Waikato Journal of Education 15, Issue 3: 2010

CONCEPTIONS, LANGUAGE, CULTURE AND MATHEMATICS AND THE NEW ZEALAND CURRICULUM

JUDY BAILEY AND MERILYN TAYLOR
Faculty of Education
The University of Waikato

ABSTRACT *In this paper we explore two conceptions of mathematics that are evident in literature. Mathematics as a static body of knowledge is one conception, and another is of mathematics being an endeavour that is constructive and creative. With the release of* The New Zealand Curriculum *(Ministry of Education, 2007), we formalised our ongoing debates about these conceptions by constructing and using metaphors to explore and refine our ideas. In some parts of the curriculum document, there is juxtaposition with a conception of mathematics being social, constructive and creative. However, other aspects of the document appear to reflect the conception of mathematics as a body of knowledge. We express a concern that this latter conception of mathematics may become privileged over other possibilities. We therefore explore what constructive, creative mathematics might look like in a classroom.*

KEYWORDS

Conceptions of mathematics, curriculum, mathematics education

INTRODUCTION

At the beginning of 2008, a revised curriculum document (Ministry of Education, 2007) was released into New Zealand schools. It is due for mandatory implementation in 2010. With the release of this document, we, as two initial teacher educators, decided to formalise our ongoing debates about conceptions of mathematics. We were mindful of Barker's (2008) contention that forming personal perceptions of curriculum documents is an important aspect of enacting curriculum change. We thought it would be worthwhile to consider these conceptions with particular reference to the revised document. It is considered that conceptions held by individuals have a significant impact on the teaching and learning that occurs in classroom settings (Dossey, 1992), and as such are important to identify and reflect upon.

Different conceptions of mathematics are evident in mathematics and associated education literature (Dossey, 1992). As part of thinking about and examining two of these conceptions, we have found it useful to create metaphors. These metaphors have enabled us to explore and refine our deliberations and have also given us a lens with which to examine the New Zealand curriculum document (Ministry of Education, 2007). In addition, we have considered language and culture as integral aspects of both conceptions.

A static body of knowledge that exists separately to one's self is one conception of mathematics (Fisher, 1990). This body of knowledge is considered to contain ideas to do with quantity, patterns, shape and space, and chance. From this perspective, the role of the learner is to try to "grasp hold of and hang on to" the mathematics ideas. For the purposes of this discussion, we will use the metaphor *suitcase mathematics* to encompass this conception.

Another conception is mathematics as a constructive, creative, experiential human endeavour (Dossey, 1992; Mason, 2008). Our metaphor for this conception is that of an open space where dancers explore ideas about quantity, patterns, shape and space, and chance with other dancers (learners of mathematics). In this discussion, we will use the metaphor *dancing mathematics* to illustrate this conception.

We recognise that these two conceptions are part of a continuum of ideas and beliefs about the nature of mathematics. We also acknowledge the possibilities for interplay between various conceptions. In this paper, however, we focus on these two conceptions as discrete entities, and have done so for two reasons. Firstly, these two conceptions were prevalent in the literature we encountered; and secondly, we have a concern that one conception (with its subsequent impact on teaching practice) may be privileged over another. We therefore think that an examination of each conception as a separate entity is more fruitful as a way of developing an understanding of possible impacts on curriculum.

SUITCASE MATHEMATICS

We have found conceptions of suitcase mathematics in a variety of literature. For example, Kline (1967) refers to the complex nature of mathematics and posits that one conception is "a body of knowledge about number and space" (p. 3). Gullberg (1997) states that mathematics is often confused with arithmetic—this might suggest that a suitcase of mathematics primarily holds ideas about number and its associated operations. Another idea is mathematics being regarded as "something to be studied as an object rather than something to be used" (Begg, 2009, p. 13). Prochazka (2008) also attempts to answer the question, "What is mathematics?", and refers to the definition of mathematics found on Wikipedia, which states, "mathematics is the body of knowledge, centred on such concepts as quantity, structure, space and change, and also the academic discipline that studies them" (p. 25).

It seems that such a conception of mathematics has its roots in the Platonist view of mathematics where it has an existence of its own and is thought to be an ideal world waiting to be uncovered (Barton, 2008; Dossey, 1992). Plato also stated that mathematics was the highest expression of human thought (Barton, 2008)—an instance where mathematics appears to be almost reified and disconnected from everyday life and people. This belief is not uncommon. By four years of age, even some children have learned that mathematics is often considered to be special, different and that others have the power and control of what mathematical ideas are to be encountered (Hughes, 1986).

We believe such descriptions could lead one to envisage mathematics as separated from the self, where mathematics is "picked up" and "put down" according to circumstance. It is not an integral or lived experience. The Platonist view appears to place mathematics on a "mountain peak", accessible to only a few. We have therefore proposed a metaphor of suitcase mathematics as a "container" of ideas that have to be "grasped" and learned. The container remains separate from the individual, and the ideas are confined within set boundaries.

DANCING MATHEMATICS

Another conception presents mathematics as an endeavour that is social, constructive and experiential (Barton, 2008; Mason, 2008; Solomon, 2009). For example, Mason suggests mathematics can be "a constructive and creative enterprise" (2008, p. 4). Barton proposes that "mathematics is created by communication" (2008, p. 144) and suggests that it emerges from communication with others. These considerations ascribe to a notion of mathematics that is not separate to one's self, nor is it finite, but a creation that is "never finished, never completed" (Barton, 2008, p. 144).

Just as our suitcase conception of mathematics appears to be aligned with a Platonic view of mathematics, this different conception could be aligned with an Aristotelian view. This view contends that mathematics knowledge involves experimentation, observation, abstraction and construction (Dossey, 1992). We have come to imagine this conception of mathematics as one of freely expressed dance where one creates and explores (dances), communicates with others (other dancers and/or an audience), and works alongside others to create expressive works. There are parameters that define this discipline of dance (i.e., mathematics) with boundaries that can be pushed. The parameters within which this mathematics is created are ideas to do with quantity, patterns, shape and space, and chance; and these ideas are considered to be in constant generation by the dancer (i.e., learner).

LANGUAGE, CULTURE AND CONCEPTIONS OF MATHEMATICS

Language and culture are an integral part of both conceptions of mathematics. Language used in mathematics encompasses many different communication modes, including oral, visual, digital, textual and symbolic forms (Zevenbergen, Dole, & Wright, 2004). Barton (2008) argues that a particular type of mathematical language is used in schools and uses the term "near-universal conventional" mathematical language. This "near-universal conventional" language has its origins in the Indo-European history of mathematics and has privileged the use of English (Barton, 2008).

The acceptance of "near-universal conventional" language and mathematics concepts has, by their inclusion, excluded others. Barton contends that mathematics could have been different and taken many alternative forms. This suggests that the currently accepted mathematics conventions were not inevitable and are "the result of a particular historical trajectory that includes many social influences, including language" (Barton, 2008, p. 24).

As mathematics evolves, language informs and constrains new ideas. Sometimes, new language has to be developed to adequately express newly created concepts (Barton, 2008). Over time some ideas will become a part of the suitcase contents, taken for granted and unquestioned. For example, early number systems did not include the concept, symbol or the language for zero. Over some hundreds of years, the symbol and meaning of zero gradually became accepted (Parameswaran, 1999). It would now be unthinkable for us to have a number system without zero. On the other hand, whilst new ideas are formed, other ideas already in the suitcase may become obsolete as the need for them ceases to exist. For example, the use of feet and inches for measurement is diminishing.

We contend that the language used in a mathematics classroom will be both a product and a reflection of the conceptions of mathematics being enacted. If the expected forms of communication are already well formed and being transmitted, there will be few opportunities to experiment, abstract and generalise (Dossey, 1992). The development of "zero" is an example of how dancing mathematicians created a significant mathematical concept. Whilst initially created by dancing, the language and concept of zero has become part of the suitcase contents. Because the need for zero is now taken for granted, learners are seldom given the opportunity to create this notion for themselves.

Language and culture are inextricably linked (Bishop, 1988). If it is accepted that mathematics and language are connected, then mathematics is also culturally bound (Barton, 2008; Solomon, 2009). One aspect of this notion is that mathematical concepts can be developed in response to differing cultural needs. For example, the use of locating methods, including using stars and detailed topographical knowledge of the sea, was important mathematics for Polynesian navigators—a key factor in their survival (Lewis, 1972). Additionally, mathematical concepts can be conceived of in different ways according to cultural understandings. For example, numbers are thought of as "actions" in one culture, whereas they can be seen as "things" in a different culture (Barton, 2008).

NEW ZEALAND CURRICULUM DOCUMENT

We will now move to an analysis of parts of the New Zealand curriculum document in relationship to the two conceptions of mathematics and include considerations of language and culture. In the document, directions for learning are provided through a "vision" statement that aims to support "young people [to] be confident, connected, actively involved lifelong learners" (Ministry of Education, 2007, p. 7). This vision is articulated through a stated set of values (for example, excellence; and innovation, inquiry and curiosity) and key competencies (for example, thinking). These values and key competencies are expected to be an integral part of all learning areas (for example, English, the arts, mathematics and statistics). Each learning area is described in a specific statement, and supported with a set of achievement objectives for years 0–13 of schooling. All of these aspects (i.e., the vision, values, key competencies and learning areas) are connected to a set of principles. These include having high expectations for learners and acknowledging the principles of the Treaty of Waitangi. We arbitrarily clustered these aspects of the New Zealand curriculum document into two groups, (principles, values and key competencies; and learning areas and achievement objectives) and considered how *some* aspects might align with the two conceptions of mathematics. Within this process we moved to also thinking about the possible implications for pedagogical practice.

Principles, values and key competencies

It is interesting to note that aspects of the principles, values and key competencies align with our metaphor of dancing mathematics where mathematics is viewed as a constructive and creative endeavour. The principles and values call for an awareness of, respect for, and a response to equity through fairness, social justice and cultural diversity. The negative impact that mathematics has traditionally had on various groups of students, including minority and indigenous learners, has been well documented (see for example, Barton, 2008; Civil, 2009; Ohia, 1995; Schoenfeld, 2008). A question that then emerges is "how do we respond to a challenge for an equitable and culturally responsive mathematics?"

As previously mentioned, it is recognised that mathematics arises in response to different contexts, and thus mathematics can vary from one cultural context to another (Barton, 2008; Bishop, 1988; Barton & Fairhall, 1995). Viewing mathematics as a constructive, creative endeavour allows us to respond to the quest for an equitable and culturally responsive mathematics by acknowledging and experiencing the mathematics of different cultures in classrooms (Ohia, 1995). For example, in some parts of Alaska and in the Maldives, the mathematics of the indigenous culture is being incorporated into the mathematics curricula (Barton, 2008). Ohia (1995) states that Māori art, crafts and cultural artefacts can be aligned with mathematical topics and concepts.

The New Zealand curriculum document (Ministry of Education, 2007) identifies innovation, inquiry and curiosity as desired values. These values are expected to be an integral part of all learning areas, including mathematics and statistics. Dancing mathematics appears to be the conception that could more easily align with this position. When mathematics is viewed as a constructive, creative endeavour, then there is space for learners to be innovative, inquisitive and curious. For example, it is possible to engender curiosity by using the calculator to explore number patterns. In comparison, a suitcase conception of mathematics might see the calculator being used as a checking (or calculating) device.

Also of interest are the key competencies. Thinking, as one of the key competencies, is described in part as "using creative, critical, and metacognitive processes to make sense of information, experiences, and ideas" (Ministry of Education, 2007, p. 12). We are presented with a picture of actively engaged learners constructing and making sense of their own ideas; and developing a level of awareness of their own thinking (McChesney & Cowie, 2008). This picture aligns with the dancing conception of mathematics. Learners would also be expected to make sense of information in a classroom where a suitcase conception is being enacted, but creative and critical processes are likely to be constrained. Learners may not have the opportunity to become aware of possible alternative views in mathematics. For example, in this setting, children would be learning pre-defined algorithms and strategies, rather than constructing or discovering these for themselves.

Learning areas and the achievement objectives

"Mathematics and statistics" is one of the eight specified learning areas in the New Zealand curriculum document. Mathematics is described as "the exploration and use of patterns and relationships in quantities, space and time" (Ministry of Education, 2007, p. 26). This statement appears to be aligned with both conceptions of mathematics. The suggestion that mathematics involves "exploration" invites a conception of dancing mathematics. The next words in the description of mathematics, "and use of", seem to indicate a mathematics already explored and waiting to be "opened"—the suitcase conception. The learning area statement then declares "by studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically" (2007, p. 26). This reiterates the notion of mathematics as a creative endeavour.

The achievement objectives "set out selected learning processes, knowledge, and skills relative to eight levels of learning" (Ministry of Education, 2007, p. 39). A statement prefaces each of the eight levels in mathematics and statistics and reads, "in a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to …" (Ministry of Education, 2007, Mathematics and Statistics achievement objectives). This statement is then linked to lists of achievement objectives arranged into three strands: number and algebra; geometry and measurement; and statistics. The emphasis of each of the three strands changes according to the level; for example, number and algebra is presented as the predominant strand at the early levels in particular.

The prefacing statement indicates that students will be engaged in thinking, solving problems and modelling situations. This presents mathematics as an active and creative endeavour. However, the achievement objectives appear to be a mixture of tightly defined and prescriptive objectives and more open, exploratory dancing statements of intent. One example of a prescriptive achievement objective at level 1 is "know the forward and backward counting sequences of whole numbers to 100" (Ministry of Education, 2007, Mathematics and Statistics Achievement Objectives, Level 1). This could be aligned with a suitcase conception of mathematics. A more exploratory achievement objective at level 1 is "create and continue sequential patterns" (Ministry of Education, 2007, Mathematics and Statistics Achievement Objectives, Level 1).

In our view, whilst there are some possibilities for creative mathematics at level 1 in the achievement objectives in the New Zealand curriculum document, the opportunities to engage in such mathematics seems to diminish as children move through primary school. We wonder if an increasing number of more prescriptive achievement objectives could perpetuate the suitcase conception of mathematics and create a difficulty for envisaging a constructive, creative mathematical world.

For the final section then, we consider what dancing mathematics could look like in a classroom.

DANCING MATHEMATICS IN A CLASSROOM: CONSTRUCTIVE AND CREATIVE MATHEMATICS

In a constructive and creative mathematics classroom, we envisage learners being engaged in open-ended investigative tasks where there are a number of possible avenues to explore. An example might be adapting the use of a question such as "Milly has three horses, and she is given three more. How many horses does Milly have?" to "there are six horses in two paddocks: where might the horses be?" For a five-year-old, such an open-ended investigative task might involve finding a family of facts for the given number; exploring the relationship between addition and subtraction and the ways a total can be formed from different groups; and the commutative property of addition and subtraction. In this situation learners would be able to explore the particular avenues of mathematics that they encounter.

When mathematics is experienced as a constructive, creative enterprise, there is a focus on "sense making". Mason (2008) proposes "sense making" is the most important and possibly neglected part of any pedagogical activity. In a dancing classroom, it is the learners who are supported by the teacher to make sense of their own explorations, rather than following particular procedures. In this setting, learners are making connections between their world and the ideas being constructed in their minds.

Another strategy to support "sense making" in a dancing classroom is creating a place where there is room for children to engage in "exploratory talk". Exploratory talk is where children are critically examining and reasoning about their ideas (Mercer, 2000). For example, in a dancing classroom, learners would be expected to explain and justify, carefully listen and thoughtfully respond to each other's mathematical ideas (rather than personalities). There also needs to be the possibility for argument and challenge between teacher and learner (Solomon, 2009). These suggestions resonate with Claxton's (2008) premise that learners need to be engaged, extended and the limits of their understanding stretched.

In order to develop such exploratory talk, teachers need to be careful of their use of language and the possible unintended messages that particular phrases can convey. For example, avoiding the use of "yes, you are right" or "no, that's wrong' (Barton, 2008). Rather, process phrases such as "tell me what you are thinking" can be used (Reinhart, 2000). Developing children's exploratory talk requires considerable input from the teacher (Hunter, 2009). Time is needed to develop such communication skills and employ this way of supporting learners to become dancing mathematicians.

In addition to the communication and exploratory talk that happens in a dancing classroom, there is also the need for silence. Knoll (2008) suggests that "silence … is what can link doing and creating with learning … in mathematics" (p. 131). Rather than teachers telling learners the answers, they need to resist the temptation to take over, and provide times for silence (Knoll, 2008). Such "silences" offer the learner an opportunity to explore his/her own ideas and find their own mathematical directions.

Another attribute of the dancing classroom' is "playing" with mathematical ideas. Barton (2008) states that we expect "exploratory and playful mathematical activity in very young children, and in advanced research mathematicians, but in between we sit students down to do exercises and listen to teachers or lecturers explain how it is" (p. 149). Our analysis of the level 1 to 4 achievement objectives in the New Zealand curriculum document would support Barton's contention. In the dancing classroom then, we advocate for physical and abstract play to occur at all levels: not just as a "starter" activity or a "game", but creating an environment focused on playing with mathematical ideas. For example, children could be investigating their ideas about the nature of mathematics by taking photographs.

Mason (2008) contends that looking back and reflecting are vital components of learning. We suggest that time needs to be provided for learners to record, generalise and abstract their own ideas. This would provide the learner with an opportunity to more formally and thoughtfully explain the dance of mathematics they have created. In the dancing classroom, teachers and children could identify the specific learning intentions at the *end* of a unit. This would be an alternative to prefacing a lesson with a teacher-chosen learning intention. Rather, the children could identify their learning after the ideas have been explored. Given our first suggestion of using open-ended investigations, each child's "I have learned to …." statement may be different.

EMERGING CONSIDERATIONS AND QUESTIONS

The conceptions of mathematics that a teacher holds, consciously or otherwise, has a significant impact on the way in which mathematics is taught (Beswick, 2006; Dossey, 1992; Neyland, 1995; Solomon, 2009). Teachers who primarily hold a conception of mathematics as a static body of knowledge will likely employ a more transmission-style of teaching (Dossey, 1992). In this situation, inflexible classroom conditions may result and will not reflect the way in which mathematicians work (Davis, Sumara, & Luce-Kapler, 2000). For those who believe that mathematics is what mathematicians do, teaching will be focused on the social creation of mathematical ideas (Neyland, 1995). According to Anthony and Walshaw (2007), quality teaching results in children being "legitimate creators of mathematical knowledge" (p. 63).

In this article, we have considered aspects of the New Zealand curriculum document (2007) with reference to two conceptions of mathematics. We have also considered attributes of language and culture with respect to mathematics. It is apparent that the suitcase and dancing conceptions can be juxtaposed with differing parts of the New Zealand curriculum document. We wonder if this could perpetuate mathematics being presented as a static entity, thereby creating a challenge for envisaging a mathematical world beyond a suitcase.

We find ourselves left with a range of questions. These include:

- What conception of mathematics is being enacted in a classroom? Is it the mathematics contained within a suitcase? Is there potential for children to dance mathematics?
- Given that the curriculum document expects certain mathematical ideas to be taught, how does this impact on the teaching and learning of mathematics?
- Can we meet Claxton's (2008) challenge to support learners to be discoverers and explorers?

REFERENCES

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy/Pangarau: Best evidence synthesis iteration.* Wellington, New Zealand: Ministry of Education.

Barker, M. (2008). The New Zealand curriculum and pre-service teacher education: Public document, private perceptions. *Curriculum Matters, 4,* 7–19.

Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. New York, NY: Springer.

Barton, B., & Fairhall, U. (1995). Is mathematics a Trojan horse?: Mathematics in Māori education. In B. Barton & U. Fairhall (Eds.), *Mathematics in Māori Education: A Collection of Papers for the History and Pedagogy of Mathematics Conference* (pp. 1–11). Auckland, New Zealand: Mathematics Education Unit, The University of Auckland.

Begg, A. (2009). Learning tasks: More than content. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice*. Wellington, New Zealand: NZCER Press.

Beswick, K. (2006). The importance of mathematics teachers' beliefs. *Australian Mathematics Teacher, 62*(4), 17–22.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publications.

Civil, M. (2009). Mathematics education, language and culture: Ponderings from a different geographic context. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing Divides: Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 129–136). Brisbane, QLD, Australia: MERGA.

Claxton, G. (2008). *What's the point of school: Rediscovering the heart of education.* Oxford, England: Oneworld Publications.

Davis, B., Sumara, D., & Luce-Kapler, R. (2000). *Engaging minds: Learning and teaching in a complex world*. Mahwah, NJ: Lawrence Erlbaum.

Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39–48). New York, NY: Maxwell Publishing.

Fisher, C. (1990). The Research Agenda Project as prologue. *Journal for Research in Mathematics Education, 21,* 81–89.

Gullberg, J. (1997). *Mathematics: From the birth of numbers.* New York, NY: W. W. Norton.

Hughes, M. (1986). *Children and number: Difficulties in learning mathematics*. Oxford, England: Basil Blackwell.

Hunter, J. (2009). Developing a productive discourse community in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing Divides: Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 249–256). Brisbane, QLD, Australia: MERGA.

Kline, M. (1967). *Mathematics for the nonmathematician.* New York, NY: Dover Publications.

Knoll, E. (2008). Silence! Reviving an oral tradition in mathematics education. In B. Warland (Ed.), *Silences in teaching and learning in higher education* (pp. 131–133). Ontario, Canada: Society for Teaching and Learning in Higher Education.

Lewis, D. (1972). *We, the navigators: The ancient art of land finding in the Pacific.* Canberra, ACT, Australia: Australian National University Press.

Mason, J. (2008). *ICMI Rome 2008: Notes towards WG2.* Retrieved July 16, 2008, from www.unige.ch/math/EnsMath/Rome2008/WG2/Papers/MASON.pdf

McChesney, J., & Cowie, B. (2008). Communicating, thinking, and tools: Exploring two of the key competencies. *Curriculum Matters, 4,* 102–111.

Mercer, N. (2000). *Words and minds: How we use language to think together.* London, England: Routledge.

Ministry of Education. (2007). *The New Zealand curriculum*. Wellington, New Zealand: Learning Media.

Neyland, J. (1995). Beliefs and values in mathematics education: An outline of Ernest's model. In J. Neyland (Ed.), *Mathematics education: A handbook for teachers* (Vol. 2, pp. 139–149). Wellington, New Zealand: Wellington College of Education.

Ohia, M. (1995). Maori and mathematics: What of the future? In B. Barton & U. Fairhall (Eds.), *Mathematics in Māori education: A Collection of Papers for the History and Pedagogy of Mathematics Conference* (pp. 33–38). Auckland, New Zealand: Mathematics Education Unit, The University of Auckland.

Parameswaran, S. (1999). A self-portrait by zero. *Mathematics Teaching, 168,* 35.

Prochazka, H. (2008). What is mathematics? *Australian Mathematics Teacher, 64(*3), 25.

Reinhart, S. C. (2000). Never say anything a kid can say. *Mathematics Teaching in the Middle School, 5*(8), 478–483.

Solomon, Y. (2009). *Mathematical literacy: Developing identities of inclusion.* Oxford, England: Routledge.

Zevenbergen, R., Dole, S., & Wright, R. J. (2004). *Teaching mathematics in primary schools.* Crows Nest, NSW, Australia: Allen & Unwin.