Experimental determination of natural convection heat transfer coefficients in an attic shaped enclosure

T.N. Anderson¹,²*, M. Duke² and J.K. Carson²

¹School of Engineering, Deakin University, Pigdons Road, Geelong VIC 3217, Australia
²Department of Engineering, University of Waikato, Hamilton 3240, New Zealand

Abstract
Heat transfer by natural convection in triangular enclosures is an area of significant importance in applications such as the design of greenhouses, attics and solar water heaters. However, given its significance to these areas it has not been widely examined. In this study, the natural convection heat transfer coefficients for air in an attic shaped enclosure were determined for Grashof Numbers over the range of $10^7$ to $10^9$. It was found that the measured heat transfer coefficients could be predicted to within 5% by Ridouane and Campo’ [1] equation (Eqn. 1) for natural convection in a triangular enclosure previously developed for Grashof Numbers in the range $10^5$ and $10^6$.

$$Nu = 0.286 A^{-0.286} Gr^{1/4}$$ (1)

As such, it is suggested that this equation may be suitable for predicting the natural convection heat transfer coefficients in full scale attic enclosures.

Keywords: triangular enclosure, heat transfer, attic, natural convection

1. Introduction
The problem of heat transfer due to natural convection in triangular or attic-shaped enclosures is quite widespread and has application to buildings, solar collectors and greenhouses. However, unlike rectangular, square and cylindrical enclosures, it has received relatively little attention.

It should be noted that the problem has not been completely ignored; in fact many of the studies undertaken in the area have relied upon the use of computational fluid dynamics to further the

* Corresponding Author: Tel.: +61 3 5227 3403; Fax: +61 3 5227 2028; E-mail address: timothy.anderson@deakin.edu.au (T.N. Anderson)
understanding the flow and heat transfer in triangular enclosures, although there are relatively few experimental studies.

In their computational study Akinsete and Coleman [2] found that for a right triangular enclosure with a cooled base, heated inclined wall and adiabatic vertical side that there was an increase in heat transfer near the apex of the heated and cooled side. In this study they observed a single convection cell as did Poulikakos and Bejan [3].

However, in a separate computational study Poulikakos and Bejan [4] found that at higher Rayleigh numbers a Benard-type instability resulted in multiple cells forming in their enclosure. They found that the formation of these cells was related to the aspect ratio of the right triangular enclosure, as did Asan and Namli [5, 6] and Aramayo et al [7, 8].

The flow behaviour and presence of convection cells has also been discussed in recent times for isosceles triangular enclosures, similar to those formed by pitched roofs. Holtzman et al [9] showed computationally that the flow in such enclosures was asymmetric and undertook flow visualisation to validate this conclusion. A similar phenomenon was computationally observed by Ridouane et al [10].

Given the large number of computational studies that have examined natural convection in triangular enclosures, there is a distinct lack of generalised correlations to predict the heat transfer in these spaces [11-13]. Al-Shariah and Ecevit [14] were perhaps the first to present a truly generalised equation for heat transfer in a triangular enclosure. In their study they examined a right triangular enclosure with one heated and one cooled side and found that the heat transfer could be expressed as a function of the Rayleigh number, the enclosure aspect ratio and the inclination of the cavity to the horizontal.

More recently Lei et al [15] conducted an experimental study of a base heated isosceles triangular enclosure and presented correlations to describe the heat transfer from the base and inclined sides for Grashof Numbers in the range from $2.95 \times 10^4$ to $1 \times 10^6$.

However the most notable experimental work for isosceles triangular enclosures is that of Flack et al [16] and Flack [17]; in these studies one side of an isosceles triangular enclosure was heated while the other was cooled and the base was treated as being adiabatic. They performed a number of experiments for varying aspect ratios and also Grashof Numbers in the range $2.9 \times$
10^6 to 9 x 10^6. Using their data they subsequently developed a series of correlations for predicting heat transfer in such enclosures.

Recently, Ridouane and Campo [1] re-examined the data from Flack’s work and presented their results as a single generalised correlation expressed in terms of the enclosure Grashof number and aspect ratio. One key shortcoming of this correlation is that it is based on data from a relatively narrow range of Grashof numbers. In two recent studies, Anderson et al [18, 19] suggested using the low natural convection heat transfer due to air inside an attic space as a means of passively insulating the rear surface of a Building Integrated Photovoltaic/Thermal (BIPVT) Solar Collector. However, for large attic spaces in a tropical or warm climate such as those of Australia or New Zealand, the Grashof numbers could exceed those previously studied by several orders of magnitude, with values of up to and possibly exceeding 10^10.

In light of the lack of experimental data relating to higher Rayleigh and Grashof number flows, it was decided to experimentally examine the issue of natural convection in an isosceles triangular enclosure and test the validity of Ridouane and Campo’s [1] correlation over a wider range of Grashof number conditions.

2. Experimental Method

To determine the heat transfer by natural convection it was necessary to build an appropriate experimental apparatus. For this study, the aim was to examine the applicability of Ridouane and Campo’s [1] correlation of Flack’s [16, 17] work at higher Grashof number conditions. As such the scale of the enclosure was increased from 152.4mm x 76.2mm x 254mm in the previous studies to an enclosure 707.1mm x 353.6mm x 1500mm as a means of increasing the characteristic length and therefore the Grashof number.

In order to create a temperature gradient within the enclosure, a flexible aluminium foil resistance heater (750W nominal) was attached to a 2mm aluminium plate and used as one of the inclined sides of the enclosure (Figure 1). The aluminium plate, having a high thermal conductivity, was used to maximise temperature uniformity on the heated surface inside the enclosure (an examination of the temperature data found that the temperature variation was less than 5% across the surface of the aluminium plate). To minimise heat loss from the rear surface of the heater, it was insulated with approximately 100mm of mineral wool fibre insulation with a nominal R-value of 2.2 and backed by a sheet of 20mm plywood.
Similarly, the base of the enclosure was made from 100mm of the mineral wool fibre insulation between two layers of 20mm plywood. The ends were fabricated from a single layer of 20mm plywood. The second inclined surface was constructed from 2mm aluminium plate to ensure that there would be minimal thermal resistance across this surface. Additionally, the inclined aluminium plate was cooled by a fan providing a free stream velocity of approximately 4 m/s. This step was taken to ensure that the majority of the heat transfer away from the enclosure occurred from this surface. Finally, to reduce unwanted heat loss, all edges of the enclosure were sealed with high temperature duct tape to ensure that no air leakage from the enclosure could occur.

In order to vary the Grashof number, it was necessary to vary the temperature difference across the enclosure. In order to achieve this, the power being supplied to the electrical resistance heater was varied using a variable power transformer (Variac) and was measured using a single phase EMU Elektronik power meter.

To determine the temperature gradient occurring from the heated side to the cooled side of the enclosure, the mean temperature of the heater was measured using a series of 6 copper-constantan (T-type) thermocouples that were placed uniformly over the heater surface, while a seventh thermocouple was used to measure the ambient temperature. Before undertaking the measurements these thermocouples were calibrated against a platinum resistance thermometer and were found to be accurate to within ±0.3K across the temperature measurement range. Subsequently, the thermocouples were connected to a Picolog TC-08 eight channel thermocouple DAQ system and recorded by a computer via the USB interface. The configuration of the power and temperature measurement system and instrumentation is illustrated schematically in Figure 2.

Finally, to accurately determine the heat transfer coefficient within the enclosure it was necessary to allow the system to reach steady state conditions. To do this, the heater power was set and the ambient and heater temperatures were monitored. When the variation of the temperature difference between the heater and the ambient was not more than 0.6K over a 30 minute period, it was assumed that the system had reached a steady state, as shown in Figure 3. Subsequently, the readings taken during this period were used to determine the natural convection heat transfer coefficient.
3. Analysis

From the experimental measurements it is relatively straight-forward to determine the overall heat loss coefficient for the experimental rig. Under steady state conditions the rig heat loss coefficient \((U)\) is represented by a function of the input electrical power \((Q_e)\) the difference in temperature between the heaters mean \((T_p)\) and the ambient temperature \((T_a)\) and the heater area \((A_p)\), as shown in Eqn. 2.

\[
U = \frac{Q_e}{A_p(T_p - T_a)} \quad (2)
\]

Although this provides a crude estimate of the heat transfer coefficient inside the enclosure, it is necessary to undertake a full heat balance to identify the heat that is being transferred by convection. For this study the heat balance could be given by Eqn. 3: where \(Q_e\) is the electrical input power, \(Q_{end,cond}\) is the heat loss though the ends of the enclosure by conduction based on the mean enclosure temp \((T_e,\) taken as the average of \(T_p\) and \(T_a\)), \(Q_{back,cond}\) is the heat loss from the heater by conduction through the insulation on its rear surface, \(Q_{base,cond}\) is the heat loss through the base by conduction based on the mean enclosure temp and \(Q_{rad}\) is the radiation heat loss.

\[
Q_e - Q_{end,cond} - Q_{back,cond} - Q_{base,cond} - Q_{rad} = Q_{\text{convection}} \quad (3)
\]

The remaining term, \(Q_{\text{convection}}\) is the heat transferred from the heated to cooled inclined wall by convection. In reality this term is the sum of the thermal resistance due to natural convection from one side of the enclosure to the other, the conduction resistance through cold aluminium plate wall and the external forced convection away from the cold plate. However, the thermal resistance of the aluminium wall and the external forced convection are relatively small and as such these terms can be neglected (testing showed that neglecting these terms resulted in less than a 3% increase in the internal heat transfer coefficient).

The calculation of heat loss by conduction through a plane wall (i.e. \(Q_{end,cond}, Q_{back,cond}, Q_{base,cond}\)) is a relatively simple application of Fourier’s law (Eqn. 4) over the area \((A)\) of the wall of interest. Where \(k\) is the thermal conductivity of the wall and \(L\) is the thickness of the wall or alternatively \(R\) (in m\(^2\)K/W) is the thermal resistance of the wall.
The radiation heat transfer in an enclosure of this nature is somewhat more complicated. However it was recognised that for this case, heat transfer by radiation between the heated and cooled sides is effectively negligible, given that both inclined sides were made from aluminium plate \(\varepsilon_p \approx 0.06\). Furthermore, the base of the enclosure is essentially adiabatic and therefore could be discounted from any radiation calculations. Finally the view factor from the heated surface to the end walls for a cavity of this size are relatively small (less than 0.3) [20] as is their area \((0.125\text{m}^2)\), thereby reducing the amount of radiation heat transfer.

To verify that radiation could be neglected, it was assumed that radiation heat transfer would only occur between the wooden ends (where there was the least insulation), and the aluminium heater. It was assumed that the radiation heat transfer could be treated as being analogous to having two parallel plates, essentially representing a “worst case” scenario. Under these conditions it was assumed that the equation for the radiation heat transfer could be simplified to Eqn. 5 for a two surface enclosure where the two surfaces are: the heater of area \((A_p)\) made of aluminium with emissivity \(\varepsilon_p\), and the other made of wood with emissivity \(\varepsilon_w\) and an area \((A_w)\) equal to the sum of the areas of the ends and base of the enclosure and where the view factor \((F)\) between the plates is equal to unity [11].

\[
Q_{\text{rad}} = \frac{\sigma(T_h^4 - T_c^4)}{1 - \varepsilon_p^2 + \frac{1 - \varepsilon_w^2}{A_p} + \frac{1}{A_p F}}
\]

From the analysis of \(Q_{\text{rad}}\) it was found that, based on the assumptions listed above, that radiation heat transfer accounted for less than 5% of the heat being transferred and so was assumed to be negligible. Similar calculations showed that using this conservative approximation of the radiation between the hot and cold sides of the enclosure accounted for less than 1% of the heat transfer. It should also be noted that the radiation and external convection term, discussed earlier, act in opposite directions and are of similar magnitude which essentially means that they cancel each other. Hence, based on the energy balance, and the outlined assumptions, it was possible to determine the heat transfer by convection between the heated and cooled surfaces from Eqn. 6.
\[ h_c = \frac{Q_{\text{convection}}}{A_h (T_p - T_a)} \]  \hspace{1cm} (6)

4. Results
Having established an appropriate experimental methodology and means of analysing the natural convection heat transfer in the enclosure a number of heat transfer coefficient measurements were performed.

In this study the power was varied such that the mean heater temperatures were between 30°C and 120°C under steady state conditions. By recording the steady state data it was then possible to determine the heat transfer coefficient using Eqn. 6. For this study, the range of plate temperatures equated to Grashof Numbers in the range of $10^7$ to $10^9$, where the characteristic length was taken to be the vertical height of the enclosure \[1\] and the physical properties were taken at the mean enclosure temperature.

In Figure 4, it can be seen that with an increasing temperature gradient in the enclosure, that there is an increase in the natural convection heat transfer coefficient. This is characteristic of natural convection flows, as at higher temperature gradients there is a higher degree of turbulence and therefore heat transfer, and also the buoyancy gradients are higher.

Although this demonstrates that the heat transfer coefficient increases for increasing temperature gradient it is not a dimensionless correlation and so cannot be readily translated to enclosures of a different size. This is readily overcome by translating the data into a non-dimensional form. In Figure 5 it can be seen that there exists a relationship between the Nusselt number and Rayleigh number that can be represented in the general form of Eqn. 7 and more specifically as shown in Eqn. 8.

\[ Nu = cRa^n \]  \hspace{1cm} (7)

\[ Nu = 1.33Ra^{0.2} \]  \hspace{1cm} (8)

This is typical of the relationship that exists for turbulent natural convection where an exponent value of $n = 0.2$ would commonly be used for surfaces of constant heat flux, as was the case here [21].

A key shortcoming of Eqn. 8 however, is that it is based on a relatively narrow range of Grashof (or Rayleigh) numbers and does not account for aspect ratio. As such, it was decided to examine
how well an existing correlation (Eqn. 9) from Ridouane and Campo [1] that accounts for aspect ratio could predict the heat transfer in the experimental enclosure, although was based on data over a relatively narrow range of Grashof numbers.

\[ Nu = 0.286 A^{-0.286} Gr^{1/4} \]  

(9)

In Figure 6 it can be seen that although Eqn. 9 was developed for Grashof Numbers in the range \(2.9 \times 10^6\) to \(9 \times 10^6\) it is actually able to provide a prediction to within 5\% of the values achieved in the experiments conducted in this study, where the Grashof Numbers were in the range of \(10^7\) to \(10^8\). This suggests that the correlation is well suited for use over a much wider range of conditions and is a suitable generalised representation of heat transfer in a triangular enclosure.

Furthermore, in Figure 6 it can be seen that there is a good relationship between the predicted values from this correlation, as well as the CFD results from Anderson et al [19], and the experimental data presented in this work.

5. Conclusion

Recently it was suggested that natural convection in the attic of a cold roof building could serve as an insulating medium for building integrated solar collectors installed in a cold roof building [18, 19]. Ridouane and Campo [1] had previously developed a correlation to describe natural convection in attic style enclosures based on Grashof Numbers in the range \(2.9 \times 10^6\) to \(9 \times 10^6\). However for a tropical or warm climate such as those of Australia or New Zealand, the Grashof numbers in an attic space could conceivably exceed those previously studied by several orders of magnitude.

In light of the shortcomings of the data relating to natural convection in attic shaped enclosures, the experimental work in this study showed that Ridouane and Campo’ [1] correlation was also valid for Grashof Numbers in the range of \(10^7\) to \(10^9\). Based on this finding it is therefore possible to utilise this correlation in determining the heat loss from the uninsulated rear surface of a roof integrated solar collector in a cold roof building.
6. References


Fig. 1: Experimental enclosure
Fig. 2: Schematic of measurement system
Fig. 3: Temperature v time showing steady state
Fig. 4: Heat transfer coefficient v enclosure temperature difference
Fig. 5: Nusselt number v Rayleigh number
Fig. 6: Relationship between Nusselt number from Ridouane and Campo’s correlation versus CFD prediction and measured Nusselt number