Supporting the Development of Number Fact Knowledge in Five- and Six-year-olds

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This paper focuses on children's number fact knowledge from a study that explored the impact of using multiplication and division contexts for developing number understanding with 34 five- and six-year-old children from diverse cultural and linguistic backgrounds. After a series of focused lessons, children's knowledge of number facts, including single-digit addition, subtraction, and doubles had improved. However, they did not always apply this knowledge to relevant problem-solving situations. The magnitude of the numbers did not necessarily determine the difficulty level for achieving automaticity of number fact knowledge.

Researchers and policy makers have focused on the needs of future citizens, identifying 'key competencies' (knowledge, skills, and dispositions) thought to be necessary for a successful life and a well-functioning society in the 21st century (e.g., Darling-Hammond, 2010; Gilbert 2005; Ministry of Education, 2007). It is vital to understand mathematics learning in the early years of school because mathematics is an important predictor of later academic success and appears to be relatively stable for individuals over time (e.g., Bynner & Parsons, 2000; Wylie & Hodgen, 2011). Longitudinal research has found that young children who had poor mathematics understanding continue to struggle with mathematics at a later age, and this pattern continues with difficulties in finding employment (Bynner & Parsons, 2000; Duncan et al., 2007).

Mathematics reform over the past few decades has led to the development of frameworks outlining progressions in number as students acquire increasingly sophisticated ways of thinking and reasoning (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, & Gould, 2005; Ministry of Education, 2008). Typically, at lower stages students solve problems using counting strategies (Baroody, 2011). As they come to appreciate additive composition, they are able to use strategies that involve partitioning and recombining quantities (part-whole thinking). The initial focus with younger children is often on addition and subtraction before introducing other domains such as multiplication and division, and proportional reasoning. However, research has shown that young children have considerable knowledge of multiplication prior to formal instruction in this domain (Bakker, van den Heuval-Panhuizen, & Robitzsch, in press).

Students are thought to need particular number knowledge in order to apply strategies for solving problems (Ministry of Education, 2008). Such knowledge includes number-word sequences, basic facts, and place value. It has long been acknowledged that number fact knowledge is a necessary component of successful achievement in mathematics. However, there is disagreement about what constitutes fluency in number fact knowledge, as well as how and when to support this aspect of children's arithmetic competency.

Distinctions have been made between counting-based and collections-based approaches to working with numbers (Yackel, 2001). Work on counting builds an appreciation of number-word sequence and fits with the idea of number as seriation – each number comes after and is one more than the previous number in the sequence (Sarama & Clements, 2009). A collections-based approach requires a focus to be put on the composition of numbers in terms of groups and fits with idea of number as inclusion – each number

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includes those that are smaller (Sarama & Clements, 2009). Yackel talks about collections in terms of partitioning numbers according to place-value units (e.g., tens & ones). Yang and Cobb (1995, p. 10) have highlighted "an inherent contradiction" in the way that American children are initially encouraged to count by ones and thus construct unitary *counting-based* number concepts, but when place-value instruction begins, are then expected to reorganise these into *collections-based* concepts involving units of tens and ones. Yang and Cobb contrast this counting-based view with the collections-based approach of Chinese mothers and teachers, who emphasize groups (units) of ten. The difference in emphasis on counting versus grouping by tens helps to explain Yang and Cobb's (1995) finding of more advanced mathematical understanding by the Chinese children relative to that of the American children.

Collections may be partitioned in a variety of ways (other than units of tens and ones) and this process is essentially part-whole thinking. Part-whole thinking strategies are also referred to as 'derived facts strategies' (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Henry & Brown, 2008), 'mental calculation strategies' (Thompson, 2010), 'decomposition strategies' (Carr, Taasoobshirazi, Stroud, & Royer, 2011), 'structuring processes' (Wright, Ellemor-Collins, & Tabor, 2012) and 'deductive strategies' (Gray, 1991). Part-whole strategies are characterised by the use of number fact knowledge to solve problems, rather than counting strategies. The focus on multiple partitioning of numbers fits with more recent approaches that recognise the importance for young children of developing an awareness of mathematical pattern and structure (Mulligan, 2011; Mulligan & Mitchelmore, 2009).

Another important distinction commonly made is between conceptual and procedural knowledge (Baroody, Feil, & Johnson, 2007; Skemp, 2006). Researchers interested in students' conceptual understanding have focused on developing children's awareness of number relationships, including subitizing, part-whole relationships, and more and less relationships (e.g., Jung, Hartman, Smith, & Wallace, 2013). Gray and Tall (1994) characterise *procedures* as "things to do", distinguishing them from *concepts* as "things to know" (p. 117). They use *procept* to refer to an amalgam of procedures/processes and concepts (e.g., 6 is the result of counting from 1 to 6, and as well as the result of adding 3 + 3). They argue that more capable children use proceptual thinking (a flexible way of thinking about numbers), whereas less able children persist in using counting procedures.

A major emphasis in New Zealand Year 1 and 2 classrooms is on helping children learn to count objects in a set by ones (Ministry of Education, no date). National Standards (Ministry of Education, 2009) specify that after a year at school, children should solve problems by using a 'count all' strategy. After two years at school, it is expected that they use advanced counting strategies (count on or back by ones, or skip count). According to National Standards, once children have been at school for three years they are expected to use knowledge of number facts to solve problems involve using part-whole thinking. They are expected to partition and combine numbers to solve whole-number problems, initially with addition and subtraction, then multiplication and division, and eventually rational numbers.

Why is Number Fact Knowledge Important?

Number fact knowledge is used in this paper to refer to the rapid recall of number combinations that result from the four operations. These comprise one aspect of the Knowledge domain within the New Zealand Number Framework (Ministry of Education, 2008). The flexible use of number facts is vital for becoming a part-whole thinker. Derived facts are also essential for developing a range of mental strategies to solve number problems (Ministry of Education, 2008). For example, children might use their knowledge that 2 + 2 = 4 to work out that the answer to 2 + 3 is one more than four. The teaching of derived facts need not wait until children regularly use counting-on strategies (Fischer, 1990; Henry & Brown, 2008; Steinberg, 1985). Even children who use counting-all strategies can learn to use derived facts to solve problems.

Research shows that low achievers in mathematics have consistent difficulties in recalling number facts and using them to solve problems (Baroody, 2011). It has been suggested that one of the reasons is that these learners often continue to rely on counting strategies, which take a lot of energy and attention, so that number facts do not become known facts (Gray, 1991). Some researchers have highlighted the importance of learners developing automaticity in mathematics (Gray, 1991; Hattie & Yates, 2014; Hopkins & Lawson, 2002). It is suggested that learning to memorise number facts and use them to solve new problems should be made more explicit (Baroody, 2011).

The project described here set out to explore the impact of using multiplication and division contexts with five- and six-year-olds on their emerging understanding of number, including number fact knowledge and part-whole relationships.

Method

This study was set in an urban school (medium socio-economic status [SES]) in New Zealand. The participants were 34 five- and six-year-olds (17 girls & 17 boys) in two classes, one designated as Year 1 and the other Year 2. The average age of the students at the beginning of the study was 6.2 years (range 5.6 to 6.9 years). The children were from a diverse range of ethnic backgrounds, with approximately one third of European ancestry, one third Māori, and other ethnicities including Asian, African, and Pasifika (Pacific Islands migrants). One third of the children had been identified as English Language Learners [ELL]. At the start of the study, the children were assessed individually using a diagnostic task-based interview designed to explore their number knowledge and problem-solving strategies (April). The assessment interview was completed again after each of the two four-week teaching blocks (June and November). The assessment tasks included: addition, subtraction, multiplication, and division problems, known facts, incrementing in tens, counting sequences, and place value.

Two series of 12 focused lessons were taught; the first phase was in May and the second in October. In these lessons, the children were introduced to groups of two, using familiar contexts such as pairs of socks, shoes, gumboots, jandals, and mittens. Multiplication and division was introduced using simple word problems, such as:

> Five children each get 2 socks from the bag. How many socks do the children have altogether?
>
> Mr B has 15 sweets. He puts five sweets in each bag. How many bags of sweets are there?

Once children were familiar in working with groups of two, groups of five were introduced using contexts such as gloves focusing on the number of fingers on each glove, and five candles on a cake, then groups of ten using the context of filling cartons with eggs. Although the emphasis of the study was on multiplication and division, the focus in this paper is specifically on children's knowledge of number facts and use of derived facts to solve problems.

A typical lesson began with all students completing a problem together on the mat, using materials to support the modelling process, and sharing ways of finding a solution. The teacher recorded children's problem-solving processes (including use of manipulatives) and their mathematical ideas in a large scrapbook ('modelling book'). The problem for the day was already written in the book and both drawings and number sentences were recorded, acknowledging individual children's contributions. The children then completed a problem in their own project books, choosing a similar or larger number, and/or selecting a new number. Materials were made available and children were encouraged to show their thinking using representations and to record matching equations.

Results

Children's performance on the tasks was examined to look for patterns and progressions. Items selected from the diagnostic task-based interview included recall of known number facts and solutions to number problems. Responses to addition, subtraction, and multiplication problems were weighted according to the sophistication of strategies (counting all = 1, counting on/back or in multiples = 2, known or derived facts = 3). Table 1 shows examples for each of the five types of addition number facts used in the assessment.

Table 1
Examples for each of the Five Types of Addition Number Facts

No. Fact Types	Examples
Doubles	5 + 5, 3 + 3, 4 + 4, 10 + 10, 6 + 6
Plus One	2 + 1, 1 + 4, 1 + 9
Doubles +/- one	2 + 3, 5 + 4, 5 + 6
Combinations for 10	4 + 6, 7 + ? = 10, 2 + 8
Place Value	20 + 7, 10 + 8

Children's correct responses on selected addition facts at the start and end of the project are shown in Table 2 as a percentage (n=34).

Table 2
Percentages of Correct Responses on Selected Addition Facts

Tasks	Baseline %	*End of Yr* %	Tasks	Baseline %	*End of Yr* %
5+5	*91*	*94*	2 + 3	*26*	*53*
3+3	*68*	*88*	5 + 4	*18*	*50*
4+4	*50*	*88*	5 + 6	*6*	*32*
10+10	*53*	*85*	4 + 6	*6*	*15*
6+6	*12*	*47*	7 + ? = 10	*6*	*44*
2 + 1	*68*	*85*	2 + 8	*12*	*59*
1 + 4	*47*	*79*	20+7	*24*	*62*
1 + 9	*47*	*74*	10+8	*21*	*53*

By the end of the project, only two children did not know some of the number facts presented. The easiest number facts were the doubles and included: 5 + 5 (94%), 3 + 3 (88%), 4 + 4 (88%), and 10 + 10 (85%). More children knew doubles such as 5 + 5 than knew the plus-one facts. This is despite the fact that the sum was 10 rather than 5 or smaller. Performance on combinations for ten varied according to the distance between the two addends (apart from the doubles: 5 + 5 which was the easiest). The most difficult fact was 4 + 6 (15%) whereas 1 + 9 was considerably easier (74%). It was easier to add a single-digit quantity to 20, than to construct a '-teen' number such as 18.

Table 3 presents the percentages of correct responses on selected problem-solving tasks, including the proportion using particular types of strategies. Children improved markedly on multiplication tasks. Not only were more children successful on multiplication than on addition and subtraction, but also more of them used higher-level strategies to solve the problems, such as skip counting or derived number facts. For example, more than three-quarters of the children used a higher-level strategy (counting on/skip counting or known/derived facts) for 6 × 2, 4 × 5, and 3 × 10, whereas just over half of them used one of these higher-level strategies to solve 4 + 3 or 8 + 5.

Table 3
Percentages of Correct Responses on Selected Problem Solving Tasks

Tasks	Baseline %	*Final %*	Tasks	Baseline %	*Final %*
Addition/Subtraction			Multiplication		
3 + 4 (screened)	74	*79*	6 x 2 (screened)	68	*91*
Counting All	32	*18*	Counting All	29	*6*
Counting On/Skip Counting	24	*29*	Skip Counting/Repeated Add'n	32	*79*
Known/Derived Facts	18	*32*	Known/Derived Facts	6	*6*
5 + 8 (screened)	32	*68*	4 x 5	47	*91*
Counting All	6	*9*	Counting All	29	*9*
Counting On	24	*38*	Skip Counting/Counting On	12	*59*
Known or Derived Facts	3	*21*	Known/Derived Facts	6	*24*
14 – 9 (screened)	21	*53*	3 × 10 (array)	32	*85*
Counting All	0	*3*	Counting All	6	*6*
Counting On/Back	18	*29*	Skip Counting/Counting On	24	*56*
Known/Derived Facts	3	*18*	Known/Derived Facts	3	*24*

By the end of the project, more children were able to remember relevant number facts (such as 3 + 3 or 4 + 4) than used derived-fact strategies to solve 4 + 3 (88% compared to 32%).

Table 4 presents the inter-correlations for responses to selected tasks at the start and end of the project. At the start of the study, knowledge of known facts was strongly related to the score on addition and subtraction problem solving ($r = 0.81$). A similar relationship was found for solutions to multiplication ($r = 0.73$), and division problems ($r = 0.69$). Correlations from the start to the end of the project (inside the bordered region) indicate that knowledge of known facts were most strongly predictive of subsequent strategies for addition and subtraction (0.83), and division (0.73).

Although children were familiar with a range of number facts, they did not appear to use them to solve problems involving operations. For example, when shown an array of 30 cakes in three rows of ten and asked how many cakes altogether, the majority of children (85%) could work out the answer by the end of the project, an increase from 32%.

Approximately one-quarter (24%) of these students used known or derived facts, and more than half (56%) used skip counting. More than half of the children were able to combine a multiple of ten (a '-teen' or a '-ty' number) with a single-digit quantity without using a counting strategy.

Table 4
Correlations among Number Facts and Operations at the Start and End of the Project

	Nov A/S	Nov Mult	Nov Div	Nov Facts	Apr A/S	Apr Mult	Apr Div
Nov A/S							
Nov Mult	0.46						
Nov Div	0.71	0.36					
Nov Facts	0.75	0.66	0.72				
Apr A/S	0.80	0.63	0.68	0.75			
Apr Mult	0.70	0.50	0.55	0.67	0.73		
Apr Div	0.65	0.52	0.65	0.71	0.62	0.61	
Apr Facts	0.83	0.63	0.73	0.86	0.81	0.73	0.69

Discussion and Conclusion

The study showed that over the course of the project, children developed a broader range of number facts but did not necessarily use that knowledge in solving problems. More children were familiar with the fact 5 + 5=10 than knew the plus-one facts for small sums (e.g., 2 + 1 = 3). The pattern of mastery (according to the size and types of facts) was contradictory to curriculum materials specifying progression (e.g., Ministry of Education, 2008). This finding suggests that teachers need to be aware that certain key number facts may be learned early, even though they involve sums greater than five. There is a strong case to be made for children to be encouraged to memorise the easiest facts, regardless of the number size. The increase in those using known or derived facts supports work showing that young children can learn to use derived facts instead of relying on counting strategies (Fischer, 1990; Jung et al., 2013; Steinberg, 1985). Children should be given the opportunity to work with multiplication and division problems and larger numbers, and be encouraged by their teachers to recognise the value of derived facts for arriving at solutions. This is consistent with the view that awareness of mathematical pattern and structure needs to be supported in young children (Mulligan, 2011; Mulligan & Mitchelmore, 2009). Awareness of pattern and structure should help to strengthen confidence, mastery, and automaticity of number fact knowledge (Hattie & Yates, 2014; Hopkins & Lawson, 2002; Wylie & Hodgen, 2011).

The introduction of multiplication and division contexts was clearly related to improvements in solving multiplication and division problems using more efficient strategies. However, its impact on addition and subtraction problem solving was less marked. In future iterations of the project, a more explicit emphasis on number relationships in the context of single-digit doubles (e.g., 4 + 3 is "one more than" 3 + 3) could provide further support for the use of derived-fact strategies for addition and subtraction, as well as strengthening children's understanding of "two groups of" within multiplication and division. It should be acknowledged that instruction during the intervening weeks between the two phases may also have contributed to the findings.

If an emphasis continues to be placed on the direct teaching of counting-on procedures (strategies), then the likely consequence is that less able children may become locked into counting as their only problem-solving strategy (Gray & Tall, 1994). This has been demonstrated in research with large cohorts of students showing that counting-on is the preferred strategy to solve addition and subtraction problems for between 14% and 8% of students in Years 7 to 9, respectively (Young-Loveridge, 2010). The teaching of counting procedures is similar to the traditional instrumental teaching that prevailed prior to mathematics education reform. Teachers with lower levels of confidence and competence in conceptual (relational) understanding of mathematics may not recognise the importance of helping young children move from a reliance on counting strategies to acquiring and applying number fact knowledge (Baroody, 2011; Skemp, 2006). New Zealand's National Standards for mathematics legitimate the teaching of counting-all and counting-on strategies during the first two years of schooling (Ministry of Education, 2009). The absence of a focus on multiple number relationships is further reinforced by the current curriculum (Ministry of Education, 2007). Ministry of Education online information for teachers under the Standards heading asserts that, at Level 3 [years 5 and 6] the key idea is "that numbers can be represented in a variety of ways" (Ministry of Education, no date). This contradicts other Ministry documents suggesting that multiple representations for (small) numbers should be emphasized after the first two years of school. The emphasis on the teaching of counting is perhaps the consequence of taking a framework that describes children's *development* (i.e., learning), and using it as a framework for *teaching*.

The findings of this study show that young children can build a repertoire of number facts when classroom instruction supports this knowledge acquisition. An explicit focus on using derived-fact strategies to solve addition and subtraction problems could be effective in deepening children's understanding of number relationships and their inter-connections.

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