The Use of Mobile Technology Apps when Teaching and Learning Geometry in Sri Lankan Secondary Schools

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Abstract

In the last decade, there has been rapid development of digital technologies (DT), and their adoption in mathematics education has been widely discussed in the research literature. The Sri Lankan curriculum stipulates the use of digital technologies in learning, but the practice differs among most teachers. The purpose of this study was to explore different perspectives surrounding the use of mobile technology applications (apps) by pre-service mathematics teachers with a focus on the pedagogical approaches used when teaching geometry to year ten students in Sri Lanka. The interpretive paradigm was selected as the most appropriate philosophical basis and paradigm for underpinning the study. It enables the interpretation of the identification of any pattern, permits insights into the pre-service teachers' beliefs and knowledge, and outlines the research setting (block-teaching) of GeoGebra being used for teaching and learning (social processes). Pre-service teachers' knowledge about the use of GeoGebra for teaching and learning geometry was analysed with the technological pedagogical content knowledge (TPACK) framework.

This study followed the mixed method approach. Two case studies were conducted in two teacher education institutes (TEIs) in Sri Lanka to examine pre-service teachers' beliefs and knowledge of the use of apps in geometry after their block-teaching experience. Quantitative and qualitative methods were used for the data collection. Two case studies conducted with the survey questionnaire were followed by semi-structured focus group interviews with pre-service teachers and individual interviews with teacher educators. The purposive sample of 163 pre-service mathematics teachers and eight mathematics teacher educators from pre-service TEIs in Sri Lanka participated in the study. Findings have revealed that pre-service teachers not only need an understanding of geometry for GeoGebra, but their beliefs about DT and their geometry content knowledge have played crucial roles as they influence both their interpretation and engagement. The study proposes four ways that apps can influence mathematics pre-service teachers' perspectives. Namely, pedagogical (which focuses on teaching and learning geometry); technological (the affordances of GeoGebra and features of the dynamic geometry environment (DGE) relevant to the curricula); epistemological (understanding about GeoGebra for the development of geometrical concepts); and political (which considers context, including policy, social, and cultural settings). These aspects were identified as being relevant to the use of apps in the selected geometry content during the block-teaching practices. Findings from this study also suggested that it may contribute to the literature in the field by the identification and consideration of metacognitive technological pedagogical content knowledge (M-TPACK) relevant to the selected geometry concepts.

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List of Acronyms

DGE	Dynamic Geometry Environment
DGS	Dynamic Geometry Software
DMK	Didactic Mathematical Knowledge
DoE	Department of Examination
DoM	Department of Mathematics
DT	Digital Technology
GCE(O/L)	General Certificate of Education (Ordinary Level)
GCE(A/L)	General Certificate of Education (Advanced Level)
ICT	Information Communication Technology
MoE	Ministry of Education
MT	Mobile Technology
NATE	National Authority of Teacher Education
NCTM	National Council of Teachers of Mathematics
NDT	National diploma in teaching
NEC	National Education Commission
NEREC	National Education Research and Evaluation Centre
NIE	National Institute of Education
PCK	Pedagogical Content Knowledge
TCK	Technological Content Knowledge
TEDS-M	Teacher Education and Development Study in Mathematics
TEI	Teacher Education Institute
TG	Teacher Guide
TPACK	Technology Pedagogical Content Knowledge
UGC	University Grant Commission
VM	Visual Manipulation
WB	World Bank
ZPD	Zonal Proximal Development

Chapter 1

1.1 Introduction

The immersive proliferation of smartphone applications (apps) and digital technology (DT) media have altered the nature of many social activities (Jepson & Ladle, 2015). Goos et al. (2020) explained the nature of DT activities (necessary, optional, social) in the physical space that changes to virtual space. Thus, the use of mobile applications has altered day-to-day activities in society as well as in education (Sangrà & González-Sanmamed, 2010). Mobile technology (MT) applications capable of running in mobile technology devices have changed the nature of activities in society by positively and negatively impacting education, day-to-day life, work, and entertainment.

MT advances have meant that MT devices (e.g., smartphones) have changed from being simply an accessory for communication to being an accessible piece of equipment for education. People communicate through social communication applications (networks), which are socially connected through virtual spaces. Despite most MT applications not being produced for educational purposes, some offer great flexibility for teaching and learning (Crompton & Burke, 2014). Recognition of the communicative abilities and other touch-based facilities offered by "mobile applications" led to the development of the term *apps* (Handal et al., 2014). The proliferation of mobile phone apps has had a critical impact on the way users organise and/or socialise (Barzel et al., 2019). In addition, mathematics apps have self-contained programs that are endowed with various technical and pedagogical challenges for mathematics education when they are used as tools.

Chapter 1 introduces the research study and the thesis, including the first section about the motivation, including personal, to undertake the study while the second section discusses the teacher education research context in Sri Lanka. The third section reviews the background relating to mathematics education and the possibility of using mathematics apps for geometry, while the fourth section describes the theoretical considerations related to the research question. The final section summarises the thesis outline.

1.1.1 My motivation

First, as a mathematics teacher and later as a teacher educator in the National Institute of Education (NIE), I am interested in understanding more about the use of MT apps in mathematics education (pre-service teacher education). I have identified different perspectives and examined the factors that may influence the pedagogy, knowledge, and beliefs of pre-service mathematics teachers. I have ensured that the investigation is realistic and relevant to the needs of junior secondary classroom practitioners in the education system.

Second, I have chosen to undertake the study as it is relevant to current social and educational needs in Sri Lanka. Mathematics education has been identified as an area of great concern and a priority for the Sri Lankan education system. It represents my discipline and is something that I am interested in investigating. The report of the National Education Commission (NEC) in 1992 identified a set of basic competencies for the Sri Lankan school system as a means of achieving the National Goals (Ginige, 2008; Mampitiya, 2014). The NEC (2003) report justified a competency-based mathematics curriculum approach for the school and teacher education systems to improve students' mathematics achievement. Ministry of Education (MoE) statistical data showed that only 1% of all Sri Lankan teachers are untrained, which is an improving trend compared to previous years. However, the Department of Examination (DoE) and Department of Mathematics (DoM) stated that during the last decade mathematics achievement had declined consistently in national examinations. The DoE (2016) explained that after an analysis of students' responses in the national examination (General Certificate of Education-Ordinary Level (GCE (O/L)) in mathematics in 2015, geometry problems had the lowest proportion of correct responses in examinations. This is particularly evident with low achievement in geometry affecting national mathematics achievement (World Bank, 2011). Thus, the fact that success rates in examination results are declining, even with the increased presence of trained teachers in the profession, is alarming and is my key motivation for this research.

1.1.2 Rationale for the study

I decided to conduct research on junior secondary pre-service mathematics teacher education in TEIs in Sri Lanka. First, I am familiar with the teacher education research context as a teacher educator, and TEIs are the only possible access point for influencing novice teachers' mathematics education practice in the Sri Lankan education system. Second, pre-service teachers are employed as secondary mathematics teachers in Sri Lankan state schools after they obtain diplomas from TEIs. Third, Athurupana (2011) indicated that junior secondary mathematics teachers may be weak in some geometry content knowledge that is specific to developing teaching practice. The National Education Commission (NEC) also suggested the need for research on DT integration for pre-service teacher education to improve secondary students' achievement (NEC, 2014).

Fourth, the local research (DoM, 2016) and international researchers (Tatto et al., 2011) emphasised the importance of upskilling the geometry component in mathematics teacher preparation (e.g., pre-service teacher education) courses. In the international literature, Tatto et al. (2011) argued that in a Teacher Education and Development Study in Mathematics (TEDS-M) of teacher preparation there were notable variations in teacher content knowledge in mathematics, with some domains, including geometry, having differences in between and within countries. However, the Ball et al. (2008) study indicated the bridge between teachers' knowledge and teaching practice is still inadequately understood. Some studies have discussed significant aspects affecting teaching and learning secondary geometry with DT (Baccaglini-Frank et al., 2009; Baltaci, 2018).

1.1.3 Significance of the study

NEC (2014) has suggested that the DT tool trend may open new avenues for pre-service teacher education. First, I consider how this new exposure to the MT app[1] has affected what pre-service teachers do with the MT available to them. Kearney et al. (2012) argued that young people behave and learn differently because of their continuous, pervasive exposure to modern MT.

[1] DT facilities available in TEIs vary, but mobile phones were available for all pre-service teachers who participated in the study. MT were not considered DTs in general.

Early researchers applied various labels to how these young, novice teachers behave, including “digital natives” (Prensky, 2001) and ‘the next generation’ (Tapscott, 1998). In addition, Kearney et al. (2012) noted that MT apps have offered numerous opportunities as well as challenges for education. The argument is that teachers can engage in a wide range of different mobile apps in mathematics with students, which may entail learning being formalised or informative ways, and even without a formal classroom (Faggiano & Mennuni, 2020; Maton, 2010). Therefore, by introducing a multi-approach for mathematics teaching and learning with MT, pre-service teachers may have the opportunity to understand different ways of acquiring mathematical knowledge in different pedagogical media in different learning environments (Drugova et al., 2021; Marriotti, 2006).

Second, Sri Lankan studies identified that secondary mathematics students have difficulties in geometry that are similar to secondary mathematics teachers (DoE, 2016). The World Bank (2011) emphasised that there may be problems in pre-service teachers’ beliefs concerning geometry teaching and learning. The debate has been on distinguishing knowledge from beliefs, a daunting task that mathematical education researchers are battling with (Sahin & Basgul, 2020). In the context of this research, pre-service teachers’ knowledge (Goos, 2005) and pedagogical beliefs have been considered regarding students’ mathematics achievement. Ertmer (2005) suggested that pedagogical beliefs act as an additional filter, specifically concerning mathematics teachers.

Third, another issue is that sparse mathematics teacher education research is available in the Sri Lankan context, and systematic research about pre-service mathematics teacher education is rarer. The Sri Lanka Association for the Advancement of Education (SLAAD) has shown that the former competency-based secondary curriculum has had many implementation problems in teacher education (SLAAD, 2010). Discussion around issues with the current mathematics education curriculum has been prevalent in much of the local literature (McCale, 2007; Perera, 2008). The World Bank (2011) informed relevant authorities about the problems in the competency-based mathematics curriculum. In 2008 national-level study for students’ mathematics achievement

(Grade 4 and Grade 8) were conducted by the NEREC[2]. Since then, no systematic studies have been published regarding mathematics education. Therefore, I feel this study is significant in the Sri Lankan context. It will be useful for my career, and it may add another viewpoint about the possibilities for mobile apps to provide support for mathematics teacher education.

1.2 Background

In the last decade, there has been a rapid development of mobile applications (apps) for mathematics, and their adoption for mathematics education has been widely discussed in the research literature (Calder, 2015; Carr, 2012; Larkin, 2016; Ng & Sinclair; 2015; Octal, 2017). Octal (2017) investigated mobile apps more concerning conceptual development than procedural or declarative knowledge in mathematics. Apps have many other possibilities to benefit mathematics education. For example, Calder and Murphy (2018) addressed the potential opportunities of app affordances that could facilitate the mathematics learning process. Crompton and Traxler (2015) focused on the potential areas of application and possibilities in the use of apps for students' mathematics achievement. However, Larkin and Milford (2017) indicated that teachers or students merely using the app did not necessarily improve the students' achievements.

Most of the empirical studies (Ertmer et al., 2012; Tondeur et al., 2016) explored the relationships between teachers' personal beliefs and their teaching practices. Evidence indicated a mismatch between teachers' beliefs and their actual practice with DT (Chen, 2008). However, a growing body of research has discussed different aspects of the use of digital tools for mathematics teaching and learning (Drijvers et al., 2008; Hillmayr et al., 2020). Drijvers et al. (2008) explained the benefits of didactical functions of DT in mathematics education and categorised them under (i) doing mathematics, (ii) practical skills to learn mathematics and, (iii) developing concepts to learn mathematics. Specifically, geometry conceptual developments have been discussed by several researchers (Kuzle, 2017; Olivero & Robutti, 2007) who indicated the possibility

[2] National Education Research and Evaluation Centre, University of Colombo

of using Dynamic Geometry Software (DGS[3]) for geometrical thinking in problem solving. In contrast, Gunbas, (2015) indicated that mathematics teaching and learning with DT tools was not an easy task. In particular, there is a lack of adequate strategies even from education policy to effectively integrate them into teaching and learning mathematics.

1.2.1 Teacher education institutes (TEIs)

Sri Lanka has a bureaucratic model for pre-service TEIs, according to the Education Act of 1986. This Act enables the MoE to administer[4] 19 TEIs to conduct initial teacher education courses leading to the award (by the NIE) of diplomas for 32 different courses. These are recognised as teaching qualifications in state schools. Sri Lankan state education is free from Grade 1 to tertiary education. The government supports TEIs to provide meals and accommodation for pre-service teachers. Therefore, the NIE and TEIs hold the principal academic power in all pre-service teacher education. These pre-service teachers have compulsory block-teaching sessions in the first two years' residence period at TEIs, and then in the third year, they are allocated an internship period at secondary schools.

There are two categories of secondary mathematics pre-service teachers in secondary schools: National Diploma in Teaching (NDT) holders (with initial teacher education qualifications from TEIs) and first-degree qualification holders who have obtained their qualifications from universities approved by the University Grant Commission (UGC). Sometimes, due to the needs of the education system, these graduates are absorbed[5] into the teacher education service. Programmes available for pre-service teacher education at TEIs coupled with the annual output of qualified mathematics teachers were still insufficient to meet the demand for qualified teachers (NEC, 2014).

Entry into pre-service institutes in Sri Lanka has age-range limitations: prospective students must be above 20 and below 26 years of age. As a result, all local pre-service institutes enrol young teachers (NEC, 2014). National

[3] Software provides the user with tools for creating Euclidean geometry (points, lines, etc..) through direct motion via a pointing device (mouse, touch pad), and to construct geometric relations among these objects.
[4] Act 30 of 1986.
[5] Circular No.95/07 absorbed Science/ Mathematics diploma holders from Peradeniya, Kelaniya, Colombo universities as trained teachers.

curriculum documents suggest that this cohort being relatively familiar with DT may open new avenues for pre-service teacher education. Administrative authorities argue that young people behave and learn differently because of their continuous, pervasive exposure to modern MT apps (Abeygunasekara, 2021). There is only one research study (Ekanayake & Wishart, 2014) on MT application for teacher education in Sri Lanka. In the research, they conducted case studies with science teachers use of MT in selected schools in the central province. However, in Sri Lanka, MT was not officially authorised pedagogical media in school education until the Covid-19 pandemic.

The international literature shows that this exposure affected novice teachers' use of available DT for teaching (Maton, 2010; Tondeur et al., 2016). The argument is that teachers can learn in a wide range of different DT/MT apps in educational institutions as separate subject modules. The challenge for teachers is to connect curriculum and technology in classroom practice (Niess, 2001). Kearney et al. (2012) discussed different pedagogical perspectives for learning with MT. In addition, they explained the benefit of MT apps is that users can generate learning contexts, with the learning occurring in different contexts and at different times and not confined to formal learning settings in educational institutions. Pre-service teachers experienced different perspectives in acquiring mathematical knowledge in different pedagogical media (Calder, 2011). Mariotti (2006) explained how teachers should make mathematical knowledge accessible to students. DT tools can also be used for such purposes as manipulatives. Patsiomitou (2018) explains that MT apps can be used as digital manipulatives, so it is necessary for teachers to recognise the affordances of apps to develop creative responses to the cultural and social needs of students.

Admission to TEIs to follow a “Pre-Service Professional Course in Teacher Education" is advertised each year through the MoE’s Gazette in Sri Lanka. The pre-service teachers are chosen based on the results of their GCE (A/L) examination and interviews and then trained for three years for a teaching diploma. The pre-service teacher education programme for mathematics is conducted in all three national languages (Sinhala, Tamil, and English) as the mathematics subject major and mathematics subject minor programmes. In these mathematics major and minor programmes, pre-service teachers receive instruction in their mother tongue or English (e.g., bilingual[6] teachers) but entry qualifications are different. In addition, the total number of students accepted into the courses depends on the number of applications received and the availability of mathematics education courses in each TEI's articulated teacher education policy.

1.2.2 Teacher education and curriculum policy

In Sri Lanka it was difficult to find official policy documents relevant to teacher education after the National Authority of Teacher Education (NATE[7]) was disestablished. NATE was a political agenda related to the establishment of the educational institutes and caused the withdrawal of power from state institutes (Alwattegama, 2020; Little, 2010). Pre-service teacher education was established through Act 5 of 1960 in Sri Lanka. This plays a crucial role in teacher education. According to Circular no. 1998/19 school curriculum developments, the in-service teacher education and academic components of pre-service teacher education have become the responsibility of NIE[8]. This study has considered only state responsible pre-service mathematics teacher education in TEIs[9]. The National Education Commission (NEC[10]), is responsible for educational policies. Apart from these state institutes, private universities also have teacher education programmes, but these are not covered in this study. According to the TEIs Act 26, pre-service mathematics teachers are mainly NDT diploma holders from the pre-service state TEIs in Sri Lanka, or (rarely) mathematics degree holders from universities. As this study is based on pre-service teacher education, the latter group of degree holders is not part of this research project.

The TE policy was changed after the reforms, and pre-service teachers were assigned to teach two specialised subjects (e.g., a mathematics major and science minor or vice versa). As a result, there are two categories of mathematics courses delivered from TEIs (mathematics major and

[6] Bilingual education was established under circular No.22 in Sri Lanka
[7] NATE was established under Act 32 of 1997
[8] Act No 28 of 1985
[9] Act No 30 of 1986
[10] Established under Sri Lankan Gazette No. 91.02.11

mathematics minor). Despite these (mathematics minor) teachers being recruited for secondary schools as mathematics and science teachers, they have the responsibility to teach mathematics for GCE (O/L) classes only from what they had learned up to GCE (O/L). It was a widely held belief that to get through GCE (O/L) mathematics, many students neglected the geometry component of secondary education. Many mathematics minor pre-service teachers (who had neglected geometry in their GCE (O/L)) have only three hours to learn the secondary geometry curriculum at TEIs. As a result, mathematics minor pre-service teachers must rely on peer support or students' textbooks and teacher guides (TGs).

Students' mathematics textbooks and TGs are highly structured and relevant to the national curriculum (intended) policy. Many curriculum studies have shown a mismatch between the intended curriculum and the implemented curriculum in Sri Lanka (Jayaweera, 2010; Perera, 2008). Liyanage (2011) indicated that mathematics teachers experienced competency-based reforms in 1998 without proper teacher training[11] on curriculum policy. In addition, Perera (2009) explained that even though competency-based reforms were not practical in the classroom, mathematics teachers are pressured to cover relevant lessons for evaluation in the given time. All state schools follow three-tier evaluation system for each year, which is targeted for national examinations (Athurupana, 2009). For example, two-dimensional figure problems need to cover all activities within a given time slot before provincial mathematics evaluations in the third term[12]. As a result of the three-layer evaluative process (school, zonal, provincial), many secondary teachers have followed the same teaching procedure (e.g., examination-dominated teaching for GCE (O/L)) using the same textbook, TGs and past GCE (O/L) examination papers. This has occurred without much consideration of varying levels of students' pre-knowledge or any other aspect. This might partly explain why secondary mathematics teachers' negative beliefs have about understanding geometry beyond a certain level as they mainly "rote learn" the subject (NEC, 2014; World Bank, 2011).

[11] Funding for teacher training workshops mainly from government projects (loans or donor assisted).
[12] Term has a similar meaning to semester.

International researchers (Abelson, 1979; Handal & Herrington, 2003; Pajares, 1992; Thurm & Barzel, 2020) have indicated that many other factors influence for the relationship between beliefs and knowledge of pre-service teachers. Handal and Herrington (2003) suggested that mathematics teachers' negative pedagogical beliefs may not support learners' understanding of mathematics content (e.g., geometric proof).

1.3 Mathematics education

Sri Lankan context mathematics education at TEIs focuses on training professionally qualified teachers only for primary and junior secondary schools. The National Diploma in Teaching (NDT) obtained from TEIs caters to the teacher requirements at the primary (Grades 1–5) and junior secondary (Grades 6–11) schools[13] in Sri Lanka. In the secondary school curriculum, mathematics is a compulsory education subject from Grade 1 (age 5 or 6 years) to Grade 11 (age 16 or 17 years). The teacher educator/pre-service teacher ratio varies from 5:1 to 12:1, although there are disparities in the distribution of teacher educators among different TEIs (NEC, 2014).

The NDT course has a two-stage evaluation system. The first stage is an institutional evaluation. At the end of the second year, the institutional evaluation is conducted by written examinations, teaching experience in block-teaching and some professional development. The evaluation also considers participation in co-curricular activities. The second stage at the end of the third year of the NDT considers the national evaluation along with the records of the internship period at school and the summative evaluation which have been carried out by the DoE. A third-year national examination is common for all TEIs in Sri Lanka.

The mathematics courses in local TEIs consist of an academic component, a professional component, and a general component with information communication technology (ICT). All these components are implemented over a two-year compulsory residence period at TEIs. The ICT module in the mathematics education curriculum has the potential to support mathematical

[13] Secondary school mathematics classrooms facilities are vary according to school category (e.g., national schools, 1AB, IC, type 2).

learning by designing tasks with available ICT facilities[14]. However, researchers have continually argued that general ICT modules at TEIs are not supportive of the needs of pre-service teachers (e.g., Edirisinghe, 2016; Liyanage, 2014). DT facilities in the TEIs classroom[15] remains under-used and teachers only rarely take advantage of the potential suggested by the TE policy (Liyanage, 2014). The academic component consists of the specialised subject modules offered by the TEIs. Pre-service teachers must complete two main components. The professional component includes 90 hours of educational psychology, educational sociology, educational guidance and counselling, educational measurement and evaluation, and school management. Despite great content in the curriculum for the development of teaching and block-teaching, local research studies have criticised subject-specified content knowledge in teacher education programmes (Liyanage, 2014; Perera,2008).

The international literature indicates the importance of novice teachers' competence in content knowledge (Herbst et al., 2017), pedagogical courses (Goos, 2005), and general cultural courses (Özden, 2005). It is critical in their pre-service teacher training programmes to enable them to carry out their teaching practice professionally. In the Sri Lankan context, pre-service mathematics teachers are highly dependent on secondary mathematics curriculum resources (e.g., mathematics textbooks and TGs) in their block-teaching practice.

1.3.1 Pre-service teachers' teaching practice

In Sri Lanka, TEIs are responsible for the block-teaching practice of secondary mathematics pre-service teachers. In local contexts, social disparities exist in TEIs, the urban TEIs (e.g., TEI1) have updated DT facilities with more social recognition in ICT-related teacher education options compared to the rural TEIs (e.g., TEI2), which have limited DT facilities and are focused more on traditional practices for teacher education. All professional development guidance is dependent on teacher educators (NEC, 2014). Compulsory pre-service teachers' practical teaching experience is evaluated in two ways. The first

[14] Some TEIs' ICT facilities were funded by doner-assisted (e.g., GIZ) or loan programmes (e.g., WB)
[15] Many TEI classrooms have a traditional lecture room environment (e.g., teacher, blackboard, students taking notes). However, only a few TEI are equipped with an interactive blackboard and multimedia projectors.

aspect starts with an action research project with a reflective journal during the block-teaching, including project assessment with or without a micro-teaching option (knowledge sharing workshops) at selected TEIs, before block-teaching starts at secondary school. The second aspect to consider is that some TEIs have related block-teaching experience sharing workshops with the guidance of some teacher educators. According to the negotiated agreements with secondary schools located close to TEIs, schools grant permission to assign groups of pre-service teachers for block-teaching with a mentor.

In addition, in TEIs' guidelines for block-teaching[16], some TEIs conducted workshops that targeted the use of tools (technology) in pre-service teachers' preparation for block-teaching or how to empower them with knowledge-sharing workshops. However, the ground-level reality (ICT facility) is different for each TEI due to social issues that are relevant to the context. For example, TEI1 had difficulty introducing GeoGebra as a pedagogical tool for geometry block-teaching. The reasons could have been because pre-service teachers had limited DT access and administration authorities had negative beliefs about the pedagogical benefits relevant to the use of DT. The national curriculum document has encouraged teachers' exposure to DT, creating new avenues for pre-service teachers' professional development at TEIs (NEC, 2014). It has suggested that DT/MT facilities be updated for pre-service teachers' professional development so that the technology is appropriate for the TEIs and relevant to 21st Century needs[17].

International literature indicates that pre-service teachers believe MT has the potential to create an enabling learning environment in formal or informal ways in their teaching. Different social disparities even exist within DT policies (such as cultural beliefs that smartphones are evil for students' education). Self-motivated professional development efforts are moving away from an emphasis on building pre-service teachers' isolated technical skills in DT to generic skills

[16] In the Sri Lankan context, block teaching is similar to the teacher practicum in the European context. Pre-service teachers going through block teaching were allocated a school timetable with a combination of major and minor subjects.

[17] 21St-century needs refer to being able to adopt to technological needs through learning skills, such as critical thinking, communication, collaboration, and problem solving through mathematics.

with their own MT devices in their pre-service teacher education courses. Yigit (2014) stated that the focus moved to developing pre-service teachers' skills in the context of designing and promoting the user to overcome their barriers. To be effective, professional development experiences must be linked to new visions for teaching and learning (made possible with MT), rather than focusing on developing user proficiency with specific software and hardware for teaching. Furthermore, Mayer (2005) described the role of instruction in the use of digital applications, using physical or digital manipulations relevant to the content to evoke challenges in knowledge and the learner's beliefs in relevant subject areas.

1.3.2 Pre-service teacher education challenges

After the curriculum reforms in mathematics education, local TEIs were allocated time for general teaching methods only, and the mathematics special component was dropped. As a result, some mathematics minor[18] pre-service teachers who had to drop geometry at their secondary school have now become "out-of-field" mathematics teachers when they are recruited to secondary schools for geometry teaching. This out-of-field teaching is referred to in the international literature (Ingersoll, 2002) as the practice of assigning teachers to teach a subject (geometry) without proper, subject-specialised training. Goos et al. (2020) discussed a similar situation among secondary mathematics teachers in Ireland. Researchers have suggested additional support mechanisms and benefits in blended learning options with DT/MT for upskilling these mathematics teachers (Burden & Hopkins, 2016; Goos et al., 2020). For example, Burden and Hopkins (2016) outlined the benefits of mobile technologies in teacher education, and the possibility of whether mathematical apps (Teixeira & Doetsch, 2020) can seamlessly support teacher education.

Pre-service teachers' knowledge of teaching subject matter (geometry) is important and differs from their general knowledge of mathematics (Ball & Bass, 2000). In addition, the possibility of pre-service teachers' knowledge about

[18] Pre-service teachers are trained in two subjects. Mathematics-major pre-service teachers' minor subject is science (and vice versa) that is with a science major the minor subject is mathematics. The block teaching is undertaken in schools under the guidance of a teacher mentor.

geometry teaching with MT[19] relevant to the Sri Lankan research context has been considered in this study. Goos et al. (2020) argued that there is a need for specific knowledge of geometry relevant to the context for teaching with apps. Ball et al. (2008) indicated the importance of geometry knowledge for teaching, stating that an understanding of geometry as well as how students learn geometry is required. Several studies (Usiskin, 1982; Yi et al., 2020) provided support for a framework of the van Hiele model in geometry understanding even though some research (Martin & Towers, 2016) did not accept sequence levels of thinking. Martin and Tower (2016) particularly focused on the notion of “folding back”, a dynamic way of thinking about the complexities of the teaching and learning of mathematics. Recognising the central role that mathematics teachers play in their students’ learning of mathematics, researchers (Chai et al., 2013; Fennema & Franke, 1992; Shulman, 1986) emphasise the importance of the pedagogical content knowledge (PCK) of mathematics education.

1.4 Conceptual framework and research questions

The interpretive paradigm is the philosophical and methodological underpinning that orients the current study. I have considered pre-service teachers’ beliefs in pedagogical practices with mobile applications associated with an interpretive epistemology as they are directed towards uncovering the meaning of pedagogy practices of pre-service teachers with technology. The study followed a mixed method research design in two phases. Several theoretical models/frameworks have been combined to address the research questions and to discuss why these models/frameworks are employed in my study. It started with the van Hiele model, which conceptualises pre-service teachers’ making sense of secondary students’ geometry learning trajectory and relevant pedagogical knowledge for geometry teaching (Clements & Battista, 1992). In the Technology Pedagogical Content Knowledge (TPACK) framework (Mistra & Koehler, 2006), pre-service teachers may have different levels of progressive development in TPACK when they experience the DT tool in block-teaching. Niess et al. (2005) suggested a five-level model to understand the progressive

[19] DT facilities available in TEIs vary, but mobile phones were available for all pre-service teachers who participated in the study. Teaching with MT was considered not DTs in general.

development of teachers' knowledge from PCK to TPACK. Lyublinskaya and Kaplon-Schilis (2022) discussed how teachers may have different levels of TPACK experiences, linking them more closely to different aspects of the teaching and learning process, based on the work of Neiss et al. (2005).

The challenges in teaching and learning geometry in social and cultural spheres in the research context, which are considered Vygotskian perspectives, include zonal proximal development (ZPD), and semiotic mediation (Vygotsky, 1978). Second, some pre-service teachers have indicated the possibility of using GeoGebra as the dynamic geometry environment (DGE) tool (e.g., dragging) which is relevant to the Vygotskian perspective of semiotic mediation (Falcade et al., 2007; Laborde et al., 2006; Mariotti et al., 2018). The methodical approach aligned with the socio-cultural perspectives surrounding the use of GeoGebra for geometry teaching and learning at block-teaching schools is considered relevant to the research question, which is discussed in the next section.

1.4.1 Statement of research

The purpose of this study was to explore different perspectives surrounding the use of MT apps by pre-service mathematics teachers with a focus on the pedagogical approaches used when teaching geometry to Grade 10 students in Sri Lanka. This statement includes three specific research context aspects. First, GeoGebra [20] is the only digital geometry environment (DGE) app authorised in curriculum documents for geometry teaching and learning in Sri Lanka. Second, according to the MoE document, pre-service secondary teachers are appointed to teach up to the junior secondary class level in secondary schools. Third, as the study focused on pedagogical approaches, pre-service teachers were selected after they had completed their block-teaching practices in junior secondary mathematics classes in state schools.

1.4.2 Research questions

As a teacher educator, I am interested in understanding the use of mobile applications in mathematics education and examining the factors that may

[20] GeoGebra was the only DGE app used for teaching and learning mathematics in this study. This is a free public domain dynamic geometry software is at www.geogebra.org

influence the pedagogical perspectives and beliefs of mathematics pre-service teachers. Therefore, the following main research question and supplementary questions were compiled.

The main research question is:

In what ways does the use of mobile technology apps (e.g., GeoGebra) influence Sri Lankan pre-service secondary mathematics teachers' perspectives and beliefs of their pedagogical practices on geometry?

Supplementary questions are:

RQ1 What aspects of using mobile technology apps (e.g., GeoGebra) might influence the geometry content knowledge for pre-service teachers involved in junior secondary mathematics education?

RQ2 In what ways might using mobile technology apps (e.g., GeoGebra) for teaching geometry in mathematics education influence pre-service teachers' pedagogy when teaching geometry?

RQ3 In what ways might using mobile technology apps for teaching geometry in mathematics education influence pre-service teachers' beliefs about teaching geometry?

RQ4 How might GeoGebra be used for teaching and learning geometry content in Grade 10 secondary mathematics?

A better understanding of the use of mobile applications in mathematics education and examining the aspects that may influence the pedagogy perspectives and beliefs of mathematics pre-service teachers is required while ensuring the investigation is realistic and relevant to the needs of the junior secondary classroom practitioners in the education system.

1.5 Thesis outline

The thesis consists of seven chapters. Chapter 1 gives an overview of the study, including the background and scope of the research problem, the research context, and the study's significance and purpose. Chapter 2 reviews the literature relating to mathematics teachers' knowledge, beliefs, and other relevant aspects to establish the theoretical foundation for the study. Chapter 3 describes the methodological considerations associated with the mixed method research approach, the research design, and the strategies of inquiries. It includes details about the methods and instruments used: the geometry test, survey questionnaire, semi-structured interviews, and case studies. It also includes detail of two phases of the study as well as data analysis and ethical approval processes. Chapters 4 and 5 analyse the results of pre-service secondary school mathematics teachers' content knowledge, pedagogical perspectives, and beliefs based on the geometry test, survey, documentary analysis, and interviews. Chapter 6 synthesises the impact of pre-service teachers' content knowledge and beliefs concerning their use of GeoGebra, based on the discussion of the findings of the relevant research literature. Finally, Chapter 7 provides a conclusion, summarises the major findings, and describes the contributions to the field, implications, and limitations of the study. It also gives recommendations for future research. A detailed description of the outcomes includes emerging challenges and indicates the inclusion of a metacognition element in the use of GeoGebra for geometry in certain appropriate instances. Finally, the study provides recommendations for policymakers to counter these challenges and it highlights some implications in the context of pre-service education in Sri Lanka.

Chapter 2

Literature Review

2.1 Introduction

The objective of this literature review is to scope the literature related to the use of digital technologies (DTs) for mathematics education relevant to the research questions. This chapter begins with a brief overview of how DTs are spreading into individuals' personal living-space experiences (Abdel-Aziz et al., 2016; Cavanaugh et al., 2014; Drugova et al., 2021; Kobylanski, 2019; Vizo et al., 2020). With the pervasiveness of digital devices in society, mobile devices (e.g., tablets and smartphones) have become popular for education. There is a range of research focused on students' use of mobile technology (MT) to support their learning with touch screen devices (Chai & Fan, 2016; Chorney & Sinclair, 2019; Larkin & Milford, 2018; Moyer-Packenham et al., 2016). In addition, researchers have discussed the possible dimensions (e.g., intellectual quality, supportive classroom environment, connectedness, and recognition of difference) through which apps can be selected in productive pedagogies. Research studies on gaming apps have indicated the possibility of supporting students' mathematics development (Calder, 2015; Carr, 2012; Fregola, 2015). Some have indicated the possibility of improving users' content knowledge, but also motivation, thinking, and creativity skills with apps (Calder & Taylor; 2010; Larkin, 2015; Kul, 2018). Numerous researchers (e.g., Jones, 2002; Masri et al., 2016; Uwurukundo et al., 2020) have examined interactive learning environments of dynamic geometry software (DGS) and affordances of apps for mathematics teacher education. The next section outlines the structure of the chapter.

The first section (2.1) covers the areas that surround the background context of the study. It provides policy relating to the teaching and learning of geometry in Sri Lanka. The second section (2.2) discusses relevant Vygotskian perspectives that consider the classroom as a social system operating within a larger communal system, that is controlled inside and outside by social influences. The third section (2.3) includes the van Hiele model and the pedagogical beliefs of mathematics pre-service teachers. The fourth section (2.4) covers the different approaches that mathematics teachers use to gain knowledge (researchers have used different approaches to understand teachers' content knowledge and pedagogical knowledge).

The fifth section (2.5) covers teaching mathematics with digital technologies. This happens in a social context where there are many interactions among teachers, learners, and digital technologies. Section 2.5 also covers the TPACK model and the use of digital technology for task design. The sixth section (2.6) reviews the affordances of DTs, and the seventh section (2.7) discusses the DGS in mathematics teaching and learning. The shift to mobile apps for mathematics from traditional learning methods, often with the aid of social elements, can happen seamlessly and subconsciously for pre-service teachers' professional development and the possibility of metacognition knowledge. Finally, the eighth section (2.8) briefly reflects on all sections and ends with a summary of the chapter.

2.1.1 Background of the current study

Sri Lanka has enjoyed a free education policy since 1947 and it achieved universal primary education by 1964 (Alwattegama, 2020). Reforms, policies, and the goals of education have varied with political leaders, and from regime to regime and are affected by social elements in the local education system (Little, 2010; Liyanage, 2014). Lack of proper teacher training, unplanned policy changes influenced by political leaders, and politicisation of recruiting procedures of teachers and administrators are common issues in curriculum reforms (Little, 2010).

Many reforms are based mainly on foreign loan agency criteria and the individual beliefs of policymakers or politicians rather than the country's labour market needs (Liyanage, 2014). For example, according to the reforms, mathematics is one of the core subjects to be offered to all students until the junior secondary levels (Grade 1–11) of school education. This compulsory nature of Sri Lankan secondary mathematics based on policy and social beliefs. This means the knowledge of mathematics consider is essential for 21st

Century needs[21] in Sri Lankan society rather than real employable mathematics skills need for students.

During the last four decades, views on learning have changed significantly to influence educational practice with digital technologies in secondary education in Sri Lanka. Even educational provision itself in Sri Lanka has been segregated by the language medium of instruction (e.g., Sinhalese, Tamil) since independence from the Britain in 1948 (Little, 2010). Moreover, post-independence reforms in secondary education were followed by learning theories of behaviourism[22] until 1997. Competency-based curriculum reforms were introduced with a constructivist[23] approach (Gunawardena, 2010; World Bank, 2013). These studies clamed, Sri Lanka has achieved a high level of literacy, but it has been unable to provide future employable learning needs for students or meet labour market needs even after many educational reforms.

Mathematics education in Sri Lanka has been influenced by competency-based curriculum reforms and DT policies (Gunawardena, 2010; Little, 2010). The competency-based teacher education curriculum included teacher competencies with the KSP (Knowledge, Skill, Practice) framework in 2007 (Nawastheen, 2021). Researchers have discussed the KSP model which has been based on a constructivist reality that represents one's knowledge of what exists, obtained through knowledge, personal skills and subjective practical experiences (Edirisinghe, 2007; Ginige, 2007). For example, Ginige (2007) described how the constructivism based educational reforms can reduce socio-cultural disparities[24] in minority culture groups. The KSP model suggests that learners can actively construct their knowledge in a learning process in familiar learning environment by linking new information acquired through prior knowledge, beliefs, and experiences in the learning environment (Ginige, 2008; Gunawardena, 2010). However, several studies have shown that the

[21] 21[St]-century needs refer to being able to adopt to technological needs through learning skills, such as critical thinking, communication, collaboration, and problem solving through mathematics.

[22] Behaviourism is underpinned by a focus on the concept that students can learn through reactions to their behaviour or by observing the behaviour of others.

[23] Constructivism is centred on the idea that students create knowledge through learning experiences.

[24] This KSP approach created more opportunity for deprived communities to access teacher education. For example, a TEI was established in a tea plantation area. Properties had given for pre-service teachers' who were coming from tea plantation workers, even with lower entrance criteria.

competency-based teacher education reforms were not successfully implemented due to the ground-level effects of socio-cultural disparities (Nawastheen, 2021; NEC, 2014).

State school teachers believe that there are many disparities between urban and rural educational institutes in reform implementation because of socio-cultural needs (e.g., accessibility) and mismanaged (e.g., politician influence) infrastructure mechanisms in the Sri Lankan education system (Liyanage, 2014). These claims often match several international studies about teachers' beliefs and the effect of the social elements in domain of learning (Albion & Ertmer, 2002; Borba, 2021; Goos, 2005; Hegedus et al., 2020). Moreover, these researchers indicated that social and cultural elements influence education reforms but were less concerned about political elements. International researchers have frequently discussed a mismatch between the intended curriculum reforms, the attained curriculum, and the implemented curriculum (Borba, 2021; Cuban, 1993). The intended curriculum means what is prescribed in curriculum reforms by policymakers, and policies (Becker et al., 1994; Bokhove et al., 2019).

Sri Lankan researchers have indicated the benefits and challenges of teacher education reforms (e.g., mathematics reform) and the use of DT tools in TEIs classrooms[25] (Edirisinghe, 2016; Liyanage, 2014). In contrast, school teachers believed that they required more classroom time, for instructional design, planning and implementation of lessons which was not practically possible in the rigid education structure and within socio-cultural beliefs[26] relevant to mathematics education (NEC, 2014; Wadanambi & Leung, 2019). International studies have indicated that teachers may have personal beliefs and relevant geometry knowledge to manage their classrooms in meaningful ways with DT tools (Albion & Ertmer, 2002; Belbase, 2015; Gumiero & Pazuch, 2021; Xie & Cai, 2020). Sometimes researchers indicated school teachers faced multiple

[25]Many TEI classrooms have a traditional lecture room environment (e.g., teacher, blackboard, students taking notes). However, only a few TEI were equipped with an interactive blackboard and multimedia projectors. These TEIs' ICT facilities were funded by doner-assisted loan programmes (WB) or international funding (GIZ).

[26]Socio-cultural beliefs relevant to mathematics education imply the way Sri Lankan society accept mathematics proficiency as a norm of intelligence. They also encourage the younger generation to admire indigenous mathematical culture in temples, irrigation sites, and many more places.

barriers integrating DT, such as institutional aspects[27] (Brush et al., 2008; Ertmer et al., 2005), cultural and social needs of the community (Ansari & Khan, 2020; Somekh, 2008). The possibilities in the use of apps for mathematic teaching in TEIs and responses to cultural and social needs have become more visible during the COVID-19 pandemic[28]. Thus, teachers' knowledge, socio-cultural beliefs, and secondary education policies can influence the use of apps in mathematics education.

2.1.2 Secondary education policy in Sri Lanka

This research investigated pre-service mathematics teachers' perspectives about the use of apps (e.g., GeoGebra) linked to the secondary education policy in secondary schools in Sri Lanka. Therefore, it is important to understand the educational policy in Sri Lanka which has been dominated by culture and religion since the colonial era Education Ordinance[29]. Later, after independence from Britain in 1948, education was segregated according to the language (Sinhalese, Tamil, and English) of instruction (Alawattegama, 2020; Little, 2010). In 1947, Sri Lanka initiated a free, no discrimination, secondary education policy for the public (Liyanage, 2014). In practice, education is free of charge, including free school uniforms and free students' textbooks up to the secondary school level. In 2001, bilingual education was introduced in Sri Lanka. In this bilingual education, English is the medium of instruction in some subjects[30] (including mathematics) from Grade 6 to Grade 11 in selected state schools.

In Sri Lanka, the school education system includes 13 years: the first 5 years (age 6 to 10 years) are compulsory primary education and Grade 6 to Grade 13 is secondary education. Secondary education (age 11 to 19 years) ends in Grade 13 with GCE (A/L) national examinations (Lekamge et al., 2008). Secondary students experience a strong private tuition-oriented culture. Indeed, many parents are expected to pay for private "cram" (supplementary) tuition classes from the level of Grade 1 students (Liyanage, 2014; NEC, 2009).

[27] Institutional aspects mean policies, and practices relevant to the institute.
[28] Online learning at TEIs during the Covid-19 pandemic meant there was a positive atmosphere in the society to understand the benefit in mobile apps for teaching and learning.
[29] 1939 Education Ordinance 31
[30] For example, subjects like Science, Mathematics, Information technology, Western music, Geography

Several researchers have discussed Sri Lanka's curriculum content load and rigid curriculum structure approach to secondary education despite several curriculum reforms since 1950 (Jayaweera, 2010; McCaul, 2007). These authors have indicated that even after a series of curriculum reforms that massive curriculum content has not reduced.

Sri Lankan schools still follow an education ordinance[31] that was introduced in the pre-independence era (Jayaweera, 2010). Necessary amendments to the policy were implemented via MoE circulars (NEC, 2014). This has been criticised for not improving learning progressively nor developing to cope with 21st Century world needs (Alawattegama, 2020; Jayaweera, 2010). In 1998, a competency-based curriculum was introduced for teacher education including integration with DT. It was based on the constructivism approach that gave more opportunities to minority cultural groups and addressed social gaps within the society (Ginige, 2007). As a result, competency-based teacher education that started in 1998 failed due to examination-oriented individual needs (Little, 2010; Nawastheen, 2021). These researchers not only discussed the dialectic between the individual and society but also radical changes in DT and political influence that have occurred during education reforms in Sri Lanka. In addition, researchers have discussed disparities in local fee education policy as well as reforms in the implementation process (Alawattegama, 2020; Little et al., 2011). For example, even though the 2007 reforms were a failure for the competency-based approach, some curriculum materials (e.g., Teacher Guides [TG]) still followed the competency structure. The MoE and NIE have the responsibility for providing free curriculum material including TG and students' textbooks up to junior secondary grades (NEC, 2014) which are legalised as government acts.

A "mathematics-for-all" education policy came into effect after educational disparities among children in 1971 led to youth unrest (Liyanage, 2014). Authorities such as the presidential task force, believed that unemployment was the main driving force behind the youth unrest. Educators believed mathematics-for-all would lead to equal opportunities for employment for all

[31] Education Ordinance No 31 of 1939

secondary students via these new educational reforms. As a result, the mathematics-for-all mantra was introduced during the 1972 reforms (Jayaweera, 2010; Perera, 2009). However, the 1972 reforms were considered an utter failure as less consideration was given to the socio-cultural needs of the people by the next political party that came into power three years later.

The mathematics-for-all mantra has remained in secondary education for the last four decades, even though the 1972 reforms were unsuccessful or ineffective (Liyanage, 2014). As a result, junior secondary students have been forced to spend more than 12% of their compulsory education time learning mathematics. Education authorities continually demand that mathematics qualifications (passes) be an essential requirement for secondary students (Jayaweera, 2010). NEC (2014) claim that the inadequacy of subject-specific (e.g., mathematics) teacher training, infrastructure and lack of curriculum resources were common problem in many TEIs, even the teacher training institutes advocated through Acts to support the educational ordinance. For example, the circular no. 2009/11 amendment allowed for free textbooks for all secondary students. As a result, all type of secondary schools has equal opportunity to access free curriculum resources in mathematics.

2.1.3 Secondary school geometry curriculum resources

The secondary school mathematics curriculum is documented in the mathematics TGs and mathematics textbooks for students from Grade 6 to Grade 11. Secondary mathematics textbooks prepared by the NIE/MoE and distributed all island through the MoE.

Secondary mathematics curriculum materials played an important role in pre-service teacher education (e.g., block-teaching practice) in Sri Lanka. Figure 2.1 shows the subject-specific time allocation for geometry content comparison in the junior secondary mathematics curriculum content by Grades 6 to 11. Time allocation for each geometry lesson per Grades was calculated from secondary mathematics textbooks.

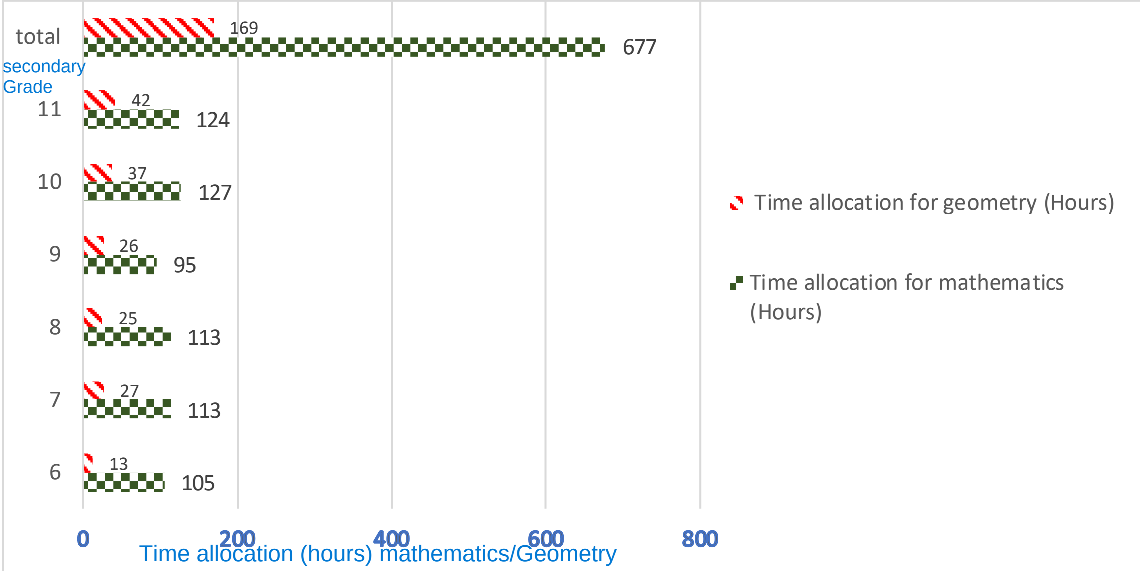


Figure 2.1. Time allocation (hours) for geometry content vs Grades

The bar graph in Figure 2.1 displays time allocation for geometry in secondary grades in Sri Lanka. It is difficult to find a proper pattern for cognitive aspects of the time allocation geometry content from Grade 6 to Grade 11 in secondary schools. However, the secondary school mathematics curriculum has more hours for geometry content (167 hours out of 677 hours) in secondary schools compared to the mathematics teacher education curriculum which allows only three hours for understanding the secondary mathematics curriculum. Figure 2.1 shows that (when pre-service teachers have been employed as) mathematics teachers, they must spend one fourth (25%) of their teaching time for geometry teaching. For pre-service teachers, textbooks are an authoritative source of content for national examinations and an important working tool for consolidating subject matter knowledge through activities.

The secondary mathematics curriculum up to GCE (O/L) officially takes a spiral (vertical curriculum) approach in the design of the geometry content across the grades (Mampitiya, 2014). This mismatch (e.g., official curriculum and actual curriculum) is practically not visible (NEC, 2014). Mampitiya (2014) explained geometry content relevant to Euclid elements that have been prioritised in the secondary curriculum and local students' textbooks, but less emphasis has been given to supporting geometric thinking. This is similar to Vincent and Stacey (2008) who analysed a sample of nine Grade 8 Australian textbooks and identified an overall absence of proof problems requiring deductive thinking. In addition, Nordstroem and Loefwall (2006) claimed that the notion of proof was not defined in a meaningful and appropriate way in two Swedish mathematics textbooks for students aged 16-18 years.

Scholars recognised the influence of curriculum material on classroom teaching, socio-cultural practices, and examined the use of mathematics textbooks (e.g., Ding & Li, 2014; Hanna & de Bruyn, 1999; McCale, 2007; Mouzakitis, 2006; Nordstroem & Loefwall, 2006). Hanna and de Bruyn (1999) who discussed a sample of Canadian textbooks for secondary students, identified that only the topic of geometry lesson guide to learn about proof but the content was controversial. Similarly, Mouzakitis (2006) indicated that mathematics textbooks influenced the curriculum knowledge of teachers and secondary students. He engaged in comparative content analysis of Euclidean geometry textbooks: an Italian textbook "*lycei*" and the official textbook in the Greek "*lyceum*". They included common sources of curricular content, a paradigmatic exposition of an organised body of knowledge and a suggestion of pedagogical style. Ding and Li (2014) found that Chinese textbooks had different representations for present tasks purposely designed for the transition from concrete representations (e.g., related to artefacts or word problems) to more abstract ones. Maffia and Mariotti (2020) concluded that the introduction of mathematical content through manipulatives and other artefacts is a widespread practice in curriculum material.

In Sri Lanka, another critical issue is the scarcity of relevant mathematics education or curriculum material analysis research over nearly the last decade, and systematic research about mathematics teachers is rarer still. Sri Lanka Association for the Advancement of Education (SLAAD) claimed that the existing secondary curriculum has many implementation problems at the classroom level (NIE, 2005; SLAAD, 2010). McCale (2007) argued the possible disparities between teacher education and adoption of the school mathematics curricula with socio-cultural needs[32]. Wadanambi and Leung (2019) discussed Sri Lankan pre-service mathematics teachers' professional beliefs, which

[32] In the context of this study, Socio-cultural needs imply the elements of what the society deems the students should learn to become an esteemed citizen.

encouraged them to adopt flexible practices under the influence of social and contextual demands in the context. To examine the relevance of these arguments (e.g., flexible practices, adoption to the socio-cultural needs) to current research, the next section will review the socio-cultural perspectives of Vygotsky's work.

2.2 Vygotskian perspectives relevant to the study

Vygotsky (1978) viewed educational practice and learning as a process of participating in a socio-cultural activity in which knowledge is constructed in a joint activity between social and cultural contexts. The fundamental concepts of Vygotsky's socio-cultural perspectives provide an overarching framework to encapsulate the social aspects of students' mathematical learning as a norm of intelligence[33].

Next, the aspects of students' learning in mathematics and visual thinking are scrutinised using Sfard's (1998) theorising drawn from Vygotsky's concepts of internalisation and the semiotic mediation. Sfard (1998) explained that Vygotsky's original work was based on child-parent interactions. However, Cazden (1979) indicated applicability for classroom student-teacher interactions. He also considered teacher-student interactions as mathematics teachers function in a socio-cultural system that has evolved.

The Vygotskian theory considers all higher human mental functions as products of mediated activity. The role of the socio-cultural environment in learning, as discussed by Voigt (1994), refers to Vygotsky's view of the environment. Specifically, the individual internalises mathematical knowledge, which can be influenced by cultural practices. Martin and Peim (2009) claimed that one's environment and cultural practices directly influence for the mathematics learning which elaborated through activity theory. Activity theory is rooted in Vygotsky's (1978) work on society's role in learning. Futher, it seeks to explain why humans develop the way they do (Engeström, 1999; Nardi, 1996); with, objects and mediated tools. Vygotsky (1978) first conceived activity as a mediated action with subject, object (goal), and artifact (Engeström, 1999). For

[33] Sri Lankan society considers knowledge of mathematics as a norm of intelligence that enhances employability. Indigenous cultural aspects (e.g., ancient temples, irrigation system) may influence mathematical knowledgeable of people in society.

an activity setting to exist, the subjects' actions must be goal-oriented and involve a set of practices and artifacts that mediate action toward the goal. Lankford (2021) claims that activity theory considers the "how" mathematics is learned, as well as how it is developed in settings through mediating tools that can transfer to other activity settings. However, activity theory alone may not provide a complementary picture of knowledge development for teaching with apps.

Vygotsky defined examples of psychological tools from mathematics as well as systems for counting. Berger (2005) argued that Vygotsky's notion implied all knowledge was semiotically mediated and necessary for understanding how teachers use mathematical signs to gain access to mathematical objects. The process of internalisation, as described by Vygotsky (1978), may transform tools into psychological tools. The internally oriented psychological tool shapes new meanings, and in this sense, a tool may function as a semiotic mediator. This theoretical perspective suggests that learning is affected and is modified by the tools used for learning (Maffia & Mariotti, 2020). Yilmaz (2018) explained reciprocally that the learning tools are modified by the ways that they are used for learning. In addition, Vygotsky's notion of the zone of proximal development (ZPD) underlines the social nature of mathematics learning. Klang et al. (2021) explained that the instruction in mathematical problem-solving can improve the student's "potential" level of understanding problem solving. For example, ZPD is the space where a student cannot solve a geometry problem alone, but where it can be done with the collaborative assistance of a teacher or a more competent peer.

2.2.1 Zone of proximal development (ZPD)

The ZPD is a concept relevant to mathematics problem solving that considers the gap between two concepts: actual development of mathematics problem solving and potential development of mathematics problem solving (Fosnot, 1996). Zolkower and Shreyar (2007) explained the ZPD as:

> ..'the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers' (p.86)

The gap between present and expected ability is not the only factor influencing teacher development and socialisation. Therefore, the Vygotskian notion of ZPD may help us understand the different interactions within a pre-service teachers' learning environment influenced by mobile apps. Researchers used the notion of ZPD to investigate relationships between novice and experienced teachers using technology as a psychological tool (e.g., Goos, 2005; Niess & Wiles, 2016; Zolkower & Shreyar, 2007). Pre-service teachers' emerging skills with technology can develop under the guidance of more experienced peers or teacher educators. The above-mentioned trends in the current study suggest a socio-cultural perspective as opposed to other learning theories.

In addition, Zolkower and Shreyar (2007) presented a Vygotsky-inspired analysis of how an Argentinean teacher mediated a group discussion for about 22 sixth-grade students' mathematics achievements in a bilingual (Spanish/ English) school. Conversely, Tall et al. (2006) focused on "thinkable concepts" related to objects in a digital learning environment, exploring and describing their properties, describing them, inferring certain properties implying others, and explaining coherent frameworks such as Euclidean geometry. Moreover, they explained objects and their properties, classified them, and (similar to a van Hiele type level) built from a primitive perception to more refined conceptions and descriptions.

Signs and tools are assumed by Vygotsky (1978) and the semiotic mediation of language provides a historically situated, socio-cultural version of the process of understanding. Both the technical and psychological tools are an integral part of the socio-cultural elements. Mildenhall and Sherriff (2019), explained the science of instruction for a task is in the symbolic or verbal form in any digital application that can be considered as a tool. In the next sections, the pre-service geometry teaching of GeoGebra is examined through the lens of semiotic mediation.

2.2.2 Semiotic mediation

The socio-cultural theory emphasises that higher mental functions indications of moving from concrete to abstract thinking, are not only mediated through tools (e.g., concepts, language, artefacts), but also through all learning. Through mediation and as learning increases, students master the use of tools and begin to internalise social practices (Vygotsky, 1978). Bartolini_Bussi and Mariotti, (2008) explained that an artifact is a tool of semiotic mediation when the teacher uses it intentionally to mediate mathematical content to students. However, Hasan (2002) argued there was a difference between mediation and semiotic mediation. Ghassemzadeh (2005) claim that mediation is the mechanism in which external, socio-cultural activities that are transformed into internal, mental functions. Maffia and Mariotti (2020) argued that mediation is a common term used in DT during mathematics teaching and learning and they discussed that in semiotic mediation how the teacher acts as a mediator using the artifact to mediate geometrical content to the secondary students. Zolkower and Shreyar (2007) explored how a teacher mediated a group discussion for bilingual secondary students' mathematics achievements, especially in the case of the artifact and signs. The current study focused on a pedagogical perspective regarding the use of GeoGebra (a DGE tool) from a socio-cultural perspective in the pre-service block-teaching context and considering DGE as a mediation tool that may be more appropriate.

Margot (2005) argued the role the mediator played as a psychological tool or sign, such as words, graphs, algebra symbols and digital or physical tools. Further, she elaborated on how forms of mediation, which are products of the socio-cultural context, do not just facilitate activity; rather, they define and shape the inner processes of concepts. Earnest (2021) claimed that it is necessary for the teacher to organise a learning environment accurately to enhance students' cognitive development through discussion, interaction, and other social elements. Earnest (2021) led to the development of an instrumental approach integrating the affordances of the instrumental perspective developed in the cognitive ergonomic (Vérillon & Rabardel, 1995), and the didactic perspective (Drijvers et al., 2010). These authors argued that the cognitive approach and the anthropological approach have different views on schemes and techniques.

Drijvers et al. (2010) considered a scheme as a less stable way to deal with specific situations or tasks and discussed a scheme as a part of an instrument. It is an instrumentation scheme and utilisation scheme related to the artefact. Artigue & Trouche (2021) explained that these are building blocks for more integrated schemes of an instrumental genesis approach.

2.2.3 Instrumental genesis relevant to DGS

The instrumental genesis approach is an instrumental perspective which clarifies the limits of a constructivist approach. The starting point of the instrumental approach has a distinction between artefact and instrument (Rabardel & Bourmaud, 2002). The artefact is very often (but not necessarily) a physical object used as a tool (Drijvers et al., 2010). For example, DGS is an artefact when the software is considered as one single artefact, or it can be seen as a collection of artefacts: construction artefact, measurement artefact, and dragging artefact (Mariotti, 2000). Drijvers (2002) discusses it as a historic view:

> As historic and epistemological analysis confirms, the development of mathematical knowledge is based on a productive dialectics between theory and practice. A key element of this dialectic relationship between theory and practice is represented by artefacts. (p. 13)

Drijvers (2002) further explained it using an example of Euclid's geometry artefacts, such as how the ruler and compass play a special role. When experts use an artefact, it becomes an instrument. The process of an artefact (instrument = artefact + schemes) becoming part of an instrument is in the hands of users (e.g., pre-service teachers). In this view they are termed instrumental genesis (Drijvers et. al., 2010; Kieren & Drijvers, 2006). The instrument also involves the techniques and mental schemes that the user develops and applies while using the artefact. In this view, instrumental genesis develops schemes and techniques (Mariotti, 2006). Mariotti (2006) argued, DGS (e.g., GeoGebra) does not itself serve as a tool for teachers. It becomes a tool, referred to as an instrument, only when the pre-service teacher forms one or more mental utilisation schemes. This interaction promotes an understanding of the factors affecting the integration of the mobile application into the mathematics classroom.

The above theoretical frameworks can be applied to analyse the pre-service teacher's role in the use of GeoGebra for a geometry task with DGE features by looking through alternative instrumental genesis lenses. The theory leads from the analysis of interactions between geometrical tasks and instrumented techniques into pedagogy. When an artefact is used to accomplish a task, drawing a diagram in a DGE may evoke the classic notion of geometric construction by ruler and compass. However, if the pre-service teacher is not an expert, the meanings emerging from the use of the artefact may not be immediately and consciously related to geometrical meanings. Instead, they are related to the specific context and the specific individual, and they have 'personal' meaning. These technical and conceptual aspects are user intertwined and co-develop. Moreover, they can be defined as a sequence of interactions between the user and the artefact leading to a goal (Drijvers et al., 2010). Users see a technique as the observable counterpart of the invisible mental scheme.

Instrumental approach task design can be defined as a "utilisation scheme" of an artefact (Laborde, 2006). A scheme is organising the activity with an artefact to realise a given task. In this view, it can be argued that "usage schemes" correspond to the management of the artefact, and "instrumented action schemes" are directed towards the realisation of the task (Pastsiomitou, 2019). These schemes can result from personal construction but also the appropriation of socially pre-existing schemes. However, Drijvers et al. (2010) argued that "instrumentation theory cannot be the complete solution to everything" (p.113) therefore different perspectives need to be considered as alternatives.

It is important to consider institutional conditions that enhance instrumental genesis in the epistemological perspectives (Patsiomitou, 2019). The term instrumental genesis denotes the process by which the artefact becomes an instrument. Examples of schemes of instrumented action include the dragging tool in a DGE, as described by Leung et al. (2008). In line with Laborde et al. (2006), a key feature of a DGE is its ability to visually represent geometrical invariants within simultaneous variations induced by dragging activities. The formation of utilisation schemes and the building of instrumented action schemes proceed through geometry teaching activities and thus have a two-sided relationship. Drijvers et al. (2010) discussed that the relationship between tool and learner can be considered a process in which the tool shapes the thinking of the learner.

Digital mathematics tools, geometry materials integrating interactive diagrams, interactive visual examples and visual demonstrations are important areas of research in mathematics education. Semiotics enable us to understand the challenges that are driven by these materials (Patsiomitou, 2019). Numerous researchers (e.g., Artigue, 2013; Patsiomitou, 2019) are interested in the instrumental genesis of teachers' learning trajectories for mathematics teaching. Mariotti et al. (2003) focused on the analysis of attributes of DGS for geometry teaching. When instruments of semiotic mediation are considered with DGS, teachers can introduce and conceptualise geometrical ideas more easily in their geometry teaching.

2.3 Teaching geometry

To analyse the quality of geometry teaching, we need to understand the geometry content, teachers' geometry knowledge, and how students learn geometry (Battista, 1999; Niyukuri et al., 2020), along with other influencing factors in the classroom (Cuoco et al.,1996). Geometry, ranging from the historical era of Euclid to today, has the basic characteristic of making an abstraction close to reality. Moreover, geometry content in the Sri Lankan school curriculum commonly includes elements of Euclid geometry. Examination-oriented mathematics curricula have included geometrical problem-based learning options for secondary school students. Students' learning has been linked to both the teacher and the geometry curriculum (Bokhove et al., 2019) and many other aspects.

There are different arguments about teaching geometry in the secondary curriculum. Learning geometry itself may assist many other units (e.g., algebra) of mathematics (Groth, 2012; Gumiero & Pazuch, 2021). For example, Euclidean geometry is used to talk about an algebraic object such as coordinates in a plane, and it can be used for geometric transformation in DGE. Another unit is geometrical proof in national examinations, in which students have more difficulties. Bokhove et al., (2019) explained that these are probably the most conspicuous of difficulties, even for geometry teachers, because geometric proof is one of the most sophisticated topics in secondary school mathematics.

The geometry thinking of pre-service teachers can be examined from two different perspectives: learning geometry and teaching geometry (Jones & Tzekaki, 2016). During the mathematics teaching process, the teacher is responsible for support for students' mathematics learning. Leikin (2019) argued that understanding mathematics concepts depends on the student's potential. For example, drawing a diagram for a geometrical problem represents the cognitive problem-solving potential of the user. Although this cognitive behaviour is different for each student's involvement in the task, the time taken for the task may not only depend on paper-and-pencil or any other tool GeoGebra used for the task. The general argument is that geometry block-teaching experiences for some geometry lessons (e.g., geometrical proof) frequently occur during short periods and may not be a successful teaching and learning experience for pre-service teachers.

Even pre-service teachers seem to be uninterested or less confident in teaching some specific topics in mathematics (e.g., geometrical proof). These teachers may know definitions, assumptions, and theorems but do not know how to teach the relevant topic. Jones and Tzekaki (2016) indicated that teaching some geometry topics (geometrical proof) by knowing about definitions, assumptions, and theorems is not sufficient. Teachers need to have specific mathematics knowledge. Researchers interpret this specific mathematics knowledge in different ways (Ball & Bass, 2000; Natalie, 2004; Pino-Fan et al., 2015; Shulman, 1986; Tatto & Senk, 2011). Pino-Fan et al. (2015) defined this as Didactic Mathematical Knowledge (DMK), which refers to specialized knowledge of mathematics teaching: the knowledge that mathematics teachers need to have on specific topics to be taught in specific grades.

Scholars have focused on the role of instruction in teaching geometry and instruction for helping students move from one level to the next in the van Hiele model (Crowley, 1987; Roubert, 2018; van Hiele, 1984). In the last six decades, numerous research studies have included the van Hiele model in their research to analyse geometry teaching and learning (Alex & Mammen, 2016; Mbusi & Luneta, 2021; Usiskin,1982, van Hiele, 1986), this will be discussed in the next section.

2.3.1 van Hiele model for geometry teaching and learning

Dutch mathematics educators Dina and Pierre van Hiele of the University of Utrecht introduced this model (Alex & Mammen, 2016) and investigated why students were having difficulties in learning geometry (Mbusi & Luneta, 2021).

The National Council of Teachers of Mathematics (NCTM) in the USA and several other countries (e.g., Idris, 2009; Mbusi & Luneta, 2021) followed the van Hiele model for teaching geometry.

The van Hiele model has four levels: The first level starts with “visualisation”, where students individually recognise and identify geometric figures according to their appearance, but they do not perceive the properties or rules of figures (Idris, 2009; Mayberry, 1983). The difference between pre-recognition and visualisation is that the students at level-one (visualisation) can name a figure, for instance “this is a square, this is a rectangle, or this is a parallelogram”, based on the appearance of the figure.

Level two is “analysis”. Students can analyse figures in terms of their components and relationships among components (Mbusi & Luneta, 2021; Usiskin, 1982). They perceive properties or rules of a class of shapes empirically, but properties or rules are perceived as isolated and unrelated. Students can also identify and name geometric figures by knowing their properties. Although the students at this level can acknowledge various relationships among the figure components, they do not perceive any relationship between them (e.g., squares and rectangles or rectangles and parallelograms).

In this model, level three is “order”. The properties of the figures are logically ordered. Students begin to see relationships and order among definitions for geometric shapes (Usiskin, 1982; Yi et al., 2020). Yi et al. (2020) described that those students can logically order and interrelate previously discovered properties and rules by giving informal arguments. Some researchers defined this level as informal deduction (Groth, 2011; Yi et al., 2020). Logical implications and class inclusions are understood and recognised. At this level, the students can see the relationships among the geometric figures. For example, they can easily say that a square is also a rectangle, and a rectangle is also a parallelogram. This simple deduction is possible but still difficult to prove.

Attainment of level four is “deduction”. At this level, students can analyse relationships of systems between figures and students can define axioms and theorems. They can prove theorems deductively, construct proofs, and they can also understand the role of axioms and definitions. In other words, proof can be written with an understanding of the problem. (Fuys et al.,1988; Usiskin, 1982). Level five is “rigour”, where students can analyse various deductive systems like establishing theorems in different axiomatic systems and the ability to make the abstract deduction (Mayberry, 1983; Mbusi & Luneta, 2021). In this last level, non-Euclidean geometry can be understood.

Properties of the above five levels of geometric thinking are sequential and hierarchical, in which students achieve lower levels before advancing to the higher levels (Mbusi, & Luneta, 2021). Each level has its language, set of symbols, and network of relations, so that students at a lower level cannot understand the teacher who reasons at a higher level (Usiskin,1982; Yi et al., 2020). As students go through levels, what is implicit at a lower level becomes explicit at the next level. Researchers have raised concerns about the numbering of each level and the linearity of the model (Fuys et al., 1988; Yi et al., 2020). Mbusi and Luneta, (2021) indicated that most secondary students only reach the first or second van Hiele level. Further, they said that secondary students’ progress from the second level to the fourth level is very slow and it takes several years for students to reach level four from level two.

Progression through the model is dependent on a teacher’s instructional activities and students’ understanding of geometry (Bleeker et al., 2013; Clements & Battista, 1992; Yi et al., 2020), The students’ levels and teacher phases are summarised in Table 2.1. In this table, a summary for each level and phase is given along with a discussion of the student’s involvement and teachers’ instructional strategies (Bleeker et. al., 2013; Fuys et al., 1988; Mason, 1988). Table 2.1 presents a summary of the instructional role relevant to the teacher during each phase of teaching geometry.

Table 2.1
van Hiele's model elaboration

*van Hiele's model **student** levels* *(Mason, 1998)*	*van Hiele proposed five phases of geometry learning experiences (Bleeker et al., 2013).* ***Mathematics teacher's role****.*
• **Level, 1 I (Visualisation)** Recognise and identify geometric figures according to their appearance, but they do not perceive the properties or rules of figures. • **Level-II (Analysis**) See figures as collections of properties. They can recognise and name properties of geometric figures, but they do not see relationships between these properties. • **Level 3 (Order):** Perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. • **Level 4 (Deduction):** Are able to construct proofs. . • **Level 5 (Rigour):** Can understand the formal aspects of deduction, such as establishing and comparing mathematical systems. Use of indirect proof and proof by contrapositive, as well as non-Euclidean systems.	• **Phase 1 (Inquiry/information):** Gains insight into the student's prior knowledge about the lesson and gets an idea of the direction. • **Phase 2 (Directed/guided orientation):** Uses a variety of carefully sequenced short tasks to help the students explore the structures characteristic of the level and to elicit specific responses. . • **Phase 3 (Explication/new idea):** Assists the learners in using appropriate and accurate language. Learners verbalise and express their thinking and observations about the topic. • **Phase 4 (Free orientation):** Explores relations within the level or 'field of investigation' so that the relations between the objects of study become explicit to them. Facilitates this process by presenting the learners with multi-step tasks, tasks with several means of solving them, and open-ended tasks. • **Phase 5 (Integration):** In this phase, it is important that no new information be presented, but that the learners summarise and review what they have learnt to form an overview of the objects and relations they have investigated. The teacher's role in this phase is to ensure that a complete (relevant to the level) summary is formulated, and the origin of this summary is reviewed.

Table 2.1 shows that the van Hiele model elaborates on the learning and teaching of geometry (Bleeker et al., 2013*;* Mason, 1998). Yi et al. (2020) argued that there are five teaching phases in the van Hiele model that represent a phase that encapsulates students' progress from one level of geometry understanding to the next. However, the five teaching phases are sequential, and consist of an information phase, a directed orientation phase, an explication phase, a free orientation phase, and an integration phase (Crowley, 1987; Fuys et al.,1988). In one way, features of the van Hiele model (e.g., the levels and

phases and their characteristics) can be considered as the interplay between teacher instruction and students' progressive development.

Students are guided by teachers using appropriate language relevant at the specific level (Fuys et al.,1988; Yi et al.,2020). In the model, teacher phases include lesson planning, scaffolding learning, introducing new mathematical language, engaging students in group discussions, and promoting problem solving. The van Hiele model was developed to improve geometry teaching by organising instruction to consider the learner's thinking (Fuys et al, 1988).

Literature indicates gaps in this van Hiele model for geometry teaching and learning. First, there are different arguments about the van Hiele model's five levels (Fuys et al, 1988). Several researchers argued that it (van Hiele) did not acknowledge the existence of a zero or pre-recognition level (Clements & Battista, 1990; Yi et al., 2020). In other words, they described level-0 (pre-recognition) where students initially perceive geometric shapes but attend to only a subset of a shape's visual characteristics and are unable to identify many common shapes. For example, students may see the difference between triangles and quadrilaterals by focusing on the number of sides but may have difficulty naming them (Mason, 1997; Li et al., 2020). Further, researchers indicated that a student may be on different levels of van Hiele's model concerning different topics within geometry (Mbusi & Luneta, 2021; Usiskin, 1982).

Second, the van Hiele model indicates that the student's level of thinking is linear and addressed in their learning process. In contrast, Martin and Towers (2016) argued that the growth of an individual's understanding of geometry concepts is not linear. They explained that the growth of understanding of geometrical concepts by learners is a continuous movement back and forth through layers of knowing which was called" folding back" (Pirie & Kieren, 1991), and argued for a dynamic way of thinking about the complexity of the teaching and learning of mathematics.

Relevant to the current research context, some of these teachers' factors are in teachers' psychological nature, such as teacher beliefs and relevant to the teacher's knowledge. Mathematics teacher beliefs are valuable for teacher education. The next section will discuss mathematics pre-service teacher beliefs.

2.3.2 Pre-service teachers' beliefs about mathematics

This study has been mainly concerned with pre-service mathematics teachers' pedagogical beliefs relevant to the block-teaching experience. However, early researchers defined beliefs as general concepts rather than the teacher or subject-specific beliefs. For example, Richardson (1996) explained beliefs as "psychologically held understandings, premises, or propositions about the world that are felt to be true" (p. 103). McLeod and McLeod (2002) suggested that there is no single definition for belief that is true and correct. A few decades ago, research literature indicated that teachers' belief systems may loosely abound with attitudes, values, dispositions, and knowledge (Abelson, 1979; Nespor, 1987). Peterson et al. (1989) conducted mathematics research with 39 teachers and students from 31 schools in Wisconsin, to understand pedagogical beliefs about mathematics, curriculum, and instruction on selected mathematics topics (e.g., addition and subtraction) in selected grades. In this early research literature (Peterson et al, 1989; Richardson, 1996), three categories of experience are described as influencing the development of beliefs and knowledge about teaching. These categories are experiences with own schooling, personal experiences, and experiences with formal knowledge.

The first category is teachers' own experience. Ertmer et al. (2010) explained that many pre-service teachers enter pre-service institutes with beliefs about teaching and learning constructed from their own experiences as students. The second category is personal beliefs, and the importance of teachers' personal beliefs and views on instructional decision-making and mathematics classroom practice were discussed by several researchers (Ball & Bass, 2000; Thurm & Barzel, 2020). The third category, the influence of the experience of formal knowledge, is considered under subject matter beliefs (Xie & Cai, 2019) and the importance of pedagogical beliefs (Pajares, 1992). The next section will describe mathematics teachers' pedagogical beliefs.

2.3.3 Mathematics teachers' pedagogical beliefs

Relevant to the block-teaching context, mathematics pre-service teachers' beliefs can be considered as an individual pre-service teacher's perspective on how one engages in mathematical tasks and pedagogical practices. A growing body of literature shows that mathematics teachers' pedagogical beliefs affect their classroom practices although the entire nature of the relationship is highly complex and dialectical (Pajares, 1992; Peterson et al., 1989; Xie & Cai, 2020). Richardson (1990) argued that the improvement of the teacher learning process requires acknowledging and building upon teachers' experiences and promoting the reflection of those pedagogical beliefs. Furthermore, Peressini et al. (2004) argued that none of the experiences of pre-service teacher training and employment in learning to teach are independent of one another, which ensures a complicated collection of influences on the nature of a teacher's pedagogical beliefs.

Ertmer et al. (2010) considered pedagogical beliefs as a key variable requiring change. Pre-service teachers may hold multiple pedagogical beliefs in teaching practices such as domain-general and domain-specific. The domain-specific aspects that influence teachers' beliefs about specific subject content (e.g., geometry) have been the focus of several researchers (Alizadeh-Jamal et al.,2018; Brush et al., 2008). Bussi et al. (2020) explained that domain-general beliefs can include the use of tools for pedagogical approach, task design, and the lesson plans or any other activities relevant to mathematics teaching with DT tools.

2.3.2 Pre-service teacher's beliefs about mathematics teaching with DT tools

Pre-service teachers' beliefs about teaching with DT tools can vary with the way they use them in the mathematics classroom. Baccaglini-Frank et al. (2009) explained that tools mean both software and tangible physical materials to support teachers' explanations. Thompson (1985) discussed how some teacher education programmes paid less attention to the impact that mathematics teachers' beliefs had on their teaching practices with DT. Becker (1994) suggested that meaningful digital application use for teacher education is more aligned with constructivist teaching philosophy. Angeli and Valanides (2005) argued that to understand the use of digital applications for teaching and learning, we should not only consider teachers' knowledge but also teachers' beliefs.

In addition, Voogt et al. (2013) suggested teachers' beliefs relevant to technology as (i) beliefs about technology (Niess, 2005), and (ii) pedagogical beliefs about teaching with technology (Thurm & Barzel, 2020). The goal of these studies is not to develop predictive indicators of teacher effectiveness, but to understand the nature of teachers' thinking and their beliefs. Xie and Cai (2020) emphasised the importance of pedagogical beliefs and epistemological beliefs in mathematics education. Ernest (2009) explained that the powerful effect of a belief is more useful in understanding and predicting how teachers make decisions. Researchers have used different research instruments for the identification of teachers' beliefs about the use of digital tools (Albion & Ertmer, 2002; Alizadeh-Jamel et al., 2018; Anderson & Piazza, 1996). More recently, qualitative methodologies have been used to inductively examine teachers' beliefs about technology. While some researchers advocated the determination of beliefs through observation alone (Rouleau et al., 2019; Thompson, 1992), other researchers used interviews in combination with technology practices (Chazan et al., 2009).

As DT continue to evolve, researchers are focusing on which teachers' knowledge is important and how these technologies can be used to enhance mathematics learning and teaching. Thomas & Palmer (2014) explained that mathematics teachers' knowledge and beliefs are basic competencies for mathematics teaching with technology. The next section discusses mathematics teachers' knowledge.

2.4 Mathematics teachers' knowledge

Among researchers focusing on mathematics teachers' knowledge, Shulman (1986) stated that "no one asked how subject matter was transformed from the knowledge of the teacher into the content of the instruction" (p. 6) which has attracted the ongoing attention of teacher educators. Shulman called it "a missing paradigm" (p. 7) and described four domains of knowledge that he considered in mathematics teaching. These domains are (i) knowledge of the materials for instruction, including visual materials and media, which is specific to curricular knowledge (Artigue, 2002); (ii) knowledge of the characteristics of the learners, including their subject-related preconceptions about learner knowledge (Amarin & Ghishan, 2013) ; (iii) knowledge of educational contexts, including classrooms, schools, district, which elaborates as context knowledge (Abdel-Aziz et al., 2016); and (iv) knowledge of educational goals and beliefs (Anderson & Piazza, 1996). These knowledge domains are important in mathematics teaching.

The context of mathematics teaching and learning has more variables, such as mathematics curricula and a knowledge of materials and resources for teaching content, including how subject matter content is structured and sequenced in different materials (Ball et al., 2005). These concepts are based on an explicit constructivist view of teaching in which researchers in other disciplines have suggested different ways of categorising teacher knowledge (Ernest, 1989; Xie & Cai, 2020). Ernest’s model (1989) of mathematics teachers’ knowledge, beliefs, and attitudes has more components: knowledge of mathematics, knowledge of other subject matter, knowledge of teaching mathematics including pedagogy and curriculum, knowledge of classroom organisation and management for mathematics teaching, knowledge of the context of teaching mathematics including the school context and the students taught, and knowledge of education including educational psychology, education, and mathematics education. There are many other views on teachers’ mathematics knowledge. For example, Cochran et al. (1993) described teachers’ mathematics knowledge as: “…a teacher’s integrated understanding of four components of pedagogy, subject matter content, student characteristics, and the environmental context of learning”, p. 266). Therefore, in the past four decades, different perspectives of teaching shifted from general teaching practice to the teaching of specific subjects such as mathematics.

Some scholars in the discipline have defined perspectives on mathematics teachers’ instructional process, an increasingly complex concept (Godino et al., 2007; Tatto et al., 2008). For example, Godino et al. (2007) explained about didactical suitability of mathematics teachers’ knowledge in the instructional process. Other researchers also indicated many challenges in mathematics teachers' instructional process especially difficult geometry content (e.g., proof development) in secondary classrooms (Jones & Tzekaki, 2016; Kunimune et al., 2010). Jones and Tzekaki (2016) explained that some mathematics teachers promote secondary students to memorise the rules without understanding the process of proof in geometry. Ding et al. (2009) explained that secondary students memorising rules is common as they can reproduce similar proofs, but they faced difficulties in applying the principles to develop a different proof. However, scholars (Baltaci, 2018; Godino et al., 2007; Kulze, 2017) indicated that some geometry topics (e.g., geometric proof development, geometrical construction) were difficult for secondary students and may need special attention in mathematics teacher education.

Many studies have considered curriculum knowledge to influence content knowledge, specifically about mathematics curriculum material (e.g., Fujita & Jones, 2014; Mouzakitis; 2006; Perera, 2008). The argument of what types of mathematics knowledge are essential for teaching geometry at secondary school has been the subject of teacher education for more than five decades. The knowledge for teaching mathematics is different from the knowledge required to learn mathematics. Researchers have discussed that knowing more mathematics does not ensure that one can teach it in ways that enable students to develop the geometry and deep conceptual understanding envisioned in the documents (Goos et al., 2020; Mewborn, 2001). Moreover, Goos et al.'s (2020) stated that (at the state level) novice teacher preparation programmes are being influenced by mandates regarding the number and domain-specific mathematics modules that teachers must complete for their pedagogical knowledge. The next section discusses pre-service teachers' geometry knowledge.

2.4.1 Pre-service teachers' geometry knowledge

Unlike other subject module (e.g., algebra) in mathematics, geometry problems include both a pictorial view of the appropriate problem as well as text (Barrantes & Blanco, 2006; Kul, 2018). Thus, it is important to minimise the extraneous process by removing unnecessary information during the instructional design for the geometry task (Bertolo et al., 2014). Therefore, pre-service teachers' knowledge is an important factor to reduce extraneous processing, manage essential processing, and foster generative processing, which are the main issues when designing instruction for tasks in geometry (Mayer, 2005). Teacher educators consider these generic instruction issues in their pedagogical approaches for pre-service teachers. Some of these pre-service teachers are digital natives who may think differently in their instructional design for a task than teacher educators.

The pedagogical support for pre-service teachers in their block-teaching sessions has been scaffolded by teacher educators in their micro-teaching sessions in the local context. The way pre-service teachers design geometrical activities is important in encouraging learners to engage in appropriate cognitive processing during learning (Barrantes & Blanco, 2006). However, it is also important not to overload their processing capacity when providing relevant text and pictorial tools for the task.

As this study focuses on pre-service teachers, it may be appropriate to investigate their use of the digital environment and their resulting learning (Olive, 2000). The development of DGS such as GeoGebra drag-mode capability has made the domain-specific (proof) understanding of geometric configurations possible and allows users to complete the work easily. Thus, Jones and Tzekaki, (2016) explained that research studies in geometry have focused on modes of understanding (visual, figural, conceptual), as well as on mental images and their manipulation while employing DGS theoretical notions.

The development of a domain-specific understanding of teachers' knowledge was reported by Tatto et al. (2008) who used a large-scale empirical study (TEDS-M) to offer a theoretical framework for teaching mathematics. They believed there are three kinds of mathematics content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Subject matter content knowledge is what a content specialist knows; for example, what a mathematician knows about mathematics. Shulman (1987) explained that PCK is the specialised knowledge needed for teaching mathematics, such as understanding how key ideas in mathematics are likely to be misunderstood by learners, and it allows multiple ways of representing important ideas in the domain.

2.4.2 Pedagogical content knowledge (PCK)

The PCK can be identified as a lens for mathematics teachers on why certain concepts are best taught in particular ways. In contrast, the dynamic nature of knowledge development was also discussed by various researchers (Cochran et. al., 1993) who conceptualised this in a much broader way than Shulman (1987) discussed PCK. Cochran et al. (1993) were exploring the relationship between pedagogical understanding and mathematics content knowledge of teachers linked with external factors like student characteristics and environmental context. Much mathematics education research literature contained different views that emphasise the importance of understanding mathematics teachers due to the significant role that PCK plays in mathematics teaching and learning (see Ball et al., 2000; Tatto, et al., 2008).

Scholars have discussed the nature of the knowledge needed by mathematics teachers (e.g., Ball et al., 2000; Unal et al., 2011). Different analytical frameworks offer some common features such as teachers' content knowledge and differences such as interactive stages of instructions (Schmidt et al., 2007) and argue for a coherent framework to probe the complexities of teacher understanding and the transmission of mathematics pedagogical content knowledge.

Table 2.2
Mathematics Pedagogical Content Knowledge

Mathematical knowledge	• Establishing appropriate learning goals • Knowing different assessment formats • Selecting possible pathways and seeing connections within the curriculum • Identifying the key ideas in learning programs • Knowledge of mathematics curriculum • Knowledge about mathematics language
Knowledge of planning teaching and learning	• Planning or selecting appropriate activities • Choosing assessment formats • Predicting typical students 'responses, including misconceptions • Planning appropriate methods for representing mathematical ideas • Linking the didactical methods and the instructional designs • Identifying different approaches for solving mathematical problems • Planning mathematical lessons
Enacting mathematics for teaching and learning	• Analysing or evaluating students' mathematical solutions • Analysing the content of students' questions • Explaining or representing mathematical concepts or procedures • Generating fruitful questions • Responding to unexpected mathematical issues • Providing appropriate feedback

Source: adapted from Tatto et al. (2008).

According to Table 2.2 (Tatto et al., 2008), PCK considers that the mathematical curricular knowledge component and the knowledge of planning for mathematics teaching and learning (pre-active) component may facilitate pre-service content knowledge. Schmidt et al. (2007) view the teachers' chosen cognitive processes during the pre-active and interactive stages of instruction as well as considering their knowledge of learners, pedagogy, and curricula.

In focusing on content knowledge for teaching, researchers have distinguished sub-categories, including subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (Fujita & Jones 2006; González & Guillén; 2008; Goos, 2006). These studies have revealed gaps in the geometry subject matter content knowledge of pre-service teachers. Fujita and Jones (2006) reported on the geometric knowledge of Scottish pre-service teachers and the ways that those pre-service teachers defined and classified quadrilaterals. Analysis of pre-service teachers' answers indicated a weak understanding of the hierarchical relationship of quadrilaterals as well as gaps between provided figural concepts and definitions. Researchers have suggested that the gaps can improve the use of digital technologies such as in dynamic geometry environments (Goos, 2006; Haja, 2005). For example, Haja (2005) studied pre-service secondary teachers and their problem-solving capabilities while they were undertaking geometrical constructions using DGS.

In addition to an understanding of mathematics content and contextual factors (including knowledge of the students, curriculum policies, and awareness of resources), pre-service teachers must be able to select from a range of pedagogical approaches, choosing those that best serve the learning needs of their students relevant to curriculum changes. Curriculum content is regularly updated with curriculum reforms, particularly in mathematics to reflect developments in understanding. Students predominantly depend on teachers for their achievement in mathematics (Graven et al., 2013). Alongside teachers' lack of adequate knowledge of mathematics for teaching, some researchers contend that the learning context and negative pedagogical beliefs about geometry have dire consequences when they start to teach (De Villiers, 1997; Goos et al., 2020). De Villiers (1997) explained that, once mathematics teachers get a clearer view of secondary students' learning difficulties in geometry and the sources of those difficulties, they can begin to adjust their teaching strategies, resources, and instructional strategies. The challenges for geometry understanding relate to the van Hiele model (see section 2.3.1) of geometry understanding in different levels. In contrast, Martin, and Towers

(2016) discussed the ring growth model moving outwards toward more abstract and generalised properties. The argument is that mathematical understanding may be neither linear (like van Hiele) nor mono-directional. Furthermore, Mayer (2005) described instruction for the design of a digital application, using manipulations of the environment that are intended to foster changes in knowledge in the relevant subject area. In this case, the digital application becomes a pedagogical tool. The complexities of pedagogies demanded by user-centred approaches may be exacerbated using DT for teaching.

2.5 Digital technologies for teaching

Scholars have identified challenges facing teachers when they attempt to integrate digital technologies into their teaching, such as infrastructure issues and lack of training. In the early stages of ICT integration, schools and authorities often purchased tangible goods such as hardware and software with their limited budgets and therefore allocated less money for professional development (Hawkridge et al.,1990; Tondeur et al., 2017). These authors suggested that human factors should be heavily weighted in DTs integration. Despite this increased access, teachers were continuing to grapple with both practical and philosophical challenges posed by the adoption and implementation processes of ICT (Dexter et al., 1999). Furthermore, Ertmer (1999) reports two type of barriers; first order barriers (e.g., technical, and organisational support) and second-order barriers (e.g., underlying pedagogical beliefs).

Although some teachers faced all these barriers, they could still implement new technological tools and strategies in their classrooms. Therefore, teachers' attitudes and beliefs about the value of digital technology for learning need to be addressed (Ertmer et al., 2012). However, some teachers still have problems understanding the pedagogical and psychological instructions relevant to digital tools (Chen, 2008). Moreover, an obstacle to teachers in creating an effective learning environment is that some infrastructure is inaccessible or cannot easily be brought into the classroom (Tondeur et al., 2017). International researchers have suggested different frameworks for how learning occurs in different aspects of digital technologies (e.g., Borba, 2021, Harasim, 2012; Hegedus et al., 2020; Mayer, 1998; Sinclair et al., 2017; Verschaffel et al., 2019). This means DT development relevant to mathematics teaching and learning processes can be analysed with different frameworks.

Several studies (Lisa & Faridi, 2021; Polly et al., 2010) indicate that pre-service teachers may feel a sense of insecurity integrating technology in their lessons. This may be due to lack of prior knowledge or experiences designing ICT supported learning activities. Some studies (Tearle & Golder, 2008; Zengin, 2017) state that group work might help mitigate these feelings of insecurity when teachers need to design technology-related curriculum materials.

Castro et al. (2021) argued that mathematics teaching has altered, mostly, regarding teachers' verbalisation of instructions with DT learning context. They have indicated possible aspects of teachers' teaching (didactic-mathematical knowledge for teaching) that are altered due to the society they live in (Pino-Fan et al., 2016). These researchers indicated a possible relationship between the teachers' instructional practice and their thinking for teaching secondary students learning geometry.

Ongoing instruction practice reflection throughout teachers' careers is seen as important to ensure that pedagogy remains aligned to content within an ever-changing curriculum. The PCK has considered the notion of the transformation of the subject matter for teaching (Shulman, 1986). TPACK can be defined as an extended version of Shulman's framework of PCK with technology knowledge added to the framework (Yigit, 2014). If teachers are strong in TPACK, they simultaneously know concepts, representation, and formulation of concepts with technology, and pedagogical techniques needed for students in the classroom.

2.5.1 Technological Pedagogical Content Knowledge (TPACK) model

In the last two decades, the TPACK framework has quickly become a widely referenced concept, especially for teachers' knowledge about the use of technological tools. Alongside the work of Shulman (1986) with PCK, Koehler and Mishra developed technological pedagogical content knowledge (TPACK), previously known as TPCK. Koehler and Mishra (2008) defined TPACK as:

> …the basis of effective teaching with technology. It requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to

> learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. (p. 17)

Researchers argued that changes in practices do not depend solely on knowledge, but many other factors influence these, such as context aspects (i.e., curriculum reforms, subject matter, grade level, student backgrounds, etc.), which are essential and overarching components of the TPACK framework (Mishra & Koehler, 2006). In the first and second columns of Table 2.3, TPACK is discussed by Niess et al. (2009) and is relevant to progressive levels of teachers (using GeoGebra instead of technology). The TPACK five levels performance indicators are in the third column for comparison.

Table 2.3

Comparison of TPACK from different perspectives

(Niess et.al., 2009)	Progressive development relevant to the DGE tool adapted from Niess et al (2009)	Teacher-related Performance Indicators from Lyublinskaya and Kaplon-Schilis (2022)
Recognizing (knowledge)	Teachers can use the technology and recognise the alignment of the GeoGebra with mathematics content, yet do not integrate the GeoGebra in the teaching and learning of geometry.	The teacher uses instructional technology for motivation only, rather than subject matter development. New ideas are presented by the teacher mostly without technology.
Accepting (persuasion)	Teachers form favourable or unfavourable attitudes toward the teaching and learning of mathematics with an appropriate DGS app.	The teacher uses instructional technology for subject matter development. However, a larger part of technology use is for teacher demonstrations, which include presentations of new knowledge.
Adapting (decision),	Teachers engage in activities that lead to a choice to adopt or reject the teaching and learning of mathematics with an appropriate DGS tool.	The teacher uses instructional technology for geometry content development. However, a larger part of technology use is for teacher demonstrations, which include presentations of new knowledge.

Exploring (implementation),	Teachers actively integrate the teaching and learning of mathematics with a DGS tool.	Teacher plans for instructional technology to be used mostly by students who explore and experiment with technology for subject matter development.
Advancing (confirmation),	Teachers evaluate the results of the decision to integrate the teaching and learning of mathematics with an appropriate DGS tool.	Advancing teacher develops instructional technology tasks for students that provide them with a deeper conceptual understanding of the subject matter.

In Table 2.3, the TPACK comparison provides different perspectives that have an impact on teachers' technology integration. It is not comprehensive enough because it does not consider what factors (hindering or assisting factors) teachers face when they want to apply their knowledge and skills in their teaching. The TPACK lens has been used to investigate some pre-service teachers' use of apps for their teaching, and it can influence secondary students' geometry learning (Handal et al., 2014). Moreover, TPACK helps teachers effectively integrate technology into their content area (Harris & Hofer, 2009). The dilemma of TPACK relevant pre-service teacher education it was not a unique body of knowledge itself.

The TPACK is constructed from other latent forms of teacher knowledge as a transformative view or thought of as a combination of other forms of teacher knowledge and enactment during teaching in an integrative view (Graham, 2011). In the current study, TPACK is a form of knowledge that makes a teacher competent with technology and can be described as the knowledge about technological applications and their affordances, pedagogy, and content (Lyublinskaya & Kaplon-Schilis, 2022). In addition, learners may realise how particular topics can be understood using technology for specific learners (Bonafini, & Lee, 2021; Villiers, 2004). According to Figure 2.2 the knowledge domains (content, pedagogy, and technology) form the core of the TPACK framework (Mishra & Koehler, 2006).

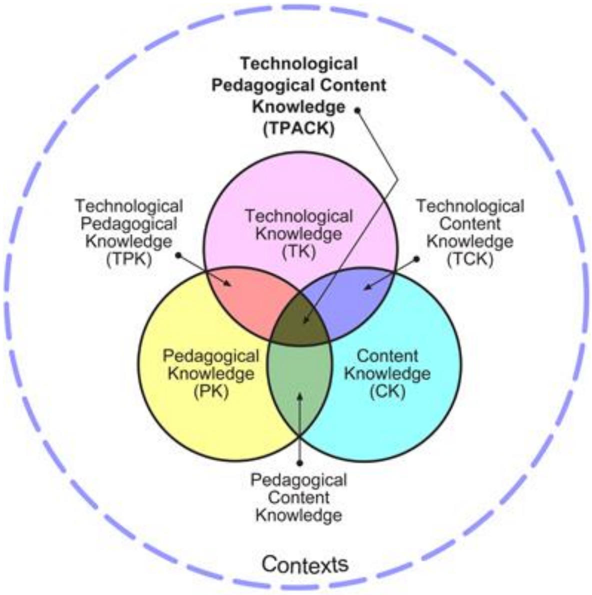


Figure 2.2. TPACK framework (Mishra & Koehler, 2006)

Figure 2.2. illustrates the three intersecting components in the Venn diagram (technology, content, and pedagogy), which are different aspects that comprise teachers' TPACK relevant to teaching and learning geometry described as follows:

- An overarching conception of what it means to teach a geometry-integrating technology in the learning process.
- Knowledge of instructional strategies and representations for teaching selected topics in geometry with technology.
- Knowledge of pre-service teachers' understandings, thinking, and geometry learning with technology; and
- Knowledge of curriculum and curriculum materials in geometry that integrates technology for learning.

Four aspects of TPACK have extended Simon's (1995) work on the components of teachers' knowledge in a mathematics teaching cycle by incorporating a focus on technology. Voogt et al. (2013) synthesised 55 journal publications relevant to TPACK and concluded that teachers' knowledge and beliefs about pedagogy and technology are intertwined. The indispensable element of teacher pedagogical beliefs for the present information age is important in the development of teaching and learning with the use of DT. Bonafini and Lee (2021) explained that the TPACK framework helps researchers to understand different domains of teachers' knowledge utilised in a mathematical lesson using DT.

Lyublinskaya and Kaplon-Schilis (2022) unpacked the overarching conception component of TPACK with the performance indicator rubric. The rubric was tested with 175 lesson plan exemplars from elementary school teachers enrolled in the education graduate teacher program. The rubric includes the TPACK level related to Niess (2005); namely: recognising, accepting, adapting, exploring and advancing (see Table 2.3). These indicators have shown the possibility of using TPACK for analysis of pre-service teachers, and geometry teaching experience with apps.

Bonafini and Lee (2021) suggested that the addition of technology does not change the way new material is presented to the students or how students understand the concepts. The way teachers adapt the materials and technology for their students' needs is the most important consideration. Scholars indicated some aspects of pre-service teachers' teaching and learning with TPACK (e.g., Angeli & Valanides 2009; Sahin, 2003) and scaffolding authentic technology experiences (Brush et al. 2003; Goktas et al. 2009). All these studies discussed different aspects of TPACK with technological applications. Handal et al. (2014) considered TPACK as a conceptual framework that is valuable for apps because it integrates three knowledge domains (content, pedagogy, and technology) in using apps for mathematics education.

2.5.2 TPACK for the apps

Looking through the TPACK lenses, apps can become powerful tools in the hands of mathematics teachers (Handal et al., 2014). The TPACK lens for apps is based on assumptions that apps may be enough to expose teachers to educational technologies and possible curriculum content.

Table 2.4

TPACK lens with apps relevance studies

TPACK definition	Example
Technological Knowledge (TK) about apps	The affordances of the app for teaching (Drennan & Moll, 2018)
Content (Geometry) knowledge (CK)	Geometry content knowledge in the secondary curriculum (Abunda, 2021)
Pedagogical knowledge (PK)	Different pedagogical approaches are relevant to the use of applets for geometry problem solving tasks (Leikin & Ovodenko, 2021)
Pedagogical content knowledge PCK	Combine the CK and PK to make the subject easy and make the content understandable. PCK does not exist as its domain, rather, teachers integrate separate ways during teaching with DT (Niess et al., 2009)
Technical Pedagogical knowledge (TPK)	Take advantage of appropriate technology for supporting certain teaching and learning pedagogical (e.g., how to use content-specific simulations), (Handal et al., 2014).
Technical content knowledge (TCK)	Represent and create content with technology using different tools in GeoGebra to do geometry construction. (Aslaner & Acikgul, 2020).
Technological Pedagogical Contentment knowledge (TPACK)	How to use a DGE app in geometry to reach geometry content with one's pedagogical approach (Acikgul & Aslaner, 2020).

The researchers (outlined above in Table 2.4) discussed challenges in apps within formal (physical classroom) and informal settings in teaching and learning. In addition, apps features, such as poor quality of apps, lack of understanding of apps (Crompton, 2015), lack of awareness of how to use mobile apps for educational purposes (Larkin, 2014), and how the conceptual model of TPACK influence the professional development of pre-service teachers (Acikgul & Aslaner, 2020; Yigit, 2014). Exploration of TPACK is relevant to the current research context in defining appraising apps in mathematics. It allows mathematics-related apps to be analysed using the

TPACK, as content knowledge (CK) stands for teachers' acquaintance with the geometry subject matter; technological knowledge (TK) which represents those operational capabilities that teachers need to deploy apps, and pedagogical knowledge (PK) which represents teachers' understanding of apps-mediated teaching (Handal et al., 2014).

In Sri Lankan context, content knowledge (CK) of geometry covers compulsory Euclid geometry that starts with 2D geometry and moves up spirally grade by grade. The pedagogical knowledge (PK) of Euclid geometry is in the vertical mathematics curriculum defined by the TG. For example, in 2007, the competency-based approach for two-dimensional geometry was introduced in which competencies for angles on a straight line, parallel lines, triangles and quadrilaterals were gauged in Grades 6 to 7. In Grade 8, those competencies were developed with symmetry and transformations. The CK of quadrilaterals articulates teachers' understanding of the subject being taught and their awareness of the requirements of the curriculum. Furthermore, the teachers' pedagogical approach represents the range of strategies which teachers can draw upon to facilitate teaching and learning. The overlap between these two knowledge domains represents PCK, the ability to select appropriate teaching strategies to support effective learning (Adler et al., 2005). The next section describes technological approaches in geometry teaching with DT.

2.5.3 DT for task design

Since the late 1990s, using DT resources as a teaching and learning tool had major benefits for teachers; this has been well documented by various researchers and organisations (Albion, 2015; Loveless, 2005). DT tools can enhance teaching-learning, design tasks and help in achieving higher-level objectives. Furthermore, digital tools are important artefacts to support new ways of teaching and learning and to develop users' skills for cooperation, communication, problem solving and lifelong learning (Voogt, 2003). Indeed, digital technological resources promote learning, motivate and empower the user, and facilitate the job of the teacher (Bussi et al., 2014). Miller (2014) argued that teachers are no longer bound to the instructional activities facilitated by schools and administrators as they have a wide array of self-directed learning opportunities that facilitate the use of digital tools.

The teacher has control of the classroom and performs the role of facilitator when interfacing with digital technologies. If the right environment is available, the teacher is the best interface between the student and the learning environment during the learning process as he/she can guide the student to filter necessary information from the enormous source of information in the online environment. Depending upon the nature of the content, the scope of content, and the level of students, teachers can find appropriate technology for task design. Bryant and Hunton (2000) suggested that when users design a task with a digital application, individual characteristics are considered in an instructional role. In contrast, Kortenkamp and Ladel (2013) argued that to understand the relationship between instruction and teaching, interaction students' needs to be considered between the context and the cognitive processes of different students.

The current study explains that some individuals learn and recall well from visually presented symbolic information with diagrams, whereas students may also learn easily and recall well from verbally presented information in the multimode presentation. In the dual coding theory, Mayer (2005) suggested that users have preferred representation styles in designing a task in a multi-media context. Furthermore, in designing a geometry task for a student, the teacher should be aware of the teacher's instructional steps appropriate for the task. Some studies considered the van Hiele levels for instruction design for the tasks (Alex & Mammen, 2016). Finally, teachers as individuals are entrusted with facilitating knowledge with DT to interact with students and design tasks (Bryant & Hunton, 2000; Yılmaz, 2018). The next section discusses the possible benefits and challenges of the DGE.

2.5.4 Digital Geometry Environment (DGE)

In geometry, DGE offer "thinking spaces" and they are considered tools that help to organise mathematical thinking and design tasks. Many of these DGE facilities focus on geometry teaching and learning for the primary and secondary levels. Early researchers like Goldenberg (1995) addressed it as follows.

> ...it is merely a new interface to Euclidean construction. Line segments that stretch and points that move relative to each other are not trivially

> the same objects that one treats in the familiar synthetic geometry, and this suggests new styles of reasoning. (Goldenberg, 1995, p. 220)

The manipulation features of geometric construction are a common feature in DGE apps, which allows the user to build a geometric model of objects. However, Pesci (2003) argued, the approach to geometrical figures as digital objects in DGE. For example, in GeoGebra, points, lines, triangles or other shapes are grouped with attributes that may relate to manipulating that object (Olivero & Robutti, 2007). Therefore, the user can manipulate the objects by moving some of their parts using a mouse, keyboard, or touch operation.

DGE has created a new way of learning via touch-operation-based smart digital devices such as tablets or smartphones. As touchscreen devices become increasingly popular in the facilitation of mathematics understanding, the importance of implementation of technology tools for teaching and learning mathematics increases accordingly (Moyer et al., 2005). The ownership, flexibility, autonomy, and ease of access of these hand-held smartphones (and their application capabilities) can benefit pre-service teacher education (Kearney & Maher, 2013). It is important to understand how pre-service teachers design tasks with apps for teaching. Important threads are as follows: (i) DGE visualisation tools for geometry teaching and learning (Moyer-Packenham & Bolyard, 2016); (ii) design of activities and tasks that are based on interactive dynamic features (e.g., dragging) (Forsythe, 2015); (iii) patterns of solving geometrical problems with interactive linked multiple representations (Castro et al., 2021), and (iv) roles of diagrams as instructional tools with DGE (Burlamaqui & Dong; 2014).The argument is that novice pre-service teachers possibly act differently with DGE tools because of continuous, pervasive exposure to modern mobile technology activities in society.

Mathematics teachers have discussed that the affordances of mathematical apps have multifaceted benefits for design tasks with digital objects in DGE. The current study explains the possibility of some individuals learning and recalling well from visually presented symbolic information with diagrams with apps.

2.6. Affordances of DT

Affordances can be considered as opportunities and constraints of the potential relationship between the user and the artefact (Brown, 2005). For example, in the current study, there is evidence for a potential relationship between the pre-service teacher and GeoGebra. Affordances implies the complementarity of the learner and the environment (Gibson,1977). According to Gibson, affordances is an action potential, the capacity of an environment or object to enable the intentions of the student within a problem situation. Gibson (1986) considered that the affordances of DT have both helping and hindering effects and defined affordance as "possibilities that the agent has for action" (p. 13), which may be more appropriate for a digital technology environment. Several researchers discussed the affordances of DT in different learning environments as well as the opportunities that the environment offers the learning process (Burlamaqui & Dong; 2014; Gibson, 1986; Greeno, 1994; Tanner & Jones, 2000). In addition, Calder (2017) discussed the haptic and multi-touch affordances of apps as a feature of mathematics learning enhancement.

2.6.1 Affordances of mathematics apps

As mobile technologies continue to evolve, researchers have become focused on how particular apps can be used to enhance mathematics teaching and learning. Importantly, for this study, a range of recent literature explores the contribution of digital technology to the teaching and learning of geometry. The touch screen features of apps are interactive and open new avenues for geometrical learning and teaching (Calder & Larkin, 2016; Sinclair & Ng, 2015). The affordance of DT was highlighted by Calder (2011) in his research on learning mathematics with digital technologies in primary schools. The literature also raises some concerns about the selection of good quality mathematics apps (Larkin & Milford, 2018; Namukasa et al., 2016). Larkin (2018), who analysed 57 mathematics apps also raised some concerns about the use of mathematics apps and suggested that there are several poor-quality mathematical apps on the market. For example, Larkin and Milford (2018) stated that the minimal information available for a digital application may be a barrier in selecting appropriate apps for the users' learning trajectories. He also explained that apps should be appropriate for users' individual needs, interests, and to their social and cultural contexts. Therefore, selecting an appropriate app is an external barrier to learning mathematics with apps. However, selecting an app is a difficult time-consuming process.

A range of literature explores the contribution of app affordances to the teaching and learning processes (Burden & Hopkins, 2016; Calder, 2017; O'Malley et al., 2013). In addition, Calder (2017) explained that affordances of digital technologies might reshape the learning process including the dynamic interaction, immediate feedback to input, and the visual nature of the learning. The apps can enhance the learning environment and foster the development of conceptual understanding (Drijvers et al, 2010). In some studies, researchers (e.g., Ball et al., 2008; Drijvers et al., 2010; Namukasa et al., 2016) argued about teachers' practices concerning their knowledge and their beliefs in the affordances of digital technology.

Scholars who explored the mathematical or affordances of geometrical apps in mobile devices may have different modalities (e.g., Calder, 2015; Larkin, 2013; Namukasa et al.,2016). Zbiek et al. (2007) explained it as attractive and effective teaching or learning tools and defined the *pedagogical fidelity* of an app which refers to the functionality of the tool to enhance further learning. In addition, it includes the extent to which users believe that a tool allows them to act mathematically in ways that correspond to the nature of mathematical learning in a teacher's practice. However, compass and straight-edge constructions can be fun, useful, and are usually required in secondary school curricula. In contrast, other researchers (e.g., Handal et al., 2014) indicated that novice teachers' lack of requisite mathematics knowledge and poor skills affordances of mathematical tools might disadvantage secondary students. For nearly five decades, the DGS, (e.g., Geometer's Sketchpad, GeoGebra) became a very widely used geometry software for computers in mathematics education (Jones, 2001; Kulze, 2017; Segal et al., 2021; Sinclair et al., 2016). Segal et al. (2021) analysed 27 Israelian mathematics pre-service teachers' experiences of geometrical tasks and concluded that the affordances of DGS may be beneficial for geometry teaching.

2.6.2 Affordances of Dynamic Geometry Software (DGS)

Dynamic geometry software (DGS) is a generic term to describe a specific type of mobile app, specially designed for plane geometry. Researchers advocate the affordances of DGS to facilitate teaching and learning geometry (Sinclair & Chorney, 2018). Some DGS (e.g., GeoGebra, Sketchpad, and Geometry Pad) for hand-held mobile devices can be downloaded free or low cost. This symbolic, visual manipulation has increased the adoption of DGS by users. Undoubtedly, this visualisation of the manipulation function has provided more advantages for understanding geometry concepts. Meanwhile, DGS in mobile devices gives a ubiquity and portability challenge for the touch-screen generation. This argument is expanded by Yigit (2014) who discusses geometry learning and teaching:

> …use any type of dynamic geometry software (DGS) to teach a geometrical concept, they must understand how to use the representations of the concept in DGS. They also must understand the related pedagogical techniques needed to best illustrate the concept in DGS, along with understanding any challenges or benefits of using DGS. (Yigit, 2014, p. 27)

Patsiomitou (2018) considered pedagogical knowledge as a key in the use of DGS that is widely embraced by novice teachers. Laborde's (2001) research suggested varying levels of task design. In the first level, teachers can consider how DGS facilitates the visual aspects of the task. Teachers use DGS during the stages of instruction, and their knowledge of learners, pedagogy, and curricula. In level two, it is possible to incorporate DGS (facilities) such as dragging. Laborde's (2001) research was a comparative international study about the knowledge that novice primary and lower-secondary school teachers acquired during their mathematics teacher education. In the light of these apps' affordances, Bülbül et al. (2020) focused on teacher education, such as how teachers can make pedagogical enhancements (e.g., dynamic and visual representation) with DGS for geometry teaching and learning.

2.7 DGS for geometry teaching and learning

Mathematicians suggest that vision, spatial sense, and dynamic sense are important in mathematic teaching and learning with DGS. However, DGS take advantage of multi-touch capabilities (Byers & Hadley, 2013), to support users to take an active role in the learning process (Crompton et al., 2018). DGS multi-touch capabilities may influence users' mathematical understandings and strategy development in unique ways (Baccaglini-Frank et al., 2010). There is potential for touch-screen technology to allow for greater embodiment than mouse-based interaction, as it can afford users more direct control over the manipulation of representations.

DGS can facilitate learning via visual ways of mathematical thinking as suggested by Aspinwall et al. (1997). They suggested three types of visual thinking: (i) pictorial; (ii) recognising patterns; and (iii) dynamic imagery. However, the use of DGS may distinguish the dynamic option from the visual option which is not available via paper and pencil methods (Sinclair & Gol Tabaghi, 2010). Burden (2017) indicated the possibility of the teachers' learning landscape shifting to convenient and available DGS providing opportunities for any-time, self-generating, and on-demand learning from a traditional classroom setting. These paradigm shifts may require different mindsets about pre-service teachers' identities in their roles as active participants within their professional development with DGS.

2.7.1 DGS for professional development

The study indicates the challenges pre-service teachers have when navigating through hypothetical geometry learning trajectories with GeoGebra, which enables them to enter and leave learning contexts when required (Sinclair & Moss, 2012). Researchers have considered that readiness to use technology and awareness of how applications can support one's learning and must become integral skills in a teacher's professional repertoire (UNESCO, 2007). Venturini & Sinclair (2017) indicated the potential of DT for sharing knowledge and facilitating self-professional development to improve learning outcomes. Yigit (2014) has considered the facilitation of independent learning pathways or trajectories for professional development as a relatively new field in the context of pre-service teacher education.

In addition, MT apps promote an authentic, ubiquitous, and personalised learning environment. They also support teachers' professional learning so that knowledge sharing with DGS/digital pedagogical practices can be used in classrooms. However, there remain different models of research into DGS use for preparing initial teacher education (Baran, 2014). Veletsianos et al.'s (2013) use of participatory pedagogies by teachers enables a greater sense of social presence and improves community building and active participation through shared discussion tools. The possibility of teacher professional development through the knowledge sharing model with peers has been discussed by researchers (Edirisinghe, 2016; Nonaka & Toyama, 2003). In this model, knowledge is defined according to the knowledge sharing framework suggested by Nonaka and Takeuchi (1995). This framework has been used for the professional development of teachers and considers two types of knowledge (tacit and explicit knowledge). This model has four stages: socialise, externalise, combine, and internalisation. Chen (2021) described how knowledge sharing works during teachers' professional development. The first stage, "socialisation", enabled teachers to share knowledge. For example, with socialisation, the teacher educator can build relationships through tacit-tacit knowledge of geometrical concepts being shared with the pre-service teacher.

The possibilities of pedagogical knowledge relevant to the geometry concept have developed when the teacher educator was supported to find the appropriate tool (DGS) needed for the purpose (Arzarello & Soldano, 2019). Then, the teacher educator can facilitate externalising (tacit-explicit) geometrical pedagogical knowledge based on their experience. Pre-service teachers with block-teaching experience can share (explicit-explicit) knowledge with their peers through professional development workshops. Pre-service teachers have engaged in an ongoing process of selecting the appropriate pedagogical approach for using DGS relevant to the geometry concept (Artigue, 2002; Villiers, 2004). These teachers have the possibilities to engage in a wide range of different digital tasks in DGS applications in mathematics teaching. Experience is valuable as many young pre-service teachers are in the process of instrumental genesis (see section 2.2.4), which indicates pre-service teachers' transition to their present occupation as self-learners with technology. Finally, both teacher educators and novice teachers can internalise (explicit-tacit) pedagogical knowledge.

In this way, initial learning phases are supported by DGS as well as traditional practices such as mentoring of teacher educators. As novice teachers build competence and confidence through their professional connections and communications, they become “knowledgeable others” integrating effective practice into their career (Agyei & Voogt, 2012). In this way, there are sustainable opportunities for self-professional development with DGS. The underlying context of mobile learning in professional development, as well as the teacher as a self-learner, means the participant controls the nature and timing of the support task (cognitive support) and services (effective support) (Acikgul & Aslaner, 2020). Tasks and services can be offered technology but are not mandated by the teacher educators’ responses since they are critical to informing decisions about further tasks and services (Goos, 2020). Future research is needed to examine pre-service teachers’ professional development with technologies when mentors are not able to seek engagement at a prescribed time in a face-to-face setting.

Learning through apps offers pre-service teachers an affordance for mathematics learning as well as for their professional development. Indeed, the use of apps has a positive influence on both the attitude to mathematics learning as well as learners’ motivation (Attard & Curry, 2012; Calder, 2015; Calder & Murphy, 2017). Apps may encourage learners to be actively engaged, but this engagement may not necessarily translate into enhanced learning.

Mobile apps in the online or offline mathematical classroom have been considered indispensable since 2020 with the COVID-19 pandemic in the Sri Lankan context. Educational institutions, including schools, granted permission for teachers and students to teach and learn with their own mobile devices during the forced lockdowns of the COVID-19 pandemic. However, misunderstanding between teachers and students became a common social aspect in asynchronous communication (Forbes & Gedera, 2019). Social aspects have become pivotal for teachers and students in untapping mathematic learning opportunities offered by the apps (Clark-Wilson et al., 2020). Sometimes the dialectic between the individual and society has been altered by the instrumental approach. Therefore, mobile learning can both complement and conflict with formal education principles (Breda & Santos, 2021; Sharples et al., 2005). Argument is that GeoGebra's enhanced geometry learning, possibly due to its dynamic learning environment.

2.7.2 GeoGebra for geometry

GeoGebra is a mathematics app that is popular among smartphone users and computer labs because it is a freely downloadable, offline DGS app (Fung & Poon, 2021; Kuzle, 2017). In the last decade, the use of GeoGebra (app) in mathematics education has grown rapidly (Ocal, 2017; Ziatdinov, 2022). The advent of mobile technology and mathematics app is having an unprecedented impact on the mathematics learning process, different aspects that have been discussed by researchers (Crompton & Burke, 2014; Handal et al., 2014; Zengin, 2017). Recio et al. (2019) stated that GeoGebra is popular with mathematics teachers because it has strengthened teachers' mathematical and pedagogical knowledge. Meanwhile, other researchers have discussed the use of GeoGebra for the construction of geometrical constructions (e.g., Bülbül et al., 2020). The current study is concerned with pre-service teachers’ teaching geometry with apps. GeoGebra app can provide teachers with three powerful tools: visualisation, dragging and the dynamic platform with touch screen option.

GeoGebra is used in teaching and learning with different pedagogical approaches. For example, GeoGebra can be used for geometrical constructions like the compass and straightedge constructions of geometry in an almost similar way as traditional paper and pencil methods. Santos-Trigo et al. (2021) discussed the seven Mexican mathematics teachers' systematic use of GeoGebra affordances contributed to thinking about the tasks in terms of the geometric meaning of the concepts (e.g., drawing a tangent circle to a given line) involved.

GeoGebra allows students to construct geometry platforms by providing them with virtual tools such as the parallel line tool, and the measuring tool (Ocal, 2017). Visualisation of abstract concepts and interactions is therefore becoming possible. For example, the use of TPACK framework for App has been discussed by Handal et el., (2014). Since the influx of MT into mathematics, educators and policymakers have considered Apps will be a powerful tool for learning, because students can easily access through handheld devices.

Several theoretical approaches have been considered in this study related to the field of geometry with apps. First, the sociocultural perspective of Vygotsky’s work was to consider the understanding of the pre-service teachers' perspectives in a mathematics classroom context. Pre-service teachers' emerging skills with apps can develop under the guidance of more ICT experienced peers or teacher educators. Second, the van Hiele model aims to organise the description and cognitive thinking of teaching complex geometric content (i.e., geometric proof or geometrical constructions). Third, the TPACK framework gave an option for understanding teachers' technical, pedagogical, and geometry-content knowledge relevant to apps. These complementary frameworks analysing different perspectives may influence pre-service teacher's affordance of GeoGebra apps and reveal different aspects about teachers' beliefs. Another aspect that emerged through the project was the possibilities for teachers' meta-cognition awareness for teaching when they selected geometry content (e.g., geometrical proof) with apps.

2.7.3 Metacognition knowledge: possibilities for teaching with apps

Teachers' metacognition knowledge indicated the possibilities of linking together all the epistemological and cognitive elements for geometry teaching with apps. It is commonly accepted that the "cognition" depends on higher-order processes of thinking that enable people to work efficiently (Verschaffel et al., 2019). More than five decades ago, Flavell (1979) defined metacognition as the knowledge that one obtains from her own cognitive experiences, "[t]his is true for adults as well as for children" (p.910), and suggested metacognition awareness. This metacognition awareness comprised three interacting categories namely, person, task, and strategy. Researchers explained components of metacognition are: (i) knowledge of cognition (e.g., knowledge about the task) strategies appropriate for solving the task and personal characteristics relevant to the task (Flavell, 1979); and (ii) regulation of control components (Garafalo & Lester, 1985). Metacognition awareness can include monitoring, control, and reflection of interrelated components.

Metacognition was adopted in different ways by mathematics researchers when they discussed metacognition knowledge of mathematics problem solving with DGS. Kuzle (2017) suggested a series of metacognitive activities with pre-service teachers in the problem-solving stages in Geometer's Sketchpad. Some researchers view self-regulation of learning as being closely related to this component of metacognition (Baltaci, 2018; Kuzle, 2017; Verschaffel et al., 2019; Yi Shen & Chuan, 2011). Baltaci (2018) discussed 46 Turkish pre-service teachers' metacognitive awareness of self-regulated GeoGebra practice, including analysing problems, planning, monitoring, evaluating, and debugging in teaching geometric locus problems.

Scholars have discussed how teachers thinking about teaching have possibilities identified when engaging the mathematical content through DGS-tangible objects, known as representamen (Breda & Santos, 2021; Richard et al., 2019). Using drawing instruments in DGS software, referred to as artefacts, and definitions, properties might also initiate metacognition (Richard et al., 2019). Lying in the epistemological plane (e.g., artefacts), all elements connected to the cognitive plane (construction, visualisation), are involved in solving mathematical proofs (Winer & Battista, M.2022). This is discussed in terms of their cognitive mathematics working space framework (Kuzniak & Richard, 2014). The use of GeoGebra (DGS) for geometry problem solving may have possibilities to improve pre-service teachers' metacognition awareness and relationship on their stages within TPACK for mathematics teaching.

2.8 Summary of the literature review

First, the literature informs the theoretical framework of the study. It also considers the literature relating to different aspects of mathematics education and DGS apps in pre-service teacher education. These theoretical aspects of a range of research and research in the review give good insights into the main research question and identify current gaps.

Second, the current research considers a socio-cultural approach as it is relevant to the research context. Several studies reviewed in this section also suggested that TPACK and the importance of mediation (tools, teacher, and peers) are relevant to the instrumental theory and to evaluating activities aided by DGS. Third, despite increased interest in the use of DGS for geometry being apparent in the reviewed literature, it is surprising to see that so little empirical research has been conducted on apps, especially in the developing-country context.

Finally, the pre-service teachers' pedagogical approach to the use of apps can vary, depending on internal (affordances) and external barriers such as distraction by engaging with social media, technical issues such as availability of network facilities, and the importance of geometry in the curriculum. Thus, this study contributes to the literature by addressing pedagogy perspectives about the use of mobile apps for geometry, which adds new knowledge to the literature researching the Sri Lankan context.

Chapter 3
Methodology

3.1 Introduction

The purpose of this chapter is to identify and discuss the methodological issues and aims of this study, which examines the use of mobile applications in mathematics education as well as factors that may influence the pedagogical perspectives and beliefs of mathematics pre-service teachers. In scientific research, the methodology can be defined as the framework that has components from epistemological and ontological assumptions as well as methodological issues (Hitchcock & Hughes 1995; Usher 1997). Researchers such as Denzin and Lincoln (2005) have described *epistemology* as the way researchers know the world or the relationship between inquiry and the known, and *ontology* is described as "the nature of reality and the nature of human beings in the world" (p. 183). The methodology is described as "the best means for acquiring knowledge about the world" (Denzin & Lincoln 2005, p. 183). These perspectives can be realistic and relevant to the needs of junior secondary classroom practitioners in the education system.

In this study, common philosophical assumptions relevant to social science research are briefly presented in introduction section 3.1 and the interpretive paradigm for the framework is identified. This philosophical assumption is discussed in the theoretical framework in section 3.2 and methodology in section 3.3. It justifies how the researcher explored the type of beliefs and evidence that are used to make claims that addressed the research questions. The fourth section (3.4) addresses the research design, which explores how an explanatory sequential mixed method design is relevant for the current study. Participants and setting in the research context (section 3.5) have given a brief description of the research methods. The rigorous procedure used to gather information to answer the research question through data collection methods is discussed in method section 3.6. The data analysis in section 3.7 expands on the overall data analysis to link how the research questions were answered. The ethical issues, especially at the data collection stage, are discussed in section 3.8. Lastly, the chapter summary is presented in section 3.9.

The main research question is:

In what ways does the use of mobile technology apps (e.g., GeoGebra) influence Sri Lankan pre-service secondary mathematics teachers' perspectives and beliefs of their pedagogical practices on geometry?

Supplementary questions are:

RQ1 What aspects of using mobile technology apps (e.g., GeoGebra) might influence the geometry content knowledge for pre-service teachers involved in junior secondary mathematics education?

RQ2 In what ways might using mobile technology apps (e.g., GeoGebra) for teaching geometry in mathematics education, influence pre-service teachers' pedagogy when teaching geometry?

RQ3 In what ways might using mobile technology apps for teaching geometry in mathematics education, influence pre-service teachers' beliefs about teaching geometry?

RQ4 How might GeoGebra be used for teaching and learning geometry content in Grade 10 secondary mathematics?

3.2 Theoretical framework

The theoretical framework covers the philosophical assumptions relevant to the research; it guides all stages of the research process, namely the research conceptualisation, research planning, research implementation, and research utilisation stages (Onwuegbuzie 2010). Awareness of philosophical beliefs helps the researcher to be aware of all assumptions underlying the research study.

The research paradigm is defined by researchers with different views in different research (Creswell, 2014; Denzin & Lincoln, 2005; Guba, 1990; Usher, 1997). Usher (1997) explained that a research paradigm acts as a model for "what and how to do research, and what problems to focus on and work on" (p. 13). Guba (1990) defined paradigms as "a basic set of beliefs that guide action" (p.17). Denzin and Lincoln (2005) stated paradigms are human constructions, and their value reflects the researchers' origin and, as such, the way their ideas are constructed and planted in the data. Creswell (2014) chose the term "world view" (p. 6) to define paradigms and discussed this as a philosophical orientation of the study. This argument claims that the philosophical underpinnings of the research give them motivation for the choice of the paradigm.

Research paradigms inherently reflect the beliefs of the world; however, these paradigms may not be appropriate for some studies. The two main philosophical assumptions are associated with the positivist (quantitative) and interpretivist (qualitative) paradigms within the social sciences (Onwuegbuzie, 2010). The positivist paradigm involves exploring social reality, which is objectively given by the study. In brief, “positivism is concerned with uncovering truth and presenting it by empirical means” (Henning et al., 2004, p. 17). Positivistic thinkers adopt scientific methods with objective reality. These objectives can be measurable using properties and quantities, which include descriptions of parameters and their interrelationships. The modified objectivist perspective is called post-positivism (Given, 2008). Given (2008) explained, the object of the inquiry exists outside and is independent of the human mind in post-positivism. It cannot be perceived with total accuracy by observations, which means complete objectivity is nearly impossible to achieve. However, this is not the case in the current study.

Assumptions are associated with what types of evidence are used to make claims (epistemology) or whether reality is multiple or singular (ontology) (Creswell, 2014). Ontology is, therefore, concerned with the nature of reality, regardless of whether the reality exists or is the product of one’s mind. Researchers can change their views according to ontological debates. Epistemology debates the nature of knowledge. These different ways of seeing the world have repercussions in this study. For example, the current study discusses the belief that stakeholders use mobile applications to teach geometry. The knowledge of these participants is relevant to the research problem. In addition, these stakeholders’ views are dependent on the situation and context. Niglas (2009) stated that it is the concrete research problem or aim, rather than the philosophical position, which determines the research paradigms. This means the realm of the philosophy associated with research can also demand the associated paradigms.

Interpretive researchers believe that reality consists of humans' subjective experience of the external world. Further, there are no methods to knowledge or correct routes to knowledge (Wills, 2000). The interpretive researchers adopt an inter-subjective epistemology and the ontological belief that reality is socially constructed. Willis (2000) clamed, interpretivists believed that the attempt to derive knowledge can be constructed from an in-depth examination of the phenomena of interest.

The current study includes an in-depth examination of two selected local teacher educational colleges (TEIs) in Sri Lanka. The beliefs and pedagogical approach to geometry were the phenomena of interest. Epistemological belief influenced stakeholders' (pre-service teachers and mathematics teacher educators) perceptions relevant to the research problems. The ontological view discusses the questions about what reality is, or being real (Creswell, 2013). The ontological view explains the reality of MT in learning geometry with the different pedagogical beliefs of pre-service teachers at the ground level. These same questions are challenged by social researchers who believe a single situation might have multiple realities in any social phenomenon (Thomas, 2009). This is reflected in how an individual pre-service teacher or teacher educator sees reality by using a different lens that is relevant to their experience, beliefs, and knowledge.

The study has analysed potential changes in pre-service teachers' beliefs about the use of mobile apps for geometry versus the traditional approach. An in-depth analysis of this phenomenon has been interpreted by pre-service teachers' pedagogical practices in the secondary classroom. Moreover, the current study also has context-dependent features. Thus, an interpretive approach is appropriate for the theoretical lens to unpack those socio-cultural factors that influence pre-service teachers' pedagogical beliefs about using mathematical apps for teaching secondary geometry. All the arguments stated here influenced the researcher to select interpretive paradigms for this research. The next section discusses the methodology relevant to the study.

3.3 Methodology

The philosophical tenets provided in the previous section are the foundation of the current research. This section explains the research method and design used in the study (including strategies, instruments for data collection, and analysis methods) while explaining the stages and processes involved.

3.3.1 Mixed method

A theoretical lens guided the researcher undertaking the study (Guba, 1990). The epistemology belief and RQ of the current study influence the type of evidence required to make a claim. For the first research question (RQ1), pre-service teachers' geometry content knowledge needs to be understood by their interpretation of it which is in turn influenced by their beliefs about social contexts. In RQ3, several stakeholders (e.g., pre-service teachers and teacher educators) provided a general explanation of the phenomenon. The reality is better determined by different individuals' perspectives that provide a clear picture of the research problem (Creswell, 2014). Thus, to provide individual perspectives for RQ2 and RQ1, in-depth interviews were conducted with pre-service mathematics teacher before and after block-teaching periods. The focus was on mixed methods rather than epistemology.

There are many debates about using qualitative methods such as interviews (Creswell, 1999; Hughes, 1997) with quantitative surveys. The current study starts with a quantitative strategy as it needs evidence for factors that may influence pre-service teachers to use mobile apps for geometry in general. First, a survey was adopted. These survey questions have been generated to link to several variables in order to help generate generalised comments for a large group of mathematics pre-service teachers. Similarly, to get an overview of pre-service teachers' geometry content knowledge and pedagogical perspectives, a geometry test was designed. These tools were combined to provide the quantitative data needed for this study. As noted by Johnson and Onwuegbuzie (2004), "mixed methods research is formally defined here as the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, and concepts" (p.17). Mathematics teachers were interviewed individually to provide qualitative evidence for the research questions of this study. These interview data were combined with pre-service teachers' beliefs about teaching geometry. Vulliamy et al. (1990) also justified the mixed method research approach.

> …qualitative researchers do sometimes use data collection techniques that result in quantification and statistical analysis, and positivist researchers use data collection techniques, such as semi-structured interviewing, which are more usually associated with qualitative researchers. Therefore, characterising research debates in terms of a quantitative and qualitative divide as so often continues to be the case (particularly in developing countries) is unhelpful. (p.5)

This study is based on a developing country context. Given this context and the need for a validity check, it was considered that mixed method research with qualitative and quantitative methods of data collection was more appropriate for this study. However, Denzin and Lincoln (2005) noted that different qualitative researchers use ethnographic prose, historical narrative, and first-person accounts. In contrast, the researcher in the current study used semi-structured interviews.

Yin (2009) also questions the opposition between qualitative and quantitative methods and suggests that, regardless of whether one favours qualitative or quantitative research, there is a useful and essential common ground between them. Mixed methods researchers acknowledge that the world is both qualitative and quantitative and seek to harness the strengths of inquiry that both approaches offer (Cohen et al., 2018; Creswell et al., 2004). In order to answer the research questions (RQs) effectively in the current study, the researcher first considered the relevant characteristics of the questions and showed that they would benefit from using both quantitative and qualitative research approaches.

The use of quantitative as well as qualitative evidence is supported by Johnson and Onwuegbuzie (2004) who stated that an integrated methodology for the social sciences has more benefits compared to a single method. The underlying rationale describes a piece of research as a qualitative, quantitative, or mixed method (Creswell, 2018; Denzin & Lincoln, 2005). As a result, mixed-method research designs are favoured as an alternative to either a positivist (quantitative) or a metaphysical (qualitative) orientation. However, other researchers are happy to mix these strategies within their research projects (Creswell, 2014; Johnson & Onwuegbuzie, 2004). These positions are taken by individual researchers; those who see the two strategies (quantitative and qualitative) as entirely separate and based on alternative views of the world.

The current study explores pre-service teachers' and mathematics teacher educators' pedagogical approaches to geometry content, which gives insights into individuals' inner-world views and focuses on qualitative aspects. The qualitative approach is described as the interaction with research content, for example, when the research problem demands in-depth interviews that require qualitative data. The research problem of this study uses both quantitative and qualitative evidence as well as an interpretive approach because it is concerned with understanding the world from the subjective experiences of individual participants selected for the study. The mixed method researcher can use measurements with variables or evidence from interviews and participant observations because they rely on a subjective relationship between the researcher and subject rather than the discipline (Denzin & Lincoln, 2005). Taking these arguments into consideration, the study uses a mixed methods research design, which has qualitative and quantitative methods of inquiry that complement each other.

In the current Sri Lankan context, the use of the mobile app as a teaching and learning tool is a subjective perspective of the meanings of events that underpin social action. It is important to understand the topologies with the rationale and different disciplines of mixing qualitative and quantitative approaches relevant to the Mobile learning (Cheon et al., 2012). Thus, to explain such events, the researcher needs to determine the benefits of different mixed method research designs that should be used. Table 3.1 summarises different topologies used by mixed-method researchers in their mixed-method research designs.

Table 3.1

Summary of some mixed-method research designs

Researcher(s)	**topology**	**Details**
Greene et al. (1989)	5 mixed methods	Triangulation, complementarity, development, initiation, expansion
Tashakkori & Teddlie (2003)	3 designs	Equivalent status, dominant–less dominant, multilevel use designs

Creswell (2018)	3 designs	Convergent, explanatory sequential, and exploratory sequential
Onwuegbuzie and Johnson (2006)	9 legitimation types	Sample integration, weakness minimisation, sequential, conversion, paradigmatic mixing inside-outside, commensurability, multiple validities, political

Table 3.1 shows how different disciplines of qualitative and quantitative approaches are combined for mixed methods research designs. Researchers have not made the same assumptions when they are combining and defining different topologies for mixed method research design (Creswell, 2018; Greene, et al., 1989; Johnson & Onwuegbuzie, 2004; Tashakkori & Teddlie, 1998). They have also described different topologies for mixed method research designs. This study, focused on Creswell's (2018) approach as it was the most recent version of Creswell's ongoing theorising in this space. For example, Creswell (2018) compares three types of mixed method research designs: “convergent design, explanatory sequential design and exploratory sequential design” (p.35). In convergent design, quantitative data and qualitative data are collected and analysed separately. The benefit of this is that it provides a quantitative and qualitative picture of the problem through the merging of results.

In general, the exploratory sequential design starts by exploring the research problem through qualitative data and in the second stage of the exploratory design, the qualitative data is used to structure the quantitative data to answer the research questions. However, the current study uses an explanatory design. It examines pre-service teachers’ perspectives of geometry teaching through a survey then their interpreted pedagogical practices and beliefs of technology integration are recorded through interview data. The integration starts with “the quantitative strand, which is analysed to determine what results need further exploration through qualitative data” (Creswell, 2018, p.39).

The strength of the explanatory design is that both quantitative and qualitative data build upon each other to answer the research questions. This is despite the current study starting with a quantitative survey and geometry test data of pre-service teachers. This is elaborated further with the qualitative interview data associated with an interpretive approach directed towards uncovering the meaning of pedagogy practices and beliefs of pre-service teachers and teacher educators. Research questions are supported by quantitative data, for instance using semi-structured focus group interviews, semi-structured individual interviews on pedagogical practices, and pre-service teachers' beliefs about learning geometry. However, the current study follows Creswell's (2018) explanatory sequential design, which will be discussed in the next section along with the rationale for using it.

3.4 Research design

The previous section discussed the rationale for mixing qualitative and quantitative research approaches to help determine the research design. This section consists of two parts: the first part justifies the different research reviews relevant to the selected design and the second part illustrates the current research design. A review of three research studies which have common research designs is summarised in Table 3.2.

Table 3.2

Summaries of studies

Purpose	Author	Research design	Participants	Strategies of inquiry
Designing mobile technology for learning purposes	Zurita, and Nussbaumw, (2004)	Mixed method	Low-income public-school students	Experiment study and interview
Evaluating the effects of mobile learning	Chen et al. (2008)	Mixed method	University students	Survey and interview
Evaluating the effects of mobile learning	Corlett, et al. (2005)	Mixed method	University students	Survey and focus group

Table 3.2 shows that surveys and interviews are common methods used in the three highlighted studies and mixed-methods research. Some studies start in the data collection phase, whereas several other researchers who explored digital technology (Onwuegbuzie & Hitchcock, 2017) used experimental

studies, including “tests”. In all these studies, the experiment results of the target group’s learning achievement were the major dependent variable. This variable was measured by a researcher’s prepared test or by a standardised test that had been given to the target group.

In the current study, the first research question is based on comparing pre-test and post-test geometry test data and the analysis of the open-ended survey questions (Q3.1 to Q3.4) “understanding the factors that influence geometry content knowledge” to uncover social factors which have positivistic epistemology characteristics. As noted by Vulliamy (1990), quantitative methods have a one-to-one correspondence with positivistic epistemology characterised by the testing of hypotheses to uncover social factors, while qualitative methods are associated with an interpretive epistemology directed towards the uncovering of meaning. Therefore, to understand pre-service teachers’ beliefs in using apps for geometry, the main study and pilot study employed semi-structured individual interviews for teacher educators and semi-structured focus group interviews for pre-service teachers.

The benefits of this mixed-method study are used in combination with a quantitative questionnaire followed by qualitative interviews, which is also called a form of alternative method (Onwuegbuzie & Hitchcock, 2017). For example, the focus is on the characteristics of traditional survey instruments, and the statistical analysis matches the objective of the first research questions. This study also employed open survey questions such as “perceptions of participant experiences on geometry at school”. In the current study, the mixed method approach was integrated with quantitative data collection methods. The mixed-method approach was chosen to understand the reality as well as the ongoing ways that persons or organisations go about their work in the mathematics teaching and learning process and to define meanings relevant to the research question. This integration was constructed to make sense of their world.

The study has followed an interpretive approach as it seeks to understand human experience by attempting to see the world through the eyes of the pre-service teacher participants (Cohen et al., 2018). The research questions (RQ1 and RQ3) are mainly based on pre-service teachers’ perceptions and beliefs of geometry teaching and learning experiences. Leedy and Ormond (2005) also agree that "to answer some research questions we must dig deep to get a complete understanding of the phenomenon we are studying" (p.133) before searching for causes and treating problems. In RQ4 it was difficult to determine the complete impact of different pedagogical approaches surrounding the use of apps for geometry.

Crompton and Burke (2015) also stated that many researchers selected experimental and case study designs when evaluating the effects of mobile learning. The current study used a case study design as it matched the research questions of the study. It has employed two case studies to explore pedagogical perceptions and beliefs of pre-service teachers about apps in the geometry teaching and learning process. It utilised two cases, bounded by different contextual factors, to get a more complete understanding of the research process undertaken. The next section discusses the research design.

3.4.1 Design of the study

This study was organised as a pilot study followed by the main study (see Figure 3). To recap, the pilot study is conducted in two selected TEIs to help validate the test instruments. The items tested in the pilot study were analysed and then modified and used in the main study. However, the design of the main study has two phases and comprised some more features that were identified through the pilot study. The data collection methods included interviews, document analysis, a survey questionnaire, and a geometry test. The main study examined pre-service teachers' pedagogical perspectives and beliefs about using mobile applications with a focus on geometry. The possible effect of other environmental variables in the two TEIs was also considered by comparing the two case studies.

Pilot study

A pilot study was used to test the usability of the research tools. To do this, data were collected with appropriate tools. A pilot study helps to show that the prepared tools are valid and reliable, that it measured what is supposed to be measured and did so with a suitable degree of accuracy (Cohen et al., 2018; Merriam, 1998). The data collection tools for the pilot study were survey questions, a geometry test, and semi-structured interview schedules.

Mathematics minor pre-service TEIs in Sri Lanka were chosen as subjects for the pilot study.

Phase 1

The first phase of the main study was designed to obtain an overview of the context variable, with particular reference to student geometry content knowledge from mathematics teacher educators and pre-service teachers at the beginning of the internship period. Therefore, phase 1 of the study had five research tools: a pre-geometry test and survey for 163 pre-service teachers from two TEIs, a semi-structured individual interview schedule (from two mathematics teacher educators), a focus group interview with eight pre-service mathematics teachers, and a document analysis schedule.

Phase 2 was designed to conduct an in-depth study of the use of mobile technologies in geometry by pre-service teachers at the end of the internship period; therefore, a post-geometry test followed by a survey questionnaire, focus group semi-structured interviews, and individual interviews. Phase 2 took place in the same two TEIs with the same participants. Thus, the above data collection methods were chosen to understand the real, ongoing ways that persons or organisations go about their work during the mathematics teaching-learning process as well as the meanings relevant to the research question, which they have constructed to make sense of the world (Merriam, 1998). Surveys and interviews were conducted in both phases of the main study to understand the factors that influence geometry content knowledge and that might also uncover social factors.

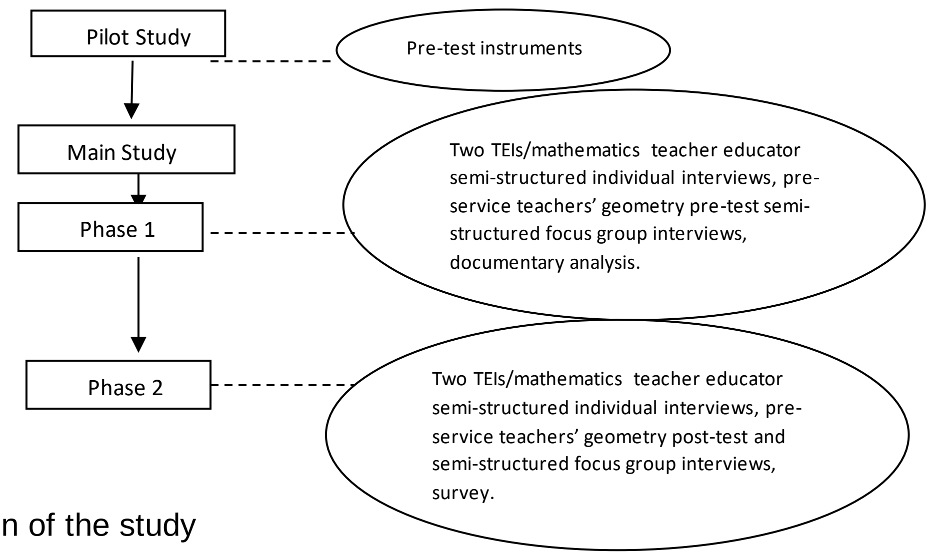


Figure 3.1 Design of the study

Figure 3.1 explains the design of the study. As the study gauged pre-service teachers' experiences in geometry, their perceptions were considered through semi-structured focus-group interviews. The study also focussed on exploring pre-service teachers' content knowledge and pedagogical approach when utilising mobile technology, which are contextually based variables, with a survey and geometry test. The study used live mathematics classroom experiences of individual mathematics teacher educators, with individual in-depth interviews also employed in the study.

To help answer the research questions in the current study, the researcher needed to better understand the pre-service teachers' curriculum (relevant to secondary geometry) which was ascertained to some extent by using document analysis. This study also compared perceptions about the use of mobile apps for geometry. The study is designed as a comparison of two case studies to get a clearer picture of the phenomena. The case study is used to analyse both qualitative and quantitative data evidence from each TEI. As Yin (2002) stated: "…the case study's unique strength is its ability to deal with a variety of evidence documents, artefacts, interviews and observations" (p.11). This study used a mixture of approaches for each case study. The next section elaborates on the case study design used for this study.

3.4.2 Case study design

The two case studies used in this study focused on the socially constructed context experiences of the pre-service teachers obtained through interaction (with mobile technologies) in which social phenomena are explored (Silverman, 2007). Cohen et al. (2018) discussed how case studies investigate and report the complex dynamic and unfolding interactions of events and human relationships. They further explained that case studies could penetrate situations in-depth. Indeed, case study design focuses on the process of gathering in-depth data (Cohen et al., 2018; Yin, 2002). The current study used a comparative case study design to understand different issues relevant to pre-service teachers' pedagogical perspectives and beliefs surrounding the use of mobile applications in geometry.

Case studies can also help us understand how pre-service teachers adapt to changes over time in their use of technology as well as their reactions to changes in their teaching, beliefs, constraints, and affordances of the professional environments in which they will work (Goos, 2005). For the research questions in the current study, we need to understand pedagogical practices with mobile applications that may be associated with an interpretive epistemology, as they are directed towards the uncovering of meaning for the pedagogy practices of novice teachers.

The two case studies were developed to understand the pedagogical identities of pre-service teachers in two different approaches (traditional methods versus mobile applications). Also, the researcher aimed to analyse changes in pre-service teachers' beliefs and knowledge and related aspects through an interview (a quantitative approach). Moreover, pre-service teachers' geometry pedagogical perspectives and beliefs both before and after using GeoGebra were explored.

In this case study, two focus-group interviews with pre-service teachers (phase 1 first year block-teaching practices and phase 2 second year block-teaching practice after they used mobile apps) were carried out to understand potential changes in the use of mobile apps. Therefore, the current study also followed the mixed-method sequential process (e.g., Creswell, 2018). However, the way that pre-service teachers used GeoGebra in secondary school classrooms for geometry was dependent on the classroom context and teachers' pedagogical perspectives and beliefs as well as other socio-cultural aspects. This suggests that the "case study approach is particularly valuable when the researcher has little control over events" (Hitchcock & Hughes, 1995, p.322). Thus, the chosen case study design for the current study gives benefits to a researcher who has little control over external events. As the study follows a case study design, two TEIs were selected purposively according to the availability of potential participants (e.g., mathematics major and minor pre-service teachers) needed for the study. In the first case study (TEI1) mathematics major and minor pre-service teachers explored traditional teaching methods and in the second case study (TEI2) some pre-service teachers used mobile apps (after phase 1 of the study). The next section discusses the participants and the research context relevant to the study.

3.5 Participants and setting

Participants of this study were pre-service teachers in the first two years at TEIs (phase 1 was conducted in the first year and phase 2 was conducted on the same students in their second year). During these two years, pre-service teachers learned through block-teaching which is considered practical teaching. In the third year, these prospective mathematics teachers at TEIs are recruited as mathematics teachers in state schools for the internship period and assigned to teach mathematics to secondary students.

3.5.1 Setting

The context of the study was limited to only pre-service TEIs in Sri Lanka. There are four mathematics TEIs in the Sri Lankan state education system. Only two TEIs conducted English medium mathematics courses in 2018 and these were selected for this study. The administrative overview of all selected pre-service teacher education institutes is vested in the MoE and TEIs, and curriculum and evaluation responsibilities are distributed among NIE and TEI principals. TEIs administer the mathematics curriculum according to the guidance of NIE. This also enables the MoE to have authority over all TEIs to conduct initial teacher education courses leading to the awarding of the National Diploma in Teaching (by the NIE) for all mathematics courses. These diplomas are recognised teaching qualifications in all mediums (Sinhala, Tamil, and English) in state secondary schools.

3.5.2 Participants

Participants for the study were pre-service mathematics teachers and teacher educators from selected TEIs who participated voluntarily. In the main study, participants were English-medium mathematics major and minor pre-service teachers, and the pilot study participants were ICT pre-service teachers who took mathematics as a minor subject. Phase 1 of the main study was conducted in March 2018 with first-year students at TEIs and the second phase started in March 2019 and was extended to 2020 (due to political unrest in Sri Lanka) with the same group of pre-service teachers.

Both TEIs had around 163 mathematics pre-service teachers. All these pre-service teachers underwent teaching practice from 2018 up to 2020. This study follows a mixed methods research approach and the process of selection of participants depends on this approach. The interview procedure for the selection of participants, sample sizes, and methods are described in detail in section 3.6.2.

Selection of participants for the pilot study

A short meeting was held with the principal of the TEI to describe the study needs and to ensure any disruption to the TEI academic calendar was minimal. A letter of invitation was then given to the principal of each college seeking their participation in the study. The TEIs were also given consent forms at this time. Depending on the principal permission, pre-service teachers were invited to participate in the study. Approximately 80, ICT major pre-service teachers from TEIs voluntarily participated in the pilot study. These pre-service teachers were informed that they had the right to withdraw their participation in the study at any time.

Selection of participants for the main study

The two TEIs were selected according to the availability of some major mathematics courses, the medium of instruction (English), and the number of pre-service teachers available in the study period. Some colleges did not have enough participants to sit the English-medium mathematics test. First, initial inquiries were made to identify mathematics teacher educators, pre-service teachers for mathematics education, and the availability of those students in selected TEIs during the time suggested for phase 2 data collection. The TEIs were provided with a consent form. The information and forms needed to recruit mathematics teacher educators and pre-service teachers were given to the principal to view.

The purpose of this geometry pre/post-test was to identify the geometry content knowledge of pre-service teachers from control and mobile apps using TEIs. Teachers for the main study followed the same assessment procedure as the pilot test. In the pilot study and the main study, consenting participants were asked to answer the geometry test on an answer sheet, which was collected after the test. After the test, pre-service teachers from the control college were invited to participate in the survey, and the mobile apps were used by the colleges after the pre-test. However, the number of students (20–30) varied due to the attendance of students and the facilities available at TEIs on that day.

It should be noted that the selection of students from selected colleges was purposive, and students were selected according to the course at the TEIs. From these pre-service teachers, a purposive sample of pre-service teachers (3–4) was selected and invited to participate in semi-structured focus group interviews in phases 1 and 2 of the main study. These interviews allowed pre-service teachers to share their pedagogical experience in geometry with or without the use of apps in geometry. Pre-service teachers had the right to refuse to answer any question.

In accordance with the principals' consent, mathematics teacher educators from the two colleges were invited to participate in the study. The mathematics teacher educators had been given the "information letter" that clearly explained research details and invited them to participate in an individual semi-structured interview. However, the number of teacher educators at each interview (1–2) varied due to the recruitment of teacher educators in selected TEIs and other external work assigned to them on that day.

Table 3.3

Data collection summary

<table>
<tr><th>Participants</th><th>Number of participants</th><th>Evidence</th><th>Stage of the study</th></tr>
<tr><td>ICT pre-service teachers</td><td>80</td><td rowspan="4">Geometry content knowledge
Belief of use of apps
Pedagogical approach for geometry
External factors, internal factors
Geometry pedagogical knowledge
Belief of use of traditional method for teaching geometry</td><td rowspan="2">Pilot
2018 March</td></tr>
<tr><td>Mathematics teacher educators</td><td>2</td></tr>
<tr><td>Mathematics major pre-service teachers</td><td>163</td><td>Main study
Phase 1
2018 September</td></tr>
<tr><td>Mathematics teacher educators</td><td>4</td><td>Phase 2
2019 (extended to 2020 due to political unrest in Sri Lanka)</td></tr>
</table>

		Belief of use of GeoGebra for geometry Pedagogical approach for use of apps for geometry	

Based on Table 3.4, out of the total sample of 163 pre-service teachers selected for the main study, Based on Table 3.4, 48% of pre-service teachers were from the urban TE1 while the others (52%) were from the rural TE2.

Table 3.4

TEIs by gender (cross-tabulation)

			Gender		
			Male	Female	Total
TEI1	Urban				
		% of Total	9.5%	38.1%	47.6%
TE2	Rural				
		% of Total	9.5%	42.9%	52.4%
Total					
		% of Total	19.0%	81.0%	100.0%

In these case studies, fewer males participated, which is a commonly seen trend in the teaching profession in Sri Lanka. This predominance of female trend is also apparent in the main study population shown in the gender pyramid in Figure 3.2

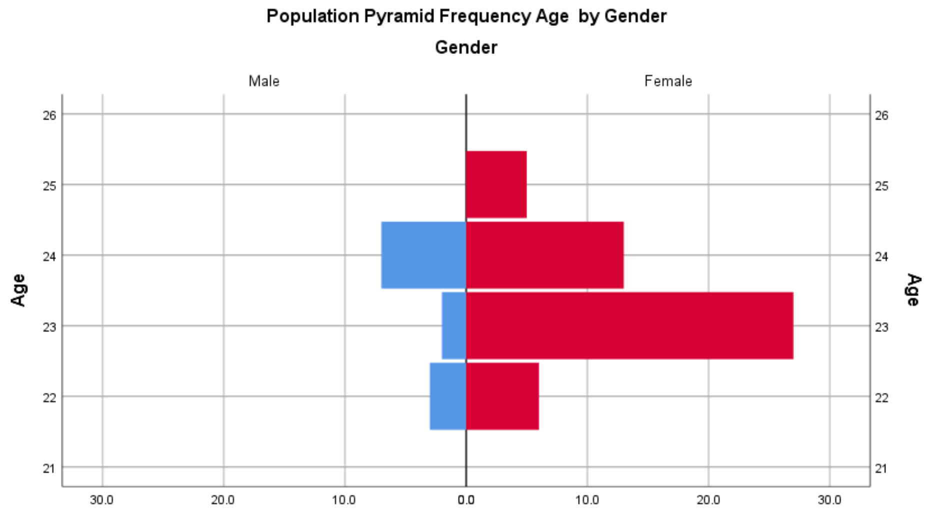


Figure 3.2. Pre-service teachers' ages distribution percentages by gender

According to Table 3.4, 81% of the teachers in the sample are female. The population pyramid (Figure 3.2) shows that all-male participants are in the 22–24-year age group while female participants are in the 22–25 year age group. There are five female teachers aged 25 years in the selected sample. Figure 3.2 also shows that most (77%) pre-service teachers are in the 23–24 year age group (including both genders).

3.6 Methods

Data collection methods for this study included surveys, geometry tests, semi-structured individual interviews, and semi-structured focus group interviews, with two focus group interviews in phase 2 being conducted online only due to a lockdown resulting from the first wave of the COVID-19 pandemic.

3.6.1 Survey

The purpose of this survey was to identify factors that influenced pre-service mathematics teachers by comparing pre-service teachers' groups not using the mobile app and using the mobile app at TEIs. However, due to the COVID-19 pandemic both TEIs had some pre-service teachers who had experience in MT. After consent, pre-service teachers for the main study also followed the same procedure as the pilot test. However, in the main study, consenting participants were asked to fill out the survey questionnaire in phase 1 and phase 2 of the study. In the survey, pre-service teachers were requested to include an index number given by the researcher for a geometry test. Pre-service teachers were informed that some demographic data were included in the survey instrument such as pre-service teachers' family background, parental income and education, and availability of personal mobile facilities. The variables were needed for quantitative data analysis relevant to the survey questions. The independent variable and dependent variables were included in closed questions. The survey questionnaire also had open questions. These open questions targeted qualitative responses from participants to record their different perceptions about pedagogical approaches and perceptions about geometry at school. All consenting pre-service teachers had the right to decline to answer any questions or withdraw from the test and the survey at any time.

3.6.2 Geometry test

The study needed to understand pre-service teachers' understanding and pre-knowledge of geometry before examining the influence of apps by providing access to mobile technology. This was important because pre-service teachers represented an important stakeholder group of the study; yet, what remains unknown is whether the implementation of apps is reflective of pre-service teachers' learning needs.

The first research question was designed to gain an understanding of teachers' geometry content knowledge before mobile application use. Several researchers (e.g., Burton et al., 2011; Kommers, 2009; Leikin & Ovodenko, 2021) have used "tests" to understand digital technology in mathematics education. For these quantitative studies, experiment results of the target group's learning achievement were the major dependent variable. This variable was measured by a researcher-prepared/standardised test or tasks that had been given to the target group. Thus, in phase 1 of the main study, some topics were selected in geometry at Grade 10 for the pre-test. The test was used to understand pre-service teachers' content knowledge relevant to geometry teaching at the beginning of the internship period.

In the Sri Lankan context, the "recruiting qualification" for pre-service teachers in mathematics courses (major subject and optional minor stream) has changed. The major stream pre-service teachers have mathematics achievement at GCE (A/L), and minor stream pre-service teachers have mathematics achievement only up to GCE (O/L). Therefore, geometry test items were prepared based on selected lessons on geometry and how to teach relevant content to the Grade 10 students. The test structure was prepared according to van Hiele's levels.

Table 3.3

Test paper structure

Topic	Assessment criteria
Triangle Quadrilateral Circle Tangents	Discriminate
	Properties
	Definitions
	Class inclusions
	Relationships
	Implications

	Proof
Assessment criteria Q1 to Q10	Is not aware of the basic concepts in geometry and has very limited knowledge in terms of geometric vocabulary
	Knows the concepts of the adjacent side, hypotenuse, opposite side and angles, congruence, perpendicular and parallelism in the plan
	Knows the concepts of symmetry, diagonal, bisector, midpoint and the concept of congruence, parallelism and perpendicular other thermoses relevant to circle, tangent

Data collection process in the geometry test

Pre-service teachers for the test were allocated index numbers and dates for the test. Index numbers were used to identify their responses. Depending on the principal's consent, logistic facilities were arranged for the test, inside the college on scheduled dates. Pre-service teachers were invited to participate in the test on the organised date. Therefore, the test items were prepared and validated with teacher educators at the TEIs. After the designated time to answer, scripts were collected and marked by one of the mathematics teacher educators at the TEIs.

3.6.3 Semi-structured individual interview

The pilot and main study involved semi-structured interviews with different stakeholders. According to the literature, several forms of interviews (structured and open) can be used. The semi-structured interview includes elements of both structured and open interviews. According to Denzin and Lincoln (1998), "the structured interview refers to a situation in which an interviewer asks each respondent a series of pre-established questions with a limited set of response categories" (p.52). In a structured interview, there is no probing or follow-up of answers. As a result, there is little room for variation in response, and the interviewer sets the pace and direction of the interview. A structured interview inadequately explores the reasons for incidents happening to the interviewee. To find out the learning process (of those who might feel nervous or find it difficult to articulate their thoughts in an open interview), the semi-structured interview is more suitable for this study. In the study, a group of pre-service teachers and mathematics teacher educators from selected colleges participated in the interview process. Thus, two types of semi-structured interviews were used:

- Semi-structured individual interviews–phases 1 and 2 for mathematics teacher educators.
- Semi-structured focus group interviews-phases 1 and 2 for pre-service teachers.

Hitchcock and Hughes (1995) described the semi-structured interview as quite valuable in the different stages of a study; that is, a short, preliminary investigative study designed to reveal issues which can be explored in greater depth later using a variety of techniques. Winwood (2019) explained that interviews empower participants to describe their teaching experiences and interpretations of real class situations. A semi-structured individual interview was undertaken with mathematics teacher educators. The semi-structured focus-group interviews were used for exploring pre-service teachers' pedagogical beliefs about geometry block-teaching. The group interview scheduled had pre-planned open-ended and closed questions and possible prompts prepared ahead of time.

The semi-structured individual interview schedule was used to conduct interviews with mathematics teacher educators in the selected pre-service teacher colleges. This group had a maximum of two teacher educators. In these interviews (each lasting approximately 20–30 minutes) teacher educators shared their ideas about pedagogical issues related to geometry at local pre-service colleges. Teacher educators had the right to decline to answer any questions. In this study, three teacher educators' interviews were recorded and transcribed. An initial briefing was given to mathematics teacher educators after phase 1 of the main study.

3.6.4 Semi-structured focus group Interviews

Focus-group interviews are a form of group interview, but it is important to distinguish the difference between individual interviews and focus-group interviews. Connolly, (2016) explained interviews are an effective tool for interpreting participants' beliefs, and feelings to understand how participants construct their realities. According to Watts and Ebbutt (1987), group interviewing encourages participants to relax and to feel at ease with other known peers with whom they can identify and relate. However, there are many definitions of a focus-group interview in the literature. Morgan (1997) states that a “focus-group relies on interaction within the group based on topics that are supplied by the researcher” (p.12). However, the main factor characterising focus groups is the insight and data produced by the interaction between participants.

Semi-structured individual interviews were used in the pilot study and focus-group interviews were used in phase 1 of the main study. The researcher chose the focus-group interview method because:

- Greater natural interaction is possible in group collaboration.
- Pre-service teachers would support each other to develop ideas.
- Time limitations and group interviews enabled the researcher to collect information from four pre-service teacher-students within 40 minutes.
- This would empower pre-service teachers; make novice teachers feel that someone is interested in their ideas.
- It reduces the potential power discrepancies between researchers and pre-service teachers.

Semi-structured focus group interview schedule

The semi-structured focus group interview schedule was used for pre-service mathematics teachers. Two groups of pre-service teachers from each of the selected TEIs participated in the process and focus group interviews were conducted as a small group discussion. The small size (four participants) of the focus group allowed pre-service teachers to discuss their perceptions and their experience in geometry (Morgan, 1997). Each pre-service teacher group interview lasted 20–30 minutes.

In the Sri Lankan context, pre-service teachers have few chances to participate in any discussions about their pedagogical experience. Therefore, semi-structured group interviews were used to encourage participation by this less-confident group (Liamputtong, 2008). The structured interview was organised to identify pre-service teachers’ perspectives and beliefs about technological pedagogical content knowledge (Mishra & Koehler, 2006), and the interviews were transcribed by the researcher and subjected to a thematic analysis to answer the research questions.

Online focus group interviews

The physical classroom context of the TEIs changed due to the sudden lockdown forced by the first wave of the COVID-19 pandemic at TEIs. Some semi-structured focus group interviews were scheduled based on the block-teaching experience of pre-service teachers and used for the online interviews in phase 2 of data collection. WhatsApp groups were set up by teacher educators at the TEIs because these used less data for pre-service teachers' smartphones and therefore it cost less for pre-service teachers. Even though the same interview schedules were used, pre-service teachers also discussed their online geometry teaching experience in secondary grades. Some block-teaching sessions of pre-service teachers were evaluated by mentors (teacher educators or senior teachers at school) through a paper-based standard evaluation sheet for TEIs' evaluation criteria. Analysis of these block-teaching phenomena was supported by pre-service teacher educators' views on their pedagogical experience at TEIs. These different sources (teacher educators, pre-service teachers) of interview text data provided evidence of how apps (GeoGebra) might be used for teaching and learning geometry content in secondary mathematics, even in the online context.

Importantly, there may be different social context aspects (online interview) that may influence mathematic pre-service teachers to engage (or not) in a different digital application (e.g., GeoGebra) in mathematics teaching. Gundumogula (2020) explained how online focus group interview text data provided evidence but maybe not like face-to-face interviews. Pre-service teachers learn in informative or formative ways in their professional practise by the end of their second year of (physical) classroom or online learning. Besides, interview data relevant to the knowledge and belief of pre-service teachers might enable them to harness the apps' affordances in a different learning context (online or physical). These data also provided evidence for the "digital divide" that was apparent among the selected TEIs.

3.6.5 Document analysis

This study was designed with the purpose of better understanding pre-service mathematics teachers' geometry teaching experience in their block-teaching. Pre-service teachers had given their teaching a concept, lesson, or topic. Some pre-service teachers provided documentary evidence of curriculum-specific pedagogical approaches (e.g., geometrical proofs) and students' assessment results to verify their focus group discussion evidence. Government acts and educational institutional settings all have a variety of documents that are generally accessible. Document analysis can involve teachers' information (such as reflective journals) or other material such as lesson plans.

In the current study, document analysis was completed to get an overview of the setting and to provide supportive evidence for the RQ. According to Yin (2002), the most important use of documents is to collaborate and augment evidence from other sources. He also explains that documents can be used for making inferences, but these inferences should be treated as clues rather than as definite findings. For example, in this study, information from the documents relevant to the mathematics curriculum was used to assist with validating the data from other sources of information such as interviews. O'Connor (2019) explained that the document analysis is a part of discourse analysis and has the main aim of investigating the social meaning of diagrams and text. A document analysis of my study ultimately enabled an in-depth analysis of interviews, which helped to update knowledge of pedagogical practices of the use of apps for mathematics education.

First, the official documentation relevant to the TEIs and secondary curriculum was analysed. For example, the official circulars, gazette, and National Education Commission reports were analysed to understand pre-service teacher education in Sri Lanka. An overall summary of data collection details is given in Table 3.6.

Table 3.6
Summary of data collection methods and participants

Research question		**Purpose**	**Instruments**	**Participants**
RQ1 RQ2 RQ3 RQ4	Pilot study	To validate test items for 40 pre-service teachers from two selected colleges	**Geometry test** **Survey questionnaire** **Focus group interview schedules.** (Two focus groups) **Semi-structured individual interview schedules**	80 pre-service teachers Selected 8 pre-service teachers for focus group One mathematics teacher educator for interview

RQ1 RQ2 RQ3	Main study (**phase 1**)	To analyse the pedagogical approach	**Semi-structured individual interview** schedules Documentary analysis	mathematics teacher educators from TEIs **(total of 8 teacher educators)**
RQ2 RQ3 RQ4		To identify pre-service teachers' pedagogy perspectives/beliefs	**Semi-structured focus group interview** schedules	8 pre-service teachers from one TEIs (2 focus groups) **(total of 16 pre-service teachers**)
RQ1		To explore pre-service teachers, Grade 10 subject content	**Geometry pre-test**	Pre-service teachers from control TEI1 and pre-service teachers experimental TEI2 **Phase 1** (total of 163 pre-service teachers) **Phase 2** (the same 163 pre-service teachers)
RQ1	Main study (**phase 2**)		**Geometry post-test**	
RQ2 RQ3 RQ4		To identify pre-service teachers' pedagogy perspectives/beliefs about the use of mobiles vs the traditional method	**Semi-structured focus group interview** schedules (2 from online)	8 pre-service teachers from each TEI (2X4) Two TEIs (2X4X2) **(Total of 16 pre-service teachers**)
RQ1 RQ2 RQ4		To analyse factors influencing outcomes	**Survey questionnaire**	Pre-service teachers from both TEIs
RQ1 RQ2 RQ3 RQ4		To analyse the pedagogical approach	**Semi-structured individual interview** schedules (online)	2 mathematics teacher educators from two TEIs **(Total of 2 lecturers)**

The study employed five data collection instruments: the survey questionnaire (pre-service teachers), pre-post geometry test (pre-service teachers), semi-structured individual interview schedule (teacher educators), semi-structured focus group interview schedule, and documentary analyses schedule.

3.7 Data analysis

The study employed five data collection instruments: a survey questionnaire (pre-service teachers), pre-and post-geometry tests (pre-service teachers), semi-structured focus group interviews (pre-service teachers), semi-structured individual interview schedules (teacher educators), and document analysis. This data analysis outcome has been described according to the methodological approach. The data analysis in phase 1 started with the pre-geometry test and the survey, followed by the qualitative semi-structured focused group interviews with participants (first-year pre-service teachers) and semi-structured individual interviews with teacher educators.

These interview data were associated with more understanding of the multiple meaning of individuals. This integration is the place in mixed method research where the quantitative data intersects with the qualitative data (Creswell, 2015).

Similarly, the data analysis of phase 2 started with the second-year pre-service teachers' survey and geometry test data. The post-semi-structured interviews in phase 2 were stimulated by a study of the pre-service teachers' experiences after the block-teaching periods. This data analysis process was summarised according to the research instruments.

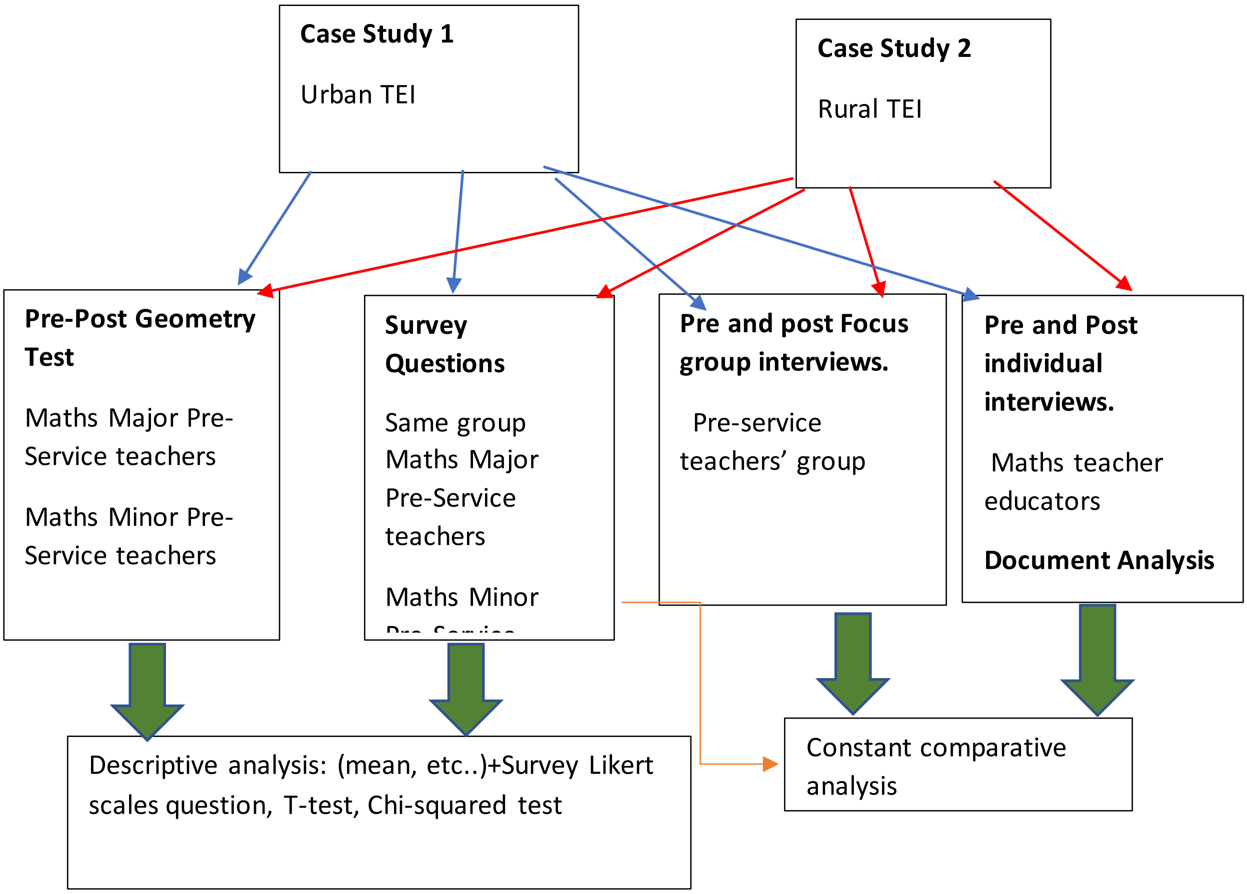


Figure 3.3 Data analysis plan

According to Figure 3.3, the research was designed as two case studies in these two TEIs. The comparative method was used to compare the two TEIs: one TEI used the traditional teaching method and the other one had to experience mobile apps. In the first year, phase 1 of the study had been completed in both TEIs. This provided the survey data, pre-geometry test data, and semi-structured interview data. In the second year after the block-teaching period, the same process was repeated in both TEIs to provide the post-test geometry data, survey data, and the post-semi-structured interview data. However, due to COVID-19 in phase 2 of the study many pre-service teachers from both TEI experienced online teaching and learning with MT and apps. The practical implementation of the data analysis plan is discussed in the next section.

3.7.1 Quantitative data analysis

The type of variable in the research instrument determined the quantitative data analysis procedures. In the survey, measurements are dependent when measurements of one group are paired with the measurements from the other group (Cohen et al., 2018). In the geometry test scores different statistical tests were used to determine the difference between groups. For example, comparing the two means of pre-service teachers' mathematics scores in a dependent design requires a paired *t*-test; if measurements are independent, an independent t-test can be used (Cohen et al., 2018). For some survey questions, factor analysis and Chi-squared test were used.

The current study explores the effect of apps on the geometry achievements of pre-service teachers, which aligned with the study's aims. When testing a null hypothesis, a p-value is calculated to determine the significance level, with 5% as the significance level. Therefore, any value smaller than that is considered statistically significant (Cohen et al., 2018). However, for some research, it is often not possible to obtain a random sample of participants (Noortgate et al., 2015). The current study had a similar issue as the number of research participants for the study were English-medium mathematics teachers who undertook major and minor mathematics courses during the year 2018. For different reasons, the selection standard for pre-service teachers for the major mathematics courses in the English medium has varied at different TEIs.

Geometry test data analysis

In the pilot study, mathematics test items were validated by the mathematics teacher educators in charge of mathematics.

Table 3.7
Test paper structure and evaluation

Question number	Level	Assessment criteria and awarded	
Q1:	1		
Q3:		5	Discriminate
	II	5	Properties
	III	5	Definitions
		5	Class Inclusions
		5	Relationships
		5	Implications

	IV	20	Proof
Q2: Q4:	I	10	Is not aware of the basic concepts in geometry and has very limited in terms of geometric vocabulary. The pedagogical approach to the geometry content
	II	10	Knowledge of the concepts of the adjacent side, hypotenuse, opposite side and angles, congruence, perpendicular and parallelism
	III	15	Knows the concepts as the axe of symmetry, diagonal, bisector, midpoint and the concept of congruence, parallelism and perpendicular
	IV	15	

The test items' evaluations and marks were awarded according to van Hiele's levels. The next section discusses the basic analysis of semi-structured interviews and open-ended survey questions.

3.7.2 Qualitative data analysis

The main study has qualitative data relevant to semi-structured interview data, and how DT or mobile apps (GeoGebra) influenced mathematics teachers' pedagogical perspectives and beliefs. This has interpreted the meaning of any pattern (Cohen et al., 2018), provided insights into the participants' motivations and reasoning, and given trends in the use of mobile technology by pre-service teachers for pedagogical approaches to geometry (social processes) in the learning context (society).

Therefore, pre-service teachers' semi-structured focus group interviews and open-ended questions about pre-service teachers' geometry experiences have generated qualitative data. Moreover, both phases of the study had more mixed method research characteristics such as exploration, and the researcher acting as the primary "instrument" of data collection. Denzin and Lincoln's (2018) work on analysing qualitative research traditions with different aspects, such as the place of the qualitative researcher and multiple material practices, supports the interpretation of meanings. Thus, this study also follows the same path. Before the semi-structured interviews, the demographical variables of these participants were analysed by the researcher. This analysis provided the ability to understand participants' experiences of a situation as in-depth as possible (Denzin & Lincoln, 2018).

The qualitative data consists of eight focus group semi-structured interviews (pre-service teachers and individual interviews for mathematics teacher educators). The interview data provided information about differences in teaching practices between teachers with GeoGebra and traditional methods. Data analysis consisted of examining, categorising, comparing or otherwise recombining the evidence to address the initial research question of the study (Campbell et al., 2013). The constant comparative method was used to analyse the data with inductive category coding. The following diagram, adapted from Maykut & Moorhouse (1994), describes the steps in a constant comparative method of analysis of qualitative data relevant to the study.

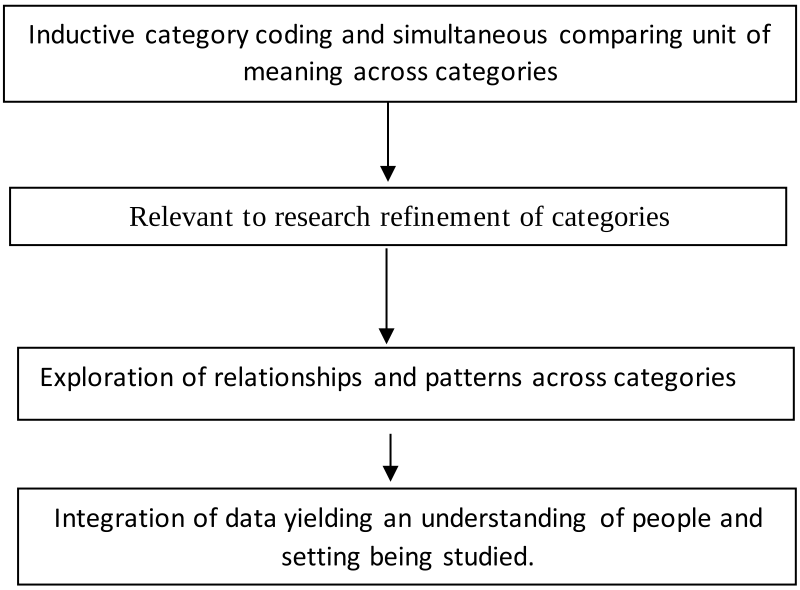


Figure 3.4 Constant comparative method of data analysis

In the interviews' data analysis, codes of meaning were generated from the qualitative data. For example, mathematics teacher educators have defined "affordances of GeoGebra" with two different aspects.

Two aspects of affordances were extracted from the teacher educators' interviews. The first aspect of the data was "…pre-service teachers need to be able to cope with the objects of the GeoGebra: they have to find out how it works, and how it might be used for geometry lesson". A new unit of code for "affordances of apps" was generated from these interview transcripts.

A second teacher educator stated, "affordances of apps is often considered in professional development. It means, the affordances of apps add value to professional development". This idea was compared to all other codes of "affordances". Understanding the proper meaning of "affordances of apps" relevant to the current study was created. In this process, some subsequently grouped (categorised and coded) data with similar units relevant to the meaning (positive beliefs of GeoGebra) were merged. In this method, initial categories were changed, merged (confidence of the useful apps for geometry teaching), or omitted (thinking of GeoGebra for geometry teaching). New categories (e.g., meta-cognition of geometry teaching) were generated. Therefore, a new relationship relevant to pedagogical perspectives and beliefs of the pre-service teachers (meta-cognition of geometry teaching) about the use of apps in geometry in their block-teaching has been identified.

3.7.3 Validity and reliability

The number of participants from each TEI varied. The participant sample size influenced the design sensitivity of the observed phenomena in the research (Cohen et al., 2018). Therefore, it is important to report the "effect size" for the sample which can be detected using statistical tests (Serdar et al., 2021). First, quantitative data were analysed using factor analysis and validated according to the test. For example, the survey had 20 statements that invited Likert-scale responses relevant to the TPACK framework. Factor analysis of the relevant dataset was validated beforehand, using SPSS v20 statistical software. (See section 5.4.1).

Pre-service teachers' beliefs were ascertained in the survey questions, including through open-ended questions that invited answers on what is taught in geometry (content knowledge), how to teach geometry (pedagogical knowledge), and the teacher's knowledge of the app used (technological knowledge). For example, pre-test and post-test geometry scores were used to test the hypothesis that there is a difference in pre and post geometry test scores. Thus, this study used hypothesis testing, which is commonly used by researchers (Serdar et al., 2021), A null hypothesis takes a form such as "no effect", "no difference", or "no correlation". In a sample, these were validated from the pilot study. As this study followed a sequential mixed methods research approach, the analysis and data validity process were dependent on the approach taken.

3.7.4 Data coding

To maintain anonymity the two pre-service teachers' education colleges were named urban TE1 and rural TE2. TE1 was in an urban area with high-speed internet and in a Wi-Fi network facility available area. TE2 was in a rural area that has weak telecommunication facilities with relatively poor network connectivity and speed. The research participants MT1, MT2 to MT8 were qualified mathematics teacher educators from selected TEIs with more than 10 years' experience as mathematics teacher educators and mentors. These teacher educators were aged 40 to 50 years and had degrees in mathematics including postgraduate qualifications relevant to mathematics education. T1 to T163 were pre-service teachers who participated in the main study (both phase 1 and 2).

3.7.4 Social and personal bias

Pre-service teachers had unexpected experiences during their block-teaching. The major bias for pre-service teachers' beliefs came with political unrest in the country (political unrest after the Easter terrorist bomb blasts at Sri Lankan churches, and pre-service teachers' physical and mental stress resulting from the forced COVID-19 lockdown). These challenges were identified through the comparison of interview data. Therefore, pre-service teachers' knowledge and beliefs about using apps for block-teaching practices were challenged by personal and social issues (e.g., prior knowledge, and socio-cultural meaning relevant to the setting).

In addition, the researcher of this study was a teacher educator who worked for the MoE for nearly 20 years. The personal bias may have influenced the focus group and individual interviews for pre-service teachers and affected teacher educators' responses. The main argument is that sometimes this bias may have influenced the negative or productive outcomes. For example, during the forced lockdown for COVID-19, some pre-service teachers use GeoGebra out of necessity as a pedagogical approach for geometry teaching. Hence, pre-service teachers' beliefs in pedagogical approaches (relevant to geometry problem solving) with apps may have changed from phase 1 to phase 2 because of this factor.

3.7.5 Methodological limitations

There is an inherent limitation in a semi-structured focus group approach when members repeat the same information (dominant voice) to other group members, and this could influence the findings reported in the study. Hughes et al. (1997) asserted that focus groups are particularly useful for researching the perspectives of groups of participants from culturally diverse backgrounds. However, when the voluntary participants were selected for phase 1 of the study to gather detailed information about their beliefs regarding geometry teaching mediated with GeoGebra, it was difficult to find voluntary pre-service teachers who were socially and culturally diverse. Moreover, some of them refused to participate in the study (an online focus group meeting) due to additional costs incurred during the COVID-19 pandemic. Therefore, a few more pre-service teachers left the phase 2 focus group as they could not afford the mobile data package cost for interviews.

3.8 Ethical issues

This research proposal was approved by the University of Waikato, Ethical committee under code 32555 on 28 February 2018. Including informal consent for all participants, the researcher gained permission to gather data from the principals of the selected TEIs. All participants were informed of the purpose of this study. Any queries by the participants about this research had been clarified and answered. Those teacher educators who agreed to respond to the interviews of this study signed consent forms. All pre-service teachers who agreed to participate in the geometry test and survey questionnaires also signed consent forms before the test was administered. Participants had the opportunity to withdraw from the research without penalty. All surveys and interview respondents were guaranteed confidentiality and protection of their identity. Every effort has been made to ensure the anonymity and confidentiality of data.

3.8.1 Participants' right to withdraw from the study or to withdraw data

Participation in contributing to this research was voluntary. Pre-service teachers and teacher educators had the opportunity to withdraw individual data at any time up until the point of analysis. However, once data had been contributed to focus group interviews, participants were unable to withdraw their responses, as this would affect the remaining data. Once the focus group interview was completed, case studies were completed and no further participation was required, and data analysis would be underway, so withdrawing was no longer an option. These issues were outlined in the letters given to the participants with the informed consent forms.

3.9 Chapter summary

This chapter starts with methodological aspects to provide an awareness of the assumptions underlying this research. All the assumptions discussed here have influenced the researcher to select a methodology for the research with the epistemological and ontological assumptions. Thus, the mixed-method research design was chosen to collect the rich data needed for the study. The research design for this study uses comparative case studies that are analysed through qualitative and quantitative methods. Survey questionnaires were used to evaluate participants' beliefs and pedagogical issues (before the block-teaching started in the first year) and to determine their changes in teaching geometry (at the end of the block-teaching in the second year).

A descriptive statistical method was used to analyse the pre-service teachers' survey and geometry test. In-depth, semi-structured, face-to-face interviews, as well as focus-group interviews, were used as qualitative data collection methods. Furthermore, the justification for each of the data collection methods used in the study and data coding was given.

Finally, to ensure the trustworthiness of the study, validity and appropriate ethics were discussed. The case study analysis gave a different perception of pre-service teachers in their geometry teaching, and pedagogical approaches to using mobile technology relevant to the research questions RQ1 and RQ4 are discussed in the next chapter.

Chapter 4
Pre-service teachers' knowledge and practices

4.0 Introduction

Chapters 4 and 5 present the two chapters of findings that address the research questions that guided this thesis. First, Chapter 4 presents findings relevant to research question 1 (RQ1) and research question 4 (RQ4). These findings are organised around pre-service teachers' different perspectives (e.g., pedagogical, technological, and epistemological) that might influence their knowledge when they are using mathematics apps (e.g., GeoGebra[34]) for mathematics education. The next chapter of findings (Chapter 5) looks at pre-service teachers' beliefs about teaching and learning secondary geometry.

This chapter is organised under six themes and the following research questions:

RQ1 What aspects of using mobile technology apps (e.g., GeoGebra) might influence the geometry content knowledge for pre-service teachers involved in junior secondary mathematics education?

RQ4 How might GeoGebra be used for teaching and learning geometry content in secondary mathematics?

Relevant to the research context, a geometry mobile application (app), known as GeoGebra, was used by pre-service teachers. In general, GeoGebra is a free downloadable mathematic app for smartphones as well as laptops/computers. In addition, it is the only DGE app recommended for secondary geometry by the curriculum authorities in Sri Lanka, where various technical and pedagogical benefits and challenges might influence geometry teaching and learning in the selected research context.

Six main themes and subthemes emerged from the data; each theme defined for this study addresses the local (Sri Lankan) context. As these themes are not mutually exclusive, they sometimes intersect and are influenced by each other, and sometimes these intersections can be seen among the subthemes.

The first section (4.1) examines the main theme regarding the pre-service teachers' geometry learning. Section 4.2 discusses the second main theme which is geometry teaching. The third section (4.3) is the third theme that

[34] The GeoGebra application in mobile and web-based versions are both defined as "app" in this study.

describes the perspectives of GeoGebra for geometry teaching. The fourth section (4.4) explains Technological Pedagogical Content Knowledge (TPACK) for apps, approaches, or perceptions in geometry teaching. Possible factors that influence the use of apps for geometry are discussed in section 4.5. Finally, section 4.6 presents the chapter summary. The coding methods used for the study are described in chapter 3 (see section 3.5.2). For anonymity, codes T1 to T163 were used to identify the pre-service teachers. For example, the code P1/T1 represents pre-service teacher #1's responses in phase I of the study. Similarly, MT1 represents mathematics teacher educator #1 at TEIs.

4.1 Geometry learning

In this study, geometry learning is defined as experiences of geometry as a secondary student or pre-service teacher. Evidence was obtained from the participants' focus group interview transcripts as well as from open-ended survey questions and pre and post geometry test data. The analysis of data indicated that the individual pre-service teachers defined themselves as "teachers" and "learners" when using the apps for teaching and learning geometry. These different perspectives are linked to pre-service teachers' beliefs, experiences, knowledge, and affordances of apps when interacting with the learning context.

The survey question (from Q 2.12) revealed that over 42% of pre-service teachers had negative responses about geometry teaching and learning with apps in the block-teaching (TEIs). The remaining pre-service teachers 58% gave positive or non-responses. Three subcategories were generated that were relevant to the perspectives of geometry learning: namely, pre-service teachers' geometry learning with apps, geometry content knowledge and challenges in the learning context.

4.1.1 Pre-service teachers' geometry learning with apps

This section presents pre-service teachers' perspectives of geometry learning with apps during block-teaching sessions. Three different categories of pre-service teacher perspectives were identified from the focus group interview data.

The first category of pre-service teachers did not know anything about GeoGebra. During the semi-structured focus group interviews in phase I of the study, pre-service teacher P1/T9 explained her perception of geometry learning at TEI:

> ***P1/T9**: Not like earlier, now we don't have a geometry module at TEIs. I think all are surviving with our secondary school geometry knowledge or [our] own geometry learning before the block-teaching practice. I had a lesson for mid-point theorem at block-teaching, mathematics is my minor. I have learned it from my roommate because now I can't remember what I learned at GCE (O/L) which was 7 years ago. I don't know about apps. Is GeoGebra easy?*

Pre-service teacher P1/T9 said that she had not participated in any geometry module at TEI. She learned the relevant content from a friend. TEI policy presumes that pre-service teachers have entered the mathematics course with appropriate secondary mathematics content knowledge. Pre-service teachers are enrolled in the mathematics courses based on an interview and results of a national examination (GCE [A/L]), or mathematics minor course students are selected from their GCE (O/L) mathematics result according to TE policy. In other words, pre-service teachers need to understand the TEI learning culture, as well as mathematics curriculum resources and other block-teaching aspects.

The second category is pre-service teachers' willingness to use GeoGebra facilities for learning. Therefore, they experienced DGE features in GeoGebra for their learning. The mathematics major course teacher, P1/ T32, explained her experience of geometry learning for block-teaching:

> ***P1/T32:** After teacher education reforms**,** we are learning geometry from the students' textbooks before teaching (geometry). Before block-teaching, I practised (a few times) all problem-solving activities given in the secondary students' textbook for geometrical proof in Grade 10. Still, I am relying on textbooks and TG. The geometrical proof is a very difficult lesson to learn for teaching. I have asked my friend how to learn proof for teaching and downloaded some activities. He taught me how to use GeoGebra for diagrams with dragging and manipulation for flexible block-teaching lesson design.*

P1/T32 demonstrates different ways of geometry learning, such as drill and practising problem-solving activities from the student textbook (curriculum materials). The participant also used additional e-resources from the internet and apps. It seems that materials are the only reliable resource in shaping pre-service teachers' learning opportunities to teach mathematics. However, P1/T9 and P1/T32 indicated that geometry learning for teaching requires one's phase of education, curriculum resources or apps, and peer support. Moreover, these pre-service teachers used textbooks and TG as the main sources for planning lessons and choosing the content to be taught. Some pre-service teachers used DT resources (GeoGebra) for DGE facilities (e.g., dragging) for planning and implementing geometry lessons in the classroom.

The third category of pre-service teachers has a negative view of DT (GeoGebra). P2/T38 indicates a negative perspective on the use of apps for geometry learning in her phase II, semi-structured focus group interview:

> ***P2/T38****: Apps at TEIs may impair pre-service teachers' cognitive and essential skills of geometrical construction. The paper and pencil method were suggested in the TG and the national GCE (O/L) exam, and that's what we have to do at TEI. Why [do] we have to use GeoGebra? Is it because of others?*

According to P2/T38, the TG and GCE (O/L) examinations are problematic institution elements for making decisions on the use of apps at the classroom level. Specifically, P2/T38 thought the TG given to schools meant that teachers are expected to conform to the rules imposed upon them, which they may not have had control over. P2/T38's perception was that apps may not support pre-service teachers' skills needed for geometrical construction. This may be due to the pressure of the exam-oriented culture or the negative beliefs about DT in Sri Lanka. It is difficult to see why P2/T38 feels apps may "impair pre-service teachers". Sometimes her block-teaching schools were under pressure to develop the essential skills needed for students in the use of the compass for geometrical construction problems in the national exam [GCE (O/L)]. In contrast, P1/T32 thought that DT (GeoGebra) has facilitated flexible teaching; it can be changed according to different geometric content and their teaching perspectives. Thus, P1/T32 and P2/T38 had different perspectives about the use of apps for geometry learning in block-teaching.

A quantitative analysis of findings provided evidence that possibly teachers' knowledge of geometry content changes with their instructional situation relevant to the curriculum in block-teaching. Some pre-service teachers indicated that geometry content knowledge is important for understanding students' needs during block-teaching. The next section discusses the geometry content knowledge of pre-service teachers.

4.1.2 Geometry content knowledge

In the tertiary mathematics education (pre-service teacher education) sector (which this study focuses on) geometry content knowledge includes not only subject matter but also the capacity to implement relevant curriculum content in the classroom context at block-teaching schools. Therefore, pre-service teachers' geometry content knowledge in selected geometry content is considered. In addition, how instructional strategies for geometry content are given by the pre-service teacher relevant to the context are also considered.

To address the geometry curriculum issues relevant to the content knowledge, a test (similar to the geometry assessments at TEI) was given to the same group of pre-service teachers in the sample on two occasions (in their first year and second year). However, it was challenging to measure all aspects of geometry curriculum content knowledge of pre-service teachers directly, given the limited time for data collection. The geometry test items (see section 3.8 in chapter 3) were selected by teacher educators during the data collection period. Each of the pre-service teachers' responses (in the test paper) was marked and a final test score was awarded.

The geometry curriculum content test scores were subjected to a paired sample *t*-test (Table 4.1) to determine whether a significant mean difference existed between the first year and second-year test scores. This compares pre-service teachers' geometry curriculum knowledge at the start of the first year (pre-test) with the end of the second year (post-test), resulting in a 0.05 level of significance (Table 4.1).

Table 4.1

Paired samples' t-test statistics

			Mean	Std. deviation
Pair 1	Geometry post-test marks		66.00	13.425
	Geometry pre-test marks		52.86	8.924

		Correlation	Significance
Pair 1	Geometry post-test marks Geometry pre-test marks	0.652	.000

Table 4.1 shows that pre-service teachers' mean score (x = 52.86) for geometry content knowledge when they entered the TEI improved to x = 66 at the end of the second year. According to the geometry pre and post-tests mean values, the selected pre-service teachers from both TEIs improved their performance on selected geometry curriculum content when they moved from the first year to the second year at TEIs. An examination of the correlation value (r=0.652) reveals that there was a statistically significant mean difference in favour of the post-test (e.g., geometry content and pedagogical approach). The significant mean improvement supports the findings, with a positive (0.652) correlation. These pre-service teachers learning in TEIs have improved their geometry knowledge to work as mathematics teachers from the first year (pre-test) to the second year (post-test) as sample correlations are significant in the second year.

Despite the geometry test being based on the pre-service teachers' curriculum content knowledge and block-teaching experience (pedagogical approach) on selected topics from the geometry curriculum, several pre-service teachers thought it was challenging in their working context. The next section will consider the pre-service teachers' learning context during the mathematical (problem solving) activities and practice in the block-teaching to address RQ4.

4.1.3 Challenges in the learning context

It is essential to know about pre-service teachers' geometry knowledge and how it relates to their learning context. Teacher educators play a significant role in mathematics teacher education and the implementation process of pre-service teacher education. Teacher educator MT8, a mentor of block-teaching in geometry, was also concerned about pre-service teachers' (lack of) experience in geometry as well as their geometry teaching and working space:

> ***MT8****: The content of the geometry curriculum is developed in different units at the secondary level, but many English medium (bilingual) pre-service teachers do not have a physical classroom at many block-teaching schools, sometimes not even a blackboard. Students learn in corridors, only on a*

chair. Therefore, some pre-service teachers were not good at their secondary geometry at school as secondary students; it is challenging to teach a subject like geometry without a visual display or at least a blackboard. These pre-service teachers can indeed learn about geometry teaching from and through teaching. But it is not easy. There are only a few (less than 10) secondary students in bilingual mathematics Grade 10 classes and many pre-service teachers used their own DT device (laptop/iPad or smartphone), which is the best option for geometry teaching when they do not have a blackboard. But its [App] is not in DT policy at TEIs. They are more reliant on e-resources than textbooks. The curriculum assessment of geometry (before the teaching practice evaluation) was successful for our pre-service teachers.

MT8 from TEI1 explained the challenges of block-teaching experience at mathematics (bi-lingual) pre-service teachers. She was explaining a curriculum policy issue in Sri Lanka with reforms, where policymakers have not been concerned about ground-level classroom infrastructure facilities. According to her, pre-service teachers were more reliant on DT as they are bi-lingual mathematics teachers. In addition, pre-service teachers were prompted to assess curriculum content. Teacher educators may tend to focus on the evaluation of pre-service teacher compliance and curriculum coverage, rather than just supporting teachers to enable quality teaching without much concern for the physical context in their secondary classrooms. However, the content knowledge of pre-service teachers was tested in selected topic comparisons before and after their block-teaching practice in the second year.

In an individual interview, other teacher educators indicated that only some pre-service teachers might gain the confidence to use DT for teaching geometry after the block-teaching sessions at the end of the second year. However, these pre-service teachers can learn about geometry teaching despite the many physical barriers, and, at the same time, it would be beneficial for them too. Fewer infrastructure facilities are available for bilingual mathematics students at block-teaching schools. P1/T11 has defined it as an opportunity to use computer labs or multimedia rooms for their secondary mathematics teaching and learning because they are bi-lingual teachers as she did not have a physical classroom:

P1/T11*: We are English-language mathematics pre-service teachers. We gain more benefits with mobile apps compared to mother-tongue mathematics pre-service teachers. Normally I think our curriculum has more space for digital applications and (English medium) e-resources for* *geometry learning. We have fewer bilingual mathematics students and sometimes we don't have physical classrooms. I am always confident to teach with geometry apps and use the multimedia stuff for my mathematics lessons in the lab. When we used DT in our block-teaching, sometimes we faced problems from our mentors. We need a proper policy, at least for bilingual teachers to use apps at TEI and schools.*

Pre-service teachers selected for this study are bilingual mathematics pre-service teachers. P1/T11's perception is that the English-language (bilingual) curriculum generates new learning spaces to use apps for geometry learning. In these transcripts, the student learning context is woven together with teaching. The learning context they explain is inseparable from teaching in Sri Lanka. The next section addresses the pre-service teachers' willingness to teach geometry.

Pre-service teachers' confidence in teaching geometry was affected by institutional and social elements outside their control because of the reforms (bilingual). There are many other factors (discussed in the next section), which appear to be external but affect pre-service teachers' negative or positive perspectives on the use of apps for geometry teaching.

4.2 Geometry teaching

This section explores the notion of geometry teaching from pre-service teachers' perspectives and evidence from findings. Their understanding of geometry and how to teach it are influenced by many factors. Pre-service teachers are at the beginning stage of their career during the block-teaching. During the block-teaching at school, they are accepted as training teachers and some schools assigned only a few geometry lessons. This section includes confidence in geometry teaching, confidence in the use of apps for geometry teaching, and teaching with reflective practises.

4.2.1 Confidence in geometry teaching

Specific knowledge about the selected geometry content as well as relevant curriculum resources that the pre-service teacher is supposed to use at block-teaching sessions (in Grade 10) is considered here as geometry knowledge. Thus, confidence in geometry teaching has been analysed with their achievement of the post-geometry test. Table 4.2. shows the analysis of mean statistics of geometry tests and responses to the open-ended survey questions (Q 2.11: "Are you confident to teach the secondary geometry curriculum?")

Table 4.2
Mean statistics of geometry test and confidence of geometry teaching

	Gender	Mean	Std. deviation	Std. error mean	Not confident to teach geometry at block-teaching)
Phase 2 of the study	Male	66.50	10.690	3.086	49%
	Female	65.88	14.081	1.972	32%
Phase[35] 1 of the study	Male	54.50	7.550	2.179	-
	Female	52.47	9.242	1.294	-

The group statistics in Table 4.2 show the common and increasing trend of mean marks for geometry post-test and pre-test of male pre-service teachers (around 55 to 67) compared to female pre-service teachers (52 to 66) may be fewer male teachers (19%) enrolled at TEIs. Analysis of open-ended survey questions provides evidence that nearly half (49%) of these male teachers were not confident in teaching geometry even though they had a higher mean score (67) on the geometry tests. Similarly, of the female teachers (81%) who had a mean score of 65 for the geometry test, 32% (one-third) were also not confident to teach selected geometry content at the secondary level. However, across both TEIs, around 49% of males and 32% of female mathematics pre-service teachers did not have sufficient confidence to teach selected geometry curriculum content at the secondary level, even after the end of the second year at TEI. The gender difference is not statistically significant at the TEIs. A potential explanation is that geometry teaching confidence may vary by gender, although the number of male pre-service teachers in the sample (due to the low enrolment of male pre-service teachers in the TEIs) was low, which may have influenced the statistical significance value.

The final argument is that around half of the male and one-third of the female pre-service teachers were not confident to teach geometry at the end of the

[35] In Phase 1, some responses "not confident to teach geometry" were not collected because after second year only pre-service teachers will be placed in state schools for geometry teaching.

second year at TEIs. This is surprising as the survey data showed that many pre-service teachers had pass grades for GCE (A/L) and GCE (O/L) mathematics. Therefore, despite having the highest level of secondary mathematics achievement, they were not confident teaching secondary geometry.

4.2.2 Confidence in the use of apps for teaching

The interpretation of pre-service teachers' focus group interview data so far has suggested more support is needed for secondary level geometry teaching with apps. However, all pre-service teachers who participated in the study did not use GeoGebra for their block-teaching. Instead, some of them were involved in limited GeoGebra-mediated self-learning or traditional teaching of GeoGebra for lesson development. The survey responses relating to pre-service teachers' confidence in geometry teaching with apps were analysed.

The target variable was "confidence of geometry teaching" (the dependent variable) described with individual subclasses (TEI) (in order to best explain the data set of categorical variables). It shows that more female pre-service teachers (19%) have the confidence to teach geometry with apps compared to male pre-service teachers (10%); this may be due to the low male participation in the sample. However, the Chi-squared test (goodness of fit) shows that gender difference did not influence the responses of secondary geometry teaching with the app (χ_2=.466, df=1, p>.05). Similarly, the Chi-squared test in Table 4.3 shows (χ_2=.101, df=1, p>.05) that TEI type (TEI1 or TEI2) also did not influence the beliefs of secondary geometry teaching with apps.

Table 4.3

Chi-squared tests (confidence of geometry teaching with apps vs TEI)

	Value	df	Asymptotic significance (2-sided)	Exact sig. (2-sided)
Pearson Chi-squared	.101[a]	1	.751	
Continuity correction [b]	.004	1	.951	
Likelihood ratio	.101	1	.751	
Fisher's exact test				.802

Note: (a) 0 cells (0.0%) have an expected count less than 5. (b) Computed only for a 2x2 table

Table 4.3 shows the contextual condition (a) satisfied for the Chi-squared test. Even though the two-tailed significant value (p) is .802, which is higher than p, this means (null hypothesis: confidence in geometry teaching does not depend on the TEI) the null hypothesis is not rejected. Therefore, a selected sample of pre-service teachers' beliefs on geometry teaching with the app at a secondary level is not influenced by the institutional-level factors. However, qualitative data shows that there is a difference (confidence in geometry teaching with apps) between mathematics minor and mathematics major students in TEI1 only. There have been many debates on the underlying interpretation of what type of views on geometry teaching is needed to teach mathematics with apps effectively.

In general, this is more evident at TEI2 (geometry teaching with apps) where some pre-service teachers have a negative impression of geometry teaching. For example, in the focus group interview, P2/T46 (mathematics major student) had discussed confidence in the geometry lesson content but had less confidence in teaching with apps:

> ***P2/T46*** *I am not confident teaching geometry with apps. I know the geometry content well; however, it is challenging to teach with apps. When I reflect on my teaching, I realised that as many secondary students like to rote-learn the geometry content. Other students did not like to learn geometry, it was difficult for me. The 40 theorems of the secondary student's textbook had an impact on the GCE (O/L) exam results and students learning.*

P1/T46 noted rote-learning of the geometry 40 theorem as a common difficulty among secondary students. She further explained that many students do not like to learn geometry, which made her less confident to teach. The survey responses also supported this argument. Different dimensions can be identified from the data to describe mathematics teachers' beliefs about teaching with apps (these dimensions of beliefs will be discussed in section 5.1). P1/T46 discussed her reflective practices in block-teaching.

In the (open-ended) survey question Q 3.12 (What is your point of view for teaching with GeoGebra?), some pre-service teachers (29%) considered GeoGebra as the support mechanism for teaching. Moreover, 25% of pre-service teachers talked about the perspective of geometry teaching with

GeoGebra, such as improving specific content knowledge (9%), or pedagogical content knowledge (PCK) (12%), and 4% did not give a reason.

Semi-structured focus group interview transcripts showed that pre-service teachers had many views about geometry teaching with apps. These views of teaching geometry provide more evidence for pre-service teachers' knowledge of teaching with DT as well as their block-teaching practice concerning geometry. For example, P2/T61 discussed his perceptions of overall geometry teaching in the interview transcript below:

> ***P2/T61:*** *Familiarisation with the language of geometry and students' experience in private tuition context play a decisive part in making possible the understanding of many conceptions (e.g., figure symmetry) in geometry block-teaching at school. Apps may have an option to present a geometry diagram as a visual presentation, but we are not exposed to the advanced topic (e.g., cognitive aspects of geometry) at TEI so sometimes I feel it misleads us.*

P2/T61 explains that the development of secondary students' knowledge about geometrical language (such as naming figures plus definitions) is essential. Identification of apps for visualised geometry diagrams has challenged his teaching. He elaborated on the support mechanism (role of external learning) as a benefit of block-teaching schools. Another factor that P2/T61 explains is the state of pre-service education at TEIs. Pre-service teachers are not exposed to advanced topics in geometry. P2/T61 described how sometimes this learning situation is compounded by not having a clear idea of the direction in which pre-service teaching of geometry is going. However, 46% of pre-service teachers did not respond to survey question Q3.1 at all. P2/T20 explains his pedagogical knowledge about geometry teaching (from the survey Q3.1):

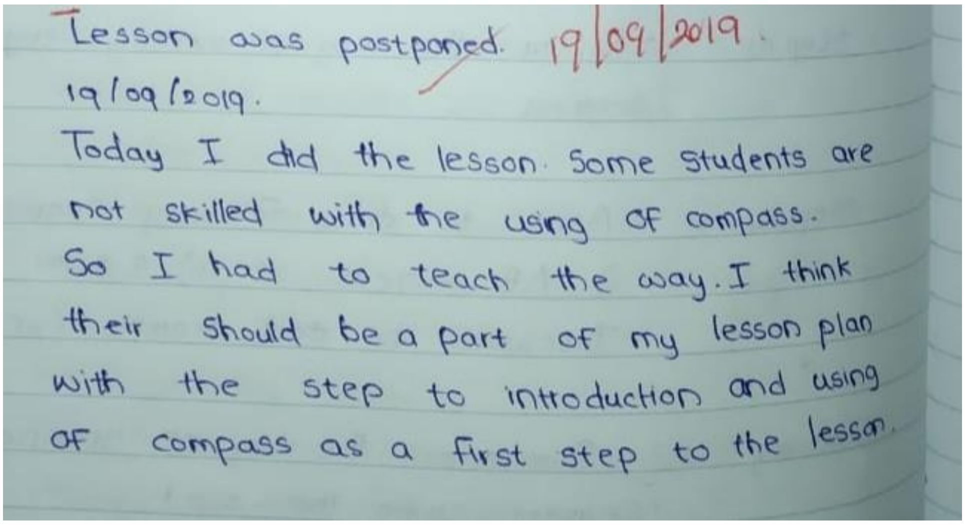
> ***P2/T20****: My teacher taught me geometry problem solving at secondary school with traditional instruction. Every day is the same: the teacher shows us several examples of how to solve a specific type of problem on the blackboard with the following directives. We absorb the teacher's instruction and practise many geometry problems [with] step-by-step instruction. It was an excellent method to cover the content of our exam. To teach geometry problem solving is not like our days. Some students do not read the problem; others do not investigate a mathematical idea or deal with the situation. Only a few students make, test, and verify their conjectures in a plethora of approaches. The way students think is different. Some do not understand problems due to the medium of instruction. It is so difficult to cover some geometry content with the given strand for the grade*

and students' learning needs. We like to use the app, but I don't know how to use [it]. I am stressed about my geometry teaching.

P2/T20 shares his perception of acquired geometry knowledge, which he states is a traditional mathematics instruction in which he is "absorbing" the teacher's guidance and following sequential steps to solve a geometry problem. P2/T20 thought that he could solve any secondary geometry problem by using step-by-step instructions, maybe due to his procedural knowledge. By *procedural knowledge*, he may mean the ability to follow sequential steps in a geometry problem. P2/T20's learning context as a student played a vital role in his teaching. However, he has a problem with geometry teaching now. He thinks some students have problems translating verbal statements of a geometry problem into a visual representation. Thus, P2/T20's view is that the nature of the geometry problems allows different cognitive processes to be exhibited for a solution. This discrepancy between what one's secondary students need and what the curriculum states must be taught is confusing for some pre-service teachers.

4.2.3 Teaching with reflective practice

Semi-structured focus group interview transcripts showed the implicit tension between complying with secondary grade-level expectations and meeting the learning needs of students. The pre-service teacher (P2/T30) wrote about the drawbacks of her instructional practices in teaching in her lesson plan reflections (See Figure 4.1). Her block-teaching practice has a reflection at the end of the lesson plan, which is considered one of the sources of the learning-through-teaching experience. The local TEIs' pre-service teachers' lesson plans include a section for their reflection on block-teaching. This gives practitioners (pre-service teachers) a challenging opportunity to learn from teaching with reflective practices.



Lesson was postponed. 19/09/2019.
19/09/2019.
Today I did the lesson. Some students are not skilled with the using of compass. So I had to teach the way. I think their should be a part of my lesson plan with the step to introduction and using of compass as a first step to the lesson.

Figure 4.1. Lesson plan reflection (P2/T30)

The teacher's lesson plan shown above (Figure 4.1) includes self-reflection providing geometry content details to analyse pre-service teachers' strategies of their interactions with students in the geometry lesson. The teacher (P2/T30) explained how she felt about her instructional practise because it provides an individual with a personal catalogue of their recent personal experience.

In addition, this written self-reflection by P2/T30 reminds the teacher of what secondary students may be experiencing during their geometry lessons. Her reflections projected the pre-knowledge needs of the geometrical construction lesson relevant to the selected grade. This experience may be beneficial for her next block-teaching sessions. However, this evidence from the semi-structured focus group interviews about block-teaching revealed that geometry pre-knowledge affects a pre-service teacher's perceptions of geometry teaching. These pre-service teachers may have limited knowledge of geometry which may, in turn, affect their use of GeoGebra for teaching.

4.3 Perspectives on GeoGebra for teaching

In this section, pedagogical perspective means pedagogical benefit or challenges of GeoGebra to teach appropriate geometry content effectively. These subthemes emerged from evidence gained from the focus group interview transcripts. Three categories of pre-service teachers have been identified. These transcripts are from the focus group interviews.

The first category is pre-service teachers' who have negative beliefs about the use of GeoGebra for geometry teaching. They described it as a pedagogical opponent aspect. The "pedagogical opponent" word is used to explain some pre-service teachers believed GeoGebra may not have any pedagogical fidelity for geometry teaching and learning. The second category of pre-service teachers is uncertain about how to use GeoGebra apps. They have inconsistent beliefs about the use of GeoGebra for geometry teaching and learning. The third category of pre-service teachers (pedagogical benefit) talked about the benefits of GeoGebra for block-teaching. They have explained their pedagogical approach with GeoGebra relevant to geometry content, especially with DGS features such as dragging (see section 5.3.3).

Pedagogical reasons

Transcripts from the focus group interview transcripts provide evidence for the diverse range of pedagogical reasons as explained by pre-service teachers at block-teaching:

> ***P1/T57:*** *Teaching geometry is not easy for us [pre-service teachers]. We have to understand content as well as different steps of the rule or algorithm in proof, while others have used a different pedagogical approach. But some of them do not like to teach geometry at all. I believed that teaching tools sometimes make each step of the lessons easy to teach and I need to get a good grade for teaching. I don't think apps are a good option, not even as a tool for teachers' practicum. GeoGebra does not have any pedagogical benefits, just has visual display properties only.*

Pre-service teacher P1/T57 discussed his experiences with geometry teaching. In his view, the schematic approach can be considered as a recipe in which you have to follow the rules step by step. P1/T57 is only worried about teaching aids (tools) to score higher marks for practicum evaluation. He sees geometry teaching as a pool of rules and algorithms, which enables a person to solve a geometrical problem by following those rules and algorithms. Many other pre-service teachers in the focus group had similar concepts. These pre-service teachers had negative perceptions of geometry content that have a different pedagogical approach. P1/T57 felt that the use of the pedagogical tool was beneficial for getting a good score in block-teaching. Also, he defined GeoGebra as a visual tool, and he stated that GeoGebra does not have any pedagogical benefits.

Uncertainty about app use

During the focus group discussion, P2/T42 explained what he had meant by 'static': bound by theories whose concepts, axioms, and theorems were fixed and unchangeable.

> ***P2/T42**: The pragmatic nature of teaching positions at block-teaching practice is so hard as we don't have a chance to understand students. Especially in Grade 11, we must teach geometry content by assuming [the] prior knowledge (grade 10) of our students. For example, sometimes these Grade 11 students knew the formal definitions of the different quadrilaterals but did not make use of the descriptions when faced with assessments using forms of visual representation of shape. Suppose we used apps when there is not enough time to cover the curriculum. In school, we haven't had time to understand students' concepts or the weaknesses of students' assessments, and we have just reproduced theories on geometry in a static way. We don't have time to use the app. I am not sure about it.*

According to P2/T42, geometry teaching means applying these theories correctly, and many secondary students in her Grade 11 class had forgotten the Grade 10 geometry content. Despite knowing the content, they were unable to do an assessment which was also difficult for pre-service teachers. The school mentor provided limited time for pre-service teachers to cover the geometry content in Grade 11, and it was also difficult for them to do the many revisions needed to Grade 10 content. P2/T42 thought either the use of apps was a waste of time, or he was not certain about GeoGebra. P2/T42's pedagogical knowledge about apps may have given him a negative belief. Indeed, the lack of awareness of pedagogical approaches with DT was often the main reason for the negative perceptions of some pre-service teachers and teacher educators.

Pedagogical benefit

P2/T4 was a second-year pre-service teacher, who had 8 weeks' experience in the use of GeoGebra as she had experience as the ICT teacher in a private institute (due to personal reasons she started her block-teaching practice later than others) in geometry block-teaching at secondary grades (Grades 6 to 11). However, she had positive beliefs about the use of GeoGebra for geometry. P2/T4 may have pedagogical experiences on teaching that influenced her beliefs about integrating GeoGebra for geometry. These experiences may be due to multiple factors such as curriculum content and her personal view of the ineffective use of the blackboard for diagrams. It may be that P2/T4 thinks that she understood the tactic knowledge concerned with their block-teaching practice.

> ***P2/T4:*** *I had the opportunity to teach geometry with GeoGebra at TEI and school allowed me to use my iPad[36]. I have taught this class last two months only five students in this bilingual class, they like when I used GeoGebra for many geometry lessons. GeoGebra makes it easy to teach geometry than blackboard. I mean I have used GeoGebra for a circle to the tangent lesson step-by-step construction. It has reduced constraints of the problem robustly. Finally, I've just moved my fingers touch the straight line with the circle and showed the tangent concept more easily than by drawing and explaining many diagrams on the board. I feel my students are really happy to learn with GeoGebra. They are good at geometry.*

However, she had a positive perception of the benefit of GeoGebra as a DT tool. This positivity may be due to multiple factors in a pedagogical approach, which may compare to P2/T42's perception of ineffective tough screen use of the DT apps. Maybe P2/T42's pedagogical experiences in teaching have influenced her perception of integrating GeoGebra for geometry teaching. The divides between pedagogical benefits, uncertainty and negative beliefs (an opponent of GeoGebra use) (P2/T4, P2/T42, P2/T7) can be considered as another aspect of GeoGebra use in secondary geometry teaching. Some pre-service teachers employed different pedagogical approaches with DT in the block-teaching environment because some block-teaching schools did not allow them to use their smartphones during block-teaching. Pre-service teachers used different instructional strategies in their block-teaching which and this will be discussed in the next section.

4.3.1 Instructional methods

The open-ended survey (Q3.13) responses are relevant to the instructional strategies for apps in geometry teaching/ geometrical tasks design. Some pre-service teachers used different instructional strategies relevant to students' difficulties pertinent to understanding geometry problems. Only a few pre-service teachers mentioned GeoGebra; others explained their instructional

[36] Due to the 2019 ethnic conflicts, mobile phones were not allowed during teaching practice session in that local school (with Muslims), at least as a tool in secondary school. Thus, T4 had used her own device (iPad) with this GeoGebra.

practices in block-teaching. However, around 45% of pre-service teachers did not respond to this survey question.

Pre-service teachers' answers (Q 3.13) about specific instructional strategies with apps at block-teaching are categorised under three themes: direct instructions, discovery learning instructions, and relevance to use manipulatives (physical, GeoGebra).

Direct instructions

Pre-service teachers (P2/T44, P2/T53) discussed the potential of direct instructional methods (e.g., double column) for geometrical proof problems:

> **P2/T44** *I have instructed that the double column instruction be used for geometrical proof in the mid-point theorem, every student can easily understand it*, even *if it may be easy with GeoGebra.*

P2/T44 instructed students to use the double-column for geometrical proof. and she said she planned lessons with GeoGebra's dynamic features (see section 5.3.3) P2/T53 explained the use of GeoGebra in teaching geometrical proof According to her, she planned lessons with GeoGebra dynamic features (see section 5.3.3). P2/T53 can be considered as using a discovery learning instructional strategy (step by step instructions with GeoGebra) that promotes students' use of their pre-knowledge relevant to the problem:

> ***P2/T53*** *I have planned the lesson with GeoGebra. I believed that without knowing geometry pre-knowledge relevant to students, it is sometimes challenging to design different instructional approaches. In my second-year block-teaching, I tried to deconstruct lessons (with students' points) into small visible components and wrote instruction separately, a more flexible approach, allowing students to I had a lesson including some geometrical proof problem solving. I deconstructed the lesson into small components such as circles, tangents, triangles, and theorems related to them, and completed the proof**.***

She identified the options to support pre-knowledge (e.g., tangents in circles). She described her pedagogical approach as the way she deconstructs the content element into more visible smaller components that allow students to explore the problem. She believed that it would help secondary students to make connections with their previous understanding and knowledge. Her beliefs about the pedagogical approach to geometry learning changed with her secondary classroom teaching experience. P2/T53 thought that she achieved a more flexible pedagogical approach. Her perceptions changed and she used different pedagogical approaches for students to think about geometry problem solving.

The use of physical manipulatives

In this research, context manipulatives are the external representations that are used as a “tool” for teaching.

> ***P2/T13:*** *I think I have tried many ways to teach congruence and solve problems; students have difficulties in identifying angles in parallel lines. Even I followed [the curriculum] as in TG. I decided to revisit angles first for geometry teaching, we need to have good awareness, especially pre-knowledge of the content in TG and teaching aids. TEI provides us with an excellent opportunity to enjoy our block-teaching practice our many ways with the use of DT or any other teaching aid. This may not be possible sometime in next year’s internship.*

In her perception, P2/T13 did not mention what she meant by her ‘many ways’. She later said it referred to the geometry content that she already taught. She used a different pedagogical approach as a teaching aid (tool). She believed that everyone could try their ways of instructional strategies for teaching. In the second-year focus-group interview, P2/T13 bought a teaching tool (see Figure 4.2) that she used to remind students of their pre-knowledge of angles related to parallel lines before the lesson.

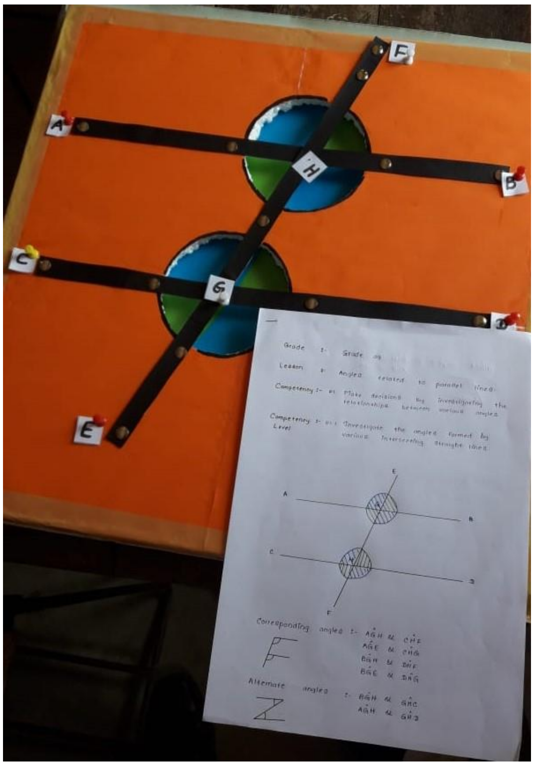


Figure 4.2. Main study, P2/T13’s teaching tool

Figure 4.2 shows that physical manipulatives are the hands-on materials and objects that can be manipulated to assist in learning about angles in geometry. P2/T3 believed that TEI had provided the opportunity to learn through teaching with a different pedagogical approach such as using a teaching aid (physical manipulatives or digital tools), even though it is a developing concept in the local context rather than a suggested lesson plan on TGs.

There is evidence showing that pre-service teachers' perceptions about the use of digital technology (DT) changed with their pedagogical approach to teaching practice.

The use of dynamic manipulative (GeoGebra)

The concept of congruency applies to DT along with curriculum, pedagogy, and assessment. It is suggested that congruency is a potential indicator of integrated DT apps. P2/T35 elaborated on his instructional approach as follows:

> ***P2/T35:*** *I thought to use the dynamic features of a dynamic geometry software (DGS) for my geometry lesson. That was the lesson on geometrical construction, which was the 12th lesson. Apps are not necessary throughout the task when I need them only for some parts. I taught some concepts with GeoGebra and completed the assessment quickly.*

The transcript above also provides evidence showing that pedagogical perceptions may have been considered when P2/T35 was selecting apps. He had ideas about the geometry content in the curriculum and the dynamic features of DGS. Dynamic features allow users to manipulate geometry elements interactively (e.g., dragging). Maybe he selected those GeoGebra features he considered relevant to some of the content.

In the selected sample, pre-service teacher P2/T35 was an ICT teacher in a private institute before he joined the TEI. Maybe his personal technological experience enhanced his selection of the geometry content when he selected the tool needed to develop his ideas**.** Indeed, previous experience in using ICT may have reshaped his geometry teaching. He used a dynamic feature of the app to develop the lesson. Perhaps his perceptions of the dynamic element of the geometry app might encourage him to use it more.

> ***P2/T8****: Some basic concepts were removed from Grade 8 but the spiral link to the next level was not modified. As an example, "visualisation of 3-dimensional (3D) shapes" in the textbook was removed and in Grade 8 it is narrowed down to a 40-minute lesson of a solid. It is challenging for students to understand cube problems. I think my experience with DGS was highly beneficial to develop the lesson with different visuals*.

Here, a pre-service teacher (P2/T8) used the app, which is associated with multiple representations of 3D shapes. In her new lesson plan, she discussed the way that she had used the visual and dynamic nature of GeoGebra to prepare her activity sheet.

In the focus group interview, P2/T40 explained that the flexible learning environment at TEI encouraged peer collaboration and he reflected on his experience:

> ***P2/T40:*** *I was not good at geometrical construction, but one day I was suddenly allocated a teaching construction lesson for Grade 10, and my friend helped me learn it with GeoGebra quickly; it opened a new learning opportunity for me. He taught me how to use GeoGebra for construction, with dragging and step by step.*

Peer collaboration also involves pre-service teachers working together, trying different ideas out in practice, and giving each other support for possible solutions while using GeoGebra. GeoGebra is an Android phone app, and DGS is designed for a laptop. GeoGebra was chosen for this study as it remains the dominant, free download app for geometry.

It was noted that the collaborative learning approach benefitted P2/T40's geometry teaching. In this way, learning with apps took place without prior intention when his need arose. For both proof and problem solving, the TEIs want teachers to possess a broad and active knowledge of fundamental principles that are standardised and not committed to subjective creations. Evidence indicated variations between pre-service teachers' perceptions of different TEIs. It mainly depends on the acceptance and pedagogical approach of apps in the teaching and learning of mathematics at the tertiary level.

This evidence can be used to address RQ4 in this study. However, some transcripts revealed that it is challenging, and some pre-service teachers were frustrated by the time needed for adaptation to the GeoGebra. Individual interview data show that few teacher educators know how apps are adapted or how the app will change in geometry teaching. However, these pedagogical approaches to geometry with GeoGebra have changed the way they teach with manipulations. Technological manipulation was another challenge for some pre-service teachers. Survey data showed that technological content knowledge was also a challenge for integrating DT into geometry; this will be discussed in the next section.

4.3.2 Technological content knowledge (TCK) for geometry apps

TCK includes information about how to employ technologies relevant to geometry content with specific apps. Table 4.6 below shows the computed univariate statistics for each perception in column 1 relevant to app integration (Q8_1 to Q8_13 of survey data). The statistical means and standard deviation of each statement are displayed in Table 4.4 of the single variable for better comparison. Here, 13 comments were relevant to pre-service technical knowledge of the apps. These perceptions are considered to be technical ability based on some attributes of the app's (technology) mathematical tools.

Table 4.4

Pre-service teachers' views on apps use (survey data)

As a pre-service teacher:	Mean	Std. deviation
01. Mobile apps manipulation, the benefit for learning geometry	1.21	0.408
02. Apps visuals can be useful for the learning of difficult geometry content	1.56	0.630
03. Manipulation (e.g., dragging) of apps is technically easy in some lessons	1.59	0.684
04. It is straightforward to design tasks using apps	1.82	0.722
05. Apps are important for self-learning geometry constructions	1.50	0.550
06. DGE app for teaching is more convenient than a compass/ruler and textbook	3.04	0.999
07. Downloading the appropriate app for geometry is easier	1.67	0.630
08. It is easier to learn technical issues of apps as a group	2.15	0.918
09. Pre-knowledge of geometry is needed for manipulating the app	2.87	1.255
10. Apps' technical design structure (objects) makes it easy for the design task	1.70	0.765
11. Technical facilities of the phone have restricted the use of apps' features	2.80	1.201
12. Social factors influence the use of mathematical apps	3.33	1.248
13. Geometry learning is motivated by dynamic features of apps	1.73	0.738

Pre-service teachers responded to these 13 statements according to a Likert scale: 1*: agree*, 2: *strongly agree*, 3: *tending [to agree*], 4: *disagree*, and 5:

strongly disagree. The highest mean value (3.33) was recorded for Q5_8_12, (Social factors influence the use of mathematical apps) which corresponds to not agreeing with the statement that the use of apps is influenced by social aspects.

According to the data, the second highest mean value was 3.04, as many pre-service teachers were disagreeing (marked Disagree = 5) with the statement in Q5_8_6, (DGE app for teaching is more convenient than a compass/ruler and textbook), meaning that more pre-service teachers were willingly using traditional tools and textbooks for geometry teaching rather than apps. Indeed, the reality is that mathematics teachers (still) prefer to use a textbook for teaching mathematics.

The lowest mean (1.21) falls under the statement (Q 8_1) "Mobile apps technically benefited me for learning geometry", which means more pre-service teachers agree with the statement. The second-lowest mean value from Table 4.6 was (Q 8_5): “Apps are important for self-learning in a teaching context”. Most of the students agreed with the statement about using apps for their self-study (as they marked Agree =1).

These findings revealed that most pre-service teachers in the sample agreed that geometry apps were beneficial for learning, but they preferred to use a textbook for geometry teaching. These responses challenged the emerging themes and pedagogical approaches that have been currently used by both groups (traditional vs apps used by pre-service teachers).

The questionnaire data also provides an overview analysis as well as a discussion of the challenges and potential uses of apps by pre-service teachers in their geometry block-teaching context. These views suggested that the context can be considered and measured in analyses as an independent variable of self-learning with the app, which may not be a fair option. From this view, the block-teaching context may affect the self-learning of pre-service teachers, but it is conceptually and analytically separate from them. This recognition also suggests that because the block-teaching context is not external and separate from pre-service teachers, it is something that is interwoven with teaching practice and can quickly develop their technical content knowledge. Both positive statements mean pre-service teachers agreed apps had benefited them in learning geometry.

Pre-service teachers have some negative perceptions regarding the technical features of apps. Even if arguments against the use of apps, such as those described in this section are overcome, some other internal obstacles to the successful integration of apps into mathematics teaching and learning remain. The next section will describe the potential thinking behind geometry apps.

4.3.3 Potential thinking of DGE features for geometry teaching

Pre-service teachers' geometry teaching with DGE in this study means mathematics block-teaching experience is facilitated with GeoGebra at secondary school. In the focus group interview, P2/T34 explained his confidence in technical knowledge:

> ***P2/T34:*** *My technical experience gave me ample opportunity to improve my teaching approach. I believed that only some apps (DGS) are embedded with technical, conceptual features relevant to the geometry curriculum.*

P2/T34 mentioned that some geometry curriculum content that he had taught with GeoGebra enhanced his professional practice. Selecting the appropriate tool or teaching aids was an essential element in a pre-service teacher's lesson plans.

These participants' geometry apps' block-teaching experiences are evaluated through a lesson plan, the way tools are used (linked to the content in the geometry curriculum, tool concepts), teachers' instructional strategies, classroom management, and students' outcomes (understanding the lesson). However, relevant to current research, evidence comes from focus group interviews or pre-service teachers who provided lesson plans or survey questions.

In phase 2 of the study, P2/T22 described her beliefs about geometry teaching with DGE apps. This transcript evidence was obtained from the semi-structured focus-group interview (after block-teaching). P2/T22 had focused on the secondary students' experiences rather than the different pedagogical approaches or issues relevant to the teaching.

> **P2/T22:** *I think geometry apps will help us to develop students' spatial imagination, practical comprehension, and logical thinking. GeoGebra will [be a] benefit especially for spatial imagination in virtual space.*

P2/T22 thought that apps will benefit spatial imagination. These pre-service teachers have different perceptions and experiences of geometry teaching. These experiences may be different because (as described in Chapter 3, section 3.4) these pre-service teachers are from two different TEIs and may represent another type of secondary school (Chapter 1, section 1.5) in Sri Lanka. This sample of pre-service teachers responded to a post-survey question. Q3.1 concerning TPACK for apps discussed in the next section.

4.4 Technological pedagogical content knowledge (TPACK) for apps

This is the third main theme, which includes the familiarisation of apps and the fidelity of geometry apps. TPACK is a lens through which the role of mobile apps (GeoGebra) in secondary mathematics education is observed.

The mathematic study app, GeoGebra, was chosen as it is recommended in secondary curriculum documents in Sri Lanka. GeoGebra has features of an interactive geometry software (DGS) and is compatible with mobile phones. In this study, pre-service teachers used GeoGebra in different ways such as for demonstrations and visualisation, as a construction tool, and for preparing teaching materials.

This theme emerged from the collection of subthemes relevant to teacher-developed pedagogical approaches. An analysis of focus group interview data showed why pre-service teachers make individual decisions and how apps work for geometry teaching, including when a user is new to an app (e.g., DGS) and takes time to explore it.

> ***P2/T41*** *I am good at playing video games, which have a natural user interface so when I start work with GeoGebra [DGS software] it has a similar interface with virtual manipulation [VM]. It's difficult to explain. I like to work with apps [rather] than mathematics textbooks.*

P2/T41's preference for using apps can be seen in his discussion, which may influence his affordances of an app or pedagogical ideologies in VM. It may also offer an interactive learning environment with a visual representation of dynamic objects. P2/T41 thinks VM has enhanced geometry learning.

Pre-service teacher comments (reflections) at the focus group interviews likewise indicated the features of a DGS in the app. A pre-service teacher P2/T41 elaborates on the way he adopts steps in the process of a construction problem, as well as collaborative instructional efforts that are used to develop the lesson. In general, when a user is new to an app (especially DGS), it takes time to explore and understand that app. Here, P2/T41 thinks his gaming experience encouraged him to self-learn a new app (GeoGebra).

The use of technology has not always enhanced pedagogical content knowledge. In the semi-structured focus group interview, P2/T2 (a pre-service teacher) was not happy about his pedagogical approach to the angles lesson with a digital presentation:

> ***P2/T2****: I thought I did the lesson well as a group activity with a presentation, but the assessment shows many students were "not there".*
> ***P2/T1****: How do you know it. Maybe the geometry vocabulary was difficult for them. Only from the assessment, were your students not there?*
> ***P2/T2****: Maybe you are correct, it isn't straightforward…I thought when I used visual manipulation, the students understood well in group activities, and I will try a paper-pencil activity tomorrow.*
> ***P2/T41****: Do you think it is your activity? Maybe your instructional strategies?*
> ***P2/T2****: Don't know. Maybe students are fascinated with my presentation or did not work as a group in the activity, or cognitively the message may not pass to them; let me try to do the old method tomorrow. Which students are more familiar with?*

This dialogue was from a group discussion after block-teaching. The group discussion had more group collaboration and discussion than direct answering of research questions. This type of dialogue may occur in any learning situation in the classroom. The use of a presentation or any other activity may not be a success in classroom teaching. The use of a presentation (P2/T2) as a tool does not mean the students' geometry learning was enhanced. It is challenging to say secondary students did not get an idea from the pre-service teacher's lesson because he used a digital presentation. In the focus group interview, other pre-service teachers (P2/T41 and P2/T1) suggested many different reasons for these students' low scores in their assessments. P2/T1 indicated that students had faced difficulty with the geometrical vocabulary needed for the assessment task. If he (P2/T2) did not use proper instructional strategies or wording for the students during his presentation, the students might not actively be engaged in the lesson and be just passive receivers. Another reason was that those Grade 7 students were relatively new to English[37] language mathematic vocabulary. These are socio-cultural factors which may influence P2/T2's negative beliefs in geometry teaching experience.

In this dialogue, P2/T2 generally addressed the classroom teachers and students' interaction in mathematics with technology integration. This interplay between technology, pedagogy, and content knowledge helps us understand how pre-service teachers try to explore the knowledge needed for the integration of apps. Even here, pre-service teachers may have a different approach to TPACK; all knowledge components here do not work for these pre-service teachers in this selected learning environment. It was revealed that some pre-service teachers had a different level of familiarisation and perceptions of work with digital resources or apps.

4.4.1 Fidelity of geometry apps for teaching

In this research context, fidelity means the extent to which pre-service teachers believe that an app allows the user to act mathematically in ways that correspond to the nature of the geometry lesson planning. This underpins pre-service teachers' block-teaching practice. Pre-service teacher P2/T6 and her peers discussed her group project and the way they used apps in geometry teaching. This response was extracted from open-ended survey question 2.9. Here, they explained the way they documented their experience (knowledge in teaching):

> ***P2/T6****: I like geometry apps. My lesson is to learn the properties of a quadrilateral with an axis of symmetry in rotation figures We did a group project report on DGS from our teaching context. It's beneficial. First, we elaborate on the way we can use the GeoGebra to develop students thinking processes. Then we discussed and developed the lesson with dragging points, and rotation of figures.*

P2/T6 and her group had different perceptions and experiences of the design activity for geometry teaching. They discussed the pedagogical approach by referring to the functionality of the apps for further learning. Later, the pre-service teachers' group collaboration facilitated concept development. In the current context, many teacher educators and pre-service teachers are still

[37] In Sri Lanka, English is the second language and many English language students are not exposed to the English vocabulary relevant to mathematics until Grade 6 (bilingual mathematics classes).

reluctant to allow widespread access to these devices in a formal classroom context at some TEIs. For example, P2/T6 explained the way they may think scaffolds the understanding of symmetry to apply to different shapes of a quadrilateral. He indicated the possibility of facilitating the learning context through the activity by manipulating the dynamic nature of the apps.

The aim of his (P2/T6's) lesson was to learn the properties of a quadrilateral and axis of symmetry when rotation figures. Zbiek et al. (2007) defined this type of functionality as pedagogical fidelity, i.e., "the extent to which teachers (as well as students) believe that a tool (app) allows acting mathematically in ways that correspond to the nature of mathematical learning that underlies a teacher's practice" (p. 1187). Apps analyse how context independently affects teachers' pedagogy or how teachers develop knowledge of context. Viewing of apps fidelity is relevant to the pedagogical context, which is combined with pre-service teachers' thinking about pedagogical knowledge.

4.4.2 TPACK for apps

In the focus group interview, P2/T24 explained how the flexible online learning environment at TEIs encouraged peer collaboration and design tasks for his students:

> ***P2/T24:*** *I was not good at geometrical construction, but I am good at gaming apps. One day, I was suddenly allocated an online teaching construction lesson for only five Grade 10 students. My friend helped me learn it with GeoGebra quickly; it opened new knowledge for me. He taught me how to use GeoGebra for construction, step by step. I reflected on each step and realised that features in this app could be used to teach the lesson. I have included my reflection journal note in the lesson development.*

Here, P2/T24 reflected on his learning experiences with GeoGebra with his friend. It may be that the friend's instructions influenced P2/T24's perceptions. The peer pre-service teacher helped T24 to understand the fidelities of the app (GeoGebra). The fidelities are defined as the degree to which app features support mathematic and pedagogical features of the geometry learning-teaching process and the way he designed task design for users.

Table 4.5

P2/T24: part of the reflective journal about GeoGebra instructions

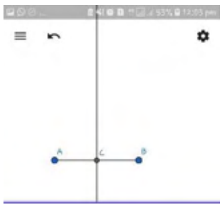
Diagram	Instructions	T24’s reflection
	First, I told them to draw a line, asked them to think and select an appropriate function that they can do with a line	They are not familiar with this app before. My experience in gaming tactile nature helped me to find [the] necessary objects easily but students need more time
	I asked them to name, bisect or move were easily found and when I asked them to [it] try out	The finger manipulation is not hard. Intuitive and encourages students to study. It’s like the game. In here to draw correct visual straight-line representation needs basic skills
	I asked them to draw circles, centre lines and manipulation.	Visual manipulation (VM) is more comfortable keeping the mobile phone on hand; this is easy to learn and is less time consuming than learning from a physical tool at a desk

Table 4.5 is about P2/T24’s GeoGebra learning experience. This strategy also involves the pre-service teachers working together, trying mathematical ideas out in practice, and giving each other support to find possible solutions by using GeoGebra. It was noted that the peer support benefitted P2/T24’s DGE app (GeoGebra) learning. P2/T24 believed GeoGebra had higher mathematical fidelity and the direct interface of the app for geometry construction opened a new learning opportunity for him. P2/T24 also explained how his five years’ experience in online gaming and VM allowed him to understand and reshape his learning experience on geometry construction problems in a short period.

P2/T24's discussion about GeoGebra indicated that each step developed with the features of a DGS in the app. In Table 4.5, P2/T24 elaborated on how he adopted steps in the process of a construction problem as well as the collaborative instructional efforts with peers that were used to develop a similar lesson plan. In general, when a user is new to an app, especially DGS, it takes time to explore the app. His preference for apps can be seen in his reflection in the discussion. It may influence his affordances of an app or pedagogical ideologies in VM. It may also offer an interactive learning environment with a visual representation of dynamic objects. VM has been used to enhance geometry learning. Other self-motivated group pre-service teacher comments at focus group interviews likewise indicated the features of DGS in the app used for geometry learning.

In summary, this section provides detailed knowledge and components with the synthesised statistical values of the different aspects that have been discussed in detail. Pre-service teachers' previous experience using DT may shape their technology pedagogy-specified knowledge use of apps for their geometry teaching. However, as noted here, some pre-service teachers had used apps in more creative and productive ways to create more engaging and rewarding activities in geometry. For example, P2/T6 further thought of the ways DGS could support students' development.

The self-motivated group of pre-service teachers, who had the affordances to use GeoGebra, had many years of experience with omnipresent game apps used for gaming. According to the data, these groups of pre-service teachers might have personal knowledge associated with gaming techniques and the manipulation of objects. They stated that their familiarity with the game user interfaces promoted easy access to geometry apps as well as professional knowledge to link TPACK.

Extended TPACK's advantage is that it has more attributes to mobile learning (such as the affordances of apps, fidelity of apps, and the learning environment) with a different user interface than TPACK. The unique features of extended TPACK strengthened the pre-service teachers' use of apps for teaching and learning geometry. The app's integration for pre-service teacher education is complex and may require a collective effort. Here, the researcher defined this as extended TPACK (more appropriate for the mobile phone apps) which addresses part of the research question RQ4. This is crucial, not only to answer the research questions but also to contribute to ongoing knowledge to ensure the relevance and ongoing development of pre-service teacher education in Sri Lanka.

During the research, some pre-service teachers were self-motivated to learn about a new app for geometry and to use it when it is relevant. It is unfair to expect that all pre-service teachers selected in this cohort will have a similar self-motivation to learn about GeoGebra as it is not a compulsory component in their teacher education syllabus. For that reason, the next section will summarise aspects drawn from the analysis of the TPACK evidence.

4.4.3 Familiarisation of geometry apps

In a developing country such as Sri Lanka, there is tremendous potential to establish links to education using the public's recent familiarisation with mobile apps. The scalability and systemic development of these findings, relevant to the research context to answer research questions (RQ1 & RQ4), may have several influencing factors (both external and internal).

Fundamental issues

In survey question Q 2.6, pre-service teachers were asked about their perceptions of geometry apps, and 27% of the responses were related to technical constraints. For example, three pre-service teachers (P2/T25, P2/T33, and P2/T5) discussed different reasons for their technical constraints:

> ***P2/T25:*** *Learning GeoGebra is sometimes tricky. Geometry apps have more instructional features with display tools, but [GeoGebra] requires basic knowledge of geometry and technical skills for manipulation. I sometimes feel it is easier to use paper and pencil for curriculum content than using GeoGebra.*

P2/T25 thinks that while GeoGebra would present an interpretive space for the user; she still believes it is difficult. The app display tools, which are shown in the menu, may be difficult to use intuitively. However, it is an individual belief about technical constraints. She believes that paper and pencil are easier to use for the local geometry curriculum due to the technical constraints of an app.

> ***P2/T33:*** *Some useful apps may have the capacity of representing complex geometrical concepts in block-teaching. For me, it takes time to learn the*

process and procedures of the tools given in GeoGebra. Something like drawing tangents with interception points A and B of two circles is too difficult for me.

P2/T33 explains her understanding of using a specific technological tool in a mathematical context. Her perception: it is difficult to use GeoGebra, such as when making tangents intersect on two points on a circle. When pre-service teachers are in block-teaching situations, they will interact with their mobile apps in a way that is more pedagogically productive. Hence, P2/T33 and P2/T25 have different beliefs about the use of apps due to some technological and pedagogical constraints. It is challenging to interpret what is meant by 'useful apps', as some pre-service teachers had different views about apps in the focus group interview.

Personal goals

These quotations are from pre-service teachers' responses to the open-ended question (Q1.17) in the survey:

> ***P2/T15:*** *No idea about geometry apps as we have never, ever used any educational apps for our mathematics education, except for communication and social media apps. My goal is to get a good grading for NDT. I am only good at the game apps on my phone. Why should we have to use GeoGebra?*

The reality for some pre-service teachers, apart from communication, is that they are using mobile phones as gaming tools, and some pre-service teachers have problems selecting apps for education.

> ***P2/T23:*** *Apart from internet browsing and grammar-checking apps, we have never used any mathematics apps. Nobody has discussed such things during my study period. I don't know how to use apps for geometry. My goal is to learn geometry with or without apps as I did not learn geometry properly for my GCE (O/L).*

Here, P2/T23 describes the way she has used internet or grammar apps; she had never used mathematic apps before. She prefers to learn geometry content with mathematical apps, maybe without apps means e-resources.

> ***P2/T56****: I am interested to learn from mobile apps, but it is better to get a traditional workshop on "what type of apps are available" or how we can select good apps for our mathematics content. For example, some apps may have facilities to support self-learning or may develop our task for geometry teaching. I have a goal to learn MT for mathematics but before that it is better if we have at least an optional component for educational apps, otherwise we are wasting data without knowing the basics of selecting good apps.*

In the local context, traditionally new concepts mean some stakeholders were expecting state-initiated workshops. State teachers' continuing professional development and incremental salary increases always depend on these professional development workshops. However, as much as technology is developed, P2/T56 still prefers traditional teaching workshops from lecturers with expertise (e.g., a lecturer who specialised in mobile apps and mathematics education) as a workshop. These pre-service teachers may need to overcome a lack of personal knowledge about how to use an app.

Instructional activities and development paths

Participants (e.g., P2/T6) explained that, without teacher verbal discussion, apps promote students' self-regulated learning. According to P2/T6, less instruction means the student may begin to see hierarchies after these are made visible with an app (for example, a square is a rectangle and a quadrilateral). Also, pre-service teachers (P2/T24, P2/T6, P2/T22) and a teacher educator (MT8) explained that less instructional design with apps helps to articulate the task with relationships (i.e., attributes between parallelograms and other 2D figures, rectangles, rhombi, and quadrilaterals). As stakeholders, TEIs involved in teacher education think differently with regard to app integration for some topics of geometry. This can potentially lead to fragmentation in both the evidence base and knowledge base decisions concerning mathematics apps.

This evidence uncovered clues to the nature and development of these pre-service teachers' apps in action, which addressed RQ4. However, some pre-service teachers had used apps for their planning processes of geometry teaching; these are discussed in the next section.

4.5 Possible factors that influence the use of apps for geometry

The responses to survey question Q2.11 showed that more than 58% of pre-service teachers explained different aspects of the curriculum knowledge relevant to the use of apps for geometry. These differences in aspects among the participants included: (i) personal factors (31%) (ii) institutional factors[38]

[38] Institutional factors are policies, infrastructure, practices, and teacher training.

(27%) (iii) social factors (10%); and the remaining 32% of the participants did not respond to the open-ended question about “the use of apps for geometry” question (Q2.11) in the survey. Some of the pre-service teachers’ responses in focus group interviews relevant to each of the above-selected aspects are explained further in this section.

4.5.1 Personal factors

A pre-service teacher (P1/T43) explained her experience selecting a geometry topic for block-teaching:

> ***P1/T43**: I hate geometry, even [when I was] at secondary school. Unfortunately, we all got geometry topics for last week’s teaching practice as a group, and we prepared lesson plans for "tangent to the circle" from TG and practised textbook activities many times. I hardly managed our block-teaching sessions. I realised that it is difficult for me to explain [to the] students. I don’t think my geometry lesson was successful, I prefer to use GeoGebra when somebody can help me.*

P1/T43 may have limited geometry curriculum knowledge which may hinder her geometry teaching. She said that it was difficult to attend textbook activities on this topic even though she had practised it many times. (Perhaps she had a bad experience with the geometry curriculum in her secondary classes.) These pre-service teachers may have different perceptions of the geometry curriculum. The argument is that despite the lesson content, medium of instruction, curriculum policy, and sub-elements of the curriculum being external, they are interwoven with some pre-service teachers’ knowledge and beliefs about the use of apps for geometry. For example, in response to survey question Q 2.11 a pre-service teacher (P2/T54) mentioned the challenges he faced at TEI:

> ***P2/T54:** I like to learn geometry because I didn’t study geometry well in my GCE (O/L). We have a DT curriculum just to teach office packages. Not anything about the way of using a DT tool for maths. I don’t feel only the use of mobile apps for geometry improves learning outcomes, even with the] self-motivation option as suggested by the TEI. It creates a social gap. Students who have money for data, experience [using] apps, and have their device (a good smartphone) can use it. What about the others? Our institute does not provide internet for free. First, we need to understand, do we need a different digital tool for geometry learning?*

P2/T54 indicated that there was a pressing need for further investigation into the DT curriculum, especially at TEI (the tertiary level). In this response, institutional and socioeconomic factors influenced pre-service teacher P2/T54’s perceptions. It is suggested that the TEI should identify the factors that affect the implementation of the DT application for relevant subjects. Perhaps P2/T54's means of assessing the technology or its use at different levels as a teaching tool for geometry might not improve learning outcomes. These personal issues prevented P2/T54 from using the apps. P1/T17 focus group interview transcripts showed that he had ignored some geometry lessons when he was a secondary student.

> ***P1/T17****: I don't know much about the curriculum for geometry construction at the secondary level. Geometry construction is a subject that needs psychomotor skills; that's why it was challenging to learn and teach. With reforms, the proportion of geometry content for GCE (O/L) has changed [so] not like what I had in my secondary school. Now GCE (O/L) students can't drop geometry test items in the exam. So many geometry assessments must be covered in a limited time. I don't think for geometry construction problems in the national exam; we can get any benefit from apps; even the curriculum reforms promote the use of DT (digital technology).*

P1/T17's knowledge of geometry content in the new curriculum revision has been vital for block-teaching in local (Sri Lanka) secondary schools. P1/T17 explained his learning experience about geometry as a secondary student, which was a pre-service teacher-dependent factor**.** Perceptions of geometry content knowledge and his (P1/T17) personal approach to the curriculum may have created a negative perception of the use of apps for selected geometry topics.

4.5.2 Institutional factors

The semi-structured focus group interviews also determined that pre-service teachers have considerable influence institutional factors such as secondary geometry curriculum and assessments. The secondary schools have received large amounts of support from the government for the implementation of reforms, but the TEI had to continue without, or with less government funding to overcome problems in the mathematics reform process.

> ***P1/T27****: We don't have an entrance exam for TEIs – no geometry paper. We have just followed the secondary geometry curriculum from textbooks and TGs; for our block-teaching, that's enough. Why do we need to worry about the geometry curriculum*? *With reforms, it seems do not worry about 'points', 'lines', and 'planes' being taken as undefined entities in secondary lessons. We learned these as Euclid elements when we were secondary students. After [the] curriculum reforms at TEIs, geometry learning [was]*

completely neglected, sometimes we get an assessment as homework only. It is better if we know how to learn a geometry app.

P1/T27 said that TEIs do not have entrance examinations as previously required in Sri Lanka. A pre-service teacher explained that TEIs do not have a geometry module after the major curriculum reforms in 2015 (see section 2.1). Several alternatives were discussed and used at TEIs, including extra assessments or the use of DT applications.

P1/T29: *Curriculum reforms with competencies and continued secondary geometry syllabus updates are difficult for us, as sometimes we are not aware of these changes until our block-teaching starts. Some curriculum topics like axioms, theorems, and procedures [are] too hard for me, even reforms suggested that DT integration apps like GeoGebra needs basic geometry. I had neglected those topics when I was a secondary student*.

In semi-structured focus group interview phase I (P1), P1/T29 and P1/T27 explained their perceptions of geometry learning at TEIs from their secondary geometry curriculum content knowledge. In the local context, curriculum reform policy is incredibly problematic for some pre-service teachers. P1/T29 has negative perceptions of the geometry curriculum; specifically, the knowledge of axioms, theorems, and procedures within the secondary classroom.

TEI curriculum

Sri Lankan teacher education curriculum is not particularly concerned about DT/apps integration for bilingual education at TEIs. A selected group of students (bilingual students) in state schools (block-teaching) are learning secondary mathematics in the English medium. All selected mathematics pre-service teachers for this study are bilingual pre-service teachers. In phase II of the study, in response to the open-ended survey question (Q2.11), P2/T28 gave more evidence of issues about the medium of instruction (English) and curriculum content. Her perception of geometry learning highlighted the power of language in understanding instruction geometry problems and curriculum assessment:

P2/T28: *I feel some lessons in the geometry curriculum are easy with apps. We had a geometry lesson for 2D geometry figures and relationships for new students taught in the bi-lingual policy. Without [the] proper language used for instructional design (English medium) it was difficult for new students to understand the relationships of 2D geometry. I aimed to teach 2D figures with the associate properties. It is easy to use GeoGebra which has a task design option with quadrilateral dragging to make the next figure* *with relevant instructional design. GeoGebra is an easy app which we can learn by ourselves. I have learned many GeoGebra functions by myself.*

P2/T28's perception was the language used by a teacher plays a central role in the development of understanding of 2D shapes and their relation to other figures (quadrilaterals) in the curriculum. However, she said that "GeoGebra is an easy app", which helps learners to understand hierarchical classifications with dragging. She did not mention the way she designed the task. It is not clear what new types of geometrical knowledge and what aspects emerge as a result of access to digital technologies (GeoGebra), particularly computational, and dynamic visualisation. P2/T28's perception is that mathematic instructional design fluency (including instructional design and understanding the task, relevant to the aim associated with 2D shapes and their associated attributes) helps students to understand lessons quickly. Her perception is that after some assessment, some English-language (the medium of instruction) pre-service teachers start to teach selected geometry topics with digital tasks along with sound instructional design.

P2/T26 described her perceptions about curriculum knowledge for block-teaching in the second year at the phase 2 semi-structured focus group interview:

> ***P2/T26****: I think self-learning is the more critical issue of our geometry, as the content of the curriculum [has] gaps…geometry content relevant to a "symmetry for transformation" has moved out, but next-grade content of the same topic has not changed. Not like last year, I have taken a different approach to complete the competency level (symmetry) at Grade 7 lesson scaffolding from GeoGebra.*

P2/T26 explained more about the way she had scaffolded the app and grade concepts from one grade to the next in the previous year (vertical curriculum). P2/T26 mentioned that curriculum content connections and linking relationships between the two grades are essential in the geometry curriculum. According to her perception, the secondary geometry curriculum has gaps. She believed that pre-service teachers' knowledge about the content in the curriculum (awareness of geometry curriculum content) was important. Finally, she believed GeoGebra can be used to scaffold the symmetry concept which is vital in the learning of geometry.

TE policy

In the Sri Lankan context, teacher education policy has been promoted to provide mathematics education in English (medium of instructions) at TEIs. These pre-service teachers are called English medium pre-service teachers (bilingual teachers). TEIs selected for this study are bilingual pre-service teacher education institutes.

> ***P1/T16****: As a bilingual teacher, at block-teaching, I had only 5 students. I used a maths lab for my lesson, but my mentor told me not to use the DT application. I think without having a proper policy for DT integration at TEI, it is an issue for us to use GeoGebra. A few teachers may have inappropriately used PowerPoint presentations in their lessons which does not mean others can't use ICT for their block-teaching.*

P1/T16 described how sometimes bilingual mathematics teachers were blamed for using PowerPoint presentations (DT application) and e-resources inappropriately for the local curriculum knowledge. However, there are many dilemmas about bilingual (English medium) mathematics teaching with DT use at secondary schools in local TEIs**.** P1**/**T16 explains that GeoGebra may become a powerful tool to design digital tasks in the hands of pre-service teachers in the curriculum policy. Pre-service teachers can use it for enacting effective curriculum experiences with creative lesson plans. However, she did not explain the curriculum policy concerning the kinds of pedagogical approaches and classroom organisation that can be employed in DT-integrated learning environment.

Social and cultural issues are important for pre-service mathematics teachers in a research context. Secondary mathematics apps (e.g., GeoGebra) are relatively new to mathematics education in Sri Lanka. For example, pre-service teachers who had positive beliefs about apps at block-teaching have experienced multiple barriers even to using their smartphones at block-teaching schools. Yet, the vast majority of pre-service teachers, still do not have enough power to overcome multiple barriers and strong cultures that exist within schools about the use of mobile apps.

> **P2/*T45***: *I think it is so hard to cover all geometry content in the GCE (O/L) exams. When we are assigned to teach those classes as pre-service mathematics teachers, we faced many issues; too much curriculum load for us. If we can use digital apps, it may [be] easy to graphically represent*

objects or concepts in 2D information within a short period of block-teaching.

In some cases, pre-service teachers had a different view on the “curriculum load” (excessive curriculum content) in secondary mathematics. Evidence from data shows that the curriculum load means the number of geometry lessons that teachers were assigned to teach each year in each timeframe. Secondary teachers have to cover these geometry lessons (syllabus) with time targets. In the phase II survey question (Q2.12), a male pre-service teacher (P2/T45) explained his perceptions of the curriculum.

P2/T45’s perception is that it is challenging to achieve time targets in geometry lessons. P2/T45 may think he should possess geometrical background knowledge of the theorems covered in the secondary curriculum at a much deeper level of understanding than his students. This background knowledge of geometry curriculum content forms a knowledge base that is specific to teachers. In the individual interviews, teacher educator MT5 emphasised that 25% of the secondary mathematics curriculum time is allocated to geometry content, but TEIs did not consider these learning context issues.

MT5 *It is sad to say the geometry curriculum load (25%)and physical facilities may act as a barrier to the confidence of geometry teaching. Some pre-service teachers do not like to select geometry lessons for block-teaching sessions. Even though English medium block-teaching schools sometimes demand more DT integration for geometry from these young pre-service teachers even with less infrastructure at school. Some pre-service teachers are very confident in the use of GeoGebra after the pandemic. Only a few students are doing English medium mathematics in upper secondary grades, so many schools do have physical classroom for these English medium mathematics students. Some pre-service teachers faced many difficulties in their block teaching.*

This teacher educator (MT5) perceived that the curriculum exists conceptually and operationally could not separate from the pre-service teachers’ needs. Since the curriculum context surrounds pre-service teachers, it can be thought of as something that independently acts as a barrier for them. In contrast, MT5 stated that the solution to overcome this problem is developing the capacity to teach geometry with apps (DT). Despite MT5 explaining that many pre-service teachers did not select geometry as a subject for their block-teaching. It looks like not only the curriculum content, but there may have many other factors influence for selecting geometry lessons for block-teaching. Although the teacher educator explained that it might be a learning context issue.

> ***P2/T58****: There is a gap in the curriculum at TEI and secondary school bi-lingual curriculum. Block-teaching has allowed us to reflect what we taught and how our secondary students learn geometry. However, we are in a curriculum cage with less infrastructure. We just have to cover curriculum content in the given time allocation [with] no time to explore apps.*

P2/T58 explained problems in the secondary curriculum. Mathematics teaching time allocated for geometry in school, and the time allocation in TEIs. TEI has only 3 hours for secondary geometry. It is difficult to understand what P2/T58 means by “the curriculum cage.”

To understand geometry better, students need more time. However, compared to the curriculum content in each timeframe, it may be difficult for pre-service teachers to explore a new pedagogical approach. It can be considered part of the teaching-learning environment, where a negative trend in the use of apps has found.

P2/T62, who had different experience as a private school teacher, had a different view:

> ***P2/T62:*** *I am confident using GeoGebra. Not like in the state national curriculum, we had enough time to cover geometry at the private school. We had a series of workshops on professional learning through apps relevant to the mathematics curriculum when I was a teacher at a private school. It is an excellent opportunity for professional learning.*

Some pre-service teachers were aware of the mathematics app before they joined the TEI. Since P2/T62 was experienced in using the app, this may have encouraged her to use it for teaching sessions. Even though she was not happy with the overloaded, rigid curriculum structure, but she had the affordances of the app, and she discussed professional learning through apps.

The results of the study highlighted several institutional factors influenced for geometry block-teaching. For example, the relevance for secondary school curriculum and of the teaching space as a whole as well as their specific needs regarding physical time allocation, and other infrastructure facilities. The next section explores issues relevant to the social factors.

4.5.3 Social factors

Survey question Q2.21 addressed perceptions of socio-cultural constraints in the learning context. About 10% of pre-service teachers in the sample had various views of learning space constraints.

> ***P2/T45:*** *We are working as a group and hardly managed to prepare teaching aids for our block-teaching. We are in temporary accommodation. Due to the power cut, we have limited facilities now. It is hard. When we [go] back to college, we would like to try this app, new apps are coming [and it] is also a complicated update with this new curriculum context as we don't know how much data we have to waste to find a good app relevant to the curriculum content. Some app content is equivalent to rote learning.*

This group of pre-service teachers (P2/T45 and group) may have the intention of learning about new mathematics apps, but the cost of data may become an issue for them. P2/T45 explained that they are working as a group for success in their block-teaching, even though their learning space is difficult. It is challenging to interpret what he meant in survey question Q 2.21 as P2/T45 did not participate in the focus group interview.

> ***P2/T5:*** *I don't think mobile apps are a good option for geometry learning; they may create a discrimination gap. The rural school where we went for teaching practice does not have at least electricity, and transport facilities fail, sometimes [there is] no signal for the mobile network. It is challenging to prepare teaching aids relevant to the lesson with the [high] cost of data.*

It is difficult to understand what P2/T5 means by "geometry learning" as a pre-service teacher or as a secondary student at school. The school where they (pre-service teacher P2/T5 and the group) went for teaching practice did not have electricity or minimum standard facilities (access to water, sanitation, and space for learning) which meant that they were placed in a rural school as a group in their block-teaching. These types of rural schools have challenging learning environments. Vast digital divides caused by issues such as insufficient reception for mobile networks is common in these areas. Sometimes this is due to geographical difficulties or being in lower socioeconomic areas (e.g., tea plantations). For instance, if pre-service teachers are assigned to a rural school in an economically disadvantaged area, there are fewer opportunities for them to upload data for mobile phones and have access to mobile networks.

Examination oriented teaching culture

This subtheme emerged from transcribed semi-structured focus group interviews and responses extracted from the open-ended question of the survey and provided evidence for their different perceptions of geometry learning. These are categorised as individual factors as the individual pre-service teachers had different perceptions of geometry learning. The first-year pre-service teachers (P1/T37, P1/T15) responded to the open-ended survey question (Q2.12): 41% of pre-service teachers had personal perceptions of geometry learning.

> ***P1/T37****: I am going to teach geometry in the second year. I like geometry teaching, but we have pressure on GCE(O/L) exam results in block-teaching school, but I think it has challenging subject content and problem solving while using the van Hiele model. In my own experience, many students may not understand the problems in their O/L geometry paper. I believe that determining appropriate problem-solving strategies are more important than hands-on experience with apps.*

P1/T37 may have been exposed to a more theoretical module relevant to teaching in the first year. P1/T37 believes that students may have difficulties in solving geometry problems in the GCE (O/L) examination. He thinks understanding the given situation in geometry problems and determining appropriate problem-solving strategies are difficult for students.

> ***P1/T15:*** *These students may not work on the same platform as suggested by the van Hiele model. I think some students have difficulties in determining appropriate problem-solving strategies in their geometry exams. I think maybe some of them are in level 2 or others are in level 3. If students are not on the same platform, [there is] no point to go for apps.*

In addition, P1/T37 and P1/T15 believed it is crucial to consider the level of geometric thinking achieved by students rather than hands-on experience with apps. Pre-service teachers are encouraged at TEIs to develop their skills and attributes for an app-mediated DT option. These skills and features include assessing GeoGebra apps, selecting appropriate geometry content, communicating, building knowledge, representing ideas, creating and developing lesson plans, collaborating, and teaching.

Some teachers were in GeoGebra apps-mediated self-learning and lesson-plan development: the role of the pre-service teacher is pivotal in designing and implementing effective teaching and learning activities that engage students in the development of such skills and attributes.

> ***P1/T5****: I think only some students can distinguish the properties of a square and triangle but not the properties of a square and trapezium in the exam. These Grade 7 students [are] not at the same level of thinking as in van Hiele's theory. Therefore, there is no point in using the apps.*

P1/T37 and P1/T5 may have struggled to understand levels of van Hiele's model from their perceptions of geometry learning at TEIs. In this stage, they (P1/T37, P1/T5) have not received any practical teaching experience. P1/T5's perception is that the van Hiele theory indicates that progress from one level to the next is more important in teaching. Maybe P1/T37 and P1/T5 believed that it is important for a pre-service teacher to understand the van Hiele levels before commencing geometry teaching. However, it is difficult to interpret what they mean by the same platform (P1/T37) or the same level of thinking (P1/T5), as these responses are extracted from the open-ended survey question in phase I of the main study.

It is important to understand why P1/T37 and P1/T5 were thinking about these levels of the van Hiele model in geometry learning and may have progressed with their geometry teaching experience. P1/T37's perception is to use an app for geometry; the students should be at the same level of geometry understanding. She may prepare her lesson plans according to the content and the pedagogical approach, which is relevant to her perception without the use of apps. Her impression of the need for the same level of geometry understanding by students may mean she avoids using apps for her teaching**.** The argument here is that the individual perceptions of geometry learning may prevent the use of the app in the first year at TEI.

Medium of instruction

Another issue is that the medium (language) of instruction played a significant role in the geometry problem solving learning context. Semi-structured focus group interview data showed that pre-service teacher P2/T19 had a positive perception of the medium of instruction in geometry teaching. Furthermore, P2/T19 argued about her helpless situation in teaching geometry construction and elaborated on the possibility of provision of DT resources (factual knowledge). According to her, the use of DT is a privilege for English-medium pre-service teachers in the local context:

> ***P2/T19:*** *I don't know much about geometry apps. I don't have much knowledge of geometry either. I don't know, how to teach students to understand the geometric construction that is the next lesson assigned to me to teach. New reforms encouraged us to use DT, which may be useful for curriculum content and task design. Many e-resources and tools are available on the internet, it is the only relief we have. English-medium instructional design is comfortable for me with iPad apps. Apps have pulldown menus and [are] easy to manipulate by hand. It's also easy to use apps for some geometry content.*

P2/T19 explained the benefits and challenges she faced in the mathematics curriculum; he did not mention his solution for the lack of geometry curriculum knowledge. His perception is that the language of instruction (English) benefited them in using the iPad for pre-planning of geometry lessons at block-teaching. He was happy with the app's "manipulation by hand" option. It means he believed that the app's affordances may be a technical element that supported him to overcome his curriculum barriers.

The mathematics teacher educator in a TEI is responsible for the pedagogical practices of pre-service teachers. In the individual semi-structured interview, a mathematics teacher educator (MT4) explains her experience and perspective of English-medium pre-service teachers:

> ***MT4****: I believed that this assessment of the geometry curriculum-specified topics was very successful for our English-medium pre-service teachers in the last few years. Some pre-service teachers never had an opportunity to study secondary school mathematics in English. Some of them have problems in instructional design for mathematics tasks [and] need e-resources for some lessons. They are learning to teach in English medium to secondary students in the block-teaching. Some of them are happy to use apps for mathematics as fewer instructions are needed.*

In the individual interview, MT4 explained her perception of block-teaching. In her view, all pre-service teachers using English were prompted to analyse the geometry curriculum content. Furthermore, she explained that the geometry curriculum developed in different units across the secondary grades from Grade 6, with plane figures, triangles, circles, and quadrilaterals, flows through a spiral structure up to level 11. From MT4's perspective, some pre-service teachers had not completed their secondary education in English when they were secondary school students. These internet resources are beneficial as some of them are not competent in the current geometry curriculum content since they were students in secondary school.

4.6 Chapter summary

The findings were relatively cohesive around the block-teaching experience of selected pre-service teachers relevant to the RQ. Geometry can be taught to secondary students in ways that: (a) support pre-service teachers' understanding of teaching and learning selected geometrical concepts; (b) are relevant to their beliefs of technology tools (GeoGebra); (c) challenges of DGE for developing geometry lessons with GeoGebra. Research questions are not mutually exclusive. Therefore, some findings relevant to the RQs are overlap.

Some pedagogical aspects (e.g., less teacher instruction) benefits of apps emerged as pre-service teachers assumed roles as users of the GeoGebra for geometry. Pre-service teachers are looking for possibilities of geometrical insights relevant to the geometry curriculum. They might have limited content knowledge, which may hinder their geometry teaching as well as their understanding and confidence to use GeoGebra for mathematics education. Several other aspects (affordances of apps, confidence, instructional method) relevant to the GeoGebra might negatively or positively affect the geometry content knowledge of pre-service teachers.

How to use apps for the selected geometry content in Grade 10 block-teaching was the pre-service teacher's responsibility; this is the first challenge that some pre-service teachers faced in mathematics education. Many pre-service teachers had low confidence to use apps as tools (for selected geometry lessons at block-teaching schools). The confidence in the use of GeoGebra for geometry teaching was not statistically significant for TEIs or the gender of pre-service teachers. However, there was a difference between mathematics major and mathematics minor course students only in TEI1. There may be a gap between pre-service teachers' beliefs of pre-geometry knowledge of students and reality in block-teaching practice. Few pre-service teachers believed that the user-friendly instructional design of apps mainly benefitted only English-language (bilingual) secondary students. Several pre-service teachers thought that apps can be easily used as tools (e.g., 2D shapes and geometrical proof) to teach some difficult concepts in the geometry curriculum. The reason is that the apps have a more visible dynamic component (e.g., diagram) which made it easier to present the geometry problems with fewer verbal instructions. However, some pre-service teachers explained personal, institutional, and social factors that might negatively or positively influence the use of mobile apps in secondary geometry. Those aspects are partly addressed RQ1 and RQ4

Finding indicated the pre-service teacher's political perspective relevant to study. For example, the gap in curriculum policy and lack of broader theoretical or conceptual frameworks for ICT modules at TEIs may have acted as an external barrier to some pre-service teachers' beliefs about the use of apps for geometry teaching. Pre-service teachers had different pedagogical beliefs about the use of mobile apps for geometry; this also influenced their confidence in geometry knowledge.

Some pre-service teachers believed that effective teaching with GeoGebra demands specialist knowledge of technology and specific expertise in geometry content. Those teachers who were weaker in basic concepts of geometry content (and/or had difficulties in technical understanding of apps) had inconsistent beliefs that it was challenging to manipulate geometry apps as pedagogical tools. Pre-service teachers' prior experience of geometry learning might have led to changes in the instructional methods regarding the use of an app for teaching and learning geometry.

These analyses also provided information on different perspectives of the geometry knowledge of the selected sample of pre-service teachers before and after teaching practice with the affordances of GeoGebra. However, the overall results show that, even after two years, one-third of female pre-service teachers and half of the male pre-service teachers in the selected sample were not confident teaching the secondary geometry curriculum with or without the use of apps. The secondary school mathematics curriculum has a substantial weighting (25%) on geometry content. A policy insight gained from this statistical analysis shows that confidence in teaching geometry was neither influenced by the factors associated with the context of the teacher education institute (TEI) nor the gender of pre-service teachers. These findings may have a link to other beliefs on pre-service teachers' use of mobile apps for secondary geometry teaching and learning, which will be described in detail in Chapter 5.

Chapter 5

Pre-service teachers' beliefs about teaching and learning geometry

5.0 Introduction

Chapter 4 discussed the perspective of pre-service teachers' knowledge in teaching and learning geometry. The researcher highlighted that those teachers not only need knowledge for digital technology (DT) use, but their beliefs play the role of a guide, and filter their actions in teaching geometry. Therefore, pre-service teachers' beliefs, and their use of GeoGebra focusing on teaching and learning geometry for secondary students, are considered in this chapter. These findings address RQ2 and RQ3.

RQ2: In what ways might mobile technology apps or DT apps (for teaching geometry in mathematics education) influence pre-service teachers' pedagogy when teaching geometry?

RQ3: In what ways might mobile technology apps (for teaching geometry in mathematics education) influence pre-service teachers' beliefs about teaching geometry?

The development of the discussion in this chapter is based on the quantitative analysis of survey responses (percentages and significant tests, and data reduction techniques). Those findings are supported by the analysis of the semi-structured interview transcripts, and open-ended questions in the survey that are relevant to the RQ. These quantitative findings are complemented by the qualitative analysis with themes identified through constant comparative analysis. Sometimes quantitative evidence was not available to support the themes or sub-themes from the qualitative analysis.

This chapter has six main sections. The first section gives a brief introduction to the chapter. Belief in geometry learning with apps is considered in section 5.1., which includes personal beliefs, and pre-service teachers' beliefs of past geometry learning, when they were secondary students. The second section (5.2) concerns beliefs about pre-service teachers' geometry teaching with apps which includes pedagogical beliefs about apps, beliefs about curriculum resources, and epistemological beliefs.

The third section (5.3) explains beliefs about pre-service teachers' affordances of geometry apps and how they described their beliefs in dynamic manipulation. The different ways of interacting with TPACK are discussed in section 5.4, including the data reduction technique factor analysis, possible interrelationships, extended knowledge about TPACK, and potential features. Policy issues are then discussed in section 5.5, including beliefs about policies, such as the mathematics curriculum policy, the DT policy update, and the TE policy. Finally, a summary of findings is given in section 5.6.

5.1 Pre-service teachers' beliefs about geometry learning with apps

Pre-service teachers' beliefs about learning with apps often have complex relationships with personal elements and epistemological elements of geometry. Several themes relevant to the beliefs of pre-service teachers' geometry learning with apps are identified through analysis of focus group transcripts and the responses to survey questions. These pre-service teachers' personal beliefs have changed with the interaction of the learning context and the particular content of the selected geometry lessons in their block-teaching (including geometry problem solving and geometrical construction). Two subthemes are identified: personal beliefs and beliefs from past geometry learning.

5.1.1 Personal beliefs

The survey Q 2.1 data show that 47% of the pre-service teachers who participated in the study believed that the GeoGebra app was not beneficial in supporting them during their block-teaching experience in the first year at TEI. However, in the second year, that 47% reduced to 38% of pre-service teachers who believed that the GeoGebra app did not support their teaching and learning at block-teaching schools. Therefore, pre-service teachers' beliefs on the value of apps for geometry teaching and learning had changed (improved) over the two years at TEI, although these findings are not statistically significant according to the Chi-squared test. These arguments are supported by the beliefs of two pre-service teachers (P2/T18, P2/T49) about geometry apps. They discussed their experience of teaching geometrical problem-solving lessons from the secondary mathematics school curriculum in Grade 10. This mainly addresses RQ3. The pre-service teacher, P2/T18, elaborates:

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> ***P2/T18:*** *In my first- year, I was scared to use GeoGebra for problem solving. Now I believed I am more comfortable with any app. My roommate helps me. He and his friends play online games at night. They said that all apps use similar figure manipulation when we practise often. It doesn't mean we are addicted to online games. I have a sense of awareness about any new app: the experience of how touch screen apps work. My goal is to learn from any mathematic apps. I want to be a good 21st-Century maths teacher.*

P2/T18 expressed his personal beliefs while discussing using individual or subjective touch screens, with the online game experience in his learning space. He did not directly mention his individual social gaming experiences, but he explained that his apps' learning processes appeared to interact with gaming app experiences and the social influences of his hostel life.

5.1.2 Beliefs about geometry block-teaching

P2/T49 shared his beliefs and experiences of geometry block-teaching over two consecutive years. He selected "geometrical problem-solving lessons" from the secondary mathematics school curriculum as was given by his mentor:

> ***P2/T49:*** *At my first-year block-teaching school, I thought that we (mathematics teachers) must teach our students how to solve geometry proof problems step by step on the blackboard. It was my goal, and even if it was not very successful, I have tried to understand student's levels with van Hiele's approach.*
>
> ***Researcher:*** *What happened in the second year?*
>
> ***P2/T49:*** *In the second-year block-teaching, schools asked me to use DT and prepare students for National exam questions. I believed van Hiele's approach is the best for the understanding of students.*

P2/T49 thought his personal beliefs depended on individual goals in his block-teaching learning context. Pre-service teachers in the focus group interview discussed how their beliefs were altered by their personal goals during the consecutive years at TEI. In this sense, personal beliefs are acting as an orienting function on teaching.

Some pre-service teachers (e.g., P2/T59) have a negative view of the use of apps such as GeoGebra for geometrical constructions:

> ***P2/T59:*** *To be honest, I don't think apps are a good option for geometrical construction problems. We are successful in teaching with paper and pencil, so it's better to restrict (ourselves) to the traditional drill and practice method than apps. I think drawing diagrams using paper-pencil is more important for our students as well as GCE (O/L) exam problems; That's how*

students get marks. I want all students to get through their OL exam. I believe that (by) learning GeoGebra (we are) wasting time in geometry teaching. Why do we have to learn it by ourselves?

P2/T59 believed that learning with GeoGebra wastes time. Her goal is for secondary students to achieve good grades in the GCE (O/L) examination. She explained the benefits of traditional examination-orientated academic practice (to follow the traditional drill and practise method for mathematics problems, which influences social beliefs). She believed that restricting the use of apps would better enable conventional teaching methods.

This evidence suggests that personal beliefs have altered the way pre-service teachers interact with institutional and social elements in block-teaching schools. The way pre-service teachers learn geometry as secondary students may influence the way they learn geometry as pre-service teachers.

5.1.3 Beliefs of past geometry learning experiences as students

Analysis of the open-ended survey question Q2.63 revealed that many pre-service teachers in this study responded to their past geometry learning experience as secondary students, and this has influenced their geometry block-teaching. Over half of the pre-service teachers, even at TEIs, agreed that memorising geometry (such as geometrical theorems and proofs) is the best learning option. Other common opinions related to understanding geometry (such as the van Hiele model or DGS) for geometry learning and the teaching related to these had low percentages. These values are summarised in Table 5.1.

Table 5.1

Past geometry learning experiences

Suggested methods	**%**
Rote learning	62
van Hiele framework/levels	13
DGS or apps/e-resources	11
No response	14
Total	100

According to Table 5.1, 62% of pre-service teachers believed that rote learning was the best option for some geometry content (e.g., theorems) in the competitive, examination-oriented evaluation structure in secondary education. Only 13% of the pre-service teachers believed that the van Hiele model for understanding geometry was more beneficial than rote learning. Only 11% of pre-service teachers thought that DGS/apps/e-resources were beneficial for difficult concepts compared to other methods, and they thought they would assist them in learning only some geometry content. According to the findings of this study, around 14% of pre-service teachers did not respond to this question. This may be because we collected data after block-teaching, when some pre-service teachers were tired from institutional evaluation stress, or perhaps they had bad memories of their experiences with learning this content from secondary school.

Many pre-service teachers highlighted in their semi-structured focus group that when they were secondary students, they had difficulty learning geometrical proofs, which are essential elements in the geometry assessments. Some pre-service teachers said that the way it appears in the curriculum is not user friendly. Other pre-service teachers argued that the makeup of the massive geometry content (secondary curriculum) without any connection led to the option of favouring rote learning. All of these past experiences about geometry learning may have influenced pre-service teachers' geometry teaching and learning.

Almost all the pre-service teachers who participated in the focus group interviews believed that as secondary students, they learned geometry just to achieve in the GCE (O/L) examination. P2/T50 explains her examination beliefs:

> ***P2/T50****: As secondary students in Grade 10, before we have learned any geometry topic, we are concerned about the marking scheme and the structure of the geometry questions relevant to that lesson. Those were important factors at the national GCE (O/L) (at the end of secondary education). Especially in the geometry proof question, by writing the same proof and procedures we can obtain a passing grade for mathematics. Therefore, our mathematics teachers always suggest that as secondary students we should be memorising geometry theorems by rote learning (if at all).*

These perspectives indicated that pre-service teachers' geometry learning beliefs are linked to personal elements (past experiences), social elements (obtained mathematic passes), and institutional-related elements (curriculum) in the block-teaching context. The next section discusses pre-service teachers' beliefs about geometry teaching with apps.

5.2 Beliefs of geometry teaching with apps

The pre-service teachers' perspectives about teaching geometry with apps that are considered in this section involve beliefs about geometry teaching only from selected domains (geometrical problem solving, geometrical construction). Pre-service teachers' geometry teaching with apps is classified under two subsections: pedagogical beliefs and curriculum beliefs. The semi-structured focus group interview transcripts showed that pre-service teachers had different pedagogical beliefs about geometry teaching with apps.

5.2.1 Pedagogical beliefs about apps

Pre-service teachers are concerned about apps' intellectual quality, connectedness to the curriculum content, and supportive environments for teaching with the app (GeoGebra) for their lessons (Larkin & Milford, 2018). Pre-service teachers P2/T11, P2/T21, and P2/T31 from the same teacher education institute (TEI1) were asked by their block-teaching schools to teach a geometry lesson titled "finding a tangent to a circle". They taught the lesson as part of their block-teaching. They discussed their teaching, teaching materials[39], and the way they had taught this geometry problem solving lesson (see Figure 5.1) relevant to the "tangent to the circle" task that they selected for their teaching practice. All these pre-service teachers participated in the same focus group interview.

[39] The materials used for the geometrical problem-solving lesson were given to the researcher by the participants when they had a semi-structured focus group discussion.

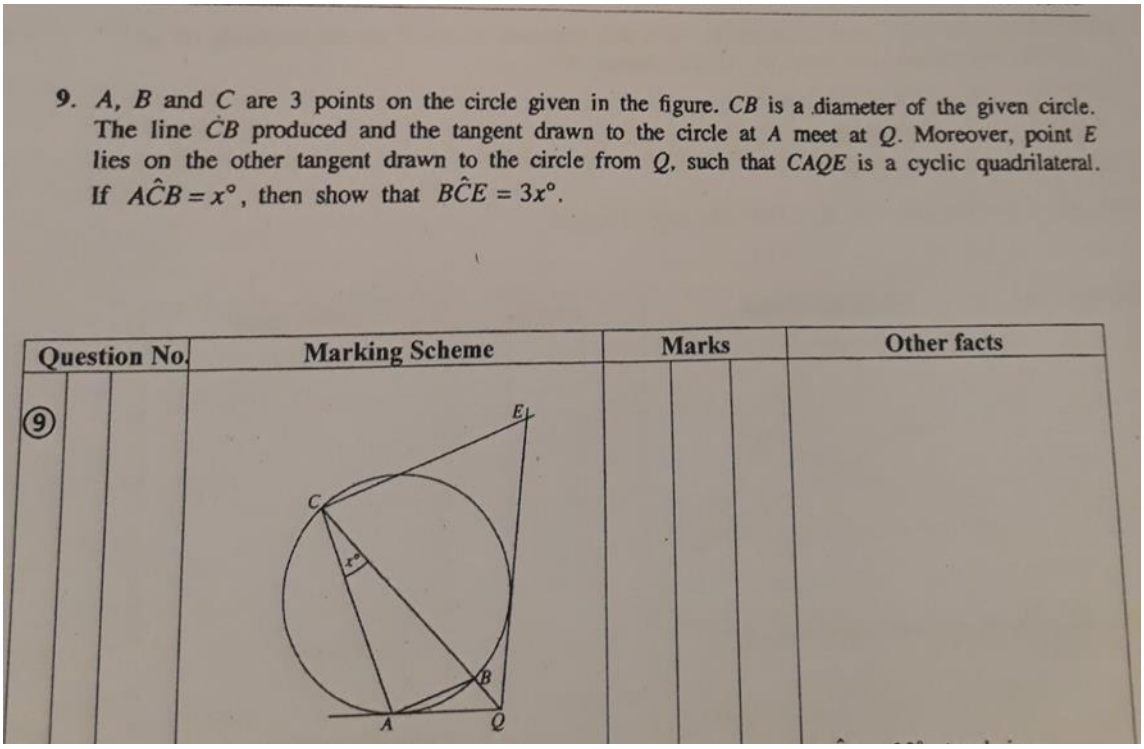

Figure 5.1, Geometry question about a tangent to the circle

The pre-service teachers discussed their teaching experiences using the same GCE (O/L) examination problem (see Figure 5.1).

> ***P2/T11:*** *I believed the lesson on circles with tangents is challenging to teach to students. Problems associated with tangents (AQ and QE) and bisector (CQ) are too difficult for secondary students at O/L.*
>
> ***Researcher:*** *How do you know it?*
>
> ***P2/T11:*** *I had experience in my block-teaching. It is not due to the inadequacy of content or facts.*
>
> ***Researcher:*** *What do you think?*
>
> ***P2/T11:*** *Students have difficulties even in the given diagram. I believed it is associated with students' inability to visual analyse the problem (see Figure 5.1) or maybe they do not understand the concept of tangent (angle between the tangents from a point is bisected by the line joining the point to the centre), theorems relevant to diameter BC (angle in a semicircle). You can see the circle, tangent, and triangle; many concepts and theorems are linked to one problem.*

P2/T11 explained her teaching difficulties in creating a supportive environment for students using the app. She believed that many concepts, such as tangent to the circle, and many theorems are linked to the same geometry problem (Figure 5.1). In her interview, she explained how unnecessary facts or misunderstandings came out of the visualised diagrams in tangents (e.g., students misunderstood that a line tangent (AQ) to a circle is perpendicular to the CQ point of Q) which was a consistent misunderstanding in this problem. The accuracy of the diagram can be observed clearly with the use of GeoGebra. One considers how the division of steps in the problem-thinking process by using colours has made it easier for students to visually understand.

The second pre-service teacher (P2/T21) discussed her pedagogical approach to the same geometry problem (Figure 5.1) in the following transcript:

> ***P2/T21:*** *Drawing the diagram is difficult for the student even in the O/L curriculum. I thought about it; maybe the different way students might think.*
>
> ***Researcher:*** *Can you simplify?*
>
> ***P2/T21:*** *I believed that an initial understanding of the problem (theorem relevant) is the main issue.*
>
> ***Researcher:*** *Why?*
>
> ***P2/T21:*** *Jot down the main ideas in the geometry problem. Then they must identify vital ideas relevant to information for solving the geometry problem. I think a concept-visualised will help students to visualise the diagram.*
>
> ***Researcher:*** *Did it help?*
>
> ***P2/T21:*** *Yes. As a student, I believe the most crucial point is to apply the theorem to the problem. They had a chance to practice with a few questions in the workbook. They have to practice the O/L exam problem (Figure 5.3.1; a circle with tangents).*
>
> ***Researcher:*** *What do you mean by "visualise"?*
>
> ***P2/T21:*** *Not only visualise with colour but I used GeoGebra for deep understanding**.** I mean to support their view and thinking with the diagram (which has a circle, tangent, and triangle with properties). I encouraged them to use a coloured pen or any other method to differentiate different objects in it.*

During the discussion, P2/T21 explained that students need to draw diagrams correctly, and that teachers need to understand the concepts related to the tangent lines (i.e., the theorems of circles and tangents identifying key properties relevant to solving the problem) and the students' ways of visualising their thinking. Further, P2/T21 elaborated that those students might need to have an extensive web of connections pertinent to the lesson's concepts to see the diagram relevant to the GCE (O/L) examination problem. Finally, she tried to explain her point of view about teaching in the given situation. This given problem includes different concepts such as a circle, tangent, and triangles. Therefore, she may be thinking about her teaching approach with GeoGebra.

In the discussion above, there is a pedagogical approach from the perspective of individual pre-service teachers; that is to say, we have taken the stance that beliefs and teaching knowledge are qualities that individuals possess.

5.2.2 Beliefs about curriculum resources with apps

Pre-service teachers' geometry knowledge and beliefs about curriculum resources (TG, students' textbooks, app), in the block-teaching practices are considered in this section. In Phase 1 of the survey questionnaire, only 44% of the selected pre-service teachers participating in the study believed that their geometry printed curriculum resources were not sufficient for block-teaching with apps. In this context, pre-service teachers' beliefs might represent basic instruction in handling geometry with DGE. However, the percentage who believed this improved was 50% in phase 2, although the difference was not statistically significant, even at the 0.05 significance level. During the focus group interview, a pre-service teacher (P2/T36) explained her beliefs about the geometry proof lesson in TG as follows:

> ***P2/T36****: I do not know how to teach geometry proofs according to TG.*
>
> ***Researcher*** *Why?*
>
> ***P2/T36:*** *We learned general theories of learning, But I don't understand how to teach geometrical proof constructively.*
>
> ***Researcher:*** *What are your options?*
>
> ***P2/T36****: I don't know. It is difficult for me to understand how students think about proof. We need support to teach geometry as I am not good at this, even during my school days.*
>
> ***Researcher:*** *What type of support?*
>
> ***P2/T36****: At least material support. It is unfair to think that every pre-service teacher knows all the secondary curriculum's content as a secondary student.*
>
> ***Researcher:*** *How about self-learning with GeoGebra?*
>
> ***P2/T36:*** *The national exam GCE (O/L) has 42 theorems in the curriculum to memorise. It challenges me to learn even van Hiele model by myself. TGs do not have any instructions to handle GeoGebra which is very difficult for me.*

Pre-service teacher P2/T36 is from TEI1 (which promotes only the traditional paper-pencil teaching method). She explained that her knowledge of geometrical proof as a secondary student was insufficient for teaching during the block-teaching. This lack of understanding of geometry teaching and learning may have created negative beliefs about teaching geometry proof at school. P2/T36 believed that she needs support to overcome barriers as a geometry teacher later in her career. As discussed at the beginning of the section, it is a pre-service teacher's responsibility to learn secondary geometry from printed resource books (student textbook, TG) or e-resources or any other methods at TEIs. TEIs are not responsible for developing pre-service teachers' knowledge about subject content (geometry) in the local TE curriculum. In the next section, beliefs of geometry learning and teaching are discussed.

5.2.3 Epistemological beliefs

Epistemological beliefs concern the nature of knowledge in geometry teaching (Thurm & Barzel, 2020). In survey question Q2.2, about the expected difficulty of geometry, 59% of pre-service teachers said they had less awareness about geometry teaching and selecting resources than was required. At the semi-structured focus group interviews, a pre-service teacher (P2/T12) explained her teaching experience.

P2/T12: *I gave supportive e-material [downloaded from the internet] for Grade 10 students to identify angle types and I explained alternative angles, corresponding angles, and vertically opposite angles.*

Researcher: *Then?*

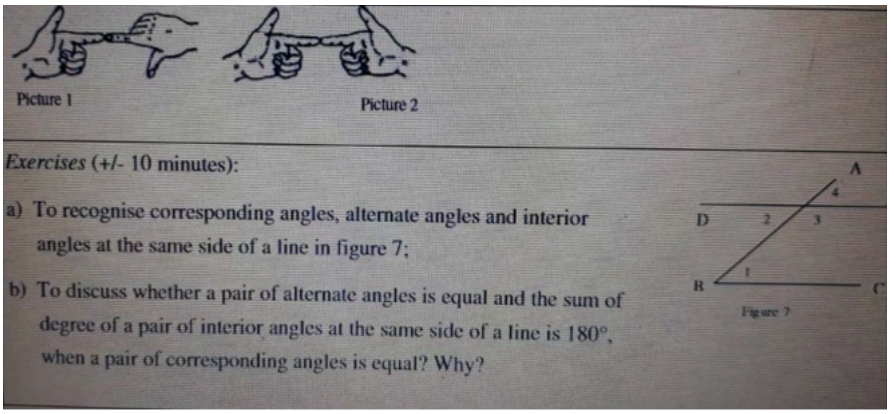

Figure 5.2, Resource material

P2/T12: *I asked a group of students to correct answers and make a note about each other's assessments.*

Researcher: *Do you have an example?*

P2/T12: *Yes, this is what one group of Grade 10 students wrote [Figure 5.2]*

Researcher: *What do you think?*

P2/T12: *I don't know, Is it a good approach in this context? Maybe it is a misconception about angles.*

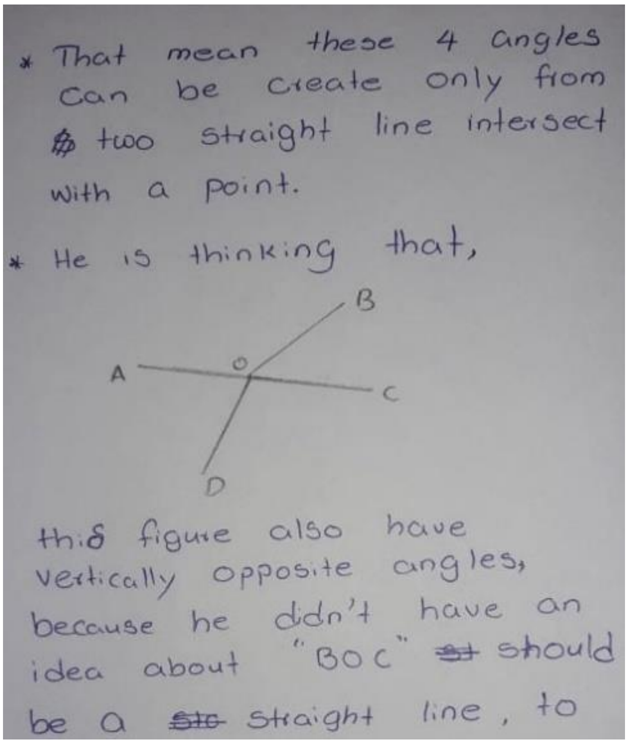

Figure 5.3. A student's note

P2/T12 used a different approach to revise various types of angles. In addition, he taught his students to understand other students' mistakes in geometry learning. His belief prompted students to learn from others' mistakes. He talked about the benefits of understanding students' mistakes as an option for learning to teach.

It is challenging to interpret what he has meant by learning from his mistakes. Perhaps he does not understand the reason for using the approach ("Is it a good approach in this context?") or maybe he is in dilemma. His knowledge of teaching originates from his thinking about teaching; however, he believed that students' geometry understanding has many challenges.

Focus group data indicated that there might be a link between pre-service teachers' beliefs about teaching secondary geometry and their pedagogical beliefs about apps (GeoGebra). P2/T14 explained her experience:

Researcher: *Have you ever used mobile apps for geometry?*

P2/T14: *Not yet. For some lessons, it has some benefits. It is just a drill and good for practice because you can work anywhere, not just on a desktop computer.*

Researcher: *Can you use mobile apps at the block-teaching school?*

P2/T14: *It was not difficult to use at my block-teaching school, but the school permitted me to do a try-out workshop for students on how to use GeoGebra for geometrical construction. I explained to students that GeoGebra is a try-and-fit app.*

Researcher: *Why?*

P2/T14: As *a secondary student, I was weak at learning geometrical construction. GeoGebra has dynamic features I learned by myself. It helps me. I thought that approach was good for the student.*

Researcher: *How do you know?*

P2/T14: *I believed it tempts less-experienced students to explore geometric drawings as an alternative to geometric construction. Many students said they tried and fit geometry jargon for GeoGebra, and some problems with geometry construction remained after the workshop.*

P2/T14 explained that geometric construction was difficult, even when he was a student. His geometry learning experiences may have influenced his beliefs. He thought the drill-and-practice method for learning geometry was useful. He thought students would like to use GeoGebra to drill and practise difficult concepts that require more than basic knowledge. He believed that some students sometimes forgot some basic rules of geometrical construction when they attempted geometry problems, but they can use apps, even when they do not understand the basic construction rules or language used in geometry. They used a try-and-fit (e.g., especially with geometrical construction) approach to geometry problem solving with GeoGebra. Such tactics rarely led to a solution when paper, pencil, and compass were used. However, some pre-service teachers' focus group interview transcripts indicated that problem solving beliefs differed and depended on the pre-service teachers' knowledge about teaching. Visual representations of a diagram in general and manipulatives with GeoGebra may be a benefit for students' understanding. Even though P2/T14 provides alternative expressions through GeoGebra for students, some have difficulties following initial instructions for geometrical construction. In brief, understanding a pre-service teacher's beliefs and the affordances of apps for teaching and learning geometry are important aspects. In the next section, pre-service teachers' beliefs about the affordances of apps are discussed.

5.3 Beliefs about the affordances of app

The affordance can be the potential for action: the capacity of an environment to enable the pre-service teacher's intentions to use apps for geometry. Moreover, it appears different attributes of pre-service teachers influence their geometry teaching with an app. Beliefs about the affordances of GeoGebra are different in this study. This may relate to the attributes of the apps in the atmosphere during an interactive activity by a user (pre-service teacher) who can manipulate and refer to an object differently. Some pre-service teachers use dynamic geometry environments (DGEs) for visual manipulation. The next section is about the pre-service teachers' visual manipulation.

5.3.1 Visual manipulation

Visualisations can be considered as a method to strengthen geometry understanding with pictorial view and are also thought of as useful to bridge gaps in difficulties of text understanding. Only 8% of pre-service teachers talked about the visual manipulation (VM) of GeoGebra. The visual affordance is representative of pre-service teachers' complementary visual status and their digital environment interaction with the GeoGebra tool. This was visible in geometrical construction problems which have always been an integral part of Euclidean geometry, from basic compass-and-straightedge problems to the constructability of a tangent.

Didactically, construction problems in geometry DGS provide a rich source of exploration at different levels of difficulty, sometimes encouraging highly creative approaches. Therefore, a substantial teaching unit on constructions in geometry was consistently included for prospective mathematics teachers. They are not just abstract physical properties but potential relationships between the pre-service teacher and the tool. A pre-service teacher (P2/T47) provided the following comments in the focus-group interview:

> ***P2/T47:*** *First, I tried the construction problems using DGS after I understood the construction process and GeoGebra. Then I did the same problems using traditional tools. I have used a hybrid model [compass- ruler and GeoGebra] to teach. It is a good option for me, solving many GCE (O/L) problems within a short timeframe. It is easy to try with GeoGebra first, my reflective journal, I have noted each step for lesson plan development.*

In the geometric constructions, P2/T47 tackled both modes. These two different modes (DGS versus traditional) of representation of geometric concepts could support the pre-service teacher's (P2/T47) constructions. Perhaps the two pedagogical media may support the development of the pre-service teachers' lesson plan development of teaching geometric constructions.

It was generally accepted by these pre-service teachers in their focus group interviews that if manipulation is used appropriately, geometry learning can be enhanced. Some pre-service teachers explained their experience of Virtual manipulation (VM) as a basic option in GeoGebra, especially for geometry construction.

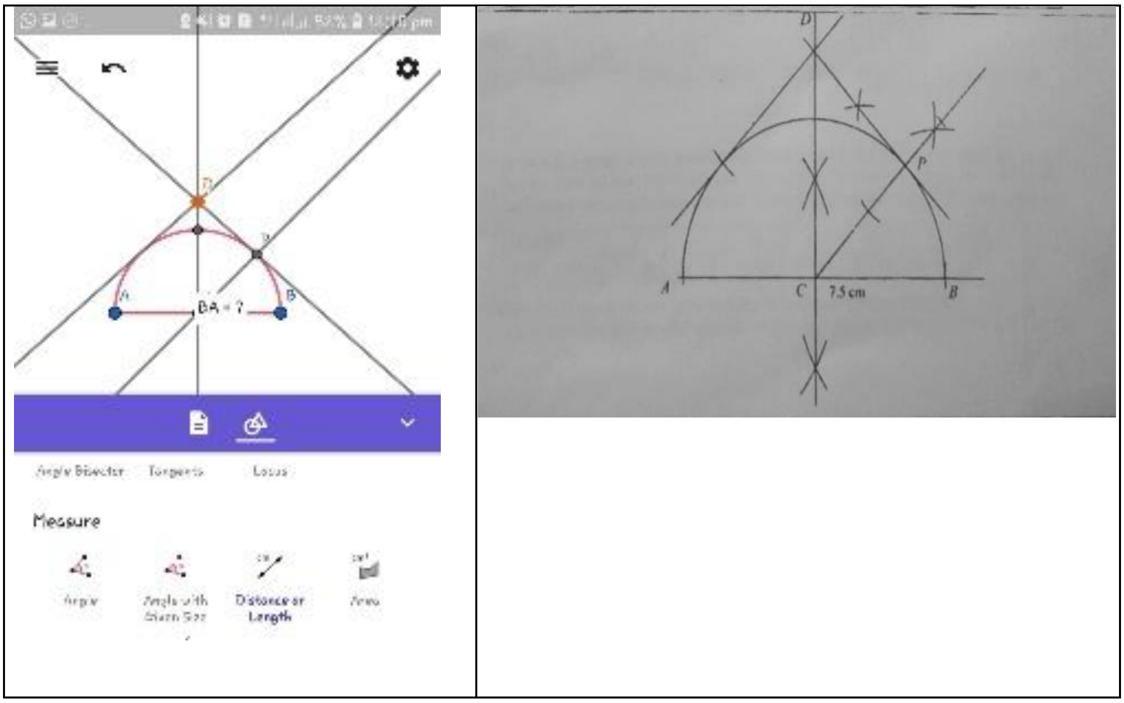

Figure 5.4. P2/T47 Geometrical construction problem

Figure 5.4 depicts how P2/T47 developed a GeoGebra task with steps for the selected construction problem to support his geometrical thinking for teaching.

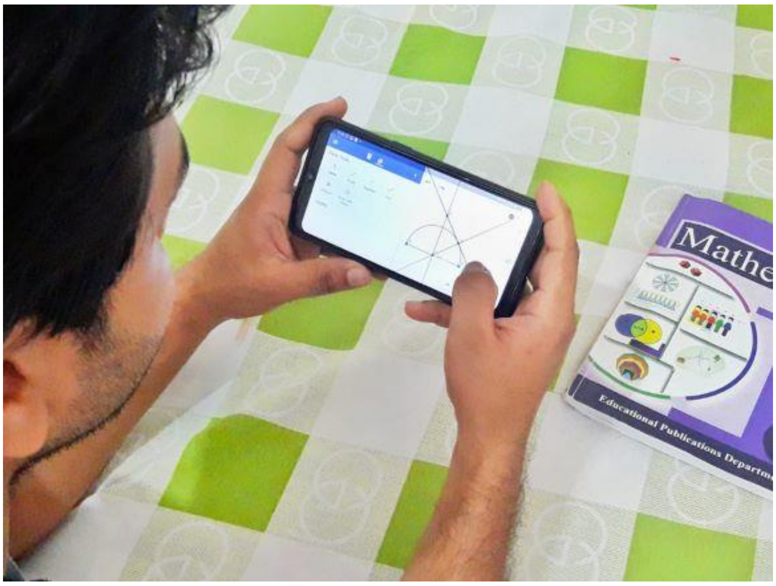

Figure 5.5. Working with GeoGebra

Figure 5.5 shows that one GeoGebra app property is for elements to stay together when a finger moves points or lines; there is also a distinction between variant and invariant properties. However, the way pre-service teachers teach geometrical construction activities is vital to encourage secondary students to engage in appropriate cognitive processing during learning which may have a pictorial view (paper) and a dynamic presentation (GeoGebra). Many other factors can influence successful learning outcomes with apps, such as the learning environment's dynamic manipulation. In the next section, the dynamic affordance of the GeoGebra application is discussed.

5.3.2 Dynamic manipulation

DGE (GeoGebra) enables the user (pre-service teacher) to start from a set of free or initial objects (e.g., points plotted anywhere) and use the drag to construct the diagram relevant to the task. The pre-service teacher can drag or change the position of the initial objects (circle, lines) and then the position, orientation, or size of the constructed objects are modified with GeoGebra.

P2/T53 explained her beliefs about GeoGebra with dynamic features. In her teaching task, she used dynamic features to manipulate the problem for secondary students:

> ***P2/T53:*** *It is just thinking of the way problem solving works with dynamic features of GeoGebra. I thought of the tangent to the circle problem and broke it down into steps, and drew the diagram; then, I felt that apps are an excellent option to develop such lessons.*

__Researcher:__ If they have to draw it manually, why do you need an app?
__P2/T53:__ Yes, I have drawn it first manually, then I thought to use the GeoGebra app to manipulate [the diagram] dynamically, which makes it easy for me as the mobile phone itself can rotate. That gave me an idea of how can use GeoGebra and the way students might think about the problem if they have geometry knowledge.
__Researcher:__ Can you explain?
__P2/T53:__ It is not an easy task as each step as a teacher. I thought about students' thinking process in tangent problem-solving tasks with DGS

She believed that students need to understand concepts such as the tangent, which requires a connection of thinking relevant to the circle in this task. Moreover, she explained the specific links with this geometry problem. Finally, P2/T53 argued that apps enabled her to structure the lesson and support the student's way of using geometrical concepts connected to the diagram (Figure 5.5). In her view of teaching, she believed that apps allowed geometry lessons to be presented to the students dynamically.

__P2/T53:__ My lesson has based on this problem: A, B, and C are 3 points on the circle, CB is the diameter of the circle. The line CB produced, and the tangent is drawn to the circle at A meet at Q. Moreover, point E lies on the other tangent drawn to the circle from Q, such that CAQE is a cyclic quadrilateral. If ACB angle is equal to X show that BCE angle is 3X.

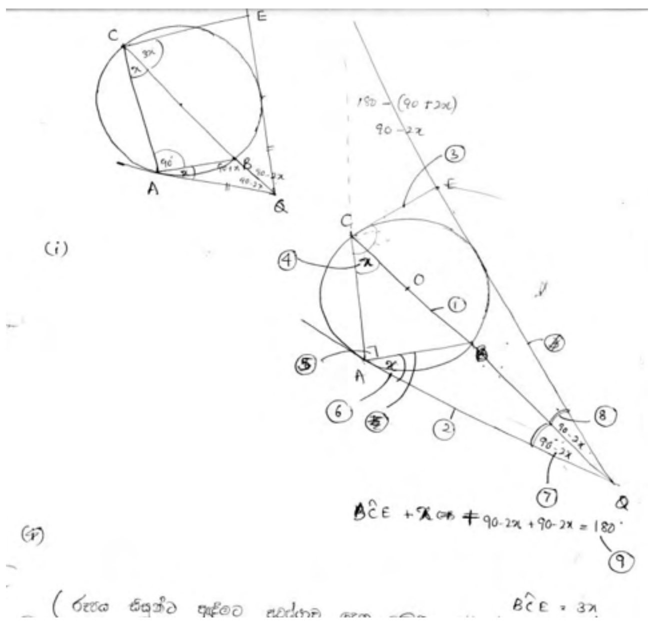

Figure 5.6. P2/T53 steps marked diagram

During a focus-group interview, P2/T53 described each of these marked steps (1–9) that were interrelated to concepts or theorems needed to solve the problem. P2/T53 talked about the importance of geometry knowledge in teaching problem solving (Figure 5.7).

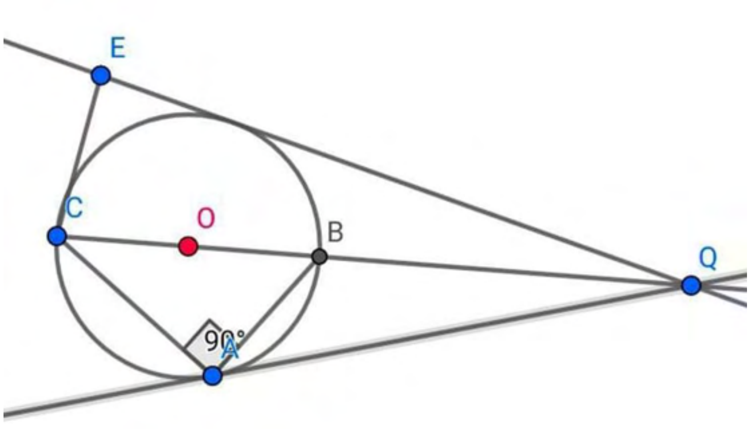

Figure 5.7. Step completed diagram (P2/T53)

Figure 5.7 indicates that the use of GeoGebra (e.g., the dragging mode) overcomes difficulties that the normal ability to remember a construction as a procedure (tangent to the circle) can cause. These manipulations and the learning can then be executed on other objects chosen. P2/T53s' explanation about her teaching process transcript has been analysed under task, person, and strategies of orientation organisation.

Pre-service teachers need to search for and appreciate the function of GeoGebra and the generality of how it might be adopted for other geometry problems. P2/T7 wanted to simply get a visual and dynamic manipulation feature in GeoGebra to teach proof related to a particular geometry problem. P2/T7 explained that problem visualisation is essential for students to solve a geometrical problem. She also discussed the importance of visualisation for pre-service teachers:

> ***P2/T7:*** *I think when I have projected a geometry diagram relevant to the problem using GeoGebra, students can easily visualise the patterns and general properties pertinent to a geometry problem.*
> ***Researcher****: You can draw the diagram on the blackboard?*
> ***P2/T7****: Unlike a blackboard, GeoGebra is interactive; when we drag the pointer, the diagram will change. It is more interactive, and less thinking is required. You can do more work in less time.*
> **Researcher:** Can you explain more?
> ***P2/T7:*** *I think, GeoGebra incorporates the freedom of a dragging and dropping point; it is easier to manipulate than the blackboard. Then easy to step back until the solution is visible.*

P2/T7 explained that the geometrical figure relevant to a geometry problem at the screen's surface could be manipulated by dragging and dropping any point or basic object in the GeoGebra application. P2/T7 believes that having sufficient degrees of freedom in object manipulation has created a new learning space. Pre-service teachers saw the value of using different kinds of manipulation with students.

Later, in the focus-group interview, she explained how students may think when they are working on a geometry problem with apps. For the O/L exam, students should draw a diagram using pencil and paper if they can understand the process of solving. Therefore, she believed that GeoGebra is just a tool for teaching and understanding the difficult concepts of a problem. At the focus group interview, another pre-service teacher (P2/T10) had a similar idea:

> ***P2/T10:*** *I expected to teach the geometry proof lesson at block-teaching that I had not learned since I was in secondary school.*
> ***P2/T4:*** *It is easy to learn with GeoGebra; you can ask from Madura?*
> ***P2/T10****: I think it is appropriate for the geometrical proofs.*
> ***P2/T4****: I have used it for construction.*
> ***P2/T10****: Really?*
> ***P2/T4****: First, he shows me that it is easy to manipulate objects; it is a similar pattern for all objects.*
> ***P2/T10****: No point in wasting time with* the *app as we are maths minor students. Our teacher educator said that we have to teach students the old way.*

Here, P2/T4 stated that GeoGebra gave him a new learning opportunity to learn a construction lesson with his friend's support. Researchers defined this type of scaffold for learning through the apps as pedagogical fidelity.

> ***P2/T48:*** *We have a smartboard at TE. Therefore, I think GeoGebra is useful for presentations of problems. But not many friends do it when we have group work; they have just left it without any interest.*

The way pre-service teacher (P2/T48) used the GeoGebra apps for geometry has changed with the affordances of the DGE features of the app. These DGE facilities (e.g., touch screen or dragging) of GeoGebra motivated pre-service teachers to use GeoGebra as a pedagogical tool. The nature of the teaching and learning content (geometrical constructions), permission to access MT (schools and TEIs not allowing the use of own smartphones as a tool), and technological facilities (multimedia classrooms or labs) were some barriers to the use of apps for geometry in secondary mathematics. However, in the individual interview, a teacher educator (MT6) from the rural TEI2 had a negative belief about learning and teaching geometry with apps:

> ***MT6:*** *Maths-minor pre-service teachers must get good hands-on experience using the compass and other instrumental box tools rather than the apps. The app can't assist the teacher's thought processes of the geometrical concepts. They have to learn geometry through teaching.*

Here, MT6 from TE2 believed that pre-service teachers should learn geometry through teaching. She thought the geometrical app did not develop cognitive fidelity which needs for geometry constructions. MT6 understood that improving mathematics pre-service teachers' mathematical knowledge through teaching experience with apps was not appropriate in the local context. In contrast, teacher educator MT8 at TEI1 allowed pre-service teachers to familiarise themselves with the apps. She believed that Apps (e.g., GeoGebra) develop conceptual and procedural knowledge of geometry. Related subthemes about teachers' knowledge that emerged from survey questions are analysed in the TPACK lens in the next section.

5.4 Different ways of interacting with TPACK

TPACK has served as one of the theoretical lenses (section 2.5.2) for this study. The qualitative data (which had different factors than quantitative data) generated three factors relevant to the knowledge domain components.

An analysis of an open-ended survey question (Q4.1) shows that in the first year of the study in TEIs, only 10% of pre-service teachers said they knew how to use geometrical apps. However, at the end of the second year, up to 68% could use them. On the other hand, 32% of pre-service teachers had negative thoughts about apps. This analysis is based on the TPACK lens of the open-ended survey responses (Q4.2), showing that pre-service teachers' knowledge about apps in the second year had different dimensions (Table 5.2) compared to the first year of study.

Table 5.2

Pre-service teachers' beliefs and knowledge about apps

Knowledge sub-theme	**Dimensions**	**Negative beliefs**	**%**	**Positive beliefs**	**%**
TK	Personal technical knowledge	Apps are not appropriate for the TEI learning space, and they facilitate entertainment only	07	A learner can easily manipulate apps with their technological knowledge	08
PK	Teaching knowledge	Apps create problems for pedagogical approaches	09	The affordance of apps facilitates different pedagogical approaches	14
CK	Learning content	The geometry curriculum is appropriate for psychomotor practice but not for apps	03	DGE Apps have a new interface for geometry learning	09
TPK	Technological cognitive	The influence of ICT promotes fewer cognitive skills	02	Apps promote visualisation	08
TCK	Technical content interface	Technical tools have content possibilities and limitations	04	DGS apps manipulate dynamically	11
PCK	Pedagogical learning space	Apps promote misconceptions about geometry	03	Apps created a new learning space for pedagogical contact	7
TPACK	TPACK apps interface	Apps have some physical limitations	04	Mobile apps promote thinking of thinking (metacognition)	11

Table 5.2 shows that pre-service teachers had different beliefs about geometry teaching relating to the content, compatibility of content activities for the lesson, and trialability of the instructional steps used to implement the task. Some of these beliefs were relevant to pre-service teachers' own experiences in geometry learning. Pre-service teachers revealed their knowledge of using apps to enhance geometry teaching or improve learning outcomes and to use apps' dynamic features to teach complicated geometry (mathematical) content to improve student learning outcomes.

5.4.1 Factor analysis

Factor analysis is a data-reduction technique. In the survey questionnaire, question number 8 had 13 items about the use of apps for geometry. Answers followed a Likert scale, with five answer choices ranging from 'strongly agree' (coded as 1) to 'strongly disagree' (coded as 5). In question 8, 10 of the remaining 13 items were positively worded, and three were negatively worded. The sequencing of items was mixed; that is, items measuring the same dimension were not placed one after another.

Factor analysis (see Table 5.3) was carried out to determine pre-service teachers' perceptions of the mobile app used for geometry. The factor analysis (based on correlation) assumed that the relationship between the variables in the statements was linear. The first step in the factor analysis is the correlation matrix's check factorability, which checks the factor analysis's suitable data set.

Table 5.3

Kaiser-Meyer-Olkin (KMO) index and Bartlett's test of sphericity (BTS)

Kaiser-Meyer-Olkin measure of sampling adequacy		0.717
Bartlett's test of sphericity	Approximate Chi-squared value	285.618
	Degrees of freedom (df)	159
	Significance	0.000

The values are presented (Table 5.4) as part of the factor analysis. Bartlett's test of sphericity (BTS) and Kaiser-Meyer-Olkin (KMO) index confirmed the validity of the statements. BST is significant at $p<0.01$ and KMO index is significant at 0.71. The factors were extracted (Table 5.4) using principal component analysis; the component rotated using direct orientation.

Table 5.4

Extraction values for each statement

Statement no: Statements in the survey	Extraction value

Q2_8_1: I GeoGebra is easy to use (menu-driven, touch screen) for geometry teaching	0.697
Q2_8_2: I can select appropriate geometry content and teach with GeoGebra	0.708
Q2_8_3: GeoGebra task design will help me to think and explain the lesson easily (easy instructional design)	0.666
Q2_8_4: I think GeoGebra will facilitate teaching by manipulating geometrical objects (Easy for students to understand the lesson)	0.590
Q2_8_5: Different colour options for objects in GeoGebra have facilitated the visualised diagram properties so that they are clear for students' learning	0.485
Q2_8_6: I may be comfortable with GeoGebra being used as a technological tool, but I prefer to use paper-pencil for geometrical constructions	0.705
Q2_8_7: I don't feel difficulty in a small touch screen in MT constructing some geometrical figures	0.545
Q2_8_8: I am thinking about my teaching with GeoGebra for learning	0.583
Q2_8_9: I think apps will create new learning options for geometry understanding	0.540
Q2_8_10: I would like to have more opportunities to learn new knowledge with GeoGebra	0.450
Q2_8_11: I am confident in using GeoGebra for secondary geometry curriculum content	0.673
Q2_8_12: I would like to learn more technical features in the GeoGebra app	0.728
Q2_8_13: I have sufficient self-confidence in the instructional design for GeoGebra activities	0.641

Extraction method: the principal component analysis.

Factor extraction involves determining the smallest number of factors (extracts) used to identify the number of underlying variables in each statement (8.1 to 8.13). The variables in the statements used an extraction technique (in Table 5.4): that is, principal component analysis. All the variables have extraction values more than 0.45 and less than 0.73 (<0.9). Therefore, none of the variables in the test item need to be removed from the factor analysis as all statements have the necessary eigenvalue.

The total variance table showed that all test items had relevant factors loaded appropriately as extraction values (any value more than 0.3 is considered appropriate for the analysis).

In factor analysis, it is important to understand the initial eigenvalues and extraction sums of squared loadings (Table 5.5) which give the cumulative frequency.

Table 5.5

Total variances

	Initial eigenvalues			Extraction sums of squared loadings		
Component	Total	Percentage of variance	Cumulative percentage	Total	Percentage of variance	Cumulative percentage
1	3.636	27.966	27.966	3.636	27.966	27.966
2	2.149	16.528	44.494	2.149	16.528	44.494
3	1.151	8.857	53.352	1.151	8.857	53.352
4	1.076	8.280	61.632	1.076	8.280	61.632
5	.954	7.342	68.973			
6	.875	6.733	75.706			
7	.679	5.220	80.925			
8	.557	4.286	85.212			
9	.459	3.529	88.741			
10	.451	3.465	92.206			
11	.421	3.239	95.445			
12	.323	2.483	97.928			
13	.269	2.072	100.000			

Extraction method: the principal component analysis.

The number of factors best describes the underlying relationship among considered statements according to the scree plot diagram (Figure 5.8). The first three factors (on the component axis) considered best to define the underlying relationships are among the component and eigenvalues.

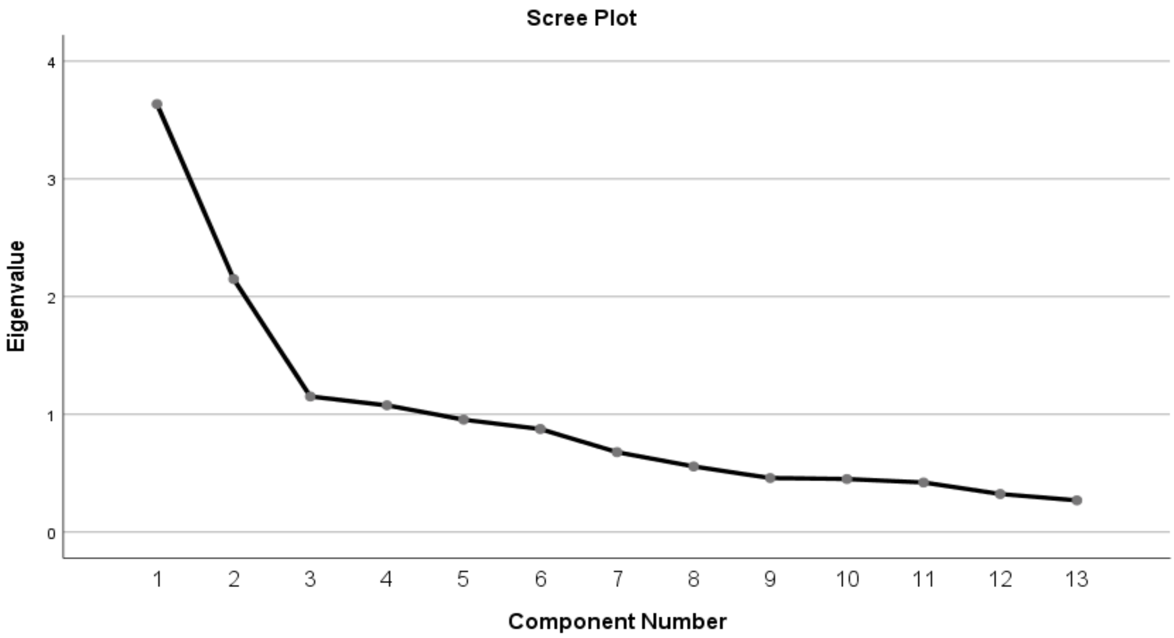

Figure 5.8, Scree plot diagram with eigenvalues

The line plotting of the first three variables in the scree plot diagram above has a steep slope; it describes the underlying relationships in the pattern matrix (Table 5.6), according to the variables clustered in three knowledge components.

Table 5.6

Component pattern matrix

	1st factor	2nd factor	3rd factor
Q2_8_3	0.782		
Q2_8_8	0.766		
Q2_8_5	0.722		
Q2_8_13	0.612		0.442
Q2_8_7	0.590		
Q2_8_4	0.569		

Q2_8_12		0.810	
Q2_8_9		0.732	
Q2_8_11		0.642	-0.432
Q2_8_10		0.431	
Q2_8_6			-0.805
Q2_8_1			0.866
Q2_8_2			0.836

Extraction method: the principal component analysis
Rotation method: oblimin with Kaiser normalisation.

These knowledge components have been given names according to clustered variables PCK (1), TCK (2), and TPACK (3). These three variables can describe much of the variance in the original data set.

Table 5.7

Component correlation matrix

Component	PCK (1)	TCK (2)	TPACK (3)
PCK	1.000	0.067	0.65
TCK	0.067	1.000	0.54
TPACK	0.65	0.54	1.000

Extraction method: the principal component analysis.
Rotation method: oblimin with Kaiser normalisation.

Table 5.7 depicts the 13 items that are positively and negatively affected and are subjected to principal component analysis (PCA) using SPSS (version 20) software. Three factors relevant to the study were determined through statements and then interpreted. To assist in this interpretation process, the factors were rotated, which does not change the solution but helps to present the loading more easily. The rotation method in Table 5.6 is direct oblimin as it gives the number of strongly loading variables. In this study, the TPACK factor is loading Q2_8_2 and Q2_8_1 positively and Q2_8_6 negatively. TPACK helps us to understand how teachers develop knowledge about the integration of apps for geometry from the statements.

5.4.2 Possible interrelationships

All the above findings are relevant to RQ2 and are summarised in the following diagram. In addition, Chapter 4 findings support the detailed analysis and conceptual design of the model shown in Figure 5.9

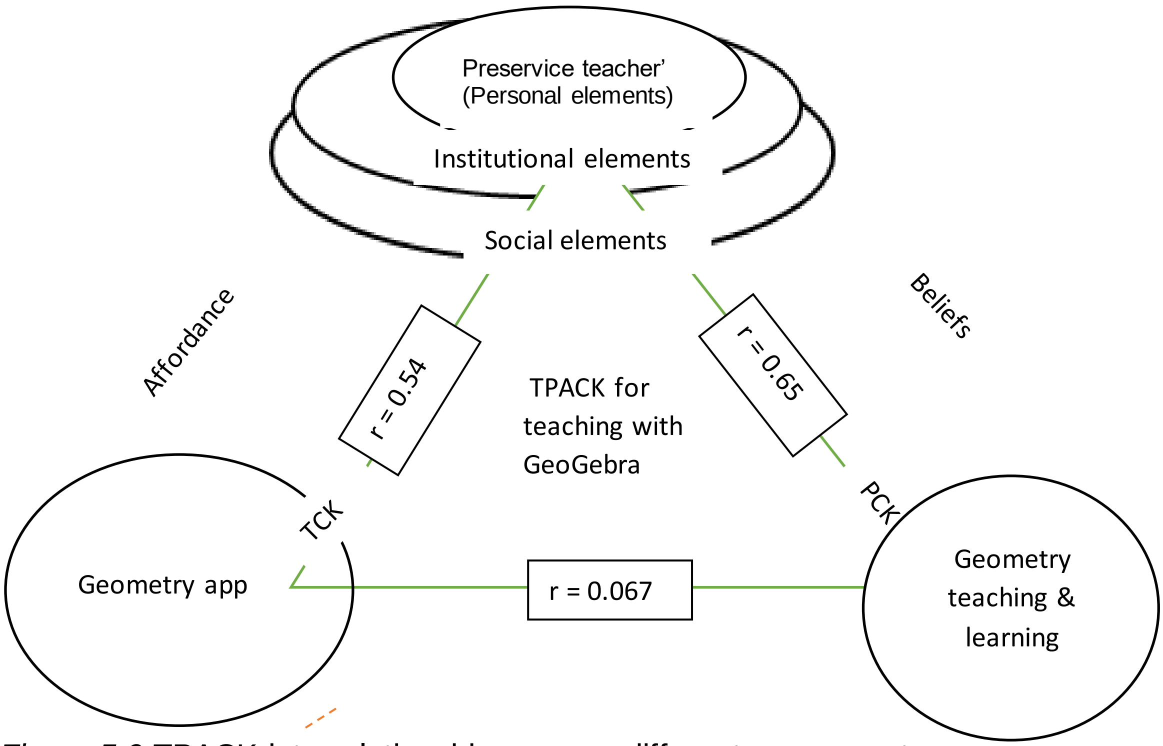

Figure 5.9 TPACK interrelationships among different components

TPACK and the interrelation correlation model served as one of the theoretical lenses (section 2.5.2) for this study. However, it unpacks factor analysis evidence from pre-service teachers' knowledge components, including three knowledge domains: pre-service teachers' pedagogical content knowledge (PCK), technological content knowledge (TCK), and specific knowledge about teaching with TPACK. The factors analysis correlation (Table 5.7) shows correlation values according to interactions between components. The model can classify beliefs and perceptions of affordances.

These apps and specific attributes have possibilities called thinking of thinking TPACK (see Figure 5.9). It is complex to understand how teachers do "thinking of thinking" about TPACK. Integrating 21st century technologies for geometry teaching and learning with apps may be a possibility.

This shows the mutual recognition between pre-service learning and geometry teaching and "thinking of teaching" as a learner in the classroom learning process as well as relationships between the academic contract (e.g., pre-service teachers' knowledge of geometry teaching with apps), pedagogical content knowledge (PCK), and geometry teaching. TPACK recognises, through the tacit acceptance of each party, that there are reciprocal relationships. For example, it quantitatively describes through statistical analysis how pre-service teachers' content knowledge has a r= 0.65 correlation to beliefs in geometry teaching.

Similarly, the factor analysis shows how pre-service teachers' affordances of GeoGebra for geometry teaching and learning had a weak relationship (r=0.067). Likewise, the thematic analysis of interview data showed that an interrelationship extended to TPACK with the PCK and TCK components. These interrelationship values (from the factor analysis) indicated possibilities of extending knowledge in TPACK. The next section describes qualitative evidence of TCK and PCK for geometry teaching, TPACK, and geometry apps.

5.4.3 Possibility of extending knowledge in TPACK with qualitative analysis

Specific knowledge of TPACK emerged as a theme for the first factor analysis (Q8) data. Moreover, metacognition can be defined as the way of thinking about thinking (e.g., pre-service teachers used dynamic features of GeoGebra to incorporate metacognitive thinking into the geometry proofs). Certain attributes (cognitive aspects) of GeoGebra can be considered as enabling an extended TPACK.

To elaborate on the transformative view of extended TPACK, metacognition can be introduced. Metacognitive activities of pre-service teachers include planning how to approach a learning task; using appropriate pedagogies and strategies to solve a problem; monitoring one's comprehension of the text and self-correcting the response to the self-assessment: and evaluating progress toward completing work. For example, creating curiosity in the solver and giving a clear sense of the argumentation process could facilitate metacognition.

In particular, visual explorations of open problem situations can foster the production of conditional statements. Expressing and defending beliefs and opinions and questioning others' ideas can help learners to recognise, clarify, and repair inconsistencies in their thinking (Ball & Bass, 2000; Webb et al., 2006). Webb et al. (2006) pointed out that giving explanations promotes learning, as the explainer reorganises and clarifies material, as she or he identifies misconceptions, fills in gaps in his or her understanding, internalises, acquires new strategies and knowledge, and develops new perspectives and experience.

5.4.4 Potential features

M-TPACK is an imaginary space where the teachers' thinking of teaching, influences the affordances of GeoGebra. A pre-service teacher (P2/T52) who thought she had a different pedagogical approach briefly discussed this:

> ***P2/T52****: In my teaching practice, the proof with geometry problem solving was the hardest lesson, I tried to approach [it] thinking of thinking in my teaching as to how to manipulate data with the app so that students can understand the proof.*
>
> ***Researcher:*** *How did you manage?*
>
> ***P2/T52:*** *In my original lesson plan, I had a goal to get a good outcome from students. I explained the properties of the quadrilateral, which is placed inside the circle, and discussed the theorem relevant to the circle again.*
>
> ***Researcher:*** *Then?*
>
> ***P2/T52*** *My plan changed for Grade 11 students because many students had forgotten the previous lessons on the quadrilateral, which their class teacher completed in the last term.*
>
> ***Researcher:*** *What happened next?*
>
> ***P2/T52*** *I changed my approach for Grade 11 students…I started thinking about my teaching approach. In my previous lesson in Grade 9, I taught quadrilateral properties, which was very successful. So, I tried the same method for Grade 11 students. I started to think of a story on theorem properties I could use that was relevant to a triangle quadrilateral with the app. I listed the properties and asked the students to consider given problems appropriate to them until they understood how to analyse a problem.*
>
> ***Researcher:*** *What about a circle?*
>
> ***P2/T52*** *I waited and gave some tips to think until they understood how to analyse a problem with a circle, then I compared and extended the process to the theorems of a quadrilateral.*
>
> ***Researcher:*** *Were you satisfied with the lesson?*
>
> ***P2/T52*** *I believed that the lesson was a success, but I couldn't finish the lesson in the given time.*

Researcher: *Why?*

P2/T52*: Many students took more time than I thought.*

Researcher: *What did you do?*

P2/T52*: I requested extra time from the class teacher and finished the lesson.*

Researcher: *What did you think about the lesson?*

P2/T52*: My "thinking about my teaching plan how to teach" made this lesson a success by dividing it into concepts. It is easy to think about and teach, although it is time-consuming. I used an app to save time and link concepts within the app. Many students finished their assessments correctly, even though the real problem was not solved completely. However, they revealed and marked facts precisely in the figure, which means they may understand the concepts.*

P2/T52 thought it was a difficult lesson, and she put a lot of effort into the teaching process. Analysing the above dialogue, she tried to foster secondary students' knowledge in separate entities relevant to quadrilateral properties inside a circle. The conceptual understanding of the content of the process was linked with quadrilateral teaching. She attempted to get her students to memorise, critically analyse, and then understand geometrical diagram properties and concepts relevant to the discipline through thinking about their thinking.

Metacognitive possibilities

It is not easy to interpret how these ideas are connected cognitively along with the processes used to establish new knowledge of a circle quadrilateral and to determine the validity of claims. Understanding the content knowledge concepts relevant to the geometry proofs problem was difficult for the pre-service teacher. Her first approach to content changed according to her experience in a block-teaching classroom (students' knowledge) and may have changed her thoughts, which led to a change in teaching practice. This experience may be evidence that pre-service teacher belief changes consist of changes in teachers' classroom behaviour and the art of teaching. Their perspectives about apps may change their teaching sessions' didactical needs. From qualitative data in the focus group interviews, some pre-service teachers described the way they developed their hypothetical learning trajectories:

P2/T32: *In general, geometry problem solving is an interactive work with apps that requires different levels of cognition. Thinking of the way I* *explained to students about geometry problem solving with the DGS app facilities, I have numbered steps in the diagram in the way students may think.*

T32 explained in her focus-group interview that the text in the given problem only includes given facts or information. Also, teachers explained that a separate set of instructions related to pedagogical knowledge building may facilitate clear and accurate diagram development, including ‘thinking about your thinking of teaching”, with GeoGebra (DGS facilitating the app). Some pre-service teachers thought about how to design hypothetical learning trajectories. They have thought about how to teach students the ways of solving the problem facilitated by apps, with the integration of prior knowledge, content knowledge (CK), and technological knowledge of the apps (TK).

5.5 Belief in policies

Pre-service teachers at TEIs experienced the interrelationship between the mathematics curriculum and block-teaching, as shown by the focus group interview data:

> ***P2/T44**: The TEI curriculum has only one module for general pedagogical skills. No time is allocated to geometry subject matter knowledge in the TEI mathematics curriculum, and only 15 hours are allocated to secondary mathematics.*

The study data evidence shows gaps in the mathematics teacher education policy, so self-assessment of areas of weakness by teachers and implementation of strategies to develop knowledge in these areas was undertaken:

> ***P2/T33** There is no proper policy for DT integration at TEI or the national level. Pre-service teachers have an ICT course to learn MS Office packages, but there is no programme to link mathematics teaching and learning with the apps.*
>
> ***Researcher:** Do you think apps will benefit pre-service teachers in their teaching of geometry?*
>
> ***P2/T33:** No point at all [in using apps]. If the national policy or national exam system did not support DT integration, then pre-service teachers are wasting their time. Even the selection criteria are not appropriate.*
>
> ***Researcher:** What do you think of the mathematics pre-service teacher selection criteria?*
>
> ***P2/T33:** The major and minor mathematics teachers have to teach mathematics at school, but they have different selection criteria.*

Researcher: *Why do you think the national exam is not supportive?*

P2/T33: *Teachers have perceptions [that the] traditional rote-learning approach is more appropriate for secondary students because of [the] exam-orientated culture. When some content is geometry in the national exam then you must do rote-learning.*

The reasons for this finding are threefold. First, mathematics pre-service teachers are selected from two different criteria for major and minor streams. However, regarding the after-school requirements, these teachers were assigned to teach mathematics on-demand at school. Second, to integrate 21st Century technology into instruction or to use mobile apps for education, an acceptable DT integration policy is needed at TEI. Third, teacher educators need relevant curriculum-based technology integration training to teach pre-service teachers DT integration into the mathematics curriculum.

5.5.1 Mathematics curriculum policy

In general, secondary mathematics curriculum policy suggests that DT integration may open new avenues in the teaching and learning processes of teachers. The teacher guide (TG) also recommends using digital technology (DT) to access information and enrich teachers' learning with teacher education apps even though policies are not updated yet for MT. During the block-teaching sessions, pre-service teachers could teach mathematics in a real classroom. In the focus-group interview, a pre-service teacher (T38) described his experience as follows:

P2/T38*: After the curriculum reforms, constructive theory (outlined in the national curriculum documents) suggested that students must personally construct mathematical ideas as they try to make sense of situations (communications from others, DT, or the textbooks).*

Researcher*: What is your experience in geometry teaching?*

P2/T38*: Rather than just drill and practice exercises from apps. DT integration for geometry teaching is more than that. We don't have a TEI geometry curriculum. [There is] no ICT policy for teacher education and the national exam system is not supported by technological tools. 21st Century technology is a total failure at TEI.*

Researcher: *Can you elaborate on the idea?*

P2/T38: *Our education system still promotes teacher-centred practice and examination-oriented culture. Even though reforms push student-centred, our curriculum content is too much. It isn't easy to memorise such a large amount of content within 2 years of study and then answer questions. The exam system demands remembered content and awards marks for*

> *calculation rather than geometry concepts. Students can get through the exam with rote learning.*

Some pre-service teachers continue to resist curriculum reform efforts to move away from a teacher-centred approach to a student-centred classroom. However, P2/T38 believed that these changes need to occur; there cannot merely be a reliance on DT tools. He felt that without a proper environment, it is difficult for pre-service teachers to develop a skilled pedagogical mindset, and content application for educational uses of 21st-century technology is paramount.

Communication via social network access increased as pre-service teachers moved to the second year; this was not directed via an institutional or TEI curriculum request. Only some pre-service teachers selected for the study were involved in this transformation of integration through their self-motivation to use mathematics apps. These pre-service teachers' beliefs showed a new trend of ownership of learning for personalised professional development. Sometimes, without optimal learning guidance from teacher educators, this group of pre-service teachers used free online resources for geometry learning and teaching. In contrast, some focus group interview data provided evidence that some pre-service teachers believe they are not well prepared to use digital applications effectively.

5.5.2 DT policy

An analysis of some open-ended questions in the survey indicates that pre-service teachers generally doubt their ability to integrate digital technology into their teaching.

> ***P2/T51****: I have 7 years of experience in mobile technology and always update my knowledge of apps used for educational purposes…We are good at online gaming, but we have less experience using this particular mobile app [GeoGebra] for geometry learning. I am interested in learning, but you know we have a lot of physical restrictions on using a mobile phone during class time.*

Pre-service teacher T51 said that they have administrative pressure (DT policy at TEI) that they spend more time on online gaming and social networks in class time. However, they have an interest in mathematics apps. Here, DT policy influences their use of smartphones. Only two social media study groups (using WhatsApp) mediated pre-service teachers' GeoGebra use at the urban TEI for

GeoGebra learning. T38 mentioned some challenges with the ICT module at TEI:

> ***P2/T38:*** *We have a separate ICT module at TEI. They just teach us how to use different applications on the computer, such as MS Word, PowerPoint, or Excel. I don't believe that I am confident using educational apps on a smartphone; maybe I don't know how to use apps for teaching-learning geometry. Sometime [in the future], TEI may authorise the use of mobile apps for teaching and learning.*

In the previous transcript, P2/T38 said that she has a problem using apps in teaching and learning geometry, and they are not an option in the ICT module. Pre-service teacher education programmes in the local context have recognised the challenges associated with ICT teacher education, so they have developed teachers' abilities to use computer applications, although there is still no training for mobile apps. As a result, pre-service teachers have a separate module for learning computer applications. According to P2/T38, these ICT programmes include MS Office software training only. It means pre-service teachers have less opportunity to learn any innovative strategies with GeoGebra to enhance student teachers' competencies to integrate technology into their teaching and learning according to the DT policy.

5.5.3 TEI policy

Additionally, the pre-service teachers indicated that they believe TEI policy needs to be changed so that basic facilities (such as free internet) for the use of mathematics apps are updated. Sometimes, teacher educators' perceptions of MT would help prepare them for teaching with apps even if it would be considered a contemporary situation.

> ***P2/T55****: Sometimes, TEIs administrators are not happy to provide a free internet facility for students or to at least let us use our mobile phones in our classes. They always think we are still doing social network entertainment. That may not be appropriate for young students. There were some cyberbullying incidents like that, but we are adults, and it is not always valid to generalise for everyone. Luckily, we have lecturers who are always encouraging us to find new ways of teaching using new technologies. As a DT policy, TEIs do not allow the use of smartphones during class time but our lecturer allows us to use them. I am sure they will change.*

P2/T55 explains that some teacher educators have supported the use of apps for pre-service teachers even if it is against the existing TEI policy. These teacher educators are encouraging and providing both formal and informal learning opportunities to develop the necessary knowledge for block-teaching. In the transcripts, P2/T55 pre-service teachers indicate administration problems that they have faced.

Pre-service teachers do not have access to internet-free facilities at Sri Lankan TEIs, so mathematical software use on computers has not boomed. Instead, the smartphone has become a virtual device for pre-service teachers at TEI for teaching and learning geometry from the first year, even though it is unauthorised in the TEI classroom. The challenge for these young pre-service teachers is to behave, teach, and learn differently with/without the permission of the TEIs.

The argument is that pre-service teachers believe that radical reforms need to be made to the mathematics curriculum, TEI, and DT policies even if they conflict with cultural and traditional norms in Sri Lankan society. The following section outlines the professional development beliefs of 21st century pre-service teachers who require increased exposure to DGE apps such as GeoGebra.

5.6 Summary of findings

The pre-service teachers had various beliefs (positive and negative) about teaching or learning specific topics of geometry content with GeoGebra. Interview findings suggest that the pre-service teacher education programme should focus on using apps and how apps can be optimised for teaching and learning the appropriate subject content. However, few teacher educators believed that strong geometry content knowledge alone is sufficient for the pre-service teacher to teach the content in new approaches.

Pre-service teachers' knowledge and their beliefs have been identified regarding the use of apps for geometry teaching and learning. A lack of curriculum content knowledge is evident in geometry mathematics-minor pre-service teachers. This brings the attention to TEI policy needs to address this issue in the curriculum, and these issues address RQ3.

Geometry learning means memorising geometry theorems by rote learning (if at all) rather than in a meaningful way. Moreover, secondary students quickly forget geometry essentials, so they must be re-taught yearly; that is what the spiral curriculum does in the local context. Thus, rote learning is necessary for geometry learning due to less communication between relevant stakeholders (TEIs, MoE) in an examination-oriented structure. Therefore, learning and communication between relevant objects may be closely connected. GeoGebra offers new opportunities for pre-service teachers' geometry teaching and digital learning as it enhances their engagement and problem solving in geometry at block-teaching schools which will address RQ2. As well as the visual and dynamic affordances of GeoGebra, touch-screen mobiles and iPads have facilitated interaction with geometrical phenomena.

This study identified a gap between what pre-service teachers are supposed to teach at block-teaching and what the school stipulated to be taught. Moreover, a lack of communication between TEIs and the relevant authorities was apparent. Pre-service teachers with experience in private schools used GeoGebra for geometry at block-teaching schools. These pre-service teachers discussed the dynamic interactions between objects, specific pedagogical knowledge, technological knowledge, and relevant app facilitation. Only some teacher educators are allowed to use apps for geometry. Apps may facilitate the differentiation of the learning associated with geometry teaching and teachers' beliefs of cognitive understanding of problem solving. This means pre-service teachers' beliefs and knowledge about tools and institutional elements played a crucial role in addressing the RQ3 research question.

Moreover, app affordances allowed some pre-service teachers to increase their ability to transfer between different learning contexts. Factor analysis revealed connections of factors extracted from pre-service teachers' geometry teaching/learning with GeoGebra. In the M-TPACK model, pre-service teachers' metacognition of problem solving with apps includes a critical awareness of the following attributes: pre-service teachers' beliefs about thinking and learning geometry, the pre-service teacher's role as a thinker about learners, and pre-service teachers' thinking about their own teaching. These pre-service teachers may have understood pedagogical reasoning about geometry problem solving with the app, and this addresses RQ4.

Many institutional elements influence pre-service teachers' teaching beliefs and knowledge about the use of apps for geometry. Pre-service teachers' personal beliefs of pedagogical approaches, such as the van Hiele model, may be more appropriate for TEI geometry teacher education. However, teacher educators are worried about evaluation criteria. The apps-integrated pedagogical approach is not recorded during the evaluation process. Even the use of apps has more benefits. Once it’s included in the evaluation process, the pre-service teachers may demand workshops. They will ask for external support to help them understand the link between technological knowledge, pedagogical knowledge, and content knowledge in the use of MT resources. According to the study’s findings, mobile apps have already been utilised by the pre-service teachers in recent years (due to COVID-19) and in the near future may be considered a viable option for teacher education in Sri Lanka.

Chapter 6
Discussion

This chapter undertakes a comprehensive review of how Sri Lankan pre-service secondary teachers' perspectives and beliefs about mobile technologies influenced their pedagogical practices in geometry block-teaching. These are based on synthesising findings from Chapters 4 and 5 to address the research questions central to this thesis. The following research questions guided this study:

The main research question is:
In what ways does the use of mobile technology apps (e.g., GeoGebra) influence Sri Lankan pre-service secondary mathematics teachers' perspectives and beliefs of their pedagogical practices on geometry?
Supplementary questions are:

RQ1 What aspects of using mobile technology apps (e.g., GeoGebra) might influence the geometry content knowledge for pre-service teachers involved in junior secondary mathematics education?

RQ2 In what ways might using mobile technology apps (e.g., GeoGebra) for teaching geometry in mathematics education, influence pre-service teachers' pedagogy when teaching geometry?

RQ3 In what ways might using mobile technology apps for teaching geometry in mathematics education, influence pre-service teachers' beliefs about teaching geometry?

RQ4 How might GeoGebra be used for teaching and learning geometry content in Grade 10 secondary mathematics?

The research questions are addressed in six sections. However, they are not addressed question-by-question, rather than they are discussed in six main themes. The first section (6.1) covers geometry teaching and learning with apps. The second section (6.2) is on app affordances. The third section (6.3) discusses pre-service teachers' geometry content knowledge. The fourth section (6.4) includes pre-service teachers' pedagogical beliefs about apps. The fifth section (6.5) discusses pre-service teachers' perspectives about the use of GeoGebra for geometry. The sixth section (6.6) covers policy gaps and use of digital tools for teacher education, and the final section (6.7) summarises the discussion.

6.1 Geometry teaching and learning with the app

Teaching geometry with GeoGebra (app) has benefits and challenges, which are synthesised into four groups according to the pre-service teachers' perspectives about block-teaching experiences.

The first group of pre-service teachers (mathematics minors) preferred to use traditional ruler-and-compass methods only when they teach geometrical constructions in the physical classroom, and they highlighted difficulties in teaching and learning geometrical proofs in Grade 10. They have no interest in GeoGebra. They believe GeoGebra is difficult to learn alone, while they also think GeoGebra is too complicated to use with the students in the classroom as they prefer to user the conventional methods. One-third of the pre-service teachers in this group have used the van Hiele approach (see section 5.1.2) for understanding geometry problems rather than using an app. This finding is consistent with international research (Kunimune et al., 2010; Noto et al., 2019). These researchers reported similar findings, namely that a challenging aspect of mathematics teaching is the development lesson for geometrical proof with van Hiele approach. For example, Kunimune et al. (2010) reported that many students in lower secondary school in Japan remain at level 1, according to van Hiele's model in terms of construction of a geometrical proof. Noto et al. (2019) discussed the learning challenges of nine Indonesian pre-service mathematics teachers when they were teaching geometrical proof. In addition, Noto et al. (2019) identified learning obstacles related to the difficulty in applying the concept for geometrical proof, such as being unable to construct the proof and not knowing how to start the proof.

The second group of pre-service teachers have used GeoGebra as a demonstration tool for teaching geometrical proof construction lessons for Grade 10 students than to the traditional paper and pencil method. Some of these teachers (see section 5.3.1) prefer to use GeoGebra for diagram visualisation of geometrical proofs with colour and spatial orientation (e.g., symmetry) techniques for construction tasks using geometrical symbols only at the beginning of the lesson as a demonstration tool. These findings are consistent with the findings of international researchers (Baccaglini-Frank & Mariotti, 2010; Castro et al., 2021). In similar pre-service teachers' practicum contexts, tools have been considered psychological, or tangible materials. For example, Castro et al.'s (2021) study was conducted among Colombian pre-service teachers in their internship period. Colombian pre-service teachers had attended Euclidian geometry and geometry methods courses, and the authors indicated the possibility of internalising technology (apps) as a demonstration tool when designing geometrical tasks involving GeoGebra. Other Colombian pre-service teachers used GeoGebra to explore the triangle and the rectangle problems with visual manipulation. Thus, DGS tools required mediation knowledge: the knowledge that a pre-service teacher should have to assess the pertinence of the use of curriculum materials with GeoGebra tools to foster the learning of a specific mathematical task.

The third group of pre-service teachers have benefitted from the dynamic geometry environment (DGE) features (e.g., dragging and VM) in GeoGebra, which makes teaching easy for them. They believed GeoGebra was a tool for teaching difficult geometry content in the secondary curriculum. For example, the use of dragging and moving possibilities for geometrical constructions with touch screen in Grade 10 (see section 5.3.3) geometry. Thus, these pre-service teachers believed GeoGebra has many benefits because it provides a close connection between the symbolic manipulation and visualisation capabilities of understanding geometrical proof for students in a physical classroom or online virtual learning environment. These findings resonate with other research on the use of DT in geometry learning, including similar international studies (Mariotti et al., 2003; Sinclair & Ng, 2015), which focused on the analysis of attributes of DGE tools. For example, Sinclair and Ng (2015) identified different aspects in their research reporting that their participants did not use the symbolic manipulation to understand geometrical proof lessons. Leung (2008) explained the drag facility or other available commands that the teacher can utilise for geometrical proofs in different ways, but the author did not discuss other factors such as the learning environment. Pre-service teachers can conceptualise dynamic features as representations of a mathematical object in GeoGebra to solve proof problems through different procedures in different learning environments. Pre-service teachers in group three indicated that the geometric figures drawn on the chalkboard or students' workbooks are static and sometime difficulty to portray and explain the underlining principles of the concept (geometric diagram) to the students. With GeoGebra, these pre-service teachers believed they could benefit from using dynamic geometric figures that embodied multiple representations of a concept in Geometry.

The fourth group of pre-service teachers, who preferred to use traditional approaches, highlighted the contrasts when comparing relationships between dynamic geometry (e.g., GeoGebra) and traditional learning environments (e.g., the use of geometric tools, such as compass and straight edge) for the construction of geometric figures (see section 5.2.2). They have negative beliefs about the use of GeoGebra for geometry. Moreover, they argued that the computational transformation of some geometry concepts is inappropriate for students. Kuzniak (2015) also discussed computational transposition with DGS. Examples of computational transposition include the construction of geometrical objects in DGS that do not exist in theory, and the distinction (not geometrical) between the points that form the vertices of a constructed triangle in DGS (Patsiomitou, 2018). Thus, this pre-service teacher group believed that the use of DGS apps (e.g., GeoGebra) may not be a solution to their difficulties in understanding certain geometrical proof concepts, as these pre-service teachers prefer to teach with traditional methods following TEIs policies.

These findings partly addressed RQ2 and RQ3 which considered pre-service teachers' positive and negative beliefs about using mobile apps for geometry teaching as an important aspect. Negative beliefs about the use of apps, as well as TEI policy favouring traditional methods, might have influenced the overall use of GeoGebra for mathematics education in Sri Lanka. This study only covers pre-service teachers' geometry learning and teaching experiences with GeoGebra based on some selected topics (e.g., geometrical proof problem solving and geometrical constructions). Thus, the next section covers the pre-service teachers' geometry block-teaching experiences.

6.1.1 Pre-service teachers' geometry block-teaching experience

Findings indicate pre-service teachers' block-teaching experiences varied according to personal aspects (confidence, past experiences) and institutional aspects (curriculum, course, infrastructure), and social aspects relevant to the block-teaching context.

First, pre-service teachers personally believed that their geometry content knowledge influenced their geometry teaching confidence. The Chi-squared test in section 5.4 analysed the correlation between pre-service teachers' geometry content knowledge and their confidence in geometry teaching. The correlation was statistically significant, which means geometry teaching confidence and geometry content knowledge are interrelated. Even gender is not a significant factor: half of the male teachers and one-third of the female pre-service teachers were not confident in teaching geometry curriculum; however, there is no statistical difference between TE1 and TE2 pre-service teachers' confidence in geometry teaching.

Pre and post-test data analysis have shown that the pre-service teachers' mean marks are over 60%, even though they were not confident in teaching secondary geometry. A potential explanation for this self-perceived lack of confidence could be that they may not have acquired the pedagogical awareness needed for teaching or their experience of secondary geometry content. This finding is consistent with the literature that many pre-service teachers' lack a conceptual understanding of mathematics influences their confidence in teaching (Meaney & Lange, 2014; Niyukuri et al., 2020). Meaney and Lange (2014) conducted a study with 104 Australian pre-service teachers and suggested that pre-service teachers limited mathematical content knowledge and previous identities as school students can be connected to institutional identities.

Second, some pre-service teachers said learning from past experiences as students assists in their professional career as mathematic teachers in block-teaching (see section 4.1.3). This was similar to the findings of Barrantes and Blanco (2006) who studied a group of pre-service teachers in Spain about the interaction between their past experiences as students of geometry, and their present conceptions about how geometry should be taught. They found that most pre-service teachers remembered their own experiences as students, and this influenced their block-teaching.

Third, this analysis has shown that some pre-service teachers in the sample were unfamiliar with vertical and horizontal integration of common curriculum content in secondary schools, which has been considered an institutional aspect. In the common curriculum, vertical integration defines as the progress of a specific topic throughout the grades. For example, a student learning triangle in Grade 6 will be learning isometric triangles in Grade 7, essentially an advanced topic from the previous year (vertical developments). The same topics advancing during the same year are considered horizontal development. However, this vertical and horizontal integration was difficult for students and pre-service teachers in their block teaching (see section 4.5). The finding is consistent with a Nigerian study (Adolphus, 2011) that revealed that junior secondary mathematics teachers (30) had difficulty in curriculum integration and performed poorly in geometry. It also mentioned that the same weakness could be seen in students' performance in geometry in Nigeria.

Fourth, mathematics minor (novice) pre-service teachers referred to the time allocation for geometry curricula, lesson plan development support from knowledgeable (GeoGebra experienced) peers. Goos (2005) discussed similar findings with the notion of the zone of proximal development (ZPD) to investigate relationships between novice and experienced teachers using technology as a psychological tool. Further, she analysed different interactions involving the pre-service teacher and their learning environment concerning the use of technology. ZPD is primarily concerned with issues of learning related to interactions with other people as well as the material and representational tools offered by the learning environment, and technology can be considered as such a tool.

Findings of this study mainly covered pre-service teachers' knowledge and pedagogical beliefs in their block-teaching (RQ3). TEIs assumed that pre-service teachers (e.g., P2/T19) are students who completed secondary education, and that they have already learned geometry content in their secondary schools.

Mathematics minor pre-service teachers state they need special support for difficulties faced by geometry teachers. Pre-service teachers believe that their lack of geometry pedagogical knowledge acts as a barrier later in their block-teaching. Moreover, some of these barriers can be categorised as internal (personal beliefs) or external (block teaching school context, or mathematics major/minor courses). For example, the mathematics minor course pre-service teachers highlighted the need for a mathematics module on geometry. Even with less knowledge of geometry, these pre-service teachers designed geometry lessons and tasks for block teaching with (and without) use of apps. Gourdeau (2019) discussed similar findings on the differences between Canadian mathematics courses and mathematics education courses for secondary level mathematics pre-service teachers. Further, they argued that pre-service teachers' knowledge about the curriculum in mathematics and beliefs of lesson plan development has an influence on teaching during their school placements. Pre-service teachers had different beliefs on lesson plan development for geometry teachings such as lesson goals, development paths and instructional strategies for hypothetical learning trajectories.

6.1.2 Hypothetical learning trajectories

The study has given evidence for different learning trajectories of pre-service teachers, each of which starts in their block-teaching contexts, especially the affordances of GeoGebra. Analysing the learning trajectories started with pre-service teachers (e.g., P2/ T7, P2/T24), whose learning goals are developed from geometric construction and geometry proof tasks with GeoGebra. For example, to attain a certain geometrical competence in a geometrical construction about tangents (the goal), secondary students learn each successive level in geometrical constructions (the developmental progression), aided by GeoGebra tasks (instructional activities) with relevant pedagogical moves designed to help students build the mental actions-on-objects that enable thinking at each higher level. In the current research context, pre-service teachers (e.g., P2/T6) believed learning trajectories are useful for thinking about instruction and instructional activities designed with DGS Geometrical construction in Grade 10 is one of the topics taught in secondary classes with DGS. These pedagogical experiences of lesson plan development with DGS can shape a teacher's knowledge and confidence to teach when they become professionally qualified mathematics teachers (Patsiomiton, 2018). Researchers have identified that curriculum interacts with the learning goal during mathematics teaching. (e.g., Mbusi & Luneta, 2021; Smith et al., 1998). Further Smith et al. (1998) explained the importance of learning goals in the design of hypothetical learning trajectories. Moreover, these pre-service teachers can actively engage with hypothetical learning trajectories of secondary students in their mathematics classroom. This partly addresses RQ4.

Some mathematics minor pre-service teachers (e.g., P1/T7) from TE1 did not have much confidence in geometry content or using GeoGebra design task for students in the first year. The pre-service teachers believed that they gained new knowledge with peers when they used apps to develop tasks for teaching and learning geometry with GeoGebra in the second year. In the current research context, these pre-service teachers have overcome difficulty in solving geometrical problems and they use a DGE tool with more knowledgeable peers.

Findings indicated that scaffolding was given by peers (mathematics major pre-service teachers) when the geometry minor pre-service teachers developed their lesson plans or tasks for block-teaching. This current study is grounded in the idea ZPD that pre-service teachers' interactions with more knowledgeable pre-service teacher peers and the practices associated with them including lesson plan development, and the benefits of DGE facilities, enable them to become more confident users of GeoGebra (digital application) in their teaching and learning.

The mediation of the digital application may create a communication channel, according to Hoyles and Noss, (2002). Drivers' et al. (2010) suggested the theoretical perspective that geometry learning is affected and modified by the DGE tools used for learning and that, reciprocally, the DGE tools are modified by the ways that teachers use them. Thus, the evidence shows that GeoGebra has been used for geometry teaching in secondary mathematics as a pre-service teacher's self-dependence personal factor and it is also influenced by many other aspects such as affordances of apps.

6.2 Apps' affordances

Apps' affordances refer to the ability of users to know what can be done and how it can be done using the apps. In addition, app affordance in this study is represented by pre-service teachers' hypothetical learning trajectory with apps during block-teaching. Namely, the use of apps for teaching was associated with pre-service teachers' (e.g., P2/T53) planning and designing of instructional tasks relevant to geometry content, pedagogical content knowledge, and technical reflection of the lesson. These processes provide teachers with new knowledge to acquire opportunities to design and understand the task, and apps provide a scaffold for mathematics understanding (Calder, 2015).

In this study, some pre-service teachers discussed their pedagogical beliefs and knowledge about the GeoGebra features in its navigation for teaching secondary school geometry. The findings relevant to geometry teaching and learning are shown in section 4.1.1. Pedagogical relationships with apps, rather than with paper and pencil, have been considered as foundation benefits for students' engagement with some geometry concepts. Further, pre-service teachers believed that they not only gained geometry content knowledge but learned about their teaching processes (see section 5.3.3) while using GeoGebra for teaching geometry problem solving during the block-teaching sessions.

Pre-service teachers may have had different teaching and learning approaches, but common points of weakness were teaching geometrical construction (e.g., P2/T4) and geometry problem solving (e.g., P2/T7, P2/T53). Pre-service teachers explained why group block-teaching was important for them and how others (e.g., knowledgeable peers) scaffolded them to help them teach difficult geometry lessons with the use of GeoGebra and/or the traditional paper-and-pencil approach. Indeed, affordances of apps, allied with differentiated learning has opened an avenue for professional development (Tomlinson & Allan, 2000).

Pre-service teachers (e.g., P2/T41, P2/T2) believed content knowledge is not the only key feature of geometry teaching and learning. Specifically, in their block-teaching, they judged what would work and then tried it out in a classroom to determine whether it was successful for themselves and their students. Some pre-service teachers were happy to work in the paper-and-pencil environment than use the DGE (GeoGebra) for geometry teaching and learning. These pre-service teachers preferred to use the paper-pencil approach even though students had difficulties understanding geometrical constructions and geometrical proof problems. In contrast, Winer and Battista (2022) studied American high school students' difficulties in geometrical proof problems and suggested that the use of DGS may help the student to understand the formal proofs easily.

In addition, the current study argues that pre-service teachers' teaching in the first year begins with core ideas related to the geometry content (e.g., tangent to a circle in section 5.2.1) to be taught. However, in the second year, some of these teachers believed that their teaching knowledge with GeoGebra in geometry improved. The tentative reason may be influence of the COVID-19 society promoted the use of smartphones for online teaching during the pandemic. The step-back process of thinking about a geometry problem-solving process with relevant instruction before solving the problem was beneficial to the students. As a result, after COVID-19 pandemic, some pre-service teachers believed that they were enriched by a fresh understanding and increased awareness of geometry instruction with apps. Similar findings relevant to teachers were discussed by Faggiano and Mennuni's (2020) analysis of mathematics teaching in a DGE environment with five Italian Grade 12 students during the COVID-19 pandemic. They suggested that DGE has created meaningful technology-rich learning spaces for students to construct meaning for rotation lessons in remote teaching and traditional educational contexts. Affordances of apps may also affect the pre-service teachers' learning and cognitive process such as the reasoning, interpreting, evaluating, and understanding of a geometrical problem.

In summary, when reviewing the data on the affordances of GeoGebra, it was notable how some pre-service teachers' beliefs suggest they are not turning away from the traditional paper-and-pencil method, but they are now using GeoGebra working with objects in DGS. Note that, when interacting with GeoGebra, some (but not all) pre-service teachers spontaneously articulated justifications for their actions while teaching geometry, even during the COVID-19 pandemic. Affordances of DGS can be quite challenging for pre-service teachers. The next section discusses the dynamic affordances of GeoGebra.

6.2.1 Dynamic affordance of GeoGebra

Pre-service teachers (P2/T47, P2/T7) stated that dynamic affordance represents their ability to link and simultaneously interact with visual, symbolic, and geometrical representations relevant to geometric construction problems. Moreover, pre-service teachers believed that dragging with finger movements in GeoGebra allows them to explain the properties of the figure more quickly than paper-and-pencil representations (see section 5.3). When using paper and pencil without having basic geometry construction knowledge (e.g., of a tangent bisector), it was difficult for some mathematics minor pre-service teachers to complete the construction diagram relevant to the given geometry problems. Similarly, Laborde (2002) pointed out that the use of DGE apps evolved from being a visual amplifier to becoming a fundamental component in a diagram that enhances conceptual understanding. It is therefore reasonable to surmise that interactions with GeoGebra provide an opportunity for pre-service teachers to work on diagrams quicker than the paper-and-pencil method.

The current study findings (e.g., P2/T48, P2/T53) indicated possibility of a pedagogical perspective regarding the use of GeoGebra (a DGE tool) from an instrumental approach. For example, some pre-service teachers have difficulty in solving geometrical problems and they use a DGE tool with more knowledgeable peers. In the pre-service block-teaching context and considering DGE as a mediation tool may be more appropriate. Drijvers et al. (2010) suggested the pedagogical perspective that geometry learning is affected and modified by the DGE tools used for learning, and that, (reciprocally) the DGE tools are modified by the ways that teachers are used. Artigue and Trouthe (2021) explained, with DGE apps, a diagram can simply be created by dragging the DGE object while maintaining all the properties according to the given instructions. These authors explained that DGE can contribute to developing pedagogical reasoning, particularly in supporting geometrical constructions. Artigue and Trouthe (2021) did not consider geometrical proof problem-solving tasks. However, didactic potential of DGE may support to understand geometry. Further research is required to gain a better understanding of the didactic potential of a DGE.

This study findings, some pre-service teachers (P2/T47 indicated the possibilities of technical perspective of GeoGebra app, technical features (e.g., touchscreen)) in GeoGebra app may tempt less experienced pre-service teachers to regard “exploring” with geometric drawings as an alternative way to understand geometry (see section 5.3.1). These findings are consistent with those of Richard et al. (2019) who argued that the cognitive process may depend on the geometrical concept, the learner, and aspects of the didactic potential of a DGE. The authors explained that DGE can contribute to developing geometrical reasoning, particularly in supporting geometrical constructions or geometrical proof problem-solving tasks, so that features of DGE is fostered to support proof understanding. Further research on this subject is required to gain a better understanding of the didactic potential of a DGE.

Visual manipulation in GeoGebra may enable pre-service teachers to experience geometric problem solving as an alternative way to understand geometry (see section 5.3.1). Pino-Fan et al., (2018) concur with the findings in this study: the pre-service teachers discussed more localised ways of pedagogical thinking about geometry problem reasoning, and they defined a specific way of acting as a dynamic of mathematical understanding. Faggiano and Mennuni (2020) argued that the cognitive process may depend on the geometrical concept, rather than the learner, and aspects of the technological potential of a DGE. The authors did not consider geometrical problem-solving tasks. However, didactic potential of DGE has supported students to understand geometry (rotation concept) for Italian secondary students during the pandemic. Hence, further research is required to gain a better understanding of the students’ perspective of apps.

This finding has indicated the possibility of pre-service teachers' epistemological perspective of geometry problem solving with affordances of GeoGebra. For example, pre-service teacher (P2/T53) who had less experience in visual manipulation of GeoGebra, used dragging and rotating features to explain tangent to the circle (Grade 10 lesson). Her block-teaching task was analysed to understand the technological perspective (e.g., tangent to the circle) about the dynamic effects of GeoGebra. Moreover, only a few pre-service teachers had done enough exploration into the dynamic features of GeoGebra to properly elaborate on the pedagogical approach of the problem-solving steps. These pre-service teachers believed that dynamic features in GeoGebra can be used to teach geometry problem-solving steps by looking deeper into the geometrical steps of problem-solving. This dynamic tool (dragging) induces a potential dialectic between the conceptual space (abstraction) of mathematical entities and the world of virtual empirical objects. These findings are consistent with Zengin (2017), who studied about mathematics pre-service teachers in Turkey. They have considered geometrical proof problem solving tasks and concluded GeoGebra was an effective dynamic tool for pre-service teachers. The authors explained that DGE can contribute to developing geometrical reasoning, particularly in supporting geometrical constructions. Because of this possibility, dragging has been one of the focus areas of research in DGE resulting in fruitful discussions on promising dragging modalities and strategies that seem to be conducive to knowledge construction.

Political perspective of pre-service teachers (see section 5.5) indicated as a policy gap in pre-service teacher education. For example, pre-service teachers'(P2/T33) lesser awareness pedagogical benefits of the GeoGebra and uncertainty of DT policy were constrained by their use of it. The survey data indicated that TEI pre-service teachers were unfamiliar with GeoGebra functions. Indeed, only two groups of pre-service teachers in the current study used GeoGebra for designing geometrical problem-solving tasks in TEIs. P2/T6 indicated the possibility of facilitating the learning context through the symmetry lesson by manipulating the dynamic facilities of the GeoGebra. Even though, in the current context, many teacher educators and pre-service teachers were still reluctant to access to MT in a block teaching classroom context at some TEI. Research concurs with the findings in the current study: the pre-service teachers discussed more localised ways of thinking about mathematics, and they defined a specific way of acting as a dynamic of mathematical understanding (Komatsu & Jones, 2020; Zengin, 2017).

Pre-service teachers defined GeoGebra in a specific way of acting as a dynamic of mathematical understanding. P2/T53 block-teaching task was analysed to understand the pedagogical approaches (e.g., tangent to the circle) with the dynamic effects of GeoGebra. Komatsu and Jones (2020) research concur with the findings in the current study: the pre-service teachers discussed more localised ways of thinking about paper and pencil activities.

In summary, when reviewing finding relevant to the affordances of GeoGebra, it was notable how some pre-service teachers' learning trajectories are moving away from the traditional paper-and-pencil method to use GeoGebra. However, learning trajectories can be influenced by pre-service teachers' geometry content knowledge.

6.3 Pre-service teachers' geometry content knowledge

Geometry content knowledge of pre-service teachers is an essential component when using the apps according to the views of pre-service teacher educators. The current study indicated that pre-service teachers' geometry content knowledge influences pre-service teachers' pedagogy in teaching geometry with or without an app (GeoGebra).

The survey question relevant to geometry content knowledge has considered only "the geometry proof" lesson in Grade 10 students. Geometrical proof has a means of verification in geometry (sometimes it does not make sense to students); the more fundamental function of explanation and discovery ought to be utilised to present proof as a meaningful activity to students (Hersh, 1993). This requires that students be inducted early in the art of problem posing to allow sufficient opportunity for exploration, conjecturing, refuting, reformulating, and explaining.

The findings of this study suggested that pre-service teachers can be divided into three groups according to their interview responses to geometry proof lessons. In the first group, pre-service teachers said that they could teach geometrical proof using a different pedagogical mediating tool (e.g., stepped task using GeoGebra) (see section 5.3.3). P2/T53 explained about how the design of stepped tasks enabled teachers to use a top-down problem-solving approach to students' positions. These pre-service teachers provided support to learners in constructing knowledge by organising the appropriate tasks. Indeed, pre-service teachers learn with peers to mediate mathematics through apps. Block-teaching context facilitate some pre-service teachers to play an essential role in mathematics teaching as semiotic mediator. However, researchers have highlighted an open relationship between the user and the mediator (Goos, 2006; Jackson, 2017). Leikin (2019) discussed stepped tasks as special design tasks for self-regulated learners. Further, he explained how stepped tasks in DGS act as a mediating tool for geometry problem-solving.

The findings suggested that the second group of pre-service teachers could solve problems in geometry proofs, but they did not know how to teach it to secondary students. The knowledge of teaching how to solve geometry proof problems is different from knowledge of solving geometrical problems. Scholars have offered a comprehensive path to represent the development of pre-service teachers' pedagogical knowledge to teach (Goos, 2020; Mewborn, 2001). The findings from my study were consistent with Mewborn's (2001) study relevant to geometry problem solving; although some pre-service teachers believed they could successfully solve geometry problems, they also believed they could not satisfactorily explain the concepts and procedures they teach. The study is unique in its focus on problem solving in geometric proofs.

The third group of pre-service teachers from both TEIs comprise a geometry minor group that has limited content knowledge about geometrical proof; they do not want to teach or learn that lesson at all because they dropped the geometrical proof lesson in their GCE (O/L) examination. This group of pre-service teachers said that teaching geometry is not compulsory for the TEIs evaluation process or one of the TEIs' needs, therefore they are not interested in geometry teaching. For them, geometry is an out-of-field subject to teach.

Previous experience in the affordances of apps benefited geometry teaching with GeoGebra. Findings indicated that pre-service teachers' pedagogical beliefs about geometry teaching change with their affordances of DGS and hypothetical learning processes. For example, P2/T6 indicated the possibility of facilitating the learning context through the activity by manipulating the dynamic nature of the GeoGebra. Even though, in the current Sri Lankan context, many teacher educators and pre-service teachers were still reluctant to access MT in a block-teaching classroom context at some TEIs, three domains affected pre-service teachers' pedagogical beliefs during their hypothetical learning trajectories in geometry with GeoGebra. The first domain was geometry content knowledge, the second was pedagogical content knowledge, and the third was teaching with GeoGebra-specific) TPACK knowledge.

Regarding the pedagogical content knowledge, teacher educators at TEIs initiated these interactions with institutional elements such as sharing pedagogical workshops on block-teaching experiences. Pre-service teachers described different pedagogical situations during their teaching and learning experiences of geometrical construction problems with or without the use of apps. A group of pre-service teachers in the study believed that they could use GeoGebra effectively, but some had difficulties with the curriculum content as they had less knowledge about the geometry content. Leikin (2006) suggested that teachers' awareness of mathematics curriculum analysis (using curriculum topics) was important for teaching.

When describing the knowledge required for teaching with GeoGebra in an MT, some pre-service teachers explained technical difficulties and manipulation of apps. (see section 4.5). Several researchers (Keskin, 2011; Loughran, 2019) differentiated different forms of teachers' technical knowledge into either basic MT or practical knowledge relevant to apps. Regarding DGS (GeoGebra-specific) pedagogical knowledge, some pre-service teachers at TEI1 changed their pedagogical approach to geometrical construction problems with the help of the dynamic features of the GeoGebra app in their second year.

Chapter 4 findings showed that pre-service teachers' pedagogical perceptions changed with the app's (GeoGebra's) affordances relevant to context and content. Pre-service teachers' metacognition experiences using GeoGebra for geometry problem solving lessons in Grade 10, a dynamic pedagogical approach called M-TPACK was identified. Possibilities of M-TPACK, used for mathematics apps in geometry teaching, requires further research. These findings have some similarities to mathematics learning and teaching research by Kuzle (2017), who showed that DGS can potentially reshape the learning experience of students. However, in contrast, the current study considered the professional learning of novice teachers with GeoGebra. These pre-service teachers' dynamic pedagogical approaches are relevant to some geometry content (e.g., geometrical proofs problem) teaching that emerged from the findings. In contrast, some pre-service teachers (while saying they do not oppose the use of GeoGebra) emphasised the possible (pedagogical) benefits of GeoGebra's use for some geometry content such as geometrical constructions.

6.4 Pre-service teachers' pedagogical beliefs about apps

Pedagogical beliefs, which may be dependent on content and context, can influence pre-service teachers' geometry teaching with apps. Section 5.2.1 indicated that pre-service teachers' pedagogical beliefs related to apps changed from the first year to the second year in their block-teaching experience at secondary schools. Findings indicated two case studies from TEI1 and TE2 have different apps-related pedagogical beliefs among pre-service teachers' groups (e.g., maths minor and maths major) in their block-teaching practice. In TEI1, mathematics minor and major pre-service teachers had different pedagogical beliefs in apps used for geometry. TE2 did not have any inconsistency between expressed pedagogical beliefs of mathematics minor and major pre-service teachers in their block-teaching practice. A possible explanation for these expressed pedagogical beliefs may be the social and contextual demands of two TEIs. A recent Sri Lankan study by Wadanambi and Leung (2019) also expressed the difference between mathematics pre-service teachers' professional beliefs. Further, they explained that differences are due to the social expectation and contextual demands embedded in an educational context. Ertmer (2005) also suggested that contextual factors[40] might cause inconsistency between expressed technology-related pedagogical beliefs and implemented technology-related teaching practices.

The teaching and learning content and teacher educator (mentor) beliefs may influence pre-service teachers' pedagogical beliefs about GeoGebra. For example, the geometry learning content suggested by the block-teaching schools may influence mathematics teachers' pedagogical beliefs. Similar to the present study, other researchers have identified different frameworks of pedagogical beliefs of teachers, including those on the nature of mathematics

[40] For example, ICT facilities (internet etc.) are available in classrooms.

and the actual context of mathematics teaching and learning (Ertmer, 2005; Handal, 2003). In addition. Katmer-Bayraklı and Erişen (2019) suggested that the beliefs of teacher educators in teacher-training programmes may also influence pre-service mathematics teachers' pedagogical beliefs.

Pre-service teachers' pedagogical beliefs about GeoGebra depend on geometry lessons (the ways apps are used for designing geometrical tasks). Teacher educators at TEI1 used GeoGebra to encourage pre-service teachers to engage in self-regulated learning in geometry tasks and instructional design for geometrical proof. Teacher educators at TEI1 believed that some pre-service teachers may have "enacted" pedagogical beliefs. The word "enact" has been used to define pre-service teachers' beliefs relevant to the certainty of the outcome and confidence about their pedagogical approach to relevant content and the use of dynamic features of GeoGebra. However, some pre-service teachers indicated the possibility of "evoking" pedagogical beliefs. The word "evoke" has been used as their beliefs are sometimes inconsistent and evoke tension in the use of GeoGebra for teaching. For example, among mathematics minors in particular, pre-service teachers even used the apps because they mentioned tension in geometry teaching (e.g., the possibility of misconceptions relevant to geometry concepts in such as geometrical constructions). These findings do not directly consist with mathematical beliefs literature; however, Ernest (2016) argued that some teachers believe in the certainty of mathematical knowledge.

An analysis of some pre-service teachers' responses to the interviews' findings indicated a weak understanding of geometrical proof and geometrical constructions. However, pre-service teachers (e.g., P2/T24) are competent in using GeoGebra for constructing geometry tasks to scaffold to students for understanding geometry. These findings have similarities with Crompton et al.'s research (2018) which suggested using DGE applications for teachers to understand geometry. In addition, Haja (2005) studied four secondary post-graduate mathematics education students from the UK and their problem-solving capabilities while they were undertaking geometrical constructions using DGS. Haja (2005) concluded that pre-service teachers were competent enough to apply the content knowledge to construct the dynamic figures using Cabri-Géomètre II software.

In addition, findings indicated that pre-service teachers had wide-ranging beliefs concerning geometry teaching and learning with apps, including pre-service teachers' mathematical knowledge, curriculum, and instructional design of mathematics tasks. Analyses of teachers' views on these subsystems led to the emergence of two major positions regarding mathematics instruction: progressive and traditional (Handal, 2003). Advocates of progressive instruction think that learning is an active construction, drawing on prior knowledge as well as a collaborative construction of socially defined knowledge and values through levels of engagement. In this case, teaching needs to be challenging students' thoughts while guiding them toward a complete understanding. Here, pre-service teachers are expected to organise the learning environment, assess students' thinking, and initiate group activities. It is important to understand the impact of pre-service teachers' block-teaching experience and their pedagogical beliefs on the local geometry curriculum for RQ3 and RQ2. Thus, the next section will discuss the pre-service teachers' inconsistent pedagogical beliefs about geometry teaching and learning.

6.4.1 Inconsistency of pedagogical beliefs

Nearly one-third of the pre-service teachers indicated there is an inconsistency in pre-service teachers' GeoGebra related pedagogical beliefs and apps related block-teaching practice in secondary schools. Section 4.6 explains different factors that influenced beliefs about geometry teaching with GeoGebra such as school context (external) and pre-service teachers' personal conflicting beliefs about apps. A similar situation was identified by Chen (2008) about the inconsistency between teachers expressed pedagogical beliefs and their use of technology. Chen (2008) explored the relations between teachers' pedagogical beliefs and technology with 12 Taiwanese high school mathematics teachers. Further, he concluded reasons for the inconsistency comprise three interrelated aspects: (i) the influence of external factors; (ii) teachers limited theoretical understanding; and (iii) teachers' personal beliefs.

The findings indicated that pre-service teachers, explained their pedagogical beliefs when they are facing problem-solving in teaching geometric construction. Sometimes pre-service teachers attempt to explain their pedagogical beliefs related to helplessness, despair, “how to teach students to understand the geometric construction” relevant to the problem or “what I do for the lack of students’ understanding of the rules of basic geometry construction previous grades”, which have changed with different pedagogical approaches. I have used the word “awoke beliefs” in the relevant pre-service teachers block teaching context about the specific content of “geometrical constructions”. Ertmer (2005) found that “beliefs vary in strength and kind and the ease with which teacher can change their beliefs” (p. 31). DGS options made them comfortable for task design with apps, as shown in section 5.3.3. Such actions may explain why pre-service teachers’ pedagogical beliefs change from the first year to the second year because apps are easy to manipulate.

The findings indicated that many factors influenced pre-service teachers’ geometry content knowledge and pedagogical beliefs during their professional development at TEIs. Moreover, pre-service teachers’ professional learning also had a personal element, such as subject content-oriented tacit knowledge shaped and formed by block-teaching. Literature on mathematics teacher education demonstrates that mathematics teachers learn through their teaching (e.g., Cobb & McClain, 2001). While pre-service teachers were learning about the teaching process during their course at TEI, the survey data indicated that institutional elements were a significant factor affecting this process. Other researchers noticed a similar trend among novice teachers and their learning during the teaching process (Chazan, 2009; Goos, 2008; Ma, 1999). For various reasons, some pre-service teachers were not confident about their geometry teaching knowledge.

Findings from section 5.4 elaborate on the professional learning needs of pre-service teachers as well as different approaches to learning through teaching. Researchers have investigated similar issues and different approaches to professional learning through teaching while focusing on classroom factors (e.g., Meaney & Lange, 2012). However, few studies systematically investigate apps in teachers’ learning (Goodwin & Highlight, 2013; Larkin, 2017), while other studies only cover mathematics teacher development and the support of digital tools. Fung and Poon (2021) highlighted teachers' role in the impact of DGS. Meaney and Lange (2012) suggested that pre-service teachers should not be considered ‘recalcitrant recipients’; rather their professional identities needed to be considered.

Some pre-service teachers completed their geometry problem solving lesson plan alone, whereas some worked in groups, such as “WhatsApp” groups where they were encouraged to collaboratively (social elements) tackle mathematics problems, thus creating a teaching resource from relevant lessons and sharing it with peers, and then suggesting modifications (from group feedback). Their work was not limited to the classroom at TEI, as all their lessons were completed using a mobile device with either an internet or data connection. These findings are consistent with Goos (2005) who investigated the pedagogical practices and beliefs of pre-service teachers in integrating DT into secondary mathematics.

The survey data suggested that the TEI curriculum focuses on pre-service teacher beliefs and aims to change traditional beliefs despite challenges in the mathematics classroom. However, from the selected sample in this study, pre-service teachers’ geometry teaching beliefs relevant to institutional elements were insignificant; this finding will be discussed in detail in section 6.3. The findings in section 4.6 indicate that teacher educators and mentors have different knowledge and beliefs about GeoGebra, especially regarding block-teaching at secondary school. Moreover, all pre-service teachers may not get the same opportunity to explore GeoGebra because it depends on the institutional elements at the TEIs.

The findings show that some TEI1 pre-service teachers understood more about geometry problem solving. Moreover, pre-service teachers at TEI1 were more involved in contingent engagement in professional learning with apps. Pre-service teachers’ contingent engagement in professional education is discussed by O’Malley (2013). In some studies, researchers have discussed teachers’ practices with their beliefs (Ball et al., 2008; Drijvers et al., 2010). Section 5.6 findings suggest that teacher educators influence pre-service teachers’ beliefs about DT in the subject matter and pedagogy for block-teaching. Mathematics teacher education focuses partly on pre-service teachers' important beliefs. It aims to change these beliefs despite challenges in the form of teacher educators' traditional practices. Beliefs are mainly influenced by personal, institutional, and social elements. Haspekian (2014) highlights how teachers are often constricted in their beliefs; he assumed that these beliefs are relevant to personal habits that are socially constructed. Pre-service teachers interacted with their beliefs during their use of apps.

Some researchers emphasised the role that teacher education programmes play in influencing pre-service teachers' DT beliefs and their subsequent pedagogical practices with DT, similar to the current study (Belbase, 2015). This indicates that teacher educators may need to change pre-service teachers' beliefs (Richardson, 1996). These personal elements incorporated during the use of apps were hardly visible among pre-service teachers in the current study. Furthermore, some researchers mentioned similar problems to our study in the access to DT, such as insufficient Wi-Fi facilities for teachers (Koh et al., 2013). Failure to use DT in education is often explained by teachers' negative beliefs about its implementation at relevant institutions (Hernández & Ramos et al., 2014; Koh et al., 2013). In the next section, the link between pre-service teachers' perspectives will be discussed.

6.5 Pre-service teachers' perspectives about the use of GeoGebra

At TEI1, mathematics education transactions occurred between pre-service teachers and teacher educators and were shaped by the patterns of teacher education activity of individuals within the mobile app teaching-learning experience. According to Kearney et al. (2012), three interrelated factors have influenced these transactions: mathematics apps, which manipulate the experience of the mathematics education course (e.g., mathematics major or minor); communication, which exists between the teacher educators or knowledgeable peers and the individual pre-service teachers. These elements cater for various educational needs of transition.

The transition of transferring geometry problem solving in Grade 10 lessons from paper and pencil to apps, and pre-service teachers' attempts in problem solving, could salvage strategies. In this process of learning, the new direction's value was calculated or a jump into the new approach was made (e.g., Schoenfeld, 2016). Findings suggested that GeoGebra does not transform learning; instead, it is pre-service teachers engaged in geometry problem solving (learning independently or with peers) who create meaningful geometry problem solving strategies and present that content using GeoGebra for teaching. These pre-service teachers believed that in teaching and learning geometry (particularly Euclidean geometry) and solving problems related to geometry construction, DGS (GeoGebra) is a better tool than paper and pencil. However, Duval (1998) argued that the paper-and-pencil-based methods are best as they dissociate the diagram from the "process of drawing".

In this study, pre-service teachers explained their transactional teaching experiences and shared reflections (on using a new approach to teaching with the GeoGebra) with peers/teacher educators at TEIs. The pre-service teachers presented their experience in using GeoGebra to problem-solve easily with their peers and mathematics teacher educators. This experience may facilitate improvement from the first to second block-teaching years. A concept described as the psychological and communications space is relevant to the problem (Horzum & Unlu, 2017; Winer & Battista, 2022). Some pre-service teachers felt confident in their transitions with apps as they always had the support of their peers who would be available virtually during specific hours. Additionally, these pre-service teachers were encouraged to use GeoGebra during their lesson development with others (when working in pairs or groups) to complete the teaching practice lessons. Hence there was a continual informal learning cycle with apps, where pre-service teachers demonstrated their personalised understanding of lesson development through more complex dynamic features of apps and collaborative processes including social interactions. The study findings indicated that mobile technology apps' specific aspects (personalisation, virtual collaboration) might have influenced the geometry content knowledge of pre-service teachers, which partly addresses RQ1.

6.5.1 Unpacking pre-service teachers' TPACK levels

A group of pre-service teachers used GeoGebra for drawing diagrams with DGE facilities in their pedagogical approaches to present a geometrical problem for secondary students in a dynamic way. Niess (2005) considered this an “acceptable level” of TPACK, a pre-service teacher direct geometry lesson where GeoGebra was considered a demonstration tool.

Another group of pre-service teachers suggested that DGE enables them to unpack TPACK through the mediating tool (GeoGebra), which may have facilitated students’ thought processes during problem solving. Through the TPACK lens, pre-service teachers have indicated how to use the appropriate instructional skills and processes to solve problems through GeoGebra.

The pre-service teachers also reflected on how to develop a plan for a relevant teaching task. Consequently, these metacognitive elements facilitate the pre-service teachers’ different approaches to the task. An accumulation of resources, however, was not sufficient for productivity but rather led the participants into unproductive directions in the absence of metacognitive monitoring. The beliefs of pre-service teachers to manage GeoGebra with TPACK at the right time was considered metacognitive behaviour for useful decisions in teaching. This study identified pre-service teachers’ professional development needs as they integrate and update DT in their teaching practice. Thus, it is hoped that teacher educators and teacher education institutions will make changes so that pre-service teachers can better develop their practice to make better use of DT in their teaching.

In these situations, metacognitive acts dealt with reflecting on the current geometry problem solving stage, which most often occurred due to pre-service teachers’ reflections. Working with DGS (GeoGebra), pre-service teachers’ problem-solving strategies may have allowed them to move in new directions. Data revealed that insufficient assessment in identifying difficulties in current problem-solving strategies for teaching at TEIs was a common reason for unproductive geometry teaching during the first year. Thus, subsequent efforts to implement new strategies were fruitless and so unproductive teaching occurred, as observed by Schoenfeld (2016). Nevertheless, findings from the current study could synthesise new metacognitive behaviour of pre-service teachers with TPACK “taking a step back” with GeoGebra. It was reflective behaviour which entailed reassessing, organising relevant knowledge, and redirecting those processes that contributed to efficient movements towards a solution. The decision of TPACK to “take a step back” was a metacognitive act that was essential for productive problem solving with GeoGebra in this study. This type of reflective behaviour may promote pre-service teachers' metacognitive awareness.

6.5.2 Relevance of TPACK

The TPACK framework considers the "what" that is to be learned by preservice teachers. TPACK's knowledge (Koehler et al., 2011) demands examining its contextualised implementation; design implies fitting for particular situations rather than generalities. It describes knowledge for teaching as knowledge of pedagogy, content, and technology and their combinations. Findings from chapter 5 factor analysis indicated only three out of seven possible knowledge domains. Other knowledge domains of TPACK did not provide a systematic and meaningful pathway for pre-service teachers. DT researchers discussed three knowledge bases (content, pedagogy, and technology) that form the TPACK framework core (Angeli & Valaniders, 2009; Voogt et al., 2013). The TPACK framework has added seven technology-related knowledge domains from Shulman's (1976) discussed pedagogical content knowledge (PCK) to technology-integrated content knowledge (TCK).

The first factor indicated that pre-service teachers' geometry teaching knowledge (PCK) correlates with the geometry teaching knowledge with GeoGebra (TPACK). The second factor indicated that pre-service teachers' technical knowledge domains (DGE apps affordances) relevant to geometry content (TCK) have a correlation in geometry teaching knowledge with GeoGebra (TPACK). However, factor analysis showed a very low correlation between TCK and PCK.

The findings relevant to the interview's transcripts show similarities with the first three progressive developments of teachers (recognising, accepting, adapting), with technological tools suggested by Niess (2005). Lyublinskaya and Kaplon-Schilis (2022) considered Niess et al. (2005) for teacher-related performance indicators for the overarching conception component of TPACK. They further described TPACK Level Teacher-Related Performance Indicators under five categories. These findings have some similarities relevant to problem solving with GeoGebra. The current study selected a TPACK framework of technological pedagogical content knowledge to build a bridge between these related variables, which emerged from the research questions. In addition, the teaching of mathematics by pre-service or novice teachers, using digital applications, had also been considered in studies by Ball and Bass (2000) and Goos (2005). The dilemma of TPACK research is to think of it as not a unique body of knowledge itself (Graham, 2011). The TPACK is constructed from other latent forms of teacher knowledge as a transformative view or thought. TPACK has a combination of other forms of teacher knowledge and enactment during a teaching in an integrative view. In addition, Drugova et al. (2021) argued TPACK no longer gives holistic view of Russian universities digital learning levels. Inadequate information in TPACK relevant to the university setting SMAR model was used to gather data with the Substitution, Augmentation, Modification and Redefinition (SMAR) levels.

Pre-service teachers' block teaching experience about the use of GeoGebra (apps for teaching and learning geometry) with existing facilities can be found in Chapter 5. Specifically, some teacher educators believed that the app's integration into pre-service teacher education is complex and requires a collective effort by stakeholders such as administrators and mentors during block-teaching. Chapter 5 highlights the interrelationships among technology, content, and pedagogy as well as different aspects that comprise pre-service teachers' knowledge relevant to perceptions of teaching and learning geometry with apps. In contrast, Alizadeh-Jamel et al. (2018) analysed teacher knowledge of DGS with TPACK with a fuzzy analysis based on Iranian teachers' beliefs and added knowledge about tools and their affordances, pedagogy, and content. Niess's (2005) analysis was that a visual description of teachers' thinking and understanding merged towards the interconnected and integrated manner sequence identified by TPACK, which was not always similar in the case of the GeoGebra app used by the pre-service teachers at TEIs. At the Recognising level, the pre-service teacher used instructional technology tools (apps) for motivation only, rather than for subject matter development. This means that the addition of technology does not change the way new material is presented to the students or how students learn the new material.

In the current study, GeoGebra was used by the pre-service teachers for problem solving. The pre-service teachers considered GeoGebra an aid in understanding the conceptual process of teaching and learning geometry and a tool for achieving geometry problem solving objectives with meta-cognition thinking. In two case studies, pre-service teachers believed GeoGebra to be an important instrument to support the instructional design of teaching and learning programmes, especially the different levels of geometry problems. Apps could be used to develop users' skills for cooperation, communication, problem solving, and metacognition of learning (Kuzle, 2017). Other scholars have discussed the nature of the knowledge needed by mathematics teachers.

6.5.3 Possibilities of M-TPACK

Literature still needs contributions from studies that focus on the uses of apps for teaching geometry, students' problem-solving preferences and the influence of pre-service teachers' pedagogy (RQ2). Section 5.3.3 findings suggest that pre-service teachers could develop alternative strategies for geometry problem solving in the GeoGebra environment and explore different pedagogies that could not be easily explored in a paper-and-pencil environment (Koyuncu et al., 2014). Further, Akyuz (2015) stressed that, when users solve problems using technology, they tend to develop different competencies based on their mathematical knowledge. He also discussed that the interaction between TPACK can promote pre-service teachers' thinking of geometry problem solving with DGS.

Study findings show that pre-service teachers believed they had similar experience in geometry content in Grade 10 but taught in different ways. Ernest (1989) discussed the effects of teachers' knowledge of mathematics and concluded that two teachers may have similar knowledge but teach in different ways of thinking in teaching. Metacognition is defined in different ways by many researchers (Baltaci, 2018; Flavell, 1979; Fung & Poon, 2021). Baltaci (2018) explained metacognition is one of the theories that enable pre-service teachers to connect new knowledge to the knowledge they have about DGS, observe their teaching, and internalise new knowledge by using them in problem solving of geometric locus. Schoenfeld (2016) defined metacognition as a person's activation of maintaining and planning processes of her thinking practices. Therefore, metacognition involves the process of individuals' thoughts about the thinking and learning mechanisms and combining these thoughts with their experiences, and planning. Fung and Poon (2021) explained the possibility of boosting mathematics understanding through dynamic activities in GeoGebra.

Metacognitive TPACK (M-TPACK) facilitates the employment of problem-solving strategies: identifying the key concepts underlying the geometry problems, recognising types of representations of GeoGebra that could be useful, and carrying out operational processes to find a solution path to a given geometry problem. Researchers have explored how metacognition is involved in geometry problem solving with DGS and how it operates during a given task (e.g., Kuzle, 2017). Study findings reveal that entities such as problems, concepts, methods, and arguments can be considered primary objects in problem solving in the DGS.

6.6 Gaps in policies and use of digital tools for teacher education

A synthesis of the findings in TE1 and TEI2 shows that pre-service teachers have contradictory perspectives about DT policies at TEIs and their experience of digital tools in block-teaching contexts. However, both TEIs' pre-service teachers emphasised the DT module at TEIs, the only opportunity for pre-service teachers to gain experience in Microsoft (MS) software applications (e.g., MS Word, PowerPoint, Excel) as traditional ICT policy.

Some pre-service teachers showed their affordances of GeoGebra (for rapidly developed DGE tools) and DGE-related pedagogies for mathematics teacher education. For example, some pre-service teachers used to drag (DGE facilities) and a touch screen as pedagogical media for geometry problem solving (e.g., tangent to circle problem solving) without any training about apps in the TEIs. Pre-service teachers indicated the possibility of using GeoGebra to develop geometrical concepts. However, 32% of pre-service teachers believed that DT tools do not have any pedagogical benefit for teaching and learning geometry.

To address RQ4, findings on the use of DGS show that pre-service teachers discussed their difficulty in finding the correct object relevant to the concept of the geometry problems. The term *object* is broadly used for any entity involved in GeoGebra practice and is identified separately. For example, the way apps are used when one performs a geometry problem solving exercise can be described in steps, and the objects involved can identify the use of languages (verbal, graphic, symbolic, and mathematics languages). This mathematics language is the ostensive part of a series of geometry concepts, definitions, and procedures involved in understanding the problem, and evaluation of objects' configurations is common in the examination. The term *configuration* is used to designate a system of objects related to each other that are relevant to the problem (Koyuncu et al., 2015). Thus, elements for understanding difficulties relevant to problems can be seen from personal and institutional elements discussed in the study.

To address RQs, the study has focused more on the geometrical problem-solving lesson experience of pre-service teachers in DGE tools at block-teaching schools. One group of pre-service teachers had pedagogical perspectives about GeoGebra: the DGE app was a valuable tool for teaching problem solving lessons visualised tasks (tangent to circle problem) relevant to secondary classes, even though it had been challenging for students to solve. The "geometrical problem" is a situation that consists of exact open questions which will challenge mathematics teachers intellectually (Schoenfeld, 2016). Further, he explained mathematical problem solving is a process of engaging in a steps task on a different level for which there is no obvious or immediate solution. The involvement of metacognitive knowledge and personal beliefs about maths, the goal of instruction, and a pedagogical imperative are also discussed in his work. The relationship between the use of paper and pencil and DGS also depends on the tasks designed by the pre-service teacher during the lesson (Koyuncu et al., 2015). The DGE activity design plays an important role in teaching and learning geometrical problem solving (Fung & Poon, 2020), and so it should not be kept apart from mathematics curricula (Richard et al., 2019)

The second group of pre-service teachers had a political perspective: that the problem-solving steps were not always linear, and they argued about curriculum and TEI policy. They argued that geometrical problem solving was difficult to consider students' achievements. These pre-service teachers believed that the structured mathematics curriculum and selected exercises in workbooks were not themed by the concepts suggested in curriculum policy.

They were more interested in standard curriculum materials provided by the state, even though they thought GeoGebra may be good for geometry teaching. They never used GeoGebra for their geometry teaching as the TEI's policy did not mandate the use of GeoGebra. However, these pre-service teachers believed that an updated students' mathematics textbook aligned with the secondary school curriculum was urgently needed. According to their beliefs, officials who are responsible for the national examination paper (GCE (O/L) should think about the mathematics understanding of students rather than being concerned about setting complicated problems for the geometry component and avoiding marks for arithmetic calculation.

During the problem-solving process in DT, teachers can realise students' difficulties in understanding mathematics and learn about their problem-solving tendencies. On the other hand, the synthesis of findings in Chapter 5 showed that only some second-year student teachers had improved the teaching of geometry self-regulated learning with GeoGebra. Their perceptions demonstrated that they had missed something they expected to learn at their TEIs, such as the integration of DT in geometry teaching. This suggested that a new module (DT integration for mathematics) is needed after collaboration to replace the existing basic computer awareness module at TEIs. This is consistent with previous research findings by Tanner et al. (2005) who suggested that successful DT integration depends on developing a shared vision with the administration.

An important finding in Chapter 4 was that the mathematics teacher education curriculum at TEIs included a DT module only covering Microsoft Office packages, presentations, and the use of digital equipment such as smart whiteboards. Therefore, pre-service teachers have demanded that a new DT module be introduced for mathematics education at TEIs, which would enable pre-service teachers to change mathematics thinking, classroom dynamics, and user roles with apps. Furthermore, TEIs intensify learning opportunities for novice teachers by focusing their attention on the students' thinking. For example, Cobb and McClain (2001) acknowledged that, while analysing a TE-designed curriculum to improve student learning, they realised how much the novice teachers themselves had been learning regarding theory and experiences. This research identified some TEI issues; namely, the existing teacher education DT was not meeting the needs of grassroots-level users.

6.6.1 Making connections to policy.

Chapter 4 shows how pre-service teachers' factors and geometry teaching beliefs relevant to the context are interrelated. It also suggests how these beliefs are linked with pre-service teachers' knowledge about apps in selected geometry curriculum content and app affordances. Moreover, it is dependent on pre-service teachers' knowledge of curriculum policy. Regardless of the environment (DT-rich or otherwise) where pre-service teachers worked, they developed their knowledge according to available resources and experience within the context, which matches the findings of Larkin (2014). Over half of the pre-service teachers thought that their teaching knowledge with apps had improved their professional development.

Challenges that occur between teacher educators and pre-service teachers are shaped by different interactions within the TEI context. Pre-service teachers' beliefs reveal that three interrelated factors have an influenced interaction: the structure of the mathematics education course (major/minor); the dialogue between the teacher educator and the pre-service teacher as a learner; and the level of autonomy of the individual pre-service teacher to have to use resources (DT policy). These are institutional elements that mathematics educators can manipulate to cater to pre-service teachers' various educational needs studying mathematics education.

Successful use of apps is much more likely when pre-service teachers share personal elements (such as understanding the implications of apps by themselves) within their mathematics teacher education institutes (Richard et al., 2019). Nevertheless, as is shown from the findings in this study, pre-service teachers' beliefs are often developed during pedagogical practice development on geometry with easy access to apps and collaborative working with the group at the block-teaching school (TE policy).

In mathematics teacher education with DT, especially virtual reality, Dynamic and Interactive Mathematics Learning Environments (DIMLEs), are still at the primary level in terms of their use in mathematics classrooms. Therefore, the exponential development of DT and mobile applications (apps) have challenged traditional educational, social, economic, and cultural habits, and is an area that needs more research. It may not be possible to ignore the subsequent demand for DGE apps for mathematics in future teacher education programmes.

To summarise, it is apparent that three domains (curriculum policy, DT policy, and TE policy) are interconnected in this study. More importantly, this synthesis has shown that some pre-service teachers believed that support from others is important in their professional development. This partly addresses RQ3.

6.6.2 Communication of institutional and personal elements

Chapter 4 showed how pre-service teachers' self-regulated learning plays a vital role during the use of GeoGebra in the education of mathematics teachers. Further connections of experience are identified in cognitive psychology studies. McCombs (1989) defined experienced teachers with self-regulated learning ability, including strategies for knowledge acquisition and procedures for problem solving. This includes pre-service teachers' innovations on why, how, and what to teach in geometry lessons. For example, some pre-service teachers developed the foundation for understanding geometric proof (van Hiele level 4) in my study. This is the hardest component of teaching (Pavlovičová & Bočková, 2021; Usiskin, 1982). The work of Goos (2008) showed a different approach to support change in the way pre-service teachers viewed the context and influence of other pre-service teachers. Goos (2008) in her case study, further analysed how novice teachers have changed to align with ZPD.

The finding indicated that the action research experiences of pre-service teachers at TEIs are important when exploring new pedagogical approaches for mathematics teaching. For example, many pre-service teachers recorded reflective journals when they were implementing lesson plans with GeoGebra, experiencing a technology-based environment in the mathematics classroom. These pre-service teachers intentionally learned technical details of GeoGebra and ways of managing student learning, and unintentionally developed sensitivity to their students and knowledge about them. Leikin and Rota (2006) performed a joint retrospective analysis of a teacher who changes their personal beliefs in teaching.

Interestingly, Chapter 5 shows that (at both TEI1 and TEI2) pre-service teacher educators' personal beliefs about the ICT module generally indicated that the teachers supported geometry teaching. However, pre-service teachers at TEI1 said that the ICT module was only for application packages, and it did not help with how to use apps for geometry learning or block-teaching. Those in TEI2 said they would like to modify the ICT module because they believed it did not support geometry teaching and learning. It appears that pre-service teachers at both TEIs reported less confidence in the DT-related module benefits at TEIs as they considered they did not support their geometry learning or block-teaching practice. These findings contrast with those of Lai and Pratt (2004), indicating teachers had considerable support from DT integration modules for mathematics education. This may be because DT modules' content at TEIs did not match pre-service teachers' professional development needs in mathematics education.

Pre-service teachers (mathematics minor course) explained that they faced difficulties regarding the content knowledge of some topics, which they had learned by themselves with the support of the apps. This argument is supported by Koyuncu et al.'s (2015) study, showing that secondary mathematics teachers' strategies for geometry proof problem-solving improved with the use of DGS. The study also indicated that around 10% of secondary mathematics pre-service teachers were motivated to learn GeoGebra by themselves and use it for geometry block-teaching. This study has considered GeoGebra as a self-learning tool without any training workshops, or any other formal request from the MoE. Moreover, in developing countries such as Sri Lanka, various social and cultural contexts influence MT apps.

Another significant aspect is at TEIs, where some teacher educators' beliefs about the use of apps for geometry are subject to the influence of institutional elements such as approval of institute administration policies. Findings indicate that some pre-service teachers improved their knowledge of some geometry concepts as well as their use of digital-mediated pedagogy. This will be discussed further in the next section.

6.6.3 Pre-service teachers' perspectives in geometry teaching and learning

Professional learning for geometry teaching requires an understanding of geometry and how students learn geometry. Some pre-service teachers believed that they were not supported properly during their professional development at TEI. In contrast, more than half of pre-service teachers indicated the benefits of collaborative support (from peers) for geometry teaching and problem solving at TEIs. This is consistent with literature underscoring the value of the geometry module with DT for pre-service teachers (Butterfield & Chinnappan, 2010), although there are many advantages of geometrical constructions made with DGS for the pre-service teachers' professional development module. In contrast, Niyukuri et al. (2020) showed that pre-service teachers in Burundi still prefer traditional professional development training methods.

A synthesis of section 5.3.1 findings shows that pre-service teachers reflected on their previous teaching experiences in traditional paper-and-pencil methods even when they were using GeoGebra to develop tasks for geometrical proof. Moreover, construction activities with paper and pencil should not be discarded because both DGS and paper-and-pencil environments may contribute to novice teachers' conceptual understanding. The findings are consistent with the Koyuncu et al. (2015) study of 33 Turkish prospective mathematics teachers who were allowed to use the GeoGebra freely during the paper-and-pencil problem-solving process. The authors concluded that GeoGebra solutions might be affected by their paper-and-pencil solutions, and they suggested more experimental studies may be needed in this area.

The cognitive processes of pre-service teachers' use DGS is relevant to the block-teaching practices, and their knowledge of learners, pedagogy, and curricula. Kulze (2017) viewed the DGS as a cognitive tool that can enhance and reorganise the geometry problem solving process. Based on this approach, M-TPACK was beneficial to novice teachers' professional development. The following section outlines the professional development beliefs of pre-service teachers who require increased exposure to DGE apps such as GeoGebra.

The impact of perceived gaps in DT policy beliefs regarding mathematics teacher education in Sri Lanka and concerning the RQs (RQ3 in particular) is discussed. Chapter 5 also indicates how mathematics achievement in secondary grades has influenced different pre-service mathematic education (major/minor) teaching courses at TEIs. Moreover, the findings suggested that the quality of geometry teaching with apps at TEIs requires an understanding of pre-service teachers' perspectives and beliefs concerning DGE apps.

The current research findings revealed that pre-service teachers' confidence in geometry teaching, their pedagogical beliefs, their knowledge of geometry teaching and how students learn geometry have some similarities to the work of Fung and Poon (2019). They worked with 38 Chinese students who studied in GCE (A.L) classes in a Hong Kong school and determined that the group of students who used GeoGebra improved their metacognition compared to the control group. Their results reveal that metacognition (for prediction and planning), in particular, can be significantly improved by using GeoGebra rather than the traditional method. In addition, Fung and Poon (2019) suggested that visualisation, dragging, dynamic platform, and immediate feedback can be considered underlying factors for such development. My study considers pre-service teachers' use of DGS for geometry problem-solving teaching, but it has similar factors that may influence their metacognition possibility of TPACK for geometry teaching.

As noted in the literature review and reveal from the findings, TPACK provides a conceptual framework for analysing pre-service teachers' knowledge and beliefs about the use of GeoGebra for their block-teaching. Figure 5.4 represented the relationships among components in a generalised knowledge component from factor analysis. In addition, section 5.3. shows the possible representations of the metacognition possibilities of geometry problem-solving analysis with TPACK experienced by some pre-service teachers (around 10%) participating in this study who are new to the field. These pre-service teachers were new to the GeoGebra app, and they understood the facilities of the GeoGebra app themselves for geometry problem-solving.

The discussion suggests the possible future usefulness of the M-TPACK approach for mathematics education with apps (DGS) with proper research work. Indeed, it will be valuable for pre-service mathematics teachers' professional development as well as the thinking of geometry understandings in selected domains.

An additional benefit includes pre-service teachers' geometry curriculum content revision; respondents mentioned that GeoGebra (DGS) was helping them gain confidence when consolidating and teaching geometry, even in the unexpected conversion to online mode during the pandemic. Pre-service mathematics teachers suggested that education DT policies may be required to support this change in the thinking of geometry curriculum tertiary education institutes (e.g., TEIs) with MT. Abeygunasekera's (2021) research argued the benefits and challenges gained from the COVID-19 pandemic in the unexpected online MT experience of tertiary education students.

Three negative aspects emerged in the study relating to GeoGebra use at TEIs that conflicted with some existing socio-cultural beliefs. First, a GeoGebra-equipped smartphone cannot be offered to every pre-service teacher and secondary student in the classroom (this is an equity issue). Second, mobile phone apps distract pre-service teachers and hinder the learning process (according to traditional beliefs). Third, mobile apps can create independency and so impoverish personal relations.

These views of traditional educators opposing mobile technology apps for teacher education were limited to only two TEIs. However, the survey responses and beliefs of pre-service teachers revealed that mobile app use in school has some controversial aspects, and the problem of integrating mobile apps for teaching and learning into the traditional Sri Lankan classroom will not be resolved easily.

6.7 Summary of discussion

This chapter contains an overview of pre-service teachers' block-teaching experience with and without GeoGebra within limited facilities. Pre-service teachers have different perspectives relating to geometry teaching with apps. These changes have already come from the mathematics classroom, but all teacher educators may not successfully exploit them. Pre-service teachers' perspectives on the use of digital technology (apps) for selected geometry content changed with the challenging nature of the block-teaching school context. Pre-service teachers and teacher educators were not the only individual elements in block-teaching practice; other elements (e.g., curriculum, learning context, social aspects) contributed to the use of apps for secondary geometry. The selected groups of pre-service teachers improved their geometry content knowledge over the year. However, only a few teacher educators believed that TEIs' use of apps may substantially assist the user pedagogically, with benefits in developing pre-service teachers' understanding of geometry concepts.

The discussion indicated that apps such as GeoGebra are useful for some pre-service teachers in their teaching of difficult geometry concepts. Hence, the different aspects of affordances of apps are inextricably linked to the way pre-service teachers interact with them. In some cases, pre-service teachers engaged in collaboration share technological aspects with peers and they can use those experiences about GeoGebra later in their independent performance as teachers. The findings have indicated apps (such as GeoGebra) facilitate teaching some curricula content. Pre-service teachers considered some symbolic instructional activities to geometrical proofs in Grade 10 students with GeoGebra. These aspects are important to consider as elements of pre-service teachers' geometrical knowledge. Mathematics education is going to remain a traditional model that involves face-to-face lecturers in systemic and systematic evaluation, or it will become mixed-mode learning with digital applications (e.g., DGS), depending on policies.

The discussion highlights the effect of existing policies on mathematic pre-service teachers' content knowledge and professional learning with apps. The synthesis of findings also identified some mathematics minor teachers' professional learning gaps as well as the urgent need to strengthen geometry content knowledge (both derived from the beliefs of pre-service teachers). These are institutional recruitment issues, related to recruitment policies. The study also suggests that the curriculum policy, DT policy, and TE policy be updated to properly include the use of apps, and this may benefit teachers in their professional development.

Metacognition TPACK (M-TPACK) is an exciting new knowledge tool for further researching students in mathematics teacher education. The results of my study indicate the possibility of hypothetical learning trajectories, that the use of the DGE apps promotes pedagogical mathematical practices at TEIs, which may impact a generation of learning for future students.

Chapter 7
Conclusion

7.0 Introduction

This chapter considers and articulates the conclusions drawn from the study. It summarises the significant findings relevant to the research questions, such as the influence of GeoGebra on mathematics pre-service teachers' perspectives and beliefs concerning their geometry teaching. The study is based on two case studies of pre-service teacher education institutes (TEI1 and TEI2) in Sri Lanka. Pre-service teachers' beliefs and knowledge pertinent to geometry content were examined, as well as curriculum tensions at the TEIs relevant to the RQs. In addition, pedagogical knowledge without the GeoGebra app and with GeoGebra through the TPACK lens was also considered to gain insights into the ways that they addressed the RQs.

The first sub-section provides an overall summary designed to address the four RQs and is presented according to the themes while considering the particular contextual factors related to the study's environment. For example, relevant to the RQ1, the study indicates that mathematics minor pre-service teachers at TEI2 have less confidence teaching some geometrical concepts than mathematics major pre-service teachers in the secondary curriculum. In contrast, some mathematics minor pre-service teachers from TEI1 preferred to use GeoGebra (DGE) features (e.g., dragging) to teach some geometry concepts (e.g., symmetry) in their block-teaching. This occurred even though their expertise was developed through informal personalised learning with peers and the use of GeoGebra was not promoted within the existing DT policy at TEIs.

The second sub-section discusses the contribution to the field that the research project makes to mathematics education and teacher education in general. The research can benefit the Sri Lankan mathematics education community as it is the first study in the local context relevant to using apps (such as GeoGebra) for geometry teaching by pre-service teachers. It also indicated some gaps in the structured, rigid mathematics education curriculum. The importance of an updated DT policy with the inclusion of relevant 21st Century technologies is also considered. However, for teacher education, according to DT policy, this is not presently an option that is promoted in the block-teaching practice.

In the third section, implications relevant to the study are considered under teachers and TEIs. Concerning teachers, their ownership of professional learning has become an important part of their internship as pre-service teachers. Teachers' beliefs about app affordances as well as the importance of pedagogical approaches for block-teaching practice are analysed through the TPACK lens and discussed in this section. Changes in teacher education relevant to mathematics minor teacher recruitment are indicated as an urgent implementation that needs to be addressed in the TEIs. Indeed, the differences between mathematics minor/major are not considered in the evaluation for block-teaching practice according to the TE curriculum.

The fourth section discusses the methodological and social limitations of the study. The methodological section discusses the challenges of the methodological approach and case study design. The social limitations include relevant social issues (e.g., political unrest and the first wave of the COVID-19 pandemic) which influenced pre-service teachers' beliefs and delayed the data-collection process of the study. Finally, some suggestions for future studies and a summary of the study are discussed.

The next section discusses the summarised responses to the research questions (RQs) in more detail.

7.1 Overall summary that addresses the RQ

This study's research questions addressed pre-service teachers' beliefs about geometry teaching and learning with GeoGebra in the Sri Lankan context, through an in-depth synthesis of two years of research findings.

Specifically, the main research question is:
In what ways does the use of mobile technology apps (e.g., GeoGebra) influence Sri Lankan pre-service secondary mathematics teachers' perspectives and beliefs of their pedagogical practices on geometry?

Supplementary questions are:

RQ1 What aspects of using mobile technology apps (e.g., GeoGebra) might influence the geometry content knowledge for pre-service teachers involved in junior secondary mathematics education?

RQ2 In what ways might using mobile technology apps (e.g., GeoGebra) for teaching geometry in mathematics education, influence pre-service teachers' pedagogy when teaching geometry?

RQ3 In what ways might using mobile technology apps for teaching geometry in mathematics education, influence pre-service teachers' beliefs about teaching geometry?

RQ4 How might GeoGebra be used for teaching and learning geometry content in Grade 10 secondary mathematics?

First, RQ1 addresses pre-service teachers' knowledge about geometry content in the secondary curriculum in selected aspects. The effect of content knowledge on pre-service teachers' confidence in geometry teaching at secondary schools was one selected aspect. As an example, in content knowledge, two selected areas (geometrical problem solving and geometrical constructions lessons) were considered. Some pre-service teachers believed that their relatively weak content knowledge negatively influenced their geometry teaching confidence. Findings were relatively consistent in that pre-service teachers had difficulty with the competency-based approach (e.g., with curriculum material such as students' workbooks and TGIs) relevant to geometry problem solving, which influences their geometry teaching. The lack of knowledge about how to use GeoGebra for the geometry curriculum was raised by several pre-service teachers at TEI2. However, pre-service teachers' beliefs about the affordances of apps (GeoGebra) were statistically significant for TEI1 pre-service teachers only. In general, other beliefs (e.g., the pedagogical approach and prior knowledge about apps) that influenced geometry teaching with GeoGebra are addressed under RQ2.

Second, concerning RQ2, pre-service teachers expressed different understanding of the use of pedagogical theories and approaches (e.g., van Hiele's theory and TPACK). The discussion provided valuable insight (e.g., professional repertoires), into the relationship between pre-service teachers’ different pedagogical approaches and teachers’ geometry knowledge. In addition, confidence in the use of apps for teaching was dependent on different elements (e.g., prior experience). The use of GeoGebra for block-teaching had different challenges, with the lack of opportunities for a new pedagogical approach within a traditional teacher education policy the main challenge. Pre-service teachers have requested upgraded technological facilities (e.g., free internet access for using apps as teaching tools) at TEIs as well as an updated DT policy. They also believed that having free internet access would motivate them to use DT more in their block-teaching. Beliefs about the use of GeoGebra for teaching geometry in selected content in secondary geometry will be addressed in RQ3.

Third, concerning RQ3, beliefs about geometry teaching were linked more with the pre-service teachers’ knowledge about lesson content and the use of pedagogical tools. However, they believed that block-teaching experience with apps at TEIs, such as presenting a problem, drawing a diagram, allocating resources, and presenting the teaching process to peers in reflective workshops, positively influenced their confidence in geometry teaching and professional development. This shows that pre-service teachers' beliefs about teaching geometry with GeoGebra (or other mobile apps) still replicate traditional and/or administrative practices but in a different way.

Pre-service teachers also believed that effective pedagogies (with apps) have a relationship with instructional design for geometry curriculum content and teaching. These beliefs also depend on personal awareness (pre-service teachers’ confidence), institutional (disciplinary context), and social elements (e.g., political unrest and the first wave of the COVID-19 pandemic in 2020) relevant to teaching. These teachers’ block-teaching placements were conducted according to TE policy, aside from technical difficulties, and some teacher educators acknowledged that pre-service teachers had limited opportunities. For example, in phase 2 of the study (2020), which occurred during the first wave of the COVID-19 pandemic, pre-service teachers had limited access to traditional classroom teaching. Therefore, several pre-service teachers who participated in the focus group interview in the second year talked about the relationship between their beliefs and pedagogical practice with apps integration for online geometry teaching. These teaching beliefs about the use of apps changed from the first year to the second year of block-teaching. The findings indicated that these beliefs may have been influenced by the approach of individual pre-service teachers, the geometry teaching required for national examinations, confidence about geometry content in the block-teaching institutional experience, or social elements within the educational system.

Fourth, concerning RQ4, pre-service teachers' block-teaching experiences from two selected lesson topics (geometrical construction and geometrical problem solving) in the Grade10 geometry curriculum were considered. The affordances of GeoGebra features were visible in their use only from pre-service teachers in TEI1. These pre-service teachers used DGS features of GeoGebra (e.g., dragging) in their teaching process. Only one teacher educator at TEI1 agreed to use GeoGebra for geometry teaching for pre-service teachers' block-teaching, although this was not mentioned in DT policy at TEIs. For instance, in the current TEI timetable, only three hours of geometry are allocated for pre-service teachers over two years. Therefore, pre-service teachers and teacher educators believed more time was generally needed for geometry teaching.

Finally, the main research question is about the perspectives of pre-service teachers' pedagogical practices and beliefs in geometry block-teaching with GeoGebra. The study has proposed four ways that apps can influence mathematics pre-service teachers' perspectives: pedagogical, technological, political and epistemological. The pedagogical perspectives have focused on teaching and learning geometry. In addition, the findings indicated that pedagogical factors (e.g., content knowledge and pedagogical knowledge) are influenced by beliefs about teaching geometry. The pre-service teachers' pedagogical thinking and planning on how to approach a learning task with apps do not have the same impact as the literature in the TPACK framework.

The technological perspective has mainly considered the GeoGebra affordances and functions of GeoGebra, the dynamic geometry environment (DGE) relevant to the geometry curricula. Technological skills in self-learning apps have considered and improved in Covid 19 pandemic. It accompanied the voice of pre-service teachers through touch-operation based device /smartphones with GeoGebra. The personal ownership of devices and peer collaboration facilitated self-regulated learning, which has become an important part of a pre-service teacher's lifelong learning. Affordances of GeoGebra have facilitated pre-service teachers' confidence in the technical manipulation of the tool. This is considered important because some pre-service teachers believed that their consistent and constant engagement with GeoGebra motivates them to improve their geometry teaching.

The political perspectives were considered, including social context, policy, and cultural settings in the block-teaching practices, that are relevant to the use of apps with the selected geometry content. Pre-service teachers' personal, institutional and social beliefs have influenced personal responses to political perspectives. For example, some pre-service teachers have used different strategies to solve geometry proof problems, monitoring one's comprehension of text, self-correcting in response to the self-assessment, and evaluating progress toward the completion of a task with GeoGebra, even when the TEIs policy had not promoted it.

The epistemological perspectives were understanding GeoGebra for the development of geometrical concepts. The geometry understanding which has emerged through pre-service teachers' involvement with DGE features was considered. Findings from this study suggested the possibility of contributing to the literature by the identification and consideration of M-TPACK (metacognitive technological pedagogical content knowledge) relevant to the selected geometry concepts (e.g., geometrical proofs). Therefore, pre-service teachers' reflections and processes suggested metacognition as an awareness of one's thought processes and an understanding of the patterns behind them. This process was another thread to the extended TPACK model with apps, which is discussed as metacognition TPACK (M-TPACK). Metacognitive activities include planning and reflecting iteratively on how to approach a learning task. Many aspects and some obstacles influence how a pre-service teacher might use GeoGebra in their block-teaching classroom, including their beliefs about geometry, the affordances of apps, as well as their perspectives on mathematical knowledge. Therefore, this study makes some contributions to the field of teacher education and mathematics education relevant to the local context, which is outlined in the next section.

7.2 Contribution to the field

As discussed in the previous section, this study contributes to the use of mobile apps (e.g., GeoGebra) for mathematics teacher education in the Sri Lankan context. This is the first research study undertaken in Sri Lanka examining the use of MT apps in the teaching of secondary school geometry. It is also the only research study covering new knowledge about DGS specific to the mathematics pre-service teacher education in the Sri Lankan education context. However, it enhances the broader field of mathematics education and teacher education through contributing insights from the particular context that was examined. In addition, the study highlighted the relationship between pre-service teachers' beliefs and their geometry teaching knowledge. These pre-service teachers talked about their teaching experience in extended TPACK, with GeoGebra in the respective domain (e.g., pre-service teachers' pedagogical thinking).

First, the study indicated the possibility of pre-service teachers' pedagogical beliefs and knowledge about the use of GeoGebra as it has changed from the first to the second year during block-teaching. Some pre-service teachers' pedagogical beliefs about the use of apps for geometry can be considered into either "enacting" or "avowing" a belief. For example, I use the term "enacting" to enact a belief equating to pre-service teaching confidence in teaching with some geometry content with the use of the dynamic features of GeoGebra. I use the term "avowing" to avow a belief which means the belief is inconsistent and evokes tension during the geometry teaching with GeoGebra. For example, one pre-service teacher explained how her beliefs link to her tension between what she thought about how students incorrectly respond to geometry problems. Specifically, pre-service teachers believed they have a tension related to pedagogical practice, related to the way they should teach, as their teaching may not address students' misconceptions about geometry concepts. In addition, students faced difficulties in geometrical problem solving in the geometrical constructions during the block-teaching.

Second, the relationship between mathematical knowledge and the use of specific tools for teaching selected geometry content depends on personalised (teacher educators' or pre-service teachers') beliefs. For example, many teacher educators believed that a ruler and compass were the most representative tools for teaching geometry in the local context in Sri Lanka. In contrast, pre-service teachers used a different DT tool (e.g., GeoGebra) and relevant pedagogical approaches for selected geometry content (e.g., geometrical proofs) to improve their specific teaching knowledge during the second year of block-teaching. They found these approaches more beneficial even though some teacher educators did not encourage them. Many pre-service teachers were aware of students' difficulties in understanding geometrical proof problems in geometrical construction without a mediating tool. Therefore, some pre-service teachers used a DGS facility (e.g., dragging) mainly for teacher demonstrations while preparing teaching material in the second year of block-teaching. In contrast, several pre-service teachers did not use DT tools for their block teaching at all, even though they stated the benefits of GeoGebra in the survey questions. This tension suggests implications for practice and policy.

Many pre-service teachers believed that the dynamic features[41] of GeoGebra do not require the manual (sketching/drawing) ability to obtain perfect step-by-step graphical outcomes of the geometrical proof problems. These pre-service teachers believed that the dynamic features of GeoGebra may positively contribute to teaching difficult geometrical concepts in mathematics education. Another aspect that contributes to the field is the notion of M-TPACK, which could be defined as the facilitation of reflective thought through construction within a problem-solving task. As well, self-regulated thinking and reflection on

[41] According to pre-service teachers this is a simple method used to draw and manipulate lines and shapes.

planning and teaching (including technological, pedagogical, and content knowledge of the problem) can be considered M-TPACK knowledge.

Some pre-service teachers are involved in the possibility of metacognition thinking of teaching actions for problem-solving with GeoGebra, before the lesson, during the lesson and after the lesson (see section 7.2.3). More second-year pre-service teachers from TEI1 showed greater confidence and better preparation in teaching geometry with MT (GeoGebra) after the first wave of the pandemic compared to their first year. This may be because they had inadequate block teaching experiences and tight academic schedules in their first year. Their learning development may be due to the COVID-19 pandemic as forced lockdowns created a good atmosphere for self-learning with apps. Moreover, some pre-service teachers won more freedom to explore and utilise apps in different thinking which may have influenced their beliefs and knowledge about geometry teaching. Even though the current study did not plan for the social impact of COVID-19, this study has given some indication of how online MT teaching benefits pre-service teachers' beliefs on mathematics education.

7.2.1 Mathematics education

Mathematics education in Sri Lanka currently places too much emphasis on a traditional, structured, and rigid curriculum rather than the flexible curriculum demanded by mathematics teachers. The current study indicates pre-service teachers' beliefs about the teacher educators' autonomy in the delivery of the mathematics curriculum with the ICT module at TEIs. This ICT module mainly focused on specific IT application content (e.g., Microsoft Word, Excel, and PowerPoint) rather than the pedagogical approaches for DT (e.g., GeoGebra) use in mathematics education. Pre-service teachers believed that TEI may facilitate the use of MT (e.g., apps) as a pedagogical tool for their block-teaching. This means that, in some cases, it was difficult to examine the relationship between pre-service teachers' (e.g., mathematics minor) pedagogical beliefs about teaching with GeoGebra and their actual block-teaching experience relevant to mathematics education.

First, without sufficient prior geometry knowledge (e.g., mathematics minor pre-service teachers may lack the required geometry knowledge), it was difficult for the pre-service teachers to master the mathematics curriculum at TEIs in the first year. This means mathematics minor pre-service teachers suggested that they need extra time to familiarise themselves with the mathematics content knowledge. Many pre-service teachers had expected at least to have extra online or hybrid[42] geometry modules at TEIs, apart from the formal mathematics curriculum.

Second, the pre-service teachers expressed less confidence in teaching geometry when they have used different instructional design approaches with or without DT tools or GeoGebra to deal with problem-based teaching and learning in mathematics education. Moreover, findings indicated they have used different approaches in the instructional design appropriate for DGE apps for geometry teaching as students faced difficulties in some geometry domains. For example, pre-service teachers believed that students have difficulties with geometrical problem solving, particularly those involving geometrical constructions. Some pre-service teachers requested after they obtained their NDT diploma, study for a further mathematics education degree part-time at TEI as an extension of their diploma programme in collaboration with local or foreign universities in DT education as it would benefit them.

Third, the pre-service teachers indicated that an updated mathematics education curriculum using apps as teaching and learning tools was also needed. GeoGebra, nor any other mathematics apps[43] are currently included in the TEIs' mathematics curriculum as resources. Moreover, pre-service teachers believe that GeoGebra is a good pedagogical tool for dealing with selected content in the mathematics curriculum as it will motivate pre-service teachers to teach more productively. However, these pre-service teachers also believed that certain system elements in the local context in Sri Lanka, such as the competitive, examination-orientated tuition culture and the first wave of the COVID-19 pandemic, influenced the need for a mathematics curriculum update.

[42] Online and face-to-face teaching.
[43] Only suggested for school curriculum in selected lessons.

Finally, teacher educators believed that the competency-based education curriculum reforms in 2007 tried to address many of the above issues, but they failed because of weaknesses in the implementation process. These teacher educators believed that pre-service teachers' traditional beliefs about teaching and assessment methods had not changed to meet the demands of competency-based curriculum reforms in teacher education. Therefore, pre-service teachers' beliefs about block-teaching were important in the pre-service teacher education programme at TEIs. Pre-service teachers' input into resourcing, pedagogical approaches, and time allocation for professional teaching were seen as critical in the teacher education process.

7.2.2 Teacher education

The study also revealed difficulties faced by teacher education with implementing the DT policy or competency-based curriculum reforms imposed by the MoE. Teacher educators preferred to have the academic freedom for teacher education to address the changes in reforms. Furthermore, they felt the need for teachers' education and to cater for individual pre-service teachers' needs.

The study revealed that pre-service teachers believed that the existing system sometimes does not have options to act according to TEIs' needs. They must teach according to the structured curriculum with a given structure and time targets suggested by the NIE or MoE. The discussion showed that some pre-service teachers experienced fewer problems teaching with the same geometry curriculum during the second year of block-teaching, particularly around new pedagogical approaches. First, TEIs and their mentors (at block-teaching schools) need to understand pre-service teachers' beliefs about geometry teaching and pedagogical approaches before the block-teaching. Concerning teacher education, the research has shown that pre-service teachers' beliefs about learning outcomes from curriculum structure, course content, and pedagogy perspectives needed to be considered.

As well, it was apparent that practitioners (pre-service teachers and teacher educators) had contrasting beliefs about the DT integration policy. Pre-service teachers had positive beliefs about the policy updates with app resources. Moreover, they believed that the relevant app resources could be shared with stakeholders, and this would benefit TEIs. For example, pre-service teachers believed that GeoGebra's best-practice materials for solving some geometry problems in the national examination can be shared with all relevant stakeholders in all TEIs.

A synthesis of the analysis showed that many pre-service teachers believed there were gaps in the TEIs' mathematics curriculum and secondary school curriculum. For example, the findings showed that Euclid's elements in the Sri Lanka secondary curriculum are mostly influenced by geometrical problem solving in local secondary students' textbooks. Still, less emphasis has been placed on the TEI's curriculum, such as how to teach via problem solving for the development of geometric thinking, which is important. The study also revealed difficulties faced by TEIs (teacher educators and pre-service teachers) implementing the DT policy in that they cannot always act according to the policy, or the structured curriculum as suggested by the NIE or MoE.

The ownership of professional learning and the affordances of these hand-held smartphones and GeoGebra (their application) capabilities will benefit teacher education in pre-service teacher progression through TPACK levels. The contribution of TPACK and its influence on the personal, institutional, and social elements at TEIs needs to be considered in policy as part of any successful mathematics teacher education innovation in Sri Lanka.

This study of mobile app use (e.g., GeoGebra) has contributed by filling a gap (DT policy implementation literature) in teacher education in Sri Lanka, especially in integrating apps for mathematics teaching and learning. However, these pre-service teachers believed that better DT infrastructure may contribute to improving general geometry teaching, as well as the use of apps. The most important part is the proper implementation strategies for pre-service teacher education.

Finally, pre-service teachers believed that teacher education programmes at TEIs continue to place a heavy focus on theoretical issues in classroom management but relatively little focus on the subject matter, especially during the block-teaching. The TPACK framework (Mishra & Koehler, 2006) classifies teachers' knowledge required for DT integration and how they might develop this knowledge. In the current study, pre-service teachers believed that this extended aspect, which I have labelled as an M-TPACK framework, supported their pedagogical approaches as well as the instructional design used. This M-TPACK model would be useful for professional development in Sri Lanka, especially for future research, as well as for consistency in using any app aspects such as developing logic and proof. The next section discusses the possibility of M-TPACK.

7.2.3 Possibility of M-TPACK

Some pre-service teachers discussed self-regulated learning support to think about how apps should manipulate dynamically without extraneous cognitive loads, such as needless animations or irrelevant information. They further explained that the interface design in the manipulation of mobile apps will promote thinking about dual-channel processing during the instructional design of the lesson. Hence, dual-channel processing has involved the use of the DGS app as a tool for geometry teaching. The first channel is visual pictorial (a visual diagram of the geometry problem), which processes images seen visually (including details displayed on a mobile screen). The other channel is the auditory-verbal channel, in which the pre-service teacher processes spoken words during the lesson. The use of DGS apps has an active processing capacity when pre-teachers actively engage in cognitive processes by preparing and selecting the relevant pedagogical approach, organising it into visual and/or verbal models, and integrating those new models with prior knowledge.

First, when the pre-service teacher becomes more experienced, a similar problem-solving aspect of thinking could mentally represent the technological manipulation of the app and the pedagogical approach to the geometrical problem. In the planning, the lesson sequence of problem-solving activities with manipulation, pedagogical knowledge, mathematical knowledge and technological knowledge of the app were combined by the students. Self-regulated thinking and reflection on planning and teaching (including technological, pedagogical, and content knowledge of the problem) are considered M-TPACK knowledge.

Second, we might regard M-TPACK as the facilitation of reflective thought through a construction within a problem-solving task (i.e., doing these construction tasks with MT, hence evoking the possibility of new knowledge emerging). This would indicate a need for integrating metacognition within an extended thread in TPACK, which is when TPACK can be conceptualised as meta conceptual awareness of the demands of the geometry problem solving teaching as well as the pre-service teachers' knowledge in the sub-domains and the respective context. Third, these claims are discussed in terms of the possibility of coherent versus multiple theories based on the pre-service teachers' metacognition of geometry teaching with TPACK in the Sri Lankan context. The next section discusses the implications of the study.

7.3 Implications

In this section, implications are discussed pertaining to three areas: practice, institutions, and policy. Implications relevant to practice emerged from aspects discussed by pre-service teachers and teacher educators including certain aspects such as Sri Lankan classroom culture influencing teachers' perspectives and beliefs about the use of GeoGebra. The implications relevant to TEIs for geometry which require attention by the relevant authorities are improvement of geometry content knowledge among pre-service teachers at TEIs. Finally, implications relevant to policies have been considered.

First, pre-service teachers indicated that secondary students are generally weak in geometry problem solving at the block-teaching schools where they do their practicum practice. They believed this was not only due to the inadequacy of geometry content knowledge but also the students' difficulty in understanding the process. These pre-service teachers believed the process meant analysing the problem and understanding or searching the diagram and other relevant steps. It was a challenge for pre-service teachers to teach better understanding to evaluate the adequacy of the given information and organise facts related to the problem goal. The pre-service teachers believed these elements of metacognition appeared when they thought about their geometry problem solving teaching (e.g., using GeoGebra) in a dynamic geometry environment. Therefore, informal learning of pre-service teachers (through the lens of TPACK with GeoGebra) with their own MT devices (e.g., touch-operation based smartphones) can be officially promoted by an action research project. This may be another project for their career development support programme at TEIs.

Second, pre-service teachers' perspectives and beliefs about the use of GeoGebra for Grade 10 did not always match with teacher educators' and some school administrators' beliefs. For example, teacher educators believed that developing problem-based learning is a good approach for apps in mathematics education as for the teacher education curriculum update at TEIs, but many pre-service teachers do not indicate that option for TE policy updates. Secondary school-imposed rules influenced pre-service teachers during block-teaching in that they perceived geometry as a rote-learning subject, and pre-service teachers should be encouraged to follow that. Meanwhile, some school administrators believed reviewing national examination problems (e.g., past exam papers) on the blackboard was more important than using DT tools for teaching practices (institutional elements). Therefore, formal introduction to DGE apps for geometry teaching and learning will be challenging, TE policy updates will be the priority for such implications at TEIs.

Third, pre-service teachers in preparation programmes are realising that using these DGE apps (GeoGebra) introduces a unique set of challenges and opportunities relevant to the social issues for the use of mobile apps for teacher education. Also, some teacher educators still believed that pre-service teachers using a mobile app as a tool was not a fair option due to personal beliefs, or socio-cultural beliefs that smartphones are evil for students. For example, in the local context, internet users (teacher educators and pre-service teachers) at TEIs used their smartphones the most (rather than a desktop or laptop computer) during the pandemic as a primary tool for accessing the online (internet) education due to the unavailability of basic IT infrastructure (e.g., wi-fi or physical internet connections) and the relatively high cost of computers.

In contrast to the teacher educators' negative beliefs, some pre-service teachers believed that using GeoGebra on smartphones enables them to teach geometry in a more dynamic way than just explaining, writing, and drawing diagrams on a blackboard. However, the teacher educator's beliefs may have changed due to personal, institutional, and social needs. For example, during the first wave of the COVID-19 pandemic, these smartphone apps became tools in the pre-service teachers' professional learning. Capitalising on pre-service teachers' extensive use of smartphone apps for block-teaching led to teacher educators at TEIs deliberately including these apps more in their online courses during the first wave of the pandemic (social and institutional elements). A key implication for these institutes is to build on this change of attitude that the COVID-19 pandemic facilitated and evaluate how this might most effectively be utilised for developing programmes that incorporate DT for the enhancement of personal content knowledge and pedagogical approaches.

All these developments are hard to put into operation, however, and do not always correlate well with large classrooms and traditional beliefs about paper-and-pencil assessments (at secondary schools and TEIs) in Sri Lanka. Pre-service teachers believed that defining the criterion of fair assessment will benefit implementation (if possible). For example, several pre-service teachers were not allowed to teach geometry with apps during block-teaching practice as it did not match the block-teaching practice assessment criteria or the policy. However, an implementation plan for individual teaching improvement with GeoGebra will be not possible without the harmonised social and cultural role of mathematics teachers.

7.3.1 Mathematics teachers

Socioeconomic factors have influenced mathematics teachers' beliefs and knowledge of mobile apps (e.g., GeoGebra) for mathematics education in the developing country context of Sri Lanka. Teachers have difficulties in economic aspects, such as the time taken for participants to acquire a smartphone and relevant Wi-Fi facilities, or the use of mathematics apps for educational purposes. Pre-service teachers believed that these aspects were important and are influences for determining whether they would successfully engage with DT in their learning and their teaching practice.

The discussion shows how pre-service teachers want to use DT more regularly in their block-teaching practice. They also believed that GeoGebra can be integrated into geometry education when it is necessary to teach difficult concepts. The pre-service teachers have shown the possibilities for metacognition of geometry problem solving with TPACK (in which geometrical concepts and dynamic features of GeoGebra are addressed together leading to a deeper understanding of geometry problems). One of the pre-service teachers' transitions is to think about geometry with a TPACK lens. The transition of pre-service teachers might suggest a significant shift in thinking and metacognition knowledge. Thus, providing opportunities for pre-service mathematics teachers to acquire such experience at TEIs and think about different pedagogical approaches is important, even if it is not consistent with TE policy.

The micro-teaching and reflective practices were expressed by groups of pre-service teachers throughout their block-teaching. These pre-service teachers have viewed geometrical constructions and problem-solving lesson plan exemplars in Grade 10 with different TPACK levels (Niess et al., 2005). The lesson plan exemplars for Grade 10 were explained by pre-service teachers, some of whom have used the DGE features of GeoGebra for geometric construction (adapting level). An approach needs to be taken that will allow these adapting level pre-service teachers to transition to the next level. Another group of pre-service teachers believed that instructional design with DT tools (for geometry problem solving with GeoGebra task) is recognised but their lesson plans are made mostly without any pedagogical tool. In contrast, some viewed it as a type of 'metacognition thinking' about their teaching process with GeoGebra features relevant to the geometry problem solving concepts. However, pre-service teachers' pedagogical understanding of apps has remained a complex and challenging aspect of professional learning (with the relationship between apps and pedagogy in TPACK). The extended framework of TPACK has highlighted the process pre-service teachers undergo in geometry teaching, including the phenomenon of geometrical problem solving supported by GeoGebra.

In addition, teachers believed that updated resources and a relevant pedagogical approach with apps were erratic due to underlying external challenges (social constraints) in Sri Lanka. For example, the final data analysis of the (phase 2) interviews of pre-service teachers' beliefs about DT tools were challenged and changed probably due to the enforced online teaching proficiency (due to the COVID-19 pandemic lockdown). During the pandemic, teacher educators have encouraged the use of mediating DT tools (GeoGebra) for pre-service teachers who do not have proper internet access. Of concern, IT infrastructure (Wi-Fi facilities) is not equally distributed among TEIs and pre-service teachers at a time when teacher educators confront challenges associated with the COVID-19 pandemic. These pre-service teachers were involved in using mobile mathematics apps as an offline geometry teaching and professional learning option for mathematics.

7.3.2 Pre-service teacher education

After synthesising the discussion related to pre-service teachers' use of GeoGebra for learning geometry, possible teacher development opportunities were identified. Some pre-service teachers learned the GeoGebra app by themselves or with peer support and explored different pedagogical approaches for geometry teaching and learning. However, during the first year in TEI2, many pre-service teachers had more negative beliefs about DT as they would be less experienced in block-teaching under the existing DT policy.

In the first year, only a small group of pre-service teachers had positive beliefs about the use of apps, and they experienced difficulties in defining tasks relevant to geometry content. Moreover, they believed the most challenging part of the implementation was selecting the cognitive approach in GeoGebra relevant to geometry content. In the second year, more pre-service teachers believed dynamic tools in GeoGebra were easier to use when interpreting (instructions for) the lesson plans with their own defined tasks. They believed GeoGebra made it easier for them to display diagrams step-by-step with relevant instructional sequences than to use the traditional ruler and set squares for drawing diagrams on the blackboard (especially for geometrical construction tasks). Hence, some pre-service teachers believed that GeoGebra can play a supportive role (organising the geometry content) in block-teaching.

Pre-service teachers also believed that GeoGebra made it easier for them to think about describing a particular geometry concept (e.g., symmetry) relevant to the geometrical problem. It facilitated symbolic manipulation (visualisation) rather than the paper-and-pencil method. This feature may have also improved pre-service teachers' opinions of GeoGebra for the teaching process.

Moreover, interview transcripts show that these pre-service teachers' metacognitive views of the teaching process with TPACK (metacognition view of dynamic features of GeoGebra for pedagogy, M-TPACK) may have better improved secondary students' conceptual understanding of geometry problems compared to verbal or written instructions. The pre-service teachers believed that teaching confidence gained from improved app knowledge had led to them thinking about teaching geometry in different ways. For example, they believed they used extended TPACK to identify the best possible teaching and learning method for geometric constructions and geometric proofs.

Pre-service teachers' confidence in teaching geometry was another significant aspect identified in this study. Moreover, low confidence among pre-service teachers was commonly reported in the sample, especially in the second-year mathematics minor group. Pre-service teachers' low confidence in geometry teaching may have strongly influenced their beliefs, knowledge, and possibly, their professional development. Few pre-service teachers believed that these gaps in professional development, even after they had graduated from the TEI could support informal learning by using smartphone apps. This implies that developing pre-service teachers' confidence in conjunction with their content knowledge is a key area for the TEIs to address. Many pre-service teachers were interested in a blended learning approach (online and face-to-face) module for geometry even after they graduated from the TEIs.

Some pre-service teachers believed that GeoGebra affordances have a link with their experience in mobile phone games. They believed that their gaming experience motivated them to apply this experience to geometrical thinking with GeoGebra. In other words, pre-service teachers can have foundational experiences: (inter)actions and reflections that may prepare them for more formalised geometry learning opportunities. This was especially evident during geometrical construction, and the subsequent manipulation of geometrical objects, including dragging lines, triangles, or other shapes as part of a geometrical problem. These pre-service teachers manipulated objects (with touch-operation) in diagrams during their teaching. They believed that this feature may facilitate secondary students' conceptual understanding of geometrical problems in the future.

This ownership of professional learning and affordances of apps in these hand-held smartphones will benefit teacher education. This should be encouraged through the TEI’s programmes, probably beyond mathematics education too. Indeed, these pre-service teachers believed that their use of GeoGebra for geometry teaching was influenced by personal, institutional, and social elements at the TEIs.

7.3.3 Curriculum policy

The TEIs should provide a clear direction for developing pre-service teachers’ DT with pedagogical components, and it is crucial for all stakeholders who have a stake in education, including teacher educators and academic presidents at each TEI. These stakeholders should assist in the drafting of DT policies at the TEIs by contributing their experience and positive beliefs to build a strong acceptance of mathematics apps among students and teachers and to improve the pre-service teacher education courses.

The discussion revealed that some pre-service teachers believed that some IT infrastructure was unreliable when GeoGebra was introduced to TEIs and schools in Sri Lanka, especially regarding Wi-fi, multimedia projectors, and free internet access. This needs to be addressed at both a policy and practical level.

Many other beliefs of pre-service teachers influenced geometry teaching, and these are discussed under the research question (RQ3). However, it should be stressed that the present study only considered pre-service teachers' content knowledge, beliefs, and perspectives regarding the use of GeoGebra for geometry teaching and learning. The actual use of other mathematical apps for geometry may have different impacts on geometry teaching. Consequently, a DT policy plan seems to be an important incentive to foster the uptake of apps used in the mathematics classroom in the total secondary school network but only when pre-service teachers are aware of the mathematical and technological content and how they work for their specific needs.

This study identifies an urgent need to change the education recruitment policy of mathematics minor subject teachers at TEIs as the existing policy enables poorly qualified or unqualified teachers to be recruited (mathematics minor teachers currently only need a 50% pass mark in the GCE (O/L) examination).

Poor content knowledge and lack of confidence among pre-service teachers can have a detrimental effect on the learning process of these teachers during block-teaching.

It is a well-established fact that the current mathematics minor subject policy offered by the MoE has meant that many skilled mathematics students (not including GCE (A/L) mathematics subject qualified students) are not able to enter teaching education due to competition from lesser qualified candidates. As a result, for nearly a decade, many weak or relatively unqualified teachers have taken over pre-service teaching.

7.3.4 Teacher education policy

Pre-service teachers believed that confidence in mathematics teaching requires a subject specified (e.g., mathematics) teacher education curriculum match with secondary school mathematics curriculum policy. Confidence in geometry teaching is not only part of TEI expectations but also those of secondary schools. During their block-teaching, pre-service teachers recognise and provide the support and resources that each student needs to achieve those expectations. However, achievement gaps in national level GCE (O/L) mathematics among some groups of students have persisted for decades after the “mathematics for all secondary education” policy was introduced. Therefore, implications for an updated “DT access and equity of mathematics education” policy at TEIs were identified by this study.

As part of the reform process in 2007, a competency-based teacher education policy was introduced in Sri Lanka. Teacher educators believe that it had been a successful policy for nearly two decades. However, pre-service teachers believe that the current teacher education policy needs to be matched with the secondary curriculum reform by the National Institute of Education. In addition, there is a conflict between pre-service teachers’ actual teaching processes and the background policy, especially regarding pedagogical approaches and geometrical content knowledge when teaching mathematics. Therefore, a teacher education policy update relevant to the block-teaching placement and process is required.

The present study highlighted the potential impact of DT policy-related factors on the actual integration of apps with pre-service teachers in TEIs. First, the study confirmed pre-service teachers' pedagogical beliefs about the use of apps in geometry in block-teaching schools within an explicit rigid curriculum structure. Some teacher educators have negative beliefs about mobile phone use for geometry content at the TEIs. However, many pre-service teachers had positive beliefs about GeoGebra for teaching and learning geometry at TEIs. It also highlighted how pre-service teachers want to use GeoGebra more regularly in learning pedagogical practice. This means some pre-service teachers would prefer to use DGE apps in their geometry teaching if this vision is shared by the teacher education/DT policy. This observation collaborates previous research findings suggesting that a successful DT policy depends on the development of a shared vision among stakeholders (Hughes & Zachariah, 2001). Therefore, updating the DT policy (to use DGE apps for geometry teaching and learning) at TEIs with a shared vision of stakeholders (pre-service teachers and teacher educators) is an urgent need.

Some pre-service teachers had negative beliefs about the geometry content in the secondary geometry mathematics curriculum materials (e.g., mathematics workbooks and TGIs) and the national examination evaluation process. Pre-service teachers mentioned that some geometry problems in the national examination were concerned more about the accuracy of the answers than the problem-solving process. Therefore, they suggested that a national examination policy needs to be matched with the secondary mathematics curriculum goals.

In this study, considering current GeoGebra trends and innovations, a wide variety of beliefs will enable teaching and learning experiences to be delivered with different pedagogies. If DT technologies are included in the policies, this may allow for future research opportunities with apps. Moreover, the availability of free mobile apps, the synchronous or asynchronous modes of eLearning, and the affordances of apps may enable pre-service teachers to receive personalised professional learning. Few pre-service teachers in this study could use GeoGebra for block-teaching in the second year.

7.4 Limitations of the study

When the study started, the pre-service teachers and teacher educators had negative beliefs about the use of the mobile phone apps for mathematics education at TEIs due to social elements in their study environment. Those beliefs and feelings during the first year of block-teaching changed in the following year during the COVID-19 pandemic as mobile phones and the use of apps for educational purposes and online learning during lockdowns replaced traditional methods. For example, one of my study settings TEI1, became the pioneer mathematics education institute conducting online learning pedagogies. Its use of mobile apps for teaching became prominent during the first wave of the COVID-19 pandemic (at the end of the second block-teaching year as everybody was using mobile phones as learning tools).

This study strove to capture the beliefs and experiences of Sri Lankan pre-service teachers via phase 2 focus group interviews but, unexpectedly, this coincided with the first wave of the COVID-19 pandemic in 2020, and also major interruptions due to political unrest in Sri Lanka, following the Easter bombing. Due to these two aspects, TEIs were closed before the data-collection process was completed. These external factors changed some pre-service teachers' beliefs as they had to switch their teaching to online only. Some pre-service teachers were much more disadvantaged than others by these changes to teaching and learning assessments during the pandemic. The post-pandemic focus interview data consistently indicated the benefits and challenges of this sudden shift to remote online instruction for teaching secondary learners using mobile apps.

7.4.1 Methodological limitations

The limitations of this study are discussed in terms of the methodology and design of the study. The study focused on two TEIs. The TEI1 which was located in an urban area had more ICT facilities. The TEI2 was disadvantaged and was the rural one. TEI2 was particularly affected by the unexpected first wave of the COVID-19 pandemic. Therefore, the study was bound by both specific contexts and pandemic times. Findings may be difficult to generalise for other settings in TEIs (see section 3.7.5). The literature has argued limitations because survey studies use self-reported data without any evidence regarding the validity of respondents' reports. Indeed, the respondents explained their own experiences and opinions subjectively according to their understandings and knowledge of mobile apps.

7.4.2 Social limitations

In the local TEIs, teacher educators are dealing with a new generation of pre-service teachers growing up with 21st-Century technology using mobile apps as a tool in their day-to-day activities. According to the study design, only the teacher educators in urban TEIs were encouraged to use GeoGebra for geometry teaching (when possible) as they had better mobile networks than the rural TEIs. Due to the aforementioned civil unrest, all mobile phone use was prohibited at TEIs for nearly two months during class time. The researcher did not request that TEIs use the GeoGebra app as it was not mandatory for the students, and no formal request was made in the second year of data collection. Initial inquiries were made with TEIs from teacher educators to select participants (pre-service teachers) and to gauge their availability at their hostels after class hours in selected TEIs during the period of data collection. Both phases of data collection were conducted after TEI teaching time at the hostel; however, some pre-service teacher ethnic groups did not participate in the second phase of data collection due to some major social issues (e.g., threats to life) and since they were no longer residents in the TEI hostel.

The second major social incident and limitation of the study was the bombing attack on three churches in Sri Lanka in Easter 2019, which killed 310 people; this stopped the data collection for one month as all TEIs were closed again for a few months. This incident delayed pre-service teachers' block-teaching at secondary schools, and lessons could only be done via social-media, mobile apps during this time. During the data-collection period of the study, sudden youth unrest was experienced, which affected use of mobile phone at TEIs during class hours. To mitigate against possible effects, the data-collection period was extended until this social issue had settled. Due to sudden and unexpected social unrest among youths, including bomb explosions and banned social media, special official permission was obtained for data collection from the Administrators at Teacher Education, Ministry of Education in Sri Lanka to conduct the data collection relevant to the mobile apps. This total process created unnecessary delays in the entire data-collection process and may have influenced the pre-service teachers' beliefs about the ways that they used GeoGebra in schools.

Third, the second phase of data collection of interviews was delayed due to political issues, then the pre-service teachers suddenly experienced mobile teaching and learning during the COVID-19 pandemic lockdown. In the first wave of the COVID-19 pandemic, government authorities suddenly announced that mobile phones would be used in the teaching and learning process. They (teacher educators and pre-service teachers) were not prepared for the sudden use of mobile apps for the mediation of education. They became acutely aware of their need to access high-speed internet, updated DT facilities, new apps and software, and knowledge and skills to effectively use these tools during teaching. They had limited opportunity to get a comprehensive picture of their placement schools and their beliefs had changed with the complexity of the sudden online teachers' role. The focus group interviews were conducted using online group meetings as these pre-service teachers were busy with secondary school teaching. Thus, the lockdown and online learning directives experienced by the pre-service teachers affected their beliefs concerning geometry teaching in the second year.

7.5 Recommendations for future research

The study covers secondary geometry education in Sri Lanka, but some aspects might apply to any education system in any country. For example, under the affordances of the apps in the study, pre-service teachers focused on system constraints in problem solving within a restricted, structured curriculum. However, restricted curriculum targets are common issues in any education system. It may open avenues for new research, such as secondary teachers' problem-based learning in mathematics with apps. Research into whether a special geometry teaching module with DT is relevant to the developing country context could be undertaken as it may benefit those countries.

The study identified a relationship between pre-service teachers' confidence in teaching geometry and their beliefs in geometry content knowledge. The researcher also suggests a follow-up study be conducted using a larger sample of pre-service teachers at mathematics TEIs, including after (second year) the beginning (first year) pre-teachers block-teaching experience in schools. This study was conducted only for pre-service teachers at TEIs. This study could be expanded as a mixed-method research survey followed by interviews and observations of a sample of mathematics major and minor subject stream pre-service teachers at all mathematics education TEIs (English, Sinhala and Tamil medium) in Sri Lanka.

A variety of technologies relevant to pedagogical approaches are better for delivering not only mathematics education but also other subjects at TEIs. A large-scale quantitative research study with a TPACK survey is suggested to recognise the appropriateness of new DT technologies (such as virtual reality) with different pedagogical approaches relevant to the teacher education curriculum. In addition, beliefs about pre-service teachers' affordances of these new technologies in block-teaching can be observed (video recorded) in multiple case studies. This might also further examine how metacognition could be developed through using DT. It may be implemented as a longitudinal study in selected TEIs.

In the discussion, there were contradictory views about the relationship between pre-service teachers' beliefs about block-teaching in geometry and teacher educators' views of classroom practices. Thus, separate surveys for all TEIs are suggested to examine pre-service teachers' beliefs and pedagogical practices with/without mobile apps relevant to the curriculum. These studies could be done in mathematics and other subjects such as science, English, and technology.

On the one hand, more availability of free mobile apps for personalised learning is possible, and synchronous or asynchronous mode and future learning initiatives will provide a more personalised experience and flexibility for the users. The suggested study could consider trends in the use of mathematics apps for teacher education; choosing educational apps relevant to the local curriculum at TEIs is important.

A synthesis of the discussion shows that new research has aroused new interest and strategies relevant to DT integrated curriculum modules for pre-service teacher education. A mixed-method study with a baseline survey for

TEIs and multiple case studies are suggested for each TEI to understand existing standards and possibly the next generation of DT integration at TEIs. Pre-service teachers believed that there are many other options for new DT policy in common platforms (e.g., LMS) relevant to the use of apps such as virtual reality and SCORM (sharable content object reference model) technology that enables tracking inside the context of the common platform. Furthermore, next-generation technology will be more accessible to pre-service teachers for uniformly evaluating other types of digital learning experiences during block-teaching. Finally, in developing countries such as Sri Lanka, hybrid teacher education (online and physical classroom) is suggested as a potential research project.

7.6 Summary of conclusions

The present study highlights the potential impact of policy-related factors on the actual integration of apps with pre-service teachers in TEIs. The use of GeoGebra in mobile devices being viewed negatively by older-generation teacher educators may be due to the difficulty of changing traditional practices or relatively hard-to-read small screens. In contrast, the younger "touch-screen generation" of many pre-service teachers were familiar with mobile apps and have different personal beliefs about the use of GeoGebra for geometry teaching.

The study confirmed that pre-service teachers' knowledge and positive beliefs on the use of apps in geometry in block-teaching schools (within an explicit rigid curriculum policy that stresses shared goals) depend on social elements in the context. The synthesis of findings suggested an extended TPACK framework for the thinking of the geometry problem solving process with GeoGebra and reflection on the planning process with GeoGebra. An extended version of TPACK with these meta-cognition elements (M-TPACK) is suggested as another potential thread for TPACK. As well, these pre-service teachers believed that the personal ownership of such professional learning has become an important part of their lives.

The current study also explored challenges in pre-service teacher education, secondary school, and DT policies relevant to geometry teaching with GeoGebra which will contribute to the Sri Lankan mathematics education field.

The study summarised implementation relevant to pre-service TEIs and pre-service teachers. In addition, the methodological and social limitations of the study were highlighted, and recommendations were made for future research. Several large-scale research projects on teacher education with mobile apps were suggested from the findings in this study.

The key findings discussed the ways that the RQs were addressed. There are also implications for accessing and adapting findings as well as suggestions for further research such as challenges in mobile apps integration for mathematics education. Sound geometry content knowledge and pedagogical content knowledge, which are prerequisites for pre-service teachers' conceptual understandings and for deciding where those understandings might be heading, were suggested as contributions to the field relevant to the study. The selected TEIs have frequently provided support for mathematics professional learning of pre-service teachers that focused on geometry content knowledge as well as on teaching based on "where pre-service teachers were" in their learning with/without GeoGebra, as opposed to teaching directly to the standard mathematics curriculum policy. However, a few important GeoGebra app characteristics are yet to be explored, such as how pre-service teachers are using mathematics apps to link geometry to algebra concepts. To be effective, professional development experiences may link to hybrid modes of teaching and learning modules (made possible with GeoGebra) rather than focusing on developing user proficiency in digital technologies only. It also means that self-learning of processes and the autonomy of pre-service teachers' development have been created through touch-operation-based smartphones with GeoGebra.

Even with several limitations, the study adds value to the existing knowledge on potentially effective geometry teaching practices with DT (GeoGebra) in secondary schools in Sri Lanka, and other countries with similar education systems. The results have been interpreted with caution due to social and methodological limitations. Thus, in phase 2 of the study, pre-service teachers talked more about the use of GeoGebra with their new adoption of online teaching with MT due to the COVID-19 forced lockdown.

In the current study, a few selected geometry topics were considered by teachers as being relevant to the Sri Lankan context with special reference to GeoGebra. However, this study also indicates that some pre-service teachers prefer to take ownership of their professional learning with apps as part of an increased interest in professional development approaches during the 21st Century. Mathematics researchers in different countries show different ways of interacting with this DGS in the mathematics classroom. Mathematics teachers' roles and different practices in DT-related professional learning become more visible through the continuous DT technologies updates and ongoing developments. Becoming aware of how these updates can be facilitated into practice seems to bring the work of researchers in mathematics education closer to the teachers' and students' reality.

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TE2

THE UNIVERSITY OF
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25/6/18

The commissioner
National College of Education (NCoE)

Dear Sir

1bis is a letter of introduction for M.N.S. Edirisinghe who is a Ph. D studen 'M.f Com'!'i oner (Teacher Education) Education. As part of the course, the PhD student is required to do a Research. nMJ'W,lt Edu tion involve working with 20 pre-service teachers individually participate in a Geometl)8eab J, . (pre / post) for about 30 minutes in a quiet place, response to a survey questionnaire for about 15 minutes, 4 pre-service teachers in focused group interviews about 20 minutes. -111.e geometry test to be used have been approved by a lecturer teaching on Mathematics II paper.

All information gathered during this study will be kept confidential. The NCoE and the pre-service teacher taking part will not be identified. The PhD student will organise the sessions at times which do not conflict with in1portant NCoEwork. Sessions with the pre-service teachers we'll be audio-taped with your consent and the pre-service teachers', *to* enable later reflection on the session. During the session, the pre-service teachers will have the right to stop the session completely or decline to do a test or survey. Tapes will be erased, and consent forms destroyed after the thesis has been submitted.

If you are willing for the Ph. D student to come to your school to work with 20preservice teachers, please fill in the consent form overleaf and return to the PhD student. If you have any questions or require more information, please feel free to contact Dr Nigel Calder or Dr Sashi Sharma, details below, or the student on +64 27 5201366.

Yours sincerely,

Dr Nigel Calder, Associate Professor in Mathematics Education.
Phone 07 5578753 nigel.calder({hvaikato.ac.nz

Dr Sashi Sharma, Senior Lecturer in Mathematics Education
shashi@waikato.ac.nz

Appendix B

President NCoE
Pl. Provide Facilities
Approved

THE UNIVERSITY OF
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Dear Sir
The commissioner
National College of Education (NCoE)

This is a letter of introduction for M.N. S Edirisinghe (ST .ID 1327571) who is a Ph.D student in Education. *As* part of the course, the PhD student is required to do a Research. This will involve working with 20 pre-service teachers individually participate in a Geometry test (pre /post) for about 30 minutes in a quiet place, response to a survey questionnaire for about 15 minutes, 4 pre-service teachers in focused group interviews about 20 minutes. The geometry test to be used has been approved by a lecturer teaching on Mathematics II paper.

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Yours sincerely,

Dr Nigel Calder, Associate Professor in Mathematics Education.
Phone 0064 07 5578753 nigel.calder@waikato.ac.nz

Dr Sashi Sharma, Senior Lecturer in Mathematics Education
shashi@waikato.ac.nz

THE UNIVERSITY OF WAIKATO
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Pilot study/ Main Study

PARTICIPANT (Pre-service teachers)
Geometry pre/post- test /Survey questionnaire

Invitation letter

Title: Using mobile technology when teaching and learning geometry in junior secondary school Mathematics Education in Sri Lanka.

Dear Participant,
My name is M.N.Shiiyama Edirisinghe and I am currently undertaking a PhD in Education at the University of Waikato. The research is designed to give us further understanding of the use of mobile technology in Mathematics Education with the focus on the geometry component in secondary education. As part of the study, the PhD student is required to collect data from pre-service teachers. I am writing to you to invite you to participate in the geometry pre/post test and survey questionnaire as part of the research project for my thesis. The survey questions will focus on geometry and pedagogical perspectives and beliefs about the use of mobile technology and demography data. If you agree to participate, I will contact you again to arrange a time and place convenient to you for the test and survey. You have the right to refuse to answer any question, or withdraw from the test at any time, or withdraw information you have provided up until the data analysis is begun, approximately 1 February 2020. While every effort will be made to ensure confidentiality, this cannot be guaranteed. However, as a participant, you can expect every reasonable effort to be made by the researcher and his supervisors to have your privacy protected and personal details kept confidential. To achieve this, pseudonyms will be used, and care will be taken to remove any identifying information in the final thesis and any reporting of findings. Similarly, your NCoE will not be identified but will be referred to as 'the pre-service institute', -or a similar term. The findings may be reported in conferences and written presentations. Consent forms data will be stored separately and securely for 5 years at the University of Waikato and then destroyed. The consent form for the geometry test/ survey is shown on a separate sheet of paper. A summary of the thesis findings will be made available to you, and the complete thesis will be published on the University of Waikato website. If you agree to participate in this study, please fill in the consent form and return it to me.

If you have any questions or require more information, please feel free to contact Dr Nigel Calder or Dr Sashi Sharma, details below, or me on +64 27 5201 366.Thank you for your time and help in assisting with this research.

Yours sincerely,

M.N.Shiyama. Edirisinghe Shiyamaj@gmail.com

Dr Nigel Calder, Associate Professor in Mathematics Education.
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Survey Questionnaire

1. Demography details

1.1 Participants code: ..../......   1.2 Date...26/03/2019....

1.3 Age ......24..........   1.4 Gender.J:1:'!-f?.......

1.5 Highest level of mathematics qualification

	Grade
1.5.1 GCE(A/L)	,3
1.5.2 GCE(O/L)	A
1.5.3 other	

1.6 \What is your academic background relevant to teachbig practice

	Major	Minor
1.5.1 Sub·ect number oflessons allocated	[illegible]	
1.5.1 Teaching experience (weeks)	2	

1.7 Do you use mobile phone?   Yes J   No

(NOTE : If your answer is "No" skip to section 2)

1.8 My mobile phones are best described as
Basic Phone   Smart Phone   I don't know

1.9 Do use you have ever use your mobile phone for leaning mathematics?
Yes J   No

1.10 Do you ever use GeoGebra application (computer or mobile)?
Yes   :No

2.0 Family details

2.1 Do you like to fill your family details   Yes, _-/   No

If your answer is "yes" please: fill section 2.2 to 2.4 otherwise move to section 3.

2.2. What is your family income approximately per month?

....

....

2.3 What is you_r parents educational back.2:round?

	Father	Mother
1.8.1 Number of years of schooling		
1.8.2 Hig,i.est educational qualification		—./

3. Teaching learning experience

3.1 What is your own experience in teaching geometry at school?

[illegible]

3.2 llat is your own experience in learning Geometryat school ??

[illegible]

3.1 What is your own experience in teaching geometry at NCoE?

[illegible]

3.2 What is your own experience in learning Geometry at NCoE?

[illegible]

. 5 Using Mobile phones for Mathematics.

Please rate how strongly you agree or disagree that each of the listed relevant to your mobile phone using for teaching and learning mathematics (Tick one box in each row.)