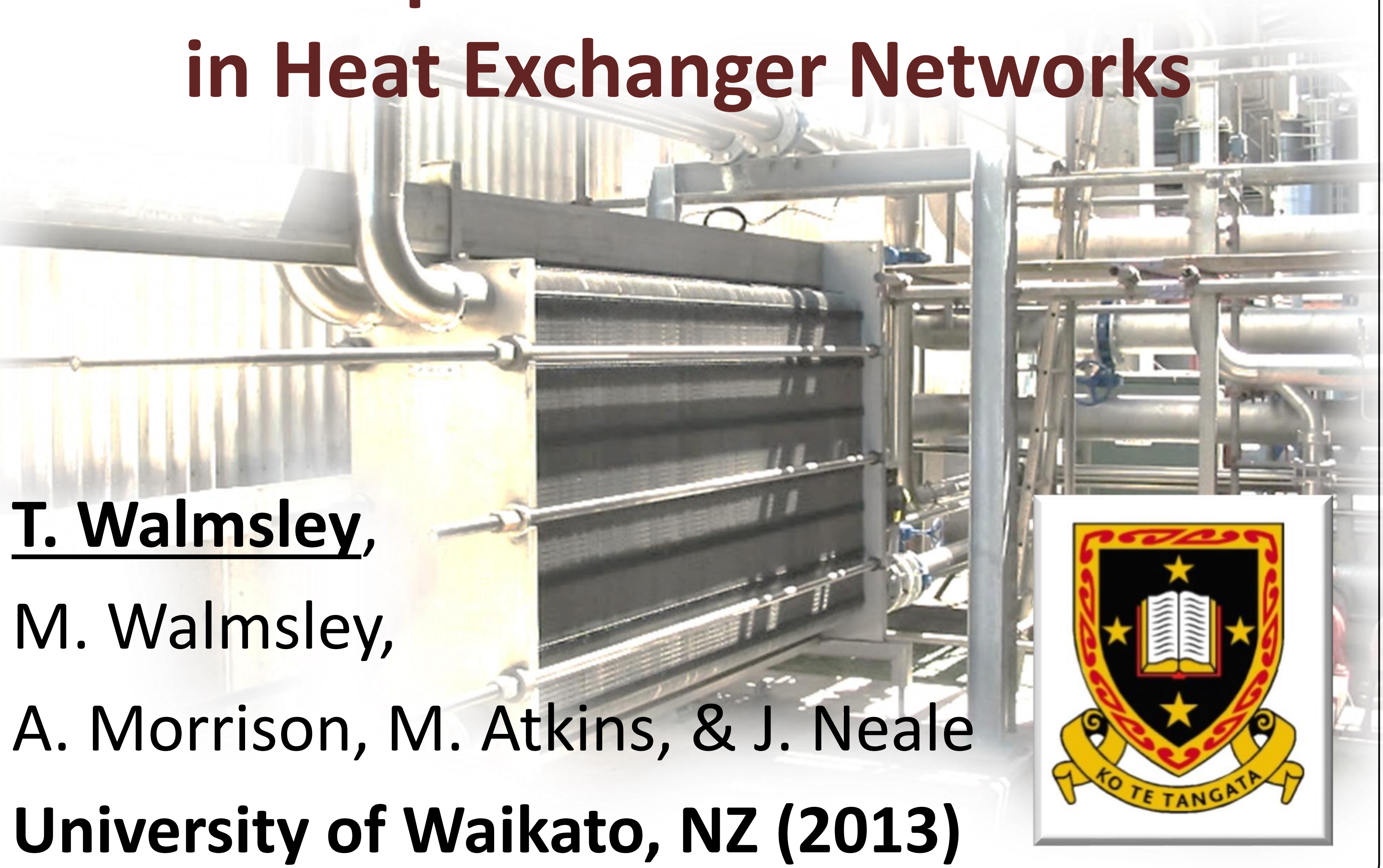
Cost Optimal Area Allocation in Heat Exchanger Networks



T. Walmsley,
M. Walmsley,
A. Morrison, M. Atkins, & J. Neale
University of Waikato, NZ (2013)

Overview

- Context & Motive
- Introduction
- Cost Derivative Method
- Application of Method
- Summary

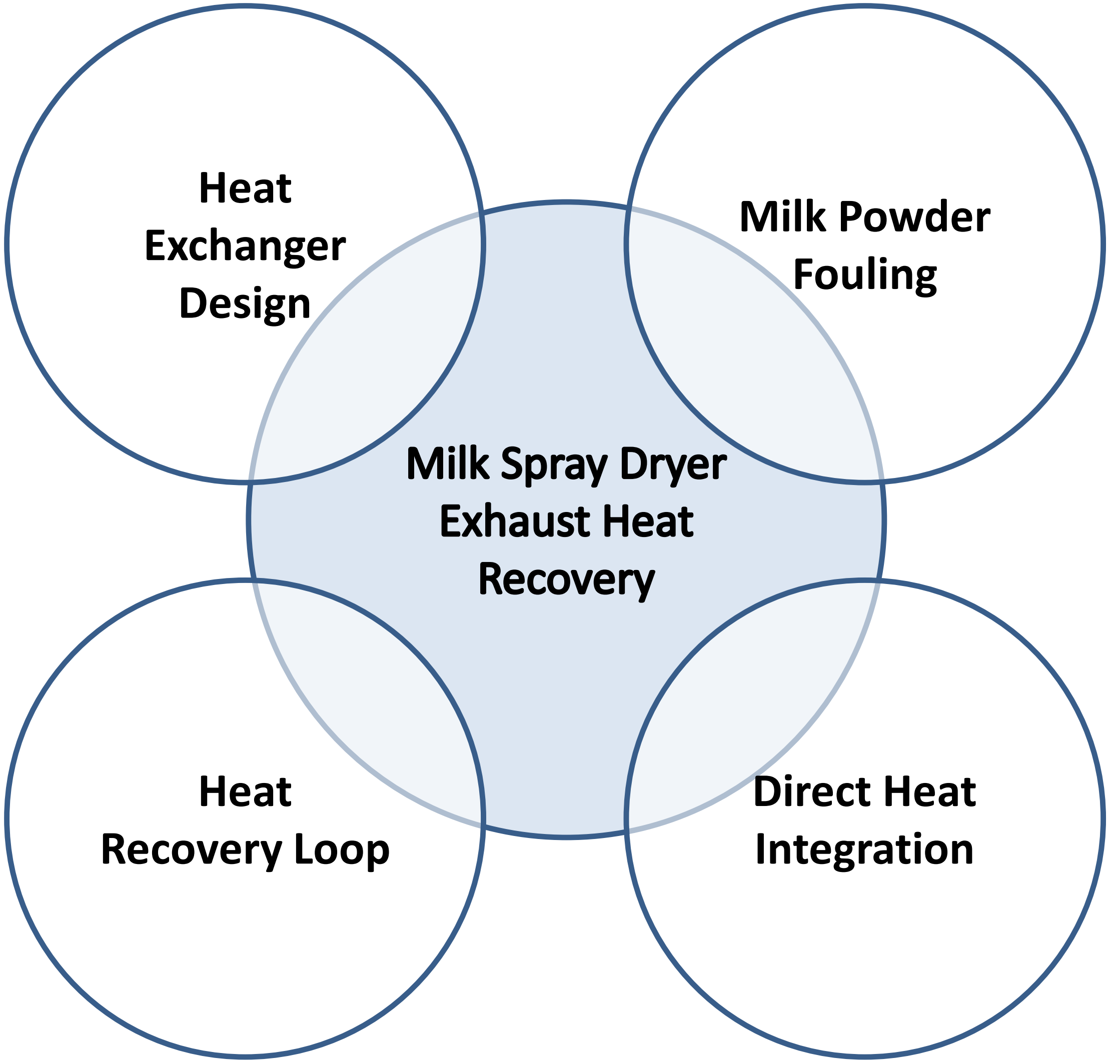
New Zealand

University of Waikato

Dairy

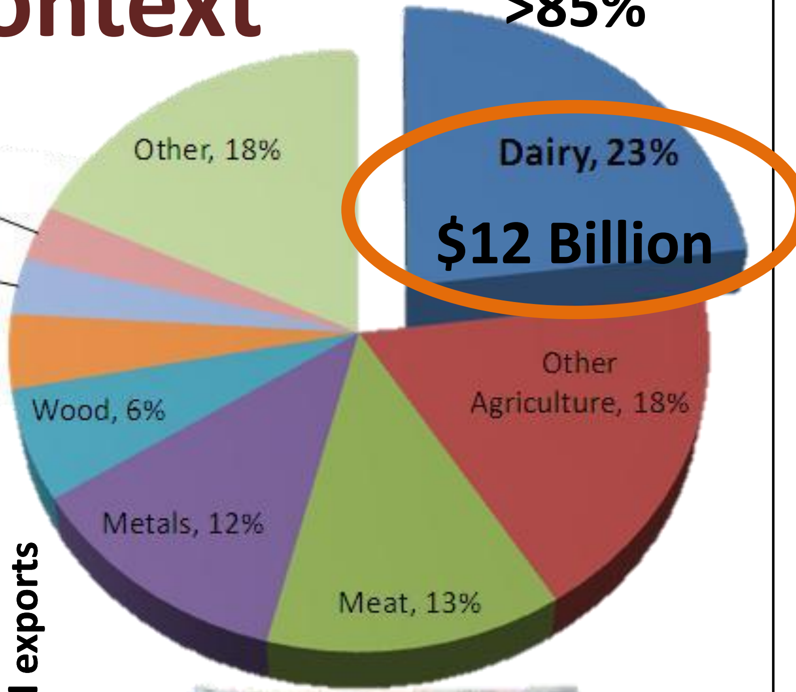
Cows

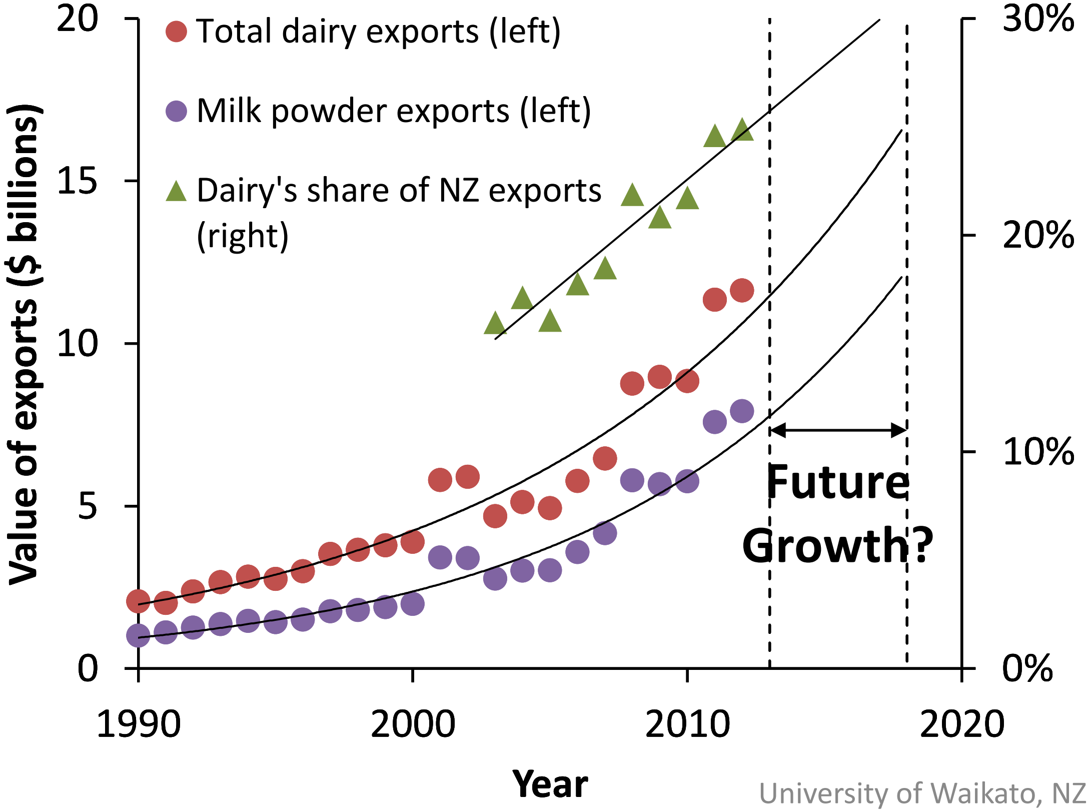
Overall Research Focus / Goal



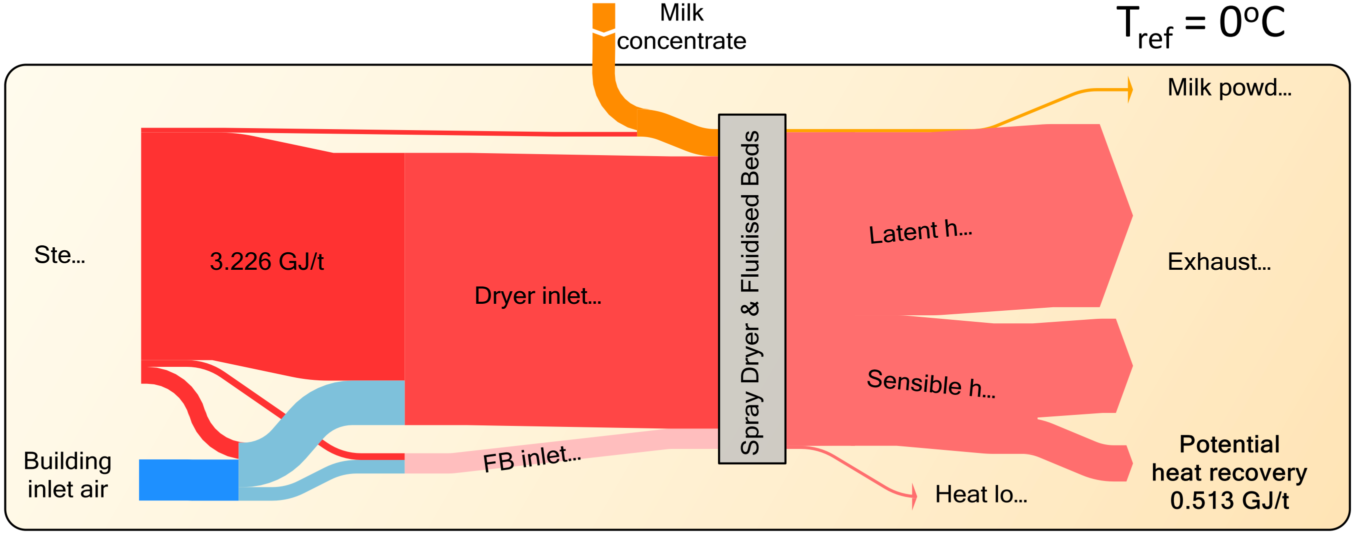
NZ Dairy in Context

Milk Powder
>85%





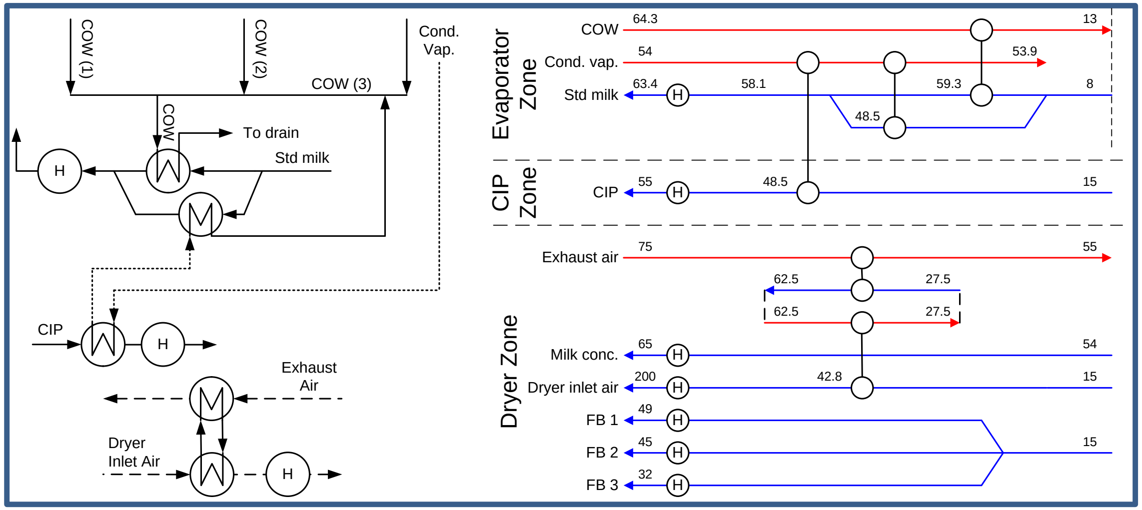
Milk Spray Drying



$T_{ref} = 0^{o}C$

Developed Networks Using ΔT_{cont}

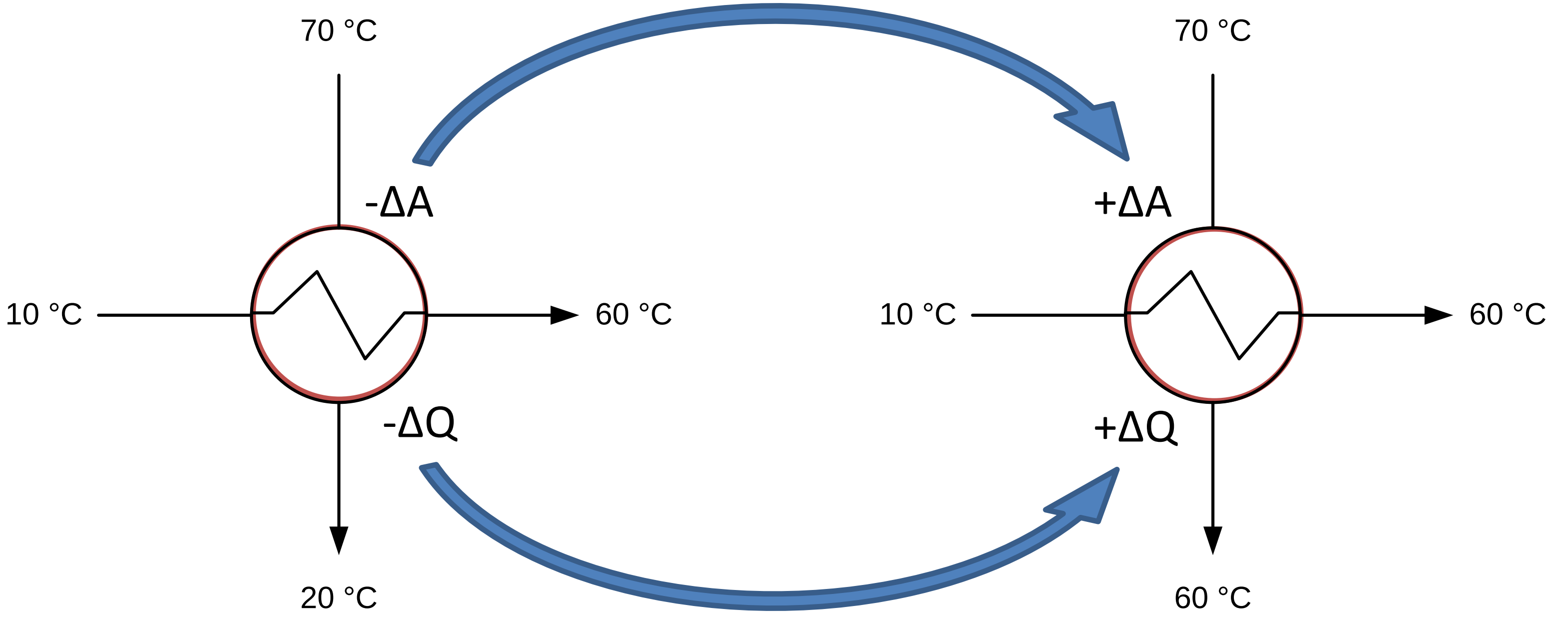
Is there a **better** method to size HE’s?



Walmsley et al. (2013) *Improving Energy Recovery in Milk Powder Production through Soft Data Optimisation*. App. Therm. Eng.

Introduction

- What is **optimal** area allocation in a HEN?
- Can area be **better** allocated in HEN's compared to the ΔT_{min} method?

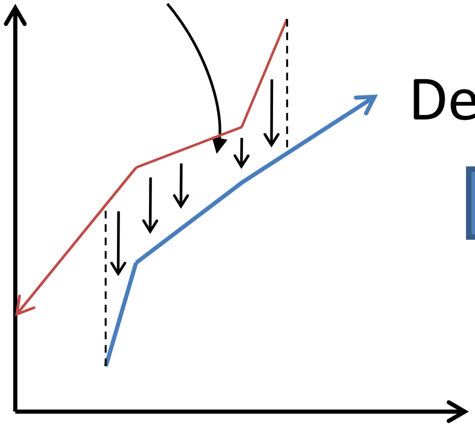


Challenges Facing Finding the Design "Optimum"

- Fluid property difference, e.g. viscosity
- Phase: latent vs senisable heat
- Heat Exchanger type: shell & tube or plate
- Flow arrangement: counter vs parallel flow
- HT film coefficient variations
- Construction material: carbon steel or stainless steel
- Pressure limitations
- Future utility price changes

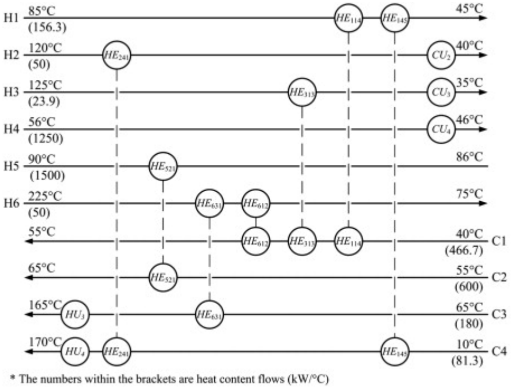
A Brief Review:
Minimising HEN Area Pre-Synthesis

Find min. area target
using CC and LMTD



e.g. Pinch
Design Method

Synthesis



* The numbers within the brackets are heat content flows (kW/°C)

Targeting "Minimum" HEN Area approaches/constraints:

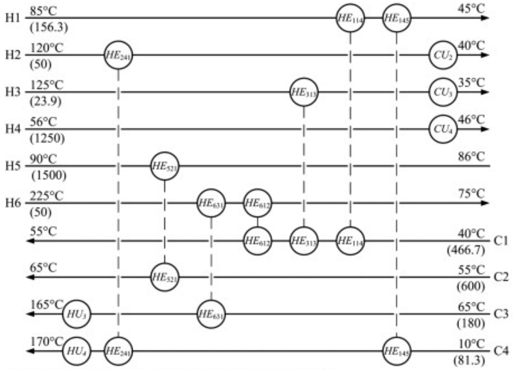
- Minimum approach temperature(s) (ΔT_{min} or ΔT_{cont})
- Minimum heat flux
- Pressure drop feasibility
- Number of exchanger shells and units

A Brief Review:
Minimising HEN Area During-Synthesis

Computer
Programming

e.g. Transhipment
models

Synthesis

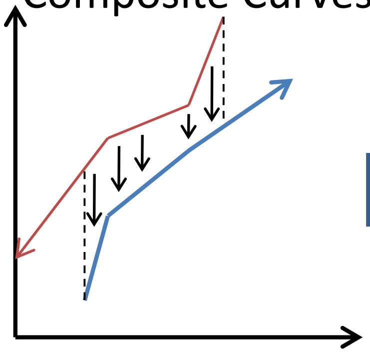


* The numbers within the brackets are heat content flows (kW/°C)

- Numerous studies in this space

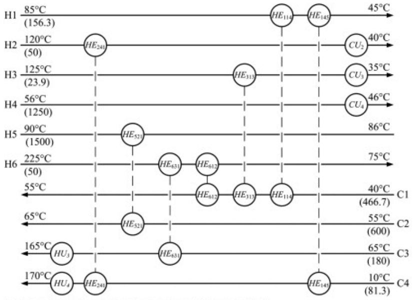
A Brief Review:
Minimising HEN Area Post-Synthesis

Composite Curves



Synthesis

HEN Structure



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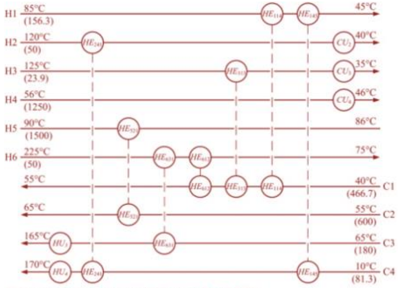
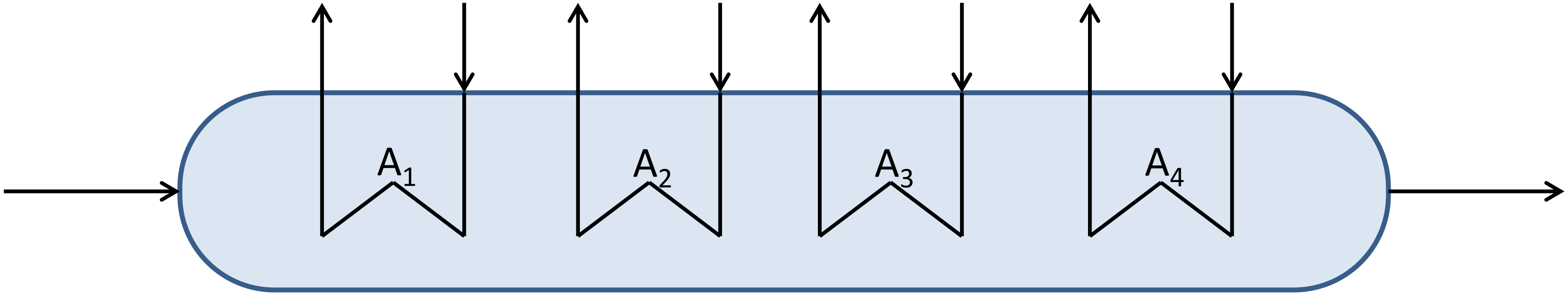
Min. Area

Complete Design

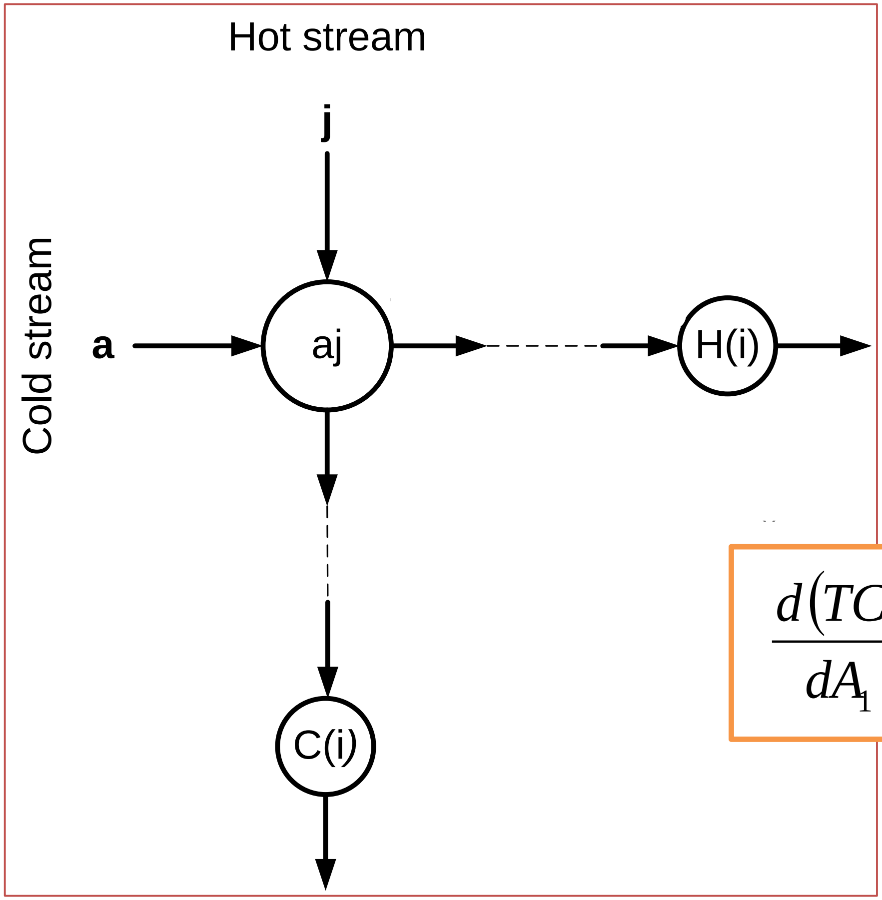


* The numbers within the brackets are heat content flows (kW/°C)

- Very few non-programming based studies in this space
- Ait-Ait & Wade (1980) derived the conditions for optimal area allocation in a multi-stage HE



What is the Cost Optimum HE Size in a HEN?



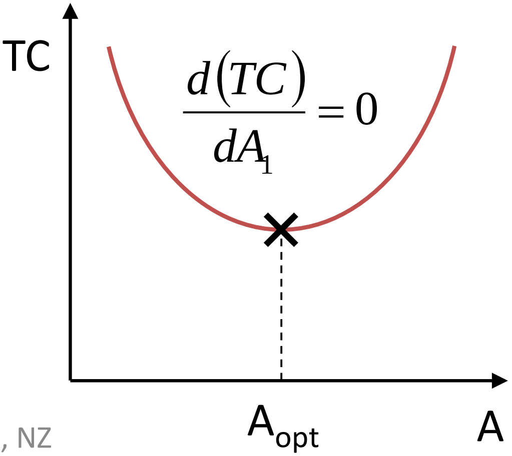
+ Δ Capital Cost on HE_1

- Σ Δ Hot/cold Utility Savings

- Σ Δ Capital Savings Utilities

Δ Total Cost

$$\boxed{\frac{d(TC)}{dA_1} = \frac{d(CC)_1}{dA_1} - \sum\left(P_{ut(i)}\frac{dQ_{ut(i)}}{dA_1}\right) - \sum\frac{dCC_{ut(i)}}{dA_1}}$$



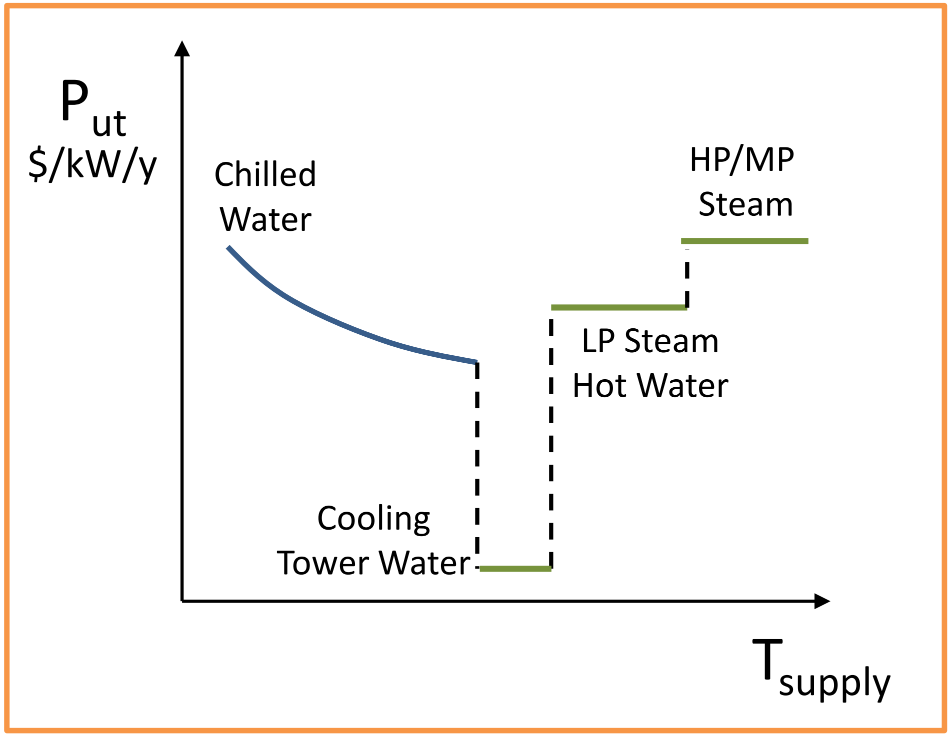
Cost Derivative Method

$$\boxed{\frac{d(TC)}{dA_1} = \frac{d(CC)_1}{dA_1} - \sum\left(P_{ut(i)}\frac{dQ_{ut(i)}}{dA_1}\right) - \sum\frac{dCC_{ut(i)}}{dA_1}}$$

$$CC = F + kA^n \quad \rightarrow \quad \frac{dCC}{dA} = knA^{n-1}$$

Cost Derivative Method

$$\frac{d(TC)}{dA_1} = \frac{d(CC)_1}{dA_1} - \sum\left(\boxed{P_{ut(i)}}\frac{dQ_{ut(i)}}{dA_1}\right) - \sum\frac{dCC_{ut(i)}}{dA_1}$$



Cost Derivative Method

$$\frac{d(TC)}{dA_1} = \frac{d(CC)_1}{dA_1} - \sum\left(P_{ut(i)}\frac{dQ_{ut(i)}}{dA_1}\right) - \sum\frac{dCC_{ut(i)}}{dA_1}$$

Chain Rule:

$$\frac{dQ_{ut(i)}}{dA_1} = \frac{dQ_{ut(i)}}{dQ_1}\frac{dQ_1}{dA_1}$$

Define:

$$\theta_1^{ut(i)} \equiv \frac{-dQ_{ut(i)}}{dQ_1}$$

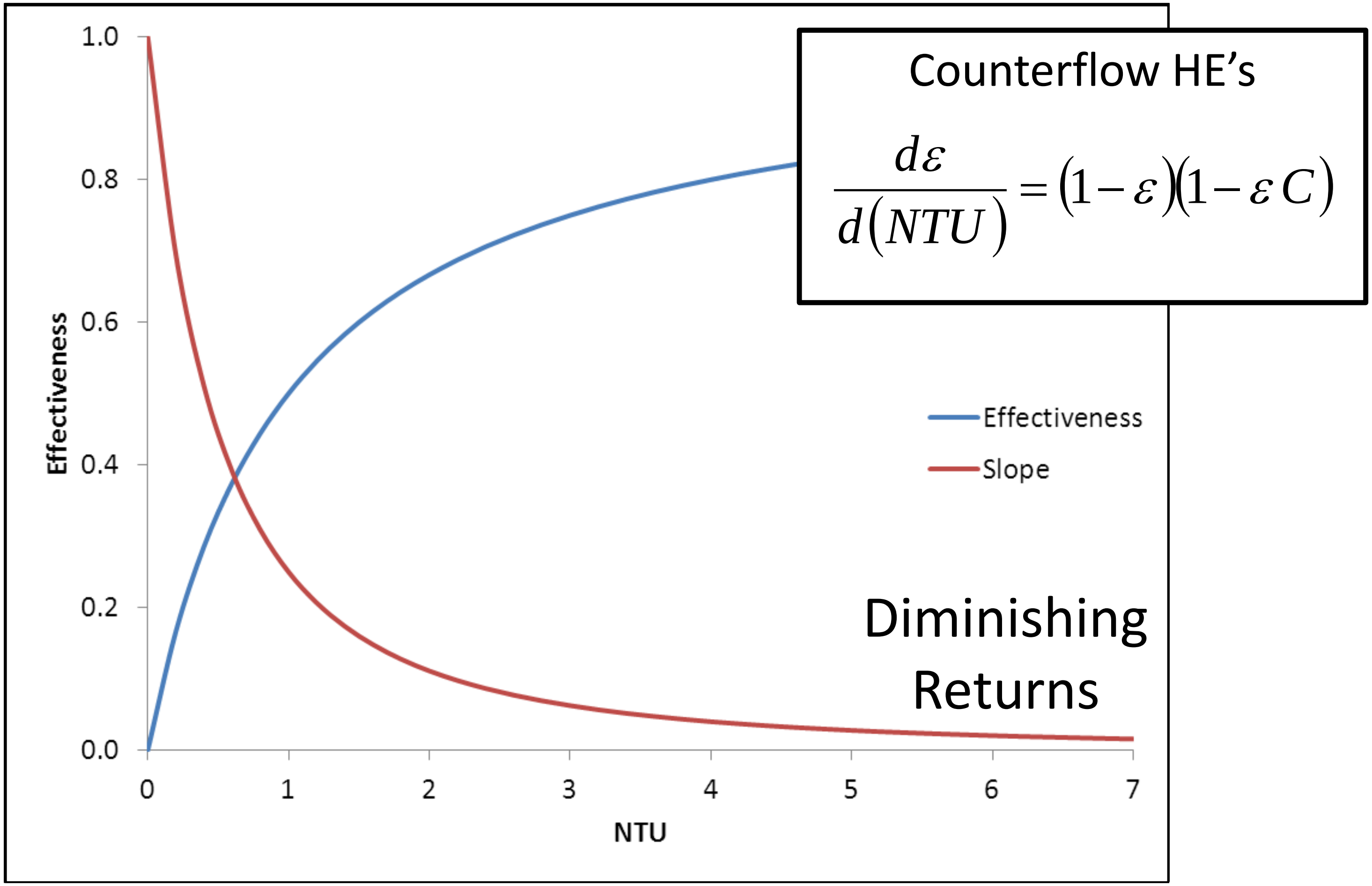
Where *Θ* is called the heat duty flow-on factor

ε-NTU method definitions

Assume C_{min}, ΔT_s, and U are constant with area

$$\frac{dQ_1}{dA_1} = \frac{d(\varepsilon C_{\min}\Delta T_{\max})}{d\left(\frac{C_{\min}NTU}{U}\right)} = U\Delta T_{\max}\boxed{\frac{d\varepsilon}{d(NTU)}}$$

ε-NTU Relationship for Counter-flow



Counterflow HE's

$$\frac{d\varepsilon}{d(NTU)} = (1-\varepsilon)(1-\varepsilon C)$$

Diminishing Returns

$$\frac{d\varepsilon}{d(NTU)} = f(\varepsilon \text{ or } NTU, C^*, \text{Arrangement})$$

Cost Derivative Method

$$\frac{d(TC)}{dA_1} = \frac{d(CC)_1}{dA_1} - \sum\left(P_{ut(i)}\frac{dQ_{ut(i)}}{dA_1}\right) - \sum\frac{dCC_{ut(i)}}{dA_1}$$

Chain Rule: $$\sum\frac{dCC_{ut(i)}}{dA_1} = \sum\left(-\frac{dCC_{ut(i)}}{dA_{ut(i)}}\cdot\frac{dQ_1}{dA_1}\cdot\frac{dQ_{ut(i)}}{dQ_1}\cdot\frac{dA_{ut(i)}}{dQ_{ut(i)}}\right)$$

$$\frac{dCC}{dA} = knA^{n-1}$$

$$\frac{dQ_1}{dA_1} = U\Delta T_{\max}\frac{d\varepsilon}{d(NTU)}$$

$$\frac{dQ_{ut(i)}}{dQ_1} = -\theta_1^{ut(i)}$$

$$\left(\frac{dQ}{dA}\right)_{ut} = \frac{U\Delta T_{\max}}{1-\varepsilon}\frac{d\varepsilon}{dNTU}, \quad \text{when } C_{\min} = C_p$$

$$\left(\frac{dQ}{dA}\right)_{ut} = \frac{U\Delta T_{\max}}{\frac{NTU}{\varepsilon} - \frac{\varepsilon C^*}{d\varepsilon/dNTU}}, \quad \text{when } C_{\min} = C_u$$

Cost Derivative Method

$$\frac{d(TC)}{dA_1} = \frac{d(CC)_1}{dA_1} - \sum\left(P_{ut(i)}\frac{dQ_{ut(i)}}{dA_1}\right) - \sum\frac{dCC_{ut(i)}}{dA_1}$$

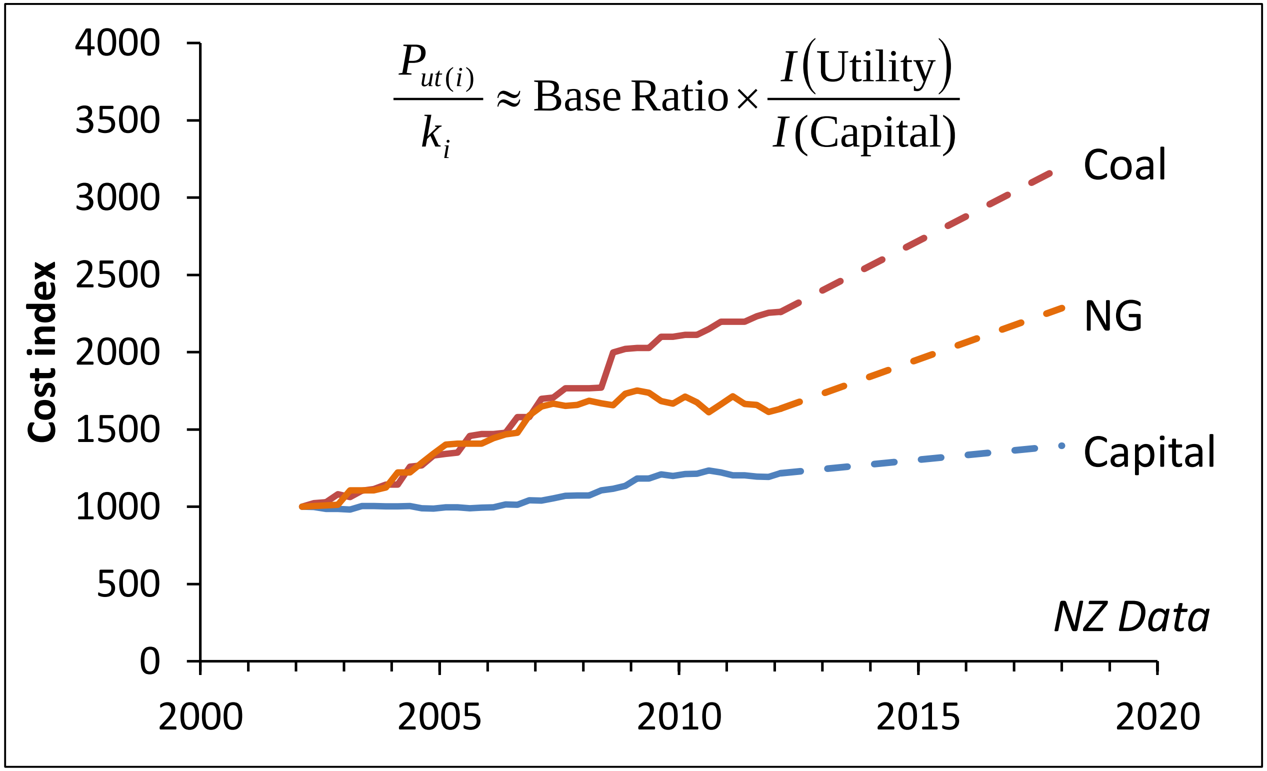
- If d(TC)/dA = 0, then cost “optimum” sizing is achieved for one HE

$$\frac{d\varepsilon}{d(NTU)} = \frac{\left(knA^{n-1}\right)_1}{U_1 \Delta T_{\max,1} \sum\left(\theta_1^{ut(i)}\left(P_{ut(i)} + \frac{\left(knA^{n-1}\right)_{ut(i)}}{dQ_{ut(i)}/dA_{ut(i)}}\right)\right)}$$

- Let’s look at the **Utility-Capital Trade-off**

Utility-Capital Cost Trade-off

$$\frac{d\varepsilon}{d(NTU)} = \frac{\left(knA^{n-1}\right)_1}{U_1 \Delta T_{\max,1} \sum \left(\theta_1^{ut(i)} \left(P_{ut(i)} + \frac{\left(knA^{n-1}\right)_{ut(i)}}{dQ_{ut(i)} / dA_{ut(i)}} \right) \right)}$$



$$\frac{P_{ut(i)}}{k_i} \approx \text{Base Ratio} \times \frac{I(\text{Utility})}{I(\text{Capital})}$$

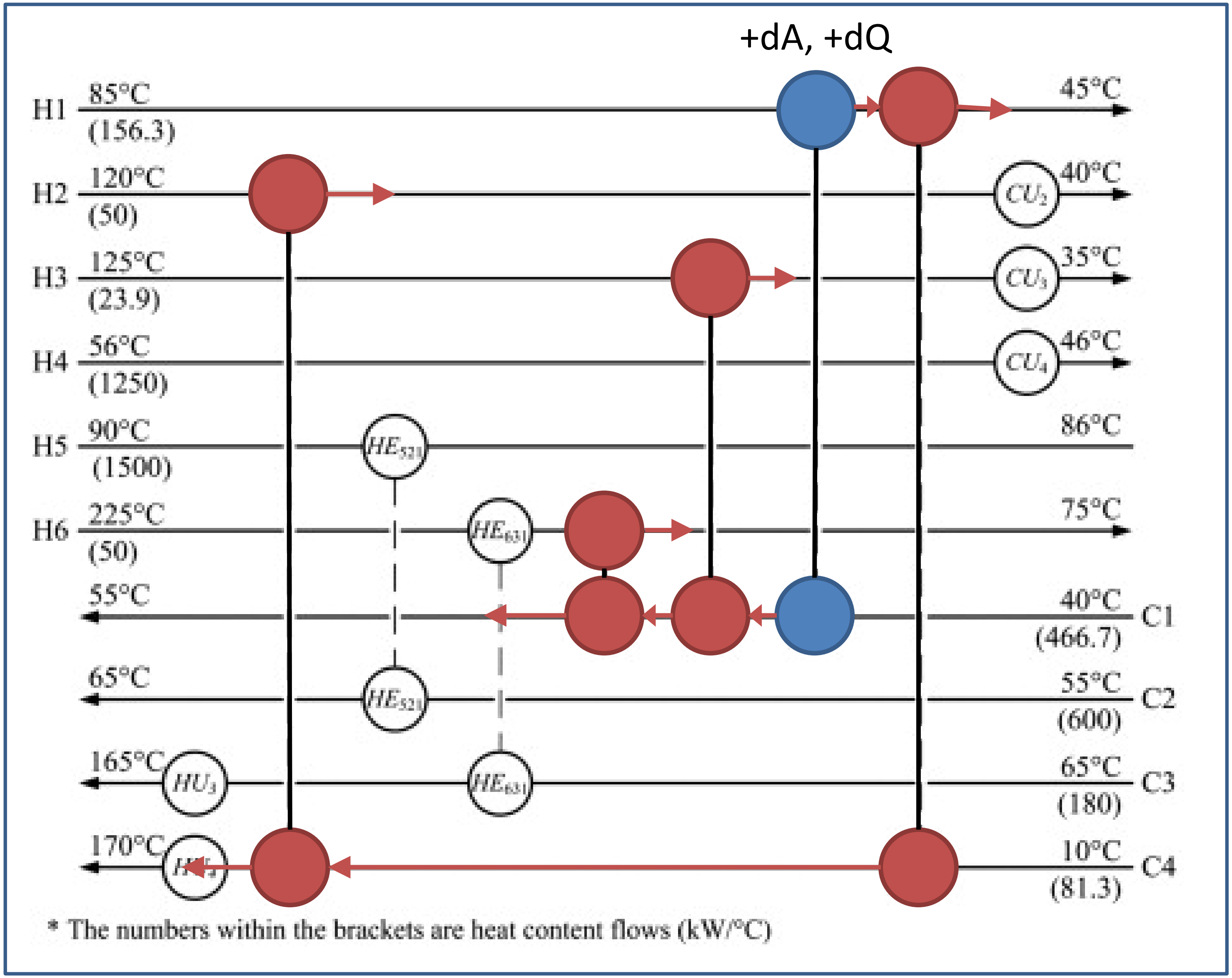
Cost Derivative Method

- If d(TC)/dA = 0 and all HE's fulfil the criteria below, then cost "optimum" sizing is achieved.

$$\frac{d\varepsilon}{d(NTU)} = \frac{\left(knA^{n-1}\right)_1}{U_1 \Delta T_{\max,1} \sum \left(\enclose{circle}{\theta_1^{ut(i)}} \left(P_{ut(i)} + \frac{\left(knA^{n-1}\right)_{ut(i)}}{dQ_{ut(i)} / dA_{ut(i)}} \right) \right)}$$

- Let's look at the **Heat Duty Flow-on Factor**

The Heat Duty Flow on Effect in HENs



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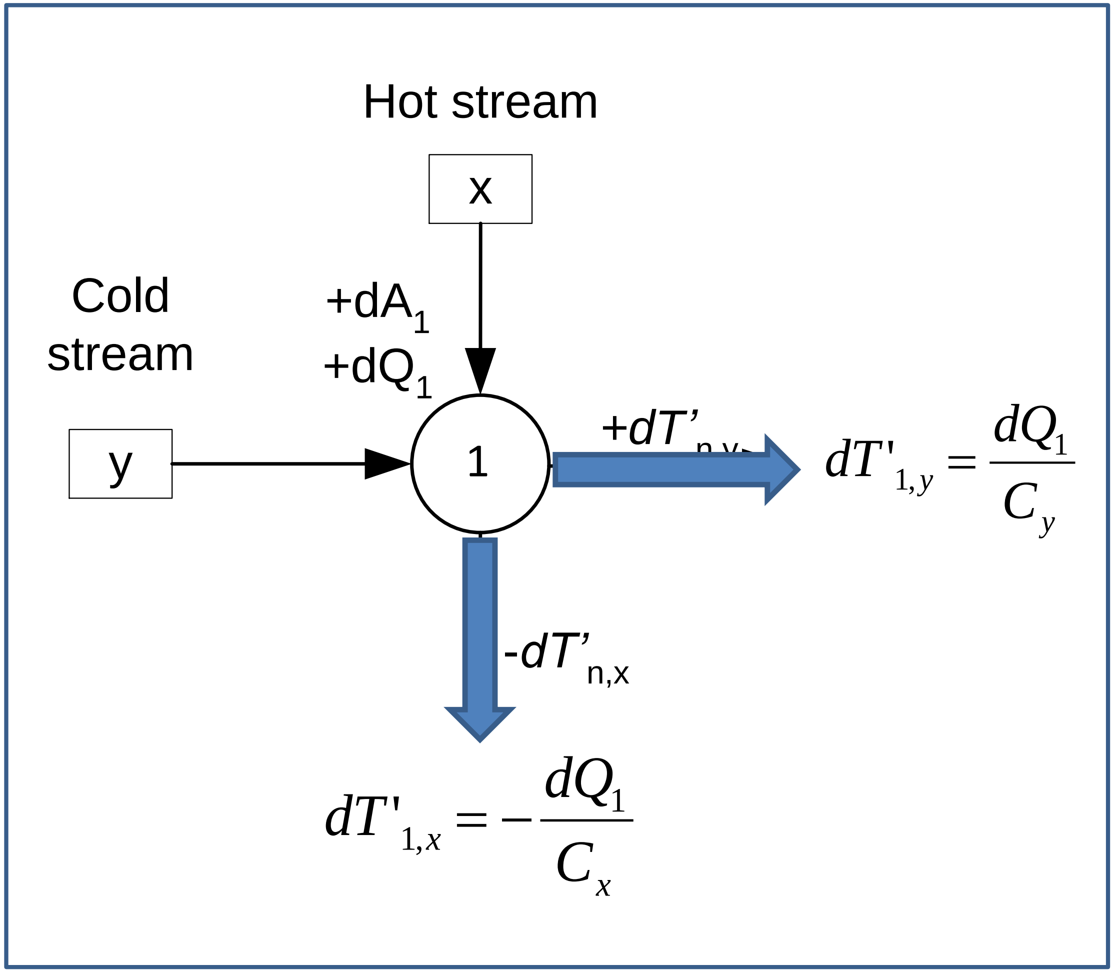
Heat Duty Flow-on: Origin

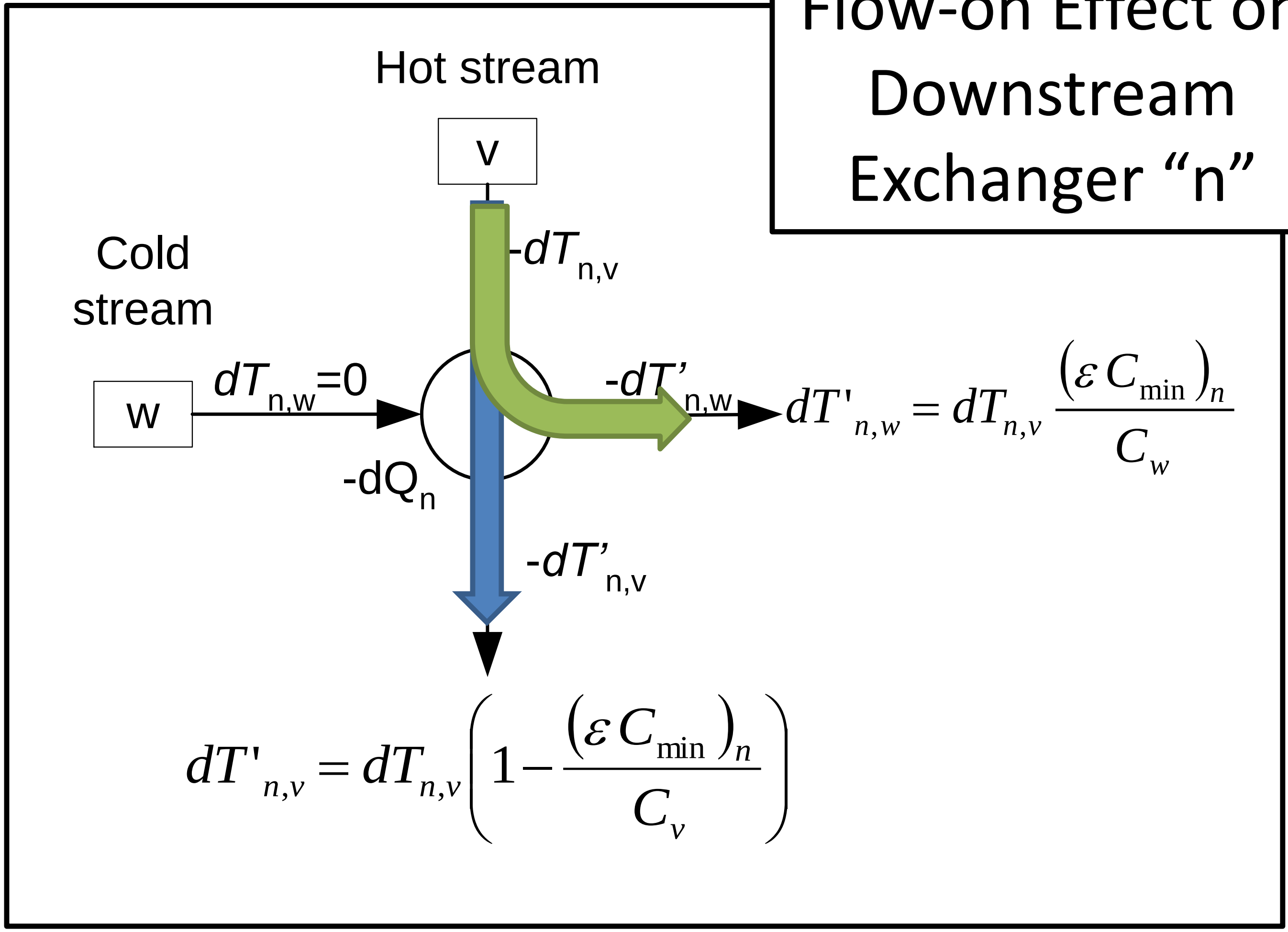


$$dT'_{1,y} = \frac{dQ_1}{C_y}$$

$$dT'_{1,x} = -\frac{dQ_1}{C_x}$$

Heat Duty Flow-on Factor: Propagation

Flow-on Effect on Downstream Exchanger "n"

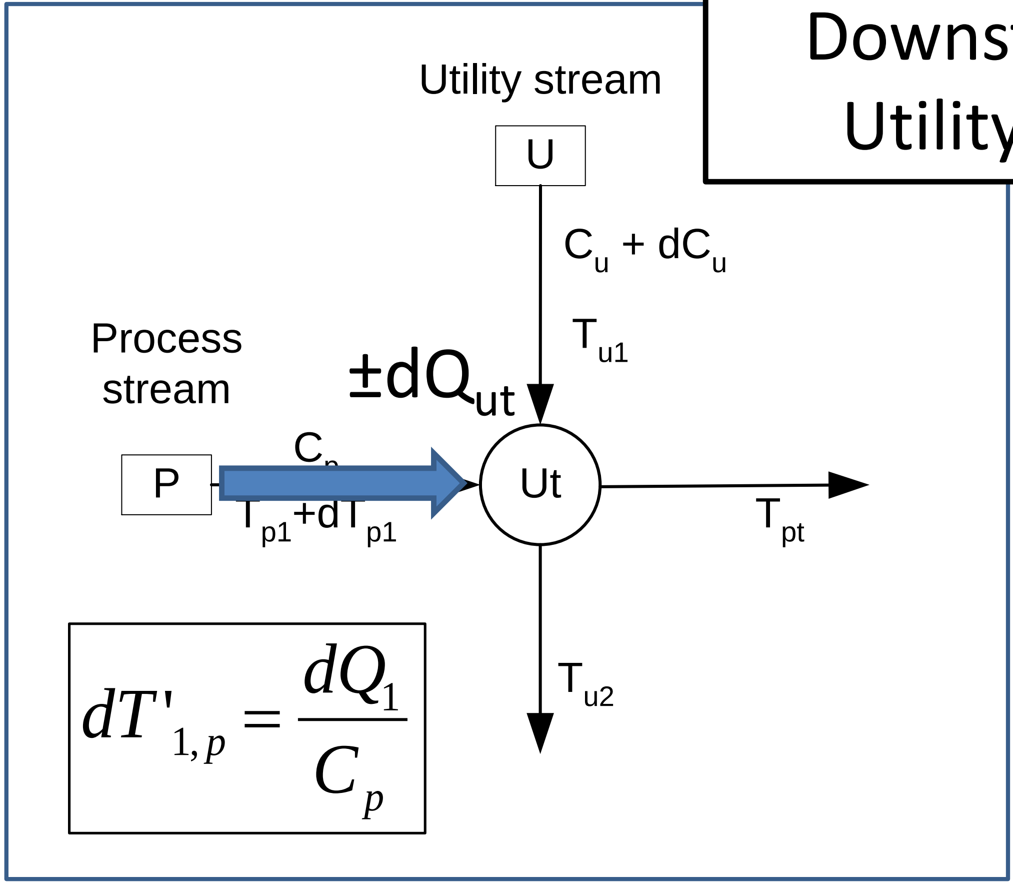


$$dT'_{n,w} = dT_{n,v} \frac{(\varepsilon C_{\min})_n}{C_w}$$

$$dT'_{n,v} = dT_{n,v}\left(1 - \frac{(\varepsilon C_{\min})_n}{C_v}\right)$$

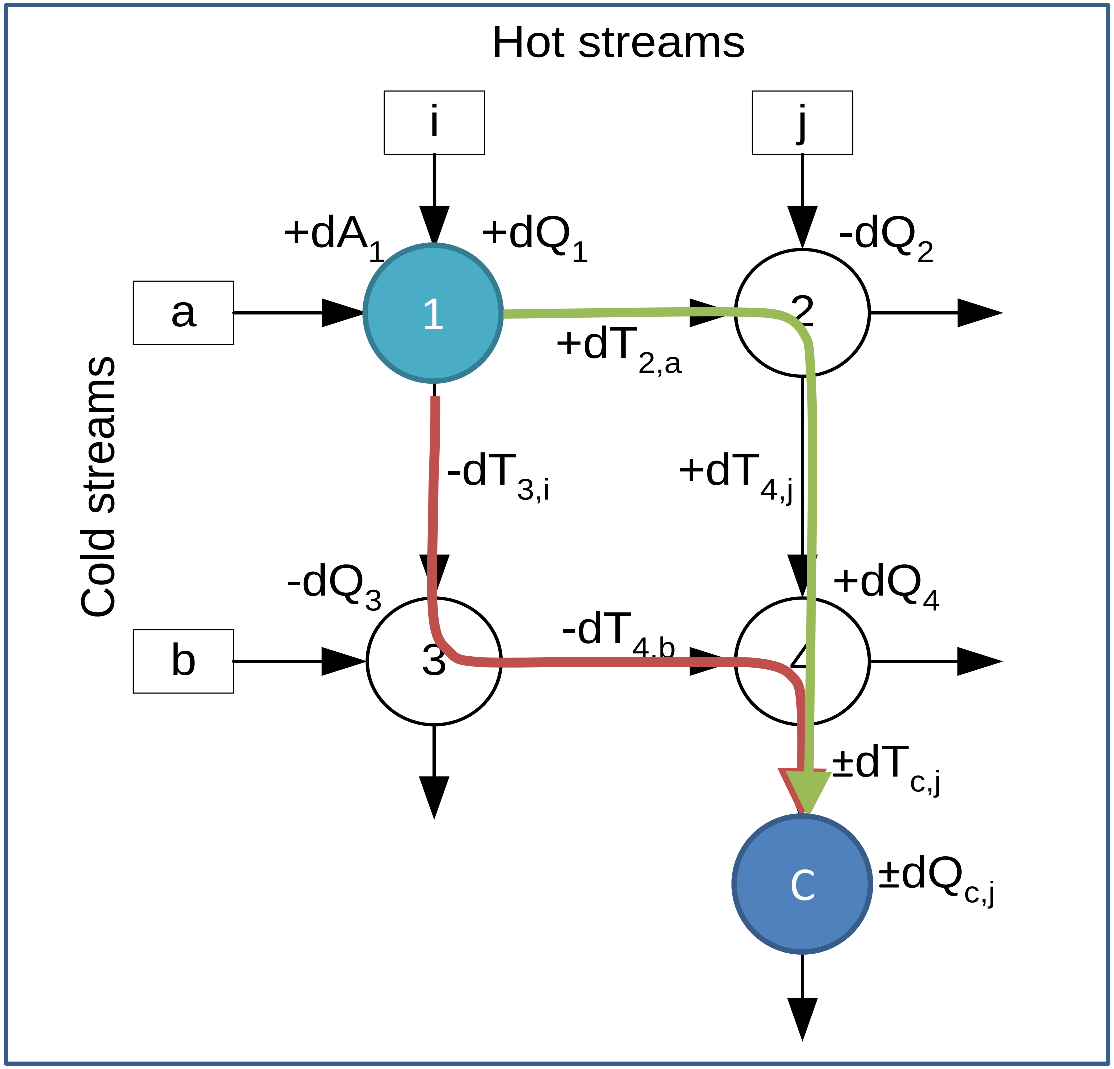
Heat Duty Flow-on Factor: Termination

Flow-on Effect on Downstream Utility Use



$$dT'_{1,p} = \frac{dQ_1}{C_p}$$

Heat Duty Flow-on Factor: Pathways



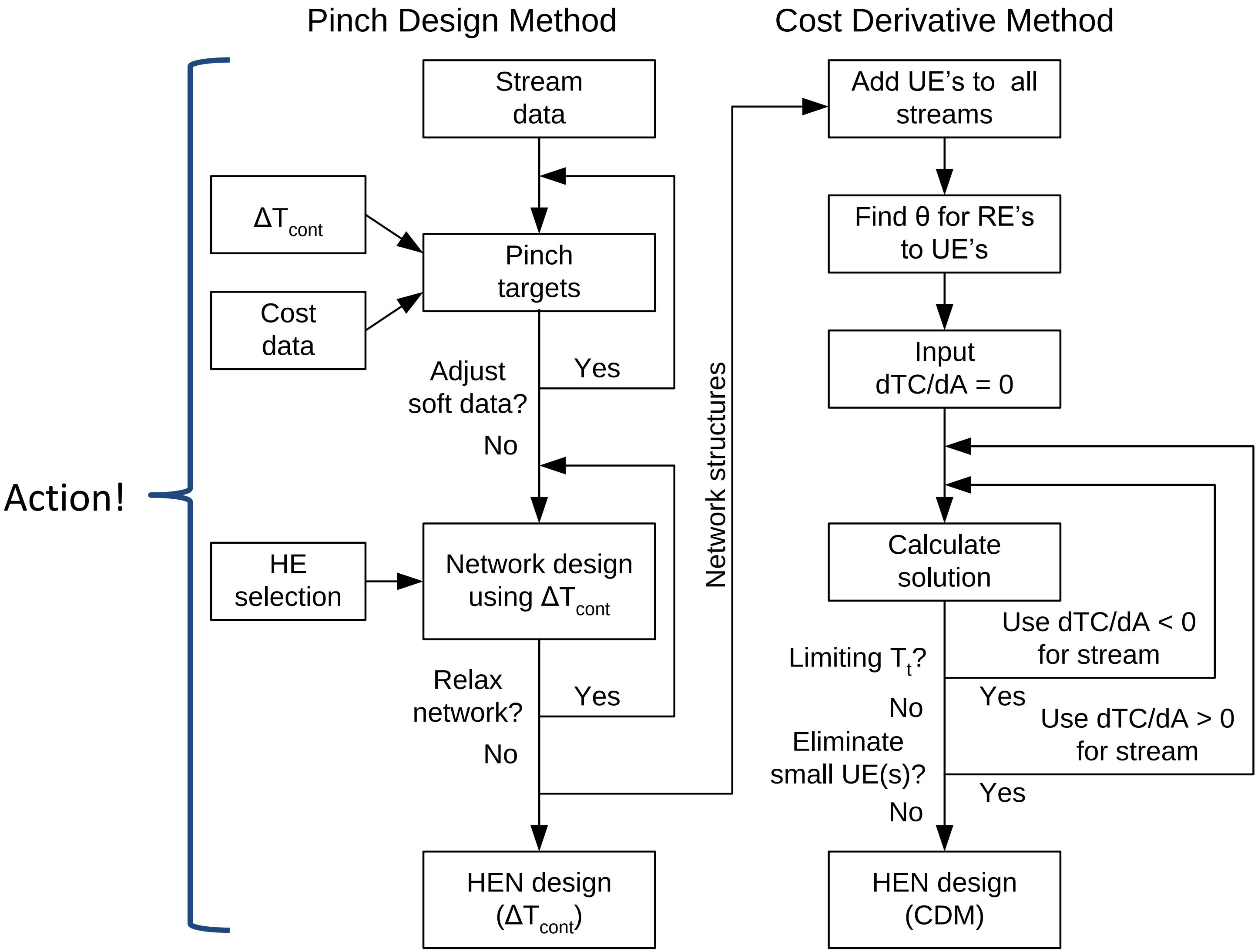
Example: Gundersen (2000)

Stream data for a simple distillation separation process

Stream	**Code**	**T_s (°C)**	**T_t (°C)**	**C (kW/°C)**	**Q (kW)**	**h (kW/m^2 °C)**	**φ ($/y/kW)**
Reactor outlet	H1	270	160	18	1980	0.5	
Product	H2	220	60	22	3520	0.5	
Feed	C1	50	210	20	3200	0.5	
Recycle	C2	160	210	50	2500	0.5	
Steam	HU	250	249			2.5	200
Cooling water	CU	15	20			1	20

*Data from *A Process Integration Primer* (2000)

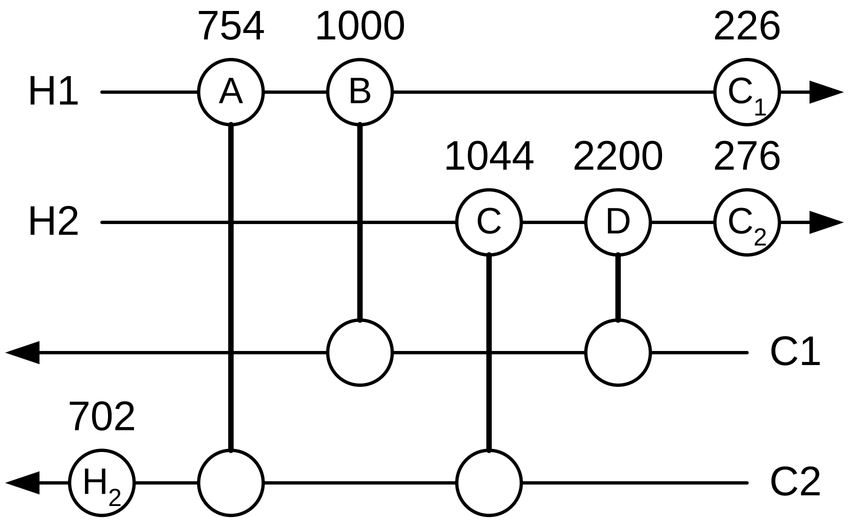
Application Algorithm



Example: HEN from PDM

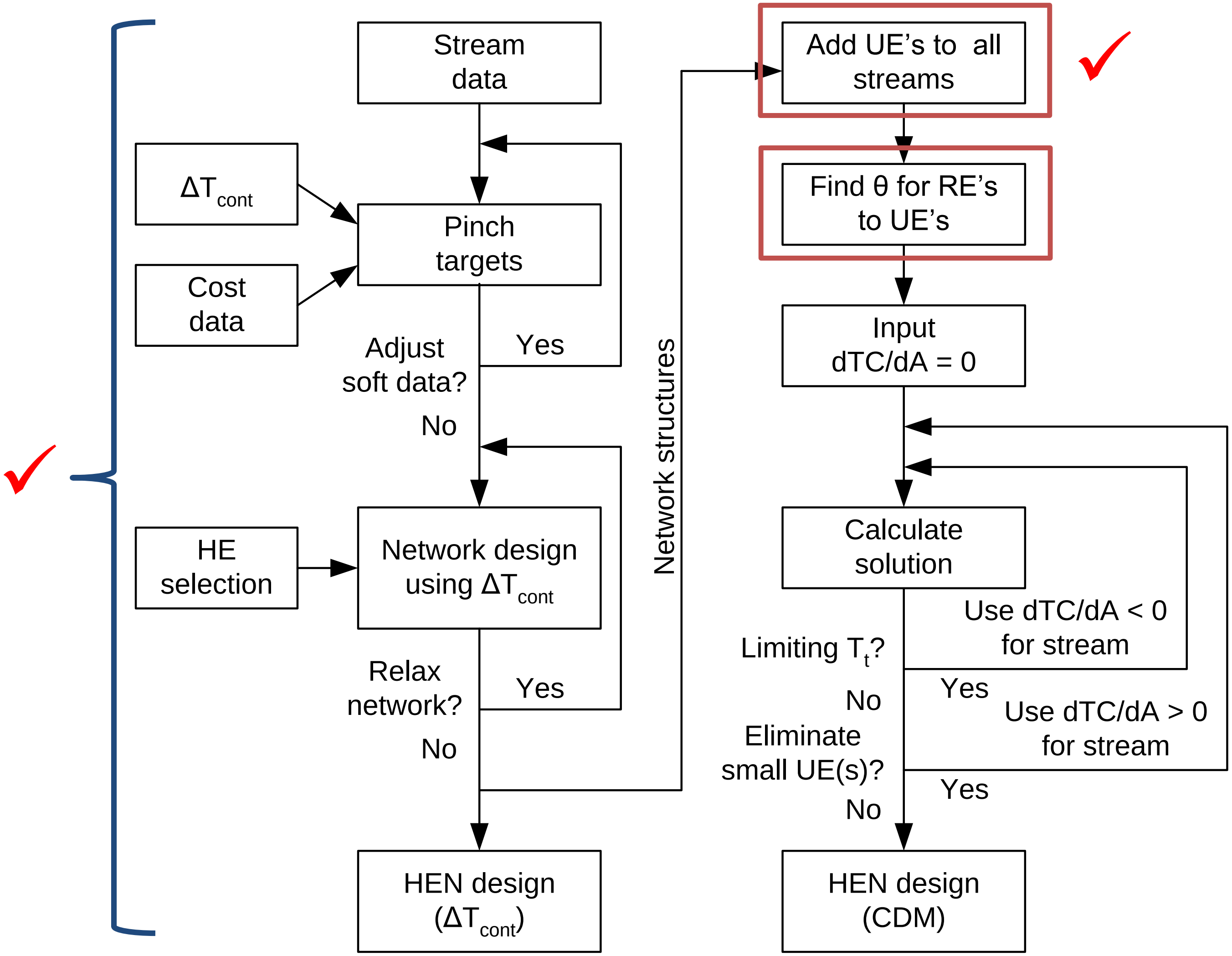
Q_r = 5.0 MW for ΔT_{min}(opt) = 12.5 °C

(a) PDM



Can **HE sizing** be improved
to achieve lower Total Annual Cost?

Application Algorithm



Pinch Design Method

Stream data

ΔT_{cont}

Cost data

Pinch targets

Adjust soft data?

Yes

No

HE selection

Network design using ΔT_{cont}

Relax network?

Yes

No

HEN design (ΔT_{cont})

Network structures

Cost Derivative Method

Add UE's to all streams

✓

Find θ for RE's to UE's

Input dTC/dA = 0

Calculate solution

Limiting T_t?

Yes

Use dTC/dA < 0 for stream

No

Eliminate small UE(s)?

Yes

Use dTC/dA > 0 for stream

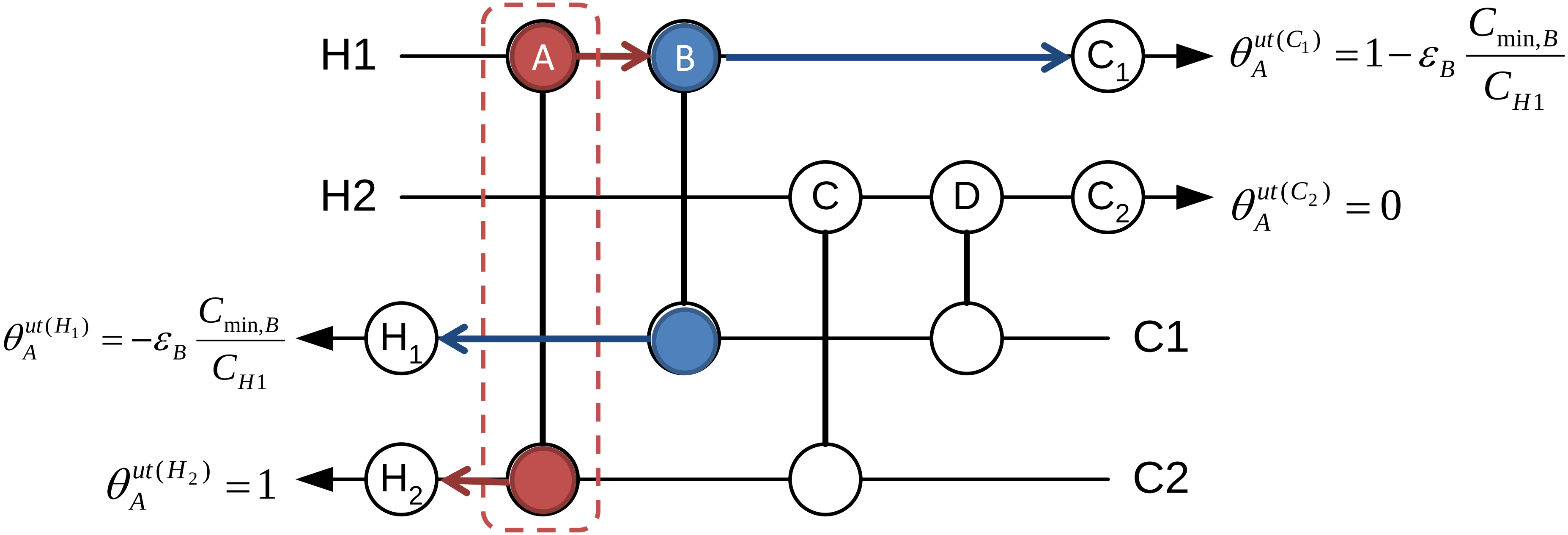
No

HEN design (CDM)

✓

Example: Flow-on effects for A

Two HX's in series



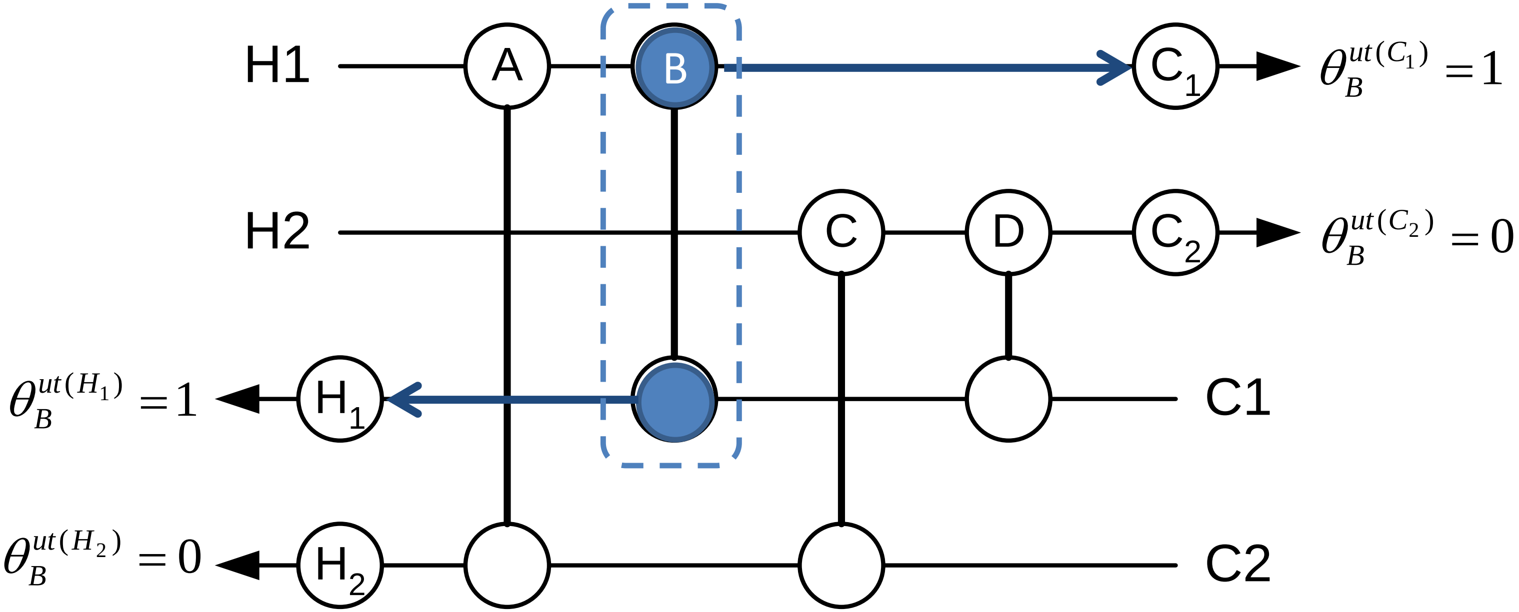
$$\theta_A^{ut(C_1)} = 1 - \varepsilon_B \frac{C_{\min,B}}{C_{H1}}$$

$$\theta_A^{ut(C_2)} = 0$$

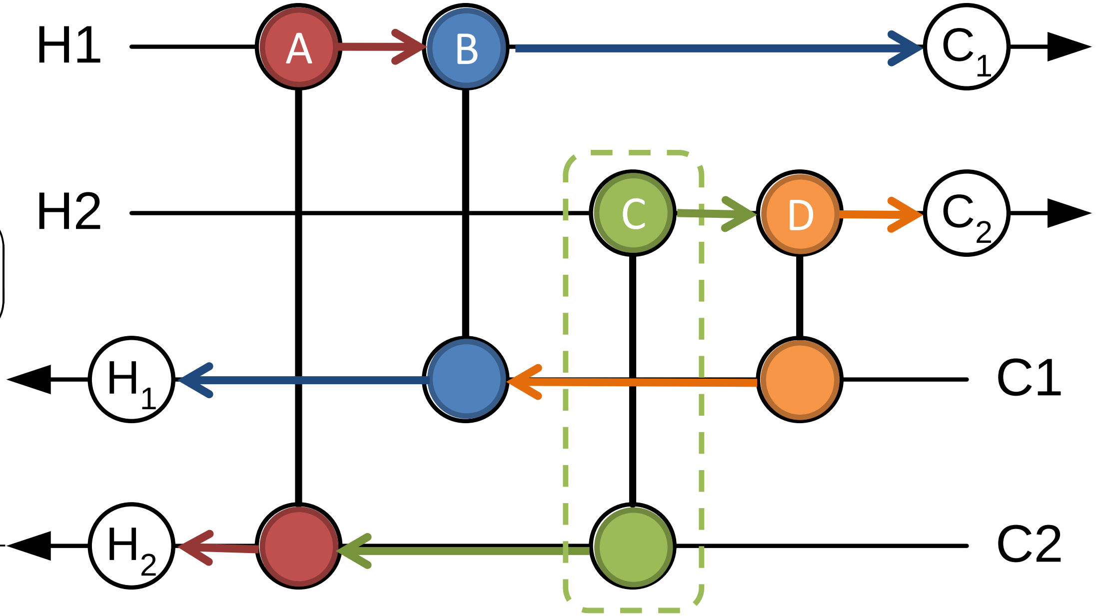
$$\theta_A^{ut(H_1)} = -\varepsilon_B \frac{C_{\min,B}}{C_{H1}}$$

$$\theta_A^{ut(H_2)} = 1$$

Example: Flow-on effects for B



Example: Flow-on effects for C



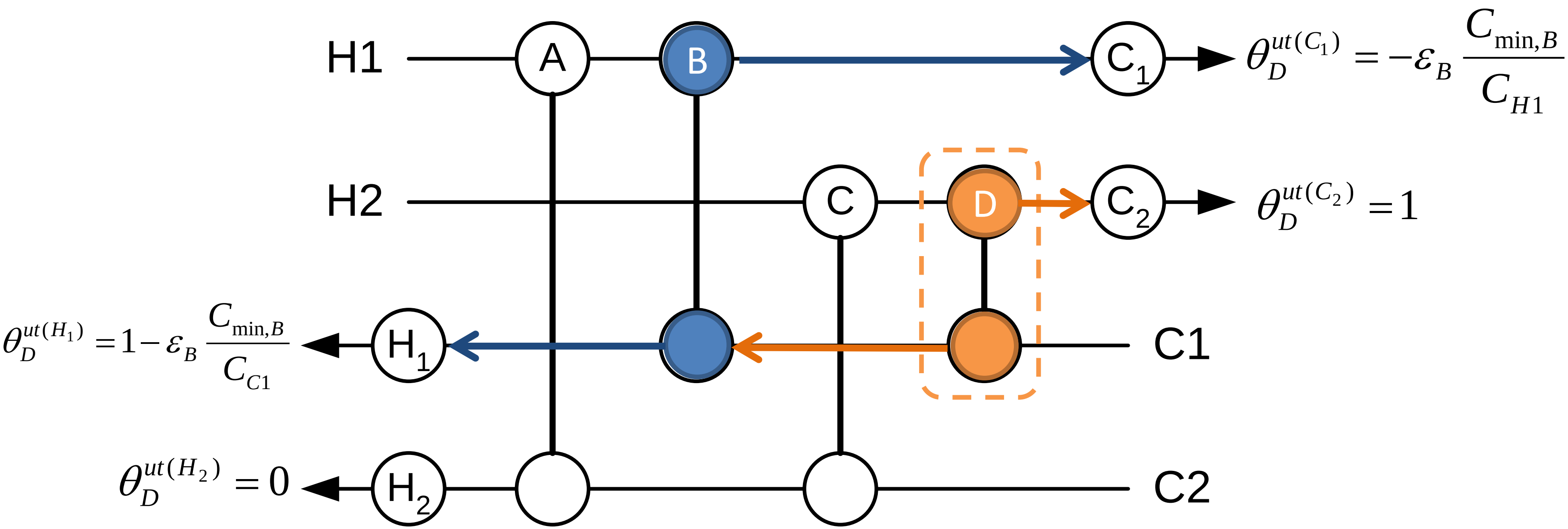
$$\theta_C^{ut(C_1)} = \left(-\varepsilon_D \frac{C_{\min,D}}{C_{C1}}\right)\left(-\varepsilon_B \frac{C_{\min,B}}{C_{C1}}\right) - \left(\varepsilon_A \frac{C_{\min,A}}{C_{H1}}\right)\left(1-\varepsilon_B \frac{C_{\min,B}}{C_{H1}}\right)$$

$$\theta_C^{ut(C_2)} = 1-\varepsilon_D \frac{C_{\min,D}}{C_{H2}}$$

$$\theta_C^{ut(H_1)} = \left(-\varepsilon_D \frac{C_{\min,D}}{C_{C1}}\right) \times \left(1-\varepsilon_B \frac{C_{\min,B}}{C_{C1}}\right)$$

$$\theta_C^{ut(C_2)} = 1-\varepsilon_A \frac{C_{\min,A}}{C_{C2}}$$

Example: Flow-on effects for D



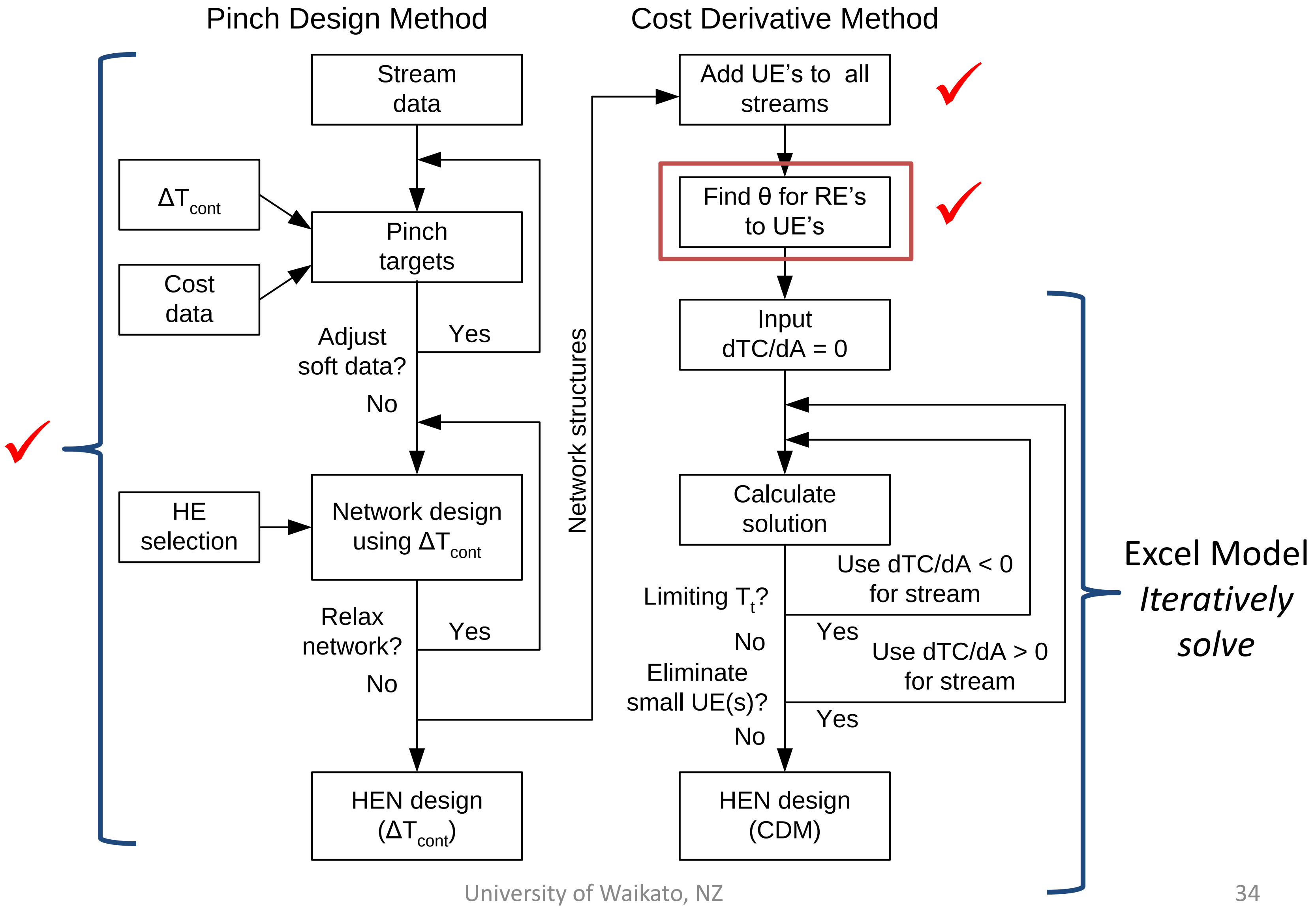
$$\theta_D^{ut(C_1)} = -\varepsilon_B \frac{C_{\min,B}}{C_{H1}}$$

$$\theta_D^{ut(C_2)} = 1$$

$$\theta_D^{ut(H_1)} = 1 - \varepsilon_B \frac{C_{\min,B}}{C_{C1}}$$

$$\theta_D^{ut(H_2)} = 0$$

Application Algorithm



Pinch Design Method

✓

Stream data

ΔT_{cont}

Cost data

Pinch targets

Adjust soft data?

Yes

No

HE selection

Network design using ΔT_{cont}

Relax network?

Yes

No

HEN design (ΔT_{cont})

Network structures

Cost Derivative Method

Add UE's to all streams ✓

Find θ for RE's to UE's ✓

Input dTC/dA = 0

Calculate solution

Limiting T_t?

Use dTC/dA < 0 for stream

Yes

No

Eliminate small UE(s)?

Use dTC/dA > 0 for stream

Yes

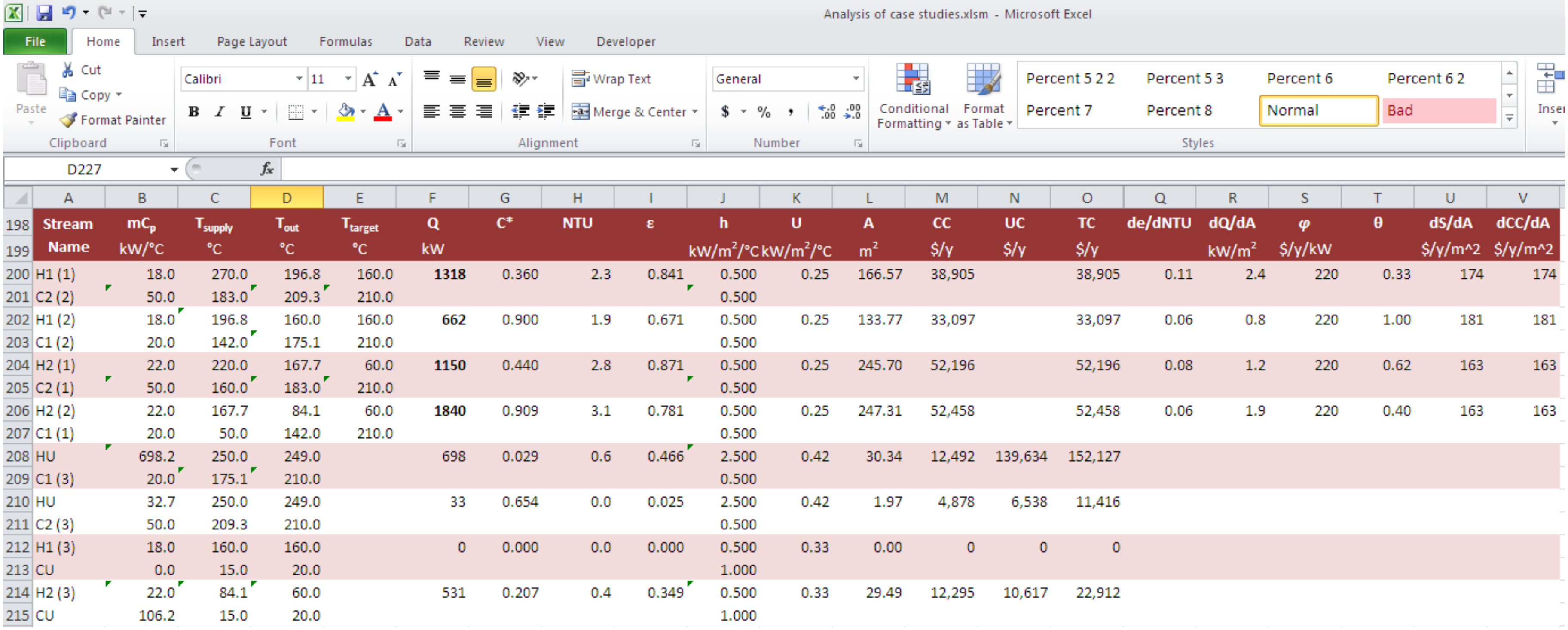
No

HEN design (CDM)

Excel Model *Iteratively solve*

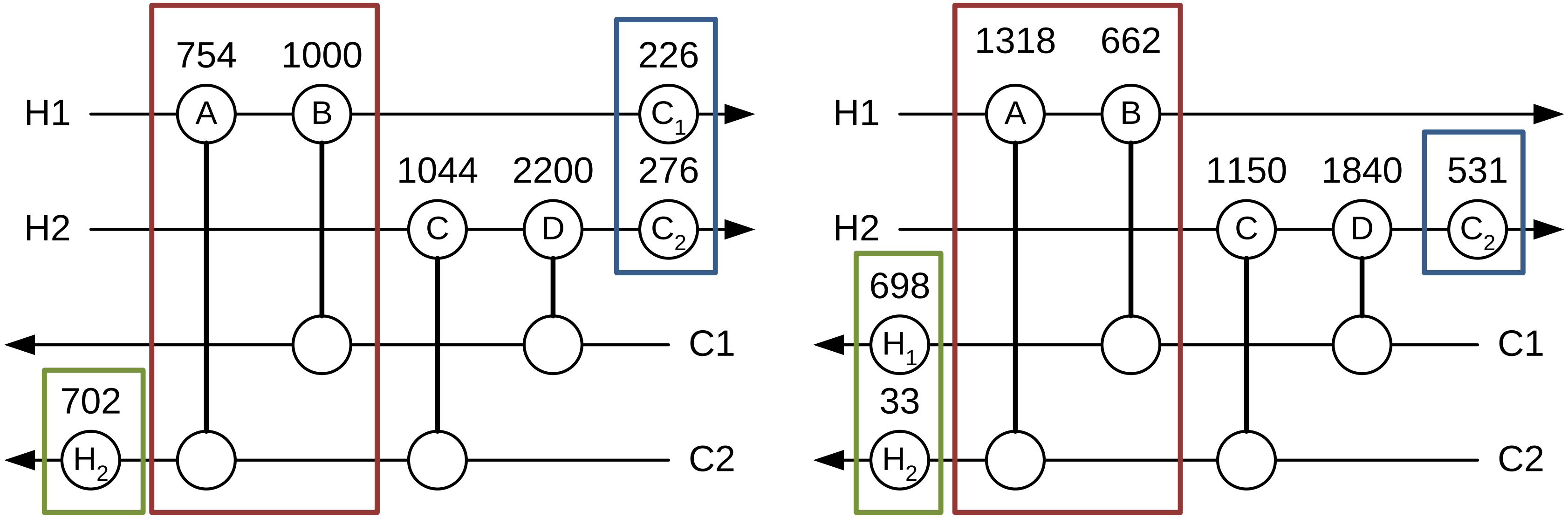
Modelling CDM in Excel

- Set-up method in a Excel Spreadsheet
- Located the cost optimal area allocation



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	Q	R	S	T	U	V
198	**Stream**	**mC_p**	**T_{supply}**	**T_{out}**	**T_{target}**	**Q**	**C***	**NTU**	**ε**	**h**	**U**	**A**	**CC**	**UC**	**TC**	**de/dNTU**	**dQ/dA**	**φ**	**θ**	**dS/dA**	**dCC/dA**
199	**Name**	kW/°C	°C	°C	°C	kW				kW/m²/°C	kW/m²/°C	m²	\$/y	\$/y	\$/y		kW/m²	\$/y/kW		\$/y/m^2	\$/y/m^2
200	H1 (1)	18.0	270.0	196.8	160.0	**1318**	0.360	2.3	0.841	0.500	0.25	166.57	38,905		38,905	0.11	2.4	220	0.33	174	174
201	C2 (2)	50.0	183.0	209.3	210.0					0.500											
202	H1 (2)	18.0	196.8	160.0	160.0	**662**	0.900	1.9	0.671	0.500	0.25	133.77	33,097		33,097	0.06	0.8	220	1.00	181	181
203	C1 (2)	20.0	142.0	175.1	210.0					0.500											
204	H2 (1)	22.0	220.0	167.7	60.0	**1150**	0.440	2.8	0.871	0.500	0.25	245.70	52,196		52,196	0.08	1.2	220	0.62	163	163
205	C2 (1)	50.0	160.0	183.0	210.0					0.500											
206	H2 (2)	22.0	167.7	84.1	60.0	**1840**	0.909	3.1	0.781	0.500	0.25	247.31	52,458		52,458	0.06	1.9	220	0.40	163	163
207	C1 (1)	20.0	50.0	142.0	210.0					0.500											
208	HU	698.2	250.0	249.0		698	0.029	0.6	0.466	2.500	0.42	30.34	12,492	139,634	152,127						
209	C1 (3)	20.0	175.1	210.0						0.500											
210	HU	32.7	250.0	249.0		33	0.654	0.0	0.025	2.500	0.42	1.97	4,878	6,538	11,416						
211	C2 (3)	50.0	209.3	210.0						0.500											
212	H1 (3)	18.0	160.0	160.0		0	0.000	0.0	0.000	0.500	0.33	0.00	0	0	0						
213	CU	0.0	15.0	20.0						1.000											
214	H2 (3)	22.0	84.1	60.0		531	0.207	0.4	0.349	0.500	0.33	29.49	12,295	10,617	22,912						
215	CU	106.2	15.0	20.0						1.000											

Initial Comparison of CDM to PDM



TC = $**386** k /y

(a) PDM

TC = $**363** k /y

(b) CDM

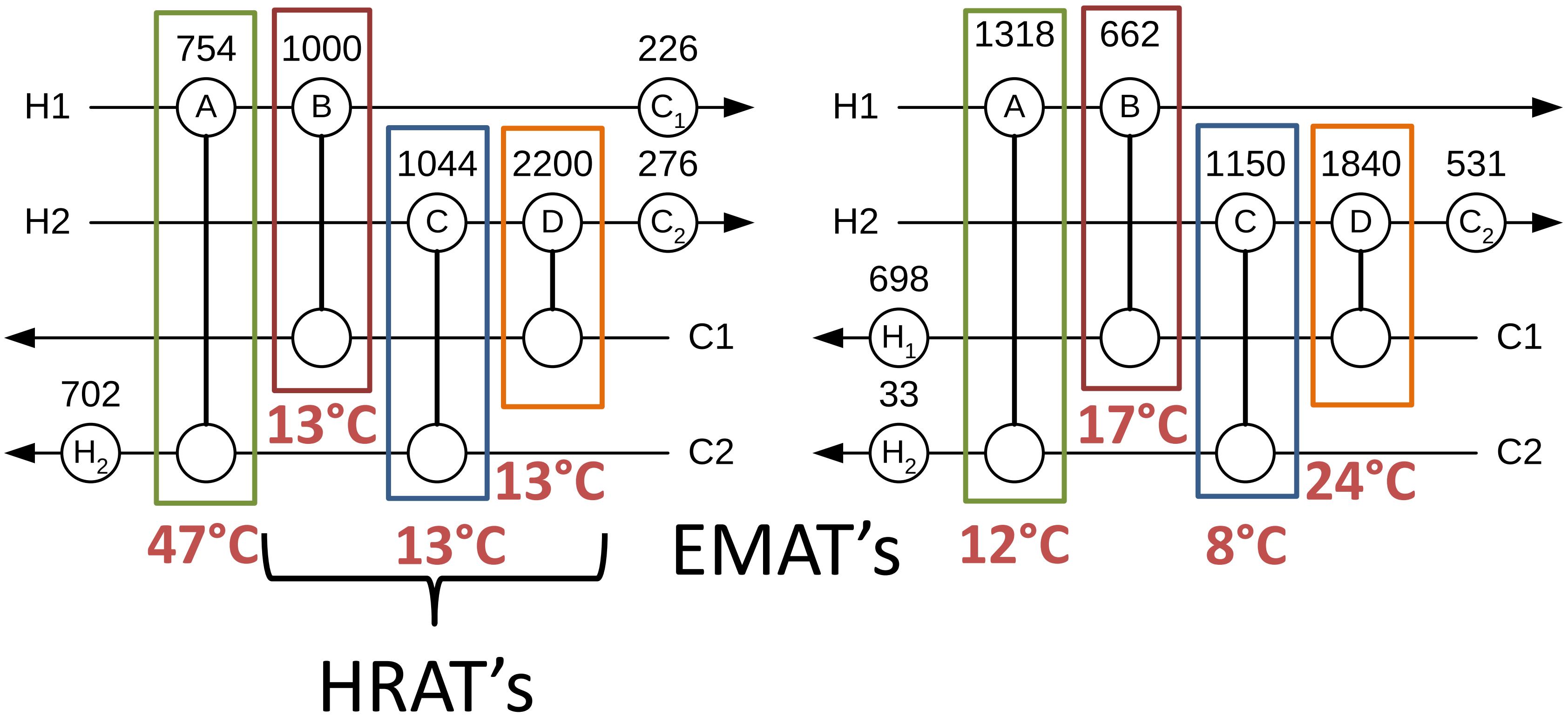
Initial Comparison of CDM to PDM

TC = $**386** k /y

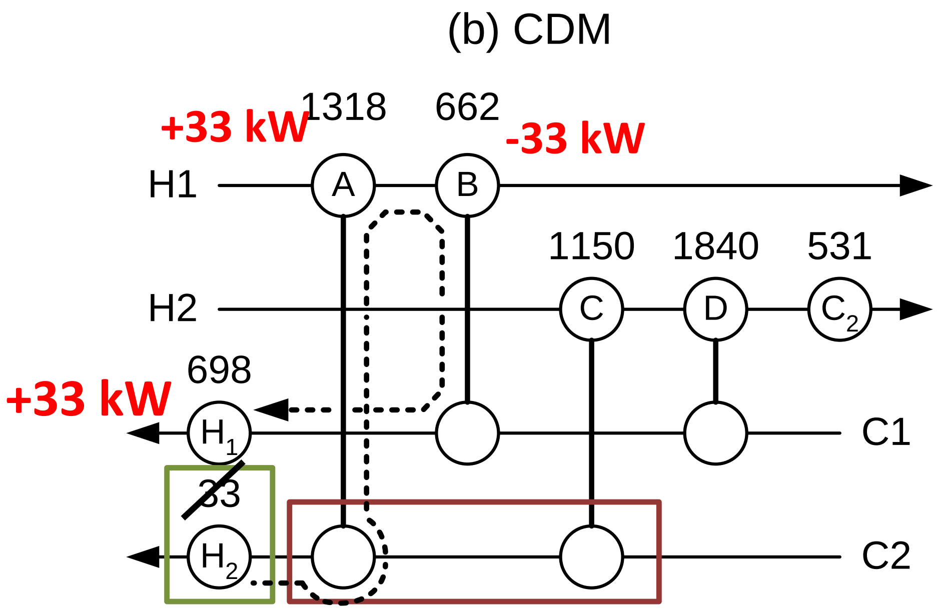
(a) PDM

TC = $**363** k /y

(b) CDM



Utility Exchanger Reduction

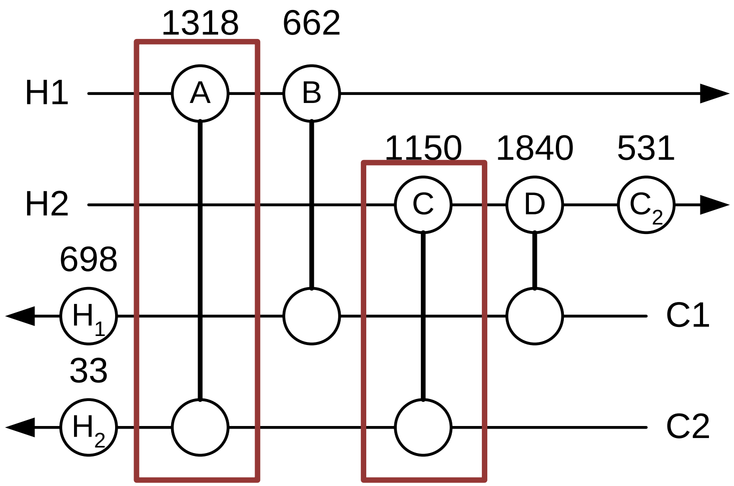


- Does this produce minimum total cost?

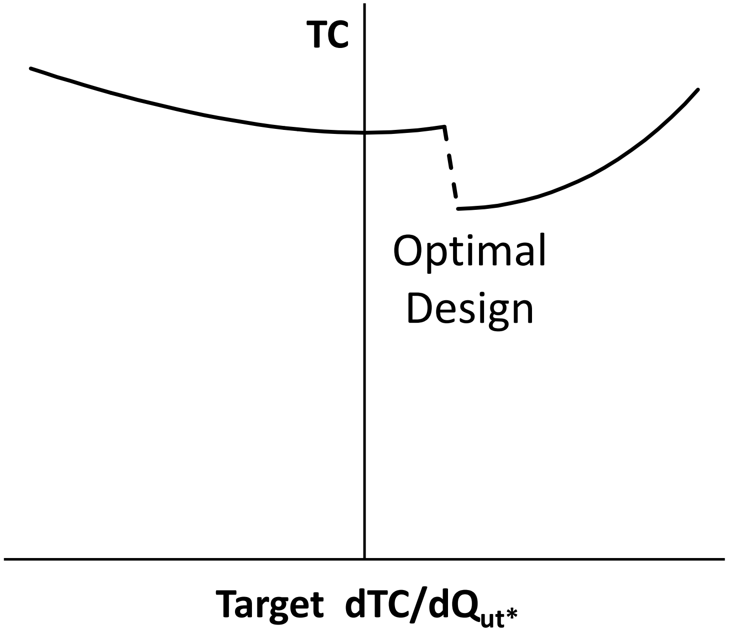
Utility Exchanger Reduction

TC = $**363** k /y

(b) CDM



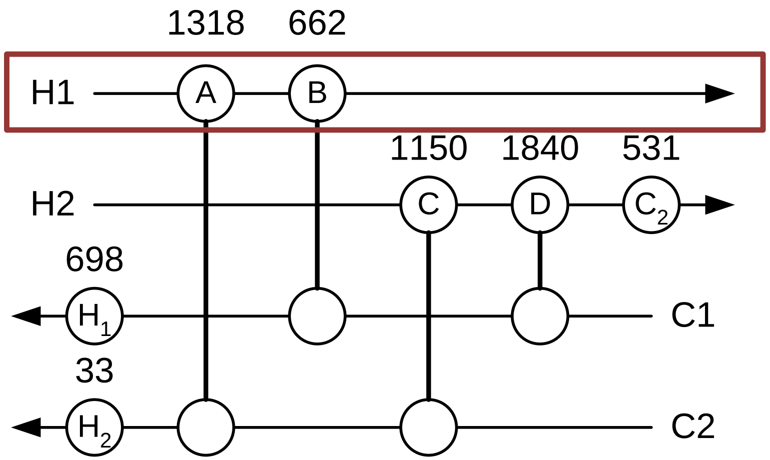
New Conditions for Heat Exchangers A & C

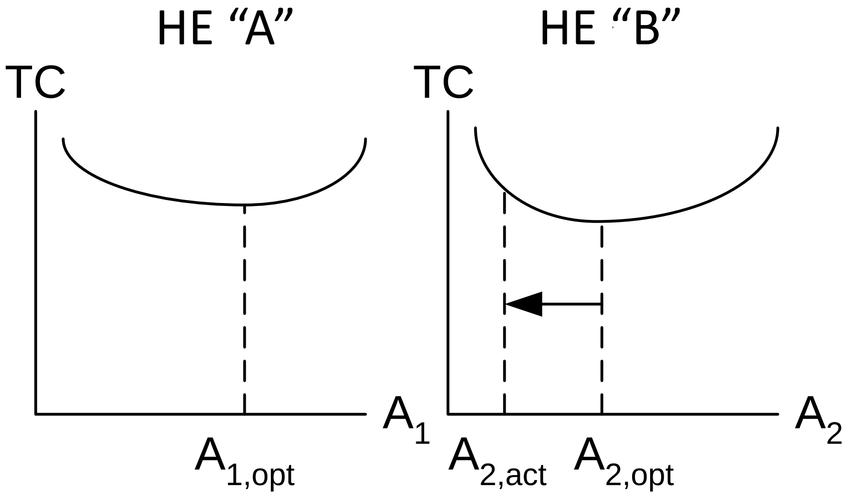


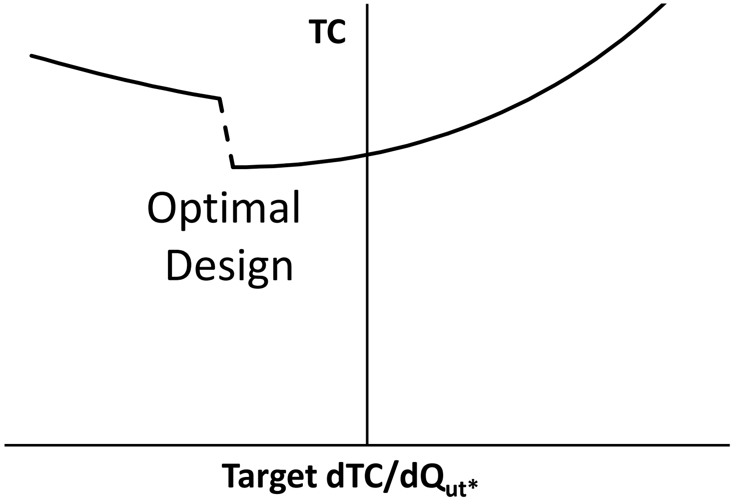
Target Temperature Constraints

TC = $**363** k /y

(b) CDM





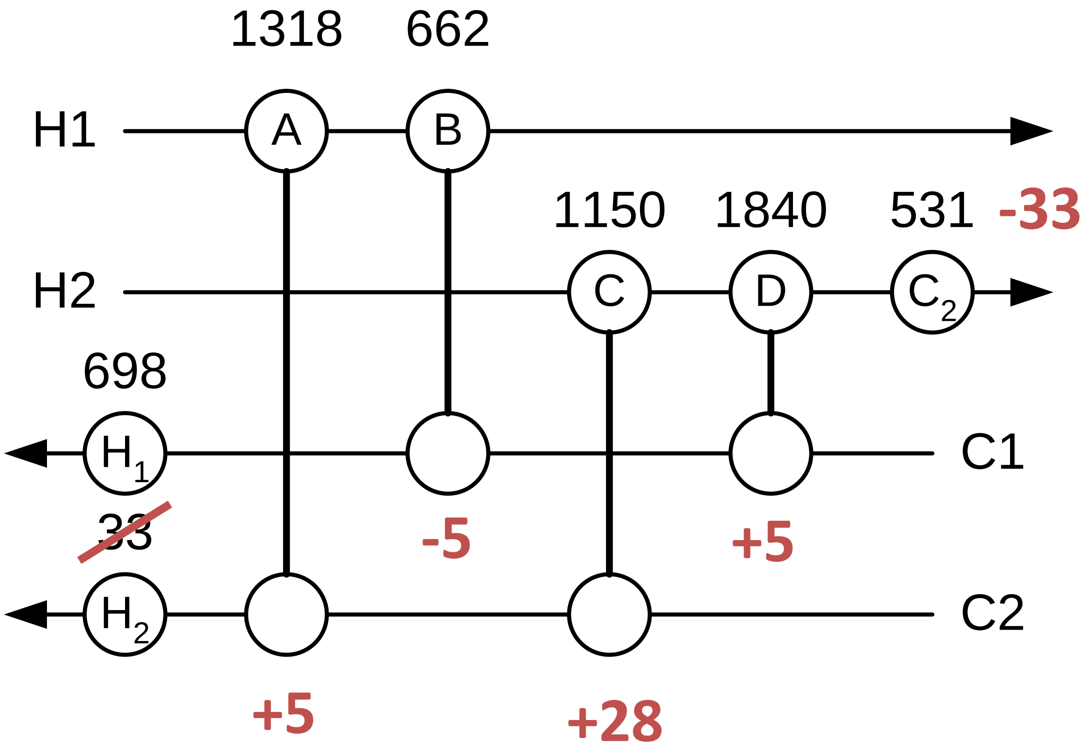


Optimise Duty/Area Allocation

TC = $**363** k /y

TC = $**358** k /y

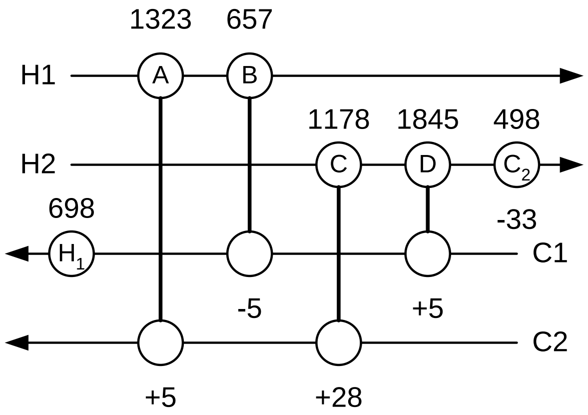
(b) CDM



Compared to Programming HEN Solutions

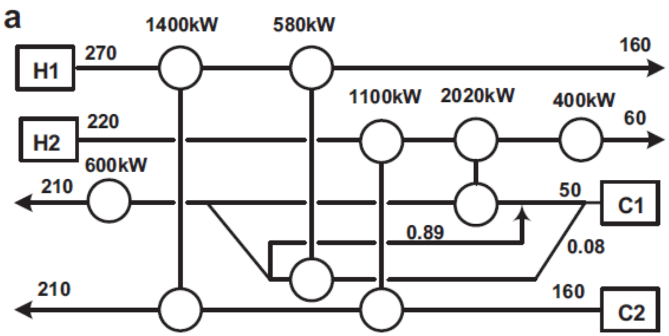
(Escobar & Trierweiler, 2013)

CDM*



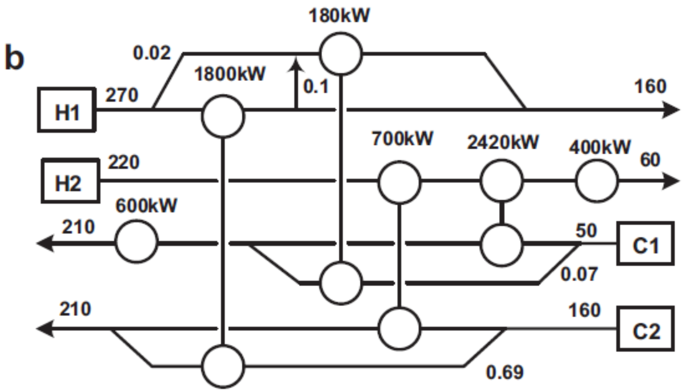
TC = $**358** k /y

Super structure



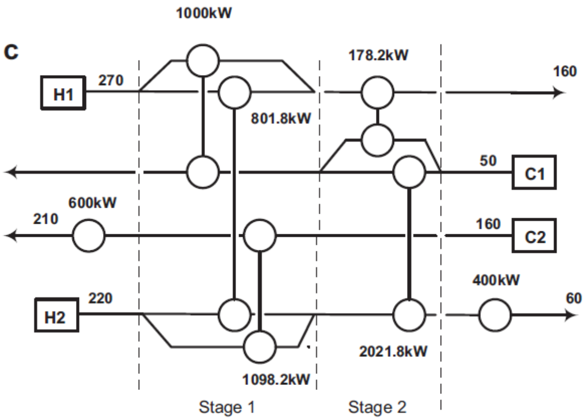
TC = $**376** k /y

Hyper structure



TC = $**361** k /y

SynHeat Model



TC = $**367** k /y

Summary

- Optimising area/duty allocation is valuable and can give 5-10% improvement beyond Pinch

- The new method accounts for different phases, HE types & arrangement, temp profiles, etc.

- In the example, the new method:
  - Saved 7% of total cost compared to the PDM
  - Similar total cost to the Hyper-structure network solution, but required no splits compared to 4 splits

Future Work

- Understand how the cost derivative could inform synthesis
- Application to dairy and indirect heat exchange systems
- Inclusion of film coefficient correlations assuming some HE selection
- Application to retro-fit analysis and design

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