical Beliefs of Adult Learners

Information Sheet for Participant Interview

My name is Damon Whitten. I am a PhD student at The University of Waikato. I am conducting a study into what adults believe about mathematics and how their beliefs influence their engagement with numeracy. You are being invited to participate in an interview about your mathematical beliefs, mathematical experiences, and mathematical learning.

If you decide to volunteer, you will be asked to participate in one interview. During the interview you will be asked a series of questions. Some of them will be about your ideas about mathematics. Others will be about your approaches to learning mathematics and some will be about your past experiences with mathematics. With your permission, I will audio record the interviews. All recordings will remain private and be erased after the research is complete. You will not be asked to state your name on the recording and your name will not appear in any published material.

The information you provide will help educational providers improve their delivery of numeracy to adult learners. It will also help adult learners overcome negative learning experiences they may have had with mathematics and improve their ability to develop numeracy skills.

The interview will take approximately one hour and take place at ABC Training in a private room.

Participation and withdrawal
Your participation in this study is completely voluntary, and you may refuse to participate or withdraw at any time (no questions will be asked). You may skip any question during the interview but continue to participate in the rest of the study.

Privacy and confidentiality
The content of your interview will be included in a doctoral thesis. However, your confidentiality is guaranteed absolutely. No individual participating in the research will be identified. All personal names, place names and organisational names will be replaced with pseudonyms. Additionally, any identifying characteristics will be removed from all published material. All collected information is kept in a secure location and will be erased following the completion of the study. Access is limited to the researcher, and the researcher's two supervisors Jenny Young-Loveridge and Diana Coben.

The data collected from you will be used to complete a doctoral thesis and may be used as the basis for articles or presentations in the future. As stated above your name will not be used, neither will any information that would identify you in any publications or presentations.

Complaints procedure

Concerns can be addressed to the researcher's supervisors, Associate Professor Jenny Young-Loveridge or Professor Diana Coben.

Further information on this study can be gained by emailing:

Damon Whitten

Email: damon@waikato.ac.nz

Mobile: 021 863 279

If you have any concerns regarding this project please contact:

Associate Professor Jenny Young-Loveridge

Waikato University

Department of Mathematics, Science & Technology Education

Email: educ2233@waikato.ac.nz

Thank you for your participation. Please complete the attached form.

Exploring the Mathematical Beliefs of Adult Learners

Interview Consent Form

I have read the information sheet and fully understand what is required for my participation in this section of the research project. I will be asked to do the following as part of the research:

Take part in an interview that explores my mathematical beliefs, experiences with mathematics, and approaches to learning mathematics.

I understand that:

- The information collected will be used in a published thesis.
- My identity will remain confidential.
- My name will not be used in the report.
- Recordings will be kept secure and private.
- Recordings will be deleted after the study is complete.
- I can withdraw from the study at any time during the data collection process.

I know that this research is being carried out by Damon Whitten at The University of Waikato as part of doctoral study. The chief supervisor is Associate Professor Jenny Young-Loveridge.

Finally, I have been informed of the complaints procedure and understand my rights as a participant. I agree to take part in the interview

Name: …………………………………………………………………………………

Age: ………………………………………… Date: ....

Signed: ……………………………………………………………………………….

Researcher's name and contact information:

Damon Whitten
Email: damon@waikato.ac.nz
Mobile: 021 863 279

Supervisor's name and contact information:

Jenny Young-Loveridge

Email: educ2233@waikato.ac.nz

Appendix C: Intervention information and consent forms

Developing Mathematical Beliefs and Skills of Adult Learners

Information Sheet for ABC Training Provider

Many learners have had such poor learning experiences that they have developed negative beliefs, attitudes and dispositions toward learning. This is particularly evident in the area of mathematics, where learners' negative beliefs about mathematics, and their ability to learn it, often act as barriers to them from fully engaging in the numeracy taught in tertiary programmes. Negative beliefs may lead to fear, anxiety and feelings of disempowerment in a mathematical learning context. Negative beliefs also prevent learners from mastering the numeracy skills necessary to gain employment or complete qualifications. Research suggests that explicitly addressing negative beliefs and developing positive beliefs leads to better engagement in numeracy, better use of strategies and higher achievement overall.

ABC Training Provider is invited to participate in a research project that explores the impact of a specialised numeracy programme with adult learners. The programme will run over two non-consecutive hours a week for eight weeks.

The findings of this study will be applicable to the broader tertiary sector and will be made available as a published doctoral thesis. However, as a participating organisation, you will receive the key findings in the form of a presentation and professional development session with teaching staff.

The numeracy programme will be facilitated by the researcher. The researcher is an experienced numeracy tutor who has many years experience working with adult learners in the tertiary sector. Participating learners are required to give consent and will be fully informed of their rights as research participants. This research complies with The University of Waikato's Ethical Conduct in Human Research and Related Activities Regulations as such learner wellbeing is a priority.

The programme is designed for learners who have struggled with numeracy, and may have had negative learning experiences at school. It is anticipated that the programme will be a positive and beneficial experience for the participating learners and the organisation.

Timeframe

The numeracy programme is an eight-week programme that includes two, two hour sessions each week (to be negotiated with tutors in regard to class schedules). The programme is designed to take place within a broader course context and align with the existing programme's numeracy outcomes. The sessions will not require the tutor to be present hence the programme will free the tutor for two hours a week to invest in other activities.

Privacy and confidentiality

The study complies with The University of Waikato's Ethical Conduct in Human Research Regulations. As such the study operates under the principles of fully informed consent and confidentiality. All collected information is confidential and kept in a secure location. All place names, organisational names and personal names will be replaced with pseudonyms. Additionally, any identifying characteristics will be removed from all published material. All recordings (video recordings and notes) will be kept private and in a secure location. Access is limited to the researcher, and the researcher's two supervisors.

Your rights during the project

During the numeracy programme your organisation has the right to withdraw at any time without giving a reason. Additionally, the researcher will inform all learners of their rights to withdraw also.

Complaints procedure

Concerns can be addressed to the researcher's supervisors, Associate Professor Jenny Young-Loveridge or Professor Diana Coben.

If you would like more information please contact:
Damon Whitten
Mobile: 021 863 279
Email: damon@waikato.ac.nz

If you have any concerns regarding this project please contact:
Associate Professor Jenny Young-Loveridge
Waikato University
Department of Mathematics, Science & Technology Education

Exploring the mathematics beliefs of adult learners

Consent Form for ABC Training

ABC Training Provider has understood the terms of the study and agrees to take part. The organisation will allow the researcher to run an eight-week numeracy programme that consists of two, two-hour sessions each week. ABC Training Provider understands that the outcomes of the intervention are to develop learners' beliefs about mathematics and learning, to develop effective learning strategies and gain a conceptual understanding of course related numeracy.

ABC Training Provider understands that:

- Learners will not be coerced to participate.
- Learners must give free and fully informed consent.
- The intervention will be video recorded.
- Recordings will be kept secure and be erased when the study ends.
- Confidentiality is guaranteed.
- The training centre can withdraw at any time during the data collection process.
- This research is being carried out by Damon Whitten at The University of Waikato as part of doctoral study.

The chief supervisor is Associate Professor Jenny Young-Loveridge.

Finally, ABC Training Provider has been informed of the complaints procedure and understands their rights as a participating organisation.

ABC Training Provider agrees to participate in this study under the conditions set out in the Information Sheet.

Name: ………………………………….....................………………………………………

Position: ……………………………………...................… Date: ....

Signed: …………………………………………….....................………………………….

Researcher's name and contact information:

Damon Whitten
Email: damon@waikato.ac.nz

Mobile: 021 863 279

Supervisor's name and contact information:

Associate Professor Jenny Young-Loveridge

Email: educ2233@waikato.ac.nz

The Adult Numeracy Project
Information Sheet for Participants

You are being invited to participate in an eight-week numeracy programme as part of a research project. The research project is exploring the impact of developing adult learners' beliefs about mathematics. The findings of the research will be used to complete a doctoral study and be written as a published thesis. If you agree to participate in the programme you will be agreeing to take part in the study. The programme is described below.

The numeracy programme

The programme is designed to develop beliefs about numeracy. Developing these beliefs may lead to better engagement and enjoyment of numeracy, result in better learning, and result in a greater ability to use numeracy in your life. This programme will be different from your normal numeracy provision as it will focus on:

- What mathematics is and how it is best learned
- How to use learning strategies and methods
- How to solve real life numeracy problems
- Understanding the mathematics you will require in your field of study.

This programme is designed for adults who may have struggled with mathematics in the past. **You do not have to be good at mathematics** to be part of the programme. In fact, if you have had difficulties with maths during school or during your current study, this programme is perfect for you. A typical learning session will be fun, present opportunities to solve problems and discuss and share ideas.

By taking part in this programme you may improve your numeracy and your enjoyment of mathematics. You may also overcome difficulties you have had in the past with mathematics, and you may even grow to love mathematics. The programme will be facilitated by an experienced numeracy tutor who is used to working with adults who dislike mathematics and have struggled with it in the past.

The research requirements

Because the programme is part of a research project, particular activities or conversations will be video or audio recorded to better learn how adults engage in numeracy tasks. Additionally, you may be asked to take part in an interview that asks you about your experiences learning mathematics and your experiences taking part in the numeracy project. Your behaviours in the programme, your progress and your

discussions are likely to be recorded and used in the research. The next session describes your privacy rights and rights to decline.

Participation and withdrawal

Your participation in this study is completely voluntary, and you may refuse to participate or withdraw from the programme at any time. You do not have to give a reason for withdrawing from the programme.

Privacy and confidentiality

The content of the sessions will used in a doctoral thesis. However, your confidentiality is guaranteed absolutely. No individual participating in the research will be identified. All personal names, place names and organisational names will be replaced with pseudo names. Additionally, any identifying characteristics will be removed from all published material. All collected information is kept in a secure location and will be erased following the completion of the study. Access is limited to the researcher, and the researcher's two supervisors Jenny Young-Loveridge and Diana Coben.

The data collected from you may be used as the basis for articles or presentations in the future. As stated above your name will not be used, neither will any information that would identify you in any publications or presentations.

Complaints procedure

Concerns can be addressed to the researcher's supervisors, Associate Professor Jenny Young-Loveridge or Professor Diana Coben.

Further information on this study can be gained by emailing:

Damon Whitten
Email: damon@waikato.ac.nz

If you have any concerns regarding this project please contact:
Associate Professor Jenny Young-Loveridge
Waikato University
Department of Mathematics, Science & Technology Education
Email: educ2233@waikato.ac.nz

Thank you for your participation. Please complete the attached form.

Developing Mathematical Beliefs of Adult Learners

Programme Consent form

I have read the information sheet and fully understand what is required for my participation in this section of the research project. I will be asked to do the following as part of the research:

Take part in an ten-week numeracy programme that has two sessions each week

Be video and audio recorded while participating in the programme

Take part in an interview that explores my mathematical beliefs, experiences with mathematics and my approach to learning mathematics

It has been explained to me what we are going to do. I agree to take part in the numeracy programme. I understand that the content of the programme will be used in a published thesis. I understand that my identity is confidential and that my name will not be used in the report. I understand that all recordings will be kept secure and private, and that recordings will be deleted after the study is complete.

I agree to participate in this study under the conditions set out in the Information Sheet.

Name: _____

Date: _____

Signed: _____

Researcher's name and contact information:

Damon Whitten
Email: damon@waikato.ac.nz
Mobile: 021 863 279

Supervisor's name and contact information:

Associate Professor Jenny Young-Loveridge
Email: educ2233@waikato.ac.nz

Appendix D: Ethics approval

MEMORANDUM

To:	**Damon Whitten**
cc:	Associate Professor Jenny Young-Loveridge
	Dr Elmarie Kotzé
From:	**Associate Professor Linda Mitchell**
	Chairperson, Research Ethics Committee
Date:	31 October 2012
Subject:	Supervised Postgraduate Research – Application for Ethical Approval (EDU093/12)

Thank you for submitting the amendments to your application for ethical approval for the research project:

**Understanding the role of low achieving adult learners' beliefs about mathematics:
The impact of an intervention designed to challenge negative beliefs**

I am pleased to advise that your application has received ethical approval.

Please note that researchers are asked to consult with the Faculty's Research Ethics Committee in the first instance if any changes to the approved research design are proposed.

The Committee wishes you all the best with your research.

Associate Professor Linda Mitchell
Chairperson
Faculty of Education Research Ethics Committee

Appendix E: Mathematical Beliefs Survey

Mathematics Beliefs Questionnaire

I am interested in your ideas about mathematics. Answers to the following questions will help me understand what you think mathematics is all about. This questionnaire is confidential. Please tell me what you *really* think. Thanks for your help!

		Strongly agree	Agree	Disagree	Strongly disagree
1	Any word problem can be solved if you know the right steps to follow.				
2	A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.				
3	If I can't solve a maths problem quickly, I quit trying.				
4	Word problems are not a very important part of mathematics.				
5	Ability in maths increases when one studies hard.				
6	Maths is a worthwhile and necessary subject.				
7	If I can't do maths problems in a few minutes, I probably can't do it at all.				
8	Memorising steps is not that useful for learning to solve word problems.				
9	Time used to investigate why a solution to a math problem works is time well spent.				
10	Calculation skills are of little value if you can't use them to solve word problems.				
11	By trying hard one can become smarter in maths.				

		Strongly agree	Agree	Disagree	Strongly disagree
12	Maths will not be important to me in my life's work.				
13	Maths problems that take a long time don't bother me.				
14	There are word problems that just can't be solved by following a fixed sequence of steps.				
15	It doesn't really matter if you understand a maths problem if you can get the answer.				
16	A person who can't solve word problems really can't do maths.				
17	I can get smarter in maths by working hard.				
18	Maths is of no relevance to my life.				
19	I feel I can do maths problems if I just hang in there.				
20	Word problems can be solved without remembering formulas.				
21	Getting a right answer in maths is more important than understanding why the answer works.				
22	Mathematical skills are useless if you can't apply them to real life situations.				
23	Working hard can improve one's ability in maths.				

		Strongly agree	Agree	Disagree	Strongly disagree
24	I study mathematics because I know how useful it is.				
25	It is not important to understand why a mathematical procedure works as long as it gives a correct answer.				
26	I find I can do maths problems that take a long time to complete.				
27	Most word problems can be solved by using the correct step by step procedure.				
28	I can get smarter in maths if I try hard.				
29	Learning mathematical skills is more important than learning to solve word problems.				
30	Studying maths is a waste of time.				
31	I'm not very good at solving maths problems that take a while to figure out.				
32	Knowing mathematics will help me earn a living.				
33	Maths classes should not make word problems so important.				
34	Hard work can increase one's ability to do maths.				
35	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.				

		Strongly agree	Agree	Disagree	Strongly disagree
36	Learning to do word problems is mostly a matter of memorising the right steps to follow.				
37	If you are going to be able to understand something, it'll make sense to you the first time you hear it.				
38	Being good at mathematics is 90% ability and 10% hard work.				
39	The only thing that is certain is certainty itself.				
40	Getting ahead takes a lot of work.				
41	I try my best to understand the connections between the subjects I learn.				
42	Smart students understand things quickly, usually the first time.				
43	You have a certain amount of intelligence and you really can't do much to change it.				
44	If scientists try hard enough, they can find the truth to almost everything.				
45	To me, studying means getting the big ideas from a text, rather than the details.				
46	Going over and over a difficult textbook chapter usually won't help you understand it.				
47	Scientists can ultimately get to the truth.				
48	If a student forgot details after reading a textbook but still came up with new ideas, that student is smart.				
49	Some people are born smarter than others and you can't do anything to change that.				

		Strongly agree	Agree	Disagree	Strongly disagree
50	I enjoy thinking about issues that experts are uncertain about.				
51	If something in maths is true it never changes.				
52	The really smart students don't have to work hard to do well in school.				
53	If a person can't understand something in a short amount of time, they should keep on trying.				
54	Being a good student generally involves memorising facts.				
55	You can learn new things but you can't really change your basic intelligence.				
56	It's a waste of time to work on problems that have no possibility of coming out with a definitive answer.				
57	Almost all the information you can learn from a textbook you will get during the first reading.				
58	Things that are believed to be 'facts' today may be viewed as just opinions in the future.				
59	When you first encounter a difficult concept in a textbook, it's best to work it out on your own.				
60	Learning is a slow process of building up knowledge.				
61	An expert is someone who has a special, natural gift or talent in some area.				
62	A teacher's job is to keep his/her students from wandering off the right track towards the answers.				
63	Often, even advice from experts should be questioned.				
64	People who challenge authority are over-confident.				

		Strongly agree	Agree	Disagree	Strongly disagree
65	If I find the time to re-read a textbook chapter, I get a lot more out of it the second time.				
66	How much a person gets out of school depends on the quality of the teacher.				
67	You can believe most things you read.				
68	I like to think about issues that authorities can't agree on.				
69	Sometimes you just have to accept answers from a teacher even though you don't understand them.				

Please take the time to write answers to the two questions below.

What is mathematics?	
If a new student started your course and wanted to learn numeracy, what advice would you give them?	

Name: _____   Programme: _____

Age: _____   Male ☐   Female ☐

Ethnicity: ☐ New Zealand Maori

☐ Pasifika: Please state island group _____

☐ Pakeha\European New Zealand

☐ Other: Please state _____

Thank you for taking the time to complete the survey

Appendix F: Interview schedule

Welcome participant
Rearticulate purpose of interview
Opening question – A little while ago you completed a survey that asked questions about your beliefs about mathematics. What did you think of the survey?
Did anything stand out for you or do you remember any of the questions?
I'm going to go through some of your responses to the survey questions and ask you to explain why you answered as you did.

Belief about mathematics as a discipline

Effort can increase mathematical ability	
In the survey you agreed/disagreed with the statement "I can get smarter in maths by working hard." Tell me about why you agree/disagree?	
Prompts	Do you still think this? Why or why not?
	How much difference does hard work make in mathematics?
	Why do you think some people succeed/fail with mathematics?

Usefulness of mathematics	
In the survey you agreed/disagreed with the statement "maths will not be important in my life's work". Explain why you answered as you did?	
Prompts	Do you still think this? Why or why not?
	How useful is the numeracy you are learning on your course?

Time spent on tasks	
In the survey you agreed/disagreed with the statement "Maths problems that take a long time don't bother me". Tell me about why you answered as you did?	
Prompts	Do you still think this? Why or why not?
	How long should a maths problem take to solve?
	What is the longest you have worked on a single problem?
	Does it matter how long a mathematics or numeracy problem takes to solve?

The importance of word problems	
In the survey you agreed/disagreed with the statement "A person who can't solve word problems really can't do maths." Explain your thinking around this.	
Prompts	Do you still think this? Why or why not?
	Which do you think is more important, being able to solve word problems or being able to solve equations? Explain why.
	What makes word problems difficult?

Understanding concepts is important in mathematics	
In the survey you agreed/disagreed with the statement "It is not important to understand why a mathematical procedure works as long as it gives a correct answer." Explain why you answered as you did?	
Prompts	Do you still think this? Why or why not?
	Is getting the right answer more important than understanding why the answer works? Explain why.

The use of step by step procedures.	
In the survey you agreed/disagreed with the statement "Most word problems can be solved by using the correct step by step procedure". Explain why you answered as you did.	
Prompts	Can word problems be solved without knowing the correct procedures? Explain.
	How important is it to listen to other peoples' solutions to word problems? Explain.
	Is being good at maths about being able to remember the right steps to solve problems? Explain why or why not.

Mathematical learning histories	
Describe your mathematics education at school.	
Prompts	Did you enjoy maths at school?
	What did you enjoy/not enjoy in particular?
	Why do you think you did/didn't enjoy maths at school?
Do you enjoy your current numeracy classes? Describe the factors that make it more/less enjoyable.	
In what ways is your current study different/similar to learning maths in school?	

Behaviours observed in class
The questions in this section will be related to specific observed 'events' in class. An example of a question may be: When the tutor had finished explaining about Pi (π), I noticed you didn't work through the workbook like he suggested, can you explain why? Or I noticed the group you were in solved the problem in a different way. What made you decide to do that?

Learning strategies	
How do *you* go about learning mathematics? What strategies do you use?	
How often do you study mathematics/numeracy outside of class time?	
Do you plan your study before you begin? Describe your planning.	
When you study numeracy or mathematics, what sort of study techniques do you use?	
Probes	Do you practice the same problems you were given in the class?
	Do you complete the problems from the workbook?
	Have you ever tried finding different ways of solving the same problem?
	Do you talk to friends about how you solved a problem?
	Have you talked to friends about how the numeracy you learn in class relates to activities outside of class?
	Have you ever made up a problem and tried to solve it?
	Have you ever thought about how the maths you are learning will be used in your job?
If you were to give a friend advice on how to best learn mathematics, what would you say?	

Epistemic beliefs	
In the survey you agreed/disagreed with the statement "Some people are born smarter than others and you can't do anything to change that." Explain why you answered as you did.	
Probe	How might a person overcome their limitations?
In the survey you agreed/disagreed with the statement "If you are going to be able to understand something, it'll make sense to you the first time you hear it." Explain why you answered as you did.	
Probe	What should somebody do if they don't understand information the first time they hear it?
In the survey you agreed/disagreed with the statement "I enjoy thinking about issues that experts are uncertain about." Explain why you answered as you did.	
Probe	Do you think knowledge changes or stays the same? Why?
In the survey you agreed/disagreed with the statement "Sometimes you just have to accept answers from a teacher even though you don't understand them". Explain why you answered as you did.	

Probe	Is it better to find out information by yourself or be told by an expert? Why?
In the survey you agreed/disagreed with the statement "Being a good student generally involves memorising facts." Explain why you answered as you did.	
Probe	Has someone who is smart worked out how everything is connected or somebody who has memorised lots of information? Why?

Do you have anything you would like to add to the interview?

Thank you for your time and input.