Third–Order Nulling Effect in Darlington Transistors

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***Abstract*—In this paper we present the first proof that a Darlington transistor has an inherent nulling effect in its third–order intermodulation distortion, similar to the well–known third-order null seen in single BJT amplifiers. It is proven mathematically and by measurement. The results suggest the null actually becomes feasible as a source of distortion reduction in a Darlington BJT amplifier.**

I. Introduction

A well–known characteristic of a single BJT amplifier, as seen in Figure 1, is that its third-order distortion current has a condition where it is cancelled out completely from the output current. The mathematics behind this has been well documented by several authors [1] [2] [3], and is relatively easy to prove. The actual mechanism in the BJT which produces this cancellation is not well documented. Only one author [4], theorises that a component of diffusion current is created within the transistor, which is opposed to the third-order current in the signal output.

The condition for nulling, from [1], is given by

$$I_C = \frac{V_T}{2R_E} \tag{1}$$

Where I_C is the DC collector current, V_T is the thermal voltage and R_E is the summation of internal and external emitter resistance. This resistance can be approximated to not include the source resistance and the intrinsic base resistance of the transistor, which will have a non-negligible effect if the current gain β of the BJT is small. Generally if the β is large, we can assume R_E is only the external emitter resistance.

This distortion reduction technique is rarely used practically in the literature because the null in a BJT occurs at a small collector current and not at a useful operating current for the device. However, closely related techniques have been published that use capacitive and inductive elements on the emitter which place the transistors bias current in the null, at a feasible collector current [5] [6] This is considered reactive nulling, whereas this papers focus is passive nulling of the third–order distortion current. Aside from using the null as a distortion reduction method there has also been interest in using it as an accurate technique to measure the emitter resistance of a BJT. One common method requires forcing base current into a single BJT and measuring the collector voltage while holding the collector current at zero [7]. Estimating the resistance using the third-order null, compared with the forced base method, has been shown to be accurate [8]. The common alternative to the forced base drive method is high-frequency measurements of the H-parameters of the device, which usually require direct contact to the BJT itself [9] [10] [11].

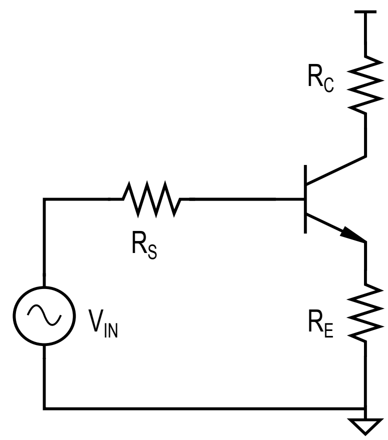

Fig. 1. Single BJT Amplifier.

II. Theory

While the proof for the third-order nulling in the BJT case is well documented and straight-forward, we will present it again using a different method which will lead into proving the null condition for the Darlington case. This is because this method is more elegant, and requires less algebraic manipulation than previous proofs.

A. Single BJT Amplifier

Firstly we preform a Kirchoff's voltage loop around the base-emitter loop of the circuit in Figure 1.

$$V_{IN} = \frac{I_C}{\beta}(R_S + R_E) + V_\pi + I_C R_E \tag{2}$$

$$V_{IN} = I_C(\frac{R_S + R_E}{\beta} + R_E) + V_\pi \quad (3)$$

It is known from the Eber-Molls model that,

$$V_\pi = V_T ln(\frac{I_C}{I_Q}) \quad (4)$$

Substituting Eq. 4 into Eq. 3 and rearranging the results gives,

$$\frac{V_{IN}}{V_T} = \frac{I_C R_{EE}}{V_T} + \ln(\frac{I_C}{I_Q}) \quad (5)$$

Where $R_{EE} = \frac{R_S + R_E}{\beta} + R_E$.

Placing a $\frac{I_Q}{I_Q}$ term into the R_E term allows us to define the equation W as a function of X,

$$W\{X\} = FX + \ln(X) \quad (6)$$

Where W = $\frac{V_{IN}}{V_T}$, F = $\frac{I_Q R_E}{V_T}$ and X = $\frac{I_C}{I_Q}$.

We now have a transfer function for the BJT amplifier in a form where we can compute the condition for first, second and third order currents. Differentiating Eq. 6 gives the gain terms of a Maclaurin Series, which relate directly to the transistor's harmonic gain terms. The series is of the form below (up to the third order only).

$$f\{X\} = A_1 X + A_2 X^2 + A_3 X^3 \quad (7)$$

A_1, A_2 and A_3 are the first, second and third–order transconductance terms respectively. Differentiating Eq. 6 and inverting to make V_{IN} the subject we get the following terms;

$$\frac{dX}{dW} = \frac{1}{\frac{1}{X} + F} \quad (8)$$

$$\frac{d^2X}{dW^2} = \frac{X}{X^2(\frac{1}{X} + F)^3} \quad (9)$$

$$\frac{d^3X}{dW^3} = \frac{X(1 - 2XF)}{(1 + XF)^5} \quad (10)$$

The third order term contains $(1 - 2XF)$ and clearly if $2XF = 1$ a condition is reached in which the third order distortion current nulls completely. This then reduces down to the condition given by Eq. 1, however R_E now contains the effects of source and base resistance as well, herein referred to as the apparent emitter resistance.

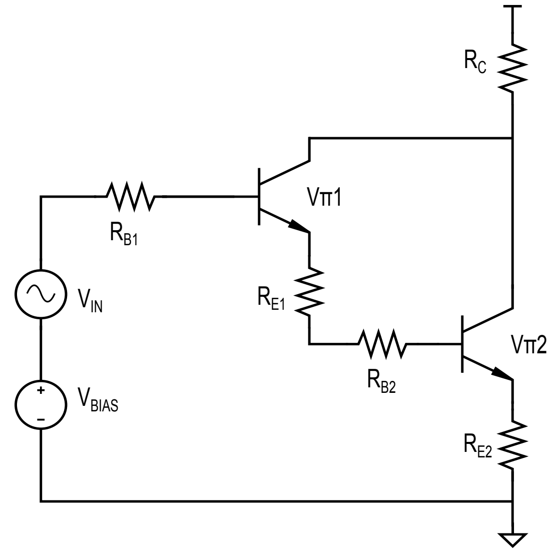

Fig. 2. Darlington BJT Amplifier.

B. Darlington BJT Amplifier

Derivation of the Darlington nulling condition follows the same method. Firstly we define the Kirchhoff's voltage loop of a Darlington amplifier. This is shown in Figure 2, where we define the voltage loop as,

$$V_{IN} = I_C(\frac{R_{B1}}{\beta_1 \beta_2} + \frac{R_{E1} + R_{B2}}{\beta_2} + R_{E2}) + V_{\pi 1} + V_{\pi 2} \quad (11)$$

Using the assumption of unity common base current gain (α = 1) we can assume $I_{C1} = I_{E1} = I_{B2}$. Following this assumption and the steps presented in equations 4 through 10, the following transfer function for a Darlington amplifier is obtained.

$$W\{X\} = FX + \ln(X^2) \quad (12)$$

Where W = $\frac{V_{IN}}{V_T}$, F = $\frac{I_{Q2} R_{EE}}{V_T}$ and X = $\frac{I_{C2}}{I_{Q2}}$.

Again we differentiate to find the transconductance terms for the first, second and third–order.

$$\frac{dX}{dW} = \frac{1}{\frac{2}{X} + F} \quad (13)$$

$$\frac{d^2X}{dW^2} = \frac{2X}{(2 + XF)^3} \quad (14)$$

$$\frac{d^3X}{dW^3} = \frac{4X(1 - XF)}{(2 + XF)^5} \quad (15)$$

It can be seen that the third–order term will null completely if XF = 1, which can be written in the form below. This gives our condition equation for a Darlington amplifier to null in the third order. Note that this doubles the total collector current at

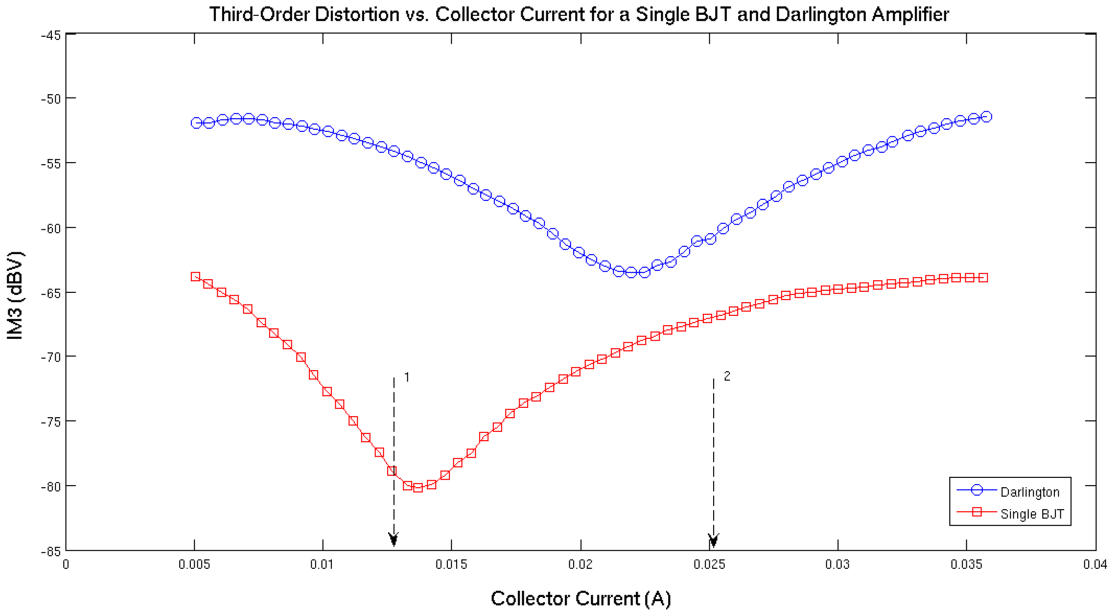

Fig. 3. Measured data of Collector Current vs. IM3 distortion in a single BJT amplifier and a Darlington amplifier

the point of nulling for a given BJT, or equivalently permits tolerance of twice the apparent emitter resistance for a given operating point to lie in the null.

$$I_{C2} = \frac{V_T}{R_{EE}} \tag{16}$$

III. Measurement Results

A CA3083 packaged transistor array was selected as the BJT for measurements to prove the third-order null. This was chosen mainly because it is thought to have a high internal emitter resistance compared to other common BJTs, but also a packaged transistor ensured close matching of thermal voltages for two transistors. Both a single BJT, and a Darlington configuration were measured with the collector-emitter voltage (V_{CE}) held constant. This was done using a LM358 op-amp and a feedback loop from the collector node back to the base node of the transistor. This approach was chosen to reduce thermal drift as the transistor cells approach higher currents and therefore higher operating temperatures. The collector voltage was then swept, allowing control over the collector current. The output frequency was then measured using an HP 3561A Signal Analyzer at a low fundamental frequency of 11kHz. To obtain a better dynamic range, the output was also filtered via a twin T notch filter with a frequency centred on 11kHz. This allowed the Signal Analyzer to gain more dynamic range in its measurements, and make the values recorded around the third-order null more accurate.

A plot of the third order distortion level versus collector current for both transistors is shown in Figure 3. Markers 1 and 2 indicate where a single BJT and Darlington should null respectively, if their emitter resistances are 1 ohm. This resistance value is chosen because the CA3083 emitter resistance is not known at this point. For the single transistor the null occurs at approximately 14.0 mA. Substituting into Eq. 1 shows the transistor as having a null resistance value of 0.914 ohms. For the Darlington transistor the null occurs at approximately 22.0 mA. Substituting in Eq. 16 shows the Darlington cell as having a null resistance value of 1.16 ohms.

To further corroborate the emitter resistance value, an Agilent E5270 Parameter Analyzer was used to measure a single CA3083 transistor using the forced base measurement method. The collector current of the transistor is held at zero and the base node is forced with a relatively large current. This produces a voltage proportional only to any internal resistance in the transistor onto the collector node. Since there is no current contributed from the collector side, the voltage V_{CE} can be estimated to be approximately the voltage across the internal emitter resistance. The maximum base current rating for a CA3083 is 20mA, so a range of 4mA to 14mA is used for the measurements. Obtained was a plot of the base current versus collector voltage for the CA3083, as seen in Figure 4. It approximates the emitter resistance of the CA3053 as 0.614 ohms, calculated from the slope of the graph.

IV. Discussion

Figure 3 shows clear nulls in both transistors, however there is some discrepancy between the measurements and the proof. Equation 16 suggests the null in a Darlington should be exactly double that in a transistor, assuming β is large. This discrepancy can be accounted for somewhat by the following factors.

Firstly, while the difference in collector current is quite large, in terms of the difference in apparent emitter resistance, it is small. It is most likely we are observing a small difference in emitter resistance between the Darlington and the Single

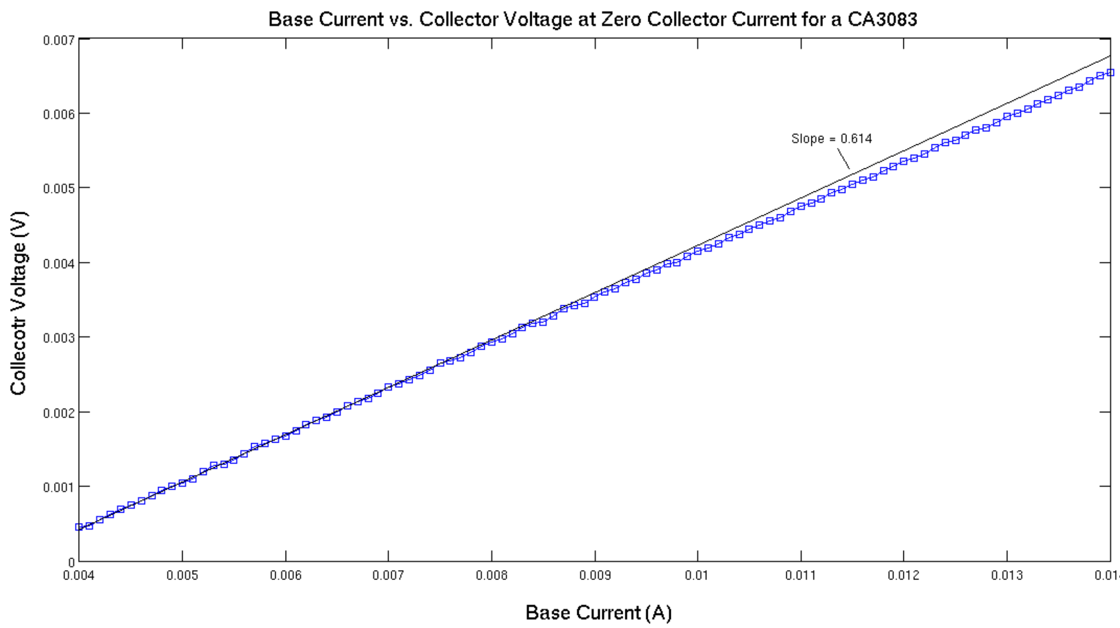

Fig. 4. Measured data of Base Current vs. Collector Voltage with collector current held at zero.

BJT, which results in a noticeable difference in the nulls position in collector current. If the percentage difference is taken from the measured apparent emitter resistances, it is 21.2% (or 0.246 ohms which is small in resistance terms). We also see a 21.4% difference in the Darlington null position, varying from where it is expected to be from the proof, two times that of the single BJT.

It is possible that the CA3053 has a fairly small β compared with other transistors, typically 50-100 at normal operating conditions. While this would not account for much of the discrepancy, it would introduce a noticeable error. This can be calculated using Eq. 16 where R_{EE} is given by the following, Eq. 17 and Eq. 18, for the Single BJT and Darlington respectively.

$$R_{EE} = \frac{R_S + R_E}{\beta} + R_E \tag{17}$$

$$R_{EE} = \frac{R_{B1}}{\beta_1 \beta_2} + \frac{R_{E1} + R_{B2}}{\beta_2} + R_{E2} \tag{18}$$

If we assume the worst case value of β to be 50, and assume $R_{B1} = R_S$ = 50 ohms and R_{B2} = 1 ohm, for the Darlington we calculate 1.20 ohms or a 0.04 ohm change This relates to a null shift of 0.736 mA. For the Single BJT we calculate 1.93 ohms or a 1.02 ohm change, which relates to a null shift of 7.38 mA. Clearly from the single BJT calculation, the β of the transistor must be much higher as we do not observe such a large shift in the measurements.

Observing the emitter resistance measurement data for the CA3083 we see again a discrepancy in the estimated value. However, if we reconcile this using Eq.17 and using a reasonable β of 100 we find if we take R_{E1} as 0.614 we obtain an apparent emitter resistance or 1.12 ohms and a null position of 11.4 mA. This would mean the Darlington null measurement would follow the mathematical proof within error.

V. Conclusions

There now exists a clear mathematical proof that a Darlington transistor has a natural third–order null at double the collector nulling current for a single BJT amplifier. While measurements show this proof to be reasonably valid, there still exists error which is unaccounted for by the proof. The measurement techniques of emitter resistances in BJTs are not rigorous so clarifying the exact emitter resistance in the used transistor requires more work. The error measured between the Single BJT and Darlington circuits emitter resistance is 21.2%. In this context the fact that the emitter resistances are 1 ohm or below, means 21.2% is not a large error at all.

This proof leaves a lot of interesting related topics to explore. Producing a technique that uses the null current to accurately estimate the emitter resistance could prove useful to a BJT manufacturer when batches of transistors are fabricated. Perhaps more interesting is the fact that a Darlington cell nulls naturally at a much higher operating current. Traditionally the single BJT null has been deemed 'not-useful' due to the fact that its nulling current is small and nowhere near a useful operating current for RF applications. A Darlington null happens at a much more useful region of transistor operation. Further development of topologies that push the nulling current even higher would become very appealing to RF power amplifier designers.

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