Mathematical Investigations: A Primary Teacher Educator’s Narrative Journey of Professional Awareness

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As a teacher educator, I used narrative inquiry to investigate my professional practice in working alongside pre-service primary teachers in mathematics education. One theme that emerged from this research was the exploration of narrative as a powerful means with which to pursue professional development. In this process I encountered, and subsequently changed, previously unknown personal beliefs about learning mathematics. A second theme focused on the value of mathematical investigations, for myself as a mathematical learner and for supporting pre-service teachers to develop their understandings of what it means to learn and teach mathematics.

Introduction

Over a period of 20 months I used narrative inquiry to investigate my professional practice as a pre-service teacher in mathematics education. Undertaking this inquiry was motivated by a desire to more closely examine my professional practice whilst simultaneously meeting the requirements for a Master of Education. Reflection, albeit informally done, has always been an integral part of my teaching practice. However, narrative inquiry provided a more structured vehicle for the ongoing critical reflection of my professional role, and resulted in significant personal learning.

Narrative inquiry, which in essence is a form of story-telling, has become a recognised form of educational research, and is regarded as a powerful means with which learners can reflect on and develop their own professional practice (Chambers, 2003; McCormack, 2002; Rushton, 2001; Smith, 2006). A goal of narrative inquiry is for participants to learn, and possibly change their thinking as a result of this learning (Clandinin & Connelly, 2000). One example of such learning that occurred for me was the discovery of changing beliefs about the nature of mathematics.

The other main theme to emerge during my research centred on the use of mathematical investigations to support pre-service primary teachers to consider what the learning and teaching of mathematics may entail. As part of this process I personally did several mathematical investigations, embedding myself in the position of mathematical learner. My conception of a mathematical investigation is of an open-ended problem or statement that lends itself to the possibility of multiple mathematical pathways being explored, leading to a variety of mathematical ideas and/or solutions. Such investigations tend to take more time than usually encountered in more traditional mathematics problems frequently used in schools.

Narrative Inquiry

Over the past two decades the practice of reflection has been recognised as a legitimate aspect of action research in education (Adler, 1993; Francis, 1995; Schön, 1983). More recently, narrative inquiry has also become a valued form of research (Chambers, 2003; Luwisch, 2001; O’Connell Rust, 1999; Smith, 2006) and can be regarded as a journey
during which researchers come to know more deeply about their lives and who they are as people. Beattie (1995) makes a particularly strong case for the use of narrative inquiry within educational research, writing, “at the heart of meaningful educational reform and change, lie the narratives” (p. 66).

A central tenet of narrative inquiry is that of change (Clandinin & Connelly, 2000). Winter (2003), a writer who draws parallels between basic principles of action research (and thus narrative inquiry) and some key Buddhist doctrines also refers to the condition of impermanence, i.e., change. Although Winter also refers to the human tendency of wishing to avoid change, Mason (2002) cautions that we cannot change others, but rather, we can work at changing ourselves.

One might suppose that because there is an inherent focus on change that there is a supposition of an initial deficit position. However, this is not necessarily the case. An alternative model of emancipatory practitioner research, based on the work of Jacques Lacan, is offered by Brown and Jones (2001). These authors suggest that rather than seeking resolution or an end-point, the research process can be regarded as the building of a narrative layer that supports and grows alongside the writer’s life as it occurs. Thus, perfection or an ideal is not sought, but a greater awareness of one’s professional practice with the possibility of instituting change if that is deemed worthwhile.

In narrative “the subject is never given at the beginning, but it unfolds as the story is told” (Ricoeur, 1986, as cited in McCormack, 2002, p. 337). McLaughlin writes that this is a part of narrative and suggests that one needs to be able to “live with the ambiguity and lack of clarity long enough to formulate a specific focus to research” (McLaughlin, 2003, p. 70). This can be an unsettling process.

Reflection is an integral part of narrative inquiry and is linked to the gaining of new understandings. Reflection can also lead to the discovery of contradictions in one’s writing. Winter (2003) suggests that seeking out such contradictions is a part of the process of narrative inquiry. A useful model for reflection is offered by Korthagen (2004). His model consists of a series of layers which seek to deepen one’s reflection, with the innermost layers including an examination of one’s beliefs.

When writing narrative, different perspectives or interpretations of situations, and the writing, are always possible. Chambers (2003, p. 412) writes, “different perspectives further open up possibilities for engaging in the process of reflection in that they offer specific and sometimes comparable or contrasting points of view”. Wilber (1998) however, warns against the extremes of post-modernism whereby all interpretations would be considered to be equally valid.

The results of narrative research are not definitive statements or generalisations about an aspect of that which is being researched (Adler, 1993; Beattie, 1995; Brown, 2001; Brown & Jones, 2001; Winkler, 2003; Winter, 2002, 2003). McCormack (2002) refers to such research as not providing a “map” but allowing “the reader to witness the process of the story’s construction and its meaning for the storyteller” (p. 337). The readers of such research might then be in a position to tell stories about how the research may connect with their own practice.

The second theme encountered in this research revolves around my story of encountering mathematical investigations, both as a learner and as a pre-service teacher. In line with the literature, I do not have a definitive statement about how to be an educator of pre-service teachers learning mathematics. Rather I share my story and what it means to
me, and this may create the opportunity for readers to make connections with their own stories and/or practice.

Procedure

During the first semester of the 20-month research period, our mathematics education team met weekly to look at what and how we taught in our two compulsory mathematics education papers. During this process it was decided that we would use mathematical investigations with our first year pre-service primary teachers (a cohort of approximately 200 students) to support them to:

1. explore and learn/re-learn some mathematical ideas; and
2. provide a means with which they could look at their attitudes and beliefs about mathematics learning.

All students were informed at the beginning of the semester that their mathematics education lecturers were doing some research that was looking at ways to improve pre-service teacher education in mathematics. They were invited to participate voluntarily in the research, which could involve a number of aspects including: being observed during class sessions at university and when working with a young child in a school; making their journals (which were an essential requirement for one of their assignments) available to form part of the research data; and being informally interviewed. Sixty-one of the 75 pre-service primary teachers (81%) in my classes agreed to be involved.

An investigative approach was a new approach for me. I had never previously taught or learned mathematics in this manner, so I initially had a lot of questions and some concerns. I began teaching three groups with this approach in the following semester. During this time I kept a journal in which I recorded my thoughts, feelings and questions. This continued a writing process (i.e., a narrative) begun in the previous semester.

The pre-service teachers also kept journals of their experiences, thoughts and mathematical thinking as they completed their investigative work. Audio-tapes of five pairs of pre-service teachers’ conversations were collected as they worked on mathematical investigations during class time in the second week of the semester at the beginning of undertaking a two-hour investigation. An observer organised and instructed each student pair in the use of the tape-recorder and then withdrew to a corner of the classroom where she observed the student pairs and the class as a whole.

An informal discussion with a pre-service primary teacher who was struggling with this investigative approach to learning mathematics was also audio-taped and transcribed. This discussion took place after I had become aware of this student’s discomfort both during class and as recorded by the student in a concurrently occurring online-discussion.

I also participated in collegial observation. One colleague observed my teaching of one class during this investigative approach, and I observed two colleagues whilst they were teaching. Notes were recorded during each observation and informal discussions took place after each observation. Reflections on these observations were recorded in my personal journal.

The pre-service teachers’ journals, transcripts of pre-service teacher conversations, observations and the transcript of my discussion with one student were written and collected during the first five teaching weeks of the first year paper. They were analysed shortly after this five-week period and this analysis became part of my ongoing narrative.
After using this investigative teaching approach I explored several mathematical investigations myself. The first of these involved looking at a hypothetical trajectory of a billiard ball on different-sized billiard tables. Following this experience, as a learner using mathematical investigations, I was keen to re-explore the approach in my role of pre-service educator, this time with one class of second year pre-service primary teachers.

To begin I created a number of “stations” where the pre-service teachers were asked to engage in an activity and identify questions that they had about geometry and/or measurement which were stimulated by the activities. I then selected five different investigations that linked to the students’ questions. The pre-service teachers were asked to choose one of these investigations to pursue over a period of approximately 6 hours in class time. They were free to work on their own or with others. The majority of pre-service teachers chose to work with one other person or within a small group.

At the end of that semester I asked if some individuals would be willing to be informally interviewed about their experiences and thoughts regarding participating in mathematical investigations. Francis (1995) suggests that the high profile of reflection in teacher education (such as my narrative inquiry) is only warranted if it impacts on more equitable and just outcomes for pre-service teachers, and ultimately on children’s learning. As such, although I had been informally monitoring the pre-service teacher’s progress and reactions, I believed it was also necessary to hear, in the pre-service teacher’s own words, how they were experiencing this process. This would ultimately also impact on my development as a pre-service mathematics educator. Four pre-service teachers, who had now experienced an investigative approach twice, volunteered to be interviewed and I proceeded to conduct the interviews over the few weeks following our investigative work in class.

Results and Discussion

Re-thinking Mathematics and What it Means to Learn Mathematics

This narrative journey resulted in significant, multi-faceted learning, including reforming my ideas about the nature of mathematics; thinking more deeply about mathematics teaching and learning in general; and more specifically, learning about my own professional practice as a pre-service mathematics educator with particular reference to the use of mathematical investigations.

As Ricoeur (1986, cited in McCormack, 2002) describes, the process of narrative is an unfolding one. As my narrative unfolded I was somewhat surprised to find myself, early in the process, deliberating about the nature of mathematics. Even after eighteen months I was still thinking and struggling with ideas of “what is mathematics?” and do “mathematical truths exist?” I wrote in my journal:

I still struggle with the notion of ‘mathematical correctness or truth’. How do my newer beliefs that mathematics is about “doing” fit with the existence of mathematical rules and proofs? Is it, that a rule or proof only exists in the “doing” or “discovering”. That is, it does not exist without or outside the mathematician, and thus must only be found in the doing? (13/08/04)

Following a discussion with a colleague regarding the issue of the validity of multiple answers in response to a mathematical problem I wrote:

My thoughts are that the answers were all correct (referring to a problem in class) given the differing sets of assumptions or interpretations that each person/group made. Usually these interpretations
have to be the teacher’s and thus teachers (and the children who think in the same way as the
teacher) have been the ones who hold the power. Thus, mathematics has not been accessible to many
people. Learners justifying their answers with their own reasoning relocates the power to the learner
(this does not allow for “shoddy” thinking however). I propose that always defining problems so
tightly as to create only one correct answer does not lead to useful life or problem-solving skills. Nor
does it lead to “real” learning, rather the “game” of “let’s guess what the teacher wants us to do/say
now”, i.e., it is the teacher’s interpretation that matters. Thus accepting multiple interpretations
supports the learner to “really” learn, and creates an expectation of learners making sense of
contradictions and a range of perspectives. (13/08/04)

These excerpts of writing represented considerable changes in my thinking about the
nature of mathematics. As Clandinin and Connelly (2000) suggest, one’s thinking can be
changed by narrative inquiry. I later wrote that I believed that there is not an absolute body
of mathematical truth that exists somewhere as a separate body of knowledge. Rather, that
one’s interpretation and understanding of the context of a mathematical problem
determines the “truth” that may or may not exist within any given context. I went on to
describe mathematics as a sense-making activity (involving discovering and doing)
involving numbers, pattern, shape and space, rather than existing as a predetermined body
of knowledge.

Changes in my Teaching Practice and Beliefs

There were several other changes that also occurred for me during this research. For
example, I initially held concerns regarding whether or not mathematical investigations
would result in mathematical learning. Following my experiences I later embraced the use
of mathematical investigations as one means with which to hopefully initiate and
encourage mathematical learning and reflection with pre-service teachers. There also
appeared to be change in what I “expected” within a mathematics lesson. Whereas I
previously would have wished for a definitive statement of learning about some
mathematical idea, there was more room for students to explore, conjecture and think.

Whilst engaged in the mathematical investigations as a learner, during which time I
was also continuing my journal writing, I discovered I held several subconscious beliefs, all
of which were contrary to what I espoused in the classroom. This discovery and examining
of beliefs illustrates Korthagen’s (2004) suggestion that deepening one’s reflection is a
worthwhile practice. I found that I believed that “real” mathematicians solve problems
quickly, do so on their own, do not get stuck and that there is only one correct
interpretation of a problem. In direct contrast to this, in my teaching I promoted social
constructivist and enactivist theories of learning (Barker, 2001), both of which propose that
learning occurs with other people. One of the readings given to our first year pre-service
teachers stated that it is “honourable” to be stuck (Collier, 1999) and we encouraged the
acceptance of multiple interpretations. To find that I did not really believe any of these
things was an eye-opener to say the least!

Changes in my teaching practice occurred as a result of discovering these unconscious
assumptions. For example, having personally experienced being “stuck” I now believe that
this really is a worthwhile part of the learning process. My practice in the classroom, with
respect to this issue, is now more congruent with what I have espoused for a number of
years. An example of this occurred whilst working alongside first year pre-service teachers
working on an algebra investigation. When they became stuck, rather than rushing in to
“relieve” them I was able to stand back if I judged that to be most helpful, or ask questions
and/or provide hints.
I found the idea, offered by Brown and Jones (2001), that a researcher is not necessarily seeking an “ideal” (e.g., becoming the perfect mathematics educator) welcome as I engaged in the narrative inquiry. However, the process of letting go of reaching for an “ideal” was not a smooth one. Interestingly, when I was first asked to write about “what makes an effective mathematics educator?” I had no problem with setting out what I thought. It was certainly evident that I had a fixed notion of what constituted an effective mathematics educator, and what was needed to reach such an ideal. If I were asked to write in response to the same question now, I am not sure I could. Like Korthagen (2004) who refers to the complexities of what makes a good teacher, I am now much more attuned to the variety and changing range of influences and factors operating in a classroom at any one moment in time.

In narrative “the subject is never given at the beginning, but it unfolds as the story is told” (Ricoeur, 1986, as cited in McCormack, 2002, p. 337). I initially found this aspect of narrative research to be very unsettling. I was sure I should have some predetermined goal or question to be researching. However, the story did unfold, despite my worst fears and enduring resistance that it would not. I now trust the narrative process, and perceive it to be a powerful and liberating one. It was certainly in the ongoing reflection and writing that I came to understand more fully the journey, with the predicting of an outcome being less important – an idea proposed by McCormack (2002). This also links with the writing of McLaughlin (2003) who suggests that the practitioner researcher needs to be able to, “live with the ambiguity and lack of clarity long enough to formulate a specific focus to research” (p. 70). Having done this I relate with McLaughlin’s suggested feelings of confusion, anxiety, frustration, doubt, feelings of inadequacy, and a desire for clarity as the research process unfolds.

**Pre-service Teachers’ Experiences of Mathematical Investigations**

I used an investigative approach with classes of pre-service teachers twice during the research period. On the second occasion I was particularly delighted by most of the students’ engagement. Indeed they chose to present what they had learned to the class (writing the mathematical ideas they had learned onto an overhead transparency and presenting this to the class with demonstrations and models as appropriate) at the end of the six hours. Although their teaching/presenting skills in such situations are still developing it was evident that they certainly had understood various mathematical ideas. This was also apparent when working alongside the pre-service teachers during the 6 hours. For some of the pre-service teachers, some of the mathematics ideas had been encountered for the first time whilst others found they developed an understanding of a particular procedure or idea for the first time. For example, one group of pre-service teachers developed an understanding of why \( \pi \) is equal to approximately 3. One student with whom I had an informal discussion, showed particular pleasure at coming to understand the meaning of \( \pi \), and appeared to have a greater appreciation of mathematics as a sense-making experience rather than an arbitrary set of rules.

Comparable and contrasting points of view provide opportunities for engaging in further reflection (Chambers, 2003). With this in mind, at the end of this semester I interviewed four pre-service teachers. As Chambers (2003) suggests, having different perspectives creates new opportunities for reflection. I found that although some of the pre-service teachers’ experiences resonated with mine, others were different and, I was able to
gain new insights and perspectives about this teaching and learning approach. These included:

1. the need for discussing, in more depth, pre-service teacher beliefs about the learning and nature of mathematics;
2. discussing the place of traditional skill teaching that might occur alongside this approach; and
3. continuing to observe carefully the learning (of mathematics) that is hopefully occurring.

The pre-service teachers were mostly positive about the investigative approach. However it would appear, for two of the four interviewees, that there was an initial period where the process of participating in an investigation was an unfamiliar experience, and created some feelings of discomfort. This was particularly evident for one pre-service teacher who later recognised that he was initially creating “barriers” to the process. This finding, of feelings of discomfort, was comparable with feelings that I too had initially experienced. I wonder what part I may have played in creating these initial feelings of unease, because of my own concerns and discomfort.

Because I now have more experience with this investigative approach as a learner and teacher I believe that I can take this awareness of possible feelings of discomfort into my teaching, and as a beginning point, be able to empathise with students who experience this. Having also experienced the learning that can result from this approach, I believe it to be pertinent to highlight the possibilities of learning that can occur if the student can be encouraged to persevere through these initial feelings of discomfort. Pre-service teacher beliefs about the learning and/or nature of mathematics could also be openly acknowledged and discussed within a supportive environment. It would also appear that using this approach more than once is productive and enables the students to make deeper connections with the issues that arise.

One of the four pre-service teachers that I interviewed described how she found investigations to be less threatening and experienced them as being less pressured compared with her previous experiences in mathematics learning situations. Another described how coming to understand why $\pi$ is equal to “3 and a bit more” was an “a-ha” moment. This appeared to be quite a pivotal experience for him in developing a new enthusiasm for the investigative process. He said:

… like that activity (referring to a practical activity where the value of $\pi$ is discovered) , when we went outside. My thinking was ‘if you want us to go outside, I’ll enjoy some sunshine and that’s about it’. Little did I know that I was going to have an a-ha moment and that was great.

Receiving such positive feedback is certainly encouraging when considering whether or not to continue using this approach with future cohorts of pre-service primary teachers.

All four pre-service teachers stated that they learned some mathematical ideas, or understood a previously learned concept for the first time, by participating in the investigations. This is congruent with my own personal experience. The four interviewees also alluded to a deeper level of learning using this approach. This level of learning could be contrasted with a more traditional approach where a teacher might impart some knowledge (e.g., telling students a piece of information, finding the value of $\pi$, or showing a particular procedure) followed by students practicing numerous examples. One pre-service teacher described her experience of the deeper learning saying:
With the traditional method, I can sometimes see there is [sic] good points to it, but then when we
did that ‘one’, obviously I would’ve been told what \( \pi \) was (referring to her past), but I never
remembered it. So when we started, I thought ‘really, what is it?’ and when I found out, it’ll be in my
head for the rest of my life. I found out for myself.

This pre-service teacher seemed to link this deeper learning with “doing it herself” rather
than being told something. Another described the difference in learning as follows, “it is
learned today, but it was taught in the old days”. This same pre-service teacher stated that
the investigations had “reignited the flame” with respect to her enjoyment of mathematics,
and also referred to the importance of being able to relate previously learned mathematical
ideas to a context. It would certainly seem that for these four that an investigative
mathematical approach had been worthwhile.

One of the pre-service teachers expressed concern about the time taken to learn
mathematical ideas by using an investigative approach. She stated that for her, “it is more
time consuming” and asked the question, “have we got more hours in the day to spend on
maths…?” I too have had that concern. However, based on my experiences both as a
learner and teacher using this investigative approach, I believe that the learning is deeper
and more meaningful and thus warrants the required time. Also, when considering my new
ideas about what the learning of mathematics may entail, I now believe that this approach
more closely captures the essence of what mathematics is actually about, i.e., a process of
making sense of situations involving number, patterns, shape and space, rather than the
finding of a particular answer using a set procedure that someone else has previously
discovered.

The pre-service teachers also perceived their mathematical behaviours to have changed.
For example, they became more thorough in their investigating, and open to the idea that
perhaps multiple interpretations are valid in the learning of mathematics. Some of the pre-
service teachers’ beliefs and/or ideas about teaching mathematics also appeared to change.
Three of the interviewees indicated they would try using an investigative approach when
they begin to teach. One pre-service teacher stated that her thinking:

… has shifted from being formula based mathematics [to] social constructivism … you are interacting
with others, you are using your previous knowledge and ideas and you are experimenting with it. I had
never been allowed to do that with maths before and I enjoyed it.

One pre-service teacher indicated she felt “frightened” that she would be unable to deal
with the possibilities that children might raise within the course of an investigation. She
stated however, that, “I believe I am now preparing myself to work through whatever their ideas are, which I think is really positive”. Once again, I can empathise with this
experience. I too, found this investigative approach to be initially somewhat unsettling with
respect to possibly not knowing the mathematics that might be encountered during the
course of the investigation. Perhaps this is part of a process of moving from viewing
mathematics as a discipline where it is important to know the answer, to an alternative view
of seeing mathematics as a process of doing and discovery. In this alternative view not
knowing the answer would be seen as an exciting and natural part of doing mathematics
and an opportunity for new learning.

It gave me a great deal of pleasure when after the second year practicum one pre-
service teacher returned to show me the results of children’s work done during an
investigation that she had used whilst on practicum. She spoke of children wanting to do mathematics and being disappointed when it was not scheduled for a particular day.

Although I value the experiences and insights that these interviewees shared, I am also aware that there will be other experiences and interpretations of the mathematical investigative approach that are not represented by these four pre-service teachers. It was not my intention in this research to gather quantitative data that is representative of all students, but I believe it is nevertheless important to remain open to the ideas and insights that others may hold. This remains a possibility for further research.

**Conclusion**

Narrative research led to a number of changes in both my beliefs and teaching practice. Personally working on a mathematical investigation was a pivotal point in the journey that led to the discovery, and subsequent change, of previously unrecognised beliefs about learning in mathematics and changes in my thinking about the nature of mathematics. I am now comfortable with the notion that mathematics learning takes time and can be aided by collaboration between students and between teacher and student. I also accept that being “stuck” can be an acceptable and helpful part of learning mathematics; and that multiple interpretations are a valid part of the learning process. Mathematics is now viewed as a sense-making activity, involving discovering and doing.

I believe this has led to changes in my teaching practice. The changes included using this approach knowing that the approach results in mathematical learning, giving students “space” to be stuck, and providing more in-depth interactions to support their mathematical learning. Earlier uncertainty about whether investigations are a useful approach to support the learning of mathematics ideas were at least partially resolved with the positive experiences encountered whilst using mathematical investigations, as a teaching approach and as a vehicle for personal mathematical learning. It was also evident from talking with some pre-service teachers that this approach is a valuable one to engage them to think more deeply about the learning and teaching of mathematics.

Although I have undergone valuable personal learning I do not wish to become a crusader advocating that using mathematical investigations will solve all challenges involved in supporting our pre-service teachers to become more skilled at teaching and learning mathematics. Rather it has been a personal journey that at this point has found mathematical investigations to be a useful learning and teaching tool. A next step would be to explore in more detail the experiences of a greater number of students and to follow their professional progress in an effort to ascertain the value of engaging in mathematical investigations during their pre-service teacher education.

**References**


