Transport of cross helicity and radial evolution of Alfvénicity in the solar wind

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[1] A transport theory including cross helicity, magnetohydrodynamic (MHD) turbulence, and driving by shear and pickup ions, is applied to the radial evolution of the solar wind. The radial decrease of cross helicity observed in the solar wind can be accounted for when sufficient driving is included to overcome the inherent tendency for MHD turbulence to produce Alfvénic states. *INDEX TERMS:* 2134 Interplanetary Physics: Interplanetary magnetic fields; 2149 Interplanetary Physics: MHD waves and turbulence; 2164 Interplanetary Physics: Solar wind plasma; 7839 Space Plasma Physics: Nonlinear phenomena; 7859 Space Plasma Physics: Transport processes. **Citation:** Matthaeus, W. H., J. Minnie, B. Breech, S. Parhi, J. W. Bieber, and S. Oughton (2004), Transport of cross helicity and radial evolution of Alfvénicity in the solar wind, *Geophys. Res. Lett.*, *31*, L12803, doi:10.1029/2004GL019645.

1. Introduction

[2] The evolution of solar wind magnetohydrodynamic (MHD) turbulence is a challenging space plasma physics problem, and one that is central in understanding various features of the heliosphere including radial temperature, solar energetic particles and modulation of galactic cosmic rays. Relatively complete formalisms for turbulence transport in a weakly inhomogeneous medium have been developed using several complementary approaches [*Marsch and Tu*, 1989; *Tu and Marsch*, 1993; *Zhou and Matthaeus*, 1990; *Matthaeus et al.*, 1994]. Frequently further approximations are imposed to achieve a physically transparent model. One of those simplifications is to the case of zero cross helicity or equivalently equal admixtures of "inward" and "outward" Alfvénic fluctuations. This is probably well satisfied beyond a heliocentric distance of a few Astronomical Units (AU), but it is marginal from 1–3 AU, and is definitely not a reasonable simplification at distances less than 1 AU from the sun [*Belcher and Davis*, 1971; *Bavassano et al.*, 1982a, 1982b; *Roberts et al.*, 1987; *Tu et al.*, 1989a, 1989b]. A transport theory is needed that supports mixed cross helicities, while retaining both nonlinear and linear transport effects in a tractable form. In this letter we present such a transport theory and find, through a simple preliminary analysis, that it predicts a threshold in the average strength of shear driving that is required to explain the well known observed feature that Alfvénicity of solar wind turbulence decreases with heliocentric distance.

[3] Solar wind transport theory is now at a stage where it can account reasonably well for the observed radial evolution of turbulence level, correlation scale, and proton temperature from 1 AU to beyond 60 AU. To achieve reasonable comparisons with Voyager and Pioneer data, it has been found that the theory must include driving, or resupply, of turbulence. Between 1 and 10 AU this driving is mainly due to stream shear, while beyond 10 AU it is mainly due to excitation of fluctuations associated with the partial assimilation of pickup ions of interstellar origin [*Isenberg et al.*, 2003]. Typically such models [*Zank et al.*, 1996] are vast simplifications of the full MHD transport formalism, which involves 16 coupled spectral equations [*Zhou and Matthaeus*, 1990; *Marsch and Tu*, 1989], even when small-scale compressive effects are neglected. This reduction involves several approximations, including spherical symmetry, constant solar wind speed, zero magnetic and kinetic helicities, structural similarity of all correlation functions, and several assumptions appropriate to the outer heliosphere, such as dropping terms of order V_A/U (Alfvén speed over solar wind speed) and, notably, zero cross helicity. Here we relax the last of these approximations—allowing for nonzero cross helicity—which represents correlation of velocity and magnetic field or "Alfvénicity."

[4] The transport equations for weakly inhomogeneous, locally incompressible MHD are typically based upon correlation functions and written in terms of the Elsässer variables $\mathbf{z}_\pm \equiv \mathbf{v} \pm \mathbf{b}/\sqrt{4\pi\rho}$, for velocity fluctuation $\mathbf{v}$, magnetic fluctuation $\mathbf{b}$, and mass density ρ. When $\mathbf{z}_+ = \mathbf{0}$, the remaining (arbitrary) $\mathbf{z}_-$ field becomes a large amplitude solution of the MHD equations that propagates at the Alfvén speed $V_A = B_0/\sqrt{4\pi\rho}$ parallel to a uniform background magnetic field $\mathbf{B}_0$. (Similarly when $\mathbf{z}_- = \mathbf{0}$, $\mathbf{z}_+$ is an anti-parallel propagating solution.) However when both fields are nonzero, they interact nonlinearly with one another, producing turbulence, and simple propagation may no longer be an accurate picture. The squared Elsässer amplitude $Z^2 = (Z_+^2 + Z_-^2)/2$ is twice the total fluctuation energy (flow plus magnetic) per unit mass, where the separate "energies" are $Z_\pm^2 = \langle|\mathbf{v} \pm \mathbf{b}/\sqrt{4\pi\rho}|^2\rangle$. The cross helicity $H_c \equiv \langle \mathbf{v} \cdot \mathbf{b}/\sqrt{4\pi\rho}\rangle = (Z_+^2 - Z_-^2)/4$ is a measure of the correlation of velocity and magnetic fluctuations, and also of the preponderance of one type of Elsässer fluctuation (and in some cases, one sense of propagation direction) over the other. It is convenient to measure cross helicity using the normalized quantity

$$\sigma_c \equiv \frac{Z_+^2 - Z_-^2}{Z_+^2 + Z_-^2}. \tag{1}$$

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Neither the presence of cross helicity nor the use of an Elsässer representation bias our description towards either a wave or a turbulence picture of the fluctuations.

[5] Simplified transport theories follow the evolution of Z^2 and a single similarity lengthscale λ (usually, the correlation scale) to describe the radial evolution in the outer heliosphere. Here we add a third simplified equation for σ_c.

2. Homogeneous Turbulence Effects

[6] We first consider approximations (one-point closure) for undriven homogeneous turbulence. When $\sigma_c = 0$ a useful model for energy decay is

$$\frac{dZ^2}{dt} = -\alpha\frac{Z^2}{\tau_{\rm nl}} = -\alpha\frac{Z^3}{\lambda}, \quad (2)$$

along with

$$\frac{d\lambda}{dt} = \beta\frac{\lambda}{\tau_{\rm nl}} = \beta Z, \quad (3)$$

where $\tau_{\rm nl} = \lambda/Z$ is the eddy-turnover time; α and β are $O(1)$ constants. For nonzero cross helicity we can similarly write [*Dobrowolny et al.*, 1980; *Matthaeus et al.*, 1994; *Hossain et al.*, 1995], based on analysis of the MHD equations, that

$$\frac{dZ_\pm^2}{dt} = -\frac{\alpha_\pm}{\lambda_\pm}Z_\pm^2 Z_\mp \to -\frac{\alpha}{\lambda}Z_\pm^2 Z_\mp, \quad (4)$$

where we do not distinguish between similarity lengthscales that appear in the two equations. Upon using the identities $Z_\pm^2 = (1 \pm \sigma_c)Z^2$ we find immediately that

$$\frac{dZ^2}{dt} = -\alpha f^+(\sigma_c)\frac{Z^3}{\lambda}, \quad (5)$$

where we define

$$f^\pm(\sigma_c) = \frac{\left(1-\sigma_c^2\right)^{1/2}}{2}\left[(1+\sigma_c)^{1/2} \pm (1-\sigma_c)^{1/2}\right]. \quad (6)$$

This Kármán–Taylor phenomenology is most appropriate when the strongest nonlinearities are those associated with anisotropic quasi–two-dimensional (quasi-2D) MHD. The paradigm for this is "2D" turbulence, where wavevectors are perpendicular to $\mathbf{B}_0$, i.e., $k_\parallel \equiv 0$, and there is no Alfvén propagation effect at all. An alternative phenomenology [*Dobrowolny et al.*, 1980] for turbulent decay is based on

$$dZ_\pm^2/dt \sim -Z_+^2 Z_-^2/(\lambda V_A), \quad (7)$$

which has the convenient property that $dH_c/dt = 0$ for homogeneous turbulence. This leads directly to an equation of the form $dZ^2/dt \sim -(1-\sigma_c^2)^2 Z^4/(\lambda V_A)$ for the energy, which shows that in the *Dobrowolny et al.* [1980] picture energy decay slows as V_A^{-1} under influence of a large-scale magnetic field. This approach is less favored [*Hossain et al.*, 1995] because it does not account for anisotropy of the spectrum. In the following we employ the Kármán–Taylor approach for modeling the evolution of Z^2, λ, and σ_c. However analogous results using the *Dobrowolny et al.* [1980] phenomenology can be readily obtained.

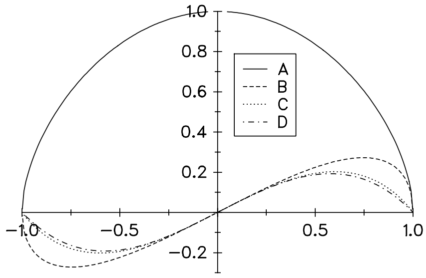

Figure 1. The functions $f^+(\sigma_c)$ (A) and $f^-(\sigma_c)$ (B) that embody the modifications to the nonlinear decay phenomenology when cross helicity is present. Curve C is the combination $f'(\sigma_c) = \sigma_c f^+(\sigma_c) - f^-(\sigma_c)$ that appears in equation (15). The numerically established approximation $f' \approx (\sigma_c - \sigma_c^3)/2$ is shown as curve D.

[7] For nonlinear evolution of the correlation scale we temporarily restore two lengthscales $\lambda_\pm$, define $2Z^2\lambda = \lambda_+ Z_+^2 + \lambda_- Z_-^2$, and find

$$\frac{d\lambda}{dt} = \frac{Z_+^2\dot{\lambda}_+ + Z_-^2\dot{\lambda}_-}{2Z^2}, \quad (8)$$

where an overdot designates a time derivative. At this point we now employ a finite-H_c generalization of equation (3) suggested by *Hossain et al.* [1995], namely that $\dot{\lambda}_\pm \sim Z_\mp$, and arrive at $\dot{\lambda} = \beta(Z_+^2 Z_- + Z_-^2 Z_+)/(2Z^2) = -\beta\alpha^{-1}\lambda Z^{-2} dZ^2/dt$, for some constant β. Using equation (5) then yields

$$\frac{d\lambda}{dt} = \beta f^+(\sigma_c)Z, \quad (9)$$

which is our required generalization of equation (3).

[8] The equation for evolution of σ_c is obtained from

$$\frac{d\sigma_c}{dt} = \frac{\dot{Z}_+^2 - \dot{Z}_-^2}{2Z^2} - \frac{\sigma_c}{Z^2}\frac{dZ^2}{dt}, \quad (10)$$

and then using equations (4) and (5) to arrive at

$$\frac{d\sigma_c}{dt} = \alpha\frac{Z}{\lambda}\left[\sigma_c f^+(\sigma_c) - f^-(\sigma_c)\right] \equiv \alpha\frac{Z}{\lambda}f'(\sigma_c), \quad (11)$$

with $f^\pm$ as defined in equation (6). For reference, plots of $f^\pm(\sigma_c)$ are shown in Figure 1. Also plotted is $f'(\sigma_c)$ the square-bracketed term in equation (11), which always has the same sign as σ_c. Thus this equation for decaying MHD turbulence *always* amplifies a seed cross helicity. This effect has been called "dynamic alignment" [*Dobrowolny et al.*, 1980; *Grappin et al.*, 1982; *Matthaeus et al.*, 1983; *Pouquet et al.*, 1986], since it favors production of Alfvénic (correlated) velocity and magnetic field fluctuations. In contrast, solar wind turbulence shows a reduction of Alfvénicity as turbulence ages with increasing heliocentric distance. This is connected with sources of uncorrelated velocity and magnetic fluctuations [*Stribling and Matthaeus*, 1991; *Roberts et al.*, 1992], such as large-scale velocity shear or pickup ions. Effects that cause reduction in cross helicity are not accounted for in this section, but will be included below in the transport phenomenology for the solar wind.

[9] Summarizing, equations (5), (8), and (11) provide a phenomenological description of turbulent decay of undriven homogeneous MHD turbulence with cross helicity and in the absence of large scale shear. Comparing this with $H_c = 0$ phenomenologies (equations (2) and (3), e.g., *Hossain et al.* [1995]), it is evident that nonzero H_c causes a reduction in the effective decay "constants," i.e., $\alpha \rightarrow \alpha f^+(\sigma_c)$ and $\beta \rightarrow \beta f^+(\sigma_c)$.

3. Spatial Transport Effects

[10] We now consider spatial transport, beginning with the steady-state equations for Z^2 and λ in the heliocentric radial coordinate r,

$$\frac{dZ^2}{dr} = -\frac{Z^2}{r} + \frac{C_{\rm sh} - M\sigma_D}{r} Z^2 + \frac{\dot{E}_{PI}}{U} - \frac{\alpha}{\lambda U} Z^3, \qquad (12)$$

and

$$\frac{d\lambda}{dr} = \frac{M\sigma_D - \beta C_{\rm sh}}{r}\lambda - \beta\lambda\frac{\dot{E}_{PI}}{UZ^2} + \frac{\beta}{U} Z, \qquad (13)$$

that have been used for outer heliospheric studies [*Zank et al.*, 1996; *Matthaeus et al.*, 1999; *Smith et al.*, 2001]. Advection (left hand side) at solar wind speed U is balanced (on the right side) by terms associated with (WKB) expansion, a parameterization of large-scale shear of dimensionless strength $C_{\rm sh}$, a "mixing" or expansion term, consisting of a geometrically determined parameter M and the normalized energy difference $\sigma_D = (u^2 - b^2)/(u^2 + b^2)$ (fluctuation energy per unit mass, velocity u^2 and magnetic field, b^2), the excitation of MHD fluctuations by pickup-ions [*Isenberg et al.*, 2003], and the Kármán–Taylor MHD turbulence phenomenology described above.

[11] To compute evolution of cross helicity we employ a separate equation for each Elsässer amplitude [e.g., *Matthaeus et al.*, 1994],

$$\frac{dZ_\pm^2}{dr} = -\frac{Z_\pm^2}{r} + \frac{C_{\rm sh} - M\sigma_D}{r} Z^2 + \frac{\dot{E}_{PI}}{U} - \alpha\frac{Z_\pm^2 Z_\mp}{\lambda U}. \qquad (14)$$

[12] Note that the driving effects of mixing, shear, and pickup ion do not drive cross helicity. These effects inject inward and outward-type fluctuations equally, although for distinct reasons. Mixing effects on Z_+ and Z_- are equal up to terms of order V_A/U for a uniform expansion and a simple fluctuation symmetry (e.g., 2D) [*Matthaeus et al.*, 1994; *Oughton and Matthaeus*, 1995]. Shear drives large-scale nonlinear Kelvin–Helmholtz instabilities that inject kinetic energy, thus having no preference for propagation direction. Finally, pickup ions in the outer heliosphere [*Isenberg et al.*, 2003] encounter an approximately transverse magnetic field and therefore couple equally to both directions of propagation. Thus in the present approximation, driving supplies energy but not cross helicity.

[13] Using the analog of equation (10), namely $d\sigma_c/dr = [d/dr(Z_+^2 - Z_-^2)]/(2Z^2) - \sigma_c(dZ^2/dr)/Z^2$, we make use of the above results to find that

$$\frac{d\sigma_c}{dr} = \frac{1}{U}\frac{d\sigma_c}{dt}\bigg|_{\rm homog} - \left[\frac{C_{\rm sh} - M\sigma_D}{r} + \frac{\dot{E}_{PI}}{UZ^2}\right]\sigma_c, \qquad (15)$$

with equation (10) defining $d\sigma_c/dt|_{\rm homog}$. For the Kármán–Taylor picture, equation (4), we obtain

$$\frac{d\sigma_c}{dr} = \alpha f'(\sigma_c)\frac{Z}{U\lambda} - \left[\frac{C_{\rm sh} - M\sigma_D}{r} + \frac{\dot{E}_{PI}}{UZ^2}\right]\sigma_c, \qquad (16)$$

where f' is defined in equation (11).

[14] We see now that the question of the growth or decay of σ_c can be examined quantitatively using this transport formalism. If the first term on the right of equation (16) dominates, then cross helicity increases in an expanding wind [*Dobrowolny et al.*, 1980; *Matthaeus et al.*, 1983; *Grappin et al.*, 1982; *Pouquet et al.*, 1986]. This term always produces dynamic alignment because $f' = \sigma_c f^+ - f^-$ always has the same sign as σ_c (see Figure 1). Alternatively, σ_c will decrease with heliocentric distance—as it does for a linear expansion [*Zhou and Matthaeus*, 1990; *Oughton and Matthaeus*, 1995] or for turbulence strongly driven by velocity shear [*Stribling and Matthaeus*, 1991; *Roberts et al.*, 1992]—if the collective effects of shear, mixing, and pickup ions dominate in equation (16). We plan further examination the conditions for increase or decrease of solar wind cross helicity evolution in the future.

[15] The remaining cross helicity-modified transport equations for Z^2 and λ can now be assembled. Summing equations (14) yields

$$\frac{dZ^2}{dr} = -\frac{Z^2}{r} + \frac{C_{\rm sh} - M\sigma_D}{r} Z^2 + \frac{\dot{E}_{PI}}{U} - \frac{\alpha f^+}{\lambda U} Z^3, \qquad (17)$$

which differs from the nonAlfvénic transport equation for Z^2 only in the modification to the strength of $\alpha \rightarrow \alpha f^+(\sigma_c)$, as in the homogeneous case. (Recall that corrections at order V_A/U are neglected here.) Similarly we can assemble an equation for λ including σ_c effects, in parallel to the steps leading to equation (9), finding,

$$\frac{d\lambda}{dr} = \frac{M\sigma_D - \beta C_{\rm sh}}{r}\lambda - \beta\lambda\frac{\dot{E}_{PI}}{UZ^2} + \frac{\beta}{U} f^+(\sigma_c) Z. \qquad (18)$$

The three transport equations (16), (17), and (18) constitute a generalization to the case of nonzero cross helicity of the previously used pair of equations for Z^2 and λ. They are expected to be useful in computing the transport of MHD turbulence throughout the heliosphere, e.g., in studies of solar modulation of cosmic rays [*Parhi et al.*, 2003].

[16] We conclude by providing in Figure 2 a sample solution, in the ecliptic plane, where the degree of Alfvénicity (cross helicity) is observed to decrease with heliocentric distance [*Bavassano et al.*, 1982a, 1982b; *Roberts et al.*, 1987; *Tu et al.*, 1989a, 1989b]. The parameters used (see caption) are comparable to values employed by *Smith et al.* [2001]. The model results are compared to observational points adapted from *Roberts et al.* [1987]. Evidently the transport theory accounts reasonably well for the observed decrease of σ_c vs. r, providing further support for the suggestion [*Roberts et al.*, 1992] that Alfvénicity is reduced by turbulence driven by shear in solar wind stream structure. Note, however, that the adapted data points are not properly sorted, e.g., by wind speed. Consequently a more careful comparison with observations is needed. The present results also motivate further examination of latitude effects

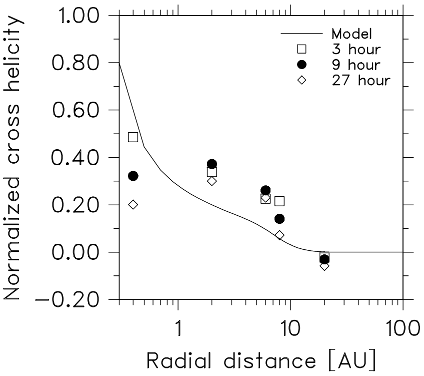

Figure 2. Radial evolution of normalized cross helicity σ_c at low latitudes, near the ecliptic plane. Observational values extracted from Helios and Voyager data are suggested by the symbols, which are adapted from *Roberts et al.* [1987] (courtesy of D. A. Roberts). Also shown is a solution for $\sigma_c(r)$ from the transport equations (see text), using parameters appropriate to low latitudes: shear strength C_{sh} = 1.5, mixing parameter $M\sigma_D = -1/9$, a standard form of the pickup driving term with f_D = 0.25 [see *Smith et al.*, 2001], and Kármán–Taylor constants $\alpha = 2\beta = 1$, with boundary data at 0.3 AU specified as Z^2 = 2000 km^2/sec^2, λ = 0.025 AU, and σ_c = 0.8. The mean magnetic field is taken to be inwards so that $\sigma_c > 0$ corresponds to "outward" fluctuations.

on solar wind cross helicity [*Goldstein et al.*, 1995; *Bavassano et al.*, 2000].

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