Forming Conjectures Within a Spreadsheet Environment

Nigel Calder
University of Waikato

Tony Brown, Una Hanley, and Susan Darby
Manchester Metropolitan University

This paper is concerned with the use of spreadsheets within mathematical investigational tasks. Considering the learning of both children and pre-service teaching students, it examines how mathematical phenomena can be seen as a function of the pedagogical media through which they are encountered. In particular, it shows how pedagogical apparatus influence patterns of social interaction, and how this interaction shapes the mathematical ideas that are engaged with. Notions of conjecture, along with the particular faculty of the spreadsheet setting, are considered with regard to the facilitation of mathematical thinking. Employing an interpretive perspective, a key focus is on how alternative pedagogical media and associated discursive networks influence the way that students form and test informal conjectures.

The study to be described was part of an ongoing research programme exploring how spreadsheets might function as pedagogical media, as compared with pencil and paper methods. As a tool for investigation, we asked, how might the study inform our understanding of the ways spreadsheets filter the learning experience? In particular, we asked how might spreadsheets influence learner's perceptions and understandings of mathematical phenomena? One aspect of this programme, to be pursued here, was to identify the ways in which participants approached mathematical investigations. We explored how they negotiated the requirements of the tasks, and how they produced their conjectures and generalisations.

We commence by outlining the three core themes and some literature upon which these themes are premised. First, we introduce a hermeneutic theoretical perspective in which the process of understanding mathematical phenomena is seen as oscillating between individual encounter and social discourse. Understanding here is considered to be a function of the learner's interpretation and reflection, where such engagement gets fixed as conceptual phenomena. These concepts, however, evolve through further cycles of encounter as understanding develops. This understanding is thus manifest in what students say, and what they do. It is our contention that through an examination of participants' social interaction and output, we will gain insight into the ways students internalise mathematical understandings.

Second, we review literature that underpins our concern with the particular qualities that spreadsheets bring to investigative mathematical processes. This enables us to differentiate patterns in the dialogue and output better, and, as a consequence, pinpoint the influence of spreadsheets on the learner's investigative trajectories. The assumption here is that spreadsheets filter engagement with the mathematical task in particular ways. Spreadsheets may distinguish the style of mathematical activity that results, from the more familiar paper and pencil methods. We explore this possibility with particular regard to how informal conjectures are formulated.

Third, we directly consider the notion of informal conjecture, the conditions that evoke their formation, and how varying the pedagogical media might fashion the social framework in which such positions or generalisations evolve. The data are then assembled about two separate, but related, stories. We used the first to illustrate differences in how investigations are engaged, when encountered through different pedagogical media. The second enabled us to illustrate how this different engagement influences the pathway through an investigation.

The first story compares the pedagogical media of the spreadsheet with pencil and paper approaches. It identifies the unique approach to investigation and generalisation that the spreadsheet lens evokes. Here participants identified several aspects that facilitated the emergence of informal theories or conjectures. These included: framing the investigation in a visual, structured manner; a tendency for more immediacy in response to generalising; and a greater interactive dialogue around such activity.

The second story builds on these characteristics. It also highlights the particular exploratory trajectory that learners tend to gravitate towards, as they negotiate and develop familiarity with mathematical investigations. We reveal how students revised their investigative pathway as the output confirmed or refuted their initial sense making of the situation and how they modified their investigative sub-goals. We show how spreadsheet activity influenced this pathway in particular ways, and how this supports our conjecture that mathematical understanding is a function of the pedagogical media through which it is encountered.

Hermeneutic Understanding

The emergence of social constructivist learning theory in mathematics education research over the last decade or so has, it might be argued, resulted in greater emphasis on inquiry methods, including investigation and problem solving, and a greater promotion of interaction between students. Teachers have increasingly encouraged students to link the content and the processes of mathematical learning. This has placed more emphasis on working in groups, verbalising interpretations of mathematical situations, and negotiating the understandings that emerge. In the New Zealand context, for example, successive evaluative reports of the New Zealand numeracy project (Higgins, 2001, 2003; Thomas & Ward, 2001) have reported that teachers place a greater emphasis on students' explanation of their mathematical thinking. The expectation that students justify their answers was also reported. This sort of approach has activated interplay between the task of the individual learner, and the way in which that is understood as an engagement within a more social frame. Cobb (1994), for example, has highlighted the pedagogical tension between the perspectives of mathematics education being perceived as a notion of enculturation, as compared with one of individual construction and sets out the theories that have been invoked in support of these different understandings. Meanwhile, Brown (1996) has sought to soften the individual/social divide with a phenomenological formulation that emphasises "the individual's experience of grappling with social notation within his or her physical or social situation" (p. 118). The social frame, in this view, is a function of how the activity is constructed and of the perceived environment within which this takes place. And conversely, the mathematical understandings are a function of the social dynamics. The mathematical activity is necessarily a result of how the pedagogical apparatus is constructed and used. Students' understandings are filtered by means of a variety of cultural forms (Cole, 1996). In this context, particular pedagogical media can be seen as cultural forms and different forms model different ways of knowing (Povey, 1997).

The hermeneutic circle presupposes that understanding emerges through cyclical engagement with the phenomenon, and the pervading discourse through which it is contextualised. For instance, we engage in mathematical activity from our mathematical fore-conceptions. This engagement alters our conceptualisation, which then allows us to re-engage from a fresh perspective. This oscillation between the part (the activity), and the whole (the pervading mathematical discourse), enables the understanding. Ricoeur's (1983) notion of the *hermeneutic circle* emphasises the interplay between understanding and the narrative framework within which this understanding is expressed discursively, and which helps to fix it. While these 'fixes' are temporary, they orient the understanding that follows, and the way this comes to be expressed. Likewise, in seeing understanding as linguistically based, dialogue and comment by students will provide the source for the interpretations that we make of students' mathematical understanding (Brown, 2001), in the domains considered in the project. Mathematical conversations and the negotiation of learning, we shall suggest, differ in respect of alternative domains. Thus, learning experiences will differ in quality. As a consequence, the pedagogical media through which the task is engaged will influence the nature of the learning experience; spreadsheets will initiate particular learning qualities.

Spreadsheets as a pedagogical medium

The prevalence of information and communication technology (ICT) media generally, has transformed the way mathematical ideas are encountered in schools. Access to many key elements of school mathematics has been altered as initially calculators, then more advanced software, offered new ways in which certain constructs are created and understood. In geometry, for example, a circle is understood differently according to whether it is constructed using a pencil and compass, a template, Cabri-geometre or LOGO. Studies involving the dynamic geometry software, Cabri-geometre, (Mariotti, 2002; Mariotti & Bartolini, 1998) employ the Vygotskian (1978) notion of semiotic mediation to link technical tools to the process of internalisation. Semiotic mediation is the way in which we learn to assign meaning and to internalise that meaning. A number of studies (e.g., Mariotti, Laborde, & Falcade, 2003) have focussed on the analysis of particular attributes of Cabri-geometre (dragging facility, commands available etc.) as instruments of semiotic mediation that the teacher might utilise to introduce and conceptualise mathematical ideas. In our study, the functionality properties of the spreadsheet (fill down, use of formulae etc.) are considered as potential tools for semiotic mediation of the mathematical concepts of patterning and generalisation. It follows that conceptualisation of mathematical phenomena, will be different when engaged through the particular software lens. Mariotti, Laborde and Falcade (2003) contend, for instance, that a function can be conceptualised differently using Cabri-geometre.

Meanwhile, spreadsheets have been found to offer an accessible medium for young children tackling numerical methods. With the potential to simultaneously link symbolic, numeric, and visual forms, they have been shown to enhance the conceptualisation of some numerical processes (Baker & Beisel, 2001; Calder, 2002). Here visualisation bridges the concrete and abstract manifestations of mathematical experiences. While some mathematicians contend that mathematics itself is evolving through its interaction with computers (Devlin, 1997; Francis, 1996), there is no consensus amongst them regarding this point. Borba and Villareal (2005) argue that ICT emphasises the visual aspect of mathematics, and changes the status of visualisation in mathematics education. The positive role visualisation plays in supporting conceptual understanding is frequently advocated (Bishop, 1989; Dreyfus, 1991; Dubinsky & Tall, 1991), but visualisation has often been considered as secondary, or supportive, of a symbolic, analytical, or algebraic conceptualisation. There is growing evidence, however, that visual reasoning is itself legitimate mathematical reasoning (Borba & Villareal, 2005). In studies (e.g., Julie, 1993; Smart, 1995; Souza & Borba, 2000; Villareal, 2000) involving students using graphic calculators and computer software, ICT mediated the mathematical understanding, and a visual approach to reasoning was identified. The researchers also contend that this visual reasoning, initiated by interacting with the mathematics through an ICT medium, extended students' mathematical conceptualisation: "they employed their visual knowledge to help make generalisations and solve any new problems. In doing so, they extended their mathematics beyond what was expected by the teacher and the textbook" (Smart, 1995, p. 203).

Researchers have identified other benefits that spreadsheets offer within investigative approaches. These include its interactive nature (Beare, 1993), its suitability for linking concepts (Funnell, Marsh, & Thomas, 1995), and its capacity to give immediate feedback (Calder, 2004). Others (e.g., (Ploger, Klinger, & Rooney, 1997) allude to this propensity to foster an investigative approach in developing algebraic thinking. They have found, significantly, that young students learn to pose problems and to create explanations of their own. Unencumbered with numerical computation involving decimal or large numbers, and using formulas in meaningful ways, the young students gained access to the predictive quality of algebraic thinking. This allowed them to pose rich "What if..." questions. Manouchehri (1997) reported similar findings, while Wilson, Ainley and Bills (2004) contend that spreadsheets give opportunities for the conceptualisation of algebraic variables.

Our study illuminated ways that these aspects, coupled with the speed of response to inputted data, appear to give learners the opportunity to develop as risk takers. Students made conjectures and immediately tested them in an informal, non-threatening, environment. This permitted the learners opportunity to reshape their conceptual understanding in a fresh manner. Improved high-level reasoning and problem solving linked with this capability have been reported in more general research into using ICT in mathematics (Baker, Gearheart, & Herman, 1993; Drier, 2000; Sandholtz, Ringstaff, & Dwyer, 1997). The capacity to provide instantaneous feedback also allows for conjectures to be immediately tested and perhaps refuted. The spreadsheet medium supported the investigation in a particular way as this attribute enabled the participants to set, then reset sub-goals, as they worked their way through the investigation. Meanwhile, Lin (2005) claims that generating and refuting conjectures is an effective learning strategy. He describes a deliberate refuting process that involves testing individual examples for sense making. The spreadsheet enables different kinds of examples to be tested, compared and contrasted, within a particular frame.

In the following section we consider the nature of conjectures, how they emerge and offer potential to influence the path of an investigation.

The Notion of Conjecture

Mathematical conjectures often have speculative beginnings and as Dreyfus (1999) implies, have elements of logical guesswork. Researchers often consider them as generalised statements, containing essences distilled from a number of specific examples (e.g. Bergqvist, 2005). They are often contextualised and constrained by defining statements, for which they hold true, unless identified as false conjectures. They can be tested for accuracy by various approaches including abstraction (e.g., algebraic or geometric proof), inference, or counter example. In their embryonic form they emerge as opinions, mathematical statements, generalisations, or positions. These can then be challenged or confirmed with explanation, leading to mathematical thinking. The development of mathematical conjecture and reasoning can be derived from intuitive beginnings (Jones, 1998, 2000). Jones and others (e.g., Fischbein, 1994; Schoenfeld, 1986), contend that deductive and intuitive approaches are not exclusive, but can be mutually reinforcing. While discussing mathematising in a geometrical context, Hershkowitz (1998,) likewise, suggests that visual reasoning is more than just a support, or catalyst for developing a proof. It can underpin the approach taken to generalisation, and be its proof and verification in one process.

Despite summaries of the literature showing that, in general, students do not provide a sound basis for proof, Dreyfus (1999) believes that even primary aged children show the seeds of mathematical reasoning. There are varying degrees of sophistication in the formation of conjectures, as they manifest in dialogue. Building on Chinn and Anderson's classroom discourse model (1998), Manouchehri (2004), described the nature of arguments offered in mathematical discourse; the simplest being an individual stating a position and a supporting explanation without any reflection, either confirmation or challenge, by other group members. More sophisticated forms of conjecture emerged through exchanges relating to the mathematical explanations. Students in the study demonstrated collective argumentation, as they negotiated the meaning of the output produced. Collective argumentation occurs when two or more individuals justify their conjecture through interactive dialogue (Krummheuer, 1995; Yackel, 2002). The study also illustrated how actions, diagrams, and notation function alongside verbal statements in an argumentation (Yackel, 2002). The students used the computer output, and their subsequent actions, to help substantiate their claims.

A more advanced form of conjecture occurs as students offer counter-examples, or when they identify similarities between two mathematical explanations (Manouchechi, 2004). Chi (1997) asserts that such exchanges need not be harmonious, and that arguments refuting others' explanations are effective learning mechanisms. The learner's perturbance, as a result of gaining immediate access to counter-intuitive outcomes to inputted data, can create a tension that might subsequently influence the investigative process. In our project this was illustrated by the data in the second setting. The students reflected on this tension, and through the discussion it evoked, reset their sub-goals (Nunokawa, 2001). We examined the data for signs that the distinct features of the spreadsheet environment were influential in the setting of sub-goals. That is, we looked at how the investigative trajectory was being shaped in a particular way.

Methodological Approach

Our project took place in two settings in a spreadsheet environment, to which the researchers were able to gain ethical access without compromising students' ongoing programmes. The first setting illustrated the differences in the investigative process between the pedagogical media. The second, given that this process was different, illustrated how the investigative trajectory unfolds. The excerpts examined are representative of the data gathered at the respective settings. The first situation located groups of three, first year, primary, pre-service students in a typical classroom setting with counters, calculators and pen and paper available. Meanwhile groups from the same class, worked in an ICT laboratory, doing the same investigation using spreadsheets. Their discussions were audio recorded and transcribed, each group was interviewed after they had completed their investigation, and their written recordings were collected. These data, together with informal observation and discussions, formed the initial basis for the research. Five weeks after the first data were gathered, a similar approach for data collection was used, with the students using the same medium, but undertaking a different investigation.

The second situation involved ten-year-old students, attending five primary schools, drawn from a wide range of socio-economic areas. There were four students from each school, who had been identified as being mathematically talented through a combination of problem solving assessments and teacher reference; eleven boys and nine girls. The students participated in four two-hour sessions, once a week, over four weeks, using spreadsheets to investigate mathematical problems. They received some instruction on using spreadsheets as well as how to use them as a tool to explore the problems. The data were produced in the same way as for the first situation. The transcripts from both were then systematically analysed for patterns in the dialogue, within and between the settings.

Results and Analysis

Two aspects were considered in the formation and testing of conjectures. Firstly, the data were examined for differences the pedagogical media may have evoked, with particular regard to the pre-service teaching students. An episode with the ten-year-old learners was then analysed with regard to the notion of sub-goals. This episode illustrated how the particular characteristics of the spreadsheet setting influenced the way the participants worked through the task. Their investigative trajectory was shaped by the medium through which the task was encountered.

Comparison of Two Pedagogical Media: Situation One.

Are the social discourses different in the two pedagogical settings? We analysed the data to see if distinct patterns emerged in the dialogue. We wondered if differences could be identified that indicated the approach taken varied according to the media employed? In this first situation, we considered their engagement with the following task:

> *Investigate the pattern formed by the 101 times table by:*
>
> Predicting what the answer will be when you multiply numbers by 101.
>
> What if you try some 2 and 3 digit numbers? Are you still able to predict?
>
> Make some rules that help you predict when you have a 1, 2, or 3-digit number. Do they work?
>
> What if we used decimals?

The dialogue in each setting demonstrated a contrast in the initial approach to engaging in the mathematics. In the classroom setting it began with a group member initiating the negotiation of the meaning and requirements of the task with a single discrete numerical example, for example, group one:

> Justin: What if we went one, two, three, four, five, six and multiply it by one hundred and one?
>
> Karl: Let’s try each number one at a time. One times a hundred and one is a hundred and one.

While group two also used this approach to begin the process of solving, the group also facilitated its understanding of the nature of the task and specifically, what the task was asking of them, for example:

Sarah: So if we had twenty three times a hundred you would have twenty three hundred...Lets say we do twenty three times a hundred and one, we would get twenty three hundred plus twenty three ones

Hemi: Does it look right?

Sarah: Yes that is what I would guess it to be. Like if it was eleven times a hundred and one it would be eleven hundreds and eleven ones.

While this is clearly the preamble to the process of forming a conjecture, the students needed to then verify these and other examples before using more recognisable language of generalisation.

Rachel: We went through one at a time and solved them. We solved them on paper and we solved them with a calculator.

In contrast, those groups working in the spreadsheet environment, tended to initially perceive that the bigger picture was most easily accessed through entering a sequential, formulaic structure into the spreadsheet and then visually analysing for patterns, for example:

Kyle: I haven't predicted. I was just going to put in A1 times 101 and drag it down (does it).

Josie: So we're investigating the pattern of 1 to 16 times 101.

This appeared to be a more direct path to the patterning approach, and several comments later this group had recognised a pattern, and explored further based on the rule for their pattern.

Josie: If you did a huge number like five hundred times 101 it would be 500500 wouldn't it?

Kyle: Let's have a look. It's 50500.

Josie: Let's try a hundred times 101.

Kyle: 10100. If you put in 800 it would be 80800.

Their discussion seemed to focus more on the pattern through a visual lens rather than an operational one. That is, the pattern of the digits in the outcome rather than how the numerical operation affects the structure of the outcome. Another spreadsheet group highlighted this aspect of visualising the whole pattern to scrutinise for general qualities.

Rita: 101, 202, 303, 404, and 505 onwards, because it is one times the number. It's straightforward in terms of doing the spreadsheet. It should continue to show that pattern throughout. Drag it down and I think it will probably pick it up.

This also introduced a difference in terms of the technical language utilised. Did this alter the way the students negotiated their informal conjecture or proceeded to analyse it? "Drag it down" is functioning language rather than mathematical, but the inference is clearly that there is a pattern, which might possibly lead to a generalisation. They assumed that the spreadsheet by nature would enable them to quickly access that pattern.

> Josh: Can't we just do it down the column? It should be the top one. A1 multiplied by 101 and then drag it down.

Whether this negotiation of procedures, and the different style of social interactions initiated, changed the approach to the mathematical dialogue is difficult to ascertain. However, considered in conjunction with other aspects, it certainly seemed to lead to a different contextualisation of the mathematical ideas. Most significantly, the social interactions appeared to shape the analysis of the patterns in distinct ways. Given that the path to, and manifestation of, the patterns differs, the dialogue indicated a different approach once the patterns were identified. Those using the spreadsheet used a more visual approach. They were observing and discussing visual aspects, for example, the situation of digits or zeros. For example:

> Rita: With two digits you just double the number. You take the zero out.
>
> David: What about when you get to the three digits?
>
> Rita: Was that 22? So the middle number is still a double?

Those using pencil and paper were more concerned with the operation aspects that generated the patterns, for example:

> Sarah: Basically, if you times your number by a hundred, and then by one, you would add them together, and get your answer.

Generalising a pattern in terms of the sequence of digits is significantly different from generalising in terms of an operation. In this aspect, the different settings have clearly filtered the dialogue. The setting influenced the students' approach to forming conjectures, and by inference their understanding.

The Influence on Sub-goals: SituationTwo

Familiarisation with the problem is a critical preliminary stage that sets the learner on an initial investigative trajectory. This action is not discrete from the solving process however, nor is it necessarily chronologically placed prior to the commencement of that process. Sub-goals are generated as part of the familiarisation and re-familiarisation of the problem, and the location of learning will influence the specificity of their production (Nunokawa, 2002). It is also noteworthy that the characteristic of spreadsheets to produce immediate responses to inputted data assisted the further development of the emerging theory. It facilitated the risk taking aspect of the investigative process (Beare, 1993; Calder, 2002). As well, it led the learner to set a new sub-goal in the investigation promptly. The second situation relates to the following task:

> *Dividing 1 by the counting numbers:*
>
> When we divide 1 by 2, we get 0.5, a terminating decimal.
>
> When we divide 1 by 3, we get 0.33333...., a recurring decimal.
>
> Investigate which numbers, when we divide the number one by them, give terminating, and which give recurring decimals.

Initially, the students negotiated to gain some initial familiarisation of the task.

Sara: One divided by one is one - it should be lower than one.

Jay: Try putting one divided by two, and that should be 0.5

They then entered 1 to 5 in column A and =A1/1 in column B to get:

1	1
2	2
3	3
4	4
5	5.

This posed an immediate tension with their initial thoughts and fostered the resetting of their sub-goal. This was also the beginning of the hermeneutic circle. Sara's pervading school mathematics discourse suggested one output, that it should be less than one, while at the micro level of the investigation the output was greater than one. This oscillation between the macro perspective (the pervading discourse) and the micro (the actual investigation), and the interpretive response that this elicits, occurs within the particular social frame, instigating a distinctive response to the investigation.

Sara: Is it other numbers divided by one or one divided by other numbers?

Jay: Let's recheck.

She entered =A1/4 and got the following output:

1	0.25
1	1.

Jay: Umm, we're not going to get change ... we'll have to change each one.

They appeared to feel intuitively that there should be a way to produce a table of values easily for exploration. The spreadsheet environment was shaping the sense making of the task, and the setting of their sub-goals. Critically, it was enabling them to immediately generalise, produce output, and then explore this visually. They explored other formula, for example, =B1/(4+1).

Jay: Oh now I see =1/A1

They generated the following output:

1	1
2	0.5
3	0.33333...
4	0.25
5	0.2
6	0.16161616...
7	0.1428514285...
8	0.125 etc.

Sara: So that's the pattern. When the number doubles, it's terminating. Like 1, 2, 4, 8 gives 1, 0.5, 0.25, 0.125.

Jay: So the answer is terminating and is in half lots. Let's try that = 0.125/2; gives 0.0625-which is there.
[Finds it on the generated output from above].

The structured, visual nature of the spreadsheet prompted the young students to pose a new conjecture, reset their sub-goal, and then allowed them to easily investigate the idea of doubling the numbers. The table gave them some other information however.

Jay: 1 divided by 5 goes 0.2, which is terminating too. [Long pause].

This created a tension with their most recent conjecture. It required them to reconcile, through an interpretive lens, the output produced with the underlying discourse. After further exploring, they reshaped their conjecture, incorporating their earlier idea.

Sara: If you take these numbers out they double and the answer halves.

Jay: That makes sense though, if you're doubling one, the other must be half. Like 125 0.008; 250 0.004.

Sarah: What's next? Let's check 500

Jay: Let's just go on forever.

They generated a huge list of output, down to over 4260. The nature and structure of the spreadsheet enabled them to seamlessly, yet intentionally, generate large amounts of relevant data, thus fashioning the emerging theory.

Jay: 500 0.002; 1000 0.001.

This indicated the relationship between the numbers that give terminating decimals and the powers of ten. It led to a conjecture couched in visual terms:

Sarah: When you add a zero (to the divisor), a zero gets added after the point *(decimal point). (Our insertions).*

Their conjecture and conceptual understanding evolved through a series of interpretive fixes as the output and subsequent dialogue influenced the setting of their sub-goals. Through the interpretive lens they evoked, their dialogue reflected the oscillation between the ascendant school discourse and the generated output. The following was also recorded on a piece of working paper, as a list of the numbers that produced terminating decimals:

1, ~~2~~, ~~5~~, 10, ~~20~~, 100, 1000

After recording two and five, it appears they noticed that these were factors of ten and subsequently crossed them out. This observation also occurred with the twenty and one hundred. Our interpretation was later verified with the children. They had made sense of, explored, and generalised aspects of the investigation, culminating in the indication of a relatively complex notion of factors and the generalisation process. The pedagogical medium through which they engaged in the task seemed to have influenced the contextualisation and approach they have taken. The children's responses in the interviews, when asked: "When you saw the problem, how did you think you would start?" were consistent with this.

Sara: Re-read to get into the maths thinking, then straight to a spreadsheet formula.

Fran: Thought of a formula.

Greg: I type what I think and try it.

The spreadsheet groups progressed more quickly into exploring larger numbers and decimals. This appears to indicate a greater propensity for exploration and risk taking engendered by the spreadsheet environment. This finding is consistent with other findings (Beare, 1993; Calder, 2002; Sandholtz et al., 1997). It is also clear from the young students' discourse and their responses in the interviews, that the spreadsheets have provided not only a unique lens to view the investigation, but have possibly drawn a distinctive response in terms of investigative practice.

Fran: Using a spreadsheet made it more likely to have a go at something new because it does many things for you. You have unlimited room. You can delete, wipe stuff out.

Chris: Columns make it easier - they separated the numbers and stopped you getting muddled. It keeps it in order, helps with ordering and patterns.

Jay: It helps when you look at patterns. You just type it in and see the whole pattern.

Conclusions

The study demonstrated that varying the pedagogical media provided distinctive responses in the social interaction that contextualised the mathematical ideas, hence framing the construction of informal mathematical conjectures in particular ways. We also contend that this subsequently conditioned the negotiation of the mathematical understanding. As Brown (1996) argued, the mathematical understanding is a function of the social frame within which it is immersed, and the social frame evolves uniquely in each environment. The data supported the supposition that the availability of the spreadsheet led the students to familiarise themselves with, then frame the problem through a visual, tabular lens. It appears that it also evoked an immediate response of generalisation, either explicitly through deriving formulas to model the situation, or implicitly by looking to fill down, or develop simple iterative procedures. The first situation highlighted this, where those students using the spreadsheets produced a table of output quickly, then analysed it for visual patterns. Their dialogue indicates this visual approach to interpretation and it echoes of the visual reasoning discussed in the literature review (Borba & Villareal, 2005; Smart, 1995).

The spreadsheet approach, perhaps due to the actual technical structure of the medium, led more directly to an algebraic process. This is evident from the language interactions that included both algebraic and technical terminology. It seemed, in fact, that the spreadsheet setting, by its very nature, evoked a more algebraic response. The participants in these groups were immediately looking to generalise a formula that they could enter and fill down. Their language reflected this, but the interactions also contained more language of generalisation, and it took them generally less interactions to develop an informal conjecture.

Meanwhile those working in the classroom setting used a discrete numerical example to engage in the problem; both to make sense of its requirements and to initiate the process of solving. They tended to try, confirm with discussion, and then move more gradually into the generalisation stage. Their conjectures were slower to emerge not only due to variation in computational time, but also because of the approach engendered by investigating in the spreadsheet environment. The way they thought about the problem was different. Their initial dialogue seemed more cautious, and contained comments requiring a degree of affirmation amongst group members before moving into developing their conjecture. The social interaction and the process undertaken differed. Their dialogue illustrated the formation of conjectures and generalisation based on their operational approach, rather than visual interpretation.

Clearly, engagement with the task differed in the two environments. The second setting provided an illustration of how the actual investigative trajectory evolved. The students almost immediately entered formula to generate data to help make sense of the problem, as well as generate possible solutions. They indicated the spreadsheet environment evoked that response. Using a hermeneutic process, their pervading mathematical discourse in this domain, enabled them to interact with the mathematical activity. They produced output that was interpreted visually. Tension, arising from differences between expected and actual output, and opportunities, arising from possibilities emerging from these distinctive processes, led to the setting and resetting of sub-goals. These, in turn, further shaped the understanding of the investigative situation, and the interpretation of mathematical conjectures. Their interpretations, from each engagement with the task, influenced their understanding, and enabled them to re-engage with it from a modified perspective. These 'fixes' are expressed discursively, and were illustrated in the students dialogue and output.

The second setting also demonstrated how the intuitive beginnings of the mathematical conjecture, were enhanced by deductive reasoning. They were mutually reinforcing (Fischbein, 1994; Schoenfeld, 1986). The student exchanges, relating to their mathematical explanations, negotiated the resetting of sub-goals, and refined the emerging conjectures. Their collective argumentation, in conjunction with the visual output, led to the formation of generalisations (Yackel, 2002). There was a distinct pathway to mathematical thinking and understanding, induced through the particular pedagogical medium.

The study was limited to two settings and two sets of conditions where students worked within their usual programmes. While this could have limitations for their findings, it also has the potential to enrich them, as the findings remained relatively consistent over these distinct settings. Given that only able mathematical students participated in the second situation, alerts us to the need to treat the generalisation of these findings with caution. Further research would need to be undertaken across a broader range of abilities.

The students identified other general attributes of using spreadsheets that were conducive to the investigative process: speed of response, the structured format, ease of editing and reviewing responses to their generalisations, and their interactive nature. This is consistent with the findings of other research discussed in the literature review (Beare, 1993; Calder, 2004; Funnell, Marsh, & Thomas, 1995).

The data were indicative of an alternative understanding of the process as produced through a different pedagogical medium. As the students' interpretation of, and engagement with, the mathematical phenomena varied with the pedagogical media through which it was encountered, it is reasonable to contend their understanding also varied. This particular medium has unfastened unique avenues of exploration. It has, as a consequence, fashioned the investigation in a way that for some learners may have constrained their understanding. The approaches and outcomes, as reflected in the dialogue, are different, but not necessarily exclusive. If the dialogue between learners shaped the mathematical thinking and formation of conjectures in alternative ways, according to the pedagogical medium, then perhaps the best opportunity to enhance mathematical understanding is through complementary pedagogical approaches.

References

Baker, J. D., & Biesel, R. W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics, 101*(1), 23–31.

Baker, E. L., Gearheart, M., & Herman, J. L. (1993). *The Apple classrooms of tomorrow: The UCLA evaluation studies* (CST Technical Report 353). Los Angeles, CA: Centre for the study of Evaluation/Centre for Technological Assessment, University of California.

Beare, R. (1993). How spreadsheets can aid a variety of mathematical learning activities from primary to tertiary level. In B. Jaworski (Ed.), *Technology in mathematics teaching: A bridge between teaching and learning* (pp. 117–124). Birmingham: University of Birmingham.

Bergqvist, T. (2005). How students verify conjectures: Teachers' expectations. *Journal of Mathematics Teacher Education, 8*(2), 171–191.

Bishop, A. (1989). Review of research on visualization in mathematics education. *Focus on learning problems in mathematics, 11*(1), 7–16.

Borba, M. C., & Villarreal, M.E. (2005). *Humans-with-Media and the reorganization of mathematical thinking: Information and communication technologies, modeling, experimentation and visualisation*. New York: Springer.

Brown, T. (1996). The phenomenology of the mathematics classroom. *Educational Studies in Mathematics, 31*(1-2), 115–150.

Brown, T. (2001). *Mathematics education and language: Interpreting hermeneutics and post-structuralism* (Rev. 2nd ed.). Dordrecht, The Netherlands: Kluwer Academic.

Calder, N. S. (2002). *Can the use of spreadsheets enhance the development of numeracy?* Tauranga, NZ: University of Waikato.

Calder, N. S. (2004). Shaping Understanding: How does investigating in a spreadsheet environment affect the conversations of initial training students doing mathematics. In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Group of Australasia, Townsville, Vol. 1, 143–150). Sydney: MERGA.

Chi, M. T .H. (1997). Quantifying qualitative analyses of verbal data: a practical guide. *Journal of the Learning Sciences 6*(3), 271–315.

Chinn, C. A., & Anderson, R. C. (1998). The structure of discussions that promote reasoning. *Teachers' College Record, 100*(2), 315–368.

Cobb,P. (1994). Where is the mind? Constructivist and socio-cultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13–19.

Cole, M. (1996). *Cultural psychology: a once and future discipline*. Cambridge, MA: Harvard.

Devlin, K. (1997). The logical structure of computer-aided mathematical reasoning. *American Mathematical Monthly, 104*(7), 632–646.

Dreyfus, T. (1991). On the status of visual reasoning in mathematics and mathematics education. In F. Furinguetti (Ed.), *Proceedings of the 15th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 33–48). Assisi, Italy: PME.

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics, 38*(1–3), 85–109.

Drier, H. S. (2000). Investigating mathematics as a community of learners. *Teaching Children Mathematics, 6*(6), 358–362.

Dubinski, E., & Tall, D. (1991). Advanced mathematical thinking and the computer. In D. Tall. (Ed.), *Advanced mathematical thinking* (pp.231–248). Dordrecht, The Netherlands: Kluwer.

Fischbein, E. (1994). The interaction between the formal, the algorithmic and the intuitive components in a mathematical activity. In R. Biehler, R.W. Scholz, R. Straber & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline*. Dordrecht, The Netherlands: Kluwer.

Francis, G. (1996). *Mathematical visualization: Standing at the crossroads*. Conference panel for IEEE Visualization '96. Retrieved September 13, 2006, from http://oldweb.cecm.sfu. ca/projects/PhilVisMaths/vis96panel.html.

Funnell, L., Marsh, T., & Thomas, M. (1995). Strategies for integrating computers into mathematics lessons: Emphasising spreadsheets. *SAME papers* (pp. 223-38). Hamilton, NZ: University of Waikato, Centre for Science, Mathematics and Technology Education Research.

Higgins, J. (2001). *An evaluation of the year 4-6 numeracy exploratory study*. Wellington: Learning Media.

Higgins, J. (2003). *An evaluation of the advanced project 2002*. Wellington: Learning Media.

Jones, K. (1998). Deductive and intuitive approaches to solving geometrical problems. In C. Mammana & V. Villani, (Eds.), *Perspectives on the teaching of geometry for the 21st century* (pp. 78-83). Dordrecht, The Netherlands: Kluwer.

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics, 44*(1–2), 55–85.

Julie, C. (1993). Graphic calculators: Beyond pedagogical and socio-political issues. In C. Julie (Ed.), *Political dimensions of mathematics education: Curriculum reconstruction for society in transition* (pp. 342–347). Cape Town, South Africa: Maskey Miller Longman.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.) *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229–269). Hillsdale, NJ: Erlbaum.

Lin, F. (2005). Modelling students' learning in argumentation and mathematics proof. In H. Chick, H. & J. L. Vincent (Eds.), *Proceedings of the 29th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3–18). Melbourne: PME.

Marriotti, M.A.(2002). Influences of technologies advances on students' math learning. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 695–723). Mahwah, NJ: Erlbaum.

Marriotti, M. A., & Bartolini B. M. (1998). From drawing to construction: Teachers mediation within the Cabri environment. In A. Oliver & K. Newstead (Eds.), *Proceedings of the 22nd annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 180–195). Stellenbosch, South Africa: PME.

Marriotti, M.A., Laborde, C., & Falcade, R. (2003). Function and graph in DGS environment. In N. A. Pateman, B. J. Doherty, & J. Zilliox (Eds.), *Proceedings of the 27th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 237–244). Honolulu, Hawaii: PME.

Manouchehri, A. (2004). Using algebra software to support a discourse community. *The Journal of Mathematical Behavior, 23*(1), 37–62.

Manouchehri, A. (1997). Exploring number structures with spreadsheets. *Learning and Leading with Technology, 24*(8), 32–37.

Nunokawa, K. (2001). Interactions between sub-goals and understanding of problem situations in mathematical problem solving. *The Journal of Mathematical Behavior, 20*(2), 187–205.

Ploger, D., Klinger, L., & Rooney, M. (1997). Spreadsheets, patterns and algebraic thinking. *Teaching Children Mathematics, 3*(6), 330–335.

Povey, H. (1997). Beginning mathematics teachers' ways of knowing: the link with working for emancipatory change. *Curriculum Studies, 5*(3), 329–343.

Ricoeur, P. (1983). *Time and Narrative*. Chicago: University of Chicago Press.

Sandholtz, J. H., Ringstaff, C., & Dwyer, D. C. (1997). *Teaching with technology: Creating a student centred classroom*. New York: Teachers' College Press.

Schoenfeld, A. H. (1986). On having and using geometrical knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: the case of mathematics*. Hillsdale, NJ: Erlbaum.

Smart, T. (1995). Visualisation, Confidence and Magic. In L Burton & B. Jaworski (Eds.), *Technology in mathematics teaching: A bridge between teaching and learning* (pp. 195–212). Lund, Sweden: Chartwell-Bratt.

Souza, T., & Borba, M. (2000). Explorando possibilidades e potenciais limiacoes da calculadora grafica. *Educacao e Matematica, 56*(1), 35–39.

Thomas, G., & Ward, J. (2001). *An evaluation of the Count Me In Too pilot project*. Wellington: Learning Media.

Villareal, M. (2000). Mathematical thinking and intellectual technologies: The visual and the algebraic. *For the Learning of Mathematics, 20*(2), 2–7.

Vygotski L. S. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.

Wilson, K., Ainley, J., & Bills, L. (2004). Spreadsheet generalising and paper and pencil generalising. In M. J. Hoines & A. B. Fuglestad. (Eds). *Proceedings of the 28th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, 441–448). Bergen, Norway: PME.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *The Journal of Mathematical Behavior, 21*(4), 423–440.

Authors

Nigel Calder, Mathematics, Science and Technology Education, University of Waikato at Tauranga, Private Bag 12027, Tauranga, NZ.
Email: <n.calder@waikato.ac.nz>

Tony Brown, Institute of Education, Manchester Metropolitan University, 799 Wilmslow Road, Didsbury, Manchester M20 2RR, UK.
Email: <a.m.brown@mmu.ac.uk>

Una Hanley, Institute of Education, Manchester Metropolitan University, 799 Wilmslow Road, Didsbury, Manchester M20 2RR, UK.
Email: <u.hanley@mmu.ac.uk>

Susan Darby, Institute of Education, Manchester Metropolitan University, 799 Wilmslow Road, Didsbury, Manchester M20 2RR, UK.
Email: <s.darby@mmu.ac.uk>