

## “The Hardness of the Iconic Must: Can Peirce’s Existential Graphs Assist Modal Epistemology?”

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### Introduction:

- My answer to the title question will (perhaps unsurprisingly) be ‘yes’. Specifically looking at *necessary truth*, following Peirce’s definition of mathematics: “**the science that draws necessary conclusions**”.

- *Modal epistemology* – I don’t mean theories which for instance explicate all knowledge as modal. Rather, consider something we arguably know, and which is necessarily true:

**M1: IF three odd numbers are added together THEN the result will also be an odd number.**

- *How* do we manage to know **M1**? Here seeking Good Old-Fashioned Epistemology, e.g. Hume’s skeptical examination of the idea of *substance* in *Treatise* (1, vi): every idea has to come either from sensation or reflection, the idea of substance can’t come from sensation as it is not seen, heard, smelt or tasted...and can’t come from reflection as “**the impressions of reflection resolve themselves into our passions and emotions: none of which can possibly represent a substance.**” Conclusion: no knowledge about substance.

- Modal epistemology relatively ill-explored in analytic philosophy, compared to the metaphysics of modality.

### Hume and his heirs: the contingency of experience:

- Hume (and empiricism downstream from him): physical objects are *observable*. Necessity is *not observable*:

**I consider, in what objects necessity is commonly suppos'd to lie; and finding that it is always ascrib'd to causes and effects, I turn my eye to two objects suppos'd to be plac'd in that relation...I immediately perceive, that they are contiguous in time and place, and that the object we call cause precedes the other we call effect. In no one instance can I go any farther, nor is it possible for me to discover any third relation betwixt these objects. (*Treatise*, III, xiv)**

- Such necessary truths as there are merely consist in *relations between ideas*. Apart from trivial definitional truths (e.g. “All bachelors are unmarried”) this covers only mathematics. And in fact *even here* the early Hume claims skepticism:

**...the ideas which are most essential to geometry, viz. those of equality and inequality, of a right line and a plain surface, are far from being exact and determinate, according to our common method of conceiving them. Not only we are incapable of telling, if the case be in any degree doubtful, when such particular figures are equal...but we can form no idea of that proportion, or of these figures, which is firm and invariable. Our appeal is still to the weak and fallible judgment, which we make from the appearance of the objects, and correct by a compass or common measure; and if we join the supposition of any**

**farther correction, 'tis of such-a-one as is either useless or imaginary...As the ultimate standard of these figures is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of...**(*Treatise*, II, iv)

- By this full-fledged assault on our knowledge of the necessary, Hume takes himself to abolish much worthless metaphysics.
- Hume's epistemological methodology is (logically) *atomist* – all our knowledge is built up (in principle) from *impressions* and the *ideas* they generate.

### Metaphysical necessity rides again

- Fast-forward to C20th. It seems in fact not all necessity can be reduced to what is 'true by definition'. Some is *discovered*. E.g. 'Water is H<sub>2</sub>O'. New category of *a posteriori* necessity (a.k.a. '*metaphysical necessity*').
- This is described as *a posteriori*, as it was scientists who discovered the chemical formula of water. However strictly speaking the scientists *did not discover that the statement is necessarily true*. This is attributed by philosophers, initially by consulting 'intuitions' (about an imaginary planet, Twin Earth).
- When such intuition came to appear rather under-theorised, modal epistemology began to be spelt out further in terms of *conceivability*.
- Yablo (1993) "**A proposition p is conceivable iff one can imagine a world that one takes to verify p**". (Then a proposition is possibly true if it is conceivable. It is necessarily true if it is not conceivable that it not be true.)
- However this imagining is to be distinguished from *imaging*.... (it is a more rationalist process). Just what it is is a little vague (Van Inwagen – actually it is question-begging → modal skepticism)
- Chalmers (2002) follows Yablo (with some extra bells and whistles) but also down on imaging (distinction between *perceptual imagination* and *modal imagination*...).

### My argument

**Key Hypothesis:** Structural articulation is the source of all necessity. Necessary reasoning is in essence just a recognition that a certain structure has the particular structure that it in fact has.

- Anne Newstead distinguishes 3 positions re. the role of diagrams in mathematical reasoning: **i)** of essential value, **ii)** mere heuristic aid, **iii)** harmful. I am arguing for **i)**.
- To do this it is helpful to draw on a Peircean 3-way distinction between kinds of sign:

### Peirce: Icon, index, symbol

These 3 fundamental kinds correspond to 3 ways in which a sign can pick out its object:

- **Icon:** by *possessing the quality* signified (e.g. maps, mathematical diagrams).
- **Index:** *via some direct unmediated relationship* (e.g. smoke/fire, "this", "∃x").

- **Symbol**: *via* some kind of (further, independent, arbitrary) *convention or rule* (e.g. “cat”, a red traffic light).

- *Icons* are the only signs whose signification (can) have *internal structure*. An *index* is just a pointer, and the convention by means of which the *symbol* symbolizes what it does is external to it. The perspicuous representation of relations *via* structural resemblance is one of the icon’s greatest strengths. (Another of Peirce’s definition of the icon: an icon’s *parts* should be related *in the same way* that the objects represented by those parts are themselves related.)

- Symbols signify **general concepts**. Indices signify **particular things**. Icons signify **neither**, what Peirce calls **a pure dream**... This is just a way of saying that for icons, the relationship between sign and object is closer than for the other 2 types. Also, only when icons and indices are embedded in symbols does one get a *true or false claim*.

- But can’t a distorted photo of Obama ‘falsely state’ that that is how he looks?

- *Qua* picture, the photo just *is what it is*. To interpret it as saying something false about Obama is to: **i)** peg it to a real-world object (*index*), **ii)** claim something general about that object’s appearance (*symbol*).

- Most real-world signs are a mixture of these 3 (e.g. ‘I’ is index insofar as it indicates its speaker, and symbol insofar as we learn it’s the word ‘I’ that plays this role. Real-world arguments draw on all 3 sign-types. E.g.:

**The Bible says Enoch and Elijah were lifted by God up to heaven.**

**Therefore, either the Bible can be wrong, or not all men are mortal.**

**“What the Bible is, and what the historic world of men is, to which this reasoning relates, must be shown by indices.”...**

**“The reasoner makes some sort of mental diagram by which he sees that his alternative conclusion must be true, if the premise is so; and this diagram is an icon or likeness....”**

**“The rest is symbols...” (Peirce, “What is a Sign?”)**



Can you write that down for me?

### Achilles and the Tortoise (a la Lewis Carroll)

- Despite Barry Stroud’s opinion that “...**there is no sound moral to be drawn from the story about the nature of validity or logical consequences as such**” (“Inference, Belief and Understanding”, p. 179), to my mind this piece highlights brilliantly both the existence and the puzzlement of what I’m here calling ‘the hardness of the iconic must’:

- *The tale*: The two mythical racers contemplate Euclid’s #1 geometric proof:

**(A) Things that are equal to the same are equal to each other.**

**(B) The two sides of this Triangle are things that are equal to the same.**

**(Z) The two sides of this Triangle are equal to each other.**

The Tortoise asks Achilles what he would say to someone claiming to accept (A) and (B), but not (Z), how he might “**force him, logically, to accept (Z) as true.**” Achilles has

surprising trouble with this. He devises another conditional to express what he sees as manifestly true and missed by the Tortoise:

**(C) If (A) and (B) are true, (Z) must be true.**

Interestingly, the Tortoise asks him to *write (C) down*. The Tortoise then asks what difference this writing on the paper makes to what he should *do* (specifically re. inferring (Z)), even if he accepts the truth of (A) and (B). Achilles is forced to resort to:

**(D) If (A) and (B) and (C) are true, (Z) must be true.**

The Tortoise of course asks him to write down (D), and then refuses to act on it...and so on to infinity.

- What is the moral here? What is the Tortoise missing?
- The Tortoise is arguably rhetorically highlighting a *gap* between a *logical diagram* and *one's seeing how it should be used*, by refusing to see and use it himself. (Interesting links here to Wittgenstein's rule-following problem).
- What kind of gap is this? A 'logical gap'? A 'practical gap'? And what might close it?
- Arguably, what the Tortoise fails to 'see' is *a structural isomorphism shared by a sign and an act*. A little mysterious perhaps, but the Achilles and the Tortoise story shows how ubiquitous and fundamental it is. It is this structural isomorphism that shows that the 'logical must' is iconic. So *how* do perceived structures lead to actions? (Deep issues about normativity at this point...)
- Here one might define an *internalism* about logic, analogous to the metaethical notion, whereby if one is not motivated to *act* by logical norms, one does not fully understand them. But then the question becomes – whence such internalism?

### Brandom: 'Positive Logical Freedom'

- One of the few contemporary philosophers to explicitly face this issue is Brandom. He writes **"...the most urgent philosophical task is to understand...the bindingness or validity...of conceptual norms"**.
- The key is a Kantian concept of *positive freedom*. **"Before Kant, freedom had traditionally been understood in negative terms: as freedom from some kind of constraint. He revolutionized our thought by introducing the idea of positive freedom: freedom to do something."** (i.e. to do something that you could not do before, even though no-one would have stopped you).

Such freedom relies on 2 apparently opposing forces which in fact work together:

- One makes individual choices to act
- One surrenders to norms of ethics which appraise those acts as right or wrong

**"...autonomy, binding oneself by a norm, rule, or law, has two components, corresponding to 'autos' and 'nomos'. One must bind oneself, but one must also bind oneself. If not only that one is bound by a certain norm, but also what that norm involves...is up to the one endorsing it, the notion that...a distinction has been put in place between what is correct and incorrect according to that norm"**

goes missing.” (Brandom, “Kantian Lessons about Mind, Meaning and Rationality”, pp. 12-13).

Together these result in a ‘bonanza’ of positive freedom – to live in civil society, buy things on Trademe...and so on. Similarly in the *realm of logic*:

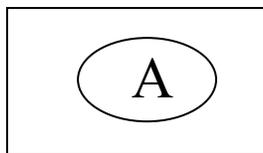
- One makes individual assertions
- One surrenders to norms of logic
- But this results in a “**bonanza of positive expressive freedom**” – a freedom to say things which would be impossible without such surrender (in fact Brandom points out “...almost every sentence uttered by an adult native speaker is radically *novel*.”)
- Only disagree with Brandom in Hegelian twist he puts on all this (p. 16 “**what maintains** [the norms of logic] **is the attitudes of others...**”). To me, the fact that these necessities are *seen* puts a more realist face on it. Something is going on here which is *more enigmatic than conventionalism* (also recall, we saw convention = *symbol*. We are here considering the *icon*.)

### Peirce’s Existential Graphs

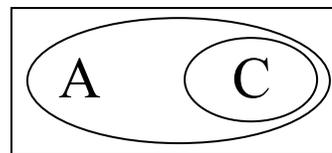
[The] purpose of the System of Existential Graphs ... [is] to afford a method (1) as *simple* as possible (that is to say, with as small a number of arbitrary conventions as possible), for representing propositions (2) as *iconically*, or *diagrammatically* and (3) as *analytically* as possible.... (Peirce, *Collected Papers*, §4.561n)



A & C



~A

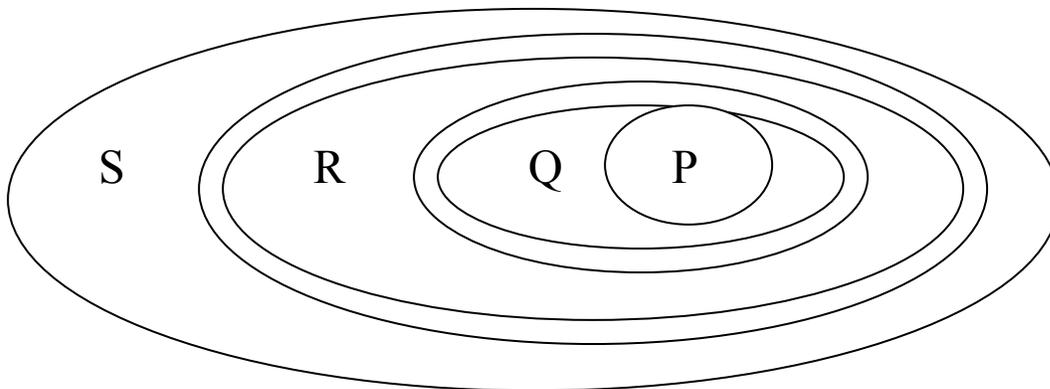


A ⊃ C

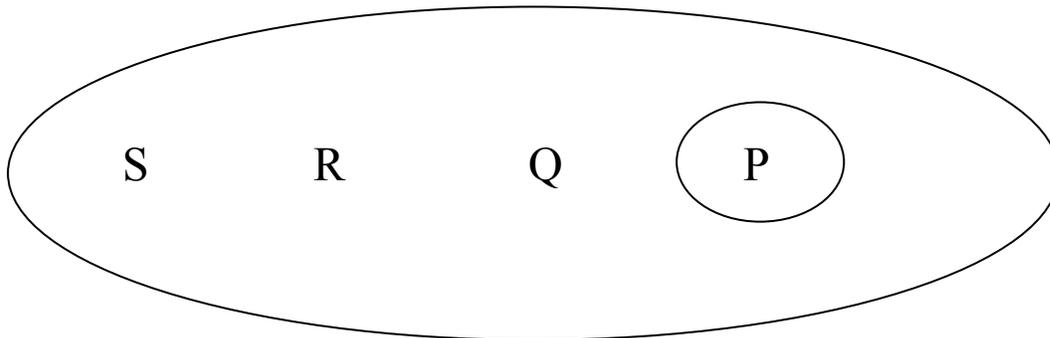
The following formulae are all logically equivalent:

- i) S ⊃ (R ⊃ (Q ⊃ P))
- ii) S ⊃ ((R & Q) ⊃ P)
- iii) (S & R) ⊃ (Q ⊃ P)
- iv) (S & R & Q) ⊃ P

By contrast to algebraic logic, in Peirce’s existential graphs their equivalence can be *seen*. For example if we graph i), we get:



However ‘double cuts’ are *visibly equivalent* to double negation and accordingly may be removed:



This clearly represents:  $(S \ \& \ R \ \& \ Q) \supset P$ .

- When one aspect of a diagram forces another aspect to be a certain way, this enables us to ‘see’ necessity.
- The more this can happen, the more *perspicuous* the formalism is (in other words, *qua* our initial definition of the icon, the more its *parts* are related *in the same way* that the objects represented by those parts are themselves related).
- Even algebraic logic when used correctly forces us to only prove valid theorems (in a sound system). *It is thus irreducibly iconic also*. It is just that it contains *more arbitrary rules* which must be learned. Thus the phrase ‘*symbolic logic*’ is a bit of a misnomer. In Peircean terms the true distinction is between *algebraic* and *graphical* logics and the distinction merely concerns how *perspicuous* the icons are.
- But surely we could devise conventions that would allow us to represent, say, the rules of algebra symbolically (e.g. call modus ponens ‘Fred’)?
- Peirce: we could, but any coherent usage is parasitic on a prior iconic understanding.
- Early Wittgenstein with his ‘Picture Theory of Meaning’ grasped this vision of iconic perspicuity in characteristically pure (and impractical / OTT?) form:

**Symbols are not what they seem to be. In “aRb” “R” looks like a substantive but is not one. What symbolizes in “aRb” is that R occurs between a and b. . . .** (*Notebooks 1914-16, p. 98.*) (*Wittgenstein claims this quote disproves ‘the reality of relations’*)

[E]very theory of types must be rendered superfluous by a proper theory of symbolism. (*Notebooks 1914-16*, p. 121)

### Conclusion:

*Necessity is observable.* You just need to *pay attention*, not just to individual things but to how those things are related in larger structures:

**It has long been a puzzle how it could be that, on the one hand, mathematics is purely deductive in its nature, and draws its conclusions apodictically, while on the other hand, it presents as rich and apparently unending a series of surprising discoveries as any observational science. The truth, however, appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts. (Peirce, *Collected Papers*, §3.363)**

- In other words, contrary to what Hume suggested, empiricism doesn't stop you from 'seeing' necessity. Only logical atomism does.
- Modern analytics wrong to disdain '*imaging*' as a faculty to guide modal epistemology. We just need a more general concept of what this might consist in.

**GREAT QUESTIONS** (that came up in question-time. Apologies to the askers of these questions if I have mis- or under-represented them.):

**Q1)** Can't icons be misleading? You can make a picture of something which is not true.  
**A1)** There is a crucial distinction in signification between i) terms, ii) propositions, iii) arguments. Peirce taxonomises it thus:

	Term	Proposition	Argument
<b>Determines Information to be Conveyed</b>	No	Yes	Yes
<b>Determines Inferences to be Drawn</b>	No	No	Yes

Necessity, being forced to believe something, only appears at level iii).

I guess this commits me to saying that insofar as 'water = H<sub>2</sub>O' is necessarily true it has to be (brief and schematic) argument. (Or not in fact be necessarily true). Need to investigate this more...

**Q2)** I don't see how a picture can be a *proof*. This is because a proof needs to have *steps* and to lead the mind through a process which starts at the premises and ends at the conclusion. Any mathematical diagram contains many, many different relations and interconnections and it is the mind's choice which to focus on. **A2)** Good point. I think the answer might be that a diagram itself is not a proof but a proof is the mind's *use* of a diagram. My point in this paper is just that every proof needs an explicit or implicit diagram, and that still holds.

**Q3)** This could be somehow related to Kant's categorical imperative.... **A3)** Agree – but how, exactly?

