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# TEACHERS' REFLECTING ON PROFESSIONAL KNOWLEDGE IN THE NUMERACY (MATHEMATICS) CLASSROOM

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## Abstract

*New Zealand primary school teachers are expected to regularly reflect on their teaching practice in order to consider the implications of past teaching on future planning. Aligned to teachers' ongoing reflection, the New Zealand Curriculum (Ministry of Education, 2007) contains a section on effective pedagogy—teacher actions promoting student learning, which includes a Teaching as Inquiry Cycle (pp. 34–35). Embedded within their inquiry, teachers consider the teaching-learning relationship and often turn to frameworks of knowledge for guidance. This article shares the implications of using a framework of teacher knowledge in research. While the framework used contained much detail for the researcher, it overlapped categories and at the same time lacked acknowledgement of some important concepts for teachers in classroom practice. Findings from using a framework in this research were combined with findings from previous research to formulate the Wheel of Professional Knowledge, which was developed for mathematics teachers to use when reflecting on their practice.*

## Keywords

Frameworks; professional knowledge; teaching as inquiry; numeracy; mathematics

## Introduction

The various components of teacher knowledge and their importance for the teaching process (as identified by Shulman in his seminal paper in 1986), have provided the basis for many knowledge frameworks in mathematics that gauge the mathematical knowledge required for teaching (Ball et al., 2008; Bobis et al., 2012; Chick et al., 2006; Hill et al., 2008). Many decades after Shulman first proposed the idea of pedagogical content knowledge (PCK), one might think that a comprehensive theory of decision-making could provide a basis for a usable classroom observation scheme. However, producing a workable framework for classroom observations that is theoretically grounded is more difficult than one would expect. As Schoenfeld (2013) explained:

Although the theoretical and practical enterprise are in many ways overlapping, the theoretical underpinnings for the observation [scheme] are sufficiently different (narrower in some ways and broader in others) and the constraints of almost real-time implementations so strong that the resulting analytic scheme is in many ways radically different from the theoretical framework that gave rise to it. (p. 607) <sup>p</sup>

Creating a classroom-based workable framework to assist the professional knowledge of teachers has thus become an ongoing interest of many teachers and researchers, with many hours having been spent formulating classroom-based workable frameworks, against which the attributes of PCK and other key knowledge areas important for teaching, can be compared.

This article shares some of the findings of classroom research investigating the professional knowledge of New Zealand primary school teachers in the numeracy classroom (Mills, 2018). An established framework (Chick et al., 2006) was used in the research, which provided detail of the teachers' teaching and their students' learning. However, a framework required for teachers as reflective practitioners has subtle differences from the detail required of subcategories of knowledge on a framework for researchers. One of the aims of this research was to develop a tool for teachers to use when reflecting

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on their practice in the mathematics classroom. It needed to be less meticulous than frameworks used by researchers, but specific enough to reflect on key elements of teaching and learning. This led to the formation of a *Wheel of Professional Knowledge* (see Figure 1 later in this article) for teachers to use alongside their *Teaching as Inquiry* cycle (Ministry of Education, 2007).

### Frameworks of teacher knowledge and teaching as inquiry

The importance of teacher knowledge required for the teaching process is recognised in many frameworks created over past years. However, categorising teacher knowledge into suitable headings for eliciting information about teacher action in practice has often been contentious. Creating a classroom-based workable framework to explore the professional knowledge of teachers has thus become an ongoing interest of many teachers and researchers (Ball et al., 2008; Chick et al., 2006; Gess-Newsome, 2015; Roche & Clarke, 2009; Schoenfeld, 2013). While many commonalities within frameworks emerge, the understanding of content as it relates to one's subject area is crucial and PCK in its entirety cannot simply be imported from one subject area to another (Gess-Newsome, 2015; Hill et al., 2008; Loughran et al., 2012; Magnusson et al., 1999; Shulman 2015). For example, mathematics teachers require an understanding of how and why a particular mathematical procedure works, while effective science teachers need to know how to best design and guide learning experiences under particular conditions and constraints in order to help their students to develop scientific knowledge and understanding (Magnusson et al., 1999). PCK is a dynamic construct that describes the process a teacher employs when teaching particular subjects, to particular learners, in particular settings (Ball et al., 2008; Shulman, 2015). Therefore, frameworks often need to be subject specific to capture the essence of the teaching practice.

During the development of a framework, many issues associated with teaching arise and it becomes obvious that to include them all would result in a framework that was cumbersome and difficult to implement. Developing a PCK framework that is applicable to the teaching of all mathematics content while capturing the many dimensions associated with teaching into a manageable observation scheme, poses many challenges. Those teachers utilising such a scheme seldom get to see the “twists and turns of plausible, but unworkable ideas, that precede the presentation of the clean final product” (Schoenfeld, 2013, p. 607). There has been a shift in mathematical research in recent years from *knowledge as a substance* to *knowledge as an activity* (Settlage, 2013). The theoretical underpinnings of PCK frameworks used by researchers, have thus led to the development of frameworks based on categories and dimensions used within classroom practice (Schoenfeld, 2013), principles of practice (Smith & Stein, 2011), and powerful ideas (Askew, 2013; Schwartz, 2008).

Shulman (2015) recognised earlier research on PCK placed little emphasis on emotions, affect, feelings and motivation—non-cognitive attributes; did not attend to pedagogical action; gave little attention to social and cultural contexts; and did not recognise the outcomes of instruction—the relationship between how teachers thought and evidence of student learning. These affective dimensions have begun to emerge in frameworks of inquiry more recently, which has resulted in a shift towards examining teachers' overall professional knowledge and actions required for teaching. This shift in knowledge was recently the basis of the *Teacher Professional Knowledge and Skill* (TPK&S) framework. The TPK&S framework recognises the relationship between teaching and learning, by acknowledging the connections between teacher professional knowledge bases (assessment knowledge, pedagogical knowledge, content knowledge, knowledge of students and curricular knowledge), topic-specific professional knowledge, classroom practice and student outcomes (Gess-Newsome, 2015).

The New Zealand Curriculum (Ministry of Education, 2007) requires teachers to inquire into the impact of their teaching on their students' progress as a component of effective pedagogical practice. Teaching as Inquiry is an investigation by a teacher, on the influence of their teaching practice on their students' learning, in order to recognise the improvement in student learning (Earl & Ussher, 2016). Teaching as Inquiry should be seen as a “cyclical process that goes on moment by moment, day by day, and over a longer term” (p. 35). Teaching as Inquiry involves gathering data from the teacher's own practice, as well as evidence from other research, to plan teaching or enact strategies in order to achieve the outcomes identified in the focusing inquiry. Teachers investigate the success of their teaching using a range of assessment approaches, and then interpret the information to plan what they should do next.

## Teaching for numeracy in the mathematics classroom today

It is generally acknowledged that it is no longer acceptable for students to leave school without the basic skills, knowledge and dispositions they need to function effectively in life (Hattie, 2009). Hence, recent reforms have seen more use of the term numeracy in education, where emphasis is on the understanding of mathematical concepts (Bennison, 2015; Coben, 2003; Goos et al., 2011). It has been suggested that the ultimate goal of mathematics education should be the development of numeracy, or mathematical literacy, where teaching should stimulate learners to develop the ability to give meaning to numbers and numerical facts in everyday life (Commonwealth of Australia, 2008; Ministry of Education, 2007; Perso, 2006; Tertiary Education Commission, 2008). Numeracy has been defined as “the ability to process, communicate, and interpret numerical information in a variety of contexts” (Askew et al., 1997, p. 6). Numeracy is often referred to in terms of what it means to be numerate and was defined for New Zealand schools as “the ability and inclination to use mathematics effectively—at home, at work and in the community” (Ministry of Education 2001, p. 1). Perso (2006) suggested that although numeracy is about the mathematics you know, it is also about having a disposition and a confidence to use it: “Knowing some mathematics is essential but not sufficient for numeracy. However, knowing some mathematics must precede the choice to use it or not” (p. 37). The Tertiary Education Commission (2008) argued similarly, that when it came to solving real-life problems, being able to do mathematics did not necessarily mean being able to use mathematics.

As a result of ongoing reforms, the role of the teacher has changed in a profound way, and teaching in today's classroom requires professionalism, skills and knowledge that teachers of previous years did not require (Hattie, 2009). The shift in focus in the numeracy classroom from reliance on memorisation of facts and procedures to one of conceptual understanding requires an understanding of the relationship between knowledge of subject matter, knowledge of teaching practice and knowledge of curriculum. As Hattie emphasised, today's teachers must be reflective, analytical and literate; they must be creative, imaginative, knowledgeable, and sensitive to the diverse needs of the children in today's classrooms. If teachers are to create classroom experiences and conditions, which prepare children for tomorrow, they need to practise the skills of inquiry and reflective thought.

It has been argued that the most productive option for improving classroom instruction is to focus directly on the *teaching* rather than the *teachers* (Hiebert & Morris, 2012; Stigler & Hiebert, 2004). Improving teaching and gaining better results begins by improving the instructional methods that are implemented in the classroom. However, differentiating between teaching and teachers can be problematic, as teaching is generally simultaneously linked with the improvement of teachers (Chick et al., 2006; Hiebert & Morris, 2012). There is a need for teachers to have and maintain sound knowledge of mathematics to underpin the structural material and intellectual choices that they make in their classrooms (Anthony & Walshaw, 2008).

## Working with a framework of professional knowledge

This paper is based on findings from the use of a PCK framework developed by Chick et al. (2006), which was the basis for the analysis of classroom observations of the teaching of multiplication, division and fractions used in a larger research study (see Mills, 2018). Four teachers were observed every two months over a period of six months and were teaching the senior classes at their respective schools: two Years 5 and 6 classes at one school, and the Years 6 and 7 class, along with the Years 7 and 8 class at another school. Multiple sources of data collection were gathered over the four sessions, for the larger project. The research focused on the relationship between the teachers' espoused PCK (gathered from an initial questionnaire), and their PCK as observed in their teaching practice. Data collected from observation of the classroom lessons included field notes and both video-recordings and audio-recordings. The audio-recordings were transcribed and analysed using the categories of the Chick et al. framework and later crosschecked against the video-recordings.

The framework used was a framework developed by mathematics researchers for use when exploring teachers' mathematical knowledge in-depth. The framework by Chick et al. provided categories for the researcher to carry out a fine-grained analysis of the teachers' classroom practice with the breakdown of three broad areas into sub-categories, aligned to the three main areas of professional knowledge

originally mooted by Shulman (1986): content knowledge, general pedagogical knowledge, and pedagogical content knowledge. Chick et al. incorporated the need for students' prior knowledge, student problems, relevant representations, teaching strategies, student activities, student thinking and curriculum knowledge into three parts considered necessary for PCK: Clearly PCK, Content Knowledge in a Pedagogical Context, and Pedagogical Knowledge in a Content Context.

## Findings

The comprehensive analysis of teacher knowledge and student learning allowed the researcher to determine a number of practices associated with teacher professional knowledge. Coding of the data was very detailed and complex and a summary of the findings is presented in this paper to demonstrate the use of a framework as part of the research process.

In the teaching of multiplication and division, in the Clearly PCK category, teachers used a range of strategies to encourage and support their students' learning. They utilised their students' thinking to promote further learning with varying frequency depending on whether the students were left to problem solve among themselves, or whether discussion was more teacher directed. Manipulatives were used to represent concepts and support students in their conceptual understanding. Initially, materials were not available for use by all students in all classes, but by observation of the final lesson, all students used them. Estimation was seldom used as a method to check for reasonableness of answers, inconsistency in the use of mathematical language caused confusion and repeated reference to the learning intention (WALT) often stifled potential opportunities to utilise unanticipated teachable moments.

In the Content Knowledge in a Pedagogical Context category, the teachers regularly deconstructed the content of what they were teaching in an effort to assist the students with their understanding of multiplication. However, the small number of times the teachers made connections with the mathematical structure of the problems was highlighted by the few instances in which they exhibited what is referred to as profound understanding of mathematics. There were times when the teachers recognised student misconceptions and corrected these. However, at other times, the teachers caused misconceptions due to in-the-moment responses.

In the Pedagogical Knowledge in a Content Context category, the teachers did not utilise the assessment data available to them to plan their lessons, and instead relied heavily on the lesson sequences in books for direction and content. The majority of questions were the lower-level supporting type, and the teachers seldom extended the students with higher expectations, going beyond initial solution methods to solve problems.

The professional knowledge of teachers when teaching fractions for understanding was similarly analysed against the Chick et al. framework. In the Clearly PCK category, the teachers did not specifically mention the curriculum and/or associated Number Framework within their lessons, although these were implicit in the chosen lesson objectives. All of the teachers used manipulatives to reinforce conceptual understanding, although two of the teachers admitted they struggled with this idea because they had never used equipment themselves. All of the teachers asked the students to discuss their ideas to consolidate understanding, although it was only one of the classes where the students conversed with each other about their solution methods: while they told each other what they did, they struggled with justification of their procedures. In the other classes, the students talked with each other about parties or biscuits, but conversation around the mathematics learning appeared limited. The cognitive demands and understanding the complexities associated with the fraction tasks were unpacked through models representing the contexts. Models included the use of paper circles, commercial fraction tiles and paper strips. The nature of the lessons determined whether misconceptions of the students were readily recognised: when children worked in pairs and discussed their problem-solving, the teachers could quickly see whether conceptual understanding was evident.

In the Content Knowledge in a Pedagogical Context category, the teachers deconstructed the meaning of fractional numbers early in their lesson when they discussed that the denominator relates to the number of equal pieces that make up the whole, while the numerator tells how many of those pieces there are. Making connections between structure and concepts were observed when connections

between the structure of fractions and division were made. Procedural knowledge came to the fore with one teacher who emphasised the rule for carrying out computations more than the other teachers.

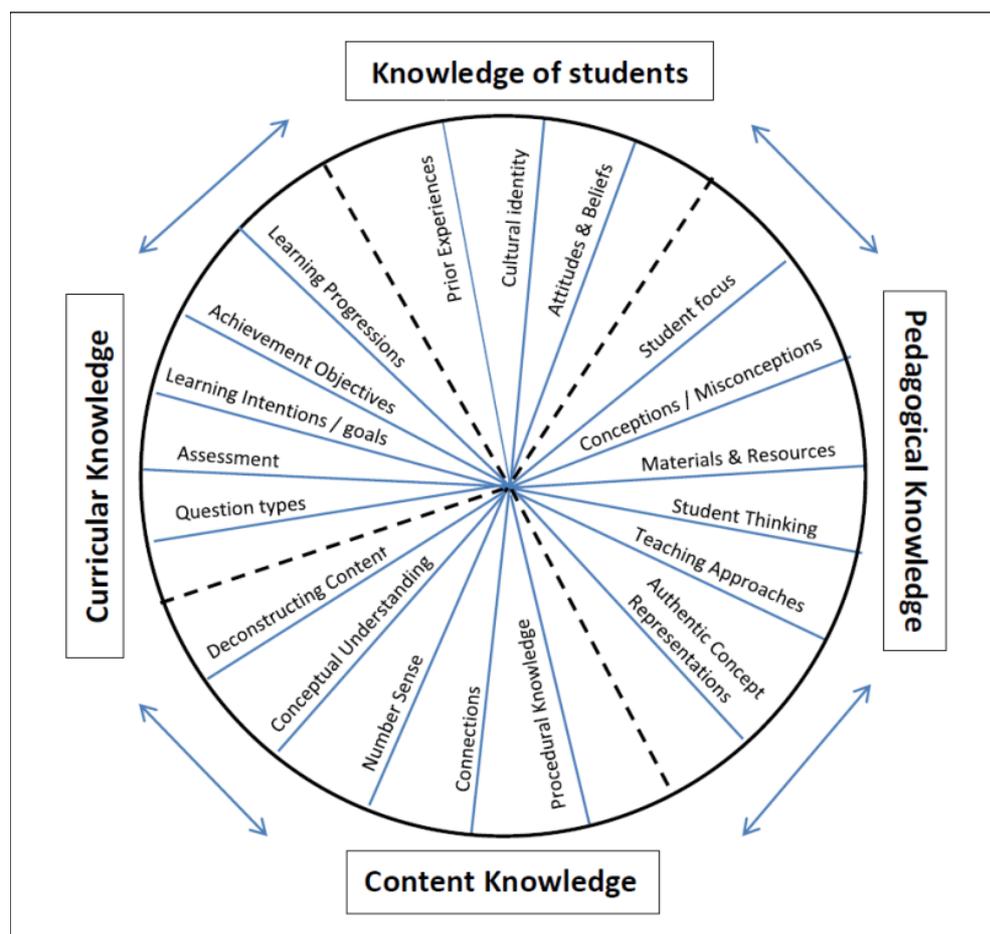
In the Pedagogical Knowledge in a Content Context category, one common technique was to begin the fraction lessons by sharing the learning intention (WALT), which was recorded in the modelling book and unpacked to ensure that the students knew the purpose of the lesson. One way the teachers maintained student focus was to create word problems in real-life contexts, which included the students' names.

While it is acknowledged use of the adapted framework of Chick et al. provided a sound basis for the analysis of data in this study, many of the categories overlapped and others were missing if the framework was to be used by teachers for reflecting on their own practice.

### The wheel of professional knowledge

After exploration of several frameworks and using the detailed framework of Chick et al. to analyse professional knowledge in practice, it was concluded that teachers could benefit from the use of a framework to reflect on their personal professional knowledge and to support their mathematics teaching practice. It was decided to incorporate the traditional PCK categories associated with teacher content knowledge and pedagogical practice, alongside categories relating to knowledge of students, including their attitudes and beliefs about mathematics and relationship with their cultural identity, which are often missing categories in formal frameworks. Shulman (2015) also acknowledged these affective notions as missing components of his original PCK thinking, which was based on theoretical underpinnings, rather than the actual practice of teachers. Observation of teacher practice revealed this gap, which may not always be evident in research carried out by other means, such as questionnaires. Having this knowledge is an important component of a teacher's professional knowledge and was included in the establishment of a *Wheel of Knowledge* (Figure 1) for classroom teachers.

The *Wheel of Knowledge* is based on four key categories of professional knowledge identified throughout this research, including knowledge of students, pedagogical knowledge, content knowledge and curriculum knowledge, and was created specifically for use by mathematics teachers in their classroom practice. A wheel was chosen to represent the cyclic, ongoing nature of acquisition and implementation of knowledge, and to sit alongside implementation of the Teaching as Inquiry Cycle (Ministry of Education, 2007, p. 35). The spokes of the wheel are not intended to represent all areas of professional knowledge required to teach mathematics effectively. However, they are a combination of aspects of professional knowledge that were identified as particularly important in the mathematics classroom as a result of this research. The sectors might widen, or narrow, depending on the characteristics of specific mathematical topics, the students concerned or lesson styles. While the sectors are separated within the diagram, the dynamic inter-relationship between the components is reflected in the meeting together at the centre of the wheel and the double-ended arrows between headings.



**Figure 1:** Wheel of professional knowledge.

It is not expected that teachers would focus on all categories in every lesson, nor on any one category (spoke of wheel) giving it greater or lesser importance. Instead, recognition of each component should be evident over a period of time: this may be a complete unit of work or may take a longer period such as a term or a year to cover. It is presented as a guide for teachers when reflecting on their own practice or discussing teaching practice with colleagues. The wheel of professional knowledge could be utilised by school leaders, classroom teachers and pre-service teachers. It might be used in staff meetings, as part of the attestation and appraisal process, or for teachers to refer to as a reminder of their professional knowledge in action.

## Conclusions

The use of a framework provided a systematic representation of categories, which provided the researcher with detailed data associated with teachers' professional knowledge, in practice in the classroom. The framework used was originally presented by Chick et al. (2006), based on Shulman's original PCK categories of pedagogical knowledge, content knowledge and pedagogical content knowledge (Shulman, 1986). Sub-constructs within the three broad categories assisted in the detailed analysis of the data associated with the teachers' professional knowledge in order to gain insight into the attributes of classroom practice.

The research indicated that while there is a need to connect subject matter knowledge with pedagogical practice, some of the affective aspects and issues relating to broader social and cultural contexts of teaching practice are often missing from frameworks. The aspects missing from the framework included students' prior experiences with mathematics learning and the impact this has on current learning, along with students' cultural background and heritage and the impact of these identities on mathematical contexts: all considered important aspects if teachers use the framework in their classroom practice. Therefore, the three main categories of the framework used in this research became a foundation for the

forming of a new framework, along with a fourth category identified as missing by the researcher and aligned to Shulman's (2015) later acknowledgement of PCK in action. Thus, the four key categories of professional knowledge utilised when designing the *Wheel of Knowledge* were knowledge of students, curricular knowledge, content knowledge and pedagogical knowledge.

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