Enhancement of the Hooghoudt Drain-Spacing Equation

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Abstract: The Hooghoudt equation is widely used as a simple means of specifying drain spacing when designing networks of parallel drains in drainage systems, based on estimating the maximum water table height between two drains. It is shown via comparison with a numerical model that the Hooghoudt equation can overestimate water table height and therefore yield drain spacings that may be too wide. This is because the Hooghoudt drain-spacing equation in fact has a concealed dependency on Van Genuchten soil-water retention curve parameters, which can bias the water table estimates unless adjustments are made explicitly. A modification of the Hooghoudt equation is presented that incorporates two new dimensionless coefficients to make allowance for this dependency. The modified expression yields improved accuracy as measured against the numerical reference model. **DOI: 10.1061/(ASCE)IR.1943-4774.0000835.** © *2014 American Society of Civil Engineers.*

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Introduction

Artificial field drainage as a means to remove excess water in the root zone is now about 200 years old (Smedema et al. 2004), but estimating optimal drain spacing remains a challenging part of any drainage-scheme design. If spacing is too wide, the water table between adjacent drains may rise to the root zone during heavy rain events or irrigation. On the other hand, needless construction costs are incurred if the drains are unnecessarily close. To find the optimal spacing, the midpoint water table height must be estimated by way of a suitable governing groundwater equation.

Laplace, Boussinesq, and Richards (1931) equations are widely mentioned in the literature as the partial differential governing equations for fluid flow in porous media, and analytical solutions have been obtained for a range of special drainage cases.

For example, Hooghoudt (1940) derived an expression based on the one-dimensional Boussinesq equation by using Dupuit-Forchheimer (DF) assumptions for flow to two parallel drains. The derivation assumes that the effect of radial flow from the center of the drain to $D/\sqrt{2}$ is significant, where $D[\mathrm{L}]$ is the thickness of the soil layer below the drains. To allow for the effect of radial flow near the drains and to reduce the DF approximation error, Hooghoudt (1940) replaced D in his equation with an "equivalent depth" d, which is an imaginary depth under the drains that is always smaller than D. Moody (1966) and Sakkas and Antonopoulos (1981) presented simple estimating expressions for d, whereas Van Schilfgaarde (1963) described a graphical estimation approach. Van Beers (1979) gave a dimensionless nomogram to avoid trial-and-error solution of the Hooghoudt equation. Mishra and Singh (2007) derived a new equivalent depth for the Hooghoudt equation by changing the assumption of the effect of the radial flow from $D/\sqrt{2}$ to a combination of $2D/\pi$ and the area of groundwater flow above the drain in the radial zone.

In related analytical approaches, Kirkham (1958) solved the 2D steady-state Laplace equation in a conceptual confined-flow domain for a constant drain discharge and ignored all flow above drain level. Van Der Molen and Wesseling (1991), Dagan (1964), Hammad (1962), and Ernst (1962) respectively used different approaches to derive some simpler solutions for the Kirkham equation. In addition Youngs (1965, 1975), Collis-George and Youngs (1958), and List (1964) presented some inequalities and equations for the upper and lower bounds of water table height between two parallel drains. Childs (1969) fit an empirical equation to data obtained from a sand tank physical model experiment devised by Collis-George and Youngs (1958). Miles and Kitmitto (1989) and Barua and Tiwari (1996a, b) derived an analytical drain-spacing formula suitable for homogenous and layered anisotropic soils. Lovell and Youngs (1984) and Kirkham (1966) gave comprehensive reviews of various analytical drainage equations.

As an alternative to analytical methods to obtain drain spacing, the governing equations of water movement in soil can be solved using numerical techniques with fewer assumptions. Gureghian and Youngs (1975) described a finite-element technique to solve the Laplace equation for drainage situations and applied it to both homogeneous and heterogeneous soils. Zaradny and Feddes (1979) used the finite-element Galerkin method to solve the 2D Laplace equation applied to the vicinity of a tile drain. Smedema et al. (1985) used a finite-element solution of the Laplace equation for homogeneous-anisotropic soil drainage and then compared the resulting drain-spacing estimates with those from the Hooghoudt equation. They concluded that the Hooghoudt equation can be used with reasonable confidence for drain spacing in both homogeneous and anisotropic soils.

Khan and Rushton (1996a, b, c) applied a finite-difference numerical approach to solve the 2D steady-state Laplace equation for drainage applications and noted that drains must be represented as an internal boundary separated from the water table. Zaradny (2001) used finite-element methods to predict groundwater flow in a drainage zone and showed that, in this context, groundwater flow theory for confined aquifers can also be used for unconfined aquifers. Zavala et al. (2007) used a numerical solution of the

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Boussinesq equation, incorporating boundary conditions by use of two types of drainage flux equation as a function of water table elevation. Their study concluded that both types of boundary condition can give reasonable results consistent with laboratory experiments.

Castanheira and Santos (2009) developed a 2D Galerkin finite-element steady-state Richards equation model for finding optimal drain spacing and then compared their results with the Kirkham and Hooghoudt analytical solutions. Shokri (2011) described a numerical surface/subsurface model that couples a 3D Richards equation with the 2D shallow-water equations, enabling design of drainage schemes that may also serve to supply irrigation water.

Fuentes et al. (2009) gave two expressions for relations between the storage coefficient and the soil-water retention curves of Van Genuchten (1980) and Fujita-Parlange (Fuentes et al. 1992) during water table drawdown between two drains. These expressions were subsequently shown by Pandey et al. (1992) to be consistent with experimental data. Chavez et al. (2011a, b) used a finite-difference method to solve a combination of the Boussinesq equation and the Van Genuchten soil-water retention curve to estimate drainable porosity, in order to study the hydrodynamics of groundwater flow near tile drains. Jiang et al. (2010) used finite elements to solve a 2D Laplace equation for both random hydraulic conductivity variation and hydraulic conductivity decreasing exponentially with depth; they obtained both the spatial hydraulic head distribution and the drain discharge. Zavala et al. (2012) compared the result of the finite-element 1D Boussinesq and 2D Richards equations in an agriculture subsurface drainage situation. This showed the limitations of the Boussinesq equation, such as the DF assumptions for between-drain water table estimation. Recently Youngs (2012, 2013) developed an analytical equation incorporating the capillary fringe to simulate steady-state flow to tile drains in a hypothetical infinitely deep soil. In addition, a number of special-purpose computer codes are available for drain-spacing estimation, including *DRAINMOD* (Skaggs 1980), *DRENAFEM* (Castanheira and Santos 2009), and *MHYDAS-DRAIN* (Tiemeyer et al. 2007).

Despite considerable activity in the field, currently there is no available analytical solution of the Richards equation for drain-spacing estimation. Most of the existing analytical and numerical drainage models are based on solutions of the partial differential Laplace and Boussinesq equations. The present paper applies a dimensionless finite element solution of the partial differential Richards equation to obtain an enhancement of the analytical Hooghoudt equation. It is shown that the coefficients of the original Hooghoudt equation are actually functions of soil-water retention parameters, although not explicitly specified. While drain spacing can be solved directly via a numerical model of the Richards equation, the analytical Hooghoudt expression still has attraction in its ease of application. In this regard, the modified Hooghoudt equation presented here has the advantage of giving more accurate results than the original expression and should find application in practical drainage design.

The following section, "Theory," classifies drain-spacing equations into those using the DF assumption and those using the Laplace equation. The steady-state Richards equation is then revisited. The system and boundary conditions are described in the section "System Definition." The enhanced Hooghoudt equation is presented in the section "Modification of the Hooghoudt-Moody Equation."

Theory

The standard drain-spacing equations and the governing subsurface-flow equation are given next. Drain-spacing equations have been widely discussed in the literature, and only a brief summary is provided here.

Common Drain-Spacing Formulas Based on DF Assumptions

The steady-state analytical Hooghoudt equation is widely used to determine drainspacing (Van der Molen and Wesseling 1991) and can be written as

$$\frac{q}{Ks} = 4\left(\frac{m}{L}\right)^2 + \frac{8d}{L}\left(\frac{m}{L}\right) \quad (1)$$

$$d = \frac{L}{(L - D\sqrt{2})^2/DL + 8/\pi \ln D/r_0\sqrt{2} + f(D, L)} \quad (2)$$

where q = recharge rate per unit surface area [LT^{-1}]; K_s = saturated hydraulic conductivity of the medium [LT^{-1}]; m = maximum water table height above the drain-level midpoint between two parallel drains [L]; and r_0 = drain radius [L]. The function $f(D, L)$ is smaller than the other terms and is generally ignored (Wesseling and Kessler 1994). Moody approximations of Eq. (2) can be written as (Moody 1966)

$$d = \frac{D}{8D/\pi L \ln(D/\pi r_0) + 1} \quad \text{for} \quad 0 < \frac{D}{L} \leq \frac{1}{4};$$
$$d = \frac{\pi L}{8 \ln(L/\pi r_0)} \quad \text{for} \quad \frac{D}{L} > \frac{1}{4} \quad (3)$$

Common Drain-Spacing Formulas Based on the Laplace Equation

An analytical solution was obtained by Kirkham (1958) for water table height midpoint between two drains, using the 2D steady-state Laplace equation. This gives the Kirkham relation

$$m = \frac{qL}{Ks}\frac{1}{\pi}\left[\ln\frac{L}{\pi r_0} + \sum_{n=1}^{\infty}\frac{1}{n}\left(\cos\frac{2n\pi r_0}{L} - \cos n\pi\right)\coth\frac{2n\pi D}{L} - 1\right] \quad (4)$$

In the derivation of Eq. (4) the simplification is made that water flow above drain level can be ignored (Kirkham 1958). Dagan (1964) gives a simplified version of Eq. (4) as

$$m = \frac{qL}{Ks}\left[\frac{L}{8D} - \frac{1}{2\pi}\ln\left(2\cosh\frac{\pi r_0}{D} - 2\right)\right] \quad (5)$$

Van der Molen et al. (1991) show that Eq. (5) gives a good approximation of Eq. (4) when D/L is small.

Steady-State Richards Equation

For saturated/unsaturated flow in porous media, the authors use the steady-state Richards equation

$$\nabla[K_w(h)\nabla(h + z)] = -qs \quad (6)$$

where h = pressure head [L]; z = elevation [L] qs = general sink/source term [LT^{-1}]; and $K_w(h)$ = hydraulic conductivity. Eq. (6) is defined in the whole pressure domain as

$$K_w(h) = \begin{cases} KsKr(h) & \text{for } h < 0 \\ Ks & \text{for } h \geq 0 \end{cases} \quad (7)$$

where $K_r(h)$ = relative permeability [-]. This study uses the relative permeability function as in the Van Genuchten (1980) model

Table 1. Descriptive Statistics for Van Genuchten Water Retention Model Parameters α and n from Carsel and Parrish (1988)

Number	Texture	α (m^{-1})			n		
		Number of samples	Mean	Standard deviation	Number of samples	Mean	Standard deviation
1	Sand	246	14.5	0.029	246	2.68	0.29
2	Loamy sand	315	12.4	0.043	315	2.28	0.27
3	Sandy loam	1,183	7.5	0.037	1,183	1.89	0.17
4	Loam	735	3.6	0.021	735	1.56	0.11
6	Clay	400	0.8	0.012	400	1.09	0.09

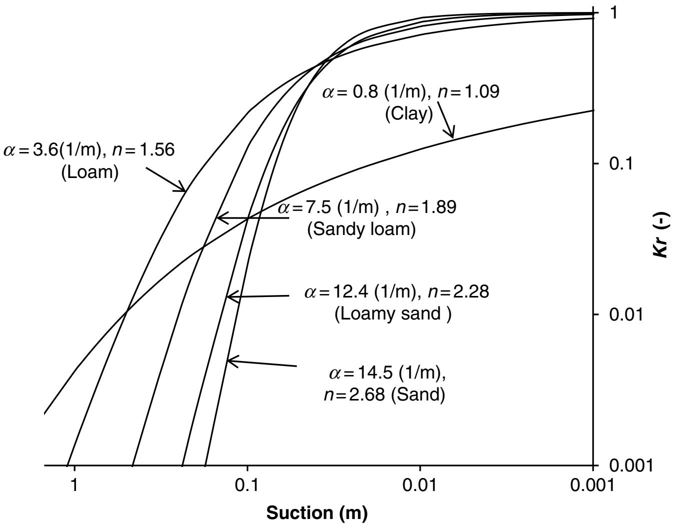

Fig. 1. Relative permeability (Kr) versus pressure head for a range of Van Genuchten soil-water retention parameters α and n in Table 1

$$Kr = Se^{l}\left[1-\left(1-Se^{\frac{1}{M}}\right)^{M}\right]^{2}; \qquad Se = \frac{1}{(1+|\alpha h|^{n})^{M}} \tag{8}$$

where Se = effective saturation [-]; l = pore connectivity parameter usually assumed to be 0.5 [-]; α [L^{-1}] and $n > 1$ [-]; = two Van Genuchten curve-fitting parameters; and $M = 1 - 1/n$. It is evident that for the special case of $Kr = 1$ the Richards equation is equivalent to the Laplace equation.

Table 1 shows the result of a study by Carsel and Parrish (1988), which provide estimates of α and n for a range of soil texture groups from their analysis of a large amount of recorded soil water retention data. Malaya and Sreedeep (2011) give a comprehensive review of factors that may influence the soil-water retention curve. Fig. 1 shows calculated relative permeability from Eq. (8), plotted on log scales against the pressure head for a range of Van Genuchten soil-water retention parameters.

System Definition

This study uses a dimensionless domain to compare maximum water table height as calculated from some analytical drainage equations and also from selected numerical simulations. Fig. 2 shows a set of horizontal parallel drains of radius r_0/L located at elevation D/L above datum and depth H/L under the ground surface. The soil is assumed to be homogenous and isotropic, having saturated hydraulic conductivity Ks and located above a horizontal impermeable surface.

A Neumann boundary condition (constant discharge boundary; CDB) $\partial h/\partial y = -q/Ks$ is applied at the top boundary. Because simplification assumes the drain to remain half-full and the condition in the drain is hydrostatic, as an internal boundary condition a Drichlet boundary condition (constant head boundary; CHB)

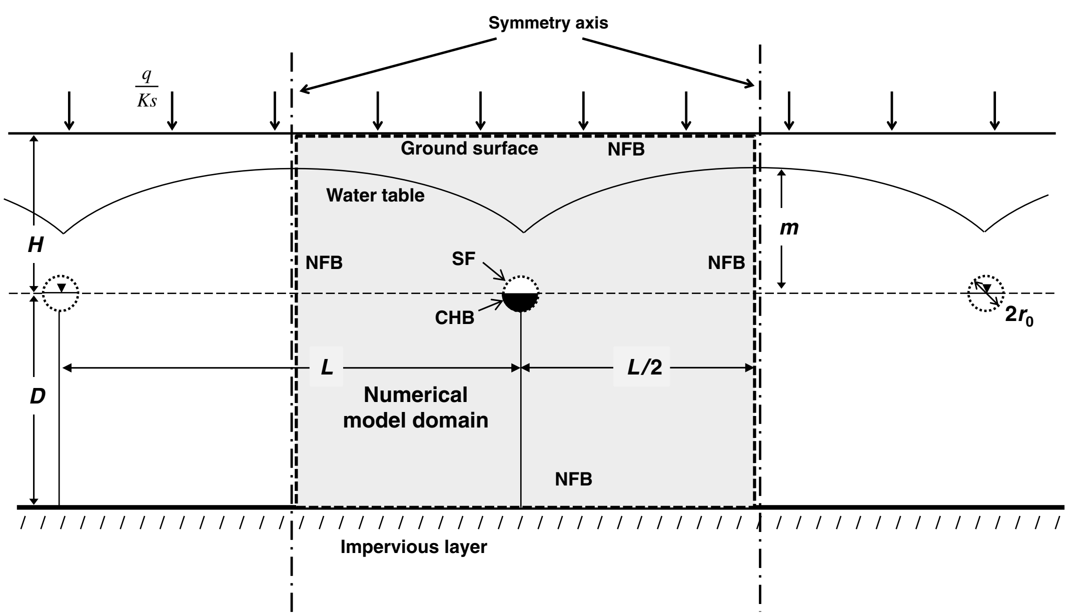

Fig. 2. Parallel drainage system and numerical model domain with a no-flow boundary (NFB), a constant head boundary (CHB), and a constant discharge boundary (CDB)

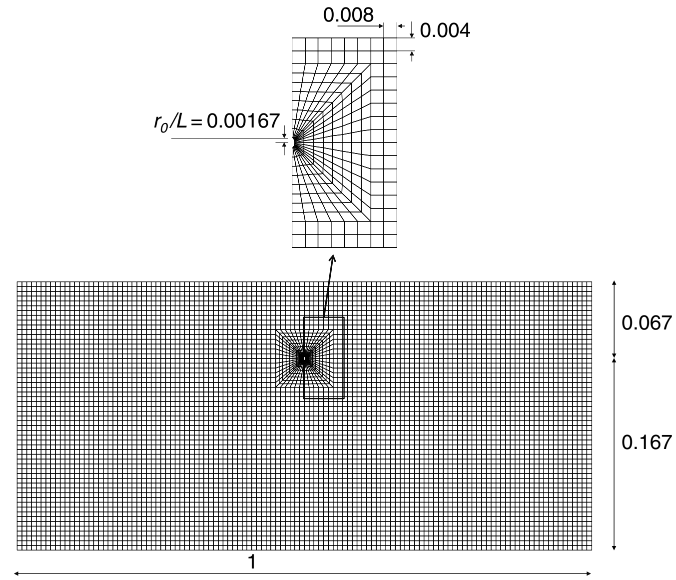

Fig. 3. Finite-element mesh applied to the model for $D/L = 0.167$, $H/L = 0.067$, and $r_0/L = 0.00167$

$h = D/L$ is applied to the lower half of the drain tube. For the upper half of the drain a Cauchy boundary condition (seepage face boundary; SF) is applied once the pressure head on the upper half of the drain boundary becomes larger than 0, which means that the drain is swamped. The SF boundary condition allows the boundary to transfer from a no-flow boundary (NFB) $\partial h/\partial y = 0$ to a CHB condition. For the rest of the boundaries, a NFB condition is implemented from symmetry or for representing the lower impervious layer.

To create the "true" reference for comparison, the Richards equation is implemented numerically using the finite-element SEEP/W code (GEO-SLOPE International 2012). A layout of the finite-element mesh used in the numerical model and the area surrounded by the drain are illustrated in Fig. 3 for $D/L = 0.083$, $H/L = 0.033$ and $r_0/L = 0.00167$. The results from Carsel and Parrish (1988) are used for the two Van Genuchten soil-water retention parameters α and n (Table 1).

To check the magnitude of the discretizing error, the total input flux of the upper boundary condition (Fig. 2) is compared with the calculated discharge of the drain by the developed numerical model. The comparison shows that the differences between input flux and calculated discharge always remain under $10^{-5}q/K$.

Results and Discussion

Fig. 4 shows two situations of equipotential lines and water table heights calculated by the numerical solution of the Richards equation for $\alpha = 0.8$ and $n = 1.09$ (clay soils) and for $\alpha = 14.5\ \mathrm{m}^{-1}$ and $n = 2.68$ (sandy soils), with $q/Ks = 0.045$, $D/L = 0.083$,

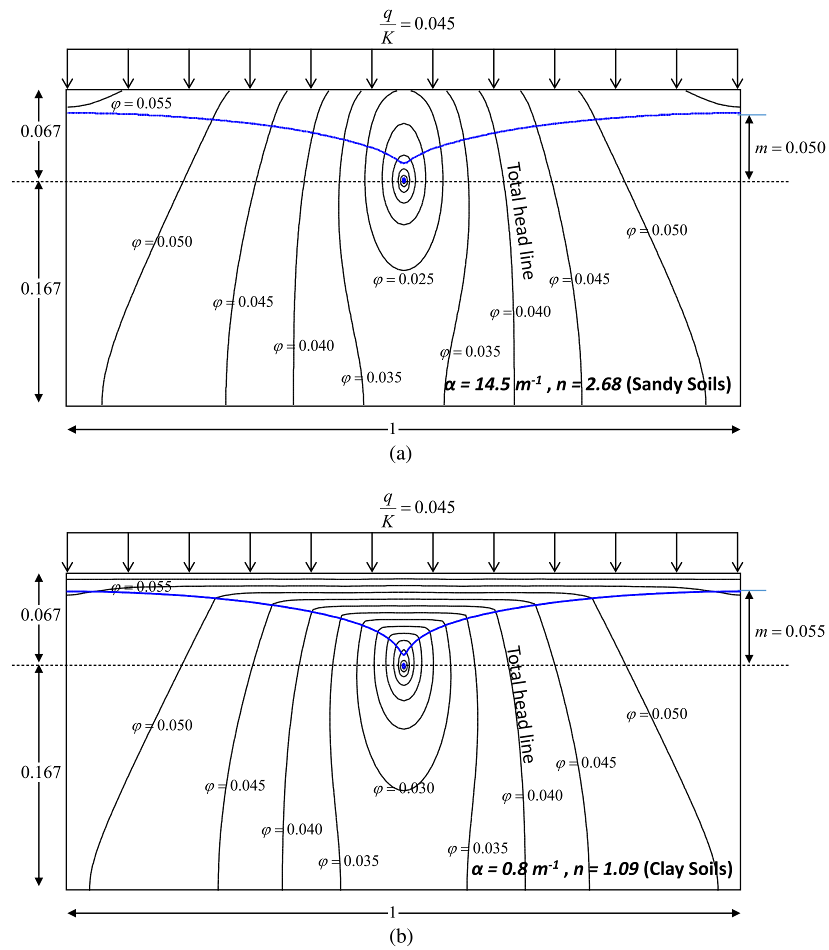

Fig. 4. Equipotential lines and water table height computed by the numerical solution of (a) the Richards equation for clay soils ($\alpha = 0.8\ \mathrm{m}^{-1}$, $n = 1.09$); and (b) for sandy soils ($\alpha = 14.5\ \mathrm{m}^{-1}$, $n = 2.68$), when $q/Ks = 0.045$, $D/L = 0.083$, $H/L = 0.033$, and $r_0/L = 0.00167$ ($\varphi = h + z$)

$H/L = 0.033$, and $r_0/L = 0.00167$. Despite the fact that the two models use the same boundary conditions and subdomain geometry, Fig. 4 indicates that flow patterns are soil-dependent. This applies particularly in the unsaturated zones, where the equipotential lines of clay soils are fairly horizontal compared to those of sandy soils. The reason is that hydraulic conductivity gradients are more readily developed in clay soils than in sandy soils. Furthermore, the maximum calculated water table height is different for the two cases. That is, m/L is obtained as 0.0195 for $\alpha = 0.8$ and $n = 1.09$ (clay soils) and as 0.0164 for $\alpha = 14.5\ \mathrm{m}^{-1}$ and $n = 2.68$ (sandy soils).

The maximum water table height midpoint between two drains m/L is computed for various dimensionless parameters q/Ks, D/L, r_0/L, and H/L, and Van Genuchten soil-water retention parameters α and n. Fig. 5 shows the relation between q/Ks and m/L for $D/L = 0.017$, 0.033, 0.083 and 0.333 when $r_0/L = 0.00167$ and $H/L = 0.033$. It is evident that the Hooghoudt equation with the Moody approximation of equivalent depth from Eq. (3) gives the closest result to the numerical Richards model among the other analytical methods. This similarity is clearly visible for small D/L with clay soils ($\alpha = 0.8\ \mathrm{m}^{-1}$, $n = 1.09$). However, the differences between the Richards numerical model and the Hooghoudt-Moody equation become greater as D/L increases. The difference is particularly evident for $\alpha = 14.5\ \mathrm{m}^{-1}$ and $n = 2.68$, which represent sandy soils (Table 1). The Kirkham and Dagan equations [Eqs. (4) and (5), respectively], always overestimate m/L compared to other methods.

Modification of the Hooghoudt-Moody Equation

Eq. (1) can be expressed as a zero-intercept quadratic expression

$$y = ax^2 + bx \tag{9}$$

where $y = q/k_s$; $x = m/L$; $a = 4$; and $b = 8d/L$. This study applied Eq. (9) as an empirical regression expression $y = a'x^2 + b'x$ (where dashes denote parameter estimates) to the output of the

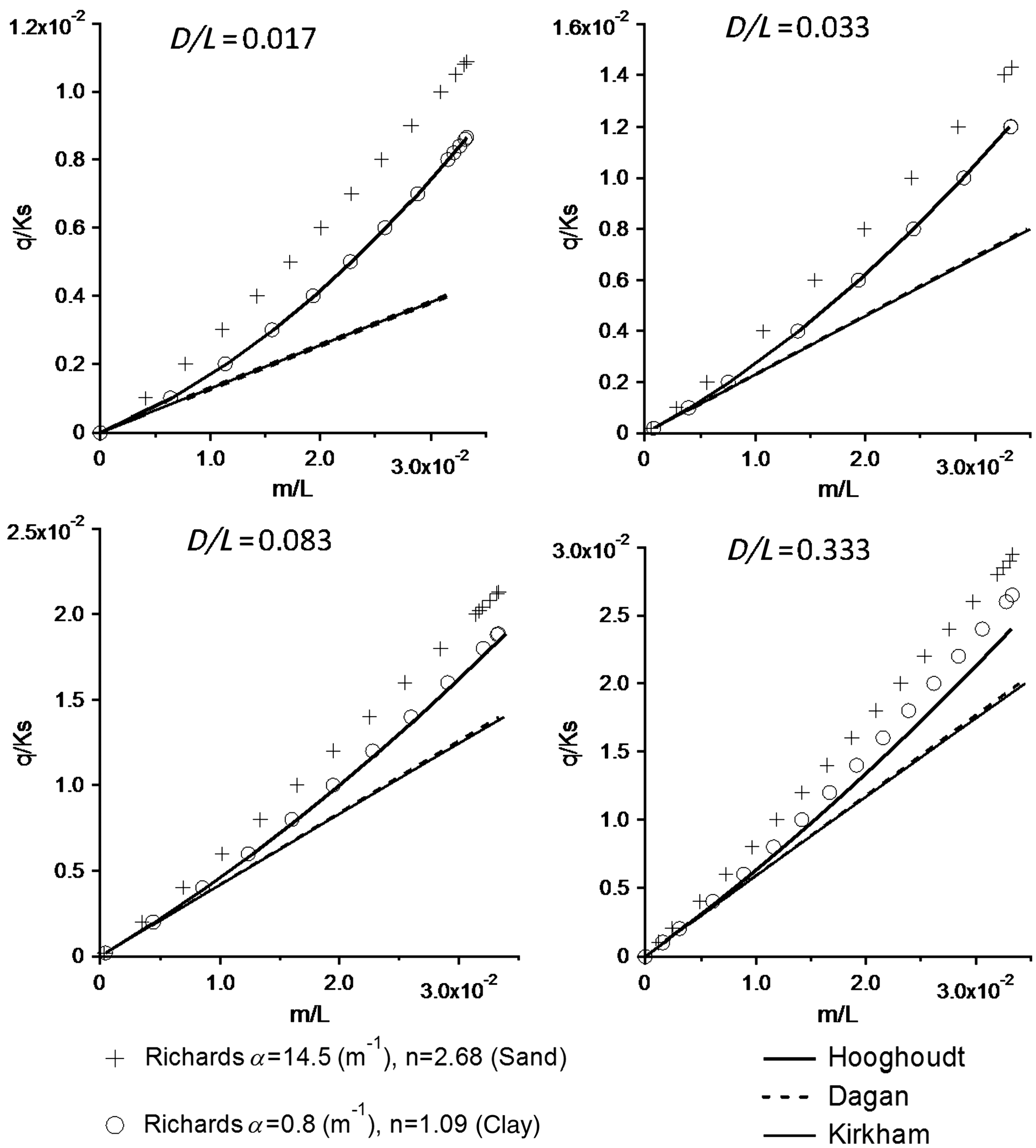

Fig. 5. Relation between q/Ks and m/L for analytical and numerical results for $D/L = 0.017$, 0.033, 0.083, and 0.333 when $r_0/L = 0.00167$ and $H/L = 0.033$

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numerical Richards model as data and obtained the anticipated good fits (Fig. 6).

From Eq. (9) it might be expected that the fit-derived coefficients a' and b' would be similar to a and b. However, there are some evident differences. In particular, the difference is considerable for sandy soils and larger D/L ratios. To illustrate this difference for a range of α and n from Table 1, the ratios of a'/a and b'/b (referenced henceforth as β_{d1} and β_{d2}, respectively) are plotted against d/L in Fig. 7.

If Eq. (9) is used as a fitting curve to the results of the numerical models, $a' = a = 4$ and $b' = b = 8d/L$ should be expected (within estimation error); in other words $\beta_{d1} = \beta_{d2} = 1$. However,

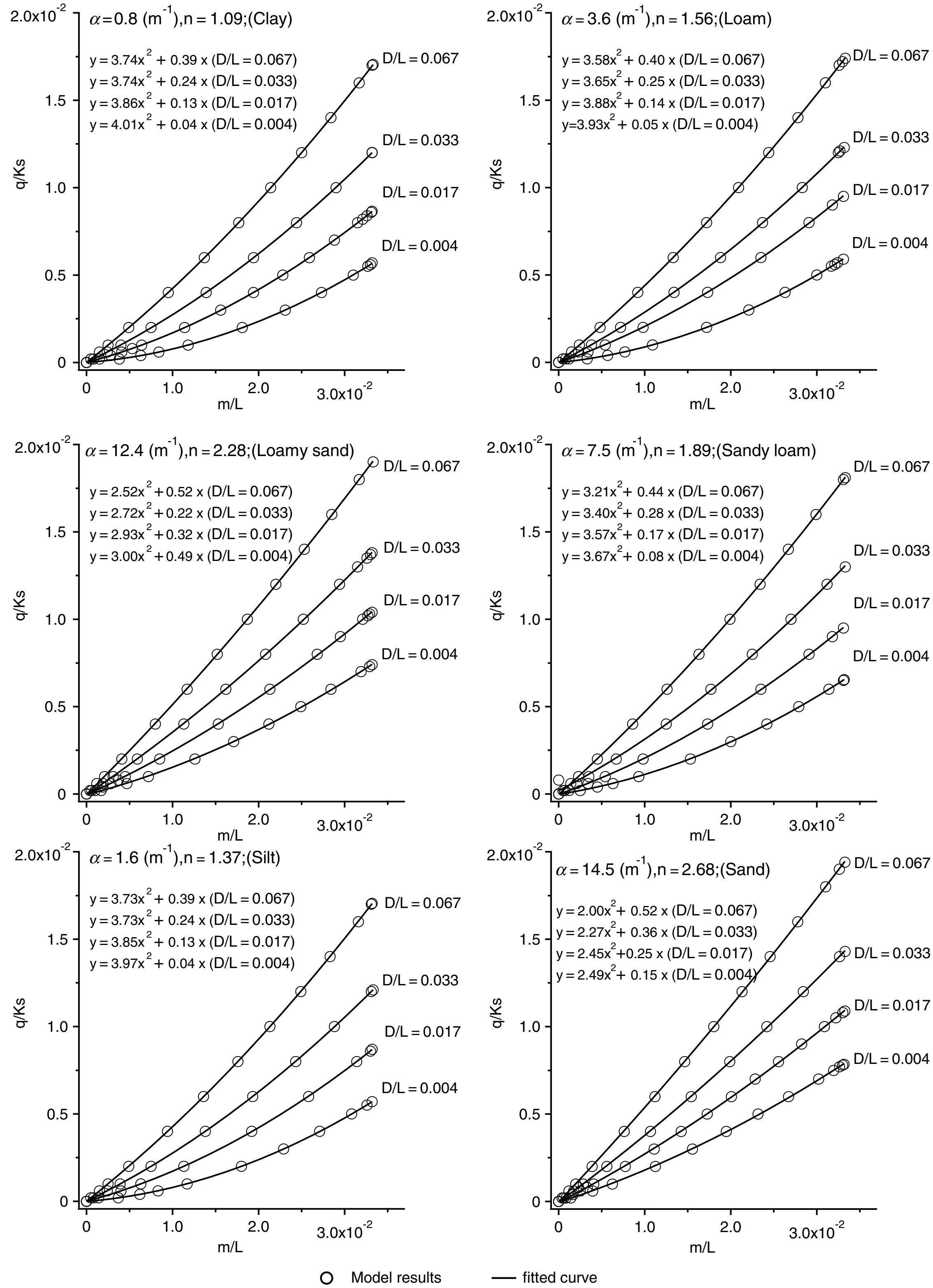

Fig. 6. Fitted curves to numerical values (open circles) as obtained from the numerical model of the Richards equation for a range of α, n, and D/L when $r_0/L = 0.00167$ and $H/L = 0.033$

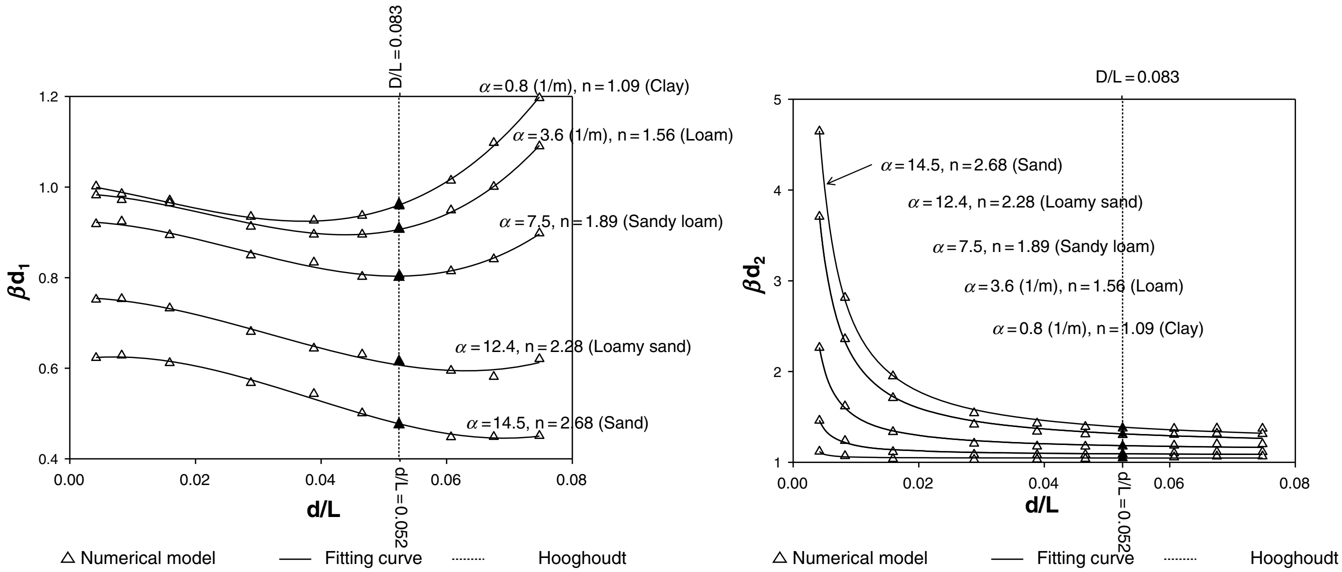

Fig. 7. Relation between β_{d1} and β_{d2} and d/L for a range of α and n when $r_0/L = 0.00167$ and $H/L = 0.033$

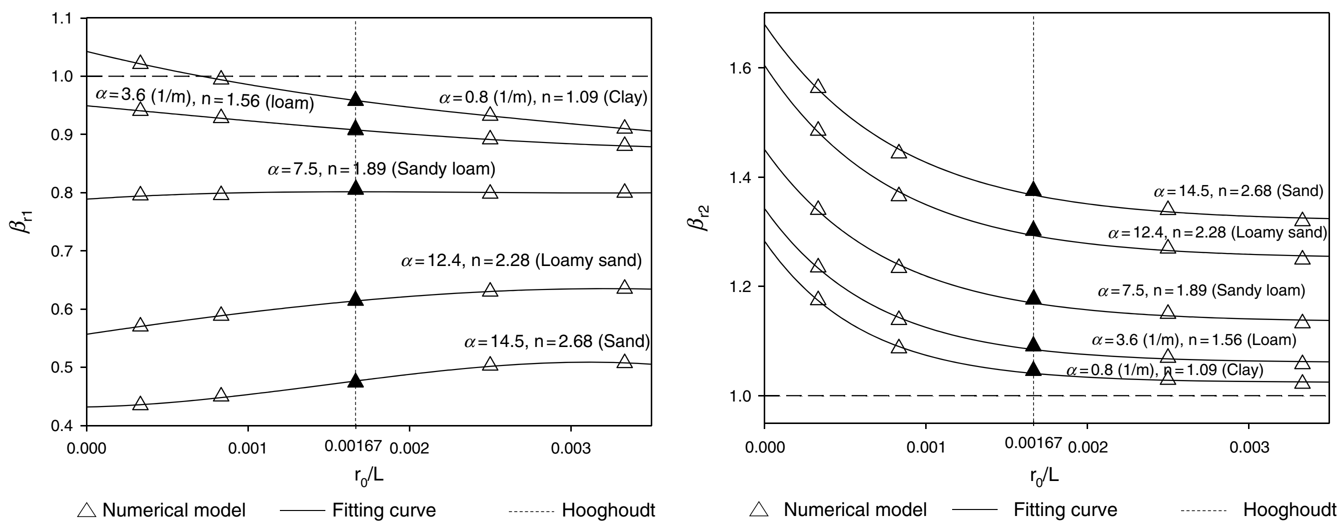

Fig. 8. Relation between β_{r1} and β_{r2} and r_0/L for a range of α and n and r_0/L for $D/L = 0.083$ and $H/L = 0.033$

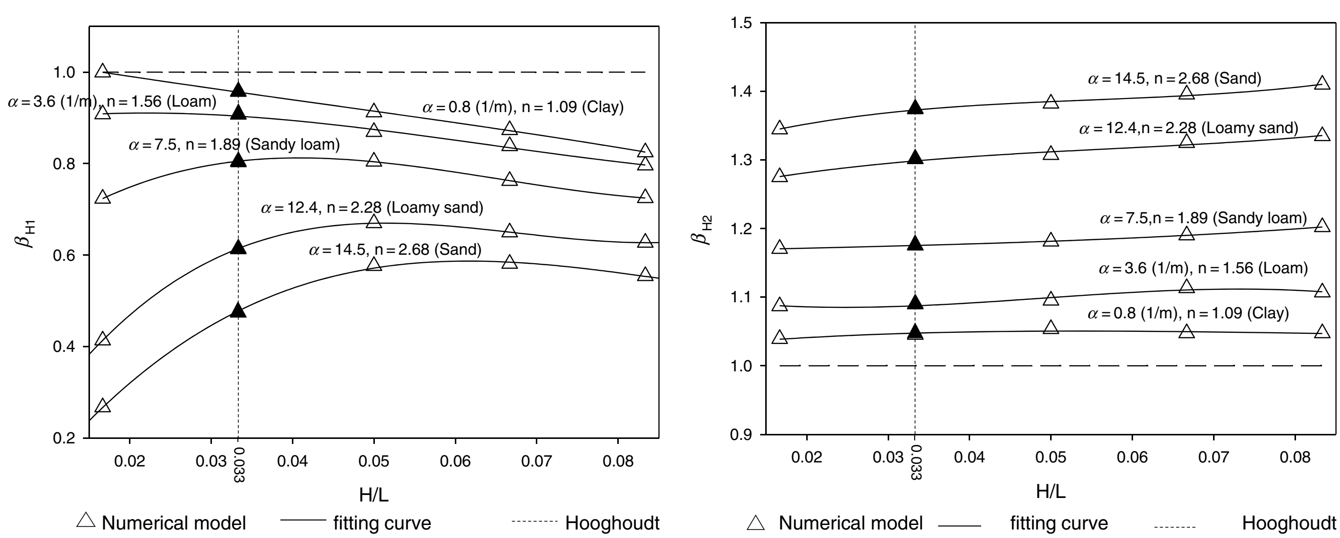

Fig. 9. Relation between β_{H1} and β_{H2} and H/L for a range of α and n and H/L for $r_0/L = 0.00167$, $D/L = 0.083$

Table 2. Calculated Value of Base Betas ($\beta 01$ and $\beta 02$) When $D/L = 0.083$, $r_0/L = 0.00167$, and $H/L = 0.033$ for a Range of Van Genuchten Soil-Water Retention Model Parameters (α and n)

Number	Texture[a]	$\beta 0_1$	$\beta 0_2$
1	Sand ($\alpha = 14.5$ m^{-1}, $n = 2.68$)	0.47	1.37
2	Loamy sand ($\alpha = 12$ m$^{-1.4}$, $n = 2.28$)	0.61	1.30
3	Sandy loam ($\alpha = 7.5$ m^{-1}, $n = 1.89$)	0.80	1.18
4	Loam ($\alpha = 3.6$ m^{-1}, $n = 1.56$)	0.91	1.09
5	Clay ($\alpha = 0.8$ m^{-1}, $n = 1.09$)	0.96	1.04

[a]Van Genuchten soil-water retention model parameters.

Fig. 7 shows that β_{d1} and β_{d2} are functions of both d/L and the Van Genuchten soil water retention parameters α and n. Fig. 7 also shows that cubic and exponential curves give good empirical fits to the numerical model output.

Similar numerical models are set up for $r_0/L = 0.0003$, 0.0008, 0.00167, 0.0025, and 0.0033 when $D/L = 0.083$ and $H/L = 0.033$; $H/L = 0.017$, 0.033, 0.05, 0.067, and 0.083 when $r_0/L = 0.00167$, $D/L = 0.083$ for a range of Van Genuchten soil-water retention values. Then, with the same methodology used for defining β_{d1} and β_{d2}, four new parameters (β_{r1}, β_{r2}, and β_{H1}, β_{H2})

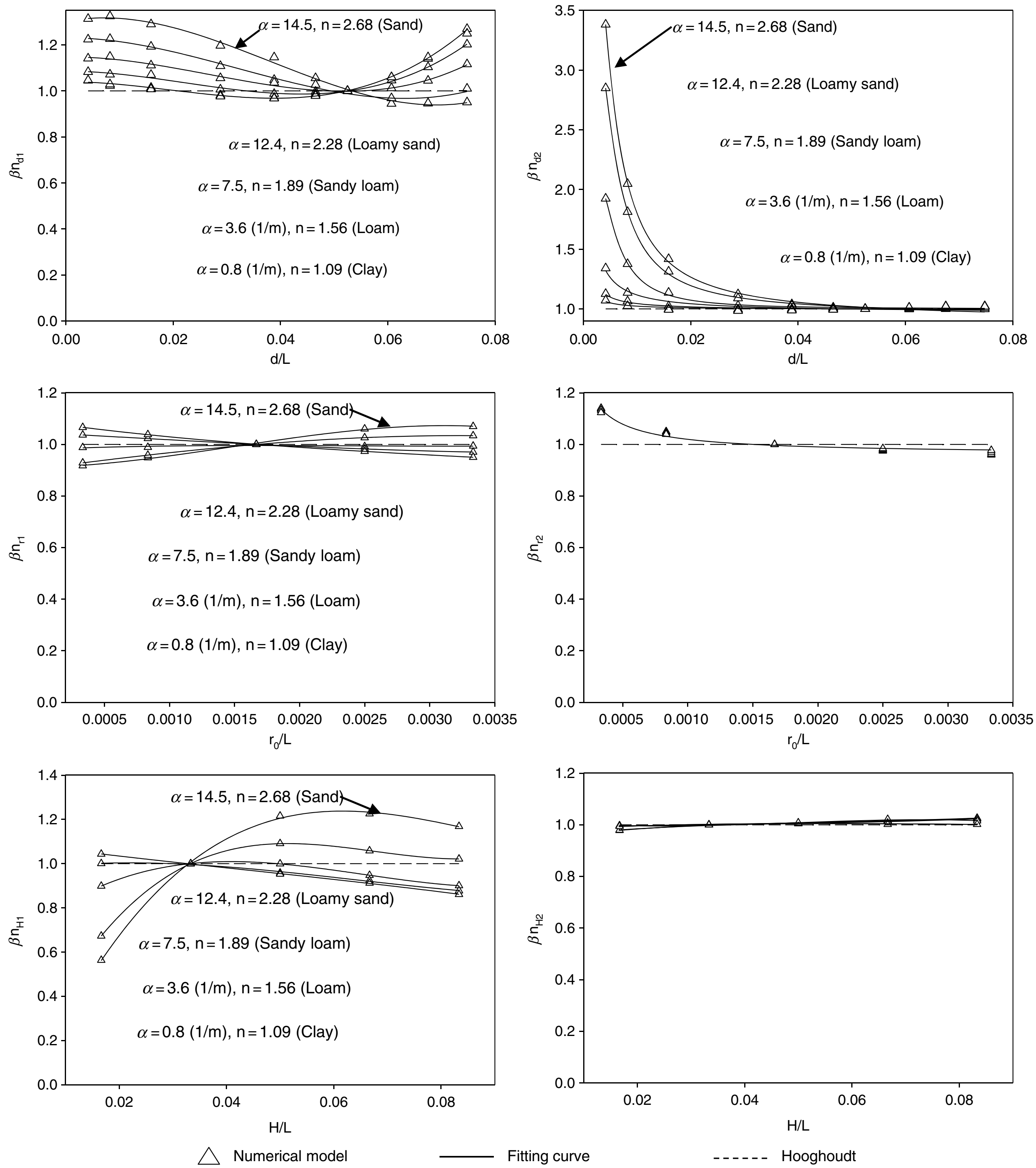

Fig. 10. Calculated βn_{d1}, βn_{d2}, βn_{r1}, βn_{r2}, βn_{H1}, and βn_{H2} for a range of Van Genuchten soil-water retention model parameters (α and n) versus d/l, H/L, and r_0/L, respectively

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are obtained. These two parameter pairs are plotted versus r_0/L and H/L In Figs. 8 and 9 respectively.

Each triangle in Figs. 7–9 represents a coefficient of parabolic fitting curves for different model geometries (D/L, H/L, and r_0/L). In Fig. 7 the numerical model has been set up for $r_0/L = 0.00167$, $H/L = 0.033$, and a range of D/L; in Fig. 8 for $D/L = 0.083$ and $H/L = 0.033$ and different r_0/L; and in Fig. 9 for $r_0/L = 0.00167$, $D/L = 0.083$, and a range of H/L.

By inspection of Figs. 7–9, it is evident that a unique and joint model has been repeated in all graphs when $D/L = 0.083$, $r_0/L = 0.00167$, and $H/L = 0.033$. These unique points (hereafter "base betas") are shown as the solid triangles in Figs. 7–9 and symbolize $\beta 0_1$ and $\beta 0_2$. Calculated values of $\beta 0_1$ and $\beta 0_2$ for a range of soils are listed in Table 2.

The values of the β_d, β_r, and β_H shown in Figs. 7–9 are normalized by dividing over the base betas [Eq. (10)]

$$\beta n_{ij} = \frac{\beta_{ij}}{\beta_{0j}} \tag{10}$$

where βn_{ij} = normalized betas; i in the index of $\beta = d$, r, and H; and $j = 1$ and 2.

Fig. 10 illustrates the calculated normalized betas for a range of d/L, r_0/L, H/L, and soil types mentioned in Table 1.

The results just given indicate that replacing the Hooghoudt a and b parameters with $4\beta 0_1 \beta n_{i1}$ and $(8d/L)\beta 0_2 \beta n_{i2}$, respectively, gives predictions of the water table height midpoint between the drains (m) that are as accurate as the numerical Richards equation model. These substitutions give the enhanced Hooghoudt equation as

$$\frac{q}{Ks} = 4\beta_1 \left(\frac{m}{L}\right)^2 + \frac{8d}{L}\beta_2 \left(\frac{m}{L}\right) \tag{11}$$

where $\beta_1 = \beta 0_1 \beta n_{i1}$; and $\beta_2 = \beta 0_2 \beta n_{i2}$.

This expression enables drain spacing to be estimated more accurately than the original Hooghoudt equation [Eq. (1)]. In particular, the use of Eq. (11) avoids the overestimation of maximum water table height associated with the Hooghoudt equation in its traditional form.

Allowing the effect of unsaturated hydraulic properties via implemention of Eq. (11) has the potential to improve drain spacing and decrease the total expenses of drainage projects, particularly for sandy soils.

Conclusion

A dimensionless finite-element Richards equation model was developed to incorporate the effect of soil-water retention parameters on the maximum water table height between two adjacent drains. This model was then used as a comparison reference to show that the well-known analytical Hooghoudt equation overestimates the maximum water table height midpoint between two drains, particularly for sandy soils. The reason is that the Hooghoudt equation ignores water movement in the unsaturated zone. However, the nonlinear Richards equation incorporates a pressure-dependent hydraulic conductivity function that includes both the saturated and the unsaturated zones. Furthermore, both of the Hooghoudt equation coefficients are shown to be functions of soil-water retention parameters. Two new dimensionless coefficients β_1 and β_2 are incorporated into the Hooghoudt equation, giving a modified expression that yields between-drain water table height estimates close to those from the numerical reference model. These new coefficients are functions of soil physical properties.

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