

A QUICK, RELIABLE SOLUTION FOR MODELLING CHEESE CHILLING PROCESS

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ABSTRACT

Optimisation of the refrigerating system for cheese processing requires an accurate prediction of chilling time, product temperature distributions and heat flow. Many existing CFD models can provide the best predictions by directly solving the three-dimensional heat transfer problem within the product and for air flow around the product; however, they are time-consuming and not suitable for routine use. A one-dimensional numerical solution proposed by Ghraizi, Chumak, Onistchenko, and Terziev (1996) has been used to provide the quick answer to food processing engineers with a good accuracy. In this method, the partial differential equation describing one-dimensional non-linear unsteady heat conduction inside the product has been solved by a finite difference technique. The method can take into account the temperature-dependence of thermal properties of foods and a general shape factor was used to reflect the product geometry. The model was applied to a single block of cheese, and agar. Predicted results are compared to experimentally-measured temperature profiles as well as to results generated by the Food Product Modeller, FPM, software.

Keywords: Modelling, chilling, heat transfer, cheese.

1. INTRODUCTION

Chilling is one of the most important part of the cold chain. Modelling heat transfer in the chilling process has a vital role in food process design and in improving food quality. Foods being processed are normally heterogeneous and have irregular shape. Thermoproperties are functions of food compositions and temperature. Therefore, the partial differential equation governing transient heat transfer is highly non-linear and difficult to solve. The accurate determination of the boundary conditions, especially the surface heat transfer coefficient, is also not easy.

The methods to model the heat transfer problem can be classified into analytical, empirical, and numerical solutions. Analytical techniques produce exact results when their underlying assumptions are fulfilled, but that is rarely the case. Their main usefulness is in providing benchmark results to verify other methods. Empirical formulas are derived with the objective of providing quick answers, with accuracy (usually $\pm 10\%$) good enough for most industrial users. However, they are limited to situations similar to those used to derive and validate the formulas. Numerical methods can, in principle, provide near-exact predictions for almost any situation, although in practice their accuracy is limited by inadequate knowledge of the problem's parameters (product properties, geometry, flow characteristics) (Pham, 2008). In this research, we propose to use the one-dimensional numerical solution of (Ghraizi et al., 1996) to model heat transfer in the chilling processes of a single block of cheese and a block of agar.

2. MATERIAL AND METHODS

2.1 Experiments

A single block of cheese from Open Country Dairy and a block of agar (5% of agar powder, and 95% water) were selected for chilling experiments. The experiments were conducted in the chilling room at the AgResearch Ltd, Hamilton facility. The samples were placed in a styrene test chamber with a variable speed fan at the downstream end, and a fine net at the upstream end to even out the air flow.

In the first experiment, a block of agar in an acrylic plastic box having dimension of 135 x 270 x 210 mm was used. In the second experiment, a block of cheese having dimension of 180 x 360 x 280 mm, was tested in two different cases: with and without the carton (still in the polyethylene bag). Test samples were kept in a controlled temperature chamber to make them a uniform initial temperature before conducting chilling trials. Temperature was measured at the surfaces, and geometric center with T-type thermocouples connected to a Keysight 24972A data acquisition unit to record every one minute. The thermocouples were calibrated with the ice-point reference before and after measurements.

The cooling air temperature was measured by two different thermocouples placed above, and below the test block. Air velocity was measured at the data sampling frequency of 1 Hz using a hot-wire anemometer (Dantec 54N60 FlowMaster). The velocity sensor was placed at 5 cm above the middle of the top surface of the test blocks. The air velocity was kept constant during the experiment.

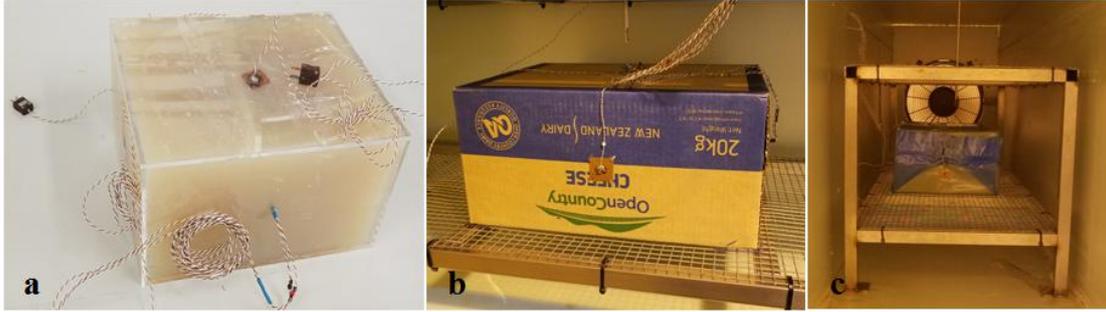


Figure 1: Experimental setup for temperature measurement
(a) agar, (b) cheese with carton, (c) cheese without carton

2.2 Numerical solution

The mathematical model of the heat transfer upon symmetric cooling was defined as a non-linear heat conduction equation with the corresponding boundary conditions, as follows: (Fikiin, 1996; Ghraizi et al., 1996)

$$C(T) \cdot \frac{\partial T(x,t)}{\partial t} = \frac{1}{x^r} \cdot \frac{\partial}{\partial x} \left[k(T) \cdot x^r \frac{\partial T(x,t)}{\partial x} \right] \quad (1)$$

$$T(x,0) = T_m(x) \quad (2)$$

$$\left. \frac{\partial T(x,t)}{\partial x} \right|_{x=0} = 0 \quad (3)$$

$$\left. -k(T) \frac{\partial T(x,t)}{\partial x} \right|_{x=R} = h \cdot [T(R,t) - T_a] \quad (4)$$

where, Eq. (1) is the Fourier heat conduction equation inside food products, Eq. (2) is the initial boundary equation, Eq. (3) is the symmetric boundary condition, and Eq. (4) is the third kind boundary condition. The shape factor, Γ , was generated based on the idea of substituting it into the governing equation of the one-dimensional solution, Eq. (1), such that, the numerical results will coincide or be satisfactory close to those obtained by solving the corresponding multidimensional problem. Fikiin (1996) proposed the formula of Γ as follows:

$$\Gamma = \frac{A \cdot R}{V} - 1 \quad (5)$$

where, the characteristic length, R , is the half thickness of the shortest dimension. A , V are the heat transfer surface and the volumetric of the object, respectively.

The finite difference scheme is shown in Fig. 2, in which the (i,j) -point corresponding to $x = x_i$, $t = t_j$ is determined as follows:

$$x_i = x_{i-1} + \Delta x, \quad i = 1, 2, \dots, N, \quad \Delta x = \frac{R}{N}$$

$$t_j = t_{j-1} + \Delta t, \quad j = 1, 2, \dots$$

where the time step, Δt , and space increment, Δx , can be variable. Because of the temperature-dependence of thermoproperties, it is necessary to average those quantities in the vicinity (efgh) of each (i,j) -point to make the numerical solution to be more stable (Onishenko, Vjazovsky, & Gnatiuk, 1991). These local average values can be obtained by integrating the governing equation in the vicinity (efgh)

$$\iint_{efgh} x^\Gamma C(T) \cdot \frac{\partial T(x,t)}{\partial t} dxdt = \iint_{efgh} \frac{\partial}{\partial x} \left[k(T) \cdot x^\Gamma \frac{\partial T(x,t)}{\partial x} \right] dxdt \quad (6)$$

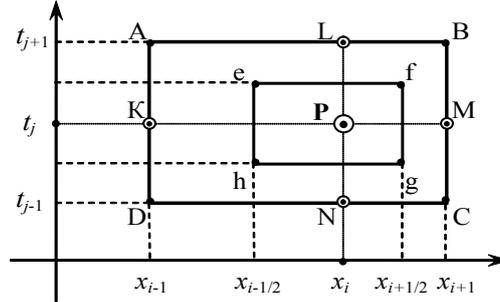


Figure 2: The finite difference scheme

More details of the finite difference solution of Eq. (1-4) are presented in (Ghraizi et al., 1996), in which, the internal nodes were discretized by the central difference formula, and the boundary nodes were approximated by the three-point backward/forward difference. The approximate system of linear algebraic equations therefore has the following form:

$$a_{ij}T_{ij} + b_{ij}T_{i-1,j} + e_{ij}T_{i+1,j} = d_{ij}, \quad i = 1, 2, \dots, N-1 \quad (7)$$

$$3T_{0j} - 4T_{1j} + T_{2j} = 0 \quad (8)$$

$$-k(T_{Nj}) \cdot \frac{3T_{Nj} - 4T_{N-1,j} + T_{N-2,j}}{2\Delta x} + h[T_a - T_{Nj}] = 0 \quad (9)$$

where

$$a_{ij} = \frac{C_{ij}}{\Gamma+1} \left[\left(x_i + \frac{\Delta x}{2} \right)^{\Gamma+1} - \left(x_i - \frac{\Delta x}{2} \right)^{\Gamma+1} \right] + \frac{\Delta t}{2\Delta x} k_{ij}^2 \left(x_i + \frac{\Delta x}{2} \right)^\Gamma + \frac{\Delta t}{2\Delta x} k_{ij}^1 \left(x_i - \frac{\Delta x}{2} \right)^\Gamma$$

$$b_{ij} = -\frac{\Delta t}{2\Delta x} k_{ij}^1 \left(x_i - \frac{\Delta x}{2} \right)^\Gamma$$

$$e_{ij} = -\frac{\Delta t}{2\Delta x} k_{ij}^2 \left(x_i + \frac{\Delta x}{2} \right)^\Gamma$$

$$d_{ij} = \left[\frac{C_{ij}}{\Gamma+1} \left\{ \left(x_i + \frac{\Delta x}{2} \right)^{\Gamma+1} - \left(x_i - \frac{\Delta x}{2} \right)^{\Gamma+1} \right\} - \frac{\Delta t}{2\Delta x} \left\{ k_{ij}^2 \left(x_i + \frac{\Delta x}{2} \right)^{\Gamma} + k_{ij}^1 \left(x_i - \frac{\Delta x}{2} \right)^{\Gamma} \right\} \right] T_{i,j-1} + \frac{\Delta t}{2\Delta x} \lambda_{ij}^2 \left(x_i + \frac{\Delta x}{2} \right)^{\Gamma} T_{i+1,j-1} + \frac{\Delta t}{2\Delta x} k_{ij}^1 \left(x_i - \frac{\Delta x}{2} \right)^{\Gamma} T_{i-1,j-1}$$

C_{ij} , k_{ij}^1 , k_{ij}^2 are the average volumetric heat capacity and thermal conductivities of the (i,j) -point, and are assumed to be the constant values

$$C_{ij} = (1/6)(C_K + C_P + C_M + C_C + C_N + C_D)$$

$$k_{ij}^1 = (1/6)(k_P + k_N + k_D + k_K + k_A + k_L)$$

$$k_{ij}^2 = (1/6)(k_P + k_L + k_B + k_M + k_C + k_N)$$

It can be noticed that the system of equations, Eq. (7-9) has the tridiagonal form, which can be solved by the tridiagonal matrix algorithm. Its solution is illustrated in the form of the recursion formula below:

$$T_{ij} = M_{i+1,j} T_{i+1,j} + N_{i+1,j}; i = 0, 1, \dots, N-1 \quad (10)$$

$$M_{i+1,j} = \frac{-e_{ij}}{a_{ij} + b_{ij} M_{ij}}; i = 1, 2, \dots, N-1 \quad (11)$$

$$M_{1,j} = \frac{a_1 + 4e_1}{3e_1 - b_1} \quad (12)$$

$$N_{i+1,j} = \frac{d_{ij} - b_{ij} N_{ij}}{a_{ij} + b_{ij} M_{ij}}; i = 1, 2, \dots, N-1 \quad (13)$$

$$N_{1,j} = \frac{-d_1}{3e_1 - b_1} \quad (14)$$

$$T_{Nj} = \frac{d_{N-1,j} - 3T_a b_{N-1,j} \left(\frac{2\Delta x h}{3k(T_{Nj})} \right) - N_{Nj} (a_{N-1,j} + 4b_{N-1,j})}{M_{Nj} (a_{N-1,j} + 4b_{N-1,j}) + e_{N-1,j} - 3b_{N-1,j} \left(1 + \frac{2\Delta x h}{3k(T_{Nj})} \right)} \quad (15)$$

In the procedure to determine the temperature profile at the j -th time layer, the algorithm needs to know temperatures at the $(j-1)$ th, j -th and $(j+1)$ th time layer to calculate the average thermoproperties of the (i,j) -point. Therefore, an iterative “prognosis-correction” regime (Fig. 3) was required (Ghraizi et al., 1996), in which, K is the number of iterations and the iterations are interrupted when the maximal difference between the temperature profiles, T_{ij} , of two consecutive iterations is less than a given tolerance, ε .

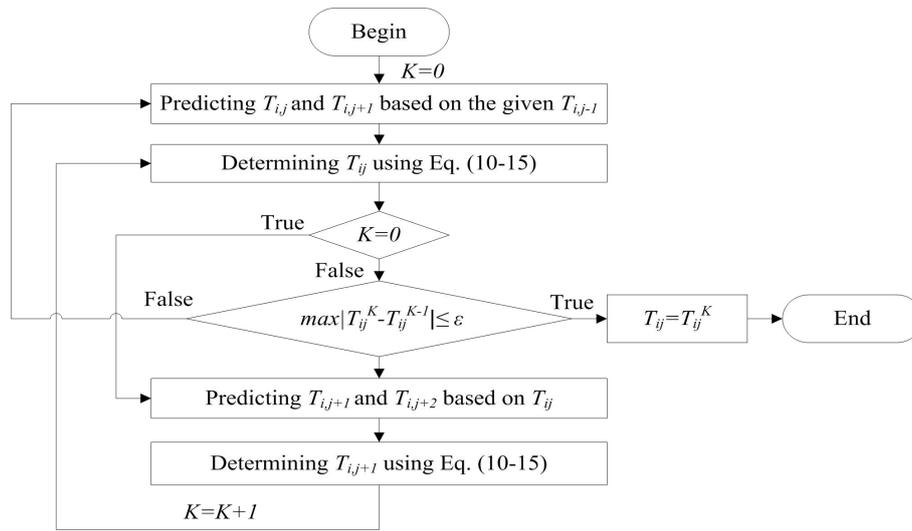


Figure 3: “Prognosis-correction” regime to determine T_{ij}

2.3 Thermal properties and heat transfer coefficient model

Thermoproperties of the test materials were determined as functions of food compositions and temperature. The thermal conductivity model was presented in (Dul'nev & Novikov, 1977; Dul'nev & Novikov, 1991), the effective specific heat model was presented in (Hoang & Nguyen, 2013), and the density model was obtained from (ASHRAE, 2002). The compositions of cheese using in this calculation were 36.3% water, 23.5% protein, 34.1% fat, and 3.6% ash, while agar was assumed as the pure water.

The heat transfer coefficient, h , was determined from an estimate of the convective heat transfer that occurs between the surface of the food item and the cooling medium, and the thermal resistance of the packaged materials, and the air gap:

$$\frac{1}{h} = \frac{1}{h_{conv}} + \frac{\delta_{air}}{k_{air}} + \sum \frac{\delta_{pkg}}{k_{pkg}} \quad (16)$$

where the convective heat transfer coefficient was determined as a mean value of the convective heat transfer coefficients proposed by Pham, Lowry, Fleming, Willix, and Reid (1991) and Cleland and Earle (1976):

$$h_{conv} = \frac{(v \times 8.6) + (v \times 4.5 + 6.8)}{2} \times 1.12 \quad (17)$$

Thermal conductivities and thicknesses of the packaging materials and air gaps are shown in Table 1, and the predicted heat transfer coefficient in each chilling trial are shown in Table 2

Table 1. Thickness of packaging materials and air gap.

Sample	Thickness, mm			
	Acrylic plastic ($k=0.2 \text{ Wm}^{-1}\text{K}^{-1}$) ^a	Polyethylene ($k=0.33 \text{ Wm}^{-1}\text{K}^{-1}$) ^a	Air gap ($k=0.026 \text{ Wm}^{-1}\text{K}^{-1}$) ^b	Cardboard ($k=0.078 \text{ Wm}^{-1}\text{K}^{-1}$) ^b
Agar	4.5	-	-	-
Cheese without carton	-	0.3	-	-
Cheese with carton	-	0.3	3	3

Source: ^aSingh, Burgess, and Singh (2008); ^b The Engineering Toolbox (n.d.)

3. RESULTS AND DISCUSSION

We conducted 4 trials with a single block of cheese and agar at the initial temperature, $T_{in} = 20^{\circ}\text{C}$, the cooling air temperature, $T_a = 0^{\circ}\text{C}$, and different air velocities. The simulated results of the one-dimensional numerical model (CFM) were compared to the predicted results of Food Product Modeller version 4 (FPM, AgResearch MIRINZ), using a three-dimensional simulation, and measured data. The surface temperature and the center temperature along the shortest axis were taken into this assessment exercise. Table 2 illustrates the mean absolute errors between measured and predicted data using CFM and FPM at different temperatures. The absolute errors were calculated by Eq. (18):

$$\Delta T = |T_{exp} - T_{mod}| \quad (18)$$

Table 2: Average absolute errors between measured data and predicted results

Chilling trials	R , m	Γ	v , ms^{-1}	h , $\text{Wm}^{-2}\text{K}^{-1}$	$mean \Delta T_{surf}$		$mean \Delta T_{center}$	
					CFM	FPM	CFM	FPM
Agar	0.063	1.100	3	16.3	0.14	0.73	0.73	0.43
Cheese without carton	0.090	1.143	2	18.2	1.99	1.11	0.64	0.58
Cheese with carton	0.090	1.143	1	4.1	-	-	0.35	0.18
			2	4.8	-	-	0.50	0.32

In general, the predicted results generated by CFM slightly worse than FPM. This is reasonable since FPM is the three-dimensional model while CFM is only the one-dimensional model. However, it was compensated by the much shorter solution time, since its number of nodes was much less than that of the three-dimensional FPM model. The maximum deviations between the measured temperatures and calculated values using CFM and FPM for single block of agar were 1.1°C and 1.0°C , respectively. For the single block of cheese, those numbers were 2.2°C and 1.4°C . Taking into account the uncertainty of the temperature sensors, those numbers are acceptable.

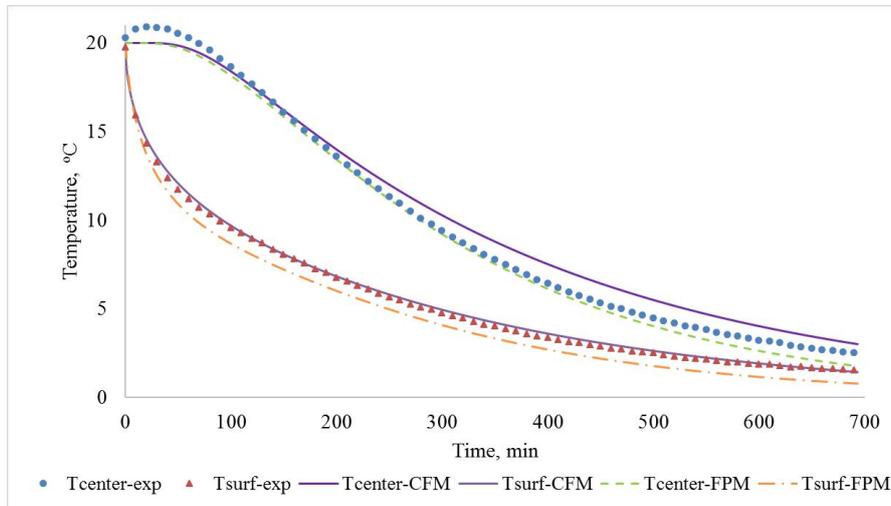


Figure 4: Plots of the temperatures predictions of CFM, FPM with experimental data for single block of agar

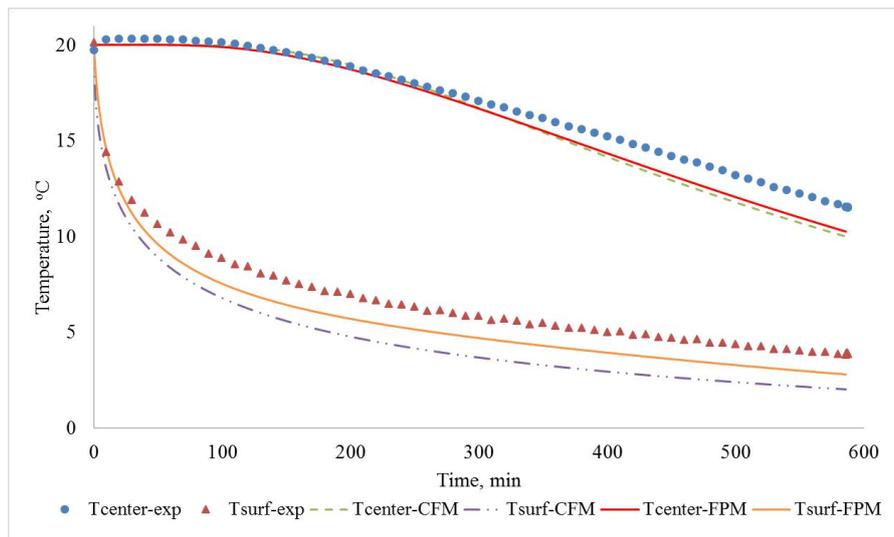


Figure 5: Plots of the temperatures predictions of CFM, FPM with experimental data for single block of cheese without carton

Fig. 4, and Fig. 5 illustrate comparisons between simulated results and experimental data for the chilling process of agar and cheese without the carton. The figures showed the good agreement between the prediction of CFM and the experimental data and the predicted values of FPM. Therefore, the one-dimensional numerical model (CFM) can be used to provide a quick answer with a sufficient confidence about the temperature distribution along the shortest characteristic dimension of a 3-D object, which represents the greatest interest for food engineering investigation. (Fikiin, 1996)

4. CONCLUSIONS

A one-dimensional numerical model for non-linear unsteady heat transfer of food products in the chilling process was presented. The model fits well with the experimental results and the predicted data of the three-dimensional model. Therefore, CFM could be a useful tool for food process design.

NOMENCLATURE

A	area of heat transfer surface (m ²)	R	characteristic length (m)
C	volumetric heat capacity (Jm ⁻³ K ⁻¹)	V	object volume (m ³)
k	thermal conductivity (Wm ⁻¹ K ⁻¹)	v	air velocity (ms ⁻¹)
h	surface heat transfer coefficient (Wm ⁻² K ⁻¹)	x	position (m)
T	temperature (K)	Δx	space increment (m)
t	time (s)	Γ	shape factor
Δt	time step (s)		
	Subscripts		
air	air gap	i	ith coordinate
a	ambient	j	jth time layer
cov	convective heat transfer	mod	model
exp	experiment	pkg	packaging
in	initial		

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