Abstract
We consider a two region growth model with vertical innovations where technical externalities in
R&D lead to a technology leading region being the most attractive location for innovative firms.
Innovations are produced in the form of quality improvements building on available knowledge and
firms choose a technologically advanced location to maximise the productivity of R&D and maintain
their niche monopoly. The partial nature of spillovers causes an additional force for agglomeration:
the clustering effect. Agglomerated locations have the benefit of local inter-varietal knowledge
spillovers for growth while peripheral locations depend on trade and regional knowledge spillovers.

Key Words
endogenous growth
innovation
knowledge spillovers
agglomeration
economic geography
quality ladders
creative destruction

JEL Classification
F12; O41; R10

Acknowledgments
The author would like to thank the Royal Society of New Zealand Marsden Fund and the University
of Waikato for financial support. Thank you to Prof Jacques Poot and Prof Les Oxley of the
University of Waikato, Prof Philip McCann of the University of Groningen, Dr Tim Denne and Dr
John Small of Covec, Prof Henri de Groot of VU University Amsterdam and an anonymous referee
for comments on earlier drafts of this paper. Also thank you to comments from participants at the 3rd
Global Economic Geography Conference in Seoul Korea, the Pacific Regional Science Conference
(PRSCO) in Seoul Korea and the North American Regional Science Conference (NARSC) in Miami
Florida. All remaining errors are my own. The views expressed in this paper are those of the author
and do not necessarily represent those of the Waikato Management School, The University of
Waikato or Covec Ltd.
1. Introduction

Endogenous growth models rely on technical externalities such as knowledge spillovers. These theoretical models have been extended by applying spatial factors to knowledge spillovers, resulting in models where growth affects location and location affects growth (Baldwin et al., 2003). Technical and pecuniary externalities influence firm location decisions, resulting in forces for agglomeration and spatial effects on growth. Existing models conclude that growth in new products is greater in agglomerated locations with more intensive manufacturing and research and development (R&D). Given the evidence that technical externalities such as knowledge spillovers are related to the spatial distribution of economic activity, and following the same Marshallian logic, it is also reasonable to assume that firm location is similarly influenced by the technology or quality levels of nearby manufacturers, not just agglomeration. Empirical evidence also shows that productivity growth is related to the spatial distribution of productivity levels (Coe and Helpman, 1995) and R&D (Keller, 2002). Yet existing spatial models of growth rely on increasing product variety and offer no avenue to consider technology or quality levels in nearby locations as a factor in firm decisions.

We introduce technological knowledge to the spatial growth model by using a quality ladders or creative destruction approach to innovation and growth. We take account of spatial externalities in knowledge spillovers such that there is greater knowledge available to R&D for innovative firms who are located in regions that are more technologically advanced. As a result there is a new force for agglomeration we term the clustering effect; firms prefer to collocate in a region where other firms are technologically advanced. Firms in these locations are more innovative than firms in less advanced locations further encouraging firms to locate in the technologically advanced location. As such, a location can emerge as technologically advanced and all firms’ favored location for R&D. The model leads to new impacts on the steady state that depend upon the level of knowledge spillovers and the relative technology levels between locations.

The model proposed here adds to the literature by using vertical innovation and creative destruction (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Young, 1998) instead of love-of-variety, within a two region growth model so that firms also make decisions based on the technology level. The model recognises the spatial characteristics of knowledge spillovers and innovation.

This type of model can help explain spatial variation in economic growth outcomes and technology levels in highly developed, integrated markets with similar institutions. These markets typically share institutions and economic policy, labour is mobile between regions, there is significant inter-regional trade, yet different regions have widely varying economic growth outcomes and production technologies. Existing models don’t describe the effects of integration on technology levels or the emergence of technologically advanced firms and locations. The model here rectifies this by including spatial technical externalities in the quality ladders model of growth.

Background

New growth theory does not provide an adequate explanation for differences in growth and technology when regions are so similar. New growth theory examines growth by making innovation endogenous in the growth process and using increasing returns as an incentive for innovation. Romer (1990), Lucas (1988), Aghion and Howitt (1992) and Barro (1990) treat investment in innovation as an additional type of capital. These investments may be in R&D, human capital, or other initiatives that assist with innovation. While Grossman/Helpman/Romer models (Romer, 1990; Grossman and Helpman, 1990) use a love of variety and Dixit-Stiglitz (1977) competition as the source of growth, Schumpeterian growth models (Aghion and Howitt, 1992) suggest creative destruction as the “norm of capitalism” (McCraw, 2007) where new versions replace old varieties. These quality ladders/creative destruction approaches might help explain the emergence and growth of technologically advanced agglomerations. But the existing literature does not adequately take account of how technology and spatial knowledge spillovers influence innovation and growth, leading to technologically advanced firms and locations.
With a product variety approach, the existing regional growth models are inadequate for describing the emergence and growth of innovative and technologically advanced locations. Both static and dynamic models of trade and economic geography have advanced our understanding of the impact of integration on firm location. Regional models of growth attempt to explain variations in economic growth between regions and nations through both pecuniary and knowledge spillover externalities due to geographic space. Several models combine horizontal innovations à la Grossman/Helpman/Romer with technical and pecuniary externalities over space (Martin and Ottaviano, 1999; Martin, 1999; Baldwin and Forslid, 2000; Baldwin et al., 2001; Fujita and Thisse, 2003; Hirose and Yamoto, 2007) predominantly due to the similarities in modelling techniques with the New Economic Geography and the fundamental use of Dixit-Stiglitz competition. The different models vary assumptions on the mobility of capital, labour and intermediate vs consumer demand to influence the forward and backward linkages. While the literature has successfully developed spatial models of growth, these models all represent growth through increasing product variety, relying heavily on Dixit-Stiglitz (1977) competition.

Knowledge spillovers are the transfer of knowledge between industries or locations where the knowledge is used as an input to new innovations. Endogenous growth models assume these occur easily and empirical studies suggest that technological spillovers between locations or industries are an important component of the growth process (Coe and Helpman, 1995). Virtually all models in new growth theory rely on technical externalities such as knowledge spillovers and production externalities. Eaton and Kortum (1999) show these externalities are related to the distribution and location of manufacturing and R&D activities. A better understanding of the economics of innovation (Nelson, 1993) and the distance characteristics of knowledge spillovers will significantly improve our understanding of economic growth.

Innovation is a predominantly local event. McCann (2007) describes the importance of face-to-face interaction to innovation and knowledge spillovers. Acs and Varga (2002) and others note the similarities between modelling techniques of new growth theory and the core-periphery model. They also recognise the economics of innovation could be incorporated into regional growth models suggesting a new model of technology-led regional economic development which combines the two fields with insights from the economics of innovation. A regional quality ladders model of endogenous growth offers significant insight into the emergence and growth of technology leading firms and locations.

Endogenous growth models usually assume a frictionless spillover of knowledge to all firms and agents. The economics of innovation shows us that knowledge is not transferred so effortlessly and innovations are spatially concentrated. While some knowledge can be codified and transferred easily, much knowledge is at least partially tacit and spillovers of tacit knowledge occur over space and time through face-to-face contact (McCann, 2007), interaction and migration (Faggian and McCann, 2009). This restriction for knowledge to be only partially codified means knowledge and innovation has space, time and cost characteristics in its spillover between agents, firms and in particular, between locations. This role of space and time in knowledge spillovers means that economic growth also has space and time characteristics, contributing to a wide variance in innovation activity between different cities, regions and nations.

Given this understanding of innovation, the concentration of economic activity also results in greater knowledge spillovers between spatially concentrated firms. There is greater interaction between agents of these firms and a greater number of transfers of employees between firms resulting in higher spillovers. Agglomeration economies in innovation are an example of the geographic nature of knowledge spillovers (Audretsch and Feldman, 1996).

The integration of economies can affect the international or regional location of industry. The new economic geography suggests that imperfect integration may create regional winners and losers (Krugman, 1991; Krugman and Venables, 1995). A particularly interesting characteristic of the new economic geography models is that the economic conditions of two regions can be exactly the same, yet yield dramatically different economic outcomes.

_A Regional Model of Growth with Creative Destruction_
The model here includes migration of knowledge workers because we are interested in understanding innovation and knowledge spillovers, where migration is also a key mechanism for knowledge transfer (Faggian and McCann, 2009). Including worker migration gives a better insight into the impact on growth of closer economic integration (beyond trade) between countries and regions. Migration of knowledge workers between locations, in response to differences in real wages, equalises wages in the long run and trade allows consumers in all locations to benefit from innovative, increasingly higher quality products. As such we can gain a better understanding of the influence on economic growth from integration in terms of competition, location, migration, trade, technology and innovation.

Existing knowledge is an input to innovation. Greater knowledge inputs reduce the cost of developing a quality improvement. In this model we assume that knowledge from the same variety is directly transferable to developing quality improvements in the following period. In addition, knowledge associated with the level of quality for other varieties is partially additive to the knowledge inputs that determine the cost of quality improvements for a manufacturer’s own variety. That is, if there is a quality improvement in one variety, it provides some knowledge input to firms producing other varieties when they develop quality improvements in future periods. Lastly, knowledge is partially transferable between locations; if a variety is produced in one location, the knowledge associated with its quality level does not fully transfer to the other location as an input to developing quality improvements. The result is a core-periphery pattern similar to Krugman (1991) with the overlap between the stable equilibria of symmetric distribution (break point) and core-periphery (sustain point) depending upon the ability for knowledge to transfer between locations. Agglomeration economies in manufacturing are strengthened due to knowledge externalities in research as in Baldwin and Forslid (2000). Knowledge spillovers provide additional growth. As in the other two region growth models, we find growth is highest when there is full agglomeration in one location, as knowledge spillovers are greater with manufacturing concentration. Agglomerated locations are more reliant on local inter-varietal knowledge spillovers for growth while less agglomerated locations rely on trade and regional knowledge spillovers.

We present an economy where firms choose their location by considering the freeness of trade and knowledge spillovers. Consider an economy with two production sectors that each has its own factor of production; unskilled and knowledge labour. In the manufacturing sector each firm produces a variety where the market is contestable through quality improvement in the R&D sector. If a firm is able to have the greatest quality improvement for variety \(i\), they are able to “take the market”, producing that variety for the following period. “Taking the market” is where a firm develops the best quality improvement of a variety and supplies the entire niche market. Niche refers to their monopolistic share of global expenditure. If they invest too little in R&D, there is an opportunity for an alternative producer to research the quality improvement for that variety and take the market from the incumbent manufacturer. Each period a quality improvement destroys the value of the existing quality level because a firm cannot maintain its niche market without R&D. This is the process of “creative destruction” as described by Aghion and Howitt’s (1992) endogenous growth model. Aghion and Howitt (1992) use a contestable monopoly market for an intermediate product. We also use the contestable market idea but with monopolistic competition and multiple manufactured varieties where each competes with other varieties through Dixit-Stiglitz preferences following the quality ladders approach of Grossman and Helpman (1991) and Young (1998). However in the quality ladders model, Dixit-Stiglitz competition is not fundamental to the mechanism of economic growth. Economic growth in this model comes from the change in the quality of manufactured varieties.

The paper is organised as follows. Sections 2 and 3 describe the mechanics of the model, equilibria and forces for firm location decisions. In Section 4 we explore the influence of knowledge spillovers and agglomeration upon economic growth and draw the growth policy implications for different regions. Section 5 provides a brief summary of the conclusions drawn from earlier sections.
2. A Model of Growth with Partial Knowledge Spillovers

We consider a model with discrete time periods. The economy has two production sectors, one for differentiated manufactured varieties and the other for traditional goods. There exists an R&D sector which produces quality improvements in manufactured varieties. A firm must obtain an innovation, with its associated quality level, produced by the R&D sector in every period, prior to the period of production at that quality level.

Model Structure

There are two types of labour: unskilled and knowledge labour. As in Young (1998) and Grossman and Helpman (1991) we have a manufacturing sector and a competitive R&D sector that employs knowledge labour. To allow for trade, migration and regional knowledge spillovers we follow Krugman (1991) by including two regions and adding a traditional goods sector employing unskilled labour. Unskilled labour is immobile between regions while knowledge labour migrates in response to differences in real wages. The two regions are referenced by home and foreign where foreign variables are noted by a "כ".

In each period manufacturing firms employ knowledge workers to produce each firms’ variety with a given highest quality level for which each firm has a patent. Firms also employ knowledge workers in the R&D sector to use existing knowledge of quality levels for all varieties and develop a quality improvement for a variety that the firm will produce in the following period. Existing technological knowledge of quality levels does not transfer perfectly between varieties or locations. Therefore firms are attracted to locate in the same location as other firms in order to have greater access to local technological knowledge. For a significant quality improvement the firm is granted a new patent to produce that variety. We assume the quality improvements are significant such that only the highest quality version of each variety is produced in any period and any former lower quality versions are no longer produced.

Firms compete for market share where consumers have Dixit-Stiglitz preferences and a preference for higher quality products. Consumers also have inter-temporal preferences. With $\rho > 0$ as the discount rate, the representative consumer has the utility function given by:

$$U = \sum_{t=0}^{\infty} \alpha^t \ln Q_t, \quad \alpha = \frac{1}{1 + \rho}$$

$$Q_t = C_{M,t}^{\mu} C_{O,t}^{1-\mu} \quad 0 < \mu < 1$$

Where $C_{O,t}$ is consumption in period $t$ of the traditional good (or other goods), $C_{M,t}$ is the consumption of $n$ differentiated varieties of manufactured goods from the home region and $n^*$ differentiated varieties manufactured in the foreign region:

$$C_M = \left[ \sum_{i=1}^{n} (A_i c_i)^{(\sigma-1)\sigma} + \sum_{j=1}^{n^*} (A_j^* c_j^*)^{(\sigma-1)\sigma} \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1$$

In equation 2, $c_i$ is the quantity consumed of variety $i$, manufactured in the home region, $A_i$ is the quality of variety $i$, $\sigma$ represents the elasticity of substitution between any two varieties of equal quality and the "כ" describes the attributes of varieties manufactured in the foreign region. This is the standard CES function with the additional factor $A_j^*$, which represents a further factor of differentiation where consumers have a taste for the quality level of variety $i$.

Intertemporal utility optimisation implies the transversality condition and the Euler equation:
\[
\frac{E_t}{E_{t-1}} = \frac{1 + r_t}{1 + \rho}
\]

where \( E_t \) is consumer expenditure in period \( t \), \( \rho \) is the rate of time preference and \( r \) is the rate of return on savings between periods \( t - 1 \) and \( t \). We normalise total expenditure: \( E + E^* = 1, \forall t \)

Manufactured varieties transported between locations incur transport costs that take Samuelson’s “iceberg” form where transport costs are incurred in the good itself. That is, the manufacturer sells more of the good than actually arrives because some portion “melts” in transit. If \( \tau \) represents the proportion of the variety that arrives at the destination, the remaining portion is used up during transportation. \( \tau < 1 \) is a measure of the freeness of trade or an inverse index of transport costs. Transport costs for the traditional goods sector are assumed zero, \( (\tau = 1) \).

By optimisation and the nature of Cobb-Douglas preferences, \( \mu \) is the share of expenditure spent on manufactured varieties and \( 1 - \mu \) is the share spent on traditional goods. The value of total expenditure on traditional goods in the home region is \( P_D C_D = (1 - \mu)E \) and on manufactured varieties is:

\[
P_M C_M = \sum_{i=1}^{n} P_i c_i + \sum_{j=1}^{n^*} \frac{P_j^*}{\tau} c_j^* = \mu E
\]

Similar equations exist for consumers in the foreign region with transport costs applied to imported manufactured varieties.

Endogenous growth is of the form in Young (1998). The model allows for multiple products and endogenous growth through creative destruction without scale effects. We choose a model without scale effects because scale would naturally encourage agglomeration. Here we demonstrate location choices and agglomeration explicitly due to the technical and pecuniary externalities of knowledge spillovers and transport costs. In each period a manufacturing firm produces for its niche monopoly and conducts R&D to ensure a quality improvement large enough to maintain its niche monopoly position for the following period. A firm must conduct R&D to ensure a quality improvement great enough to remain the producer of variety \( i \). Production of an individual variety involves a fixed cost of innovation and a constant marginal cost. Mobility of knowledge labour between locations and sectors equalises wages for knowledge workers between regions and between manufacturing and R&D sectors. The knowledge labour requirement in the previous period, \( t - 1 \), and the fixed cost of achieving the targeted quality level \( A_{i,t} \) in period \( t \) is:

\[
F_i(A_{i,t}, \bar{A}_{i,t-1}) = \begin{cases} 
\gamma e^{\eta A_{i,t}/\bar{A}_{i,t-1}} & \text{if } A_{i,t} \geq \bar{A}_{i,t-1} \\
\gamma e^{\eta} & \text{otherwise,}
\end{cases}
\]

where \( \gamma \) and \( \eta \) are fixed parameters and \( \bar{A}_{i,t-1} \) is an index of technological opportunity representing the knowledge spillover that is an input to innovation production. The innovation cost can be thought of as two components: a fixed cost of \( \gamma e^{\eta} \) and a research cost of \( \gamma e^{\eta A_{i,t}/\bar{A}_{i,t-1}} - \gamma e^{\eta} \) which determines the size of the firm’s quality improvement.

In a product variety model a firm produces the same variety forever so the source of knowledge spillovers is always all other manufactured varieties. In the creative destruction approach the firm innovates by improving an existing variety. While in Young (1998) and Grossman and Helpman (1991) firms only use their own variety’s technological knowledge to develop the next quality improvement, this index can easily be modified so firms can also use knowledge from other varieties. There is also an opportunity to differentiate between the externalities present in different sources of knowledge spillovers. The knowledge spillover input to innovation is therefore made up of two components: the knowledge associated with the quality level of the firm’s own variety and secondly, a portion of the knowledge associated with the quality level of all other varieties. We weight the knowledge input from all manufactured varieties according to a related variety approach.
where the relatedness of products describes how useful knowledge is to innovation in the firm’s own variety. As such the knowledge of a firm’s own variety is fully understood and carries a weighting of one while knowledge from other varieties is weighted by its relatedness $\lambda_V$. In this model we assume the relatedness of knowledge is constant for all pairs of varieties $\Lambda$. We also use the approach of Baldwin and Forslid (2000) where knowledge transfers imperfectly between firms that are geographically separated, so the location of all manufacturers becomes an important factor in a firm choosing its own location.

We define $\overline{A_{w,t-1}}$ as the observed worldwide weighted average of the technological knowledge of all manufactured varieties in the period prior to period $t$, where the weight of quality in the foreign region is the level of the inter-regional knowledge spillover, $\lambda_R$:

$$
\overline{A_{w,t-1}} = \frac{1}{n + n^*} \left[ \sum_{j=1}^{n} A_{j,t-1} + \lambda_R \sum_{k=1}^{n^*} A_{k,t-1}^* \right]
$$

If the variety has been produced before, then index of technological opportunity is the firm’s own quality level plus an imperfect portion of knowledge from other varieties:

$$
\overline{A_{l,t-1}} = \max(A_{l,t-1}, \lambda_R A_{l,t-1}^*) + \lambda_V \overline{A_{w,t-1}}
$$

$A_{l,t-1}$ represents the period $t-1$ quality level of the firm’s manufacturing sector and $\lambda_V$ represents the relative weight of the inter-varietal knowledge spillover from all varieties. If the variety has never been produced the index of technological opportunities is:

$$
\overline{A_{l,t-1}} = \overline{A_{w,t-1}} + \lambda_V \overline{A_{w,t-1}} = (1 + \lambda_V) \overline{A_{w,t-1}}
$$

It is a weighted average of existing quality levels plus the inter-varietal spillover. The function is such that symmetry is maintained even when a new variety is invented (and another variety disappears due to the endogenous constant number of varieties).

After the investment in R&D during period $t-1$ firms may produce any quantity at a constant marginal cost; $w\beta$ (labour) in period $t$.

The innovation production function used in the model here distinguishes between the externality effects on knowledge spillovers depending on their source and separates the technical externality in the product space from the geographic space. The related variety weighting $\lambda_V$ describes the ability to use knowledge from other varieties when a firm conducts R&D to improve the quality level of their own variety. $\lambda_R$ describes the ability of knowledge to transfer between manufacturing firms that are geographically separated.

**Short Run Equilibrium**

Unskilled and knowledge workers provide one unit of labour per period. Let $L_O$ and $L_O^*$ be the supply of unskilled workers in both the home and foreign regions respectively. We set the worldwide stock of unskilled workers at $(1-\mu)$ shared equally between regions.

$$
L_O = \frac{1-\mu}{2}, \quad L_O^* = \frac{1-\mu}{2}
$$

The traditional goods sector is perfectly competitive, with 1:1 technology and constant returns to scale. Unskilled workers provide one unit of production per period, i.e. $C_O + C_O^* = (1-\mu)$. Total production of traditional goods is shared equally across both regions. Free trade ensures the same nominal price of traditional goods in both locations and equal nominal wages for unskilled workers. With full employment total wages equals total revenue which we set as the numéraire.
Knowledge workers may move between locations. Let $L_K$ and $L_K^{*}$ be the supply of knowledge workers in both the home and foreign regions respectively where the worldwide stock of knowledge workers is specified as $\mu$.

\[ L_K + L_K^{*} = \mu \]

In the manufacturing sector we find the demand functions for home consumers of individual local and imported manufactured varieties $i$:

\[ c_i = p_{A_i}^{(\sigma-1)} p_i^{1-\sigma} p_{M_i}^{(\sigma-1)}, \quad c_i^{*} = p_{A_i}^{* (\sigma-1)} \left( \frac{p_i^{*}}{\tau} \right)^{1-\sigma} P_M^{(\sigma-1)} \]

where $P_M$ is the price quality index in the home region:

\[ P_M = \left[ \sum_{j=1}^{n} A_j^{(\sigma-1)} p_j^{(1-\sigma)} + \sum_{k=1}^{n^*} A_k^{* (\sigma-1)} \left( \frac{p_k^{*}}{\tau} \right)^{(1-\sigma)} \right]^{1-\sigma} \]

We treat transport costs as a cost of production for the export market. Similar demand equations exist for the consumers in the foreign region with isomorphic local prices, and a local price quality index.

In the period prior to production, potential investors/firms choose whether to enter, and if they do enter, they select a product and a level of quality improvement. No firm can appropriate the intertemporal knowledge spillover and a fixed cost investment must be made in the period prior to each production period. Since all industries are symmetric, quality will be the same for each variety. Firms face the same costs whether they improve an existing variety or introduce a new variety. We assume no two firms can choose to produce the same variety and we assume no economies of scope. Since $n$ and $n^*$ are determined endogenously, each manufacturing firm is infinitesimally small and there is no strategic interaction, production takes place under symmetric monopolistic competition. Manufacturing firms invest in R&D, paid for by future sales revenues. Firms choose price and quality in order to maximise the discounted monopolistically competitive profits:

\[ \max_{P_{i,t},A_{i,t}} \pi = \left( P_{i,t} - \beta \right) c_{i,t} + \left( P_{i,t}^{*} - \frac{1}{\tau} \beta \right) c_{i,t}^{*} - F_i \left( A_{i,t}, A_{i,t-1} \right) \]

where we have used labour as the numéraire, $\pi$ is firm profit, $\beta$ is the marginal cost of producing one more unit of variety $i$ and is the per unit labour requirement. The form of competition does not matter; firms’ prices are the same under both Bertrand and Cournot competition if there is a sufficiently large number of firms. Optimisation is subject to the local demand function in each location.

Each firm selects in period $t-1$ a period $t$ quality improvement with its associated cost of innovation and its period $t$ price so as to maximise the above profit equation (14). It is assumed that each firm takes price setting behaviour of other firms as given and therefore firms ignore the effects of their pricing decisions on the price index, i.e. we treat $P_{M}$ and $P_M^{*}$ as fixed when differentiating. This assumption is plausible with a sufficiently large number of firms. Similarly, firms have rational expectations of quality increases in other varieties and the effect on price. $F_i \left( A_{i,t}, A_{i,t-1} \right)$ is the number of knowledge workers required by the firm in the R&D sector to achieve a target quality level of $A_{i,t}$, and $A_{i,t-1}$ is an index of technological opportunity representing the intertemporal knowledge
spillover. The firm treats $A_{t-1}$ as given when making decisions about quality, price and production in period $t$. Differentiating, we find the first order conditions:

\begin{align*}
(15) & \quad c_{t,t} + (P_{t,t} - \beta) \frac{\partial c_{t,t}}{\partial P_{t,t}} = 0 \\
(16) & \quad c_{t,t}^* + (P_{t,t}^* - \frac{1}{t} \beta) \frac{\partial c_{t,t}^*}{\partial P_{t,t}} = 0 \\
(17) & \quad \frac{\partial \pi}{\partial A_{t,t}} = \frac{(P_{t,t} - \beta) \frac{\partial c_{t,t}}{\partial A_{t,t}} + (P_{t,t}^* - \frac{1}{t} \beta) \frac{\partial c_{t,t}^*}{\partial A_{t,t}} - \frac{\partial F_i}{\partial A_{t,t}} = 0}
\end{align*}

As in the usual Dixit-Stiglitz model free entry is assumed and the number of varieties is endogenous to the model. While $n$ is endogenous and consumers have a love of variety, the number of varieties is not the source of growth. Long-run growth in this model comes from quality improvements. Free entry means that in equilibrium the marginal firm just breaks even, i.e. that profits will be zero because if profits were positive there is opportunity for a marginal knowledge worker to shift to the R&D sector and produce either greater quality improvements OR an additional variety. The free entry condition is:

\begin{align*}
(18) & \quad \left(\frac{P_{t,t} - \beta}{1 + r_t} \right) c_{t,t} + \left(\frac{P_{t,t}^* - \frac{1}{t} \beta}{1 + r_t} \right) c_{t,t}^* = F_i(A_{t,t}, A_{t-1})
\end{align*}

From the first two first order conditions we find expected pricing behaviour:

\begin{align*}
(19) & \quad P_{t,t} = \frac{\sigma}{\sigma - 1} \beta w_t, \quad P_{t,t}^* = \frac{\sigma}{\sigma - 1} \frac{\beta w_t}{\tau} = \frac{P_{t,t}}{\tau}
\end{align*}

where we have included wages (which are the numéraire) for illustrative purposes only. Price in export market reflects a direct pass through of transport costs.

Returning to the first order conditions; dividing equation 17 by the free entry condition (18) and rearranging we find equilibrium behaviour in the innovation sector such that firms select the size of product quality improvements to equate the elasticity of the research cost with respect to quality with the elasticity of demand with respect to that variable. For constant elasticity demand:

\begin{align*}
(20) & \quad \varepsilon_{A_{t,t}}^{c_{t,t} + c_{t,t}^*} = \varepsilon_{A_{t,t}}^{F_i} \\
& \quad \varepsilon_{A_{t,t}}^{c_{t,t} + c_{t,t}^*} = (\sigma - 1) = \eta \frac{A_{t,t}}{A_{t-1}} = \varepsilon_{A_{t,t}}^{F_i}
\end{align*}

Substituting $A_{t-1} = \max(A_i, \lambda R_i^*) + \lambda \nu A_{w,t-1}$ and rearranging

\begin{align*}
(21) & \quad \frac{A_{t,t}}{\eta} = \max(A_i, \lambda R_i^*) + \lambda \nu A_{w,t-1}
\end{align*}

Equation 21 describes the willingness of firms to invest in R&D. The fixed cost of innovation (willingness to pay for innovation) per period is equal to $F_i(A_{t,t}, A_{t-1}) = \gamma e^{(\sigma-1)}$. By rearrangement, the level of quality improvement is equal to:

\begin{align*}
(22) & \quad \frac{A_{t,t}}{\max(A_i, \lambda R_i^*)} = \frac{(\sigma - 1)}{\eta} \left[ 1 + \frac{\lambda \nu A_{w,t-1}}{\max(A_i, \lambda R_i^*)} \right]
\end{align*}
We choose appropriate parameters to meet the assumption that equation 22 is greater than one so that there are always quality improvements in equilibrium.

Quality improvement includes an additional multiplier to Young (1998), \(1 + \frac{\lambda_V\lambda_{}\max(A_t,A_{t-1})}{\lambda_{R}A_{t}}\), due to the intervarietal knowledge spillover. Intuitively, quality improvement has two parts; it is the firm’s willingness to pay for R&D plus the quality improvement due to the technological spillover. For an individual variety the knowledge spillover is a function of its level of quality relative to the average level of quality. Otherwise variety growth and world growth is entirely determined by parameters of the research and production functions, \(\eta\) and \(\sigma\) respectively. Note for quality improvement, this relationship need only be true in one region, as is the case when all manufacturing is agglomerated in one location. Notably, in the symmetric equilibrium \(\frac{A_{t+1}}{A_t} = \frac{n + \lambda_{R}n^*}{n + n^*}\). Therefore, the condition that ensures quality improvements reduces to:

\[
\frac{(\sigma - 1)}{\eta} \left[ 1 + \frac{\lambda_V(n + \lambda_{R}n^*)}{n + n^*} \right] > 1
\]

**Labour Market Clearing and Endogenous Variety**

We assume no unemployment such that all local knowledge labour is used in manufacturing \((L_M)\) and investment/research \((L_R)\). The knowledge labour requirement in manufacturing in the home region equals the expenditure on all home varieties divided by the price received \(^3\) (per unit of actual production) times the unit marginal cost.

\[
L_{M,t} = \frac{\mu(s_tE_t + (1 - s_t^*)E_t^*)}{P_t} \beta = \frac{(\sigma - 1)}{\sigma} \mu(s_tE_t + (1 - s_t^*)E_t^*)
\]

Where \(s_t\) is the domestic market share of home region manufacturers and \((1 - s_t^*)\) is their foreign market share. i.e. \(s_t = \sum_{i=1}^{n} s_{i,t}\) and \((1 - s_t^*) = 1 - \sum_{j=1}^{n^*} s_{j,t}^* = \sum_{t=1}^{n} s_{i,t}^*\). The knowledge labour requirement in research equals the number of entrants in the next period, times the research investment per firm:

\[
L_{R,t} = n_{t+1} \gamma e^{\eta A_{t+1}/A_{t}}
\]

Similar equations exist for the foreign region. Knowledge labour \((L_K)\) market clearing in the home region requires \(L_K = L_M + L_R\):

\[
L_{K,t} = \frac{(\sigma - 1)}{\sigma} \mu(s_tE_t + (1 - s_t^*)E_t^*) + n_{t+1} \gamma e^{\eta A_{t+1}/A_{t}}
\]

The free entry relation (equation 18) multiplied by the number of firms, and substituting functions for price, costs and quantities can be rearranged to

\[
n_t = \frac{1}{\gamma} \frac{\mu(s_tE_t + (1 - s_t^*)E_t^*)}{e^{\eta A_{t}/A_{t-1}}(1 + \gamma)}
\]

And in the foreign region:

\[
n_t^* = \frac{1}{\gamma} \frac{\mu((1 - s_t)E_t + s_t^*E_t^*)}{e^{\eta A_{t}/A_{t-1}}(1 + \gamma)}
\]

The rearranged free entry relation describes the number of firms as a function of income, demand elasticity and the cost of innovation. Note the possible situations where there is zero manufacturing in one location. If there is manufacturing in both locations, both equalities must hold, but if there is
manufacturing only in one location, only one equality is relevant (note with agglomeration the market shares simplify significantly).

Substituting equation 27.1 (advanced one period) $n_{t+1} = \frac{1}{\sigma} \mu (s_{t+1} E_{t+1} + (1-s_{t}^*) E_{t+1}^*)$, into equation 26 we have:

$$L_{K,t} = \frac{(\sigma - 1)}{\sigma} \mu (s_{t} E_{t} + (1-s_{t}^*) E_{t}^*)$$

(28)

$$+ \frac{1}{\sigma} \mu (s_{t+1} E_{t+1} + (1-s_{t+1}^*) E_{t+1}^*)$$

$$\frac{1}{(1+r_{t+1})}$$

Using the first order condition for the growth of consumer expenditure (equation 3):

(29.1) \[ \frac{E_{t+1} + E_{t+1}^*}{E_{t} + E_{t}^*} = \frac{1+r}{1+\rho} = \alpha (1+r) \]

(29.2) \[ \frac{E_{t+1} + E_{t+1}^*}{1+r} = \frac{E_{t} + E_{t}^*}{1+\rho} = (E_{t} + E_{t}^*) \alpha \]

allows us to rearrange the free-entry relation to solve for the value of consumer expenditure as a function of the model’s parameters.

$$\mu (s_{t} E_{t} + (1-s_{t}^*) E_{t}^*) = \frac{L_{K,t}}{[(\sigma - 1) + \frac{\alpha}{\sigma}]}$$

(30)

Short run equilibrium requires the economy move to a steady state level of consumer expenditure, with a constant interest rate $1+r = 1+\rho = \frac{1}{\alpha}$. Substituting into the solution for $n_{t}$ and $n_{t}^*$ (equations 27.1 and 27.2):

(31.1) $n_{t} = \frac{L_{K,t} \alpha}{[(\sigma - 1) + \alpha] \gamma e^{\eta A_{t,1}/A_{t-1}}} = \frac{L_{K,t} \alpha}{[(\sigma - 1) + \alpha] \gamma e^{(\sigma-1)}}$

And

(31.2) $n_{t}^* = \frac{L_{K,t}^* \alpha}{[(\sigma - 1) + \alpha] \gamma e^{\eta A_{t,1}^*/A_{t-1}}} = \frac{L_{K,t}^* \alpha}{[(\sigma - 1) + \alpha] \gamma e^{(\sigma-1)}}$

3. Long Run Equilibrium

For long run labour market clearing, the condition must be true for both locations and real wages of mobile workers are equalised between locations. Even if there is manufacturing concentration in one location, the condition holds for both locations since there would be zero knowledge labour in the peripheral location in the long run. In every period manufacturing firms invest in R&D, improving quality by a growth factor $\frac{q}{\eta}$ plus some factor due to the intertemporal knowledge spillover. In the following period firms compete monopolistically charging a price of $P_{l,t+1} = \frac{\sigma}{\sigma-1} \beta$ where export prices include transport costs $P_{l,t+1}^* = \frac{\sigma}{\sigma-1} \frac{\beta}{\tau} = \frac{P_{l,t+1}}{\tau}$. The long run steady state is where real wages of mobile workers are equalised between locations.

The perfect price quality index describes the price index of utility and therefore includes traditional goods such that $P \equiv P_T^{1-\mu} P_M^{\frac{\mu}{\beta-\mu}}$ where $P_M$ is the local manufacturing price quality index. The real wage for a worker is defined as $\omega_K = \frac{\mu}{\beta}$. If there are differences in real wages at the beginning of a period there will be migration of knowledge workers. Migration of knowledge
workers due to wage pressure leads to the long run equilibrium. In this model we use a static migration equation where knowledge workers respond to wage differences at the start of each period. Migration of knowledge workers occurs as follows:

\[ L_{K,t} = L_{K,t-1} + M, \quad L_{K,t}^* = L_{K,t-1}^* - M \]

where \( M \) is the migration of knowledge workers shifting from the foreign region to the home region in response to differences in wages. We define the average real wage as \( \bar{\omega} = \omega_{K,t} + \omega_{K,t}^* \). We assume the ad hoc dynamics from the standard core-periphery Model in Fujita, Krugman, and Venables (1999)

\[ M = \delta (\omega_{K,t} - \bar{\omega}) L_{K,t} \]

Unskilled workers cannot migrate between locations.

**Long run Location**

The Long run steady state is defined by a constant number of manufactured varieties, constant growth in the level of quality of manufactured varieties and its regional division as well as the prices and quantities defined by short run equilibrium above. We consider any interior distribution equilibrium and consider the forces for manufacturing concentration or dispersion. Analysis of the location forces in manufacturing follows Krugman (1991). Consider the value of sales for a firm in the home region under any manufacturing distribution.

\[ V_H = \left( \frac{\mu}{n + n^*} \right) (E + E^*) \]

Beginning with an example, a single firm is considering switching location from home to foreign (or an alternative firm is set up in the foreign region). We are testing whether there is incentive to shift from a larger region to a smaller region, the name of the region is arbitrary, but we have selected home for the purpose of the example. The value of the foreign manufacturer’s sales in the home region will be the value of sales of a home manufacturer times \( \left( \frac{w^*}{w_H} \right)^{-(\sigma-1)} \). Their sales in the foreign region will be the value of sales of a home firm times \( \left( \frac{w^*}{w_H} \right)^{-(\sigma-1)} \). The total value of the switching manufacturer’s sales is

\[ V_H^* = \left( \frac{\mu}{n + n^*} \right) \left( \left( \frac{w^*}{w^* + \tau} \right)^{-(\sigma-1)} E + \left( \frac{w^*}{w} \right)^{-(\sigma-1)} E^* \right) \]

Transport costs work against the firm selling in the home region, but to the firm’s advantage when selling to the now domestic foreign region. The ratio of value for a home firm that chooses to switch to the foreign region, \( V_H^* \) as defined by equation 35, to the value of a home firm that remains at home (\( V_H \) as defined by equation 34) simplifies to:

\[ \frac{V_H^*}{V_H} = \frac{\tau^{(\mu+1)}(\sigma-1)E + \tau^{(\mu-1)}(\sigma-1)E^*}{E + E^*} \]

The firm must encourage knowledge workers to follow by paying equal real wages to the home region. They must compensate the new foreign region workers for the additional cost of importing all other manufactured varieties. We must have

\[ \frac{w^*}{w} = \left( \frac{1}{\tau} \right)^{\mu} \]

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It is profitable for a firm to switch location if \( \frac{V_H^*}{V_H} > \frac{w^*}{w} = \tau^{-\mu} \) to cover the higher nominal wages as workers require at least equal real wages:

\[
\frac{V_H^*}{V_H} = \frac{\tau^{(\mu+1)(\sigma-1)}E + \tau^{(\mu-1)(\sigma-1)}E^*}{(E + E^*)\tau^{-\mu}} > 1
\]

The fraction describes the transport cost ratio per unit of production that must be paid to workers to encourage them to shift. Inspecting the ratio, this is only greater than 1 if the firm is considering switching from a smaller region to a larger region, or if trade freeness is sufficiently low. That is, at a low level of trade freeness there is a symmetric equilibrium due to the market crowding effect, but at a high level, firms prefer to be grouped together because the market crowding effect is diminished and is now dominated by the greater home market effect.

Now we take the same approach to consider the ratio of research costs. In period \( t - 1 \) home manufacturers have the research cost for production in period \( t \):

\[
F_t(A_{i,t}, A_{i,t-1}) = \eta e^{\frac{A_{i,t-1}}{}}
\]

\[
A_{i,t-1} = A_{i,t-1} + \frac{\lambda_v}{n_{t-1} + n^*_{t-1}} \left[ \sum_{j=1}^{n} A_{j,t-1} + \lambda_R \sum_{k=1}^{n^*} A_{k,t-1} \right]
\]

With symmetrical firms the index of technological opportunity \( A_{i,t-1} \) is simplified significantly. The switching manufacturer has the research labour requirement equal to

\[
F_t^*(A_{i,t}, A_{i,t-1}) = \eta e^{\frac{A_{i,t-1}}{}}
\]

\[
A_{i,t-1} = \lambda_R A_{i,t-1} + \frac{\lambda_v}{n_{t-1} + n^*_{t-1}} \left[ \lambda_R \sum_{j=1}^{n-1} A_{j,t-1} + \sum_{k=1}^{n^*+1} A_{k,t-1} \right]
\]

Whether the firm switches or not, in order to maintain its niche monopoly on variety \( i \), the firm has the same targeted quality level. Since we have assumed the home region is equal or larger than the foreign region, the switching manufacturer will require equal or more knowledge workers to achieve the same quality target. This clearly already suggests that if there is an unequal distribution of workers and firms between regions the cost of R&D will encourage agglomeration, the forces must later be considered in conjunction with the other market forces described above.

Considering the research cost ratio (equation 40 divided by 39) we have:

\[
\frac{F_{i,t}}{F_{i,t}^*} = \frac{\eta e^{\frac{A_{i,t-1}}{}}}{{\eta e^{\frac{A_{i,t-1}}{}}}}
\]

\[
A_{i,t-1} + \frac{\lambda_v}{n_{t-1} + n^*_{t-1}} \left[ \lambda_R \sum_{j=1}^{n-1} A_{j,t-1} + \sum_{k=1}^{n^*+1} A_{k,t-1} \right]
\]

We assume \( n_{t-1} \) and \( n^*_{t-1} \) are sufficiently large such that a firm moving has no effect on the intervarietal portion of the knowledge spillover. Further since we are considering the long run equilibria, we assume that \( n_{t-1} \) and \( n^*_{t-1} \) are constant and we can drop the time subscript. Once again the research labour must be encouraged to move by being paid at least the same real wage they would have earned in the home region.
Given the number of varieties in each location depends upon the distribution of labour, as described by equations in 27 and 31, the fraction is only equal to one if there is an equal distribution of labour between locations. If there is an unequal distribution, R&D always has an agglomerating force towards the larger location or the location with higher quality levels. We refer to this as the “clustering effect”. Under both these fractions, if trade freeness is great enough, both forces are for agglomeration but the clustering effect only occurs when there is an unequal distribution of labour. If trade freeness is low however the clustering effect may be.

In the core-periphery model (Krugman, 1991) firms balance the pecuniary externalities from transport costs. The centripetal force is described as the “home-market effect” where firms attempt to locate in the biggest market in order to reduce transport costs and export to the smaller markets. At the same time this is balanced with the “market crowding effect” where imperfectly competitive firms prefer to be located in locations with fewer competitors. In addition, knowledge workers prefer the location with the lowest cost of living and as a result consumers try to minimise transport costs. This is known as the “cost of living effect”. There is an overlap between the core-periphery and symmetrical equilibria due to the cost of living effect. The cost of living effect depends upon the distribution of firms and only occurs with unequal equilibria. These three forces from pecuniary externalities determine the equilibria in the standard core-periphery model. In addition to these forces the model here adds another force for agglomeration, the clustering effect, where firms prefer to locate alongside other firms due to knowledge spillovers that reduce their cost of innovating.

The effect of partial agglomeration in the model is also not static because an agglomerated location grows faster than the periphery. The symmetric equilibrium is less stable in this model than the core-periphery model because of this dynamic effect. In the core-periphery model and starting from a symmetric equilibrium, if for some exogenous reason one location is now larger than another the “market crowding” effect may be dominant and the distribution would return to the symmetric equilibrium in the following period. However in the model here the larger location would achieve a higher level of quality such that the “clustering effect” is magnified and full agglomeration may be triggered. Firms could remain in the larger location, despite the market crowding effect because it reduces their cost of innovation and increases their quality target. This hysteresis effect suggests that once agglomeration is triggered possibly by rising freeness of trade, reducing trade freeness may not return the location to the previous spatial equilibrium as easily as implied by the core-periphery model. Because of partial knowledge spillovers, there is a larger overlap between the core-periphery equilibria and the symmetric equilibrium for some range of trade freeness. The range of the overlap decreases with higher regional knowledge spillovers as greater knowledge spillovers reduce the impact of the clustering effect.

**Stability analysis**

The Tomahawk diagram given in Figure 1 describes the range of equilibria at different levels of trade freeness and inter-regional knowledge spillovers. The two possible equilibria, manufacturing concentration or the symmetric equilibrium are described by the solid lines. The dotted lines describe unstable internal equilibria while the dashed line at (τ = 1) describes equilibria that are only stable under specific conditions.

Starting from a symmetric equilibrium with low trade freeness (τ = 0), as freeness of trade increases τ → 1, the break point, τ_B, is the level of trade freeness where the home market effect dominates the market crowding effect. There is no cost of living or clustering effects as these are dependent on the initial distribution. The break point, τ_B, is the level of trade freeness where the
distribution of economic activity reverts from the symmetric equilibrium to the core-periphery outcome. Alternatively, starting with free trade ($\tau = 1$), and the manufacturing concentration outcome, as freeness of trade decreases $\tau \to 0$, the break point point is where the market crowding effect dominates the home market effect. This means the symmetric equilibrium is now stable, but only if firms and workers migrate between locations the clustering and cost of living effects decrease and the equilibria could switch back to the symmetric equilibrium. Both types of equilibria are possible for some range of trade freeness. As freeness of trade declines further, the sustain point, $\tau_s$, is where the market crowding effect increases and eventually also dominates the cost of living and clustering effect. That is, the market crowding effect dominates all the forces for agglomeration. The range of the overlap depends upon the level of interregional knowledge spillovers $\lambda_R$.

Figure 1. Tomahawk diagram with inter-regional knowledge spillovers

Considering the ratio of sales less research cost, between alternative locations, it is profitable to switch location if $\frac{V_H^* - F_{1,t}^*}{V_H - F_{1,t}} > \tau^{-H}$ (equation 35 minus 40, divided by equation 34 minus 39). We can use this ratio to find the break point $\tau_B$ and sustain point $\tau_S$. Due to symmetry the sum functions can be simplified to be equal to the firm’s own quality level. We make the assumption that $n$ and $n^*$ are large enough that switching location has no noticeable effect on the relative size of $n$ and $n^*$, it is only the size of externalities in different locations that has an effect on research cost. This is similar to the assumption that a firm switching location has no effect on the price index.

To find the break point ($\tau_B$) we start with the symmetric equilibrium ($E = \frac{1}{2}E_w = E^*$) and $n = n^* > 0$ and find the critical level of trade freeness where it is profitable to switch location. The ratio is:

$$
\frac{V_H^* - F_{1,t}^*}{V_H - F_{1,t}} = \frac{e^{(\mu+1)(\sigma-1)}E + \tau(\mu-1)(\sigma-1)E^* - (n + n^*)e^{\eta A_{lt+1}e}}{(E + E^*) - (n + n^*)e^{\eta A_{lt+1}e}} \left[ \frac{\lambda_R A_{lt} + \lambda V}{n + n^*} \right]^{1/2} \left[ \sum_{j=1}^{n} A_{jt} + \lambda R \sum_{k=1}^{n^*} A_{kt} \right]^{1/2}
$$
\[ \tau_R = \frac{1}{\tau} E_w \tau^{(\sigma-1)} \left( \tau + (\sigma-1) \right) - 2 n e^{\eta A_{l,t+1} e} \frac{1/\lambda_R A_{l,t}^{\lambda_R A_{l,t}^{1/2} A_{l,t}}}{1/\lambda_V^{1/2} A_{l,t}} \]

While the formula appears complicated, the research cost part of the ratio is almost the same for both numerator and denominator because the regions are symmetrical. Knowledge spillovers have no effect on the forces for agglomeration because both locations are equally attractive to firms in terms of the cost of innovation. \( \lambda_R \) still appears as a multiplier in \( F_{l,t} \) on the variety's existing quality level because a firm suffers some loss of knowledge regarding its own quality level if it switches location. This is the hysteresis effect rather than the agglomeration effect and simply describes that a firm prefers to stay in its current location to maintain its technological advantage. The remaining effects are the usual effects present in the core-periphery model.

To find the sustain point (\( \tau_s \)) we start with the manufacturing concentration equilibria in the home region and find the critical level of trade freeness where it is profitable to switch location. In the manufacturing concentration equilibria all skilled workers are located in the home region and there are only unskilled workers remaining in the foreign region (\( E = (1 - \frac{\mu}{2}) E_w, E^* = \frac{\mu}{2} E_w, n > 0 \) and \( n^* = 0 \)). Since regions are identical the sustain point is identical for the foreign region. The ratio is

\[
\frac{V_F - F_{l,t}^*}{V_H - F_{l,t}} = \tau^{(\mu+1)(\sigma-1)} E + \tau^{(\mu-1)(\sigma-1)} E^* - (n + n^*) e^{\eta A_{l,t+1} e} \frac{1/\lambda_R A_{l,t}^{\lambda_R A_{l,t}^{1/2} A_{l,t}}}{1/\lambda_V^{1/2} A_{l,t}} \left[ \Sigma_{j=1}^{\lambda_R} A_{l,t} + \lambda_R A_{l,t} \Sigma_{k=1}^{n^*} A_{k,t} \right]
\]

\[
(\tau - E) - (n + n^*) e^{\eta A_{l,t+1} e} \frac{1/\lambda_R A_{l,t}^{\lambda_R A_{l,t}^{1/2} A_{l,t}}}{1/\lambda_V^{1/2} A_{l,t}} \left[ \Sigma_{j=1}^{\lambda_R} A_{l,t} + \lambda_R A_{l,t} \Sigma_{k=1}^{n^*} A_{k,t} \right]
\]

\[ \tau_S = \frac{\tau^{(\sigma-1)} E_w \tau^{(\mu+1)} \left( 1 - \frac{\mu}{2} \right) + \tau^{(\mu-1)} \frac{\mu}{2} - n e^{\eta A_{l,t+1} e} \frac{1/\lambda_R A_{l,t}^{\lambda_R A_{l,t}^{1/2} A_{l,t}}}{1/\lambda_V^{1/2} A_{l,t}}}{E_w - n e^{\eta A_{l,t+1} e} \frac{1/\lambda_R A_{l,t}^{\lambda_R A_{l,t}^{1/2} A_{l,t}}}{1/\lambda_V^{1/2} A_{l,t}}}
\]

By inspection note that \( \lambda_R \) only affects the numerator (the value of sales less research cost when the firm switches location) because there are no regional knowledge spillovers occurring in the manufacturing concentration outcome. The sustain point is crucially dependent upon the level of knowledge spillovers while the break point is unaffected by knowledge spillovers. This is why the size of the overlap (the distance between break and sustain points) varies with the level of regional knowledge spillovers.

When \( \tau = 1 \) there are no pecuniary externalities from transport costs, only knowledge externalities due to knowledge spillovers. If \( \lambda_R < 1 \) at the same time the clustering effect will ensure the core-periphery outcome. However if \( \lambda_R = 1 \) there are full knowledge spillovers and no gains in R&D attached to specific locations. Therefore any distribution is possible. This is described by the dashed line as it is only possible for these specific conditions.

If \( \lambda_R = 1 \), there are no clustering or hysteresis effects and the result is the same as the standard core-periphery model. If \( \lambda_R < 1 \) inter-regional knowledge spillovers are partial and provide an additional force for agglomeration that is dependent upon the distribution of manufacturing firms. If there is a symmetric equilibria there is no clustering effect because there is no advantage to switching location, but if there is concentration in one location there is a strong clustering effect because firms prefer the larger location as there are greater local knowledge spillovers. This creates

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the overlap between symmetric and concentration equilibria. If $\lambda_R = 0$ the clustering and hysteresis effects are much stronger and there is a larger overlap between equilibria; Agglomeration is maintained even if trade freeness declines massively, $\tau \to 0$, because firms still prefer to be located in the larger location, alongside other firms to take advantage of the knowledge spillovers. The size of the overlap is determined by inter-regional knowledge spillovers.

**Forces for Manufacturing Concentration**

The distribution of knowledge labour between locations is dependent upon migration, driven by the real wage difference. Following Krugman (1991) we consider a situation in which all workers are concentrated in the home region. We consider whether it’s possible for a firm to begin manufacturing in the foreign region. The total value of the switching manufacturer’s sales is

$$V_H^* = \left( \frac{\mu}{\tau} \right) \left( \frac{W^*}{W} \right)^{1-(\sigma-1)} E + \left( \frac{\tau W^*}{\nu} \right)^{1-(\sigma-1)} E^*.$$

Transport costs work to a disadvantage in home region sales but to an advantage in the foreign region sales. The ratio of home region sales (43) to foreign region sales(44) is as in Krugman (1991)

$$\frac{V_H^*}{V_H^*} = \frac{\tau}{\tau + \nu} \left[ (1 + \mu)\tau^{(\sigma-1)} + (1 - \mu)\tau^{-(\sigma-1)} \right].$$

It is profitable for a firm to switch location if

$$\frac{V_H^*}{V_H^*} \geq \frac{w_H}{w_H} = \tau^{\mu}$$

to cover the higher wages, including fixed costs. The requirement for agglomeration in one location is that if $v = \frac{1}{2} \tau^{\mu} \left[ (1 + \mu)\tau^{(\sigma-1)} + (1 - \mu)\tau^{-(\sigma-1)} \right] < 1$, it is unprofitable for a firm to begin production in the foreign region if all manufacturing production is in the home region. Therefore if $v < 1$, manufacturing concentration is a possible equilibrium and if $v > 1$, it is not because equilibrium would revert to the equal distribution outcome.

Departing from Krugman (1991), consider the costs of innovation, as an additional factor for manufacturing concentration, beyond the pecuniary externalities of transport costs. Research by a manufacturer in the foreign region is also more expensive, since he must hire more researchers to cover the reduction in knowledge spillovers (due to distance with other firms), and ensure a quality level at least enough to “take the market”. The additional research cost is the research cost of the difference in knowledge spillovers. The sales ratio is found in Krugman (1991) but in addition, a switching firm must overcome the additional cost of research in the foreign region. The ratio of foreign to home research workers is

$$\frac{F_{lt}^*}{F_{lt}} = \frac{\eta A_{lt}}{\eta A_{lt}} \left[ \lambda_R \sum_{j=1}^n A_{jt-1} + A_{kt-1} \right].$$

If we make an additional assumption, that $n$ is large enough, such that a firm’s own quality has no noticeable effect on the intervarietal spillover effect, (similar to a firm’s own price having no noticeable effect on the price index) and with symmetry all firms have the same $A_{lt}$, this ratio simplifies to

$$\frac{F_{lt}^*}{F_{lt}} = \frac{e^{\lambda_R(1+\lambda_R)}}{e^{(1+\lambda_R)}}.$$

Since $0 \leq \lambda_R \leq 1$ this will always be greater than 1 (and equal to 1 where $\lambda_R = 1$) and describes a firm’s preference to be agglomerated with other firms due to knowledge spillover externalities. Furthermore, the switching firm must pay higher wages in the foreign region to make up for the
higher cost of living. The ratio \( \frac{F_{lt}^*}{F_{lt}} > \tau^{-\mu} \) describes a firm’s preference to change region due to research costs.

\[
\frac{F_{lt}^*}{F_{lt}} \tau^{-\mu} = \frac{e^{\frac{\eta}{\gamma}(1+\lambda_V)}}{e^{\lambda_V}} \tau^{-\mu} > 1
\]

This emphasizes the additional force for manufacturing concentration, beyond the traditional core-periphery model. The forces for agglomeration found in Krugman (1991) still exist, plus this additional force for manufacturing concentration due to the firm’s preference for agglomeration because of knowledge spillovers.

4. Economic Growth

So far we have considered the static equilibria of prices, quantity, investment, distribution of economic activity and forces for agglomeration. In the steady state quality is increasing due to the firm’s investment in the R&D sector to maintain their niche monopoly. This section explores the growth rates under different states of agglomeration (whether steady states or not) and considers the impact of knowledge spillovers upon growth.

In equation 22 we found the level of quality improvement. With symmetrical manufacturers, firms choose a quality target of:

\[
A_{lt} = \left(\frac{\sigma - 1}{\eta}\right) \left[ 1 + \frac{\lambda_V \left(n + \lambda_R n^*\right)}{n + n^*} \right] A_{l,t-1}
\]

That is, the level of quality improvement depends upon elasticity of substitution, the parameters of the innovation production function and the level of agglomeration. Elasticity determines a firm’s margin and therefore its willingness to pay for research. The parameters of the innovation production function determine the effectiveness of knowledge labour in research. \( \eta \) defines the effectiveness of knowledge labour to improve product quality while \( \lambda_V \) and \( \lambda_R \) describes the ability for knowledge to transfer between firms.

The growth rate of quality per period is \( g_A = \frac{A_{l,t+1} - A_{l,t}}{A_{l,t}} \), substituting equation 49 (advanced one period) yields the growth rate of quality:

\[
g_A = \left(\frac{\sigma - 1}{\eta}\right) \left[ 1 + \frac{\lambda_V \left(n + \lambda_R n^*\right)}{n + n^*} \right] - 1
\]

As capital enters the model in the form of quality level, \( g_A \) is the growth rate of the knowledge capital stock, even though the user of that capital may change from time to time through knowledge spillovers, innovation and contestability.

Per capita growth is equal to the rate at which utility improves between periods. That is we need to find \( \frac{\partial q_t}{\partial t} = \frac{q_{t+1} - q_t}{q_t} \). Alternatively the growth rate of capital is directly linked to real income growth. That is, while wages, expenditure and prices keep the same nominal value (the numéraire), the price index changes over time such that the ability to purchase quality increases as consumers purchase higher quality products. The increase in purchasing power or the decline in the perfect price index, is the economy’s growth rate.
Taking the price quality index for manufactured varieties, \( P_M = \left[ \sum_{j=1}^{\eta} A_j \right]^{-1} p_j^{1-\sigma} + k=1 n_k A_k \sigma - 1 p_k \sigma r_1 - \sigma \Pi_1 - \sigma \). With \( A_j \) and/or \( A_k \) increasing at a rate of \( g_j \) the manufacturing price quality index is changing at a rate of \( (g_A^{(\sigma-1)}) \frac{1}{(1-\sigma)} \). But given that this is a price quality index for manufactured varieties only, the overall perfect price quality index is actually to a power of \( \frac{\mu}{(1-\sigma)^\eta} \). Thus the cost of living price quality index is falling at a rate of \( \frac{\mu}{(1-\sigma)^\eta} \) times \( g_A^{(\sigma-1)} \). The growth rate of utility is:

\[
 (50) \quad g_Q = \frac{\mu}{(1-\sigma)} \left[ (\sigma - 1) \left[ 1 + \frac{\lambda \nu (n + \lambda_R n')}{n + n'} \right] - 1 \right]^{\sigma - 1}
\]

Notably the growth rate is the same in both locations whether we have a symmetric outcome or the manufacturing concentration outcome. This is because while the price index for manufactured varieties falls, the equivalent for traditional goods remains constant, since consumers still spend the same portion of their earnings on traditional goods. The overall price index for both manufacturing and traditional goods decreases at the same rate because producers of traditional goods can trade for manufactures.

In equilibrium the number of varieties is constant. Long run growth is driven by increases in quality. Since the elasticity of demand with respect to quality is independent of the population size, investment in R&D is not affected by the scale of the economy. But since in this model \( n_k \) and \( n'_k \) are affected by \( L_k \) the growth of utility is affected by migration. Changes in the size of a region lead to important transitional effects in growth rates. Growth separates into the long-run component of increasing quality, and a short-run transitional component which is influenced by changes in the distribution of labour. While we don’t investigate this transitional component further here these transitional effects could help explain the varying growth rates in recently integrated markets. Once we include these spillover and regional effects there are scale effects to growth in the short run. That is, larger or agglomerated locations increase the growth rate due to the higher knowledge spillovers between a greater number of varieties in the same location until we reach the steady state where both regions benefit equally from innovations in the agglomerated region.

The Impact of Knowledge Spillovers upon Economic Growth

Knowledge spillovers in this form are essentially a free portion of additional growth. They do not crowd out or form a portion of a firm’s spending on growth and knowledge spillovers do not discourage investment in innovation since the market is contestable and firms attempt to make a return on their investment in the following period only.

Inter-varietal spillovers represent the knowledge spillover from other varieties as an input to innovation. In particular inter-varietal spillovers have a greater influence on economic growth if they are not also diminished by inter-regional spillovers. Inter-varietal spillovers have a greater impact on quality improvements when inter-regional knowledge spillovers are perfect, or there is manufacturing concentration.

In Figure 2 if \( \lambda_R < 1 \) and there is manufacturing concentration, growth (equation 51) is represented by the line between growth of \( \frac{\mu}{(1-\sigma)^\eta} \left[ (\sigma - 1) \left[ 1 + \frac{\lambda \nu (n + \lambda_R n')}{n + n'} \right] - 1 \right]^{\sigma - 1} \) if \( \lambda \nu = 0 \) and \( \frac{\mu}{(1-\sigma)^\eta} \left[ 2(\sigma - 1) \left[ 1 + \frac{\lambda \nu (n + \lambda_R n')}{n + n'} \right] - 1 \right]^{\sigma - 1} \) if \( \lambda \nu = 1 \). Alternatively, if it is the equal manufacturing outcome, growth is subject to inter-regional knowledge spillovers. Growth ranges between \( \frac{\mu}{(1-\sigma)^\eta} \left[ (\sigma - 1) \left[ 1 + \frac{(1+\lambda_R)}{2} \right] - 1 \right]^{\sigma - 1} \) when \( \lambda \nu = 0 \) and \( \frac{\mu}{(1-\sigma)^\eta} \left[ (\sigma - 1) \left[ 1 + \frac{(1+\lambda_R)}{2} \right] - 1 \right]^{\sigma - 1} \) if \( \lambda \nu = 1 \).
Inter-regional spillovers describe how knowledge does not fully transfer between locations for manufactured varieties. Under the core-periphery outcome the inter-regional knowledge spillover has no effect on growth because all manufacturing and R&D are in the same location. Alternatively, under the symmetric outcome, partial inter-regional knowledge spillovers reduce growth. Figure 3 shows how growth varies with different levels of inter-regional knowledge spillovers. There is no variation in growth when there is manufacturing concentration because the inter-regional spillovers have no effect. Under the symmetric equilibria, increasing inter-regional knowledge spillovers has a positive effect on growth because R&D can take advantage of knowledge in both locations.

**Agglomeration and Economic Growth**

First we consider economic growth under the symmetric equilibrium. With an equal distribution of manufacturing between locations quality improves by a factor of \( \frac{(\sigma - 1)}{\eta} \left[ 1 + \lambda_v \right]^{-1} \). Conversely with the core-periphery outcome the agglomerated location sees quality improvement by a factor of \( \frac{(\sigma - 1)}{\eta} \left[ 1 + \lambda_R \right]^{-1} \).
With $\lambda_R < 1$ the quality improvement under a core-periphery outcome is always higher than under the symmetric equilibria (or any other interior solution). Growth is highest in the core-periphery setting (which is also a steady state) as partial inter-regional knowledge spillovers are no longer present. Since all firms are local, knowledge spillovers do not diminish due to distance but due to inter-varietal factors. For the periphery the quality of manufactured varieties consumed (by importing) also improves by the same factor because they are able to trade traditional goods for manufactured varieties.

Figure 4 shows the levels of growth with different levels of agglomeration. Agglomeration in the home region is shown by its market share of manufactured varieties. If all manufactures are in the home region, or all manufactures are in the foreign region, growth is highest. Growth is at its minimum when the home region has half the manufactures, and the foreign region has half. The reason the periphery region experiences growth, even with no manufactures (and no innovation) is because trade allows the periphery to trade traditional goods for manufactured varieties. While the real income level of the home region is lower when there are no manufactures because all manufactured varieties must be imported, the growth level is the same as the agglomerated location.

![Figure 4. Economic Growth with Varying levels of Agglomeration](image)

**Policy Implications for Economic Growth**

The policy implications from endogenous growth theory such as R&D subsidies were dismissed by Young (1998) as ineffective on long-run growth rates. The model here also recognises that R&D subsidies are ineffective at increasing long-run growth rates. Subsidies only change the pool of rents, shared by knowledge labour. However this result is because of the choice of innovation production function without scale effects. A function with scale effects would support the implementation of R&D subsidies. It is the distribution of labour that decides the number of varieties in each location. Without scale effects models support the use of an R&D subsidy that is based on the intensity of R&D effort rather than as a percentage of R&D expenditure. Such a subsidy changes the elasticity of R&D cost to quality and therefore changes the R&D investment behaviour of manufacturing firms.

By including technology in the firm's location, investment and production decisions, the model here helps explain the emergence of technologically advanced firms and locations. R&D subsidies in a single location could initiate and speed up the agglomeration process, by increasing the wage differential. Alternatively they could slow down the agglomeration process if a subsidy in the periphery decreases the wage differential, and therefore reduces long run growth. The regional effect of knowledge spillovers is particularly interesting. Policies which attempt to retain manufacturing in the periphery have a detrimental effect on long run growth.
Subsidies in one location, the periphery, are less effective than in the core. Furthermore, if they prevent the agglomeration of industry both locations miss out on the quality improvements due to knowledge spillovers that could have occurred within an agglomerated location and the region emerging as technologically advanced. Subsidies in the periphery may have a short term effect of retaining some industry and maintaining higher wages in the periphery, but are ultimately detrimental to global economic growth and incomes.

The quality ladders or creative destruction approach to growth has some interesting implications because it particularly highlights the effect of historical “accidents”. That is, events which have affected growth for a period in the past, have permanent effects on income and technology levels, but not permanent effects on growth.

Consider an unequal internal equilibrium. Regional knowledge spillovers are a greater proportion of quality improvement in a low manufacturing location than a high manufacturing location. In the high manufacturing location, varietal spillovers are more important to growth. To encourage quality improvements, the low manufacturing location may be better off supporting regional knowledge spillovers than supporting R&D. An agglomerated location on the other hand may support growth by encouraging relationships between the location’s firms.

By supporting inter-regional knowledge spillovers a location will have greater quality improvements that occur through knowledge spillovers. This could lead to the region emerging as technologically advanced and the preferred location for firms to conduct R&D. If these inter-regional knowledge spillovers are supported equally in both directions the equal distribution outcome becomes more stable. These conclusions are similar to those found by Baldwin and Forslid (2000) who show that integration policies which lower the cost of trading information are stabilising. The model here is similar but is able to differentiate between the source of knowledge spillovers. In addition there is a hysteresis effect where firms typically prefer to continue to conduct research and manufacturing in the same location.

5. Summary and Conclusion

This paper brings the quality ladders approach of endogenous growth theory to the two region setting and provides a framework for dealing with knowledge spillovers.

In the core-periphery outcome, growth is the same in manufacturing locations as in traditional goods locations even though one region is technologically advanced because traditional goods locations benefit from trading for innovative products that achieve higher quality improvements in the agglomerated region. Real earnings in the traditional goods locations may be lower, but they will grow at the same rate as in the core. Perhaps the observation of varying growth rates described in the Section 1 is more a symptom of the transition between separate economies to highly integrated markets. In the longer run, growth rates in both locations would be expected to equalise. That is, growth rate differences are both a reflection of less innovation in a less agglomerated location, and the transition to long-run income levels. Closer integration for trading goods causes a shift of industry from one location to another, and in the long run, growth is the same in both locations, but one location will have higher real earnings than the other.

From a policy perspective, a number of interesting implications arise. We have the same results from Young (1998) that R&D subsidies as a proportion of R&D costs do not increase the R&D expenditure of each firm. These types of policies increase the pool of rents available to the entrepreneur and therefore increases wages but has no effect on innovation effort, the number of varieties, the level of quality improvement and therefore economic growth. R&D subsidies connected to research intensity have a greater effect on economic growth because they increase a firm’s willingness to spend on research.
Under the spatial model R&D subsidies in a low manufacturing location reduce growth rates by slowing the transition to the agglomerated outcome because it reduces, or even reverses the real wage differential which drives migration. On the other hand subsidies in a high manufacturing location speed up the transition to agglomeration. For an internal equilibrium, a low or equal manufacturing location may be better off focussing on policies which encourage inter-regional knowledge spillovers rather than policies for R&D. A location with manufacturing concentration depends less on inter-regional linkages and more on local spillovers for economic growth.

As in Baldwin and Forslid (2000) knowledge spillovers are a stabilising force. However because the break point is unaffected by these spillovers the range of transport costs where both equilibria are stable is quite different in this quality ladders model. Transport costs reach the same threshold where agglomeration is the only possible equilibrium. However, with partial knowledge spillovers there is a greater range of transport costs where both equal distribution and the core-periphery are possible stable equilibria. This highlights the importance of hysteresis in integration and the effects on technology levels. Sudden and incomplete integration could result in drastic and irreversible changes in the location of economic activity, with catastrophic consequences for incomes and production technology levels in some regions.

Notes

1 This is not an exhaustive list of two region endogenous growth models.
2 Workers in the traditional goods sector are not necessarily unskilled. The important property of the factor of production in this sector is that it is immobile between regions and sectors.
3 The choice of units \((1 - \mu)\) unskilled workers and \(\mu\) knowledge workers) follows Krugman (1991) and ensures prices and wages in the traditional goods sector are the numéraire, and that the nominal wage rate of knowledge workers equals that of unskilled workers. If the number of knowledge workers were specified differently the wages of knowledge workers are a constant multiple of the wage rate of unskilled workers. We maintain simplicity by avoiding this additional multiple. A scaling factor could also be used to calibrate the model to any arbitrary growth rate as described in Baldwin and Forslid (2000).
4 These are the demand functions from the consumers point of view. We are optimizing firm behavior for a local market (12.1) and an export market. \(P_l\) and \(P^e\) refer to the local prices. The export demand function from the firm's point of view is the same as the local demand function with a foreign price index and transport costs are instead treated as a cost of production in equation 14.
5 The price received is the domestic price only. If a firm exports, the firm produces more of the good than actually arrives. When foreign consumers pay a price of \(\frac{P^e}{t}\) for each of the \(\tau c_{l,t}\) units consumed, the manufacturer receives \(P_{l,t}\) for each of the \(c_{l,t}\) units produced.
6 In other texts the “home market effect” is sometimes referred to as the “market access effect” and the “market crowding effect” is sometimes referred to as the “local competition effect”.
7 While the ratio is undefined for \(\lambda_p = 0\) this is a special case where the innovation opportunity cost of switching region is not only the cost of the quality improvement but the cost of all previous quality improvements. In effect this cost is so large that it is undefined.
References


