

# Volume Measurement Using 3D Range Imaging

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**Abstract:** The use of 3D Range Imaging has widespread applications. One of its applications provides us the information about the volumes of different objects. In this paper, 3D range imaging has been utilised to find out the volumes of different objects using two algorithms that are based on a straightforward means to calculate volume. The algorithms implemented successfully calculate volume on objects provided that the objects have uniform colour. Objects that have multicoloured and glossy surfaces provided particular difficulties in determining volume.

**Keywords:** range imaging, volume measurement

## 1 INTRODUCTION

The Range Imaging Cameras are a class of ranging device that acquire a full-field image of range data simultaneously of the scene from one vantage point. Currently available commercial cameras, such as the Mesa Imaging SwissRanger, enable efficient and easily obtainable low-resolution (up to  $200 \times 200$  pixels) range data with one or two centimetre precision in ranging. In this paper we explore the use of the Mesa Imaging SwissRanger SR4000 [1] to measure the volume of objects in the field of view (FOV) of the camera.

The use of range data acquired via mechanically scanned laser range finder for volume measurement was reported by Zhang et al. [2]. They apply a meshed surface to the point cloud data and from that estimate the volume. We, in contrast, are interested in exploring straightforward methods to volume measurement. For example, if one captures a single view of the scene, then it is relatively straightforward to estimate the volume enclosed by the camera sensor origin to the viewed surface expressed in the camera coordinate system. By subtracting off the volume of the scene with the object of interest removed gives the volume of the object provided there are no concavities in the object that are out of view.

In the following we present two algorithms for calculating the volume of an object in the FOV. All algorithms require a background scene with the object removed. Testing is performed on a number of rectangular solid objects of known volume.

## 2 ALGORITHMS IMPLEMENTED

The basic principle behind the algorithms investigated is that by subtracting the volumes occupied by the surfaces measured from the range images with the object and without the object the volume of the object is determined. This has the advantage that no segmentation of the object is needed. The description of the two algorithms follow.

### 2.1 Algorithm 1

The SwissRanger SR4000 provides range data and pixel coordinates calibrated in metres with the camera sensor at the origin of the coordinate system. It is therefore relatively straightforward to calculate the volume enclosed by the observed scene (treated as a three-dimensional surface) back to the camera sensor. If two views of the scene are captured, one with the object and one without, then the difference between the two volumes calculated is the volume of the object under view (under certain assumptions such as it has no concavities out of view).

To calculate the volume encompassed by the scene and the camera origin adjacent pyramids were formed by choosing three neighbouring pixels in the scene and the camera's origin. By choosing four points the pyramids are all tetrahedra. Three pixels in the viewed scene that form the base of the tetrahedra were always chosen so that the sides of the tetrahedra abut each other so that the full volume can be exactly filled with tetrahedra. In Figure 1 the way three pixels were chosen to make sure that the tetrahedra fill the volume is shown. The total volume is the sum of the volumes of the tetrahedra.

A systematic approach was used to position the tetra-

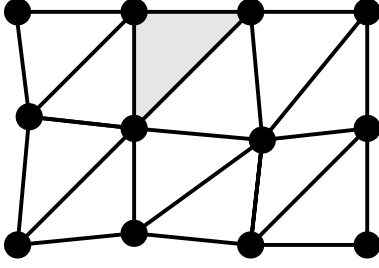


Figure 1: The grid pattern used to form the bases of the tetrahedra for volume measurement. The black dots are the pixels (which are not exactly on a regular grid but are ordered) and the lines represent the base of each tetrahedron.

hedra bases on the scene surface. First two points were selected from the first row in the horizontal direction and a third point was taken as the point in the row below the first point. In this way the selection was done moving in both directions along the  $x$ -axis and along the  $y$ -axis. This way of taking the points ensured that the tetrahedra completely fill the volume with no overlap and no gaps.

The volume  $V$  of each tetrahedron is given by

$$V = \frac{A_b h}{3}. \quad (1)$$

where  $A_b$  is the area of base and  $h$  is the height of the tetrahedron. The area of base here corresponds to the triangle formed by the three points taken on the range image and can be calculated using Heron's (or Hero's) formula.

Consider a triangle whose sides are of length  $a$ ,  $b$  and  $c$ , then the area of the triangle formed by the three points using Heron's formula is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \quad (2)$$

where  $s$ , the semi-perimeter of the triangle, is

$$s = \frac{a+b+c}{2}. \quad (3)$$

The height  $h$  of the tetrahedron is given by the perpendicular distance of the origin from the plane formed by the three scene points forming the base of the tetrahedron. The plane formed by the three base points is

$$Ax + By + Cz + D = 0 \quad (4)$$

with  $A$ ,  $B$ ,  $C$  and  $D$  calculated as follows. Let us take the three points from the range image to be  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$ , then  $A$ ,  $B$ ,  $C$  and  $D$  are

$$A = y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2), \quad (5)$$

$$B = z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2), \quad (6)$$

$$C = x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2), \quad (7)$$

$$-D = x_1(y_2 z_3 - y_3 z_2) + x_2(y_3 z_1 - y_1 z_3) + x_3(y_1 z_2 - y_2 z_1). \quad (8)$$

The height of the tetrahedron, namely the perpendicular distance  $h$  of the point  $P(a, b, c)$  from the base plane is,

$$h = \left| \frac{Aa + Bb + Cc - D}{\sqrt{A^2 + B^2 + C^2}} \right| \quad (9)$$

Summing over all tetrahedra gives the total volume between the camera sensor and the scene. The volume of the object of interest is the difference of the volume calculation with the object in the scene and the volume calculation of the scene without the object.

## 2.2 Algorithm 2

The objective of the second algorithm is to find the dimensions of the object and to calculate the volume from the object's dimensions.

In order to find the dimensions of the object two range images were taken, one with and one without the object. The idea is that only the portions of the range image which changes appreciably from the first to the second range contribute to the volume. This idea works best for the objects with no bulging protrusions.

With the two range images, the  $z$  coordinates of the range image with the object was subtracted from the corresponding  $z$  coordinates of the range image without the object. This results in a new range image, with the same values for the  $x$  coordinates and the  $y$  coordinates but subtracted  $z$  coordinates.

A threshold is applied to the data so that if the  $z$  coordinate's difference is less than the threshold, then it is set to zero, otherwise it is left unchanged. If the  $z$  coordinate of a particular pixel is non-zero after thresholding that means that pixel of the object is sufficiently high and together with the neighbouring pixels contributes to the overall volume of the object.

It is important to choose a threshold not too high else pixels of the object contributing to the overall volume may be missed, likewise it cannot be too small, else it will take background points into the object. A threshold of 0.15 m was used in this project.

The difference image was processed pixel by pixel. Any pixel whose  $z$  coordinate difference is non-zero and whose neighbours have non-zero  $z$  coordinate contribute to the volume calculation. On finding such a pixel a cuboidal block formed by the four pixels (i.e. the pixel and its neighbours) contributes to the volume of the object. The volume of all such cuboids are summed together to give the total volume of the object. Note that this method copes with holes in the object.

The volume of each cuboid is calculated in the following manner. With four neighbouring pixels that have non-zero  $z$  coordinates differences, their average was taken as the height  $h$  of the cuboid. The length  $l$  and breadth  $b$  were taken in the same way by taking the average of the dimensions along the  $x$ -axis and the  $y$ -axis. The volume  $V$  of

the rectangular cuboid is then simply given by

$$V = lbh. \quad (10)$$

### 3 MATERIALS AND METHODS

First the algorithm’s correctness was checked with simulated data for which the volume could be exactly calculated. Range images of surfaces and objects were simulated on Matlab R2010 and then the algorithms were tested using those data as inputs. These confirmed that the algorithms were correct for precise and uncontaminated range data (see Figure 2).

The MESA SwissRanger SR4000 range imaging camera used produces calibrated range data in metres over a  $176 \times 144$  pixel FOV. The camera was mounted on a stand about 800 mm above the table to point directly downwards at the table on which objects were placed. A total of 12 rectangular cuboid objects varying in size from  $4 \times 10^{-4} \text{ m}^3$  to  $0.019 \text{ m}^3$  were imaged and their volumes calculated. The object’s volume were determined first by measurement with a ruler and are listed in Table 1. Some of the objects (being hardcover books) had glossy multi-coloured covers which, as discussed below, caused problems for volume measurement. A second set of images were made of the objects when they were covered in a uniform coloured matt paper.

Table 1: List of the objects imaged.

Object	Volume ( $\text{m}^3$ )
1	$3.93 \times 10^{-4}$
2	$7.17 \times 10^{-4}$
3	$7.60 \times 10^{-4}$
4	$9.55 \times 10^{-4}$
5	$1.1 \times 10^{-3}$
6	$1.1 \times 10^{-3}$
7	$1.5 \times 10^{-3}$
8	$2.2 \times 10^{-3}$
9	$2.5 \times 10^{-3}$
10	$8.02 \times 10^{-3}$
11	$1.62 \times 10^{-2}$
12	$1.89 \times 10^{-2}$

The camera used is an early version of the SR4000 and unfortunately has a calibration error resulting in pixels being misplaced by up to 4cm at the edges of the FOV due to radial lens distortion [3] hence the objects were always placed in the centre of the FOV of the camera to reduce error in volume measurement.

### 4 RESULTS

The plot obtained when the real covered data (i.e. the objects with uniform matt colouring) was given as the input

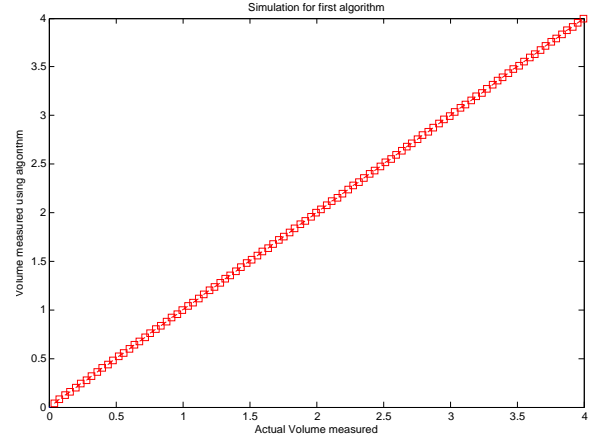


Figure 2: Volume calculation on simulated data using algorithm 1.

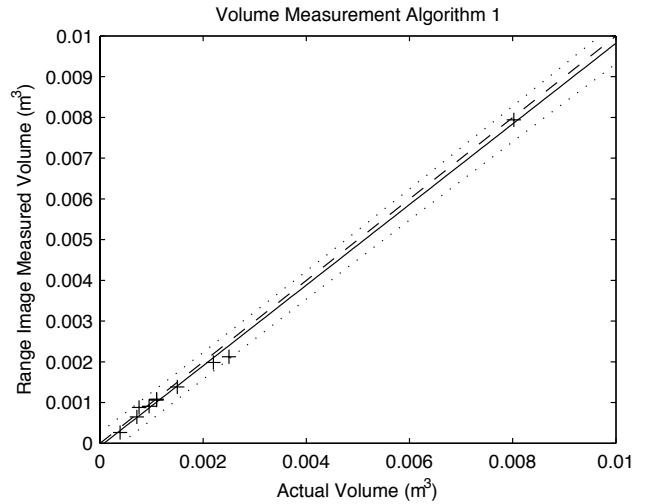


Figure 3: Volume calculation on real covered data using algorithm. 1.

to the first algorithm is shown in Figure 3. The best fit line of the points obtained using the first algorithm is given by

$$y = 0.990x + -7.80137E-05 \quad (11)$$

The second algorithm was tested using covered data and the results are presented in Figure 4. There were some of the points that didn’t lie on the line specially as the size of the objects increased. The behaviour of the algorithm showed similarity with the previous algorithm’s behaviour. The line of best fit to the data is

$$y = 0.944x + 4.17017E-05 \quad (12)$$

From the above obtained results, on comparison of the two algorithms for the covered data, it was seen that the second algorithm gave closer results to the actual volume as compared to the first algorithm. The slope of the best fit

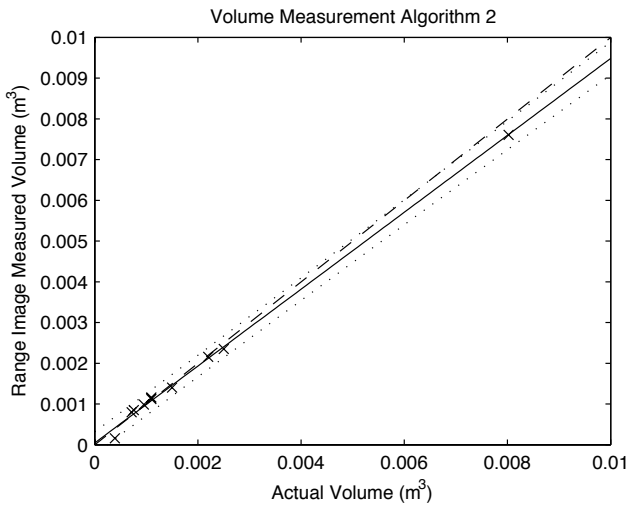


Figure 4: Volume calculation on real covered data using algorithm. 2

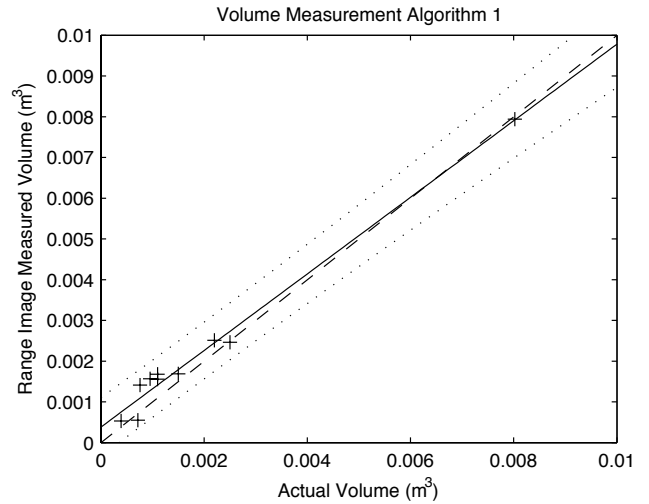


Figure 6: Volume calculation on real covered data when taken from larger height using algorithm. 1.

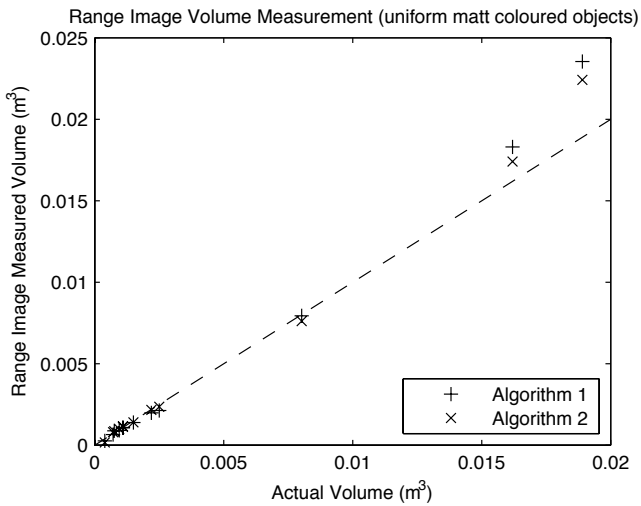


Figure 5: Comparison of two algorithms for uniform matt coloured objects for lesser height of the Camera.

line and offset in second case was found to be more closer to the ideal result than the first algorithm's results. Plot of the comparison for the two algorithms is presented in Figure 5.

To test the robustness of volume measurement, the height of the camera above the table was increased and the captures of the objects repeated. Results were obtained for the uniformly coloured objects and are presented in Figure 6 for the first algorithm. The best fit line of all the points obtained was

$$y = 0.940x + 0.000379772 \quad (13)$$

which shows a little deterioration from the original measurements.

For the second algorithm with camera at higher height,

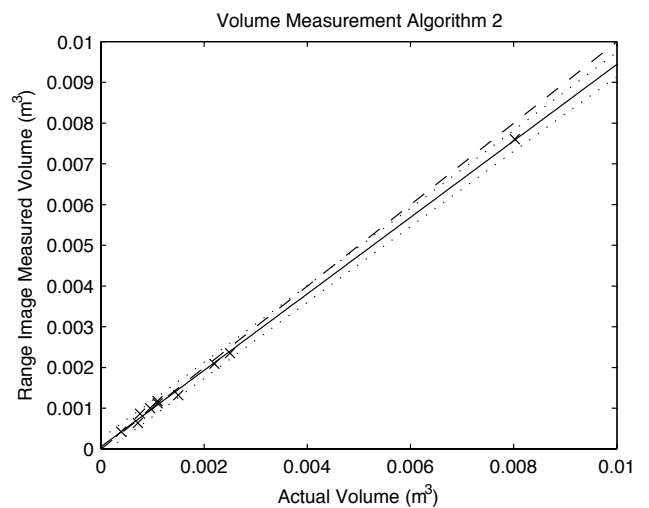


Figure 7: Volume calculation on real covered data for larger height using algorithm. 2.

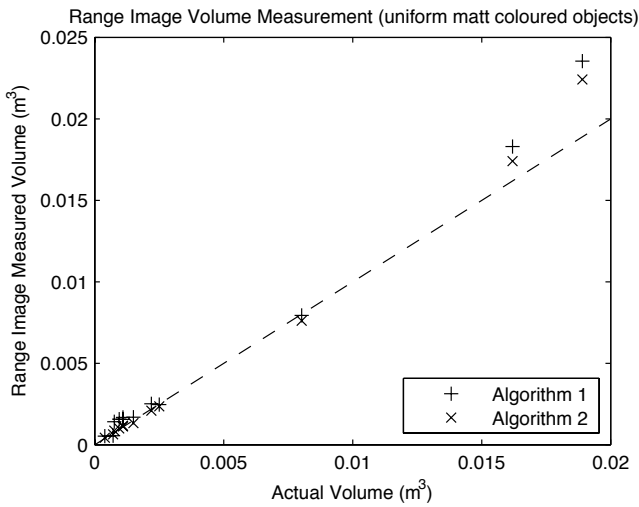


Figure 8: Comparison of algorithms for uniform matt coloured objects for higher height of the Camera.

using the covered data, the behaviour obtained is shown in Figure 7. The equation of best fit line for all the data points is close to that obtained for the lower height of the camera and is given by Eqn. 14.

$$y = 0.939x + 5.18871e - 05 \quad (14)$$

On comparison for the algorithms for higher height of the camera, it was again found that algorithm. 2 gave much accurate results than algorithm. 1. Plot of the comparison is presented in Figure 8.

The volume measurement was repeated on the image data captured of the objects when their somewhat glossy and colour surfaces were exposed. The volumes obtained using both the algorithms are considerably noisy as shown in Figure 9. The results depended on the colour of the surface and so the behaviour gave quite a lot of error in the calculation.

As seen in the Figure.9, the error is high and the results show quite a lot of deviation. It is seen that the algorithms seem to be following the theoretical predictions as expected earlier for the covered data quite nicely. But when it came to uncovered data, the results varied drastically and contained high errors.

## 5 DISCUSSION

From the above practical experiments and the results observed, quite a number of conclusions can be made. It is seen that the algorithms developed worked quite nicely and gave results quite close to the actual volumes when the range images were taken of objects that are of uniform colour and of matt surfaces. Moreover it is seen that the colour of the sheets used to cover the objects had to be white in order to reduce the errors. In case of multi-coloured and

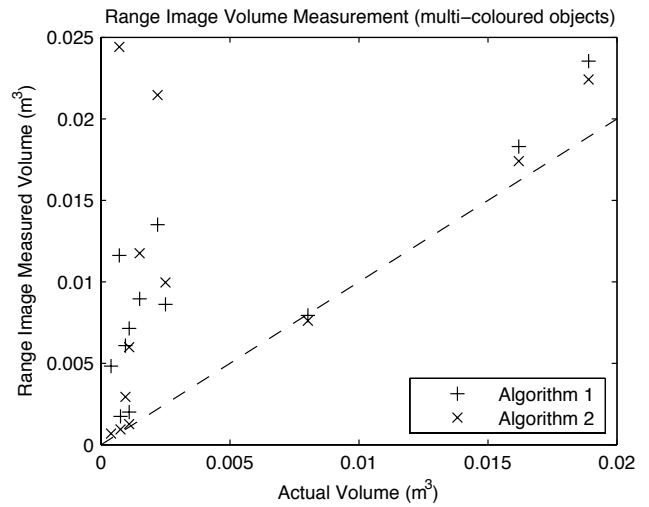


Figure 9: Volume calculation comparison on real uncovered data taken from larger height of the Camera.

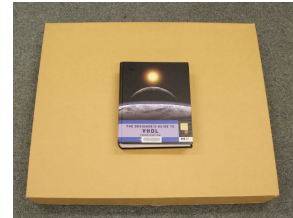


Figure 10: Book without shiny surface.

somewhat glossy surfaces, the results deviated far from the ideal and volume measurement was unreliable.

Range imaging data captured with full-field range imaging cameras is subject to multipath effects. This is where light from some other part(s) of the scene makes its way into the wrong pixels, whether by multiple scattering of light within the scene, or scattering of light within the camera optics. This contamination causes phase errors in the active light modulation received back at the camera. The phase error leads to a misdetermination of the range of the object in the view of the pixel. If the view of the object is bright in the pixel then the error introduced into the range is negligible, but if the view of the object is dark in the pixel then the error in ranging can be appreciable, even centimetres in extreme cases. It is likely that this is occurring in the scenes analysed, however the authors do admit they are surprised by the magnitude of the effect on volume measurement and wonder whether other factors are in play.

The errors in volume measurement are reduced immensely when the surface of the objects are covered in uniform white paper. It was also observed that the light coloured books or the less shiny ones gave results closer to the actual volume than the darker and the shiny ones. For example, the experiments performed on the book shown

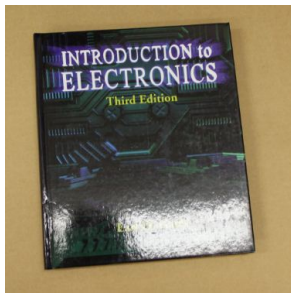


Figure 11: Book with specular reflection due to shiny surface.

in Figure 10 gave the volume that was correct up to three decimal places when using the second algorithm.

In contrast the book shown in Figure 11 with darker shade and noticeable specular reflection of light due to a glossy surface had a large error as measured using the second algorithm. Erroneous volume measurements occur because of the specular reflections occurring when the light from the camera strikes the surface of the object. So the reduction of specular reflections using white sheets as the surface of the objects helps in the reduction of errors.

One more interesting thing can be observed in the above experiments. It can be seen that the algorithms may not provide the correct results for the objects with certain parts coming out of it and not touching the surface like the protrusions bulging out. In such situations the volume measured using the second algorithm will also include the volume of the space between that part and the surface which may introduce a lot of error in the results.

When the object has a hole in it, then again there will be inclusion of some more errors arising due to the boundaries of the hole. But the volume calculation yields closer results. The error in second algorithm again is less and provides closer results.

Therefore, it can be concluded that the above developed algorithms works best with very less error when the surface of the objects are covered with sheets having white colour without any shine. Presence of any one of them will deviate results and will include quite a lot of error in them.

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