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The influence of porosity on the thermal diffusivity of foods

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Abstract

Thermal diffusivity is an important parameter in unsteady-state heat transfer processes. Compared to other physical properties, there appears to be very few predictive models for thermal diffusivity in the literature. Most experimental studies have focussed on the influence of moisture content and temperature on thermal diffusivity and there is very little about the influence of porosity. In this study, the effects of porosity on thermal diffusivity are examined for selected foods and thermal diffusivity models from the literature are compared against measured data. The thermal diffusivity model proposed by Choi and Okos based on a weighted arithmetic mean of the components' thermal diffusivities is not suitable for porous foods, resulting in large prediction errors. Applying a similar approach based on the harmonic mean rather than the arithmetic mean provided closer estimates of thermal diffusivity for both types of porous foods considered, but still produced unacceptable prediction errors (e.g. greater than 100 % when applied to model the thermal diffusivity of rice). Thermal diffusivity predicted from effective thermal conductivity, effective density and effective specific heat capacity data also resulted in unacceptable error. It is clear that there is scope for more work to be done in this area.

Keywords

Thermal diffusivity, thermal conductivity, porosity

1. Introduction

Thermal diffusivity is an important parameter for modelling transient thermal processes [1], such as cooking, chilling and freezing, and is defined as the ratio of thermal conductivity to volumetric heat capacity. The porosity of a food item is known to have a significant influence on thermal conductivity [2-4], and therefore it necessarily influences thermal diffusivity as well. The literature contains a large quantity of thermal conductivity data and thermal conductivity models [2,3,5]; however, there is less data on thermal diffusivity and very few effective thermal diffusivity models. Of the effective thermal diffusivity models that have been applied to food [2], the majority are purely empirical, correlating thermal diffusivity as a function of temperature and moisture content [6-10]. Empirical models have limited value beyond data reduction, since the can only be used with foods for which measured data also exists. Choi and Okos [11] appear to have been the only researchers who have applied a model based on the components' thermal diffusivities when they used the volume-weighted arithmetic mean (known as the Parallel model when applied to thermal conductivity [5]) to model effective thermal diffusivity of foods:

$$\alpha_e = \sum_i \alpha_i v_i \qquad (1)$$

where α is thermal diffusivity, v is volume fraction, the subscript e refers to the effective property, and the subscript i is the summation index. Alternatively, the definition of thermal diffusivity has been used to calculate effective thermal diffusivity from effective thermal conductivity data, effective density data and effective specific heat capacity data, or, alternatively thermal conductivity from thermal diffusivity (e.g. [12]):

$$\alpha_e = \frac{k_e}{\rho_e c_e} \qquad (2)$$

where k is thermal conductivity ρ is density and c is specific heat capacity. However, it has been claimed that Eq. (2) only provides approximations of effective thermal diffusivity [13,14], and since it involves measurement as well as, in some cases, modelling uncertainties from three physical properties it can result in the propagation of measurement errors [15]. The aim of this study was to investigate the manner in which porosity in food affects thermal diffusivity and how best to model it, and to evaluate the model proposed by Choi and Okos [11] and the approach of predicting effective thermal diffusivity from effective thermal conductivity, effective density and effective specific heat capacity.

2. Thermal diffusivities of the major food components

Choi and Okos [11] measured thermal diffusivities of the main food components as a function of temperature (T). Table 1 shows their correlations corrected by applying a factor of 10^{-6}, to each term as there was clearly a typographical error in their manuscript (as can be seen by performing a simple calculation for thermal diffusivity from their data for thermal conductivity, density and heat capacity at 0 °C).

Table 1: Thermal diffusivities of major food components [11] with correction factor of 10^{-6} applied to source data

Food component	α (m^2 s^{-1}) (T in °C)
Liquid Water	$1.38 \times 10^{-7} + 6.2477 \times 10^{-10}T - 2.4022 \times 10^{-12} T^2$
Ice	$1.1756 \times 10^{-6} - 6.0833 \times 10^{-10}T + 9.5037 \times 10^{-12} T^2$
Protein	$6.8714 \times 10^{-8} + 4.757 \times 10^{-10}T - 1.4646 \times 10^{-12} T^2$
Fat	$9.877 \times 10^{-8} - 1.2569 \times 10^{-10}T - 3.8286 \times 10^{-12} T^2$
Carbohydrate	$8.0842 \times 10^{-8} + 5.3052 \times 10^{-10}T - 2.3218 \times 10^{-12} T^2$
Fibre	$7.3976 \times 10^{-8} + 5.1902 \times 10^{-10}T - 2.2202 \times 10^{-12} T^2$
Ash	$1.2461 \times 10^{-7} + 3.7321 \times 10^{-10}T - 1.2244 \times 10^{-12} T^2$

For porous foods, air is also a significant food component. For temperatures between 0 and 40 °C the thermal diffusivity of air may be modelled by [1]:

$$\alpha_{air} = 1.818 \times 10^{-5} + 1.244 \times 10^{-7}T + 1.926 \times 10^{-10}T^2 \tag{3}$$

It is worth observing that at 0 °C the thermal diffusivity of air is 2 to 3 orders of magnitude higher than the other major food components.

3. Testing the thermal diffusivity model of Choi and Okos [11] ('Averaging method')

In order to test the model of Choi and Okos [11], Eq. (1) is used to model the thermal diffusivities of sucrose solutions. Table 2 shows thermal diffusivities modelled by Eq. (1) compared against the data of Bhowmik and Hayakawa (as cited in [3]) along the discrepancies (ε) between the two:

Table 2: Comparison between measured thermal diffusivities and thermal diffusivities for sucrose solutions at 33 °C and different concentrations.

x_w	x_{carb}	v_w	v_{carb}	α_{meas} (m^2 s^{-1})	α_{model} (m^2 s^{-1})	ε (%)
0.6	0.4	0.783	0.217	1.35×10^{-7}	1.39×10^{-7}	-3.11
0.7	0.3	0.849	0.151	1.39×10^{-7}	1.43×10^{-7}	-3.13
0.9	0.1	0.956	0.044	1.47×10^{-7}	1.49×10^{-7}	-1.21

The mass fractions (x) were converted to volumetric fractions (v) using Eqs. (4) and (5):

$$x_i = \frac{v_i}{\rho_e} \tag{4}$$

$$\rho_e = \sum_i \rho_i v_i \tag{5}$$

The % discrepancy or error (ε) is defined by Eq. (6):

$$\varepsilon = \frac{(\alpha_{meas} - \alpha_{model})}{\alpha_{meas}} \times 100\ \% \tag{6}$$

Table 2 shows that the predictions of Eq. (1) are close to the measured data, which is unsurprising since Choi and Okos [11] tested their models on two liquid foods. However, since none of the foods considered in their study contained significant porosity, it is important to test their model on porous foods.

The literature does not contain many data for thermal diffusivity measured over a range of porosities, since most attention has been paid to the influence of moisture content and temperature [3]. One source of suitable data is the work of Zanoni et al. [16], who measured the porosity of bread crust and bread crumb over a range of porosities, and, importantly, provided the composition data of their doughs. Table 3 shows the data measured by Zanoni et al. along with volume fractions calculated using Eqs. (4) and (5) and effective diffusivities predicted using Eq. (1).

Table 3: Comparison between measured thermal diffusivities and predicted thermal diffusivities for bread crumb at different porosities (dough composition 46 % water, 7 % protein, 0.2 % fat, 44.8 % carbohydrate, 2% ash)[16].

v_{air}	v_w	v_{prot}	v_{fat}	v_{carb}	v_{ash}	$\alpha_{measured}$ ($m^2\ s^{-1}$)	$\alpha_{e,Eq\ 1}$ ($m^2\ s^{-1}$)	$\varepsilon_{Eq.\ (1)}$ %	$\alpha_{e,Eq\ 7}$ ($m^2\ s^{-1}$)	$\varepsilon_{Eq.\ (7)}$ %
0	0.649	0.0758	0.00303	0.263	0.0094	2.5×10^{-7}	1.39×10^{-7}	45	1.34×10^{-7}	47
0.24	0.493	0.0576	0.00230	0.200	0.0069	2.9×10^{-7}	6.08×10^{-6}	-1996	1.75×10^{-7}	40
0.37	0.409	0.0478	0.00191	0.166	0.0056	3.3×10^{-7}	9.29×10^{-6}	-2716	2.11×10^{-7}	36
0.5	0.325	0.0379	0.00152	0.132	0.0043	4.0×10^{-7}	1.25×10^{-5}	-3028	2.65×10^{-7}	34
0.71	0.189	0.0220	0.00088	0.076	0.0021	5.0×10^{-7}	1.77×10^{-5}	-3441	4.54×10^{-7}	9
0.79	0.137	0.0160	0.00064	0.055	0.0013	5.3×10^{-7}	1.97×10^{-5}	-3614	6.21×10^{-7}	-17

Table 3 shows that the thermal diffusivities predicted by Eq. (1) differ from the measured data by several orders of magnitude. Table 3 also includes thermal diffusivities predicted based on the harmonic mean of the components' diffusivities (analogous to the Series thermal conductivity model):

$$\alpha_e = \frac{1}{\sum_i \frac{v_i}{\alpha_i}} \tag{7}$$

The harmonic mean (Eq. 7) provides much more accurate predictions of the thermal diffusivity; however, they still differ from the measured values by greater than 30 % on average.

Bread can be described as having internal porosity (i.e. pores contained within a continuous matrix) [17]. It is worth considering a type of food containing external porosity (e.g. particulate foods). Morita and Singh [18] measured the thermal diffusivity of rough rice, and although they did not state the porosity explicitly they did measure the bulk density (ρ_{bulk}) and moisture content of the rice. The porosity (which in most cases corresponds to the volume fraction of air, v_{air}) of the rice may be estimated from the bulk density using Eq. (8):

$$v_{air} = \frac{\rho_{solid} - \rho_{bulk}}{\rho_{solid} - \rho_{air}} \approx 1 - \frac{\rho_{bulk}}{\rho_{solid}} \tag{8}$$

The densities of the major food components as functions of temperature were also measured by Choi and Okos [11]. Assuming a typical solids composition for rice of 10 % protein and 90 % carbohydrate, estimates of porosity could be obtained from the bulk densities and moisture contents measured by Morita and Singh [17] using Eqs. (5) and (7) and the density relationships of Choi and Okos [11]. Table 4 shows the data measured by Morita and Singh, along with the estimated porosities and the thermal diffusivities for rice predicted using Eq. (1).

It is clear from Tables 3 and 4 that the model proposed by Choi and Okos (Eq. 1) is not suitable for porous foods since it overestimates the thermal diffusivity by a factor of 10^3 or more. This error cannot be attributed solely to uncertainty in the estimation of v_{air} or the thermal diffusivities of the major food components measured by Choi and Okos [11], and it seems unlikely that it can be attributed to measurement error in the work of either Zanoni et al. [16] or Morita and Singh [18], since many other thermal diffusivity data for porous foods are of similar order of magnitude [3]. The thermal diffusivities predicted by Eq. (7) are much closer to the measured data than the predictions from Eq. (1); however, errors are typically greater than 30 % for bread, and greater than 100 % for rice.

Table 4: Comparisons between measured effective diffusivities of bulk rice at 26 °C with predictions based on estimated porosities using Eqs (1) and (8)

x_w (measured) [17]	$\rho_{measured}$ (kg m^{-3}) [17]	$\alpha_{measured}$ (m^2 s^{-1}) [17]	v_{air} (estimated)	$\alpha_{Eq(1)}$ (m^2 s^{-1})	ε Eq. (1) %	$\alpha_{Eq(7)}$ (m^2 s^{-1})	ε Eq. (7) %
0.1	632	1.42×10^{-7}	0.68	1.53×10^{-5}	-10732	3.15×10^{-7}	-122
0.13	642	1.35×10^{-7}	0.67	1.50×10^{-5}	-11024	3.06×10^{-7}	-127
0.16	656	1.28×10^{-7}	0.65	1.45×10^{-5}	-11304	2.96×10^{-7}	-131
0.19	664	1.22×10^{-7}	0.63	1.42×10^{-5}	-11572	2.89×10^{-7}	-137

4. Thermal diffusivity prediction based on effective thermal conductivity

It is difficult to test Eq. (2) since thermal diffusivity and thermal conductivity are seldom measured independently of each other for a particular food in a given study. Instead, it is very common for one property to be derived from measured data of the other property using Eq. (2) (as is the case in the work of Zanoni et al. [11]). In particular, thermal conductivity data are commonly derived from transient heat transfer experiments in which the thermal diffusivity is measured directly rather than thermal conductivity. Ideally, it would be possible to take thermal conductivity for a particular food from one source, use that data as an input for Eq. (2) and then compare the predicted thermal diffusivities against measured thermal diffusivities of the same food from a different study. However, due to differences in measurement temperatures and compositions of similar foods (or relevant composition data not being provided in the study), no suitable data for comparing thermal diffusivities derived using Eq. (2) against measured thermal diffusivity data was found in the literature. In the absence of suitable measured data, it is possible to make use of thermal conductivity models for the sake of comparison. Theoretical upper and lower bounds have been derived for thermal conductivities of heterogeneous materials [17]. Specifically, the Parallel (Eq. 9) and Series (Eq. 10) thermal conductivity models represent the upper and lower bounds respectively for any material:

$$k_e = \sum_i k_i v_i \tag{9}$$

$$k_e = \frac{1}{\sum_i \frac{v_i}{k_i}} \tag{10}$$

Since any measured thermal conductivity should lie between the values predicted using the Series and Parallel models, they can be used to establish the upper and lower values of thermal diffusivity predicted using Eq. (2), with effective density modelled using Eq. (5) and effective specific heat capacity modelled using Eq. (11):

$$c_e = \sum_i x_i c_i \tag{11}$$

Table 5 shows the thermal diffusivities of bread crumb measured by Zanoni et al. [16] along with measured effective (apparent) densities, effective specific heat capacities calculated using Eq. (11) and the specific heat capacities for the major food components measured by Choi and Okos [11], as well as effective thermal diffusivities calculated using Eq. (2) both for thermal conductivity calculated using Eq. (9) and for thermal conductivity calculated using Eq. (10).

Table 5: Comparison between measured thermal diffusivities of bread crumb at different porosities [16] and thermal diffusivities predicted using Eq. (2) based on the Series and Parallel thermal conductivity models

v_{air}	$\rho_{measured}$ (kg m^{-3})	$c_{e,Eq.\,(11)}$ (J kg^{-1} K^{-1})	$\alpha_{measured}$ (m^2 s^{-1})	$k_{Parallel}$ (W m^{-1} K^{-1})	k_{Series} (W m^{-1} K^{-1})	$\alpha_{Eq.(2)\,Parallel}$ (m^2 s^{-1})	ε Parallel %	$\alpha_{Eq.(2)\,Series}$ (m^2 s^{-1})	ε Series %
0	979	2992	2.5×10^{-7}	0.510	0.427	1.74×10^{-7}	30	1.01×10^{-7}	59.
0.24	741	2992	2.9×10^{-7}	0.394	0.095	1.88×10^{-7}	39	2.24×10^{-8}	92
0.37	613	2992	3.3×10^{-7}	0.331	0.065	1.81×10^{-7}	45	1.57×10^{-8}	95
0.5	484	2992	4.0×10^{-7}	0.269	0.051	1.96×10^{-7}	54	1.21×10^{-8}	97
0.71	276	2992	5.0×10^{-7}	0.167	0.037	2.03×10^{-7}	59	8.89×10^{-9}	98
0.79	197	2992	5.3×10^{-7}	0.128	0.034	2.28×10^{-7}	59	8.06×10^{-9}	98

Comparison of the error values in Table 3 and Table 5 shows that Eq. (2) based on either the Series or Parallel model does not provide more accurate predictions of effective thermal diffusivity than Eq. (7).

5. Discussion

Figure 1 shows a plot of the measured data for bread crumb along with the predictions of Eqs. (1) and (7), the two predictions based on Eq. (2), and the empirical model that Zanoni et al. fitted to their data [16]. Note that the thermal diffusivities are plotted on a logarithmic scale.

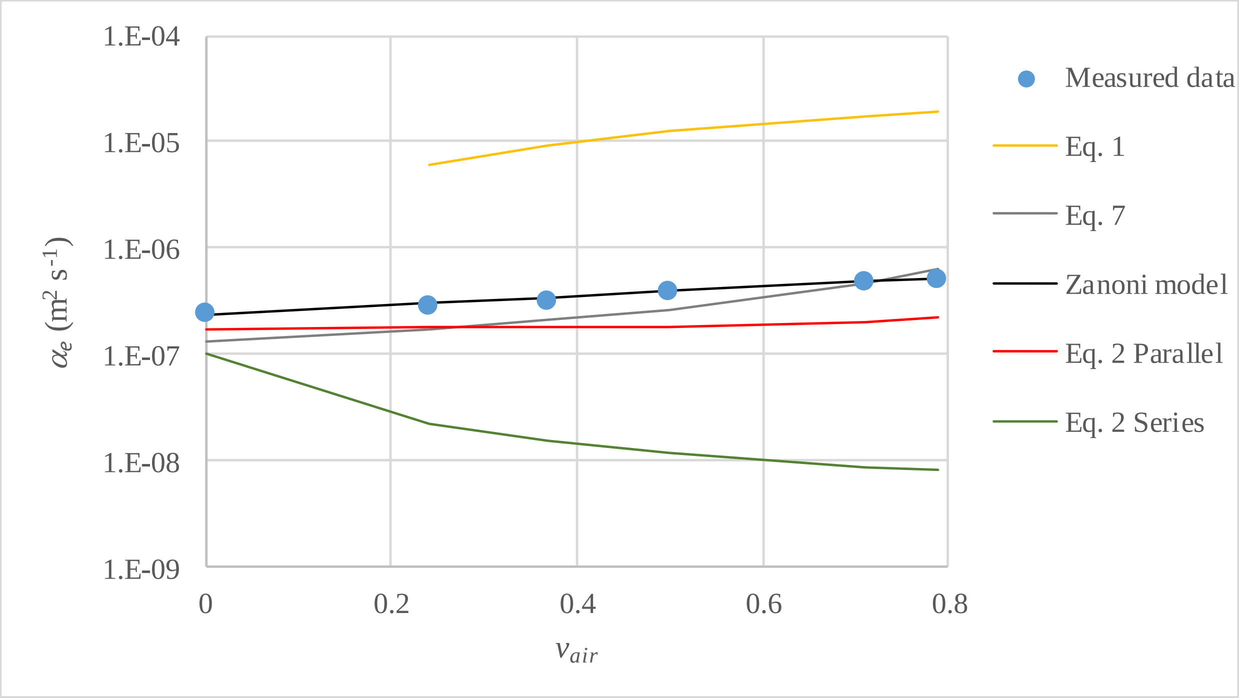

Figure 1: Comparison of effective diffusivity models with measured data for bread [16]

It is clear from Figure 1 that the two different approaches to modelling thermal diffusivity based on composition data produce widely differing results, and neither method models the measured data well, other than Eq. (7) at higher porosities. Apart from some overlap at lower porosities, the predictions from the method of averaging component thermal diffusivities (Eqs. 1 and 7) are significantly higher than those based on Eq. (2). Another observation is that according to the ‘averaging method’ the thermal diffusivity only increases with increasing porosity, whereas Eq. (2) based on the Series thermal conductivity model suggests it is possible for thermal diffusivity to decrease with increasing porosity. In general, the averaging method predicts a stronger dependence of thermal diffusivity on porosity than Eq. (2).

Given that no thermal conductivity model should produce higher thermal conductivity values than those of Eq. (9) (since it represents the upper bound of possible thermal conductivities), it appears that Eq. (2) would consistently under-predict effective thermal diffusivity of bread. This observation would support the claim that Eq. (2) should only be used to estimate effective diffusivity [13,14].

Given that the thermal diffusivity of air is 10^2 to 10^3 times higher than that of the other main food components, it might reasonably have been expected that porous foods would have noticeably higher thermal diffusivities than non-porous foods, especially for porosities greater than 0.5. However, the vast majority of measured thermal diffusivities of foods are in the region of 10^{-7} m^2 s^{-1} (compared to 10^{-5} m^2 s^{-1} for air) both for porous foods and for non-porous foods [3]. This observation would suggest that porosity does not affect thermal diffusivity in a manner that can be accounted for by taking a mean value of the components' thermal diffusivities. It appears therefore, that there is plenty of room to improve prediction accuracy of effective thermal diffusivity models.

6. Conclusions

The thermal diffusivity model proposed by Choi and Okos based on a weighted arithmetic mean of the components' thermal diffusivities was not suitable for the two types of porous foods (bread and rice) considered in this study. Applying a similar approach based on the harmonic mean rather than the arithmetic mean (Eq. 2), provided closer estimates of thermal conductivity for both types of porous foods considered, but still produced unacceptable prediction errors, greater than 100 % when applied to model the thermal diffusivity of rice. Predicting thermal diffusivity from effective thermal conductivity, effective density and effective specific heat capacity also resulted in unacceptable error. It is clear that there is scope for more work to be done in this area.

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