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**A Yield Curve Perspective on
Uncovered Interest Parity**

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Abstract

This article uses a dynamic multi-factor model of the yield curve with a rational-expectations, general-equilibrium-economy foundation to investigate the uncovered interest parity hypothesis (UIPH). The yield curve model is used to decompose the interest rate data used in the UIPH regressions into components that reflect rationally-based expectations of the cyclical and fundamental components of the underlying economy. The UIPH is not rejected based on the fundamental components of interest rates, but is soundly rejected based on the cyclical components. These results provide empirical support for suggestions in the existing theoretical literature that rationally-based interest rate and exchange rate dynamics associated with cyclical inter-linkages between the economy and financial markets may contribute materially to the UIPH puzzle.

Keywords

uncovered interest parity
forward rate unbiasedness hypothesis
yield curve
term structure of interest rates
ANS model
Nelson and Siegel model

JEL Classification

E43; F31

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1 Introduction

This article uses a dynamic multi-factor model of the yield curve with a rational-expectations, general-equilibrium-economy foundation to investigate the uncovered interest parity hypothesis (UIPH), i.e the proposition that exchange rates should appreciate/depreciate at a pace that offsets the interest rate discount/premium available between the underlying currencies. The broad motivation is to provide an empirical perspective on the potential role that rationally-based cyclical interest rate dynamics may play in the puzzling results from prior empirical tests of the UIPH. To put this motivation adequately into context, it is useful to provide a brief overview of the existing UIPH literature, and that of its parallel specification as the forward rate unbiasedness hypothesis (FRUH).¹

Firstly, it is well established that the UIPH/FRUH is typically rejected based on the standard regression of lagged changes in exchange rates on the corresponding interest rate differentials or forward exchange premia.² Indeed, rather than yielding the expected slope coefficient of 1, such regressions frequently produce significantly negative estimates, implying that exchange rates move contrary to the predictions of the UIPH/FRUH. That said, recent empirical investigations based on longer horizons/maturities, rather than the weekly, monthly, or quarterly data often used, have been more supportive of the UIPH/FRUH. For example, Alexius (2001) generally does not reject the UIPH using 10-year interest rates and exchange rate changes over the corresponding 10 year horizon. Meredith and Chinn (2004) reports similar results using 5- and 10-year interest rates over the corresponding horizons, while rejecting the UIPH based on 3-, 6-, and 12-month maturities/horizons. Similarly, Razzak (2002) generally does not reject the FRUH on a 1-year horizon, but rejects it for the 1-month horizon.

These mixed empirical results have prompted further bodies of literature, as summarised in the survey of Sarno (2005), on how the UIPH/FRUH might be reconciled with the data. For example, Fama (1984) originally proposed that deviations of the data from the UIPH/FRUH might reflect time-varying risk premia, although subsequent investigations using standard finance/economic models with plausible parameter values have not been able to establish satisfactory sources of those risk premia or the required magnitude of variation.³ A second class of proposals with some empirical support is that failures of the UIPH/FRUH might reflect departures from the rational expectations assumed in the formulation of the UIPH/FRUH.⁴ Another strand of the literature suggests that the puzzling results from the standard regression tests of the UIPH/FRUH might be largely a statistical artifact arising from the time-series properties of the data over finite samples.⁵

The strand of literature most closely connected to this article suggests that rationally-based interest rate and exchange rate dynamics associated with cyclical interlinkages between the economy and financial markets may be an important factor contributing to the UIPH/FRUH puzzle. McCullum (1994) originally illustrated this concept by augmenting the UIPH relationship with a

¹The FRUH proposes that exchange rates should appreciate/depreciate at a pace that matches the forward exchange premium/discount. Section 2 shows that the FRUH is equivalent to the UIPH because the forward exchange premium/discount is derived directly from interest rate differentials under covered interest parity.

²The frequently-referenced surveys summarising those results are Hodrick (1987), Froot and Thaler (1990), and Engle (1996). Recent examples of empirical investigations that reject the UIPH/FRUH are Liu and Maynard (2005), Wu (2005), and Zhou and Kutan (2005).

³See Sarno (2005) pp. 676-678. Wu (2005) is a recent addition showing that time-varying interest rate risk alone cannot feasibly explain the failure of the UIPH. The latter result is also implied in the discussion in section 4 of this article.

⁴See Sarno (2005) pp. 678-679. The heterogenous agent model of the exchange rate market in Grauwe and Grimaldi (2006) is a recent theoretical addition to this literature.

⁵For example, Baillie and Bollerslev (2000) shows via simulation that persistent time-varying volatility in the data would lead to a diffuse distribution for the FRUH regression slope coefficient, and Maynard and Phillips (1998) considers the implications of I(0) changes in the exchange rate and I(1) forward exchange premia. Empirically, Sarno (2005) pp. 679-683 discusses that there is better support for the FRUH based on long-run cointegrating relationships between the levels of the exchange rate and the forward exchange rate, and Delcoure, Barkoulas, Baum and Chakraborty (2003) is a recent addition to that literature.

simple monetary policy reaction function to represent the smoothing of the path of interest rates and exchange rates by the central bank. Solving the two-equation stochastic system analytically under rational expectations then produces an expected negative slope coefficient for the standard UIPH regression. Meredith and Chinn (2004) expands the McCullum (1994) approach into a more realistic macroeconomic model that includes the UIPH, a Taylor rule monetary policy reaction function, a Phillips curve inflation relationship, an investment-savings output equation, and a short-maturity and long-maturity interest rate to represent the yield curve. Applying the standard UIPH regression to artificial interest rate and exchange rate data generated from stochastic simulations of that calibrated model under rational expectations reproduces the typical empirical results discussed above; i.e. negative slope coefficients and rejections of the UIPH for short horizons, but slope coefficients near 1 and non-rejections of the UIPH for longer horizons. Lim and Ogaki (2003) obtains a similar pattern of results by simulating a rational-expectations open-economy model that includes an exogenous domestic interest rate process with temporary and persistent innovations.

While these theoretical and simulation results illustrate that deviations of exchange rate and interest rate data from the UIPH can occur without recourse to time-varying risk premia and/or non-rational expectations, that strand of literature currently has no direct empirical support. Indeed, the illustrative model of McCullum (1994) has been rejected in a subsequent empirical investigation by Mark and Wu (1996). The models of Lim and Ogaki (2003) and Meredith and Chinn (2004) could in principle be tested empirically, but the estimation of rational expectations models is practically challenging, and the number of parameters in the Lim and Ogaki (2003) and Meredith and Chinn (2004) models would hinder inference in any case.

An alternative empirical approach to investigating how material rationally-based cyclical interest rate dynamics might be for the UIPH puzzle is to use the augmented Nelson-Siegel (ANS) model of the yield curve from Krippner (2006). The ANS model is an intertemporally-consistent and arbitrage-free version of the popular, parsimonious, and easy-to-apply Nelson and Siegel (1987) approach that represents the yield curve via component functions of maturity. Krippner (2005) provides an economic foundation for the ANS model by explicitly relating it to a generic dynamic general-equilibrium economy that embeds rational expectations and constant risk premia. That foundation therefore provides the basis for using the ANS model to decompose the interest rates of any maturity into components that reflect the cyclical and fundamental factors in the underlying economy under rational expectations. The component data can then be used directly in the empirical tests of the UIPH to assess the relative contributions that rationally-based cyclical and fundamental components of interest rates make to the UIPH puzzle.

The article proceeds as follows: section 2 outlines the UIPH/FRUH and the ANS model of the yield curve, and then details how the ANS model can be used to investigate the UIPH. Section 3 describes the data and discusses points relevant to the empirical estimation. Section 4 discusses the empirical results, and section 5 summarises and concludes.

2 The UIPH/FRUH and the ANS model of the yield curve

This section firstly outlines the theory underlying the UIPH/FRUH. Section 2.2 proceeds to discuss the essential aspects of the ANS model relevant to this article, and section 2.3 then discusses how the ANS model is used in this article to investigate the UIPH/FRUH. Note that all of the notation and examples refer explicitly to the Canadian (CA) and United States (US) data subsequently used in the empirical application of section 3. Of course, the approach applies quite generally to the exchange rates and yield curves of any currency pair, with the potential exception (as discussed in section 2.2) when the interest rates of one or both of the currencies are close to zero.

2.1 The UIPH and the FRUH

The UIPH and the FRUH both originate from the covered interest parity relationship, which defines the forward exchange rate as:

$$e_{t,m} = e_t + m (R_{t,m}^{\text{US}} - R_{t,m}^{\text{CA}}) \quad (1)$$

where e_t is the natural logarithm of the nominal exchange rate between the Canadian dollar (CAD) and the United States dollar (USD) at time t (defined as the number of USDs per CAD, so a rise in e_t is an appreciation of the CAD against the USD); $e_{t,m}$ is the natural logarithm of the forward CAD/USD exchange rate at time t for settlement at $t+m$ years; and $R_{t,m}^{\text{CA}}$ and $R_{t,m}^{\text{US}}$ are respectively the annualised continuously-compounding zero-coupon interest rates for Canada and the US at time t for maturity $t+m$ years.

Equation 1 precludes outright arbitrage opportunities between the forward exchange market and the interest rates of the two currencies. That is, if covered interest parity did not hold, then it would be possible to arbitrage between the forward exchange rates $e_{t,m}$ and the equivalent alternative of directly borrowing and investing at the prevailing exchange rate and interest rates on the underlying currencies.⁶ Empirically, covered interest parity is well-supported by the data, as noted in the survey of Sarno and Taylor (2003) chapter 2.

Assuming rational expectations, the ex-ante relationship between e_t and $e_{t,m}$ is $E_t[e_{t+m}] = e_{t,m}$, where E_t is the expectations operator conditional on information available at time t . Substituting $E_t[e_{t+m}]$ for $e_{t,m}$ in equation 1 and re-arranging then gives the UIPH, i.e:

$$E_t[e_{t+m}] - e_t = m (R_{t,m}^{\text{US}} - R_{t,m}^{\text{CA}}) \quad (2)$$

i.e the expected change in the exchange rate over the horizon m should equal the prevailing difference in interest rates with a time to maturity of m . Alternatively, re-arranging equation 1 to express $m (R_{t,m}^{\text{US}} - R_{t,m}^{\text{CA}})$ as the forward exchange premium $e_{t,m} - e_t$, and substituting that into equation 2 gives the FRUH specification, i.e $E_t[e_{t+m}] - e_t = e_{t,m} - e_t$.

The UIPH is typically tested by estimating the following equation using ex-post exchange rate and interest rates data:

$$\Delta e_{t,m} = a_m + b_m \cdot m (R_{t,m}^{\text{US}} - R_{t,m}^{\text{CA}}) + v_{t,m} \quad (3)$$

where $\Delta e_{t,m}$ is $e_{t+m} - e_t$ (i.e the change in e_t from t to $t+m$) lagged m years; a_m is the estimated constant which allows for any systematic risk premia; b_m is the estimated slope parameter; and the innovation terms $v_{t,m}$ represent unanticipated differences between expected and realised exchange rates, which should be distributed with mean zero. The estimation of a_m and b_m is typically the primary consideration in empirical tests of the UIPH/FRUH, and this article follows that precedent.⁷ Hence, if the UIPH holds, then a statistical test on the estimated parameter b_m should not reject the theoretical value of 1, while the estimated parameter a_m may be non-zero to allow for any systematic premia that may arise because the exchange rate and interest rate data are observed in a non-risk-neutral environment. Similarly, the FRUH is typically tested by estimating the equation $\Delta e_{t,m} = a_m + b_m \cdot (e_{t,m} - e_t) + v_{t,m}$ using lagged ex-post exchange rate and forward exchange rate data.

As discussed in the introduction, the empirical estimates of b_m based on the UIPH and FRUH specification over short horizons are typically significantly less than the theoretical values of 1 and are often significantly negative, while the estimates of b_m based on data over long horizons are not significantly different from the theoretical value of 1. Given these mixed empirical results, the following two sections discuss the ANS model of the yield curve and how it might be applied to provide an empirical perspective on the UIPH/FRUH. Note that from this point onward, the article will work exclusively with the UIPH specification.⁸

⁶That is, equivalent under the typical assumptions in the literature that capital markets are unconstrained, returns are not distorted by tax considerations, and transactions costs are negligible.

⁷The additional test of whether the information available at time $t-m$ was used efficiently is that $v_{t,m}$ should exhibit no serial correlation beyond the moving-average correlation induced when the horizon m is greater than the frequency of the data, but that aspect is not tested in this article.

⁸Testing the FRUH would require an additional transformation of the interest rate differentials into forward

2.2 The ANS model of the yield curve

The yield curve model used for this article is the ANS model from Krippner (2006). At any point in time t , the ANS model represents the annualised continuously-compounding zero-coupon interest rate $R_{t,m}$ (ANS) as a function of time to maturity m , i.e:

$$R_{t,m}(\text{ANS}) = \frac{\sigma_1 \theta_1 m}{2} + \sum_{n=1}^3 \beta_n(t) \cdot s_n(m) - \sum_{n=1}^3 \sigma_n^2 \cdot u_n(m) \quad (4)$$

where the core of the ANS model is the sum of the time-varying coefficients $\beta_n(t)$ applied to the time-invariant modes $s_n(m)$. The latter are simple functions of maturity, i.e:

$$s_1(m) = 1 \quad (5a)$$

$$s_2(m) = \frac{1}{\phi m} [\exp(-\phi m) - 1] \quad (5b)$$

$$s_3(m) = -\frac{1}{\phi m} [2\phi m \exp(-\phi m) + \exp(-\phi m) - 1] \quad (5c)$$

where ϕ is a constant parameter that alters the rate of decay in the exponential functions. Figure 1 plots these three $s_n(m)$ functions, which Krippner (2006) names the Level, Slope, and Bow modes based on their shapes. Also following the terminology in Krippner (2006), the coefficients $\beta_1(t)$, $\beta_2(t)$, and $\beta_3(t)$ are called the Level, Slope, and Bow coefficients, and the coefficients $\beta_n(t)$ multiplied into the modes $s_n(m)$ are called the Level, Slope, and Bow components of the yield curve. The parameters θ_1 and σ_n and the functions $u_n(m)$ account for the market prices and quantities of risk for each component of the yield curve model. These are required to make the ANS model intertemporally-consistent and arbitrage-free, but a detailed understanding of these elements is not required for this article. Readers requiring more detail are referred to Krippner (2006).

[Figure 1 here]

Given an observation of yield curve data (i.e the market-quoted yields or prices of a group of fixed interest securities with a span of maturities but otherwise similar characteristics, all observed at time t), applying the ANS model therefore decomposes that observation into estimated Level, Slope, and Bow components. There will also be a series of estimated yield residuals, i.e the differences between the actual market-quoted yields and the yields derived from the ANS model, which are typically very small. For example, anticipating the discussion of the data in section 3, figure 2 illustrates the US yield curve data observed for February 2004, the associated yield curve estimated using the ANS model, and the estimated yield residuals for the non-coupon-paying securities.

[Figure 2 here]

For the purposes of this article, the essential intuition of how Krippner (2005) relates the ANS yield curve model to a generic dynamic general-equilibrium model of the economy is best illustrated within a deterministic environment, followed by a brief discussion of the role and effect of the stochastic components. The economic model in Krippner (2005) is based on J real factors of production (e.g capital, labour, etc., potentially by industry sector), each with its own associated inflation rate. Each real factor and inflation rate follows a standard Vasicek (1977) mean-reverting process, with the following deterministic component:

$$E_t [dq_j(t)] = -\kappa_j [q_j(t) - \mu_j(t)] dt \quad (6)$$

where E_t is the expectations operator as at time t ; $q_j(t)$ for $j = 1$ to J are the real state variables representing instantaneous growth on returns to the factors of production in the economy at time t ; κ_j are positive constant mean-reversion parameters; and $\mu_j(t)$ are the steady-state (i.e long-run) values of $q_j(t)$ as at time t . $q_j(t)$ for $j = J+1$ to $2J$ are the inflation state variables, which have the

exchange rate data, and the subsequent empirical estimations for the FRUH would be identical to those for the UIPH in any case.

analogous parameters to the real factors. The instantaneous short rate at time t is the sum of all the real and inflation state variables, i.e $r(t) = \sum_{j=1}^{2J} q_j(t)$, and that quantity also equals nominal output (given it is the sum of returns to the factors of production and their rates of inflation).

Equation 6 is an ordinary differential equation, with the solution $E_t[q_j(t+m)] = \mu_j(t) + [q_j(t) - \mu_j(t)] \cdot \exp(-\kappa_j m)$, where m represents a future horizon from time t . Hence, the current state variables and their steady-state values define the expected path of the state variables and therefore the expected path of the short rate $E_t[r(t+m)]$ as a function of future time m . In a deterministic environment, $E_t[r(t+m)]$ equates to the instantaneous continuously-compounding forward rate curve as a function of maturity m , i.e:⁹

$$f(t, m) = \sum_{j=1}^{2J} \mu_j(t) + \sum_{j=1}^{2J} [q_j(t) - \mu_j(t)] \cdot \exp(-\kappa_j m) \quad (7)$$

Using the definition $R_{t,m} = \frac{1}{m} \int_0^m f(t, m) dm$ then gives the continuously-compounding zero-coupon curve as:

$$R_{t,m} = \sum_{j=1}^{2J} \mu_j(t) - \sum_{j=1}^{2J} [q_j(t) - \mu_j(t)] \cdot \frac{1}{\kappa_j} \exp(-\kappa_j m) \quad (8)$$

Comparing equation 8 back to the deterministic version of the ANS model (i.e setting the market prices and quantities of risk to zero) shows that:

$$\beta_1(t) \cdot s_1(m) = \beta_1(t) = \sum_{j=1}^{2J} \mu_j(t) \quad (9)$$

Hence, the Level component of the ANS yield curve at time t reflects the market's current assessment of the long-run equilibrium nominal interest rate consistent with underlying economic fundamentals (specifically, long-run nominal output growth, which is in turn the sum of the steady-state variables for the real factors of production and their long-run inflation rates).

The “remainder” of the ANS yield curve at time t is:

$$\varepsilon_{t,m} + \sum_{n=2}^3 \beta_n(t) \cdot s_n(m) = - \sum_{j=1}^{2J} [q_j(t) - \mu_j(t)] \cdot \frac{1}{\kappa_j} \exp(-\kappa_j m) \quad (10)$$

where $\varepsilon_{t,m}$ is the estimated yield residual for the zero-coupon security of maturity m . This “non-Level” component of the ANS model at time t reflects the market's current expectation of the future path of the instantaneous short rate (as a function of future horizon m from time t) relative to the long-run equilibrium interest rate. Or in other words, the non-Level component of the ANS yield curve represents the expected cyclical component of interest rates consistent with the economy returning to its underlying economic fundamentals (specifically, the path of nominal output growth returning to long-run nominal output growth).

Regarding the stochastic components of the economic model, Krippner (2005) allows the steady-state values $\mu_j(t)$ to evolve as low-variance Brownian motions. This allows for gradual unanticipated changes to the expected long-run values of the state variables over time, and the aggregation of innovations in $\mu_j(t)$ matches the Gaussian dynamics assumed for the Level component of the ANS model. Similarly, the state variables $q_j(t)$ evolve as mean-reverting Gaussian stochastic processes, which allows for unanticipated “shocks” to the prevailing state variables, and the aggregation of the innovations in $q_j(t)$ match the Gaussian dynamics assumed for the non-Level components

⁹In a stochastic environment, additional terms involving the market prices and quantities of risk for each component of the economy are required to make the forward rate and interest rate curves intertemporally-consistent and arbitrage free. Krippner (2005) shows that those additional terms match the parameters and functions of the ANS model relating to the market prices and quantities of risk for each component of the yield curve.

of the ANS model.¹⁰ Note that, because the dynamics of the ANS model and economic model are both Gaussian, this implies a non-zero probability of negative interest rates. That aspect can safely be ignored in practice, as is done when models based on the Vasicek (1977) specification are applied to interest rate data, unless the interest rates of some maturities are already materially close to zero.

Applying the ANS model to the yield curves of two economies therefore provides the basis for decomposing the interest rates for those economies into rationally-based fundamental and cyclical components. More formally, appendix A details how two economies linked by a bilateral exchange rate under conditions more general than those assumed in Krippner (2005) will result in ANS yield curves for both economies. Generically specifying an economic model in this way and condensing it into just the three coefficients and several parameters of the ANS model of the yield curve for each economy has two distinct advantages over the structural economic models proposed by Lim and Ogaki (2003) and Meredith and Chinn (2004). Firstly, it avoids the need to explicitly specify and model the myriad of dynamic inter-relationships that may exist within the economy (such as Phillips curve relationships, monetary policy reaction functions, monetary policy credibility effects, exchange rate influences on inflation and/or the real economy, etc.). In effect then, the ANS model allows the decomposition of the interest rate data into its fundamental and cyclical components while remaining agnostic about the precise dynamics that generate those components in the underlying economy. The second advantage of the ANS model approach is parsimony, which makes the ANS model very straightforward to apply in practice (like the Nelson and Siegel (1987) model on which the ANS model is based). That said, one disadvantage of the ANS model approach is that it offers no direct means of decomposing the exchange rate data used to test the UIPH into its cyclical and fundamental components. This is unfortunate, because if such a decomposition were possible, it would enable a more comprehensive series of UIPH tests based on the cyclical and fundamental components of both interest rates and exchange rates.¹¹

2.3 Investigating the UIPH using the ANS model

The first use of the ANS model to investigate the UIPH is simply as a convenient means of generating zero-coupon interest rate data from market-quoted yield curve data that are typically coupon-bearing for maturities of one year and beyond. That is, once estimated from the available yield curve data (as in the example of figure 2), the ANS model provides a continuous zero-coupon interest rate function for any maturity over the interval $0 \leq m < \infty$. Tests of the UIPH can then be undertaken using a time series of estimated zero-coupon interest rates for an arbitrary given maturity m , i.e:

$$\Delta e_{t,m} = a_m + b_m \cdot m [R_{t,m}^{\text{US}}(\text{ANS}) - R_{t,m}^{\text{CA}}(\text{ANS})] + v_{t,m} \quad (11)$$

Note that the use of estimated zero-coupon interest rate data is common practice in the literature.¹² However, given both market-quoted and estimated zero-coupon interest rates are available for the 3- and 6-month horizons/maturities investigated in this article, it is worthwhile undertaking the UIPH tests with both sets of data to ensure that using estimated interest rate data does not materially influence the empirical results.

¹⁰Both sides of equation 10 decay asymptotically to zero by maturity, which is consistent with the expectation that nominal output growth will converge to steady-state nominal output growth in the long-run.

¹¹Such tests may be revealing in cases where much of the cyclicity of the underlying economy is reflected in deviations of the exchange rate from its fundamental value, or as alluded to in the discussion of section 3, where monetary authorities deliberately attempt to influence the exchange rate away from its fundamentals. For example, Mark and Wu (2004) shows that unanticipated exchange rate interventions within a rational expectations framework can produce deviations from the UIPH without recourse to time-varying risk premia and/or non-rational expectations.

¹²For example, the analysis in Soto (2001), Schmidt and Kalemánova (2002), and Fang and Muljono (2003) is based on interest rates estimated using the Nelson and Siegel (1987) approach. The method of “bootstrapping” (e.g see Hull (2000) p. 150), is an alternative method of estimation that precisely replicates the market-quoted yields, but longer maturity yields are subject to distortions due to “errors” in the data (e.g bid-ask bounce or stale quotes).

The estimated ANS model of the yield curve can then be used to decompose the zero-coupon interest rates used to test the UIPH. The first step of the decomposition is to remove the ANS-model-estimated market prices of risk and volatility components, i.e. $\frac{\sigma_1 \theta_1 m}{2} - \sum_{n=1}^3 \sigma_n^2 \cdot u_n(m)$, from the interest rate data. The UIPH tests based on risk-neutral volatility-adjusted (RNVA) zero-coupon interest rates can be expressed as:

$$\Delta e_{t,m} = a_m + b_m \cdot m \left[\left\{ \varepsilon_{t,m}^{\text{US}} + \sum_{n=1}^3 \beta_n^{\text{US}}(t) \cdot s_n^{\text{US}}(m) \right\} - \left\{ \varepsilon_{t,m}^{\text{CA}} + \sum_{n=1}^3 \beta_n^{\text{CA}}(t) \cdot s_n^{\text{CA}}(m) \right\} \right] + v_{t,m} \quad (12)$$

where zero-coupon estimates of $\varepsilon_{t,m}^{\text{US}}$ and $\varepsilon_{t,m}^{\text{CA}}$ will only be available for the 3- and 6-month maturities in this article (given only those securities are non-coupon-bearing). In the absence of zero-coupon estimates for the other maturities, $\varepsilon_{t,m}^{\text{US}}$ and $\varepsilon_{t,m}^{\text{CA}}$ are simply set to zero, and the time-varying component of equation 12 becomes the RNVA ANS model interest differential $\sum_{n=1}^3 \beta_n^{\text{US}}(t) \cdot s_n^{\text{US}}(m) - \sum_{n=1}^3 \beta_n^{\text{CA}}(t) \cdot s_n^{\text{CA}}(m)$.¹³

The RNVA interest rates for any given maturity can now be decomposed into components representing the rationally-based fundamental and cyclical components in the underlying economy. With reference to the discussion in section 2.2, the fundamental component is simply the Level component of the ANS model, and the cyclical component is the non-Level component of the ANS model (i.e the ANS Slope plus Bow components, and the yield residuals $\varepsilon_{t,m}$ for the 3- and 6-month securities).

As an example of using the ANS model for the decompositions discussed above, figure 3 illustrates the RNVA ANS interest rate curve and the Level and Slope plus Bow components of that curve for the February 2004 US yield curve in figure 2. The estimated RNVA ANS interest rate curve is $R_{\text{Feb-2004},m}^{\text{US}}(\text{ANS}) = \sum_{n=1}^3 \beta_n^{\text{US}}(\text{Feb-2004}) \cdot s_n^{\text{US}}(m)$, the Level component of that curve is $\beta_1^{\text{US}}(\text{Feb-2004}) \cdot s_1^{\text{US}}(m) = \beta_1^{\text{US}}(\text{Feb-2004})$, and the Slope plus Bow component is $\beta_2^{\text{US}}(\text{Feb-2004}) \cdot s_2^{\text{US}}(m) + \beta_3^{\text{US}}(\text{Feb-2004}) \cdot s_3^{\text{US}}(m)$. Figure 3 also highlights the RNVA ANS interest rate for the 2-year maturity; i.e $m = 2$. This has the value $\sum_{n=1}^3 \beta_n^{\text{US}}(\text{Feb-2004}) \cdot s_n^{\text{US}}(2) = 1.70\%$, with the Level component $\beta_1^{\text{US}}(\text{Feb-2004}) = 6.47\%$, and the Slope plus Bow component $\beta_2^{\text{US}}(\text{Feb-2004}) \cdot s_2^{\text{US}}(2) + \beta_3^{\text{US}}(\text{Feb-2004}) \cdot s_3^{\text{US}}(2) = -4.77\%$.

[Figure 3 here]

Continuing the example, figure 4 then illustrates the difference between the estimated RNVA ANS yield curves for the US and Canada as at February 2004, and the difference between the ANS Level and non-Level components of the US and Canadian RNVA ANS yield curve. As highlighted in figure 4, the 2-year RNVA interest rate differentials and the Level and non-Level components of those differentials are just the respective function values at $m = 2$.

[Figure 4 here]

The example above illustrates how an interest rate differential and its Level and non-Level components are generated at a single point in time. Repeating the estimation of the ANS model for each observation of the Canadian and US yield curve data over time allows the generation of a time series of interest rate differentials of the required maturity, and the Level and non-Level components of those interest rate differentials. That data can then be used in conjunction with changes in the exchange rate over the horizon corresponding to the interest rate maturity to test the UIPH for that horizon.

The different estimates of interest rate data and the decomposition of those interest rate into components provides many different permutations of UIPH tests, particularly for the 3- and 6-month maturities where estimates of the yield residuals $\varepsilon_{t,m}$ are available. Hence, for the 3- and 6-month horizons, tests of the UIPH are undertaken for: (1) the market-quoted zero-coupon interest rate (equation 3); (2) the interest rate from the ANS model (equation 11); and (3) the RNVA

¹³The yield-to-maturity of a coupon-bearing security is effectively an internal rate of return on the coupons and principle. Similarly, the estimated yield residual for a coupon-bearing security will be on an internal rate of return basis, which is not zero-coupon.

interest rate (equation 12). The additional UIPH tests on the underlying interest rate components are as follows: (4) the Level component and non-Level components, with the latter separated out as the Slope plus Bow components and the yield residual components, i.e:

$$\begin{aligned} \Delta e_{t,m} = & a_m + w_m \cdot m [\beta_1^{\text{US}}(t) - \beta_1^{\text{CA}}(t)] + y_m \cdot m \left[\sum_{n=2}^3 \beta_n^{\text{US}}(t) \cdot s_n^{\text{US}}(m) \right. \\ & \left. - \sum_{n=2}^3 \beta_n^{\text{CA}}(t) \cdot s_n^{\text{CA}}(m) \right] + z_m \cdot m [\varepsilon_{t,m}^{\text{US}} - \varepsilon_{t,m}^{\text{CA}}] + v_{t,m} \end{aligned} \quad (13)$$

(5) the Level component and non-Level components, i.e:

$$\begin{aligned} \Delta e_{t,m} = & a_m + w_m \cdot m [\beta_1^{\text{US}}(t) - \beta_1^{\text{CA}}(t)] + x_m \cdot m \left[\left\{ \varepsilon_{t,m}^{\text{US}} + \sum_{n=2}^3 \beta_n^{\text{US}}(t) \cdot s_n^{\text{US}}(m) \right\} \right. \\ & \left. - \left\{ \varepsilon_{t,m}^{\text{CA}} + \sum_{n=2}^3 \beta_n^{\text{CA}}(t) \cdot s_n^{\text{CA}}(m) \right\} \right] + v_{t,m} \end{aligned} \quad (14)$$

(6) the Level component only, i.e:

$$\Delta e_{t,m} = a_m + w_m \cdot m [\beta_1^{\text{US}}(t) - \beta_1^{\text{CA}}(t)] + v_{t,m} \quad (15)$$

(7) the non-Level components only, separated out as the Slope plus Bow components and the yield residual components, i.e:

$$\Delta e_{t,m} = a_m + y_m \cdot m \left[\sum_{n=2}^3 \beta_n^{\text{US}}(t) \cdot s_n^{\text{US}}(m) - \sum_{n=2}^3 \beta_n^{\text{CA}}(t) \cdot s_n^{\text{CA}}(m) \right] + z_m \cdot m [\varepsilon_{t,m}^{\text{US}} - \varepsilon_{t,m}^{\text{CA}}] + v_{t,m} \quad (16)$$

and (8) the non-Level component of the RNVA ANS model only, i.e:

$$\Delta e_{t,m} = a_m + x_m \cdot m \left[\left\{ \varepsilon_{t,m}^{\text{US}} + \sum_{n=2}^3 \beta_n^{\text{US}}(t) \cdot s_n^{\text{US}}(m) \right\} - \left\{ \varepsilon_{t,m}^{\text{CA}} + \sum_{n=2}^3 \beta_n^{\text{CA}}(t) \cdot s_n^{\text{CA}}(m) \right\} \right] + v_{t,m} \quad (17)$$

For horizons/maturities of one year and beyond, estimated residuals are not available. Hence, equations 3, 13, and 16 cannot be estimated, and the estimation of the other equations proceeds with $\varepsilon_{t,m} = 0$.

3 The data and empirical estimation

The data used for the analysis in this article are the month-end CAD/USD exchange rates, and month-end Canadian and US yield curve data. These data were chosen for the investigation for several reasons. Firstly, the CAD/USD exchange rate is set within a relatively unhindered floating regime and is the only currency pair within the Group of Seven (G7) currencies that has been relatively untainted by major currency market events in recent decades. Regarding the other G7 currencies, Germany, France, and Italy used the European Monetary System and subsequently adopted the euro currency in 1999, Japan has been subject to some degree of exchange rate management including occasional large interventions as recently as 2003,¹⁴ and the United Kingdom

¹⁴Also, in reference to the discussion in section 2.2, short-term Japanese interest rates have been held at zero almost continuously since the late-1990s, which would invalidate the application of the ANS model.

(UK) was subjected to major foreign exchange speculation and subsequent withdrawal of the UK pound from the European Monetary System in 1992. Secondly, the US and Canada central bank websites readily provide long time series of detailed market-quoted yield curve data (as detailed below), while the data for other currencies is limited. That is, long time series of market-quoted data generally consist of only two points on the yield curve (e.g a 90-day rate and a 10-year bond yield), while more comprehensive market-quoted yield curve data is only available for relatively short periods.¹⁵

The CAD/USD exchange rate data is taken from the online Federal Reserve Economic Database (FRED) available on the Federal Reserve Bank of St. Louis website. The US yield curve data are constant maturity interest rates obtained from the FRED database, with the specific series being the federal funds rate, the 3-month and 6-month Treasury bill rates (both zero-coupon securities), and the 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, and 20-year or 30-year constant maturity bond rates (all semi-annual coupon-paying securities).¹⁶ The Canadian yield curve data used in the empirical application are constant maturity interest rates obtained from the Bank of Canada website, with the specific series being the Bank of Canada policy rate, the 3-month and 6-month Treasury bill rates (both zero-coupon securities), and the 2-year, 3-year, 5-year, 7-year, 10-year, and 25-year or 30-year constant maturity bond rates (all semi-annual coupon-paying securities).¹⁷ The sample period is January 1985 to December 2005, giving 252 monthly observations of the yield curve. The start of this period was chosen to be beyond the late-1970s/early-1980s structural change for US yield curve data that has been previously documented in the literature.¹⁸ December 2005 was the last month available at the time the analysis was undertaken.

The method for estimating the ANS model from a time series of yield curve observations is detailed in appendix C of Krippner (2006). An example of the output from that estimation has already been discussed in section 2, and the estimated ANS parameters for the US are $\phi = 0.51$, $\theta_1 = 1.48\%$, $\sigma_1 = 0.81\%$, $\sigma_2 = 1.72\%$, and $\sigma_3 = 1.32\%$, while those for Canada are $\phi = 0.44$, $\theta_1 = 1.67\%$, $\sigma_1 = 1.12\%$, $\sigma_2 = 3.01\%$, and $\sigma_3 = 2.25\%$. As an example of the data used for testing the UIPH in this article, figure 6 illustrates the time series of the RNVA 1-year interest rate differential and annual changes in the CAD/USD lagged one year (i.e $\Delta e_{t,1}$). Figures 7 and 8 respectively illustrate the Level and non-Level components of the RNVA ANS 1-year interest rate and $\Delta e_{t,1}$. For the 3-month, 6-month, and 1-year horizons, figure 8 plots the difference between lagged ex-post changes in the exchange rate, and the ex-ante changes predicted by the UIPH.

[Figures 5, 6, 7, and 8 here]

While economic theory would suggest that the data should be stationary in the long-run (because exchange rates cannot appreciate or depreciate indefinitely, and interest rate differentials should be bounded by relative economic fundamentals), an inspection of figures 6, 7, and 8 suggests that the data did have relatively high persistence over the sample period. Indeed, the results contained in table 1 for the 1-year horizon data suggest that the hypothesis of stationarity is frequently rejected, or alternatively the hypothesis of a unit root frequently cannot be rejected over the sample period. However, the results in table 2 indicate that the data is at least cointegrated.

[Table 1 here: 1-year unit root and KPSS tests]

[Table 2 here: 1-year cointegration tests]

The unit root and cointegration results for the 1-year horizon are typical for the other horizons investigated in this article. Hence, this article follows the advice in Hamilton (1994) p. 447 and tests the standard UIPH regression assuming both stationary data and cointegrated data to ensure

¹⁵For example, comprehensive yield curve data for Germany is only available on Datastream from 1996. Note that the Bundesbank and Bank of England websites offer zero-coupon yield curve data obtained using curve-fitting methods applied to market-quoted yield curve data, but not the market-quoted data itself.

¹⁶20-year data are unavailable from January 1987 to September 1993, and so 30-year data (with a 30-year maturity) are used during this period for the estimation of the ANS model.

¹⁷30-year data are available from January 1991, and 25-year data are available before then.

¹⁸The empirical evidence is based on changing relationships between output growth and inflation. See, for example, Estrella, Rodrigues and Schich (2003), Jardet (2004), and Krippner (2005), and the contextual discussions contained therein.

that the results are not sensitive to the persistence of the data over the sample period.¹⁹ The UIPH regression allowing for cointegrated data uses the method of Stock and Watson (1993), which essentially results in the estimated equations being augmented with the leads and lags of changes in the interest rate differential.²⁰ For example, equation 3 becomes:

$$\Delta e_{t,m} = a_m + b_m \cdot m (R_{t,m}^{\text{US}} - R_{t,m}^{\text{CA}}) + \sum_{\tau=-m}^m \Delta [m (R_{t-\tau,m}^{\text{US}} - R_{t-\tau,m}^{\text{CA}})] + v_{t,m}^* \quad (18)$$

Unfortunately, this augmentation rapidly reduces the degrees of freedom as the horizon being tested increases, and the implications are discussed in the following section in light of the empirical results.

Finally, note that all of the horizons tested are greater than the monthly frequency of the data, and so the order of moving-average serial correlation induced in all of the equations to be estimated will be the horizon in months less 1. Hence, the Newey and West (1987) method with a window of the horizon in months less 1 is used to correct the standard errors of the regressions for the expected autocorrelation (and will at the same time correct for any heteroskedasticity that is a typical feature of exchange rate data).²¹

4 Results and discussion

The upper section of table 3 contains the results from estimating equation 11, for which data is available for all maturities. Assuming stationary data, the estimates of b_m are negative and significantly different from 1 for horizons up to two years, and are positive and insignificantly different from 1 for horizons from three to five years. This pattern by horizon is consistent with the results in the existing literature, as referenced in section 1. The estimates of a_m are insignificantly different from zero for all horizons. This result is common to all of the subsequent estimations in this article, and so is not discussed again.²²

[Table 3 here]

The lower section of table 3 contains the results from estimating equation 11 assuming cointegrated data. The results for the 3 and 6-month horizons confirm the results for the stationary versions of the regressions; i.e the estimates of b_m are negative and significantly different from 1. The remaining results confirm the pattern of results by maturity assuming stationary data, except the estimates of b_m become positive from the 2-year horizon onward (albeit the r-squared statistics show increasing evidence of overfitting, as the degrees of freedom drop rapidly with increasing horizon).

The remainder of this article focusses on the UIPH tests for the 3-month, 6-month, and 1-year horizons, given those horizons unambiguously reproduce the typical puzzling result of negative estimates of b_m regardless of the estimation method. Hence, tables 4, 6, and 8 respectively contain the complete set of results for the series of UIPH tests on the 3-month, 6-month, and 1-year horizons assuming stationary data, and tables 5, 7, and 9 contain the parallel estimations assuming cointegrated data.

[Tables 4, 5, 6, 7, 8, and 9 here]

¹⁹This article does not consider mixed integration, i.e where exchange rate changes are I(0) and the interest rate differential data is highly persistent or indistinguishable from I(1). While Maynard and Phillips (1998) shows that mixed integration does have the potential to distort critical values in finite samples, Liu and Maynard (2005) finds in practice that the effect is relatively modest compared to the standard FRUH regression (and the FRUH is still rejected).

²⁰Following Hamilton (1994) pp. 608-613, the window width for the Stock and Watson (1993) method is determined by the correlation between residuals $v_{t,m}$ in the original regression and leads and lags in the changes in the right-hand side data. Because changes in the exchange rate data are lagged m years, the expected window of correlation is m (a result that was also confirmed empirically), and so the appropriate symmetric window width is $2m$.

²¹See, for example, Baillie and Bollerslev (1989) and Huisman, Koedijk, Kool and Palm (2002).

²²This result probably arises from the use of government-risk interest rates in the analysis. This means that the interest rate differentials should not reflect any differences in default-risk, which would be a factor when non-government interest rates are used.

Before discussing the component results, it is worthwhile highlighting some aspects of the 3- and 6-month UIPH regressions using the various estimates of interest rate data. Firstly, the estimates of b_m are immaterially different whether market-quoted or ANS zero-coupon interest rates are used. Similarly, whenever the yield residuals are included as a separate explanatory variable, the estimated coefficients z_m are statistically insignificant. This suggests that, even if market-quoted zero-coupon data were available for horizons from one year and beyond for the UIPH estimations in table 3, is unlikely that the empirical results would be materially different from the results based on the ANS interest rate data. Secondly, note that the results using the RNVA interest rates are immaterially different from the results based on market-quoted or ANS zero-coupon interest rates. Indeed, for any given maturity m , the function $\frac{\sigma_1 \theta_1 m}{2} - \sum_{n=1}^3 \sigma_n^2 \cdot u_n(m)$ is time invariant, and so the adjustment of the data to RNVA interest rates only affects the estimate of a_m .²³

The UIPH estimations using the individual components for the RNVA interest rates shows that the coefficients w_m for the Level component of interest rates are positive and insignificantly different from 1, and the coefficients y_m for the Slope plus Bow component of interest rates are negative and significantly different from 1. These results suggest that the negative estimates of b_m are due to the influence of the Slope plus Bow components of interest rates (i.e the rationally-based cyclical component of interest rates when the ANS model is related back to a generic general-equilibrium economy).

The remaining tests of the UIPH use the Level and non-Level components of interest rates independently. Using just the Level component and omitting the non-Level components from the UIPH estimation effectively filters out the cyclical components of the original interest rates before applying the UIPH regression. Similarly, using just the non-Level components and omitting the Level components effectively filters out the steady-state or fundamental components of the original interest rates before applying the UIPH regression.

Once again, the estimated parameters w_m are all positive and insignificantly different from 1. In other words, the UIPH is not rejected when the cyclical component of interest rates is excluded from the UIPH regression. Conversely, the estimated parameters x_m are once again all negative and significantly different from 1. In other words, the rationally-based cyclical components of interest rates again appear to be responsible for the negative coefficients obtained in the UIPH regressions.

5 Conclusion

This article applies the ANS model of the yield curve from Krippner (2006) to investigate the UIPH. After decomposing the interest rate data used in the UIPH regressions into components that reflect rationally-based expectations of the cyclical and fundamental components of the underlying economy, it is found that the UIPH is not rejected based on the fundamental components of interest rates, but is soundly rejected based on the cyclical components. These results provide empirical support for suggestions from the theoretical models of McCullum (1994), Lim and Ogaki (2003) and Meredith and Chinn (2004) that rationally-based interest rate and exchange rate dynamics associated with cyclical interlinkages between the economy and financial markets may contribute materially to the UIPH puzzle.

²³As an aside, the immaterial differences in the estimates of a_m after the adjustment to RNVA interest rates confirms the result in Wu (2005) that time-varying interest rate risk alone cannot plausibly explain the deviation of the data from UIPH. More specifically, an inspection of figure 8 shows that a time-varying term premium would at times have to account for persistent prediction errors of $\pm 2\%$ in the 3-month horizon, and $\pm 5\%$ in the 1-year horizon. Noting that $\frac{\sigma_1 \theta_1}{2} = 0.0094\%$ for Canada and 0.0060% for the US, and assuming volatility σ_1 does not vary by several orders magnitude across time (the data in figures 5 to 7 does not suggest otherwise), then the market price of risk θ_1 would have to vary by a minimum of around 1000 times around its long-term average (i.e $4\% \div 0.0094\% \div 4 = 1705$ for the 3-month horizon, and $10\% \div 0.0094\% = 1066$ for the 1-year horizon).

A Generalising the economic model underlying the ANS/VAO model

This appendix proceeds in three parts to formally establish the basis for applying the ANS model to the yield curves of two economies to decompose the interest rates for those economies into their fundamental and cyclical components for the UIPH tests. As background to the first two parts, the economic foundation proposed in Krippner (2005) for the ANS model of the yield curve is based on an explicit comparison to the yield curve derived from an augmented version of the Berardi and Esposito (1999) (hereafter BE) model of the economy. The BE model and the augmented BE (hereafter ABE) model developed in Krippner (2005) both assume for mathematical and expositional convenience that all factors and inflation rates in the economy are independent; i.e that the correlations between the stochastic components are zero, and that the mean reversions for each real factor and inflation rate in the economy have no interdependencies. Such a structure does not explicitly allow for arbitrary dynamic inter-relationships that may exist within the economy (such as Phillips curve relationships, monetary policy reaction functions, etc.), and so it cannot be taken as given that an economy with such inter-relationships will still compare directly to the ANS model of the yield curve.

Hence, section A.1 firstly establishes that a completely general version of the BE model allowing for correlated stochastic components and interdependencies between mean-reversions still produces a yield curve with the ANS functional form. Similarly, section A.2 establishes that a completely general version of the ABE model (which is the BE model with an allowance for the steady-state variables to follow low-variance Brownian motions) also produces a yield curve with the ANS functional form.

Finally, section A.3 provides extends the principles from section A.2 to the key results that underlie the analysis in this article; i.e a model with two completely general ABE economies and a bilateral exchange rate will produce yield curves of the ANS functional form for both economies. Hence, applying the ANS model to the yield curve data of two economies enables the direct decomposition of the interest rates for those economies into their fundamental and cyclical components for the UIPH tests.

B The generalised BE economy

In its most general form, the BE economy may be expressed as the following stochastic vector differential equation:

$$ds(t) = -\kappa[s(t) - \theta] dt + \sigma dz(t) \quad (19)$$

where $\mathbf{s}(t)$ are the state variables $[s_1(t), \dots, s_{2J}(t)]'$; θ are the steady-state variables $[\theta_1, \dots, \theta_{2J}]'$; $dz(t)$ are Wiener increments $[dz_1(t), \dots, dz_{2J}(t)]'$; κ is the mean-reversion coefficient matrix

$$\begin{bmatrix} \kappa_{1,1} & \cdots & \kappa_{1,2J} \\ \vdots & \ddots & \vdots \\ \kappa_{2J,1} & \cdots & \kappa_{2J,2J} \end{bmatrix}; \text{ and } \sigma \text{ is the standard deviation coefficient matrix } \begin{bmatrix} \sigma_{1,1} & \cdots & \sigma_{1,2J} \\ \vdots & \ddots & \vdots \\ \sigma_{2J,1} & \cdots & \sigma_{2J,2J} \end{bmatrix}.$$

The outer product of the stochastic terms is $\Omega = \sigma dz(t) [dz(t)]' \sigma' = \sigma \mathbf{I} \sigma' = \sigma \sigma'$. The matrix Ω will in general be non-diagonal (which allows for relationships between innovations in the growth rates of the factors of production and their rates of inflation), but it may be rotated into a diagonal representation. That is, using the notation of Greene (1997) pp. 35-38 for characteristic vectors and values (or eigenvectors and eigenvalues): $\Omega = \mathbf{C} \Lambda \mathbf{C}'$, where $\mathbf{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_{2J}\}$ is a matrix of order

$$2J \times 2J \text{ (i.e a } 2J\text{-row vector of } 2J\text{-column eigenvectors); and } \Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_{2J} \end{bmatrix} \text{ is a } 2J \times 2J$$

$2J$ diagonal matrix of eigenvalues. Pre-multiplying equation 19 by \mathbf{C}' then gives an orthogonal basis, i.e:

$$\mathbf{C}' ds(t) = -\mathbf{C}' \boldsymbol{\kappa} [\mathbf{s}(t) - \boldsymbol{\theta}] dt + \mathbf{C}' \boldsymbol{\sigma} dz(t) \quad (20)$$

which is orthogonal given that $\mathbf{C}' \boldsymbol{\sigma}_1 d\mathbf{z}_1(t) [d\mathbf{z}_1(t)]' \boldsymbol{\sigma}_1' \mathbf{C} = \mathbf{C}' \mathbf{C} \boldsymbol{\Lambda} \mathbf{C}' \mathbf{C} = \mathbf{I} \boldsymbol{\Lambda} \mathbf{I} = \boldsymbol{\Lambda}$.

Applying the expectations operator as at time t to equation 20 gives:

$$E_t [\mathbf{C}' ds(t+m)] = -\mathbf{C}' \boldsymbol{\kappa} \{E_t [\mathbf{s}(t+m)] - \boldsymbol{\theta}\} dt \quad (21)$$

and noting that $\mathbf{C}\mathbf{C}' = \mathbf{I}$, the right-hand side of equation 21 may be re-expressed, giving:

$$E_t [\mathbf{C}' ds(t+m)] = -\mathbf{C}' \boldsymbol{\kappa} \mathbf{C} \{E_t [\mathbf{C}' \mathbf{s}(t+m)] + \mathbf{C}' \boldsymbol{\theta}\} dt \quad (22)$$

With the exception of an extraordinary coincidence, the matrix $-\mathbf{C}' \boldsymbol{\kappa} \mathbf{C}$ will not be diagonal, and so the solution will not be as straightforward as solving the scalar differential equation for separate elements of the vector $\mathbf{C}' ds(t+m)$. However, Rainville and Bedient (1981) pp. 247-273 shows how to obtain a solution in the general case of $d\mathbf{X} = \mathbf{A}\mathbf{X} + \mathbf{B}$ using eigenvectors and eigenvalues. Hence, substitute $\mathbf{P}\mathbf{Q}\mathbf{P}' = -\mathbf{C}' \boldsymbol{\kappa} \mathbf{C}$, where $\mathbf{P} = \{\mathbf{p}_1, \dots, \mathbf{p}_{2J}\}$ is a matrix of order $2J \times$

$2J$ (i.e a $2J$ -row vector of $2J$ -column eigenvectors), and $\mathbf{Q} = \begin{bmatrix} q_1 & 0 & \dots & 0 \\ 0 & q_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q_{2J} \end{bmatrix}$ is a $2J \times 2J$

diagonal matrix of eigenvalues. Following Rainville and Bedient (1981), the solution is then:

$$E_t [\mathbf{C}' \mathbf{s}(t+m)] = \mathbf{C}' \boldsymbol{\theta} + \sum_{j=1}^{2J} w_j \mathbf{p}_j \exp(q_j m) \quad (23)$$

where w_j are constants. The constants w_j may be identified using the boundary condition at $m = 0$, i.e:

$$\begin{aligned} \mathbf{C}' \mathbf{s}(t) &= \mathbf{C}' \boldsymbol{\theta} + \sum_{j=1}^{2J} w_j \mathbf{p}_j \\ &= \mathbf{C}' \boldsymbol{\theta} + \mathbf{P} \mathbf{w} \end{aligned} \quad (24)$$

where $\mathbf{w} = [w_1, \dots, w_{2J}]'$. Hence, $\mathbf{w} = \mathbf{P}' \mathbf{C}' [\mathbf{s}(t) - \boldsymbol{\theta}]$, given the property of eigenvectors that $\mathbf{P}^{-1} = \mathbf{P}'$.

Under the very mild requirement that the eigenvalues q_j are real and negative (as is effectively assumed in BE), this establishes the key result that the functional form for each state variable will be a time-invariant constant plus a summation of exponential decay terms.²⁴ Hence, the expected path of the short rate will be the summation of a time-invariant constant and time-varying exponential decay terms. Following Krippner (2006), the expected path of the short rate may therefore be approximated to arbitrary precision by a time-invariant constant and time-varying exponential-polynomial terms. Also following Krippner (2005), the application of the Heath, Jarrow and Morton (1992) framework to the expected path of the short rate with exponential-polynomial terms will lead to forward rate and interest rate curves of the ANS form.

²⁴With reference to the economic interpretation of chapter 3, the requirement of real and negative eigenvalues implies that nominal GDP growth will be stationary and without regular cycles (i.e without the damped sinusoidal cycles that would result from the presence of any complex eigenvalues). These properties are readily evident from the casual observation of realised historical data.

C The generalised ABE economy

In its most general form, the ABE economy may be expressed as the following stochastic vector differential equation:

$$\begin{bmatrix} ds(t) \\ d\theta(t) \end{bmatrix} = - \begin{bmatrix} \kappa_1 & \kappa_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}(t) - \boldsymbol{\theta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix} dt + \begin{bmatrix} \boldsymbol{\sigma}_{11} & \boldsymbol{\sigma}_{10} \\ \boldsymbol{\sigma}_{01} & \boldsymbol{\sigma}_{00} \end{bmatrix} \begin{bmatrix} d\mathbf{z}_1(t) \\ d\mathbf{z}_0(t) \end{bmatrix} \quad (25)$$

where $\mathbf{s}(t) = [s_1(t), \dots, s_{2J}(t)]'$ are the state variables; $\boldsymbol{\theta}(t) = [\theta_1(t), \dots, \theta_{2J}(t)]'$ are the steady-state variables; $d\mathbf{z}_x(t) = [dz_{x,1}(t), \dots, dz_{x,2J}(t)]'$ for $x = 1$ and 0 are Wiener increments; $\kappa_x =$

$$\begin{bmatrix} \kappa_{x,1,1} & \cdots & \kappa_{x,1,2J} \\ \vdots & \ddots & \vdots \\ \kappa_{x,2J,1} & \cdots & \kappa_{x,2J,2J} \end{bmatrix} \text{ for } x = 1 \text{ and } 0 \text{ are the mean-reversion sub-matrices; and } \boldsymbol{\sigma}_{xy} =$$

$$\begin{bmatrix} \sigma_{xy,1,1} & \cdots & \sigma_{xy,1,2J} \\ \vdots & \ddots & \vdots \\ \sigma_{xy,2J,1} & \cdots & \sigma_{xy,2J,2J} \end{bmatrix} \text{ for the combinations of } xy = 11, 10, 01, \text{ and } 00 \text{ are the standard}$$

deviation coefficient sub-matrices. Note that the $\mathbf{0}$ sub-matrix entries in the deterministic coefficient matrix ensure that the steady-state variables evolve as unit root processes, as assumed in Krippner (2006). Other restrictions may also be introduced for compatibility with economic theory (e.g $\boldsymbol{\sigma}_{01} = \mathbf{0}$, so that short-run dynamics cannot influence long-run dynamics), but that does alter the mathematical nature of the exposition given here.

Following the approach used for solving the generalised BE model in section B.1, the outer product of the stochastic terms is then:

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\sigma}_{11} & \boldsymbol{\sigma}_{10} \\ \boldsymbol{\sigma}_{01} & \boldsymbol{\sigma}_{00} \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}_{11} & \boldsymbol{\sigma}_{10} \\ \boldsymbol{\sigma}_{01} & \boldsymbol{\sigma}_{00} \end{bmatrix}' \quad (26)$$

As with the generalised BE model, the matrix $\boldsymbol{\Omega}$ will in general be non-diagonal, but it may be rotated into a diagonal representation. That is, $\boldsymbol{\Omega} = \mathbf{C}\boldsymbol{\Lambda}\mathbf{C}'$, where $\mathbf{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_{4J}\}$ is a matrix of order $4J \times 4J$ (i.e a $4J$ -row vector of $4J$ -column eigenvectors), and $\boldsymbol{\Lambda}$ is a $4J \times 4J$ diagonal matrix of eigenvalues. Pre-multiplying equation 25 by \mathbf{C}' then gives an orthogonal basis, i.e:

$$\mathbf{C}' \begin{bmatrix} ds(t) \\ d\theta(t) \end{bmatrix} = -\mathbf{C}' \begin{bmatrix} \kappa_1 & \kappa_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}(t) - \boldsymbol{\theta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix} dt + \mathbf{C}' \begin{bmatrix} \boldsymbol{\sigma}_{11} & \boldsymbol{\sigma}_{10} \\ \boldsymbol{\sigma}_{01} & \boldsymbol{\sigma}_{00} \end{bmatrix} \begin{bmatrix} d\mathbf{z}_1(t) \\ d\mathbf{z}_0(t) \end{bmatrix} \quad (27)$$

Applying the expectations operator as at time t to equation 27 gives:

$$E_t \left\{ \mathbf{C}' \begin{bmatrix} ds(t+m) \\ d\theta(t+m) \end{bmatrix} \right\} = -\mathbf{C}' \begin{bmatrix} \kappa_1 & \kappa_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} E_t[\mathbf{s}(t+m)] - \boldsymbol{\theta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix} \quad (28)$$

and equation 28 may be re-expressed as:

$$E_t \left\{ \mathbf{C}' \begin{bmatrix} ds(t+m) \\ d\theta(t+m) \end{bmatrix} \right\} = -\mathbf{C}' \begin{bmatrix} \kappa_1 & \kappa_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{C} \begin{bmatrix} E_t[\mathbf{C}'\mathbf{s}(t+m)] - \mathbf{C}'\boldsymbol{\theta}(t) \\ \mathbf{C}'\boldsymbol{\theta}(t) \end{bmatrix} \quad (29)$$

As with the generalised BE model, the matrix $-\mathbf{C}' \begin{bmatrix} \kappa_1 & \kappa_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{C}$ will in general not be diagonal, but the solution is obtained using the eigenvector and eigenvalue approach already outlined in section B.1. Hence, substitute $\mathbf{PQP}' = -\mathbf{C}' \begin{bmatrix} \kappa_1 & \kappa_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{C}$, where $\mathbf{P} = \{\mathbf{p}_1, \dots, \mathbf{p}_{4J}\}$ is a matrix of

order $4J \times 4J$ (i.e a $4J$ -row vector of $4J$ -column eigenvectors), and $\mathbf{Q} = \begin{bmatrix} q_1 & 0 & \cdots & 0 \\ 0 & q_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & q_{4J} \end{bmatrix}$ is

a $4J \times 4J$ diagonal matrix of eigenvalues. Note that many of the eigenvalues q_j will be zero, given the unit root processes assumed for the steady-state variables. Following Rainville and Bédient (1981), the solution is then:

$$E_t \left\{ \mathbf{C}' \begin{bmatrix} \mathbf{s}(t+m) \\ \boldsymbol{\theta}(t+m) \end{bmatrix} \right\} = \sum_{j=1}^{4J} w_j \mathbf{p}_j \exp(q_j m) \quad (30)$$

where w_j are constants, and when $q_j = 0$, $\exp(q_j m) = \exp(0 \cdot m) = 1$. The constants w_j may be identified using the boundary condition at $m = 0$, i.e:

$$\begin{aligned} \mathbf{C}' \begin{bmatrix} \mathbf{s}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix} &= \sum_{j=1}^{4J} w_j \mathbf{p}_j \\ &= \mathbf{P} \mathbf{w} \end{aligned} \quad (31)$$

where $\mathbf{w} = [w_1, \dots, w_{4J}]'$. Hence, $\mathbf{w} = \mathbf{P}' \mathbf{C}' \begin{bmatrix} \mathbf{s}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix}$, and so the functional form for each state variable will be a summation of time-varying constants (which result from the zero eigenvalues q_j) plus a summation of time-varying exponential decay terms (which result from the terms with non-zero eigenvalues q_j , again under the mild assumption that the latter are real and negative). Hence, the expected path of the short rate will be the summation of time-varying constants and time-varying exponential decay terms, and following Krippner (2006), this may be approximated to arbitrary precision by a time-varying constant and time-varying exponential-polynomial terms. Again following Krippner (2005), the application of the Heath et al. (1992) framework to the expected path of the short rate with exponential-polynomial terms will lead to forward rate and interest rate curves of the ANS form.

D Two generalised ABE economies with an exchange rate

In its most general form, two ABE economies with an bilateral exchange rate may be expressed in the same form as equation 25, i.e:

$$\begin{bmatrix} ds_1(t) \\ d\boldsymbol{\theta}_1(t) \\ ds_2(t) \\ d\boldsymbol{\theta}_2(t) \\ de(t) \\ d\theta_e(t) \end{bmatrix} = -\boldsymbol{\kappa}_{1,2,e} \begin{bmatrix} \mathbf{s}_1(t) - \boldsymbol{\theta}_1(t) \\ \boldsymbol{\theta}_1(t) \\ \mathbf{s}_2(t) - \boldsymbol{\theta}_2(t) \\ \boldsymbol{\theta}_2(t) \\ e(t) \\ \theta_e(t) \end{bmatrix} + \boldsymbol{\sigma}_{1,2,e} dz_{1,2,e}(t) \quad (32)$$

where $\mathbf{s}_1(t)$ and $\mathbf{s}_2(t)$ are respectively the vectors of the state variables for economy 1 and 2; $\boldsymbol{\theta}_1(t)$ and $\boldsymbol{\theta}_2(t)$ are respectively the vectors of the steady-state variables for economy 1 and 2; $e(t)$ and $\theta_e(t)$ are respectively the state variable and steady-state variable for the bilateral exchange rate; $dz_{1,2,e}(t)$ is the Wiener variable vector for the two economies and the exchange rate; $\boldsymbol{\kappa}_{1,2,e}$ is the mean-reversion coefficient matrix for the two economies and the exchange rate; and $\boldsymbol{\sigma}_{1,2,e}$ is the standard deviation coefficient matrix for the two economies and the exchange rate.

The terms $\boldsymbol{\kappa}_{1,2,e}$ and $\boldsymbol{\sigma}_{1,2,e}$ are, respectively, generalisations of the mean-reversion and standard deviation coefficient matrices in the generalised ABE model of section B.2. These will allow for dynamic dependencies between the state variables and steady-state variables of the two economies and

the exchange rate. Suitable zero restrictions will be required to ensure that the steady-state variables evolve as unit root processes, and other restrictions may also be introduced for compatibility with economic theory (e.g so short-run dynamics cannot influence long-run dynamics).

The orthogonalisation of the state variables and the steady-state variables will then follow the processes already outlined for the generalised ABE model in section B.2. The solution of the expected paths of the short rate in both economies will follow from that orthogonal representation, as for the generalised ABE model in section B.2. The result will again be a functional form for each state variable that is a summation of time-varying constants (that result from the zero eigenvalues q_j associated with the unit root processes for the steady-state variables) plus a summation of time-varying exponential decay terms (that result from the terms with non-zero eigenvalues q_j , again under the mild assumption that the latter are real and negative). Hence, the expected path of the short rate in each economy will be the summation of time-varying constants and time-varying exponential decay terms. Following Krippner (2006), the expected path of the short rate in each economy may therefore be approximated to arbitrary precision by a time-varying constant and time-varying exponential-polynomial terms; i.e an ANS model of the yield curve for each economy.

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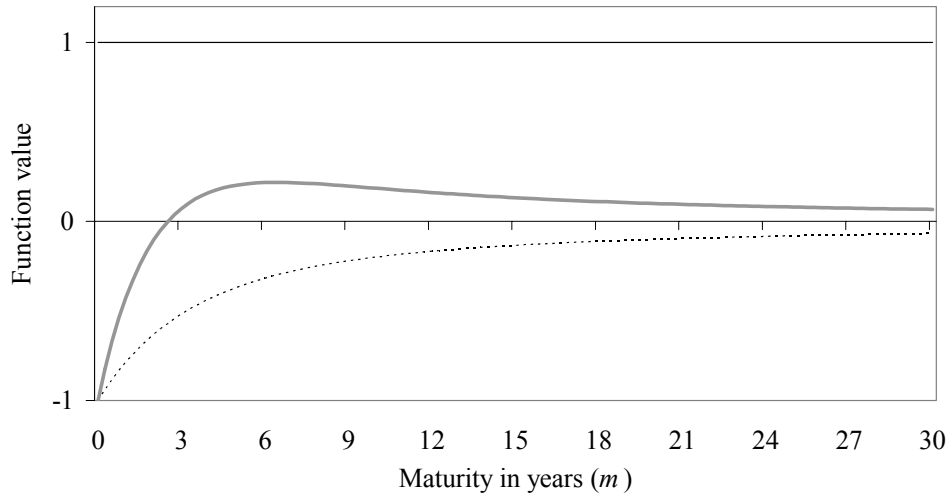


Figure 1: The Level, Slope, and Bow modes, i.e $s_1(m)$, $s_2(m)$, and $s_3(m)$ respectively, for the ANS model of the yield curve. $\phi = 0.5$ for this illustration.

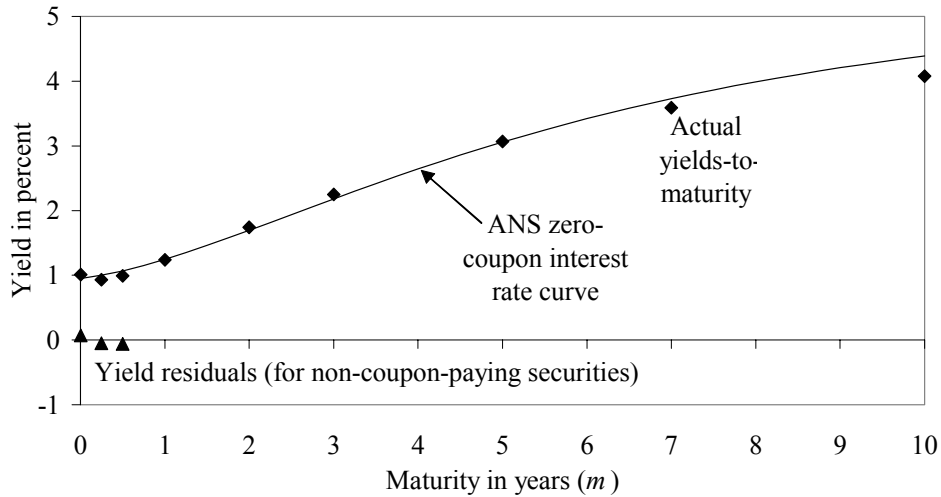


Figure 2: The US yield curve for February 2004. The diamond points are the market-quoted yields-to-maturity of the different securities, the triangle points are the estimated yield residuals for the non-coupon-paying securities, and the line is the continuously-compounding zero-coupon interest rate curve for the estimated ANS model with β_1^{US} (Feb-2004) = 6.47%, β_2^{US} (Feb-2004) = 7.98%, β_3^{US} (Feb-2004) = -2.46%.

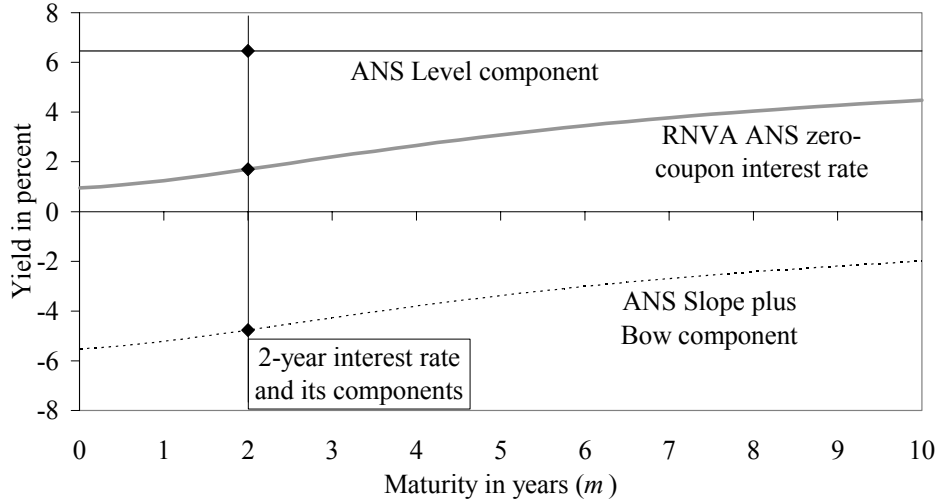


Figure 3: The RNVA ANS zero-coupon interest rate curve and its components for the February 2004 US yield curve observation. The RNVA ANS zero-coupon interest rate curve is $\sum_{n=1}^3 \beta_n^{\text{US}} (\text{Feb-2004}) \cdot s_n^{\text{US}}(m)$, the ANS Level component is $\beta_1^{\text{US}} (\text{Feb-2004}) \cdot s_1^{\text{US}}(m)$, and the ANS Slope plus Bow component is $\beta_2^{\text{US}} (\text{Feb-2004}) \cdot s_2^{\text{US}}(m) + \beta_3^{\text{US}} (\text{Feb-2004}) \cdot s_3^{\text{US}}(m)$.

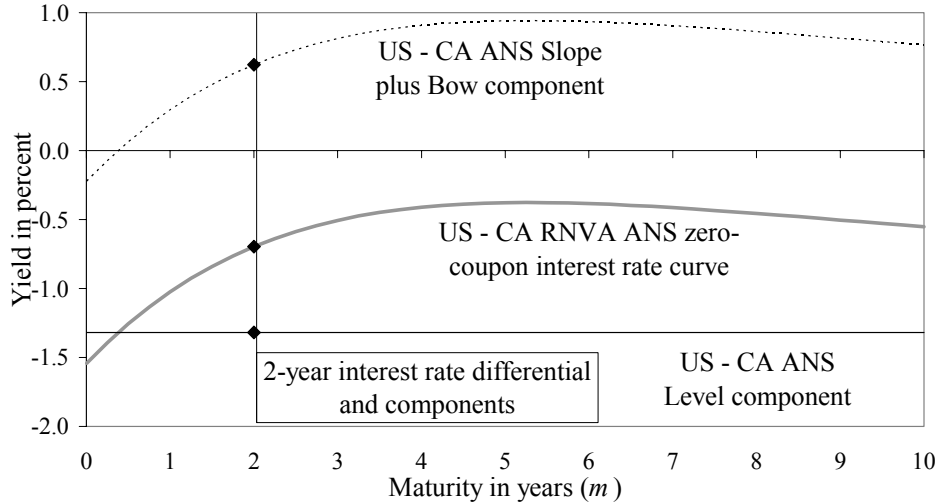


Figure 4: The RNVA ANS zero-coupon interest rate differential and its components for the February 2004 US and Canadian yield curves. The RNVA ANS zero-coupon interest rate differential is $\sum_{n=1}^3 \beta_n^{\text{US}} (\text{Feb-2004}) \cdot s_n^{\text{US}}(m) - \sum_{n=1}^3 \beta_n^{\text{CA}} (\text{Feb-2004}) \cdot s_n^{\text{CA}}(m)$, the Level component is $\beta_1^{\text{US}} (\text{Feb-2004}) - \beta_1^{\text{CA}} (\text{Feb-2004})$, and the non-Level component is $\sum_{n=2}^3 \beta_n^{\text{US}} (\text{Feb-2004}) \cdot s_n^{\text{US}}(m) - \sum_{n=2}^3 \beta_n^{\text{CA}} (\text{Feb-2004}) \cdot s_n^{\text{CA}}(m)$.

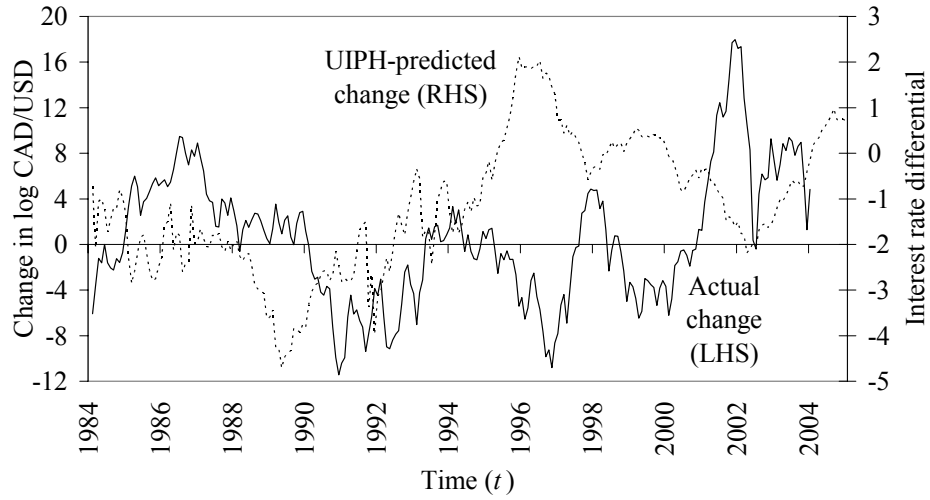


Figure 5: UIPH data for the 1-year horizon using the ANS model 1-year interest rate. The series are the lagged annual change in the CAD/USD exchange rate (i.e $\Delta e_{t,1}$) and the US less Canadian interest rate differential from the RNVA ANS model for the 1-year maturity (i.e $m = 1$).

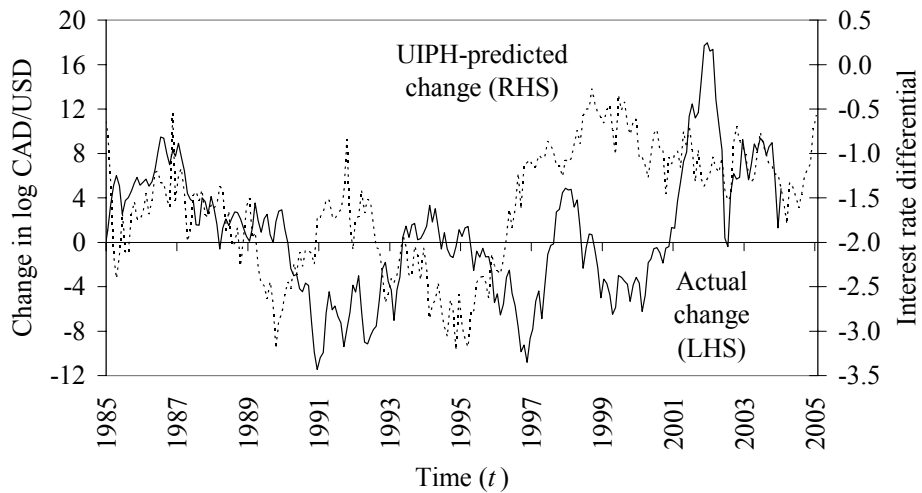


Figure 6: UIPH data for the 1-year horizon using the Level component of the 1-year interest rate. The series are the lagged annual change in the CAD/USD exchange rate (i.e $\Delta e_{t,1}$) and the US less Canadian interest rate differential for the Level component of the ANS model for the 1-year maturity (i.e $m = 1$).

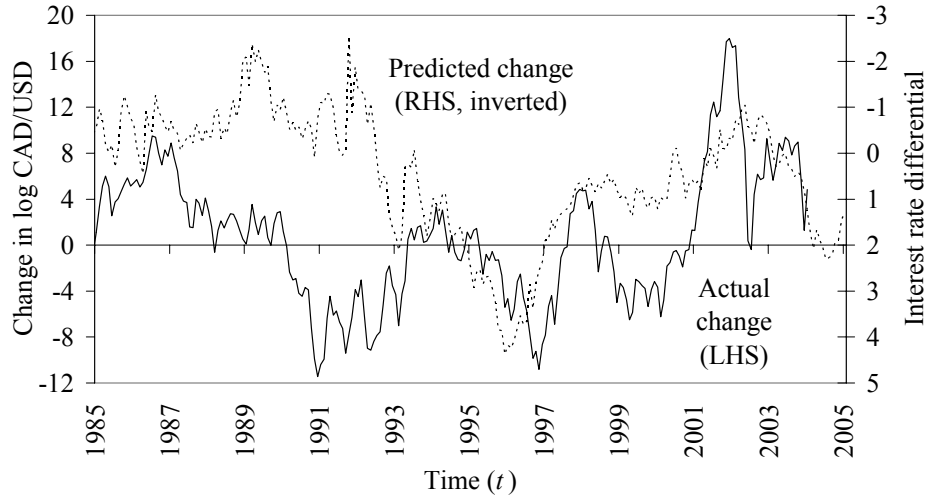


Figure 7: UIPH data for the 1-year horizon using the Slope plus Bow component of the 1-year interest rate. The series are the lagged annual change in the CAD/USD exchange rate (i.e $\Delta e_{t,1}$) and the US less Canadian interest rate differential for the Slope plus Bow component of the ANS model for the 1-year maturity (i.e $m = 1$). The latter has been inverted to illustrate the inverse relationship with $\Delta e_{t,1}$.

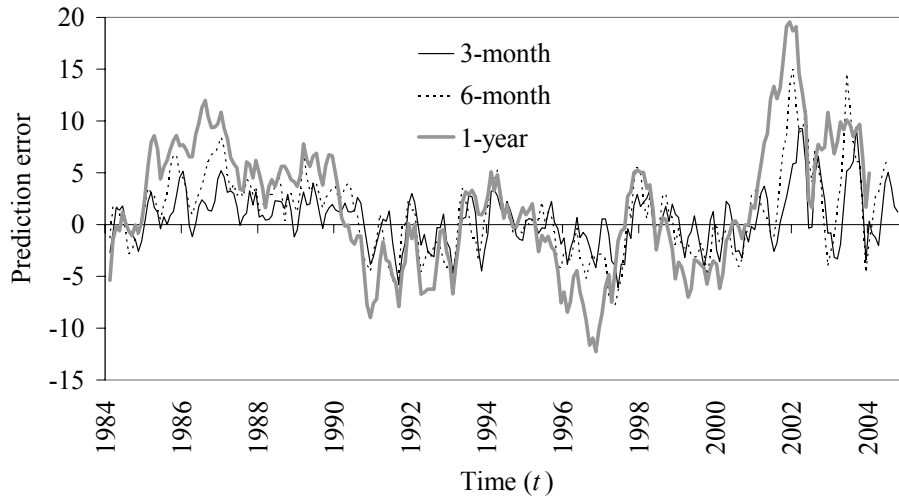


Figure 8: UIPH prediction errors. The series are the lagged changes in the CAD/USD exchange rate (i.e $\Delta e_{t,m}$) less the interest rate differential for the 3-month, 6-month, and 1-year horizons.

Unit root or stationarity test	Change in LN USD/CAD	ANS interest rate differential	RNVA ANS interest rate differential	Level	Non-Level
				component of ANS interest rate differential	component of ANS interest rate differential
PP fixed window	-2.81 * 12	-1.78 12	-1.78 12	-2.85 * 12	-2.26 12
PP auto window	-2.82 * 11	-1.78 12	-1.78 12	-2.81 * 11	-2.26 12
ADF fixed lags	-1.87 12	-1.86 12	-1.86 12	-1.86 12	-2.03 12
ADF auto lags	-1.87 12	-1.52 7	-1.52 7	-2.60 * 1	-2.41 0
KPSS fixed window	0.28 12	0.95 *** 12	0.95 *** 12	0.53 ** 12	0.66 ** 12
KPSS auto window	0.30 11	0.95 *** 12	0.95 *** 12	0.57 ** 11	0.66 ** 12

Table 1: Unit root and stationarity tests on the data used to estimate the 1-year horizon UIPH. PP is Phillips-Perron, ADF is augmented Dickey-Fuller, KPSS is Kwiatkowski-Phillips-Schmidt-Shin, and the window width/number of lags is given below each statistic. *, **, and *** respectively denote a 10, 5, and 1 percent level of significance.

Cointegration tests versus USD/CAD	Change in LN USD/CAD	ANS interest rate differential	RNVA ANS interest rate differential	Level	Non-Level
				component of ANS interest rate differential	component of ANS interest rate differential
PP fixed window	n/a n/a	-2.98 ** 12	-2.98 ** 12	-3.09 ** 12	-2.91 ** 12
PP selected window	n/a n/a	-2.97 ** 11	-2.97 ** 11	-3.10 ** 11	-2.91 ** 11
ADF fixed lags	n/a n/a	-2.39 12	-2.39 12	-2.38 12	-2.32 12
ADF selected lags	n/a n/a	-2.39 12	-2.39 12	-2.38 12	-2.32 12

Table 2: Cointegration tests on the 1-year horizon UIPH data. These are unit root tests on the interest rate differential measures less the change in the exchange rate data for the 1-year horizon UIPH. PP is Phillips-Perron, ADF is augmented Dickey-Fuller, and the window width/number of lags is given below each statistic. ** denotes a 5 percent level of significance.

Estimates assuming stationary data							
	3 months	6 months	1 year	2 years	3 years	4 years	5 years
a_m	-0.51%	-0.42%	-0.50%	-0.17%	0.47%	0.44%	0.38%
s.e	1.13%	1.05%	1.10%	1.20%	1.43%	1.36%	1.03%
P(0)	0.652	0.687	0.652	0.889	0.743	0.746	0.712
b_m	-0.85	-0.81	-0.89	-0.63	0.25	0.71	0.99
s.e	0.39	0.40	0.58	0.87	0.99	0.96	0.70
P(0)	0.028	0.045	0.126	0.472	0.804	0.462	0.157
P(1)	0.000	0.000	0.001	0.063	0.446	0.763	0.992
R^2	0.021	0.031	0.050	0.026	0.004	0.043	0.102
DF	247	244	238	226	214	202	190
Estimates allowing for cointegrated data							
a_m	0.06%	0.06%	0.36%	0.83%	1.92%	3.62%	2.53%
s.e	1.13%	1.20%	1.32%	1.80%	0.95%	0.19%	1.96%
P(0)	0.960	0.961	0.787	0.647	0.045	0.000	0.199
b_m	-0.45	-0.49	-0.24	0.73	3.49	5.84	4.92
s.e	0.42	0.51	0.76	1.35	0.87	0.29	1.63
P(0)	0.280	0.333	0.752	0.591	0.000	0.000	0.003
P(1)	0.001	0.004	0.103	0.840	0.005	0.000	0.017
R^2	0.099	0.079	0.157	0.391	0.716	0.951	0.996
DF	236	224	200	152	104	56	8

Table 3: UIPH test results using ANS model interest rates. P(0) and P(1) are p-values for the respective hypotheses that the parameters equal 0 or 1. The UIPH (i.e $b_m = 1$) is rejected at the 5 percent level of significance for short horizons, but is not rejected for longer horizons. Note that the estimates allowing for cointegrated data show increasing evidence of overfitting beyond the 1-year horizon.

	Actual	ANS	RNVA	L+SB+R	L+SBR	L	SB+R	SBR
a_m	-0.53%	-0.51%	-0.53%	0.73%	0.81%	1.05%	0.71%	0.81%
s.e	1.14%	1.13%	1.14%	2.34%	2.26%	2.29%	1.03%	0.94%
P(0)	0.639	0.652	0.639	0.755	0.719	0.648	0.491	0.391
b_m	-0.85	-0.85	-0.85					
s.e	0.37	0.39	0.37					
P(0)	0.022	0.028	0.022					
P(1)	0.000	0.000	0.000					
w_m				0.01	0.00	0.23		
s.e				1.07	1.07	1.09		
P(0)				0.993	0.999	0.834		
P(1)				0.356	0.349	0.479		
x_m					-0.97			-0.97
s.e					0.40			0.40
P(0)					0.016			0.017
P(1)					0.000			0.000
y_m				-0.85			-0.85	
s.e				0.49			0.50	
P(0)				0.087			0.090	
P(1)				0.000			0.000	
z_m				-3.47			-3.47	
s.e				5.44			5.43	
P(0)				0.524			0.523	
P(1)				0.412			0.411	
R ²	0.023	0.021	0.023	0.028	0.027	0.000	0.028	0.027
DF	247	247	247	245	246	247	246	247

Table 4: UIPH test results for the 3-month horizon assuming stationary data. P(0) and P(1) are p-values for the respective hypotheses that the parameters equal 0 or 1. L is the ANS Level component, SB is the ANS Slope plus Bow component, R is the yield residual component, and SBR is the Slope plus Bow plus yield residual component. The UIPH (i.e $b_m = 1$) is strongly rejected, which is attributable to the Slope plus Bow component of interest rates (i.e $y_m \ll 1$).

	Actual	ANS	RNVA	L+SB+R	L+SBR	L	SB+R	SBR
a_m	0.00%	0.06%	0.00%	2.80%	2.97%	2.21%	0.48%	0.77%
s.e	1.14%	1.13%	1.14%	2.48%	2.41%	2.41%	1.25%	0.97%
P(0)	0.997	0.960	0.997	0.261	0.220	0.360	0.700	0.424
b_m	-0.47	-0.45	-0.47					
s.e	0.39	0.42	0.39					
P(0)	0.230	0.280	0.230					
P(1)	0.000	0.001	0.000					
w_m				1.49	1.38	0.93		
s.e				1.18	1.19	1.17		
P(0)				0.210	0.248	0.429		
P(1)				0.680	0.749	0.953		
x_m					-0.83			-0.63
s.e					0.44			0.40
P(0)					0.056			0.120
P(1)					0.000			0.000
y_m				-0.51			-0.29	
s.e				0.68			0.68	
P(0)				0.459			0.672	
P(1)				0.028			0.059	
z_m				-8.76			-7.44	
s.e				9.31			9.22	
P(0)				0.348			0.420	
P(1)				0.295			0.361	
R^2	0.101	0.099	0.101	0.142	0.124	0.015	0.094	0.081
DF	236	236	236	220	228	236	228	236

Table 5: UIPH test results for the 3-month horizon assuming cointegrated data. The notation and results are as for table 4.

	Actual	ANS	RNVA	L+SB+R	L+SBR	L	SB+R	SBR
a_m	-0.48%	-0.42%	-0.48%	1.64%	1.10%	1.48%	1.36%	0.65%
s.e	1.15%	1.05%	1.15%	2.59%	2.22%	2.26%	1.82%	0.95%
P(0)	0.679	0.687	0.678	0.528	0.621	0.513	0.457	0.491
b_m	-0.72	-0.81	-0.72					
s.e	0.37	0.40	0.37					
P(0)	0.049	0.045	0.049					
P(1)	0.000	0.000	0.000					
w_m				0.19	0.28	0.52		
s.e				0.98	1.02	1.02		
P(0)				0.847	0.781	0.614		
P(1)				0.409	0.483	0.637		
x_m					-0.86			-0.88
s.e					0.37			0.36
P(0)					0.021			0.017
P(1)					0.000			0.000
y_m				-1.09			-1.10	
s.e				0.56			0.56	
P(0)				0.054			0.050	
P(1)				0.000			0.000	
z_m				1.81			1.86	
s.e				4.06			4.08	
P(0)				0.655			0.649	
P(1)				0.841			0.834	
R^2	0.028	0.031	0.028	0.046	0.038	0.002	0.046	0.038
DF	244	244	244	242	243	244	243	244

Table 6: UIPH test results for the 6-month horizon assuming stationary data. The notation and results are as for table 4.

	Actual	ANS	RNVA	L+SB+R	L+SBR	L	SB+R	SBR
a_m	-0.03%	0.06%	-0.03%	4.00%	4.08%	3.11%	1.21%	0.72%
s.e	1.30%	1.20%	1.30%	2.90%	2.75%	2.72%	2.50%	1.01%
P(0)	0.981	0.961	0.981	0.170	0.140	0.254	0.630	0.477
b_m	-0.48	-0.49	-0.48					
s.e	0.22	0.16	0.22					
P(0)	0.032	0.003	0.032					
P(1)	0.000	0.000	0.000					
w_m				2.25	2.14	1.50		
s.e				1.51	1.40	1.31		
P(0)				0.138	0.127	0.251		
P(1)				0.410	0.415	0.700		
x_m					-0.96			-0.84
s.e					0.56			0.42
P(0)					0.088			0.048
P(1)					0.001			0.000
y_m				-0.97			-1.10	
s.e				0.86			0.82	
P(0)				0.261			0.182	
P(1)				0.023			0.011	
z_m				-2.60			0.35	
s.e				6.84			6.36	
P(0)				0.704			0.956	
P(1)				0.599			0.919	
R ²	0.070	0.079	0.070	0.164	0.130	0.036	0.108	0.074
DF	224	224	224	196	210	224	210	224

Table 7: UIPH test results for the 6-month horizon assuming cointegrated data. The notation and results are as for table 4.

	ANS	RNVA	L+SB	L	SB
a_m	-0.50%	-0.51%	1.73%	1.98%	0.99%
s.e	1.10%	1.10%	2.17%	2.35%	1.18%
P(0)	0.652	0.647	0.427	0.401	0.403
b_m	-0.89	-0.89			
s.e	0.58	0.58			
P(0)	0.126	0.126			
P(1)	0.001	0.001			
w_m			0.48	0.88	
s.e			1.02	1.02	
P(0)			0.637	0.389	
P(1)			0.613	0.905	
x_m			-1.15		-1.19
s.e			0.58		0.56
P(0)			0.049		0.033
P(1)			0.000		0.000
R ²	0.050	0.050	0.087	0.012	0.084
DF	238	238	237	238	238

Table 8: UIPH test results for the 1-year horizon assuming stationary data. P(0) and P(1) respectively represent tests that the parameters equals 0 or 1. L is the ANS Level component, SB is the ANS Slope plus Bow component. The UIPH (i.e $b_m = 1$) is strongly rejected, which is attributable to the Slope plus Bow component of interest rates (i.e $x_m \ll 1$).

	ANS	RNVA	L+SB	L	SB
a_m	0.36%	0.35%	5.71%	5.28%	1.04%
s.e	1.32%	1.32%	3.76%	3.50%	1.35%
P(0)	0.787	0.789	0.130	0.133	0.440
b_m	-0.24	-0.24			
s.e	0.76	0.76			
P(0)	0.752	0.752			
P(1)	0.103	0.103			
w_m			3.07	2.89	
s.e			2.01	1.78	
P(0)			0.128	0.105	
P(1)			0.305	0.289	
x_m			-0.73		-1.00
s.e			1.02		0.74
P(0)			0.475		0.175
P(1)			0.091		0.007
R ²	0.157	0.157	0.276	0.187	0.128
DF	200	200	174	200	200

Table 9: UIPH test results for the 1-year horizon assuming cointegrated data. The notation and results are as for table 8.