"What is a Logical Diagram?"

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Robert Brandom's expressivism argues that not all semantic content may be made fully explicit. This view connects in interesting ways with recent movements in philosophy of mathematics and logic (e.g. Brown, Shin, Giaquinto) to take diagrams seriously – as more than a mere 'heuristic aid' to proof, but either proofs themselves, or irreducible components of such. However what exactly is a diagram in logic? Does this constitute a semiotic natural kind? The paper will argue that such a natural kind does exist in Charles Peirce's conception of iconic signs, but that fully understood, logical diagrams involve a structured array of normative reasoning practices, as well as just a 'picture on a page'.

1. Introduction: 19th Century "Picture Shock"

20th century mainstream analytic philosophy was almost entirely neglectful of diagrams in its theorizing about *semantic content*, and *reasoning*. It is worth understanding the historical background to this arguably contingent state of philosophical affairs.

The trend began in mathematics. In the 19th century this field was revolutionized by an arithematization movement, and some of the key developments foregrounded ways in which our "visual expectations in mathematics"[1] might deliver the wrong answer about mathematical fact. A famous example is the claim that a function which is everywhere continuous must be differentiable, which is in fact false. Attempting to evaluate this using visual imagination, one may imagine that if a function is continuous then it contains no 'gaps' or 'breaks', and then one seems to 'see' that at some sufficiently fine-grained level it must present a smooth surface, which would have a gradient, and thus a derivative. However, to the surprise of many, Weierstrass and Bolzano proved that certain functions are infinitely finely jagged, yet still gap-free in a way that fits the formal definition of continuity.[2] Another example is whether a 1-dimensional line might fill a 2-dimensional region. Any attempt to mentally picture something resembling an infinitely thin thread unspooling into a finite area and thereby 'filling it in' seems to show that the claim is

[1] This phrase is taken from Marcus Giaquinto (Giaquinto, 2007, p. 3)
[2] This example is nicely discussed in (Giaquinto, 2007, pp. 3-4), and (Mumma, 2010, pp. 3-4).

false, but Peano proved it true.[3] Such examples prompted some of the most influential mathematicians of the 19th century to draw strong morals about the potential for error in diagrammatic reasoning. As Marcus Giaquinto writes:

> Such cases seemed to show not merely that we are prone to make mistakes when thinking visually…but also that visual understanding actually conflicts with the truths of analysis (Giaquinto, 2007, pp. 4-5).

Hilbert famously wrote, "a theorem is only proved when the proof is completely independent of the diagram" (Giaquinto, 2007, p. 8), drawing on an almost identical remark by Moritz Pasch in his influential *Lectures in Modern Geometry* (1882)**.** So, remarkably, even the field of *geometry*, it came to be seen, needed to be purged of diagrams.[4] The end result was a "prevailing conception of mathematical proof" which John Mumma describes as "purely sentential":

> A proof…is a sequence of sentences. Each sentence is either an assumption of the proof, or is derived via sound inference rules from sentences preceding it. The sentence appearing at the end of the sequence is what has been prove (Mumma, 2010, p. 1)

This suspicion of 'visual expectations' then flowed into Frege's work on the foundations of mathematics. Cognizant of the errors which his fellow mathematicians had learned to skirt, Frege attempted to entirely remove 'intuition' from the logic with which he set to put mathematics on an entirely new and more rigorous foundation. Famously, he remarked of his own concept-script:

> So that nothing intuitive could intrude here unnoticed, everything had to depend on the chain of inference being free of gaps. (*Begriffsschrift* IV, Beaney p. 48)

Frege argued against the empiricism of John Stuart Mill that numbers were not properties abstracted from the physical world, but definable purely analytically.

Frege in turn was an enormous influence on *logical positivism* (Carnap studied under him, for instance), which in turn set the scene for philosophy's aims and methodologies in many ways that are still being worked out today. The movement's early strict focus on

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[3] Discussed in (Giaquinto, 2007, pp. 4-5).

[4] "A body of work emerged in the late 19th century which grounded elementary geometry in abstract axiomatic theories…This development is now universally regarded as a methodological breakthrough. Geometric relations which previously were logically free-floating, because they were understood via diagrams, were given a firm footing with precisely defined primitives and axioms" (Mumma, 2010, p. 6).

clarifying *meaning* owed much to Frege's vision of an ideal language all of whose inferential steps are explicitly stated, and use a set of rules specified in advance.[5] Thus A.J. Ayer laid down a strict definition of "literal significance" as confined to claims which have "factual content" by virtue of offering "empirical hypotheses" (Ayer, 1952, p. 2). Thus, to illustrate by way of a simple example, "The cat is on the mat" is literally significant because there is a cat-being-on-the-mat type *experience* which might be had – or not – in the relevant situations.

Claims which lack "literal significance" fall into two camps. Either they can be "literally false" but somehow "the creation of a work of Art" which is gestured towards as valuable, though Ayer is somewhat vague about how. Or, worse, claims might be "pseudo-propositions" – disguised nonsense. Any claim lacking literal significance is not the purview of philosophy (Ayer, 1952, p. 2). It is hard to see how a diagram could offer an empirical hypothesis, and thus have literal significance in Ayer's sense. And he briskly dismisses the idea that a philosopher might be "endowed with a faculty of intellectual intuition which enabled him to know facts that could not be known through sense-experience" (Ayer, 1952, p. 1). Likewise, the early Carnap (1932) claimed that statements were meaningful if syntactically well-formed and their non-logical terms reducible to observational terms in the natural sciences.

It is well-known that crisp, clean criteria for what constitutes a genuine empirical hypothesis were much more difficult to find than Ayer imagined. Carnap dropped back from verifiability to "partial testability" (1936-7), and confirmation became a more and more holistic affair, until finally Quine acknowledged that what meets the tribunal of experience is in an important sense the whole of science. By way of consolation for thus sounding verificationism's death-knell, he offered a new criterion of what might be called 'factuality': if we could imagine our science collated and regularized into a single theory expressed in first-order logic, its bound variables would have values. In a pseudo-science such as witchcraft they would not (Quine, 1953). Now we can say that "The cat is on the mat" is factual because in the logical formula **∃x(Cx & Oxm)** suitably interpreted, the variable **x** binds to Fluffy.

[5] although transposed into a rigidly empiricist setting which truth be told sits oddly with Frege's thinking– and arguably has caused significant problems in the philosophy of mathematics.

Philosophers' banishment of diagrams from semantics and theory of inference arguably reached a high-water mark in the late '60s with the publication of Quine's colleague Nelson Goodman's *Languages of Art*. Here Goodman made an influential argument that resemblance plays no interesting or important role in signification. Rather, he claimed that denotation, "is the core of representation and is independent of resemblance" (p. 5). His reasoning was that while the resemblance relation is symmetric (if **X** resembles **Y** then **Y** resembles **X**), the representation relation is not.[6]

However, a profound challenge to this more than century-long neglect of diagrams is 'in the air'. It seeks to reconceive diagrams as more than a mere 'heuristic aid' to proof in mathematics and logic. Rather diagrams may be understood as capable of serving either as proofs themselves, or irreducible components of such. Thus James R. Brown writes:

> …the prevailing attitude is that pictures are really no more than heuristic devices…I want to oppose this view and to make a case for pictures having a legitimate role to play as evidence and justification – a role well beyond the heuristic. In short, pictures can prove theorems (Brown, 2008, p. 96).

John Mumma writes:

> In the past 15 years, a sizeable literature consciously opposed to [the attitude that pictures do not prove anything in mathematics] has emerged. The work ranges from technical presentations of formal diagrammatic systems of proof (e.g. (Shin, 1994)) to philosophical arguments for the mathematical legitimacy of pictures (e.g. (Brown, 1997), (Dove, 2002)) (Mumma, 2010, p. 8).

Meanwhile Marcus Giaquinto has produced an interesting new book in mathematical epistemology, where he writes:

> ….a time-honoured view, still prevalent, is that the utility of visual thinking in mathematics is only psychological, not epistemological….The chief aim of this work is to put that view to the test (Giaquinto, 2007, p. 1).

Other authors have returned to ancient Greek mathematical texts to argue that one cannot understand them fully without taking diagrams more seriously (Catton and Montelle, 2010).[7]

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[6] Randall Dipert has argued against this that it no more follows that resemblance is 'entirely independent of' representation because the former relation is symmetric and the latter is not, than that the brother relation is 'entirely independent of' the uncle relation as the former is symmetric and the latter is not (Dipert, 1996).

[7] See also, from a more philological perspective, the work of Reviel Netz, e.g. "The Limits of Text in Greek Mathematics" (2005).

Meanwhile, in logic, Sun-Joo Shin argues that although, "[f]or more than a century, symbolic representation systems have been the exclusive subject for formal logic" (Shin, 2004, p. 1), this should be widened to also consider "heterogeneous systems", which "employ both symbolic and diagrammatic elements" (Shin, 2004, p. 1). This is an influential term which derives from Jon Barwise (1993). Shin argues that symbolic and heterogeneous reasoning systems have different strengths and weaknesses, and we should do a thorough study to get the best out of both, bearing in mind that different disciplines which might draw on such systems (such as logic, artificial intelligence and philosophy of mind) might have different needs.

This paper seeks to join these authors while at the same time to put this goal in a broader context, namely a movement which is also aimed at unbuilding the simple picture of "literal significance" that has been so influential in the 20th century – *expressivism*.

Expressivism: Saying, Doing and Picturing

Expressivism has a metaethical incarnation, as a view that, "…claims some interesting disanalogy between…evaluations and descriptions of the world" (Chrisman, 2011, p. 1). By contrast, Robert Brandom has put forward a *semantic* expressivism, whose main point is that not all semantic content may be made fully *explicit*. This view contrasts with a widespread view often thought to be intuitively obvious, and arguably a downstream spectre of Ayer's notion of literal significance. I will call it a *metaphysical realist semantics*. The juxtaposition here is deliberately somewhat controversial, given that many metaphysical realists take great pains to make a clear separation between metaphysical and semantic questions, and to claim that their view lies firmly on the metaphysical side. An argument will be put forward later in the paper that this self-assessment is problematic.

A metaphysical realist semantics holds that the purpose of language is to state "facts" which, if the propositions stating them are true, are part of language-independent reality. Thus, to return to our earlier example, "The cat is on the mat" (suitably disambiguated as to cats and mats) is thought to present a 'content' which it is sufficient to know the meaning of the statement's words to fully understand. Brandom calls the view *representationalism*. By contrast, he argues that the primary purpose of language is to transform what we *do* into something that we can *say*:

> By expressivism I mean the idea that discursive practice makes us special in enabling us to make *explicit*, in the form of something we can *say* or *think*, what otherwise remains *implicit* in what we *do*. (Testa, 2003, p. 561, cited in Price, 2011)

Crucially, this renders the explicit statement semantically parasitic on the implicit practice, in that one cannot fully understand the statement without antecedently understanding the practice which it "expresses". Thus Brandom writes:

> …we need not yield to the temptation…to think of what is expressed and the expression of it as individually intelligible independently of consideration of the relations between them…And the explicit may not be specifiable apart from consideration of what is made explicit (Brandom, 2000 , pp. 8-9).

Consider for example, the invention of *musical notation*. This freed musicians from having to learn music by directly copying a live musician's *actions*. Instead a musical score substitutes dots on a page for string-pluckings, key tappings, and all other actions which might produce a note. In this way a musical score can *say what musicians do* (with added bonuses such as that the score can be indefinitely copied, survive longer than any living musician, and be readily compared and contrasted with other scores). However, it is not possible to fully understand a musical score without having some antecedent understanding of the practices of music which it is expressing. For instance, if aliens were to stumble upon the score for Beethoven's 5th symphony, it is highly unlikely they could perform it without some anthropological observation of human musical actions.

This commitment to a parasitism of the explicit statement on the implicit practice renders expressivism a form of *pragmatism*. It claims that certain practices are not fully explicated in language, but presupposed by it. Pragmatism is frequently seen as a form of antirealism, merely internal realism[8], non-cognitivism[9], non-factualism[10], or as some would put it "quasi-realism"[11]. But the conclusion of this paper will consider other views on this.

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[8] (Putnam, 1992) , (Blackburn, 1998).

[9] Price suggests Rorty approaches a global non-cognitivism in (Price, 2008).

[10] This term derives from (Boghossian, 1990).

[11] The term was coined by Simon Blackburn, see in particular (Blackburn, 1984). Its links with pragmatism are explored in (Price, 1998), though (Macarthur and Price, 2007) argues that the two views share important similarities *and* differences.

Such an expressivism may make sense for music, but might it be generalized? For Brandom wishes it to be a global view, concerning all language. In particular, might expressivism be applied to talk about *logic*? Surely the matters of truth-preservingness and validity are a paradigm of practice-independent fact? Not so, according to Brandom. He claims that logic also should be seen as a way of *saying* what we are *doing* when we actually make inferences, in ways that can guide our reasoning in systematic and useful ways. In fact he self-consciously highlights the practice of philosophy itself as a particularly sophisticated pulling of unselfconscious implicit *practices* into explicit *statements* that might be critically appraised (Brandom, 2000, pp. 56-7).

Brandom's expressivism may be linked in interesting ways with Wittgenstein's Picture Theory of Meaning[12]. In the *Tractatus* Wittgenstein drew a famous distinction between what is *said* (namely atomic facts, and their truth-functional combinations) and what is *shown* (the laws of logic, the limits of the world and, interestingly in the expressivist context, ethics). In a Brandomian spirit we might describe the former as "explicit" and the latter as "implicit". However, having drawn this distinction between saying and showing, Wittgenstein made the further claim that what is shown *cannot be said* (and thus for instance logic is transcendental). Early Wittgenstein and Brandom stand out amongst mainstream semantics in sharing the bold claim that not everything true can be made explicit, or stated. They are also different in two ways, however. Firstly, where Wittgenstein suggested that "the said" and "the shown" consist in two irrevocably sundered 'camps' of content, Brandom allows any implicit practice to be made explicit (an example would be noticing a pattern in one's reasoning and naming it *Modus Ponens*). He merely notes that this can only happen against a background of further implicit practices (in the example just cited – argument categorization). Secondly, although a 'semantic parasitism' exists in both views, it apparently points in opposite directions. For where we saw that for Brandom the *explicit* is parasitic on the *implicit*, for Wittgenstein it appears that what is *shown* is parasitic on what is *said*.

I will now develop an expressivist view of logical diagrams.

[12] I have argued this previously elsewhere: *****

Defining the Diagrammatic

What exactly *is* a diagram in logic? Can we give a definition which would cover all cases which we would want to call logical diagrams, and not cover any cases which we would not? Let us begin by trying to define a diagram more generally. Figure 1 would appear to be a paradigm case – so what is 'diagrammatic' about it?

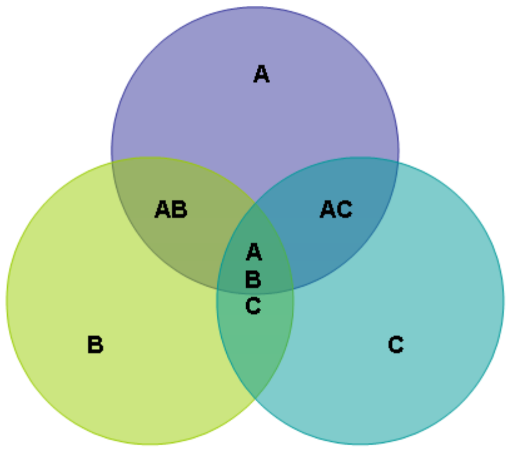

fig. 1

First of all, it seems capable of conveying some kind of meaning, as it is so structured. But whatever meaning it has certainly seems different to that which would be conveyed by a piece of prose. How exactly? Most notably by the absence of any words. Therefore we might attempt the following definition of diagrams:

i) *"X is a diagram iff there are no words on the page"*.

However *fig. 2* fails this criterion. And yet it is arguably a diagram:

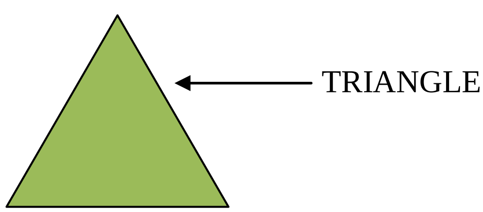

fig. 2

One might protest that the 'part on the left' of fig. 2 is the truly diagrammatic part, while the 'part on the right' is just a label. But one would not wish to say that the part on the right is not part of fig 2. It is pointing directly at it and we have conventions for interpreting such arrows. Thus having a 'truly diagrammatic part' seems to be sufficient for being a diagram, at least in this case. So perhaps we can capture this in a new definition:

ii) *“X is a diagram iff there are pictures on the page”*.

But what is a picture? Can we give a definition which would cover all cases which we would want to call pictures, and not cover any cases which we would not? Perhaps we could say that a picture, unlike a piece of prose, is made of joined-up lines? This produces another possible definition:

iii) *“X is a diagram iff there are joined-up lines on the page*

However *fig. 3* has no joined up lines, only letters. Yet it too is arguably a diagram.

TRI
TRIANGLE
TRIANG: TRIANGLE
TRIANGLE TRI TRIANGLE
TRIANGLE TRIANGLE TRIANGLE

fig. 3.

Although it is composed solely of words, they are arranged in a *structure*, and this seems to render it diagrammatic. So maybe we could capture this with a definition something like: iii) *X is a diagram iff there is some ‘non-word component’ on the page*.

This however seems awfully vague, and even so it is probably not a sufficient criterion. (Punctuation? Page numbers?)

One might wonder at this point if we are attempting to define something too basic and fundamental to be put into words. Or perhaps we are attempting to define something too heterogeneous – maybe the concept of a diagram is not a semiotic natural kind? However this would be to give up too easily. I will now offer a definition which draws on Charles Peirce’s concept of an *iconic sign*. Our problem so far has in fact been trying to craft our definition merely by inspecting *the sign itself*. Peirce believed the key issue for clarifying this matter is the sign’s *relationship to its object*.

Peirce's Icon: The Sign which Resembles

Peirce's 'philosophy of language' falls within a much wider theory of *signs*, or *semiotics*. Including pictures and diagrams is part of the point of this broader disciplinary purview. Peirce defined a sign very broadly as any irreducibly triadic relation between a *representation,* an *object*, and an *interpretation.* As well as attributing triadic structure to the sign itself, he taxonomised signs using a series of three-way distinctions. The icon is part of a triad comprising *icon*, *index* and *symbol*, corresponding to the three different ways in which Peirce believed a sign could be associated with an object and thus gain meaning.

Symbols symbolize what they do via some arbitrary *convention which must be learned*. Thus we must learn that the word "banana", in English, means bananas. All words are symbolic to some degree, as they belong to shared public language. However as (Perry, 1979), and others showed, language also includes signs which pick out an object not by learned convention but *via* some direct *'indicating'* or *'pointing'* relationship (e.g. "here", "now"). Peirce called these signs *indices*. Finally *icons* are signs which *resemble what they signify* (e.g. Peirce, *Collected Papers*, henceforth *CP*, 2.304). This category is distinct from symbols, as resemblances need not be established by convention but can be perceived anew (e.g. "That cloud looks like a frog"). Examples include maps, paintings and but also, crucially, mathematical diagrams which function by *mimicking the structures they signify*.

This definition of the icon immediately raises skeptical concerns in the minds of many. "Resemblance is cheap", it is thought. Anything can be argued to resemble any other thing in *some* respect. For instance, a photograph of Richard Nixon might be thought to resemble other objects *qua* male (e.g. Brad Pitt), *qua* brunette (e.g. Elizabeth Taylor), *qua* oval-headed (e.g. an egg), or in many other more recondite ways (e.g. "Something in his eyes reminds me of Mt Everest…") To top it off, the Löwenheim-Skolem theorem is often vaguely invoked at this point, following (Putnam, 1981), to suggest that all the points on one object can be mapped onto all the points on any other object to produce an 'isomorphism', so in some suitably impressive mathematico-logical way, a triangle could be 'like' a square, a cow could be 'like' a flock of birds and at that point the whole business of likeness dissolves into arbitrariness.

How is a serious theory of language, not to mention reasoning, to lean on such an apparently subjective pillar? There seems to be an unarticulated yet profound intuition in contemporary analytic philosophy that this is why semantics should rest squarely on *reference,* as to make room for the contingencies of the mind's ability to creatively notice likenesses would introduce theoretical chaos.

The worry is very understandable, and properly addressing it would require excavating and settling a number of very deep issues. I will mention three. i) One will need to argue for a *realism about structures* which is arguably underappreciated since the Quinean equation of ontological commitment with the bound variable noted above.[13] For Peirce claims that the parts of an icon bear the same relationship to one another as do the parts of the object the icon represents (Peirce, *CP*, 3.363). This seems to be a good way of explicating *structure*, and it is worth highlighting that icons are the structural signs par excellence, in terms of *their means of signification*. For although indices may be words and to that degree possess internal complexity (in the individual letters), their *signifying* function is to serve as a pure pointer. The word 'here' indicates a location in space – it does not 'say' anything else. And although the convention by means of which the symbol symbolizes what it does may have structure, that structure is not *internal to the sign itself*. For instance the word 'dollar' represents what it does in NZ society by a complex set of conventions involving different coloured notes, numbers on a screen in internet banking, and so on, but none of these 'convention-parts' are related in the same way as are the parts of the word 'dollar' itself (once again: its individual letters). However in a map of New Zealand we discover that Hamilton is West of Napier by observing the relevant spatial relations *in the map itself*.

ii) The second issue concerns properly understanding the role of the icon, which is not to generate ontological commitments in the simple denoting way that many in the Quinean tradition envisage. Denotation is the role of the index – but it can only peform this function when appropriately supported by the other two sign-types. In fact Peirce believed that icons, indices and symbols play three different functional roles whose co-presence and coordination is vital for language to function as it does. It is worth explaining these roles. Firstly, the symbol's conventional nature means that it signifies

[13] Pace the recent *structuralist* movement in philosophy of science

general properties, because conventions are “general rules” (Peirce, *CP*, 3.360) which can be applied any number of times in situations which display the appropriate (general) features. Secondly, due to their pure pointing function, indices designate *particular existence* – Peirce writes, “[a]n indexical word…has force to draw the attention of the listener to some hecceity common to the experience of speaker and listener” (Peirce, *CP*, 3.460). Finally, icons designate *neither general facts nor particular existences*. Rather they signify hypotheses, possible situations (Peirce, *CP*, 3.362). Relatedly, it is important to note that these three sign-types are not mutually exclusive in that a single sign can serve as icon, index and symbol in different respects all at the same time.[14]

iii) As for the re-introduction of the mind’s creative powers to semantics and the theory of inference – the issue here is to embrace it, unnerving as it may be – this is long overdue. There is a long tradition of debate in philosophy over whether the thinking mind is essentially *active* or *passive*, with rationalists preferring the former camp, empiricists the latter. It is a major difference in the thought of Kant and Hume. Although it must be conceded that Hume did highlight the imagination, seemingly an active faculty, to a degree unmatched by other British empiricists, nevertheless he explicated this faculty within a naturalistic perspective with strong determinist implications (and when he discussed the problem of free will arguably offered a compatibilist “cop-out”). At any rate, achieving this re-introduction will require mounting some deep challenges concerning what a semantics or theory of inference is *for*. The fields have arguably strayed from giving an *account* of the phenomena in question, too far towards providing an *algorithm to predict* them.[15]

Lacking time to properly pursue these three large inquiries, I will mount something of a preliminary phenomenological argument for the worth of the icon in the robustly mind-independent field of logic by demonstrating Peirce’s iconic logic *in use* (not, I hope, a terrible argument for a pragmatist to use.)

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[14] An example is a shadow-clock, which is iconic insofar as it represents the 24 hour structure of our day, indexical insofar as it relies on the sun physically casting a shadow to tell the time, and symbolic insofar as the numerals on the clock-face have meanings which must be learned.

[15] This might have something to do with the fact that key researchers in semantics and logic in the 1960s and 70s also worked in artificial intelligence.

Peirce's Existential Graphs

Later in his career Peirce developed a diagrammatic logic which he called the Existential Graphs (henceforth: EG), claiming that "all necessary reasoning without exception is diagrammatic". His next remark about how these diagrams are to be used is interesting – he writes, "…we construct an icon of our hypothetical state of things and proceed to observe it" (Peirce, *CP*, 5.162). I will demonstrate this system in its simplest, Alpha Graph form, which is provably equivalent to modern propositional logic. But first, here is a traditional (natural deduction) proof for purposes of comparison:

Proof I "Symbolic Logic": ⊢ (P ∨ ~P)

1	$\neg(P \vee \neg P)$	H
2	P	H
3	$P \vee \neg P$	I∨ 2
4	$\neg(P \vee \neg P)$	IT 1
5	$\neg P$	I¬ 2,3,4
6	$\neg P$	H
7	$P \vee \neg P$	I∨ 6
8	$\neg(P \vee \neg P)$	IT 1
9	$\neg\neg P$	I¬ 6,7,8
10	P	E¬ 9
11	$\neg\neg(P \vee \neg P)$	I¬ 1,5,10
12	$P \vee \neg P$	E¬ 11

This proof *seems* to be purely sentential in Mumma's terms. Observe the length required to establish the simplest of tautologies!

By contrast, the EG uses no connectives between sentence letters. Rather it represents *conjunction* by placing two sentence letters on the same "sheet of assertion", and *negation* by drawing a line or 'cut' around the proposition ('graph') in question (see *fig.*

4). A benefit that has been claimed for logical diagrams is so-called *free rides* (Barwise and Shimojima, 1995). One can immediately ‘see’ certain logical equivalences.[16]

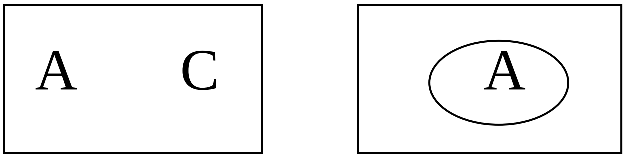


fig. 4

A & C **~A**

Thus, *fig 5* may be observed to elegantly represent: $\sim(A \,\&\, \sim C)$, $(A \vee \sim C)$ and $(A \supset C)$ simultaneously:

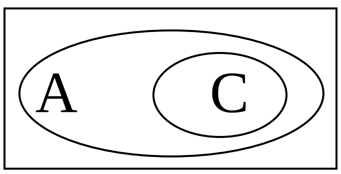


fig. 5

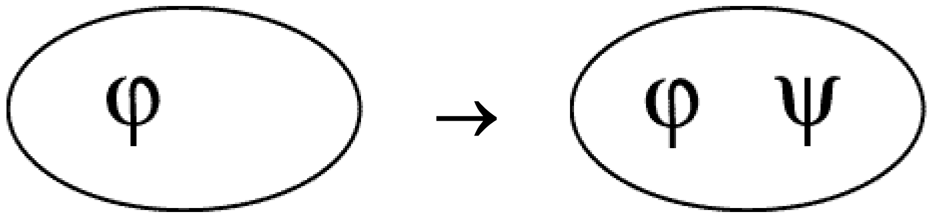
This is an immediate perception. How about a proof? First we need some rules.

Alpha Graph rules are 6, each of which consists in a permission to either *write* or *erase* a graph on the sheet of assertion. I here use (with permission) the beautiful presentation of the rules by Jay Zeman. Single arrow rules are to be read from left to right. Double arrow rules consist of two rules – the first read left to right, and the second right to left.

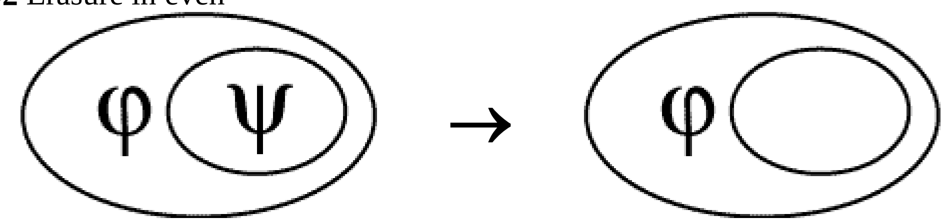
[16] Shin also calls this the “multiple carving principle” (Shin, 2004, p. 77).

Proof Rules for Alpha Graphs (Zeman, 2002)

0.01 Insertion in odd:

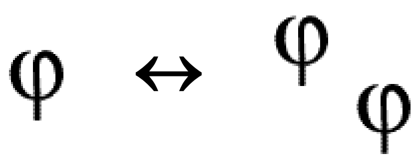


0.02 Erasure in even

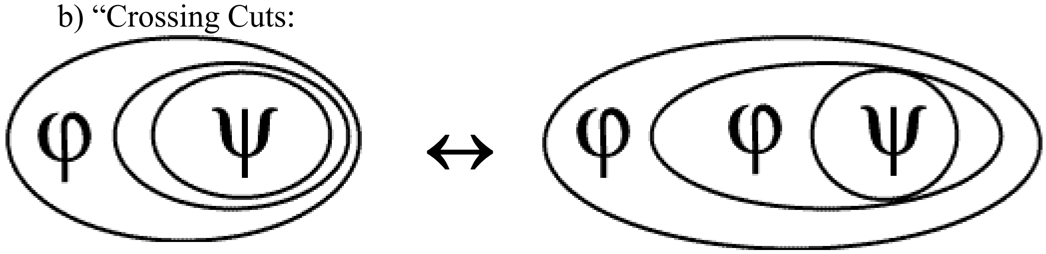


0.03 Iteration and **0.04** Deiteration

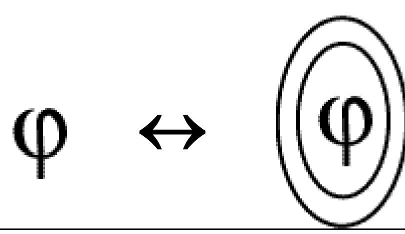
a) In the same area:



b) “Crossing Cuts:



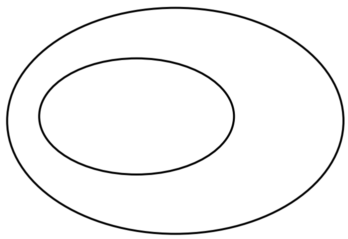
0.05 and **0.06**: Biclosure



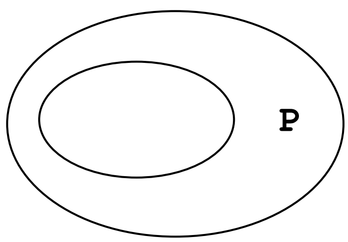
I will now perform two proofs, beginning with the simple tautology proven above:

<u>Proof II, EG: ⊦ (P ∨ ~P)</u>

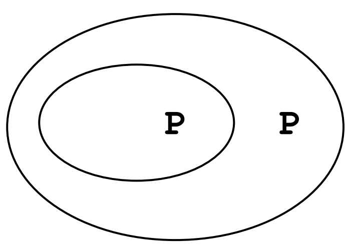
First, rule **0.05** allows the adding of a double negation anywhere (this is frequently the first step):



Rule **0.01** allows any graph whatsoever to be added in an oddly enclosed area. Not very surprisingly, we choose **P**:



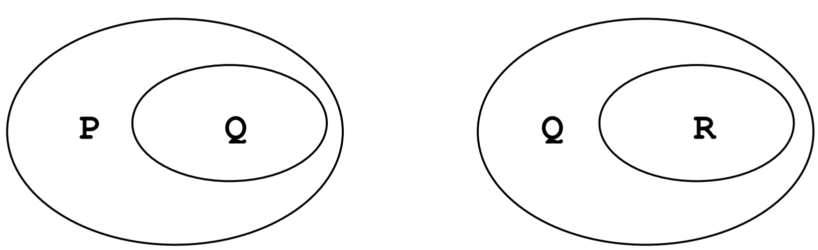
Rule **0.03b)** (Iteration "Crossing Cuts") allows **P** to be re-scribed into the evenly enclosed space within:



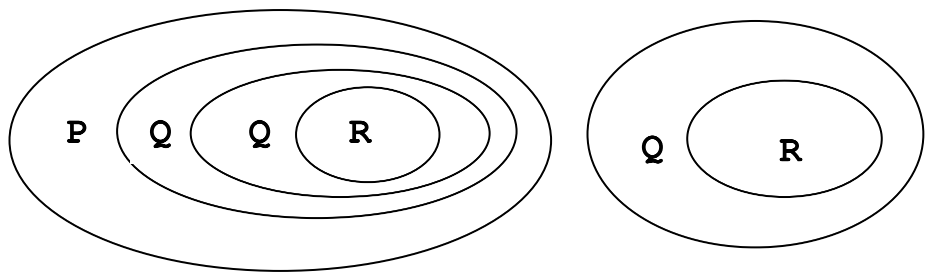
Our tautology is now most easily proven.

<u>Proof III, EG: (P ⊃ Q), (Q ⊃ R) ⊦ (P ⊃ R)</u> (Hypothetical Syllogism)

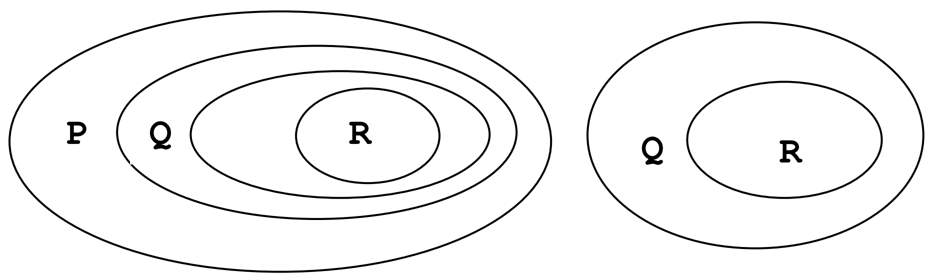
We begin by scribing the premises on the sheet of assertion:



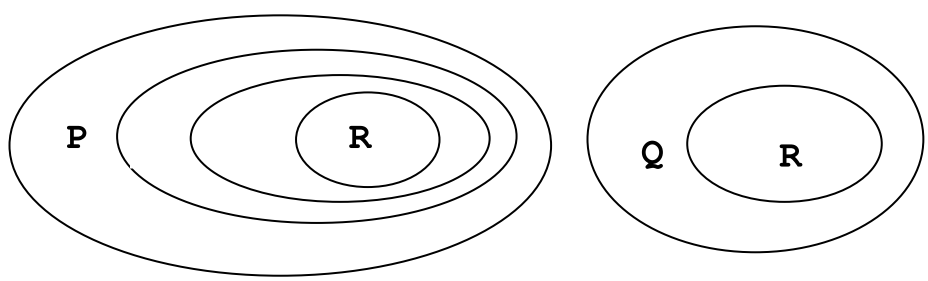
Next we iterate the right-hand portion of the diagram inside the left-hand portion, using Iteration Crossing Cuts (**0.03b**):



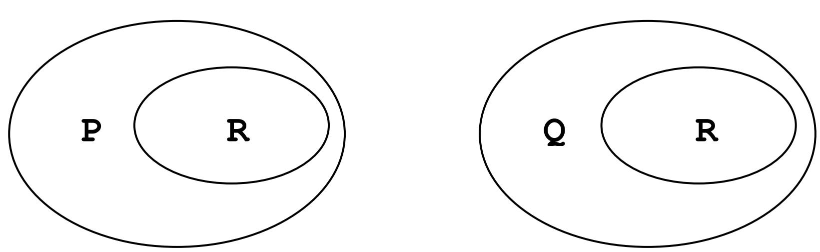
We now deiterate the innermost Q, by (**0.04b**):



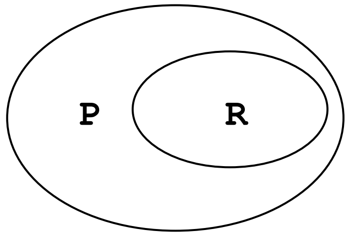
We now deiterate the other Q on the left-hand side, using Erasure in Even (**0.02**):



We may now remove the double negation ("biclosure") around R on the left-hand side (**0.06**):



And the final result is achieved by using **0.02** to erase the right-hand side (functionally equivalent to conjunction elimination):



Now think about how this procedure might work if we attempted to prove the invalid: **(P ⊃ Q),(Q ⊃ R) ⊢ (R ⊃ P)**. It is not possible as our proof puts **P** in the outermost (odd) enclosure, and there is no way consistent with the rules that it can be removed. Graphs in odd enclosures may be iterated inwards, but never *removed*.

Logical Diagrammatic Forcing.

We may now see that a "logical diagram" includes its rules of use, not just pictures on the page. It is no accident that the EG rules were presented before the diagram was constructed to furnish the proof. These rules are of course explicit representations of implicit reasoning practices[17], to use Brandom's terminology. We do not understand the rules without understanding the *actions* (literally adding and removing graphs from the diagram) which they represent. Once we understand the rules and most importantly – use them, we *directly experience* the impossibility of rendering a contradictory proposition or invalid argument on an EG. We are *forced* to recognize how some part of what we are trying to realize 'has to give'. We thereby 'see' (in some arguably metaphorical, but powerful sense, which means something like 'structurally perceive') logical necessity. This is the "faculty of intellectual intuition" so facilely dismissed by Ayer. It is crucial to note that this intuition occurs not by having epistemic contact with any further 'necessary object' (whatever that might be), but merely by fully *grasping* the relationships amongst the diagram's different parts, already present on the page.

At the same time it is important to note that not all aspects of Peirce's logical diagrams are forced by their structure, and thus iconic. Some aspects are *symbolic* – for instance one must learn the convention that letters correspond to propositions, and not to, say, predicate letters applied to an object represented by the larger circle. Some aspects

[17] Which is not to say that ordinary reasoners would necessarily recognize them as such. This is theoretical not applied logic (what Peirce called *logica docens,* opposing it to *logica utens*: a distinction that medieval logicians drew).

of the graphs are also *indexical*. For instance the sentence letters serve to indicate particular propositions (which are indicated in somewhat vestigial form – as propositional logic is characterised by abstracting away from atomic propositional content – yet remain as crucial place-holders). This is how Peirce's semiotics works – all three kinds of sign need to be present and to work together to create significance.

We have seen that so-called 'iconic logic' is not purely iconic. At the same time, so-called 'symbolic logic' is not purely symbolic. Natural deduction, used in *Proof I* above, also has rules, forces certain results and forbids others, and is iconic *to that degree*. We therefore do not have symbolic logic and iconic logic, strictly speaking. We have logical systems whose iconicity is more or less *perspicuous*. If we bear in mind our initial definition of the icon, that its parts are related in the same way that the objects represented by those parts are themselves related, perspicuity consists in as many of the relationships on the diagram as possible representing logical, as opposed to arbitrary relationships. Peirce believed his system was more perspicuous than the 'algebraic' logic he had worked in prior to developing the EG, and that it would be useful, not for proving results the other couldn't, but for studying logical form more clearly and minutely.

Conclusion

This paper has presented an expressivist view of logical diagrams. Thus Brown is vindicated in his claim that, "pictures hav[e] a legitimate role to play as evidence and justification". In fact we now see that *all* formal logic is essentially diagrammatic, although we have seen that the diagrams may be more or less perspicuous. We can also see that Mumma's definition of the 'purely sentential proof' he wishes to argue against ("[a] proof…is a sequence of sentences. Each sentence is either an assumption of the proof, or is derived via sound inference rules from sentences preceding it") wears incoherence on its sleeve. For once one adds 'sound inference rules' to the mix, one has *more* than just a sequence of sentences.

One might ask: But what about the visual-expectation-derived mistakes which 19th century mathematicians learned to avoid? If we embrace diagrammatic reasoning, how do we know we won't be led into error in this field? This is a good question. The short answer is: Get better diagrams. The long answer (which would involve formulating a principled account of which structural features of a diagram represent necessary truths, and which do not[18]) will take much further work to determine.

Through diagrams such as Peirce's EG, logical necessity is presented to the human mind in such a way that it can be understood and learned. And what more could we ask in order to say that a system of signs represents something, or has genuine content? But at the same time the diagrams do not *state* logical necessity in anything like Ayer's sense of literal significance. The graphs do not put forward an empirical hypothesis. They provide the means for us to exercise our rational intuition. Moreover, if we turn once again to Quine's criterion for ontological commitment, the basis for the metaphysical realist semantics pervasive today, we can see the graphs do not fit this model either. They do not gain their content by denoting further objects (in the way that "The cat is on the mat" is commonly understood to denote a cat, and a mat). Rather, as icons, everything needed for logical insight is already internal to them as signs – one merely has to attend to their structure.

We might pause in closing and ask: what are the implications of this expressivism for *realism about logic*? For instance, does it show that the discourse of logic does not 'talk about real mind-independent things'? My answer will be: No, but that our notion of 'real mind-independent things' requires some surgery.

It was noted that a great deal of recent metaphysics is semantics-driven,[19] although metaphysical realists sometimes feel a little guilty about this, as it would seem that their very realism should lead them to try to keep semantics and metaphysics separate. I would say, actually it is fine to derive one's metaphysics from one's semantics – just please, please get a less simplistic semantics! We may understand Quine's criterion of ontological commitment in Peircean semiotic terms as an attempt to place the full burden of representing reality onto indexical signs. This leads philosophers with realist sympathies to feel they need to ask a raft of questions of the form: "Does term X [e.g. ethical or aesthetic predicates, number-terms…] denote a real object?" If we recall that indexical signs pick out independent particulars, it often seems hard to answer "yes" to

[18] The work of Ken Manders may be understood as seminal in this regard. See for instance his (2008/1995).
[19] Thus for instance, Chrisman sums up much recent metaethics by writing, "The realism debate has been pursued (mostly) by investigating the appropriate semantic account of ethical statements" (Chrisman, 2008, p. 334).

this question for manifestly important human discourses (such as ethical or aesthetic predicates, number terms…). On the other hand, those who are unsatisfied with metaphysical realism's problematization of such areas, and those who wish to recognize a manifest social input to human language-games, often oppose metaphysical realism with some form of conventionalism which argues that term X does not denote a real object but has some other socially sanctioned and taught *function*. We may understand such a conventionalism in Peircean terms as trying to understand all signification as performed with symbolic signs. (We saw that the symbol is the sign whose meaning is derived via convention.)

Metaphysical realism and conventionalism are assumed to be polar opposites. So many dialectics in so many papers in so many areas of philosophy revolve around this, so that an argument against metaphysical realism is more often than not assumed without question to be an argument for conventionalism, and an argument against conventionalism is more often than not assumed to be an argument for metaphysical realism. But this is a false dichotomy. A third kind of signification exists which does not consist in brute denotation *or* in arbitrary convention, but which presents structure directly to the mind's eye. It is barely glimpsed in formal semantics today. And yet it is this kind of sign that represents *logical form* – hardly a trivial part of our conceptual scheme. If we could only recognize that the symbol, the index and the icon all have a unique and irreducible semantic role to play, and that reality correspondingly is comprised of *real conventions*, *real particulars* and *real intrinsic structures*, we would make some progress towards understanding that most contested concept.

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