Erratum: Lie theory and separation of variables. 3. The equation $f_{tt} - f_{ss} = \gamma^2 f$ [J. Math. Phys. 15, 1025 (1974)]

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Dr. Charles Boyer has kindly pointed out an error in the computation of the spectrum of the operator $L_0 = \partial_x^2 + \gamma^2 e^{\alpha x}$ on $L_1(R)$, which corresponds to the Bessel function basis of solutions for the Klein–Gordon equation. This error, which is the responsibility of the second author, consists in the assertion that the self-adjoint extensions $L_{0,n}$ have only discrete spectrum $\lambda = (2n + \alpha)^2$ and an orthonormal basis of eigenfunctions

$$f_n(x) = \sqrt{2/(\alpha + 2n)} J_{\alpha}(\alpha x), \quad n = 0, 1, 2, \ldots,$$

where $0 < \alpha < 2$. In fact, as is shown on pp. 93–95 of the book Eigenfunction Expansions. Part One. (Oxford U.P., Oxford, 1962), 2nd ed., by E.C. Titchmarsh, these operators also have continuous spectrum.

Taking the case $\alpha = 2$ for simplicity, we find that the operator $L_0$ has discrete spectrum $\lambda = (4n + 1)^2$, $n = 0, 1, \ldots$, and continuous spectrum $\lambda < 0$ with generalized eigenfunctions

$$\tilde{f}_n(x) = [J_{\alpha}(\alpha x) + J_{\alpha}(\alpha x)]/2\sinh(\sqrt{\lambda} \alpha),$$

$$\langle \tilde{f}_n, \tilde{f}_m \rangle = 0 \quad (\lambda \neq \lambda').$$

Here, $(\cdot, \cdot)$ is the usual $L_2(R)$ inner product. The functions $\tilde{f}_n, \tilde{f}_m$ together form a complete set for $L_2(R)$.

The separable solutions of the Klein–Gordon equation corresponding to the continuum basis are

$$\tilde{F}_n(s, t) = \frac{\sinh(\sqrt{s + \lambda} \alpha)}{\sqrt{s + \lambda} \alpha} [J_{\alpha}(\gamma u) + J_{\alpha}(\gamma v)] \times K_{\alpha}(\gamma v),$$

where $s = (u^2 + u^2 \alpha^2 + v^2)/2uv$, $t = (u^2 - u^2 \alpha^2 + v^2)/2uv$, $v > u > 0$.

There are similar expressions for other regions of the $(u, v)$ plane.

The error, while regrettable, in no way affects the principal methods and conclusions of this paper and of following papers in our series.

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