Mathematics in Workplace Settings:

Numeracy in the Mechanical Engineering Trades

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Dedication

This thesis is dedicated to my father:

Stanley Mostyn Mills

1920 - 2008

Mechanical engineering tradesman

Ship’s engineer

Maintenance engineer

Toolmaker

Mentor of Apprentices

Friend

Abstract

While mathematics is an essential tool for both professional and trades mechanical engineers, little is known about how mathematics is used and learned in the mechanical engineering trades. Using an interpretivist paradigm and informed by a social constructivist epistemology, this mixed methods study aimed to identify key features of mathematical learning in the New Zealand mechanical engineering trades: specifically, the nature of the mathematical knowledge and skills, and how they are applied and developed.

A purposive sample of 199 apprentices, skilled tradespersons and mechanical engineering trades educators completed a questionnaire about the mathematics and numeracy skills they used, how they used and learned those skills, and the role of ancillary skills such as higher-order thinking and social interaction. Seventeen of these participants also took part in semi-structured interviews. The data were analysed thematically using Engeström’s (1987) Cultural Historical Activity Theory (CHAT) and Lave and Wenger’s (1991) Situated Learning as theoretical frameworks.

Regarding the mathematics skills employed in the mechanical engineering trades, the study found that a thorough knowledge of, and proficiency in, basic mathematics and numeracy skills were essential. In addition, those basic mathematical skills were frequently used in sophisticated, real-life contexts involving higher-order thinking skills such as problem-solving, creativity, and extended reasoning, as well as metacognitive skills, such as critical thinking, learning to learn, working in teams, and planning. However, many engineering decisions were made not on mathematical considerations alone, but using non-formal heuristics and engineering judgment following particular rules generated and accepted by the engineering communities.

Regarding developing the mathematical skills, learning at both individual and community levels appeared to be done eclectically. Learning and knowledge creation took place both formally and informally, whether in the classroom or on-the-job, and hence by acquisition and participation as well as by individual reflection.

This study contributes to our knowledge of the role of mathematics in mechanical engineering trades. It does this through its demonstration of the importance of basic mathematics and numeracy skills and the new insights gained into the interconnectedness of these basic skills with higher-order thinking and metacognitive skills. Moreover, this study contributes to our knowledge of the influences of social interaction, collaboration, and communication as important tools for learning, problem-solving, and creating new knowledge in workplaces’ communities of practice. Therefore, learning is revealed as an iterative process involving developing relationships between tools and subjects as part of an evolving historical process where communities play a central role.

The study should be of interest to mechanical engineering communities and other vocations that are high users of mathematics because of the interconnections the study makes between physical tools and higher-order thinking skills situated in real contexts, the learning needed to change school habits and perspectives regarding well-developed numeracy and mental calculation skills for the workplace, ongoing professional development of mathematics knowledge skills related to real contexts, and conceptual understanding of the holistic interconnectedness of mathematics within workplace contexts.

The study also has implications for other vocations because it demonstrates that developing workplace mathematics knowledge and skills is a much more complex process than a simple transference of school mathematics skills. Successful practice depends on combining technical skills with higher-order thinking, metacognitive skills, social interaction, collaboration and communication.

Acknowledgements

"No man is an island …", the English poet, John Donne, wrote in 1624. Donne's meaning was that we all depend on each other, and that our individual achievements are dependent on the help and wisdom we have received from those around us.

I have many people to thank. First, I remember daily my late parents, Stan Mills and Jean Mills (née Paterson). They first encouraged me in engineering by buying me a meccano set, and Stan shared his engineering knowledge with me during my childhood and adolescence. Jean encouraged me in mathematics and first used my interest in mechanical devices such as clocks to teach me how to tell the time. Together, Stan and Jean instilled in me a love for valuing things in life that will last.

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> As the rain and the snow come down from heaven, and do not return to it without watering the earth and making it bud and flourish, so that it yields seed for the sower and bread for the eater, so is my word that goes out from my mouth: It will not return to me empty, but will accomplish what I desire and achieve the purpose for which I sent it. (KJV, Isaiah 55:10,11)

Close family have also offered me the benefit of their wisdom. I wish to thank my brother-in-law, Tony Rooke, and sister-in-law, Margaret Saunders Rooke, for the many hours of proofreading and helpful advice they have given to make this document more comprehensible and to flow more easily.

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I am truly dependent on the expertise, love and support of others.

May God bless you all.

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Chapter 1. **Introduction**

This study investigates key features of the mathematical learning of mechanical engineering tradespersons as they progress from beginning apprentice to skilled tradesperson and then possibly to expert engineer. It focuses on the nature of the numeracy and mathematical processes the engineers use and how they use, develop, learn and transmit those numeracy and mathematical tools.

The broad context of this thesis is mechanical engineering, which may be described as the design, construction and use of physical tools and machines. Tools and machines are usually made of metal, but sometimes wood, plastic and many other materials may be used. There are many branches of mechanical engineering, ranging from the fine, intricate work of machinists and toolmakers making dies for moulding plastic items, to the design, construction and installation of componentry in massive turbines for power stations. Mechanical engineers may be skilled tradespersons who have completed an apprenticeship or professional engineers who have completed a university degree. This thesis focuses on skilled tradespersons. A glossary of technical terms is included in Appendix A.

The remainder of this chapter explains my personal motivations for doing this research and my researcher positionality. I introduce the research questions and also discuss the differences, similarities and overlaps between the concepts of 'mathematics' and 'numeracy'. I provide a background to the mechanical engineering trades in New Zealand and describe the context of the New Zealand mathematics curriculum. I then explain the nature of mechanical engineering trades mathematics and its relevance to this study. I end the chapter with an overview of the whole thesis.

1.1. **Personal motivations for doing this research**

My personal motivations for doing this research have emerged from the practical nature of my childhood mathematics experiences. My interests were also influenced by the more theoretical experiences of mathematics and physics I learned at school and later at university, and which I then taught in New Zealand secondary schools for more than three decades. These experiences resulted in various tensions, or contradictions, in my life, especially regarding the cleft between abstract mathematics and the workplace. For many years I have observed those same contradictions in mechanical engineering tradespersons and the students I taught.

My introduction to mathematics and engineering began with my childhood interactions with my father who was a mechanical engineer. I grew up surrounded by physical tools such as spanners, screwdrivers and feeler gauges. Later I learned about non-physical, intellectual tools like thousandths of an inch to measure spark plug gaps, tolerances, scale diagrams and weights measured in pounds and ounces. Through observation, discussion and practice, I developed a feel for certain physical tools, what they were used for and how they worked. While I was conscious of the influence of pragmatism in my life, unconsciously I was also absorbing intellectual tools that were necessary to solve problems and allow new physical tools and techniques to be constructed. Using Cultural Historical Activity Theory (CHAT) (see Section 3.2) as a lens to investigate the associations between intellectual and physical tools is an important theme that runs through this thesis, and has ramifications for practical problem solving and learning, both for apprentices and skilled tradespersons.

I also joined in the discussions with my father and his engineer friends and became accepted as the smallest member standing on the periphery of a “community of practice” (Lave & Wenger, 1991, p. 29). The community members became inventive as a result of their social interaction and strong communication. With time, I learned the jargon of their work and how to share my theories and stories, complete with diagrams and began to move from the periphery to the centre of the community of practice. Therefore, communication was an integral part of my learning. Several processes involving practicing, reflection and social interaction were operating in my life. I gradually learned to live in and feel comfortable in several different worlds where my learning was both socially and personally influenced (Cole & Wertsch, 1996).

My exposure to the engineering world included both pragmatic and intellectual activities. Pragmatically, pulling bikes to bits and putting them together again, and helping out with valve grinds on cars gave me useful skills using physical tools. Intellectually, I began to develop an understanding of when mathematics should be used instead of near enough, approximate heuristical methods (Gigerenzer & Gaissmaier, 2011), or engineering judgment (Gainsburg, 2007, 2013). In time I would have likely moved progressively towards the centre of the engineering community had I not decided to become a mathematics and physics teacher. This decision suspended my pragmatic inclinations in favour of more theoretical pursuits and steadily increased two important contradictions in my mind.

The first concerned conceptual understanding and procedural knowledge. I discovered that conceptual understanding often took time to develop and could follow the development of procedural knowledge after reflection and discussion with others (Lamberg, 2013; Rittle-Johnson & Schneider, 2014; Sfard, 1998; Skemp, 2006). These processes worked iteratively together to eventually develop considerable growth in my theoretical knowledge, or what Engstrom (1987, 1999) calls expansive changes in conceptual learning and understanding. Also, my interactions with engineers taught me the limits of procedural knowledge; that attempts to replace conceptual understanding with procedural knowledge had only limited effectiveness, and progress in developing practical engineering projects could be limited by a corresponding lack of development in mathematical understanding. The second contradiction was between the real (being useful, in my view) and the abstract. This contradiction is widespread and is an important and recurring theme in this study. The difficulty arises because gaining greater conceptual understanding usually involves theorising, or verticalising thinking, which frequently involves greater abstract, higher-order thinking (Treffers, 1993). While various mathematical tools may have pragmatic origins, appreciating verticalisation and its connections to reality may take time to develop. This led to a strong contradiction in my mind, one shared by many engineers, that much of mathematics was useless, either because the mathematics was too abstract to be helpful in a practical sense, or because the links between abstraction and reality were not yet fully appreciated (Hernandez-Martinez & Vos, 2018; Marr & Hagston, 2007; Ridgway, 2002).

This tension appeared in my father's community of practice - both basic mathematical skills and metacognitive and higher-order thinking skills used in practical contexts were part of their discussions. The value of "critical thinking, learning to learn, planning and problem-solving" (FitzSimons, Mlcek, Hull, & Wright, 2005, p. 4), and the ability to question and reflect on one's thinking, were tacitly, even if not always explicitly, acknowledged. Listening to my father's later experiences as an instructor of toolmaking apprentices frequently gave me insights not only into the use of numeracy skills in sophisticated contexts but also of the role of social interaction in its development (Steen, 2001; van der Kooij & Strässer, 2004).

In conclusion, my father has left me with a rich heritage. The practical side of mathematics still appeals very strongly to me. In an important sense this thesis is taking me back to my roots, but this time with a wider understanding of the issues around the philosophical debates, especially the role of higher-order thinking in developing practical skills. In the next section, I relate how my motivations for doing the research influenced my researcher positioning.

1.2. **Researcher positionality**

In this section, I explain my researcher positionality and how it has influenced the outcomes of this research study. Philosophical assumptions may be consciously or unconsciously held and are influenced by issues like age, upbringing, cultural values, politics, social class, ethnicity, religion, education, and career and life experiences (see Section 1.1). Given the increasing ethnic and religious diversities in our communities (Msoroka & Amundsen, 2018), a corresponding and increasing awareness of the variety of possible interpretations of social data is becoming evident. In order to accommodate, rather than trying to eliminate the effect of these diversities of interpretation, some scholars have adopted the stance that "researchers should acknowledge and disclose their selves in their work, aiming to understand their influence on and in the research process" (Holmes, 2020, p. 3). However, carefully examining, reflecting on, and then openly stating the researcher's positionality is a long-term process, possibly without end, where the researcher may change perspectives and view the data in a renewed way with each perspectival change (Stetsenko, 2005). Such processes of continual social-individual changes are linked to the Cultural Historical Activity Theory (CHAT) framework (see Section 3.2) and activated by agency whereby people "co-create their world and themselves so that each individual person makes a difference and matters in the totality of social practices" (Stetsenko, 2020, p. 5). The focus on aspects of change and development, whether in apprentices, skilled tradespersons, or in my own perspectives, provides a unified philosophical approach to this study (see Section 2.5). Regarding my own positionality, I now discuss my philosophical assumptions, then the necessity for adopting a reflexive attitude, my attitude to the participants in this study, and the insider-outsider aspects of my relationship with them.

My worldview as a researcher contains certain beliefs and philosophical assumptions that have influenced the way I have approached this study. Many of these are attributable to the experiences and interaction with others during my childhood and schooling, and later as a university student and secondary school teacher (see Section 1.1). Positivist belief assumptions, often associated with the notion of *exactness* in the physical sciences, were important factors at school in the formation of my philosophical beliefs about intellectual endeavour and attempts to understand reality (Oliveira, 2020). Positivist assumptions were gradually abandoned in my late teens as a young mathematics and physics student. Among several issues that influenced my thinking on physics was Heisenberg's Uncertainty Principle which states there are fundamental limits to the accuracy with which values of certain pairs of physical quantities of a particle can be measured (Hawking, 2002; Heisenberg, 1927; Smolin, 2013; Young, 1992). My earlier regard for the efficacy of mathematics was also shaken by an introduction to mathematical logic where paradoxes abounded. For example, Kurt Gödel, who ironically subscribed to Platonism, theism and mind-body dualism, became famous for proving the so-called Incompleteness Theorem. This states that it is impossible using the axiomatic method to construct a mathematical system that is simultaneously complete and consistent, and that mathematical theories in any branch of mathematics cannot contain all of the truths in that branch of mathematics (Balaguer, 2021; Gödel, 1992). Such encounters left me with a profound scepticism about grand, all-encompassing theories and an emerging understanding that multiplicities of viewpoints and paradoxes would continue to abound. The abandonment of a positivist stance, however, also granted me release. As Geertz (1973a) has written, I too became unimpressed with the view that "computer engineering, or some other advanced form of thought is going to enable us to understand men without knowing them" (p. 10). In consequence, I regarded later debates on the relative merits of qualitative and quantitative research as meaningless because I had already come to terms with the duality of metaphors in the so-called *exact sciences* and the limitations of dealing with just one metaphor (Sfard, 1998).

The acceptance of a plurality of views was the result of my ongoing process and development of a reflexive approach. Reflexivity also involved interpretivism, which manifested itself in studying history and languages. As with my childhood and school experiences, my interests in history and languages still continue to influence my positionality as a researcher. According to Holmes (2020), reflexivity in research informs positionality and requires "an explicit self-consciousness and self-assessment by the researcher about their views and positions and how these might, may, or have, directly or indirectly influenced the design, execution, and interpretation of the research findings" (Holmes, 2020, p. 2). Reflexive self-assessment involves examining the preconceptions brought to the study. Those preconceptions may be far-reaching and sometimes only vaguely understood.

Positionality also overlaps with researchers "locating themselves about the participants" (Holmes, 2020, p. 3). This includes considering how the participants view themselves and appreciating the cultural values of their local and broader communities. I believe that the mechanical engineering tradespersons who participated in my research are highly skilled and worthy of respect, not only for their contribution to society but also for their understanding and practical application of highly technical matters. Although the mathematics is seldom advanced, the way they use mathematics is often highly sophisticated (Steen, 2001). This necessitates a potential change in a researcher's perspective to appreciate both the basic and the higher-order mathematical skills that engineers use in practical contexts.

As a former mathematics teacher with no direct expertise in the finer skills of mechanical engineering trades, I was to some extent an outsider in the mechanical engineering trades communities. This necessitated having a listening attitude. The researcher may have an insider (emic) or an outsider (etic) relationship with the participants of the study (Holmes, 2020). Some advantages of being an insider are that the researcher already has some knowledge and experience of the context of the study and its participants. This allows the researcher to ask insightful questions that reflect and draw out the community's understandings of the context and to write thick descriptions about them, perhaps involving interconnections between apparently unrelated issues (Drew, 2019; Geertz, 1973a, 1973b). On the other hand, the emic researcher may be too close to the community and fail to see beyond the assumptions already made about the context and, consequently, allow the participants' responses to pass without further clarification. Hence, the emic position is suited to situations where reality is viewed within a cultural relativist perspective where actions are reasonable and meaningful in that culture. The researcher actually becomes part of the research process (Holmes, 2020).

In contrast, the advantages of etic accounts are that they attempt to be culturally neutral and are written independently of culturally specific terminology or references. Hence, outsiders attempt to act as external scientific observers. Another advantage is that outsiders may ask questions, or make interconnections, that insiders with their long exposure to the context would possibly not consider. However, outsiders may not have sufficient background knowledge to probe and their insights may consequently lack depth. Moreover, outsiders' lack of understanding of the culture may lead to their interpretations of the data being greatly different from those of the members of the community being studied (Holmes, 2020).

It is important to appreciate that the insider-outsider status is dynamic and not static, and perhaps changes from moment to moment. There may be a dichotomy between the two positions and the emic-etic debate may even be regarded as a continuum (Holmes, 2020; Zhu & Bargiela-Chiappini, 2013). Hence, in my study, I was regarded by the participants as both insider and outsider. As someone who knew something of the mechanical engineering background, who could ask sufficiently deep, probing questions and who had come from a mathematics and physics background, I was partially accepted as an insider. I was also simultaneously an outsider because I was open about my need to deepen my engineering contextual knowledge. Therefore, as someone learning about engineering and also inquiring about the engineers' views at the same time I effectively became part of the research process (Holmes, 2020).

1.3. **The research questions**

There has been minimal research examining and reporting on mechanical engineering trades, the mathematics they use and how they develop their mathematical thinking. There appears to be a large literature on professional engineering, but very little on mechanical engineering trades mathematics, with none in the New Zealand context. This gives another key rationale for the project and its contribution to the field.

Therefore, the main, overarching research question of my study was:

> What key features of mathematical learning characterise the pathway from beginning apprentice to skilled tradesperson and then possibly to expert engineer in mechanical engineering?

The keywords ‘mathematical learning’ and ‘pathway’ are intended to encompass not only an investigation of the mathematics skills that are developed but also how they are applied by mechanical engineers. Mathematical learning and pathway also signal the potential inclusion of multiple influential factors that could emerge as the study progressed.

To support the main research question, three specific associated sub-questions were defined:

1. What is the nature of the mathematics skills employed in the mechanical engineering trades?

2. How do apprentices and skilled tradespersons in mechanical engineering trades apply mathematics skills in their work?

3. How do apprentices and skilled tradespersons in mechanical engineering trades develop the mathematics skills necessary for their work?

The word “skills” in the first sub-question refers both to knowledge about mathematical content topics (such as understanding number and performing calculations) and also to the ancillary skills needed for successfully employing that content knowledge in practice. These ancillary skills may not technically be strictly mathematical but may include skills such as metacognitive and higher-order thinking, working in teams, and so on.

The second sub-question sought to explore topics such as technical ingenuity and how higher-order thinking or problem solving are used in mechanical engineering contexts. Together, sub-questions one and two sought to capture the mathematical demands of the mechanical engineering trades. Therefore, these demands delineate the mathematical learning that is required of those on “the pathway from beginning apprentice to skilled tradesperson and then possibly to expert engineer in mechanical engineering” (see main research question).

The third sub-question refers to the ways the engineers *learned* and *developed* this body of mathematical and related skills. The role of the mechanical engineering context is introduced here because mathematics is applied in workplaces in different ways and from different perspectives from those at school, such as the use of innovation, creativity and imagination (FitzSimons, Mlcek, Hull, & Wright, 2005; Steen, 1990). In the next section, I discuss the interrelationships between mathematics, numeracy, and the mechanical engineering trades.

1.4. **Mathematics and numeracy**

While there is a substantial literature surrounding numeracy, there is no general agreement about its terminology, nature or definition. Thus, the terms *numeracy, mathematical literacy,* and *quantitative literac*y have been commonly used at various times and convey subtle differences in meaning. However, there is widespread agreement that numeracy is different from mathematics and that it includes more than skill in performing calculations (Anthony, 2020; FitzSimons et al., 2005; Karaali, Villafane Hernandez, & Taylor, 2016; Liljedahl, 2021; National Numeracy, 2020a, 2020b; Wright, 2007). Commenting on this difference, Steen (2001) contrasts the abstract structures of mathematics with numeracy that is "often anchored in data derived from and attached to the empirical world" (Steen, 2001, p. 5). Hence, mathematics and numeracy have important similarities and differences and engage with issues that may either overlap or be mutually exclusive.

Definitions chosen for numeracy tend to be influenced by the actualities of the context. In my study, the actualities frequently include both the numerical aspects of numeracy and a complex set of ancillary skills that govern both mathematics and numeracy usage. These ancillary skills include "authentic problem-solving in real or simulated tasks in small groups with shared responsibilities … [and] the development of metacognitive skills, such as critical thinking, learning to learn, planning and problem-solving" (FitzSimons et al., 2005, p. 4). I shall use FitzSimons' description above of metacognitive skills throughout the study. Another ancillary skill is being confident and comfortable in judging whether to use mathematics in a particular situation, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context (Coben, 2000; Tout et al., 2017).

Definitions of numeracy may be narrow or wide. Wide definitions may be helpful when focussing on numeracy skills of workers where numeracy involves much more than mastering certain basic mathematical skills, or where recognition needs to be given "to the changing socio-political landscape that positions numeracy as part of reflective, discriminating, and responsible citizenship" (Anthony, 2020, p. 352). A wide definition may also emphasize the integration of mathematics with communication, culture and personal aspects of individuals in context (T. Maguire & O'Donoghue, 2003). Therefore, applying mathematics in the workplace involves using numerical, cognitive and metacognitive skills and social interaction in context. All these ideas are pertinent and referred to throughout this study.

Throughout the study, the important place I give to the numerical aspects of numeracy is consistent with Skills Matter New Zealand which takes its definition of numeracy from OECD (2016a) as "the ability to use numerical and mathematical concepts" (p. 1) and with

Te Kete Ipurangi where numeracy is defined as “the ability to understand numbers and calculations” (Ministry of Education, 2019, n.p.). It also resonates well with FitzSimons et al.’s (2005) description of numeracy in workplaces as “the practical application of rational numbers [sic][1] and the metric measurement system with contextualised approximations and estimations in critical calculations” (p. 6). Further, specifically acknowledging the numerical aspects of numeracy reflects Dutz’s (2021) comment that “discussing and learning about numeracy is especially important in times when there is a great need not only to understand numbers and graphs, but also to think critically about figures and information” (p. 1), and the Scottish government statement that numeracy is having “the confidence and competence in using number which will allow individuals to solve problems, analyse information and make informed decisions based on calculations” (Smarter Scotland, 2021, p. 1; my emphasis).

Therefore, since numeracy is much more than the sum total of its numerical aspects, and involves a plethora of understandings and definitions in the literature (Geiger et al., 2015), for the purposes of this thesis, I will use the term *numeracy* in a broad perspective:

> ‘Numeracy is a term used to identify the knowledge and capabilities required to accommodate the mathematical demands of private and public life and to participate in society as informed, reflective, and contributing citizens’ (Geiger, Goos, & Forgasz, 2015).

‘Numeracy’ therefore refers to the application of mathematics in real-life contexts, including the workplace, and taking cognizance of its numerical, social, cognitive, metacognitive and political aspects.

Moreover, since *mathematics* and *numeracy* overlap, neither is subsumed in the other. I see aspects of both in many topics. As a result, I regard a topic such as trigonometry primarily as mathematics, but misunderstanding arising over decimal point placement in solving a trigonometry problem as a numeracy issue. Similarly, substituting in formulas involves both algebraic and numeracy understanding. Hence, I treat topics like trigonometry and substituting in formulas as mathematics topics with important numeracy connotations.

Regarding how I refer to mathematical learning, since learning about trigonometry or substituting in formulas involves both mathematical and numeracy understandings, it is simpler to describe the learning as mathematical learning, with the links to numeracy being taken as understood. Therefore, as is sometimes found in the literature, I sometimes use the terms mathematics and numeracy interchangeably in this thesis.

1.5. **Background to the mechanical engineering trades in New Zealand**

This section discusses the various specialised branches of mechanical engineering and some aspects of the background of mechanical engineers. There is a wide range of different titles

[1] The rational numbers can all be written as fractions. Their decimal equivalents either terminate or repeat. There are some important numbers, such as π and $\sqrt{2}$, which cannot be written as fractions, and which have decimals that are infinite and non-repeating. It would be more correct to say ‘real numbers’, rather than rational numbers in this case.

for the mechanical engineering trades specialisations. Competenz, the Industry Training Organisation (ITO) responsible for mechanical engineering trades in New Zealand, retains the more traditional and well-known names: fitters, fitters and turners, fitters and welders, maintenance and diagnostics engineers, maintenance fitters, toolmakers, and precision machinists. Engineers work in various settings, such as general engineering workshops as general engineers, manufacturing companies as maintenance engineers, or highly specialised precision engineering workshops as toolmakers or machining engineers (Competenz, 2013, 2021).

Statistics New Zealand allows limited public access to census data. However, their published occupation categories are constructed too broadly for the data to be used by researchers investigating particular engineering specialisation groups and access to more detailed information is expensive. Allowing free public access to detailed data would considerably enhance many studies, including this one, especially when investigating such key elements as age distribution, the likely effects of retirement on the trades' human resource needs, ethnicity, gender, school qualification, and trades qualifications. Moreover, the data from the New Zealand 2018 census were controversial (B. Edwards, 2019). The best information available to me for this study has been obtained from Competenz and based on the 2006 census:

- Around 14,520 workers across more than 50 industries identified themselves as mechanical engineers, their average age was 43 years and more than 17% were over the age of 55 years.

- Around 97% were male and 3% female. About 61% identified as New Zealand European, 13% New Zealanders, 9% Māori and 5% Pacific Peoples.

- Around 68% left secondary school with a qualification compared with 75% of the New Zealand workforce.

- Around 65% gained a post-secondary school qualification compared with 47% of the New Zealand workforce. The majority of those with post-school qualifications had gained a Level 4 certificate (63%).

(Source: Competenz, 2013)

Statistical data is important to the trades and concern has frequently been expressed in recent years over the aging of the engineering workforce and the loss of skills to New Zealand as they retire. In 2011, for example, only 12 people were training to become toolmakers (Competenz, personal communication, 2011), which prompted someone to ask "Is toolmaking a dying trade?" (Competenz, 2013, p. 13). Such concerns at least partly explain the efforts being made by Competenz and other groups to persuade young people to take up a trade (BCITO et al., 2015; Competenz, 2015, 2018; Sole, 2015).

An important issue regarding the current relevance of the 2006 census data is the likelihood of substantial shifts in the demographic data over time. Toolmaking is an example of a shift within an occupational subcategory. The ethnicity balance may also have substantially shifted with recent immigration trends in New Zealand. One point of interest, for which data is also currently not available, is the number of skilled tradespersons for whom English is a second language and the effects this might have on workplace communication, relationships, efficiency and mathematics skills.

To become a qualified tradesperson in New Zealand it is necessary to complete an appropriate apprenticeship and pass all the requisite Unit Standards (Careers New Zealand, 2015; Competenz, 2021; O'Leary, 2014; Tertiary Education Commission, 2015). An apprenticeship is an agreement and commitment between an apprentice, a trainer such as a polytechnic, and an employer who serves as a mentor. During the years of the apprenticeship, study is combined with practical work experience. Each trade requires a set number of hours of theory and practice to be completed before the apprentice can become formally qualified in that trade. The training and completion requirements are vested in the New Zealand Qualifications Authority (NZQA). The corresponding Industry Training Organization (ITO) for each trade which has various statutory obligations such as maintaining proper standards of performance and ensuring apprentices receive appropriate supervision and support (Careers New Zealand, 2015). Competenz, the ITO for mechanical engineering trades in New Zealand, offers two sets of nationally recognised qualifications, the older National Certificates and their reviewed and updated replacements, the New Zealand Certificates.

Competenz also provides teaching resources, oversees and administers the assessment, liaises with engineering departments in secondary schools, and promotes interest among young people in engineering and other trades. Apprentices have traditionally done their theory studies in block courses, night classes and by correspondence. However, Competenz has also instituted a programme of eLearning which is accessible to apprentices at any time they find convenient. The programme contains a number of practice sessions for each unit of learning which the apprentice does before gaining confidence to attempt the online assessment (Competenz, 2021).

While prospective mechanical engineering apprentices are encouraged to study drawing and design, and mathematics and science subjects in their final years at secondary school, there is no minimum entry-level standard required in mathematics to enter an apprenticeship (Competenz, 2021; Competenz: Skills for Industry, 2018). As far as mathematics for completing a mechanical engineering apprenticeship is concerned, the current qualification required in most specialisation branches is *Unit Standard 21905: Demonstrate knowledge of trade calculations and units for mechanical engineering trades* (NZQA, 2010). There is considerable overlap between the mathematics content required for mechanical engineering apprenticeships and the NCEA Level 1 Achievement Standards involving Number and Measurement - *Achievement Standard 91026: Apply numeric reasoning in solving problems* (NZQA, 2019a) and *Achievement Standard 91030: Apply measurement in solving problems* (NZQA, 2019b). A similar overlap exists between US 21905 and the mechanics section of the

NCEA Level 2 physics curriculum, *Achievement Standard 91171: Demonstrate understanding of mechanics* (NZQA, 2011). Thus, most of the NZQA documentation that details the relevant mathematical skills and numeracy goals needed for this study can be found in AS 91026, AS 91030, AS 91171 and US 21905, together with their associated supporting documents.

It is important to note that, while the mathematics and numeracy requirements for the mechanical engineering trades are contained in US 21905, not all of the topics in this Unit Standard are used extensively in all the mechanical engineering trades specialisation branches; each branch has its own emphases and ways of doing things. Toolmaking, for example, requires fine measurement, often with tolerances of thousandths of a millimetre, which is not usually required in an area such as sheet metal working. Sheet metal working, however, has its own challenges in problem solving that are not necessarily encountered often in toolmaking. It is not uncommon for apprentices to express an inclination to eventually specialise in one particular branch. Moreover, many mechanical engineering tradespersons appear to move quite freely from one branch specialisation to another throughout their careers. My father, for example, was apprenticed as a fitter and turner, then became a ship's engineer, then a maintenance engineer, then a toolmaker, before becoming an instructor of toolmaking apprentices.

1.6. **The New Zealand mathematics curriculum**

This section is included to help international readers understand how references throughout this study to the New Zealand education system might correspond to those in their own countries. New Zealand children typically begin school at Year 0 aged about five years. There is a national curriculum, the New Zealand Curriculum (NZC) that defines several Learning Areas, one of which is Mathematics and Statistics (Ministry of Education, 2007) for all thirteen years of schooling. Most students advance to Year 11 aged about fifteen years when many will enter a nationally organised assessment system, known as the National Certificate for Educational Achievement (NCEA). The NCEA comprises three levels, Level 1 in Year 11, Level 2 in Year 12 and Level 3 in Year 13. Each NCEA level has various Achievement Standards attached to the Learning Areas, and these are either internally or externally assessed, or both. Throughout the thesis, I will refer to senior secondary school mathematics as the more academically-oriented New Zealand Year 12 and Year 13 mathematics courses students might study during their final years of secondary schooling before entering university.

The mathematics required for almost all mechanical engineering trades applications is part of NCEA Level 1: AS 91026 *Apply numeric reasoning in solving problems* that pertains to numeracy, and Achievement Standards in geometry, measurement, Pythagoras and trigonometry (NZQA, 2019a, 2019b, 2019c, 2019d). The focus of learning is on real-life contexts.

The topics for AS 91026 are the following:

> Reason with linear proportions; use prime numbers, common key elements and multiples, and powers (including square roots); understand operations on fractions, decimals, percentages, and integers; use rates and ratios; know commonly used fraction, decimal, and percentage conversions; know and apply standard form, significant figures, rounding, and decimal place value; apply direct and inverse relationships with linear proportion; extend powers to include integers and fractions, and apply everyday compounding rates. Students are expected to be familiar with methods related to ratio and proportion; key elements, multiples, powers and roots; integer and fractional powers applied to numbers; fractions, decimals and percentages; rates; rounding with decimal places and significant figures, and standard form (NZQA, 2019a, n.p.) (see Appendix B).

These topics relate very closely to the mechanical engineering trades requirements (NZQA, 2010) which are discussed further later (see Appendix C and Section 2.1.1). Given the international character of numeracy and engineering, the New Zealand numeracy topics are likely to be similar to those of other countries, for example, the Scottish system which includes "estimation and rounding; number and number processes; fractions, decimal fractions and percentages; money; time; measurement; data and analysis, and ideas of chance and uncertainty" (Smarter Scotland, 2021, p. 1).

The Program for International Student Assessment (PISA) studies are also relevant to the mathematical needs of mechanical engineering trades apprentices. PISA claims to be "not only the world's most comprehensive and reliable indicator of students' capabilities, it is also a powerful tool that countries and economies can use to fine-tune their education policies…" (Schleicher, 2019, p. 2). PISA studies draw attention to numeracy performance in a global context and give insights into New Zealand trends. While PISA is not perfect (K. Mills, 2014), I believe it has accurately traced long-term declines in numeracy performance in New Zealand and some other countries. Presently, young people are less prepared to meet the mathematical demands of trades such as mechanical engineering than they were prior to the 1990s (May, Flockton, & Kirkham, 2017). As a result, my study focuses on the nature of numeracy and mathematical processes that engineers use, their links to problem solving, and how they use, develop, learn and transmit those numeracy and mathematical tools.

1.7. **Mathematics and mechanical engineering trades**

This section discusses the nature of mechanical engineering trades mathematics and other factors that are important to its application in the trades context. This thesis focuses on two key features of the pathway of mathematical learning that take place during and beyond the apprenticeship years: how skilled tradespersons use mathematics in their work, and how they develop those mathematical skills as they progress to become expert engineers near the centre of Lave and Wenger's (1991) community of practice. It is important to appreciate that the mathematical components of an apprentice's pathway are quite different from those intending to become professional engineers. In particular, professional engineers need much higher cognitive skills in algebra and calculus in their senior school mathematics courses (Alpers, 2010; Alpers et al., 2015; Dubibsky, 1994; Gravemeijer & Doorman, 1999; Holtzapple & Reece, 2008; Kaput & Roschelle, 1997).

The lack of more senior secondary school mathematics in trades training preparation does not mean that trades mathematics applications are unsophisticated. Two important key factors are involved. First, applying mathematics in the workplace requires well-developed fluency with numbers (see Section 2.1.1) (Atkinson & Mayo, 2010; Henderson & Broadbridge, 2009; Lomas & Mills, 2013a, 2013b; K. Mills, 2011, 2012; Steedman, 1997) (see Section 1.3). Numeracy is therefore an important theme throughout this thesis since many of the applications of mathematics in mechanical engineering trades involve calculation and other numeracy skills (see Section 2.2). Second, many studies have demonstrated that numeracy in the workplace requires more than basic number skills, such as personal and social qualities, and higher-order thinking. Examples of such studies include chemical spraying (FitzSimons et al., 2005); nursing (Coben & Weeks, 2014; Galligan, 2011; Hutton et al., 2010); paramedicine (Bell, Galligan, & Latham, 2020); professional engineering (Berkaliev & Kloosterman, 2009; Carr et al., 2014; Deans, 1999; Gainsburg, 2006, 2007), and boat building (Zevenbergen & Zevenbergen, 2009).

Personal and social qualities are sometimes referred to as Key Competencies in the New Zealand school curriculum and include skills needed for working cooperatively in teams. These exist in various forms in many countries (Hipkins, 2007; Ministry of Education, 2005; OECD, 2009b). Another area comprises problem solving, planning, critical thinking, creativity, conceptual understanding, employing engineering judgment in decision-making processes, and the development of metacognitive skills. This study investigated the interactions of these multifarious factors in the successful application of mathematics in mechanical engineering trades workplaces. The study should therefore have implications nationally and internationally for mechanical engineering trades workplaces as well as other vocations.

1.8. **Rationale**

Following the discussion in the last section of the nature and importance of mathematics and numeracy in the mechanical engineering trades, I now justify the study. Three potential contributions to the academic literature will be discussed: mathematics in mechanical engineering workplaces; correspondences between mathematics in mechanical engineering trades and other vocations and workplaces, and adult numeracy.

There appears to be a worldwide paucity of knowledge of the training of apprentices in mechanical engineering trades workplaces. I mention four important exceptions: first, a study by Akor, bin Subari, binti Jambari, bin Noordin, and Onyilo (2019) on Nigerian engineering apprentices which calls for a greater emphasis on “critical thinking, innovation and creativity, problem solving, teamwork, life-long learning, and communication skills” (p. 1279), second, a study by Audu (2014) on the employability skills of graduates from a Nigerian engineering trades programme, third, a study by Audu, bin Kamin, bin Musta’amal, and bin Saud (2014) on comparing the efficacy of various teaching methods with the acquisition of practical mechanical engineering skills in Nigeria, and fourth, a study on the mathematical needs of engineering apprentices in the United Kingdom by Ridgway (2002). However, among these four studies, only Ridgway's study focuses specifically on the role and usage of mathematics and numeracy.

The meagre literature on engineering trades mathematics is in contrast to the large literature on mathematics in professional engineering workplaces (Alpers, 2010; Engelbrecht, Bergsten, & Kågesten, 2017; Gainsburg, 2006, 2013; Harlim, 2014; Horowitz, 1999; Sobek & Jain, 2004; van der Wal, Bakker, & Drijvers, 2017). There is also a large workplace mathematics literature on trades' areas such as chemical spraying (FitzSimons et al., 2005), boat building and pre-apprenticeship plumbing (LaCroix, 2010, 2014; Zevenbergen & Zevenbergen, 2009). These latter studies provide strong indications of the importance of both the social and technical aspects of workplace mathematics. Given the clear shortage of literature on mechanical engineering trades mathematics, the present study offers an original contribution to the role played by mathematics in the mechanical engineering trades workplaces.

There are three areas where the present study is likely to provide insights and understandings of mechanical engineering trades of interest internationally and in New Zealand. First, one focus of the present study is on the nature of the mathematics knowledge and skills in mechanical engineering trades contexts. This extends existing literature because, while finding the skills is relatively straightforward (NZQA, 2010; Ridgway, 2002), little is known about the ways the knowledge and skills are used in the workplace. Second, the study focuses on the ancillary skills needed to apply mathematics in workplace contexts. While studies like FitzSimons et al. (2005) on chemical sprayers acknowledge the importance of cooperating in small groups with "shared responsibilities … [and] the development of metacognitive skills, such as critical thinking, learning to learn, planning and problem-solving" (p. 5), little is known about how these skills are used in mechanical engineering workplaces. Third, there are few studies on how learning takes place in the trades area. A notable exception is Wake's (2014) study of apprentice locomotive drivers, although even here the connection with mechanical engineering is not direct. The present study contributes to this area through the third research sub-question, which includes an examination of the metacognitive processes and socio-constructivist aspects involved in mechanical engineering trades training and skilled practice, and how learning takes place.

My study contributes knowledge to address these three significant gaps in our understanding and is likely to help apprentices, engineers and expert engineers to develop these skills beyond their current levels to become more effective skilled tradespersons or expert mechanical engineers. While this thesis focuses on the mechanical engineering trades, vocational numeracy concerns are not limited to the trades areas but are spread internationally across all levels of society, vocations and workplaces, including university students intending to become professional engineers and doctors (FitzSimons et al., 2005; Henderson & Broadbridge, 2009; Hoyles, Noss, Kent, & Bakker, 2010, 2013; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; Marr & Hagston, 2007; Parsons, 2008; Satherley, 2012; Tariq, 2002). However, while workplace applications of mathematics may differ, their understandings are often transferable across vocations, which suggests that a study of the role of mathematics in mechanical engineering trades may possess similarities to those in other workplace contexts (see Section 1.3). Therefore, lessons learned here may have applications in a wide range of different vocations and workplaces internationally.

It is possible that this study on mathematics and numeracy in mechanical engineering trades could have wider significance in adult literacy and numeracy. Despite its specific focus on the workplace, it could contribute to the developing literature on numeracy skills worldwide, similar to the *Adult Literacy and Lifeskills Survey* (ALL), the *Programme for the International Assessment of Adult Competencies* (PIAAC) survey and other studies (Alkema & Rean, 2013; Carnevale, 2013; Coben, Miller-Reilly, Satherley, & Earle, 2016; Earle, 2013, 2015; Jonas, 2018; Jones & Satherley, 2017; Lane, 2010; Marr & Hagston, 2007; OECD, 2012a; Satherley, 2012, 2014; Satherley & Lawes, 2009).

Finally, there are two issues where this study may contribute to the academic literature. First, this study focuses on participation in communities of practice that may be of relevance to adults in their everyday life and workplace experiences (S. Harris & Shelswell, 2005). Second, this study may contribute to our understanding of the formal and informal ways that engineers develop and communicate their knowledge and skills. This includes how their skills are learned both socially and through personal reflection, how basic skills are applied in subtle and sophisticated contexts using creativity and imagination (Steen, 1990, 2001), and how individual and group problem-solving skills contribute to the workplace and other environments (Jonas, 2018; Ministry of Education and Ministry of Business Innovation and Employment, 2016; OECD, 2012b, 2013, 2016b; Satherley, 2014; Tertiary Education Commission, 2008).

1.9. **Overview of the thesis**

Following on from this introduction, Chapter 2 provides a critical review of the literature. The review highlights the importance of numeracy to the study and its relationship to the mechanical engineering trades context. It also discusses the important themes of conceptual understanding and procedural knowledge, higher-order thinking skills and the ways mathematics is learned formally and informally. Chapter 3 develops the methodological considerations of the study such as the interpretivist paradigm, mixed-methods research, the CHAT framework and the study's associated activity systems, the methods used to conduct the research, how the data were analysed, and ethical considerations. Chapter 4 focuses on presenting and interpreting the findings from the questionnaires and the semi-structured interviews regarding mathematics knowledge and skills. This chapter includes their important applications in sophisticated contexts that require problem solving, creativity and flexible thinking, extended reasoning, and integrating multiple skills. Chapter 5 presents the findings regarding the way mathematics skills and knowledge are learned through childhood, school and workplace experiences. I provide formal and informal learning examples of social interactions and report the influence of modern technology, such as electronic calculators and computer software. Chapter 6 contains the discussion of the findings about the nature of the skills, how they are applied in context, and how they are developed both formally and informally. Finally, Chapter 7 draws conclusions about the study, provides a discussion of the contribution the study makes to new knowledge, outlines its implications and limitations, and presents suggestions for future research.

1.10. **Chapter summary**

In this introduction chapter, I have expressed my personal motivations and reasons for doing this research, alongside how I see my researcher positionality influencing the study. The main research question was introduced and then followed up by clearly defining key terms for this study like *numeracy* and *quantitative literacy*. To provide background and context for this study, I then gave some background of the mechanical engineering trades and the mathematics curriculum in New Zealand. Finally, to provide a justification for this study, I indicated the worldwide paucity of knowledge of the role and usage of mathematics and numeracy in mechanical engineering trades workplaces that are high users of mathematics and numeracy.

Chapter 2. **Literature Review**

Introduction

This chapter evaluates the literature which substantially influences mathematics and numeracy in mechanical engineering trades. It is aligned with the research questions of the study which focus on the various associated areas that impinge upon the pathway from beginning apprentice to skilled mechanical engineering tradesperson. There are three main areas of interest outlined in the research questions: the nature of the mathematical knowledge and skills in engineering trades, how the mathematical knowledge and skills are applied, and how the knowledge and skills are developed and learned (see Section 1.3). These areas will be followed throughout the thesis and notably, here, in this literature review chapter.

Contextual and ancillary issues are important to the ways mathematics is used in mechanical engineering trades workplaces. Therefore, I begin in Section 2.1 by analysing international studies of numeracy in society as a whole and in particular, the literature related to the data concerning adult and young adult numeracy skills in New Zealand from the Programme for International Student Assessment (PISA), the Adult Literacy and Life Skills Surveys (ALL), and the Programme for International Assessment of Adult Competencies (PIAAC). Physics provides important contexts for applying mathematics in mechanical engineering trades workplaces, especially mechanics. Therefore, Section 2.2 examines the links between school mathematics and physics curricula regarding the mathematics and mechanics topics used for mechanical engineering trades in New Zealand. In Section 2.3, I focus on ancillary and higher-order skills such as conceptual understanding and problem solving because these are substantial influences when applying mathematics in mechanical engineering trades contexts. This leads to an investigation of the role played by higher-order skills in the Science, Technology, Engineering and Mathematics (STEM) programme that has been designed to encourage more young people to enter technological vocations.

Concerning developing mathematics skills for mechanical engineering trades, Section 2.4 discusses various modern theories and controversies of how mathematics should be taught and learned. This is because the ways mathematics skills are developed in mechanical engineering trades have been influenced by historical debates over mathematics education philosophy. Finally, Section 2.5 discusses the influence of Situated Learning (SL), which is an important and widespread part of apprentice learning. SL also provides a framework to investigate how collaboration and communication, problem solving and conceptual understanding enhance the engineering workplace context. Section 2.6 summarizes the chapter.

At the outset, it is important to acknowledge the dearth of literature on mathematics in mechanical engineering trades contexts. However, there is a very large, diverse range of academic literature on mathematics in workplaces. Some studies feature mathematics and numeracy in specific vocations, such as chemical spraying, cabinetmaking and boatbuilding (FitzSimons et al., 2005; Saló i Nevado & Pehkonen, 2018; Zevenbergen & Zevenbergen, 2009). These studies are valuable for the contributions they make to understanding those particular workplaces and some broad principles of workplace mathematics. There is also a plethora of studies on mathematics and STEM, such as the contributions made by mathematics researchers to STEM education (Anderson, English, Fitzallen, & Symons, 2020; Anthony, 2020), the role of "big ideas" in STEM (Chalmers, Carter, Cooper, & Nason, 2017, p. S25) and the role of mathematics in interdisciplinary STEM education (Maass, Geiger, Ariza, & Goos, 2019). I have located one PhD dissertation on vocationally-oriented mathematics tasks given to secondary school students following vocational courses. Although it does not focus on mechanical engineering trades, it does contain a section on technical and industrial production (Sundtjønn, 2021). A Google search for "mathematics+mechanical engineering+trades" yields many results. One result is entitled "Mechanical Engineering Education: Not Just About the Math" (Foroudastan & Saxby, 2004), but unfortunately the mechanical engineering is at the professional level, not trades as in my study. The same applies to a STEM study of higher-order thinking skills in senior high school students, which despite its title, is not oriented to the trades area (Subia, Marcos, Pascual, Tomas, & Liangco, 2020).

To summarize, I have been unable to locate more than a few scholarly articles, and no doctoral studies, that relate specifically to mechanical engineering trades mathematics contexts. Therefore, in important areas that influence this study, such as problem solving, I have referred to the extensive literature on professional engineering contexts.

2.1. **Mathematics and numeracy in New Zealand society**

I begin with a discussion of mathematics and numeracy from a societal perspective. Mechanical engineering trades are among the highest users of mathematics and numeracy (OECD, 2016a, 2016b). While all the mathematics and numeracy skills they use can be found in other vocations, they are almost unique in the breadth of skills they use, the way they use them, and the variety of their contextual applications. Initial consideration of mathematics and numeracy from a societal perspective is therefore helpful in understanding mathematics and numeracy in mechanical engineering trades.

2.1.1. **Lack of numeracy skills and their economic consequences**

Numeracy impinges on most aspects of our personal lives and particularly on our ability to perform efficiently in the workplace. Most western governments now recognise this and in recent decades efforts have been made to enhance both school and post-school numeracy education (Bynner & Parsons, 2006; Evans, 2000; FitzSimons, Coben, & O'Donoghue, 2003; Kane, Patel, & Rawiri, 2006; Martin & Hunter, 2021; Satherley & Lawes, 2009; Voss, Lynch, & Herbert, 2021; Wedege & Evans, 2006). However, the general levels of adult numeracy continue to remain low at a time when numeracy skills are becoming increasingly important in the workplace and for everyday living.

This problem exists in many countries. Concerning life skills, David Blunkett, when Minister of Education in the United Kingdom, described the fact that seven million adults in England lacked even basic numeracy skills as a "silent scandal" (Coben et al., 2003, p. 36). This situation does not appear to have improved in the meantime. According to Westwood (2021), data from a survey by the Organisation for Economic Cooperation and Development (OECD) "suggests that England is the only country in the developed world where the older generation approaching retirement is *more numerate* than younger adults" (p. 67). The situation in New Zealand regarding numeracy skills is similar, with the PISA survey of 2012 reporting that 23% of young people were unable to show competencies to enable them to participate actively in mathematics-related life situations (May, Cowles, & Lamy, 2013).

Low numeracy skills have consequences for both society and its individual members. While only anecdotal evidence of the current numeracy skill levels in the mechanical engineering trades in New Zealand is available, the literature at a societal level indicates that low numeracy skill levels reduce the potential of workers to contribute to the economy and they may require more training. Reduced financial rewards, career advancement and job satisfaction are important disadvantages at an individual level (Grotlüschen, Mallows, Reder, & Sabatini, 2016; OECD, 2016a, 2016b). However, mitigating these consequences requires more than developing basic skills. For example, Carnevale (2013) writes of a changing situation where critical thinking, problem solving and other higher-level skills are needed for most workers and not only senior management, and Skagerlund, Lind, Strömbäck, Tinghög, and Västfjäll (2018) trace numeracy and emotional attitudes towards numbers as impediments to financial literacy development. In response to these concerns, some universities have begun to include more broad-based, numeracy-oriented courses alongside formal university programmes in mathematics, especially for liberal arts students (Lovric, 2017).

Low mathematics skills have implications for workplace vocations. Concerns about insufficient mathematics skills have been expressed in the United States (Atkinson & Mayo, 2010; Wu & Atkinson, 2017), Australia (Henderson & Broadbridge, 2009), the United Kingdom (Office for Standards in Education, 2011; Steedman, 1997), and in New Zealand (Martin & Hunter, 2021; Radford, 2012). In the United Kingdom universities, Tariq (2002) reports the results of a mathematics and numeracy entrance test for university biology students where "a high proportion … (42 - 63%) encountered difficulties with … questions that required an understanding of fractions, indices, logarithms, or units of measurement [and that] only 6% of students answered all 15 questions correctly" (p. 76). Regarding medical students, in a United States study, Sheridan and Pignone (2002) reported deficiencies in medical students' ability to interpret numeric data and probabilities. The problem still appears to exist because a recent survey of medical students' dosing calculation skills recommended that their "student training and assessment should include both extraction of embedded dosage information from guidelines and use of the equipment used in dosing" (Harries & Botha, 2021, p. 487).

Concerns about low levels of mathematics and numeracy attainment were expressed in the debates during the latter half of the twentieth century on how mathematics should be taught in schools and workplaces. These issues have extensive literature, some of which focuses on workplace numeracy and literacy in low paid work (Higgins, 2016). There were many reasons why these debates took place. For example, the loss of labouring jobs to mechanisation led to apprentices remaining longer at school, which led in turn to attempts to make mathematics more easily understood and more relevant to their needs, and consequently more motivating to them.

Before discussing those debates in greater detail, I discuss another debate that has been taking place for many years, particularly involving trades, concerning broad versus minimal mathematics skills. Minimalism holds that sufficient workplace mathematics learning can be achieved solely by cultivating skills directly related to the specific context of the trade itself. This view seeks to avoid having to relate learning to broad principles that are regarded as abstract, and therefore, unreal and irrelevant. The debate manifests itself in the attitude that school mathematics is useless and that things change once apprentices are out on the job and in the real world (Marr & Hagston, 2007; Steen, 2001). The case for a broad mathematics education to prepare students for the flexibility of thinking required in the workplace has been made by Ridgway (2002). Reporting on the mathematical needs of engineering apprentices, Ridgway observed that the mathematical challenges of engineering differed from the mathematics taught in school, especially in the demand for great precision and the need to do a good deal of practical problem solving. Most importantly, concerning predicting future success in the trade, the conventional measures of educational attainment had high predictive validity, whereas a test created to sample the mathematical skills directly involved in engineering had low predictive validity. Ridgway concluded that

> high-level skills required for a successful educational career generalise to practical work, whilst the acquisition of mathematical technique does not …, that ‘basic skills’ are not a foundation but rather are a component of mathematical education …, and [that] practising the deployment of [a broad range of] skills in a range of contexts should be encouraged (p. 189)

Despite the debate recurring from time to time, greater emphasis has been placed in more recent decades on high-level skills for workplace application rather than trade-specific training. This has been accompanied by an ever-widening interest in such issues as what constitutes a good school (OECD, 2020), gender (OECD, 2019b; Zeldin & Pajares, 2000), ethnic; gender; and socio-economic equity (Easton, 2013; Mackay, Fawcett, & Cadzow, 2018; OECD, 2019b), mathematics avoidance (Hoffman, 2010), mathematics anxiety (Dunkels, 1995; Frankcom-Burgess, 2017), self-efficacy (Bandura, 1994, 2012; Bandura, Barbaranelli, Caprara, & Pastorelli, 2001; Hekimoglu & Kittrell, 2010; Zimmerman, Bandura, & Martinez-Pons, 1992), and the dispositions of adult learners to education (FitzSimons, 2002a, 2002b; Zevenbergen, 2011; Zevenbergen & Zevenbergen, 2004). In short, mathematics education at school and in the workplace has become strongly influenced by key social and economic elements.

2.1.2. **Numeracy skills of children, young adults and adults in New Zealand**

From a governmental standpoint, international surveys of mathematics provide insights into the way governments might plan official policy regarding programmes of mathematical learning. New Zealand primary school students take part in the Trends in International Mathematics and Science Survey (TIMSS) and secondary school pupils take part in the PISA studies.

The PISA studies are directly relevant because they focus on the mathematical literacy levels of fifteen-year-old secondary school students and provide an indication of the likely proportions of people having numeracy and problem solving skills that will enable them to begin apprenticeships. The mathematical literacy levels of beginning mechanical engineering apprentices is unknown, even from the PISA studies. However, due to the spiral approach of New Zealand education, trends in primary school attainment are likely to compound into secondary school performance and then into the workplace. Therefore, in this section, I examine the contributions of TIMSS, PISA, ALL and the more recent PIAAC surveys to adult numeracy in workplaces.

2.1.2.1. **The TIMSS surveys of primary school students**

The International Mathematics and Science Survey (TIMSS) targets mathematics attainment levels of students at Grade 4 and Grade 8 levels in the United States (Years 4 and 8 in Australia, and Years 5 and 9 in New Zealand and the United Kingdom). Concerns have recently been expressed in the media in New Zealand about trends in the TIMSS results which show declining mathematics performances of both Grade 4 and Grade 8 students (Collins, 2020a; Sutcliffe, Marshall, Rendall, & Medina, 2021). I first consider the 2019 advanced, intermediate and low benchmark scores for Grade 4 students for the United States, Australia, England, Ireland, and New Zealand (see Table **1**).

Table 1 *TIMSS results for benchmarks in the 2019 survey, Grade 4 students*

Cumulative percentage benchmarks for five countries in TIMSS 2019 survey, Grade 4 students				
Country	Advanced Benchmark Score (625)	High Benchmark Score (550)	Intermediate Benchmark Score (475)	Low Benchmark Score (400)
United States	14	38	66	87
Australia	11	36	68	90
England	11	35	69	90
Ireland	7	38	76	94
New Zealand	6	22	53	82

(Mullis, Martin, Foy, Kelly, & Fishbein, 2020, p. 175)

These countries might be considered similar to New Zealand. The table represents cumulative scores. Therefore, 14% of United States students were advanced compared with 6% of New Zealand students. Similarly, 76% of Irish students were intermediate *or above* compared with 53% in New Zealand. Comparing the New Zealand percentages for each benchmark level with those of other countries, it can be seen that New Zealand students are significantly behind all the other countries even at Grade 4 level in their schooling.

I now consider the TIMSS data for Grade 8 students' performance in mathematics over four-yearly intervals from 2003 to 2019. The average scores for the same five countries are shown in Figure 1. New Zealand Grade 8 (Year 9) students' mathematics scores are below all of the other countries in each of the five surveys. Also, New Zealand students have shown a long-term decline in their TIMSS average mathematics scores while each of the other four countries has shown long-term increases. This has resulted in the gap between Year 9 students in New Zealand and the other four countries increasing.

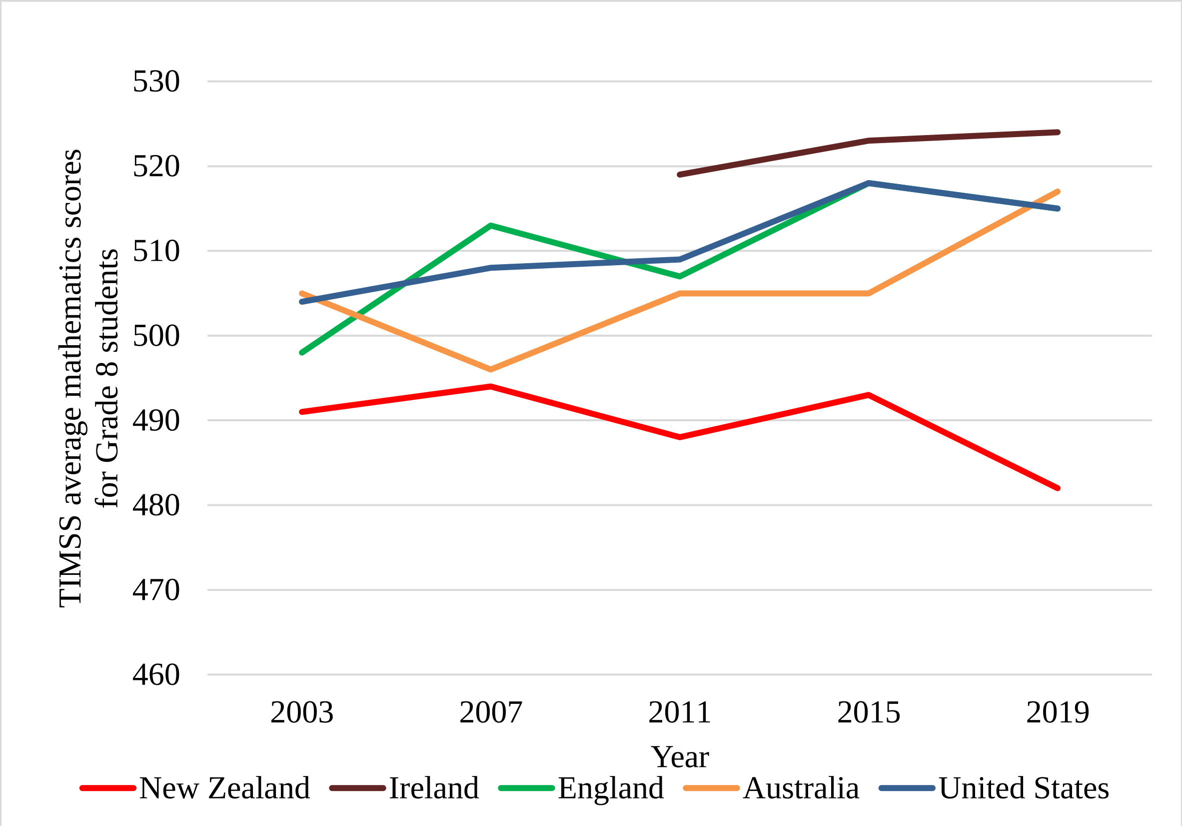

Figure 1 Average TIMSS scores for Grade 8 students in five countries, 2003 - 2019

Note. Data obtained from The International Mathematics and Science Survey 2019 (Mullis et al., 2020, pp. 160-163)

The primary school years are foundational in developing numeracy and mathematical skills. Since mathematics learning is cumulative with one level of knowledge and understanding being built on another, the TIMSS surveys which are taken several years before young people enter the workplace nevertheless give an early indication of limitations to future learning. In particular, the Grade 8 results suggest that mathematics learning in New Zealand secondary schools and later in the mechanical engineering trades may be adversely affected. In the next section, I focus attention on the growth of mathematical skills during the secondary school years.

2.1.2.2. **The PISA surveys of secondary school students**

PISA is an international assessment programme for secondary school students. The results are important to this study because the PISA surveys measure numeracy skills with test items that reflect both real-life contexts and problem solving at a time shortly before young people will leave school and take up apprentice training. Problem solving is important in the mechanical engineering trades context. Almost 6200 fifteen-year-old New Zealand students took part in the 2018 PISA survey (May, Jang-Jones, & McGregor, 2019; Medina & Sutcliffe, 2020). Two earlier New Zealand reports specifically related to the PISA studies are the *PISA 2009: Our 21st Century Learners at Age 15* (Telford & May, 2010) and *PISA 2012 Summary Report* (May et al., 2013). These reports enable trends in mathematics performance of New Zealand fifteen-year-olds to be established.

PISA Mathematics Literacy scores are published as Levels on a 1 to 6 scale, with Level 6 representing the highest achievement. In the *PISA 2012 Summary Report*, several tables of data were devoted to low-achieving students who were defined to be those who performed below Level 2 in Mathematics Literacy. Level 2 is the baseline at which students are considered to begin to show competencies that will enable them to participate actively in “mathematics-related life situations” (May et al., 2013, p. 10). Students below Level 2 can complete only relatively basic mathematical tasks and their lack of skills is a barrier to learning. These students have probably yet to develop numeracy skills likely to be suitable for entry into a mechanical engineering apprenticeship. In New Zealand, 23% of students in 2012 performed at below Level 2, almost one quarter, the same as the OECD average (May et al., 2013).

With respect to the long-term trends reported from the PISA assessment in 2015,

> the change in average score since 2003 for New Zealand reflects a larger proportion of New Zealand students performing below Level 2. … In 2015, 22% of New Zealand students were below Level 2 compared with 15% in 2003, (May et al., 2017, p. 22).

There has been a steady decline in PISA mathematics proficiency in New Zealand between 2003 and 2018 across all levels, though the downward trend seems to have stabilised from 2015. The percentage of those performing below Level 3 has increased from 34% to 45%, and the percentage of Level 5 or Level 6 students who can perform at the highest mathematical level is 9% below the 2003 figure (see Figure **2**).

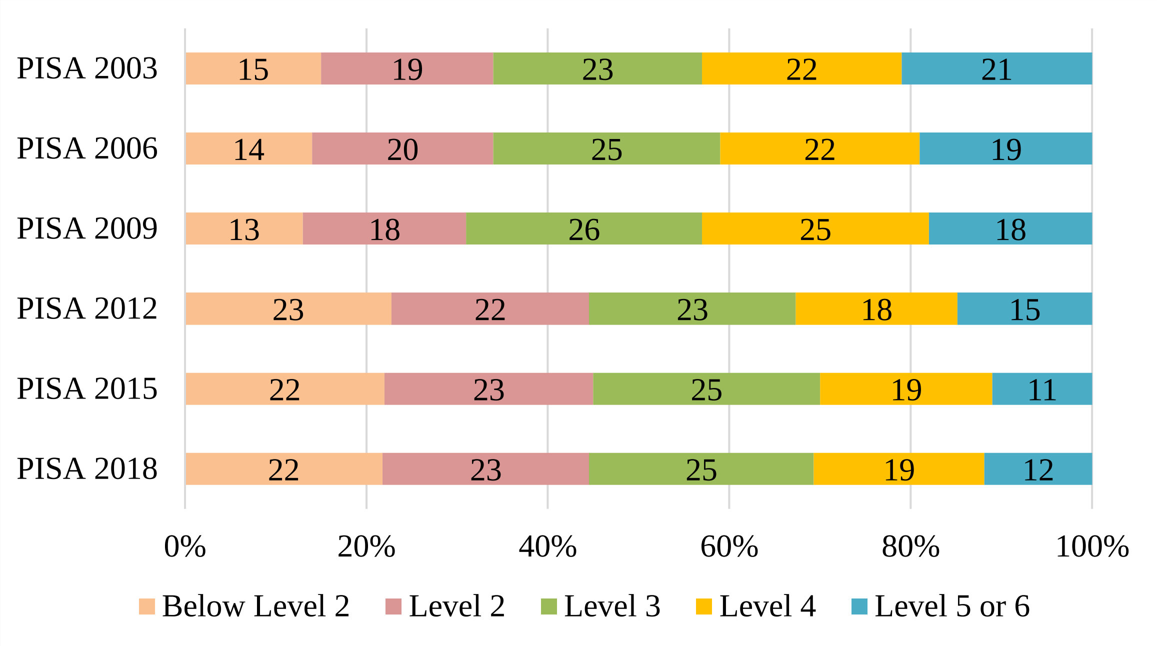

Figure 2 Percentage of New Zealand students achieving at PISA mathematics proficiency levels, 2006–2018

Note. Figure used under permission from Creative Commons 3.0 BY, New Zealand Ministry of Education (May et al., 2017, p. 23; May et al., 2019, p. 15).

It is possible that some PISA Level 2 students may have numeracy skills suitable to begin a mechanical engineering apprenticeship, even though Level 2 understanding seems to be confined to whole numbers. However, since PISA Level 3 is more closely aligned with the mathematical skills outlined in a required assessment known as Unit Standard 21905 (see Section 2.2), Level 3 may therefore be a more suitable PISA indicator of mathematics skills needed by apprentice engineers. According to Kelly et al. (2013), this is because

> Level 3 students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning (p. 3).

In New Zealand, 45% of students in 2012 performed below Level 3, and these students presumably demonstrated few, if any, of these skills (May et al., 2013). The results for the PISA 2018 cycle were released in December 2019 (OECD, 2019a). The OECD publication outlining what students know and can do commented that New Zealand's performance in mathematics had been steadily declining from 2003 to 2018 from initially high levels of performance (OECD, 2019a). Moreover, the proportion of top-performing students in mathematics (scoring at Level 5 or 6) decreased in mathematics between 2012 and 2018.

Since this study is concerned mainly with the proportion of students likely to have mathematical skills suitable for entry into an engineering trades apprenticeship, the statistic of most interest is the percentage of students who scored Level 3 or higher in mathematics. Between 2003 and 2018 the proportion of New Zealand students who scored PISA Level 3 and above in mathematics declined from 66% to 55%; a difference of 11%. While the proportions of prospective mechanical engineering apprentices scoring Level 3 or higher in mathematics are not known for either 2003 or 2018, trades apprentices tend to be drawn from those whose mathematical skills are near the middle of the proficiency range. Therefore, it is likely that there is a smaller proportion of current prospective apprentices who have the necessary PISA Level 3 mathematics skills to begin mechanical engineering apprenticeship training.

Two points need to be considered concerning the use of PISA data for 15-year-olds as indicators of apprentices' numeracy skills when they begin their trades apprenticeships after Year 12 at school in New Zealand, at perhaps 17 years of age. First, it is possible that further PISA Level 3 numeracy skills could be developed in the interim. Second, Year 12 mathematics courses contain no numeracy as such. We are left to consider Steen's (2001) comments that "more mathematics does not necessarily lead to increased numeracy" (p. 108), and that when students follow mathematics courses, seldom do they "gain parallel experience in applying quantitative skills in subtle and sophisticated ways" (p. 108). Steen concludes that "mathematics and numeracy should be complementary aspects of the school curriculum" (p. 108).

Consequently, indications from the declining TIMSS and PISA scores and other sources have led to public concern about school mathematics and the setting up of a commission by the Royal Society of New Zealand to make recommendations for change (Collins, 2020a, 2020b; Royal Society Te Apārangi, 2021). From the viewpoint of this study, the long-term declines in TIMSS and PISA scores strongly suggest that young people beginning mechanical engineering trades apprenticeships are less well prepared in terms of mathematics, numeracy and problem solving in 2018 than in 2003. I next consider the ALL and PIAAC surveys of mathematics needs and attainment related to workplaces.

2.1.2.3. **The Adult Literacy and Life Skills survey**

The 2006 ALL survey studied the numeracy demands of New Zealand workplaces and reported attainment results on a 1 to 5 scale. While the workplace activities may now have a greater orientation to Information Technology than in 2006, Satherley (2012) believes the activities "are very likely to be still relevant in a wide range of workplaces" (p. 3). He displays graphical information of the frequency of the use of numeracy in the workplace for several vocational groups that show that tradespersons are the highest of the vocation groups for measuring and weighing things, and second in counting, reading numbers, or keeping track of things (Satherley, 2012). Satherley also presents overall statistics for the frequency of numeracy activities at work by different occupation groups, where tradespersons occupy second place. More recent studies have shown that the need for numeracy is a long-term and growing phenomenon with important ramifications for the workplace (Jones & Satherley, 2018; OECD, 2016a, 2016b; Redmer & Dannath, 2020). Moreover, numeracy skills are important for mechanical engineering tradespersons because compared with some other trades vocations, mechanical engineers use a wide range of mathematical skills with high frequency (Satherley, 2012).

2.1.2.4. **The Programme for the International Assessment of Adult Competencies survey**

The PIAAC survey conducted in New Zealand from 1 April 2014 to 31 March 2015 targeted some 6177 adults aged 16 - 65 years old. Like PISA, PIAAC focuses on both real-life contexts and problem solving. In contrast to PISA, the PIAAC test items focus directly on workplace quantitative situations that are often more specialised than those in everyday life. Representative of these are "completing purchase orders; totalling receipts; calculating change; managing schedules, budgets and project resources; using spreadsheets; organising and packing different shaped goods; completing and interpreting control charts; making and recording measurements; reading blueprints; tracking expenditures; predicting costs, and applying formulas" (OECD, 2012b, p. 35). PIAAC reports its results in levels where PIAAC Level 2 numeracy skills include "the application of two or more steps or processes involving calculation with whole numbers and common decimals, percents and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables and graphs" (OECD, 2016a, p. 18). Unfortunately, PIAAC uses a scale of 1 to 5 for reporting attainment levels, while PISA uses a scale of 1 to 6. According to Gal and Tout (2014), different reporting levels are just one of several difficulties that hinder comparing PISA and PIAAC numeracy scores.

However, PIAAC also surveys adults' attainments in problem solving in technology-rich environments (OECD, 2016a). The New Zealand data revealed that:

- 4.9% of adults indicated that they had no prior experience with computers or lacked basic computer skills, one-third the size of the OECD average (14.7%)
- 45.3% scored at or below Level 1 in problem solving in technology-rich environments, slightly above the OECD average (42.9%)
- around one in three adults (34.0%) attained proficiency Level 2 in problem solving compared with the OECD average of 25.7%
- 10% were proficient at Level 3, the highest proficiency level for problem solving in the technology-rich environments survey; this figure being the largest proportion of adults scoring at this level among all participating countries and almost twice as large as the OECD average of 5.4%.

Based on (OECD, 2016a, p. 2)

Regarding the two highest levels of problem-solving proficiency (Level 2 or 3) in 2016, the top five countries were "New Zealand (44.2%), Sweden (44.0%), Finland (41.6%), the

Netherlands (41.5%) and Norway (41.0%)" (OECD, 2016b, p. 54). Therefore, it would seem that New Zealand adults are among the most technologically aware people in the world, and this may have relevance to the mechanical engineering trades and the attitudes of young apprentices, in particular, to new technology.

However, international rankings can mask important features. In this case, while New Zealand's international rankings in the 2014 PIAAC study were high, about a third of the working-age population was assessed as having overall skill Levels 1 and 2 (Alkema, 2020; Coben & Earle, 2014).

2.1.3. **Section summary**

In this section, I have demonstrated that there is widespread concern about inadequate numeracy levels in many countries, especially basic number skills. TIMSS conducts international surveys of primary school age students and the PISA studies assess numeracy levels in fifteen-year-olds in many OECD countries. PISA uses test items that focus on contextually based, everyday scenarios that often require multiple mathematical skills. The test items are therefore suitable to investigate higher-order skills such as problem solving. The TIMSS and PISA studies reviewed in this section show there has been a steady, long-term decline in numeracy levels of young people in New Zealand and some other countries. The PISA data, in particular, suggests apprentices may not be as well prepared in mathematical skills to enter the workplace compared with 2003. Therefore, the full ramifications for the training of apprentices are unknown, but may include factors such as the time taken, the extra physical and human resources needed, and the inefficient use of resources which results in both short-term and long-term economic loss.

Numeracy is an important issue for mechanical engineering tradespersons because of their high ranking as numeracy users. While the statistics reviewed in this section do not provide conclusive evidence of numeracy deficiencies among mechanical engineering tradespersons, they are consistent with the view that conceptual difficulties understanding basic number skills like percentages, fractions, ratios and decimal place value are lacking in students coming out of high school in New Zealand (Lenz, Dreher, Holzäpfel, & Wittmann, 2020; K. Mills, 2011; Resnick, Rinne, Barbieri, & Jordan, 2018). These statistics also suggest that a sizeable proportion of mechanical engineering tradespersons may be below PISA Level 3 in mathematics. Finally, PIAAC also assesses adult problem solving in technology-rich environments, which are directly related to mechanical engineering trades skills such as Computer Aided Design (CAD) and Computer Numeric Control (CNC). Technology-rich environments are further discussed in connection with problem solving in Section 2.3.

2.2. **Mathematics knowledge and skills**

This section compares school and workplace mathematics and reviews the New Zealand mechanical engineering trades topics and their assessment.

2.2.1. **School and workplace mathematics compared**

The differences in philosophy and approach between school and workplace mathematics have been noted frequently (Harth & Hemker, 2013; Herheim & Kacerja, 2019; Hoyles et al., 2013; K. Mills, 2011). I shall refer to the differences as a school and workplace mathematics tension. In engineering trades, the tension manifests in mathematical challenges that "differ from the mathematics taught in school. In particular, great precision is required, applied to a variety of mathematical techniques; a good deal of practical problem-solving is necessary, too" (Ridgway, 2002, p. 189).

Mathematics application in mechanical engineering trades workplaces bears similarities with chemical spraying. The calculations may be mathematically straightforward, but the way they are done differs according to conditions, such as temperature and humidity (FitzSimons & Boistrup, 2017; FitzSimons et al., 2005). Moreover, the calculations must be completely accurate and checked for both calculation errors and for choosing the right method. Hence,

> learning in the workplace differs from school mathematics education in that workers are always reminded to check their calculations for reasonableness, to ask repeatedly if they are not sure, and to consider their own and others' personal safety (FitzSimons et al., 2005, p. 16).

Concerning how knowledge and processes for making calculations in chemical spraying are accumulated, it appears that experience and practicing are involved; a process of embedding knowledge in ongoing practices and repeated, if necessary, until the particular competence is fully acquired (FitzSimons et al., 2005). This knowledge is directed towards specific and immediate goals that are relevant to life contexts (Bernstein, 2000). FitzSimons et al. (2005) find a theoretical framework for this method of learning in Bernstein's concepts of vertical and horizontal discourses. Vertical discourses are coherent, explicit and systematically principled while horizontal discourses are everyday or common sense (Bernstein, 1999; FitzSimons & Boistrup, 2017). Distinctions can therefore be drawn between rote learning and learning by practicing. Parallels can also be drawn between learning by practicing and a (possibly) behaviourist teaching approach that is integrated with socially constructed knowledge gained on-the-job (K. Mills, 2011). How these discourses are worked out in the mechanical engineering trades context is currently unknown.

Another school and workplace tension involved electronic calculators and computer technology. Calculators made their introduction into New Zealand primary and secondary schools several decades ago. Many people were sceptical of their introduction at the time, suggesting that their use would lead to poorer performance in mental calculation skills. In the intervening years, calculators have become broadly accepted, as is reflected in the NZQA policies for their use in assessment (NZQA, 2013, 2019d). Many other electronic calculation and graphical aids have been developed and are used extensively. These include smartphones, CAD and CNC in engineering, as well as sophisticated statistical analyses done by SPSS computer software in the social sciences. All of these software programmes are like "black boxes", where the mathematics is hidden and unknown to most users (Black & Wiliam, 1998; Guidotti et al., 2019; Williams & Wake, 2007).

Moreover, as educational software has become more sophisticated, so has experimentation with its use as a teaching device with diverse groups of students. For example, Calder and Campbell (2014, 2016) studied how using technologies in real-life contexts was likely to interest and motivate young people. Their study reported on two aspects: Māori and Pasifika students, and reluctant learners. Both aspects of the study reported enhanced learning involvement and attainment. Thus, electronic calculating technology may be an effective means of promoting learning for calculating, problem solving, and developing understanding in mechanical engineering trades contexts.

However, while modern calculation technology provides release from time-consuming mental calculation tasks that increase the likelihood of errors, it is not a substitute for mental calculation skills, especially in some vocations. On that account, from a STEM perspective, "We must be able to do mental calculations in the absence of pencil and paper; further, we must be able to calculate with pencil and paper, in the absence of a calculator" (Ochkov, 2020, p. xvii). An Australian study reveals that paramedics need to do mental calculations in emergency situations with perfect accuracy and without the use of calculators (Bell et al., 2020). Such skills are pertinent to mechanical engineering trades workplaces where mental calculations need to be done more quickly than can be done on calculators and where sometimes calculators may not be available. Therefore, there is a place for both modern calculation technology and profound understanding of fundamental mathematics (PUFM) with its emphasis on mental skills. Calculator and non-calculator components are needed in both school and workplace contexts (Dabell, 2018; Daher & Baya'a, 2009; Ma & Kessel, 2001; Roble, Tandog, & Maglipong, 2017; Tandog, Roble, Maglipong, & Luna, 2019). The roles that both mental skills and modern calculation technology play in the mechanical engineering trades is currently unknown.

A proposal for resolving the school and workplace mathematics tension and the difficulties it poses for many people has been made by Grootenboer, Edwards-Groves, and Kemmis (2019), who suggest that the school mathematics curriculum should be reconceptualised and its primacy located in practices. Their argument is framed around the core purpose of education; to help people "live well in a world worth living in" (p. 1). Thus Grootenboer et al. (2019, p. 1) state that:

> Living well and learning about what this means is typically guided by epistemologically based curricula, and conversely, school curricula determine the substance of education. We argue that this understanding of education is too narrow, and as a consequence, it severs the relationship between knowing and practising. We propose that a curriculum of mathematical practices is required for human flourishing, where the focus is on mathematical practices rather than predominantly on knowledge.

While the authors express the hope that a practice approach to mathematics curriculum might better equip individuals and societies to respond to conditions that disrupt our everyday circumstances, such as Covid pandemics, it might also help apprentices and tradespersons adjust to new situations as they occur in workplaces.

2.2.2. **The mechanical engineering trades mathematics topics**

Investigating the requisite official government documents is an initial step in identifying the mathematics and numeracy requirements for mechanical engineering trades. These are contained in the NZQA Unit Standard *US 21905 Engineering core skills - Demonstrate knowledge of trade calculations and units for mechanical engineering trades*. The Unit Standard is currently delivered by Competenz through eLearning. Competenz is an official organisation that develops and assesses national trades qualifications throughout New Zealand. The summary below outlines the mathematics topics and their contextual applications to be studied to complete engineering trades qualifications (see Appendix C):

- Arithmetic and algebraic operations for mechanical engineering
- Trigonometry
- Tables and graphs in mechanical engineering
- Define and apply quantities and units of measure in a mechanical engineering environment

These topics closely parallel the New Zealand secondary school mathematics NCEA Achievement Standards AS 91026 and AS 91030. Similarly, the mechanics requirements for mechanical engineering trades overlap with the *Level 2 Physics Achievement Standard 2.4: AS 91171 Demonstrate understanding of mechanics*, which contains the topics of motion, force, and momentum and energy. Indeed, many of the applications of mathematics in mechanical engineering are motivated by physics contexts, which have conceptual difficulties for students at least as challenging as those in mathematics. Therefore, physics and mechanics are part of the apprentices' mathematical learning because they provide contexts to apply mathematics in the mechanical engineering trades (Ates & Cataloglua, 2007; McDermott, 1984; Saiman Mat & Puji Wahyuningsih, 2017).

2.2.3. **Assessment of mechanical engineering trades mathematics**

The assessment of mathematics for mechanical engineering tradespersons in New Zealand includes numeracy and mathematics skills, such as calculating and understanding decimal place value and practical problems related to engineering contexts. However, the assessment regime has limitations. Although it can be sat online, the assessment format is nevertheless equivalent to traditional pencil-and-paper testing because there is no attempt to integrate the mathematical skills with the practical skills needed in the engineering workplace. The assessment does not guarantee the successful application of numeracy skills in daily engineering practice. The lack of satisfactory numeracy skills among apprentices has been shown to be a major source of concern and frustration to mechanical engineering trades' educators (K. Mills, 2011, 2012; K. Mills & Lomas, 2013). The assessment also does not include higher-order skills or the ability to use mathematical skills in a team situation. Moreover, US 21905 is a one-off performance in an assessment that is recognised permanently and does not ever need to be updated.

While there appears to be no academic literature regarding the assessment of US 21905, there is substantial literature critiquing the efficacy of assessment systems in general. For example, the PISA studies of fifteen-year-olds have been criticised for assessing only pencil-and-paper skills and lacking assessment of important societal skills such as entrepreneurship (Duru-Bellat, 2011; Fuhrmann & Beckmann-Dierkes, 2011; Helsingin Sanoma, 2007; Kreiner, 2013; Kreiner & Christensen, 2014; K. Mills, 2014; Stewart, 2013; Zhao & Meyer, 2013).

One attempt to make assessment more realistic and related to actual workplace conditions is provided by nursing. Among the vocations that are high users of mathematics (see Section 1.8), nursing and paramedicine seem similar to mechanical engineering trades both in the breadth of the mathematical content required and in their multi-faceted applications, including stringent requirements for accuracy (Bell et al., 2020). A study of how various aspects of mathematical education might be brought together and carried out in the nursing workplace is described in ‘Meeting the mathematical demands of the safety-critical workplace: Medication dosage calculation problem-solving for nursing’ (Coben & Weeks, 2014). They devised a model for competence in medication dosage calculation problem solving (see Figure **3**). The model recognises three competencies regarding medication dosage calculation problem solving - conceptual, calculation, and technical measurement (Coben & Weeks, 2016; Weeks, Clochesy, Hutton, & Moseley, 2013; Weeks, Hutton, Coben, Clochesy, & Pontin, 2013). Conceptual competence involves the correct interpretation of the medication dosage calculation problem and accurately setting up dosage and rate equations. Calculation competence involves the correct calculation of accurate numerical values for the medication dose and its rate of administration. Technical measurement competence involves the selection of appropriate measurement vehicles and the accurate measurement of the dose and rate of administration. Satisfactory performance in all three competencies is necessary when considering safety. Given the importance of conceptual understanding, calculation accuracy and the use of physical tools and instruments in engineering, it is possible that a similar model for satisfactory calculation practice could well serve the needs of the mechanical engineering trades (see Section 7.4.1.2).

The apparently widely accepted view in the numeracy literature of the desirability of focusing on a wider range of skills than just calculations calls for assessing skills in their practical context (Anthony, 2020; FitzSimons et al., 2005). A more holistic picture of students’ abilities is required that could include actively taking part in cooperative activities in small groups involving problem solving, and employing metacognitive skills such as the ability to be creative, critical and self-reflective in situations requiring the use of mathematics. Moreover, such a system would be compatible with the aims of promoting equity and social justice and allowing individuals to be empowered.

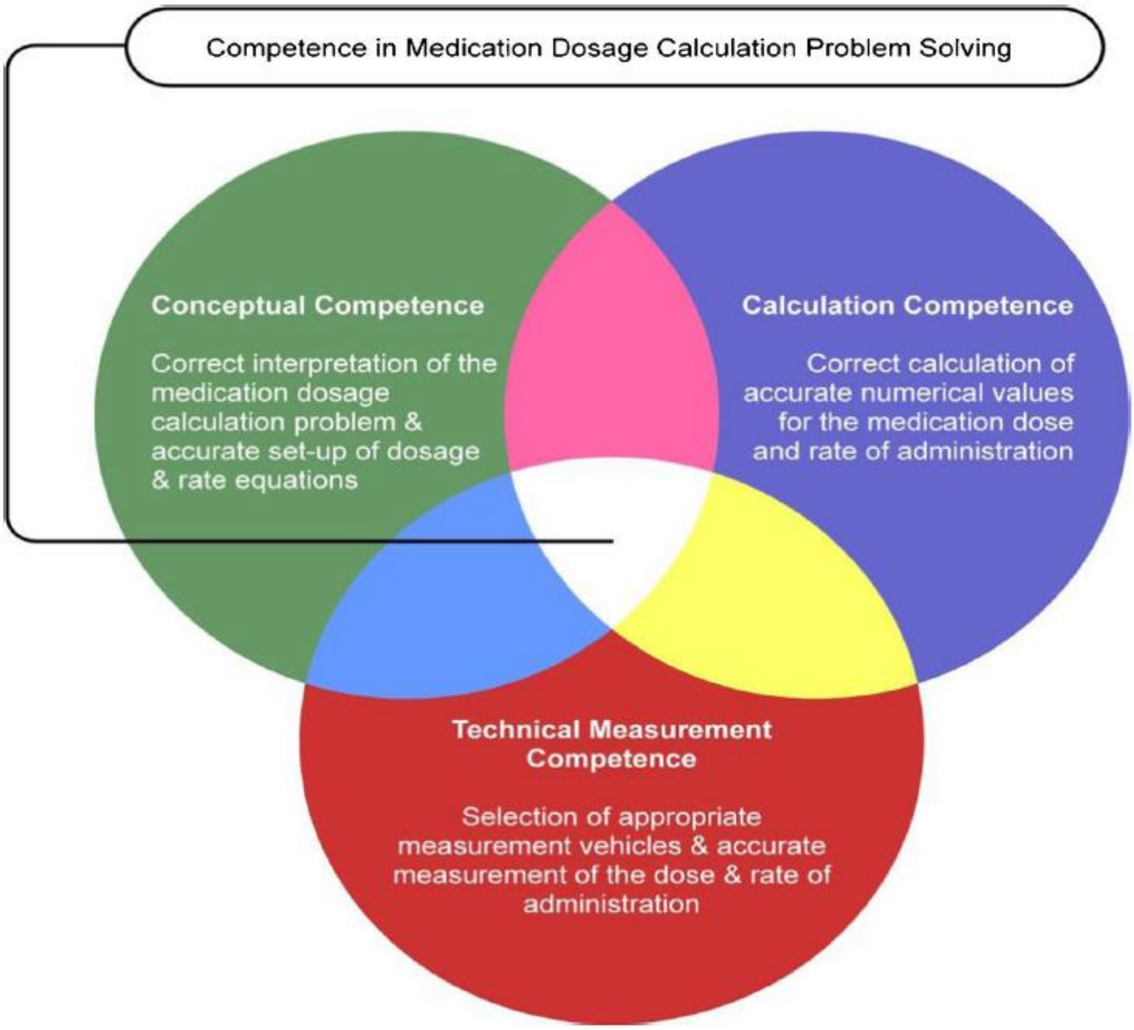

Figure 3 Competence in medication dosage calculation problem-solving

Note. Figure used with permission from Springer Nature (Coben & Weeks, 2014, p. 262)[2]

2.2.4. **Section summary**

Analysis of official government documents showed that school mathematics and numeracy topics related well to those required in the mechanical engineering trades. However, apprentices needed to make an adjustment to meet workplace mathematics and numeracy requirements. In addition, the mathematics assessment systems in engineering situations relied on pencil-and-paper tests, without recourse to practical, higher-order, social and physical considerations. Hence, there is a need to integrate mathematics and numeracy skills with their wider mechanical engineering trades contexts.

[2] Permission applied for but not yet received

2.3. **Ancillary and higher-order skills**

This section continues the review of the literature about the mathematics knowledge and skills that are required in the mechanical engineering trades. In Section 2.1, the societal effects of mathematics and numeracy were considered, and in Section 2.2 there was a focus on school mathematics and physics curricula relating to mechanical engineering trades. In this section, the literature concerning ancillary and higher-order skills is reviewed for its connections to the relevance and application of mathematical skills in the mechanical engineering trades. Higher-order skills are important in the mechanical engineering trades because they are needed to analyse complex information, establish connections between ideas, and solve problems. Therefore, they go beyond basic knowledge which simply recalls facts, and are connected closely to conceptual understanding and critical thinking, creativity and extended reasoning. Problem solving with its associated skills of creativity and extended reasoning is especially important in the mechanical engineering trades.

2.3.1. **Conceptual understanding**

This section deals with a fundamental issue in mechanical engineering: the role of conceptual understanding (J. Mills & Treagust, 2003; Sobek & Jain, 2004). While many mechanical engineering tasks require the application of standard routine procedures, there are also the needs for:

- flexible thinking to make multiple decisions, such as what mathematics to use, and how to access the necessary information
- choosing an appropriate method to calculate a numerical answer
- interpreting the answer, and applying the answer in the context of the problem with an appropriate number of decimal places
- communicating the results to other people

(Coben, 2000; OECD, 2012b; Star, Rittle-Johnson, Durkin, Shero, & Sommer, 2020).

Fundamental to the successful completion of these tasks is the issue of conceptual understanding of the numeracy and mathematical context, and also of the subtle complexities of the engineering ethos, which Gainsburg (2007) refers to variously as the engineering disposition, using engineering judgment, and having an attitude of “skeptical reverence” (p. 477).

Concerns about numeracy issues in the workplace often focus on the inability of people to perform particular numeracy procedures. This sometimes ignores the important issue of the tension between school numeracy and real life (Roth, 2010; Steen, 1990, 2001). For example, the use of averages in the workplace is often more complicated than the simple average at school, as with Wake’s (2014) apprentice locomotive drivers who tried to find train stopping distances using gradients of the railway tracks. The apprentices needed to consider the different lengths of each slope of railway track and therefore needed to apply the concept of weighted averages; something they had not encountered at school. Procedural knowledge alone was not sufficient for the apprentices to find the stopping distances, and success in solving the problem could not be addressed until the conceptual framework had first been established. Therefore, the apprentices required all three competencies - conceptual, calculation and technical measurement - to become successful locomotive drivers (see Coben & Weeks's (2014) model depicted earlier in Figure 3).

The example of Wake's locomotive drivers illustrates three important workplace issues with numeracy: procedural knowledge, conceptual understanding, and the different ways of applying numeracy in the workplace. While views emerging from the workplace tend to focus on the lack of numeracy skills and ignorance of numeracy procedures, the lack of conceptual understanding is often a more fundamental cause (Engelbrecht, Bergsten, & Kågesten, 2009; Engelbrecht et al., 2017). Some authors suggest that the development of conceptual understanding and procedural knowledge are an iterative process (Devlin, 2007; Rittle-Johnson & Schneider, 2014; Rittle-Johnson, Siegler, & Alibali, 2001). This runs counter to some on-the-job training (OJT) approaches where it is assumed that the apprentice can be shown "how to do" the mathematics and learns solely by observing and then repeating the procedure. It seems then that procedural knowledge alone can only go so far before conceptual difficulties prevent further progress in learning.

2.3.2. **The nature of problem solving and creativity in engineering**

Having identified problem solving as a key theme in mathematics curricula and in other important areas of our lives (see Section 2.1 and Section 2.2), this section discusses the nature of problem solving and creativity, how they interact, and the fundamental role each plays in mechanical engineering trades workplaces.

First, the importance of problem solving is now recognised by the OECD in PISA and PIAAC surveys, where the initial focus is on the types of problems people encounter when using information and communication technologies, such as obtaining information by searching websites (Ministry of Education, 2017; OECD, 2016a, 2019a, 2019c; Redmer & Dannath, 2020). The numeracy practices in the workplace that PIAAC focuses on include: reading financial statements, diagrams, maps or schematics; calculating costs or budgets; using and calculating fractions and percentages; using calculators; preparing charts, graphs or tables, and using simple algebra or formulae. New Zealand is consistently ranked in the top four placings alongside Australia, Finland and the United States as countries with the highest frequencies of social engagements at work, problem solving at work, and using numeracy at work (Jonas, 2018; OECD, 2019c).

Second, according to Pillaya (1998), employers in a wide variety of vocations now require higher-order thinking skills in all their employees to enable them to cope with the changing demands of the workplace. Regarding professional mechanical engineering, there is widespread agreement in the literature that problem solving and creativity skills are necessary. This perception is expressed in statements like: "solving open-ended problems is arguably the cornerstone of the engineering endeavour" (Sobek & Jain, 2004, p. 1), and engineers are "hired, retained, and rewarded for solving problems" (Jonassen, Strobel, & Lee, 2013, p. 139). On the other hand, regarding *trades engineering*, the literature on problem solving and creativity is sparse. However, in a previous study one toolmaker told me that you do not know "what's coming in the door next … [and there's] no formula that pops into your brain straight away… [so you have to] sit down and think about a way of doing it ... [you have to find] a method" (K. Mills, 2011, p. 46). Therefore, problem solving in an engineering context incorporates many key cognitive, cross-disciplinary and collaborative elements. Hence, "workplace problems often have conflicting goals, multiple solution methods, non-engineering success standards, non-engineering constraints, unanticipated problems, distributed knowledge, and collaborative activity that rely on multiple forms of problem representation" (Jonassen et al., 2013, p. 148). Taken as a whole, scholars are in agreement that the engineering community views problem-solving skills as vital to their work. Therefore, given its importance, the next section focuses on the development of problem solving (see Section 2.4).

Three important thinkers, de Bono, Elkjaer and Pólya, have advocated problem solving and creative thinking to be more broadly included in mathematics education. De Bono (1969) did this to counter what he saw as creative and independent thinking being stifled by the formal education system (Elkjaer, 2018). While Pólya's (1945) examples sometimes seem more in keeping with more abstract contexts favoured by mathematicians, some engineers have found his thinking to have important practical applications (e.g., Horowitz, 1999). Pólya commented on the controversial question of how far removed from the student's current life experience the problem had to be for it to be considered a discovery, as opposed to recalling and applying some similar problem the student had seen previously or had been solved by someone else. In a widely acclaimed classic, *How to solve it: A new aspect of mathematical method*, Pólya (1945) wrote that if you find the solution to some problem by your own means, then you may "experience the tension and enjoy the triumph of discovery" (p. v). Therefore, it is not the originality of the solution to a problem that is important, but rather, the ongoing inculcation of a discovery and problem-solving mentality in students and workers. This finding in turn carries the mindset to powerfully motivate learning and innovation in the workplace.

Encouragement to build up a problem-solving attitude is found in PISA, where students are to "analyse, reason and communicate ideas effectively as they pose, formulate, solve and interpret mathematical problems" (OECD, 2009b, p. 105). Such exercises in discovery are intended to build up greater insight and a "fruitful set of techniques" (Confrey & Kazak, 2006, p. 307). The New Zealand Curriculum recognises this but adds extra recognition of the influence of a creativity component to problem solving. Therefore, it calls for thinking to involve "creative, critical, and metacognitive processes to make sense of information, experiences and ideas" (Ministry of Education, 2007, p. 12).

In engineering and in other contexts, the path to a solution may have many imaginative and ingenious methods of solution, some of which may be quite radical. These solutions arise from many individuals sitting down and thinking about ways of doing things or finding their own method, as with the toolmaker above, and ending up with multiple acceptable possibilities (K. Mills, 2011). However, "with learning and practice, some activities that were initially experienced as problem-solving may become routine activities" (OECD, 2009a, p. 7). What is unfamiliar, and therefore counts as problem solving for one engineer, may be routine for another engineer who has seen the problem before (or one like it) and who may perhaps have memorised a way to solve it. Therefore, there is a conflict between genuine problem solving and tricks of the trade. With experience, engineers develop a repertoire of techniques, or even more powerfully, classes of techniques, which they can quickly draw on and decide which approach might best lead to a solution. Ideally, the test for genuine creativity and problem solving should involve the engineer being confronted by "cross-disciplinary situations where the solution path is not immediately obvious" (Kolovou, van den Heuvel-Panhuizen, & Bakker, 2009, p. 35).

Perhaps paradoxically, in an engineering context, when problem solving becomes solving a problem, or vice versa, there is a transition between levels of thinking and therefore a verticalisation process, which is not a prime consideration for engineers (see Section 2.4.3). Earlier, the engineer may have needed higher-order thinking skills to think through a solution but now the development and memorisation through experience have made the problem instrumental, even if full conceptual understanding had not been attained (see Section 2.4.2). This experience can now be taught to others to add to their repertoire of experiences, and as long as they can recall the appropriate method of solution from their repertoire, they too need no more than instrumental knowledge to solve such problems in the future. However, there is a difficulty here in determining which process is taking place - instrumental thinking or higher-order thinking. Without questioning the engineer in detail, it cannot be ascertained which approach they have used (Ernest, 1989; Kolovou et al., 2009). Therefore, an engineer who successfully solves a problem may be young, inexperienced, but very creative. Alternatively, the engineer may be speaking from many years of experience. Without further questioning, it is simply not possible to tell.

Regarding engineering contexts, there are debates about how some engineers acquire problem-solving and creative abilities and how they can be taught to others. The work of Pólya and de Bono was mentioned in connection with this (see also Section 2.4.4). While de Bono's writings and ideas on lateral thinking are widely acknowledged and applied, for example, in the business world (de Bono, 2013), Pólya's influence has tended to be less well known to the general public. This may perhaps be a result of Pólya's examples often appearing obviously mathematical in both context and notation, and reminiscent of school mathematics textbook problems of an earlier age (Pólya, 1945). However, in more recent times, creativity has become acknowledged in engineering with direct links made to the writings of de Bono and Pólya (Adams, Stefan, Picton, & Demian, 2008; M. Othman & Bamasood, 2021; Sharp, 1991).

Similarly, in a PhD thesis on creative problem-solving in professional engineering design, Horowitz (1999) acknowledges both Pólya and de Bono as being crucial to the development of problem solving. In particular, according to Horowitz, Pólya produced pioneering work when he offered his “four-stage process for solving mathematical problems and puzzles: understanding the problem; devising a plan; carrying out the plan, and, finally looking back” (1999, p. 5). In addition, Pólya demonstrated that this process could be applied in areas previously thought not to be amenable to methodical treatment. Horowitz also acknowledged the work of de Bono in lateral thinking, distinguishing between routine thinking where thoughts are allowed to drift in existing channels and creative thinking where thoughts are directed or when they accidentally drift laterally across channels. When drifting across channels occurs, it results in what is frequently called “surprising ideas” (Horowitz, 1999, p. 15). Therefore, often quite abstract ideas eventually come to find acceptance in practical activity.

Regarding stimulating creativity among engineers, stating the problem is easier than defining possible solutions. Therefore, Horowitz (1999) states that

> engineers are expected to be creative, but most of them seldom are. The fact that innovative engineering products appear almost on a daily basis is due to the fact that companies employ very few highly creative engineers and inventors that ‘do the thinking’ while the others are occupied in routine engineering (p. 11)

There have been formal attempts to teach problem solving to engineers, as there have been in schools. However, workplace engineering problems differ greatly from the kinds of problems that engineering students most often solve in the classroom, so learning to solve classroom problems does not necessarily prepare engineering students to solve workplace problems (Adams et al., 2008; Jonassen et al., 2013; Sharp, 1991). This is one further confirmation of the school and workplace tension. Similarly, attempts have been made to systematise the problem-solving process. One such system comprises six steps: (1) Identification; (2) Synthesis; (3) Analysis; (4) Application; (5) Comprehension; and (6) Solution. Steps 2 to 5 are “Optional Iterations” (Holtzapple & Reece, 2008, p. 88), meaning that the problem-solving process can be interrupted and its strategies altered if a person finds the solution not suited to the context of their problem, or they wish to consider other possibilities or models. Reductionism is also identified as being important, where the problem is split into separate parts that will be integrated holistically into the final solution, but which can be dealt with separately and independently in finding the solution.

In many vocations in more recent times, especially in the mechanical engineering trades, problem solving requires a large range of tools and artefacts. These include information gathering, design, and calculation resources, including computer technology, such as spreadsheets, and design software, such as CAD and CNC. The technology is constantly being developed, requiring the professional development of workers (Bzymek, Vahidi, & Spottiswoode, 2007; Engineering Technology Group, 2018). However, as remarked earlier in this section, an essential ingredient of problem solving is that it is impossible to achieve the goal through routine actions alone. If a person has to often solve the same problem or a similar type of problem, then it becomes part of their stock of routine activities, perhaps even a trick of the trade. Therefore, the boundary line between problem solving and routine activity becomes blurred, and the satisfaction disappears that might otherwise have resulted from challenging novel contexts requiring mental stimulation and higher-order thinking.

2.3.3. **Learning problem solving and creativity in engineering trades contexts**

Higher-order skills such as problem solving and creativity are closely connected with knowledge and learning as important intellectual tools in mechanical engineering trades contexts. Therefore, knowledge can be regarded as a tool that combines data and information, expert opinion, skills, and experience to aid decision making. Learning, however, refers to the way knowledge and understanding are generated, so that a learning community is “skilled at identifying, creating, storing, sharing, and using knowledge, and then modifying its behaviour to reflect new knowledge” (Serrat, 2010, p. 1059). Therefore, increasing knowledge levels in one area may have the effect of increasing the ability to learn in other areas, especially when problem-solving is involved. This may partly explain why mature engineers are so adamant about the need to know certain facts which they regard as precursors to conceptual understanding.

2.3.4. **Science, Technology, Engineering and Mathematics (STEM)**

The recognition of the need for higher-order thinking skills in the workplace has influenced school mathematics education programmes. For example, Anthony and Walshaw (2009a) write of effective pedagogical practices intended to enhance mathematics learning outcomes for a diverse range of students that will help them in their individual lives. The STEM programmes focus on developing higher-order thinking skills of students, especially in their later years of secondary schooling. STEM has resulted from a debate over falling student numbers in science, technology, engineering and mathematics subject areas, particularly of females (Attard, Grootenboer, Attard, & Laird, 2020; Klymchuk & Thomas, 2020; Osman, 2020; Owen, 2018; Struthers & Strachan, 2019). However, a contrary view about the shortage has been put forward by Xue and Larson (2015), who claim that there is no overall shortage, but rather an oversupply of graduates in academia and an undersupply entering industry.

STEM emphasizes higher-order thinking skills through its use of concepts and procedures obtained from mathematics and science in problem-solving situations to promote creativity in technological design processes. Its approach also incorporates social elements of teamwork in developing creativity (Attard et al., 2020). However, while there is an extensive literature regarding STEM and future professional engineers, STEM is also aimed at future tradespersons for which the literature appears to be much less. Some authors have introduced objectives into STEM outside of STEM’s academic area. For example, Bennison and Geiger (2020) believe numeracy across the curriculum may integrate mathematical and scientific concepts, while Kohen and Orenstein (2021) believe STEM’s use of authentic real-world problems may reflect the applied nature of mathematics which is not prevalent in formal secondary school settings. Others have added other components, such as improving the personal scientific literacy of citizens, enhancing international economic competitiveness and links with business, and laying an essential foundation for responsible citizenship, equity and social justice, including the ethical custodianship of our planet (Maass et al., 2019). The primary viewpoint of this study which focuses on engineering trades, therefore aligns well with STEM which also supports a broad range of objectives, such as the principles of dialogue and communities of inquiry, and interconnections with other disciplines, especially science (Anthony, 2020; Attard, Edwards-Groves, & Grootenboer, 2018; Maass & Engeln, 2019; Maass et al., 2019).

STEM's emphasis on both technological and ancillary skills in the workplace also aligns well with this study. In a study by Anderson et al. (2020), engineers were found to generally value solving problems, learning, and working in a team more than other aspects of their jobs. They also saw clear communication as the most important skill. Similarly, Li and Schoenfeld (2019) emphasized the integrative and problem-solving side of engineering versus engineering as a mathematical discipline, and Fan and Yu (2017) concurred with the importance of the integration of concepts and higher-order skills, especially in engineering design. They claimed there is a disconnect between school mathematics and school knowledge, yet both conceptual understanding and procedural knowledge were necessary.

STEM programmes have also been found to be effective in promoting student interest in following a technological career (Roberts et al., 2018). With regard to the effectiveness of the STEM programme, in a meta-analysis of studies Zeng, Yao, Gu, and Przybylski (2018) claim to have demonstrated the effectiveness of STEM's teaching methods over other methods in improving higher-order thinking and cognitive skills.

2.3.5. **Section summary**

This section has focused on conceptual understanding, problem solving and creativity in real-life situations such as meeting the non-routine demands of the workplace. Together, conceptual understanding, problem solving and creativity are essential and mutually interacting elements in technological workplaces. Horowitz applied the ideas of de Bono and Pólya to engineering situations where problem solving requires changing levels of thinking. The advocacy and encouragement to study STEM subjects in schools is one attempt to solve a perceived shortage of school students studying science, technology, engineering and mathematics subjects. It uses teaching approaches that align with problem solving and creativity to provide training in engineering skills that is integrated with mathematics and science perspectives. STEM also emphasizes the interconnectivity between the various science, technology, engineering and mathematics specialist teams which is consistent with the ancillary skills, such as communication, which are a focus throughout this study. Therefore, STEM principles and programmes are likely to have implications for the use of mathematics in the mechanical engineering trades.

2.4. **How mathematics is learned and taught**

This section reviews the literature about how mathematics knowledge and skills are learned in the mechanical engineering trades. The discussion of STEM in Section 2.3.4 prefigured the important question of how mathematics is learned and taught in the mechanical engineering trades context. In this section, I discuss the historical, philosophical and political influences and debates on school and workplace mathematics education, and the theoretical and practical contributions made by Ernest, Freudenthal, Realistic Mathematics Education, and authentic mathematics to these debates. I also consider the implications for pedagogy, formal and informal learning, and the complexity of workplace mathematics.

2.4.1. **Historical, philosophical and political issues**

To understand the current situation regarding mathematics in schools and workplaces it is necessary to investigate a series of debates throughout the 1990s known as the *Maths Wars* (Schoenfeld, 2004). These wars were a series of debates over such issues as which mathematics pedagogy would be most effective in equipping young people to meet modern workplace and social challenges, the perceived mutual exclusiveness of mathematical excellence versus social equity, and the place of mathematics as a social and political democratizing force versus a vehicle for maintaining the status quo (Schoenfeld, 2004). One social equity issue may have been circumvented to some extent in New Zealand during the late 1960s with the abolition of core mathematics, which prevented students taking technical and other non-academic courses in secondary schools from studying algebra, geometry and trigonometry, and hence made it difficult for them to change from trades to professional vocations (K. Mills, 2011). From the late 1960s, every secondary student studied full mathematics while they were at school, thereby removing one source of disempowerment and discrimination against many students who had earlier opted to enter technical courses in secondary school, including many future mechanical engineering trades apprentices (Ernest, 2002).

From a pedagogical perspective, the ongoing debates over several decades were between traditionalists who feared that reform-oriented curricula would “undermine classical mathematical values”, and reformers who wanted curricula that reflected “a deeper, richer view of mathematics than the traditional curriculum” (Schoenfeld, 2004, p. 253). This included the need to consider and incorporate into mathematics education curricula other issues, such as problem solving, group cooperation, and the importance of context. The debates were also the result of a perceived need for modernisation to meet the future needs of the workplace as “technologies [became] more sophisticated, and the demands of the workplace … more complex” (Ministry of Education, 2007, p. 4).

The faults of traditionalist pedagogical approaches eventually led to questions being asked about the usefulness of core mathematics programmes for New Zealand secondary school technical students. For example, my father received a traditionalist mathematics education in the late 1930s, where students learned formulae and algorithms, and how to apply them to specific contexts. However, they were unable to transfer their thinking beyond that context, sometimes not even to closely related contexts, let alone to completely different contexts, which Brookhart (2010) regards as a higher-order skill. Requests from students for explanations were almost always met with the comment, “I’ll show you again”, which revealed that the student wanted to understand the concepts, but instead received a repetition of rules and procedures without understanding, or “rules without reasons” (Skemp, 2006, p. 89) (see Section 2.4.2).

The reformers who wanted a deeper, richer mathematics curriculum were relatively few in number, but very enthusiastic for change. Therefore, the much larger middle ground tended to be masked. This group was eclectic in their philosophy and practice, acknowledged the strengths, limitations and usefulness of each perspective, and sought to employ the best of both according to the particular context of the teaching situation (Schoenfeld, 2004; Sfard, 1998).

There is a parallel here with engineers’ eclectic approaches to mathematics and problem solving. Acknowledging the usefulness and limitations of a perspective is an important aspect of engineering, where pragmatic considerations quickly lead to the replacement of one perspective (or mathematical model) by another, simply because the model does not adequately describe the physical reality of their work (Gainsburg, 2006, 2007, 2013).

The middle-ground approach advocated by Sfard (1998) involved the acceptance of a “patchwork of metaphors rather than a unified, homogeneous theory of learning” (p. 12) - Sfard characterises these metaphors as “the most primitive … objects of analysis” (p. 4) as they often:

> cross the borders between the spontaneous and the scientific, between the intuitive and the formal … [and when] … conveyed through language from one domain to another, enable conceptual osmosis between everyday and scientific discourses, letting our primary intuition shape scientific ideas and the formal conceptions feed back into the intuition. (p. 4)

Philosophical considerations and the development of grand, unified theories were also part and parcel of the maths wars debates (Goenner, 2004). However, grand theories are incompatible with metaphors; adopting metaphors tends to create dualisms in thinking. Sfard (1998) found a precedent for the contradiction between grand theories and dualisms in the well-known wave and particle theories of light. In some situations, light appears to be particles (or photons) and in others, it appears to be waves. Trying to produce a single, grand theory for light has proven to be fruitless, and a dualism has been created. Einstein and Infeld (1938) described the dualism this way

> It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do. (pp. 262-263)

Therefore, there appears to be no answer to the question of whether a photon of light is a particle or a wave. The resulting conundrum has never been resolved one way or the other and is known as the wave-particle duality of light.

Dualities are also shown in the writings of Piaget and Vygotsky, two key theorists who analysed “the growth of knowledge in the process of learning … in terms of concept development” (Sfard, 1998, p. 5). Both processes are important in the mechanical engineering trades. Sfard (1998) describes their perspectives in terms of the Acquisitionist Metaphor (AM), which stresses the processes that take place in the individual mind, and the Participationist Metaphor (PM), which focuses on the relationships between the individual and others. The PM model is shown strongly among “structural engineers [who] exemplify that people make subjective decisions about how and what mathematics to use with socially constructive aims, for example, to design buildings that maximize safety and cost” (Gainsburg, 2007, p. 503). The metaphor chosen to describe any given case, therefore, depends mainly on its context. Sfard (1998) states that if:

> one's purpose is to build a computer program that would simulate human behaviour, then the *acquisition metaphor* is likely to be chosen as one that brings forward the issue of representations - something that has to be constructed and quite literally put into a computer. If, on the other hand, one is concerned with educational issues - such as the mechanisms that enable successful learning or make its failure persistent, then the participational approach may be more helpful as one that defies the traditional distinction between cognition and affect, brings social key elements to the fore, and thus deals with an incomparably wider range of possibly relevant aspects (p. 11). (emphasis in the original)

Sfard’s metaphors of acquisition and participation can both be identified in workplaces, including mechanical engineering trades. Recognition of both the conceptual understanding of individuals and the corporate collaboration within the group are needed to explain different aspects of their activities, without unifying them into a single grand scheme. Despite this, strict rules may be laid down by communities of practice on what constitutes an acceptable or unacceptable departure from orthodoxy.

To summarize, the pathway of apprentices to skilled tradespersons is influenced not only by engineering considerations but also by wider historical, philosophical and political issues. Debates over political issues like social equity and democratisation impinge on mathematics topics and the ways mathematics is used. Therefore, these political debates impact on young people’s opportunities to learn before beginning their apprenticeships. Regarding debates about pedagogy, traditionalists and reformers clashed over the philosophy of mathematics education, with reformers wanting broader and richer approaches to mathematics topics and the way they were taught. Their philosophical differences were reflected in the dualistic acquisitionist and participationist viewpoints, which in turn were linked to personal and social aspects of learning and to higher-order skills, such as problem solving.

2.4.2. **Influences of Ernest and Freudenthal’s views on mathematics education**

Understanding the ideas of two other thinkers, Paul Ernest and Hans Freudenthal, helps establish a bridge between school and workplace, and between traditional and modern ways of thinking about mathematics education. Ernest (1989) identifies three philosophies of teaching mathematics in common use: instrumentalist; platonist; and problem solving, which he arranged hierarchically. Instrumentalism is viewed as an "accumulation of facts, rules and skills" for some external end; platonism as a "static but unified body of certain knowledge … [that is] discovered" rather than created; and problem solving as a "dynamic, continually expanding field of human creation and invention, a cultural product … a process of enquiry and coming to know, [and] not a finished product, for its results remain open to revision" (Ernest, 1989, n.p.). While the examples cited by Ernest and other advocates of problem solving may appear to have limited practicability, an appreciation of the transference role of problem-solving skills is important to understanding how mathematics is applied in the workplace (see Section 2.3).

Hans Freudenthal is associated with Realistic Mathematics Education (RME); (van den Heuvel-Panhuizen, 2001), a theory of mathematics education where mathematics is regarded as a human activity. Real-world contexts are systematically investigated to promote progressive increases in conceptual understanding through both horizontal and vertical processes of mathematization (Zulkardi, 1999). The following list summarizes some of the main characteristics of RME (Treffers, 1987):

- RME uses contexts derived from everyday situations familiar to students as starting points for learning
- RME constructs models whose function is to form a bridge between the abstract and the real to help students learn mathematics at different levels of abstraction
- Students are encouraged to create strategies to solve problems as a result of investigating the contexts
- Social interaction between students and teachers is seen as an important means of assisting learning in mathematics
- Connections are sought between mathematics and meaningful problems in other learning disciplines.

The above discussion reveals parallels between the ideas of RME, the New Zealand Curriculum and workplace mathematics. One example of this is forming and then solving simultaneous equations, which is relevant to both trades and professional engineering contexts (Schukajlow, Kaiser, & Stillman, 2018). From a trades perspective, an NCEA Level 1 Achievement Standard entitled 'Apply linear algebra in solving problems', students are to form and solve simultaneous equations in two unknowns; form and then use an appropriate mathematical model in a real-life context; apply a chain of logical reasoning to convey their mathematical ideas and conclusions to others, and explain the differences between graphical and algebraic approaches in problem-solving situations (NZQA, 2014). Such an approach also involves the important mechanical engineering skill of substituting in formulas (see Chapter 4 for both problem solving and mathematical models).

In conclusion, Skemp, Ernest and Freudenthal all advocated philosophies and learning methods that were relevant to the workplace. Their focus on constructivist methods incorporated both the individual and social aspects of mathematics education. From a workplace perspective, an important attribute of RME is the way attitudes of investigation and problem solving are engendered in apprentices and constitute a feature of both learning and practice in mechanical engineering trades workplaces (see Section 2.5).

2.4.3. **Realistic Mathematics Education and authentic mathematics**

Here I provide examples of two mathematics education philosophies with applications to the workplace. They both acknowledge the importance of context, problem-solving skills and conceptual understanding. They also both encourage students to participate actively in learning mathematics (Zakaria & Syamaun, 2017). However, they also differ; authentic mathematics employs strictly real-life contexts while RME places more emphasis on the abstract processes of verticalisation and generalisation of thinking (Confrey & Kazak, 2006). From an engineering perspective, interest tends to be focused more on horizontal investigations and experimentation, especially if this leads to an algorithm that engineers can use in their work. They are not interested usually when the verticalisation process reaches a level of abstraction beyond what they perceive to be useful. There is therefore a disjunction between the aims of school classrooms and practice in the workplace. Despite this, many apparently abstract (and *ipso facto*, impractical) ideas do eventually find practical application.

In contrast to the above discussion of RME and verticalisation processes, authentic mathematics is applied most frequently in vocational mathematics because of its exclusive adherence to real-life settings. According to Roth (2010, p. 307), authentic problems "are messy, ill-defined and call for true problem-solving". Authentic mathematics also requires a "fidelity [to] the task and the conditions under which the performance would normally occur" (Gulikers, Bastiaens, & Kirschner, 2004, p. 69). The intention is to imitate the work of professionals working in the field. Therefore, authentic mathematics includes conceptual understanding (Koh & Low, 2010; Lamberg, 2013; Vosniadou, 2006), mastery (Ranellucci et al., 2013), open-ended enquiry (Ben-Hur, 2006), thinking skills (Chappell & Killpatrick, 2003), and critical thinking, reflection, communication and collaboration (Gulikers et al., 2004).

The PIAAC survey questions follow a similar philosophy and are therefore relevant to the present study (see Section 2.3.2). One PIAAC numeracy test item shows a thermometer and the instruction "Fill in the temperature shown on the thermometer in degrees Fahrenheit (ºF)" (OECD, 2012b, p. 41). This test item involves identifying an appropriate scale, reading the scale and then interpolating between values. These skills are relevant to mechanical engineering trades (see Section 2.2). The sample item has a Level 3 difficulty on a 1-to-5 scale. However, like the PISA surveys, the PIAAC studies reflect performance only in the artificial situation of a formal test where the question and all the data are provided. They have nothing to say about workplace actualities where the worker must create the question, deal with several complex issues, and interact with other people to solve problems (see Section 2.2).

For many years, Competenz has used standard assessment items when assessing mathematics competency that are similar in style and structure to typical mathematics assessments found in schools and the NCEA (Glaeser, 2006; Glaeser, Harrington, & Watson, 2006). While the details of the assessment under Competenz's eLearning system for curriculum delivery and assessment remain unpublished, the assessment system continues to be limited to the objectives and scope of calculations as reflected in the title of US 21905. However, while mathematics assessment may be limited, it is quite possible that other objectives such as the ability to work in small groups and interpret numeric data in context are assessed in other Engineering Unit Standards.

2.4.4. **Implications for pedagogy - conceptual understanding and social learning**

Arguments over philosophy were accompanied by debates over the lack of emphasis in traditional mathematics teaching on the development of conceptual understanding and the social aspects of learning. Traditional approaches were often labelled "mechanistic" (van den Heuvel-Panhuizen, 2001, p. 1), but other classifications were also possible, such as instrumentalist, Platonist and problem solving (Ernest, 1991). The focus that PISA and the New Zealand Curriculum now place on problem solving and conceptual understanding is one outcome of these debates. Conceptual understanding is also important in mechanical engineering trades workplaces (see Sections 2.3.1 and 6.2.3) and is reflected in the increased emphasis placed on the way mathematics is now taught in schools (Agaç & Masal, 2017; Ministry of Education, 2007; Yuanita, Zulnaidi, & Zakaria, 2018).

The advantages and disadvantages of conceptual and mechanistic approaches and the circumstances where one approach is to be preferred over the other have been set out by Skemp (2006). Skemp elucidated the differences between two types of understanding - relational understanding, which he describes as "knowing both what to do and why", and instrumental understanding, or "rules without reasons" (Skemp, 2006, p. 89). While relational understanding was usually considered superior to instrumental understanding, Skemp found the following advantages for instrumental mathematics: it is usually "easier to understand"; the "rewards are more immediate, and more apparent"; "one can often get the right answer more quickly and reliably by instrumental thinking"; and it is "easier to remember" (Skemp, 2006, p. 92). These considerations are important in workplace situations where calculations have to be repeated many times. Once the method for a calculation has been created the worker simply needs to check that the first calculation has been done correctly to feel confident that the method is reliable.

The workplace considerations outlined above have influenced the reforms of mathematics teaching methods in schools, for example, problem solving and cooperating in small groups with "shared responsibilities … [and] the development of metacognitive skills, such as critical thinking, learning to learn, planning and problem-solving" (FitzSimons et al., 2005, p. 4). The overall intention is to provide students with formal and informal mathematical experiences to work cooperatively together and to know "why [something is] true" (Steen, 1990, p. 5). The outworking of problem solving in small groups in the mechanical engineering trades environment is currently unknown.

Those wanting to reform traditional mathematics teaching practices also placed an emphasis on the social aspects of learning. One approach was Collaborative Learning (CL), which according to Hakkarainen, Paavola, Kangas, and Seitamaa-Hakkarainen (2013, p. 20) takes place productively;

> … in mediated *interaction between personal and collective activities*. In many cases, individual agents may have a key role in knowledge-creation processes but are not, in fact, acting individually; their activities rely on a fertile ground provided by collective activities. Becoming a collaborative inquirer is a developmental process of its own.

Therefore, while CL accepts the role of collective processes, it also highlights the contribution made by individuals who exhibit creative abilities. These individuals, however, are still reliant on the ideas of others, both for their initial inspiration and for ongoing critique, which follows the participationist ideas of Vygotsky (1930). Therefore, in an engineering trades community, CL views the solutions that are eventually adopted as a complex series of interactions between individuals participating together in collaborative problem solving, but with an expert engineer providing breakthroughs in thinking at crucial times.

To summarize, the reforms of mathematics education in New Zealand and some other countries have been strongly influenced by philosophical considerations connected with the need for conceptual understanding (Ministry of Education, 2007). Developing conceptual understanding has been linked to problem solving and a broad range of social objectives. The need to prepare students for the workplace is recognised as an important factor in these reforms and social interaction in classrooms is now an important objective of mathematics learning in the New Zealand Curriculum.

2.4.5. **Formal and informal learning**

The process of learning begins with informal experiences. I begin with informal childhood experiences of mathematical and practical experiences in places such as the home, early childhood centres and kindergartens. I then discuss formal and informal learning in the workplace.

2.4.5.1. **Childhood learning experiences**

Formal mathematical learning begins in informal settings long before the child begins school. According to Anthony and Walshaw (2009b), children become immersed in mathematics learning experiences that begin at birth. At a very young age, they can demonstrate skills relevant to engineering contexts such as arithmetic, measurement and problem solving. This is regardless of their socioeconomic and cultural contexts. This view is supported by Downton, MacDonald, Cheeseman, Russo, and McChesney (2020) who state that children are often capable of mathematical thinking at a very young age. They perform mathematics holistically compared with school approaches that can become formalised, segmented and less richly involved in context.

The home cultural milieu can therefore be an important influence. A study of the home experiences of six children by Young-Loveridge (1988) of varying levels of socioeconomic status showed that children entered school with greatly different kinds of experiences and concepts of numbers. Moreover, exposure to domestic activities such as baking and shopping; playing games like Monopoly and dominoes; using calendars, clocks, and car speedometers; and handling calculators and money seemed to increase number skills. Therefore, a family culture of valuing and informally promoting numeracy skills, even with games, aided numeracy development in young children.

In a kindergarten study of mathematics development in young children, children in an experimental group were taught mathematics according to the principles of RME, and a control group was taught mathematics following the basic pedagogical principles of curriculum for kindergarten students. The study found that the RME technique contributed significantly to the development of mathematical competence of young children, regardless of gender, age and nonverbal cognitive ability (Papadakis, Kalogiannakis, & Zaranis, 2016).

In another study of kindergarten children, children in China and the United States were tested on a variety of mathematical tasks (Siegler & Yan, 2008). The problems involved arithmetic and numbers, or games like Snakes and Ladders. It was found that compared with the children in the United States the Chinese children were more exposed to mathematics problems at home and showed greater numerical knowledge for both arithmetic and number-line estimation problems. The authors concluded that analysing everyday activities may induce concept formation and “understanding of cross-cultural, individual, developmental, and social-class differences in knowledge and learning” (Siegler & Yan, 2008, p. 762).

To summarize, both informal and formal mathematical experiences are important for young children’s short-term intellectual growth. The nature of the mathematical skills and their conceptual understanding of them may be primitive, but emulation of others may lead to increasing procedural knowledge. The children may also be building up a store of historical experiences and knowledge and how the knowledge is used in context. Moreover, the attitudes instilled by the involvement with, and the approval of, others may be long-term and formative in their development.

2.4.5.2. **The workplace**

There is an ongoing debate about the relative merits of formal and informal teaching and learning methods in the workplace. On-the-job training is one example of informal learning, and this may consist of a variety of methods, such as observation followed by practicing, or discussions with mentors and peers. Formal teaching and learning may take the form of block courses held in classrooms or doing online assignments.

There has been much research into formal methods, but less into informal methods. However, according to Clardy (2018), it is now widely accepted that informal learning plays a critical role in all workplace learning. Moreover, Clardy refers to a so-called “70% rule” (p. 153) that states informal learning dominates workplace learning at the expense of formal and other methods. Clardy doubts the accuracy of this rule because its evidential basis is weak, and based on poor scholarship and inconsistent conceptualizations. Clardy's views are reiterated by Jeong, Han, Lee, Sunalai, and Yoon (2018), who suggest that further research needs to be done to synthesise the current literature, particularly on how informal learning is to be conceptualised and measured, and the empirical identification of factors influencing informal learning in the workplace. Therefore, the relative efficacy of formal and informal teaching and learning methods remains an open question, including in the mechanical engineering trades.

An important aspect of mechanical engineering trades involves knowledge and skills in measurement. Measurement skills can be learned both informally through involvement in practical scenarios and formally in classroom settings. However, in common with all scientific disciplines, measuring and machining are never perfect in engineering situations. Therefore, there is a need for tolerances, which for the purposes of this study, I shall define as the maximum allowable differences between the product specifications and the finished product (Kent, Bakker, Hoyles, & Noss, 2011; Velling, 2020). In addition, engineering contexts often involve small measurements which require a detailed conceptual understanding of numeracy, especially decimal place value and a feeling for the size of measurements (Tout et al., 2017). Since gaining this understanding requires school experience to be deepened, then recognising how tolerances express the differences between engineering trades contexts and specialisations is a key feature of mathematical learning.

Tolerance is associated with other words such as precise, precision, fit, fits, margin, margins, within, limit, limits, thou, micron, microns, and the symbol ±. While tolerances refer to variation in the lengths in a finished product, they are not associated with human mistakes or blunders while calculating or measuring. They are "unavoidable imperfections of workmanship" (Oberg & Jones, 1964, p. 1337). There are many sources of variation, among which are tool wear and the impossibility of reading a scale to more than just a few decimal places. Temperature change between a cold morning and a warm afternoon can create crucial changes in lengths where fine, accurate work is required in machining. Therefore, machinist engineers must always be involved in a constant process of thinking, measuring, checking and resetting their machines to keep their work within the tolerances.

Toolmakers and surgical instrument makers employ fine tolerances for much of their work and as a result have different perspectives on tolerances from other mechanical engineering trades branches, such as jig making and boiler making. Exceeding the acceptable tolerances could easily render an item useless, with serious consequences such as waste of time, materials, and money, and impact negatively on safety. In contrast, producing an item with unnecessarily fine tolerances also wastes time and money. The strategies engineers use in their work depend on the acceptable tolerances, the tools available, and the skill of the tradespersons (Marr & Hagston, 2007).

Therefore, deciding on permissible tolerances becomes an important numeracy issue, involving careful consideration of decimal place value, and is a delicate blend of art and science. Also, the ways apprentices develop this skill and relate it to decimal place value is an important aspect of their mathematical learning on the path to becoming a skilled tradesperson.

2.4.6. **Workplace mathematics is complex**

Regarding the way mathematics is learned and taught, it is important to understand how engineers (whether apprentices or skilled tradespersons) develop and apply mathematics in their work. This literature review has shown that modern mathematics learning is multifaceted, whether in the school or the workplace environment. Moreover, both school and mechanical engineering trades workplaces acknowledge that mechanical, rote-learned skills are insufficient to meet modern workplace demands. Therefore, modern demands for contextualised learning, problem solving, development of higher-order thinking skills and lateral thinking all require a wider view of mathematical education that incorporates a greater emphasis on understanding, language and communication in mathematics teaching and assessment. Problems in context have been carried over into the classroom from the real-life world of business and the workplace. However, the transition is not simple, since contexts in real life are often much more sophisticated than the classroom; the learner needs to rely less on understanding abstract concepts and more on finding mathematical solutions to problems that are open-ended, technology-dependent and multi-step in nature (FitzSimons et al., 2005; Steen, 2001; van der Kooij & Strässer, 2004).

Therefore, mathematics education reflects a contradiction between two essential and not necessarily mutually exclusive needs - conceptual understanding and problems to be set in the real world. This is illustrated in the case of authentic mathematics (see Section 2.4.3), which seeks to ensure competence in mathematics in specific *real* contexts that are relevant to life and the workplace. With authentic mathematics, however, there is also an acknowledgement of the necessity to acquire understanding at the highest conceptual level.

Therefore, there is a paradox. The distinction that was made once between theoretical mathematics and mathematics you can use has now largely disappeared. Workplace mathematics has now become an area of study in its own right and is no longer an adjunct to school mathematics. Moreover, the socio-cultural aspects of workplace philosophy and practice reflected in Sfard's (1998) acquisition and participation metaphors are now part of the classroom. The result is that "learning a subject is now conceived of as a process of becoming a member of a community … with the learners being newcomers and potential reformers of the practice, [and] teachers [being] the preservers of its continuity" (Sfard, 1998, p. 6). Consequently, newcomers have the potential to influence the old-timers[3], as well as vice versa. This may be relatively new for the classroom, yet engineers and other

[3] The terms *expert* and *old-timer* have been uplifted from the Situated Learning (SL) context of Lave and Wenger (1991). Both are understood subjectively in this thesis. Experienced engineers are jocularly referred to as old-timers by apprentices. Experts are simply those people respected in the community of practice for having high-level all-round engineering skills or perhaps advanced skills in one area, such as welding. There is no formal mathematics requirement to be an expert.

tradespersons have been using socio-constructivist methods of learning for centuries (Lave, 1977, 1989; Lave & Wenger, 1991) (see Section 2.5).

2.4.7. **Section summary**

To summarize, traditional ways of teaching mathematics have often been based on a teacher speaking and writing, with minimal, if any, learner input or participation. Learners were asked to observe and then reproduce what the teacher, or apprentice instructor, had done. The problems were closed and predictable, and their solutions could sometimes be learned by rote. Student questions were often answered by the transmission approach with the teacher having to frequently repeat what had already been said.

In contrast, more modern approaches focus on group learning with learner interaction and involve problem solving with open questions whose outcomes have not been predetermined. In this scenario, more than one mathematical outcome can be regarded as an acceptable solution. RME and authentic mathematics are two learning systems that attempt to put modern approaches into practice. RME does this by emulating real-world scenarios using verticalisation, by which it produces generalised mathematical abstraction. In contrast, authentic mathematics attempts to create scenarios as close as possible to real-world scenarios. Authentic mathematics is interested in horizontalisation, or finding practical applications of mathematics, without necessarily being interested in a verticalisation process.

2.5. **Social interaction and the workplace environment**

In the last section, I began reviewing the literature regarding how people learn the mathematics knowledge and skills used in the mechanical engineering trades. Social interaction was mentioned in the discussion there but now needs further analysis to understand more fully its influence in the mechanical engineering trades and other workplace contexts. Fortunately, there is a greater literature on social interaction than on other aspects of the trades workplace environment (FitzSimons, 2001; FitzSimons & Mlcek, 2004; FitzSimons et al., 2005; Zevenbergen, 2002). An important Nigerian study by Audu et al. (2014) found that constructivist methods were superior to earlier methods of learning problem-solving skills, and retention of knowledge of mechanical engineering apprentices. Therefore, in this section social interaction becomes the main focus, not only in learning but also in daily workplace practice involving decision making, communication and problem solving in mechanical engineering trades contexts.

One theory that incorporates social interaction in learning is Situated Learning (SL) proposed by Lave and Wenger (1991). The situatedness of SL means it is able to incorporate a specific focus on the needs, actions, and social interactions of learners. Therefore, SL is particularly relevant to apprentice learning. This thesis will use two theoretical frameworks; Cultural Historical Activity Theory (CHAT) as the main framework and SL as a second framework. The reason SL is introduced here before the discussion of CHAT is that SL synthesises the overarching principles of the main CHAT framework approach with respect to workplace learning (see Section 3.2).

I first review the reasons that led Lave and Wenger (1991) to develop their theory of SL and its nature. I then discuss the roles that communication plays in human interaction in the workplace and the eclectic nature of learning in mechanical engineering trades workplaces.

2.5.1. **Situated Learning**

SL is defined by Lave and Wenger (1991) as a socio-cultural theory where learning is situated in activity. It emphasizes how people's thoughts and actions are negotiated socially and culturally through their social interactions (Johri & Olds, 2011). Lave and Wenger developed their theory in response to the various assumptions and limitations they identified when conventional theories of learning were applied to workplace situations. First, there was the issue of transfer of knowledge, which they believed oversimplified learning as an "unproblematic process of absorbing the given, as a matter of transmission and assimilation" (1991, p. 47). Second, they believed that conventional explanations underestimated the process of learning within "the broader context of the structure of the social world" (1991, p. 48). A third issue was informal learning, which they believed involved observation and imitation. Therefore, informal everyday numeracy activities like shopping were opportunities for learning where people followed strategies that appeared to be self-made. This stood in contrast to following algorithms taught in school (Greiffenhagen & Sharrock, 2008; Lave, 1988; Lave, Murtaugh, & de la Rocha, 1984). A fourth issue was the decontextualization of knowledge. Lave and Wenger (1991) believed that abstractness needed to be made specific to the situation at hand. Therefore, difficulties may be caused for some learners who come to regard mathematics as useless, irrelevant and without practical application. An example is the transposition of formulas, where I found that two of the toolmakers I interviewed claimed to have learned to transpose formulas in chemistry, physics and engineering classes (K. Mills, 2011). Similarly, Astrop (2020) attempted to resolve the difficulties of teaching the transposition of simple engineering formulas to prisoners by using illustrations and drawings. In the cases above, the discussions while shopping or studying chemistry, physics or engineering provided real contexts that led to students internalizing understanding. It would seem then that conventional contextless approaches to mathematics teaching have limited effects on the development of either conceptual understanding or procedural knowledge.

Lave and Wenger (1991) proposed an alternative approach to the difficulties they identified in conventional learning, particularly the social role in learning, which they outlined in an important theoretical treatise, *Situated learning: Legitimate peripheral participation* (1991). They define learning as a situated activity whose central defining characteristic is legitimate peripheral participation (LPP). LPP is a process by which newcomers join a community of practice, which can be regarded as a "set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice" (Lave & Wenger, 1991, p. 98). Recent newcomers are initially on the periphery of the community of practice, but with increasing experience, knowledge and skill, they gradually move towards the centre of the community (Matusov, Bell, & Rogoff, 1994). Therefore, learners must participate in communities of practitioners to master knowledge and skills which enable them to participate fully in the sociocultural practices of the community.

SL is important to this study because of the emphasis it places on community and its ability to analyse human interactions in workplaces. For example, LPP enables discussion about the relations between newcomers and old-timers, and about activities, identities, artifacts, and communities of knowledge and practice. In this way, the social process "includes, indeed it subsumes, the learning of knowledgeable skills" (Lave & Wenger, 1991, p. 29; Matusov et al., 1994). SL is therefore viewed by Lave and Wenger (1991) as fundamentally a social process, as opposed to the cognitive development and understanding of the individual. The central role of the community of practice stands in marked contrast to other methods of learning, especially those employed in much formal schooling. Consequently, Lave and Wenger (1991) regard conventional theories about school learning as too restrictive to provide "the historical-cultural breadth to which [they] aspire" (p. 61). Moreover, SL affects the relationship between teachers and learners because SL "points to a richly diverse field of essential actors and, with it, other forms of relationships of participation" (p. 56).

2.5.1.1. **Social relationships and identity**

Relationships of participation and the relations between individuals in a community of practice are important in SL because development and change in individuals and the community are dependent on participation in the community. Change in individuals implies changes in identity. Lave and Wenger view identity as the "way a person understands himself, and is viewed by others" (1991, p. 81). Therefore, since SL is a theory of dynamic change involving movement from the periphery to the centre of the community of practice, the roles and hence the identities of individuals change. Under SL, the novice apprentice gradually gains skills, which can be viewed as learning a trade. Eventually, the mechanical engineering apprentice will arrive at, or close to, the centre of the community of practice and hence adopt the identity of a skilled mechanical engineering tradesperson (Chan, 2020).

However, the process of novice learners moving centripetally from the periphery towards the centre is not without its contradictions. Lave and Wenger (1991) see contradictions between new ideas and long-established practices, different methods of learning, conflicting values, power relationships, and family and school traditions. An example of this is older members of the community being replaced by those who have more recently arrived at the centre, and who perhaps have more energy, more creative ideas they wish to experiment with, more modern skills, or are able to adapt more easily to new conditions and technologies such as smartphones. Workplace learning is therefore shown to be an increasingly complex process with increasing potential for intergenerational contradictions. These contradictions can occur when groups of newcomers begin to innovate by searching things out on the internet, finding new information or designing new techniques. Such changes challenge existing power relationships about who has the knowledge and the authority to problem solve in new and perhaps unusual ways. The nature of these intergenerational contradictions and the specifics of how they are resolved in the mechanical engineering trades context are unknown.

2.5.1.2. **Situated Learning and this study**

Lave and Wenger (1991) applied SL to several vocations. The cases of apprentice tailors and US Navy quartermasters are likely to have relevance for this study. Lave and Wenger saw apprentices becoming skilled and respected master tailors without formal teaching, assessment or merely copying everyday tailoring tasks. The tailors began as learners in each community and gradually learned by observing what masters and journeymen already did. They made simpler items before moving on to formal garments and then "Higher Heights" suits (p. 71). Lave and Wenger also identified a similar process with apprentice quartermasters in the U.S. Navy. They too began with rather limited tasks and progressed to more complicated challenges under the supervision of more experienced tutors. However, while quartermaster training was primarily on the job, "some of the experience aboard ship is a bit like school with workbooks and exercises", and apprentices who had gone to specialized schools before joining a ship sometimes had to have "bad habits [broken which they had] acquired in school" (p. 73). This suggests the existence of a background discussion within the community, and perhaps even tension, between formal and informal methods of learning.

SL has been used in a variety of contexts. In a New Zealand study, Vaughan (2017) used SL to investigate general practice medicine, carpentry, and engineering technician work and their workplace mentors and teachers. She concluded that not only were soft skills specific to fields and were learned rather than being general, abstract and fixed, but that their development was strongly influenced by workplace mentors and teachers. In a study of teaching process skills to pre-engineers, it was found that opportunities to engage in formal public speaking helped in overcoming fear of making mistakes or disseminating false information (Maher, Bailey, & Tucka, 2018). Given the applicability of SL to analysing social components of engineering contexts, it is also able to focus on the social aspects of the pathway from apprentice to skilled tradesperson and expert engineer. Hence, SL is an appropriate frame of reference for this study.

In summary, Lave and Wenger (1991) discovered that the tailors' and quartermasters' apprenticeship learning was in marked contrast to school learning (within developed countries). Since it is possible that engineering trades apprenticeship learning might also occur in the same way, then this study has set out to explore how apprentices learn and develop mathematics skills necessary for their work by taking into account Lave and Wenger's situated learning theories and examples.

2.5.2. **Language and communication**

According to Lave and Wenger, language is a part of practice, and "it is in practice that people learn" (p. 85). Being able to talk with and gain access to the community's collective knowledge, skills and wisdom is, therefore, an important factor in determining how well apprentices succeed as learners. This success in turn is more dependent on access to "peripherality than [to] do with knowledge transmission" (Lave & Wenger, 1991, p. 105). One illustration of the importance of language, not only in the workplace communication context but also in the development in apprentices' centripetal movement to insider status, is the strong influence of sharing stories when skilled tradespersons and apprentices talk about their work. In this process, newcomers not only listen to and learn to tell stories themselves, but through the stories, they learn the more important and difficult skills of the trade. Therefore, learning the art of storytelling is part of their becoming “legitimate participants in the community of practice” (Lave & Wenger, 1991, p. 109).

Communication was therefore important, both written and verbal. Talking allowed both apprentices and skilled tradespersons to share together in the practice of the community, to signal their desire to become accepted at new levels in the community, or rather, closer to the insider status at the centre of Lave and Wenger’s community, and to engage and focus on the general problem solving and lore of the community. This fostered social acceptance as well as knowledge and skill acquisition.

As outlined in Section 2.4.4, Collaborative Learning (CL) also involves interaction between members of a community. Therefore, talking and dialogue within a community are crucial to promoting CL among its members. Attard et al. (2018) studied pedagogical practices in mathematics classrooms. They concluded that rich and robust dialogic interactions were necessary to develop reasoning abilities in learners and for them to explain their processes of mathematical thinking to others. Nerona (2019) reported the results of an important study in the engineering context which measured pre-test and post-test scores of respondents. She found that the experimental groups engaged in CL obtained significantly higher post-test scores than their control group counterparts, who had been “exposed to the traditional lecture-discussion and individual learning methods” (p. 114).

The effectiveness of CL methods in developing technical skills has been corroborated by Archer (2008), who argues that dialogue is necessary to transform engineering practice in response to changing global realities where a top-down approach needs to be replaced by educators and engineers needing to learn by drawing on each other’s knowledge and experiences rather than imposing knowledge in a top-down process.

However, perhaps the most unexpected source of CL is story-telling, which Lloyd (2000) believes is taken for granted. Therefore, Lloyd views engineering design as cohering only as a social activity mediated by a common language, the existence of which is regarded as indicating good design. Maslen and Hayes (2020) further develop the symbiosis between social and technical understandings regarding strategies engineers adopt when reasoning through disaster scenarios, such as the Überlingen mid-air aircraft collision. They first reasoned with abstract principles and then sought to appreciate the events through stories. The stories not only applied to making value judgments but also altered their professional engineering practices, which older members of the community quickly adapted to through communicating with younger people. A further use of storytelling in workplace contexts is made by Swap, Leonard, Shields, and Abrams (2001) who focus on leveraging the knowledge within an organisation and elucidating how mentoring and storytelling can be most effective in spreading knowledge. The roles of communication and storytelling in the mechanical engineering trades context are currently unknown (see Section 5.2.5).

To summarize, language is an essential skill in an effective workplace. Language is not only useful for conveying mathematical and other information but it also changes people and the way they regard the technical aspects of their work. Therefore, the social aspects of work impact workplace efficiency and effectiveness.

2.5.3. **Eclecticism of learning strategies**

This section reviews various learning theories that may appear to be mutually exclusive, but which some learning theorists believe can be used eclectically. Eclecticism is a pragmatic approach that does not resolve the dualities between different theories but attempts to create an alternative method which when applied, can sometimes lead to theoretical progress. This is currently the situation in some other disciplines, such as physics, which is often described as an ‘exact’ science (Hawking, 2002; Smolin, 2007, 2013).

I have already discussed one example of dualism - the Acquisitionist and Participationist metaphors and noted the danger of using just one (Sfard, 1991, 1998, 2009) (see Section 2.4.1 and Section 2.4.6). These metaphors encapsulate a major contradiction between the individual and social aspects of educational theories. I asked Anna Sfard if dualities in education philosophy would ever be resolved into one grand theory. She replied in the negative (private conversation, University of Auckland, 30 June 2017). Instead, with dualities of metaphors, there are times and situations when using one metaphor is appropriate, and others when an alternative metaphor is appropriate.

Other learning theorists adopt a similar view. For example, Illeris (2018) regards all learning as integrating external interaction processes such as the learner’s social, cultural or material environment, and internal psychological processes like elaboration and acquisition. Neither process covers the whole field of learning, and both processes must be actively involved if learning is to take place. Then, perhaps surprisingly given his association with Lave and SL, Wenger states that the kind of social theory of learning he proposes does not seek to replace other theories of learning, nor that his social perspective “says everything there is to say about learning” (Wenger, 2018, p. 226). Rather, there is a complex relationship between the individual and the community where the individual learns by engaging in and contributing to the practices of the community, yet individuals within the community can contribute as individual agents. This implies that society can no longer be understood without the agency of individuals who use and produce artifacts (Engeström, 2018; Wenger, 2018).

By way of contrast, Engeström (2001) finds major differences between the workplace and other learning environments. For example, standard theories of learning focus on learners acquiring some identifiable knowledge or skills so that their behaviour is altered. However, this approach assumes that the knowledge or skill is stable, well-defined and that there is a knowledgeable teacher capable of imparting what is to be learned. The difficulty is that much time is spent in workplaces on tasks that are “not stable, not even defined or understood ahead of time” (p. 137), and that transformations are “literally learned as they are being created. There is no competent teacher. Standard learning theories have little to offer if one wants to understand these processes” (p. 137). Therefore, workplace learning is firmly situated in novel scenarios that require problem solving, creativity and communication. In some engineering trades contexts, especially those involving routine procedures, standard learning theories will often suffice and often competent co-workers are available to act as teachers. However, this is not always the case in non-routine contexts.

One final factor in workplace learning requires mention - apprentices and qualified engineers are adults who have left school and therefore tend to have different perspectives from school-age students. Amongst other things, adults tend to be

- conscious of their need to know and why they need to learn it
- conscious of being responsible for their own decisions
- exposed to greater and different life experiences than younger people
- sufficiently mature to learn those things they need to know to cope with real-life situations
- concerned with problem-centred approaches to learning because they see its relevance to real-life situations
- motivated by practical considerations such as better jobs, more money, job satisfaction, self-esteem
- engaged in personal reflection and mutual discourse to identify and assess assumptions made by the teacher

(Knowles, Holton, & Swanson, 2011; Merriam, 2018; Mezirow, 1994, 2018).

In particular, adult engineers see learning as a transformative process that, combined with their maturity raises them to a greater height than recent school leavers can currently attain or appreciate. The question of maturation processes and their effects on the learning of apprentices is also unknown.

2.5.4. **Section summary**

In summary, there are aspects of SL that are relevant to this thesis and to the mechanical engineering trades context. From the apprenticeship perspective, this includes legitimate peripheral participation which novices use to advance in knowledge, skill and status in the community. From the community's perspective, SL acknowledges and describes the nature of relationships within the community, how these relationships change over time as people mature, and technology changes. In the next chapter, consideration is given to the theoretical frameworks for this thesis. SL has an important contribution to make to this study because it synthesises the overarching principles of a CHAT approach to workplace learning with a focus on the needs and actions of learners.

SL is a theory of learning involving legitimate peripheral participation (LPP) where newcomers join a community of practice that already comprises others who exercise varying degrees of responsibility depending on their skills and competence. The community of practice also takes responsibility for the newcomers' learning, which is mainly done informally, although the United States quartermasters were an exception because they also learned by formal teaching methods (Lave & Wenger, 1991). SL is appropriate to apprenticeship learning with the apprentices being initially on the periphery and moving towards the centre as they become more skilled. Since SL is a socio-cultural theory, communication is crucially important, for both day-to-day work and the ongoing learning of the total community - apprentices, skilled tradespersons, and experts. The stories the community members tell are important learning devices because they raise the learning level of the community, and transmit the folklore and culture of the community to the next generation.

2.6. **Chapter summary**

There are two issues to be considered here: how the relevant literature informs the research questions of the study that surround the pathway of mathematical learning from beginning apprentice to skilled tradesperson and to expert engineer, and the limitations and gaps that have been identified in the literature.

First, the literature has helped answer the research questions directly regarding the mathematics and numeracy content. The mathematical content mechanical engineering trades apprentices require is contained in the NZC and NCEA curricula statements, and in US 21905. Indirectly, the literature review has identified the importance of ancillary skills in mathematics learning. The NZC and NCEA documents mention problem solving, creativity, but do not specify in detail what these skills involve. There is a large body of literature on problem solving and creativity in mathematics (see Section 2.3). In addition, the NZC emphasizes the socio-cultural aspects of learning which also has a large body of literature (see Section 2.4). Moreover, there is a growing literature surrounding mathematics and numeracy usage in specific workplaces, and these often relate to higher-order skills and socio-cultural theoretical frameworks (see Section 2.3.4). Each of these areas directly, or indirectly informs the research questions for this study.

Second, regarding the gaps in the literature, while there is a growing literature of workplace mathematics studies there is also a lack of specific workplace studies on mechanical engineering trades. This perhaps tempts us to make the dangerous assumption that mathematics is applied in the mechanical engineering workplace in a similar way to those reported in the literature involving other trades workplaces, or even in other workplaces in general. Moreover, studies on professional engineering by writers such as Gainsburg (2006, 2007, 2013) and Horowitz (1999) might perhaps lead us to a further doubtful assumption, viz., that mathematics and numeracy in the professional and trades engineering areas will be similar. This assumption is not valid regarding mathematical content, because professional engineers require much higher levels of school mathematics, notably algebra and calculus. Nor do Gainsburg and Horowitz stress the widespread role and importance of numeracy. Furthermore, given the hands-on nature of the mechanical engineering trades, little appears to be known of how tradespersons employ problem solving and creativity in practical contexts, except among the tradespersons themselves. Therefore, there are likely to be substantial differences of emphasis, and perhaps even of the relative importance of the various issues between my study and those of Gainsburg and Horowitz.

In conclusion, the literature informs us in detail about the mathematical topics that mechanical engineers need for their work. The literature also recognises the importance of ancillary skills in applying mathematics in workplaces, and the ways that mathematics skills are learned and developed in the workplace. However, there are only a few studies relating to the details of how mathematics is applied, learned and developed in mechanical engineering trades workplaces, and this creates a niche for the contribution this study makes to the field of knowledge.

Chapter 3. **Methodology**

Introduction

This chapter gives an account of the methodology and particular methods used in this study. The discussion of the methodology includes the interpretivist paradigm employed, the mixed methods methodology, Engeström's (1987) Cultural Historical Activity Theory framework (CHAT) and Situated Learning (SL); (Lave & Wenger, 1991). Later sections describe the methods employed in the study: the development of the questionnaire items and the questions for semi-structured interviews, the sampling procedures, procedures for recruiting participants, the data collection techniques, how the data were analysed and presented, and the presentation of findings. A chapter summary follows the final section on ethics.

3.1. **Methodology**

This section sets out the methodology for the study, the purposes of which include evaluating research decisions before implementing them, examining the study's underpinning theoretical frameworks, justifying why certain approaches have been taken, explaining the logic behind the methods and techniques, as well as the reasons why some methods were found to be appropriate to the study and other methods rejected (Kothari, 2004; Morgan, 2007). I first discuss general paradigmatic considerations and the reasons for choosing interpretivism as a paradigm for this study in preference to critical social science, positivism, and post-positivism. I then provide further detail around the interpretivist paradigm as well as the mixed methods methodology chosen for the study.

3.1.1. **Paradigmatic considerations**

Adopting and discussing paradigms is necessary for research because a paradigm describes the way the researcher thinks, their worldview and basic assumptions, their epistemological and ontological positions, the questions they consider to be important, the techniques they use to perform their research, and what good scientific research looks like (Alharahsheh & Pius, 2020; Neuman, 2003; Punch, 2009; Willis, 2007).

The choice of paradigm is dependent on how the research questions are framed. In this study, the overarching research question concerned the nature of mathematical learning that characterises the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering. Three associated sub-questions relate to the nature of the mathematics skills employed in the mechanical engineering trades, how they are applied, and how they are developed (see Section 1.3). An important first step, therefore, is to find a paradigm that best suits the context and nature of the research question. In this section, I examine three paradigms currently in usage, and the reasons I have not selected them: critical social science, positivism and post-positivism. In the next section, I then discuss interpretivism which best fits with the nature of this study.

Critical social science was not selected as a paradigm for this study. Critical social science tends to suit studies oriented towards political and social action, as would be the case if the focus of this study was on political philosophy and its effects on apprenticeship training schemes, or “with conflict theory, feminist analysis, and radical psychotherapy” (Neuman, 2003, p. 81). Positivism was not considered appropriate for this study because it has been criticised as offering an incomplete picture of human beings. This study incorporated a large focus on human beings who cannot be reduced to numbers, abstract laws or formulas. Moreover, human beings exercise freedom, individuality and moral responsibility (Cohen, Manion, & Morrison, 2000; Neuman, 2003). Post-positivism was also not considered appropriate for the present study. While post-positivism differs from positivism in that its knowledge claims are no longer absolute, but imperfect, tentative and therefore fallible (Creswell, 2009), it still retains features of determinism, reductionism, measurement, and develops numeric measures of observations. Hence, while tentative in making known its knowledge claims, post-positivism still lacks the ability to examine situations where humans make decisions according to personal preferences.

The research questions of this study were framed with the intention of allowing the mechanical engineering tradespersons to express their voices on the nature of mathematical learning in the mechanical engineering trades, the mathematics skills that were employed, how they were applied, and how they were developed (see Section 1.3). Thus, the research questions guiding this study called for a paradigm that relates to real-life situations in which human beings make choices, have the freedom to act and exercise individual preferences (Crotty, 1998). Interpretivism was chosen as the paradigm for this mixed methods study because it could interpret, analyse and deepen the broad data obtained from human testimony, their various experiences, and personal preferences.

3.1.1.1. **Interpretivism - knowledge, reality and truth**

I now discuss the assumptions of interpretivism regarding knowledge (epistemology), reality (ontology), and truth. First, since interpretivism employs a social constructivist epistemology, knowledge is understood to be contextually situated. One consequence of this is that epistemology has no absolute character, but focuses on trying to “get as close as possible to the participants being studied” (Creswell, 2007, p. 20). Therefore, the constructivist worldview constructs meanings on a social basis and that are located in various specific contexts. The knowledge so constructed is influenced by human beings making personal choices interacting with their world, and is therefore subjective, active, individualistic, personal, and founded on previously constructed knowledge (Punch 2009).

Second, the socio-constructivist ontology of interpretivism means that realities are seen as local, specific, social and experimental. Moreover, reality is constructed through the interaction between language and aspects of an independent world which is culturally derived and historically situated (Crotty, 1998; Scotland, 2012). Therefore, interpretivism is particularly well-suited to the engineering context where the plethora of constantly changing scenarios forces engineers to rethink the nature of reality and how it needs to be modified to suit different circumstances and contexts. In the engineering context, reality is not regarded as unique because different engineers have different life experiences, cultural values and workplace exposures that influence their view of a problem and how it should be treated. Therefore, the ontological position of interpretivism is relativism where reality subjectively differs from person to person, and reality emerges when consciousness engages with objects that have meaning (Creswell, 2007; Crotty, 1998; Guba & Lincoln, 1994). Moreover, as language intervenes to label objects and actively shape and mould reality, reality is constructed as interaction between language and an independent world (Frowe, 2001).

Third, because interpretivism involves the subjective standpoints of the participants, the researcher also influences the study and hence is a participant in the study (Crotty, 1998). Therefore, with regard to truth, the standpoint is subjective because truth emerges from the life experiences of the study's participants. The researcher needs to be sensitive to the setting, what happens and how people involved see things so that the participants' multiple perceptions of their realities are reflected in particular contexts (see Section 1.2 and Section 7.6.7). It behoves researchers to make their own values known in a study (Creswell, 2007; Punch, 2009).

The above characteristics of the interpretivist paradigm support its selection for the present study. However, it is important to acknowledge that the freedom offered by the interpretivist paradigm may come at the expense of generalisability, which is not normally strongly characteristic of interpretivist studies (Crotty, 1998). Since there are multiple socially-constructed realities that tend to diverge in an interpretivist study, an enquiry must be studied holistically because the realities may interrelate strongly, or clash as with dualisms (Sfard, 1998). Therefore, 'truth' statements - in the positivist sense of enduring, context-free truth statements and grand theories - need to be abandoned since human behaviour is bound both by time, context, and other factors, such as age and culture. Instead, interpretivist studies produce 'working hypotheses' relating to a given and specific context. These studies may be applicable to other contexts, but they require a detailed examination of the given context of the original study with the context of a receiving study before accepting the "thick descriptions" provided by the original study into a receiving study (Lincoln & Guba, 1986, p. 75).

In conclusion, interpretivist research seeks to find meaning, relate experience, and provide rich data that carry the potential to identify new themes. It is ideally suited to this study's mechanical engineering workplace contexts where human beings make decisions based on factors such as personal choice, cultural norms and what best suits the needs of the current context.

3.1.1.2. **Interpretivism and this study**

Interpretivism sits comfortably with my philosophical position (see Sections 1.2 and 1.3). The consciously or unconsciously held deep beliefs of the researcher are revealed even in choosing the original research questions and their wording, the foci of the study, and the way questionnaire items and interview questions are chosen and worded. Interpretivism also applies well to understanding engineers' experiences because of the multiple social interactions involved in their learning and workplace practice. Moreover, constructivism acknowledges that “reality is socially constructed and can be understood only in context” (Willis, 2007, p. 24). Therefore, one focus of the study was on achieving deep understandings of that reality: the context and the culture from which it sprang. This is reflected in how the data were gathered and how the findings were interpreted in relation to the mechanical engineering trades context. The paradigmatic stances of interpretive social science according to Neuman (2003) are summarized in Table 2 together with correspondences with this study.

Table 2 *Properties of interpretive social science and correspondences with this study*

		Interpretive Social Science properties	Correspondences with this study
1	Reason for research	To understand and describe meaningful social action	This research seeks to understand and describe mechanical engineering trades mathematics, its application and development in individuals and in the community
2	Nature of social reality	Fluid definitions of a situation created by human interaction	The research acknowledges and seeks to investigate fluid definitions of multifarious technical and social situations created by human interaction
3	Nature of human beings	Social beings who create meaning and constantly make sense of their worlds	Engineers are recognised as social beings who create meaning within the clearly defined context of their workplaces to make sense of their workplace world
4	Role of common sense	Powerful everyday theories used by ordinary people	Powerful everyday mathematical theories, heuristics, and engineering judgment are used by ordinary - or sometimes extraordinary - people in conjunction with the sophisticated application of mathematics and numeracy
5	Theory looks like	A description of a group’s meaning system is generated and sustained	A description of a group’s meaning system is generated and sustained in such things as its pragmatic approach of relating meaning to context, and group participation in problem solving
6	An explanation looks like	Resonates or feels right to those who are being studied	See Section 7.6.4
7	Good evidence	Is embedded in the context of fluid social interactions	The qualitative data for this study is embedded in the context of fluid social interactions and tasks in the workplace context.
8	Place for values	Are an integral part of social life; no group’s values are wrong, only different	Values are accepted that are consistent with engineering practice.

Note. Adapted from Neuman, W. (2003). *Social research methods: Qualitative and quantitative approaches*. Boston: Pearson Education, Inc., p. 91

To summarize, Table 2 demonstrates the suitability of using an interpretivist paradigm in the context of the research questions as applied to the mechanical engineering trades context. In particular, the subjects' understandings of social reality, the emphasis on the interactions between human beings, and the use of common sense to produce sound decision making allow the researcher to pursue multiple aspects and investigate their interactions. An interpretivist approach has also been shown to work well in conjunction with the flexibility of a mixed methods methodology (McChesney & Aldridge, 2019) to which I now turn my attention.

3.1.2. **Mixed methods methodology**

In this section, I discuss mixed methods research, my reasons for using it in this study and how I use it, and how interpretivism provides a single paradigm suitable for analysing and integrating the quantitative and qualitative parts of the study.

Mixed methods research has been defined broadly as research in which elements of qualitative and quantitative research approaches are combined in the one study for the broad purposes of adding breadth and depth of understanding, and/or corroboration (Johnson, Onwuegbuzie, & Turner, 2007; Schoonenboom & Johnson, 2017; Tashakkori & Creswell, 2007). The rationale often stated behind using both qualitative and quantitative approaches is that each approach may reinforce their complementary strengths and mitigate the weaknesses in the other (Johnson & Onwuegbuzie, 2004). Therefore, one major purpose of using mixed methods designs is to enable expansion of understanding (Lopez-Fernandez & Molina-Azorin, 2011), which seeks to analyse and explore different facets of a phenomenon to achieve richer and more detailed understanding.

This study employed an explanatory sequential mixed methods design, which is one of several ways of performing mixed methods research. Explanatory sequential mixed methods designs typically involve two phases (Creswell, 2014). In the first phase, the researcher collects and then analyses quantitative data from a large number of participants. In the second phase, a much smaller group of participants is chosen to provide more detailed qualitative data. The first phase frequently involves conducting a survey, and the second phase semi-structured interviews, as in this study. The intended purpose of the first phase is to obtain data on a wide variety of factors thought to be relevant to the study, whereas the purpose of the second phase may be to "explain quantitative results (significant, nonsignificant, outliers or surprising results)" (Creswell & Plano Clark, 2011, p. 32). This was the approach taken in the present study.

In the sub-sections that follow, I provide further discussion of three key aspects related to the use of mixed methods methodology: the integration of methods, the choice of methods, and the strengths and weaknesses of mixed methods research.

3.1.2.1. **Integration of methods**

Integration of the qualitative and quantitative aspects of mixed methods research is considered to be a defining strength of mixed methods research (Creswell & Plano Clark, 2011; Greene, 2007; Guetterman, Molina-Azorin, & Fetters, 2020; Johnson et al., 2007). It can also be a source of controversy, such as in relation to the potential incompatibility of paradigms between qualitative and quantitative approaches . However, integration means more than collecting two separate sets of data as in two separate studies; it requires careful juxtaposition of related themes within the one study so that the insights gained by each approach are able to be examined and revealed together. Moreover, the effect on data integration should be considered at each stage of the inquiry, not only with methods, methodology and paradigm, but also when designing the research questions (Creswell, 2002; Creswell, Klassen, Plano Clark, & Smith, 2013; Schoonenboom & Johnson, 2017). The intention of integration is to exploit the value of mixed methods methodology to maximise the insights obtained from the data (Guetterman et al., 2020).

I now discuss the nature and rationale for integrating findings from sets of qualitative and quantitative data sets. The intention of integration is to achieve a more nuanced picture that deepens and elaborates on understandings so that insights might be gained that would otherwise be missed (Bryman, 2007; Woolley, 2009). Johnson and Onwuegbuzie (2004) adopt a similar stance when they discuss the “fundamental principle of mixed research” (p. 18) where multiple data sets are collected with different strategies, approaches, and methods so that the result brings out the complementary strengths and non-overlapping weaknesses. They believe this principle is a major source of justification for mixed methods research and one which will be superior to mono-method studies. Therefore, integration should be taken to mean relating the various components of the investigation to each other and to be more than just a short commentary as an addendum to various sections of a quantitative study, or a few tables and graphs as an addendum to text in a qualitative study. In this way, whether or not a study is integrated could be defined as the extent that the qualitative and quantitative “components are explicitly related to each other within a single study and in such a way as to be mutually illuminating, thereby producing findings that are greater than the sum of parts” (Woolley, 2009, p. 7). However, the need to integrate quantitative and qualitative findings is not universally acknowledged or followed. Even although integration of data is crucial to mixed methods research, it is seldom seen even though the potential of the mixed methods approach depends on this (Bryman, 2007; Woolley, 2009).

In my study, the qualitative and the quantitative data had equal footing, even although the quantitative data collection began first. Each source of data was intended to “illuminate” or “complement” the other, that is, “seeking elaboration, enhancement, illustration, clarification of the results from one method with results from the other method” (Johnson et al., 2007, p. 115). The intention of taking a mixed methods approach was to make use of both quantitative and qualitative methods to provide a more complete picture of the apprentices’ and engineers’ learning, and to tap into participants’ perspectives and meanings. Good qualitative data thus has the potential to bring out the meaning behind statistical data (van Teijlingen, 2014). Therefore, in this study, the goal of mixing the types of method was “not to search for corroboration but rather to expand one’s understanding” (Johnson & Onwuegbuzie, 2004, pp. 18-19).

The study was set up in the belief that both types of data would yield insights into the research question under investigation. Accordingly, the findings must be integrated at some point, and since the quantitative and qualitative phases were undertaken at least partly concurrently then the findings must, at a minimum, be integrated during the interpretation of the findings (Johnson & Onwuegbuzie, 2004). That was the approach taken here. Where both quantitative and qualitative data were available, the quantitative is presented first and then the qualitative. This allows the surrounding discussion and interpretation to integrate the findings from the two sources of data and demonstrate their separate yet complementary insights. However, mixed methods methodology has for many years been the subject of controversy within the academic community, especially the lack of philosophical rigour surrounding integration of qualitative and quantitative approaches. I address these in the next section.

3.1.2.2. **Mixed methods, interpretivism, and choice of methods**

In this section, I discuss how an interpretivist stance applied to mixed methods studies has been proposed as a response to criticism over lack of philosophical rigour. This criticism came from the school of thought that believed the strong associations between paradigms, methodologies and methods consequently rendered “different methodologies and methods to be philosophically incompatible, making their combination logically impossible” (Bazeley, 2002, p. 3). Various approaches were made to resolve this incompatibility and achieve greater flexibility, such as pragmatism and dialectical pluralism. For example, Schoonenboom and Johnson (2017) believe that the “incompatibility thesis does not always apply to research practice” (p. 113). However, they do not apply this universally, but to the restricted area of equal-status studies.

The alignment of methodologies with methods lies at the heart of the incompatibility issue, and hence with the type of data being collected. Therefore, there could never be a resolution without changing perspective. Willis (2007) changes the perspective by changing the focus on the type of data collected to the foundational assumptions and underlying beliefs of each of the qualitative and quantitative paradigms. He attributes this approach to Teddlie, who recommends multiple paradigm use, “criticizes the ‘paradigm purists’ who work only within one paradigm and proposes, instead, that we all become mixed-method researchers” (Willis, 2007, p. 29).

Crotty takes flexibility further when he says, “Certainly, if it suits their purposes, any of the theoretical perspectives could make use of any of the methodologies, and any of the methodologies could make use of any of the methods” (Crotty, 1998, p. 12). Therefore, since the interpretivist approach looks for culturally derived and historically situated interpretations of the social world, it has been freed from the idea of employing positivist research methods and paradigms and allows the relevance and reality of a diversity of research methods to be used (Crotty, 1998; Willis, 2007; Yin, 2006).

In more recent times, McChesney and Aldridge (2019) have proposed a single-paradigm interpretivist approach which they apply to both qualitative and quantitative aspects of mixed methods studies, thereby favouring “flexible (but intentional) integration of any research method with any research paradigm” (p. 225). In this way, interpretivism, which relates well with rich data sets obtained from disparate sources such as questionnaires and interviews, provided a single paradigm for the mixed methods research in my study.

Moreover, a single interpretivist standpoint signifies that the data sources, the methods used and the results were regarded from the same standpoint because the data emerged from the participants’ various “conceptions of reality” (McChesney, 2017, p. 22), and the study was therefore freed from the quantitative-qualitative dichotomy.

Hence, I conclude that there is a body of scholarly opinion that finds it unnecessary to maintain the traditional alignment of qualitative methods with interpretivist paradigms and quantitative methods with positivist paradigms. In the case of my study, just one paradigm, interpretivism, was used for both parts of the mixed methods study employing both quantitative and qualitative methods.

3.1.2.3. **Strengths and weaknesses of mixed methods research**

Mixed methods research has both strengths and weaknesses. Johnson and Onwuegbuzie (2004) list various advantages and disadvantages of mixed methods research. One advantage is that mixed methods research can “provide a broader and more complete range of research questions because the researcher is not confined to a single method or approach” so that additional insights and understandings can be added that “might be missed when only a single method is used” (p. 21). According to Punch (2009), mixed methods research employing quantitative methods is able to contribute the strengths of “conceptualizing variables, profiling dimensions, tracing trends and relationships, formalizing comparisons and using large and perhaps representative samples” (p. 290). In contrast, employing qualitative methods brings the strengths of “context, local groundedness, the in-depth study of smaller samples, and great methodological flexibility which enhances the ability to study process and change” (p. 290). Another advantage of the questionnaire lies in its brevity, which means that it can be given to a large number of participants and then analysed relatively quickly.

Johnson and Onwuegbuzie (2004) provide a list of suggested weaknesses of mixed methods research which appears below:

- Mixed methods research involving qualitative and quantitative methods can be difficult for a single researcher to carry out, especially if two or more approaches are expected to be used concurrently; it may require a research team.
- The researcher has to learn about multiple methods and approaches and understand how to mix them appropriately.
- Methodological purists contend that one should always work within either a qualitative or a quantitative paradigm.
- Mixed methods research is more expensive.

- Mixed methods research is more time-consuming.
- Some of the details of mixed research remain to be worked out fully by research methodologists (e.g., problems of paradigm mixing, how to qualitatively analyse quantitative data, how to interpret conflicting results).

Note. Adapted from Johnson, R., & Onwuegbuzie, A. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, *33*(7), p. 21.

Despite the alleged weaknesses listed above, not all of which apply to this study, the advantages of using a mixed methods approach outweigh the disadvantages, especially by enriching the statistical data with interviews. Moreover, the objection of paradigm mixing is obviated by employing a single interpretivist paradigm (McChesney & Aldridge, 2019).

To summarize Section 3.1, using an interpretivist paradigm together with a mixed methods methodology allows flexibility of approaches to the collection and presentation of data, their interpretation and meaning. Therefore, statistical data concerning various issues from questionnaires and associated interview data can be used to construct a rich data set that will provide a composite picture of the phenomenon under study. In the next section, I discuss the two theoretical frameworks selected for this study, which will further explain how this composite picture will be obtained.

3.2. **Theoretical framework considerations**

This study uses two theoretical frameworks - Cultural Historical Activity Theory (CHAT) as the main framework, and Situated Learning (SL) as a sub-framework. According to Zevenbergen and Begg (1999), the importance of a theoretical framework lies "in providing the overarching framework for the project, in its conceptualisation, analysis and writing …" (p. 170). First, I outline and discuss CHAT, define its terminology, introduce and define the terms of Engeström's (1987) expansive learning model, and discuss studies that employ a CHAT theoretical framework. SL was introduced in Chapter 2 because of the need to review the literature surrounding the impact of social interaction in workplace environments (see Section 2.5). Therefore, in Section 3.2.4, I compare the CHAT and SL theoretical perspectives. I leave the discussions of how the CHAT and SL theoretical frameworks applied to this particular study to Section 3.3.

3.2.1. **Cultural Historical Activity Theory**

In this section, I discuss the CHAT theoretical framework used in this study. Together with the interpretivist paradigm, the CHAT and SL frameworks guided the research design, the collection of data, the presentation of data, and interpretation of results. CHAT is a theoretical frame of reference, or set of research perspectives, and has been defined as "a cross-disciplinary framework for studying how humans purposefully transform natural and social reality, including themselves, as an ongoing culturally and historically situated, materially and socially mediated process" (Roth, Radford, & LaCroix, 2012, p. 1). The CHAT framework has been successfully applied in a wide variety of vocational and other studies, such as teaching and education settings (Ahmed, 2014; Roth, 2004; Wilson, 2014), information systems (Crawford & Hasan, 2006), public health systems (Engeström, 2001), workplaces (Engeström, 2000; Hoyles et al., 2013), differing school and workplace perspectives on mathematics (Williams, Wake, & Boreham, 2001), and the learning of specific workplace mathematics topics (Kent et al., 2011; LaCroix, 2011a; Roth & Lee, 2004). The purpose of CHAT is to help understand and analyse the relationships between the human mind and activity, or between what people think or feel and what people do (Nardi, 1996; Roth & Lee, 2007).

CHAT derives from the ideas of Vygotsky (1930, 1978) and subsequent theorists such as Leont'ev (1978, 1981), Cole and Wertsch (1996), and Engeström (1987, 1999). Vygotsky's wider work relates to this study for several reasons. His work arose from his rejection of claims that maturation alone leads to adult intellectual functions. Vygotsky's proposed solution was that learning was strongly influenced by social interaction, which could lead to change or expansive learning. The importance of social interaction in workplace learning and change, which are directly related to this study, would later become the focus of research by Engeström (1987, 1999). Vygotsky recognised the importance of tools developed and used by humans as mediators to achieve some purpose. Also, Vygotsky's (1978) notion of the zone of proximal development (ZPD) connects learning to problem solving, which is important in the engineering context of this study. The ZPD is defined as "the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving … in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Vygotsky's approach, therefore, emphasizes dynamic, social, and cultural factors in learning in answer to his dilemma about maturation alone being sufficient to explain adult development.

I now outline the historical development of and rationale for CHAT. According to Engeström, three distinct generations of CHAT can be identified. The first concerns Vygotsky's idea of mediation, which is expressed as a triangular model of subject, object and mediating artefact (Engeström, 1999; Vygotsky, 1978) (see Figure 4).

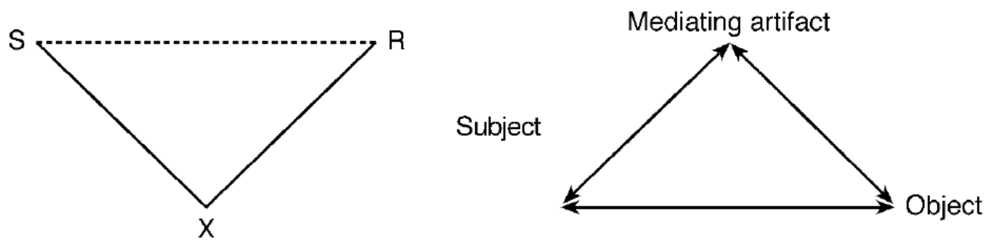

Figure 4 (A) Vygotsky's model of mediated act and (B) its common reformulation

Note. Reprinted from Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*(1), p. 134. [4]

[4] Permission received

The mediating artefact may be a physical tool, as is common in mechanical engineering, or it may be intellectual and socio-psychological, such as mathematics or language. Any devices such as graphs, diagrams, and written language may be regarded as tools. Learning is assumed to be influenced by such physical and psychological tools (Cole & Wertsch, 1996; Nygård, 2010). One difficulty of this simple model is that the unit of analysis is focused on the individual, so the portrayal of complex behaviour involving several key elements or several people is restricted and the generalisability is therefore limited. Therefore, Vygotsky’s model needed to be extended to display and investigate complex interrelationships and social interactions within a community, such as apprentices and the skilled tradespersons they work alongside and initially learn from, or even the globalised marketplace where skills are transferable and subject to competition.

Therefore, the second generation of CHAT, attributed to Leont’ev, introduced the idea of collectivity. This resulted in the addition of a focus on the division of labour, or roles, to Vygotsky’s model shown in Figure 4. Conceptualisations of collectivity, division of labour and roles evolved and necessitated differentiating between an individual action and a collective activity (Engeström, 1999). These changes in the division of labour can create specialisations within a community of labour, which are one source of potential tensions, or contradictions. In a negative sense, contradictions may be unhelpful. Viewed positively they can be seen as important motivating forces in creating conditions for progress; therefore promoting the evolution of (mainly) gradual change in response to changing political and economic conditions, or the impact of new technology that renders some skills obsolete (A. Edwards, 2011; Engeström, 1987, 2000; Gedera, 2015; Williams & Wake, 2007). Such changes can also render those people who specialised in those skills redundant, as can be seen in New Zealand where the mechanical engineering branch of toolmaking has declined as a result of globalisation and companies moving industry offshore in search of cheaper labour.

A third-generation in the development of CHAT is represented by Engeström’s learning model, and especially its notion of expansive learning which Engeström regards as “a historically new type of learning which emerges as practitioners struggle through developmental transformations in their activity systems, moving across collective zones of proximal development” (Engeström, 1987, p. 7). This iteration of CHAT involved the development of conceptual tools to understand dialogue, multiple perspectives and voices, and networks of interacting activity systems (Engeström, 1999, 2008; Nygård, 2010), including the replacement of static teams with “fluid knotworking around runaway objects that require and generate new forms of expansive learning and distributed agency” (Engeström, 2008, p. i). Therefore, each development in the evolution of CHAT can be viewed as an attempt to better describe and synthesise increasingly complex understandings of the nature of workplace activity.

Engeström (2010) distinguished his theory of expansive learning from other models of learning because it focuses on the learning of communities. Therefore, expansive learning and practice in communities become central notions. Moreover, expansive learning is important to understanding the workplace because “learners learn something that is not yet there. In other words, the learners construct a new object and concept for their collective activity, and implement this new object and concept in practice" (p. 74). This is relevant to the workplace because learners are frequently confronted with unfamiliar problems whose solutions are messy and undefined (Roth, 2010). In this way, Engeström considers traditional mentalist theories to be inadequate. Moreover, like Sfard's (1998) two metaphors of acquisition and participation (see Sections 2.4 and 2.5), Engeström considers that traditional theories have little to say about the transformation and creation of culture (Engeström, 2010). In the next section, I define and explain further the terminology of the CHAT framework and its relationship to my study.

3.2.2. **Definitions of terms in Engeström's CHAT framework**

This section discusses Engeström's third-generation CHAT framework which comprises four questions, seven elements and five principles (Engeström, 2001) and how well they resonated with the mechanical engineering trades context of this study (Moffitt & Bligh, 2021). I first discuss Engeström's questions, elements and principles in turn.

Engeström's four questions "Who are the subjects of learning?", "Why do they learn?", "What do they learn?", and "How do they learn?" (Engeström, 2001, p. 133) were all relevant to the mechanical engineering context, and hence had some relevance to this study, although to varying extents. The first two questions, "Who are learning?" and "Why do they learn?" are not related directly to the research questions for this study (see Section 1.3) and can be answered briefly; viz., all the engineers in a positive workplace environment were learning mathematics, and the reason they learned mathematics was that they acknowledged its value in their daily work. In contrast, Engeström's third and fourth questions corresponded directly to the aims of the research questions. They concern the nature of mathematical learning, what mathematics mechanical engineering tradespersons learn, and how they use and learn it.

Regarding Engeström's principles, an activity system is the central concept. An activity system typically consists of subject(s), object or objective, outcome, tools, rules, community and roles (see Figure 5). Engeström's principles are the activity system, multi-voicedness, historicity, contradictions, and expansive cycles of learning. Engeström's elements are tools, rules, roles and community. Tools may be regarded as physical or mental and are developed by human beings to achieve some purpose. The community is the group of people associated in some way with the activity (Wenger, 2011). The community of this study not only comprised other mechanical engineers with whom an individual may have had contact from time to time, but also bosses, designers, draughtspersons, and clients and company representatives who interact directly with them. Each of these groups is drawn together to cooperate in order to achieve a common object or objective. To be successful they must resolve their often conflicting multi-voices and even competing aims, which have historical roots leading to contradictions, or tensions. Contradictions, in turn, are seen as stimuli for expansive learning. Roles, or the division of labour, refer to the contributions members of the community make to the overall achievement of the objective of the activity system. The rules are a set of expectations that the community makes for the achievement of its objectives.

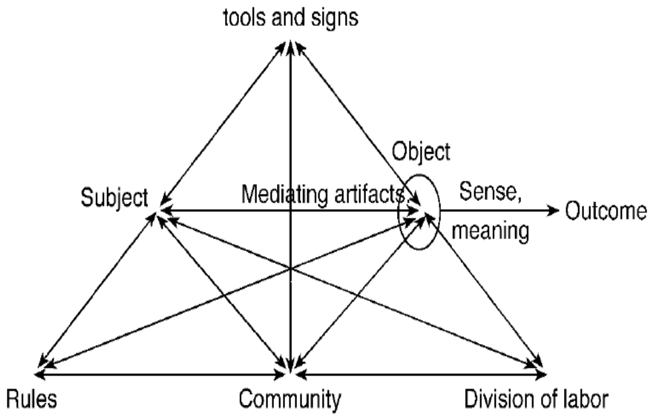

Figure 5 The structure of a human activity system

Note. Reprinted from Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. Retrieved from https://www.tandfonline.com/doi/pdf/10.1080/13639080020028747, p. 135.[5]

It is important to understand that in the context of a particular study, these elements are fluid and are constantly changing depending on which aspect is currently the focus of attention. For example, communities may be large or small, and the rules and the roles played by individuals may change according to time and circumstances. A strength of Engeström's triangle in Figure 5 for my study is that its simple geometric properties allow interconnections to be made between the elements and how they relate to the principles. The triangle also allows an easy transition of thinking to be made when the focus of attention changes. In this way, CHAT provides a dynamic rather than a static lens to examine the engineering workplace.

There is variation and possible confusion in the definitions of elements and principles and possible confusion because much of the earlier literature about CHAT and its developments were written in Russian, and some words translated into English were not exact equivalents (Yamagata-Lynch, 2010). Therefore, in this thesis, "subjects" are always people of interest, as opposed to topics of discussion, and "object" is interpreted as a noun, meaning "objective" - that is, the goal or aim being pursued. The questions and fundamental concepts are discussed in greater detail later in this section and used extensively throughout the thesis. A major advantage of CHAT is that focus can be placed on each element and principle separately, or, even more importantly, on their multiple interconnectedness.

I now discuss each of the principles of Engeström's activity systems in turn - multi-voicedness, historicity, contradictions and expansive cycles of learning. It is important to note that these are interconnected with each other and with the elements, and that building in division of labour (roles), rules and community transformed Vygotsky's model (see Figure 4) into the CHAT framework diagram in Figure 5. Also, Engeström believed that activity

[5] Permission received

systems were subject to long-term, historic contradictions, or tensions, which were accompanied by many voices involving differing opinions and perspectives (Engeström, 1987). Contradictions have the potential to eventually lead to change, which he calls expansive learning.

The first principle within Engeström's expansive learning model is a "collective, artifact-mediated and object-oriented *activity system*, seen in its network relations to other activity systems" (Engeström, 2001, p. 136) (emphasis added). An activity system is taken as the prime unit of analysis. The actions of individuals and groups within this system as well as its automatic operations are interpreted as subordinate units of analysis within the background of the entire activity system. The activity system contains a community representing multiple standpoints that were sometimes created by historical influences and divisions of labour. These often conflicting factors within the activity system are frequent sources of trouble, yet are also potential sources of negotiation and change (Engeström, 2001). The discussion of the activity systems in this study is found in Section 3.3.2.

The second principle within Engeström's expansive learning model is *multi-voicedness,* which included the wide spectrum of views, perspectives and traditions of families, schooling, curriculum influences and teaching styles, and governmental education policies (FitzSimons, 2003). However, it is important to note that multi-voicedness can emphasize voices within the total system, as in Engeström's study on Finnish hospitals (Engeström, 2001), or voices within an individual in the system, as in FitzSimons' (2003) paper on Marja's learning. This allows Engeström's learning model to be adapted to apply in situations where the activity system as a whole is the focus, or where the focus is on an individual within the system.

The third principle within Engeström's expansive learning model is *historicity,* which refers to the history of individuals or organisations with various procedures and tools. Therefore, historicity may apply to large organisations and the evolution of their particular procedures and tools (Engeström, 2001), or alternatively, it may refer to an activity system containing one individual subject with a "unique history of life experience, work experience and education experience" (FitzSimons, 2003, p. 53). Activity systems develop and transform over long periods of time, and to some extent, the problems they face need to be understood in the light of that history. This includes the history of its theoretical ideas and tools.

The fourth principle within Engeström's expansive learning model is *contradictions,* which were seen by Engeström as sources of change and development, but different from problems or conflicts. Contradictions are understood to be structural tensions that have built up historically within or between activity systems (Engeström, 1987). For example, when an activity system adopts some new technology or object, the result often is "an aggravated secondary contradiction where some old element (for example, the rules or the division of labour) collides with the new one. Such contradictions generate disturbances and conflicts, but also innovative attempts to change the activity" (Engeström, 2001, p. 137). As an illustration, FitzSimons (2003) lists several contradictions she sees in Australian education policy. These include:

- Discrepancies between policy and practice in adult education
- The incompatible aims of lifelong learning versus neoliberal ideas of ‘user pays’
- Politicians’ exhortations to raise numeracy standards versus a chronic lack of recognition by policymakers of the need for discipline-based professional development for tutors and literacy teachers teaching numeracy
- The side-by-side existence of new and old curricula and pedagogical practices in mathematics education
- The discipline of mathematics and its related pedagogical practices which do not necessarily encourage learners

Note. Adapted from FitzSimons, G. (2003). Using Engeström's expansive learning framework to analyse a case study in adult mathematics education. *Literacy and Numeracy Studies, 12*(2), p. 54.

This last contradiction sometimes comes to the fore when attempting to explain to educators in schools and elsewhere that mathematics in the workplace is much more than a mere extension of classroom exercises and practices, and that contextual approaches are required for learning to take place.

A historically-based contradiction can also arise between generations as a new generation of workers gains experience and skill and attempts to change the system to suit their own innovations and ideas. Therefore, there is a link here with Lave and Wenger’s (1991) ideas embedded in their SL model (see Section 2.5). In this case, the contradiction emerges as younger members begin to move closer towards the centre of Lave and Wenger’s community of practice. CHAT does not analyse this phenomenon in detail, thereby providing one rationale for a second framework (see Section 3.2.4).

Finally, the fifth and key principle within Engeström’s model is *expansive learning* (see Section 3.2.1) which is a key feature of CHAT’s five principles and involves developmental transformations in activity systems and moving across collective ZPD’s (Engeström, 1987). Expansive transformations are “accomplished when the object and motive of the activity are reconceptualised to embrace a radically wider horizon of possibilities than in the previous mode of the activity” (Engeström, 2001, p. 137). Therefore, an expansive cycle in individuals can be exemplified when they acquire and then put new learning into practice.

Expansive transformations are performed in a stepwise manner and have relatively long-term cycles. They come about as a result of contradictions in the activity system becoming aggravated to the point where certain individuals within the system start questioning accepted practice. These people analyse the contradictions and model “a vision” (Engeström, 2000, p. 960) for a zone of proximal development of the system which they then examine and incorporate in practice against the traditionally accepted norms of the activity system. Sometimes the result is a deliberate collective effort to promote change. Expansive learning processes are important to this thesis because they relate how new learning takes place in engineers and their activity systems to establish higher levels of development of understanding and skill.

3.2.3. **Engeström's expansive learning model matrix**

Engeström's learning model of an activity system is associated with the four questions, seven key elements and the five principles (see Section 3.2). Engeström's expansive learning model matrix is presented in Table **3**.

Table 3 *Engeström's expansive learning model*

	Activity system as a unit of analysis	Multi-voicedness	Historicity	Contradictions	Expansive Cycles
Who are learning?					
Why do they learn?					
What do they learn?					
How do they learn?					

Note. Adapted from Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*(1), p. 138. [6]

The general function of this matrix is to present Engeström's questions and principles together. This enables both brief descriptions to be made of how each question and each principle relate, and to gain an overall view of a study. In my study, this matrix also provided a structure for the analysis of the findings and allowed the systematic interrogation and analysis of the interactions of a range of factors that may have contributed to the learning of mechanical engineering apprentices along their pathway to becoming skilled tradespersons.

There is a large number of workplace studies that employ CHAT frameworks (Ahmed, 2014; Batiibwe, 2019; Engeström, 1990, 1993, 2000, 2001, 2013; FitzSimons, 2005; FitzSimons & Mlcek, 2004; Meyers, 2007). Engeström (2001) applied his expansive learning model to analyse a health care treatment system in Finland for children with multiple illnesses (see Appendix D). The matrix was able to illustrate how various decisions were made and implemented involving transfer between and coordination of multiple patient care activity systems. It is important to note that some cells in the matrix for the Finland hospital study were left empty, which indicates that the researchers chose particular questions and principles as being relevant to their study and ignored the others. Therefore, while the expansive cycle involved only the "What do they learn?" and "How do they learn?" questions of the activity system, Engeström discussed each of the five principles in detail. A similar approach of leaving certain cells blank was adopted in a study on student-teacher perceptions of effective ways for promoting critical thinking through asynchronous discussion forums (Mwalongo, 2016). Another study used CHAT frameworks to illustrate two different objectives regarding school and workplace transition of graphical skills; adapting from the ways graphs are used in

[6] Permission received

a school setting to an industrial context and interpreting the graphical output of an experiment (Williams et al., 2001).

Similar patterns of focus and analysis were followed in this study. In particular, while Engeström's "Who is learning?" and "Why do they learn?" questions were relevant to the mechanical engineering context, the research questions in this study focused particularly on Engeström's "What do they learn?" and "How do they learn?" questions (see Section 3.2.1). Hence, no emphasis was placed on the "Who?" and "Why?" questions. The correspondences between Engeström's elements and principles displayed in Engeström's triangle and matrix and those in the studies cited above align well with similar correspondences in my study. A discussion and corresponding table of Engeström's expansive learning model for my study are found in Section 3.3.2 and Section 3.3.3. Therefore, Engeström's expansive learning model for CHAT is adaptable to many different situations. In the next section, I discuss Situated Learning and its relationship to CHAT as frameworks for this study.

3.2.4. **Situated Learning and Cultural Historical Activity Theory compared**

In this section, I outline SL as a second theoretical framework for this study (see Section 2.5.1) and then compare the contributions SL and CHAT frameworks have made to this study.

SL is defined by Lave and Wenger (1991) as a socio-cultural theory in their important theoretical treatise, *Situated Learning: Legitimate Peripheral Participation*. The notion of legitimate peripheral participation (LPP, see Section 2.5) is central to SL which is seen as a situated activity in which newcomers become part of a community of practice as members on its periphery. As newcomers increase in experience, knowledge, skill and learning, they gradually become full participants in a sociocultural practice (Lave & Wenger, 1991; Matusov et al., 1994). One strength of SL is that Lave and Wenger have applied their theoretical ideas of learning to a large number of rich, specific contexts, such as midwives in Mexico, tailors in Liberia, butchers in US supermarkets, quartermasters (i.e., assistants to navigators) in the US Navy, and non-drinking alcoholics in Alcoholics Anonymous. The tailors' and quartermasters' training bear particular relevance to this study: the tailors because of their apprentice-mentor relationships and the quartermasters for their mathematical training.

SL provides an approach to learning that integrates agents, world and activities, and where experience is associated with meaningfully structured situations (Lave & Wenger, 1991). In order to reformulate thinking and learning away from mentalist approaches, Lave and Wenger make use of the notion of practice, which is regarded as an integral part of the lived-in world and experienced through social practice. Knowledge is regarded as a way of acting within a community of practice, and this means that social perspectives play a primary role in shaping and constituting reality and social practice. Experience also applies to learning and is linked with relationships among people involved in activities associated with the socially and culturally structured world. Indeed, human relationships are a feature of SL. Therefore, actions are situated in their local and immediate social contexts and are not something external to activity (Arnseth, 2008; Engeström, 2010; Lave & Wenger, 1991).

CHAT and SL have similarities and differences. Both enable creative theorising about learning and thinking, which they regard as integral parts of practice in a world influenced by social and cultural norms. They both employ the notion of practice to overcome the limitations of educational theories which prioritise mind and mental processes. They also agree that learning and teaching are placed in historical and material contexts and involve the integration of practice in a socially and culturally influenced world. Most importantly for the purposes of this study, in their theoretical frameworks, they also acknowledge the factors of history, development, transitions and change, social interaction in the learning process, and human activity mediated by physical and especially non-physical tools that are situated in context (Arnseth, 2008).

There are also differences between SL and CHAT. While CHAT regards practice in a broad sense, Lave and Wenger regard practice as predominantly social and relational (Arnseth, 2008). Regarding learning, SL views learning as "an integral part of generative social practice in the lived-in world" (Lave & Wenger, 1991, p. 35), while Engeström regards expansive learning as being produced in societal practice by "mental and material extension and transformation in time, as an integral aspect of activity" (Arnseth, 2008, p. 291). For the purposes of this study, this contrast in approach to learning was demonstrated in the relational components of workplace social interaction provided by SL and the transformation of individuals and communities of practice as a result of expansive cycles of learning. Another contrast is that Lave and Wenger seem to grant a privileged position in their framework to how people make sense of, interpret and constitute their world through practical action. Therefore, for Lave and Wenger, "learning is an integral part of generative social practice in the lived-in world" (Arnseth, 2008, p. 291), so that SL tends to pay closer attention to what people do in concrete situations and the resources they employ in their activities, as with the tailors and the quartermasters. Also, Engeström seems to focus more on the instrumentality of activity, so that while CHAT regards practice in a broad sense, Lave and Wenger regard practice as predominantly social and relational (Arnseth, 2008). Therefore, while CHAT allows detailed focus to be placed on change and development, it "makes it more difficult to study how the things happening here and now is [*sic*] structurally related to wider patterns of human activity" (Arnseth, 2008, p. 301). CHAT offers a more external perspective than SL. CHAT demonstrates how activities change, develop and interconnect with social and material structures. It does not focus on the internal perspectives of how the participants themselves actually make sense of their surroundings, which is apparent in Lave and Wenger's (1991) accounts of tailors and quartermasters.

Another distinguishing characteristic of CHAT is that learners are learning something that is not there yet, which requires problem solving in contexts where much of the information and thinking has still to be developed (Engeström, 2010). However, this has not appeared explicitly in my earlier discussion of Engeström's diagram because the discussion focuses on the key elements of the *process* of apprenticeship learning: its elements and its principles. As

Engeström explains, one advantage of CHAT over some other situated action or sociocultural theories is that CHAT asks what connected the activity to the historical transformations of people's lives and societies (Ploettner & Tresseras, 2016). Therefore, CHAT enables us to investigate how activities evolve and "interconnect with social and material structures" (Arnseth, 2008, p. 301). That was important in this study because forces inside and outside the communities of practice keep the mechanical engineering workplaces in permanent states of flux.

To summarize, in Section 3.2 I have described how the elements and principles of the CHAT framework relate to various aspects of learning and practice. However, while CHAT tends to be useful in providing a wider focus on groups, this research study also required an investigation that focused on individuals. Also, while CHAT remained the principal theoretical framework for this study, SL made important contributions, especially in specifying the details of the context. Therefore, employing both SL and CHAT in my study gave a more comprehensive and composite picture containing complementary perspectives of the breadth of skills needed to apply mathematics skills in the mechanical engineering workplace. In the next section, I discuss how the theoretical implications of this section relate to my own study on mechanical engineering trades mathematics.

3.3. **Research frameworks and this study**

In the previous sections, I discussed the CHAT and SL frameworks, and Engeström's expansive learning model. In particular, I explained that employing CHAT and SL together would give a more comprehensive and composite picture of mathematics in the mechanical engineering workplace than with just a single framework. I now explain how the CHAT and SL frameworks were applied in this study. I begin with an overall view of the research design of this study, followed by a discussion of its activity systems, how Engeström's matrix of expansive learning corresponded with the research questions, and how the CHAT and SL frameworks were applied.

3.3.1. **Research design of this study**

In this section, I discuss the research design, which comprises all the issues involved in planning and executing a research project, including its relationship with the research questions (Punch, 2009). I first consider the two broad questions of what I need to know, and whether I should use a qualitative, quantitative, or a mixed methods design (Creswell, 2009; Punch, 2009). I discuss each question in turn.

What I wanted to know is defined by the research questions which concern the key features of mathematical learning that characterise the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering trades, the nature of the mathematics skills, how the mathematics skills are applied, and how they are developed (see Section 1.3). The study was also designed to investigate how the mathematical content is learned and used in both formal and informal settings involving social interaction.

A mixed methods approach (see Section 3.1.2) was chosen because it had the potential to combine both quantitative and qualitative approaches, and allowed one method to help and inform the other method (Creswell, 2009). Mixed methods research “attempts to consider multiple viewpoints, perspectives, positions, and standpoints” (Johnson et al., 2007, p. 113), and is appropriate in situations where it is assumed that collecting diverse types of data best provides an understanding of the research problems. It is particularly well-suited to investigating research problems that require “an examination of real-life contextual understandings, multi-level perspectives, and cultural influences”, and has “an objective of drawing on the strengths of quantitative and qualitative data gathering techniques to formulate a holistic interpretive framework for generating possible solutions or new understandings of the problem” (University of Southern California Library, 2018, n.p.).

In the next section, I discuss the activity systems and the relationships between an individual’s activity system and their wider activity systems within a CHAT framework.

3.3.2. **Activity systems for this study**

This section defines and discusses the relationships between larger activity systems in this study and smaller activity systems, including individual activity systems, and how they relate to Engeström’s other principles (See Figure 5). In the largest activity system under consideration, the subjects were those who were learners of mechanical engineering, and therefore obviously apprentices, but also skilled tradespersons and educators, supervisors and any others engaged in formal or informal learning connected with mechanical engineering. It is important to note that these groups overlap where subjects could occupy multiple positions in the activity system, such as both educators and experts, or as both educators and learners (see Section 5.2.2.1).

It is possible to define multiple objects and outcomes for any activity system depending on the focus of attention. A community comprises people who are, whether directly or indirectly, connected to the activity, and is “left largely as an intuitive notion” (Lave & Wenger, 1991, p. 42). In this study, a community’s members were connected directly with the workplace and could comprise the apprentices, skilled mechanical engineering tradespersons, educators, mentors and polytechnic tutors, or indirectly, such as employers and clients. Tools and signs are broadly interpreted to mean any physical object, language, sign, or person who directly or indirectly promotes learning. Tools also included people who acted as role models involved in conversation or mentoring, communication and social interaction, and intellectual tools such as problem solving. The influences of these tools are discussed later where appropriate and within their contexts. With regard to the division of labour, or roles, it was possible for a person to have multiple roles, as with the discussion of tools above. It was also possible for the nature of the roles to change for both apprentice and educator as the apprentice grew in skill and experience.

The participants in this study were each involved in many different and sometimes non-overlapping activity systems. Each activity system had its own four key elements of tools and signs, rules, community, and roles or division of labour (see Figure 5). Therefore, a change in an activity system could be made by changing any one of the four key elements, especially subjects and community.

Moreover, activity systems are flexible and can be changed according to the focus of interest. In this case, a change of focus will require a change in subjects and possibly also in the community, tools, roles, and rules. There will also be qualitative changes when the focus is changed from larger to smaller activity systems, such as at the local workplace level. The community would be much smaller and probably less culturally diverse than if the community were a country taken as a whole. Similarly, expanding the community to include family or whānau would result in a different activity system with different rules and roles and possibly tools than if the community were restricted to workplace groups only.

The inclusion of individual activity systems containing just one subject is important to this study. While Engeström accepts that an individual may be part of many activity systems, his own focus was often on large organisations where expansive cycles related to changes in the largest activity system being considered rather than individuals within that system. Also, Engeström's applications in large organisations involved complex, interrelated issues which were often novel, and hence there was no competent and experienced 'teacher' available who knew what had to be learned. In these cases, knowledge needed to be constructed, an example of problem solving (Engeström, 2001). However, activity systems may be small as well as large. Therefore, referring to third-generation activity theory where the unit of analysis is expanded to at least two interacting activity systems, Engeström cites the example of schooling, which is "analysed as dynamics within and interplay between the activity systems of the student and the teacher, possibly also including other relevant activity systems" (Engeström & Glăveanu, 2012, p. 516). In my study, the interplay within and between activity systems existed at multiple levels, for example, a large company with multiple departments, a group of engineers coming together to work on a particular project, or student-teacher and apprentice-mentor interactions.

Therefore, it is important to note that considering two or more activity systems does not necessarily imply a large number of subjects. The references to the student and the teacher having their own "activity systems", and attention being paid to the dynamics of the "subject" suggest that activity systems may have one subject, and where the focus of the learning is on that one individual subject. However, the community of that individual activity system may still contain many people. Therefore, Engeström's emphasis that the individual cannot be understood apart from the activity system still applies. Other scholars have adapted Engeström's learning model to apply to individual activity systems where the focus of interest may be just one subject rather than any of their much larger activity systems (FitzSimons, 2003; Williams et al., 2001). In such cases, individuals not only belonged to large activity systems, but also belonged to an individual activity system where they also experienced multi-voicedness, historicity, contradictions, and individual expansive cycles of learning.

A further consideration is that expansive cycles in an individual's activity system may (or may not) result in an expansive cycle in their larger activity systems. If the largest activity system being considered is very large, as with mechanical engineers, an expansive cycle in an individual may not be identifiable in the total activity system, even if it was known to have taken place. The question of the interaction of individual activity systems with larger activity systems is therefore important. According to Lautenbach (2011) in a study of university lecturers' engagement with educational technologies, individual expansive cycles of learning were unique to the individuals, but individual lecturers provided new insights to others in their larger activity systems derived from the emerging expansive cycles within their individual activity systems. Moreover, further development in expansive cycles in the larger activity system depended on "a strong and extended community in which lecturers can share, think and grow" (Lautenbach, 2011, p. 713). Therefore, in these cases, individual expansive cycles of learning and social interaction are necessary components of long-term expansive cycles of learning in the larger activity systems being considered.

In summary, consideration of both individual and larger activity systems were relevant to my study, since the engineers worked at both individual and group levels, especially in problem-solving situations.

3.3.3. **Engeström's matrix of expansive learning and the research questions**

This section relates the research questions to Engeström's matrix of expansive learning. Each section of the findings, discussion and interpretation is also related to the main research question and associated sub-questions of this thesis which concern the key features of mathematical learning, the nature of the mathematical skills used, how they are applied, and how they are learned and developed (see Section 1.3). There are two foci in these research questions - learning, which in this context refers to an emphasis on mathematical aspects of learning in a mechanical engineering context, and pathway, which suggests the process of development of the apprentice during the apprenticeship years into a fully qualified tradesperson.

Table **4** shows brief outlines of how Engeström's "What do they learn?" and "How do they learn?" questions, elements and principles relate to this study. The notes are intended to be illustrative only and are not intended to be exhaustive.

Table 4 *Adaptation of Engeström's matrix for this study, Chapter 4 and Chapter 5*

	What do they learn? How is it applied? Chapter 4 Main Research Question, Sub-question 1, Sub-question 2	How do they learn? Chapter 5 Main Research Question, Sub-question 3
Activity System	Defined according to Engeström's diagram (see Figure 5) and where the subjects were either individuals or groups	Defined according to Engeström's diagram (see Figure 5) and where the subjects were either individuals or groups
Multi-voicedness	What voices representing different points of view were based on branch specialisation, or generation?	How frequent were voices of different points of view dependent on branch specialisation, generation?
Historicity	What skills and attitudes to mathematics might have resulted from childhood, school and other experiences?	To what extent were attitudes to learning instilled from the school approach to mathematics learning?
Contradictions	To what extent were contradictions due to school experiences of mathematics regarded as irrelevant to the *real world*, recognition of only minimal procedural knowledge concentrating on particular skills only *versus* the need to foster creativity etc., and wide education?	Were contradictions caused between formal and traditional approaches to learning mathematics versus informal constructivist approaches?
Expansive Cycle	How difficult was the transition from the school approach to mathematics learning to a workplace approach and demands?	How much more attention was given in the workplace approach to direct practical application of mathematics than at school?

To summarize, Table **4** outlines the application of Engeström's matrix to my study. In the next section, I discuss how the discussion above is applied to the methods used in the study.

3.3.4. **CHAT and Situated Learning theoretical frameworks in this study**

In this section, I outline the contribution CHAT made to this study and discuss correspondences between my study and the CHAT framework. I then discuss the reasons for adopting a CHAT framework. I follow this by explaining the advantages of using both the SL and CHAT frameworks and how they were used.

With regard to the CHAT framework and this study, an important contribution CHAT made to the study was to allow the analysis, understanding and interpretation of the various mechanical engineering activity systems, some aspects of which are illustrated in Table 5.

Table 5 *The seven key elements in Engeström's learning model and my study*

Elements of Engeström's model	Identification of Engeström's elements in this study
Subjects	Mechanical engineering trades apprentices and tradespersons
Object	The object is related to identifying key features of mathematical learning, the nature of the mathematics skills, and how they are applied and developed (see Section 1.3)
Tools	Schooling and apprenticeship training which were intended to provide numeracy, physical tools, language, and mathematical competence
Community	Mechanical engineering apprentices, skilled tradespersons, experts, educators, and employers
Rules	Formal legal regulations and implicitly agreed-on understandings on what constituted proper mechanical engineering practice and training
Roles	Mutual responsibilities of apprentices, skilled tradespersons, experts, educators and employers
Outcome	The promotion of competent and confident mechanical engineers

The activity systems are portrayed as interacting elements whose combined operations produce one or more outcomes. Moreover, CHAT shows how people's roles can change depending on specialist skills younger members and other new arrivals had obtained from prior learning. Therefore, younger members of the community appeared as subjects, tools, and even having roles as mentors for enhancing the learning of more experienced engineers (see Section 5.2.2.1).

While Engeström's principles and elements can be examined singly, the arrows in Engeström's triangle (see Figure 5) indicate that CHAT can identify, describe and analyse many complex relationships in an activity system. In this way, CHAT reveals the interconnectedness and complexity of learning, the ongoing, non-linear social nature of the learning process, and the roles of relationships within a community of practice. The interconnectedness of Engeström's elements and principles is therefore particularly relevant to learning and practice in the mechanical engineering trades context. In addition, the CHAT framework adopted in this study allowed an efficient and clear description, analysis and integration of the data, which came from questionnaire items and semi-structured interviews.

With regard to justifying a CHAT theoretical frame of reference for this study, I discuss four further reasons below, which related to the research questions as well as how the data from questions and surveys were analysed. First, there was the identification of appropriate tools to develop mathematical and numeracy competence among mechanical engineering apprentices. In this case, school and apprenticeship training, mentoring, block courses, discussions with already qualified tradespersons were all examples of tools. Second, a CHAT theoretical frame of reference provided an appropriate and effective way to analyse and therefore gain understanding of (1) the key features of mathematical learning that characterised the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering trades (see Section 1.3), and (2) to analyse how the mechanical engineering tradespersons' school and apprenticeship training served as means of developing mathematical and numeracy competence among mechanical engineering apprentices. Third, a CHAT theoretical frame of reference assisted in understanding the *social structure* of the mechanical engineering community; in particular, how individuals interacted with the community and with each other, and how these interactions may have assisted, or perhaps hindered, the development of mathematical and numeracy competence among mechanical engineering apprentices. Fourth, a CHAT theoretical frame of reference enabled the researcher to *manage and make sense of the data* obtained from the real world by creating themes that could be described, analysed and interpreted. This applied in particular to the complexities of the engineers' work and social interactions.

This study employed two theoretical frameworks. It is not unusual to choose a second framework as a sub-framework contained within a broader framework. For example, Galligan (2011) chose Valsiner's theory to focus on adults' development in mathematics, and FitzSimons et al.'s (2005) discussion of mathematics and numeracy in chemical-spraying-situated workplace numeracy tasks within a broader framework of social-historical and cultural practice. Similarly, LaCroix's study of pre-apprenticeship plumbers learning imperial units employed the sub-framework of Radford's Theory of Knowledge Objectification (TKO) within a CHAT workplace (LaCroix, 2009, 2011a, 2011b, 2014). LaCroix's studies on the development of awareness of imperial units of Canadian apprentices show links with similar development in the apprentices and engineers in this study.

The frameworks in this study needed to incorporate such important factors as the engineers' daily work which employs socially and historically evolving practices, the engineers' cognitive understandings which are also influenced by key social elements, and the importance of the change in thinking processes involved along the pathway from school to workplace (Williams & Wake, 2007). However, while CHAT acknowledges the primacy of social and cultural key elements in learning, a satisfactory theoretical framework for my study needed explicit connection with those aspects of the research questions concerning who is learning, how the skills are applied, and how the skills are learned. It also needs to show how cognitive aspects of the mechanical engineering workplace are developed and applied, and how experienced people influence the formal and informal development of younger people. Therefore, especially for the formulation of how mathematics skills are applied and developed, issues that are crucial to this thesis, a second framework was employed, SL (Lave & Wenger, 1991).

Since the engineers and apprentices often worked closely together to complete their projects, then social interaction and communication were important components in the development, learning and transmission of mathematical and other tools. SL was chosen as a second theoretical framework because it emphasizes social interaction in the learning process.

Particularly pertinent points of SL for this study were the concepts of community of practice and legitimate peripheral participation (LPP) where a newcomer joins a community on its periphery and then gradually moves towards its centre with experience, increasing knowledge, skill and expertise (Lave & Wenger, 1991).

To summarize, I saw CHAT as a broader framework, one of whose strengths was generalisability of context, while the strength of SL was its ability to fill in contextual detail. For example, the concept of LPP enables focus to be placed on the community of practice with insights to be gained on the developmental changes in both individuals and their communities. Of course, these are also part of CHAT, but only implicitly so. Therefore, an important difference is that SL allows important trends to be investigated in detail. Similarly, while CHAT acknowledges contradictions produced by historical factors and rules in general terms, SL helps the researcher focus on issues such as breaking bad habits “acquired in school” (Lave & Wenger, 1991, p. 73) and the whole process of increasing maturity accompanied by growing independence from mentors. Another contrast is the role played and the development of higher-order thinking and problem solving overlaps in both CHAT and SL, with each contributing different insights and perspectives. In the next section, I discuss the details of the methods used in this study.

3.4. **Methods**

This section describes the data generation including the development of the questionnaire items and interview questions, the sampling procedures used for recruiting participants, quality considerations, and how the data were analysed. The data were in the form of questionnaires and semi-structured interviews. These methods were deemed to be efficient and unobtrusive ways of forming a rich data set in a mixed methods design. The participants were mechanical engineering apprentices, tradespersons and educators, and others (see Appendix E). A total of 199 people responded to the questionnaires and 17 were interviewed.

3.4.1. **Data collection tools**

The data were obtained from questionnaires and semi-structured interviews incorporating a mixed methods design and were understood in the context of the interpretivist paradigm (Crotty, 1998; McChesney & Aldridge, 2019). The data are described in more detail later in this section. Elements of qualitative and quantitative research approaches were combined to add breadth and depth of understanding (Johnson et al., 2007; Schoonenboom & Johnson, 2017; Tashakkori & Creswell, 2007). In this way, using both qualitative and quantitative approaches was so that they may each reinforce the complementary strengths and mitigate the weaknesses in the other (Johnson & Onwuegbuzie, 2004). Therefore, the analyses explored different facets of a phenomenon to achieve richer and more detailed understanding (Lopez-Fernandez & Molina-Azorin, 2011). In this study, the purpose of the quantitative part of a study was to gather statistical information about how mathematics was used and learned in the mechanical engineering workplaces, while the interviews sought to delve into the engineers’ views about its dynamic and process aspects. Also, the questionnaires were used to inform the direction of the flexible semi-structured interviews, both during and before the interviews commenced. Interviews with participants, while they were away from their machines and not working, were chosen in preference to direct observation because of the potential to cause distraction, which could cause expensive mistakes, machine damage, or even personal injury. Interviews also enabled me to ask more sophisticated follow-up questions which would not have been possible during observations.

3.4.1.1. **Development of the questionnaire items**

Three separate questionnaires were written; the first for pre-apprentices in avionics, a second for apprentices, and a third for skilled tradespersons, educators and employers (see Appendix F, Appendix G and Appendix H). The avionics pre-apprentices follow a 36-week course involving mechanical engineering and electronics components related to flying aircraft. Those who successfully complete the course may then consider further specialisation in either area. They have been included in this study because the avionics pre-apprentices tend to be better mathematically qualified than apprentices in other mechanical engineering specialisations.

The questionnaire items produced data on the participants' demographic details, attitudes to mathematics at school and in the workplace, their school qualifications in several subjects, how confident they felt about mathematics, and their views on the best ways of learning mathematics. The different questionnaires for each group contained similar questions with appropriately modified wording, depending on the experience of the participants. For example, in the series of questions with the wording "How often do you think most mechanical engineering tradespersons use the following mathematics topics?", the wording for the pre-apprentices was changed to "How often do you think your school experiences have prepared you in the following mathematics topics?" The alteration of wording, in this case, was because the pre-apprentices tended to have little or no direct mechanical engineering workplace experience. The responses to the items were either constructed on a Likert scale with a range of one to four (to avoid non-committal midpoint responses), or contained open-ended responses (Garland, 1991; Nadler, Weston, & Voyles, 2015).

A thorough examination of the mathematics, physics and mechanics skills contained in Appendix C was made to decide on the topics and possible wording for the questionnaire items. This process gave me an overall idea of the mathematics, physics and mechanics applications involved. I also prepared some questionnaire items about social interaction and learning. The initial draft of the questionnaire items was then discussed with industry representatives. This resulted in some repeated and irrelevant items being omitted, and other items being added.

Since it was not at all apparent how detailed investigation of social interaction in the mechanical engineering trades workplaces could be comprehensively covered in the questionnaire items, this was left as a major focus of the semi-structured interviews. However, the questionnaire items did contain sections on the engineers' preferred methods of learning, including aspects of communication and socio-cultural learning. The questions and the wording were then rechecked by several experienced engineers in the training organisations and my supervisors. Several changes were made to the wording in some items to remove ambiguity. The engineers also strongly supported my suggestion to include questions relating to how easy people had found mathematics at school and how they had liked it, how well they coped with mathematics in their workplace situation, and how concerned the tradespersons were about numeracy capabilities among both apprentices and tradespersons. Their advice was incorporated into the final version of the questionnaires (see Appendix F, Appendix G, and Appendix H).

3.4.1.2. **Development of the interview questions**

Interviews are one means of understanding meaning in the subjects' lived worlds. Since the interviewer registers and interprets the meanings of what the subject says, then interviews are consistent with an interpretivist paradigm (Kvale, 2008). In particular, semi-structured interviews were chosen for this study because they allowed flexibility to explore new facets of the participants' experiences and views that arose, sometimes unexpectedly as passing comments, during the interviews. Such flexibility enriched understandings already known by the researcher but also had the potential to identify previously unknown issues.

Regarding developing the questions for the semi-structured interviews, a series of proposed questions and appropriate probes were prepared for ethics approval (see Appendix K). These questions centred around “What mathematics skills and knowledge are used in mechanical engineering?”, “How are skills in mechanical engineering mathematics developed and used?”, “What comparisons do you identify between school and mechanical engineering mathematics?”, “How are problem-solving skills and extended reasoning in mathematics used in mechanical engineering?” and “What is the role of electronic aids in calculations, and the design and control of machines?”

As with the questionnaire items, these questions were submitted to the engineering educators for comment. The statement in the ethics proposal regarding participants bringing a model or a drawing of a chosen project to the interview was not followed, mainly because of time and physical size considerations (see Appendix K). Instead, I asked them to tell me about their experiences of various projects on which they had been involved, complete with sketches drawn on paper. This added to my understanding of their means of communication.

Flexibility in both the style and wording of the questions proved to be essential when conducting the interviews. With the skilled tradespersons and especially the educators, an open-ended question would likely lead to a lengthy response, often coupled with reminiscence and reflection on the meaning of their response. I attribute this to the confidence that often came with maturity (Knowles et al., 2011; Merriam, 2018; Mezirow, 1994, 2018). With the apprentices, especially beginning apprentices, interview responses were often quite brief and contained little elaboration. Mentors tended to give apprentices tasks commensurate with their engineering and mathematical skills, and therefore the apprentices may not have had the necessary experience and knowledge to provide detailed, ongoing responses. However, where the apprentices did have the knowledge and my question was sufficiently specific, they were able to provide good information and commentary. Even beginning apprentices were particularly forthcoming on their latest projects, the way they were being trained on-the-job, and their relationships and communication with the other apprentices and tradespersons who frequently went out of their way to help them. Semi-structured interviews enabled me to quickly focus my questions on areas the participant was likely to be qualified to answer.

3.4.2. **Procedure for recruiting participants**

This section describes the procedures followed for recruiting participants for the questionnaires and the semi-structured interviews.

3.4.2.1. **The questionnaire participants**

My initial contacts with mechanical engineers and the organisations where they worked were through Competenz, the official organisation that develops national trades qualifications throughout New Zealand. Competenz offered to make my study known to all their affiliated organisations. I also contacted the mechanical engineering educators of every polytechnic in New Zealand, and using the internet, some companies within travelling distance in the substantial light industrial areas near where I live in south-east Auckland. The result was that I was able to gain 199 responses from a wide range of different types of engineering branches and companies.

My contacts felt it would be easier for them to photocopy the questionnaires themselves, scan their responses and then send them back to me electronically. As far as the timing of the interviews was concerned I fitted in with whatever was most convenient to the participants and employers. Negotiation with the relevant authorities proved easy to conduct as most people were as keen to interview me as I was to interview them. I entered the questionnaire responses as soon as they were returned, and once patterns in the data had begun to emerge in the questionnaire data collection, I made a decision to begin interviewing a few carefully chosen educators and company training officers (Creswell et al., 2013). These interviews closely followed the questions in Appendix K. These early interviews confirmed that it was appropriate to include social aspects of the engineers' work in the interviews because the engineers frequently emphasized its importance in their interview responses. From time to time while I was updating the quantitative data I would reflect on and review the interview process in line with emerging statistical trends.

Flexibility in the interview situation that is characteristic of semi-structured interviews allowed quick changes of direction during the interview to suit the knowledge strengths or weaknesses of the participant. Most importantly, I was able to concentrate attention on previously unsuspected fruitful lines of inquiry. Creative engineers and problem solvers were two groups that were of particular interest to me because these engineers occasionally suggested ideas for discussion that interested them. This had the advantage of uncovering unusual and unexpected insights into how they felt about their work and their understanding of it. Therefore, the study followed an Explanatory Sequential Mixed Methods Design, but with two important modifications (S. Othman, Steen, & Fleet, 2021). First, it was impossible to achieve Creswell's ideal of a random sample that might guarantee representativeness. Second, while the study contained the two phases described above by Creswell, the collection of the qualitative data from the semi-structured interviews began at a suitable stage before the quantitative data collection and analysis from the questionnaires were completed.

To summarize, the participants were recruited using a snowball approach to seek out informed participants. The interviews were begun before the questionnaire data were all received, and the flexibility of the semi-structured interviews approach enabled me to use the questionnaire data and interview participants' responses to redirect the interviews to explore issues raised by participants. In the next section, I discuss how the sample of participants for the interviews was selected.

3.4.2.2. **The interview sample**

For the semi-structured interviews, a "purposive sample" (Punch, 2009, p. 162; Suri, 2011) of 17 people were selected initially of so-called "key informants" (Sarantakos, 1993, p. 183) from people in the industry assumed to represent the various points of view and perspectives. They came mainly from the Canterbury and Auckland regions. Some of the people interviewed had multiple roles and skillsets - 5 were apprentices, 13 were skilled tradespersons, 6 were educators, and 3 had roles in entrepreneurship or apprentice training. Therefore, it is highly probable that educators, being a very small minority of mechanical engineering tradespersons, were overrepresented in the interview sample. However, with a purposive sample, this was intentional, due to educators' special experience and perspectives, communication and teaching skills, perceptiveness, and knowledge of the overall mechanical engineering process, current issues and course development. Brief biographical details are given of each of the participants interviewed in Appendix E.

Another reason for using a purposive sample was that I wanted to focus part of the interviews on educators' views regarding apprentice learners and on the knowledge and perspectives of engineers with known reputations for innovation and creativity. This was consistent with using an interpretivist paradigm and I sought these individuals by asking throughout the community, especially through Competenz; hence the sample was both purposive and snowball.

Boyce and Neale (2006) provide a general heuristic for determining the number of people to interview in qualitative studies; "that when the same stories, themes, issues, and topics are emerging from the interviewees, then a sufficient sample size has been reached" (p. 4). However, this criterion for terminating the interview process was subjective and was likely to be achieved very early on when the questions were highly structured. Moreover, an additional complication about the subjectivity of criteria for choosing a suitable sample size has been made by Braun and Clarke (2021c) who emphasise that meaning is generated through interpretation of the data, and that determining how many data items should be collected cannot be completely determined before the analysis is begun.

On the other hand, in the semi-structured framework of this study, new topics were able to keep emerging from contextual examples given by the engineers from their wide range of experiences in multifarious branches of engineering. Attempts have been made to find a heuristic for selecting a suitable sample size by studying how long it took for the same stories and so on to emerge from a series of interviews. In one such project involving two different sets of interviews, it was found that 14 and 17 interviewees gave suitable data saturation criteria for "normative" beliefs without new ideas emerging (Francis et al., 2010, p. 1229).

My study involved interviews with a purposive sample of 17 people chosen partly by snowball principle and partly for their likelihood of representing various points of view and experiences. Therefore, the way the sample size was selected, the design and wording of the interview questions and the way the interview was conducted influenced the course of the interviews and, in turn, the analysis of the results. In the next section, I discuss the quality criteria employed in the procedures undertaken to design the questionnaires and interviews.

3.4.3. **Quality considerations**

This section discusses quality considerations surrounding the interpretivist paradigm used in this study. However, an important consideration was the assessment made by various scholars that while the choice of a research paradigm may not determine the choice of methods employed, the question of quality should be determined by the criteria within those methods. First, criteria for quality considered appropriate when employing positivist paradigms, such as objectivity, reliability, and internal and external validity were thus not suitable for the interpretivist stance of this study (Creswell & Miller, 2000; Crotty, 1998; Lincoln & Guba, 1986; Willis, 2007).

In this regard, various sets of quality criteria for qualitative studies have been proposed. One such example is discussed by Shannon and Hambacher (2014) who outline five dimensions of authenticity to consider when evaluating a constructivist inquiry. They are fairness (balanced approach to all sides), ontological authenticity (for example, participants' rights to know), educative authenticity (leading to greater knowledge and understanding for participants), catalytic authenticity (involving change), and tactical authenticity (power relationships) (citing Lincoln & Guba, 1985).

Methodological rigour in constructivist inquiry is therefore established through an assessment of trustworthiness and authenticity. Trustworthiness parallels the positivistic concepts of internal and external validity, focusing on an assessment of the inquiry process, while authenticity involves an assessment of the meaningfulness and usefulness of interactive inquiry processes and social change that results from the above processes. Authenticity is unique to constructivist inquiry and has no parallel in the positivistic paradigm (Shannon & Hambacher, 2014).

A further set of criteria proposes eight key markers of quality in qualitative research; a worthy topic, sincerity, credibility, resonance, a significant contribution, ethics, and meaningful coherence (Tracy, 2010). Another proposal for quality in qualitative studies has been proposed by Lincoln and Guba (1986). They give the following advice for credibility - prolonged engagement, persistent observation, triangulation, peer debriefing, negative case analysis, and member checks, by which they include soliciting participants' views concerning the researcher's current understanding and interpretations of the data. For transferability, they recommend thick descriptive data so that others can judge the degree of fit or similarity between the study and their own contexts where they may wish to apply all or part of the findings. For dependability and confirmability, Lincoln and Guba recommend establishing audit trails that can be carried through by others.

The overall purpose of adopting any such criteria is to ensure that the research data and methods followed are transparent to the reader, and the results are embedded in the data. Thus, following these criteria will lessen some errors and misinterpretations and allow readers to more fully and quickly understand an author's standpoint. However, the question of differences between individuals, philosophical schools of thought, interpretations of methods, data and interpretation will not be obviated by adopting these criteria.

With regard to quality criteria appropriate to this study, I have followed the approach developed by McChesney (2017), who, given the lack of a clear, pre-existing set of quality criteria suitable for interpretivist mixed methods research, combined considerations previously put forward by Willis (2007) and Creswell and Miller (2000). As in McChesney's (2017) study, I have documented the ways in which my study aligned with Willis' six techniques through which interpretivist researchers "conduct research in such a way that the consumer has some confidence in what you say" (Willis, 2007, p. 220). These six techniques are: member checks, participatory research methods, extended experience in the environment, peer review, researcher journaling, and audit trails (Willis, 2007). My study reflected five of these six techniques, with the exception of member checks (see Section 7.6.4). I have also documented the extent to which my study aligned with the nine common "validity procedures" adopted by Creswell and Miller (2000, p. 126) according to the paradigm they represent. My study incorporated disconfirming evidence, prolonged engagement in the field, and thick, rich qualitative description, which are the three of Creswell and Miller's nine procedures that are relevant for constructivist research. Table 6 details how each of Willis's (2007) and Creswell and Miller's (2000) points were enacted in my study.

Table 6 *Alignment of my study with the quality considerations for interpretivist and constructivist research identified by Willis (2007) and Creswell and Miller (2000)*

Research consideration	Recommended by Willis (2007) for interpretivist research	Paradigm classification by Creswell and Miller (2000)	Incorporation in my study
Extended experience in the research environment	Yes	Constructivist	I taught mathematics and physics in New Zealand secondary schools from 1972 to 2004. This gave me an extended, in-depth experience of young people about to enter the trades and other workplaces as apprentices. My

			experience of mechanical engineering came from interactions with my father and lengthy contact and discussions with engineering teachers in the secondary school environment over many years. I made frequent checks throughout the study, especially during the interviews, to ensure I was reflecting current engineering practice.
Member checks	Yes	Post-positivist	See 7.6.4 for the discussion of member checks.
Participatory research	Yes	Critical	My frequent involvement in interviews as a participant was designed to increase my own understandings of the engineering context and to elicit further thinking by the engineers.
Peer review	Yes	Critical	Questionnaire items and interview questions were extensively reviewed for comprehensiveness, suitability and clarity of language by my research supervisors, current practicing engineers, and one member of Competenz. Also, I frequently asked participants for their interpretations and general views of the questionnaire items and interview questions. Conference presentations and published papers relating to my master's dissertation on mathematics in the mechanical engineering trades context provided feedback from academic researchers.
Researcher journaling	Yes	Critical	Extensive notes were kept of important factors such as theoretical frameworks, paradigms, literature references, debates on philosophy and approach, research methods, decisions, and thoughts regarding emerging trends from the questionnaire data analysis. This journaling influenced the way the interviews were approached.
Audit trails	Yes	Post-positivist	Extensive documentation of participants' audio recordings, emails, analysis spreadsheets and SPSS files containing coded questionnaire data were retained for subsequent consultation. Thesis drafts were saved regularly and stored daily under the date for that day so that past versions remained available for review. Raw data, including hard copy questionnaires, data spreadsheets, interview notes, interview audio recordings, interview transcripts, and signed consent forms were retained and stored following the University of Waikato data management policies. Anonymised individual participant codes were used in presenting results (see Chapters 4 and 5) to demonstrate the way that the conclusions were grounded in the qualitative data obtained from the interviews, and then integrated with the questionnaire data.

Disconfirming evidence	Constructivist	Disconfirming evidence was frequently sought during the interviews, often with a view to teasing out deeper knowledge or exceptions, and then reported in the presentation of results (see Chapters 4 and 5).
Thick, rich description	Constructivist	Extended descriptions of engineering contexts in the findings extended and clarified the engineers' views to enrich readers' understanding of their worldviews. Direct quotations from the engineers' dialogues further enriched the findings and my understanding of them. This was to facilitate interpreting the results and how they might be transferred to other contexts.

To summarize, this section identified issues surrounding quality considerations and how they were addressed. Quality considerations associated with the interpretivist paradigm included such factors as fairness, authenticity, trustworthiness, credibility, and resonance. Therefore, while my prolonged engagement in teaching mathematics to teenagers and contact with mechanical engineers helped achieve these goals, it was also necessary for me to continually engage closely with the participants to ensure that my background knowledge was up-to-date. I also had the questionnaire items and interview questions peer-reviewed by current engineers and I frequently asked the interview participants for their views on the interview questions. In addition, the supplementary questions I asked were designed to improve my knowledge of the engineering environment and check for accuracy and comprehensiveness. I attempted to create a rich and thick description of the data that would allow others to judge how well my study might be transferred to their own contexts. The next section discusses how the data were analysed.

3.4.4. **Data analysis**

This section discusses the analysis of both the quantitative and qualitative data. The data consisted of two types: responses to questionnaire items and the transcripts of the semi-structured interviews.

3.4.4.1. **Quantitative data from the questionnaires**

As mentioned above, the quantitative data collection began first. As the data arrived, the participants' questionnaire responses were entered into an EXCEL spreadsheet. Some item numbers needed to be realigned from the original questionnaires since the three different questionnaires had variations in the numbering of the items. These data then formed one large table with numeric codes in the first column denoting the participant identification number. Each row in the table contained the data for each of the rating responses for each participant. At a convenient point, the data were transferred from EXCEL to SPSS to take advantage of its formatting and graphical procedures. Simple one-factor tables were drawn up, initially to get a feel for the data. Splits of some of the data sets were then performed to identify differences between groups of engineers, such as apprentices and skilled tradespersons, and two-factor tables were constructed of relevant pairs of factors, such as how easy the apprentices found mathematics at school and in their work. The overall intention was to identify and describe broad trends from the quantitative data.

3.4.4.2. **Qualitative data from the semi-structured interviews**

Both thematic analysis and CHAT were used to analyse the qualitative interview data. Thematic analysis (TA) has been described as " a method for identifying, analysing, and interpreting patterns of meaning ('themes') within qualitative data" (Braun & Clarke, 2006, p. 297). More accurately, TA is not just a single method, but rather a collection of methods. TA employs a series of steps: preparing the data for analysis, transcribing the data, becoming familiar with the data, memoing the data, generating initial codes, searching the initial codes for categories and then themes, reviewing the themes, redefining the themes, and then writing up (Braun & Clarke, 2006; Lester, Cho, & Lochmiller, 2020; M. Maguire & Delahunt, 2017).

Reflexive TA was chosen for this study because it

> captures approaches that fully embrace qualitative research values and the subjective skills the researcher brings to the process … Analysis, which can be more inductive or more theoretical/deductive, is a situated interpretative reflexive process. Coding is open and organic, with no use of any coding framework. Themes should be the final 'outcome' of data coding and iterative theme development (Braun & Clarke, 2021a, p. 333).

Reflexive TA can be approached in various ways, for example, inductive, deductive, semantic, latent, essentialist, and constructionist. It is important to understand that these orientations are neither fixed nor mutually exclusive, but are continua with many variations being possible. Moreover, the "separation between orientations isn't always rigid. What is vitally important is that the analysis is theoretically coherent and consistent" (Braun & Clarke, 2021b, n.p.).

The advantages of TA are its flexibility which allows new insights to be obtained and different ways of interpreting meaning to be performed. TA also facilitates investigations of phenomena across and within interview transcripts to be performed, and the interrogation of data. One disadvantage is that some data may be overlooked. Another disadvantage is that TA requires the researchers to be aware of their own subjective interpretations of the data. A summary of the process I followed in analysing the data from the semi-structured interviews is shown in Table 7.

I transcribed the recorded interviews and compiled them into a single compendium, in both paper and electronic form, with one chapter for each interview participant. I read through the transcripts from the interviews and identified patterns in meaning within and across the data. Successively reading through all of the transcripts and using a system of coloured highlighters allowed certain themes to be identified, marked and annotated in the margins of the transcripts. Some of the themes had been decided in advance, for example, the mathematical topics they needed to learn such as decimals and measurement, and what the apprentices needed to learn about other necessary skills, such as communication and problem solving. Other themes emerged either throughout the interview process or as I was reading the transcripts. Each transcript was then examined in more detail "for themes within individual issues, or between them, or running through the entire set of interviews" (Davidson & Tolich, 1999, p. 239) and immersing myself in the data reading "to make sense of the whole set of data and to understand what [was] going on" (Azungah, 2018, p. 383).

Table 7: *Phases of research showing cycles of iteration for emerging themes*

Phase 1: Preparation and identification of initial themes	
Research meeting 1	Discussion with supervisors of the aims of the project and potential semi-structured interview questions.
	Preparation of potential questions for semi-structured interviews.
Research meeting 2	Submission of potential questions to supervisors.
	Revisions made according to supervisors' comments; submission of questions to engineers; revisions made according to engineers' comments.
Research meeting 3	Meeting with supervisors; decision made to approve questions.
Data collection	Interviews conducted.
Data analysis	Interviews transcribed and read many times; NVivo codes created. Transcripts examined for emerging themes; memos and widespread annotations made to the compendium of transcripts.
Phase 2: Reflection and identification of further themes	
	Review of data to refine themes and identify further emerging themes, especially interconnectedness of themes. Themes redefined according to newly understood interconnections.
Research meeting 4	Themes discussed with supervisors
	Themes further refined and results written up, paying particular attention to interconnections

Identifying and inserting CHAT elements and principles also formed part of the reading and annotation process. This allowed me to investigate the effects of individual elements, such as physical or intellectual tools, and how they interacted with principles such as multi-voicedness, historicity, contradictions, and especially, expansive cycles of learning. This exercise was done iteratively by hand and eventually gave me a feel for the data, particularly who said what, and sometimes, why they said it. Engeström's elements and principles differed from theme to theme, for example, tools featured prominently when analysing knowledge and skills while roles and community became much more important when discussing learning and mentoring.

The transcripts were also given initial NVivo codes regarding general issues which were then used to investigate the themes that had arisen, with a view of identifying and understanding their interconnections (Lester et al., 2020). The main advantages of using NVivo in my study were to identify and highlight masses of detail, which created a hierarchal and cross-sectional picture of how the data had been developed, to read individual stories easily, or to look at the data via theme and then subtheme. Therefore, once the codes had been assigned, I was able in a short time to review all the comments made by the interview participants on any particular theme, and then compare different groups, such as younger and older engineers' views.

Some sections of the tagged passages overlapped several different themes. For example, an engineer's use of decimals in the context of his work might easily become part of an old-timer's yarns about the various stages of his own pathway, but now within the context of reminiscence and reflection (see Section 6.3.5). In this way, I could compare a series of snapshots reflecting the differences in outlook of engineers over several decades and age groups. This was particularly effective in comparing the old-timers with the young apprentices. However, one problem of using this approach was deciding what was unchanging and what could be ascribed to generational difference, including the impact of new people with innovative ideas and the desire to make change happen (Lave, 1977, 1985, 1989; Lave & Wenger, 1991).

3.4.4.3. **Integrating the quantitative and qualitative findings**

This section discusses the integration of the quantitative and qualitative findings. In order to make mixed methods research superior to mono-method studies, it is necessary that the integration process relates the various components of the investigation to each other. Without proper integration, the study can degenerate into two separate and unrelated studies (Yin, 2006). In my study, the qualitative and the quantitative data have equal footing and are intended to complement one other or to enhance or enrich the results from the other method. Accordingly, in order to allow the reader to follow more readily the integration, I decided to present both the quantitative and qualitative data pertaining to each topic investigated, their analysis and interpretation side-by-side in one section, rather than in separate sections as is sometimes the approach. Therefore, the quantitative data of mainly Likert Scale values simply provides a distribution of numbers of ticks for each box. This is undoubtedly useful for ascertaining the views of the group as a whole but tells us nothing about why an individual person ticked a particular box. Similarly, having the interview transcripts without the quantitative data would provide no information on the group taken as a whole, and while the individual information may be powerful, we would have no idea of its generality. However, having the interviewed participant's questionnaire responses available before and during the interview allowed me to draw out the detail of the individual's choices. Moreover, these quantitative data could influence the way I approached the interviews (see Table 6).

The study was set up in the belief that both types of data would yield insights into the research question under investigation. Accordingly, integration of the findings took place when placed side by side for interpretation and to benefit the reader's understanding (Johnson & Onwuegbuzie, 2004). Where both quantitative and qualitative data were available, the quantitative is presented first and then the qualitative. This allows the surrounding discussion and interpretation to highlight the aspects of the findings from the two sources of data and demonstrate their separate yet complementary insights.

In summary, this section focused on the development of the two data sources of this mixed methods, interpretivist study. Questionnaire items and structured-interview questions were prepared in line with the research questions and in consultation with supervisors and representatives from the mechanical engineering trades. Participants for the questionnaires were selected from apprentices, skilled tradespersons and educators by approaching contacts through Competenz and polytechnics in New Zealand. Interview participants were selected according to purposive sampling to include a preponderance of views of experts with detailed knowledge and experience of the mechanical engineering trades as well as tradespeople and apprentices. The main quality considerations surrounding the research were that it should be carried out in such a way that confidence could be placed in its conclusions and according to the quality considerations for interpretivist and constructivist research identified by Willis (2007) and Creswell and Miller (2000). Finally, regarding the integration of the quantitative and qualitative phases of the data, these were undertaken at least partly concurrently and during the interpretation of the findings.

3.5. **Ethics**

Ethics is an important issue in all areas of research and alludes to questions surrounding moral issues and decisions confronting participants and their organisations. Ethics concerns moral ideals, character, policies and relationships between people, and their related issues (Barry & Herkert, 2017; Miles & Huberman, 1994; Punch, 2009; Starrett, Lara, & Bertha, 2017). The following discussion is about ethics issues relevant to the context of my study. These are based on the New Zealand Association for Research in Education Ethical Guidelines 2008 (Smith, 2010), and the University of Waikato's Ethical Conduct in Human Research and Related Activities Regulations (University of Waikato, 2008) (see Appendix L).

The emphasis given to particular ethical issues varies according to context. Therefore, while confidentiality is important in child studies to protect vulnerable children, in my study confidentiality is an important issue to guard participants' personal relationships within the workplace, their knowledge of others' relationships in the workplace, and knowledge of company secrets. Therefore, it was emphasized that while every effort would be made to ensure both individual and organisation confidentiality, this could not be guaranteed. To achieve this, pseudonyms were used for individuals and care was taken to remove any identifying information in the final thesis and any reporting of findings. Similarly, while the findings could be reported in conference and written presentations, organisations were not identified as such, but were referred to as "the organisation".

Regarding the ethics of power relationships in interview situations, Corbin and Morse (2003) state that in semi-structured interviews, the “researcher determines the structure of the interview and agenda through the questions asked, [while the] participant controls the amount of information provided in responses” (p. 340). In the case of my study, an important ethical issue regarding power relationships was participants feeling compelled to give information that could breach their own privacy and anonymity, with potential social, financial, legal and political consequences for the participant, the participant’s workplace relationships, the participant’s family, and for the company. Accordingly, at the beginning of each interview, I reminded the participants of the ethical requirements surrounding their rights and my study (see Appendix I and Appendix J).

With regard to why this study is worthwhile and to whom, the study focuses on mechanical engineering mathematics, how it is applied in practice and how it is learned. Issues identified in the study are the importance of basic mathematics and numeracy skills, higher-order thinking such as problem solving, and social interaction. At the present time, only anecdotal evidence is available on how sophisticated engineering skills are developed and applied, the specific mathematics difficulties mechanical engineers face, and how many engineers continue to have difficulties even after they become qualified. Therefore, this study is likely to shed light on an underdeveloped area of workplace mathematics and to contribute to the overall understanding of how mathematics is applied and learned in one workplace context. The findings here may possibly provide pointers for understanding the application of mathematics in other workplace contexts.

With regard to who benefits from this research, this research is very relevant to the current situations in New Zealand and worldwide where there are shortages of skilled tradespersons and mathematics skills need to be enhanced. Moreover, the research has shown the importance of social interaction and communication in determining how learning takes place in the workplace situation. Therefore, this research may be of immediate benefit to the mechanical engineering trades community and in other vocations that are high users of mathematics.

The time commitment of participants to complete the questionnaires was about 15 minutes. The semi-structured interviews typically lasted 30 minutes and were scheduled at times and places convenient to the participants and, if relevant, their employers, who were also given appropriate information sheets and consent forms (see Appendix M and Appendix N). A small amount of time was also needed on the part of educators and company administrators to photocopy the questionnaires, which they agreed to do without any suggestion on my part, and to return them to me either by post or electronically.

Informed consent was required for all participants. However, since they were all 16 years of age or older parental or caregiver consent was not necessary. Information sheets were given to individual participants and their companies, where appropriate, to advise them about the research and to gain permission for participation, including site access. It was stressed at the beginning of the interview that there was no coercion being placed on participants to take part in the study and that the researcher was not involved in either a teaching or commercial relationship with the participants.

Concerning the protection and storage of data, it was explained that the information and data obtained from participants could be used in the thesis and other scholarly publications and/or publications. Further consent from the participants would be sought if dissemination of the information was to go beyond this. In all cases, pseudonyms were used to protect the identity of participants and the organisations they represented. However, it may be that certain of the participants and the organisations they represented may be known to each other, especially in the case of those who have been contacted as a result of being recommended by another participant.

Power differentials between the researcher and the participants were important. It was emphasized to the participants that they had the right to refuse to answer any question, or withdraw from the interview at any time, or withdraw information they had provided up until the data analysis began in June, 2017. In addition, the participants were treated with respect during the interviews. Actions on my part, such as listening carefully, not treating as inferior any lack of academic success, and speaking in a manner that was confidence-building and non-threatening, were all likely to help safeguard participants' rights and put them at ease.

There was also a wide range of ethnic and cultural backgrounds among the participants, including some recent immigrants. If social and cultural considerations were to become apparent during the research, then appropriate advice would be sought from Ngarewa Hawera, Associate Director Māori Education - Te Hononga School of Curriculum and Pedagogy who agreed to act as a cultural adviser. Accordingly, ethics approval for the study was submitted to the University of Waikato Ethics Committee and approved on 10 August, 2016.

3.6. **Chapter summary**

This chapter has detailed the methodology of the study and the methods employed to investigate the main research question and the three research sub-questions defined in Section 1.3. An interpretivist paradigm was chosen for this study to reflect an emphasis on the standpoints and personal choices of the participants. A mixed methods design was used with data being collected from questionnaires and semi-structured interviews. The hazardous nature of the mechanical engineering workplace ruled out interviewing engineers when they were working (see Section 3.4.1). CHAT and SL were chosen as theoretical frameworks for the study. CHAT tended to provide a wider focus on groups, while SL was useful for filling in the detail of the dynamics in the mechanical engineering workplaces (see Section 3.3.4).

The methods used to design the questionnaire items and interview questions in conjunction with practicing engineering educators were reported in Section 3.4.1. In total, 199 questionnaire responses were received and 17 people were interviewed. The manner in which the samples were chosen were described in Section 3.4.2. Contacts within the mechanical engineering trades were initially made through Competenz. A purposive sample was chosen to include the views of apprentices, skilled tradespersons, and educators, as well as people likely to be experts with detailed knowledge and experience. The sample was then extended using the snowball principle on the recommendations of these new participants.

The quality considerations and methods of data analysis were reported in Section 3.4.3 and Section 3.4.4. Questionnaire and interview data were integrated to form a rich data set that offered complementary viewpoints. The qualitative data from the interviews and the quantitative data from the questionnaires needed to be integrated to avoid the study becoming two separate studies (Bazeley, 2002; Johnson & Onwuegbuzie, 2004; Yin, 2006). Therefore, it was decided to present the two sets of data side by side for each section and sub-section of the findings. In this way, they together formed a rich data set to make their mutual contributions to understanding.

Ethics considerations were described in Section 3.5 and were based on the NZARE Ethical Guidelines 2008 (Smith, 2010), and the University of Waikato's Ethical Conduct in Human Research and Related Activities Regulations (University of Waikato, 2008) (see Appendix L).

In chapters 4 and 5 of this thesis I discuss and interpret the findings according to the methodology outlined in this chapter. Chapter 6 then discusses and analyses the findings in the light of the literature review, and Chapter 7 draws conclusions about the study.

Chapter 4. **Findings - The nature and application of mathematics knowledge and skills**

Introduction

The previous chapter explained the methodology and methods used to seek answers to the main, overarching research question: What key features of mathematical learning characterise the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering? In the next two chapters, I present the findings that emerged from the data collection and analysis. The findings are organised according to significant themes that relate to the overarching research question and the three sub-questions about the nature of the mathematical skills, how they were applied, and how the skills were learned and developed.

I report the findings in two chapters because the *nature and application* of mathematical knowledge and skills fit naturally together in one part (Chapter 4), while the *learning and development* of mathematical knowledge and skills fit naturally into a separate second part (Chapter 5). Chapter 4 is divided into two sections. Section 4.1 focuses on analysing the mathematics skills and knowledge, their importance, and the calculations performed in engineering contexts. Section 4.2 focuses on how the skills and knowledge are applied in mechanical engineering trades contexts, often requiring higher-order skills such as problem solving, creativity and extended reasoning. Section 4.3 summarizes the chapter.

The findings are seen through the lens of the CHAT theory as introduced in Chapter 3. In this chapter, an intersection with Engeström's third question "What do they learn?" is evident, while in the next chapter, the presented findings align with Engeström's fourth question "How do they learn?" Engeström's elements (tools, rules, community, and roles) and principles (activity system, multi-voicedness, historicity, contradictions and expansive cycles) feature in both chapters (Engeström, 2001; FitzSimons, 2003).

Since this study used mixed methods that involved multiple data sets obtained from questionnaires and interviews, I present the data in sections and subsections around common themes. The questionnaire and interview data and my interpretations of them are analysed side by side, thus linking and integrating the questionnaire and interview data together and with the themes in an ongoing way. I use vignettes from the interview data to illustrate the complex and rich detail of the engineers' thinking as they apply mathematics in practical contexts.

It was important to explore the nature of the mathematics knowledge and skills used by the participants because of their application in engineering workplaces. This includes not only basic mathematics and numeracy skills, but also ancillary, non-mathematical skills such as higher-order thinking, problem solving, creativity and extended reasoning. Ancillary skills are inherently woven into the nature of workplace mathematics; these findings are included because they were expected to form an integral part of applying mathematics in mechanical engineering trades contexts (see Section 1.3).

4.1. **Mathematics knowledge and skills**

The key factors that emerged from the data about mathematical knowledge and skills were the engineers' awareness of their importance, and the skills they needed with numbers, calculation in context, and mental calculation (see Figure 6). I begin by considering the engineers' views of how frequently they use mathematics skills in their work. This established a connection with the mathematical skills the engineers consider important and one possible answer to Engeström's second question, 'Why do they learn?'. Regarding Engeström's elements, this section is concerned mainly with intellectual or physical *tools* or how intellectual and physical tools interconnect. However, the community of engineers also played an important role, especially in deciding the rules governing how, and in what circumstances, the tools should be applied.

4.1.1. **The importance of mathematics knowledge and skills**

It is necessary to establish the importance of mathematics knowledge and skills to the mechanical engineering trades. One indication of this is given by analysing the ten questionnaire items relating to 'How often do you think most mechanical engineering tradespersons use the following mathematics topics?' (see Figure 6).

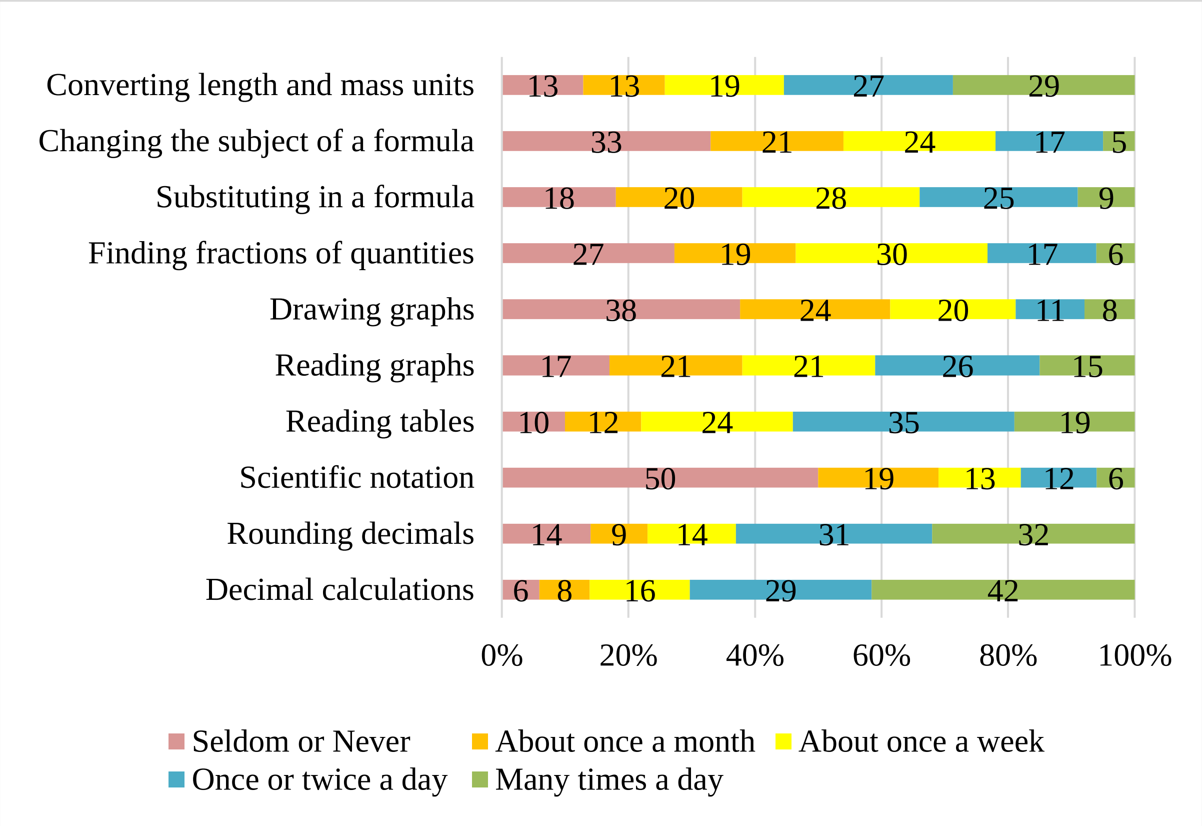

Figure 6 Percentages of educator/tradespersons and apprentices indicating how often mathematics topics were used, n = 172-175 participants per item

The data for these items were collected from approximately[7] 175 educator/tradespersons and apprentices (excluding the avionics pre-apprentices). Figure 6 shows that converting length and mass units, substituting in a formula, reading graphs and tables, and rounding and calculating with decimals were all used by at least one-third of the educator/tradespersons and apprentices at least once every day. Therefore, these are high-frequency skills that need to be highly developed in all engineers.

In contrast, changing the subject of a formula, drawing graphs and using scientific notation were seldom or never used by more than one-third of participants. While these skills may be low frequency and dependent on branch specialisation, they nevertheless are necessary and require high development in engineers in those branches. From the interviews, apart from the specialised welding branch, all engineers stressed the importance of mathematics and numeracy skills. For example, Murray[8] (engineer) specifically identified mathematical skills, such as reading graphs and scales, reading and interpreting numbers, measurement and reading scales as being important considerations of workplace efficiency.

The mathematical and higher-order skills can be regarded as tools in an Engeström activity system that are used to solve mechanical engineering problems. The data here briefly answer Engeström's (2001) second question, "Why do they learn?" and establish the importance of all ten topics for mechanical engineering trades because they were used extensively by at least some engineers on a regular basis. Reference to the importance of these skills and the complexities of their application will be made regularly throughout this thesis. Important skills were understanding and applying numbers and performing calculations, which I now discuss.

4.1.2. **Skill with numbers**

Appreciating the importance of number and its role in influencing the judgments and decisions that mechanical engineering tradespersons continually make is a key feature of mathematical learning. I examine here the need for calculation and measurement accuracy, and relate this to precision and tolerances.

4.1.2.1. **The need for calculation and measurement accuracy**

The views of three experienced engineers are analysed here on calculation errors. Calculation errors sometimes have serious consequences and are therefore a major contradiction in engineering communities. Robert (expert engineer, educator and entrepreneur) believed that a thorough grounding in number as he was taught in primary school, the "basics" as he put it, was "ABSOLUTELY essential" because they were used daily. Paul (training officer) expressed a similar view. He felt that apprentices correctly converting units for areas and volumes would be "about 50 : 50". When I asked if getting the decimal point wrong on the machine, or making something ten times too big or too small was important, Paul initially spoke quietly, but then became emphatic; "getting the decimal point in the correct place on

[7] The number of responses for each item differed because not all participants answered every item.
[8] Pseudonyms were used throughout this thesis for participants.

the machine is CRUCIAL … you can crash a machine and do some serious damage". However, he added that some machines allow simulations to check "if it's gonna shoot off one metre instead of ten centimetres". Paul revealed a contradiction when he said, "having that basic understanding allows you to be a better machinist, [but] technology means that knowing all the basics isn't a requirement and that some amazing stuff could be made without necessarily knowing the basics".

Incorrect decimal point placement usually has serious consequences, which the engineering communities wish to obviate. Stephen (avionics educator) acknowledged that "the problem of getting the wrong decimal place is probably one of the commonest problems we would have". One way to minimise errors is to build machines that simulate the action to be taken before actually performing it. This is a fail/safe mechanism. Another way is to employ checks and balances, such as tradespersons checking each other's work before machining begins. Calculation errors had important consequences, such as financial loss, either by destroyed material, or time loss. In some situations, inaccurate calculations could become safety issues, especially in the aircraft and other transport industries. However, Stephen also felt that where there were important situations involving safety, such as refuelling, where the checks and balances put in place were sufficiently rigorous that errors "couldn't really happen".

The engineers I interviewed regarded getting the right answer as very important. The need to calculate accurately is therefore a key feature of mathematical learning that needs to take place along the apprentice's pathway. Community involvement and influence were shown in the unity among the engineers about the need for calculation accuracy and about the serious consequences of errors such as financial loss, time loss, and safety.

4.1.2.2. **Precision and tolerances**

This section reviews the data on tolerances which refer to the permissible variation in the lengths in a finished product (see Section 2.4.5). They are related to mathematical conceptual and calculation issues, decimal place value, and hence to number sense and a feeling for size.

Paul (training officer) and many others regarded tolerances as crucial to their work. Paul's company made very large magnets for medical application and these required very fine tolerances expressed in microns (thousandths of a millimetre). However, tolerances in fabrication, for example, may be two or three millimetres, so it is important to note that the wide divergence of tolerances may be partly due to branch specialisation.

Fine tolerances are an important aspect of decimal use and involve number conceptualization as well as the appropriate degree of accuracy required for any given situation. Thus, Murray (engineer) spoke about specifications on limits on tightening bolts to avoid stripping threads and how to find the specifications to avoid this. Paul (training officer) emphasized the need for beginning apprentices to understand "immediately from day one … that tolerances are very important" in precision machining where Computer Numeric Control was used to make medical-grade equipment. Thus, tolerances are important to engineers because insufficient precision produces a faulty and perhaps dangerous product. Conversely, unnecessary precision leads to a waste of time and money.

As mentioned above, the use of decimals and other mathematical topics in engineering appears to be strongly context-based and influenced by the rules of the community and practical considerations. This was illustrated by Robert (expert engineer, educator and entrepreneur) who described how he used several different mathematical principles to find the volume of metal to make a small bolt with a hexagon-shaped head. Robert's problem was complicated by the metal needing to be strongly heated so that it expanded, and then (hopefully) would contract down to the correct size once it had cooled. This illustrated the interplay between theory and practice: how the engineers sought mathematics that they could use exactly, but then relied on engineering judgment to make the final decision about what fitted best in the real situation (see Section 2.3.1).

Sometimes very small measurements were crucial. Arthur related how the length of a piece of steel could alter by around 50 microns (fifty thousandths, or one-twentieth of a millimetre) due to a 10-degree Celsius temperature rise, and how ball bearings had tolerances down to 1 micron (.001 mm). Henry said that similar tolerances applied to the production of aircraft parts and Paul (training officer) said that four microns tolerance was required for cancer treatment magnets used in brain scanning. Thus, apprentices needed to learn to use a micrometer gauge and be able to machine[9] to those requirements (Courtney, mature engineer). On the other hand, Howard felt that maintenance engineering tolerances were often less precise than in other branches. Courtney said the same applied to fabrication, perhaps ± 3 mm, and then added that different branches required different skills. In fabrication, he needed "Speed!" to gain an edge over his competitors.

Based on the interviews with the mechanical engineers, it was clear that fluency, confidence, understanding and having a feel for numbers and number operations were important to them. This was especially true when fine precision was required. I now discuss how these issues worked out in context.

4.1.3. **Calculation skills in context**

The data showed that calculation was a frequently used skill in the engineering workplace. Converting within and between systems of units, substituting in and transposing formulas, finding data from tables and graphs, and using Pythagoras and trigonometry were all important mathematical tools that engineers used frequently. They were also used in conjunction with physics, which played an important role by providing contextual settings to apply mathematics in mechanical engineering workplaces.

[9] To machine to those requirements = to use a machine to produce those requirements

4.1.3.1. **Conversion of units**

Several engineers related how successfully converting units of quantities such as length, mass and volume posed challenges. This applied to converting between systems of units, such as metric and imperial, where 1 inch is equivalent to 25.4 mm. It also applied to converting within a system of units; for example, 36 mm is equivalent to 3.6 cm, and 1073 mm is equivalent to 1.073 m.

The challenge became even greater when area units were involved. For example, 1 m is equivalent to 100 cm but 1 m^2 is equivalent to 10000 cm^2. Thus, the challenges were both procedural (knowing how to move a decimal point a given number of spaces, and in which direction), and conceptual (understanding why a decimal point should be moved that number of spaces and in that direction). I first consider how the data illustrated converting length and mass units within the metric system, such as metres and millimetres, grams and kilograms. I then consider converting between the metric and the imperial systems, such as metres and feet, kilograms and pounds.

Metric conversions were performed frequently by the mechanical engineers. Thus, as shown in Figure 6, over half (56%) reported converting length and mass units at least once or twice a day, while one in eight reported seldom or never converting length and mass units. Therefore there was a wide variation in how often engineers converted units. Warren (educator) said that apprentices struggled with changing between metric units, such as mega-, kilo- and micro-. Owen (educator) concurred that those involved in electrical engineering were continually converting from milli- to centi- to micro- to deci- to kilo- and mega-. However, Nikau (apprentice) said that he always used millimetres in his fabrication work, so he did not have to change units.

With regard to imperial units, many engineering applications still use feet and inches for length, and pounds for mass or weight. Thus, Howard (engineer) confirmed that there were times when converting metric and imperial units were required for locomotives because the newer German engines were metric, “but on the old American engines we’re still using imperial”. Similarly, Paul (training officer) confirmed that metric-imperial conversions were still relevant because of their race shop that built American cars from American parts; Charlie said that Boeing aircraft were in imperial; and Arthur talked about marine pistons of, “say 2 foot in diameter and piston rings that are … ¾ inch-thick … big equipment”.

However, in contrast to some other engineers, Paul thought that people quickly adapted to using both systems of units by hands-on experience; “they just do it”. His view was backed up by Courtney (mature engineer), who frequently converted inches, eighths and sixteenths in his head. However, lack of exposure to imperial units in New Zealand schools has led to conceptualization and conversion problems for apprentices. Thus, Donald (mature engineer), who specialised in heavy-duty transport, felt that “it’s more the inches to millimetres … yeah! …” that caused difficulties for young apprentices these days. This was in contrast to his own school experiences involving familiarity with imperial. He was then introduced early to metric during his apprenticeship years, so “I sort of learned inches [at school] and then I started my apprenticeship and I learnt metric, so I sorta got the best of both worlds". Analysis of how apprentices adapted to imperial conversions is made in Section 5.2.3.

In summary, this section has shown that metric to imperial conversions were among the most frequently used mathematical skills among engineers (see Figure 6). Modern-day apprentices have been exposed to conversions between metric units for many years at school, but getting the decimal point in the right place could still be a problem for some of them. Conversions between units are examples of using formulas, which I now discuss.

4.1.3.2. **Formulas and transposition**

The findings showed that substituting in formulas and transposing formulas were among the most powerful calculation tools possessed by engineers. They were also the cause of a great deal of angst, and hence contradictions, in the engineering community. Murray (engineer) felt this angst led to a "fair bit" of mathematics avoidance. From Figure 6, over one-third of the participants were substituting in a formula at least once or twice a day, and just over one-fifth were changing the subject of a formula at least once a day. These statistics alone would suggest the importance of substituting in and transposing formulas in the mechanical engineering trades. In contrast, almost one-fifth of the educator/tradespersons and apprentices seldom or never substituted in a formula, and one third never changed the subject of a formula. Therefore, there was a wide range of usage, which may have been due to mathematical qualifications obtained at school, or possibly to branch specialisation.

Robert (expert engineer, educator and entrepreneur) explained how he would "short circuit" things by writing out formulas that he intended to use many times rather than going back to first principles for each calculation. He thought that this saved time and money, and probably led to fewer errors being made too. Once the formula had been developed, and then checked on one or two examples, the numbers only needed to be "locked in" to solve further examples. The difficulty here was that developing the formula from scratch was easier said than done. Robert used formulas for calculating heat treatment, lifting loads and finding the weights of bars.

There was widespread agreement among the educator/tradespersons and apprentices that they found algebra difficult. I had expected them to express difficulties transposing formulas, but their difficulties with substitution, which is arguably easier than transposition, surprised me. Ben said that he had passed US 21905 (see Appendix C) in his first year, but had not found it easy, especially transposing formulas. Ben seemed to be so strongly influenced by his negative experiences with transposition of formulas that it coloured his attitude to mathematics learning in general.

Henry (avionics educator) let out a long, drawn-out sigh when asked about apprentices' transposition of formulas involving just three variables. He replied, "some of them are really good at it … depends on how much exposure they've had to it". The difficulty existed despite avionics apprentices needing a higher level of mathematics knowledge than other engineering specialisations, suggesting they had had insufficient exposure in the school context. Arthur and Courtney (educators) agreed and illustrated Henry's point by describing how they calculated a suitable revolutions per minute speed for a rotating drill. This also involved extended reasoning, incorporating a series of engineering and mathematical steps.

On the other hand, modern calculating technology had made certain complicated tasks much simpler. Arthur gave the example of making machinery involving splines[10]. This involved extended reasoning using mathematical processes and substituting in complicated formulas. Modern calculating technology had removed the necessity to do this by hand, and Arthur now did the whole process very quickly, easily and accurately on the internet. He explained that using the internet was a definite advantage of modern technology because, before the internet, many engineers would have avoided doing spline calculations.

In summary, substituting in formulas and transposing their subjects both appeared to be major sources of difficulty for many apprentices and skilled tradespersons. The reasons for this are unclear.

4.1.3.3. **Finding data from tables and graphs**

Making decisions in the mechanical engineering context often depends on finding and using appropriate data. This can be done from technical books, but today is frequently done on the internet. Either way, the information is often in the form of tables and graphs, which the engineer has to find and then interpret. Finding the density of steel is an example where there may be a range of values depending on other metals alloyed with the iron. Among the educator/tradespersons and apprentices, reading and drawing tables and graphs had varied responses with over one in four (27%) reading or drawing graphs at least once each day, and about one in six (17%) less than once per month (see Figure 6).

Ben (apprentice) said he was reading tables of "itemized lists of material and stuff like that" fairly frequently to determine the materials he needed for each job. Arthur (educator) identified the gap between reading basic tables and graphs at school and the workplace as problematic. Thus, apprentices had a lot of difficulty with tables, although the practical context meant that they could actually work things out since it was relevant to what they were doing.

Owen (educator) held similar views to Arthur because graphs and tables were important for electronic circuits. For example, graphs showing diode characteristics with forward bias and reverse bias characteristics were drawn together on the same graph with two vertical axes and with strikingly different scales. This was new to apprentices and required adaptation from school graphs. With locomotives, Howard (engineer) frequently used a combination of table reading and graphical skills to determine "what's the pressure against the actual torque[11] you wanted to torque that particular bolt to".

[10] There are many different types of splines. On example of splines are long ridges on a drive shaft that mesh with grooves in a mating piece to transfer torque and power.
[11] Torque is the turning effect of a force. Thus, a large torque would mean that the nut was tight on the bolt.

Graph drawing was used weekly by 39% of the educator/tradespersons and apprentices (see Figure 6). Regarding reading or interpreting a graph, there seemed to be a diverse range of views, with educator/tradespersons and apprentices who felt elementary skills were sufficient through to those who needed to read graphs with two axes. Arthur and Owen (educators) both confirmed the challenge apprentices had in making the transition from school graph reading skills to those in the workplace.

4.1.3.4. **Pythagoras and trigonometry calculations**

These two skills were used frequently in the workplace, and many of the educator/tradespersons and apprentices agreed that solving Pythagoras and trigonometry problems in two dimensions were strengths of young apprentices. Thus, Warren (educator) felt that his students were "not too bad on that sort of two-dimensional (2-D) geometry", but they found using extended reasoning in three dimensions to be more challenging. He preferred to establish their abilities with "the lower level stuff, and build their confidence" before giving them more challenging geometry. In this way, Warren was consolidating and then extending the knowledge gained at school and applying it to more complicated three-dimensional (3-D) situations.

Henry (educator) talked about Pythagoras and trigonometry applied to analysing "phase angles and … loadings" in electronics. This differs from 2-D and 3-D geometry because the angles have no physical existence as in geometry. The resulting abstractness caused conceptual as well as the usual computational challenges.

Ben (apprentice) gesticulated with his hand as he described how he used Pythagoras on a fabrication job:

> when you know you've got to go so far up, and you have to go so far out, but they don't give you a measurement on the angle … and you know you've got to get a bit of steel to go from there to there … but they don't give you that.

Ben meant that the task was to find the length of a hypotenuse. Ben continued his explanation, complete with gesticulations, to include elements of angle calculation from trigonometry that were part of the same scenario:

> So, they give you straight up, that measurement, and straight across, that measurement, but they don't give you that one … so I can work that out, like I can work angles out for bending stuff, and work the angles out down.

I asked Ben how the "angles" were worked out. He replied, "Ah, just the drawing … whatever the drawing tells you … yeah". Another engineer had prepared the drawing for Ben, as was the case with other workers in the same company.

Fred, Arthur and Howard (engineers) acknowledged that some apprentices may have come from school with strong exposure to trigonometry and Pythagoras' Theorem. However, they also said it did not necessarily follow that they could quickly adapt this knowledge to workplace applications. This may have been the result of failing to recognise situations where trigonometry and Pythagoras principles might be applied, and hence involved the question of transfer of knowledge. Consequently, one group of apprentices was unable to make a correct judgment call to use mathematics in building a conveyor system ramp (see Section 5.2.2.2).

The references to electronics in this section form part of a wider application of physics in mechanical engineering contexts. I consider the important role of physics concepts in the next section.

4.1.3.5. **The intersection with physics**

Many of the mechanical engineering mathematics topics require physics knowledge, especially mechanics. It is important for apprentices to understand that physics formulas are based on the Système International (SI); therefore, it is essential to convert all lengths to metres, and masses to kilograms before beginning a calculation. I analyse three examples from the interview data that reveal the interconnectedness of mathematics, physics and higher-order skills with engineering contexts.

First, Henry (educator) related how avionics engineers applied mathematics to pressures for “running an engine”. If the air pressure changed, or the day was very hot and humid, Henry and his colleagues had to alter the pressure ratio through the engine, otherwise, they “couldn’t get enough grunt out of the engine … couldn’t get enough pressure to get the aircraft off the ground”. His explanation of running an engine demonstrated that integrating his physics understanding of pressures with mathematics, higher-order skills and a willingness to step beyond surface-level understanding were important in flying aircraft.

A second avionics example of the interconnectedness of physics and mathematics understanding was given by Stephen (avionics educator), who spoke about refuelling a plane on a hot day. While the tanker supplying the fuel indicated how many litres had been taken off, knowing the number of kilograms taken off was also necessary because the weight determined how much “energy you get out of the fuel”. The amount of expansion of the fuel was usually a calculation done by an engineer and involved considering several different factors simultaneously; a skill that Stephen and Henry both felt was a major part of apprentice learning.

A third example of how physics and mathematics knowledge combined with flexible thinking and problem solving applied in engineering trades contexts was described by Robert (expert engineer, educator and entrepreneur). He related how he installed new bronze bushes into a big 8-tonne flywheel on an old press. The task was complicated because the new bushes had to be shrunk on. This involved warming up the outer metal object, or cooling down the inner metal object, or perhaps doing both. However, since bronze bushes expanded more with heat than the steel, there was a problem with “upsetting”, or distortion. Robert was afraid that when things got back to their normal temperature, the new bush would “just fall out”. Robert considered cooling down the inner rather than heating up the outer rim, and eventually decided on using dry ice mixed with methylated spirits, which lowered the temperature sufficiently for the needs of his job. Robert's task involved integrating a long series of physics facts and reasoning with problem solving. There also needed to be a logical progression of ideas, inputs, and thoughts. Robert explained that it was important not to get hung up on one particular solution, but rather to say, "'No, let's park that, anything else?' because often we find that the solution turns out to be something that you never… thought".

To summarize, mathematical formulas from physics were often applied directly in mechanical engineering trades contexts, as with Henry's and Stephen's examples. On the other hand, Robert's example was quite different because the task was done very seldom and required conceptual understanding of the context and extended thinking from first principles. Therefore, these three examples are important because they all link to conceptual understanding and learning (see Chapter 5). They also signpost links with the technical aspects of deciding to use mathematics or engineering judgment, problem solving, and knowledge creation.

4.1.3.6. **Calculating volumes**

Calculations of volumes involved both practical and conceptual issues. They were important, partly for their own direct application, and partly for calculating the mass of an object when its volume and density are known. Many objects have formulas for their volumes. Approximately one-third of participants reported substituting in formulas at least once per day (see Figure 6). However, practical and conceptual difficulties arose when substituting in formulas because of the inconsistency of the units. Murray (engineer) recounted calculating the rate of airflow in a building, measured either in m^3 per hour, or litres per hour. This involved extended reasoning, beginning with doing a rough calculation of the volume of the building. Murray had heard of one engineer who could not "calculate the volume of the factory" using length multiplied by width multiplied by height. Moreover, confusion also surrounded converting m^3 and litres, an important and widely used numeracy skill.

As an example of block course training, Simon (apprentice) related how his polytechnic tutor asked students to calculate the volume of a silo, being a cylinder with a conical cap on top. Simon used the familiar formulas, $V = \pi r^2 h$, to find the volume of a cylinder, and $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone, and then "you just added the two answers together and that gave you the volume". Simon said, "Once I learn stuff, it's all good, but then it only stays in there for [a short while]", so when he had to use those formulas again, he would go to his phone. Simon often used his phone in preference to his memory. However, he well understood the need for consistency of units, "Ah… oh, you have to convert them to metres … yeah, that was something we had to do in night class actually … yeah". Therefore, Simon had learned the need to maintain consistency of units when calculating volumes.

Sometimes only estimates of volumes were required, so heuristics[12] were used. Heuristics can be regarded as an Engeström tool. Thus, when finding the total weight of a steel drum and the

[12] Heuristics = a mental shortcut or rule-of-thumb that allows people to quickly solve problems and make judgments quickly and efficiently without the need for long calculations.

steel inside it, Arthur multiplied the volume of the drum containing the steel scrap by the density of steel, and then divided by three, and Chris multiplied the diameter of a circle by 3 (instead of π) to find its circumference.

To summarize, learning to calculate areas and volumes, and relate them to a physical context were key features of the apprentices' learning. Measuring requires accurate reading of a scale, but sometimes an estimate will provide a sufficiently accurate answer. However, if it is decided to use a mathematical formula, then the question of consistency of units arises. Simon's reference to something he "had to do in night class" shows that the educators were mindful that apprentices needed to keep units consistent, and to do a sensible estimate of the conversion before using the calculator. Investigating this crucial skill was included in calculating the volume of the box problem (see Section 5.2.3).

4.1.3.7. **Modern calculation technology and mental calculation skills**

Calculation technology like calculators, smartphones, and the internet were controversial issues among mechanical engineers. On the one hand, they were regarded as useful, and perhaps even essential, tools in the modern-day engineering workplace. On the other hand, some engineers regretted the decline of appreciation of magnitude, and estimation and mental calculation skills. In this section, I analyse the questionnaire data on how often engineers reported using calculators in their work and their perceptions of the advantages and disadvantages of modern technological aids. In the next section, I analyse the importance they attached to mental calculation skills. These sections reveal important generational differences between the educators/skilled tradespersons and the apprentices.

Regarding the frequency of calculator use, the data in this section are based on educator/tradespersons' and apprentices' responses to the question: '*How often do you think most mechanical engineers use scientific calculators in their work?* Figure **7** shows the percentage of participants reporting each frequency category.

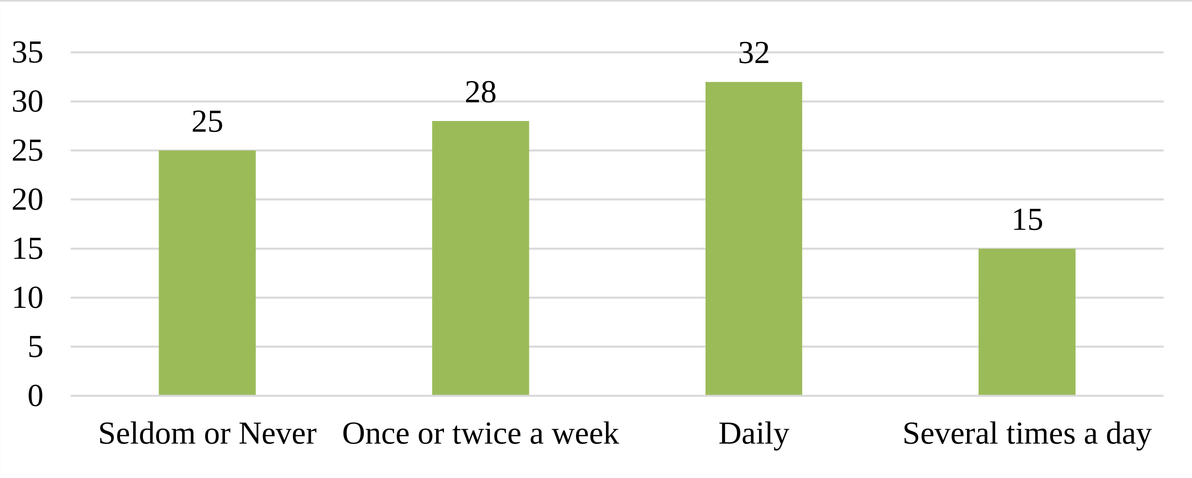

Figure 7 Percentages of educator/tradespersons and apprentices reporting how often they used scientific calculators in their work (n=175)

One quarter of the educator/tradespersons and apprentices reported using scientific calculators in their work seldom or never, while almost half said that calculators were used at least daily. It follows then that calculator use was widespread in mechanical engineering workplaces. However, there appeared to be debates in the mechanical engineering trades community regarding calculator use, the importance of mental calculation skills, and concerns about numeracy and mathematical skills in beginning apprentices and skilled mechanical engineering tradespersons. These issues are discussed now and in the next section.

Regarding the advantages of using modern calculating technology, there were some situations where modern calculating technology was superior to reliance on mental calculation skills and the traditional ways of doing things. Thus, in contrast with some other engineers, Robert (expert engineer, educator and entrepreneur) had positive views on the roles of calculators, smartphones and the internet. He even had an intranet system installed in his factory which he encouraged his colleagues to consult regularly for technical information and standard procedures. Information was also written on the forklifts, so that when "you go to lift a hunk of steel, well you can just straight away … look and can go oh, yeah and whip out your phone with the calculator on it, whack it in, yep, I can lift that".

Robert fostered a culture of integrating technology and mental skills. Thus, modern computer technology was a "magnificent toolbox, it's a magnificent toolbox" where all sorts of information could be instantly found. Robert often used Google, and even had engineering apps, like the Heat Treater's Guide[13], which had "all the alloys and their temperatures and all that in there, so ... yeah, how did we ever manage without them?" Later in the interview, Robert admitted, "I hate looking up books".

Arthur (educator) agreed that there were both advantages and disadvantages in using modern calculating technology, saying, "all you have to know is basically understand where to put the numbers in … and push [the button to get] the answer". In reply to those who criticised the reliance on electric power, Arthur admitted that a power cut would leave them all "stuck", but he then added that, while the computer wouldn't work during a power cut, it was also true that the power machinery wouldn't work either.

Arthur also commented on the difficulty inherent in understanding much of what happens behind most modern technology. He likened it to a car where "people don't understand … the connection between the key and the electrics, the motor, the fuel, everything that goes behind it". Thus, there was a contradiction between the desirability of having a conceptual understanding, and the ease and accuracy with which modern technology could perform tasks. In addition, another advantage of using software like Computer Aided Design is that the mathematics used is formidable and therefore allows tasks to be done that previously would not have been attempted.

[13] https://www.asminternational.org/home/-/journal_content/56/10192/06400G/PUBLICATION

To summarize, it appears that modern calculating technology was a widely used and essential tool in mechanical engineering trades communities. However, attitudes to the role and appropriate use of calculators and computers tended to be ambivalent. On the one hand, modern calculating technology could reduce errors in calculation, enable engineers to find information and perform calculational tasks previously thought impossible, and save time. On the other hand, there was still a need for a fundamental understanding of number and feeling for size in the engineering context. Also, there were situations where calculation technology was slower and less effective than mental calculation skills, which I now consider.

4.1.3.8. **The importance of mental calculation skills**

Several educators spoke passionately about the importance of mental calculation and estimation skills in the workplace. They associated this with being able to assess situations and make decisions quickly without recourse to using calculators, which they saw as time-consuming. They also believed that over-reliance on calculators and cell phones had produced a long-term decline in numeracy skills. In this section, the questionnaire items around concerns about numeracy and other mathematical skills were answered by educators and tradespersons only; the item about the importance of mental calculation skills was answered by educators/tradespersons and apprentices. The findings are presented in Figure **8**.

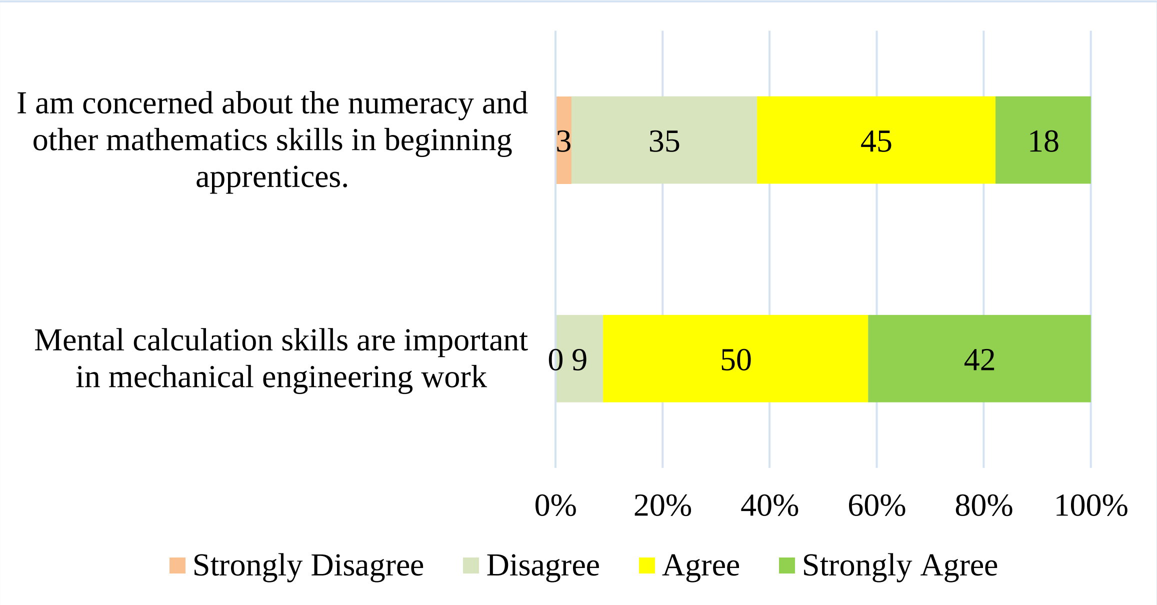

Figure 8 Percentages of educators/tradespersons and apprentices expressing the level of importance of mental calculation skills (n=199) and the percentage of educators/tradespersons expressing their level of concern about numeracy and other mathematical skills (n=101)

It appears that mental calculation was considered to be very important by the participants and that educators/tradespersons were concerned at the lack of sufficient mathematics skills in both apprentices and skilled tradespersons. More than 90% of educator/tradespersons ‘agreed’ or ‘strongly agreed' that mental calculation skills were important to their work. Nobody ‘strongly disagreed’.

More than 60% of the skilled tradespersons and educators ‘agreed’ or ‘strongly agreed’ that they were concerned about numeracy and other mathematical skills in beginning apprentices. Almost one half (45%) ‘agreed’ or ‘strongly agreed’ that they had concerns about the numeracy and other mathematics skills of skilled mechanical engineering tradespersons. Therefore, while it is possible that some growth in mathematics and numeracy skills may take place during the apprenticeship years, there still appeared to be concern about the standard of mathematics and numeracy skills of skilled tradespersons.

Regarding the views of the educators of the avionics pre-apprentices, who required a higher standard of mathematics entry qualification than some other mechanical engineering branches, Henry (educator) said that finding ¼ as a decimal would be a challenge for probably half his pre-apprentices. Henry felt that it was really important for apprentices and tradespersons to do certain calculations quickly in their heads because it was:

> an efficiency thing, ’cause if I can sit down, oh look, I’ve got ¾ of a tank of fuel left, … I can see it straight away … a lot of our meters and stuff are all gauges, so you don’t get a digital readout … if you see a number, you’ve got to go, hey hang on, what’s that?

Henry demonstrated how issues surrounding decimals, fractions, and estimating were connected with what constituted a reasonable answer and getting the answer quickly. Henry would sometimes say to his pre-apprentices, “Don’t use your calculators, do it in your heads”. This was important in avionics, because calculators were not allowed by certain licencing authorities in Europe and the USA. Warren agreed with Henry, and emphasized estimation skills, especially when apprentices believed absurd answers from their calculators. He also believed that there had been a long-term decline in the ability of school leavers to remember essential facts because the next day, “it’s like, we’re teaching them the development of the subject again”. It would appear then, that the avionics educators were concerned about mental calculation skills because they used them frequently in their everyday work.

Estimations were also important to Chris (engineer) in fabrication engineering contexts. He used estimations frequently, particularly when making good decisions about quantities of materials. Thus, in working out the circumference of a circle, Chris “would multiply [the diameter] by 3”. The apprentices would say it’s 3.14, but Chris would reply that “you estimate how much steel, how much steel you’ll want… general knowledge helps, you know”.

Stephen agreed with Henry and Warren (educators) that “you do need to be able to quickly figure out whether what you’ve figured out is anywhere near right or not” and investigate why it’s wrong. Stephen compared this with an expert mechanical engineer who appreciated “how all the systems interact and also [had] an understanding … being able to estimate and figure out what should be happening versus what is happening”. He agreed that integration of all the skills was required, as well as intuition. However, even some well-trained and skilled people might “find out some information and they’ll use that … take that at face value without realizing that a mistake’s been made and it can’t be true”. Thus, “the guys that really are good are the ones that actually understand when information is bad”.

Stephen (engineer) regarded an expert as someone who understood how systems interact, thus demonstrating the importance of multi-step thinking, conceptual understanding and integration of ideas and inductive thinking to quickly detect errors. The ability to recall basic factual information and estimation skills was therefore regarded as being important to ascertain if an answer was reasonable.

To summarize, it appears that modern calculating technology has not removed the need for mental calculation and estimation skills, which were important in relating numbers to contexts and identifying errors. There was a widely held view that mental calculation skills needed improvement for apprentices and skilled tradespersons.

4.1.4. **Section summary**

In this section, I have presented and interpreted the findings relating to the nature and use of the mathematics and numeracy skills employed in the mechanical engineering trades contained in the main research question and sub-question 1. Mathematics knowledge and skills can be regarded as tools or artefacts of an Engeström activity system. The knowledge and skills included formal mathematics topics and numeracy. Rules emerged from the data about the way mathematics should be used; in particular, there was widespread agreement of the need for high numeracy skills that were related to the need for accuracy of calculation as well as quickly performing mental calculations to establish suitable estimates. This led to an important contradiction that was expressed as an ambivalence to the perceived overuse of calculators, as well as support for using the new technology to perform tasks like spline calculations that might previously have been avoided. The knowledge of the nature of the mathematical tools and how to apply them in context was a key feature of mathematical learning and hence an expansive cycle of learning that formed part of the apprentices’ pathway to becoming skilled tradespersons. In these cases, expansive cycles of learning corresponded with Lave and Wenger’s (1991) legitimate peripheral participation (LPP) principle and enabled individuals to move closer to the centre of community of practice.

Conceptual understanding and procedural knowledge emerged as being important to developing knowledge and skills, such as the ability to understand and apply different systems of units. There were found to be conceptual and procedural challenges, especially difficulties surrounding area and volume units. Conceptual understanding underlines the need for ancillary, higher-order skills because knowledge about a mathematical tool is only a starting point; knowledge needs to be related to the engineering context and the physical tools being considered. Therefore, obtaining higher-order skills is an expansive cycle of learning that is crucial to applying mathematics in the mechanical engineering workplace. Higher-order skills are now considered in the next section.

4.2. **Application in context and higher-order skills**

In this section, I analyse the data about the second research sub-question which concerns how the mathematics skills presented in Section 4.1 are applied. The application of mathematical tools frequently involves conceptual understanding. Therefore, to understand how mathematics and numeracy are applied in the mechanical engineering trades context, it is important to consider the roles of higher-order skills - intellectual and non-mathematical tools such as problem solving, creativity, extended reasoning, and the integration of skills. The connections between these ancillary skills and applying mathematics and numeracy skills are the focus of this section. While tools are a focus in this section, Engeström’s (1987) other elements of rules, and especially roles and community, also become important. Moreover, although problem solving is frequently used in response to contradictions, resolving those contradictions may not be straightforward and may create further contradictions, also with long historical roots. Hence, the progress to expansive cycles of learning may involve complex interactions in the whole activity system and its associated communities.

4.2.1. **Problem solving in mechanical engineering contexts**

Problem solving is important in all branches of mechanical engineering trades, especially in fault-finding and maintenance engineering. The questionnaire data showed that engineers regarded problem solving as a very important skill for their trade and therefore essential to becoming a skilled tradesperson. Moreover, problem solving is frequently performed in unfamiliar contexts beyond the experience of most engineers. While the application of problem solving frequently involves mathematics, sometimes the mathematics may be hidden.

4.2.1.1. **The nature and importance of problem solving**

The engineers had different understandings of and perspectives on problem solving, but almost all of them ‘agreed’ or ‘strongly agreed’ that problem solving was important (see Figure 9).

Of the 199 questionnaire participants, almost all (98%) ‘agreed’ or ‘strongly agreed’ that problem-solving skills were useful in mechanical engineering work, and more than half ‘strongly agreed’. However, this almost unanimous result does not inform us as to why they agreed, what they understood by problem solving, or how they applied problem solving to practical contexts. Thus, I questioned several engineers about how useful they thought problem solving was. Robert’s response was typical of many when he said, “Oh … YES! It’s so much!”

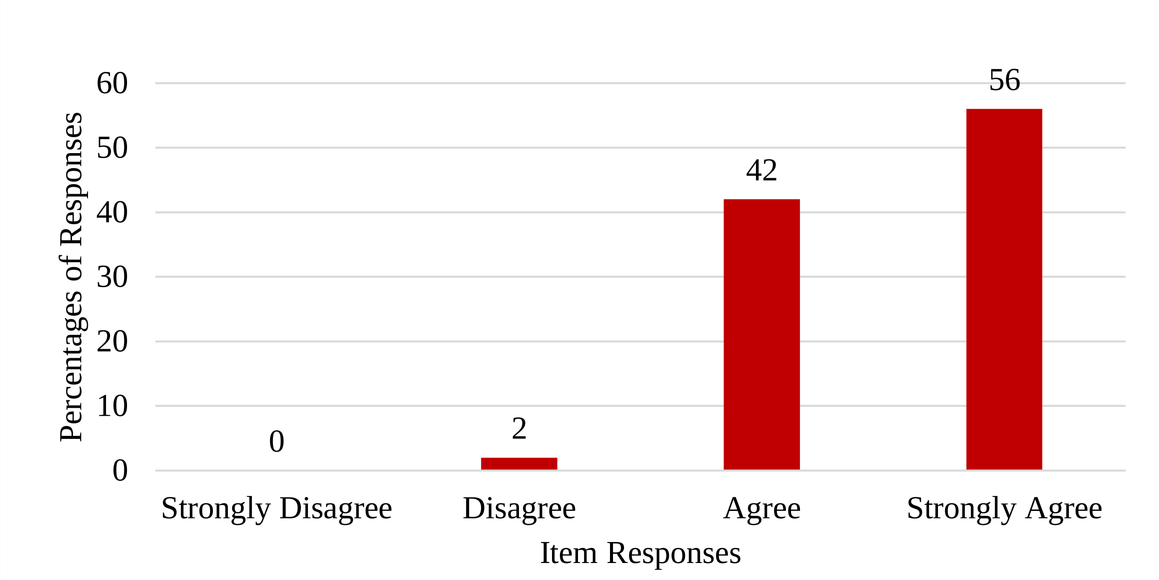

Figure 9 Percentages of all participants reporting levels of agreement that problem solving skills are useful in engineering (n = 199)

From the interviews, regarding various understandings of and perspectives on problem solving, Owen (educator) outlined the limitations of computer technology, which had not replaced the need to develop highly trained, independent and free-thinking engineers. He felt that computers were not "able to solve the problem, 'cause it's outside what they were programmed to do", and that apprentices needed to be encouraged to think outside the square; "I compare it with a horse with blinkers. If you want the horse [to do something], the horse starts looking around and is curious to know what the hell's going on here".

Henry developed Owen's viewpoint by stressing the benefits of experience in "manual training" as opposed to mathematical training in developing problem-solving skills. He felt that this led to "problem-solving efficiency, if you like". He described problem solving this way:

> Ah, I'd just see an issue, and then I'd go, 'Righto, how am I going to fix that issue? How do I come up with solutions to it? How do I come up with multiple solutions? And what's the most cost-efficient way of doing it?'

In contrast, Irene (apprentice) mentioned different aspects of problem solving she used almost every day in her work with refrigeration and cooling systems. Problem solving tended to arise in potentially hazardous situations. Safety and financial considerations required proper maintenance schedules and repair procedures to be carried out. The result was that fault-finding came into play often whether the engineers were "conscious of it or not".

To summarize, when educators exposed the apprentices to fault-finding, many hypothetical and practical problem-solving scenarios could eventuate, and many strategies for dealing with them. However, many situations were not standard, and in addition, extended reasoning was required to deal with them (see Section 4.2.2). Therefore, in many cases, a decision was made not to use mathematics, and an acceptable solution was found on pragmatic grounds. I now differentiate between tricks of the trade and genuine problem-solving scenarios.

4.2.1.2. **Tricks of the trade and problem solving**

Tricks of the trade are skills that are learned over time and become part of the standard skillsets of the engineering community. Tricks of the trade may originally have been developed in situations demanding genuine problem solving, but repeated usage has removed the need to go back to first principles.

I now discuss a well-known trick of the trade that shows how using a different measuring strategy obviates making incremental measuring errors and improves the accuracy of the final result. Chris and Courtney (experienced engineers) worked in separate workplaces, but both used the following trick of the trade to illustrate good measuring technique. Courtney described how nine lugs at equal spacing could be placed on a 3300 mm beam with 50 mm "in from each end". First, Courtney subtracted 100 mm from the 3300 mm and divided by 8, to give 400 mm spacing between the lugs. He then took all his measurements from one end of the beam, and positioned the lugs at 50 mm, 450 mm, 850 mm, and so on. Following this procedure, he stated that he:

> would be very, very accurate because I don't get an incremental error …. A lot of the guys here will go 50, then they'll get their measure and go 400 and 400 and 400, and they'll get to the end and they'll wonder why they've got only … got 20 mil left …

Courtney's views were shared by Chris, who thought that judgment skills would take time for apprentices and even tradespersons to develop and that some engineers were not "wired up" to understand the effects of incremental measuring errors. Courtney also connected incremental errors with tolerances which were:

> quite often an issue with things. You know in engineering, you'll make this one this size and this and this and this one, but everyone's a little bit different, and when they become too much, then the thing doesn't fit together, and that's why we have fits and tolerances on things.

However, he added that while "a lot of things have changed in time, especially the introduction of calculators and stuff like that, you know", using calculators "simplifies things, not solves it", so thinking was still required.

Mature engineers had learned to weigh up the relative effects of many conflicting factors. This formed a contradiction that required resolving to create an expansive cycle of learning. Moreover, in these cases, expansive cycles of learning followed Lave and Wenger's (1991) LPP and mature engineers moved closer to becoming experts at the centre of the community of practice. When making a decision, it was important to understand that mathematics was just one factor. Tricks of the trade may eventually become recognised as standard procedures.

Problem solving originally developed those procedures, which led in turn to further procedures not previously invented to suit the needs of new situations. Thus, problem solving in the engineering trades context often involves recollection of standard procedures, tricks of the trade, and seeking new, innovative adaptations. I now discuss problem solving with real and artificial scenarios.

4.2.1.3. **Problem solving with real and artificial scenarios**

Real problem solving involving engineering-specific contexts and hypothetical non-engineering contexts requiring thinking outside the square are both important in mechanical engineering trades. I now analyse two conversations with avionics educators, Stephen and Owen, about their views on problem solving and how they apply it (see Table 8 and Appendix O).

Table 8 *Summary of avionics educators' comments on problem solving*

	Comments on problem solving and fault-finding	
Purpose	To develop trouble-shooting skills and come to a conclusion	
	Stephen	Owen
Aims and actions of educators	• Current courses presented mathematical problem solving as scenarios, to find mathematical methods and solutions • Troubleshooting is looking at the evidence & coming to a conclusion • Strategies given for fault-finding; halving the system and testing each half to reach a conclusion • Problem solving featured later in apprenticeship training • Checks and balances to mitigate mistakes	• Non-engineering scenario problems given as in school • To foster aptitude for fault-finding, analysis, extended reasoning skills, to examine different points of view • Adaptation of apprentices to everyday scenarios • Extended reasoning with 8–10 steps, so write things out and set out properly
Pre-apprentices were required to …	• Appreciate the importance of getting all the right information • Be mentored into harder tasks, sit and observe initially, help out with calculations • Add and subtract and multiply … especially with decimals and things like that … pretty early on for fuel loads	• Apply principles learned in class and relate to the context • Not accept the calculator result without thinking as they often did, often couldn't apply principles taught in class • Understand that the world was not mathematically ideal • Not be content to just press the buttons on the calculator • Understand the physics behind a calculation, and not just have procedural knowledge

While problem solving was regarded by both educators from a pragmatic perspective, they nevertheless employed hypothetical, and therefore artificial contexts when giving examples to their pre-apprentices. In doing this, their intention was to foster attitudes of flexible thinking so that apprentices would look at things from different viewpoints.

Practical problem-solving scenarios, such as fault-finding and maintenance work often required extended reasoning. This required many steps being lined up in a logical order and then systematically examined to reach a conclusion. Therefore, for pre-apprentices, extended reasoning marked a major development of thinking from that learned in the school environment. A further development in thinking was the need for pre-apprentices to appreciate that the world was not mathematically ideal and that the results from a calculator needed to be critically examined to place the numerical value in context and to decide if the mathematical model or formula used was appropriate to the physical context

To summarize, the pre-apprentices were given tasks that reflected their current state of engineering preparedness. Some of their tasks directly involved mathematics, but others involved strategies for solving problems. In these situations, developing and following logical chains of reasoning were important tools that the pre-apprentices needed to learn. Moreover, they were introduced progressively to more complex problem-solving scenarios as they grew in experience. Problem-solving development was also linked to extended reasoning development (see Section 4.2.2). While using formal strategies to solve workplace problems in unfamiliar situations was a new episode in many pre-apprentices' experiences, the mathematics involved was often hidden and did not necessarily feature as prominently as the integrated employment of problem-solving skills.

4.2.2. **Creativity and flexibility of thinking in context**

Creativity and flexibility of thinking are important aspects of mechanical engineering practice. I discuss three issues of creativity and flexible thinking in turn: essential ingredients in problem solving, extended reasoning, and risk-taking.

4.2.2.1. **The link to problem solving**

The data showed that many of the educators thought that creativity and flexibility of thinking were important for problem solving. For example, Henry said, "Yeah, we want creativity, because that's gonna give you the different paradigms to come up with for your solutions". Proposing multiple solutions was encouraged, even when they were radical. This may have been because the exchange of ideas between engineers promoted the development of the critical faculty that led to creating new solutions. Exchanging ideas is linked directly to social interaction, communication, and learning (see Chapter 5). Moreover, since problem solving is concerned with finding solutions to problems that are often unfamiliar to engineers, then the answers are usually not found in standard procedures or tricks of the trade. Thus, successful problem solving often calls for imaginative, creative and innovative approaches, which engineers may also refer to as thinking outside the square.

Being aware that there might be different understandings about what constituted creativity and innovation, I asked Paul (training officer) about engineers he considered to be creative. He replied that all their good machinists were very creative and could find multiple ways to make products more efficiently, such as jigs that would reduce six operations to three. While creativity and innovation did not necessarily require mathematics, Paul felt that for their specific creative requirements, a better understanding of the mathematics that went into their work would "absolutely" help in producing creative products. Murray (engineer) made a similar point when he spoke about the need for deep conceptual understanding, and talked about how a senior mechanical maintenance person who had started off on machinery repairs became a supervisor, but then experienced difficulty trying to "physically design something, and calculate it". Murray concluded that there were situations where intuition alone was insufficient.

Referring to creativity, imagination and problem solving, Robert (expert engineer, educator and entrepreneur) related how he was frequently asked to do jobs that customers knew were too big for their gear, especially furnaces. Robert focused on finding creative solutions, so he would evade customers' doubts about the gear and ask them, "How big is your job? (*laughter*)", knowing that the size of the press was irrelevant. Similarly, when steel had to be heated "red hot, but only inches away [from something] that can't exceed 50 degrees, we'll come up with a creative solution".

Robert went on to describe installing a very large press that put heads on bolts and flanges on ships' propeller shafts 20 metres long. He explained that he decided to go ahead and install the press vertically, regardless of what others thought. He would then deal with any problems as they arose. "So, we've had lubrication problems, we've got some gravity problems, we've got a few other bits and pieces, but actually, it's worked extremely successfully". This example illustrates how new ideas are frequently viewed with scepticism because engineers frequently do not know in advance what will transpire. In such cases, they sometimes rely on fixing problems one at a time as they arise.

Creativity and flexibility of thinking were linked and performed in conjunction with problem solving; they are the result of contradictions that demand an expansive cycle of learning. Moreover, the engineers' creative impulses and their long experience created solutions that contributed to the ever-increasing body of standard procedures available to be passed on to the wider engineering community. The mathematics involved here may have been hidden or replaced by engineering judgment.

4.2.2.2. **Extended reasoning and fault-finding**

Before beginning the analysis of the data in this section, I first explain the inclusion of fault-finding and maintenance engineering in this study, and their links with mathematical learning along the pathway to becoming a skilled mechanical engineering tradesperson. In these two branches, mathematics may sometimes be used directly, as with Howard who consulted tables and measured thicknesses of pieces of lead sheet to test engine wear (see later in this section). At other times, measurements and calculations may not be necessary, the mathematics may be hidden, or engineering judgment is used to reach decisions. However, fault-finding and maintenance engineering almost always involve diagnostic skills, and logical thinking to identify problems. They also require problem-solving skills for finding solutions to those problems. Therefore, the link between fault-finding and maintenance engineering and mathematics may be direct through specific skills, such as measurement and attention to tolerances, or indirect through ancillary skills, such as logic and problem solving.

Extended reasoning is also a higher-order skill and is combined with creativity and flexibility of thinking. It plays an important role in all branches of the mechanical engineering trades, especially in maintenance engineering and fault-finding, where it is often used in conjunction with creative approaches to problem solving. The main focus of mathematics here is logical reasoning. Mathematics and numeracy considerations may not be relevant or may be partly hidden. This section forms three parts: the importance of fault-finding and maintenance engineering, introductions to maintenance engineering early in apprenticeship training, and the importance of quick and correct problem diagnosis.

First, fault-finding and maintenance are important because breakdowns and potential breakdowns pose large financial and safety issues in mechanical engineering. The engineers regarded fault-finding as extended reasoning because multiple-step thought processes and problem solving are needed to identify what has gone wrong and to devise solutions to fix it. Since problem solving is involved, then a contradiction exists and an expansive cycle of learning is involved in both the current issue as well as in the long-term creation of knowledge in the community. Hence, Chris (engineer) said, “Yep … that’s where problem solving is a major area, especially in engineering, especially where we get breakdowns …. You gotta use the old grey matter”. Chris commented on the process he followed when problem solving:

> Well, it’s usually my experience [that] you fall back on your knowledge. If I don’t know the answer, I have the ability maybe through books either today or the internet … that’s what I was brought up with, go back to the library again, it’s just knowing how to access the information. Again, if a guy is out there and he can’t find … and he’s got a problem, they can get on the internet. If there’s no internet, what does he do? (*laughter*)

Another important engineering consideration is to lengthen the life of machines and to prevent serious damage when they break down. Companies often schedule regular inspection programmes to examine, and if necessary, replace worn components. Howard (engineer) described how there was a checklist to go through when a locomotive was brought in for maintenance. Checking for wear on pistons involved removing the engine side covers, taking measurements and doing bump clearances. “You put three bits of lead on the top of the piston, wind the engine over and take the lead out … show if the piston’s not gone off to one side”. When the lead was taken out, its thickness was measured using a micrometer gauge to “make sure the con rod’s[14] not

[14] A con rod is a piston engine component which connects the piston and the crankshaft.

put off to one side". The mathematics involved here was reading and interpreting a measurement on a Vernier scale[15] with high precision.

Second, fault-finding and fixing faults are key skills in the mechanical engineering trades workplace and both appear to be among apprentices' earliest tasks. It is allied with both diagnosis and problem solving. I asked Ari (a young apprentice), what he understood by problem solving. He replied, "diagnosing". You have to be able to look at something, recognise the symptoms from what has happened and "tell what's wrong with it". However, the diagnosis was not always straightforward, as with a dump truck whose bucket would not tip. Ari said, "you assume it's something to do with the hydraulics", but this was complicated by how far the blockage was along the line. If "it's right back at the controls … so pretty much your problem solving will come in … test the pressure at the main pumps, and then yeah, pretty much work your way back". Ari acknowledged that there could be more than one solution and that there might be more than one fault, "cause sometimes you wouldn't know until [you] actually take it out and have a look at it and … multiple". He also understood that the final solution decided upon was dependent on "which one's better for the client". Ari's experiences of fault-finding were thus exposing him to fault-finding and working in team-building environments.

Fault-finding in avionics is an area where logical and systematic thinking skills were applied often. Owen (educator) believed that inculcating habits of logical and sequential thinking into his students was more important than mathematics in his course. Therefore, the emphasis was on determining,

> what is happening here? Is this engine running properly? And if it's got a fault, what kind of fault is that? Where? How is [it] showing up in the way the engine is performing? … the physics and the maths of that … bit of experience… and then you have a fault-finder.

Henry (avionics educator) gave a detailed response about assessing fault-finding skills. The avionics educators built a large board with the electronic map of an aircraft on it. The apprentices then had to learn their board, wire it all up, make it all go, so that the little lights flashed. They also had logic operating so that "you have to turn the nav[igation] light on, and then you can turn the taxi light … on". The intention was to learn the whole system. Then, the night before the assessment, the educators would cut all the wires, pull fuses, chop capacitors in half, and generally destroy the board so that it was "totally dead". They might also pull the mains fuse. The pre-apprentices' task for the assessment was to work out what had gone wrong, and then "… methodically work their way through" and fix it. Since they worked in groups, communication skills were also particularly important. This task closely paralleled authentic mathematics assessment methods (see Section 2.4.3).

[15] A Vernier scale is a visual aid designed to enable a more accurate measurement reading between two graduation markings.

Extended reasoning and its application to fault-finding and maintenance engineering are essential skills to be learned along the apprentice's pathway. Watching and emulating skilled tradespersons seemed to be major factors in the apprentices' experience and in knowledge transfer in informal settings. Moreover, fault-finding was intertwined with common sense and higher-order thinking skills.

The third important part of apprentice development is learning how to diagnose problems. This requires a logical progression of ideas, inputs, and thoughts, and to not get hung up on one particular solution, but to suspend judgment. Robert explained this was because "the solution turns out to be something that you never… thought" (see Section 4.1.3). Murray (engineer), on the other hand, while acknowledging that the aim was to try "to find different ways, or trying to look at causes", emphasized that "you need other people. You can't do it on your own", and that some people tended to "follow a straight line" and did not involve others. Thus, both Murray and Robert wanted to use eclectic approaches and to avoid the rigid thinking that formal approaches might impose.

Howard (engineer) illustrated the importance of a correct diagnosis. A locomotive compressor "wasn't building up the air [pressure] correctly". The maintenance engineers' initial reaction was to look for a mechanical fault but this did not lead to a solution. Having changed all the mechanical parts, Howard and his colleagues "had to go a step further, and say, why is it still doing it?" After much time and effort, it transpired that the fault was electrical. A component had not been installed correctly and was the wrong component anyway. Finally, the cause of the problem was "just a loose connection"; a wrong diagnosis and wasted time.

To summarize, fault-finding found major application in maintenance engineering where diagnosing faults and fault-finding required flexibility of thinking. Apprentices appeared to be put into this area fairly early in their engineering workplace experience. Sometimes, as in Howard's example, the wrong assumption could be made initially and much time and money could be wasted as a result. Murray spoke about the usefulness of formal methods to identify faults. However, these were effective only when used in conjunction with a team approach with many people working together, and exercising flexibility of thought and thinking outside the square. Thus, the cumulative experience of the engineers with malfunctioning equipment systematically built up expansive learning in maintenance engineering and fault-finding skills. It also moved the group and its individual members closer to the centre of Lave and Wenger's (1991) community of practice.

4.2.2.3. **Creativity and risk-taking**

Adopting creative and flexible approaches to problem solving involves going beyond the limits of known and widely accepted engineering practices. This carries a certain amount of risk because the exact outcomes are not known beforehand. This was illustrated in Robert's description of replacing the bronze bushes and the versatility he showed in attacking various unfamiliar projects with their multiple possible lines of approach (see Section 4.1.3.5).

Therefore, engineering contradictions had personal consequences. He related his thinking on risk-taking in a conversation:

> Robert: I like to think, yes, I take risks, and before I hit the trigger that first time, I always need to go and have a nervous pee, and I always say ‘I’m too old for this s***’, but [I think] the risks … I take are manageable, and I’ve always got a back-out.
>
> Kelvin: Yes, OK. So, it’s calculated beforehand?
>
> Robert: Ah-hah
>
> Kelvin: And essentially all you lose is some time?
>
> Robert: That’s it, which is not lost if you’re learning
>
> Kelvin: No, not if it’s successful
>
> Robert: No, no ... even if it wasn’t successful, the LEARNING from that …
>
> Kelvin: Oh, the LEARNING from it?
>
> Robert: So, the training course that I do, it’s packed full of all of my cock-ups … because those are learning moments ... they’re only wasted if you don’t learn

Robert’s story about replacing the bronze bushes on a press illustrates how engineers combine mathematics and physics knowledge with engineering skill, imagination and creative problem solving. Engineering judgment was important here too, because the expansion or shrinkage of the bronze and steel were time-dependent and temperature-dependent. Therefore, any calculations Robert could have made would not have told the full story of how the steel and the bronze would behave while they were changing in size. Thus, while the mathematics was hidden, Robert’s actions were governed by his intuition of what would work in practice.

4.2.3. **Section summary**

In this section, I analysed the skills needed to apply mathematics successfully in the mechanical engineering trades workplace. Higher-order skills emerged as important motivations for applying mathematics in practical contexts. This view was held by 98% of the educators and tradespersons, who ‘agreed’ or ‘strongly agreed’ that problem solving skills were useful in mechanical engineering work. Problem solving was extensively used even by young apprentices who were involved in fault-finding and maintenance engineering. Therefore, ancillary or higher-order skills are important tools in engineering activity systems and key features of mechanical engineering practice and learning (see Chapter 5).

The skills of creativity, flexibility of thought and extended reasoning were associated with problem solving and were regarded highly by the engineers. Thus, successful engineering practice involved integrating well-developed mathematics and numeracy skills with engineering skills and problem-solving capabilities. However, problem solving had to be carried out according to the rules set down by the wider community of practice. Unusual solutions and multiple potential solutions to problems could lead to multi-voiced, and long-term historical contradictions. If the confrontations between tradition and innovation were resolved over time, then a new level of accepted practice would be attained: an expansive cycle of learning, which also allowed individuals and the community to move closer to the centre of Lave and Wenger's (1991) community of practice.

4.3. **Chapter summary**

This chapter focused on Engeström's (2001) third question, What do they learn? It reported on the analysis of the data pertaining to the first two research sub-questions on (1) the nature of mathematics knowledge and skills used by the engineers, and (2) the application of that knowledge and those skills in the workplace context (see Section 1.3). The analysis revealed how higher-order skills, such as problem solving, and its associated skills of creativity, flexibility of thinking, extended reasoning, and logical thinking were used extensively in conjunction with mathematics and numeracy in mechanical engineering workplaces.

The main research question was "What key features of mathematical learning characterise the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering?" The results reported in this chapter indicated that the learning involved both new mathematical content and consolidating mathematical content previously learned at school (see Section 4.1), and developing accompanying ancillary skills such as problem solving that were considered necessary for the successful application of mathematics in the workplace (see Section 4.2).

Regarding the first research sub-question, "What is the nature of the mathematics skills employed in the mechanical engineering trades?", the mathematical knowledge and content closely aligned with mathematics skills already introduced at school, such as Pythagoras, trigonometry, and graph reading (see Sections 4.1.2 and 4.1.3, and Appendix C). However, applying mathematics in the mechanical engineering trades context was a complex process. Thus, converting between systems of units, mental calculations, substitution in formulas, and transposing formulas were considered to be challenges for apprentices and skilled tradespersons alike, possibly because of their more abstract nature and conceptual issues surrounding understanding (see Section 4.1.3).

Among the skills that were necessary to apply mathematics in the mechanical engineering context were accuracy in calculation and measurement, a feeling for size, making sensible estimations, deciding whether or not to use mathematics or rely on heuristics or engineering judgment, understanding and using tolerances, understanding how mathematical models relate to the mechanical engineering context, and interpreting numerical results in context. Ancillary skills, such as problem solving and its associated skills of creativity, flexibility of thinking, and extended reasoning were also found to be important and these also required development beyond school requirements (see Section 4.2).

The second research sub-question was “How do apprentices and skilled tradespersons in mechanical engineering trades apply mathematical skills in their work?”. The study found that mathematical skills were used in real contexts involving both routine and non-routine tasks. The study also identified higher-order thinking, problem solving, creativity, extended reasoning, and conceptual understanding as being important for the successful application of mathematics in the workplace (see Sections 4.2.1 and 4.2.2). Decisions regarding when to use mathematics, how mathematics should be used, and the acceptability of new ideas and innovation, were strongly influenced by accepted practice and community rules.

To summarize, this chapter has focused on the findings relating to the main research question and the first two research sub-questions. It has analysed the nature of the mathematics and ancillary skills employed in the mechanical engineering trades and how those skills are used. In the next chapter, I analyse the data to answer the third sub-question, which concerns how the mathematics knowledge and skills are learned and developed.

Chapter 5. **Findings - Learning and developing mathematical knowledge and skills**

Introduction

This chapter is the second of two chapters that present and interpret the data from the present research using CHAT and SL lenses and with reference to the research questions. Chapter 4 focused on Engeström's third question and the first two research sub-questions, what apprentices and other engineers learn, including specific and ancillary mathematics skills, and how they were applied in engineering contexts. In contrast, this chapter focuses on the third research sub-question: how do engineers learn and develop mathematical skills? It relates directly to the main research question and Engeström's fourth question "How do they learn?" (see Section 1.3 and Section 3.2).

Engeström's elements of physical and intellectual tools featured strongly in Chapter 4. In this chapter, Engeström's other elements of rules, roles and community feature more prominently as I analyse how the mathematical and associated tools are developed and learned. Apprentice engineers are in the process of learning the technical and social rules that are strongly determined by the community of engineers. Their roles change according to their developing skill levels and experience. The changing roles resonate well with Lave and Wenger's (1991) theory of legitimate peripheral participation where beginning apprentices are initially set relatively simple tasks but which steadily become more difficult as they gain in skill, understanding and confidence. This chapter demonstrates how apprentices in the mechanical engineering trades progress from the periphery of a community of practice towards the centre, along with the associated complexities and contradictions.

The chapter is in two parts. A smaller part, Section 5.1, focuses on childhood and school formative experiences and a larger part, Section 5.2, focuses on workplace-related learning and mentoring. Section 5.2 is broken into five sections – the qualities sought in prospective mechanical engineering apprentices, views on apprentice training and mentoring, conceptual issues surrounding calculation skills and their contexts, learning higher-order skills, and social interaction and communication. Section 5.2.6 summarizes the chapter.

I begin the analysis of the processes of new knowledge and skill development with the influences of childhood and school experiences.

5.1. **Childhood and school formative experiences**

An examination of childhood and school experiences is important to this study. The data showed that many of the participants could trace their historical exposure to engineering and other practical experiences to their childhood; their development was similar to my own (see Section 1.1). The data also showed that their learning was linked strongly to the Cultural Historical Activity Theory (CHAT) frame of reference (see Section 3.2). As children, they would have become members of an Engeström community where older family members were role models for them, they became familiar with physical tools, the rules the community accepted for their appropriate use, and the limitations of those tools. Later, they would learn to maintain and make their own machines. They were, therefore, introduced to problem solving and other intellectual tools, such as the language, culture, and communication styles of the community. The childhood links to mathematics may have been less obvious, but as children, they may have been exposed to the discourse surrounding measurements and weights where they gained a feeling for size that was related to practical application and context.

I begin with childhood experiences of engineering and other practical activities involving family and whānau before relating the influences of school experiences in engineering and mathematics.

5.1.1. **Childhood experiences**

Seventeen engineering tradespersons or apprentices were interviewed. They had a wide range of childhood experiences in engineering and other practical activities, and some had studied engineering at school. From their biographical data in Appendix E, three of the engineers reported strong engineering and other practical experiences during their childhood and teenage years. Among the influences drawing them towards a practical world were the voices of older members of their family or whānau; particularly, their fathers, and in some cases, their grandfathers. These influences, combined perhaps with a natural curiosity about mechanical and other practical things, provided them with an already long exposure to engineering practice and culture by the time they reached their teenage years. Thus, the engineering experience of an emerging skilled tradesperson could already go back many years. This indicated the influence of their background experiences on their learning, and how these formative experiences might have mediated and hastened the transition from the periphery to the centre of the engineering community of practice.

5.1.2. **School and workplace mathematics**

I now analyse the questionnaire data provided by the apprentices concerning their experiences of school mathematics. I first compare how easy the apprentices found mathematics at school and in their work, and then how easy they found mathematics at school with how helpful they found mathematics in their work. The data is from the responses to the questionnaire items: Item 6 *I found mathematics easy at school*, Item 8 *The mathematics I learnt at school helps me with the mathematics in my apprenticeship,* and Item 9 *Overall, I find that mathematics for mechanical engineering work is easy*.

First, examining the row totals in Table 9, 41% of the apprentices disagreed or strongly disagreed that they found mathematics easy at school. Examining the column totals, 44% disagreed or strongly disagreed that the mathematics in their work was easy. This suggests that mathematics in the workplace is a challenge for a significant proportion of apprentices.

Almost 60% (42/71) of the apprentices gave the same response to both items, suggesting that they found a similar level of mathematical challenge at work as they had done at school. However, 12 apprentices found mathematics at school easy and disagreed that they found mathematics easy in the workplace. Similarly, 10 apprentices found mathematics easy in the workplace but disagreed that they found mathematics easy at school. One could speculate that the difference between school and workplace mathematics may be partly due to general difficulties in mathematics learning at school and/or in the workplace.

Table 9 *How easy the apprentices found mathematics at school and in their work*

		Overall, I find that mathematics for mechanical engineering work is easy Item 9					
		Strongly Disagree	Disagree	Agree	Strongly Agree	Totals	Composite totals
I found mathematics easy at school Item 6	Strongly Disagree	1	3	1	0	5 (7%)	29 (41%)
	Disagree	0	15	9	0	24 (34%)	
	Agree	1	11	22	2	36 (51%)	42 (59%)
	Strongly Agree	0	0	2	4	6 (8%)	
	Totals	2 (3%)	29 (41%)	34(48%)	6 (8%)	71(100%)	
	Composite totals	31 (44%)		40 (56%)			

Second, turning to Table 10, the column totals show that 74%, about three-quarters of apprentices in this study, agreed or strongly agreed that school mathematics helped them in their apprenticeship. However, 26%, about one quarter, disagreed or strongly disagreed that the mathematics they learnt at school helped them with the mathematics in their apprenticeship. The statistical data alone is unable to reveal why they made these responses.

In the main body of the table, 32/71 (45%) of the apprentices gave the same response to both items. Also, 33/71 (46%) agreed or strongly agreed that they found mathematics easy at school and that their school mathematics helped with their apprenticeship.

Table 10 *How easy the apprentices found mathematics at school and how helpful in their apprenticeship*

		The mathematics I learnt at school helps me with the mathematics in my apprenticeship Item 8					
		Strongly Disagree	Disagree	Agree	Strongly Agree	Totals	Composite totals
I found mathematics easy at school Item 6	Strongly Disagree	3	2	0	0	5 (7%)	29 (41%)
	Disagree	0	4	20	0	24 (34%)	
	Agree	1	6	23	6	36 (52%)	42 (60%)
	Strongly Agree	0	2	2	2	6 (8%)	
	Totals	4 (6%)	14 (20%)	45 (63%)	8 (11%)	71 (100%)	
	Composite totals	18 (26%)		53 (74%)			

The most important differences between the apprentices' responses were the 20 apprentices who disagreed that they found mathematics easy at school, yet agreed that they found that the mathematics they learnt at school helped them with the mathematics in their apprenticeship.

To summarize, there was considerable variation in the questionnaire data concerning how easy the apprentices found mathematics at school and in the workplace, and how well their school mathematics helped in their engineering work. I now turn to the interview data to investigate these statistical results in more depth.

5.1.3. **The school and workplace mathematics tension**

In the last section, I analysed the questionnaire data from apprentices on their views concerning how easy they found mathematics at school and in their workplaces, and how helpful they found their school mathematics in the workplace. I now supplement this statistical data with comments from three of the five apprentices and seven of the 12 skilled tradespersons and educators who took part in the interviews. The views of these ten participants were chosen because they broadly represented the range of views in the group of 17 participants interviewed.

There were two main opinions expressed. First, all participants spoke approvingly of the mathematics skills they had learned at primary school. Second, while most of the participants expressed criticism of senior secondary school mathematics programmes, usually because they were considered too abstract, some of the experienced educators and tradespersons had come to appreciate the applicability of senior school mathematics much later in life.

While the participants did not object to mathematics per se, they held strong views that mathematics should be relevant to, and applicable in, practical situations. For example, Ben (apprentice) enjoyed practical mathematics and figuring things out for himself or with others. However, he could not see the use of statistics, or even "the computer research stuff" of graphics and design, and English was "just a waste of time". Ben was unable to see the important connection between the broader skills he had learned at school and the skills he could now use in engineering. Therefore, Ben's approach to learning was pragmatic and was restricted within the immediate boundaries of his current worldview. Ben's views were reiterated by Ari (apprentice), who said that his college experiences in mathematics "just felt a little bit useless. You know what I mean?" Ari could not see the relevance of calculus because the "really complicated, um, equations. I don't think they would ever come up personally in my future". These comments reflected Ben's and Ari's current views of their present and future practical applications of mathematics in real-life contexts.

In contrast, Irene (apprentice) had a wider perspective. She believed her physics knowledge helped very much, but "up to a point", which helped her with airflow restrictions in refrigeration. Irene thought that mathematics in the workplace had a "purpose behind what we do now", and that "there's a specific outcome" they were seeking, which was unlike going through the motions at school to pass exams. Therefore, there was a consistent and very firmly held view among these apprentices about the inadequacy of much of their senior school mathematics experiences in preparing them for the engineering workplace. They were still to experience expansive cycles of learning to adapt to workplace mathematics requirements. However, these views might change in the future as they mature and gain more exposure to engineering, as with Courtney (engineer), Robert (expert engineer, educator and entrepreneur) and Murray (engineer) (see later in this section).

The skilled tradespersons and educators' comments during the interviews showed that they wanted more than just competence in calculation. For example, Owen (educator) believed that one of the differences between mathematics in the workplace and at school was that "mathematics in school tends to be a lot of rules, whereas in the workforce you try to get them to apply their maths". Some of his apprentices were good at mathematics and could "do the handstands as required" and could "storm through [to] use the calculator". However, they needed to know that "it's not just a calculation". Consequently, Owen gave his avionics pre-apprentices mathematics challenges to help them "understand the physics and the science of the aeroplane technology to maintain the aeroplane". In other words, Owen wanted people who could think through a problem in context and use appropriate mathematics to find a solution, which they could then link back to the original engineering problem. That is, Owen wanted creative thinkers and problem solvers. However, some apprentices had a more limited perspective, thinking that the task was finished once they had obtained a numerical answer.

Commenting on the applicability of his school experiences to his work, Stephen (educator) said he never used certain school mathematics topics in the workplace, such as simultaneous equations. However, trigonometry could be useful for structures, and transposition of formulas was used frequently. Thus, Stephen considered that only some school mathematics topics were relevant to his job. Paul (training officer) had pursued a different school pathway and had studied business mathematics instead of the academically-advanced senior secondary school mathematics. He thought his school mathematics courses helped him in his apprenticeship because they were oriented towards “day-to-day use”, and therefore were more applicable to engineering. A similar view was taken by Howard (maintenance engineer), who emphasized the generality of school mathematics and the specificity of workplace mathematics “to the job on that particular day”. While his schooling had prepared him well for on-the-job mathematics, “it wasn’t really targeted to my trade, [but it] gave me a good foundation to build on. Yeah, it was more so when I went to tech. Mathematics was more related to the particular trade I was doing”.

However, the difference between school and workplace mathematics, between the abstract and the real, also extended into workplace mathematics education where theoretical and practical considerations met. Hence, in contrast to the approach of some other engineers, Chris (educator) acknowledged the importance of theory in his workplace training. Therefore, towards the end of his apprenticeship, Chris started “to realize that both [theory and practical] gelled. They have to”. In Chris’ experience, an appreciation of the role of mathematics developed with growing experience and maturity. Hence, Owen, Stephen, Paul and Chris acknowledged that some aspects of school mathematics helped them in their work. However, the more abstract topics were either not applicable, or only marginally applicable to their work situations.

The lack of application and real-world contexts in school mathematics courses were frequently mentioned, even by those who had studied senior secondary school mathematics. For example, Henry (avionics educator) said that schools were “not really worried about real-world situations”, nor did they have any perception of what the results meant in practice. Henry was more interested in informal processes, such as problem solving and thinking outside the square than in formal mathematics. He described how he got his students to design a project, build it and then fault-find it. He justified this teaching approach because his students would “get the experience. They’ve used their brains to design it, they used their hand skills to build it, and then they fault-find it”. Mathematics was thus subsumed within the practical context.

I now consider the views of mature engineers on school and workplace mathematics and who modified some of their views later in life. For example, Courtney (engineer), Robert (expert engineer, educator and entrepreneur) and Murray (engineer) said they found no application for their school calculus studies. Robert said that calculus “used to drive [him] nuts” because he “wasn’t applying it”. He claimed that things immediately improved when he could find an immediate practical application for the mathematics he was studying at his polytechnic. Murray (engineer) also emphasized the importance of practical, real-world contexts. He thought the equation for a straight line as algebraic and “very abstract … you know”. Murray summed up his objections by saying that nobody at school

> tells you WHY it's needed. You learn that, but the WHY [is] needed. WHAT is that? And WHY do I have to know that? And I find this is where I got totally flummoxed, at school.

However, Murray's attitude to powers of numbers changed when he was first introduced to spreadsheets in the workplace. Without outside help, he quickly transferred his knowledge of powers, exponents and the compound interest formula to calculate future values of annual increases.

Courtney, Robert and Murray all had senior secondary school mathematics skills and had studied mathematics successfully at tertiary level as well. They all criticised the abstract nature of school mathematics, because its application was hidden. Nevertheless, they subsequently made connections between the abstract and practical engineering contexts, and expansive cycles in their mathematical learning took place in the workplace long after they had completed their tertiary training. It is unlikely that they would have made those connections without the theoretical knowledge they had learnt in school.

5.1.4. **Section summary**

The formative influences of childhood and school experiences were the focus of this section. Several of the engineers described how, even as small children, they had become members of communities of practice, which gave them early experiences of engineering and other practical activities. They may also have been exposed to some theoretical aspects of mathematics in practical contexts, albeit at an elementary level. Family members adopted the role of mentors as the children informally learned the importance of both physical and intellectual tools. Informal learning processes also continued at school, although more emphasis was placed there on formal mathematics development. The result was that a person beginning an engineering apprenticeship possibly already had many years of historical exposure, not only to engineering and other practical contexts, but also to school mathematics, all of which may have contributed to their development. However, there were differences between the requirements of the various communities of family, school and workplace. These posed challenges and contradictions for young people as they sought to find expansive cycles of learning as they adapted to the differing approaches to, and perspectives and requirements of, mathematics at each stage.

An important contradiction from the viewpoint of this study were the differences between school and workplace mathematics. This was evidenced in the questionnaire data which revealed that 41% of the apprentices 'disagreed' or 'strongly disagreed' that they had found school mathematics easy, and 26% 'disagreed' or 'strongly disagreed' that school mathematics helped them in their apprenticeship. These views were investigated further in the interviews. Here, some participants spoke approvingly of their primary school mathematics experiences but were critical of the mathematics courses in their later years in secondary school which they regarded as abstract and removed from real-world contexts. The world of formal school and classroom learning contrasted with how mathematics is learned and practised in workplace settings, which I now investigate. Expansive cycles of learning took time to take place, as I discuss in the next section.

5.2. **Workplace-related learning and mentoring**

The engineers I interviewed all emphasized the importance of workplace-related learning and mentoring. These were seen as essential to adapting to workplace requirements, and to establishing the apprentice's pathway from beginning apprentice to skilled tradesperson. Learning relates to the third research sub-question about the development of mathematics and its associated ancillary skills of problem solving and metacognition, which were necessary for engineering work (see Section 1.3 and 1.4). Engeström's (1987) elements of community and its accompanying rules and roles are important in this section because of the influence of others in the development and application of mathematics skills. Similarly, Engeström's principles of historicity and multi-voicedness reflect long-term influences and differences of opinion and approach in the succession of communities, such as family, school and workplace, which affect an apprentice's development.

Because the mathematical aspects of engineering are not the whole of becoming an engineer, this section begins with a section on the general qualities looked for when selecting apprentices for training. This is followed by an analysis of apprentices' and educators' views about the nature of apprentice training. I then analyse and consider conceptual and higher-order skills before considering the roles of social interaction and communication in learning.

5.2.1. **Qualities sought in prospective mechanical engineering apprentices**

All the educators I interviewed believed that certain qualities were influential in how well people might adjust to the requirements of the mechanical engineering trades. I discuss these in four categories: the right attitude, a broad range of interests, an inquiring mind which indicated a willingness to learn, and mathematical knowledge. While some of these qualities were non-mathematical, they do impinge on mathematical aspects of learning and practice in the mechanical engineering workplace.

Regarding attitude, Paul (training officer) explained that his company's policy was to give someone an opportunity to join a pre-apprentice programme if they show "all of the right credentials and [have] a good attitude. They've approached me, gone to the effort to maybe door knock". The emphasis here was on a person's "attitude", which was valued highly by Paul and several other educators whom I cite below because they believed attitude was important to the apprentices' future engineering development.

Displaying a broad range of interests was taken as a sign of a willingness to learn. For example, avionics educators, like Warren and his colleagues, wanted to know applicants' hobbies and interests, and why they wanted to do the course. Broad interests included experience in practical things because Warren also wanted "kids who have got hands skills where they are physically doing stuff, and using tools, [and] interested in aviation". A good attitude was also displayed in having "a bit of drive and a bit of passion". A similar view was expressed by Owen (educator) and Howard (engineer). Howard emphasized the need to have "a passion to become a mechanical engineer … a passion for mathematics and understanding how things work … Yeah" as well as "hands-on" experience and a "feel" for engineering. Therefore, Warren, Owen and Howard wanted well-rounded apprentices with wide intellectual and social interests, and a passion to learn about engineering.

Other engineers specifically emphasized having an inquiring mind which also demonstrated a willingness to learn. Owen said that being able to discuss how the theory of flight works, how radio waves are propagated and whether the navigation of a plane was affected by the curvature of the earth were significant in building up a general knowledge useful to avionics engineers later in their careers. It was not necessarily the factual knowledge that was important, but the attitude of inquiry that was being fostered in the process. On the other hand, Paul felt that young apprentices displayed a variety of strengths and weaknesses; some were very good at general engineering principles, but lacked creativity and problem-solving skills, while the reverse was the case for others. He believed that these skills could be developed by engineering experience, especially informally through interactions with mentors during the apprenticeship years and having conversations with mature engineers. Hence, Paul acknowledged the importance of communication in the learning process.

The avionics educators placed greater emphasis on specific mathematical knowledge than educators in other branches of mechanical engineering. For example, Henry (avionics educator) said he needed students with calculus knowledge of differentiation and integration "'cause we use that a lot with AC and DC theory"[16]. Stephen (avionics educator) concurred but added that they would make exceptions in certain cases based on the results of a pre-selection mathematics test paper which was interpreted holistically. They took into account the applicant's age, maturity, and length of time away from study. Thus, even in avionics, social and personal factors could overrule the lack of a suitable mathematics qualification in accepting a prospective apprentice.

To summarize, mechanical engineering trades educators looked for a variety of personal qualities and abilities in prospective apprentices. These included having an inquiring mind, practical skills, prior experiences with machines, as well as a passion and intuitive feeling for engineering. The multi-voicedness concerning what constituted proper engineering practice, acceptable practical skills level, and hobbies and interests were evidenced in the philosophies described by educators like Warren and Owen, who would be impressed by previous historical efforts made by prospective apprentices to gain engineering and other practical experience. Skill in mathematics was just one element. The educators' judgment on how well the young apprentice might fit in with the community was often decisive, and good personal skills could override even inferior mathematics skills.

[16] Alternating current and direct current.

5.2.2. **Views on apprentice training and mentoring**

Having investigated the desired qualities and abilities of prospective mechanical engineering trades apprentices in the last section, I now analyse the views of educator/tradespersons and apprentices on apprentice training and mentoring. I first describe the educators' views of apprentice training and their maturation. A significant feature of this was a cultural attitude where apprentices' mistakes were looked upon in a positive light, and even as learning experiences. I then examine apprentices' views of their training and the value they attached to being mentored.

5.2.2.1. **Educators' views on apprenticeship training and mentoring**

This section analyses the views of educators on mentoring programmes. I first discuss the philosophy behind the programmes, and then the contributions made to technical and mathematical learning in conjunction with the roles played by mentors in the communities of practice.

Regarding philosophy, several educators were keen to emphasize the quality of their apprenticeship training and mentoring programmes. From a technical perspective, these programmes were designed to expose apprentices to a wide range of engineering experiences with mentors who were good communicators. Paul's company was typical in this regard and appointed team leaders and apprentice mentors who were not only experts in their trade, but were also "approachable people" who could give apprentices a wider appreciation of the work. The apprentices responded with "… just question after question". In addition, the philosophy of mentoring programmes also included paying attention to apprentices' personal welfare.

Regarding technical and mathematical learning, the avionics educators, Warren, Stephen, Henry and Fred said that their mentoring systems focused on both technical training and apprentices' social development. Warren said that while "90% of what [apprentices] gain is on-job experience", the apprentices also attended special block courses in the first three years of their traineeship to receive "those extra skills". Paul (training officer) said his company had a policy of focusing on the "best of the best", and then offering them exposure to a wide range of quality engineering experiences. The company put the young apprentices through a six-month programme where "they're exposed to everything … in the machine shop with machinists pushing buttons … in the coil shop and … in assembly". Every part of their training was "hands-on". Calculation skills were developed with "time on the job" and practical contexts were used to teach the importance of tolerances and problem solving. Paul strongly preferred to keep problem-solving learning as an informal activity. In this way, watching and talking were important communication features of this company's training policy. A comprehensive system of ongoing development for apprentices and skilled tradespersons combined with an ongoing strategy to foster individual initiative and innovation were backed up with an annual award of "a big sum of money".

Like Paul's company, Warren's avionics pre-apprentices were also exposed to a wide variety of experiences; they "don't just sit in one area like for two or three years". This meant that the training programmes had to be "very well-coordinated", both from a logistical viewpoint and integrated with the teaching programmes taught by colleagues throughout the respective communities of practice.

In contrast, Stephen (educator) talked about how avionics educators mentored apprentices on the technical aspects of their training on the job. This helped recent school leavers to adapt to workplace demands. They adapted "relatively slowly, so they're sort of mentored into it. So these guys helping out, they'll sit in the right-hand seat", observe the gauges and the tools and the torquing and "help out with the calculations, and get a bit of a handle on it that way". In this way they were learning the interaction between tools and context, "because it's quite physical. It's nice to know why the temperature of the day is important and what the temperature of the day does to affect the calculations". Stephen's example demonstrates the integration of physics skills, mathematics and numeracy skills, and the understanding of what they mean in an avionics context. Also, while the apprentice was involved in certain tasks with which they might be familiar, at the same time they were having the opportunity to observe how experienced practitioners were performing more advanced tasks.

Regarding the counselling role of mentors, Warren (avionics educator) emphasized a supportive environment where someone would talk to apprentices who slide "off the rails a little bit" and say, "right, we're one-on-one, let's sit down and see how you're going". He also explained that "people are constantly monitoring to see how these guys are going" with their Unit Standard tasks, which needed to be checked and signed off by on-job assessors. Apprentices who completed their Unit Standards in good time were given a "pay rise". Reflecting on his own time as an apprentice, Chris (authority on fabrication engineering) attributed his own success to the tradesmen who trained him. They took an interest in his welfare, as well as being willing to share "their skills … they were willing to part with their knowledge, and that is one major factor in becoming a good tradesman". Therefore, the influences of personal aspects in people's lives were recognized as influential in their learning and performance in the workplace.

Regarding the involvement of the communities of practice in apprentice learning, Paul described an ongoing, company-wide professional development programme. Paul said that the company's owner "always bought the latest and greatest … so the next machine will come in and it will be the latest technology so they'll [the engineers] get a chance to work on it." These professional development programmes had two aims; first, to foster a broad and detailed understanding of the company's products, and second, to make people aware of the importance of the mechanical engineering tradespersons' contribution to high-level research and the parts that are "being assembled here in the assembly plant". The programmes were reinforced by creating a shared community culture of encouraging people to ask questions and seek answers from colleagues, and using a buddy system to answer physics questions. Warren (avionics educator) brought yet another perspective when he described how school leavers and mature pre-apprentices in their 20s and 30s, who had recently joined the community of practice, helped each other. Mature people brought life experience and tended to admit their lack of knowledge, and say,

> Hey, I don't understand this. 'You young fellas, bright whippersnappers, you picked it up really quick. Can you show me? Can you put it in your own words?' And what they demonstrate to the younger ones is [that] you don't know it all, and there's a way of where they can, you know, discuss things.

In such environments, educators, mature and young apprentices engaged in mutual learning of technical skills where they invited the help of each other, thereby reinforcing the building of relationships, as well as mutually constructing learnings and understandings of the mathematics and physics course material.

5.2.2.2. **Educators' views on apprentices' maturation**

The interview data also revealed the educators' awareness of delayed maturity and its implications for apprentice training. It was an important factor in educators' attitudes to mentoring young apprentices and their acceptance of apprentices' mistakes. Thus, Donald (skilled tradesperson) commented on apprentices adjusting to fine measurements when they came from school: "Ahhhh … well, they get taught that I suppose … yeah, they make a lot of errors but that's all part of training". Stephen (educator) had a similar attitude when he said, "they're expected to make some mistakes ... there's plenty of checks and balances in place" Similarly, Paul (training officer) believed that doing things until they can be done correctly was important because it "gives the apprentice the opportunity to make a mistake and then learn from it".

By way of contrast, a retired engineer with whom I discussed the future course of this study in 2017 related to me an incident where a group of unsupervised apprentices were asked to build a ramp for an elevator in a kiwifruit packing shed. This could have been done accurately and easily using Pythagoras and trigonometry, yet the apprentices decided to use trial and error. Their lack of maturity and guidance resulted in wasted time and materials. Becoming mature in joining theory and practical application may be a long process for some apprentices.

Delayed social and intellectual maturation was also linked to the development of social and higher-order thinking skills. Fred (a leader in an Industry Training Organisation), observed that apprentices often had

> all the pieces [of the jigsaw puzzle], but they haven't necessarily formed the full picture, and once they start to make those connections, then what we find is that they all of a sudden, after two to five years, they all of a sudden are way better.

The fitting together of the jigsaw pieces was also alluded to by Robert (expert engineer, educator and entrepreneur) who commented on teaching an apprentice to operate one of his huge presses - "Oh, I can teach them in a matter of minutes how to use a press… but it takes years to learn what you can do and how to do it". Thus, in a short time, the apprentice would be able to do the basic aspects of his job, but would still lack the higher-level skills to go beyond the strictly supervised tasks given to him by his boss. Therefore, Robert and the other educators were emphasizing that the apprenticeship years were an initial training phase only. The newly-qualified skilled tradesperson would still need to learn many skills, especially higher-order skills, over many years by practicing and social interaction.

5.2.2.3. **Apprentices' views on effective learning**

In Section 4.1, I identified a number of significant issues apprentices faced when adjusting to the mathematical requirements of the workplace. These included skills with numbers, performing calculations in context, the use of calculators, and making mental calculations and estimates. I begin this section by investigating how frequently the apprentices used the mathematics, physics and mechanics skills taught in block courses (see Figure **10**). This provides one indication of the relevance of these topics to the mechanical engineering context and the apprentices' current readiness to use mathematics in the workplace.

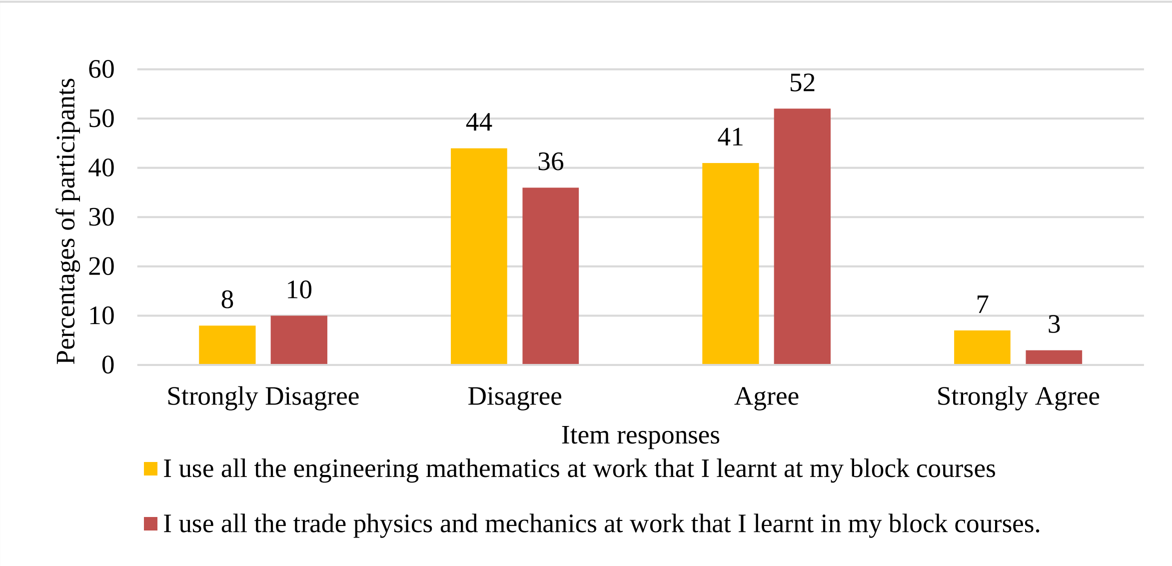

Figure 10 Percentages of apprentices reporting how often they used the topics of block course (n=61/62)

From Figure **10**, almost one-half of the apprentices (48%) 'agreed' or 'strongly agreed' that they used all the engineering mathematics at work that they had learned in their block courses, and 55% 'agreed' or 'strongly agreed' that they used all the trade physics and mechanics at work that they had learned in their block courses. Thus, apprentices were relatively evenly split in their views about how often they used mathematics, physics and mechanics skills at work. This may be attributable to some skills not being directly relevant to some branch specialisations, or to some apprentices having not yet been given the opportunity to use the skills they had learned during block courses in the workplace.

I now turn to how effective apprentices found eight methods of learning relating to: personal learning; such as doing exercises and examples, working online or consulting textbooks, thinking things through for themselves, and interactive learning; such as watching and discussing with others either on the job or in the classroom. They were asked to rate the effectiveness of eight statements about learning, such as, *‘ [Method] helps modern day apprentices to learn to solve mechanical engineering problems using mathematics.’* Figure 11 presents their responses.

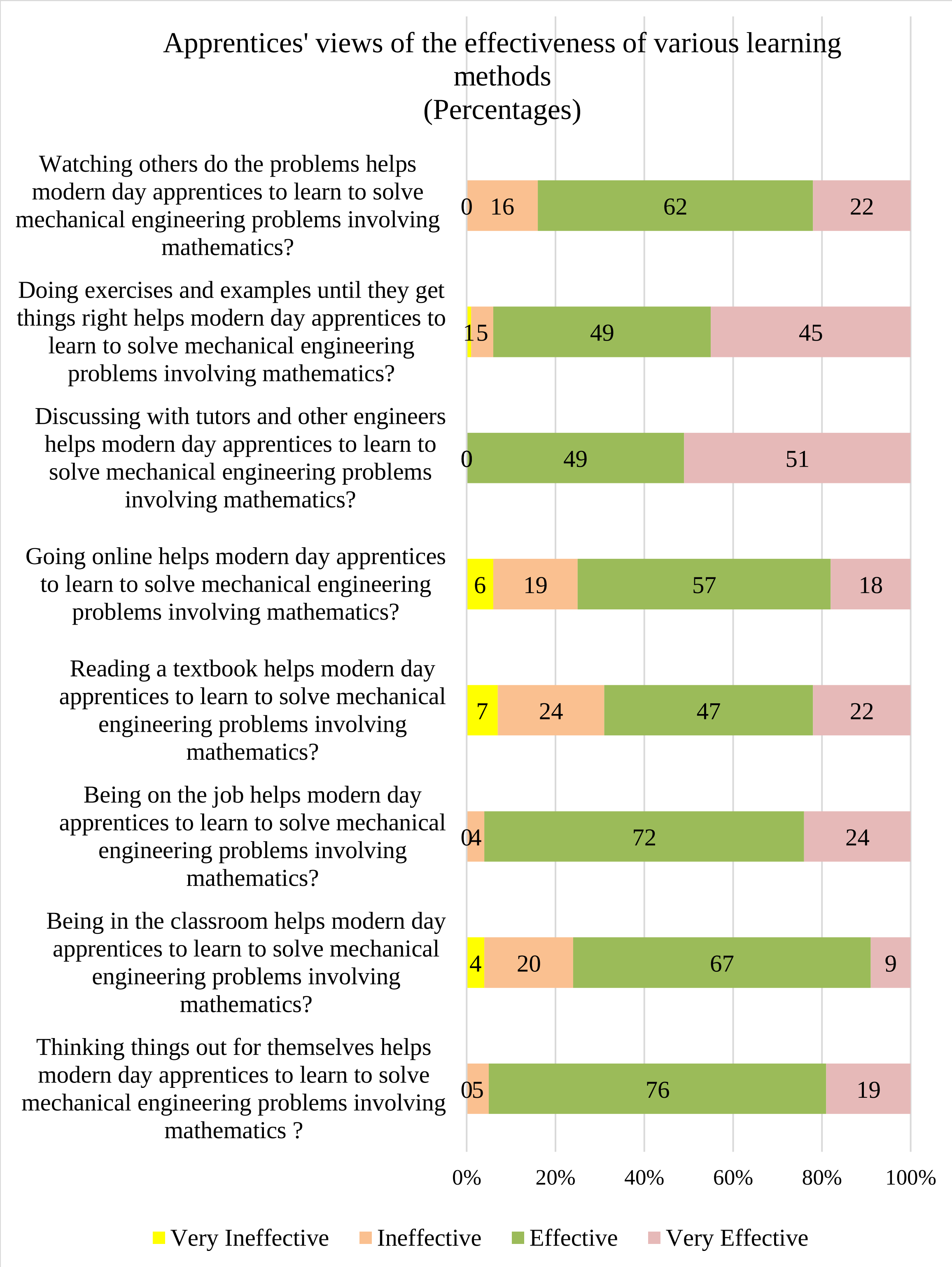

Figure 11 Percentages of apprentices reporting the level of effectiveness of different learning methods to solve mathematical mechanical engineering problems

All of the learning methods were thought to be effective by at least some of the apprentices. However, some methods of learning had particularly high ratings (‘effective’ or ‘highly effective’), such as discussions with tutors and other engineers (100%), and being on the job (96%). These supported comments from Ben, Ari and Nikau about the influence of discussion in the workplace of mathematics and its relationship to engineering, and Warren (educator) organising a lot of group interaction in the classroom. Therefore, a substantial majority of apprentices regarded social learning methods as effective or very effective.

Other highly-regarded learning methods may reflect an emphasis on more individual study, such as thinking things out for themselves (95%), doing exercises and examples until they got things right (94%), and going online (75%) which could be done by individuals at home. These learning methods perhaps reflect a focus on problem solving in modern mathematics learning.

The importance of doing many examples was emphasized by Murray (engineer) when he said:

> Ah, you don’t do it by saying to somebody, ‘Well, just do it once.’ … You’ve got to actually do a lot of exercises … to actually get it ingrained, and it becomes a habit, and I think that maybe therein lies the problem that ah … you don’t move on until that is embedded and ingrained.

A very high percentage of apprentices (88%) also thought watching others do problems helped them to solve mathematics problems. This finding contrasts with Murray’s (engineer) previous comment that “you’ve got *to actually do* a lot of exercises … *to actually get it ingrained*” (my italics), and that observing someone else does not necessarily lead to understanding the fine nuances of what is taking place at each step of the problem.

The three methods of learning that the apprentices gave the fewest effective or very effective ratings were: reading a textbook (31%), going online (25%), and being in the classroom (24%). However, these methods were also considered to be ‘effective’ or ‘very effective’ by substantial majorities of apprentices (69%, 75% and 76%, respectively).

To summarize, the apprentices very strongly endorsed the approaches to learning where communication and collaboration between teachers and students, and between students, created a learning environment where questioning and experimenting had the potential to raise levels of conceptual understanding. The apprentices also appreciated the nature of contextualised knowledge that enabled them to see what was happening, ask questions and thus make the transition to deeper understanding. The social interactions that characterised these learning methods featured strongly in workplace mentoring, which I consider in Section 5.2.5.

5.2.2.4. **Apprentices’ views on workplace training and mentoring**

This section analyses vignettes of the interviews with four apprentices, Ben, Ari, Simon and Nikau (see Appendix P) about their early experiences of life and engineering learning, the tasks given to them later in their apprenticeship training, including indications of their development of higher-order skills and the extent to which they were supervised and mentored. These are presented in Table 11.

I then analyse their comments for workplace social interaction, such as mentoring. These apprentices were all currently involved in sheet metal (or fabrication) engineering tasks where the mathematical tools frequently involved calculation and measuring skills. From the comments made about their workplace social interaction with mentors and skilled tradespersons, their views on workplace mentoring are likely to reflect apprentice sheet metal engineering experience in a positive workplace culture.

Table 11 *Summary of conversations with apprentices Ben, Ari, Simon and Nikau*

	Ben	Ari	Simon	Nikau
Early experiences of life and engineering learning	• Family tradition in engineering • Engineering at school, built a windmill and a steam train • Focus on practical mathematics • Measuring and cutting steel, "plus or minus a mil" under the guidance of a supervisor	• Worked on his own cars • Repairing and maintenance on heavy-duty diesel trucks • Familiar with reading torque wrench • Gaining familiarity with replacement parts	• Learning "materials and stuff …" • Knowledge of hydro testing for pressures	• Worked on own cars • Realigning forks on a forklift • Learning to check measurements often • Awareness that heat causes bending on welding jobs • Replacing brackets for wheels
Later tasks	• More complicated tasks, harder and bigger • Pythagoras used often • Found transposing formulas difficult • Good at mental calculations • Made a venturi in his third year[17]	• Logical thinking and finding information • Simple mathematical conversions • Accuracy required for fabrication	• Checked calculations often • Drawings and calculations - units, volumes, welding times • Familiarity with materials such as which welding wire was needed	• Awareness of problem-solving skills • Possibility of multiple solutions • Making large 6-metre steel moulds, used problem solving to fix mistakes
Workplace Interaction	• Supervision less as time went on • Asked for help with problems	• Supervised by tradesperson • Enjoyed interaction with tutors and other engineers	• Consulted tradesmen or night school tutor • Tasks signed off at each step	• Boss checked all his work • Confident to ask for help

[17] A venturi is a device that causes an incompressible fluid's velocity to increase as it passes through a constriction. It's static pressure therefore decreases. By measuring the difference in pressures, the flow of the fluid can be calculated.

First, regarding the projects in which they were engaged during their apprenticeship training, the tasks they were required to perform were aligned with their current level of mathematical application. Hence, taking measurements and measuring lengths to within specified tolerances were important. However, Ben may have had to multiply the measurements on a scale diagram by a scale factor when constructing larger items. On the other hand, Ari demonstrated a developing appreciation of what constituted acceptable margins, tolerances, required measurement accuracy for the task, and calculation rounding. These are significant features of apprentice development. Ari also demonstrated that he appreciated the fluctuating errors in his measuring devices and the acceptable limits of those measurements in practical situations. He appreciated how the mathematics tools and the physical tools interacted in the workplace. Simon spoke about more advanced mathematics when he said he had to “figure out [the rest] by yourself”. This involved extended reasoning, which would have been an important step up from Simon’s historicity experiences from school. He was training himself in how mathematics was employed in a variety of situations, including the challenging task of calculating the time jobs might be expected to take to complete. Like the other apprentices, Nikau had also learned measuring skills and an appreciation of the need to work within tolerances. His mentors had shown him how tricks of the trade could help him do this. Therefore, apart from Simon, the mathematics involved was basic but graduated in difficulty to suit the apprentices’ current level of knowledge.

Second, regarding the development of higher-order skills, apprentices undertook problem-solving activities in accordance with their levels of skill and understanding. Like the tailors and quartermasters in Lave and Wenger’s (1991) study, Ben was assigned tasks of graduated difficulty according to his experience and capabilities. He eventually constructed a venturi meter, which was able to accurately measure the flow rate of a fluid by reducing the cross-sectional flow area in the flow path which generated a pressure difference. Simon realised that the success of his work depended on high quality mathematical and physical data, and he used his phone as a sure source for finding out what he needed to know. Nikau also had to use problem solving in his work when the steel moulds did not “come out right”, and while he was using elementary-level mathematics, he acknowledged and reflected on the possibility of multiple acceptable solutions to problems and the impact of using other engineers to help find them. In each case, the apprentices’ problem-solving capabilities were nascent and probably in a continuous process of development through listening, observing, discussing, and individual reflection, as with other skills.

Third, the apprentices had a good understanding and appreciation of the value of social interactions in the workplace. Ben enjoyed his apprenticeship experience and the relationships he was forming with others, and was happy with his progress in becoming an engineer. Like the other apprentices, Ben had the confidence to ask for help, which suggests that mentors understood the role of communication/discussion as a learning tool. However, Ben still needed the mathematical skills of others to guide him in the construction of the venturi. In addition, all four apprentices spoke about the detailed guidance that supervisors/mentors gave them in the early stages of their learning. For example, Nikau's work was regularly supervised by his mentors and the tradespersons around him. The guidance was gradually withdrawn as the apprentices progressed.

Most tasks in the welding specialization did not use formal mathematics. Courtney (engineer) described how one welder's work could achieve high precision without relying on measuring with precise instruments or performing calculations. Courtney said that, "visually, [his work] looks like a robot's welding", and that the welder had a very good understanding of accuracy, and how things bent under heat. Thus, big frames on gangways for wharves required two beams to be pre-stressed, and placed back to back so that, when they were welded, they would "stay pretty much straight". The welder's skill had been very carefully developed over many years and appeared to be the result of practice and experience; similar to the tailors and quartermasters in Lave and Wenger's (1991) study. He either did not need to use formal mathematics at all, or it was subsumed within the practical context. Indeed, the whole scenario appeared to be completely reliant on engineering judgment, hand-eye coordination, and highly developed intuition. The requirement for precision was there; it was just hidden.

To summarize, the focus of this section was on mentoring and training programmes that fostered apprentices' engineering and other skills to allow them to adapt to the requirements of the mechanical engineering trades workplace. The ideal candidate for entry into the training programmes had broad intellectual and social interests, an inquiring mind and a passion to become an engineer. While sound mathematics skills were desirable, educators were prepared to relax this requirement in the case of a candidate who was strong in other areas and was well motivated to succeed. Several engineers had no childhood or school background in engineering at all but met the other criteria. Educators and apprentices also spoke about the efforts made to ensure apprentices' wellbeing, which suggests that they recognised the importance of a positive workplace culture. Hence, there was widespread recognition of the importance of teamwork, social skills and communication skills. I discuss these in Section 5.2.5.

The ways mathematical and ancillary skills were developed related to the third research sub-question. The apprentices placed high value on several of the methods of learning and applying mathematics, especially informal methods where social interaction was a feature. However, other attributes were equally significant, such as attitudes to learning, a passion for engineering, problem solving, curiosity, and practical skills. Much of the apprentices' early work involved observing and emulating experienced educators, tradespersons and engineers in making measurements, for example, and then cutting to within certain limits. They were given more challenging tasks as their practical and mathematical skills developed. In this way, the culture of the workplace was important to the comprehensive development of apprentices. Their interactions with mature, skilled tradespersons willing to pass on their knowledge were also important and are discussed in Section 5.2.5.

5.2.3. **Conceptual issues surrounding calculation skills and their contexts**

In Section 4.1, I presented the study's findings about the nature of the mathematics knowledge and skills required in the mechanical engineering trades context and how they were used. However, using mathematics knowledge and skills is a complex process requiring conceptual understandings of numeracy, mathematics, and the physical context. In this section, I focus on conceptual issues around tolerances, the metric and imperial measuring systems, and rates.

Conceptual development is an important fundamental learning component of successful engineering practice (see Section 2.3). In the mechanical engineering trades environment, conceptual development appeared to develop as a result of many processes. These included listening to the explanations of others, observing others working, discussions, formal classroom teaching, practice, exposure to workplace contexts, individual experiences and reflection, and the creation of individual understandings and heuristics that work. In this section, I illustrate the conceptual aspects of these learning processes by examining mainly the interview data relating to tolerances and finer measurements, the metric and imperial systems of units, and rates and densities (see Sections 4.1.2 and 4.1.3). In Section 5.2.4, I then focus on the development of higher-order skills, such as problem solving which requires paradigm shifts in thinking and solving multi-step problems. Difficulties with acquiring either of these skills impeded the development of practical skills for some people.

5.2.3.1. **Working within tolerances**

Different engineering specialisations worked with different tolerances. In performing fabrication tasks, Nikau (apprentice) was required to work within tolerance requirements of perhaps ± 2 mm. He received all the specifications for his jobs from the company office, including the required tolerances. Initially, "a lot of the stuff wasn't coming out when I cut something, measured it, [and] cut it". In other words, he could not keep within the tolerances. This was because his ruler was not properly lined up, or the end was worn. He decided the solution was "to not go … right off the end … [but] to go off, say, the 100 mm mark … it's more accurate, and stuff like that". Nikau's learning was linked to developing his conceptual feel for size and his measuring techniques, which he was learning from experience and his interactions with others.

Paul (training officer) explained how apprentices were introduced to the need for fine tolerances early in their experience. He linked learning this with mentoring, where team leaders or apprentice mentors talked to the apprentices about the product and "the reason we have to have 0.01 [mm] tolerance on this part". A tolerance of 0.01 mm is just 10 microns, and apprentices initially found it very challenging to deal with such small measurements. I asked Paul how they adjusted to this. He replied, "again, it all depends on previous exposure". When the apprentices heard the fine tolerances expected of them, their reaction was "Wow, that's d***** near impossible to achieve! But our equipment makes it possible". The fact that this group of apprentices was amazed suggests they may have already acquired an appreciation of the rudiments of measurement sizes, which helped them transfer their knowledge and skills to fine measurements. Mentoring and discussions were also part of the learning process. The progressively increasing involvement of Paul's apprentices with tolerances was therefore similar to the way the tailors and quartermasters in Lave and Wenger's (1991) study were introduced to increasingly complex tasks.

The conversation moved on to how well apprentices understood the notion of microns. Paul said, "Well they probably don't understand what one micron is". However, they appeared to learn that on the job:

> the more time they spend throughout their apprenticeship, the more conversations they have … or when something goes wrong … they say, 'that's four microns out' … So, how much is that? How much did we get it wrong by? Then they have a conversation, or something … explains that … or they figure it out; they go on Google or something.

In this example, Paul acknowledged that apprentices initially had difficulties with conceptualising very fine sizes and linked their learning about tolerances with social interactions on the shop floor. However, the apprentices used their initiative and formed their own community where they interrogated their understandings and misunderstandings, and sought out information relevant to their work. Therefore, important means for apprentices to learn how to adapt to fine measurements were discussion, exposure to the context, and repeated practice.

Learning about tolerances was also done in formal block courses. Henry (educator) described how apprentices learned to cope with very fine measurements during a three- or four-week block course, by just making measurements. When they had to measure using "micrometers, verniers, all different tools, … we do go down to 1 thou of an inch" (approximately 25 microns). Stephen (educator) gave the example of apprentices having to measure a pair of mating parts to make sure each was within limits, and then to make sure that the limits fitted each other. He felt that the pre-apprentices got "through that fine". In Henry's and Stephen's examples, the formal block course learning was integrated with informal discussions and repeated practice.

To summarize, different branches of engineering had different requirements for tolerances and different techniques for achieving them. Appreciation of small measurements and learning the limits of how closely things needed to be machined was an essential feature of an apprentice's development. Paul's comment that the machinery made achieving much smaller tolerances possible than the apprentices had previously imagined, suggested that apprentice learning integrated both conceptual understanding with the engineering techniques needed to achieve the precision required (see above). Moreover, ideas of tolerance were linked to an increasing awareness of decimal place values and accuracy, as well as a feeling for size.

5.2.3.2. **Working with metric and imperial systems**

The imperial system of units is no longer part of the school curriculum of many countries, including New Zealand. Consequently, apprentices come into the workplace with little or no conceptual appreciation or practical experience of measuring and calculating in lengths given in inches and feet, or weights given in pounds. Here, I compare the experiences of old-timers learning imperial and metric units with those of this younger generation.

Courtney (engineer) had grown up in his father's factory, and unlike most apprentices today, had probably received a good grounding in measuring with imperial units from his father. He also remembered using imperial units at school and was still converting halves, quarters, eighths and sixteenths of inches in his head to metric "all the time". He claimed to have learned to do this by practice and experience, because drills and bolts could be in imperial or metric sizes, "and you need to know the difference between them". Courtney appeared to have learned by exposure, practicing, discussions with his father, and experience.

Imperial units are still used widely in mechanical engineering. Paul (training officer) related how metric–imperial conversion was still relevant because his company had "a race shop that builds American cars from American parts – all in inches, quarter inches, feet". In common with Courtney, Paul believed people quickly adapted to using both systems of units by practical experience.

However, Donald (engineer) who specialised in heavy-duty transport, believed difficulties lay in the mathematics of converting "inches to millimetres … Yeah!" It took time for apprentices to adjust to fine measurements, like hundredths of a millimetre or thousandths of an inch. He admitted that "they make a lot of errors but that's all part of training". This did not seem to bother Donald, because he believed apprentices learned through interactions with others, practice, and making errors (see Section 5.2.2.2).

Ari (apprentice), who worked with Donald (engineer), understood the connection between imperial units and fractions. He quite liked the fractional approach of the imperial half inch, three quarters (¾) of an inch, nine sixteenths ($^{9}/_{16}$) of an inch, which different sized bolts on the Caterpillar equipment from the USA had, but felt it was "obviously not as straightforward as 10 mil, 20 mil, 30 mil". Ari was also skilled in comparing imperial lengths, such as $^{5}/_{8}$ inch and $^{9}/_{16}$ inch. He knew that $^{5}/_{8}$ inch was bigger than $^{9}/_{16}$ inch, "'Cause you double it … it'll give $^{10}/_{16}$". I was interested to find out if Ari had developed the skill of converting to an equivalent fraction by himself, or if he had learned it at school, or from talking with tradespersons. He replied, "Yeah, probably really back in primary, you learn that kind of stuff". He explained that he also figured it out for himself, "because obviously, you want to make the same so you can compare them". In addition, Ari felt that the size of the spanner gap was a visual aid that helped apprentices learn over time. Ari's learning also involved social interactions, formal schooling and making the transference from physical tools, like spanners.

However, Ari's view was not shared by some educators. Arthur said that today's students had difficulties understanding imperial units because some machines mixed both imperial and metric sizes. Stephen (educator) agreed, and added that while Air Bus is French and all the dimensions of the aircraft are in millimetres, "all of their fasteners are in inches 'cause they use American hardware … everybody uses American hardware or a derivation of it". This makes conceptual understanding very important.

Stephen also linked making conversions between the two unit systems to developing a feeling for size, numeracy, estimation, and mental calculation skills:

> They struggle to estimate as well as somebody who grew up with them … sometimes, if they write down a ridiculous answer, they don't realise they've written down a ridiculous answer because they haven't got a good bearing on what an inch is, or what a thousandth of an inch is.

Therefore, both Arthur and Ari emphasized that mixing imperial and metric units was a conceptual issue with significant practical consequences.

Murray (engineer) had a different perspective. He had developed his own method of converting metric units to imperial units by always converting everything to a decimal. He could do this in his head; a skill probably built up with practicing over several decades. He was also able to consult a chart of conversions on the wall, or even use trial and error. However, converting diameters of nuts and bolts was not always straightforward, as it involved conceptual and engineering understandings of rounding decimals. The bolt size must be rounded downwards, and the nut diameter must be rounded upwards to obtain a satisfactory fit. I recall my father's discussions of metric charts on toolroom walls in the 1960s, which caused plenty of confusion for some engineers then too.

To summarize, conversions between metric and imperial units required conceptual and numeracy understandings of relative sizes, conversion formulas, and relationship to a workplace context. Apprentices needed to learn how to use formal mathematics approaches, using conversion formulas or consulting a chart, and simultaneously be flexible enough to understand that even an experienced engineer like Murray sometimes resorted to trial and error. In each case, final decisions were made using engineering judgment, and the learning was done by many different means, especially exposure and practicing.

5.2.3.3. **Working with rates, volumes and densities**

Rates have both conceptual and practical consequences for the mechanical engineering trades. Examples include pressure measured in pounds per square inch (lb/in^2) or Pascals, speeds measured in ms^{-1} (metres per second) or km/h, cutting rates for drills, dilution rates, and rotational speeds measured in revolutions per minute (rpm). I illustrate the difficulties in understanding and learning rates with the example of density. Density is important to engineers because it is used to calculate the mass (or weight) of objects after the volume has been calculated in appropriate units (see Section 4.1.3). Robert (engineer) illustrated this with an example where density was used to ascertain if a sling was strong enough to lift a hunk of steel (see Section 4.1.3.7).

According to Arthur (educator), some apprentices found density challenging, partly because of the conceptual notion of density and partly because of conceptual difficulties with its units; g/cm^3 or kg/m^3. Arthur explained:

> One thing they do have a problem with that comes up pretty quickly is a cubic metre of water, like 1m by 1m by 1m is one tonne … They pick that up very well, and something that actually they don't sort of know very well from school. They've heard about volume, but they don't actually use it in practical terms … that a metre by a metre by a metre weighs one tonne. They go, 'how did you know that? Did you weigh it?' No, I just worked it out. I tell them that if you've got 10 cm by 10 cm by 10 cm, that's a kilogram.

Hence, Arthur used a formal teaching method to teach the basic concept of density. However, he first needed to establish that they understood how two variables interacted; how weight was related to volume. This was complicated though because the apprentices had heard about volume, but they had not used volume in practical situations. Moreover, difficulties with conceptualising volume and calculating weights using density were exacerbated by the concept of density and its awkward units (g/cm^3, g/cc, or kg/m^3). To illustrate the conceptual issues of two variables in connection with density, Arthur continued his formal approach and appealed to practical contexts. He explained:

> If this, say, pound of butter is actually steel, how much would it weigh? They'd go, 'I don't know.' So, you tell them about density [and] volume a bit, and they can quite [easily] work it out. It's like a light bulb situation, and next minute, Ding! The light goes on. They go, 'Oh, I've got this now. I can actually understand it'.

The formal approach used by Arthur, probably accompanied by class questioning and discussion, was designed to encourage conceptual understanding. However, how well the understanding was developed by these formal means was open to question because apprentices often found dealing with more fundamental issues, such as multiples of ten to be challenging. When using density to calculate weight, Arthur used the formula: $Mass = Density \times Volume$, so the mass (or weight) was the product of density and volume. Arthur went on to create his own heuristic; "to find the weight of a block of steel in tonnes, first find the volume in m^3, and then multiply by 8. This is because the density of steel is between 7,750 and 8,050 kg/m^3". The advantage of this heuristic was that an engineer could simply follow a simple procedure to calculate an answer in a practical situation.

This heuristic can be learned as a procedure and no conceptual understanding need be involved. However, the challenge of calculating the volume still remains. To investigate the ability of apprentices and pre-apprentices to calculate simple volumes and maintain consistent units, I gave them a multi-choice problem as part of the questionnaire. They were asked to calculate the volume of a box 200 mm by 200 mm by 200 mm in m^3. Their responses are presented in Figure 12.

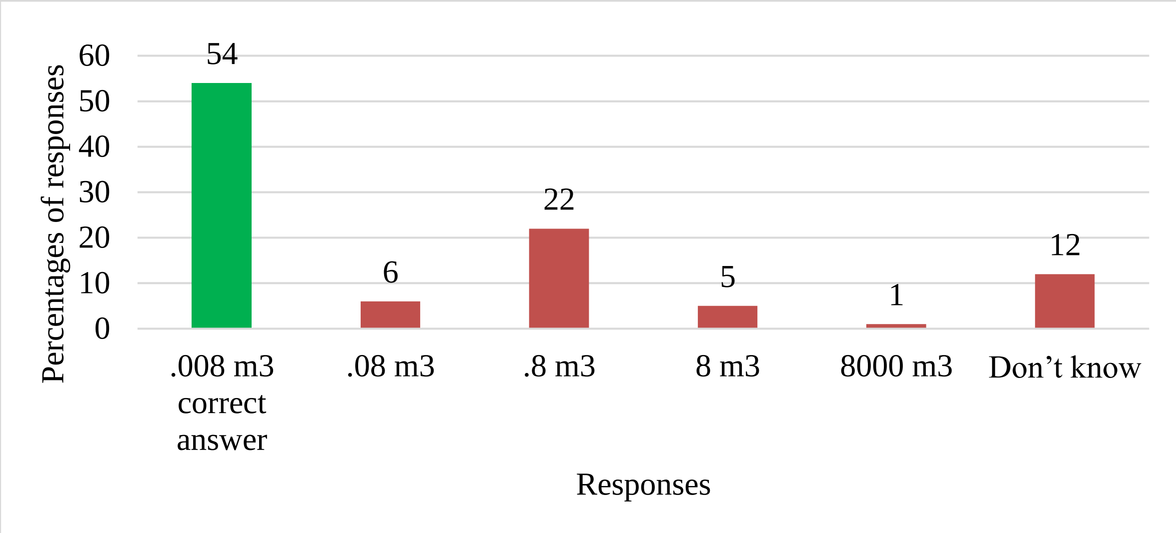

Figure 12 Percentages of apprentices who reached each answer in calculating the volume of a box *(n=93)*

From Figure 12, just over half of the apprentices (54%) obtained the correct answer of 0.008 m^3. Therefore, it was a relatively difficult calculation for the apprentices. One difficulty in doing this calculation was that the answer was required in different units (m^3) from those given in the original wording (200 mm). It was necessary to make the units consistent. If everything was converted to metres at the beginning, the calculation becomes easier to perform; volume is $.2 \times .2 \times .2 = .008\ m^3$. Owen (educator) emphasized to apprentices that they have to understand the units they "are working with pretty well, [otherwise] the maths that follows can be quite a challenge". Irene (apprentice) correctly calculated the volume, and said that she converted units very often in her work and had been trained to think carefully about the units "early on in school". Therefore, both Owen and Irene found calculating the volume of the box was straightforward because they understood the necessary concepts and the need to maintain consistency of units. Moreover, their initial learning had been reinforced over time by practical exposure and practicing.

To summarize, conceptual understandings and their application to engineering problems, such as allowing for tolerances, making conversions between metric and imperial unit systems, rates, and volumes, were found to be important issues for apprentices when they adapted to the mathematical requirements of mechanical engineering trades workplaces. Conceptual issues were also found in gaining a feeling for size and what constituted a reasonable answer, especially when fine measurements were required. There were many methods of learning used: interactions with others by discussion and observation, formal classroom teaching, practice, exposure to context, experience, individual reflection, and creating heuristics. Conversions between the metric and imperial systems had the added difficulty of young people not having been exposed to imperial units at school. Maintaining consistency of units when calculating volumes was an issue for nearly half of the apprentices, and perhaps some skilled tradespersons. Conceptual difficulties in calculating volume will almost certainly prevent the accurate calculation of mass, because calculating volume is the first step in calculating mass. Moreover, it follows that these conceptual difficulties are also likely to manifest themselves when applying the concept of rates in other contexts. Conceptual difficulties with rates are linked to higher-order skills, such as problem solving and problems involving several steps, which I now consider.

5.2.4. **Learning higher-order skills**

As mentioned earlier, engineers need mathematics and accompanying ancillary higher-order skills to successfully apply mathematics in workplace situations (see Section 1.4 and Chapter 4). These higher-order skills frequently require paradigm shifts in thinking and multi-step problem solving. Developing problem-solving skills is an essential part of the pathway from beginning apprentice to skilled tradesperson and then to expert engineer.

The engineers in this study had strong beliefs about the importance of problem-solving skills in mechanical engineering trades. However, creating the necessary expansive cycles of learning problem solving was a complex process, and was sometimes entered into reluctantly. For example, while Robert was a noted innovator, there were times when he and his colleagues felt that they should focus on what they did best and get other people to install a machine, but whenever they did this, “every single time we’ve been incredibly disappointed”. The reason to subcontract work was that problem solving in an area where they lacked expertise involved a long, mentally and physically exhausting, expensive, and potentially fruitless learning process.

Some engineers did not feel comfortable with facing the unknown. Henry (educator) commented that some apprentices simply stopped, rather than try to work things out for themselves. I asked Henry if apprentices who were better at problem solving were better prepared academically. He replied, “No, I would say they are better prepared manually because they’ve done other things mechanically, and had to work them out, and that translates into problem-solving efficiency, if you like”. This suggests that the culture of a workplace can create experience in and exposure to unknown environments, which can then build confidence and expansive learning cycles. Over time, interactions with others and growing confidence to step outside boundaries can aid the development of a problem-solving attitude. Henry’s views were supported by Murray (engineer), who cited an example of an engineer who ran meetings where everybody was encouraged to express their ideas. This led to finding creative solutions and individual workers developing confidence to express their views and contribute to company profitability. In contrast, Arthur (educator) felt that the formal education offered by schools was inimical to promoting creativity.

Confidence to go beyond the boundaries meant that apprentices needed to develop and consider multiple potential solutions to problems. This was a significant aspect of Henry’s teaching philosophy and reflected his view that apprentices would considerably improve their ability to cope with unusual avionics situations during their apprenticeship years. This also involved developing the ability to consider multiple perspectives which Henry believed was "gonna give you the different paradigms to come up with for your solutions". When I asked if there might sometimes be more than one solution, Henry replied, "Could be, yeah. If I get three engineers in a room, [and] say, 'right, this is what I want to build. I need one of these to happen.' I guarantee there'll be three different solutions". I asked if those solutions could differ widely.

> They could do! 'Cause you might have someone who's got a real radical way of doing something, but yes, it still works. You know, when I crack an egg, I can throw it on the ground, I can crack it gently… (laughter)

Henry's comments indicated that fostering problem-solving capabilities was influenced by both private reflection and exchange of ideas accompanied by critique and social interaction.

Many problem-solving situations required the ability to solve multi-step mathematics problems. Apprentices were asked how easy they thought it was to solve multi-step mathematical problems. Educator/tradespersons were asked this about the apprentices at the beginning of their training and at the end of their training. Their responses are presented in Figure 13. Most educator/tradespersons (79%) 'agreed' or 'strongly agreed' that apprentices coped well with solving multi-step mathematical problems at the end of their apprenticeships, compared with 38% at the beginning. Seventy-seven percent of apprentices 'agreed' or 'strongly agreed' that they found multi-step mathematics problems were easy.

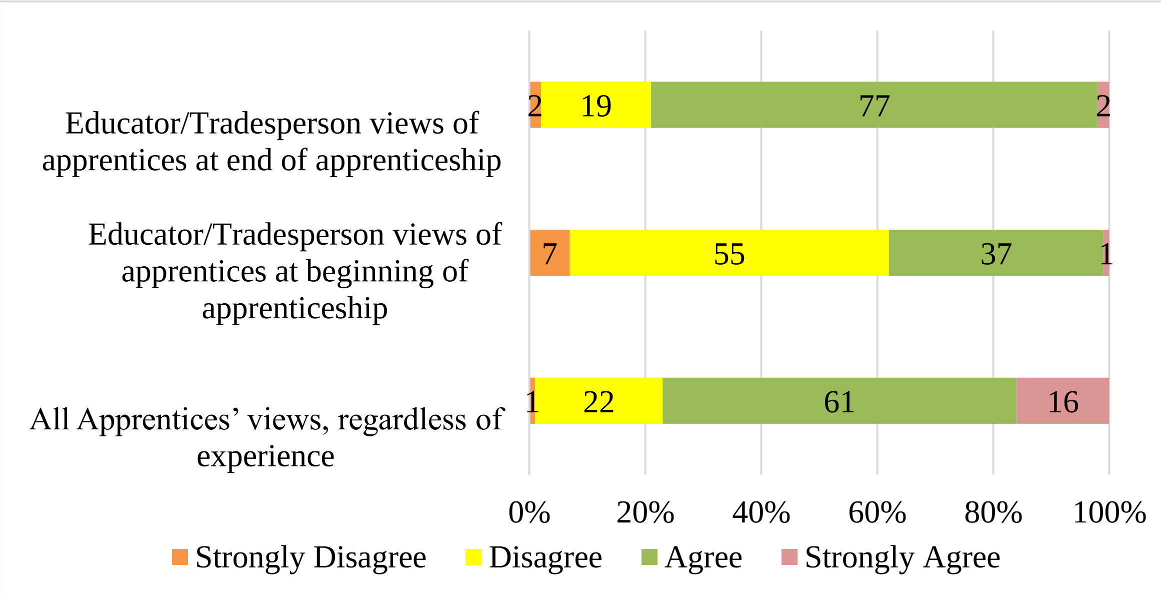

Figure 13 Percentages of educator/tradespersons' views of how easy apprentices found multi-step mathematical problems (n=93)

This increase may have been due to several factors, such as block courses, interactions with tradespersons and educators, on-the-job experience of using mathematics in real-world contexts, and growth in individual and community maturity. Therefore, the data appeared to support the conclusion that educators/tradespersons thought apprentices had improved their capability to cope with multi-step mathematics problems during their apprenticeship.

While it is not possible to definitively conclude how learning higher-order skills took place, learning appeared to follow the formal and informal patterns outlined earlier in this section. Therefore, learning higher-order skills included a process of osmosis involving interactions and discussions with others, and informal observations.

5.2.5. **Social interaction and communication**

In this section, I analyse the views of educator/tradespersons who emphasized the importance of good social interactions and communication as key factors in the workplace. They related these to such areas as learning, leadership, safety, outlining a problem, and persuading others of the soundness of their ideas. I begin by discussing the importance of good social interactions and communication. I then relate how the engineers' informal stories played an important role as a learning tool.

5.2.5.1. **The importance of social interaction and communication**

The interview data showed that social interactions and communication made important contributions in mechanical engineering workplaces as one of Engeström (1987) tools for teaching and learning, and as an element of teamwork.

First, Paul (training officer) identified communication skills as being important for mentors and hence for apprentice learning. He said, "we've identified [our mentors] because they listen, and then they hear what the problem that people are having [is], and then they'll hopefully have an answer. If it's a technical question, then there [are] reasoning conversations" that take place. Paul was thus emphasizing the role empathetic mentors played in apprentice learning. His views were reiterated in Warren's (avionics educator) description of how mature pre-apprentices demonstrated to the younger pre-apprentices that "you don't know it all, and there's a way of where they can, you know, discuss things" (see Section 5.2.2). All the apprentices rated discussions with tutors and other engineers as an 'effective' or 'very effective' means of learning (see Figure **11**). Moreover, from the interviews, Ben, Ari, Simon, and Nikau all expressed their appreciation of being able to discuss problems with older tradespersons, tutors and mentors (see Appendix P). Thus, communication was seen to be an effective component of learning and some companies actively promoted this.

Second, Henry (avionics educator) and Murray (engineer) identified communication as an essential element of teamwork, which they regarded as being crucial for effective workplace functioning. For example, Henry considered communication was "absolutely" important in the group assessment where the students had to work in teams to fix the boards the educators had made "totally dead" (see Section 4.2.2). He then elaborated, "if we've got no communication, then you end up with nobody knows what to do, everybody walks around … What do I do now?" This meant that there would be no discipline and that if, "you're asking the guy to do a job, he must do that job in accordance with the manuals, and in accordance with the system".

Henry incorporated teamwork training as part of his apprentice training programme. For example, he set apprentices the task of building a set of shelves for themselves:

> Henry: Normally what we do is, we say, ‘right, here’s your box, here’s your tools.’ Then we walk away. We don’t tell them anything.
>
> Kelvin: And what happens in that group? Do you find that one person takes control, and everybody else follows?
>
> Henry: Yep, one person goes, ‘all right, bom, bom, bom. Yeah, I’ve done some of this before. What do you know? Oh, OK, Right, you organize this. You organize that.’ Others just sort of just stand there and all do their own thing in different directions.

Henry described the teams that had no leadership or cooperation as “the ones that end up with screws missing and brackets on the wrong side”. Thus, Henry emphasized the need for good communication, leadership, and teamwork; skills he believed were essential in the workplace.

Murray (engineer) also had strong views about the importance of social interactions and communication in the workplace. “Do you really want to get me started on that one? *(both laugh)* It’s non-existent very often”. Murray linked communication with teamwork because “if you’ve got teamwork, you get communication and regular meetings. I think people tend to avoid that, and to get at cross purposes. I think communication is probably nine-tenths of our problems”.

Murray also believed that planning with the help of others was important. He recalled one engineer who led meetings and got “input from everybody, and I find that engineers don’t get input from everybody”. Therefore, encouraging communication was aimed at helping problem-solving situations which, in turn, helped maximise the efficiency and smooth running of projects; essential components in promoting workplace learning and change. Conversely, when there was a non-communicative workplace culture, Murray said that the apprentices “will just learn to not communicate”. Therefore, Murray emphasized the role of communication as a tool for getting good ideas from everybody in the workplace, and ideas that would have implications for workplace efficiency and financial profit.

Many of the examples of communication I found were initiated and led by skilled tradespersons in the community instructing the younger ones. However, apart from apprentices asking questions, there were other times when apprentices led the dialogue, for example, Henry’s attempts to get apprentices to work as a team, either by building a box or repairing an electronic board. In these cases planning and problem solving both required communication initiated by apprentices. Demonstrating appropriate communication skills was included in their group assessment. Another example of apprentices initiating dialogue was provided by Warren where older apprentices asked younger apprentices about things the younger ones had learned at school or elsewhere - “… Hey, I don’t understand this. ‘You young fellas, bright whippersnappers, you picked it up really quick. Can you show me? Can you put it in your own words?'" Hence, dialogue could be initiated and led either by the more experienced members of the community or by the apprentices.

To summarize, Paul, Henry, and Murray all emphasized the need for effective communication. Communication was a key feature of the apprentices' and tradespersons' development and workplace effectiveness. Most importantly, communication was central to confronting workplace contradictions. Therefore, communication was an essential tool in promoting expansive cycles of learning in individuals and in their larger workplace activity systems.

5.2.5.2. **The stories the engineers tell**

Similar to the situations in some other workplaces where collaborative learning is important, the stories that engineers tell are important Engeström tools for communication in the mechanical engineering trades workplaces (see Section 2.5.2). They enhance social relationships within the community and play a transformational role in the developmental pathways of apprentices and tradespersons. The stories do not usually involve complicated mathematics, but only quick mental estimates made as the storyteller speaks.

The data showed the importance of the stories engineers tell as one of Engeström (1987) tools for learning. When I asked Courtney (engineer) about how engineers discussed their work and alternative ways of doing things, he replied "drawings on the back of tobacco packets were good ways to get a point across, …[of] trying to get what's in your head into someone else's head". Then, to emphasize the financial implications of making mistakes, Courtney spoke of hiring $6,000,000-cranes at $30,000 a day, so you do not want to "muck it up". The amounts of money involved in the stories frequently impress listeners.

Arthur's (educator) reminiscences about racing around his local cemetery to test the latest modifications to his go-kart may not seem serious but they nevertheless provided insights into his early engineering learning experiences. Apprentices may pick up the art of telling stories when young. For example, Ari (apprentice) could coherently describe the logical processes in fault-finding blockages in dump truck hydraulic systems using the colloquial slang of his generation. Hence, Ari's process of learning storytelling began early in his career, and this was typical of other apprentices I interviewed. In each case, the stories revealed the experiences and levels of technical knowledge of the storyteller and what they considered to be important (see Section 4.2.2.2).

Robert (expert engineer, educator and entrepreneur) spoke about two 400-tonne presses designed to produce CNG cylinder systems in New Zealand motor vehicles during the early 1980s.

> Their manufacture was heavily protected from foreign competition, partly by import tariffs. A massive and very expensive plant was built. It had been running about three weeks when "the government removed the import tariffs (*roars of laughter*) and the business died". Robert's father was offered two huge presses for $200,000 each

> (perhaps over $700,000 each in today's currency), but could not afford them. A little while later, he was offered them for $50,000 each. Robert's father really wanted those presses, so when he heard they were going to be scrapped, he rang up and said, "What the hell are you doing? One day, New Zealand's gonna need those presses, and you will have cut them up for scrap. It's not that I don't want them, it's just that I don't want to pay for them (*laughter*)".
>
> Robert continued: "And they did the deal for a bottle of gin."
>
> Kelvin (with amazement): "Oh, really?"
>
> Robert: "A bottle of gin!"
>
> Kelvin (still incredulously): "Literally?"
>
> Robert: Literally, but the flip side was [that] we had to take all of the gear that no one else wanted.

And so, Robert's father got the two presses for a bottle of gin, and Robert would spend several months helping clear up the mess.

Robert's story of the presses can be interpreted on different levels. At a surface level, the story might simply be regarded as entertainment. However, like the earlier stories, Robert's story also had important functions of conveying factual information, observations and understandings. Moreover, by arousing the listener's interest through entertainment, learning opportunities were created where the technical aspects of the message were conveyed and so increased the understanding of the mechanical engineering context. Through storytelling, bonding occurs within the context of an activity system, social relationships develop, and teams are able to operate more effectively. Storytelling can be regarded as one means of promoting expansive cycles of learning.

There were many Engeström tools involved in my conversations with the engineers, such as communication, the stories themselves, and knowledge. Sometimes, an engineer acted as a mentor, especially when describing creativity. I would then change my role from listener to interlocutor; asking for clarification or suggesting my own perspective. This frequently stimulated other avenues of discussion. Engeström's contradictions and their historical nature pervaded our discussions, particularly mental calculation skills and problem solving. From time to time, the contradictions drifted on to controversies; an example of multi-voicedness. Importantly for me, the discussions were expansive cycles of learning that refined my earlier understandings, brought my knowledge more up to date, and enhanced my feel for the way mathematics was used in the workplace.

5.2.6. **Section summary**

In summary, training apprentices to adapt to the mathematical and other demands of the mechanical engineering trades workplace was a long and complex learning process involving maturation. It comprised the development of technical competence as well as social and personal qualities, all of which were influenced by the community and its rules. Therefore, the educators looked for potential apprentices with sound mathematical skills, intellectual curiosity, certain personal qualities, and flexibility of approach that would enable them to confront new challenges.

Apprentices experienced many different types of formal and informal learning that could potentially lead to individual expansive cycles of learning. The learning methods incorporated listening to the explanations of others, observing others, discussions, formal classroom teaching, practice, exposure to workplace contexts, individual experiences, and reflecting on and creating individual understandings and heuristics. Apprentices were often strongly influenced by experienced and empathetic mentors and skilled tradespersons. Apprentices established sound personal relationships with mentors from whom they wanted to learn knowledge and skills.

Two areas of conceptual difficulty emerged that hindered apprentices' learning progress and being able to achieve expansive cycles of learning as an engineer. The first difficulty related specifically to mathematics and the small numbers surrounding precision and fine measurements, tolerances, converting between systems of units, and conceptualising volume, mass, and rates. The second difficulty was related to higher-order skills, such as problem solving, which required apprentices to make paradigm shifts in the ways they thought about and solved multi-step mathematics problems.

Mechanical engineering apprentices and skilled tradespersons learned mathematical skills by several different methods. These methods included observing, emulating, listening, and questioning. Both formal and informal learning methods were used, such as family influence, schooling, block courses, interacting with others, language, communication and mentoring. These processes can be regarded as Engeström tools for learning and were used in accordance with rules accepted by the community.

Social interactions and communication served important functions in mechanical engineering trades communities where collaboration, conveying information, planning, and organizing teams led to successful problem solving and hence to expansive cycles of learning. Moreover, telling good stories also served as teaching and learning opportunities. This applied not only for developing mathematical and engineering skills, but also for establishing workplace cultures where questioning, reflection, and discussion of problem solving could take place.

Communities also played a crucial role in the learning and development of mathematical skills by encouraging opportunities for learning through dialogue and social interaction. The multi-voices of mentors, including family members and educators, were important in apprentices' development, especially by sharing their historical knowledge and experience. Therefore, situated learning was important to the learning and development of mathematical skills. Apprentices valued social interactions, including the sometimes divergent approaches that resulted in contradictions. Contradictions were found in school and workplace perspectives on mathematics, gaining conceptual understanding, and intergenerational differences of values and approaches. Resolving these contradictions enabled apprentices to adjust to the requirements of the workplace and participate in important expansive cycles of learning.

In short, social interactions and communication are important means of informally developing higher-order skills in communities of practice, and can potentially resolve contradictions and usher in new cycles of expansive learning. In the case of individuals, expansive cycles of learning correspond with Lave and Wenger's (1987) LPP and allowed the individual to move closer to the centre of community of practice.

5.3. **Chapter summary**

This chapter focused on Engeström's (1987) fourth question, How do they learn? It reported the findings pertaining to the third research sub-question on how apprentices and skilled tradespersons in mechanical engineering trades developed the mathematics skills necessary for their work (see Section 1.3).

The findings in this chapter demonstrated that engineers learned using both formal and informal means. Informal learning took place during childhood, while school and engineering workplace experiences incorporated both formal and informal learning (see Section 5.1). Mentoring, experience, social interactions and communication were also important means of developing higher-order skills, such as problem solving and its associated skills of creativity, flexibility of thinking, extended reasoning, and logical thinking (see Section 5.2). In some workplaces, conscious efforts were made to create environments that encouraged the development of higher-order skills. The results reported in this chapter indicated that social interaction and communication were key to promoting learning and workplace effectiveness (see Section 5.2.5).

Moreover, workplace learning was done eclectically using both acquisitionist and participationist models. Thus, individual reflection and practicing, as well as social interactions, communication and storytelling emerged as important learning tools. The need for a focus on conceptual understanding, as opposed to procedural knowledge, was demonstrated in practical examples involving conversions of units and rates (see Section 5.2).

Regarding the development of higher-order skills, learning was regarded as lifelong, and expert engineering skills involving higher-order elements of numeracy took many years to develop. Adjusting to sophisticated engineering settings added to the conceptual difficulties with mathematics experienced by many apprentices. These adjustments included making judgments on whether to use mathematics, what mathematics should be used, how the calculation should be performed, the degree of accuracy required in relation to the required tolerance, and what the numerical answer meant regarding the original context (see Section 1.4), problem solving, and team participation.

Problem solving also strongly interacted with physics and some old-timers considered more physics background from school would help learning. In addition, formal teaching accompanied by practicing was widely used. Informal situated learning occurred when apprentices and engineers worked in small teams with good communication. Learning experiences from childhood, family and whānau, school, apprenticeship, mentoring, figuring things out for themselves, and informal storytelling also featured strongly in apprentice and tradesperson development (see Section 5.2.5). In all, this chapter found that apprentices and skilled tradespersons in mechanical engineering trades developed the mathematics skills necessary for their work through a combination of formal and informal learning. The findings are discussed in Chapter 6.

Chapter 6. **Discussion**

Introduction

This chapter discusses the findings of the study reported in chapters 4 and 5 regarding the main research question and three research sub-questions.

The main research question was:

> What key features of mathematical learning characterise the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering?

The three sub-questions were:

1. What is the nature of the mathematics skills employed in the mechanical engineering trades?
2. How do apprentices and skilled tradespersons in mechanical engineering trades apply mathematics skills in their work?
3. How do apprentices and skilled tradespersons in mechanical engineering trades develop the mathematics skills necessary for their work?

I first provide a short answer to the research questions, which I then expand in more detail in the discussion of the three research sub-questions (see Sections 6.1, 6.2, and 6.3 respectively. The findings reported in chapters 4 and 5 revealed three key features of mathematical learning that characterise the pathway from beginning apprentice to skilled tradesperson and then to expert engineer in mechanical engineering.

First, simply stated, the mechanical engineering tradespersons need to learn mathematics skills and apply them in complex, interrelated, real-life contexts (see Chapter 4). Second, these mathematical skills can be regarded as intellectual tools which are integrated with other intellectual tools such as higher-order thinking, problem solving, extended reasoning, conceptual understanding, and procedural knowledge (as discussed in Section 4.2). Together, these intellectual tools govern both the creation of physical tools and how those physical tools are used. Third, mechanical engineers learn, use, and develop their mathematical skills formally and informally, using acquisitionist and participationist means, and employing practicing and communication (see Chapter 5).

However, before discussing each of the three research sub-questions, I discuss the relationship between each research sub-question and its relationship to the CHAT and SL frameworks. Near the beginning of the discussion of each research sub-question, I include a summary of the major contributions of the CHAT elements and principles to the analysis and interpretation of that section. These are intended to be illustrative of the main points only.

6.1. **Nature of the mathematics skills**

This section focuses on answering and discussing research sub-question 1 regarding the nature of the mathematics skills employed in the mechanical engineering trades, their meaning and importance. From the findings, those skills include purely mathematical skills (see Appendix C), as well as various ancillary skills needed for their successful application. After linking this section to the CHAT framework, I then answer research sub-question 1 by summarising the mathematical and ancillary skills that have emerged from the findings. I do this with reference to Engeström's elements and principles. I then consider the importance and interconnectedness of Engeström's elements and principles in the subsequent discussion of the research sub-questions.

6.1.1. **Links to CHAT**

This section provides a summary of the findings about the nature of the mathematical knowledge and skills used in the mechanical engineering trades (see Section 4.1). The bullet points below summarize the links between the CHAT framework and Section 6.1.

Engeström's Elements

- Tools: Physical and intellectual tools are constantly changing due to new thinking and technology. Tools include formal mathematics and numeracy skills pertinent to application in the mechanical engineering trades context as well as higher-order thinking skills such as problem solving, creativity, extended reasoning, reflexivity, and metacognitive skills.
- Rules: Rules include the decision to use mathematics and how it is to be used, and how ancillary skills such as engineering judgment and heuristics are related to the needs of the engineering context. Verticalisation and abstraction of thinking are used when problem solving demands them.
- Community: There are differing views on many issues, including the role of mathematics and its use.
- Roles: Educators, skilled tradespersons, and newcomers all bring knowledge to the community. They have different views on many issues, especially the mathematics required. These views can depend on age and branch specialisation. Roles strongly influence views on what mathematical knowledge is required and how it is to be used. The roles of educators and learners can be interchanged. There are gradual changes in individuals' roles due to movement from the periphery to centre of the community of practice.

Engeström's Principles

- Multi-voicedness: multi-voicedness is reflected in the debates within the community of practice over what mathematics is needed and how it is to be used. Multi-voicedness is often dependent on branch specialisation and generational perspectives.

- Historicity: Prior experiences from childhood and school are often formative but usually require adaption to engineering practice.
- Contradictions: For example, school knowledge and ways of doing things needing to be adapted to the workplace, new knowledge challenging traditional practice, individuals adapting to workplace mathematics requirements, school experiences of mathematics being regarded as irrelevant in the real world, debating minimal procedural knowledge that concentrates on particular skills only versus the need to foster creativity etc., gaining a wide experience and education.
- Expansive Cycles: These are closely linked to increasing conceptual understanding and procedural knowledge in individuals and the wider communities of practice. Various stages of mathematical development and maturity can be successfully achieved.

Regarding Engeström's tools, the findings showed that mathematics was regarded by the engineers as an important tool for engineering purposes because they used it often and widely to calculate accurately and accordingly to produce effective products. The engineers also recognised many other tools: physical and intellectual, and even human, because interaction with other engineers also serves as a tool for involvement in problem solving, and even for learning. Moreover, Engeström's matrix is significant in understanding the mathematical tools in this section because it illustrates the complexity of tool usage and its interconnectedness with Engeström's other elements and principles in the engineering workplace. Thus, contradictions regarding innovation in engineering technology could interconnect with contradictions involving community rules for engineering practice, multi-voicedness and historicity factors inherited from tradition and school education, and the struggle of some individuals to promote their ideas of what constitutes an acceptable expansive cycle of learning.

The findings, therefore, indicated that formal mathematics knowledge and skills were one significant main feature pertaining to the nature of the skills (see Appendix C) (see Section 4.1). Moreover, other non-mathematical ancillary tools were identified that were essential to successfully applying mathematics in the workplace. These included adaption to workplace requirements, higher-order cognitive and metacognitive skills, all of which I now discuss in turn.

6.1.2. **Mathematics knowledge and skills**

This section discusses the mathematics and numeracy skills pertinent to the mechanical engineering trades, ongoing concerns about mathematical skill levels, and the division of opinion over broad versus minimalist attitudes to mathematics (see Section 2.1.1 and Section 3.3.3 and Section 4.1).

Several significant findings emerged from the data. The study revealed that the mechanical engineers used the mathematics and numeracy skills outlined in Appendix C frequently in their work. Fluency with numbers was regarded as being crucial, especially having a feel for number size and being able to perform mental calculations quickly and accurately. However, despite US 21905 being the official New Zealand formal mathematics qualification for mechanical engineering trades (see Appendix C), there were long-running community concerns, and hence contradictions, regarding mathematics skill and knowledge levels among both apprentices and skilled engineering tradespersons (see Section 5.2). Therefore, the engineers regarded a one-off performance in a formal qualification as not being sufficient to ensure mathematical competence long-term. Moreover, with the requirement to demonstrate competence only in a certain proportion of the assessment, then the objective of attaining profound understanding of fundamental mathematics is missing (Roble et al., 2017; Tandog et al., 2019).

The data also revealed widespread concern about mathematical skills and knowledge among even skilled tradespersons throughout their careers and was reflected strongly in the discussions on the need for accuracy in calculation and measurement (see Section 4.1). Insufficient mathematical skills and knowledge no doubt hampers further progress in integrating physical and intellectual tools after the apprenticeship has been completed.

Viewed in this light, it is important to note that the inclusion of more mathematics content in the major revisions to the mathematics requirements for mechanical engineering trades requirements planned for December 2022 should help address these concerns (NZQA, 2019e, 2019f, 2019g). Nevertheless, the important questions of the extent of future learning and ascertaining how well an engineer or apprentice can apply mathematics skills in context remain unresolved.

In another response to the inadequacies above, some companies run professional development programmes for their engineers (see Section 5.2.2.1). However, the high percentage of engineers expressing concerns about numeracy would suggest that ongoing programmes of mathematics education and assessment tasks directly integrated with engineering scenarios might benefit the engineering trades, such as is found in authentic mathematics assessment (Drake, Wake, & Noyes, 2009; Gulikers et al., 2004; Gulikers, Bastiaens, Kirschner, & Kester, 2006, 2008; Gulikers, Bastiaens, & Martens, 2005; Gulikers, Kester, Kirschner, & Bastiaens, 2008; McCoy, 2007) (see Section 2.4.3). In this way, the community might allow debates to develop, and historical as well as recent contradictions to emerge into the open. Most importantly, new expansive cycles of expansive learning could be established that would extend the capabilities of individuals and the community as a whole.

An important debate in the mechanical engineering community concerned broad versus minimal mathematics skills that were only applicable to the basic tasks of an engineer's current engineering specialisation (see Section 4.2.1). Owen was one of several engineers who understood the short-sightedness of the minimalist position because it inhibited essential problem-solving development, understanding of aeronautical principles and fault-finding. The debate has long-term historical roots, and conflicts with the need for a broader mathematical appreciation to help prepare apprentices for future technological and specialisation change. The tension between broad and minimalist views was reflected in the multi-voiced debates over mathematics topics reform. Several engineers felt that while apprentices could calculate using a formula, they still lacked the important skills of applying the principles and relating the answer back to its engineering context.

The tension between broad and minimalist views was also reflected in the emphasis engineers placed on mathematics in context, which means that they were not usually interested in the more generalised and more abstract verticalisation processes that feature in school programmes (Gravemeijer & Doorman, 1999; Treffers, 1993; van den Heuvel-Panhuizen, 2001) (see Section 2.4.3). Instead, they prefer to focus on the practical application of numbers (FitzSimons, 2005). The debates on mathematics content are a major source of contradiction within the community. Minimalists reflect a purely pragmatic worldview that indicates a widespread inability to accept the less tangible aspects of education as being useful, or to see beyond the immediate and current needs of their area of specialisation.

Another feature of mathematics skills and knowledge was a unanimous concern and community contradiction regarding calculation and measurement accuracy (see Section 4.1). The reasons cited for accuracy concerned finance and safety, similar to nursing (Coben et al., 2010; FitzSimons et al., 2005; Gillham & Chu, 1995; Wright, 2007). Linked with the need for accuracy was the community-wide emphasis on making a step-up from school mental calculation and estimation skills and fluency when using numbers. The engineers considered fluency with numeracy to be important in gaining a feeling for size and what constituted a reasonable answer in context, and which they then frequently used for both thinking and communication.

The participants' unanimously agreed requirements for accuracy in measurement and computation are, therefore, an important part of the rules of the community and a key feature of mathematical learning. Moreover, the agreement about the need for accuracy suggests unanimity of voicedness, although what constituted an acceptable answer may depend on the context of the immediate problem, the particular engineering specialisation, and local community rules. In addition, there appeared to be historicity issues with some apprentices coming from school where the consequences of obtaining a wrong answer did not have the high-stakes consequences as they do in the workplace. For this reason, an important issue for young apprentices was to break some bad habits "acquired in school" (Lave & Wenger, 1991, p. 73).

Regarding the contribution of this study to new knowledge, the literature identifies numeracy and mathematics as important workplace issues as well as the modern importance attached to higher-order thinking skills, as in the PISA and PIAAC studies. This study enriches that knowledge by describing real-life scenarios in the New Zealand mechanical engineering trades context where these skills are widespread and used frequently. The study also demonstrates that the mathematical topic knowledge required for mechanical engineering trades mathematics corresponds closely with the New Zealand NCEA Level 1 mathematics requirements. This appears to be new knowledge.

To summarize, regarding the first research sub-question about the nature of the mathematics skills employed in the mechanical engineering trades, fluency in numeracy skills and calculation accuracy were significant contradictions for the mechanical engineering trades community. They do not seem to be solved by learning minimal skills only. Gaining suitable workplace numeracy skills is discussed in Section 6.3. In the next section, I move beyond strictly mathematical considerations to discuss ancillary skills that are necessary when applying mathematics in the mechanical engineering workplace.

6.1.3. **Ancillary skills - adaptation to workplace requirements**

In the previous section, I discussed various aspects surrounding mathematics knowledge and skills. I focus the discussion in this section on various ancillary, non-mathematical skills that are essential to applying mathematical knowledge and skills in mechanical engineering trades contexts. The discussion of the ancillary skills is linked to deciding whether or not to use mathematics, higher-order skills, and the tension between school and workplace. Incorporating ancillary skills results in numerous contradictions, among which is the tension between school and workplace mathematics.

6.1.3.1. **Deciding to use mathematics**

In this section, I discuss a widespread contradiction that emerged from the findings concerning when to use mathematics, what mathematics to use and how to use it (Coben, 2000). This was illustrated by the apprentices making a ramp in a fruit packing shed where an inappropriate decision was made to use trial and error rather than the accurate methods of formal mathematics (see Section 5.2.2.2). Applying mathematics in this workplace situation would have avoided time and financial wastage because the mathematics involved was simply substitution in trigonometry formulas studied at school. This is consistent with the multifarious factors involved in applying mathematics to workplace contexts, as outlined by FitzSimons et al. (2005). However, since issues of time and finance were significant in engineering decision-making, situations could also arise where obtaining unnecessary accuracy would have wasted time and money. This is important when assigning tolerances to a job where a satisfactorily accurate job needs to be produced in the minimum time and as cheaply as possible. In other situations, heuristics and engineering judgment were important skills that could override mathematical considerations in the decision-making process. In such cases, satisfactory approximations could simplify and speed up decision-making and the time it took to do the job. Community rules that governed acceptable practice were also multi-voiced, because what constituted acceptable practice depended on engineering specialisation, individual preference, and longstanding historical tradition. The decision to use mathematics, what mathematics was appropriate, and how to use mathematics was a complex process, and consistent with Coben’s (2000) definition of numeracy (see Section 1.4).

A further factor in the decision-making process was that workplace mathematics involved constructing mathematical models which are approximations of the underlying contextual reality. These models are based on assumptions which are also often simplifications of reality, and which, therefore, remain acceptably accurate over only a limited range of values before breaking down. Apprentices presumably learn to make these important judgment calls over time, with growing maturity, and with experience. A key feature of mathematical learning is appreciating the limitations of mathematical models (see Section 4.2.1.3). This is a crucial element in moving closer to the centre of Lave and Wenger's community of practice. It is also linked to community rules concerning what, when and how mathematics is to be used, and to community discussions about employing engineering judgment and heuristics. These discussions become potential contradictions between theory and practice, between individual engineers, within engineering communities, and especially between engineering branches.

Regarding the contribution of this study to new knowledge, the literature records the importance of ancillary skills in the workplace. This study provides practical application from New Zealand mechanical engineering trades workplaces of the importance of contextual and ancillary considerations such as higher-order thinking when deciding if and how mathematics should be used in workplace situations.

To summarize, learning the skill of deciding whether or not to use mathematics takes place over time. It involves social exchange of ideas and interaction, multi-voicedness and the rules of the community of practice. The decision-making involves weighing up the effects of various mutually-conflicting factors which may overlap with higher-order thinking. In the next section, I discuss certain other aspects of higher-order thinking, and how they affect workplace processes.

6.1.3.2. **Higher-order skills**

The importance of higher-order skills such as problem solving, creativity and flexible thinking were acknowledged as important by about 98% of the participants (see Section 4.2.1). In this connection, several engineers said that creativity and flexibility of thinking provided the paradigm changes in perspective that lead to successful problem solving. This resonates well with NZC objectives which call for broad mathematical understandings and social skills, as well as the basic skills of estimating with reasonableness, and calculating with precision (Ministry of Education, 2007). A significant result was, that for the engineers, basic knowledge, creativity and social skills were not mutually exclusive choices because they wanted people who could integrate basic skills with deeper mathematical understandings, and in social contexts.

The engineers also linked creativity to the logical thinking required for diagnosis in fault-finding and maintenance engineering (see Section 4.2.2). Even young apprentices were involved in small teams doing maintenance and were learning to make decisions based on several criteria, including what was better for the client. Consequently, early in their careers, apprentices were developing higher understandings and strategies, and gaining experience at applying problem solving in real contexts. Growing conceptual understanding and procedural knowledge of how machinery functioned, such as Ari working on hydraulic systems in a small team was, therefore, one important way of fostering progress towards the centre of Lave and Wenger's community of practice (Lave & Wenger, 1991).

Higher-order skills were also specifically mentioned by those engineers whose specialisations required an aptitude for fault-finding and analysis (see Section 4.2.1 and Section 4.2.2). Thus, it was important for apprentices to develop an attitude of applying reasoning to problem-solving situations in everyday working life. This included extended reasoning skills and examining different points of view. Significantly, getting the right answer and moving on without reflection, and passing examinations, did not necessarily develop people who could fix faulty pieces of equipment. Moreover, educators like Owen presented apprentices with mathematics problems to help them understand the physics and the science behind aeronautics technology. In this way, the important skill of taking a broad perspective was being fostered with calculations being related to context. This is consistent with the findings of other studies, including verticalisation associated with abstraction (Treffers, 1993; van den Heuvel-Panhuizen, 2001) (see Section 2.4.3). Accordingly, minimal skills would not be sufficient for aeronautical engineering needs, and so apprentices were challenged with non-aeronautical contexts to help develop higher-order thinking by practising a broad range of skills in a range of contexts (Ridgway, 2002).

However, the emphasis on higher-order skills may mean further abstraction, a contradiction which is reminiscent of school mathematics, such as Real Mathematics Education, which consciously generalises more basic understanding to create deeper levels of conceptual understanding (Treffers, 1993; van den Heuvel-Panhuizen, 2001). Verticalisation of thinking and its accompanying abstraction lie at the heart of the school and workplace mathematics tension which I discuss in the next section.

Regarding the contribution of this study to new knowledge, the literature identifies higher-order thinking skills as essential to the modern workplace. This study extends that knowledge by demonstrating how higher-order thinking skills are essential in modern New Zealand mechanical engineering trades workplaces and how they are integrated with practical applications.

6.1.3.3. **The tension between school and workplace**

This section discusses the tension between school and workplace mathematics (see Section 5.1.3). The findings from this study clearly revealed that there are different perspectives formed by contextual requirements that govern community rules for accepted practice. This results in a tension between school and workplace attitudes to mathematics, the skills needed, and the way they are used. There are important ramifications for workplace practice, attitudes and efficacy, as well as the adaption from school to workplace requirements.

First, since many of the mathematics skills and higher-order thinking objectives required for mechanical engineering trades are part of both the school curriculum and engineering requirements, it might be thought there would be a seamless and relatively straightforward transition from school to workplace mathematics. Thus, while this appears to be true for around 81% of the questionnaire participants, there was a significant minority of 19% of participants who disagreed or strongly disagreed that they found their school mathematics useful in their engineering work. This view was particularly pertinent to the upper secondary school where the interviews demonstrated that even engineers with senior secondary school mathematics qualifications could be "flummoxed" by anything they could not relate to practice and the real world (see Section 5.1.3). This important contradiction was expressed in their perceived generality and abstractness of school mathematics compared with workplace mathematics. This criticism was made even by those engineers who had later constructed their own applications for their senior secondary school knowledge.

A significant disjunction reported frequently in the literature was the failure of students and adults to see the relevance of their mathematics studies at school to life (Lave & Wenger, 1991; Marr & Hagston, 2007; K. Mills & Lomas, 2013; Ridgway, 2002). Many people tend to regard mathematics as a formalised process, unreal, "useless, abstract, and taught without relevance", and with rules made up by the teacher that do not apply in the real world (Marr & Hagston, 2007, p. 9). This view runs counter to the notion implicit in the main research question that mathematical learning is relevant to the workplace, and is consequently a contradiction for individuals and throughout many communities of practice. Murray's comment (see Section 5.1.3) summed up the attitudes of many participants when he said that nobody told you "why it's needed", and suggests that while the mathematics may be useful, the lack of contextualization in its presentation may be allied to the lack of student appreciation of its practical application. This contradiction is consistent with the view that schooling is "predicated on claims that knowledge can be decontextualized" (Lave & Wenger, 1991, p. 40). Moreover, since the New Zealand Curriculum emphasizes both mathematical content and higher-order thinking objectives, then there is a disjunction between curriculum requirements and classroom practice.

However, while motivating students to step beyond their current conceptual understanding can be uncomfortable and mystifying, those mature engineers who did so eventually found their school mathematics useful and empowering. For example, Courtney, Robert, and Murray (engineers), each of whom had progressed along the path from contradiction to expansive cycle of learning and had come back to examine the concepts many years later, made connections between the abstract and the real, had found a use for the mathematics, and then gone out of their way to personally develop themselves (see Section 5.1.3). As a result, they could now use those mathematics techniques and understandings in practical situations (Knowles et al., 2011). However, while they were still pragmatists at heart, they would nevertheless accept abstraction only if it was motivated contextually beforehand.

Regarding the contribution of this study to new knowledge, there is an extensive literature on the tension between school and workplace attitudes to mathematics and the way mathematics is used. This study has strongly confirmed that tension in the New Zealand mechanical engineering trades workplaces. Moreover, the study also demonstrated that some engineers eventually found their senior secondary school mathematics to be useful. This appears to be new knowledge.

To summarize, long-term benefits to engineers could be achieved by wider mathematics learning and relation to context. Their current attitudes to mathematics were partly due to negative historical experiences from school mathematics, and partly related to a growing awareness of how their understanding of engineering mathematics could have been enhanced much earlier. Wider mathematics learning and relation to context might help resolve the long-term ambivalence to mathematics exhibited by even skilled tradespersons and experts. This was consistent with the efforts of several educators, as well as with Ridgway's (2002) recommendation to encourage the deployment of a broad range of skills in a range of contexts. Moreover, some educators had incorporated these strategies into their teaching programmes, and also seen the importance to learning of both social interaction and personal reflection working in tandem (see Section 6.3).

6.1.4. **Section summary**

To summarize Section 6.1, the first research sub-question referred to the nature of the mathematics knowledge and skills. These skills included development of understanding and proficiency in specific mathematics topics and important ancillary skills that enabled mathematics to be applied in the workplace context. Development of some of these skills had already begun at school and needed to be further enhanced and then adapted to meet workplace requirements. The ancillary skills included the higher-order skills and the complex decision-making involved in whether or not to use mathematics, or what mathematics to use, and how it should be used. Heuristics and engineering judgment frequently overrode purely mathematical decisions.

The discussion of the nature of the skills is now complete. However, how those skills are used in mechanical engineering contexts is also a complex matter that requires close examination. In the next section, I focus on what the findings of this research revealed about how participants applied their mathematical learning in mechanical engineering trades contexts.

6.2. **Applying the mathematics skills in context**

This section continues discussing key features of mathematical learning, but from a mechanical engineering application rather than a mathematical content perspective. The data revealed that engineers frequently saw problem solving in engineering contexts as fundamental to the successful application of mathematics in their work. However, problem solving is linked to the logical sequences of the procedures they adopt to solve those problems, which are linked in turn to how the engineers understand the context of the problem and how it might be approached. Thus, understanding successful application of mathematics in the mechanical engineering trades workplace depends on three fundamental areas - first, the primacy of context, second, conceptual understanding, and procedural knowledge, and third, problem solving, and creativity. In this section, I illustrate these areas with applications drawn from routine and non-routine mechanical engineering contexts. I then address the application of mathematics in contexts requiring extended reasoning.

6.2.1. **Links to CHAT**

This section provides a summary of the findings about how the mathematical knowledge and skills are applied in the mechanical engineering trades (see Section 4.2). The bullet points below summarize the links between the CHAT framework and Section 6.2.

Engeström's Elements

- Tools: Tools are often applied with interaction between acceptable engineering practice, conceptual understanding and procedural knowledge, higher-order thinking mathematical skills, extended reasoning, problem solving, creativity, reflexivity, and transfer of knowledge between contexts.
- Rules: Rules are frequently governed by currently accepted standard engineering practice, the need for sensible tolerances, and time and financial restraints.
- Community: The many communities are influential in defining acceptable practice, experimenting with, and evaluating new ideas.
- Roles: Mentors, educators and other skilled tradespersons are active in developing apprentices and sharing knowledge and skills.

Engeström's Principles

- Multi-voicedness: There are varieties of practice between individual engineers and communities of practice, and many different ways of solving problems depending on accepted practice in local and wider communities of practice.
- Historicity: Traditional ways of doing things develop over time due to new challenges, new thinking and new technology. Attitudes to mathematics learning are instilled from family culture, school approaches to mathematics learning, and workplace culture.
- Contradictions: Contradictions exist over adapting to workplace demands for deciding on and performing mathematics, acceptance of or resistance to new ideas, and between formal and traditional approaches to learning mathematics versus informal approaches.
- Expansive Cycles: Expansive cycles of learning are often dependent on increased knowledge and skills. Much more attention is given in the workplace to direct practical application of mathematics than at school.

This section summarizes the links of Section 6.2 with Engeström's elements and principles in the CHAT framework (see Section 3.2 and Section 3.3) with the data and some fundamental issues identified in the data: the primacy of context, problem solving, conceptual understanding, and procedural knowledge, and problem solving and creativity. Problem solving, conceptual understanding, procedural knowledge, and creativity can be regarded as tools from Engeström's elements (1987) because they help in understanding and guiding engineering practice, including what intellectual and physical tools will be used and how they will be used. Therefore, there is a symbiotic relationship between intellectual and physical tools. While intellectual tools govern both the creation of physical tools and how those physical tools are used (see Chapter 6 Introduction), physical tools are also needed to create new physical tools. Moreover, while physical and intellectual tools both have their separate rules for usage, the way they interact is also governed by rules set by local and wider engineering and other communities in accordance with currently accepted practice. The interactions between Engeström's principles and applications derived from engineering contexts provide pointers to expansive cycles based on developing problem-solving skills.

The progress from contradiction to expansive cycle is a complex process for several reasons. First, problems often have a variety of novel solutions which may, or may not, receive immediate community endorsement. This can provoke further contradictions. Second, the process may stimulate the evolution of both physical and intellectual tools and these may be assisted in the meantime by the development of new, outside technologies. Third, the process can then be prolonged by multi-voicedness and historicity which can then operate as constraints to achieving further expansive cycles of learning within the community of practice. Indeed, in many engineering contexts, intellectual tools such as thinking are used to create new physical tools, as in jig making and toolmaking. In many cases, resulting new expansive cycles are then quickly challenged by new contradictions within the engineering community as the engineers evaluate the efficacy of a new and innovative engineering practice and how it might apply in different situations.

6.2.2. **The primacy of context**

Context governed the attitudes of the engineers to mathematics itself, what mathematics the engineers regarded as 'useful', and how it should be taught and learned. Analysis of the data determined that context governed multiple aspects of daily decision-making such as whether mathematics should be used and how it should be used (Coben, 2000). Therefore, the data confirmed the principle reported by FitzSimons et al. (2005) that context guides mathematical application in workplaces. Moreover, because mathematics constructs models that are only approximations of reality, and are hence incomplete, then a major question and contradiction for all engineers is the extent to which mathematical models reflect reality.

Apprentices face two other contradictions regarding context: extended reasoning, and social and metacognitive skills. Each involves adaption from a school perspective to a workplace perspective. I discuss the issues surrounding extended reasoning here, and leave the development of extended reasoning skills, and social and metacognitive skills to Section 6.3 where it forms part of a larger discussion on learning. From the interview data, apprentices appeared to have little or no involvement with planning tasks that involved extended reasoning. Either the task was considered too difficult for their current capabilities, or other more experienced engineers did the thinking for them and the apprentices simply followed their detailed instructions. Developing extended reasoning skills, especially integrating mathematics and engineering considerations, appeared to take time to develop. Thus, while contexts of school mathematics involving just one or two steps were almost always neat, but artificial, workplace contexts were frequently messy and ill-defined, but real (Roth, 2010). This may partly explain why beginning apprentices and even newly qualified tradespersons lacked these higher-order skills.

Regarding the contribution of this study to new knowledge, the role played by context in mathematics applications in the workplace has received comparatively little emphasis in the literature. This study reveals that context is a major factor in applying mathematics in New Zealand mechanical engineering trades workplaces. Moreover, this study extends workplace knowledge of the integration of thinking between intellectual and physical tools .

To summarize, contextual considerations determined every stage of applying mathematics to the mechanical engineering trades workplace. This was particularly important to the way the engineers saw and responded to problem-solving situations in their daily work. In the next section, I discuss how conceptual understanding and procedural knowledge impact on these situations.

6.2.3. **Problem solving, conceptual understanding and procedural knowledge**

The previous section discussed how the primacy of context governed engineers' attitudes to mathematics itself, but that alone does not sufficiently explain how engineers apply mathematics in their workplace contexts. We also need to consider problem solving and its links with conceptual understanding and procedural knowledge, which are important when something unexpected or unfamiliar has happened. This section focuses on problem solving, conceptual understanding and procedural knowledge to identify how apprentices and skilled tradespersons in mechanical engineering trades apply mathematics skills in their work. These areas arose naturally from practical examples in conversations with the engineers, such as fault-finding and maintenance, and the logical thinking required to identify what, and why, something had happened (see Section 4.2.1 and Section 4.2.2).

6.2.3.1. **Problem solving**

Problem solving can be regarded as an intellectual tool used by engineers. The purpose of the tool is to solve problems, which may be short-term or long-term, and which contain contradictions. The engineers' daily work frequently requires problem solving because the non-routine tasks they encounter are unfamiliar to them and they do not know "what's coming in the door next" (see Section 2.3.2) (FitzSimons et al., 2005; K. Mills, 2011). From the engineers' perspective, when a previously unencountered problem appears, they first consider if their current knowledge and understandings might provide an answer (see Section 4.2.2.2). If that fails, then deeper conceptual understanding is required. If problem solving in non-routine situations is to be successful, then it is important that the engineer is able to find or construct a satisfactory engineering technique, or procedure. This in turn depends on the engineer's conceptual understanding of the task and hence may lead to an expansive cycle of learning.

Regarding straightforward, routine tasks where the overall solution may be obvious, the application of mathematics in the mechanical engineering trades context may be straightforward, because the engineer has seen the task before, knows the concepts involved, or, that it involves applying tricks of the trade as procedures. Nevertheless, even with routine tasks, unexpected complications may arise that call for sophisticated problem solving, creativity and extended reasoning, as was seen in Robert's (engineer) replacement of a bronze bush (see Section 4.1.3). This example is also consistent with the literature where mathematics is used both descriptively and precisely as one of several key elements considered in the decision-making process (Alpers, 2010; Bakker, 2014; Gainsburg, 2007; D. Harris et al., 2015; Hoyles et al., 2010; Kent & Noss, 2002).

In other situations, mathematical applications may become complicated by deficient mathematical knowledge, or under-developed higher-order skills such as conceptual understanding, problem solving and extended reasoning (Brookhart, 2010; King, Goodson, & Rohani, n.y.). It would appear that many communities of practice acknowledge the challenges created by these deficiencies. They are major causes of contradictions and limitations on performance in the workplace. These deficiencies affect both the effectiveness of individual engineers and the community as a whole, thus motivating a quest for expansive cycles of learning and practice. Nonetheless, problem solving requires both conceptual understanding and procedural knowledge. I discuss these in the next section, together with their relationships and interactions, to further explain how the engineers and apprentices in this study applied their mathematical skills in various workplace contexts and situations.

Regarding the contribution of this study to new knowledge, there is an extensive literature on problem-solving from both theoretical and practical perspectives. This study provides new knowledge from the New Zealand mechanical engineering trades workplaces context where problem solving is used extensively in everyday situations. Problem solving is an important tool in fault-finding, maintenance engineering, planning, and design.

6.2.3.2. **Conceptual understanding and procedural knowledge**

In the last section, I outlined the importance of conceptual understanding in mechanical engineering trades workplaces (see also Section 2.3.4). However, conceptual understanding is not always complete or even necessary, as is shown in the case of interpreting a calculator screen output of 2.314^{-07}, where one engineer told me conceptual understanding was not necessary because the -07 simply indicated moving a decimal point seven places to the left, to give .0000002314. This was consistent with Skemp's instrumental procedures because the right answer was obtained quickly and reliably, and the rule was easy to remember (Skemp, 2006) (see Section 2.4.4). Moreover, using procedural knowledge in this case also had the advantage of quickly releasing the engineer to concentrate on problem solving in the actual engineering situation.

The widespread use of computer technology in the mechanical engineering trades provided another significant example of the use of procedural knowledge. For example, Computer Aided Design (CAD) is essentially a black box that allows 3-D models to be built up which can be examined and then modified if necessary (Williams & Wake, 2007). Such software programmes were highly attractive to some engineers (see Section 2.3.2), partly because the engineer is released from having to understand the sophisticated mathematics behind the calculations the machine performs. Hence, there were many situations where practical considerations favoured using technology.

The findings also revealed that CAD provided major help in product design, in reducing production time, was easily modifiable, and reduced mistakes. The introduction of black boxes like CAD allows engineering tasks to be performed that would not normally be attempted. Hence, one person using CAD can influence the multiple individual activity systems of each member of a community of practice and, consequently, foster the creation of expansive cycles of learning in the community of practice as a whole. But although conceptual understanding of the mathematics involved is no longer necessary, conceptual understanding is still necessary if the engineer is to use CAD effectively. This is because they still need to have enough knowledge to 'drive' the CAD equipment to produce what is needed, to know whether the CAD model that has been produced is accurate and not nonsensical, to relate the ideas behind the project to the machine and then interpret and implement the results. In such situations, being released from the need to justify conceptual understandings in the mathematics context has a positive effect on implementing conceptual understanding in the engineering context.

Although computer technology has brought real power to the engineering workplace in recent decades, a generational contradiction had ensued with some engineers' expressing reservations that computer calculation technology might lead apprentices to never conceptually understand the mathematics hidden in the black box (Black & Wiliam, 1998; Williams & Wake, 2007). Consequently, there were significant contradictions involving multi-voicedness between the generations.

Performing the calculations by hand was also seen by some old-timers as reinforcing conceptual understanding, and perhaps an indication that conceptual understanding was complete. In the case of being able to accurately perform the complicated minutiae of spline calculations by hand, for example, the mathematics was advanced well beyond the knowledge of most mechanical engineering tradespersons. Successful spline calculation was grounded on following an algorithm and hence on procedural knowledge (see Section 4.1). On the other hand, the old-timers in my study who acknowledged the need to get things done cheaply, accurately and efficiently, were leaning towards adopting procedural approaches. Thus, there was a paradox, with some old-timers tending to continue their historical ways of doing things, while others welcomed the contributions being made to change by the younger generation of engineers and apprentices, especially the latest innovations in computer technology.

Conceptual understanding and procedural knowledge also appeared to distinguish expert engineers from others. One engineer, Stephen, characterised an expert mechanical engineer as someone who combined technical engineering skills with an understanding of how the systems interacted, could figure out what was happening, had intuition and could identify bad information. For example, an expert engineer had procedural knowledge but could go beyond procedural knowledge of rules, algorithms and formulas of how to perform something to display metacognitive thinking and increasing levels of sophistication of numeracy conceptualization in contextual settings (Engelbrecht et al., 2017) (see Section 2.3.1). This was consistent with the qualities of higher-order thinking frequently needed in the workplace as outlined by FitzSimons et al. (2005), FitzSimons and Wedege (2007), Lave and Wenger (1991), T. Maguire and O'Donoghue (2003) and Zevenbergen (2002). Expert engineers tended to have well-developed higher-order thinking skills, but not necessarily more senior secondary school mathematics skills. Possessing higher-order thinking skills may, therefore, be a defining characteristic that moved them close to the centre of the community of practice (see Section 2.5). The result was that there was widespread agreement that conceptual understanding should be preeminent in many practical situations because it had practical consequences (Engelbrecht et al., 2009, 2017; Lamberg, 2013; Rittle-Johnson & Schneider, 2014; Schoenfeld, 1992; Vosniadou, 2006).

Regarding the contribution of this study to new knowledge, this study establishes the importance of both conceptual understanding and procedural knowledge in New Zealand mechanical engineering trades workplaces. It enriches the literature by providing examples of where conceptual understanding and procedural knowledge work in tandem in practical situations. The study also demonstrates that expert engineers appear to be comfortable in both conceptual and procedural scenarios.

In this section, I have discussed various contradictions regarding the roles of conceptual understanding and procedural knowledge in the quest for problem solving. There was a paradox here because while many engineers regarded understanding concepts as the ideal, at least in a general sense, time and other constraints meant that finding a suitable procedure often took precedence. However, there were limits to the successful application of procedural knowledge without further conceptual development leading to expansive cycles of learning. One reason for this conundrum is the importance of creativity to problem solving, which I discuss in the next section.

6.2.4. **Creativity**

The discussion focuses here on how the frequent unfamiliar situations in mechanical engineering contexts that give rise to problem solving and conceptual understanding, lead logically to creativity in finding a solution. This was graphically illustrated by Robert (engineer) installing a press vertically, replacing bushes on a press, and modifying his furnaces to accommodate larger jobs (see Section 4.2.2). In these cases, unusually creative solutions may go beyond the rules developed historically by the community regarding departures from the norm. Creativity also involves transfer of knowledge which Brookhart (2010) identifies as a higher-order skill and is consistent with other understandings of numeracy that incorporate higher-order skills (FitzSimons et al., 2005; FitzSimons & Wedege, 2007; Hattie & Donoghue, 2018; T. Maguire & O'Donoghue, 2003). Each of Robert's tasks required employing conceptual understanding of the problem to find a genuinely creative solution. In this way, the connection between problem solving and creativity has emerged as a key feature of mechanical engineering practice.

In time, a consensus about the efficacy of a departure from the norm may be reached and a new problem-solving technique become accepted practice (OECD, 2009a); in which case an expansive cycle in both individual and community learning has been completed. This can be accompanied by a corresponding development in the community's procedural knowledge, even if not all members achieve the same level of conceptual understanding. Developing one's own new techniques, and critiquing and adapting other engineers' proposed new techniques are both key features of mathematical learning, and are integral to the expansive learning process.

It would also appear that conceptual understanding and procedural knowledge are not mutually exclusive. On the contrary, they have a symbiotic relationship where conceptual understanding exists on multiple levels related to context and appears to be developed iteratively in conjunction with procedural knowledge. This relationship is recorded in the literature (Devlin, 2007; Hattie & Donoghue, 2018; Rittle-Johnson & Schneider, 2014; Rittle-Johnson et al., 2001) (see Section 2.5.3). However, while procedural knowledge may be used at length successfully, it nevertheless has limits as a substitute for conceptual understanding. Once these limits have been reached, engineers must address the question of conceptual understanding. This applies, in particular, to the problem-solving process where intellectual perception and creativity require the engineer to go beyond the limits of procedural knowledge. Hence, creativity is linked to problem solving, which is linked in turn to conceptual understanding. Establishing conceptual understanding, in turn, may lead to the construction of procedures. Hence, creativity can be regarded as an intellectual tool that motivates and guides the application of mathematics and conceptual understanding in many engineering contexts. This suggests that there is a synergy between problem-solving skills employing conceptual understanding and creativity, and well-developed numeracy, communication and organisation skills. One potential consequence of this synergy is that problem solving accompanied by the multi-voices of individuals within the community of practice, including those on the periphery, can work together to resolve contradictions and take the community to new expansive cycles of learning.

Regarding the contribution of this study to new knowledge, there is an extensive literature on creativity. This study enriches that knowledge. The study also demonstrates the interconnectedness between creativity and problem solving in New Zealand mechanical engineering trades workplaces. Creativity is linked with finding new perspectives to solving problems.

6.2.5. **Extended reasoning, integrating multiple skills**

In the last section, I discussed the role creativity plays in problem-solving contexts. Since creativity involved the interaction of conceptual understanding and procedural knowledge, the solutions found became increasingly complex, which led naturally to extended reasoning. In this section, I discuss creativity and extended reasoning in both routine and non-routine engineering contexts. First, routine tasks require recalling and then applying skills already learned. As illustrated in the findings, when the apprentices made mistakes with the concrete moulds that did not "come out right", problem solving was needed to fix things up (see Section 5.2.2.4). Fixing mistakes was regarded by mentors as good learning opportunities for apprentices, because the costs of the mistakes were small, the mistakes were easily rectifiable, and exposed the apprentices to fault-finding and problem solving. They also involved extended reasoning, because apprentices had to figure out several steps in logical thinking for themselves in an unfamiliar context. With increasing skill and experience, the apprentices would be shown how to do more complicated tasks and then be left to work independently. This is consistent with Lave and Wenger's apprentice tailors who were given the easier parts of garments to make before attempting more complicated tasks. Increasing independence marks progression towards the centre of the community of practice (Lave & Wenger, 1991).

It is important to understand that non-routine, problem-solving tasks involve more than an extended series of operations. An example is found in Keith Rucker's (2013) video on replacing a small bush. Keith's task was similar to Robert's (see Section 4.1.3.5) as he also was replacing a bush. Robert's task would normally be described as non-routine as it was done very seldom, the operations were not standard and, therefore, involved genuine problem solving. In contrast, Keith's task was regularly performed by engineers and posed its own challenges - no replacement bushes were available commercially so Keith had to machine them up himself, and he had to make multiple careful measurements, calculations, and complex decisions. However, despite the complexity of Keith's operations, the operations were routine. Therefore, his task did not involve significant problem solving.

Because of their ability to integrate skills, both engineers were very close to the centre of Lave and Wenger's communities (see Section 2.5). The tasks had both similarities and dissimilarities. They were required to deal with many factors simultaneously, among which were:

- understand the nature of the problem
- perform diagnosis to find a cause
- recognize what action needed to be taken as well as the engineering techniques required
- know how to integrate the use of the physical and mental tools available to them
- consider several potential methods of solution and their sequencing
- relate the operations to the complex contexts of the problems
- know how the materials involved might react when operated on

This was consistent with the outline of how tasks were dealt with in the workplace cited by FitzSimons et al. (2005). Both examples of replacing bushes demonstrate workplace practices that incorporate elementary mathematics in sophisticated settings, engineering judgment, and metacognitive skills, such as critical thinking, and planning (FitzSimons et al., 2005; Steen, 2001). Moreover, Keith and Robert brought many years of personal expansive learning cycles, and interaction with other activity systems and communities of practice to bear in performing the tasks. Robert also brought creativity, which was consistent with the literature where mathematics is used both descriptively and precisely as one of several key elements considered in the decision-making process (Alpers, 2010; Bakker, 2014; Gainsburg, 2007; D. Harris et al., 2015; Hoyles et al., 2010; Kent & Noss, 2002). However, only Robert's task required genuine authentic problem solving.

Regarding the contribution of this study to new knowledge, this study aligns extended reasoning and the integration of multiple skills with creativity and problem solving. Moreover, developing the ability to integrate multiple skills appears to be a long-term process. In New Zealand mechanical engineering trades workplaces, the ability to combine extended reasoning and creativity with problem solving skills appears to be indicative of people who are close to the centre of one of Lave and Wenger's communities.

To summarize, mathematics, engineering judgment and heuristics (see Section 6.1) were used in tandem in engineering situations with problem solving, creativity and extended reasoning. The sign of an expert was to be able to extract ideas and information from the multi-voicedness of their historical experiences, and to combine this with creativity and conceptual understanding of the disparate connections to create solutions and expansive cycles of learning.

6.2.6. **Section summary**

To summarize, Section 6.2 regarding the second research sub-question about how the mathematical skills are applied in mechanical engineering trades, mathematics was used by mechanical engineers in real situations where context was of primary importance. The situations were both routine and non-routine. While routine situations could often be solved using procedural knowledge, non-routine situations that were unfamiliar to the engineer necessitated problem solving, creativity and higher-order skills. Problem solving by its very nature produced intellectual contradictions because engineers had differing views on how solutions might be constructed, and produced community contradictions because of historical rules set by the community as accepted standard practice.

Problem solving also involved integrating conceptual understanding and creativity with physical tools to find successful outcomes. This was often characterised by an intricate interplay between the mathematics, heuristics and engineering judgment in situations when approximate estimations only were possible, or needed. Hence, problem solving being a tool in its own right, stimulated contradictions regarding the historical practices of engineering communities, by challenging its rules and the roles its members played in the outcomes.

Having established the significance of problem solving as a tool in its own right to resolve contradictions in the engineering community, the findings of this study made it evident that on some level, problem solving also entails a degree of learning and communication skills. The next section discusses this last, important finding, of *how* engineers develop the necessary mathematics skills. In particular, communication is not only an essential ingredient of engineering workplace effectiveness but also serves as a tool for individual learning and hence the development of expansive cycles of learning in mechanical engineering trades communities.

6.3. **Developing the mathematics skills, learning and communication**

The discussion so far has focused on the first two research sub-questions relating to two key features of mathematics in the mechanical engineering workplace - the nature of the mathematical tools, and applying mathematics as an intellectual tool in complex real-world contexts. In this section, I focus attention on the third research sub-question of how apprentices and skilled tradespersons in mechanical engineering trades learn and develop the mathematics skills necessary for their work. A major finding is the complementary roles that communication and individual reflection play in this process (see Section 5.2.2, Section 5.2.3, and Section 5.2.5). I discuss here four interrelated issues pertaining to learning – the movement of apprentices and skilled tradespersons from the periphery towards the centre of Lave and Wenger's community of practice, formal and informal learning, higher-order thinking skills, and the role of communication in the workplace and in learning (FitzSimons et al., 2005; Lave & Wenger, 1991; T. Maguire & O'Donoghue, 2003). But first, I discuss the links of this section with the CHAT and Situated Learning frameworks.

6.3.1. **Links to CHAT and Situated Learning**

This section provides a summary of the findings about how mathematical knowledge and skills are developed and learned in the mechanical engineering trades (see Section 5.2). The bullet points below summarize the links between the CHAT framework and Section 6.3.

Engeström's Elements

- Tools: Informal learning was done by observing, emulating, listening and questioning, language and communication. Formal experiences such as schooling, block courses, reading and doing formal exercises were prevalent, as were visual aids, informal social interaction with others, mentoring programmes and personal reflection.
- Rules: Various technical expressions and jargon were used that were relevant to the needs of the engineering context.
- Community: Communities had the potential to provide extensive communication opportunities for the exchange of ideas and hence to encourage innovation and learning. Informal experiences gained during childhood, dialogue and debate through interaction with family, teachers, and mentors were also important means of learning.
- Roles: Educators and skilled tradespersons shared knowledge, experiences and skills, often through their stories. Roles could be fluid depending on the skills of newcomers to the community. Mentors, educators and others could sometimes exchange their roles depending on the recognised expertise of others.

Engeström's Principles

- Multi-voicedness: Various philosophies existed about teaching and learning, and different ways and techniques of performing engineering tasks.

- Historicity: Divergent ideas of mentoring and company environments influenced attitudes to mathematics learning. Values instilled from family culture, school and workplace approaches to mathematics learning were also influential.
- Contradictions: There was recognition of multiple methods of learning, both social and individual. Contradictions existed between conceptual understanding and procedural knowledge, and between formal, traditional approaches to learning mathematics versus informal, constructivist approaches.
- Expansive Cycles: Learning success was dependent on appropriately blending formal and informal pedagogies. The culture of the workplace could contribute to increased learning, especially where apprentices and old-timers interacted socially. Communication was seen as an essential component of learning. Much more attention was given in workplace practice to direct practical application of mathematics than at school. Transfer of expansive cycles in individuals' learning could take place in the wider communities of practice.

In this study, Situated Learning (SL) was also used as a theoretical frame of reference (Lave & Wenger, 1991). CHAT and SL have both similarities and differences. Both theories focus on practice which they regard as socially and historically influenced. Nevertheless, a significant difference between CHAT and SL is that CHAT can focus on situations where answers are not known. This makes CHAT useful in this study when describing problem-solving situations where finding answers has the potential to change the activity system, whether in individuals or communities. On the other hand, the focus of SL on social interactions between people makes SL an appropriate frame of reference in analysing learning in the apprenticeship situation and the movement from the periphery of a community of practice towards its centre (Arnseth, 2008). Therefore, while CHAT and SL may appear to be different, their differences in focus are helpful to analysing the data and interpreting the results in this study (see Section 3.2.4). I now discuss the movement of members of a community from the periphery to the centre that takes place with growing skill, knowledge levels and experience.

6.3.2. **Moving from periphery to centre**

The movement from the periphery to the centre of a community of practice incorporates aspects of both Lave and Wenger's theory of legitimate peripheral participation (LPP) and Engeström's expansive learning model. LPP views new-coming apprentices as joining the periphery of a community of practice, and then moving progressively towards the centre of the community of practice as they gain experience, knowledge and skills (see Section 2.5) (Lave, 2012; Lave & Wenger, 1991). Moreover, their developing experience, knowledge and skills may be regarded as Engeström tools to achieve the outcome of competently using mathematics in engineering contexts. Learning implies transition from a lower to a higher level of understanding and competence, and hence to changes in individuals and in the community of practice which marks expansive cycles of learning (Engeström, 1990, 2001, 2010). Significant contradictions leading to expansive cycles were seen in apprentices adapting from a school to a workplace environment, in the school and workplace mathematics tension, and in the responses of various multi-voices to potential change.

I focus the discussion here on the development of beginning apprentices in mechanical engineering trades from the perspective of movement from the periphery of Lave and Wenger's community of practice towards its centre. The data indicated the significant influence of mentoring programmes to assist apprentices transition to the new demands of the workplace. This was reflected in an important result involving social interaction, where, in some workplaces in my study, newcomers and old-timers shared the benefit of their skills with each other (see Section 5.2.2.1). Consequently, old-timers contributed maturity and experience while the younger people sometimes brought attitudes and skills from their prior learning experiences at school with calculators and technology.

In such a manner, the data confirmed the existence of communities of practice comprising newcomers and old-timers engaged in situated learning. Here, newcomers, or apprentices, joined the periphery as legitimate participants. With mentoring and developing skill they began to migrate towards the centre. This is consistent with Lave and Wenger's model where there is a continual interchange of personnel occupying the centre of the community of practice (Lave & Wenger, 1991). This migration was mainly due to older members of the community retiring and being replaced by younger people contributing new ideas. On the other hand, some apprentices quickly become recognised as experts in certain areas, such as computer technology. In this way, these apprentices occupied the centre of the community regarding their highly-developed computer technology skills but remained on the periphery for the rest of their learning, which still needed development. From a community perspective, several engineers and apprentices spoke warmly of this intergenerational, interactional learning, because they had yet to catch up and exploit the new skills. The result was that what constituted accepted practice changed in this process, an indication that expansive cycles of learning were taking place throughout the wider community.

Expansive cycles of learning in both individuals and the community may come about by innovative apprentices and mature skilled tradespersons introducing new ideas, techniques, or by gaining competence using new technology. An individual's migration to the centre can therefore be influenced by several factors, such as learner motivation, exposure to an environment where innovation is sponsored, the quality of training programmes, and encouragement given by mentors (Bandura, 1994; Knowles et al., 2011).

Having completed their apprenticeship, apprentices joined a large group called skilled tradespersons. This group ranges from recently qualified tradespersons who have yet to understand the "full picture" of engineering contexts to those whom the engineers call experts (see Section 5.2.2.2). It would appear that recently qualified skilled tradespersons have mathematics skills beyond Maguire's and O'Donoghue's (2003) formative phase, and display elements of the mathematical phase. This is in contrast to the important group of experts who exhibit special engineering skills that gain them enormous respect in the community. Experts are likely to exhibit elements of Maguire's and O'Donoghue's integrative phase which is a complex, multifaceted sophisticated construct, incorporating mathematics, communication, cultural, social, emotional and personal aspects of each individual in context (T. Maguire & O'Donoghue, 2003) (see Section 1.4).

The findings demonstrated that there were intergenerational contradictions between the natural conservatism of most people as they get older and the energy of apprentices and younger tradespersons to introduce their ideas. The contradiction may also be due to different school experiences where apprentices had been exposed to school curricula promoting creative and critical thinking to make sense of information, rather than the experience of the old-timers to learn facts (Ministry of Education, 2007). However, over time each group influenced the other, resulting in creating an expansive learning cycle from what the community was prepared to accept, or considered best.

The data also demonstrated that many engineers continued to have difficulties with mathematics even though they had become skilled tradespersons. Despite their mathematics deficiency, they may still have been regarded as expert engineers, such as Tim's expert with specialised skills in welding. Thus, expertise could be specialised, or global, as with Stephen's broad criteria of knowing how systems interacted. An expert could figure out what was happening because he had intuition and could identify bad information (see Section 4.1.3.8).

It would appear that progress in adapting to the mathematical requirements of the mechanical engineering context was also an important feature of migration to the centre of the community in the sense of global expertise. Conversely, lack of progress in mathematics skills hindered or even stifled a tradesperson's progress to the centre, and hence to become an expert at the centre of the community. In these cases, and no doubt for many possible reasons, their individual expansive learning cycles had not taken place, and the contradictions they experienced in learning remained unresolved.

Regarding the contribution of this study to new knowledge, this study enriches the extensive literature on Lave and Wenger's theory of legitimate peripheral participation. Apprentices and skilled tradespersons in New Zealand mechanical engineering trades workplaces migrated from the peripheries of communities of practice to their centres as they developed knowledge and skills. Moreover, experts were near the centre of communities of practice and tended to have multiple mathematical and ancillary skills developed over many years.

In summary, contradictions involving individuals and their wider activity systems in mechanical engineering trades contexts do not necessarily lead to expansive cycles of learning. Instead, deliberate intervention is often necessary, which is dependent on both formal and informal learning, and the crucial roles played by communication (see Section 6.3.4 and Section 6.3.6).

6.3.3. **An eclecticism of learning methods**

This section discusses the formal and informal methods of mathematics learning for apprentices and skilled tradespersons (see Section 2.4.5). The data indicated the significant finding that both formal and informal learning methods were part of apprentices' childhood and youth experiences. Their family or other influential mentors provided informal exposure to engineering contexts and perhaps a limited feel for measurement sizes, while schooling and apprenticeship training combined both formal and informal learning. During their training, apprentices experienced formal and informal features of apprenticeship learning, consistent with New Zealand legal apprenticeship requirements (Tertiary Education Commission, 2015, 2020). Hence, as illustrated in the findings, off-the-job block courses sometimes involving workbooks and exercises, and on-the-job training involving situated learning with mentors and social interaction, were similar to the combination of formal and informal learning experienced by US Navy quartermasters (Lave & Wenger, 1991).

Traditional block courses recalled by old-timers who were exposed to the strictly formal approaches of night classes and learned from them, had been partially replaced by a much more socially-interactive approach (see Section 5.2.2.1). This suggests there is a deep, ongoing discussion within the community about the relative effectiveness of formal and informal methods of learning. The resulting eclecticism of teaching methods attempted to combine the best of both approaches. For this reason, engineering educators believed social interaction promoted learning the theoretical and practical applications of mathematics in engineering contexts, and the successful completion of problem-solving tasks in small groups (FitzSimons et al., 2005; FitzSimons & Wedege, 2007).

Since context is significant in workplace mathematics, then an important transition for learners was appreciating how broader contextual workplace factors might affect the way they used their school mathematics knowledge. In the case of fine measurements that were temperature-dependent, one engineer had to wait 24 hours for temperatures to stabilise the lengths of metal components before making fine measurements (see Sections 4.1.2 and 4.1.3). The community had strict rules for taking these measurements, as it also did with safety requirements. This practice was also consistent with measures required to be taken when temperature and humidity critically altered the dilution rates in chemical spraying (FitzSimons et al., 2005).

Some educators thought apprentices took time to appreciate the implications of not having all the relevant information supplied in advance, and of making quick estimations to identify unreasonably wrong answers. Young apprentices may have met this type of thinking at school, but they needed to adapt from school requirements to quite different and important workplace requirements. This involved understanding many complicated interrelations and community rules that not only shaped mathematical applications but also involved adapting to different attitudes of what constituted a reasonable answer and how that answer might be obtained and interpreted. This demonstrated the importance of being confident and comfortable in judging whether to use mathematics in a particular situation, what mathematics to use, how to do it, what degree of accuracy was appropriate, and what the answer meant in relation to the context (Coben, 2000).

I next discuss two means of learning that apprentices considered helpful to their movement towards the centre of the community of practice - block courses, and thinking things out for themselves.

6.3.3.1. **Formal and informal learning**

Formal learning is usually associated with block courses, attending lectures and practicing doing exercises (Eshach, 2007). Somewhat surprisingly, the data showed that more than ninety per cent of apprentice participants thought that doing exercises and examples until they got things right was the best formal method to develop the expertise characteristic of those near the centre of Lave and Wenger's community of practice (see Section 2.5.1.1). Also, avionics educators used exercises and examples outside of engineering contexts to extend knowledge and foster flexible thinking. Murray (engineer) said that it was important for apprentices to do a lot of formal exercises until the skills were ingrained (see Section 2.2.1).

Regarding thinking things out for themselves, apprentices and educators very strongly endorsed the approach to learning found in Realistic Mathematics Education (Treffers, 1993; Yuanita et al., 2018). In such a learning environment, where communication, collaboration, questioning and experimenting between teachers and students had the potential to raise levels of conceptual understanding, mistakes could be made and ideas that did not work could still be respected (see Section 2.4.3). Hence, any resulting contradictions would then be proven as useful learning devices when discussed in a non-threatening manner. In this way, the progression to expansive cycles of learning could be made naturally. The apprentices appreciated the nature of contextualised knowledge that enabled them to see what was happening, to ask questions and thus develop deeper understanding (see Section 5.2.2.1).

The engineers strongly supported practicing as an essential means of mathematical learning. In practical engineering contexts, knowledge and techniques must often be recalled and applied without continual, time-consuming recourse to basic principles. Hence the need to ingrain certain procedures. This resonates well with Skemp's (2006) differentiation of relational and instrumental understanding which, while recognizing the importance of both, acknowledges that in certain scenarios instrumental understanding has advantages, such as more quickly and reliably obtaining the correct answer (see Section 2.4.4).

In this regard, it is important to note that the engineers strongly distinguished practicing of skills from rote learning. This was demonstrated in the way they designed exercises and examples that were different from each other in significant details and which forced learners to consider carefully what they needed to do, how they should do it, and what the answer meant in the context (Bernstein, 2000; Coben, 2000; FitzSimons et al., 2005; K. Mills, 2011). However, while doing exercises may be regarded as formal learning, some educators combined this with a socially-oriented approach (see Section 5.2.2). A significant consequence was that class discussion could identify and rectify gaps in understanding, and procedural knowledge and conceptual understanding could be mutually reinforced by iterative processes involving practicing and social interaction (Devlin, 2007; Hattie & Donoghue, 2018; Rittle-Johnson & Schneider, 2014; Rittle-Johnson et al., 2001).

Iterative processes often involved practicing, which the educators wanted to distinguish from mere rote learning. For this reason, some educators used mathematics problems that were not directly linked to engineering contexts to encourage the development of thinking skills (see Appendix O). Moreover, exercises and examples could also be used to foster the integration of mathematical, technical and social skills consistent with T. Maguire and O'Donoghue (2003), and Hattie and Donoghue (2018).

In summary, the engineering educators tended to favour an amalgam of approaches, rather than employing just one teaching method. Therefore, educators fostered classes where mutual learning took place, with the educator sometimes acting formally as a teacher, and sometimes informally as a facilitator of learning.

6.3.3.2. **Informally thinking things out for themselves**

Informal learning is closely linked with thinking things out for themselves. As illustrated in the findings, it was evident that engineers practiced this regularly in the course of their daily work, and was regarded as effective or very effective by 95% of participants (see Section 5.2.2). Thinking things through could reflect individual self-reflection or community interchange of views, not just with solving an immediate problem, but also with the long-term development of habits of higher-order skills (Eshach, 2007). With both individual and community involvement, it is possible that Sfard's acquisitionist and participationist metaphors were operating, with individual thought and social interaction mutually influencing internalization of understanding (see Section 2.4) (Sfard, 1998, 2009).

The findings also indicated that informally thinking things out was often associated with developing higher-order thinking skills such as conceptual understanding, transfer of concepts between contexts, critical thinking, learning to learn, planning, creativity, flexible thinking, and especially, problem solving (see Section 4.2). These resonated with the interconnectedness of skills outlined by Brookhart (2010), the self-directed learning principles of Knowles et al. (2011), and contrast well with the numeracy levels of T. Maguire and O'Donoghue (2003).

Regarding the contribution of this study to new knowledge, this study demonstrates that while apprentices and skilled tradespersons in New Zealand mechanical engineering trades were perhaps temperamentally oriented more to informal learning, they also acknowledged the role of formal learning. Socially-oriented learning was also strongly endorsed by the participants.

In summary, the learning of both apprentices and skilled tradespersons encompassed both formal and informal means. Educators and mentors used formal teaching methods designed to develop skills and understanding which they integrated with social interaction to encourage team building and cooperative problem solving. Apprentices appeared to enjoy both approaches. The formal and informal means of learning also operated in developing higher-order skills, which I discuss in the next section.

6.3.4. **Higher-order skills**

Higher-order thinking and problem solving emerged from the findings as another significant key feature of mechanical engineering practice, and were frequently important stimuli to apply mathematics in engineering contexts. From the perspective of CHAT, contradictions in activity systems may originate when some individuals start questioning and then deviate from the traditionally accepted norms of the activity system (Engeström, 2001). Nevertheless, within this study, contradictions may also be produced when engineers are forced to come to terms with their current understanding of a situation being insufficient to provide a solution. This required engineers, both individually and collectively, to seek solutions that were mainly, but not necessarily exclusively, within the informally agreed community rules regarding engineering practice. I now consider how extended reasoning skills may be developed, the roles of mentoring and practicing in developing higher-order skills, and some qualities possessed by engineers regarded as experts.

6.3.4.1. **The development of extended reasoning skills**

One contradiction concerned the difficulties in developing extended reasoning skills among apprentices and even tradespersons. Extended reasoning is understood in this study to be combining several steps of thinking and operating in an appropriate sequence to produce a satisfactory outcome. The difficulties are linked to developing mathematics concept understanding in individuals and may have long roots in historical difficulties, for example, from school. One strategy to deal with this was private reflection and informally thinking things out for themselves, which apprentices thought was far more effective than skilled tradespersons (see Section 5.2.2.3). The reasons for this discrepancy are unclear. It may be due to apprentices overestimating their own cognitive abilities, or a feature of the school environment that fosters imagination and creativity in problem solving (Anthony, 2016; Kohen & Orenstein, 2021; Mason, 2003).

Another approach that had widespread support in some communities was fostering an environment of social interaction where ideas could be discussed and an opportunity given to allow contradictions to emerge into the open. In this way, the total skills and understandings of the community could be brought to bear on the problem at hand. This was consistent with the literature where extended reasoning became part of heightened levels of social interaction and metacognitive skills such as critical thinking, learning to learn and problem solving (FitzSimons et al., 2005; Ministry of Education, 2005, 2007; Roth, 2010; Zevenbergen, 2011; Zevenbergen & Zevenbergen, 2009). Among the skills that apprentices needed to develop are: considering what best suits the context, to what extent a proposed model works, whether a suitable model can be found or created quickly, what degree of accuracy is needed, time and money constraints, the acceptable tolerance, the tools available, and the skill of the tradespersons. These considerations are consistent with the reports of Marr and Hagston (2007). Extended reasoning, and social and metacognitive skills are interconnected. Lack of any one of them creates significant contradictions for individuals and the engineering community which are exacerbated by slow maturation processes that affect apprentices' learning and adaption to workplace perspectives (see Section 6.3). The result is that engineers continually need to assess the level of social, metacognitive criteria that should underly their approach to mathematical decision-making.

The data revealed that resolving contradictions to achieve expansive cycles in learning often involved developing more sophisticated mathematical models. This applied particularly to apprentices who were given tasks commensurate with their mathematical ability. More sophisticated tasks led naturally to extended reasoning, which is needed to combine several steps of thinking and operating in an appropriate sequence. This in turn may require verticalising conceptual understanding and hence greater abstraction (Confrey & Kazak, 2006; Treffers, 1993; Zulkardi, 1999) (see Section 2.4.3). As illustrated in the findings, many engineers were reluctant to move towards greater abstraction, consistent with the long historical roots throughout the engineering trades community (see Section 6.1.3.3). Thus, ironically, when one expansive cycle was attained, it could do so at the possible expense of enhancing yet another contradiction.

Successfully applying extended mathematical reasoning in the real world was interpreted through the CHAT lens as an expansive cycle formerly characterised by a series of contradictions. For example, some engineers found applications of school mathematics many years after leaving school (see Section 5.1.3). They also acknowledged the influence of growing maturity accompanied by light bulb moments. This suggests that expansive cycles involved in developing extended reasoning skills culminate after a long period of personal reflection and interaction with others, exposure to engineering contexts and growing maturity, consistent with Sfard's acquisitionist and participationist models (Sfard, 1998, 2009). Well-developed extended reasoning skills also indicate increasing proximity to the centre of Lave and Wenger's community of practice.

Learning was also reflected in adults being able to engage in personal reflection and mutual discourse to identify and assess their own and others' assumptions (Knowles et al., 2011). Moreover, in each case, while the mathematics may have been unsophisticated, its application was consistent with Steen's view of using quantitative skills in subtle and sophisticated contexts (Steen, 2001). Being able to see the connections that transfer abstract mathematical knowledge to real contexts with real-life complicating factors is a major feature of mathematical learning and an important step towards working independently and moving closer to the centre of Lave and Wenger's community of practice.

Social interaction and hence good communication were also emphasized by some engineers as essential to successful problem solving. In this respect, Murray (engineer) spoke of the need to plan with others, and was highly critical of non-existent communication in the workplace which frustrated planning, and therefore efficiency (see Section 5.2.5.1). Nevertheless, the end result of good communication over time could be that learning and practice might enable some activities originally experienced as problem solving to "become routine activities" (OECD, 2009a, p. 7), and lead to an expanded cycle of learning throughout the community.

In summary, the development of extended reasoning skills was based on the primacy of context and involved a complex series of developments and interactions involving individuals and the community. Extended reasoning skills can be viewed as an intellectual tool in Engeström's elements that operate in concert with Engeström's other elements, especially the rules and the community. The accomplishment of cycles of expansive learning leading to mature and sophisticated extended reasoning skills is based on both individual and community growth and may take many years to fully complete. However, cycles of expansive learning do not necessarily develop automatically but require intervention. In the next section, I discuss the roles of mentors and practicing of tasks in fostering higher-order skills.

6.3.4.2. **Mentoring, practicing and higher-order skills**

As illustrated in the findings, methods of teaching apprentices used strong social interaction, which mentors combined with more traditional means such as the practicing of tasks until they got things right. Apprentices had to demonstrate competence at one level before advancing to the next, which was consistent with Lave and Wenger's (1991) model for training apprentice tailors and quartermasters.

As discussed in Section 2.3 and Section 2.4, there are many studies of problem solving in school and adult learning settings (Anthony, 2016; de Bono, 1972; OECD, 2012b; Stylianides & Stylianides, 2014; Tertiary Education Commission, 2008; Yuanita et al., 2018). However, there appears to be some doubt over how well problem solving skills demonstrated in the classroom transfer to the professional engineering context (Engelbrecht et al., 2009; Harlim, 2014; J. Mills & Treagust, 2003; Sobek & Jain, 2004). Some mechanical engineering trades educators attempted to incorporate higher-order thinking into their teaching programmes to develop fault-finding skills, an example of problem solving (see Section 5.2.4). They also combined mathematics exercises and class discussion with mentoring programmes that concentrated on the whole person. The fact that educators continued to do these things over many years suggests they were convinced they were all effective.

While this research study found that practicing and mentoring were believed by educators to be effective means of fostering higher-order skills, it is not possible for this study to definitively state all the factors involved in higher-order skills development. Higher-order skills, especially problem solving, appear to be qualities of those whose development has successfully completed successive expansive learning cycles and who now stand at, or near, the centre of the community of practice. This is probably strongly associated with interpersonal skills, especially with educators and mentors who discussed with me the cognitive aspects of mathematics as well as the social aspects of apprentices' experiences.

Concerning the third research sub-question, the four ingredients of mentoring, practicing, social interaction, and personal reflection, were all involved in developing higher-order skills.

Workplace learning was consistent with Sfard's acquisitionist and participationist models (Sfard, 1998, 2009). These four ingredients are important because they appear to be highly influential in engineers moving towards the centre of the community of practice. They also appear to be especially important in the development of experts and their roles in problem solving, which I discuss in the next section.

6.3.4.3. **Experts and problem solving**

The findings demonstrated that the mechanical engineering trades communities have long recognised and respected a small group of expert skilled tradespersons for their ability to perform both basic and higher-order technical skills as well as understand how systems interact (see Section 4.1.3.8). Understanding the interactions is an indication of well-developed higher-order skills such as problem solving, metacognitive thinking, and in some cases, increased levels of sophistication of numeracy conceptualization in contextual settings. Hence, it is possible that experts have progressed beyond Maguire's and O'Donoghue's 'mathematical phase', similar to Freudenthal's Realistic Mathematics Education (see Section **2.4.3**), to integrate their mathematical knowledge with other higher-order thinking skills, social interaction and communication (FitzSimons et al., 2005; T. Maguire & O'Donoghue, 2003; Treffers, 1993). However, in the context of this research, it is possible to give only an indication of the qualities the participants thought applied to expert engineers. As illustrated in the findings, they had superior skills in creativity which they brought to problem-solving situations. They were often good communicators and interested in people. Consequently, these experts played an innovative role in project development, and their ideas might over time become widely accepted and complete an expansive learning cycle of new techniques throughout the community. Most significantly, they were noted for their creativity and problem-solving contributions to community practice (see Section 6.2.3).

An important possible exception to these comments was an expert welder who required different skills from other branches, especially an intuitive sense of what would work, and how things would distort under extreme heat (see Section 5.2.2). Therefore, although the design of big welding projects like wharf gangways is highly mathematical, their actual construction also depends heavily on utilizing subjective factors, such as engineering judgment, heuristics and a feeling for size, which was this welder's strength. These skills are learned by practicing and experience and are respected among peers even although formal mathematics skills are not apparent.

However, in branches other than welding, it was less clear how experts gained their expertise. Robert was certainly strongly influenced by his father who shared his knowledge and expertise as well as habits of questioning, taking calculated risks and experimenting (see Section 4.2.2.3). Robert and his father formed their own Engeström activity system, with Robert as the only subject and his father assuming the role of mentor. Robert was introduced to physical and intellectual tools, and consequently experienced and understood certain workplace contradictions at a young age. It is probable that these contradictions were sometimes beyond his current level of maturity, which may have stimulated his own personal reflections and communication with others. The outcome was that when Robert later joined other engineering communities, he already had many historical experiences and expansive cycles completed. Moreover, he was used to linking contradictions to problem solving, and no doubt added his own theories to his father's attempts to problem solve. The mature Robert still learns from "cock ups", eagerly embraces new technology, and is passionate about health and safety. Similar strong paternal influences were reported by most of the other engineers, but not by all. It is therefore possible that expertise is a latent trait that can be fostered in some individuals, or it may be that it arises from historical childhood or school experiences.

There is a large literature on higher-order thinking skills, including its application to workplaces. This study provides contextual examples from New Zealand mechanical engineering trades workplaces where apprentices initially found extended thinking difficult and were gradually introduced to more complex tasks of increasing length and complexity as they gained skill and experience. Effective communication and mentoring, as well as practicing appeared to be important factors in developing higher-order skills. Experts stood at the centre of the communities of practice. Thus, regarding new knowledge, this study provides added contextual examples to the literature from a New Zealand mechanical engineering trades perspective.

In summary, the development of higher-order skills is dependent on socially situated learning as outlined by Lave and Wenger (1991). Initially, parents and other family members and friends assume the role of models whom children emulate. They provide communication and dialogue which act as artefacts that promote skill and conceptual development as in Engeström's model. Later mentors continue these processes. However, it is not clear what all the factors are, or how they interrelate, especially regarding expert engineers and extended reasoning skills. Nevertheless, communication plays a crucial role in the learning process of all engineers, including the development of higher-order skills. I discuss the role of communication in the next section.

6.3.5. **The crucial role of communication**

I focus the discussion in this section on the importance of communication in the workplace, in apprentice development, and the role of the stories that engineers tell. The role of good communication emerged from the interviews in this research as a key feature of workplace practice and was consistent with many studies in the literature, as was discussed in the literature review (see Chapter 2) (FitzSimons, 2005; FitzSimons & Wedege, 2007; Gulikers et al., 2004; Lave & Wenger, 1991; Ministry of Education, 2007; OECD, 2012b, 2016a, 2016b; Rule, 2006).

The data showed that engineers understood communication as a multi-faceted and global issue that involved fostering a workplace-wide culture of discussion. This was important in choosing mentors who were among those closest to the centre of Lave and Wenger's (1991) community of practice. Therefore, good communication can also be regarded as an essential tool for teaching and learning (see Section 5.2.5).

Some engineers encouraged multi-voicedness and communication as tools to promote problem solving. The multi-voices also exhibited strong historicity, often being the latest stage of development of old problems in possibly new guises, and emerging at the end of a long period of ferment during which issues were discussed and debated. In this sense, many good ideas emerged in artefact form, as diagrams sketched on the back of tobacco packets (see Section 5.2.5). These developments began, either with a problem to solve, or with somebody questioning the efficacy of the status quo. Communication was crucial here. If a problem can be solved quickly, then perhaps some individuals may experience an expansive cycle of learning. If the problem is not solved quickly but requires community discussion, research, and perhaps a paradigm shift in thinking, then the possibility exists of an expansive learning transformation in the activity system of the whole community, with the new knowledge and techniques establishing new norms of accepted practice, consistent with Engeström's learning model (1999, 2001).

The findings showed that in this environment, even the young apprentice participants enjoyed hearing the wisdom and experience of the old-timers, and then contributing their ideas too (see Section 5.2.2.1). It was at such times that youth and old-timers might both modify their positions. Youth might admit that some of their ideas were not practicable, and the old-timers would come to accept that the new was necessary to keep up with progress, and would let go of things that had in the meantime become obsolete.

Social interaction, discussion and communication were essential components in this process of change. This could become complicated, especially since companies could not call a halt to production for any length of time while changes were made because most days they must make a profit. Nevertheless, some employers made encouraging change a deliberate policy, like Paul's (training officer) boss who had always bought the latest and greatest technology to allow the engineers to work with it. In this case, it was not just the technology acquisition policy that was important to the firm's success and the engineers' satisfaction, it was the whole system of professional development, including ongoing communication that contributed to the well-being of individuals and the company (see Section 5.2.2.1). For this reason, effective discussion and communication are significant as part of transition experiences, because they allow both apprentices and tradespersons to receive suitable mentoring support and to gain a sense of belonging within the engineering community. In the next section, I discuss the roles of bosses and mentors in apprentice learning and promoting social interaction.

6.3.5.1. **The community, apprenticeship, mentoring**

Concerning apprenticeships and informal mentoring, many members of the community had a role to play in the development of apprentices' mathematics skills. Here, inculcating understanding and proficient use of physical and intellectual tools and signs became the object of their endeavours, but always according to the rules accepted by the community. This informal learning was similar to situated learning in everyday life situations like grocery shopping, midwifery, tailoring, butchering, and Alcoholics Anonymous where the apprentices acquired the specifics of practice through observation and imitation (Lave & Wenger, 1991).

From the findings, bosses and mentors consciously assigned tasks to apprentices that were commensurate with their current skills and understanding, such as measuring and cutting metal. As they progressed, apprentices were moved on to more complicated projects (see Section 5.2.2). This strategy resonates well with Lave and Wenger's (1991) description of apprentice tailors. A significant result is that apprentices were progressively exposed to increasingly sophisticated tasks as they developed in skill and knowledge during their five-year apprenticeships. Consequently, they moved from near the periphery towards the centre of the community of practice.

When apprentices lacked appropriate mathematical skills or the ability to integrate them within the context, their bosses acting in their roles of mentors, usually made a decision for them about how to use mathematics (see Section 2.5). This applied well to constructing the concrete moulds. However, the apprentices building a ramp in the fruit packing shed were given too much freedom, and produced a job that broke fundamentally important rules of the community to use time, resources, and money efficiently (see Section 2.4.5 and Section 4.2). The apprentices either lacked the necessary mathematical expertise or failed to recognise that using mathematics was appropriate in this situation (FitzSimons et al., 2005). On the other hand, Robert (experienced engineer) used sophisticated mathematical strategies for calculating small volumes for bolts (see Section 4.1.2.2). These tasks illustrate an important and continual interplay between mathematical theory and engineering practice, which are part of the learning development process. The interplay of multiple factors in New Zealand mechanical engineering trades workplace practice is part of the new knowledge provided by this study.

To summarize, the apprentices learned on the job by many methods, such as observing and imitating, discussing, questioning and listening to the stories of the community. In each case, communication was confirmed as a crucial part of their learning process. One significant informal form of communication was the stories they tell which have intrinsic entertainment value that perhaps disguised their important role in learning. I discuss these in the next section.

6.3.5.2. **The stories the engineers tell**

Important aspects of language and communication mentioned in the previous section were the stories, or 'good yarns' the engineers tell. The data provided stories that combined both entertaining narrative and explanation. Stories served several significant functions, both technical and social. The importance of dialogue, in general, was confirmed by 100% of apprentices who felt discussing with tutors and other engineers was either effective or very effective, and multiple times in the interviews (see Figure 11). This is consistent with Lave and Wenger's important observation that language is part of practice because "it is in practice that people learn" (1991, p. 85).

People need to gain access to the community's collective knowledge, skills and wisdom, including learning the more important and difficult skills of the trade. One function of stories is to convey technical information about engineering issues and acceptable ways of doing things. Another function is to illustrate how problem-solving skills are employed in a context. This is best done when the listeners are relaxed. Another significant feature is that stories are developed in such a way as to have human interest and consequently be entertaining, yet lacking giveaway clues that restrict the listeners' ability to guess what could happen next, or why. Moreover, stories are great informal teaching devices for apprentices because apprentices quickly pick up the community's lore and use it themselves. Finally, being let in on a community's stories is a bonding measure, and conveys to the newcomer that they too have become accepted within the outer periphery of Lave and Wenger's community (1991).

Robert's story about the redundant presses his father procured (see Section 5.2.5.2) is typical of many stories the engineers have told me over the years. A significant finding is that stories provide strong confirmatory evidence of the importance of language in the engineering workplace, including situations involving routine instructions, finance, time and safety (Gal & Tout, 2014; T. Maguire & O'Donoghue, 2003; OECD, 2003, 2009a, 2012b; PIAAC expert group in problem solving in technology-rich environments, 2009). Communication is also important when creating and exchanging ideas during the design process where it is linked with creativity and problem solving. Hence, story-telling is established in the literature as an important communication and learning tool (see Section 2.5.2). While other studies emphasize the learning functions of story-telling and may touch on its teaching functions, as opposed to learning (Archer, 2008; Lloyd, 2000; Maslen & Hayes, 2020; Swap et al., 2001), this study has explicitly identified story-telling as a significant informal teaching tool (see Section 5.2.5). Therefore, the role of stories as a teaching method appears to be a new contribution to knowledge as opposed to communication of information or as an aid to social interaction in the context of workplace learning.

The stories and the art of storytelling contain Engeström's principles of multi-voicedness and contradictions, both of which exist at several levels. In Robert's story (see Section 5.2.5.2), governmental economic policies of the day were strongly interventionist and fluctuations in import tariff policy created contradictions that fundamentally affected all company financial decisions. This raised multiple voices, either agreeing or protesting. Then there was the voice of Robert's father, who experienced the contradiction between a love for machines and trying to expand a business with insufficient capital. Another voice belonged to the press owners who undoubtedly were torn between the need to recoup at least some of their huge financial loss and an emotional desire to keep the presses in economically viable activity. This one story encapsulates the qualities of those near the centre of Lave and Wenger's community of practice, where broader societal and political issues interact with and influence engineering considerations. Thus, Robert's story is memorable and instructive because it combines financial and engineering perspectives with the human touch of the owners' generosity in exchanging the presses for a bottle of gin and disposing of the remaining scrap steel.

Regarding the contribution of this study to new knowledge, this study enriches the extensive literature on the roles of communication in workplaces. Moreover, the stories that engineers tell have emerged as an important device when constructing new knowledge, and as a teaching device to transmit engineering culture to others.

In summary, apprentices imbibe stories easily. Telling stories is therefore a powerful tool for communication and learning. Even their entertainment value contributes to the development of social relationships and hence to expansive cycles in engineers' continuing development, both individually and collectively. The findings revealed that the engineers' tools were both physical and intellectual, including communication. The engineering community had various rules governing what constituted acceptable practice and innovation. The roles of engineers were complex and changing, depending on prior historical experience, so that even some beginners were able to contribute from their prior learning. On that account, storytelling is a powerful means of constructing knowledge and skills in the community of practice, of transmitting its culture to the next generation, and providing a mechanism for expansive cycles of learning.

6.3.6. **Section summary**

In conclusion, concerning the third research sub-question on how apprentices and mechanical engineering tradespersons develop their mathematics skills, both formal and informal means were used in an eclectic manner. This included classroom settings and formal written assessments as well as social learning, on-the-job learning employing observation, interaction between tutors and apprentices, and especially stories about the folklore of the community. Social interaction is thus important both as a means of communication and as a tool for learning. An individual's movement towards the centre of the community is partly dependent on how well these social interactions develop mathematics and numeracy skills. However, expert engineers are those closest to the centre of the community of practice, and they often have superior higher-order thinking skills, such as conceptual understanding and problem solving. These are probably developed iteratively over many years as a result of experience and exposure, personal reflection, and social interaction. In addition, metacognitive skills are also important to learning, because they help in organising thinking, performance and teamwork.

6.4. **Chapter summary**

In this section, I summarize the discussion of this study with reference to my research questions, and the CHAT and Situated Learning frameworks. I also discuss the interconnectedness and integration of mathematics skills in the mechanical engineering context. Finally, I indicate ways the study may be applicable to wider contexts beyond the mechanical engineering trades.

6.4.1. **Nature of the mathematics skills**

There were four significant issues in the engineering community regarding mathematics knowledge and skills. The first related to fluency with number, accuracy in making mental calculations and measurements, and quickly finding rough but useful estimates within the rules of the community. The second related to the importance of context, which was emphasised by many participants, especially in conjunction with the complex decision-making to use mathematics, what mathematics should be used, how it should be used and how an answer related to the engineering context. Financial and time considerations were important in the engineering context and engineers needed to consider if heuristics and engineering judgment should prevail over formal mathematical methods. The third contradiction concerned the role of higher-order thinking skills, which can be regarded as an Engeström intellectual tool. Higher-order skills arose because engineering contexts frequently involved problem solving and its interconnected issues of creativity, innovation, and extended reasoning. These in turn were linked to the intellectual issues of conceptual understanding and procedural knowledge, which were found to be mutually reinforcing. Higher-order skills were used frequently by the engineers, especially in fault-finding and maintenance situations, but also in other problem-solving scenarios. The fourth issue was the school and workplace tension where many of the participants regarded school mathematics as abstract and removed from reality. However, this was ameliorated in some cases where mature engineers later found real-world contexts for their school mathematics. In this light, increases in procedural knowledge could sometimes be promoted by expansive cycles of learning in conceptual knowledge.

Contradictions were frequently found and could be the result of proposed change or innovation. These could clash with rules for the use of skills determined by long-term community historical tradition of what constituted acceptable practice. The emergence of expansive cycles of learning in these situations could create further contradictions, especially over new technology where the apprentices frequently had a considerable advantage over old-timers.

6.4.2. **Applying the skills in context**

Mathematics skills involved in the mechanical engineering trades were regarded as tools in Engeström's activity systems. The way the mathematical tools were conceived and used differed from school settings and were governed according to community rules for accepted practice. This was in contrast to school mathematics and required a change in apprentice thinking to adapt to the requirements of the engineering workplace. Intellectual contradictions arose here because solutions have to be constructed and clashed with historical rules set by the community as accepted standard practice.

Problem solving also involved integrating conceptual understanding and creativity with physical tools to find successful outcomes. This was often characterised by an intricate interplay between the mathematics, heuristics and engineering judgment in situations when approximate estimations only were possible or needed. Thus, problem solving, being a tool in its own right, stimulated contradictions regarding the historical practices of engineering communities, by challenging its rules and the roles its members played in the outcomes.

6.4.3. **Developing the skills, learning and communication**

This required an adaption of attitude and application for apprentices who were used to school contexts, and was consistent in significant aspects with Lave and Wenger's model of joining a community of practice with mentors guiding the apprentices' development. Some firms recognised the importance of this model and had well-developed mentoring systems that sought to develop apprentices' and tradespersons' whole engineering and mathematics perspectives, as well as metacognitive capabilities. In this way, the engineering community tacitly recognised that successful application of mathematical skills required a holistic approach, that mathematics skills were complex and interrelated, applied in real-life contexts, and required personal and social skills. The purpose of mentoring systems was to resolve contradictions, whether engineering, mathematical or otherwise, in apprentice development.

The mechanical engineers learned, used, and developed their mathematical skills by formal and informal means, and by employing practicing and social interaction. Communication thus emerged as a crucial tool for learning because problem solving requires learning and group communication skills. Successful problem solving enhances individual learning and hence potentially the development of expansive cycles of learning in mechanical engineering communities. Expert engineers are those possessing superior higher-order thinking skills and as a result were closest to the centre of the community of practice. Such skills are probably the result of many years' experience, personal reflection and social interaction.

In conclusion, engineers learned by both formal and informal means. However, formal classroom activities such as doing mathematics exercises and written assessments were combined with social learning, where there was much interaction between tutors and apprentices. Apprentices valued the interaction they had with the old-timers, and the stories engineers told about the folklore of their community were especially significant. In this way, social interaction emerged as an important means of communication and as a tool for learning.

The mechanical engineering trades are high users of mathematics. This study has shown that they also use a very wide range of mathematical skills in conjunction with higher-order thinking in social settings. A study of the content, use and learning of mathematics in the mechanical engineering trades, therefore, has significant lessons for understanding mathematics in other workplace settings. In the next chapter, I draw conclusions about the study, including its contributions to new knowledge, its limitations and directions for future research.

Chapter 7. **Conclusions**

Introduction

The main, overarching research question of my study was to identify the key features of mathematical learning that characterise the pathway from beginning apprentice to skilled tradesperson and then possibly to expert engineer in the mechanical engineering trades.

The associated sub-questions were:

1. What is the nature of the mathematics skills employed in the mechanical engineering trades?
2. How do apprentices and skilled tradespersons in mechanical engineering trades apply mathematics skills in their work?
3. How do apprentices and skilled tradespersons in mechanical engineering trades develop the mathematics skills necessary for their work?

These were discussed in turn using CHAT and SL frameworks and an interpretivist paradigm in Chapter 6 (see also Chapter 3).

In this chapter, I first discuss conclusions regarding the research questions. These include the nature of the mathematical and ancillary skills (Section 7.1), applying the skills in mechanical engineering contexts (Section 7.2), and how the skills are developed (Section 7.3), including the movement of engineers to the centre of their community as they develop communication and higher-order thinking skills. Then, I discuss the contribution this study makes to the field in terms of new knowledge and insights (Section 7.4). This includes contributions to new knowledge specific to the mechanical engineering trades context, numeracy in other workplaces and some nuanced contributions to CHAT theory and methodology.

Next, I discuss the implications of the study for the mechanical engineering trades, the continuing development of engineers once their apprenticeships have been completed, the teaching of mathematics and adult numeracy, and implications for policy (Section 7.5). Following this, I discuss the limitations of the study (Section 7.6), some reflections on the study (Section 7.7) and then suggestions for future research (Section 7.8). These include further possible research into learning within workplaces, the differences in learning mathematics and how it is applied between mechanical engineering specialisations, the continuing development of higher-order skills in trades workplaces, and the changes in attitude to mathematics during the apprenticeship years. Section 7.9 provides a summary of the chapter and concluding remarks for the thesis.

7.1. **Nature of the mathematics skills**

This section summarizes the conclusions regarding the mathematical knowledge and skills required in mechanical engineering trades. Regarding mathematical content, the mathematical tools required frequently paralleled the mathematics topics outlined in Appendix C *Mathematics and physics topics US 21905*, such as arithmetic, algebra, trigonometry, tables and graphs, and units of measure. However, my research has found that the key features of mathematical learning included not only the mathematical content, but also various ancillary understandings and skills such as the decision to use mathematics, employ heuristic approaches (Gigerenzer & Gaissmaier, 2011), or use engineering judgment (Gainsburg, 2007), which could override purely mathematical considerations. What distinguishes my study from the above studies is the comprehensive treatment of these features in the particular mechanical engineering trades context, as well as their interconnections with social aspects of engineering practice and learning.

The mathematical content and ancillary tools mentioned above can be regarded as tools in an Engeström framework. The interaction between the use of the content tools and other ancillary skills with physical tools was dependent on various "rules" determined by both the local and larger communities. This in turn depended on the skill level of the engineer, what physical and other tools were available, and the particular requirements of the current task. The rules were also determined by engineering and other considerations, such as contextual influences, time and finance.

There were two significant consequences of these constraints. First, a decision to use mathematics, heuristics or engineering judgment was dependent on many considerations, which had the potential to cause contradictions long-term and conflicts of interest in daily practice. Second, the engineers' attitudes to mathematics were pragmatic and frequently ambivalent, an indication of long-term contradictions within individuals and between members of the whole community. This was revealed in their rejection of mathematical abstraction in favour of what they considered to be real, and their desire to move their knowledge horizontally rather than vertically. Many mechanical engineering tradespersons viewed senior secondary school mathematics as useless, although a few found a use for their senior secondary school mathematics later in their careers. Consequently, discussions about the trades mathematics topics were oriented to practical problems in context, not to hypothetical, verticalised abstractions found in Freudenthal's philosophy of Realistic Mathematics Education (see Section 2.4.2) (Treffers, 1993; van den Heuvel-Panhuizen, 2001; Zulkardi, 1999).

The engineers' pragmatism was also reflected in their multi-voiced debates on mathematics topics where a contradiction existed between studying a broad range of mathematics skills versus minimal skills required just for the needs of one branch specialisation. It is important to understand that these contradictions are genuinely long-term, and even although the mathematics requirements for mechanical engineering trades in New Zealand are about to be broadened (NZQA, 2019e, 2019f, 2019g), in essence, the tensions between school and workplace approaches and broad versus minimal approaches remain unresolved. Moreover, in my opinion, the lack of resolution is likely to remain, at least for the foreseeable future.

Contradictions in the knowledge and skills required were also historical, such as the strong intergenerational contradictions between old-timers and younger members of the community of practice, including apprentices, about using modern calculation technology, and the need to measure and calculate accurately. Old-timers also felt that young apprentices lacked estimation and mental calculation skills, and particularly a feeling for what constituted a sensible answer. This may have been due to contemporary apprentices not having hands-on experience with tools and machinery from a young age, or equally likely, a lack of emphasis on mental calculation skills in schools. Learning a feeling for size and to work within tolerances were also important for apprentices, and while most seemed to adapt well to this, it would appear that in the perceptions of the participants, a profound understanding of fundamental mathematics would help many beginning apprentices. Other research on primary school teachers' understanding of mathematics indicated similar perspectives e.g., (Ma, 2010; Roble et al., 2017; Tandog et al., 2019).

From a mathematical perspective, the focus of this section has rested mainly on the nature of basic skills, such as fractional and decimal numbers, Pythagoras and trigonometry, and measurement. Problem solving and its associated ancillary skills are higher-order skills. I discuss them in the next section in conjunction with the ways that mathematical skills are applied in mechanical engineering trades contexts.

7.2. **Applying the mathematics skills in context**

When applying mathematical skills in mechanical engineering trades situations, the role of context emerged as a central feature of workplace practice. The centrality of context, in turn, carried a second key feature of engineering mathematics practice - the integration and interaction of the multiple skills of problem solving and creativity with both procedural knowledge and conceptual understanding. This applied to both old-timers and apprentices.

The role of context is central to all applications of mathematics in the mechanical engineering trades workplace. It affects the processes of forward planning and the multiple decisions made by engineers throughout their daily work, including the important decisions of whether to use mathematics or not, and what a mathematical answer might mean in terms of the engineering application being considered. The mathematics required in the workplace is always situated in the specific context of the task. Moreover, pragmatic and contextual considerations strongly influenced what engineers regarded as useful and the mathematics problems they felt were important, as well as problem solving and creativity.

Concerning procedural knowledge and conceptual understanding, most routine tasks involved procedural knowledge, such as straightforward machine maintenance or performing an engineering task according to previously accepted community rules. In these situations, knowledge and fluency using basic number facts were viewed as important, especially by the old-timers. This allowed apprentices to be involved under supervision, making repeated use of the same mathematics calculations, such as Pythagoras, until proficiency was attained. Routine tasks were usually physically visible and, consequently, viewed as being real. However, being routine did not mean that the task was found to be easy, as with converting between imperial and metric units, or substitution in formulas (see Section 4.1.3 and Section 5.2.3). In contrast, non-routine tasks almost always involved problem solving which required conceptual understanding. Sometimes, if the task was not physically visible, then it could be regarded as more abstract. Even apparently routine tasks could involve interactions and contradictions between the real and the abstract, and the lines between routine and non-routine, conceptual understanding and procedural knowledge, could become blurred.

A significant conclusion was that most non-routine tasks involved higher-order skills, which called for problem solving, creativity and extended reasoning. They could also involve understanding the links between the various necessary routine engineering tasks and their mathematical treatment. Thus, non-routine contexts could still be unsophisticated, such as calculating the volume of the block, a necessary first step in calculating the mass of the block when its density is known. However, volume calculations revealed widespread difficulties in dealing with consistency of units, such as converting millimetres to metres. This was compounded by conceptual difficulties with volume units, such as cubic metres (m^3), cubic centimetres (cc or cm^3), and litres. In contrast, some engineers were able to combine both conceptual understanding and procedural knowledge to perform calculations correctly and to identify contexts where such knowledge should be applied.

While basic mathematical knowledge and skills were important to mechanical engineers, much of their work indirectly involved higher-order thinking. The result was that without basic mathematics knowledge and skills, and higher-order thinking, individuals and the wider engineering community were limited in their ability to produce effective results, from both engineering and cost-effectiveness perspectives. In the next section, I draw conclusions regarding the way the mathematics and ancillary skills are learned, and the role played by communication.

7.3. **Developing the mathematics skills, learning and communication**

I focus the discussion in this section on drawing conclusions about how the engineers learn the mathematical skills that they apply in context. Central to learning were the participants' developmental experiences, which reflected the progress from tensions and contradictions to expansive learning cycles in both individual and community effectiveness. In this study, participants found various tools to achieve these developments more effectively; formal and informal learning, communication, and especially the stories that engineers tell. Finally, I draw conclusions about the paradox of the school and workplace mathematics tension.

Successful solutions to engineering problems had important social aspects, not only through communication and interaction to deal with the immediate issue at hand, but also in the long-term development of problem-solving skills at both individual and community of practice activity system levels. Moreover, the inability to solve problems created contradictions, often long-term. Behind the search for expansive cycles of learning, many voices may have contributed ideas that had varying degrees of success or failure. Thus, unresolved problems had strong elements of historicity, and the eventual individual and group expansive cycles of learning may have come from social interaction, new technology or heightened levels of extended reasoning or conceptual understanding. Being able to integrate these various factors and make connections between them was a sign of resolving tensions and contradictions which indicated expansive cycles of learning taking place in both individuals and communities.

7.3.1. **From periphery to centre**

Apprentices needed to make various adaptions and step-ups from school mathematics to meet the increased mathematical demands of the workplace. Especially significant was developing fluency with using number in an engineering context. Fluency with number was closely integrated with specific skills such as higher standards of accuracy in measurement and calculation, especially placing the decimal point correctly, working within tolerances, estimating, and having a feel for what represented an acceptable answer in an engineering context (see Section 6.1).

Another development in apprentices' earlier performance levels involved strengthening higher-order skills like critical thinking, learning to learn, planning and problem solving, and extended reasoning (FitzSimons et al., 2005). These were done in practical contexts which became a major motivation for the engineers to use mathematics and took many years to develop. This research study has found another important feature of mathematics learning - that while mechanical engineering mathematics employed mainly mathematics skills (see Appendix C), these skills needed to be understood and used fluently in sophisticated workplace settings. This resonates well with the need for basic skills in the literature (Roble et al., 2017), and the sophisticated nature of their use (Steen, 1990, 2001). There was consequently a contradiction with school mathematics that frequently uses sophisticated mathematics in simple settings.

A third development involved cultural, social, emotional and personal maturational aspects of individual engineers as was seen in the influences of mentors and mature engineers on each other and on the apprentices, which was consistent with T. Maguire and O'Donoghue (2003). Similarly, this study also identified metacognitive maturity such as learning to learn, critical thinking, planning, problem solving and integrating understandings as indications of increasing maturity (FitzSimons et al., 2005). These skills are not mathematical as such but were emphasized repeatedly by the participants as crucial elements of effective mathematics application to engineering practice. They present an opportunity for future research.

In this way, the passage from periphery to centre is overlaid with mathematical as well as ancillary considerations, which also have strong social components. In the next section, I consider an aspect with social components, the influence of formal and informal learning on this passage.

7.3.2. **Formal and informal learning**

This study revealed that learning for apprentices and skilled tradespersons was achieved eclectically through both formal and informal means. Methods of learning included both of Sfard's (1998, 2009) acquisitionist and participationist metaphors (see Sections 6.3.3 and 6.3.4). Moreover, the study revealed learning was regarded as a lifelong process by old-timers who had significant influence on apprentices' development, thinking and understanding. There was an important contradiction here because even although newly-qualified tradespersons had completed the formal learning requirements of US 21905, they were regarded by experienced mechanical engineers as still having many connections to make between conceptual understanding and practical expertise before potentially reaching expert status. This may explain some of the concerns within the mechanical engineering community about lack of mathematical skills in experienced skilled tradespersons. This research showed that informal on-the-job experience does not automatically remedy certain gaps in mathematical understanding. It is possible that this situation is partly attributable to US 21905 assessments having no connection with using physical tools in real-life situations. Research into other trades and professions indicated a similar issue (Gulikers, Kester, et al., 2008; Hutton et al., 2010; Weeks, Clochesy, et al., 2013). Pertinent examples were both Robert (engineer) and Paul (training officer) who regarded ongoing informal professional development as being helpful. Thus, it is likely that developing opportunities in both school and workplace settings for dialogue that is contextually related to engineering, including higher-order skills, would be beneficial. This contains implications for how numeracy is taught formally and informally as part of workplace practice (see Section 7.5).

7.3.3. **Communication, higher-order thinking**

Apart from practical situations where conveying technical information and safety warnings were essential, informal communication also featured strongly as an essential tool for teaching and learning. Hence, storytelling, or the "yarns" engineers informally exchanged, complete with diagrams drawn on the back of tobacco packets, emerged as powerful means of communicating ideas and creating informal learning opportunities. In this way, the stories engineers tell provided an environment where the completion of expansive cycles of learning in individuals and their small activity systems in a specific workplace could lead to expansive cycles of learning in wider mechanical engineering activity systems. Relating the entertaining and humorous nature of this style of communication to context probably had the strength of fixing the important engineering aspects in the listeners' minds, and perhaps contributed to a new level of expansive learning. Such communication was also indicated in Lautenbach's (2011) study of university lecturers adopting new technology (see Section 3.3.2), where both individuals and the community were caught up in mutual expansive cycles of learning. Individual lecturers were able to provide new insights to others in their larger activity systems which had been derived from the emerging expansive cycles within their individual activity systems. The crucial aspect of this informal but effective communication was the relational aspect of social interaction within a strong and extended community (Lautenbach, 2011).

This research suggested a similar phenomenon operates in some mechanical engineering communities too. In this case, communication acts as a tool not just to solve particular problems, but also to stimulate the development of higher-order skills such as problem solving, conceptual understanding, procedural knowledge, creativity and extended reasoning. The findings demonstrated that learning is not a linear process; a culture of social interaction was needed to promote step-ups in higher-order thinking, as with Freudenthal's (1973) verticalisation process. The findings were relatively cohesive in that innovation and creativity involved individuals and communities being willing to confront the contradiction of stepping outside their comfort zones. In such situations, communication became an essential tool to encourage thinking to accidentally "drift laterally across channels" (Horowitz, 1999, p. 15) and allow surprising ideas to emerge (see Section 2.3.2). In this manner, there may be an increase in conceptual understanding, a deepened appreciation of previously acquired procedural knowledge, and an increase in the fields where knowledge can be applied. If this occurs then an expansive cycle of learning will have taken place, whether in individuals or the wider community. However, drifting across channels, even if producing viable solutions, also carries the possibility of greater verticalisation and therefore mathematical abstractness, a source of contradiction for many people. If there is no corresponding increase in conceptual understanding of the mathematics, then the mathematics may still be regarded as abstract and useless. On the other hand, if conceptual understanding does increase, then the new verticalised understanding will now be regarded as real and useful, and hence less abstract. Thus, confronting contradictions is at the heart of producing adventurous, innovative and creative thinking which, in turn, completes new expansive cycles of learning. Communication is a vital ingredient in this process because successful problem solving frequently depends on group interaction. The co-construction of solutions was identified as a feature of learning mathematics in the mechanical engineering trades in this study.

7.3.4. **The school and workplace mathematics tension**

The study demonstrated an important tension and contradiction between school and workplace mathematics that had long historical roots. On the one hand, mathematics was regarded by all the participants as an essential tool in the mechanical engineering trades. On the other hand, there was a perception held by many of the participants that senior secondary school mathematics was abstract because the connections between the mathematics and its potential contextual applications were not explained. Some participants went as far as saying that school mathematics was useless. They used the words "abstract" and "useless" to contrast what they regarded as real and therefore useful. This contradiction therefore lay at the heart of the school and workplace mathematics tension.

Mechanical engineers were sometimes forced to construct their own mathematical solutions to problems, and indeed, much mathematics has traditionally owed its origins to problems in engineering and physics. Nevertheless, while professional engineers have more highly developed mathematics skills than tradespersons, even they acknowledge that mathematical solutions may sometimes be difficult to find, or not be constructible by known methods. Engineers are forced to rely on engineering judgment and heuristics, but like procedural knowledge, these too have their limitations. Consequently, it is unavoidable that successful problem solving in engineering requires a basis of conceptual understanding, and this, in turn, involves a certain measure of verticalisation, and hence greater abstraction. Paradoxically, the pragmatic approach many engineers crave will always be dependent on concepts, and therefore on a certain level of abstractness. This is true no matter how well or how poorly the concepts approximate reality. In this light, non-routine problems create contradictions that challenge current conceptual understanding, call for problem solving, and may perhaps be solved by verticalised thinking, research and communication.

In conclusion, although the literature frequently records sceptical views on the usefulness of mathematics (Lave & Wenger, 1991; Marr & Hagston, 2007; Ridgway, 2002), in this study the findings revealed that even seemingly unvalued mathematics topics could eventually enhance mathematical thinking (see Section 2.1 and Section 6.1). In the next section, I review the contribution the study makes to the field and new knowledge.

7.4. **Contributions to new knowledge**

In this section, I discuss three areas where this study contributes to the field by generating new knowledge: the mechanical engineering workplace context, numeracy in other workplaces, and a contribution to methodology.

7.4.1. **Contribution to the mechanical engineering trades workplace context**

This thesis is almost certainly the first in New Zealand, and possibly also internationally, to investigate the application of mathematics and numeracy in mechanical engineering trades workplaces. While there is a range of literature on specific issues related to mathematics and professional engineering, the literature on mathematics and the engineering trades area is sparse and confined mainly to relatively short studies (Alpers et al., 2015; Kent & Noss, 2002). Significant examples of the very few mathematics studies related to non-professional vocations are doctoral theses by LaCroix (2010) and Sundtjønn (2021). On the other hand, there is a wide literature related to some aspects of professional engineering, for example, problem solving and higher-order thinking skills (Adams et al., 2008; Fan & Yu, 2017; Horowitz, 1999; Jonassen et al., 2013; J. Mills & Treagust, 2003).

This study appears to be unique in that it focuses not only on the technical aspects of mechanical engineering trades mathematics required in the workplace, but also on their application in context, the ancillary skills, such as problem solving, needed for those applications, the social and communication aspects involved in how mathematics is applied and developed, and how those multifarious aspects interact. This study includes contributions to interpreting and understanding learning in the mechanical engineering trades field and other workplaces; new knowledge specific to the mechanical engineering context, which also enhances the broader field of learning in trades and professions through contributing insights from the particular context that was examined; and some nuanced contributions to CHAT theory and methodology.

While the scope and the nature of this study constrains the ability to generalise the findings to international contexts, this research gives insights and understanding of the particular New Zealand context that have not been researched before. It therefore enhances overall understanding of mathematics in mechanical engineering trades by enriching our understanding within the New Zealand context as well as the overall international context.

7.4.1.1. **Knowledge and skills in the mechanical engineering trades**

This study contributes to the literature in the areas of the knowledge and application of mathematics skills in New Zealand mechanical engineering trades workplaces. First, the study revealed that mechanical engineers used mathematics and numeracy skills frequently in their work. Many of those mathematical skills were similar in content and level of understanding to those in the New Zealand Curriculum (NZC) for Level 1 NCEA. In addition, strong numeracy skills and calculation accuracy were required and a feeling for number size was also revealed as being significant. These skills are also part of the NZC. Second, the study revealed that the ways engineers used mathematics and numeracy depended strongly on engineering contextual considerations. Hence, engineering decisions were often made using mathematics in conjunction with engineering judgment. In addition, while some engineering mathematical applications involved making routine calculations, other applications required applying mathematics in genuine problem-solving situations that involved higher-order thinking and creativity. Third, the study revealed that problem solving in the mechanical engineering trades, whether using mathematics or not, often involved communication and cooperative learning in small groups of engineers working on projects together. Fourth, this study contributes to knowledge of how basic skills are used in the workplace. The literature acknowledges using problem-solving skills in conjunction with ingenuity, extended reasoning and creativity, and hence the need for higher-order thinking skills (FitzSimons et al., 2005). However, this study on mechanical engineering trades adds to the literature on workplace mathematics by giving rich insights into the ways engineers employ mathematics both in standard and creative ways. Hence, a major contribution of this study to the literature is not only the focus on particular mathematical skills, but also on the sophisticated ways they are used in conjunction with higher-order and metacognitive skills, and social interaction.

7.4.1.2. **Mathematics learning in the mechanical engineering trades**

This study contributes two major features of knowledge about how learning and developing mathematics skills takes place in New Zealand mechanical engineering trades workplaces.

First, the study found that engineers learned and created knowledge eclectically using both formal and informal learning models, such as individual reflection, social interaction and communication. Innovative and creative engineers (see Section 6.2 and Section 6.3) used unsophisticated mathematics in sophisticated settings (Steen, 2001). Hence, this study contributes to the literature on workplace learning by adding detail to the mechanical engineering context of social interaction and individual reflection. It also confirms the continuing cycles of learning as described by Sfard’s acquisitional and participational metaphors.

This study also contributes to the literature on storytelling in mechanical engineering trades contexts. The vocational literature on storytelling records pragmatic contexts such as communicating information (Lloyd, 2000; Swap et al., 2001), or is dominated by professional engineering examples (Moffitt & Bligh, 2021; Nerona, 2019) where the mathematical topics and perspectives are often quite different from the trades. In my thesis, storytelling has emerged distinctively as playing a significant role in promoting learning (see Section 5.2.5.2 and Section 6.3.5.2). Learning takes place when the listener becomes attentive to the entertainment value of the story, and is therefore open to absorbing the technical explanations given by the storyteller. The listener also begins to establish links between the various ideas expressed and becomes closer to the other members of the community of practice. The mechanical engineering tradespersons in my study therefore used stories as a device to teach others the technical and cultural aspects of mechanical engineering.

Second, this study contributes to our knowledge of the interrelation between physical and intellectual tools. This is acknowledged in nursing mathematics where steps have been taken to integrate conceptual, calculational, and technical measurement skills to achieve total medicinal competence. This study adds to the literature and theorising (for example, Coben & Weeks, 2016) about the need to integrate multiple mathematical tools with their appropriate ancillary processes in the mechanical engineering trades workplace to achieve satisfactory outcomes (see Figure 14).

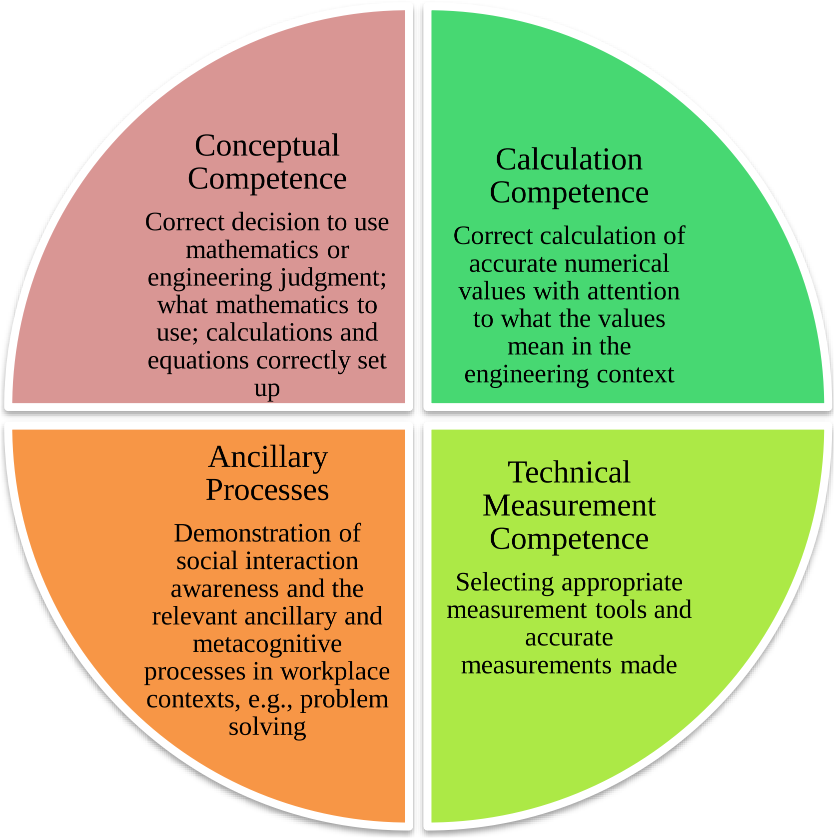

Figure 14 Mechanical engineering trades calculation competencies

(Adapted from Coben et al., 2010)

Figure 14 shows a new contribution to the literature where calculation competency components in the mechanical engineering trades could be integrated in conjunction with their practical application using physical tools and ancillary processes (see Section 2.2.3). In this way, the study carries a significant contribution to knowledge that achieving workplace numeracy and mathematical competency requires more than attention to just one factor – the integration of conceptual, calculational, technical and social interaction factors are all required.

Finally, this study is able to confirm the importance and mutually iterative nature of both conceptual understanding and procedural knowledge reported elsewhere in the literature (Engelbrecht et al., 2017; Rittle-Johnson & Schneider, 2014; Rittle-Johnson et al., 2001). The study increases our knowledge by providing several rich practical examples from the mechanical engineering trades contexts, such as fault-finding and calculating volumes.

7.4.2. **Numeracy in other workplaces**

This study also enhances our understanding of numeracy in other vocational areas. Since mechanical engineers are high users of mathematics, this study adds to the body of research on levels of numeracy among adults in New Zealand and other countries about effective performance in the workplace. Other studies have been undertaken on adult numeracy, such as the ALL and PIAAC studies (Jones & Satherley, 2017, 2018), but this study is different as it focuses on workplace scenarios involving integrated use of physical and intellectual tools. Moreover, the use of physical and intellectual tools in this study is being directly and continually influenced by social interaction and complex extraneous factors. Since it is known that difficulties with numeracy are widespread in the workplace, then lessons learned here may have application to understanding numeracy issues in a wide range of different vocations and workplaces.

This study also contributes to the role of informal communication in using and learning numeracy. The research identified that this is significant both for the immediate concerns of the day as well as a long-term consideration as a learning device. Workplaces where little social interaction takes place consequently suffer inefficiencies in day-to-day operation, as well as having difficulties creating and adapting to new ideas, both mathematical and more generally. Deficiencies in social interaction prolong the lifespan of contradictions because the total potential creative power of the community is lessened. Hence, contradictions become more difficult to resolve, and successful expansive cycles of learning in individuals and the community are inhibited. Newcomers to the community of practice might therefore remain below their natural potential to contribute to the workplace, negatively impacting the development of both themselves as individuals, as well as the company and workplace community.

7.4.3. **Contribution to methodology**

From a theoretical perspective, the CHAT framework with Engeström’s five principles has been employed by some scholars to take into consideration activity systems where individual participant subjects mature over time as part of their own personal expansive learning cycles, such as the single case study involving Mara's learning (FitzSimons, 2003). This study has demonstrated a similar conclusion because some tradespersons may require many years of exposure to all the pieces of the jigsaw puzzle, but without forming the "full picture" (see Section 5.2.2.2). Such individual cases do not necessarily appear to specifically indicate long-term processes where changes may steadily manifest themselves as they occur, as in some learning situations, but rather, the relatively short-term culmination and drawing together of many experiences, lessons and understandings emerging from multiple learning opportunities in a long-term process.

While the long-term process is an example of Engeström's historicity principle, this does not apply to the sudden maturation of individuals for three reasons. First, historicity refers specifically here to individual development rather than a change in any of a subject's larger activity systems. Second, historicity refers to activity systems taking shape and being transformed by contradictions over long periods. However, in this case, an individual's bringing together of the "full picture" is a relatively rapid process at the end of a much longer period of little apparent development. Moreover, the changes in both understanding and personal responsibility appear to take place without formal intervention. This is in contrast to CHAT historicity, where learning occurs through dynamic and continuous interactions among individual, societal, and cultural mediations with sociocultural contexts (Engeström, 1987, 2001; Leont'ev, 1978; Vygotsky, 1978). Third, the relatively rapid change may be due to factors independent of the workplace activity system, such as personal circumstances, variability in the maturation process, cultural and ethnic factors, and personal decisions (Amundsen, 2019). Hence, the sudden change may be independent of the CHAT historicity factors. This gives rise to a possible fifth question that could be added to Engeström's learning model - *When does learning take place?*

In summary, the findings of my study contribute to new knowledge because they provide further unique evidence to similar findings in other existing studies and because they relate to the unique mechanical engineering trades contexts.

7.5. **Implications and recommendations**

This section discusses four implications drawn from the study concerning mathematics and numeracy in New Zealand and for mechanical engineering trades workplaces: social interaction in mechanical engineering workplaces, continuing development of numeracy skills, the teaching of mathematics and adult numeracy, and implications for policy.

7.5.1. **Social interaction and the mechanical engineering trades**

This study has identified the significant role of social interaction in mechanical engineering trades workplaces. This applied to both mathematical and non-mathematical aspects of workplace culture. However, such interaction is not universal. Workplaces should therefore develop and extend workplace cultures where problems can be openly discussed, the corporate knowledge of the community shared, and creativity and resourcefulness encouraged and facilitated to benefit both individuals and the community. Some firms already organise formal professional development for their tradespersons, especially when new technology arrives, such as the company where Paul was a training officer (see Section 6.3.5). However, other companies need to extend their vision of mentoring so that discussions on mathematics and problem solving become a natural part of daily dialogue rather than an adjunct, or simply avoided. It is also possible that informal online digital social networks might provide a forum to share mathematical problems in engineering contexts.

7.5.2. **Continuing development of numeracy skills**

There is a need to institute professional development programmes to develop numeracy among qualified mechanical engineering tradespersons whom the participants reported as having challenges with mathematics (see Section 4.1.3 and Section 5.2). While the learning processes for tradespersons are usually informal, instilling confidence may have ramifications for leadership development within the workplace, and allow workers to contribute to the economic efficiency of the business (see Section 2.1). Such programmes need to focus on correcting gaps in conceptual understanding of number, and the integration of technical numeracy skills and communication within the engineering context. Thus, it is likely that developing opportunities in both school and workplace settings for dialogue about mathematics that is contextually-related to engineering, including higher-order skills, would be beneficial in mathematical and other aspects of workplace effectiveness.

7.5.3. **Teaching of mathematics and adult numeracy**

The study revealed the overall consistency between the contextual nature of workplace numeracy and the New Zealand Curriculum aim that mathematics should have practical applications in everyday life. Therefore, one important implication for practitioners in schools, workplaces and polytechnics is to teach understanding of mathematics concepts within authentic real-life contexts. This should be combined with an appreciation of the relevance and power of mathematics, and together with the limitations of mathematical models.

The development of new technology is tending to make certain mathematical skills redundant. For example, drawing graphs and solving equations can now be done online very easily, even for equations with multiple roots. Therefore, creating a mathematical model or set of equations of the context is a very important skill, not only in mechanical engineering workplaces, but also many places elsewhere. In this regard the current emphasis placed in the New Zealand mathematics curriculum on creating mathematical models will serve workplace realities well (see Section 2.4.2). More time should therefore be spent in the classroom learning to create mathematical models and equations than actually learning to apply the lengthy traditional methods for solving them (Schukajlow et al., 2018).

Another implication is the need to accentuate mathematics elements in student-centred inquiries and projects to develop individual thinking skills and the ability to communicate results in written form. Student-centred projects could also enhance verbal communication skills, promote social interaction and teamwork skills, and especially develop a culture of discussing mathematics as part of everyday conversation.

Yet another significant implication is the need for skills of mental calculation, estimation and feeling for number size. Courses need to be created, or parts of each lesson set aside, that feature mental calculation skills without the use of calculators. These courses should require students to relate their numerical answers to the context of the problem. Such a change would reverse the order of traditional mathematics teaching, with abstract and verticalised mathematics given first, and horizontalized examples given later. Instead, abstract and verticalised thinking should emerge naturally from concrete real-life scenarios, as in Realistic Mathematics Education (Treffers, 1993) (see Section 2.4.3).

The result would be that mathematical aspects would not be diminished, but would increase students' appreciation of how mathematics applies to life. Hence, charges that mathematics is useless might diminish, the development of a critical attitude to the limits of mathematical models would be fostered, and the social and intellectual aspects of mathematics and numeracy mutually reinforced. These would all be beneficial to engineering and other vocations, as well as to everyday life.

7.5.4. **Implications for policy**

Given that numeracy skills do not necessarily increase as a result of exposure to more senior secondary school mathematics programmes, there is a need for mathematics courses focusing on applying quantitative skills in subtle and sophisticated ways. This requires a recognition among teachers and curriculum designers that more mathematics does not necessarily lead to increased numeracy, that students need to gain parallel experience in applying quantitative skills in subtle and sophisticated ways, and that mathematics and numeracy should be complementary aspects of the school curriculum. Therefore, these courses should link numeracy directly with a broad range of contexts in secondary school and tertiary curricula, such as science, wood technology, food technology, and finance (Lovric, 2017; Steen, 2001). Some assessments could involve mental calculation without electronic aids. Other assessments should have greater emphasis placed on student-centred integrated learning.

As a result, the importance of numeracy in the workplace would be given more widespread recognition and extended into senior secondary school programmes along with features of Realistic Mathematics Education. Together with part-time jobs, doing the shopping, working on projects at home, this would help bridge the gap between the world of the classroom and the real world the students are likely to meet once they leave school. Incorporating social interaction and diverse mathematical skills in problem-solving contexts together with a system of authentic assessment may also assist in helping apprentices to see the relevance of mathematics to their lives and equip them to more adequately cope with the numeracy demands of daily life.

7.6. **Limitations**

This section discusses some limitations of the study: related to the changing New Zealand bicultural context, the interpretivist paradigm, the sampling method, member checking, ethics, interviewing, and researcher positioning.

7.6.1. **The changing New Zealand bicultural context**

Since this doctoral thesis began, there have been substantial developments in biculturalism and partnership in Aotearoa New Zealand. In particular, te Tiriti o Waitangi and the use of te reo have become progressively more integrated and prominent in our political, social, and everyday lives. Several participants in the interviews identified with both Māori and Pākehā cultures and while there is no emphasis on a Māori perspective in this thesis on mathematics in mechanical engineering trades, it is possible and desirable that Māori perspectives on workplace mathematics should emerge in the future.

7.6.2. **The interpretivist paradigm and generalisations**

The interpretivist paradigm employed in this study provides several limitations. Interpretivism’s social-constructivist ontology and epistemology mean that its insights are contextually situated, relativistic, local, social, and experimental. This means that reality differs from person to person in a subjective way. The insights are therefore also subjective, personal, based on previously constructed knowledge, and influenced to some extent by my own perspectives and values. Subjectivity pervades all aspects of the study including the choice and formation of questions, how the responses are interpreted and chosen for focus, and then discussed as findings.

Since the interpretivist paradigm produces results that are “local” then the question arises to what extent the results and conclusions can be generalised. This study collected and analysed data from a range of educational institutions and firms throughout New Zealand. Its results are likely to be generalizable to the New Zealand context as a whole. Moreover, this study’s results concerning the mathematics used in engineering, metacognitive and higher-order thinking are also likely to resonate well with other countries with education systems and engineering trades’ contexts similar to New Zealand. However, differences in workplace culture are likely to vary widely and interpreting the conclusions of this study beyond the current field should be done with caution.

7.6.3. **Sampling method**

Purposive samples for both questionnaires and interviews were selected first, and then supplemented using the snowball principle. Also, a larger number of the small minority group of mature educators were sought because of their more community-wide perspective and in-depth technical knowledge of their specialist engineering branch. Moreover, while the sample of participants included some representatives of important subgroups, for example, engineering specialisation subgroups, the overall sample size of 199 questionnaire respondents and 17 interviewees meant that the numbers of participants in these subgroups quickly became small. Among these subgroups were sizeable minorities of recent arrivals from non-English speaking backgrounds, and atypical cases, such as those who experienced difficulties learning mathematics, or outliers who had developed higher-order skills such as problem solving. Hence, the sample was not representative of the New Zealand context.

7.6.4. **Member checking**

The questionnaire data were carefully entered into Excel. The transcripts were not returned to the participants for checking as mentioned in Appendix D. Explanations were therefore dependent on the author's interpretations of the interview comments. The interview data were carefully checked for transcription accuracy. They were also checked for internal consistency of the themes for each participant, and externally for consistency between participants. Unclear verbal comments on the recorded conversations were either not accepted as data for the study, or enclosed in brackets [ ] if words were missing. Some reasons for not returning transcripts for member-checking were: (1) nobody contacted the researcher before 1 June, 2017 to withdraw their data from the study, (2) the length of time between the interviews and the completion of the transcribing, and (3) the difficulties contacting some members who had changed jobs or completed apprenticeships and moved on.

Problems recorded in the literature regarding the efficacy of member-checking of transcripts as a means of ensuring trustworthiness are: (1) a long time delay between the interview means some participants may have changed their stance in the meantime and want to alter the data, (2) some participants may want to change their data because they are ashamed of their ignorance of the subject or their lack of polished language, and (3) the way the researcher has expressed their interpretations may seem to carry a different meaning than in the participant's original everyday language (Birt, Scott, Cavers, Campbell, & Walter, 2016; Carlson, 2010; Harvey, 2015).

7.6.5. **Ethics**

Regarding ethics, an important consideration was to avoid creating conflicts within workplaces I visited, and asking questions that may have led to the disclosure of confidential company secrets. Similarly, ethical considerations prevented me from gathering data from observations. Interrupting engineers while they are working can be a major safety issue, and lead to very costly mistakes involving tens of thousands of dollars. Hence, I used interviews that took place while the engineers were not working.

7.6.6. **Interviewing participants**

The interviews also had limitations. The reticence of apprentices and the limited amount of time available prevented establishing a rapport that could have led to greater expansiveness in their replies. Also, the apprentices were assigned relatively low-level mathematical tasks by their employers that inhibited gaining information on how they were progressing with the more challenging aspects of mathematics application, including extended reasoning.

With experienced engineers, there was a limitation in knowing when to interrupt with supplementary questions or let the participant continue speaking. Not interrupting could easily lead to a lack of clarification, while interrupting too early could end the line of investigation with consequent loss of many fruitful ideas, richness and variety of mechanical engineering practice previously unknown to me.

7.6.7. **Researcher positioning**

Every researcher has some sort of positioning. In my case, those cultural values have been shaped by my life experiences, especially my time as a secondary school teacher of mathematics and physics, but also through my personal experiences and relationship with my father who was a mechanical engineer. The role and importance of conceptual understanding was one area that has influenced my thinking for many years. Another area was mental calculations, rough estimations, and relating answers to context, especially in physics where physical contexts were of immediate importance. These experiences and the values behind them are reflected both consciously and unconsciously in the choice and the design of the research questions, the methodology, and the interpretation of results. Another researcher, perhaps from another culture, could make different value judgments on what should be studied, how the study should be designed and carried out, and how the participants' responses should be interpreted.

Mitigating inappropriate influences of my teaching background on the study necessitated consulting with engineers and adopting a reflexive attitude to my interpretations and understandings. I discuss these issues in the next section.

7.7. **Reflexivity**

Through examining the data, and especially in the conversations with the engineers, I was continually challenged about my understanding of how mathematics related to the engineering context and about new applications I had not previously considered. Therefore, I regularly took a reflexive approach and checked and then often modified my own understandings and interpretations of the participants' responses within the engineering context. I now comment on three specific issues regarding reflexivity.

First, reflexivity meant examining the knowledge I was gaining about the engineering context and testing it against modern developments in mechanical engineering practice and knowledge. I did this through lengthy discussions with engineers. I also submitted the questionnaire items and interview questions to engineers for their comments about relevance, comprehensiveness of coverage, and topics that might be missing. In other words, I adopted the position of a learner refining his knowledge. Moreover, the fact that certain questions were included, or excluded, from the research design, as well as their wording, were also indications of subjective judgments being made on their relevance to the mechanical engineering trades context.

Second, I needed to make changes in my own perspective of workplace mathematics as a result of my involvement in secondary school teaching. This included the ways mathematics topics were learned and especially how they were applied in real contexts. This was in contrast to the frequently artificial contexts of the classroom. The artificiality also applied to school assessments being oriented towards general mathematics rather than numeracy skills applicable to workplace environments. While my mathematics and physics teaching experience allowed me to appreciate the importance of skills such as mental calculation and a feeling for number size, I was not experienced in how these worked out in workplace practice.

Third, reflexivity implied identifying and then questioning the unconscious assumptions made from my experiences as a mathematics teacher about how learning took place. In particular, the workplace process of embedding knowledge in ongoing practices and repeated, if necessary, until the particular competence was fully acquired implied a synergy between theory and practical application in real contexts. This was often a long process that was quite different from my own experiences (FitzSimons et al., 2005).

In summary, I learned that the full process of apprentices adapting mathematics and other skills to the workplace environment could not be rushed and that research was still needed on how engineers developed the intellectual components of their skills.

7.8. **Future research**

As indicated in the literature review, there appear to be relatively few international studies on mathematics and numeracy in mechanical engineering trades, and none in the New Zealand context. Consequently, the field for future research is very wide. Consequently, I shall confine my remarks here to future research in five areas that have emerged from the current study: (1) the means of learning within workplaces, (2) the differences in mathematics approach and application between mechanical engineering trades specialisations, (3) the development of higher-order thinking skills, which could be related specifically to engineering contexts and generically to the wider workplace contexts, (4) the continuing development of numeracy skills once formal qualifications and training have been completed, and (5) changes in attitudes to mathematics during the apprenticeship years.

7.8.1. **Learning within workplaces**

This study has shown the importance of communication in learning. Employers and others, therefore, need to recognise that dialogue in the workplace can be a significant tool in fostering learning, both in apprentices and experienced tradespersons. It is possible that the solutions to many problem-solving situations may already be hidden in the workplace, and that encouragement of dialogue would bring out that knowledge and expertise to foster expansive learning cycles in both individuals and the community as a whole.

Similarly, those workplaces that already operate professional development programmes should recognise the importance of encouraging all members of the community to participate in the discourse, both for their own benefit and the benefit of others. An ethnographic study could be set up to examine informal communication in specific workshops and how it influences on-the-job learning.

7.8.2. **Differences between engineering specialisations regarding mathematics**

There is a need to perform larger-scale studies of the branch specialisations. Future researchers could focus on the very “different skills and abilities” between branch specialisations (see Section 4.1 and Section 4.2). This would apply in particular to the aviation area, in maintenance engineering, and in engineering design, where educators strongly supported problem solving and thinking beyond the square. While the low tolerance demands in sheet metal engineering and high tolerances required in machining and avionics may partly explain differences in skill and ability, there must also be differences in approach to mathematics and its application. This in turn may help explain the broad versus minimalist debate on mathematics topics, with branches requiring higher precision supporting greater formal mathematics training (see Section 4.2 and Section 6.1). The result is that while problem solving is important to all branches, research needs to be done to ascertain how it differs between branches.

7.8.3. **Development of higher-order thinking skills**

There is a need to investigate the attributes of experts to ascertain how higher-order thinking develops in mechanical engineering tradespersons. In particular, there is a need to understand how trades engineers develop and apply problem solving. While the considerable literature for professional engineering suggests that this is a complex and poorly understood process with only limited success in being developed by formal means (Horowitz, 1999; Jonassen et al., 2013; Sharp, 1991), the literature for tradespersons is probably non-existent.

Nevertheless, this study has provided many examples of stages of developing problem-solving sophistication, beginning with childhood experiences before graduating to formal engineering practice (see Section 5.1). Since mathematics and numeracy are important in this process, future research could investigate how understanding deepens when the mathematics and problem-solving skills are used and developed in situ.

7.8.4. **Continuing development of numeracy skills during adulthood**

More research is required on the numeracy needs of both apprentices and skilled tradespersons whom the participants reported as having challenges with mathematics. It is not known to what extent these concerns are correlated with negative experiences with school mathematics or learning difficulties that may have manifested themselves early in primary schooling. Such research may have ramifications for developing numeracy skills (see Section 4.1, Section 4.2 and Section 5.2).

7.8.5. **Changes in attitudes to mathematics during the apprenticeship years**

There is a need to investigate the changes in attitude to mathematics that take place during the apprenticeship years. This is associated with maturation processes. A longitudinal study could therefore comprise several different strands, such as growth in mathematical capability, changing attitudes to using mathematics, confidence and capability in using technology, and problem solving. A study of successful problem solvers and how they developed their problem-solving skills may assist in understanding how others can be helped. It is also possible that apprentices are selected to become machinists partly based on their mature attitudes to mathematics.

7.9. **Chapter summary and concluding remarks**

This chapter has presented conclusions about the findings of the nature of skills, applying the skills in context, and developing the skills, learning and communication. As part of this concluding chapter, I also pointed out the contribution to new knowledge this study has made and outlined the implications and recommendations arising from my findings. I made readers aware of the limitations of this study and used my researcher reflexivity to explore how this study has continually challenged my own understanding and considerations of how mathematics related to the engineering context. Lastly, I indicated areas for future researchers to consider. I now offer my concluding remarks for the study taken as a whole.

My study has focused on the mathematics knowledge and skills that are used, applied and learned in mechanical engineering trades workplaces. Unsurprisingly, one major finding was that engineers need a thorough working knowledge and understanding of mathematical content such as graph reading skills, units of measurement, Pythagoras and trigonometry, and algebra. Most significantly, they need well-developed numeracy skills to respond to the uncertainties and challenges they encounter within their workplace contexts.

A second major finding was that the engineers use mathematical and numeracy skills in multifarious ways. This is because the application of mathematics in the engineering workplace is frequently motivated by practical problems. Such problems demand higher-order thinking, such as problem solving, and detailed attention to the engineering context, which may necessitate increased verticalised mathematical thinking and therefore greater abstractness. The learning is hinged on the specificity of the context. Thus, an important skill is recognizing when current mathematical knowledge and procedures have reached their limits, what mathematics should be used, how it should be used, and what the answer means in the context. Applying mathematical skills in engineering has emerged as a rich amalgam of cognitive and metacognitive ingredients, among which proficiency in basic mathematics is just one.

A third major finding of my study was that learning mathematical skills is not a matter of simply emphasizing the basics, or even becoming proficient in using mathematical procedures. A major feature of this learning was integrating the mathematical components with higher-order thinking. Moreover, basic and higher-order skills were learned by both acquisitionist and participationist means which necessitated social interaction. In this way, not only was cooperation in solving particular current issues fostered, but formal and informal learning opportunities were created for engineers to develop higher-order skills. These included flexible thinking, drifting across channels, creativity and extended reasoning. The conceptual understanding engineers developed extended beyond making links between mathematical considerations to linking mathematics to particular engineering contexts. Mathematics considerations were interconnected and integrated to embrace the whole of the engineering context.

The key to making interconnectedness and integration effective was communication. This was because communication promotes the exchange of ideas which increases the skill levels and understanding of individuals, and hence potentially of the wider engineering community. Among the means of communication, my study found that the stories the engineers told were a major source of mathematical learning. As a result, my study strongly supported the role of both higher-order thinking skills and social interaction as means of mathematical application and learning.

Finally, the writing of this thesis coincides with the publication of a report from New Zealand's Royal Society Te Apārangi expressing concerns about declining mathematics standards among New Zealand school students (see Section 2.1.2.2). The report calls for radical change in the way mathematics is taught in schools. This study is relevant to the debate because its conclusions about mathematics and numeracy skills in mechanical engineering trades workplaces are similar to the aims of the New Zealand mathematics curriculum. Both this study and the NZC identify the importance of using mathematics skills in real contexts involving problem solving, planning, critical thinking, creativity, conceptual understanding, employing judgment in decision-making processes, the development of metacognitive skills, and the roles of communication and social interaction in the learning and design processes. There are many vocations where the mathematical skills are similar to the mechanical engineering trades workplaces in this study and employ unsophisticated mathematics in sophisticated settings (Steen, 2001). It is my very strong conviction that improving mathematics standards of young people about to enter the workplace or tertiary study needs urgent attention. Hence, while the focus of this study may appear to be on a narrow field of mathematics specialisation, nevertheless, its focus on the broader aspects of mathematics usage might well have ramifications for society as a whole.

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Appendices

Appendix A: Glossary of terms and acronyms

Achievement Standard, AS.

Bush – the metal lining of an axle-hole or other circular orifice.

Collaborative Learning, CL, (see Section 2.4.4).

Competenz – the Industry Training Organisation (ITO) responsible for mechanical engineering trades in New Zealand.

Computer Aided Design, CAD – a software programme.

Computer Numeric Control, CNC – a software programme.

Cultural Historic Activity Theory, CHAT, (see Chapter 3).

Engineering judgment – a process in decision making where an engineer's experience overrides mathematical considerations about what fits best in a practical situation, (see Section 2.3.1).

Fabrication engineering – similar to sheet metal engineering.

Heuristics – a mental shortcut or rule–of-thumb that allows people to quickly solve problems and make judgments quickly and efficiently without the need for long calculations.

Industry Training Organization, ITO, (see Section 1.5).

Machining – a branch of mechanical engineering requiring fine tolerances.

Maintenance engineering – an engineering branch specialisation dealing with fixing malfunctioning machines, or taking proactive steps to avoid malfunction.

Mechanical engineering – defined in this thesis as the design, construction and use of physical tools and machines.

National Certificate for Educational Attainment, NCEA – the official New Zealand Government constituted assessment system for Years 11, 12 and 13 secondary school students.

On-the-job training, OJT, (see Section 2.3.1).

Organisation for Economic Co-operation and Development, OECD, (see Chapter 2).

Realistic Mathematics Education, RME, (see Section 2.4.3).

Sheet metal engineering – a mechanical engineering branch specialisation often involving relatively thin sheets of metal.

Situated Learning, SL, (see Section 2.5).

Spline – There are many different types of splines. Some splines are long ridges on a drive shaft that mesh with grooves in a mating piece to transfer torque and power, (see Section 4.1.3.2).

Science, Technology, Engineering, and Mathematics, STEM, (see Chapter 2).

Structure of Observed Learning Outcomes, SOLO – a taxonomy of learning outcomes, (see Section 2.5.3).

Statistical Package for the Social Sciences, SPSS – a computer software programme, (see Section 3.4).

Tertiary Education Commission, TEC, (see Chapter 1).

The Adult Literacy and Life Skills (ALL) Survey, (see Section 2.1.2.3).

The New Zealand Curriculum, NZC.

The New Zealand Qualifications Authority, NZQA.

The Programme for International Assessment of Adult Competencies, PIAAC.

The Programme for International Student Assessment, PISA.

The Système International, SI – the of units widely used in engineering and science disciplines. SI uses metres for length, kilograms for mass, and seconds for time, (see Section 4.1.3.5).

The Trends in International Mathematics and Science Study, TIMSS, (see Section 2.1).

Thematic Analysis, TA, (see Section 3.4).

Tolerances – defined in this thesis as the maximum allowable differences between the product specifications and the finished product.

Toolmaking – a branch of mechanical engineering requiring fine measurements, but which has declined in New Zealand in recent times.

Torque – Torque is the turning effect of a force. Thus, a large torque would mean that the nut was tight on the bolt.

Unit Standard 21905: Demonstrate knowledge of trade calculations and units for mechanical engineering trades, US 21905, (see Section 1.5).

Appendix A: Glossary of terms

Zone of Proximal Development, ZPD, (see Section 3.2).

Appendix B: Achievement Standard 91026 Apply numeric reasoning in solving problems

The following is a list of achievement objectives taken from the Number Strategies and Knowledge thread of the Mathematics and Statistics learning area, and which are included in this achievement standard:

- reason with linear proportions
- use prime numbers, common key elements and multiples, and powers (including square roots)
- understand operations on fractions, decimals, percentages, and integers
- use rates and ratios
- know commonly used fraction, decimal, and percentage conversions
- know and apply standard form, significant figures, rounding, and decimal place value
- apply direct and inverse relationships with linear proportion
- extend powers to include integers and fractions
- apply everyday compounding rates.

1 *Apply numeric reasoning* involves:

- selecting and using a range of methods in solving problems
- demonstrating knowledge of number concepts and terms
- communicating solutions which would usually require only one or two steps.

Relational thinking involves one or more of:

- selecting and carrying out a logical sequence of steps
- connecting different concepts and representations
- demonstrating understanding of concepts

- forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements.

Extended abstract thinking involves one or more of:

- devising a strategy to investigate or solve a problem
- identifying relevant concepts in context
- developing a chain of logical reasoning, or proof
- forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight

2 *Problems* are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. The situation will be set in a real-life or mathematical context.

3 The phrase 'a range of methods' indicates that evidence of the application of at least three different methods is required.

4 Students need to be familiar with methods related to:

- ratio and proportion
- key elements, multiples, powers and roots
- integer and fractional powers applied to numbers
- fractions, decimals and percentages
- rates
- rounding with decimal places and significant figures
- standard form.

5 Conditions of Assessment related to this achievement standard can be found at http://ncea.tki.org.nz/Resources-for-Internally-Assessed-Achievement-Standards.

(NZQA, 2019c)

Appendix C: Mathematics and physics topics US 21905

Different countries and jurisdictions have different mathematics requirements. The list of mathematics topics below is based on the New Zealand Qualification Authority's Unit Standard 21905: Demonstrate knowledge of trade calculations and units for mechanical engineering trades.

> **Arithmetic and algebraic operations for mechanical engineering:** Perform basic arithmetic operations (such as addition, subtraction, multiplication, and division of whole and decimal numbers), fractions are converted to decimals and percentages (and vice-versa), multiples are expressed to the power of 10 (and vice-versa using the prefixes mega; kilo; unit; deci; centi; milli and micro); engineering calculations using calculators (involving addition, subtraction, multiplication, division, square, square root, cube, sine, cosine, tangent); area and volume calculations are carried out for two and three dimensional shapes (using given data involving areas of squares; rectangles; triangles and circles, and volumes of boxes; cylinders and cones).
>
> **Trigonometry:** Carry out Pythagoras and trigonometric operations for mechanical engineering to find lengths and angles in right-angled triangles.
>
> **Tables and graphs in mechanical engineering:** Graphs are sketched from tabular data on graph paper, and tables are constructed by reading values from given graphs.
>
> **Define and apply quantities and units of measure in a mechanical engineering environment:** Unit names and symbols are matched to the corresponding quantities such as SI base quantities (including length; mass; temperature and time) and derived quantities (such as area; volume; speed; velocity; acceleration; angular velocity; force; torque; energy; work; power; efficiency and pressure where other quantities may include rotational speed; torque and efficiency); elementary quantities are defined and applied (such as speed; velocity; area; volume; force; pressure; work; power; rotational speed; torque and efficiency); the difference between mass and weight is demonstrated (demonstration includes calculations and an explanation); quantity values are re-stated using different SI prefixes; typical conversions performed (such as 2049 mm = 2.049 m; 0.055 mm = 55 µm, and 234 Pa = 0.234 kPa), and quantity values expressed in imperial units are converted to metric and vice versa (such as conversions between feet and millimetres; inches and millimetres; inches and µm; lb and kg, lb/in^2 and Pa, and °F and °C).

(Adapted from NZQA, 2010, pp. 2,3)

Appendix C: Mathematics and physics topics US 21905 for mechanical engineering trades

Table 12 *Proposed changes to US 21905 for 2022*

US 29397	Demonstrate knowledge of basic trade calculations and units of measure for mechanical engineering trades	2	4	New
US 29398	Apply knowledge of basic trade calculations for mechanical engineering trades	2	4	New
US 29399	Demonstrate and apply knowledge of trade calculations to solve problems for mechanical engineering trades	3	4	New

https://search.nzqa.govt.nz/apps/search/?q=mechanical+engineering&btnG=Search

Appendix D: Engeström's expansive learning model for hospital treatment

Table 13 *Engeström's expansive learning model for a hospital study of treatment of children with multiple illnesses in Finland*

	Activity system as a unit of analysis	Multi-voicedness	Historicity	Contradictions	Expansive Cycles
Who are learning?	Interconnected activity systems: hospital, health centre, patient's family	Voices of specialised health care, general primary care and lay home care			
Why do they learn?			Historically emerging pressures: patients move between primary care and hospitals	Contradictions between new object and available tools and rules in the three activity systems	
What do they learn?	A new pattern of activity: knotworking based on the instrumentality of care agreement		Historical layering and co-existence of old and new concepts: pathways and care agreement	Struggle between old and new concepts: critical pathway vs care agreement	Expansion of the object from visit to trajectory: from doctor-patient dyad to network of care
How do they learn?		Dialogue and debate between positions and voices, focused on a vital object		Contradictions converted from need state to double bind to resistance to realignment	Learning actions from questioning to analysis, modelling, examining, implementing, reflecting

(Engeström, 2001, p. 153)

Appendix E: Biographical notes on interview participants

This section gives brief biographical summaries of the backgrounds of the 17 people who took part in the semi-structured interviews, which ranged in length between 40 and 80 minutes. They came from a range of ethnicities; experience; specialisation involvement, and role or capacity as skilled tradespersons, educators and tutors. Many of the more experienced participants were capable specialists in more than one branch of engineering. Pseudonyms were used for each of the interview participants in accord with the undertaking given to them in the Participant Information Sheet (see Appendix I). In most cases, a pseudonym was chosen with first letter the same as the participant's surname. There were two exceptions to this rule; in the cases of those who had indicated on their questionnaire form that they identified with Māori ethnicity. One of these participants had a Pākehā first name and a Māori surname, while the other's names were both Pākehā. It was decided to give Māori pseudonyms to both of these participants, chosen from a New Zealand Government online source for the most popular first names given to Māori boys born in 2013. It was not considered necessary to choose pseudonyms suited to the cultural, family and linguistic ties of those participants interviewed who had been born outside of New Zealand.

Ari (21 years) had also begun his apprenticeship recently, and had spent much time sweeping floors and so on. However, even at this early stage, he had been involved in real work, "Um, … on the 773 dump truck my first week of the apprenticeship, the engine, completely pull out the engine coolers, oh the oil coolers sorry, and clean and then replacing which is a bit of a big job, took us couple of days, and …". He also related how the other men helped in his training.

Arthur (42 years) was an educator. He seems to have had early experiences of practical things where he used the local rubbish dump as a source of treasured bit and pieces, and the cemetery as a test strip for trialling his latest go-karts. He had wide experience in mechanical engineering, "…worked on fishing boats, pleasure boats, forestry work, horticulture, … [the local laundry company] … right down to sewing machines … like fixing bits and pieces of sewing machines they used to repair for the laundry equipment … I used to repair anything and everything mechanical … I'd give it a go and I suppose that's why I'm where I'm at today".

Ben (21 years) was beginning his apprenticeship, and enjoyed fabrication and welding. He could not see the use of Graphics and design at school, nor English, except to be able to read plans and drawings. Nor could he see the carry-over value of doing research in graphics and design at school to the research skills needed in his present work. However, Ben (apprentice) could see the importance of Pythagoras, trigonometry and finding volumes because they had practical application, and were especially related to the work he was currently doing. He felt confident in mathematics for his engineering work.

Chris (65 years) was also born in the United Kingdom and had widespread influence in the direction and the development of curricula in several branches of mechanical engineering throughout New Zealand. Like some other engineers, Chris did not pursue mathematics studies into Year 11, so had no formal school qualification in mathematics. He was currently involved mainly in fabrication engineering, where he is an acknowledged authority. Chris' knowledge of current engineering trends and the political developments of engineering apprenticeships provided important background to the wider context of engineers and engineering in New Zealand and United Kingdom society.

Courtney (43 years) could trace his engineering pedigree back at least five generations to his forebears building massive ships' engines in late 19th or early 20th century England. He described the work that the company he inherited from his father does as general engineering. This modest assessment of his own work tended to mask his specialist expertise in several engineering branches, from the fine work of the machinist to repairing and maintaining massive cranes "worth $6 million so you know, you kind of … yeah, you don't want to muck it up … and they're charged out at a ridiculous amount of money too, so they're big 450, I think it charges out at about $30,000 a day … Mmmm … so you only have to delay them for a day, and yeah, it's big money that's sort of involved …". Courtney gave several examples of how he had used his knowledge of school and polytechnic mathematics in engineering design and to solve engineering problems. He was an innovator.

Donald (57 years) had grown up in a practical environment where his father was an electrician. As a boy he pulled bikes to bits, as well as lawnmowers. Donald almost drifted into mechanical engineering by accident, because when he left school, "we all went to a polytech or trade school and we all sat a[n] aptitude test … and this was the trade they recommended for me … yeah, that's how it started, yeah". Donald's current role was mainly in maintenance engineering of heavy trucks, but he also had a role in apprentice training.

Henry (58 years) grew up in a practical world where his father's influence in teaching him measuring and calculation skills from a young age would hold him in good stead for his engineering work, especially avionics. Henry had a great interest in heat, which he described in detail as being very important in aircraft flying. He also knew a great deal of the physics involved in his work.

Howard (55 years) was born in the United Kingdom and was currently involved mainly in maintenance engineering in the heavy transport sector; in this case locomotives. He once owned his own business maintaining and repairing fork hoists. Howard's comments on the role of mathematics in his work revealed that the decisions the engineers make are both mathematical and non-mathematical, and that maintaining locomotives had many physics and mechanics aspects. The mathematics was often hidden. Howard's comments were frequently related to practical contexts involving measurement; in particular, use of specialised tools, fluid volumes, and engine wear.

Irene (23 years) was in her fourth year as an apprentice. She had attended an all girls' secondary school where engineering and wood technology were phased out after her first year. Thus, Irene had little or no school experience of engineering, but did enjoy graphics and design which she studied together with mathematics to NCEA Level 3. Irene felt that workplace mathematics “had more purpose” than school mathematics, that graphics and design was very useful not only for functionality and aesthetics but also for instilling generic skills and attitudinal perspectives which she now showed in the way she approached her work. Irene (apprentice) said that physics knowledge came in useful in both electricity and airflow in her present field of specialisation. She said that she had always wanted to do a trade but that apart from her science teachers, her teachers at school had “definitely not” encouraged her to do this, although they had not actively discouraged her either. Irene said there were no other women engineers in the company where she worked, and she knew of probably only five or six qualified women engineers in her field of specialisation throughout New Zealand. Given that Irene felt that there would be at least 1000 people in her field, then this would call into question the 3% figure quoted from Competenz (see Section 1.3). Irene’s grandfather, father and brother had all been engineers, but she claimed not to be following a family tradition and was “not influenced by others”. Irene’s great enjoyment was the variety of tasks and environments she experienced in her work.

Murray (63 years) came from South Africa and had wide industrial engineering experience, but not directly in mechanical engineering. Murray also had the equivalent of NCEA Level 3 Mathematics, and like several other engineers, he commented that he found formulas and algebra “totally abstract”. In spite of this, he was quite happy to share with others the way he developed his school mathematics when he found a real context for it. Murray was very much aware of the need for highly-developed and interconnected mathematics skills related to context. Thus, referring to his early experiences in textiles, he said that “you’ve got to measure up textile dyes, and of course, you’ve got to … once it has been dyed, once the dyes have been measured, heating patterns for the dye baths, heating times’ durations, and things like that”. The information for this came from tables which the engineer had to consult. Murray also commented at length on the importance of effective communication in the workplace situation.

Nikau (23 years), an apprentice, belonged to a whānau that also had an engineering heritage, “Yeah, yeah, my old man, he’s a diesel mechanic”. Like the other apprentices, Nikau’s actual engineering experience was still limited, but he did give me a detailed account of altering the distance between the forks on a forklift. Already he has integrated the skills of measuring and squaring something up with the engineering skills required for welding.

Owen (66 years) was an avionics educator. He commented on his questionnaire, “mathematics in school tends to be a lot of rules, whereas in the workforce you try to get them to apply their maths”. We kept coming back to this theme throughout the interview. Owen’s approach to teaching mathematics was to present students with “maths challenges, they have to try and figure how maths [works] that we’re asking them to do … helps them understand the physics and the science of the aeroplane technology we are trying to get them to do … to maintain the aeroplane … they can misunderstand why are they doing this calculation, it’s not just a calculation …”. Problem-solving and thinking beyond the square were important to Owen, because “we want people who actually have an aptitude for fault-finding and analysis”.

Paul was the training officer of a large company, one of whose specialities is manufacturing high-tech equipment for the medical sector. Paul pursued business style mathematics courses in years 12 and 13 at school, rather than the more academic courses. He said his school mathematics courses “did [help] in my apprenticeship absolutely, yep”. Paul has a broad, yet detailed, knowledge of several different branches of engineering, and how they relate. However, perhaps his greatest contribution to this study was his knowledge and experience of how apprentices were mentored from the moment they began their training.

Robert (50 years) has inherited both his father’s engineering business and innovative spirit which he applied to problem-solving, creativity and entrepreneurship. He said that he never understood things like calculus at school which “drove [him] nuts”, and never really appreciated mathematics until he was able to apply it at polytechnic. Then it became real for him. He was also an influential person in the development of engineering policies in New Zealand, and was a major contributor of the application of physics to engineering in this study.

Simon (23 years) was a young apprentice and thus had limited actual engineering experience. Nevertheless, he was able to talk about the mentoring process and the projects on which he had been involved. Thus, insight was gained into the tasks apprentices are given, and the increasing level of independence and responsibility given to apprentices as they grew in experience and confidence. At an early stage of his apprenticeship, Simon had been involved in working on a “job [outside of New Zealand] … it was like the ramps which were made out of the like 40 millimetre thick plate ... there were like 10 or so that all added up into a circle … like big bowls or ladles that sat … sat inside those ramps”. Simon also spoke about his night class and the problems the apprentices would be set - “And they gave you the formulas but you had to work them through with your um ... with your tutor at night class, and that … yeah … help you out, yeah … you’d work out like how much weld you’d need and how much welding time it would take to weld it all up, the seam ends and stuff and what kind of materials made out of and what kind of welding wire you’d need”. Simon’s father and at least one grandfather had been engineers.

Stephen (43 years) had completed an apprenticeship in mechanical engineering, and had worked in aviation, in airframes systems and as an engine mechanic. He spoke at length about “running an engine” where various pieces of information would be supplied, like temperature, air pressure, and a torque output. Coming to terms with dealing with several variables simultaneously was a major part of the apprentice’s learning, and “you sort of get all those, and put them together, and you can either use a graph or a calculator and you come out with a percentage …”. Thus, the mathematics was embedded in the practical demands of the task in hand. Stephen also spoke about mentoring apprentices and communication issues on the job.

Warren (62 years) was also involved in avionics, with an emphasis on the electronics side of things. He too regretted the lack of numeracy skills in some school leavers. Peer learning was important for Warren, because “we actually get a lot more interaction, and it’s worked a TREAT!” This was especially useful for bringing out learning problems into the open that students might otherwise have said nothing about. Warren expressed very strong views on the need for mental calculation and estimation skills and felt that the brain had to be used to estimate before using a calculator, "… well, they don't trust their brain, their skill, or they haven't got the skills". Warren also spoke about the mentoring support given to apprentices.

Appendix F: Mechanical engineering pre-apprenticeship questionnaire

Please place a tick in the appropriate box (or boxes), and write comments in the spaces where appropriate.

1. Date of birth ______________________
2. Gender: ☐ Male Female ☐
3. Ethnicity: ☐ European/ Pākehā ☐ Māori ☐ Pasifika

   ☐ Other (Please specify)

   ____

4. I might specialise in the following branch(es) of mechanical engineering:

   ____

For the following statements, please tick the box which you most agree with:

	Strongly Disagree	Disagree	Agree	Strongly Agree
5) I found mathematics easy at school.	☐	☐	☐	☐
6) I enjoyed mathematics at school.	☐	☐	☐	☐
7) Most of the time I cope well with doing maths problems that involve <u>several steps</u> to find the answer.	☐	☐	☐	☐
8) <u>Problem-solving skills</u> are useful in mechanical engineering work.	☐	☐	☐	☐
9) <u>Mental calculation skills</u> are important in mechanical engineering work.	☐	☐	☐	☐

10. What mathematics, science, physics and Graphics and Design qualifications did you obtain while still at school? (You may tick more than one box in each row)

Subject		Level 1 NCEA (previously School Certificate)	Level 2 NCEA (previously Sixth Form Certificate)	Level 3 NCEA (previously Bursary)	None of these
1	Mathematics				
2	Science				
3	Physics				
4	Graphics and Design				

11. How well do you think your school experiences have prepared you in the following mathematics topics? (Please tick one box in each row)

Topic		Poor	Satisfactory	Good	Very Good	Excellent
a	Decimal calculations					
b	Rounding decimals					
c	Scientific notation e.g. 0.0002 = 2 × 10^{-4} or 2.56 × 10^4 = 25600					
d	Reading tables					
e	Reading graphs					
f	Drawing graphs					
g	Finding fractions of quantities e.g. Find $\frac{2}{5} of\ 440\ mm$					
h	Substituting in a formula, e.g., Find the area of a triangle using Area = 0.5 × base × height					
i	Changing the subject of a formula e.g., make "**d**" the subject of the circumference of a circle formula $C = \pi \times d$					
j	Converting length and mass units, e.g. 2.37 m = 2370 mm and 0.04 kg = 40 g					

12. How well do the following methods help you to solve mathematics problems? (Please tick one box in each row)

Method		Very Ineffective	Ineffective	Effective	Very Effective
a	By thinking things out for myself				
b	In the classroom				
c	On the job				
d	By reading a textbook				
e	Online				
f	Discussing with tutors and other engineers				
g	By doing exercises and examples until I get things right				
h	By watching others do the problems				
i	Others (please specify)				

13. Please read and answer the following problem:

Stephen has a block of steel on his workbench which he feels is too heavy for him to lift alone or to carry safely.

- He wants to calculate its mass so that he can choose a strong enough sling to support the block properly.
- To do this he first needs to calculate the volume in m^3 (cubic metres), and then multiply by the density of the steel, which is in kg/m^3.
- Stephen correctly measures the block as 200 mm by 200 mm by 200 mm.

Please circle the option below that correctly shows the volume in m^3 (cubic metres).

(a) 0.008 m^3 (b) 0.08m^3 (c) 0.8 m^3 (d) 8 m^3 (e) 8000 m^3 (f) Don't know

Thank you for completing this questionnaire.

Appendix G: Mechanical engineering apprentices questionnaire

Please place a tick in the appropriate box (or boxes), and write comments in the spaces where appropriate.

1. Which best describes you? I am a mechanical engineering apprentice in my:

   ☐ 1st year ☐ 2nd year ☐ 3rd year ☐ 4th or later year

2. Date of birth ______________________

3. Gender: ☐ Male Female ☐

4. Ethnicity: ☐ European/ Pākehā ☐ Māori ☐ Pasifika

   ☐ Other (Please specify)

   ____

5. I might specialise in the following branch(es) of mechanical engineering:

   ____

For the following statements, please tick the box which you most agree with:

	Strongly Disagree	Disagree	Agree	Strongly Agree
6) I found mathematics easy at school.	☐	☐	☐	☐
7) I enjoyed mathematics at school.	☐	☐	☐	☐
8) The mathematics I learnt at school helps me with the mathematics in my apprenticeship.	☐	☐	☐	☐
9) Overall, I find that mathematics for mechanical engineering work is easy.	☐	☐	☐	☐
10) I use all the engineering <u>mathematics</u> at work that I learnt at my block courses.	☐	☐	☐	☐
11) I use all the trade <u>physics and mechanics</u> at work that I learnt in my block courses.	☐	☐	☐	☐

Questions 12 and 13 refer to how easy you have found passing the mathematics, physics and mechanics requirements for mechanical engineering trades (for example, Unit Standards 21905, 15847, 21908, 16955, or 16956).

	Strongly Disagree	Disagree	Agree	Strongly Agree
12) I have personally found engineering trade mathematics and calculations easy.	☐	☐	☐	☐
13) I have personally found engineering trade physics and mechanics ideas easy to understand.	☐	☐	☐	☐
14) Most of the time I cope well with doing maths problems that involve several steps to find the answer.	☐	☐	☐	☐
15) Problem-solving skills are useful in mechanical engineering work.	☐	☐	☐	☐
16) Mental calculation skills are important in mechanical engineering work.	☐	☐	☐	☐

	Seldom or Never	Once or twice a week	Daily	Several times a day
17. How often do you think mechanical engineers use scientific calculators in their work?	☐	☐	☐	☐

18. How often do you think most mechanical engineering tradespersons use the following mathematics topics? (Please tick one box in each row)

Topic		Seldom or Never	About once a month	About once a week	Once or twice a day	Many times each day
a	Decimal calculations					
b	Rounding decimals					
c	Scientific notation e.g. $0.0002 = 2 \times 10^{-4}$ or $2.56 \times 10^{4} = 25600$					

d	Reading tables					
e	Reading graphs					
f	Drawing graphs					
g	Finding fractions of quantities, e.g. Find $\frac{2}{5} of\ 440\ mm$					
h	Substituting in a formula, e.g., Find the area of a triangle using Area = 0.5 × base × height					
i	Changing the subject of a formula e.g., make "**d**" the subject of the circumference of a circle formula $C = \pi \times d$					
j	Converting length and mass units, e.g. 2.37 m = 2370 mm and 0.04 kg = 40 g					

19. What mathematics, science, physics and Graphics and Design qualifications did you obtain while still at school? (You may tick more than one box in each row)

Subject		Level 1 NCEA (previously School Certificate)	Level 2 NCEA (previously Sixth Form Certificate)	Level 3 NCEA (previously Bursary)	None of these
1	Mathematics				
2	Science				
3	Physics				
4	Graphics and Design				

20. How well do the following methods help you to solve mechanical engineering problems involving mathematics? (Please tick one box in each row)

Method		Very Ineffective	Ineffective	Effective	Very Effective
a	By thinking things out for myself				
b	In the classroom				
c	On the job				
d	By reading a textbook				
e	Online				
f	Discussing with tutors and other engineers				
g	By doing exercises and examples until I get things right				
h	By watching others do the problems				
i	Others (please specify)				

21. Please read and answer the following problem:

Stephen has a block of steel on his workbench which he feels is too heavy for him to lift alone or to carry safely.

- He wants to <u>calculate</u> its mass so that he can choose a strong enough sling to support the block properly.
- To do this he first needs to calculate the volume in m^3 (cubic metres), and then multiply by the density of the steel, which is in kg/m^3.
- Stephen correctly measures the block as 200 mm by 200 mm by 200 mm.

Please circle the option below that correctly shows the volume in m^3 (cubic metres).

(a) 0.008 m^3 (b) 0.08m^3 (c) 0.8 m^3 (d) 8 m^3 (e) 8000 m^3 (f) Don't know

Thank you for completing this questionnaire.

Your answers will form an important contribution to the research.

OPTIONAL: If you would be agreeable to a conversation about your experiences and views of mechanical engineering, then could you please write your contact details below:

Name: ____

Telephone: ____

Email: ____

Appendix H: Mechanical engineering educators, tradespersons and employers questionnaire

Please place a tick in the appropriate box (or boxes), and write comments in the spaces where appropriate.

1. Which best describes you? (Please tick more than one box if appropriate)

   ☐ Tradesperson ☐ Mechanical engineering educator ☐ Technician/Designer
   ☐ BE graduate ☐ Apprentice Supervisor
   ☐ Other (please specify) ________________________________
2. What is your date of birth? _______________

3. In which branch(es) of mechanical engineering have you had significant direct experience? ____

4. Gender: ☐ Male Female ☐

5. Ethnicity: ☐ European/ Pākehā ☐ Māori ☐ Pasifika

   ☐ Other (Please specify)

   ____

For the following statements, please tick the box which you most agree with.

	Strongly Disagree	Disagree	Agree	Strongly Agree
6) I found mathematics easy at school.	☐	☐	☐	☐
7) I enjoyed mathematics at school.	☐	☐	☐	☐
8) The mathematics I learnt at school helps me in my mechanical engineering work.	☐	☐	☐	☐
9) Overall, modern apprentices adapt well to the mathematics for mechanical engineering work.	☐	☐	☐	☐
10) I am concerned about the numeracy and other mathematics skills in beginning apprentices.	☐	☐	☐	☐
11) Overall, I am concerned about numeracy and other mathematics skills in skilled mechanical engineering tradespersons.	☐	☐	☐	☐

Questions 12 and 13 refer to how easy you think modern day apprentices find passing the mathematics, physics and mechanics requirements for mechanical engineering trades.

12) Overall, I think modern day apprentices find engineering trade mathematics and calculations easy.	☐	☐	☐	☐
	Strongly Disagree	Disagree	Agree	Strongly Agree
13) Modern day apprentices find engineering trade physics and mechanics ideas are easy to understand.	☐	☐	☐	☐
14) Modern day apprentices at the beginning of their apprenticeship cope well with doing maths problems that involve several steps to find the answer.	☐	☐	☐	☐
15) Modern day apprentices near the end of their apprenticeship cope well with doing maths problems that involve several steps to find the answer.	☐	☐	☐	☐
16) Problem-solving skills are useful in mechanical engineering work.	☐	☐	☐	☐
17) Mental calculation skills are important in mechanical engineering work.	☐	☐	☐	☐
	Seldom or Never	Once or twice a week	Daily	Several times a day
18) How often do you think most mechanical engineers use scientific calculators in their work?	☐	☐	☐	☐

19. How well do you think the following methods help modern day apprentices to learn to solve mechanical engineering problems involving mathematics? (Please tick one box in each row)

	Method	Very Ineffective	Ineffective	Effective	Very Effective
a	By thinking things out for themselves				
b	In the classroom				
c	On the job				
d	By reading a textbook				
e	Online				
f	Discussing with tutors and other engineers				
g	By doing exercises and examples until they get things right				
h	By watching others do the problems				
i	Others (please specify)				

20. How often do you think most mechanical engineering tradespersons use the following mathematics topics? (Please tick one box in each row)

Topic		Seldom or Never	About once a month	About once a week	Once or twice a day	Many times each day
a	Decimal calculations					
b	Rounding decimals					
c	Scientific notation e.g. $0.0002 = 2 \times 10^{-4}$ or $2.56 \times 10^{4} = 25600$					
d	Reading tables					
e	Reading graphs					
f	Drawing graphs					
g	Finding fractions of quantities e.g. Find $\frac{2}{5} of\ 440\ mm$					
h	Substituting in a formula e.g. Find the area of a triangle using Area = 0.5 × base × height					
i	Changing the subject of a formula e.g., make “**d**” the subject of the circumference of a circle formula $C = \pi \times d$					
j	Converting length and mass units, e.g. 2.37 m = 2370 mm and 0.04 kg = 40 g					

21. What mathematics, science, physics and Graphics and Design qualifications did you obtain while still at school? (You may tick more than one box in each row)

Subject		Level 1 NCEA (previously School Certificate)	Level 2 NCEA (previously Sixth Form Certificate)	Level 3 NCEA (previously Bursary)	None of these
1	Mathematics				
2	Science				
3	Physics				
4	Graphics and Design				

22. The apprentices' version of this survey contained the problem in the box below.

Stephen has a block of steel on his workbench which he feels is too heavy for him to lift alone or to carry safely.

- He wants to calculate its mass so that he can choose a strong enough sling to support the block properly.
- To do this he first needs to calculate the volume in m^3 (cubic metres), and then multiply by the density of the steel, which is in kg/m^3.
- Stephen correctly measures the block as 200 mm by 200 mm by 200 mm.

<u>The apprentices were then asked to:</u>

Circle the option below that correctly shows the volume in m^3 (cubic metres)

(a) 0.008 m^3 (b) 0.08m^3 (c) 0.8 m^3 (d) 8 m^3 (e) 8000 m^3 (f) Don't know

Please state roughly what you think are the percentages of apprentices at various levels of their training who appreciate that the correct volume is 0.008 m^3.

_______ % At or near the beginning of their apprenticeship?

_______ % During their apprenticeship?

_______ % Do not develop this skill during their apprenticeship?

Thank you for completing this questionnaire.

Your answers will form an important contribution to the research.

Appendix I: Participant information sheet

THE UNIVERSITY OF WAIKATO *Te Whare Wānanga o Waikato*

PARTICIPANT INFORMATION SHEET

Interviews (this sheet likely to be given to apprentices, skilled tradespersons, educators, employees and employers)

Title: Mathematics in workplace settings: Numeracy in the mechanical engineering trades.

My name is Kelvin Mills and I am currently undertaking a PhD in Education at the University of Waikato. The research is designed to give us further understanding of workplace mathematics. I am writing to you to invite you to participate in an interview as part of the research project for my thesis. The interview will be audio recorded and will last approximately 60 minutes.

The interview questions will focus on mechanical engineering mathematics and school mathematics, and problem-solving in mechanical engineering. You will be asked to bring and discuss a drawing of a project you have been involved in, preferably with a physical model, if possible. If you agree to participate, I will contact you again to arrange a time and place convenient to you for the interview.

You have the right to refuse to answer any question, or withdraw from the interview at any time, or withdraw information you have provided up until the data analysis is begun, approximately 1 June, 2016. You will also be given opportunity to review and, if necessary, have the transcript of your interview amended before the data analysis is begun.

While every effort will be made to ensure confidentiality, this cannot be guaranteed. However, as a participant you can expect every reasonable effort to be made by the researcher and his supervisors to have your privacy protected and personal details kept confidential. To achieve this, pseudonyms will be used and care will be taken to remove any identifying information in the final thesis and any reporting of findings. Similarly, your organisation will not be identified, but will be referred to as ‘the organisation’. The findings may be reported in conference and written presentations.

Consent forms and audio data will be stored separately and securely for 5 years at the University of Waikato and then destroyed. The consent form for the interview is shown on a separate sheet of paper. A brief summary of the thesis findings will be made available to you, and the complete thesis will be published on the University of Waikato website.

If you wish to receive further information about this project then please email me: mills.kr@xtra.co.nz or telephone 09 535 0241.

If you have any queries regarding ethics considerations, then in the first instance please contact: Professor Diana Coben, 07 838 4466 ext. 8748, dccoben@waikato.ac.nz, or, if the matter is not resolved to your satisfaction, then Professor John Williams, 07 838 4466 ext. 4769, jwilliam@waikato.ac.nz

Thank you for your time and help in assisting with this project.

Appendix J: Participant consent form

PARTICIPANT CONSENT FORM

Interviews (this sheet likely to be given to apprentices, skilled tradespersons, educators, employees and employers)

- I have been given and have understood an explanation of this research project.
- I have had an opportunity to ask questions and have them answered.
- I understand that the interview questions will centre on mechanical engineering and mathematics.
- I understand that the interview will be audio recorded and will last approximately 60 minutes.
- I understand that the audio data, consent forms and any transcripts and summaries will be stored securely for 5 years at the University of Waikato and then destroyed.
- I understand that my name will not be used in any written or oral presentation.
- I understand that the findings may be used for publication and conference presentations.
- I understand that every reasonable effort will be made to ensure my privacy is protected.
- I consent to participating in the study with the understanding that my participation is entirely voluntary and I can withdraw personally at any stage and have information I have contributed during the interview withdrawn up until the point of data analysis - approximately 1 June 2016.

I agree to participate in the interview.

Signed: ____

Name: ____

Date: ________________________

Appendix K: Proposed questions for semi-structured interviews

1. What mathematics skills and knowledge are used in mechanical engineering?

   Participants will be shown a copy of the mechanical engineering mathematics curriculum topics, *Unit Standard 21905: Demonstrate knowledge of trade calculations and units for mechanical engineering trades* (see next attached document). The various topics can then be followed through in order.

   Possible probes for each section are the following:

   - What mathematics do you use as a mechanical engineer?
   - In what contexts, or situations, would you use mathematics?
   - How do you use those skills? Could you relate to me an example of how you would use ………?
   - How often do you use these skills? Could you relate some examples?
   - Which topics from US 21905 do you find the most difficult? Which topics do other engineers and apprentices find the most difficult?
   - How often do engineers discuss the mathematics aspects of their work? How do they reach a decision on when to use mathematics or how to do it? Which mathematics topics do they discuss the most often? Ask the participant to relate some examples.
   - What other branches of mechanical engineering might use the mathematics skills from US 21905 that you do not use? Ask the participant to relate some examples.

2. How are skills in mechanical engineering mathematics developed and used?

   Possible probes are the following:

   - Can you relate examples from your experience and projects you have worked on where you have used mathematics in your work?
   - How do you learn your mathematics? Do you expect to continue learning mathematics for mechanical engineering once you have completed your apprenticeship? Are you continuing to learn mathematics for mechanical engineering now that you have completed your apprenticeship? Ask the participant to relate some examples.
   - How much engineering mathematics do apprentices learn on the job? Ask the participant to relate some examples.

3. What comparisons do you identify between school and mechanical engineering mathematics?

   Possible probes are the following:

Appendix K: Proposed questions for semi-structured interviews

- What differences do you see between school mathematics and mechanical engineering mathematics? - focus on both content and approach
- What similarities do you see between school mathematics and mechanical engineering mathematics? - focus on both content and approach
- What adjustments do you think have to be made to move from school mathematics to mechanical engineering mathematics? Ask the participant to relate some examples.
- What challenges did you experience coming to grips with engineering mathematics?
- Were some of the challenges you experienced to do with the physics applications of mathematics? Ask the participant to relate some examples.

4. How are problem-solving skills and extended reasoning in mathematics used in mechanical engineering?

   Possible probes are the following:

- What do you understand by problem-solving in mechanical engineering? How did you learn (are you learning) to do this? Ask the participant to relate some examples, perhaps from work currently being done.
- How often do you have to use extended reasoning skills and problem-solving techniques in mechanical engineering? What connection do you see between them and mathematics?
- How do apprentices and skilled tradespersons develop and then use extended reasoning and problem-solving techniques? Ask the participant to relate some examples.
- How important are maturity and experience in developing and learning problem-solving skills?
- What do you think distinguishes a skilled mechanical engineer from an expert? How do they become experts?

5. What is the role of electronic aids in calculations, and the design and control of machines?

   Possible probes are the following:

- How often do you use electronic calculators in your work? Do you use them for long or involved calculations, or almost constantly? Ask the participant to relate some examples.
- To what extent do you think proficiency in mental calculation skills is still important? Ask the participant to relate some examples of contexts where this might apply.

Appendix K: Proposed questions for semi-structured interviews

- How often do you use Computer Aided Design (CAD) in your work? Do you use CAD to create and design, or is your involvement just to interpret and construct what others have designed? What challenges do you see in learning CAD?
- How often do you use Computer Numeric Control (CNC) in your work? What sorts of tasks do you use CNC for? What challenges do you see in learning CNC? What engineering considerations influence the choice of procedures and tools when using CNC? What mathematics skills do you think are necessary to be successful in learning CNC?

6. Participant chosen project - participants are asked to bring a drawing of a project they have been involved in, preferably with a physical model, if possible. Discuss these in the light of the above points and from a mechanical engineering mathematics perspective.

   Possible probes are the following:

- Tell me about your experience of the project you have chosen. What was the motivation for this project? Was the project complete in itself, or part of some larger project?
- What engineering considerations were involved in designing and planning the project? Were these done for you already, or did you have to use extended reasoning and problem-solving?
- What features of this project gave engineering challenges for you to face? How did you overcome them? Did you have to problem solve to find the engineering answers? Did you work with others in seeking solutions? Ask the participant to relate some examples.
- What mathematics do you see in this project? Was mathematics involved in finding solutions to your engineering problems? Did you have to problem solve or consult with colleagues to find the mathematics answers to them? Ask the participant to relate some examples.

Appendix L: Ethical conduct in human research and related activities regulations

University of Waikato

https://www.waikato.ac.nz/research-enterprise/ethics/human-ethics

During the engagement

• Act professionally at all times: be polite, courteous, prompt and dependable.

• Adopt appropriate standards of dress, behaviour and language that signal your commitment to the successful conduct of the meeting.

• Arrive on time for appointments. If lateness or late cancellation is unavoidable, ring and apologise (preferably before you are due to arrive).

• If negotiating entry into a setting without prior arrangement, seek permission appropriately from those with the right to grant it and express your gratitude to all those who facilitate the visit.

• If activities are related to coursework requirements, adhere to agreed arrangements and do not change plans without the formal approval of a staff member with responsibility for the assignment.

• Use appropriate language for introducing yourself, based on your own position and the position(s) of those with whom you are meeting. Normally introduce yourself by your own full name (first name and surname, and title if appropriate) and address others using their full name and titles as appropriate (e.g., Dr, Professor, Your Worship) until they instruct you otherwise.

• Follow appropriate etiquette (e.g., do not sit until invited) and become familiar with cultural variations (e.g., regarding the exchange of business cards).

• Do not take things for granted: attention (or lack of it) to even apparently trivial conventions or protocols can significantly influence the outcome of encounters.

• Regardless of information sent in advance, restate or further explain your purpose, intention, what you want or expect from the meeting, how you wish to use any information obtained, and what you can do for the individual(s) or institution(s) participating (e.g., share reports, offer a presentation).

• Ensure there is mutual agreement regarding the way any information discussed may be used and disseminated. Formalise this agreement in writing when there are conditions.

• Ensure any financial reimbursement arrangements are professionally and ethically appropriate and that payments have been properly organised.

• Follow practices consistent with the University's commitment to the Treaty of Waitangi. Be aware of Māori protocol, where appropriate, and behave accordingly. If you are in doubt, ask an appropriate person.

• In all contexts, be aware of and respect the cultural practices of others.

Following and ongoing relationships

• Always explicitly thank the contact person/placement supervisor/ organisation before and after the interaction. Be sincere in expressing your appreciation for their time and effort, even if the meeting failed to achieve everything you hoped for.

• As appropriate, sustain healthy and collaborative working relationships with individuals and/or organisations.

• Adhere to agreed arrangements for confidentiality or anonymity. Check any issues that were not explicitly clarified during discussions.

• Do not take advantage of people's willingness to divulge sensitive or proprietary or trivial information. You are in a position of trust: do not share information around, even informally.

• Implement the principle of reciprocity in relationships. As far as possible follow through on anything you promised to undertake or provide.

• More generally, try to ensure through your conduct that individuals and organisations will be likely to assist other University staff or students in similar ways in the future.

Appendix M: Employer information sheet

THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

EMPLOYER INFORMATION SHEET

Employee Interviews (this sheet likely to be given to employers only)

Title: Mathematics in workplace settings: Numeracy in the mechanical engineering trades

To the CEO,

CCC,

Dear ________,

I am writing to you for your consent to interview _______________ on site about how mathematics is used in the workplace. The data obtained will be used as part of a PhD in Education at the University of Waikato. The research is designed to give us further understanding of workplace mathematics.

The interview will be audio recorded and will last approximately 60 minutes. The interview questions will focus on mechanical engineering mathematics and school mathematics, and problem-solving in mechanical engineering. _______________ will be asked to bring and discuss a drawing of a project s(he) has been involved in, preferably with a physical model, if possible.

While every reasonable effort will be made to ensure confidentiality, this cannot be guaranteed. However, no reference to CCC will be made in the final thesis, and every reasonable effort will be made to preserve the confidentiality of CCC and the participants. The name of CCC will not be made in any written or oral presentation, but the findings may be used for publication and conference presentations. A brief summary of the thesis findings will be made available to you, and the complete thesis will be published on the University of Waikato website.

If you wish to receive further information about this project, then please email me: mills.kr@xtra.co.nz or telephone 09 535 0241.

If you have any queries regarding ethics considerations, then in the first instance please contact: Professor Diana Coben, 07 838 4466 ext. 8748, dccoben@waikato.ac.nz, or, if the matter is not resolved to your satisfaction, then Professor John Williams, 07 838 4466 ext. 4769, jwilliam@waikato.ac.nz

I look forward to your reply and working with you in the near future.

Yours faithfully,

Appendix M: Employer information sheet

Kelvin Mills

Appendix N: Employer access consent form

EMPLOYER ACCESS CONSENT FORM

Employee Interviews (this sheet likely to be given to employers only)

- I have been given and have understood an explanation of this research project.
- I have had an opportunity to ask questions and have them answered.
- I understand that the interview questions will centre on mechanical engineering and mathematics.
- I understand that while it is not possible to guarantee confidentiality, every reasonable effort will be made to ensure the privacy of CCC and the interview participants is protected.
- I understand that the findings may be used for publication and conference presentations.
- I understand that the name of CCC will not be used in any written or oral presentations.
- I understand that the interview will be audio recorded and will last approximately 60 minutes.
- I understand that audio data, consent forms and any transcripts and summaries will be stored securely for 5 years at the University of Waikato and then destroyed.

I agree for Kelvin Mills to have site access and grant consent for him to conduct the interview with

____

Signed: ____

Name: ____

Date: ____________________________

Appendix O: Stephen and Owen on problem solving

Stephen (educator)

I asked Stephen what he understood by problem-solving. He mentioned first of all their current mathematics courses had mathematical problems that were presented in live situations or in words, or as a problem that you might come across as a scenario. The pre-apprentices had to find a mathematical solution. Second, he mentioned trouble-shooting, "… which is looking at the evidence and either using a book or using prior knowledge to come to a conclusion based on that". The pre-apprentices were presented with some strategies to perform trouble-shooting, such as "… halve the system and test each half and then use the results of that and then come to a further conclusion … and dividing it in half until you find the area that's at fault". I asked how quickly the apprentices adapted to the strategies. Stephen replied, "Well, I'd probably say it's new to them but … they don't struggle too much with the course in general". Stephen later mentioned that while problem-solving did not feature highly at the beginning of apprenticeship training, "it's very common to have to do that sort of thing later on … to have a problem that involves solving a few other problems to get all the right information …"

To summarize, apprentices were given tasks that reflected their current state of engineering preparedness. Some of their tasks directly involved mathematics, but others involved strategies for solving problems. In these situations, developing and following logical chains of reasoning were important tools the pre-apprentices needed to learn. Moreover, they were introduced progressively to more complex problem-solving scenarios as they grew in experience.

Owen (educator)

A non-engineering scenario was described by Owen who set apprentices mathematical problems that were similar to many asked in secondary school. In one such problem, someone shouts at a wall some distance away, and after a few seconds they hear their echo coming back. Given the distance to the wall, and the time for the echo to return, what is the speed of sound? This is a straightforward problem involving the formula $\text{Speed} = \text{Distance} \div \text{Time}$. However, none of the apprentices appreciated that the distance to the wall is half the distance the sound they have just heard must have travelled. Owen explained that "… so many of them just put a very simple formula into their calculator, out comes an answer, and they write that down". He gives the pre-apprentices many examples like the one above, because

> … we're looking for people who can stop and think before they fire off answers like that because we will happily note that they can press the buttons, or the keys on the calculator, they can do the formula, but they can't apply the principles we've raised with them when we've discussed it with them …

This type of difficulty was linked to what I shall call minimal skills, by which I mean the minimal skill level to perform a job, but without necessarily having the ability to perform the same task competently when its terms or context have been altered and extra thinking is required. This was consistent with Owen's statement that avionics required people "who actually have an aptitude for fault-finding and analysis". This required extended reasoning skills, including examining different points of view, which involved more than getting the right answer and moving on. He explained that was why they aimed at "developing not only able people from the exam point of view but we are also developing people who fix faulty pieces of equipment … that's what we are here about …" Sometimes he needed to explain to the apprentices that they had left the school phase of their life behind, and were now starting to figure how to "apply this to the problem-solving of the type we're gonna confront you with in your working day and this is … this is the next step in that particular development journey you're on".

To summarize, problem-solving development was linked to extended reasoning development in apprentices (see Section 4.2.2). While formal strategies for solving workplace problems in unfamiliar situations were a new episode in many apprentices' experiences, the mathematics involved was not necessarily as important as the integrated employment of problem-solving skills.

Appendix P: Ben, Ari, Simon, and Nikau on apprentice learning

These vignettes are more detailed descriptions with the workplace learning experiences narrated by apprentices Ben, Ari, Simon, and Nikau in Section 5.2.2.4.

Ben (apprentice)

Ben, like many other engineers, expressed a preference for practical mathematics, such as calculating "volumes, yeah stuff like that", and "working stuff out, like find the shaded area of such and such, I dunno, something like that …" In the workplace, Ben used Pythagoras often. I asked Ben, who would complete his apprenticeship in about six months, about what sorts of things he did during his first year under the guidance of a supervisor. He replied, "Cutting steel and easy welding jobs…" This involved using a metre rule and measuring and cutting to within "one mil … plus or minus a mil". During the second year, Ben would have just been "given a drawing and see you later …" The project would have been planned beforehand in the office together with a list of all the materials he needed, and Ben would generally be unsupervised. A project during his second year was to "… make guarding", which consisted of "big angle iron frames, and then mesh in the inside of it, to cover [conveyor belts at] the steel mill …". I asked Ben what he had done last year, and he said, "… it just gets harder really, you just get bigger jobs". A major project during Ben's third year was making a venturi, about three or four meters high. It was "just a big cylinder thing, with heaps of ports coming off it, sort of thing…" and which involved welding. This now posed few if any problems for Ben, including cutting the holes for the pipes which served as ports. As was company policy for all the engineers, Ben received all the instructions, plans and materials from the office, but supervision became less as time went on until finally "you just get the drawing and then they say 'Here's the material', and you just go do it". If Ben had problems, then "Yeah, I would ask for help and stuff if I need it". Even with his mathematics learning, Ben felt confident enough to approach others for help, "Yeah, I always ask if I need help". He added that he was encouraged to do this.

Ari (apprentice)

Ari, claimed to be doing “a lot of sweeping … tidying the yard, forklift work …”. However, he had also been involved in repairing things such as completely pulling out and replacing oil coolers. These were big jobs during which time Ali was supervised by another tradesperson. Ari was developing a sense of how replacement parts for Caterpillar (CAT, American heavy machinery vehicles) equipment were identified, although he was not yet entrusted with the responsibility for doing this. He described how the tradesmen had manuals to identify which spare part they needed. Therefore, Ari demonstrated developing skills in logical thinking and finding information that would stand him in good stead for fault-finding and maintenance engineering.

From a mathematics perspective, Ari did not need to know about units for torques or whether they were in imperial or metric because the units on the tool were the same as the units in his reference book. However, some big bolts needed a torque converter, “I guess you have to have a bit of maths… it’s just times everything by 20”. Doing such conversions was just part of Ari’s work culture.

Ari also spoke at length about the need for accuracy as a beginning apprentice specialising mainly in fabrication. He understood that we were “just talking just about basic maths …” He also understood the importance of numeracy and being “quite accurate, for something, … um, so obviously, like say, if you don’t torque up bolts to within certain ranges, they could come out”. This could be a safety issue, and “cause a bigger problem there than what you started with …”

Ari did not keep a calculator on him while he was working, which led me to ask him about how accurate something had to be in a given context. He replied, “Ah, yeah, you’ve got to be pretty accurate …”, often to within one decimal place. When I asked him how he learned what would be a good enough figure, he said, “Um, I guess just experience, and ... seeing it fail, or …”

Ari also spoke positively about the discussions he had with tutors and other engineers who “... who generally know what they’re doing, cause [there are] a lot of guys around here with a lot of knowledge … and yep, you’ve just got to ask them, they’ll generally help you out …” He added that sometimes “there’s the odd thing that some people don’t know ... if it’s a quite obscure topic, … but generally these guys here are quite clued up”.

Simon (apprentice)

Simon gave a detailed description of the procedures in the company where he worked. I asked him how he checked to see that he had got the right answer. He said he would go to a tradesman, or at "night school I'll go to my tutor … and he'll help me out with it". He explained that every job came with a check sheet with perhaps five different steps. The first one was checking for the right materials. The second one was getting someone to check all the materials had been cut to the right length. Each step was signed off when completed. Even the tradesmen went through this process because "yeah, you assume that everyone stuffs up (*laughter*) …"

Simon also talked about his courses at the polytechnic. He said that for his first year, "it was more like learning about the materials and stuff … like the different kinds of steel and all that … this year's more drawings and that calculation that I had to do … yeah". In the Unit Standard he had recently done there had therefore been more mathematics than previously.

> Simon: Ah, in my last one, yeah, before I did the drawings, it was a maths … maths thing, and they gave you a drawing and there was only like two random measurements and the rest you had to figure out by yourself ... And they gave you the formulas but you had to work them through with your um ... with your tutor at night class, and that … yeah … help you out, yeah … you'd work out like how much weld you'd need and how much welding time it would take to weld it all up, the seam ends and stuff and what kind of materials made out of and what kind of welding wire you'd need.
>
> Kelvin: There's a lot of mathematics in that, isn't there?
>
> Simon: Yeah, for sure, man, there was heaps of formulas, aye, like … in my head … um …I could probably find it on my phone …

Nikau (apprentice)

Nikau described a job about some forklifts where the forks were too close together for the pallets. They need to cut them off, take them out 75 mm each side, and then weld them back on again. I was interested in the accuracy that would be required for this job, and the supervision that Nikau was under. Nikau had to place his rule "on the exact… [because] that's what the customer wants…" Nikau had to then tack things together and then ask someone to come and check the measurements were correct. Nikau described how squaring up the job was done with a square ruler. Nikau checked his work regularly, "…every time you tack it, you measure it again to make sure there's nothing's moved".

Nikau was developing welding skills he had learned from his father with mentoring within the company where they gave him "… little jobs … sort of jobs to… um to practice on and stuff". One such job involved making upfront brackets for a forklift that had wheels on them. Nikau appreciated how much things moved during the welding process and that this might cause the wheels to jam. Nikau had been taught tricks of the trade such as making extra brackets to hold things in place while he welded the main part of his job. Making sure that the wheels could still spin once he had welded it all up was done by using brackets across the back so that it wouldn't bend when it got quite hot.

Nikau's work may still have been at an elementary apprentice level, especially using mathematics, but already he was involved in, acknowledging and reflecting on developing tricks of the trade and problem-solving skills in his work. He also acknowledged the possibility of multiple acceptable solutions and the influence of others in finding them.

Another of Nikau's projects involved making steel moulds for concrete motorway barriers. The moulds were six meters long. There had been a large number of people involved in this project, including skilled tradespersons. Nikau described how he and some other apprentices "made a few of them up, and stuff, made sure they come out right, but a couple of them didn't (*laughter*) …" so they had to be cut up and then welded "back up and stuff … bit of problem-solving". He could laugh about his mistakes, and he and his fellow apprentices used them as learning opportunities to develop problem-solving and other engineering skills.