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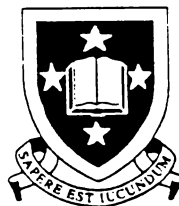
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Lateral Electron Disequilibrium
in
Radiotherapy Treatment Planning

A thesis
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of the requirements for the Degree
of
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at the
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by
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Abstract

Lateral electron disequilibrium on the central-axis of small-diameter circular beams is investigated using the EGS4 Monte Carlo system, for 10 MV photons. Monte Carlo results generated using the reciprocity technique are presented. In water, disequilibrium is found to occur for beams with diameters of less than approximately 4 cm. In lung of density 0.25 g cm^{-3} , disequilibrium occurs for diameters less than approximately 16 cm. The *lateral electron equilibrium factor* is defined and used to quantify the degree of disequilibrium.

Convolution dose calculation for 10 MV photons, using polyenergetic energy deposition kernels, gives a slight error in central-axis dose, due to beam hardening with depth. Primary and scattered polyenergetic kernels formed for a surface primary photon spectrum contain less and more fractional energy respectively than primary and scattered kernels formed for a beam hardened spectrum. A beam hardening correction to the kernels used in the convolution is proposed. This correction improves the agreement of convolution results with Monte Carlo predictions.

A superposition algorithm is developed and used to calculate dose in a lung phantom for 10 MV photons. Results show an overestimate in dose in the lung for a 5 cm square field when compared with experiment and Monte Carlo results. Disagreement is due to the assumption made in the density scaling method that secondary electrons travel in straight lines. A modification to the interaction voxel weighting in the density scaling procedure improves the result. The superposition method for 4 MV and 10 MV photons has been incorporated into the GRATIS treatment planning system.

Two and three-dimensional electron pencil beam algorithms are used to calculate the dose distribution in a water medium containing a small cavity for a 12 MeV electron beam. The measured hot spot beneath the cavity is 122%. The 2-D method predicts 114% and the 3-D method predicts 127%. Both profile and depth dose curves produced by the 3-D method are superior to those produced by the 2-D method. The 3-D pencil beam algorithm has been incorporated into the GRATIS treatment planning system.

Preface

In the treatment of tumours using radiotherapy, it is important that the dose delivered to the tumour is uniform throughout its volume, and that the dose delivered to healthy tissue is as low as possible. It is thus important to be aware of the distribution of dose throughout the patient that arises from irradiation with a particular beam or set of beams. If this distribution is known, the treatment procedure can be optimised. This optimisation process is called radiotherapy treatment planning, and is normally carried out using a computer. The accuracy with which a dose distribution is calculated, is dependent on how rigorously the dose computation algorithm models the beam. In general, the accuracy of an algorithm increases with the computing time required to run it. Since it is important that the time spent waiting for a particular dose distribution to be calculated is minimal, a compromise is usually made between speed and accuracy when developing an algorithm.

For high energy X-ray (photon) beams, the secondary electrons set in motion by primary photons can travel a significant distance before stopping. The long range of these electrons can give rise to situations of lateral electron disequilibrium — particularly when the photon beam has a small cross section or the irradiated medium contains a low density region such as lung. Most currently used dose computation algorithms do not account for situations of lateral electron disequilibrium, and consequently give inaccurate results in situations where disequilibrium exists. Two algorithms which do account for this phenomenon are the Monte Carlo method and the superposition method. The Monte Carlo method is potentially the most accurate algorithm available, but the time required to generate a treatment plan is too long for routine use. The superposition method is also very computationally intensive, but with powerful enough computer hardware it is a viable method for routine use. Treatment planning is also carried out for electron beam radiotherapy. Accurate modelling of an electron beam requires that the processes of both energy loss and scattering are accounted for. Apart from the Monte Carlo method, the most accurate electron beam models are based on the *pencil beam algorithm*, where the electron beam is treated as being comprised of many narrow ‘pencil’ beams. This thesis is the result of work carried out in developing a radiotherapy treatment planning system, which uses superposition for photon beam dose calculations and a 3-D electron pencil beam method for electron beam dose calculations.

In chapter 2, The EGS4 Monte Carlo system is used to investigate the incidence of lateral electron disequilibrium for 10 MV photon beams of circular cross section. The degree of disequilibrium which occurs for different diameters is described by the lateral electron equilibrium factor. In Monte Carlo simulations, the reciprocity technique is employed to reduce the computing time required to generate results with a low uncertainty. The increase in efficiency which this technique provides is also investigated. Results of the Monte Carlo simulations are compared with measured dose distributions for radiosurgery beams.

The convolution method of dose computation is introduced and applied to circular beams in chapter 3. The use of polyenergetic convolution kernels is investigated, with emphasis on how their validity is affected by beam hardening. A simple correction for beam hardening is proposed and results are compared with those generated using EGS4.

The superposition method is a generalisation of the convolution method which can be used in situations where inhomogeneities are present in the irradiated medium — the convolution kernel is scaled to account for density variations. A superposition method is described in chapter 4 and applied to the problem of dose computation in a medium containing lung of density 0.2 g cm^3 . Results are compared to those generated using EGS4.

The superposition algorithm has been incorporated into the GRATIS treatment planning system. This system runs on a Sun workstation and uses transputers for the computationally intensive parts of the superposition algorithm. The GRATIS system is described in chapter 5, along with a description of the superposition algorithm installed on that system.

The development of a pencil beam algorithm for electron beam dose calculations is covered in chapter 6. Two and three-dimensional versions are described, and the accuracy of these algorithms in calculating the dose distribution in a medium containing an air cavity is compared. In chapter 7, the incorporation of a three-dimensional pencil beam algorithm into the GRATIS system is described.

Publications and presentations with which the author has been involved during the course of this project include the following:

Publications

Hoban P.W., Murray D.C., Metcalfe P.E., Round W.H. (1990). Superposition dose calculation in lung for 10MV photons. *Aust. Phys. Eng. Sci. Med.*, **13**(2), 81-92.

Murray D.C., Hoban P.W., Metcalfe P.E., Round W.H. (1989). 3-D superposition for radiotherapy treatment planning using fast Fourier transforms. *Aust. Phys. Eng. Sci. Med.*, **12**(3), 128-137.

Metcalfe P.E., Hoban P.W., Murray D.C., Round W.H. (1989). Modelling high energy photon beams by superposition. *Aust. Phys. Eng. Sci. Med.*, **12**(3), 138-148.

Murray D.C., Hoban P.W., Graham I.D., Round W.H., De Vel O.Y. (1989). Radiotherapy treatment planning using transputers. *New Zealand Journal of Computing*, **1**(2), 30-38.

Metcalfe P.E., Hoban P.W., Murray D.C., Round W.H. (1990). Beam hardening of 10MV x-rays: Analysis using a convolution/superposition method. *Phys. Med. Biol.*, **35**(11), 1533-1549.

Metcalfe P.E., Hoban P.W., Harper N.R., Murray D.C., Round W.H. (1990). The production of body analogs for use in radiation physics. *Aust. Phys. Eng. Sci. Med.*, **13**(3), 117-128.

Murray D.C., Hoban P.W., Graham I.D., Round W.H., Metcalfe P.E. (1991). Superposition on a multicomputer system. *Med. Phys.*, **18**(3), 468-473.

Presentations

Hoban P.W., Metcalfe P.E., Round W.H. Dose spread arrays for high energy photon beams. *EPSM Conference, Brisbane Australia 1988.*

Hoban P.W., Murray D.C., Metcalfe P.E., Round W.H. X-ray dose calculation in lung using superposition. *EPSM Conference, Hamilton N.Z. 1989.*

Hoban P.W., Metcalfe P.E., Murray D.C., Round W.H. A 3-D electron pencil beam method. *Joint ARPS/EPSM Conference, Adelaide Australia 1990.*

Hoban P.W., Murray D.C., Round W.H. Lateral electron disequilibrium in small diameter 10 MV photon beams. *EPSM Conference, Sydney Australia 1991.*

Metcalfe P.E., Hoban P.W., Round W.H. Improved Accuracy of High Energy Computerised Dose Calculations Using Superposition. *EPSM Conference, Brisbane Australia 1988.*

Murray D.C., Graham I.D., Round W.H., Metcalfe P.E., Hoban P.W. Transputers in radiotherapy planning. *EPSM Conference, Brisbane Australia 1988.*

Murray D.C., Hoban P.W., Graham I.D., Metcalfe P.E., Round W.H., De Vel O.Y. Radiotherapy treatment planning using transputers. *New Zealand Transputer Users Group Meeting, Hamilton N.Z. 1989.*

Murray D.C., Hoban P.W., Round W.H., Graham I.D., Metcalfe P.E. Transputers: A parallel approach to radiotherapy computations. *EPSM Conference, Hamilton N.Z. 1989.*

Metcalfe P.E., Harper N.R., Hoban P.W., Murray D.C., Round W.H. The production of body analogue materials and their use in radiation dosimetry. *EPSM Conference, Hamilton N.Z. 1989.*

Round W.H., Hoban P.W., Murray D.C., Metcalfe P.E., Graham I.D. An advanced radiotherapy planning system. *New Zealand Society for Oncology, 18th Annual Scientific Meeting, Hamilton N.Z. 1989.*

Murray D.C., Hoban P.W., Metcalfe P.E., Round W.H. Treatment planning simulations using EGS4 Monte Carlo. *Joint ARPS/EPSM Conference, Adelaide Australia 1990.*

Metcalfe P.E., Hoban P.W., Murray D.C., Round W.H. The effect of beam hardening and inhomogeneities on 10MV x-rays: Analysis using a superposition method. *Joint ARPS/EPSM Conference, Adelaide Australia 1990.*

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Symbols and Abbreviations

Symbols

A	Atomic mass.
A_0, A_1, A_2	Scattering moments in Fermi-Eyges theory.
B_x, B_y, B_z	Axis directions in beam coordinates.
C_p	Beam hardening correction factor for polyenergetic primary EDKs.
C_s	Beam hardening correction factor for polyenergetic scattered EDKs.
D	Absorbed dose.
D_p	Primary dose.
D_{abs}	Absolute dose.
D_{sup}	Unnormalised superposition dose.
d, z	Physical depth.
d'	Radiological depth.
d_{max}	Depth of maximum dose.
ΔR	Radial shell thickness in a cylindrical voxel array.
ΔZ	Layer thickness in a cylindrical voxel array.
DISTZ	Distance from the beam source to the surface (the SSD).
ϵ	Efficiency of a Monte Carlo simulation.
E_k	Kinetic energy.
E_s	Scattered photon energy.
$\overline{E}_{ab}$	Mean energy deposited by secondary electrons.
$\overline{E}_{tr}$	Mean energy transferred to secondary electrons.
$\sigma \overline{E}_{tr}$	Mean energy transferred in a Compton interaction.
$\kappa \overline{E}_{tr}$	Mean energy transferred in a pair production interaction.
E_n	Mean energy of the n th spectral component.
EEF_{lat}	Lateral electron equilibrium factor.
E_p	Primary energy deposited in an EDK voxel.
E_s	Scattered energy deposited in an EDK voxel.
E_{tot}	Primary photon energy removed at an interaction site.
E_{max}	Maximum secondary electron energy.
$\bar{E}(0)$	Mean electron energy at the surface.
F_n	Relative fluence of the n th spectral component.
f	Factor to convert from dose in water to dose in bone.
g	Depth factor.
g_{H_2O}	Depth dose value in water.
H_p	Fractional primary energy deposited in an EDK voxel.
H_s	Fractional scattered energy deposited in an EDK voxel.
int_wt	Interaction voxel weighting.

$ISQR$	Inverse-square correction factor.
K	Kerma.
K_c	Collision kerma.
L_0	Air gap between the applicator face and a plane at 100 cm SSD.
μ	Linear attenuation coefficient
μ_{ab}	Linear energy absorption coefficient
μ_{tr}	Linear energy transfer coefficient
N_A	Avogadro's number.
N_{CT}	CT number.
$NCASE$	Number of histories in a Monte Carlo simulation.
P	Dose calculation point.
P_{cal}	Calibration point.
P_x, P_y, P_z	Axis directions in patient coordinates.
Φ	Particle fluence.
Ψ	Particle energy fluence.
P_A	Angular probability density.
P_L	Location probability density.
R_γ	Regression line gradient for CT numbers.
ρ	Mass density.
ρ_{ave}	Average density between interaction and deposition sites.
ρ_e^w	Electron density.
$\mathbf{r}$	Vector representing a dose calculation point.
$\mathbf{r}'$	Vector representing a primary interaction point.
$\Delta \mathbf{r}$	Vector between $\mathbf{r}$ and $\mathbf{r}'$.
R_{BEAM}	Beam radius in a Monte Carlo simulation.
R_{BOUND}	Detection region radius in a Monte Carlo simulation.
r_{lat}	Lateral electron range.
r_{long}	Longitudinal electron range.
R_p	Practical range.
R_{50}	Depth at which dose is 50% of the maximum dose.
$\sigma_{\theta x}$	x-component of angular standard deviation.
σ_{air}	Standard deviation due to σ_{θ_0} at the applicator face.
σ_{MCS}	Standard deviation due to multiple Coulomb Scattering.
σ_{maz}	Maximum value allowed for σ_{MCS} .
s	Statistical uncertainty.
s^2	Statistical variance.
$STAT$	Number of batches in a Monte Carlo simulation.
S_{col}	Linear collision stopping power.
S_{rad}	Linear radiative stopping power.
S, S_{tot}	Linear stopping power.

$S_{H_2O}^{med}$	Linear stopping power ratio of the medium to water.
S	Beam drift factor.
T	Terma; Calculation time.
$d\bar{\theta}^2/dz$	Linear scattering power.
vsp	Virtual source position.
X_0, X_1	Limits of a rectangular beam in the x-direction.
Y_0, Y_1	Limits of a rectangular beam in the y-direction.
Z	Atomic number.
$\bar{z}$	Effective longitudinal secondary electron range.
z_{eff}	Effective depth.

Abbreviations

AAPM	American Association of Physicists in Medicine.
CT	Computed Tomography.
EDK	Energy Deposition Kernel.
EGS4	Electron Gamma Shower Monte Carlo system – version 4.
ETAR	Equivalent Tissue-air Ratio algorithm.
IAEA	International Atomic Energy Agency.
keV	kiloelectronvolts.
MCS	Multiple Coulomb Scattering.
MV	Megavolts.
MeV	Megaelectronvolts.
PEDK	Polar Energy Deposition Kernel.
SAR	Scatter-air ratio.
dSAR	Differential scatter-air ratio.
SSD	Source Skin Distance.
SAD	Source Axis Distance.
TAR	Tissue-air ratio.

Chapter 1

Radiation Transport and Photon Beam Models

1.1. Introduction

Determining the distribution of energy deposited throughout a medium irradiated by beams of high energy photons or electrons, has become an increasingly important area of medical physics research. This is due to the widespread use of linear accelerator-based radiotherapy in the treatment of cancerous tumours, and the associated need to optimise the treatment procedure. The optimum procedure is that which does the required damage to the tumour and at the same time minimises the damage to healthy tissue. This optimisation process is referred to as *radiotherapy treatment planning*.

In radiation treatment planning, the principles of radiation transport are used to determine the distribution of energy deposition due to each beam. In this chapter, the important principles are discussed and the essential quantities used are defined. The discussion concentrates on photon beam irradiation, since the bulk of this thesis is concerned with this treatment modality, although the transport of both photons and electrons is discussed.

1.2. Megavoltage Photon Beams

High energy photon beams used for tumour treatment may be produced by a radioactive source such as cobalt-60, or by a linear accelerator. It is those produced by a linear accelerator which are of interest in this thesis. Photon beams are produced in a linear accelerator by bombarding a tungsten target with high energy electrons accelerated along a wave guide (Johns and Cunningham, 1983). In interacting with the target, the electrons lose their energy and cause *bremsstrahlung* photons to be emitted (see section 1.4). The beam of photons which emerges from the linear accelerator is referred to as the *treatment beam*. The region over which the treatment beam is defined, is determined by the settings of the collimation system in the *treatment head* of the linear accelerator (Johns and Cunningham, 1983). This region is also referred to as the *field*, and is normally rectangular in shape.

The beam axis may be rotated through 360 degrees, where the intersection of the beam axis and the axis of rotation is called the *isocentre* of the beam — the beam axis is always in the same plane and always passes through the isocentre. The rotatable structure which contains the waveguide and treatment head is called the *gantry*. The photon beam emerges from a *virtual source* in the treatment head — the dimensions of the field increase in proportion to the distance from this source. The increase in the size of the field with distance from the virtual source is referred to as *beam divergence*. The distance from the virtual source

to the isocentre is called the *source axis distance* (SAD), and is normally 100 cm. The *field size* set by the operator represents the dimensions of the beam at the SAD. The direction of motion of the photon beam is sometimes referred to as the *downstream* or *longitudinal* direction. Similarly, the opposite direction is referred to as the *upstream* direction. The direction perpendicular to the beam, is the *lateral* direction.

A particular beam contains a spectrum of photon energies, which range from zero MeV (Megaelectronvolts) up to approximately the energy of the electrons which struck the target. The quoted energy of the beam is the maximum energy in the spectrum, and is expressed in MV (Megavolts). For example, the Varian Clinac 18/10 10 MV beam has a maximum energy of approximately 10 MeV and a mean energy of approximately 3 MeV. Typically, linear accelerator-produced photon beams have a maximum energy of between 4 MeV (a 4 MV beam), and 25 MeV (a 25 MV beam). The spectrum of several beams has been obtained using *Monte Carlo* simulations of the linear accelerator beam production system (Mohan and Chui, 1985). The spectrum of the Varian Clinac 18/10 beam obtained using this method is shown in figure 1.1.

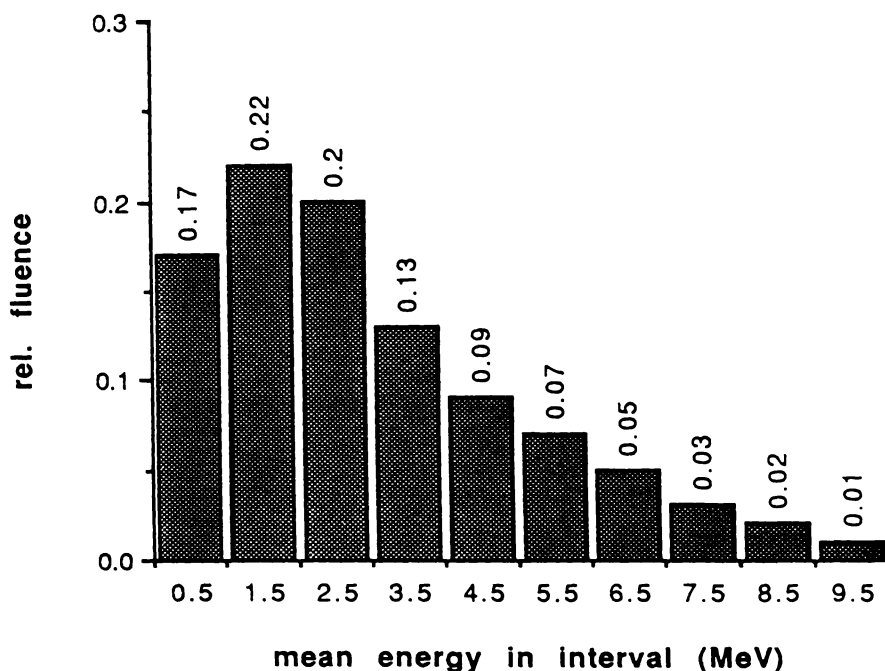


Figure 1.1. Energy spectrum of the Varian Clinac 18/10, 10 MV photon beam. This spectrum was obtained by Monte Carlo simulation of the linear accelerator treatment head.

1.3. Photon Transport

When a high energy photon enters a medium, it undergoes interactions with atomic nuclei and electrons. A photon which has not interacted in the medium, is referred to as a *primary photon*. When such a photon interacts, the interaction is called a *primary photon interaction*. A photon which has interacted, is known as a *scattered photon* and subsequent

interactions by that photon are *scattered photon interactions*. The likelihood of a particular type of interaction, depends on the energy of the photon and on the composition of the irradiated medium. The possible interactions are outlined below.

1.3.1. Photon Interaction Processes

1.3.1.1. Coherent Scattering

This type of scattering is only important at low photon energies, where the photon can be considered as an electromagnetic wave. In a coherent scattering event, the incident wave causes an atom to oscillate at the same frequency as the wave, thereby emitting radiation with the same frequency as the incident radiation. Since the incident and emitted radiation is of the same frequency, no energy is lost in a coherent scattering event. The probability per unit pathlength that a photon will undergo coherent scattering is given by the *coherent attenuation coefficient*, μ_{coh} . This type of interaction becomes insignificant for photon energies above approximately 100 keV (Johns and Cunningham, 1983), hence it is not important for megavoltage photon beams.

1.3.1.2. Photoelectric Absorption

In a photoelectric process, a photon of energy $h\nu$ is absorbed by an atom causing a bound electron to be ejected. The ejected electron has a kinetic energy $E_k = h\nu - E_s$, where E_s is the binding energy of the electron – the energy required to eject the electron from the atom. The atom is left in an excited state and consequently emits *characteristic radiation*, whose energy depends on the atomic shell from which the electron was ejected. The probability per unit pathlength that a photon will undergo a photoelectric interaction is given by the *photoelectric attenuation coefficient*, μ_r . In body tissue, this type of interaction is predominant for incident photons with energies up to approximately 20 keV but is insignificant for energies above approximately 100 keV (Johns and Cunningham, 1983).

1.3.1.3. Compton Scattering

Compton scattering is the interaction process which both primary and scattered photons are most likely to undergo. In fact, for photons with energies between 200 keV and 2 MeV, this is the only interaction which occurs in soft tissue (Johns and Cunningham, 1983). The probability per unit pathlength that a photon will undergo a Compton interaction is given by the *Compton attenuation coefficient* μ_σ .

In a Compton interaction, the incident photon is scattered elastically by an atomic electron. The incident photon of energy $h\nu$ transfers some of its energy to the electron, which is called a *recoil electron*. If the electron receives a kinetic energy E_k , then the scattered photon retains an energy $h\nu' = h\nu - E_k$. To a good approximation the electron can be considered to be unbound when considering the dynamics of a Compton interaction. Using the principles of energy and momentum conservation, and using quantum mechanical reasoning, the probability of the incident photon scattering through a particular angle can be determined, along with the corresponding energy of the recoil electron. If the scattering

angle of a photon of initial energy $h\nu$ is θ then the fraction of energy transferred to the electron is (Johns and Cunningham, 1983)

$$\frac{E_k}{h\nu} = \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)} \quad (1-1)$$

where $\alpha = h\nu/m_0c^2$ — the ratio of the photon energy to the electron rest mass energy. From (1-1), it can be seen that for very low energies ($\alpha \ll 1$) the energy transfer approaches 0 and for high energies ($\alpha \gg 1$) the energy transfer approaches unity.

The mean kinetic energy of Compton recoil electrons for incident photons of energy $h\nu$ is given by

$${}_{\sigma}\overline{E}_{tr} = h\nu \frac{\sigma_{tr}}{\sigma} \quad (1-2)$$

where σ_{tr} is the Compton energy transfer cross section and σ is the total Compton cross section (Johns and Cunningham, 1983). Similarly the mean energy of scattered photons is

$${}_{\sigma}\overline{E}_s = h\nu \frac{\sigma_s}{\sigma} \quad (1-3)$$

where $\sigma_s = \sigma - \sigma_{tr}$ is the scattered photon cross section. Note that ${}_{\sigma}\overline{E}_{tr} + {}_{\sigma}\overline{E}_s = h\nu$. A plot of ${}_{\sigma}\overline{E}_{tr}$ and ${}_{\sigma}\overline{E}_s$ versus $h\nu$ is shown in figure 1.2.

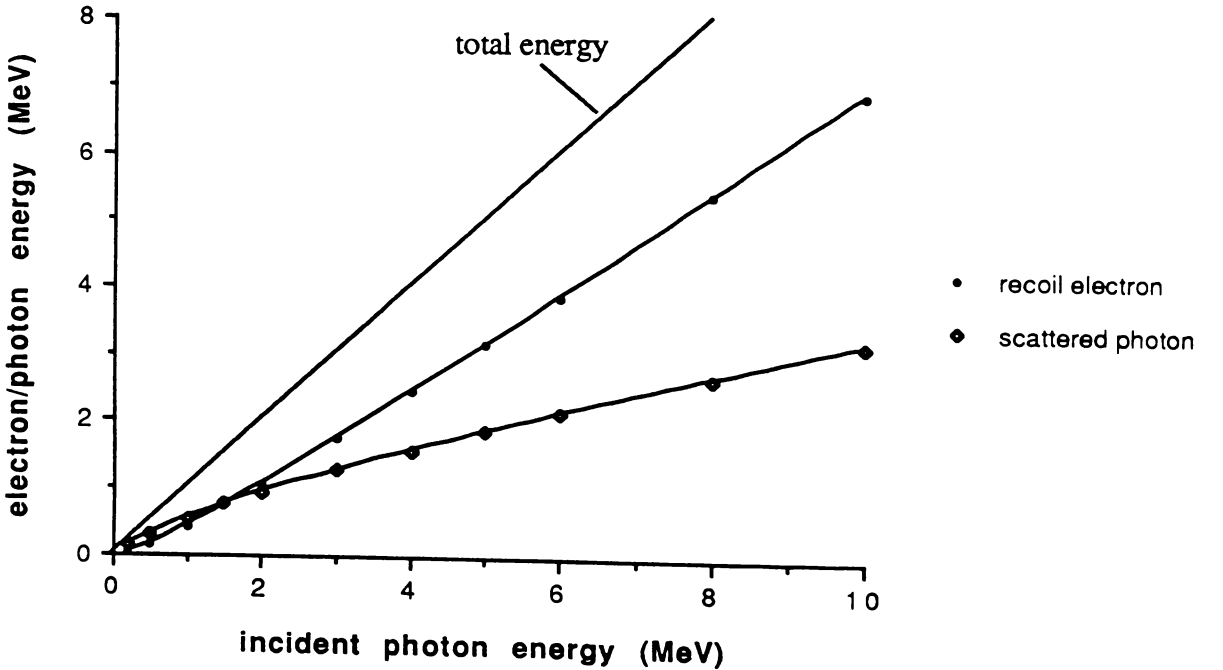


Figure 1.2. Mean energy transferred to secondary electrons, ${}_{\sigma}\overline{E}_{tr}$, and mean energy retained by scattered photons, ${}_{\sigma}\overline{E}_s$, when photons of energy $h\nu$ interact in a Compton process.

Of interest in this work is the maximum energy transferred to a Compton recoil electron. The maximum energy transfer occurs when the incident photon makes a direct hit on the

electron and is scattered through 180° — straight back along its incoming path. In this case the recoil electron is set in motion in the direction of the incident photon. From (1-1), the maximum kinetic energy of the recoil electron is

$$E_{tr,max} = h\nu \frac{2\alpha}{1 + \alpha}. \quad (1-4)$$

For the 10 MV beam, the maximum photon energy is approximately 10 MeV. For this energy, $\alpha = 19.57$ and $E_{tr,max} = 9.75$ MeV.

1.3.1.4. Pair Production

If the incident photon has an energy $h\nu$ of at least twice the rest mass energy of an electron ($2 \times 0.511 = 1.022$ MeV), a pair production event may occur. The probability per unit pathlength that a photon will undergo a pair production interaction is given by the *pair production coefficient* μ_κ . In tissue, pair production begins to be important for photon energies above about 5 MeV — above 50 MeV it is the most important interaction (Johns and Cunningham, 1983).

In these interactions, a photon subjected to the strong field of a nucleus has its energy converted to the mass of an electron-positron pair. If the photon has energy in excess of 1.022 MeV, the excess energy is shared between the electron and the positron as kinetic energy. The kinetic energy transferred to the electron-positron pair is always ${}_\kappa \overline{E}_{tr} = h\nu - 1.022$ MeV. Since this energy is divided evenly, the initial kinetic energy of each charged particle is ${}_\kappa \overline{E}_{tr}/2$ (Johns and Cunningham, 1983). For the maximum photon energy in the 10 MV beam, (10 MeV), each charged particle produced in a collision will have a kinetic energy of $(10 - 1.022)/2 = 4.49$ MeV.

When the positron has lost nearly all its energy, it combines with an electron and both particles *annihilate*. On annihilation, the energy of the electron and positron is converted to the energy of two 0.511 MeV photons which are ejected in opposite directions (Johns and Cunningham, 1983).

1.3.2. Photon Fluence

The photon *fluence* is the number of photons crossing a sphere of unit cross sectional area. If dN photons cross a sphere of very small cross sectional area da , then the fluence at the position of the sphere is given by

$$\Phi = \frac{dN}{da}. \quad (1-5)$$

For a spectrum of photon energies, the fluence can be split into a component for each energy interval i , Φ_i . The total fluence is then

$$\Phi = \sum_{i=1}^N \Phi_i. \quad (1-6)$$

The photon *energy fluence* at a point is the total photon energy crossing a sphere of unit cross sectional area at that point, per unit time. For photons of energy $h\nu$, the energy

fluence is given by

$$\Psi = h\nu\Phi. \quad (1-7)$$

The energy fluence for a spectrum of photons is

$$\Psi = \sum_{i=1}^N h\nu_i\Phi_i = \sum_{i=1}^N \Psi_i \quad (1-8)$$

These quantities can be divided into *primary* and *scattered* components. The primary component is that due only to primary photons, and the scattered component is due to photons which have interacted in the medium.

1.3.3. Linear Attenuation Coefficient

The probability per unit pathlength that a photon will interact by any process is the *linear attenuation coefficient*, μ . This coefficient is equal to the sum of the attenuation coefficients due to coherent scattering, photoelectric effect, Compton scattering, and pair production:

$$\mu = \mu_{coh} + \mu_{\tau} + \mu_{\sigma} + \mu_{\kappa}. \quad (1-9)$$

For example, the linear attenuation coefficient in water for primary photons with the mean energy in the 10 MV beam, 3.0 MeV, is 0.0397 cm^{-1} . The proportions of this coefficient due to the four processes listed above are 0.00, 0.00, 0.97, and 0.03 respectively. For photons with an energy of 10 MeV, the linear attenuation coefficient is 0.0222, with the relative proportions being 0.00, 0.00, 0.77, and 0.23. It is obvious that Compton scattering is the dominant interaction process for this beam, although pair production is important for photons at the high energy end of the spectrum.

The value of μ determines the rate at which the primary photon fluence decreases with depth of penetration. If the fluence at a depth z is $\Phi(z)$, then since the probability of a primary photon being lost out of the beam per unit pathlength is μ , the rate of change of the primary fluence is

$$\frac{d\Phi}{dz}(z) = -\mu\Phi(z). \quad (1-10)$$

Solving this equation for $\Phi(z)$ gives

$$\Phi(z) = \Phi(0)e^{-\mu z} \quad (1-11)$$

where $\Phi(0)$ is the primary photon fluence at the surface of the medium.

Media of a certain elemental composition, have a linear attenuation coefficient which is proportional to their density. For example, in compressing a gas to twice its original density, its linear attenuation coefficient will double. Therefore, the linear attenuation coefficient divided by the density of the gas is a constant quantity. This quantity is known as the mass attenuation coefficient (μ/ρ) and has units of cm^2g^{-1} . Different soft tissues in the body have a different density but have a similar elemental composition (Woodard and White,

1986), therefore the mass attenuation coefficient for soft tissues is almost constant (Johns and Cunningham, 1983).

The inverse of the linear attenuation coefficient is the *mean free path* λ . This quantity is equal to 25.2 cm for 3 MeV photons in water. Hence, the mean distance which a photon in the 10 MV beam will penetrate in water before interacting is approximately 25 cm. This means that some photons will not interact at all in the irradiation of a patient.

1.3.4. Energy Transfer and Energy Absorption Coefficients

In photoelectric and Compton interactions, atomic electrons are set in motion. In pair production events an electron-positron pair is created and set in motion. These charged particles are all referred to as *secondary electrons*. The kinetic energy of these particles is obtained from the energy of the incident photon. If the mean energy transferred to kinetic energy of secondary electrons by an incident photon of energy $h\nu$ is $\overline{E}_{tr}$, then the fraction of incident photon energy transferred to kinetic energy per unit path length is given by

$$\mu_{tr} = \mu \frac{\overline{E}_{tr}}{h\nu}. \quad (1-12)$$

This quantity is the *linear energy transfer coefficient*. For 3.0 MeV photons in water, the mean energy transferred in a primary interaction is $\overline{E}_{tr} = 1.74$ MeV (Johns and Cunningham, 1983). Therefore, since $\mu = 0.0397$ for 3.0 MeV photons, $\mu_{tr} = 0.0397(1.74/3.0) = 0.0230 \text{ cm}^{-1}$.

Most of the energy transferred to secondary electrons is subsequently deposited in the medium (see section 1.4). Some however is converted to bremsstrahlung photons in interactions with atomic nuclei. If the mean energy transferred to secondary electrons which is subsequently absorbed by the medium is $\overline{E}_{ab}$, then the fraction of incident photon energy transferred to the medium per unit pathlength which is subsequently absorbed is

$$\mu_{ab} = \mu \frac{\overline{E}_{ab}}{h\nu}. \quad (1-13)$$

This quantity is the *linear energy absorption coefficient*. For 3.0 MeV photons, $\overline{E}_{ab}$ is 1.73 MeV, making μ_{ab} equal to 0.0229 cm^{-1} . Note that $\mu_{ab} \approx \mu_{tr}$ for this energy — very little secondary electron energy is converted to bremsstrahlung.

As with the linear attenuation coefficient, the linear energy transfer and absorption coefficients have associated mass energy transfer and absorption coefficients, (μ_{tr}/ρ) and (μ_{ab}/ρ) . These quantities are again approximately constant for soft body tissues.

1.3.5. Kerma

The kinetic energy imparted to secondary electrons per unit mass by photon interactions is the *kerma* — kinetic energy released per unit mass. This is the product of the energy fluence Ψ and the mass energy transfer coefficient (μ_{tr}/ρ) . For a spectrum of photon energies, the kerma is given by

$$K = \sum_{i=1}^N \Psi_i \left(\frac{\mu_{tr}}{\rho} \right)_i. \quad (1-14)$$

If the spectrum includes only primary photons, this quantity is the *primary kerma*.

Kerma can be partitioned into collision and radiative parts (Attix, 1978). The *collision kerma* is the energy imparted per unit mass which is subsequently absorbed by the medium. This is given by

$$K_c = \sum_{i=1}^N \Psi_i \left(\frac{\mu_{ab}}{\rho} \right)_i. \quad (1-15)$$

If the spectrum includes only primary photons, then K_c is the *primary collision kerma*. The *radiative kerma* is the energy imparted per unit mass which is subsequently converted to bremsstrahlung, and is equal to the difference in the total kerma and the collision kerma.

1.3.6. Terma

A similar quantity to the primary kerma, is the *terma* — *total energy released per unit mass* (Ahnesjö *et al*, 1987). For a spectrum of primary photons, the terma is given by

$$T = \sum_{i=1}^N \Psi_i \left(\frac{\mu}{\rho} \right)_i. \quad (1-16)$$

This is in fact the energy removed from the primary beam per unit pathlength.

1.4. Electron Transport

In photon beam irradiation, energy is deposited throughout the medium by secondary electrons. The electron travels along an irregular path, on which energy is deposited, until all of the kinetic energy of the electron is lost. The two most important quantities in the transport of these electrons are the *stopping power* and the *scattering power*. These are discussed in this section.

1.4.1. Stopping Power

The electron loses most of its kinetic energy by ionization and excitation of atoms. Ionization occurs if the secondary electron passes close enough to an orbital electron to eject the orbital electron from the atom. If the secondary electron passes an atom at a larger distance, an electron may be elevated to a higher energy state, thereby exciting the atom. The amount of energy lost by the secondary electron per unit pathlength due to ionizations and excitations is called the *linear collision stopping power*, S_{coll} .

If the secondary electron passes close enough an atomic nucleus, the electron is decelerated and deflected by the electrostatic force between itself and the nucleus. The energy lost in this interaction is converted to the energy of a bremsstrahlung photon. The amount of secondary electron kinetic energy converted to bremsstrahlung per unit pathlength is called the *linear radiative stopping power*, S_{rad} . As stated in section 1.2, the primary photon beam consists of bremsstrahlung produced when the electron beam accelerated along the waveguide is decelerated by the tungsten target.

The sum of the linear collision and radiative stopping powers is the *total linear stopping power*, $S_{tot} = S_{coll} + S_{rad}$. In water, the collision stopping power for 1.0 MeV electrons

is $1.852 \text{ MeV cm}^{-1}$ and the radiative stopping power is $0.013 \text{ MeV cm}^{-1}$, giving a total stopping power of $1.865 \text{ MeV cm}^{-1}$. At 10 MeV, these values are 1.994, 0.182, and $2.176 \text{ MeV cm}^{-1}$ respectively. There is a slight increase in the collision stopping power with energy for energies greater than approximately 2 MeV. The radiative stopping power increases more rapidly with energy, but remains a relatively small component of the energy loss for the electron energies of concern here. The units for linear stopping power are MeV cm^{-1} .

As with the photon attenuation and energy loss coefficients, the stopping power is approximately proportional to density for materials of similar elemental composition. Therefore, the mass stopping power (S/ρ) (units of $\text{MeV cm}^2\text{g}^{-1}$) is almost constant for soft tissue. The term stopping power used in this thesis should be taken to mean total linear stopping power unless stated otherwise.

The approximate range of an electron can be obtained from its initial energy and the stopping power. Electrons of energy E , have an approximate range E/S_{tot} . For electrons of energy 1.74 MeV (the mean energy transferred by 3.0 MeV photons), the total stopping power in water is 1.87 MeV cm^{-1} . Therefore, the range of these electrons in water is a little less than 1 cm. There are however secondary electrons set in motion by the 10 MV beam with kinetic energies up to almost 10 MeV (see section 1.3.1.3) — these electrons have a range of over 5 cm. This long range means that secondary electron energy can be deposited at a considerable distance from the primary photon interaction site — this fact is central to the work in this thesis.

1.4.2. Scattering Power

Electrons are continuously deflected from their current direction of motion. These deflections are mainly due to electromagnetic interactions between the electron and atomic nuclei, and are elastic (Klevenhagen, 1985).

Since these deflections are small and independent, the net angular deflection of an electron after having travelled a certain distance can be predicted statistically (Andreo, 1985). In crossing a thickness Δz of material, there is an approximately Gaussian distribution of angles through which the electron could have been deflected. This Gaussian distribution has a standard deviation σ_θ given by the root mean square angular deflection

$$\sigma_\theta = \sqrt{\frac{d\bar{\theta}^2}{dz} \Delta z}, \quad (1-17)$$

where $d\bar{\theta}^2/dz$ is the *linear scattering power*. Physically, the linear scattering power is the increment in the mean square angle of deflection ($\bar{\theta}^2$) per unit thickness traversed. The scattering power decreases with an increase in electron energy (Werner *et al*, 1982; Andreo, 1985). For 1 MeV electrons, the scattering power in water is approximately $0.35 \text{ radian}^2\text{cm}^{-1}$, and for 10 MeV electrons is approximately $0.08 \text{ radian}^2\text{cm}^{-1}$.

Usually of more importance than the angular deflection, is the lateral displacement. This

is also a Gaussian distribution, and has a standard deviation

$$\sigma_{lat} = \sqrt{\int_0^{\Delta z} \frac{d\bar{\theta}^2}{dz} (\Delta z - z')^2 dz'}. \quad (1-18)$$

This quantity is used in *electron pencil beam* models such as that discussed in chapter 6 to predict the dose distribution due to an electron beam.

As with the mass stopping power, the *mass scattering power* $d\bar{\theta}^2/\rho dz$ (units of $\text{radian}^2\text{cm}^2\text{g}^{-1}$) is almost constant for soft tissue.

1.5. Absorbed Dose

The absorbed dose at a point P in the medium, is defined as the energy deposited per unit mass at that point. If an energy $d\epsilon$ is deposited in a very small volume with mass dm at P , then the dose at P is

$$D(P) = \frac{d\epsilon}{dm}. \quad (1-19)$$

The SI unit of absorbed dose is the Gray (Gy), where 1 Gy is equal to 1 J/kg. Dose is also referred to in cGy (1 cGy = 0.01 Gy), or in rads (1 rad = 1 cGy).

The measurement of absorbed dose is called *dosimetry*. The most common dosimetry method is that employing an *ionization chamber* which is a small cavity placed in the medium at the position where a dose measurement is required. The cavity contains a gas (normally air), which is ionized by secondary electrons. The amount of ionization which occurs is measured by collecting the charge liberated in ionization events. This measured charge is converted to the absorbed dose in the medium using a *dosimetry protocol*. Currently, the most widely used protocols are the AAPM (TG21) and IAEA protocols. A comparison of these two protocols is given by Huq and Nath (1990).

The rate at which dose is delivered is the *dose rate*, which has SI units of Gy s^{-1} . Since the dose rate can fluctuate, the beam is normally turned on for a set number of *monitor units* (MU) rather than for a set time. The time per monitor unit depends on the dose rate measured by special *monitor chambers* in the treatment head. The dose rate at a *calibration point* in a water medium is normally expressed in cGy MU^{-1} .

1.6. Primary and Scattered Dose

The absorbed dose due to secondary electrons set in primary photon interactions is termed *primary dose*. Similarly, the dose arising from scattered photon interactions is termed *scattered dose*. Note that photons may interact more than twice — the scattered dose component is hence sometimes divided into *first scattered* and *multiply scattered* dose (Mackie, 1985). The path of a photon which has undergone three Compton interactions is shown in figure 1.3, where ionizations giving rise to primary, first scattered, and multiply scattered dose are shown.

For high energy photons, it is important to note that the kinetic energy transferred to secondary electrons in one of these interactions may be deposited at a considerable distance

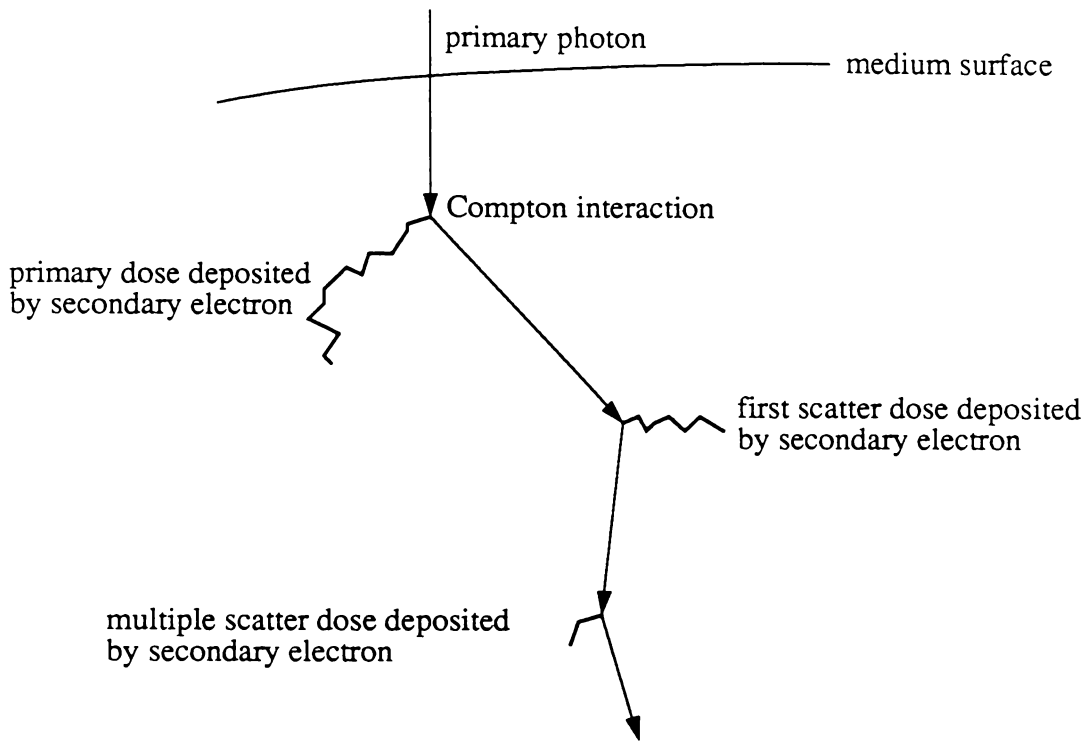


Figure 1.3. Photon undergoing three Compton interactions with atomic electrons. Recoil electrons resulting from the first, second, and third interactions give rise to primary, first scattered, and multiply scattered dose respectively.

(a few cm) from the interaction site, as discussed in section 1.4. This means that for a photon *pencil beam* whose cross sectional area approaches zero, primary dose is delivered within a channel about the pencil beam axis as illustrated in figure 1.4. As the primary photon energy increases, and hence the secondary electron energy increases, the width of this channel increases due to the increased range of the secondary electrons. It is the non-zero width of this channel that gives rise to *lateral electron disequilibrium* (see section 1.12), which is the focus of this thesis.

1.7. Depth Dose Curves

When dose measurements are made at a set of depths, a *depth dose curve* can be created. Such a curve is shown in figure 1.5 for a 10 MV, 10×10 cm beam. The dose is normalised to 100% at the depth of maximum dose, d_{max} , which, for this energy, is 2.1 cm. Since this curve has a maximum of 100%, it is referred to as a *percentage depth dose curve*.

The reason for the maximum dose occurring at this finite depth is that as secondary electrons are set in motion, they predominantly travel to a greater depth than that at which they originated. Therefore, for depths up to approximately the maximum range of the secondary electrons in the direction of the beam, the number of secondary electrons in motion increases — there is a *buildup* of secondary electrons. The region between the surface and

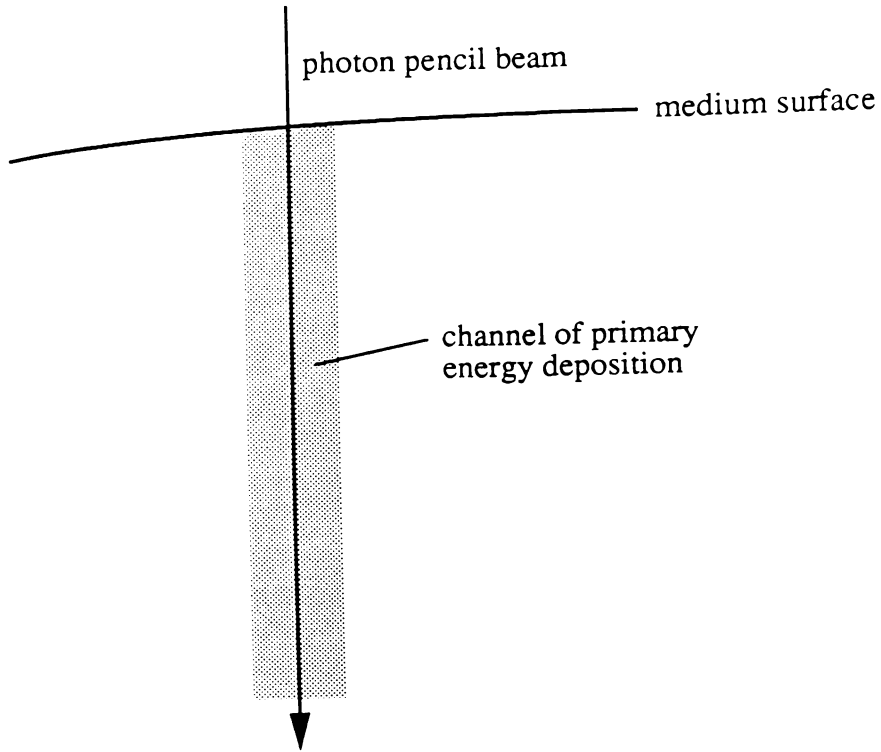


Figure 1.4. Channel about the axis of a pencil beam inside which primary energy is deposited. The width of this channel increases with primary photon energy.

d_{max} is therefore called the buildup region. For depths greater than d_{max} , the curve falls off exponentially with the primary photon fluence (see section 1.3.3).

The shape of the depth dose curve depends on the beam energy, the size of the field, and the distance from the virtual source to the surface of the irradiated medium (Johns and Cunningham, 1983). This distance is called the *source skin distance* (the SSD). Without attenuation, the primary photon fluence has an inverse-square dependence on distance from the virtual source. This decrease in fluence contributes to the reduction in fluence with depth of penetration in the medium, and hence to the reduction in dose with depth. As the SSD increases, the proportional increase in distance from the source as depth increases, is reduced. Therefore the rate at which the dose decreases with depth, decreases with an increase in SSD.

1.8. Dose Profile Curves

A set of dose measurements across the beam can be used to form a *dose profile*. A dose profile obtained from measurements with an ionization chamber is shown in figure 1.6. This curve is for the 10 MV beam, at 10 cm deep in water for a 30×30 cm field.

The profile is quite flat from the beam *central axis* out to the beam *penumbra*. The penumbra is so called because the shape is characteristic of the fluence profile which exists

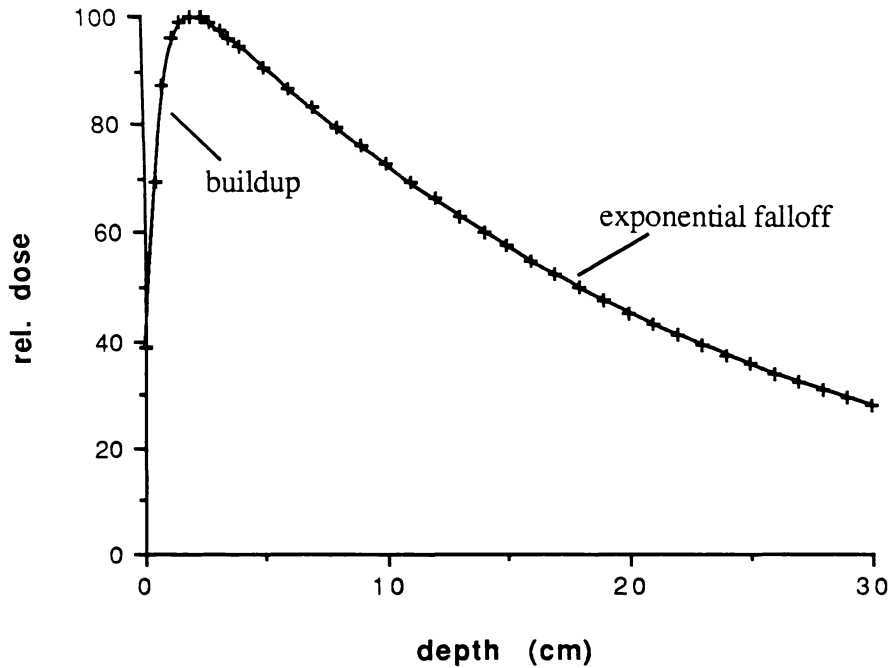


Figure 1.5. Percentage depth dose curve for a 10×10 cm, 10 MV beam. The depth of maximum dose, d_{max} , is 2.1 cm. The increase in dose down to this depth due to a buildup of secondary electrons is indicated by the buildup region. The falloff in dose past d_{max} follows the exponential decrease in primary photon fluence with depth.

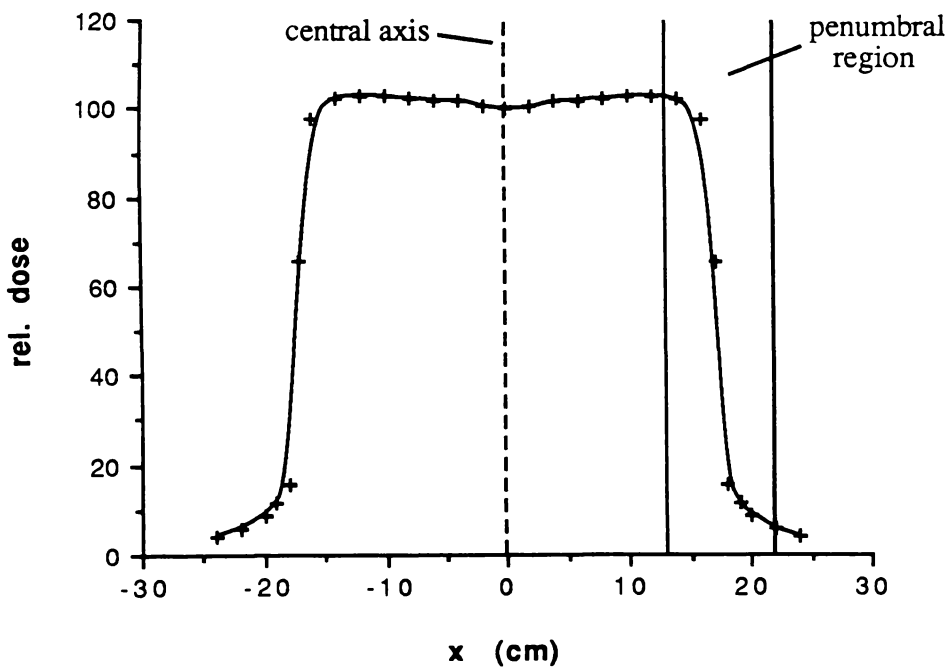


Figure 1.6. Profile curve at 10 cm deep in water for a 30×30 cm, 10 MV beam.

due to a radiation source of finite width, emitting radiation through an aperture. Models exist which predict the shape of the penumbra using the widths of the source and aperture

as well as the distance from the source to the aperture and distance from the aperture to the plane at which the profile is defined — these are called extended source models (Johns and Cunningham, 1983). For high energy photon beams, the shape of the penumbra is in fact mainly due to the long range of secondary electrons, as pointed out by Mohan and Chui (1985).

The distance from the central axis at which the dose is equal to half that at central axis, should be approximately to the half-width of the field, where the width increases with depth due to beam divergence. The profile shown in figure 1.6 is at 10 cm deep for an SSD of 100 cm: the half-width of the field at this depth is $15 \times 110 / 100 = 16.5$ cm.

1.9. Tissue-air Ratios and Scatter-air Ratios

The ratio of the dose at a point in tissue to the dose at the same point in air, is known as the *tissue-air ratio* (TAR). A tissue-air ratio versus depth curve is produced by measuring the dose in air with an ionization chamber, and then measuring the dose at that same point but with a range of depths of overlying tissue. The ratios of the dose for each depth to the dose in air then gives a TAR curve. This curve is similar to a depth dose curve, except that since there is no inverse-square falloff in dose with depth, the falloff in dose is slower. Tissue-air ratios are independent of distance from the source, for distances greater than approximately 50 cm (Johns and Cunningham, 1983).

Tissue-air ratios are useful for determining the variation in dose with field size as well as with depth — there is an increase in the TAR with field size. This increase is because more photons are scattered by the medium to the point of dose measurement in tissue as field size increases, and there is no such increase at the same point in air. For a particular depth is tissue, a TAR versus field size curve can be plotted. An illustration of such a curve is given in figure 1.7.

The points marked on the curve in figure 1.7 indicate measured TARs. For high energy photon beams, there is a lower limit on the field sizes for which dose can be measured accurately by ion chamber dosimetry. This limit is based on the secondary electron range — for the 10 MV beam the smallest field size for which dose is normally measured is approximately 5×5 cm. For field sizes smaller than this limit, TAR values are found by extrapolation of the measured points. If the curve is extrapolated back to a field size of zero, a *zero-area TAR* is obtained. This quantity represents the TAR when no scattered photons contribute to the dose in tissue, and is indicated in figure 1.7 as $TAR(0)$. The ‘extra’ dose contributed by scattered photons, is represented by the difference in the TAR and $TAR(0)$, and is called the *scatter-air ratio* (SAR). Thus, for a beam of radius r , the TAR at a depth d can be expressed as the sum of the zero area TAR and the SAR:

$$TAR(d, r) = TAR(d, 0) + SAR(d, r) \quad (1-20)$$

As a beam of circular cross section is increased in radius, the SAR increases. This increase for a unit increase in beam diameter is referred to as the *differential scatter-air ratio* (dSAR).

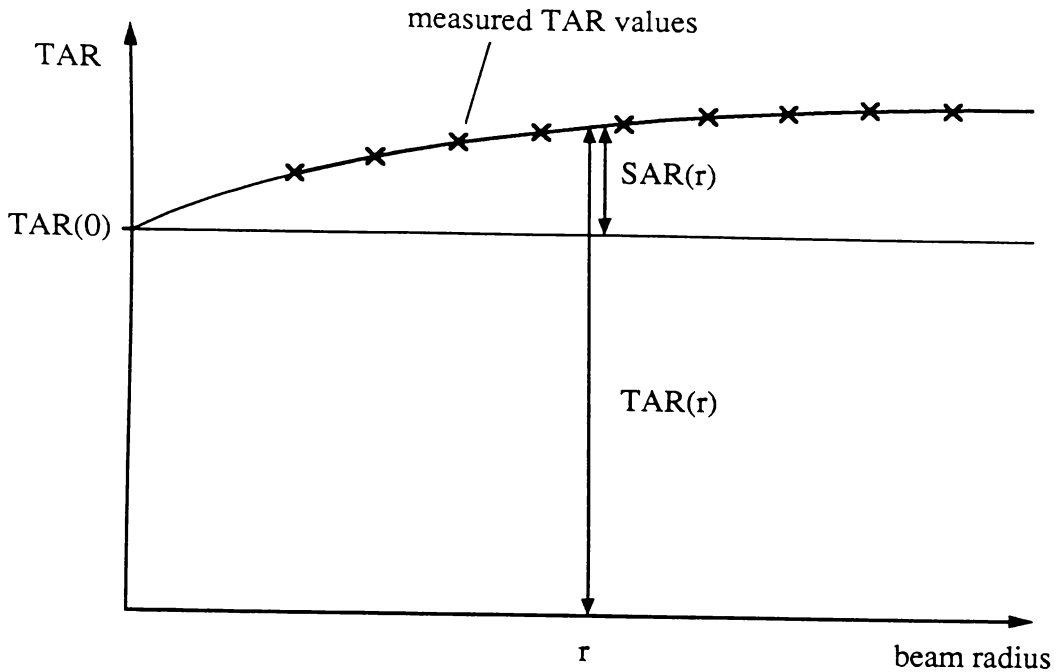


Figure 1.7. Illustration of a tissue-air ratio (TAR) versus field size curve. Measured TARs are indicated by the points on the curve. The zero-area TAR, $TAR(0)$, is given by the intersection of the curve with the TAR axis. The difference in the TAR for a particular field size and $TAR(0)$, is the scatter-air ratio (SAR) for that field size. The TAR and SAR for a field size r is indicated.

1.10. Electron Density

The electron density of a medium is defined as the number of electrons in a unit volume. For a substance consisting of a single element, the electron density is given by

$$\rho_e = N_A \rho \frac{Z}{A} \quad (1-21)$$

where N_A is Avogadro's number ($6.023 \times 10^{23} \text{ mol}^{-1}$), ρ is the mass density of the substance relative to water, and Z/A is the ratio of atomic number to atomic mass.

Electron density is usually given as a fraction of the electron density of water ρ_{ew} , which is $3.343 \times 10^{23} \text{ cm}^{-3}$. The electron density relative to water is therefore

$$\rho_e^w = 1.80 \rho \frac{Z}{A}. \quad (1-22)$$

Note that for most elements, Z/A is approximately $1/2$ — this makes ρ_e^w equal to 0.90ρ . For lead, $Z/A = 0.40$, making ρ_e^w equal to 0.72ρ . The electron density of water compared with its mass density is higher than for most substances because of the hydrogen it contains. Hydrogen contains only one nucleon (a proton) for each electron, whereas for other elements there are at least 2 nucleons for each electron.

For a substance consisting of a mixture of elements, the relative electron density is given by

$$\rho_e^w = 1.80\rho \left(\frac{Z}{A} \right)_{eff} \quad (1-23)$$

where $(Z/A)_{eff}$ is the effective ratio of atomic number to atomic mass. For water, $(Z/A)_{eff} = 0.555$ (Huizenga and Storchi, 1985), so $\rho_e^w = \rho = 1.0$ as should be the case. For bone, $(Z/A)_{eff} < 0.555$, so $\rho_e^w < \rho$. Physically, as $(Z/A)_{eff}$ decreases, the number of electrons per nucleon decreases, causing a reduction in the ratio ρ_e^w/ρ .

It is the electron density that is the important quantity in determining the energy imparted and absorbed by the medium, since both the Compton attenuation coefficient and linear stopping power are approximately proportional to electron density. This means that the linear attenuation coefficient for both bone and soft tissue is closely approximated by multiplying the linear (or mass) attenuation coefficient for water by the electron density relative to water. Similarly, the linear stopping power can be obtained by multiplying the mass stopping power for water by the electron density relative to water. The same is not true for the linear scattering power however, as this quantity is dependent on the charge of the nucleus rather than on the electron density.

Relative electron density can be obtained from CT scans. The *CT number* N_{CT} at each pixel in the scan can be converted to relative electron density using the relationship (Metcalf and Beckham, 1988)

$$\rho_e^w = R_\gamma \frac{N_{CT}}{1000} + 1. \quad (1-24)$$

For soft tissue $R_\gamma = 1.0$. For bone, Metcalf and Beckham (1988) found R_γ to be 0.465. Physically, a reduction in R_γ means that the ratio of CT number to electron density has increased. This occurs for bone because of the increased attenuation of the CT scanner photon beam (peak energy 125 keV) due to photoelectric interactions, over that which occurs for water. This is in contrast to photons in the 10 MV beam where the attenuation is proportional to electron density.

1.11. Radiological Depth

At a point P in a medium, there is an equivalent depth in water at which the attenuation of the primary photon beam is the same. This equivalent depth is called the *radiological depth* and is found from the electron density along a ray-line joining the beam source and point P . If P is at a position $\mathbf{r}_p$ and the line from the source to P intersects the medium at $\mathbf{r}_0$, then the radiological depth at P is given by

$$d_p' = \int_{\mathbf{r}_0}^{\mathbf{r}_p} \rho_e^w(\mathbf{r}') d\mathbf{r}'. \quad (1-25)$$

The *physical depth*, d_p , is simply the distance from $\mathbf{r}_0$ to $\mathbf{r}_p$. Note that d_p'/d_p is the mean electron density between $\mathbf{r}_0$ and $\mathbf{r}_p$.

1.12. Lateral Electron Disequilibrium

Within an irradiated medium, secondary electrons travel both longitudinally and laterally away from the site of primary photon interaction — this is termed *electron ranging*. The non-zero range of these electrons can cause *electronic disequilibrium* to occur in some situations. Electronic equilibrium exists at a point if the dose at that point is equal to the collision kerma (Attix, 1978). If the secondary electrons originating at the point have a maximum range R , then this situation exists only when (Dutreix *et al*, 1964):

- (i) the photon fluence is uniform, and the attenuation of photons is negligible within a radius R of the point.
- (ii) the composition of the medium does not change within a radius R of the point.

Strictly, the first of these conditions is not satisfied at any point within a medium irradiated by photons of energy above about 1 MeV because of the long secondary electron range. However, as long as the point is not within a distance R of the edge of the medium or the beam boundary, a state of *transient equilibrium* exists. Note that this distance R increases with a decrease in medium density. When transient equilibrium does not exist because of the proximity of a beam boundary, a state of *lateral electron disequilibrium* exists.

The underlying reason for lateral electron disequilibrium can be explained by referring to figure 1.8, where a primary photon interaction site at a depth s and distance y from the boundary of a non-divergent beam is shown, along with the region in which secondary electron energy is deposited. The primary interaction site is represented by the point P , and the shaded area represents the region in which energy is deposited due to secondary electrons originating at P . A point Q at a distance t directly downstream from P is also shown. This point is closer to the beam edge than the maximum lateral displacement of secondary electrons having travelled a longitudinal distance t . Therefore, electrons originating at the beam edge at depth s , may also travel a lateral distance greater than y on reaching depth $s + t$. If the beam boundary shown was shifted to the left (the field size increased), there would be an increase in the number of secondary electrons originating at depth s which reach Q . This means that lateral electron disequilibrium exists at Q with respect to primary photon interactions at depth s . If primary photon interactions at all depths are considered, lateral electron equilibrium exists at a point Q at depth d and at a distance y from the beam edge, as long as y is greater than the maximum lateral displacement of secondary electrons reaching depth d . Thus, if d is larger than the maximum longitudinal electron range, then the requirement for lateral electron equilibrium at Q is that y is greater than the maximum lateral electron range.

At the central axis of a circular beam, electronic equilibrium exists for depths greater than the maximum longitudinal electron range, as long as the beam radius is greater than the maximum lateral electron range. At the central axis of the beam depicted in figure 1.9(a), lateral electron equilibrium exists because secondary electrons originating at the

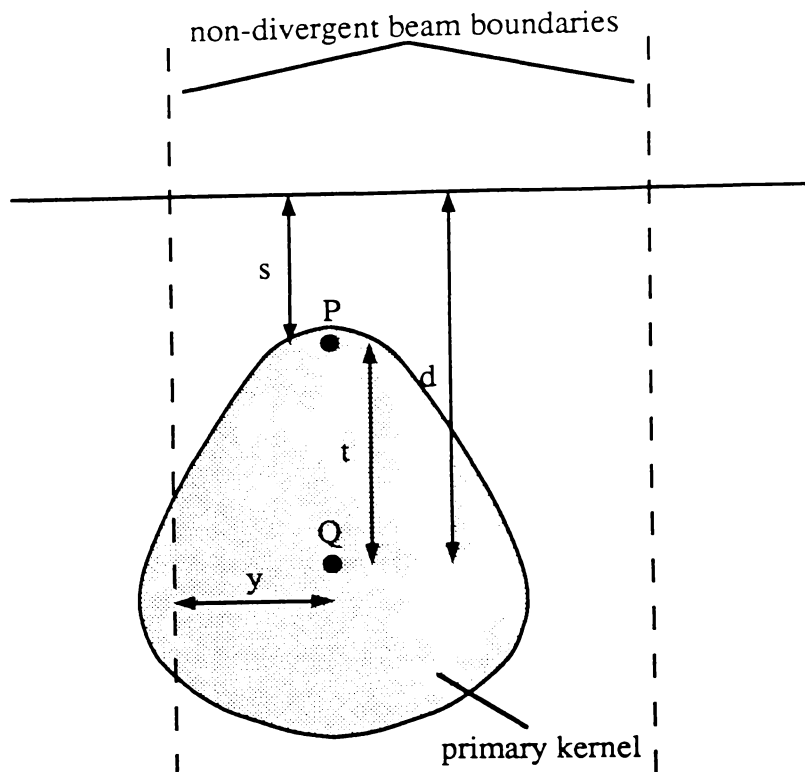


Figure 1.8. Diagram indicating lateral electron disequilibrium at point Q , at a distance y from the boundary of a non-divergent beam, due to primary photon interactions at depth s . The shaded area represents energy deposited by secondary electrons originating at point P directly above Q . Since this region extends outside the beam boundary, an increase in beam width would cause an increase in energy deposited at Q due to photon interactions at depth s : hence lateral electron disequilibrium exists at Q with respect to photon interactions at depth s .

beam boundaries do not reach the axis, hence a further increase in beam diameter would not cause an increase in primary dose. In figure 1.9(b), where the beam diameter is smaller, lateral electron equilibrium *does not* exist at the central axis because secondary electrons originating at the beam boundaries do reach the axis, hence an increase in beam diameter *would* cause an increase in primary dose. Note in figure 1.9(b) that near the surface, lateral electron equilibrium *does* exist because no secondary electrons reaching that depth have undergone a lateral displacement of greater than the beam radius. It is apparent that lateral electron disequilibrium exists at depth d on the central axis of a circular beam, if secondary electrons originating anywhere on the beam axis reach the beam edge at depth d .

1.13. Photon Beam Models

Before a patient with a tumour is treated using radiotherapy, the treatment procedure must be carefully planned. The objectives in planning a course of radiotherapy are to ensure that the target tumour is uniformly irradiated and receives the dose prescribed by the radiotherapist, and to minimise the dose delivered to surrounding healthy tissue.

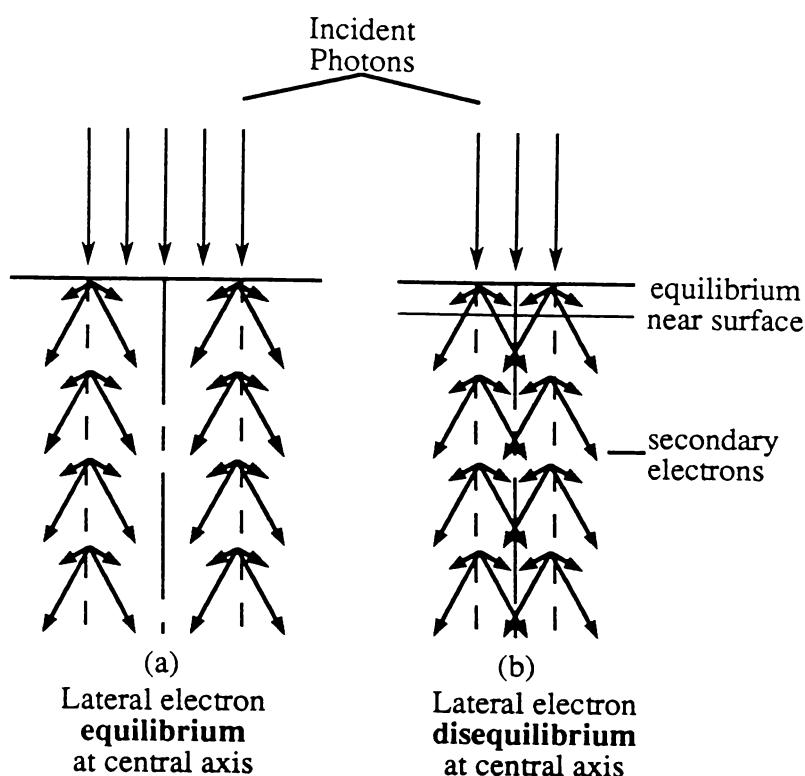


Figure 1.9. In (a), lateral electron equilibrium exists at central axis since the radius of the beam is larger than the maximum lateral range of the secondary electrons. It can be seen that electrons originating at the beam boundaries do not reach the central axis, hence a further increase in beam radius would not cause an increase in the central axis primary dose. In (b) lateral electron equilibrium does not exist at central axis since the beam radius is less than the maximum lateral electron range: an increase in beam radius would cause an increase in the central axis primary dose. At shallow depths, such as in the region above the horizontal line, lateral electron equilibrium does exist at central axis since electrons with a lateral range exceeding the beam radius do not reach this region.

An important aspect of modern radiotherapy treatment planning is accounting for internal density variations in predicting the dose distribution within the patient. This requires an accurate electron density map of the patient, and a dose calculation method which uses this map to obtain the dose distribution. Most algorithms can use data from CT scans to define the patient electron density distribution, as discussed in section 1.10. This data is either used directly during the calculation, or anatomical structures are identified and a mean electron density is assigned to each structure. If the data is used directly, the algorithm is referred to as a *pixel-by-pixel* method, otherwise the algorithm is a *bulk* method. In treatment planning systems where CT data is not employed, the bulk method is used and the electron density associated with each structure is entered manually. A medium whose density varies is referred to as an *inhomogeneous* medium. An *inhomogeneity*, is a region of different density to its surroundings.

Several different dose computation algorithms which account for density variations exist, where each uses a different calculation procedure. Generally, the accuracy of an algorithm increases with its complexity and the computation time required to produce a dose distribution.

Most photon beam dose calculation algorithms use tables of measured dose data such as depth doses, tissue-air ratios and beam profiles to generate dose distributions. The most common method of generating a dose distribution which accounts for inhomogeneous media is to modify a measured dose distribution in water. Other methods directly calculate the relative dose at each point. An outline of some of these methods is given in this section.

1.13.1. Effective SSD and Ratio of Tissue-air Ratios Methods

These methods use the radiological pathlength along a line from the beam source to the calculation point, to obtain a correction factor for that point. Tables of depth doses (Effective SSD) and TARs (Ratio of TARs) pertaining to the current field size are used. The position and shape of inhomogeneities are not taken into account in making the inhomogeneity correction.

The dose at a point P , $D(P)$, is determined by multiplying the dose in water at P , $D_w(P)$, by a correction factor CF . As with all correction methods, this factor approximates the ratio $D(P)/D_w(P)$. With the effective SSD method, this correction factor is given by (Cunningham, 1982)

$$CF = \frac{D(d_p', r)}{D(d_p, r)} \left[\frac{SSD + d_p'}{SSD + d_p} \right]^2 \quad (1-26)$$

where d_p is the physical depth of point P , d_p' is the radiological depth of point P , and r is the radius of the beam. $D(d_p, r)$ and $D(d_p', r)$ are the central axis depth dose values at these depths in water for a beam of radius r . An inverse square correction is included to account for the difference in the distance from the virtual source of P and the point at depth d_p' .

In the ratio of TARs method, tissue-air ratios are used instead of the depth dose values. In this case, the inverse square term is not required because the tissue-air ratios at depths d_p and d_p' are recorded at the same distance from the virtual source.

1.13.2. Power Law Tissue-air Ratio Method

The position as well as the density of each inhomogeneity along the line from the source to the calculation point is accounted for by this method. The shape of the inhomogeneities is still not accounted for — they are assumed to be infinite slabs whose surfaces are perpendicular to the beam axis.

The original power law method proposed by Batho (Batho, 1964) applied to two layer geometries, where a correction factor was determined at a point in tissue beneath a slab inhomogeneity. If the electron density of the inhomogeneity is ρ , then the dose at P is

obtained using the correction factor

$$CF = \left[\frac{TAR(x_2, r)}{TAR(x_1, r)} \right]^{1-\rho}, \quad (1-27)$$

where x_1 is the distance from P to the medium surface (the top surface of the inhomogeneity), and x_2 is the distance from P to the bottom surface of the inhomogeneity.

The technique has been extended (Webb and Fox, 1980) so that the correction factor at any point in a multi-layer geometry can be found. With this generalisation the correction factor is expressed a product of correction factors due to each overlying layer. If P is within the N th layer then

$$CF = \prod_{n=1}^N TAR(x_n, r)^{(\rho_n - \rho_{n-1})/\rho_0} \quad (1-28)$$

where x_n is the distance from P to the top surface of the n th layer, and ρ_0 and ρ_n are the electron densities of water and of layer n . This method yields satisfactory results for many geometries (Sontag and Cunningham, 1977), but it has been found that thick inhomogeneities and large fields can give rise to significant errors (Sontag and Cunningham, 1977; Cunningham, 1982).

1.13.3. Equivalent Tissue-air Ratio (ETAR) Method

In calculating the correction factor for a point, this method determines the equivalent depth and field size in a water medium which would give the same dose to that point. The equivalent depth is the radiological depth to the point, and the equivalent field size is calculated by considering the amount of scattered radiation reaching the calculation point from the entire irradiated volume corresponding to this equivalent depth and field size. This algorithm takes account of the position and shape of inhomogeneities, and is the most accurate of the correction methods (Sontag and Cunningham, 1978). This method is known as a *scatter function model* because it considers the primary and scattered dose components separately. For a point P in an inhomogeneous medium, the correction factor is

$$CF = \frac{TAR(d_p', \tilde{r})}{TAR(d_p, r)}, \quad (1-29)$$

where $\tilde{r}$ is the equivalent beam radius. This equivalent radius is the product of r and a weighted average electron density $\tilde{\rho}$. The numerator of the correction factor is found using the zero-area TAR for depth d_p' , and the SAR for depth d_p' and beam radius $\tilde{r}$:

$$TAR(d_p', \tilde{r}) = TAR(d_p', 0) + SAR(d_p', \tilde{r}). \quad (1-30)$$

The major task of the ETAR algorithm is to determine $\tilde{\rho}$. In the averaging process, each density element in the medium, ρ_{ijk} , is weighted by a factor W_{ijk} which depends on the contribution of that element to the scattered dose at P . In practice, the 3-D density array is first coalesced into an equivalent 2-D array to reduce computation time. This 2-D array

contains density elements $\bar{\rho}_{ij}$ and is considered to be at an effective distance Z_{eff} away from the plane of calculation, where Z_{eff} is found using scatter-air ratio data (Sontag and Cunningham, 1978). The effective density is then found from a weighted average over the 2-D slice where the weighting for $\bar{\rho}_{ij}$ is $W_{ij}(Z_{eff})$.

This method has been found to be more accurate than the simpler correction methods for a variety of situations (Sontag and Cunningham, 1978). Of interest for this work is that for small fields, dose in a low density medium such as lung can be overestimated by the ETAR method (Mackie, 1985; Metcalfe, 1990). The cause of this overestimate is that electronic equilibrium is assumed to exist at all points, and as is discussed in chapters 2 and 4, a state of lateral electron disequilibrium often occurs in lung. The reason why the assumption of electronic equilibrium causes an error in the calculations when disequilibrium exists, is discussed in chapter 4.

1.13.4. Superposition and Convolution Methods

If a dose calculation algorithm is to be accurate for all configurations, it must use a physically rigorous approach in determining the distribution of absorbed dose. One such method is the superposition algorithm, which explicitly considers the dose distribution resulting from primary photon interactions in each volume element (voxel) in the irradiated medium. The energy removed from the primary photon beam in each voxel is given by the term in that voxel (see section 1.3.6). This energy then exists as the kinetic energy of secondary electrons and scattered photons, and is deposited throughout the medium in a distribution described by the *energy deposition kernel* (EDK). The name ‘superposition’ comes from the concept of overlaying EDKs to obtain the total dose distribution. The superposition method *does not* assume the existence of electronic equilibrium. As well as accounting for the position and shape of inhomogeneities, this method accounts for the way in which the transport of scattered photons and secondary electrons is modified by inhomogeneities.

Several researchers have developed methods for dose calculation in inhomogeneous media using variations on this method (Mackie *et al*, 1985; Mohan *et al*, 1986; Ahnesjö, 1989). A superposition algorithm has been developed here and applied to lung dose calculation for small 10 MV beams, as described in chapter 4. The superposition algorithm has also been installed on a comprehensive treatment planning system running on a Sun workstation, as described in chapter 5.

For dose calculation in an inhomogeneous medium, an EDK which varies dynamically throughout the calculation is required. If the same EDK is used throughout the calculation, the kernel is said to be *invariant* and the superposition reduces to a *convolution* (Boyer and Mok, 1985). The convolution method is applied to dose calculation in a water medium for beams of circular cross section in chapter 3.

1.13.5. Monte Carlo Simulation

The most rigorous method of dose calculation is to follow the histories of individual photons

and electrons using Monte Carlo modelling (Jenkins *et al*, 1988). Electronic disequilibrium is inherently accounted for by this method since energy is deposited along the physically realistic path followed by each secondary electron. The positions at which photons interact are determined using a probability distribution based on the mean free path of the photon, and the type of interaction which occurs is determined from the relative probabilities of each interaction type (Murray, 1990). The selection of these positions and interaction types is made using random numbers — this is similar to rolling a dice, hence the name ‘Monte Carlo’.

The user can define an array of voxels, where the energy deposited in each is recorded. Energy is predominantly deposited in *electron transport steps*. These are short steps with a defined position, length, and direction, where an energy equal to the product of the step length and the linear collision stopping power is ‘scored’ in the voxel containing the step. In general, the length of an electron step is short in comparison with the voxel dimensions. The change in direction in each successive step is determined using a probability distribution based on *multiple scattering* (Berger and Wang, 1988). An electron is transported in these steps until its energy falls below a preset cutoff value.

Unfortunately, the computation time required to produce a dose distribution using the Monte Carlo method currently prohibits its use in treatment planning systems. This method of dose calculation is however an extremely useful tool for generating realistic dose distributions for comparison with other (faster) methods and has been used extensively in the work covered in this thesis. In chapter 2, the EGS4 Monte Carlo system (Nelson *et al*, 1985) is used to investigate lateral electron disequilibrium for a range of small 10 MV beams of circular cross-section. Monte Carlo calculations are also used as a comparison with convolution/superposition results in chapters 3 and 4.

Chapter 2

Monte Carlo Dose Calculation for Small Circular Photon Beams

2.1. Introduction

In chapter 1, the incidence of lateral electron disequilibrium in lung and the consequent error in dose calculation which this can cause was mentioned. This type of disequilibrium can also exist in a unit density medium if the field size is small enough. In this chapter, the effect of lateral electron disequilibrium for a non-divergent 10 MV beam, in both unit density and low density material, is investigated using the EGS4 Monte Carlo system (Nelson *et al*, 1985). Simulations were performed using the primary photon energy spectrum in figure 1.1. Beams of circular cross section were simulated because of the radial symmetry which exists about the beam central axis, and because small circular beams are used in *stereotactic radiosurgery* (Pike *et al*, 1987; Kubsad *et al*, 1990).

In radiosurgery, small circular photon beams are used to treat brain tumours. The beams used in this procedure are of a small enough diameter for lateral electron disequilibrium to exist on the central axis of the beam. It is important that the dose rate and shape of the depth dose curve is known accurately for each beam diameter used, in order to plan the radiosurgery procedure. Central axis dose for radiosurgery beams with a range of diameters was obtained using EGS4 and is compared here with measurement.

In general, Monte Carlo simulations used to calculate dose require many hours of computing time to produce results with an acceptably low uncertainty. The lower the uncertainty in the quantity being recorded, the more reproducible is that quantity in successive simulations. Methods have been devised by which to minimise the uncertainty for a given amount of computing time, thereby minimising the time required to produce results with the required uncertainty. These methods are known as *variance reduction* techniques (Bielajew and Rogers, 1988) — the variance being the square of the uncertainty. The effectiveness of a particular variance reduction technique is measured by its *efficiency*. An extremely useful variance reduction technique, used in finding the dose on the central axis of a non-divergent circular beam, is the *reciprocity technique*, which is based on the geometry-equivalence or reciprocity theorem (Bielajew and Rogers, 1988). The efficiency of this technique in calculating the central axis dose for a 2 cm diameter, 10 MV photon beam is investigated.

The reciprocity technique has been modified so that the central axis dose for divergent beams, such as those used in radiosurgery, can be calculated. This has required minor

modifications to the user code. These modifications are described, and the effectiveness of the technique is investigated.

The Monte Carlo user code employed in these simulations is `RTPCYL.MOR` (Murray, 1990). This code is based on `INHOM.MOR` (Rogers, 1984), and uses the `PRESTA` algorithm for electron step control (Bielajew and Rogers, 1987).

2.2. Efficiency of a Monte Carlo Calculation

The efficiency of a particular simulation is inversely proportional to both the variance of the results and to the time taken for the simulation (Bielajew and Rogers, 1988). If the uncertainty of a result is s and the computing time used in the simulation is T , then the efficiency ϵ is defined as

$$\epsilon = \frac{1}{s^2 T} \quad (2-1)$$

where the variance is s^2 . In simulating a particular situation, the variance is inversely proportional to the computation time for a particular type of simulation (Bielajew and Rogers, 1988). The efficiency obtained using (2-1) is therefore invariant with simulation time for a particular simulation type.

The variance of a result is estimated in an EGS4 simulation as follows. The number of incident particles simulated, `NCASE`, is specified by the user. These `NCASE` particles are simulated in `STAT` separate *batches*, where in `RTPCYL`, `STAT` = 10. Each batch includes the histories of `NCASE/STAT` incident particles. In these simulations, energy deposition is recorded in each voxel of an array representing the irradiated medium, where there is a separate array for each batch. The energy deposited in a particular voxel due to the particles simulated in batch i is E_i . A further array exists to store the mean energy $\bar{E}$ for each voxel. This mean energy is computed when the particle simulation is complete. The energy distribution over the batches is normal with a standard deviation given by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{STAT} (E_i - \bar{E})^2}{STAT}}. \quad (2-2)$$

If the simulation were repeated many times then $\bar{E}$ would also be distributed normally with a standard deviation in the mean

$$\sigma_m = \sqrt{\frac{\sigma^2}{STAT - 1}}. \quad (2-3)$$

This is the value assigned to the uncertainty s . The array containing the mean energy in each voxel is used to produce the appropriate output, which in this case is the dose in each voxel. A further array stores the uncertainty for each voxel, which may be output along with the dose.

If simulations performed with and without the use of a variance reduction technique have efficiencies ϵ_1 and ϵ_0 respectively, then the gain (or loss) in efficiency brought about by the

technique is ϵ_1/ϵ_0 . Using (2-1), the efficiency gain in terms of the variances s_1^2 and s_0^2 , and simulation times T_1 and T_0 , is given by

$$\frac{\epsilon_1}{\epsilon_0} = \frac{s_0^2 T_0}{s_1^2 T_1}. \quad (2-4)$$

In the following work, this expression will be used to quantify the efficiency gained by using the reciprocity technique.

2.3. Monte Carlo Simulation using the Reciprocity Technique

2.3.1. Principles of the Reciprocity Technique

When the dose on the central axis of a non-divergent circular beam is to be calculated, the obvious approach is to distribute incident photons uniformly within the beam boundaries, and to record the energy deposited in narrow *detection voxels* centred on the beam axis. So that the dose calculated is truly central axis dose, the detection voxels should be of a small enough diameter so that the dose does not decrease significantly from the axis to the voxel boundary. This type of simulation will be referred to as a *full simulation*.

During a full simulation, most of the computing time is spent calculating the trajectories of particles which do not contribute any dose to the detection voxels. It is this 'waste' of computing time that is remedied using the reciprocity technique, where a modified beam and detection voxel configuration is used. With this technique, the same energy is deposited in detection voxels at each depth as when a full simulation is performed, but with greatly reduced number of incident particles. A simulation which uses the reciprocity technique will be referred to as a *reciprocity simulation*.

In figure 2.1(a), a uniform photon fluence Φ incident on a water medium within a beam of radius 1.0 cm is illustrated. An energy E is deposited in a detection voxel of thickness 0.5 cm and radius 0.1 cm on the beam central axis. Note that the thickness refers to the dimension of the detection voxel in the direction of the beam. By the reciprocity theorem, if there is a fluence Φ incident within a beam of radius 0.1 cm, the energy deposited within a detection region of thickness 0.5 cm and radius 1.0 cm will also be E as shown in figure 2.1(b). In this case, the same energy is deposited using the reciprocity technique but with 1/100th the number of incident photons since the ratio of the cross-sectional areas of the beams used in the reciprocity and full simulations is 1/100.

2.3.2. Efficiency Gained by Using the Reciprocity Technique

An estimate of the efficiency gained by using the reciprocity technique for this situation can be obtained by assuming that if the energy recorded by the two methods is the same, then the variance is the same. Also assume that the computing time required per incident photon is the same for both methods. The gain in efficiency is then 100 using (2-4).

The assumption of an equal variance for an equal energy is not necessarily valid, since this is based on the assumption that the energy deposition recorded in both methods is due

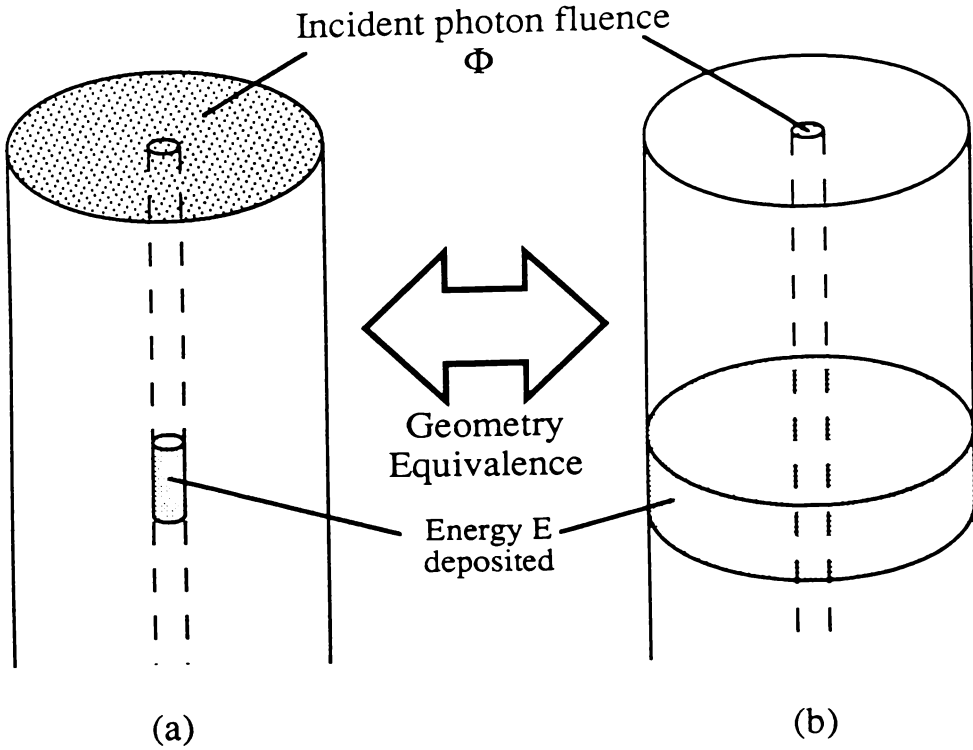


Figure 2.1. (a) Non-divergent beam of radius 1.0 cm, with incident photon fluence Φ . An energy E is deposited in a cylindrical detection voxel of radius 0.1 cm on the central axis using a full simulation. (b) Non-divergent beam of radius 0.1 cm, with incident photon fluence is Φ . The same energy E as in (a) is deposited in a cylindrical detection voxel of radius 1.0 cm using a reciprocity simulation.

to an equal number of independent secondary electrons. If the energy deposited in one method is due to more electron tracks, then that method will give a lower variance since the result is due to a greater number of independent energy deposition events. The actual efficiency gain for the situation depicted in figure 2.1 is investigated here by comparison of the variances of the results obtained in simulations performed with and without the use of the reciprocity technique.

For this comparison, simulations performed using the reciprocity technique used $1/100$ th the number of incident photons that were used in the full simulation, so that equal energies are deposited in the detection regions. This is not necessary but it adds to the 'symmetry' of the problem. In figure 2.2(a) and 2.2(b), central axis dose in 60 depth intervals, each of thickness 0.5 cm, obtained using full and reciprocity type simulations is shown. 500000 and 5000 incident photons were used in the full and reciprocity simulations respectively. The depth values in the graphs are the depths in cm at the centre of each detection voxel. The interaction voxel indices, k , run from 0 to 59, and the depth at the centre of voxel k is denoted by d_k . The values given for the dose are the number of pico-Grays (pGy) per unit incident energy fluence. The units for this quantity are $\text{pGy MeV}^{-1} \text{ cm}^2$. Relevant user

specified parameters for these simulations are given in table 2.1, where **RBEAM** is the radius of the incident beam and **RBOUND(2)** is the radius of the detection voxels.

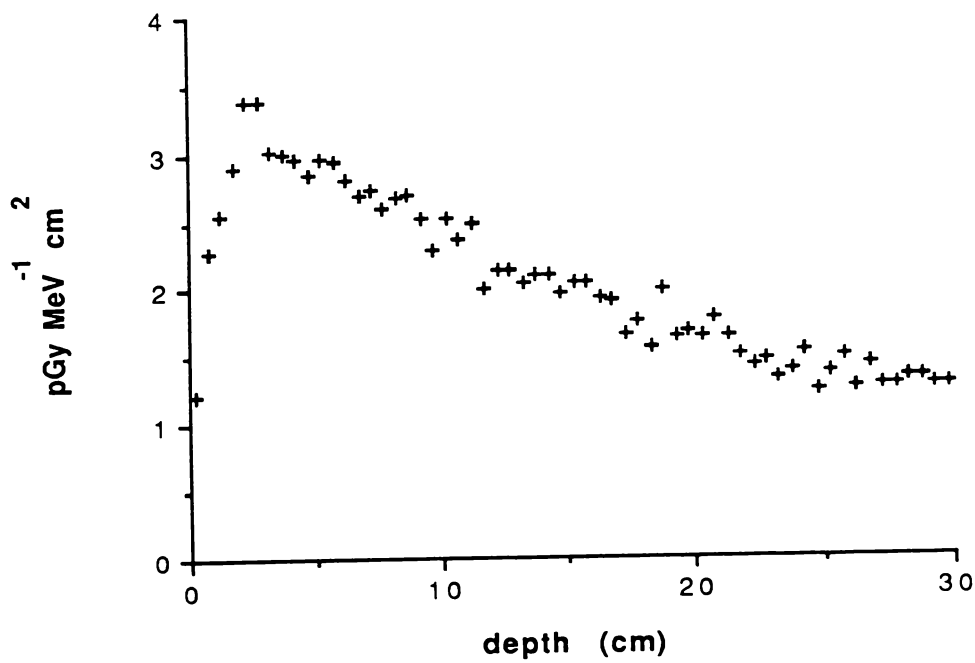
Both sets of results in figure 2.2 have a high uncertainty as evidenced by the unsmoothness of the data points. To halve the uncertainty (divide the variance by four) in each of these results, the number of incident photons simulated must be quadrupled. Note that this also quadruples the computation times. Results of simulations using 2000000 and 20000 photons for the two techniques are shown in figure 2.3(a) and 2.3(b). To further halve the uncertainty, the incident photon number must be quadrupled again to 8000000 and 80000, which gives the results shown in figure 2.4(a) and 2.4(b).

	full	reciprocity
NCASE	500000	5000
RBEAM	1.0	0.1
RBOUND(2)	0.1	1.0

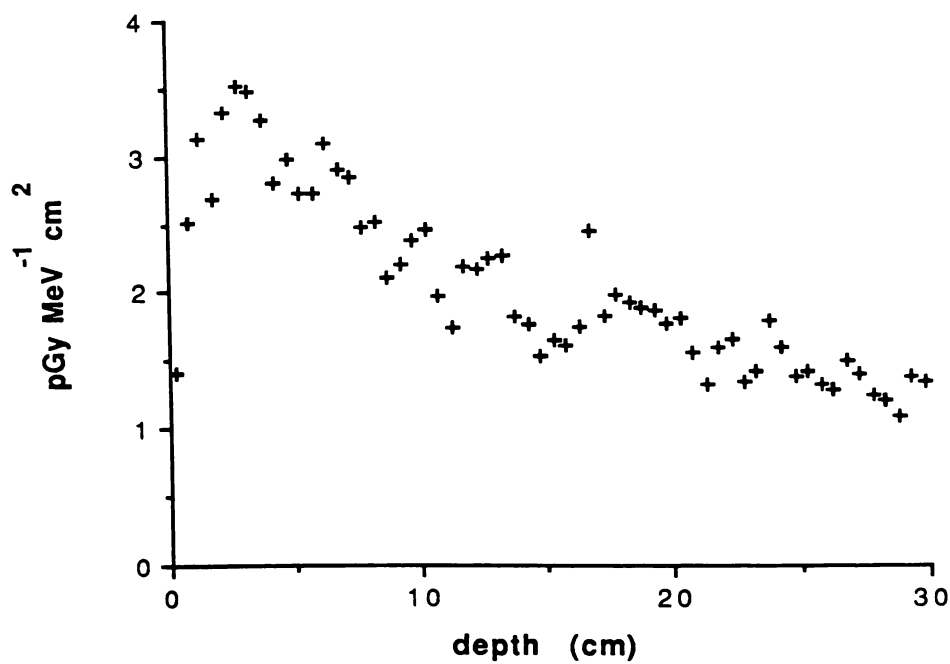
Table 2.1. User specified parameters for full and reciprocity type simulations of a 2.0 cm diameter beam, when 500000 and 5000 incident photons were used for the respective simulations. Computation times for the full and reciprocity simulation were 7060.2 seconds and 65.0 seconds respectively, on a VAX 6000-430. **RBEAM** is the radius to which the beam is collimated at the surface; **RBOUND(2)** is the radius of the detection region on the central axis. Note that **RBOUND(1)** = 0.0.

It is obvious that the uncertainty of the reciprocity simulation data is higher than that of the full simulation data for each pair of graphs, indicating that the assumption of equal variance for equal energy as suggested above is incorrect in this case. The uncertainty associated with each value as a percentage of that value is given in the results of the simulation and may be squared to obtain the variances for each depth, $s_0^2(k)$ and $s_1^2(k)$. In figure 2.5(a), the variance is plotted for each calculation depth for both techniques (scattered points), when 500000 and 5000 incident photons are used. There is a large variation in the uncertainty for each depth, hence an even larger variation in the variance. It can be seen that the variance of the dose calculated using the reciprocity technique is normally the highest, as expected from figure 2.2(a) and 2.2(b). A more informative comparison of the variances is obtained by taking an average of the variances down to and including each depth, $\overline{s_0^2(k)}$ and $\overline{s_1^2(k)}$. As seen in figure 2.5(a), these plots are quite smooth, with a slight increase with depth. This increase is expected since as the dose decreases with depth, less electron tracks contribute to the dose thereby increasing the uncertainty.

To evaluate the gain in efficiency using (2-4), the ratio of the variance for the full simulation to the variance when the reciprocity technique is used is required. In figure 2.5(b), $(s_0^2(k)/s_1^2(k))$ is plotted and is seen to have a large variation. An average of this ratio down to each depth, $(\overline{s_0^2(k)}/\overline{s_1^2(k)})$, is also plotted and is seen to have a very small gradient. A



(a)



(b)

Figure 2.2 (a) Depth dose calculated using a full simulation as depicted in figure 2.1(a), 500000 incident photons were simulated which took 7060.2 seconds on a VAX 6000-430. (b) Depth dose using a reciprocity simulation as depicted in figure 2.1(b), 5000 incident photons were simulated which took 65.0 seconds on a VAX 6000-430.

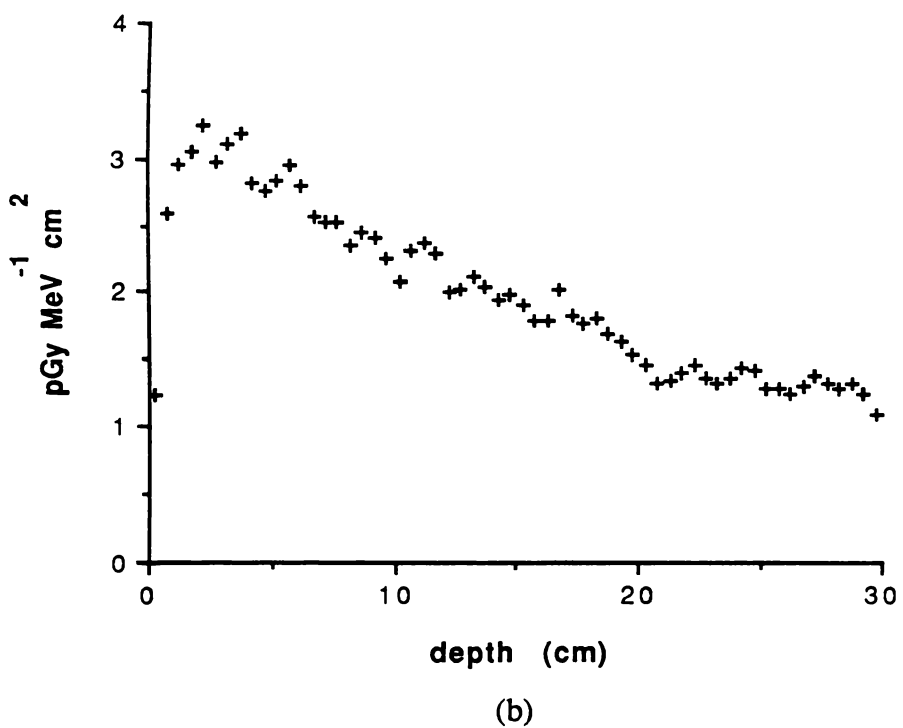
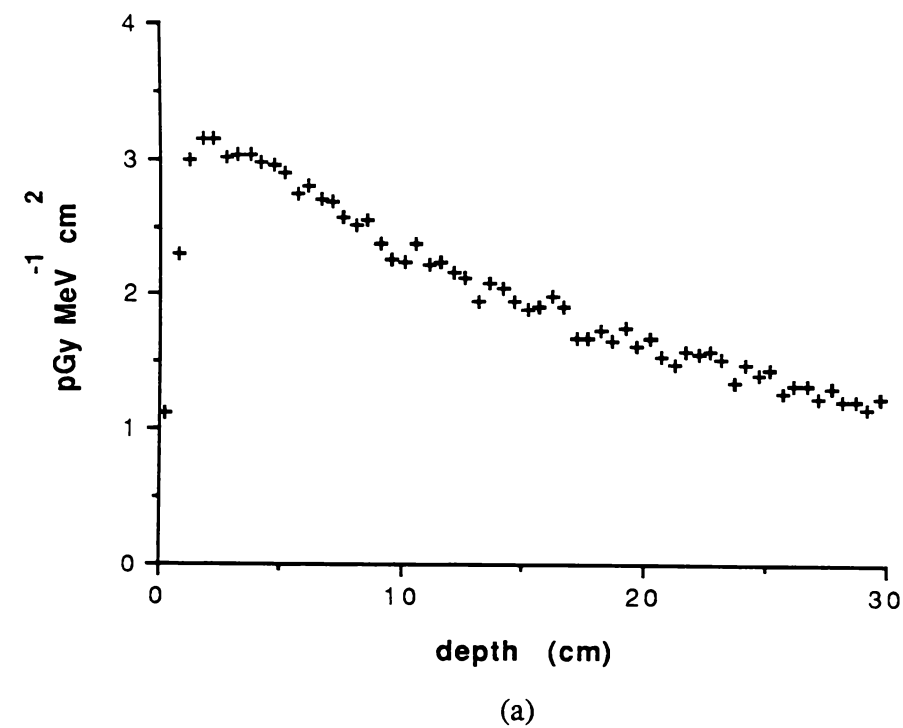
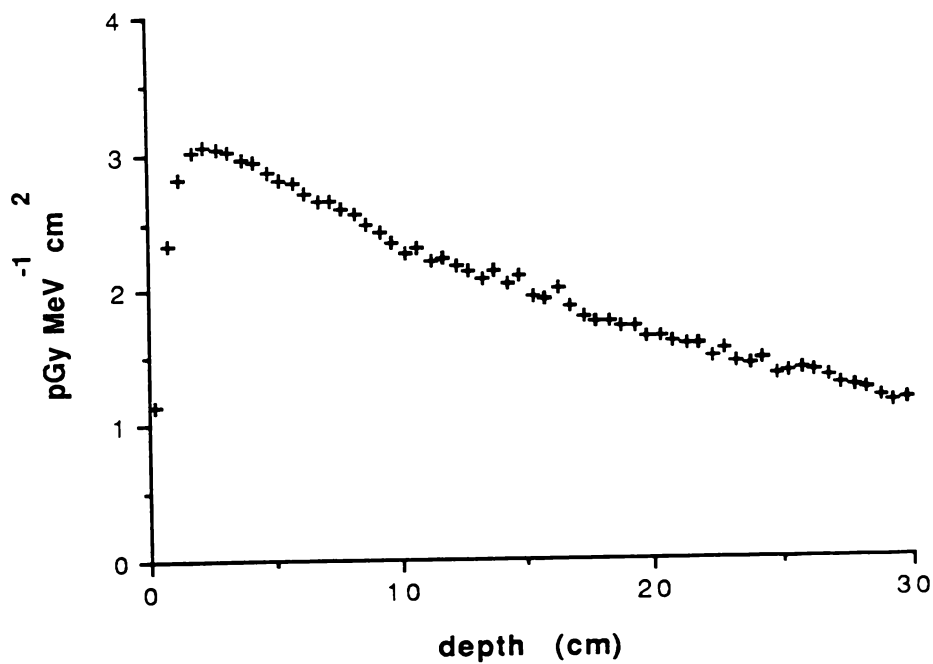
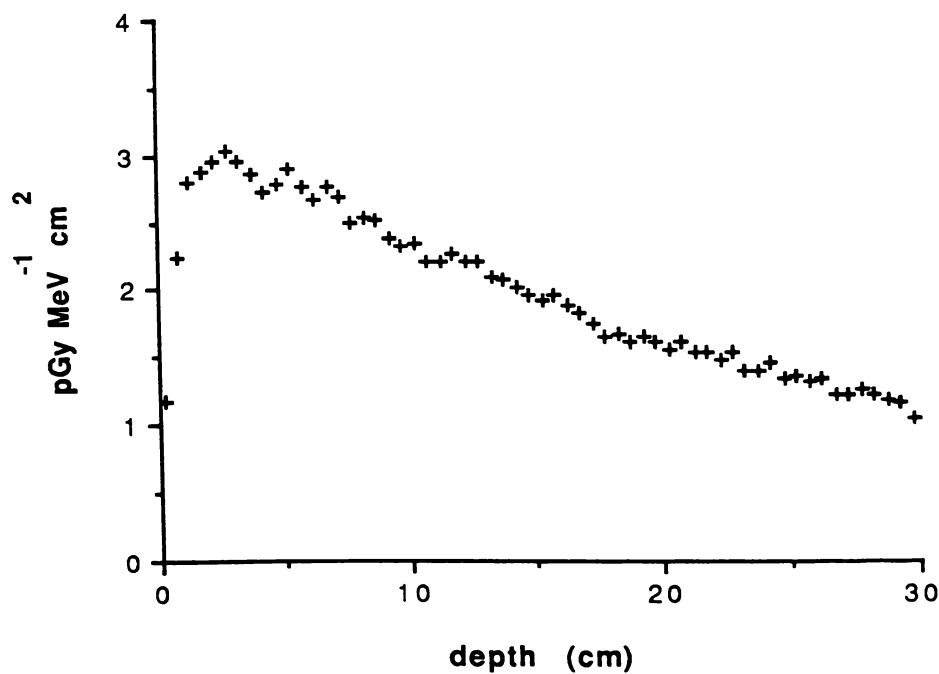


Figure 2.3. (a) Depth dose calculated using a full simulation as depicted in figure 2.1(a), 2000000 incident photons were simulated. (b) Depth dose calculated using a reciprocity simulation as depicted in figure 2.1(b), 20000 incident photons were simulated.

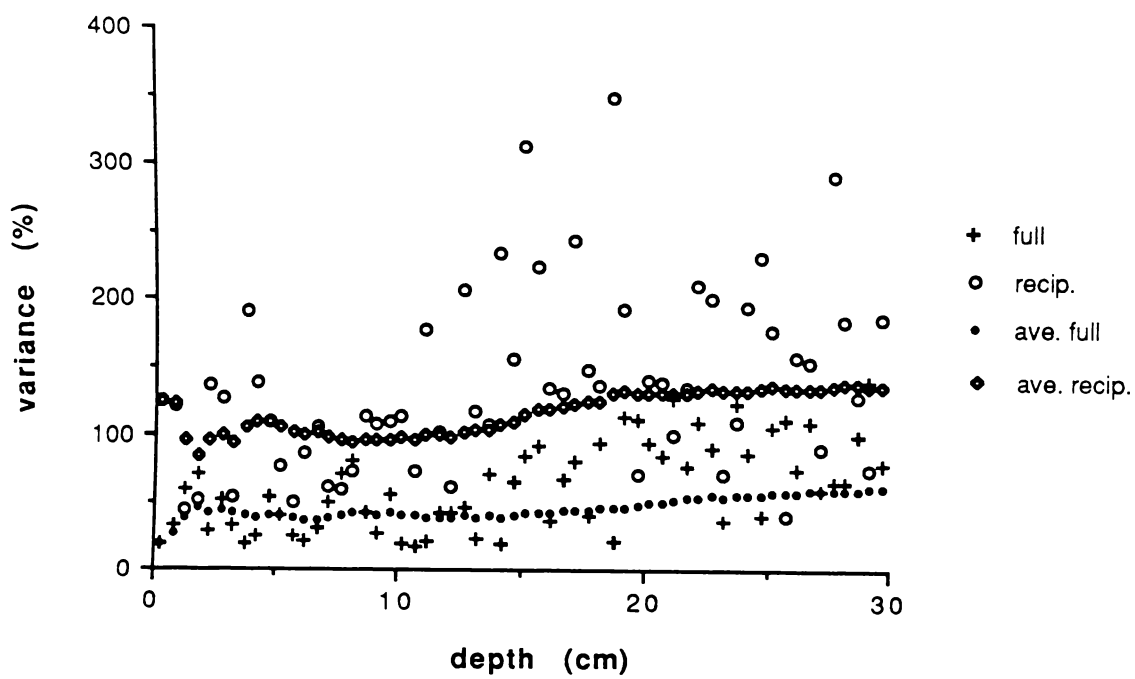


(a)

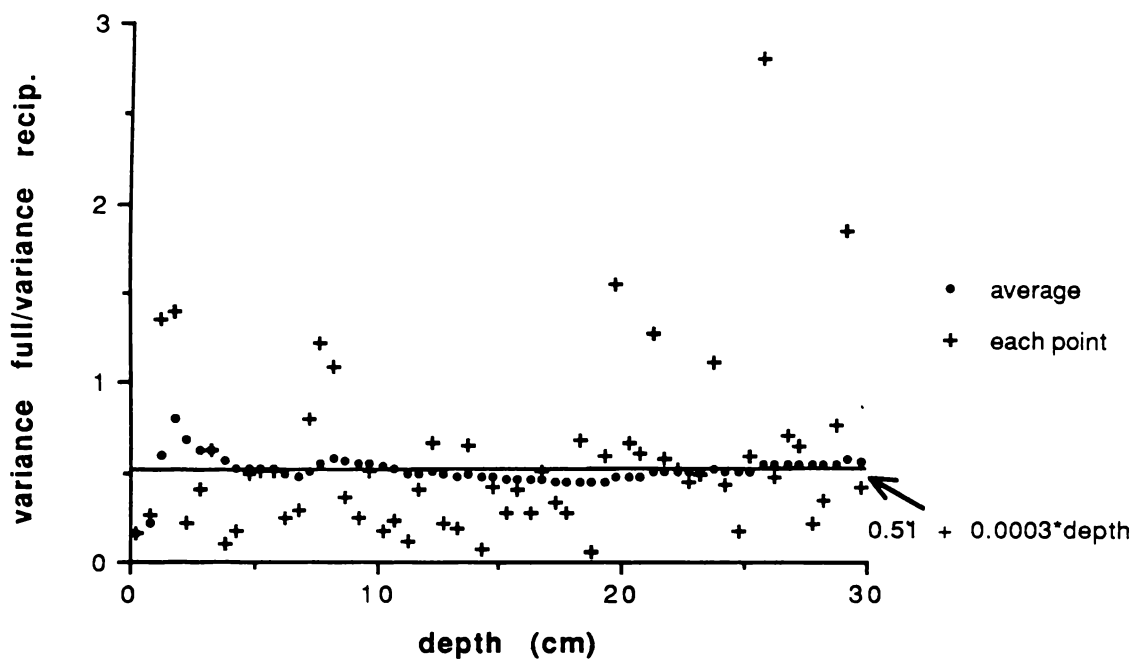


(b)

Figure 2.4. (a) Depth dose calculated using a full simulation as depicted in figure 2.1(a), 8000000 incident photons were simulated. (b) Depth dose calculated using a reciprocity simulation as depicted in figure 2.1(b), 80000 incident photons were simulated.



(a)



(b)

Figure 2.5. (a) Variance in the dose at each depth for the full and reciprocity type simulations when 500000 and 5000 incident photons are used (full and recip.). Detection voxels are 0.5 cm thick and have radii of 0.1 cm for the full simulation and 1.0 cm for the reciprocity simulation. Also shown is the mean variance down to each depth (ave. full and ave. recip.). (b) Ratio of the variance of the full simulation to that of the reciprocity simulation at each depth (each point), and the mean ratio down to each depth (average). The mean ratio is 0.52 at 30 cm deep.

line of best fit through these points has equation

$$\overline{s_0^2(k)/s_1^2(k)} = 0.51 + 0.0003d_k. \quad (2-5)$$

To calculate the efficiency gain, a value for the variance ratio is required. This value is taken from the line of best fit at 30 cm deep, which from (2-5) is 0.52. The ratio of the simulation times, T_0/T_1 , is also required. The full simulation of 500000 photons took 7060.2 seconds and the simulation using the reciprocity technique with 5000 photons took 65.0 seconds, where these times were obtained from an average over 5 runs of each type. This gives a simulation time ratio of

$$\frac{T_0}{T_1} = 108.6 \quad (2-6)$$

Both simulations were performed on a VAX 6000-430. Since 100 times the number of incident photons were used, the computation time per photon for the full simulation is 1.09 times that of the reciprocity simulation. Substituting the values for the variance ratio (0.52) and simulation time ratio (108.6) into (2-4) gives an efficiency gain of

$$\frac{\epsilon_1}{\epsilon_0} = 56.5. \quad (2-7)$$

The efficiency gain is the ratio of the simulation time required for a full simulation to that required for a simulation using the reciprocity technique, where the variance of the results obtained using both methods is the same. This ratio is denoted by

$$\left(\frac{T_0}{T_1}\right)_{s^2} = \frac{\epsilon_1}{\epsilon_0}. \quad (2-8)$$

Note that this is also the ratio of the variance using the full simulation to that when the reciprocity technique is used, when the same simulation time is used for both methods. The ratio of the number of incident photons required to give equal variances is found using (2-8) and the number of histories simulated per second, $(\frac{\Delta N}{\Delta t})$:

$$\left(\frac{N_0}{N_1}\right)_{s^2} = \left(\frac{T_0}{T_1}\right)_{s^2} \frac{(\frac{\Delta N}{\Delta t})_0}{(\frac{\Delta N}{\Delta t})_1} \quad (2-9)$$

where

$$\left(\frac{\Delta N}{\Delta t}\right)_0 = \frac{500000}{7060.2} = 70.8 \quad (2-10)$$

and

$$\left(\frac{\Delta N}{\Delta t}\right)_1 = \frac{5000}{65.0} = 76.9. \quad (2-11)$$

Therefore, since $(T_0/T_1)_{s^2} = 56.5$,

$$\left(\frac{N_0}{N_1}\right)_{s^2} = 52.0. \quad (2-12)$$

Note that this result could have also been obtained directly from the fact that the variance using the full simulation is 0.52 times that using the reciprocity technique, when 100 times

the number of incident photons are simulated in the full simulation. The result of the reciprocity simulation has $1.0/0.52$ times the variance of the full simulation, therefore since the variance is inversely proportional to the incident photon number for a particular simulation type, the number of photons in the full simulation would have to be decreased by a factor of 0.52 in order to give the same variance as the reciprocity simulation ($0.52 \times 100 = 52$).

The appearance of the points plotted in figure 2.4(a) and 2.4(b) indicates that using 8000000 and 80000 incident photons for the two methods does not yield a smooth depth dose curve in either case. Computation times for these two simulations were approximately 40 hours and 21 minutes respectively. Note that a full simulation taking approximately 21×57 minutes (20 hours) is required to obtain results with the same variance as those obtained when the reciprocity technique is used.

A simulation using the reciprocity technique with 1280000 incident photons was performed, reducing the variance by a factor of 16 over the variance of the results obtained with 80000 photons. These results, which took approximately 5.5 hours to produce, give a relatively smooth curve as shown by the plot in figure 2.6. To obtain results with the same variance using a full simulation would require a computing time of over 300 hours!

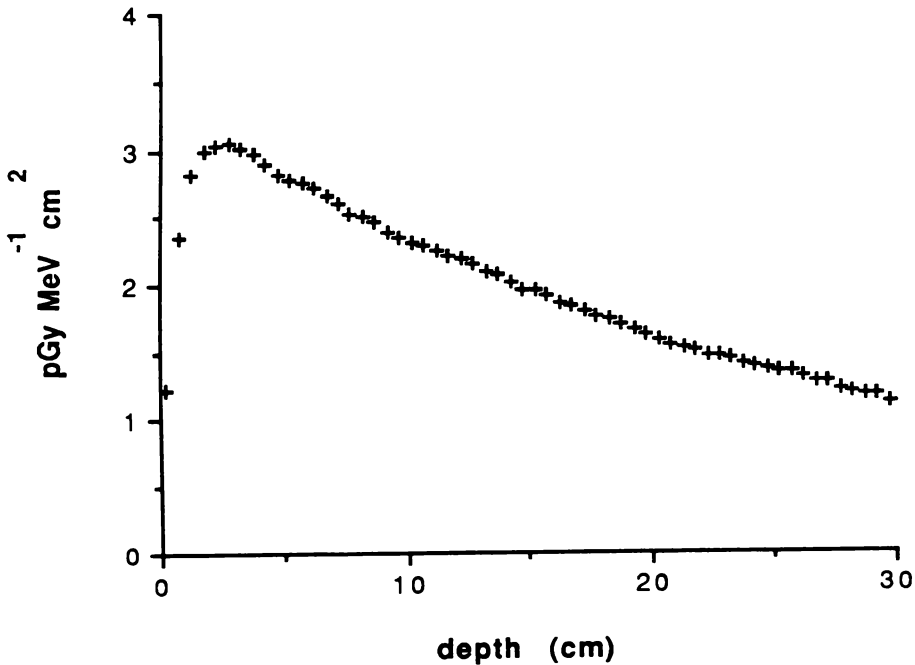


Figure 2.6. Depth dose calculated using a reciprocity simulation. 1280000 incident photons were simulated which took approximately 5.5 hours. To obtain results with the same variance using a full simulation would require over 300 hours.

2.3.2.1. Dependence of Efficiency Gain on Detection Voxel Thickness

In the configuration discussed so far, the detection voxels for the full simulation are 0.5 cm thick and have a diameter of 0.2 cm (see figure 2.1(a)). Energy is deposited in these regions

by secondary electrons which either (i) enter the cylinder and pass out again before stopping; (ii) enter the cylinder and stop within it; (iii) originate within the cylinder and pass out of it; or (iv) originate within the cylinder and stop within it. In a Monte Carlo simulation, the amount of energy deposited in a detection region by an electron is proportional to the pathlength of that electron within the region. The pathlength of an electron within a detection region in the full simulation will normally not exceed 0.5 cm, since this is the maximum dimension of the detection voxels, although pathlengths greater than this are possible also. The mean pathlength will be considerably less than 0.5 cm because a considerable proportion of the secondary electrons will pass through the sides of the cylindrical voxels.

The detection voxels for a simulation using the reciprocity technique are 0.5 cm long with a diameter of 2.0 cm (see figure 2.1(b)). The mean pathlength of electrons depositing energy within these regions will be longer than that for the detection regions of small radius used in the full simulation, since electrons do not enter and exit the detection regions through the side walls as is the case in the full simulation.

The total energy deposited within the respective regions is approximately equal by the reciprocity theorem, therefore since the mean electron pathlength in these regions is less for the full simulation, the number of electron tracks responsible for the energy deposited must be greater. Since each electron track is independent, the uncertainty associated with the total energy deposited will decrease as the number of tracks contributing increases. This is why there is less uncertainty in the results generated using the full simulation than in the results generated using the reciprocity technique, even though the energy deposited is the same. This phenomenon is illustrated in figure 2.7, where a side view of the detection regions is shown. By symmetry, the energy deposited in a small volume QF1 due to primary photon interactions at PF1 in the full simulation is equal to the energy deposited in a small volume QR1 due to primary photon interactions at PR1 in the reciprocity simulation. Also, the energy deposited in a small volume QF2 due to primary photon interactions at PF2 in the full simulation is equal to the energy deposited in a small volume QR2 due to primary photon interactions at PR1 in the reciprocity simulation, where QR2 is situated on an extension of the line joining PR1 and QR1. First, the pathlength of the electron tracks in the detection region can be seen to be shorter for the full simulation. Another important observation is that the energy deposited in QR1 and QR2 due to PR1 in the reciprocity simulation is due to the *same* electron track, whereas the energy deposited in QF1 and QF2 in the full simulation is due to *different* and therefore independent tracks, originating from PF1 and PF2 respectively. This illustrates how more separate electron tracks contribute to the dose recorded in the full simulation than in the reciprocity simulation, even though the energy recorded is the same.

This explanation accounts for the difference in uncertainty in the full and reciprocity simulations when the cylindrical detection regions used in the full simulation are long and

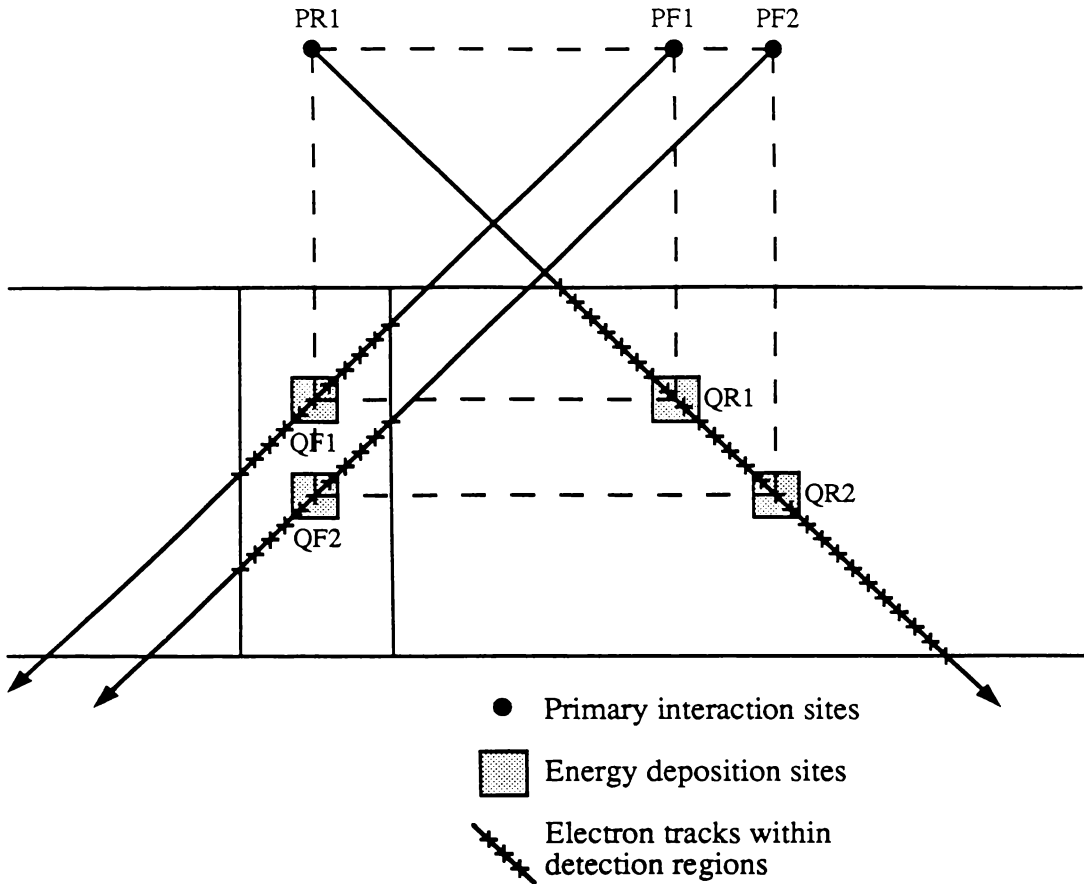


Figure 2.7. Side view of detection regions used in full and reciprocity type simulations (shown overlaid). By symmetry, the energy deposited in a small volume QF1 due to primary photon interactions at PF1 in the full simulation is equal to the energy deposited in a small volume QR1 due to primary photon interactions at PR1 in the reciprocity simulation. Also, the energy deposited in a small volume QF2 due to primary photon interactions at PF2 in the full simulation is equal to the energy deposited in a small volume QR2 due to primary photon interactions at PR1 in the reciprocity simulation, where QR2 is situated on an extension of the line joining PR1 and QR1.

narrow. If the detection regions for both simulation types were not as thick, there would be a smaller difference in the mean electron pathlength in the respective regions. This is because a smaller proportion of the electrons depositing energy in the full simulation detection voxels would then enter and/or exit the voxels through the side walls. This should result in a smaller difference in the uncertainty of the results when an equivalent energy is deposited. To check this reasoning, simulations were performed with detection regions of the same radius as used previously (0.1 cm), but of thickness 0.2 cm. The number of incident photons used was 1 million and 10000 for the full and reciprocity simulations respectively. Note that so that the energy deposited in the detection regions is approximately the same as with the previous geometry, twice as many photons were used since the volume of the detection regions is approximately halved. Plots of variance and variance ratio in 60 depth intervals, each of thickness 0.2 cm, are shown in figure 2.8(a) and 2.8(b) respectively. It can be seen that the variance of the full simulation results is a larger fraction of the variance of the reciprocity results than with the 0.5 cm thick voxels, as expected. The value of the

variance ratio, taken from the line of best fit at 12 cm deep in figure 2.8(b), is 0.60 — this is greater than the value obtained for the 0.5 cm thick voxels (0.52). Simulation times for the full and reciprocity simulations were 21170.2 and 183.9 seconds respectively. Using (2-4), this gives an efficiency gain of 69.1 — considerably higher than the value obtained with the previous geometry (56.5). Note that the simulation times for 1 million and 10000 incident photons when the 0.2 cm thick voxels are used, are greater than twice the times required for 500000 and 5000 photons when 0.5 cm thick voxels are used. This is because electron step sizes are shortened by the *PRESTA boundary crossing algorithm* in the vicinity of voxel boundaries, and there are more boundaries when the thinner voxels are used.

It has been shown here that the efficiency gained by using the reciprocity technique is increased with a decrease in the thickness of the detection regions. It should be noted however, that this increase in efficiency gain is with respect to the results obtained using a full simulation. In decreasing the thickness of the detection regions in the full simulation, the energy deposited in each detection region is decreased, with a corresponding increase in the uncertainty. Similarly, the uncertainty in the reciprocity results will increase with a decrease in detection voxel thickness. The detection regions should therefore be as thick as the required depth resolution will allow, for either simulation type. The conclusion reached here is that the shorter the detection regions, the greater the efficiency *gained* by using the reciprocity technique, but the detection regions should still be as thick as possible.

2.3.2.2. Dependence of Efficiency Gain on Detection Voxel Radius

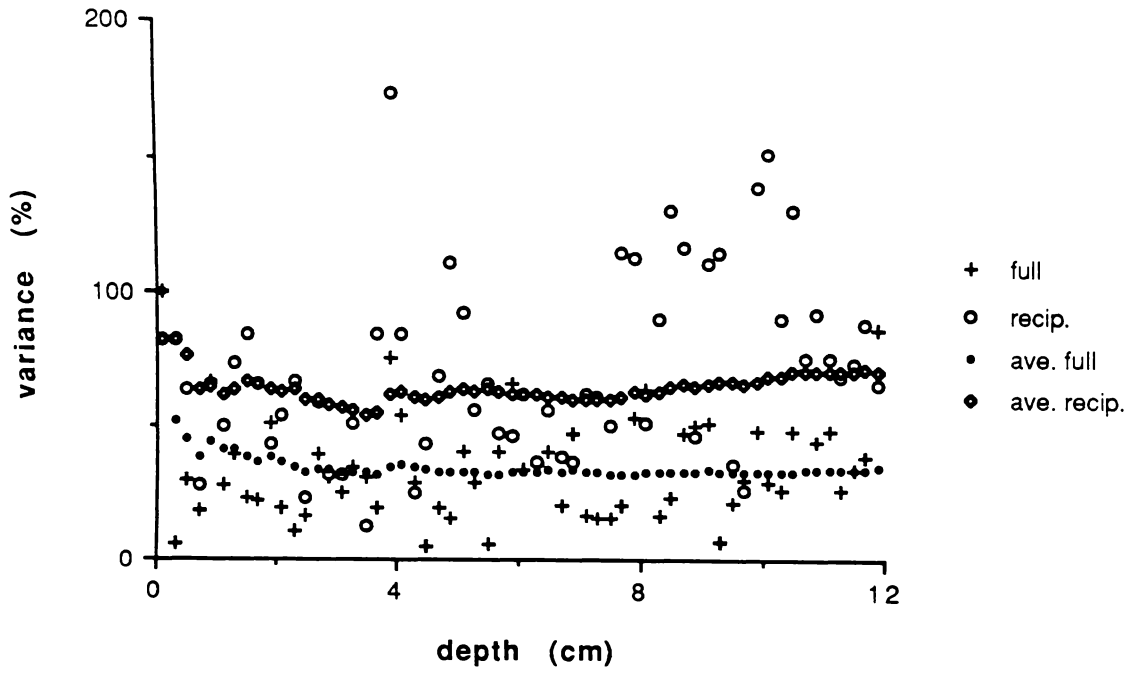
The gain in efficiency obtained by using the reciprocity technique is strongly dependent on the *radius* of the full simulation detection voxels. (Bielajew and Rogers, 1988).

In the full simulation, this radius r_d should be as large as possible to minimise the uncertainty, but not be so large that the dose recorded in these regions is not truly representative of the central axis dose. Since the sharpness of the peak in the dose profile decreases with an increase in beam radius (Kubsad *et al*, 1990), a larger detection voxel radius may be used as beam radius increases. The volume of these detection voxels is proportional to the square of their radius, and if this radius is small in relation to the beam radius r_b , then the energy deposited within the detection voxels will also be approximately proportional to the square of their radius. Therefore, since the variance is inversely proportional to the energy deposited for a particular simulation type, the variance is inversely proportional to the square of the radius ($s^2 \propto 1/r_d^2$). Using (2-1), the efficiency of a full simulation is proportional to the square of the detection voxel radius:

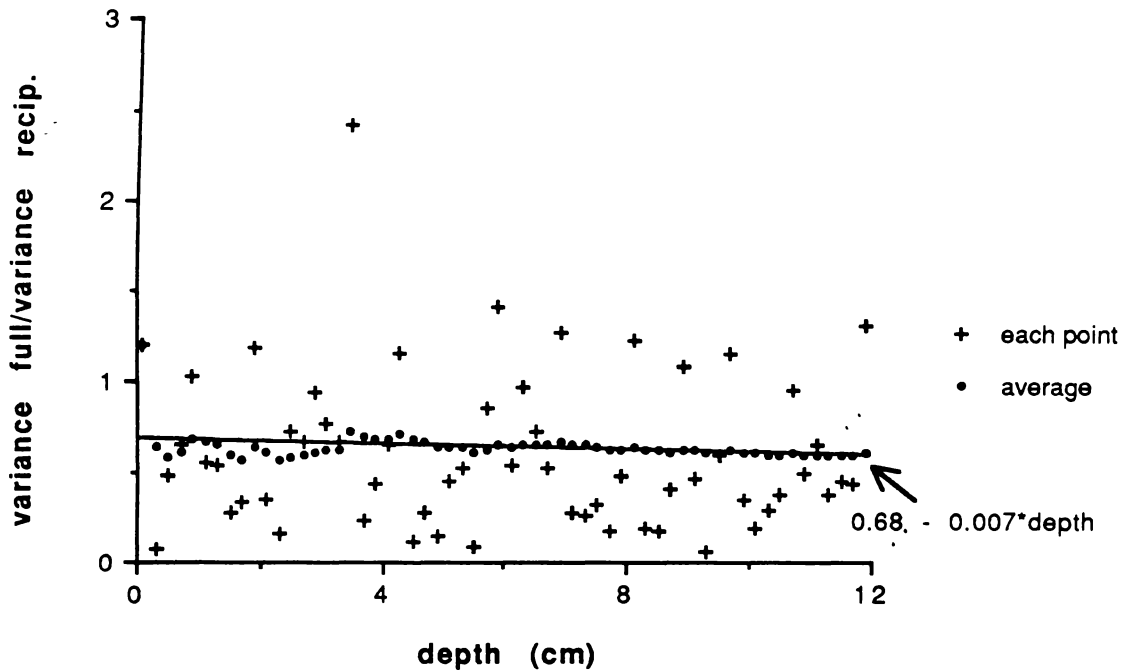
$$\epsilon_0 \propto r_d^2 \quad (2-13)$$

since the simulation time is independent of r_d .

When the reciprocity technique is used, the radius of the incident beam used is the radius of the detection voxels in the full simulation, r_d . Also, the incident fluence in the narrow



(a)



(b)

Figure 2.8. (a) Variance in the dose at each depth for the full and reciprocity type simulations when 1000000 and 10000 incident photons are used (full and recip.). Detection voxels are 0.2 cm thick and have radii of 0.1 cm for the full simulation and 1.0 cm for the reciprocity simulation. Also shown is the mean variance down to each depth (ave. full and ave. recip.). (b) Ratio of the variance of the full simulation to that of the reciprocity simulation at each depth (each point), and the mean ratio down to each depth (average). The mean ratio is 0.60 at 12.0 cm deep.

beam is equal to that of the broad beam in the full simulation. The incident photon number is therefore proportional to the square of the narrow beam radius. As long as this radius is much smaller than the broad detection voxel radius r_b , the energy deposited in the detection region is proportional to the incident photon number and hence to the square of r_d . The variance in this energy is hence inversely proportional to the square of r_d ($s^2 \propto 1/r_d^2$). The simulation time is proportional to the incident photon number and therefore to the square of r_d ($T \propto r_d^2$), so using (2-1), the efficiency of the reciprocity simulation is independent of the narrow beam radius. Therefore, using (2-13), the gain in efficiency is inversely proportional to the square of r_d :

$$\frac{\epsilon_1}{\epsilon_0} \propto \frac{1}{r_d^2} \quad (2-14)$$

Note that the decrease in efficiency gain with an increase in r_d is due to an increase in the efficiency of the full simulation, thereby lessening the advantage of using the reciprocity technique. The technique is at its most useful when the radius of the central axis region in which the dose is required is much smaller than the radius of the beam, since then a very large number of incident photons is needed in a full simulation to obtain results with an acceptably low variance. Using the reciprocity technique, the dose in an infinitesimally narrow central axis region can be obtained without any loss of efficiency, since ϵ_1 is independent of r_d . Note that from (2-14), the efficiency gain when $r_d \rightarrow 0$ is infinity. In obtaining the central axis dose for any circular beam using reciprocity, the incident narrow beam is usually defined to be very small (say 0.01 cm radius) so that true central axis dose is calculated.

2.3.3. Dose Calculation for Multiple Beam Diameters

Another advantage of using the reciprocity technique to calculate central axis dose is the ability to obtain results for several beam diameters in a single simulation. It is apparent from figure 2.1 that if detection regions of different radii were defined for the reciprocity simulation, the energy deposited within each radius would be equal to the energy deposited in a full simulation if the beam were of that radius. In RTPCYL, central axis dose for a range of beam diameters can be obtained using this principle. For the investigation of lateral electron disequilibrium in section 2.4, a cylindrical array containing 60 layers of thickness $\Delta Z = 0.5$ cm and 12 radial shells of thickness $\Delta R = 0.25$ cm is used (see figure 2.9). Using this array, central axis dose from a depth range of 0.25 to 29.75 cm in steps of 0.5 cm can be obtained for beams with diameters ranging from 0.5 to 6.0 cm in increments of 0.5 cm.

In RTPCYL, the arrays ZBOUND and RBOUND contain the positions of successive depth and radial boundaries. Voxel layer i includes the depth interval between ZBOUND(i) and ZBOUND($i + 1$). At each voxel level, annular ring j includes the radial interval between RBOUND(j) and RBOUND($j + 1$). The incident beam is of a small radius RBEAM. In the energy scoring subroutine AUSGAB, the energy EDEP deposited in an electron step is assigned to the voxel in which that step occurs (Nelson *et al*, 1985). When the particle simulation is complete, voxels on each layer are stepped through from the innermost to the outermost and

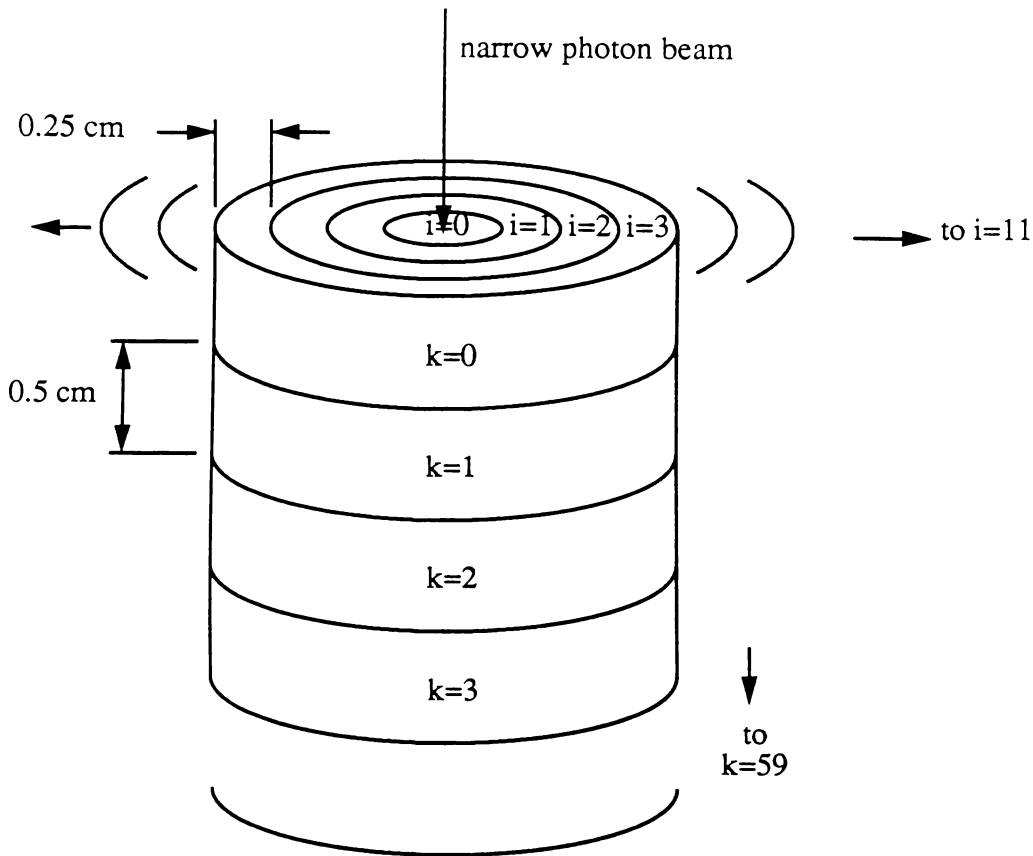


Figure 2.9. Cylindrical voxel array used for obtaining depth dose results using the reciprocity technique. The array contains 60 layers of voxels, each of thickness 0.5 cm. On each layer there are 12 radial shells with increments in radius of 0.25 cm. The beam is directed along the beam axis and has a radius of 0.01 cm.

a running-sum is performed to obtain the energy contained within each radial boundary. The energy value in each voxel (i, j) is then normalised using the incident energy fluence to obtain the dose in $\text{pGy MeV}^{-1} \text{cm}^2$ at the mean depth of voxel layer i for a beam of radius $\text{RBOUND}(j + 1)$.

2.3.4. Applying the Reciprocity Technique to Divergent Beams

The reciprocity technique as it has been described so far is valid only for non-divergent beams. A method is suggested here by which to adapt the technique to account for beam divergence. For beams of small enough diameter for lateral electron equilibrium to exist, this requires consideration of the variation in disequilibrium as the beam diverges. Also, the inverse square falloff in dose with distance from the beam source must be included. This is a useful technique because it allows the central axis dose for divergent beams such as those used in radiosurgery to be calculated using reciprocity.

Using RTPCYL, a full simulation of a divergent beam is initiated by specifying the distance 'upstream' of the surface at which the point source of the beam is located and the radius to which the beam is collimated at the surface. A 1.0 cm radius beam at an SSD of 100 cm is shown in figure 2.10(a). The incident photon fluence at the surface is Φ , and an

energy E is deposited in a detection voxel of radius 0.1 cm and thickness 0.5 cm, centred at depth z . The dose at depth z for a divergent beam of radius R_0 and fluence Φ_0 at the surface can be found using the reciprocity technique by modelling the divergent beam with a non-divergent beam whose radius and incident fluence is dependent on z as follows.

For a divergent beam, the primary photon fluence distribution in a plane at depth z is approximated by that for a non-divergent beam of radius

$$R_z = R_0(1 + z/SSD) \quad (2-15)$$

which has a primary photon fluence at the surface

$$\Phi_z = \Phi_0/(1 + z/SSD)^2. \quad (2-16)$$

This approximation is exact except for the slight increase in pathlength in the medium caused by beam angulation. With the following reasoning, there is the same similarity in the dose distributions at depth z for these two beams, even if the effect of long-range secondary electrons is taken into account. For any ray-line from the beam source to a point at depth z in the divergent beam case, there is a corresponding ray-line in the non-divergent beam approximation, which passes through the same point at depth z , but is aligned with the beam axis. Since both these ray-lines pass through the same point at depth z , and the beam angulation is very small for these small beams, the distributions of energy deposited at depth z due to narrow beams directed along these ray-lines should be very similar. This equivalent non-divergent beam corresponding to the divergent beam at depth z , is shown in figure 2.10(b). Once the radius and incident fluence of the equivalent beam at depth z is known, the reciprocity technique can be applied to obtain the dose at central axis at that depth, as depicted in figure 2.10(c).

2.3.4.1. Modification to the User Code

The non-divergent beam approximation described above has been implemented in RTPCYL, where the dose due to multiple beam diameters can be obtained in a single simulation.

In initiating a divergent beam simulation (either full or reciprocity), the distance from the beam source to the surface, DISTZ, is specified. To account for beam divergence in a reciprocity simulation, the radius of the voxel boundaries could be made to increase depth so that the dose due to an equivalent non-divergent beam is recorded at each depth. An alternative to making entries in the RBOUND array dependent on the voxel layer, is to scale the radial component of the position of each electron step, so that the distribution of energy deposition in the voxels on a particular layer is that which would occur if the radial boundaries were depth dependent. If the transport step occurs at depth z and radius r , then r is scaled inwards:

$$r = r/(1 + z/DISTZ). \quad (2-17)$$

The energy recorded within each radial boundary RBOUND(j) on layer i is then equal to that which would be deposited within a radius RBOUND(j)(1 + z_i /DISTZ), where z_i is the mean

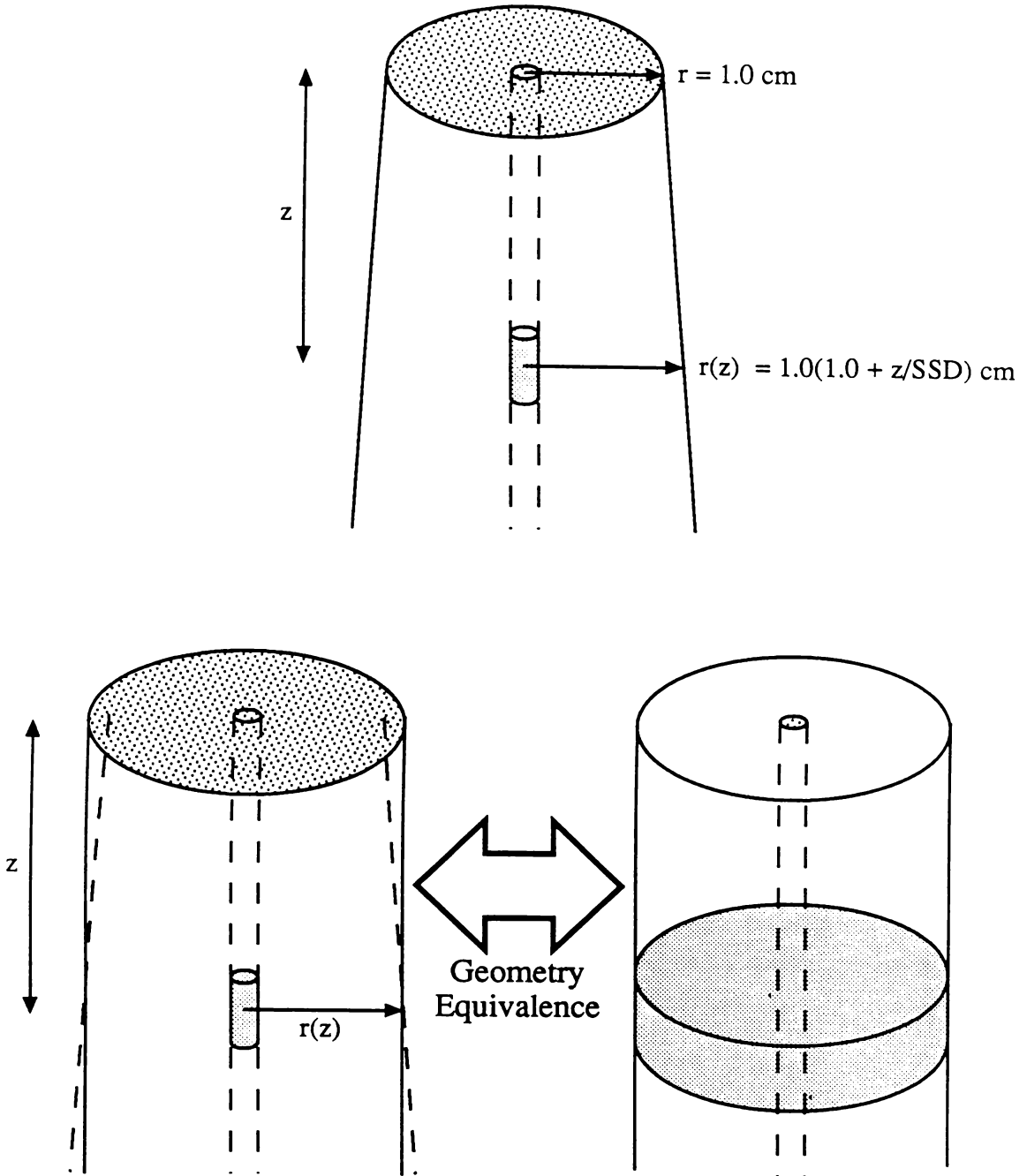


Figure 2.10. (a) Divergent beam of surface radius 1.0 cm with an SSD of 100 cm. Incident photon fluence is Φ . An energy E is deposited in a cylindrical detection voxel of radius 0.1 cm at depth z on the central axis. (b) Non-divergent beam of radius $1.0(1 + z/\text{SSD}) \text{ cm}$. Incident photon fluence is $\Phi/(1 + z/\text{SSD})^2$. Energy deposited in a cylindrical detection voxel of radius 0.1 cm at depth z on the central axis is approximately E , since both the increase in beam diameter and inverse-square falloff with depth is accounted for. (c) Non-divergent beam of radius 0.1 cm. Incident photon fluence is $\Phi/(1 + z/\text{SSD})^2$. Energy deposited in a cylindrical detection voxel of radius $1.0(1 + z/\text{SSD}) \text{ cm}$ at depth z is equal to that deposited in the detection voxel in (b).

depth of layer i . In the equivalent full simulation, this corresponds to the energy deposited in the central axis detection region at that depth due to a non-divergent beam of radius $R_{BOUND}(j)(1 + z_i/DISTZ)$.

At the end of the simulation, the energy within each radial boundary is then multiplied by an inverse-square factor $ISQR_i$ to account for the variation in incident fluence with equivalent non-divergent beam radius. This factor is given by

$$ISQR_i = 1/(1 + z_i/DISTZ)^2. \quad (2-18)$$

This procedure is depicted in figure 2.11, where the array is the same as that in figure 2.9.

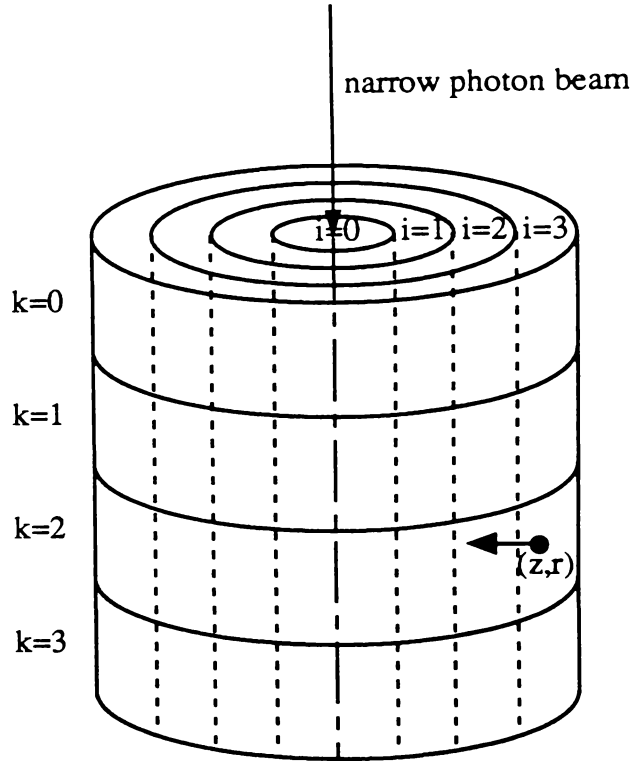


Figure 2.11. Procedure for simulating a divergent beam using the reciprocity technique. The array is the same as that in figure 2.9. When energy is deposited in a charged particle transport step at depth Z , the radial position of that step, R , is scaled by the factor $1/(1 + Z/DISTZ)$ where $DISTZ$ is the distance from the beam source to the medium surface (the SSD). At the end of the simulation, an inverse square factor is applied at each depth.

Using this technique, two aspects of beam divergence are accounted for: the increase in beam radius with depth, and the decrease in photon fluence as beam radius increases. Accounting for the increased beam radius is important for small beams where lateral electron disequilibrium exists, since central axis dose increases rapidly with radius until equilibrium is established, as discussed in section 2.4.

2.3.4.2. Dose Calculation Results

The validity of this technique was tested by comparing the central axis dose calculated using a full divergent beam simulation, with the dose calculated using the modification to RTPCYL described above. Also compared is the dose obtained by simply correcting the dose calculated for a non-divergent beam with an inverse-square factor.

Full simulations were performed for divergent beams of surface diameter 0.5 and 2.0 cm. The parameters defining the geometry of these simulations are given in table 2.2. A narrower detection region (0.05 cm radius) than that used in the previous simulations (0.1 cm diameter) is used here because the dose profile of the 0.5 cm beam is peaked at the central axis due to lateral electron disequilibrium (Kubsad *et al*, 1990).

	0.5 cm diam.	2.0 cm diam.
NCASE	8 million	8 million
DISTZ	100.0	100.0
RBEAM	0.25	1.0
RBOUND(2)	0.05	0.05

Table 2.2 User specified parameters for full divergent simulations of 0.5 and 2.0 cm diameter beams. Computation time for 8 million incident photons was approximately 40 hours on a VAX 6000-430 for each simulation. DISTZ is the distance of the point source from the surface (the SSD); RBEAM is the radius to which the beam is collimated at the surface; RBOUND(2) is the radius of the detection region on the central axis. Note that RBOUND(1) = 0.0.

Simulations using the reciprocity technique were performed for both non-divergent and divergent beams of surface diameter 0.5 and 2.0 cm. In each case, dose for both beam diameters is calculated in the same simulation. The parameters defining the geometry of these simulations are given in table 2.3. Note that the values of RBEAM for the respective beam radii in table 2.2 correspond to RBOUND(2) and RBOUND(3) in table 2.3, and that RBOUND(2) in table 2.2 corresponds to RBEAM in table 2.3.

Figure 2.12(a) compares three sets of depth dose results for a 0.5 cm diameter beam: divergent beam using a full simulation; non-divergent beam using the reciprocity technique; non-divergent beam using the reciprocity technique but with an inverse-square correction. In figure 2.12(b), the result of the divergence correction described above is shown. In comparing these two graphs, it is obvious that the inverse-square falloff with depth has a much larger effect on the depth dose curve of a divergent beam than does the physical increase in beam diameter with depth. The curve obtained using the inverse-square correction in figure 2.12(a) becomes significantly lower than the full simulation curve for depths greater than approximately 15 cm — this is due to the neglect of the increase in diameter with depth. The curve obtained using the divergence correction in figure 2.12(b), does not diverge from the full divergent dose as depth increases. Physically, as the diameter of a divergent beam

	non-divergent	divergent
NCASE	1 million	1 million
DISTZ	—	100.0
RBEAM	0.05	0.05
RBOUND(2)	0.25	0.25
RBOUND(3)	1.0	1.0

Table 2.3. User specified parameters for reciprocity simulations of non-divergent and divergent 0.5 and 2.0 cm diameter beams. For each simulation type, results for both beam diameters are generated at once. Computation time for 1 million incident photons was approximately 5 hours on a VAX 6000-430 for each simulation. RBEAM is the radius to which the beam is collimated at the surface; RBOUND(2) and RBOUND(3) are the radii of the detection regions for the 0.5 and 2.0 cm beams respectively. Note that RBOUND(1) = 0.0.

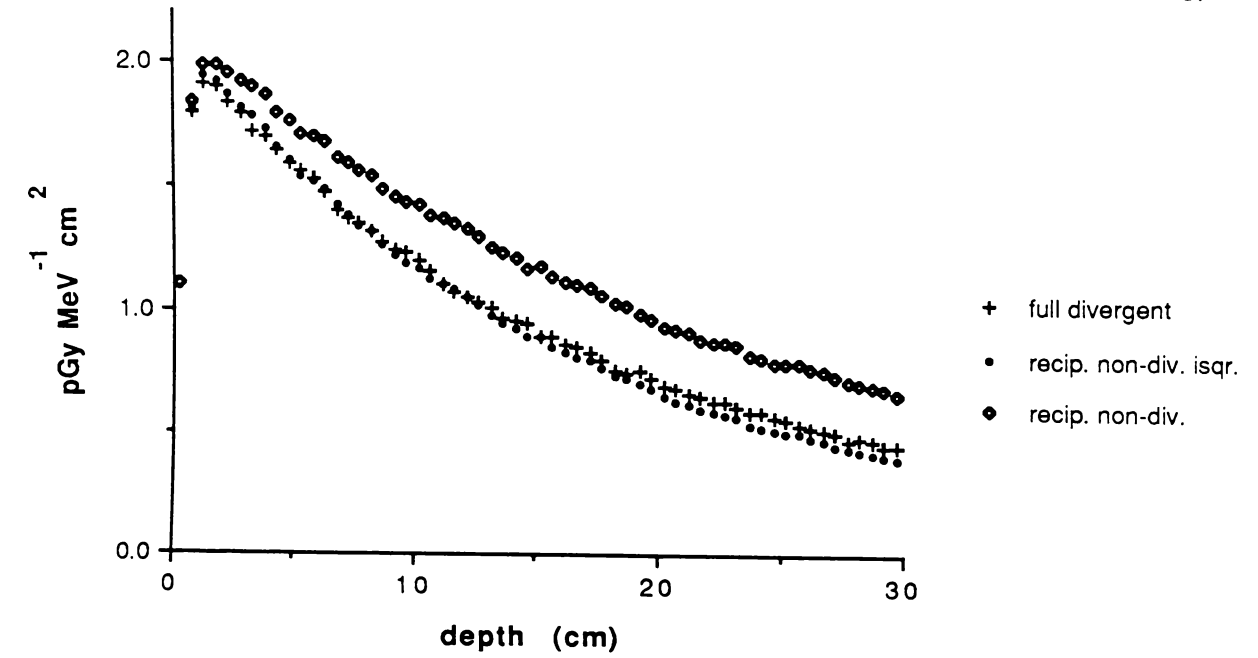
increases with depth, the amount of lateral electron disequilibrium decreases. This means that the falloff in dose with depth is not as rapid as predicted by simply inverse-square correcting a non-divergent beam depth dose curve.

Results for the 2.0 cm diameter beam are shown in figure 2.13(a) and 2.13(b). For this larger diameter, the increase in beam width with depth does not have such a significant effect since lateral electron equilibrium is more closely approached. The inverse-square corrected curve in figure 2.13(a) only diverges slightly from the full simulation curve as depth increases, however the curve predicted by the divergence correction in figure 2.13(b) is still in noticeably better agreement with the full simulation results. It is apparent from these plots that the variance of the full simulation results is larger for the 2.0 cm beam than for the 0.5 cm beam. This is because the same number of incident photons were used but less reach the detection region for the larger beam.

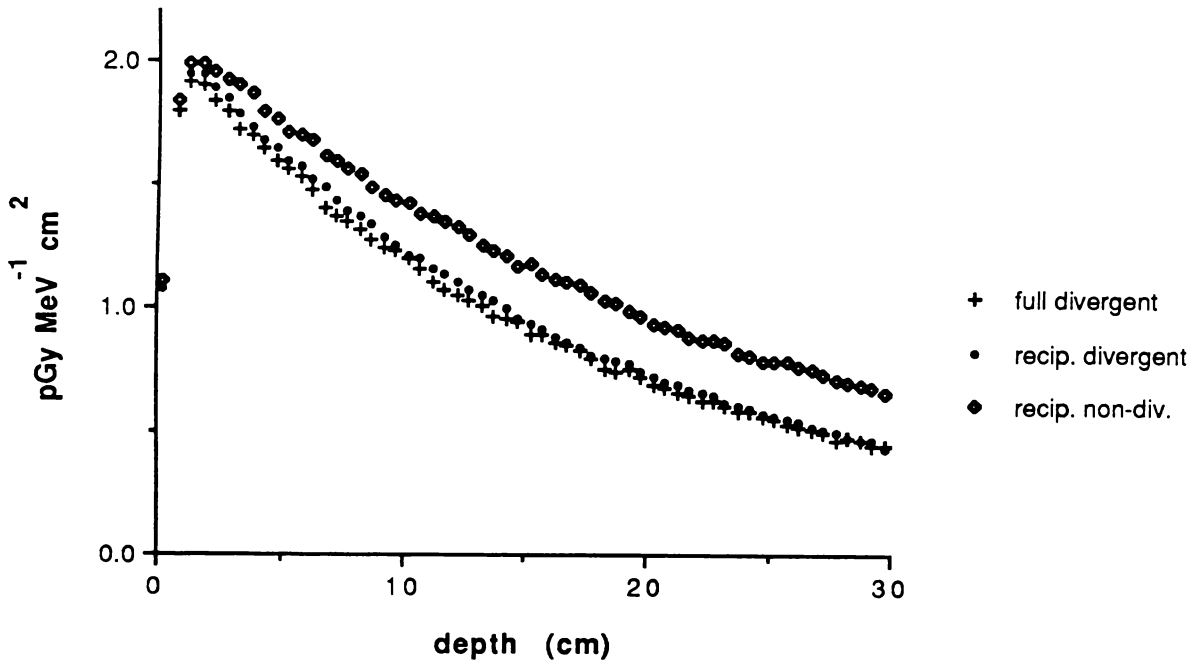
2.3.4.3. Calculation Efficiency

To compare the efficiency gained in using the reciprocity technique for divergent beams with the gain for non-divergent beams, full and reciprocity simulations for a 2.0 cm diameter divergent beam were performed. Again, 500000 and 5000 incident photons were used for the full and reciprocity simulations respectively. Parameters specifying the geometry of these simulations are given in table 2.4.

Plots corresponding to those in figure 2.5(a) and 2.5(b), but for the divergent beam, are shown in figure 2.14(a) and 2.14(b). Comparing figures 2.5(a) and 2.14(a), it can be seen that the variance in the results of the full simulation is slightly larger for the divergent beam. This is because the dose is reduced by divergence, hence giving greater uncertainty. On the other hand, the variance in the reciprocity results is less for the divergent beam. This is because the energy deposition recorded at depth z as being within a radius RBOUND(2) = 1.0, includes energy deposited within a radius $R_z = 1.0(1 + z/DISTZ)$, hence lowering the uncertainty. These factors cause the variance ratio in figure 2.14(b) to be higher for

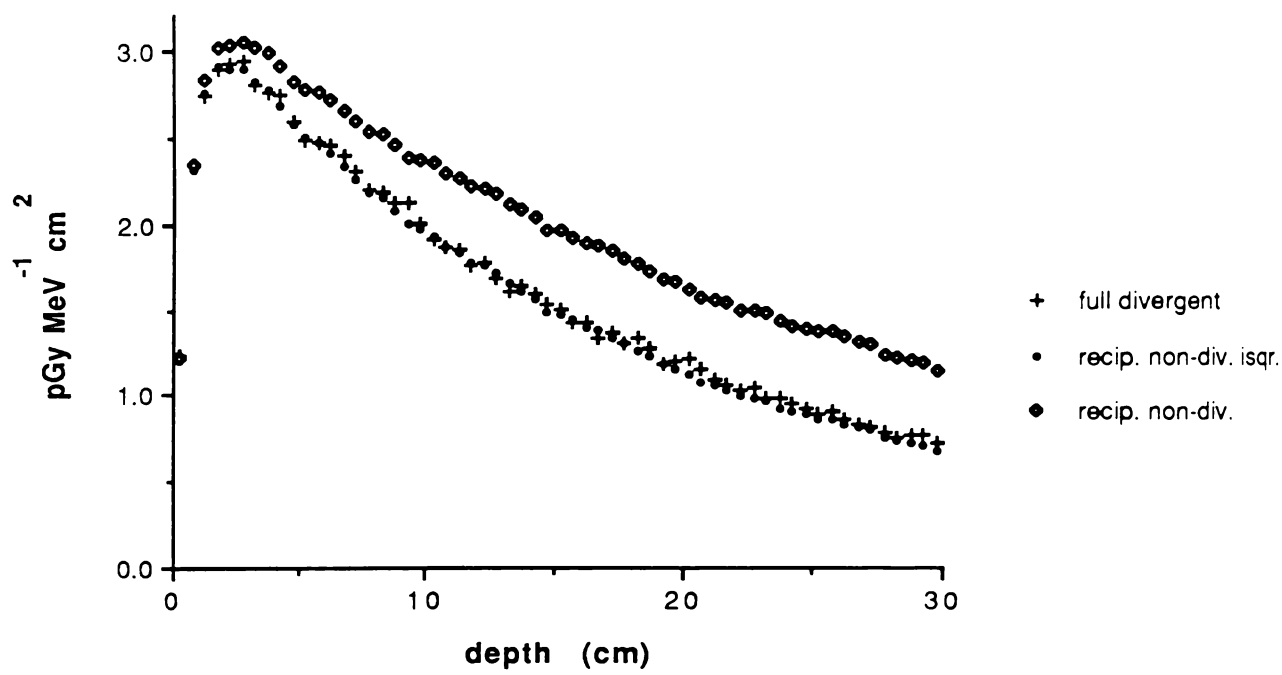


(a)

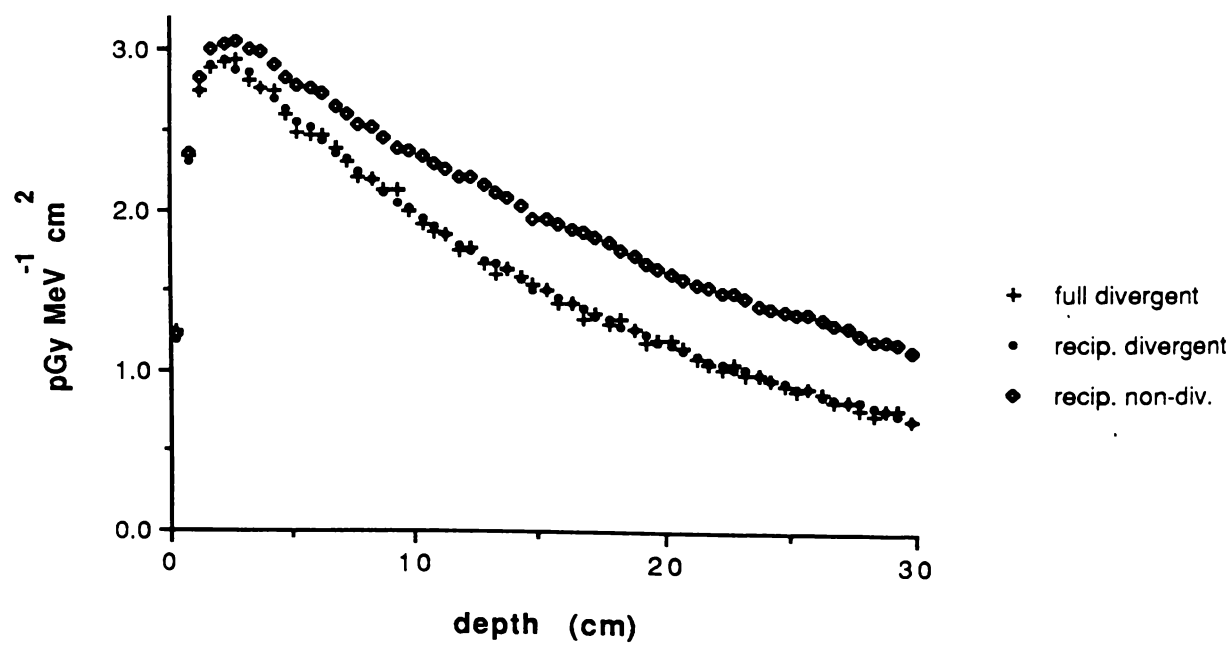


(b)

Figure 2.12. (a) Calculated depth dose results for a 0.5 cm diameter beam: 100 cm SSD divergent beam using a full simulation (full divergent); non-divergent beam using the reciprocity technique (recip. non-div.); non-divergent beam using the reciprocity technique, with an inverse-square correction (recip. non-div. isqr.). The full simulation used 8000000 incident photons and the reciprocity simulation used 1000000. The recip. non-div. isqr. dose diverges from the full divergent dose as depth increases. (b) Same as (a), except dose calculated using 1000000 photons for a divergent beam using the reciprocity technique is shown (recip. divergent), instead of the simple inverse-square corrected dose. The recip. divergent dose does not diverge from the full divergent dose, as does the recip. non-div. isqr. dose in (a).



(a)



(b)

Figure 2.13. Same as figure 2.12, but for a 2.0 cm diameter beam. In this case, the recip. non-div. isqr. dose in (a) is in better agreement with the full divergent dose than in the 0.5 cm case, due to the reduced degree of lateral electron disequilibrium. The recip. divergent dose in (b) is still in better agreement than the recip. non-div. isqr. dose with the full divergent dose.

	full	reciprocity
NCASE	500000	5000
DISTZ	100.0	100.0
RBEAM	1.0	0.1
RBOUND(2)	0.1	1.0

Table 2.4. User specified parameters for full and reciprocity type simulations of a 2.0 cm diameter divergent beam, when 500000 and 5000 incident photons were used for the respective simulations. RBEAM is the radius to which the beam is collimated at the surface; RBOUND(2) is the radius of the detection region on the central axis. Note that RBOUND(1) = 0.0.

the divergent beam. The line of best fit for the mean variance ratio in figure 2.14(b) has a value of 0.61 at 30 cm deep — considerably higher than the value obtained for the non-divergent beam (0.52). No significant difference in computation time was found between the non-divergent and divergent beams. The gain in computational efficiency obtained by using the reciprocity technique for the divergent beam is therefore greater by a factor of $0.61/0.52 = 1.17$ than that for the non-divergent beam.

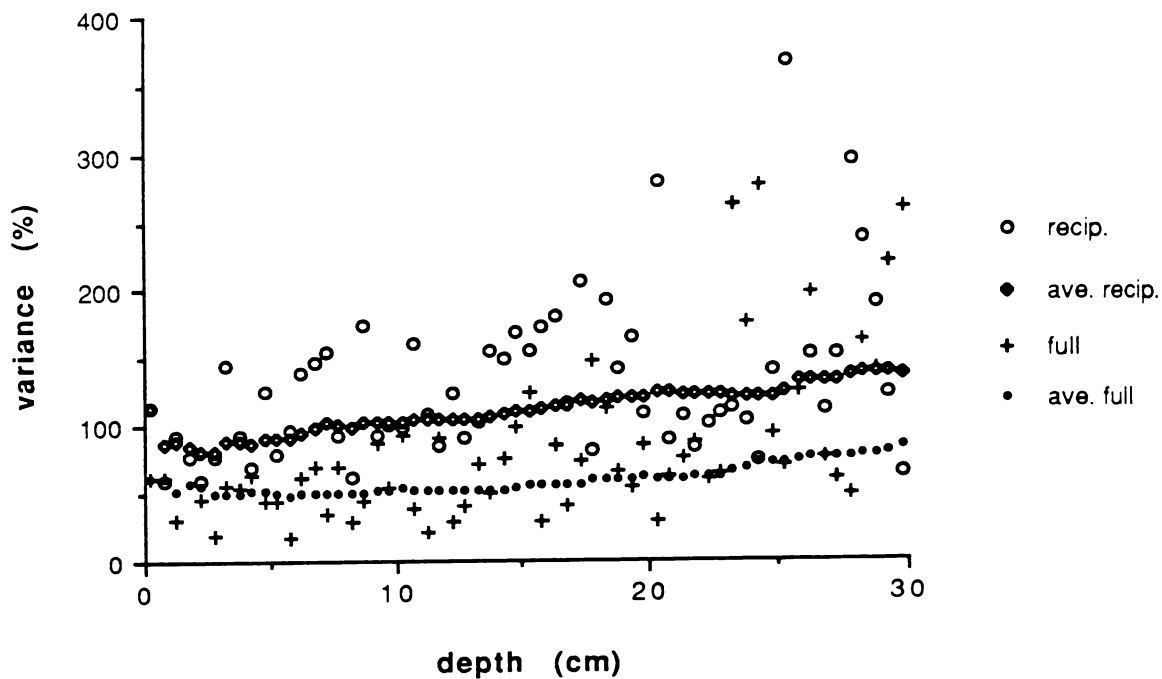
2.4. Lateral Electron Disequilibrium for Non-divergent Circular Beams

2.4.1. Secondary Electron Ranging

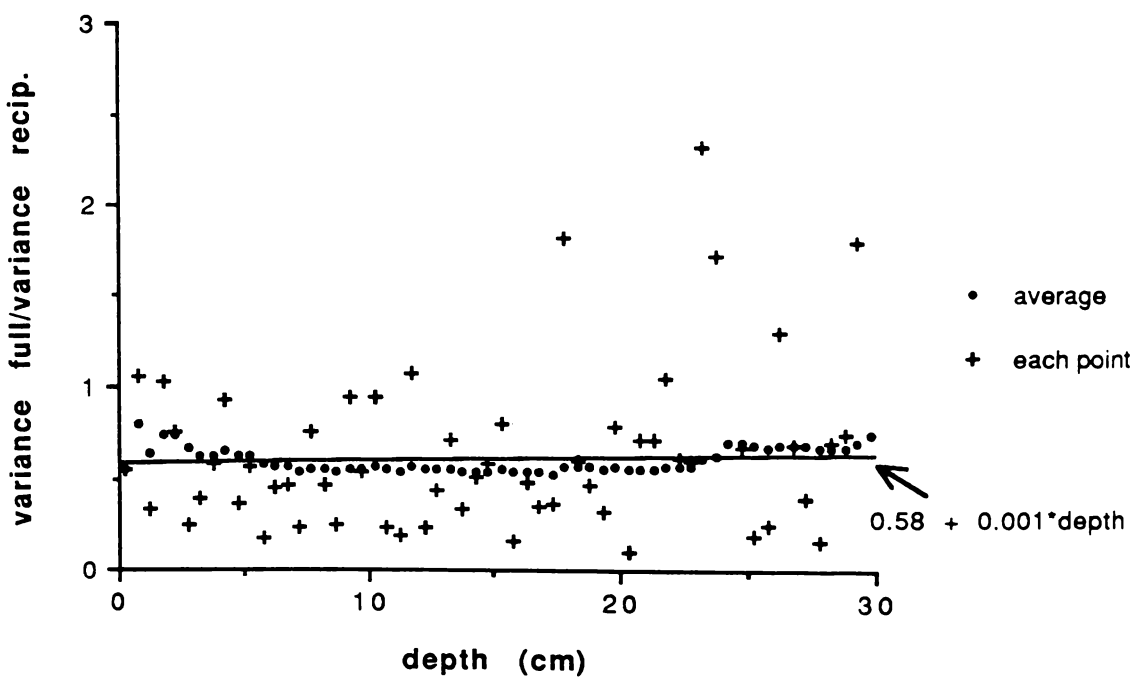
To illustrate how secondary electrons range away from the primary interaction site, a 10 MeV pencil beam (radius 0.01 cm) of 300 photons was simulated using EGS4. The maximum photon energy in the 10 MV beam was used since these photons give rise to secondary electrons with the longest range. The beam is incident on a water medium. Using the DOT-PLOT option of the user code RTPCYL (Murray, 1990), the depth in the medium Z , and distance from the pencil beam axis R , of each electron transport step was output to a file. Only the steps of secondary electrons originating from primary photon collisions were included — the method of distinguishing these electrons from electrons set in motion by scattered photons is explained in section 2.4.2. By plotting these points, a set of electron tracks is obtained as shown in figure 2.15. Note that the Z axis is more compressed than the R axis which gives the impression that the electron paths are more oblique to the incident photon direction than is really the case. Only three electron tracks exceed a radius of 2 cm from the pencil beam axis, which suggests that lateral electron equilibrium will exist for beam diameters greater than approximately 4 cm (see section 1.12).

2.4.2. Disequilibrium in a Water Medium

The degree to which the central axis dose for a 10 MV beam is reduced by lateral electron disequilibrium was investigated by producing depth dose curves for non-divergent beams with diameters ranging from 0.25 to 6.0 cm. The reciprocity technique was used to reduce the computation time required to produce results with a low variance. The calculation array used is that shown in figure 2.9.



(a)



(b)

Figure 2.14. (a) Variance in the dose for a divergent 2.0 cm diameter beam at each depth for the full and reciprocity type simulations when 500000 and 5000 incident photons are used (full and recip.). Detection voxels are 0.5 cm thick and have radii of 0.1 cm for the full simulation and 1.0 cm for the reciprocity simulation. Also shown is the mean variance down to each depth (ave. full and ave. recip.). (b) Ratio of the variance of the full simulation to that of the reciprocity simulation at each depth (each point), and the mean ratio down to each depth (average). The mean ratio is 0.61 at 30 cm deep.

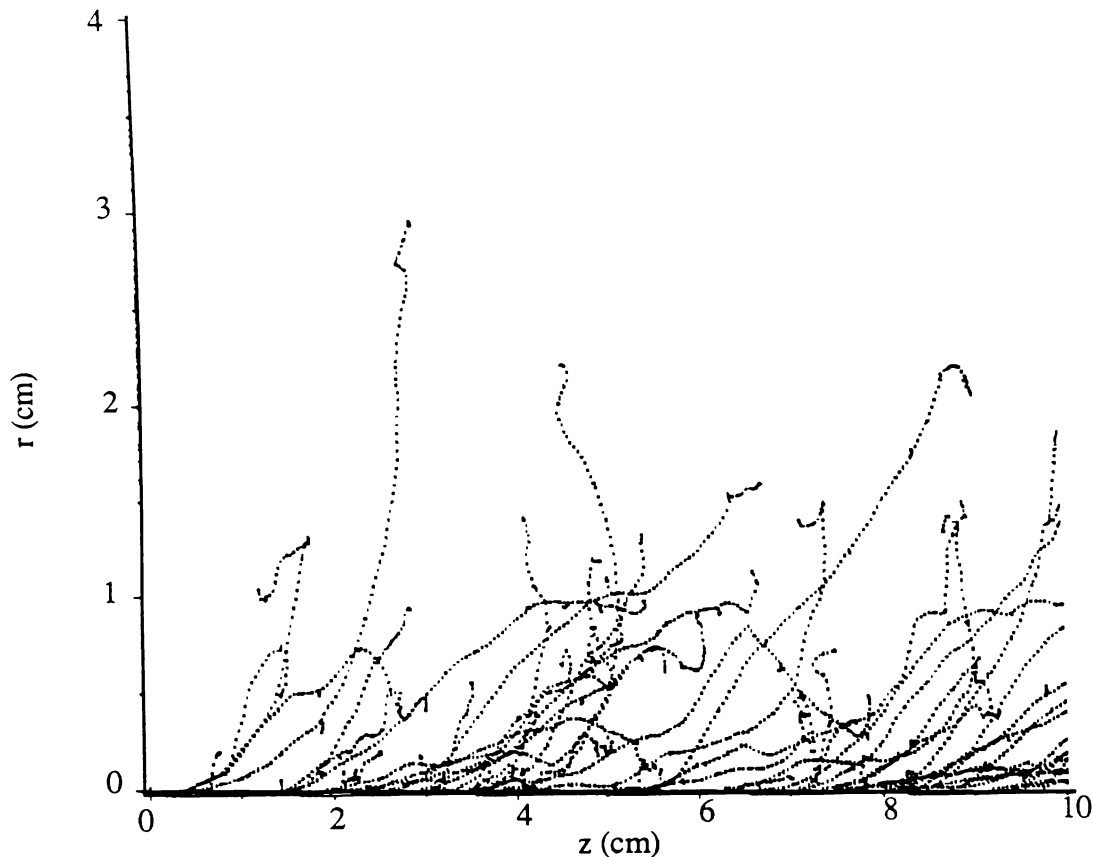


Figure 2.15. Tracks of secondary electrons due to a 10 MeV pencil beam directed along the Z axis, produced with an EGS4 Monte Carlo simulation. Distance from the pencil beam axis is plotted against depth for each electron step occurring in AUSGAB, using the DOTPLOT option in the user code RTPCYL. Maximum lateral electron range appears to be approximately 2 cm, indicating that lateral electron equilibrium exists for beam diameters exceeding approximately 4 cm.

Separate curves were produced for the primary and scattered dose components. This separation was performed as follows. Each particle has a set of *flags* associated with it which are used as 'tags' to identify certain properties of the particle. In RTPCYL, the value of FLAG4 associated with a particle is used to determine whether the energy deposited by that particle contributes to the primary or scattered dose. The value of FLAG4 for primary photons is 0. When a primary photon interacts, FLAG4 for secondary electrons set in motion in that interaction is set to 1; FLAG4 is set to 2 for the scattered photon. Further particles which are set in motion by these tagged particles then have the value of FLAG4 associated with them set to be the same as that of the particle which caused them to be transported. Energy deposition is recorded in a primary or scattered dose array depending on whether the value of FLAG4 for the particle depositing the energy is 1 or 2 respectively. All dose due to bremsstrahlung and annihilation photons is stored as scattered dose, since the value of FLAG4 for these photons is set to 2 as soon as the photon is transported.

In figure 2.16, primary dose per incident photon energy fluence is plotted to a depth of 30 cm, for beam diameters of 0.5, 1.0, 2.0, 4.0 and 6.0 cm. A steady increase in primary dose with diameter occurs up to 4.0 cm but the increase in dose from the 4.0 to 6.0 cm beams is

negligible — indicating that lateral electron equilibrium is established for the 4.0 cm beam. This result is as expected from the range of the electron tracks in figure 2.15.

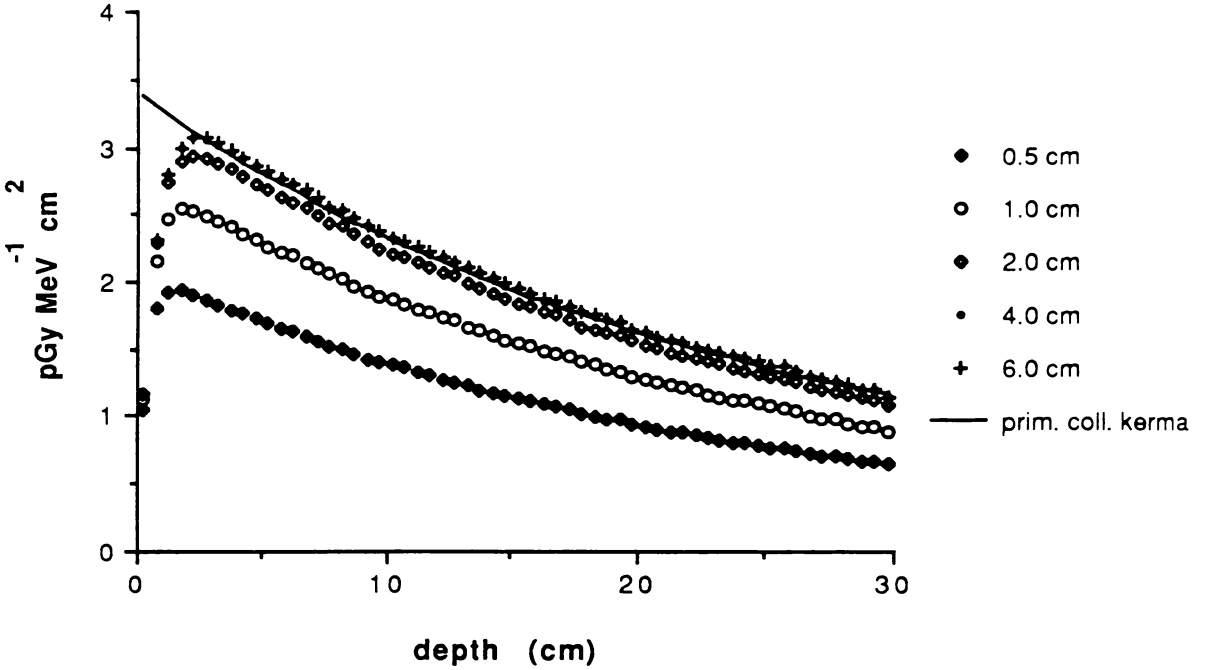


Figure 2.16. Primary dose at central axis for parallel beams with diameters of 0.5, 1.0, 2.0, 4.0, and 6.0 cm. Nine million incident photons were simulated which took 33 hours on a VAX 6000-430. Dose increases with beam diameter until lateral electron equilibrium is established for the 4.0 cm beam. Primary collision kerma, calculated using the photon spectrum in figure 1.1, is also shown.

Also included in figure 2.16 is the primary collision kerma, K_c (Attix, 1978), calculated using the primary photon spectrum in figure 1.1. If the fraction of primary photons in the n th energy component, at the medium surface, is $F_n(0)$, then the primary collision kerma at depth z is

$$K_c(z) = \sum_{n=1}^N F_n(0) e^{-\mu_n z} E_n \left(\frac{\mu_{ab}}{\rho} \right)_n \quad (2-19)$$

where E_n is the mean photon energy in the n th spectral component. Values for $(\mu_{ab}/\rho)_n$ corresponding to energy E_n can be obtained from tables (Johns and Cunningham, 1983). As discussed in section 1.3.5, the partitioning of the primary kerma into collision and radiative parts distinguishes the energy that is transferred to secondary electrons and subsequently deposited in the medium, from that which is transferred and subsequently converted to bremsstrahlung. In future, primary collision kerma will be referred to simply as collision kerma.

The mean position of energy deposition is 'downstream' from the site where they were set in motion by primary photons (Mackie *et al*, 1988). Therefore, in conditions of lateral electron equilibrium, the primary dose at depth z should equal the value of K_c at a depth

slightly shallower than z (Attix, 1978). The primary dose at depth z , $D_p(z)$, is thus related to K_c by

$$D_p(z) = K_c(z - \bar{z}). \quad (2-20)$$

Because of the exponential falloff of the collision kerma with depth, the primary dose is slightly larger than K_c for depths greater than d_{max} .

The displacement $\bar{z}$ of the dose curve from the collision kerma curve can be interpreted as the effective distance downstream from the primary photon interaction site at which energy is deposited by secondary electrons. This is referred to as the *effective longitudinal range*. In terms of the collision kerma at the surface, $K_c(0)$, the collision kerma at depth z for a non-divergent monoenergetic beam containing photons with a linear attenuation coefficient μ is given by

$$K_c(z) = K_c(0)e^{-\mu z}. \quad (2-21)$$

The ratio of primary dose to collision kerma at depth z is therefore approximated by

$$\begin{aligned} D_p(z)/K_c(z) &= K_c(z - \bar{z})/K_c(z) \\ &= e^{-\mu \bar{z}} \\ &\approx 1 + \mu \bar{z}. \end{aligned} \quad (2-22)$$

The effective longitudinal range of secondary electrons was found by Metcalfe *et al* (1990) to be 0.57 cm for the 10 MV beam. For the mean energy of primary photons in the 10 MV spectrum (3.0 MeV), the linear attenuation coefficient for water is approximately 0.04 cm^{-1} . Using this value for μ and setting $\bar{z}$ equal to 0.57 cm in equation (2-22) gives $D_p(z)/K_c(z) = 1.023$. This gives a primary dose for equilibrium conditions approximately 2% higher than the collision kerma for all depths greater than d_{max} .

The scattered dose for the same range of beam diameters is shown in figure 2.17, where the dose is seen to increase steadily with diameter, but to be of a much smaller magnitude than the primary dose. As beam diameter increases further, the scattered dose will continue to increase due to the long photon range, and consequently become an increasingly significant component of the total dose. This is evidenced by the continuous increase in the scatter-air ratio part of a tissue-air ratio versus beam diameter curve (Johns and Cunningham, 1983).

The total dose is shown in figure 2.18, where the dose for each diameter is slightly greater than the primary dose due to the addition of the scattered dose component. Since equilibrium is established for the 4.0 cm beam, the small increase in dose from the 4.0 to the 6.0 cm diameter beam is due entirely to the scattered component. The collision kerma is included in the graph as a reference for comparison between the magnitudes of the primary and total dose. The total dose may be related to the total collision kerma as in (2-22) if modified values of μ and $\bar{z}$ which account for the effect of scattered photons are used (Attix, 1978). The total collision kerma on the central axis depends on beam diameter since there is an increase in scattered photons reaching the central axis as beam diameter increases.

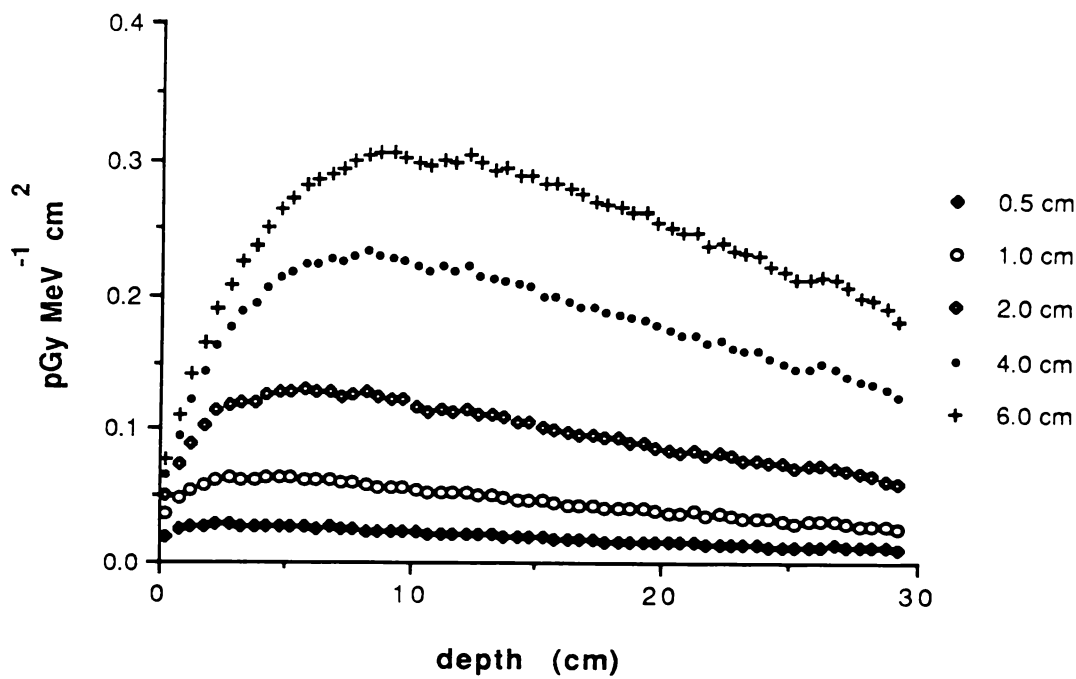


Figure 2.17. Scattered dose at central axis. Dose increases with beam diameter for all diameters shown, because of the long range of scattered photons. Note that for these beam diameters, the scattered dose is an order of magnitude less than the primary dose.

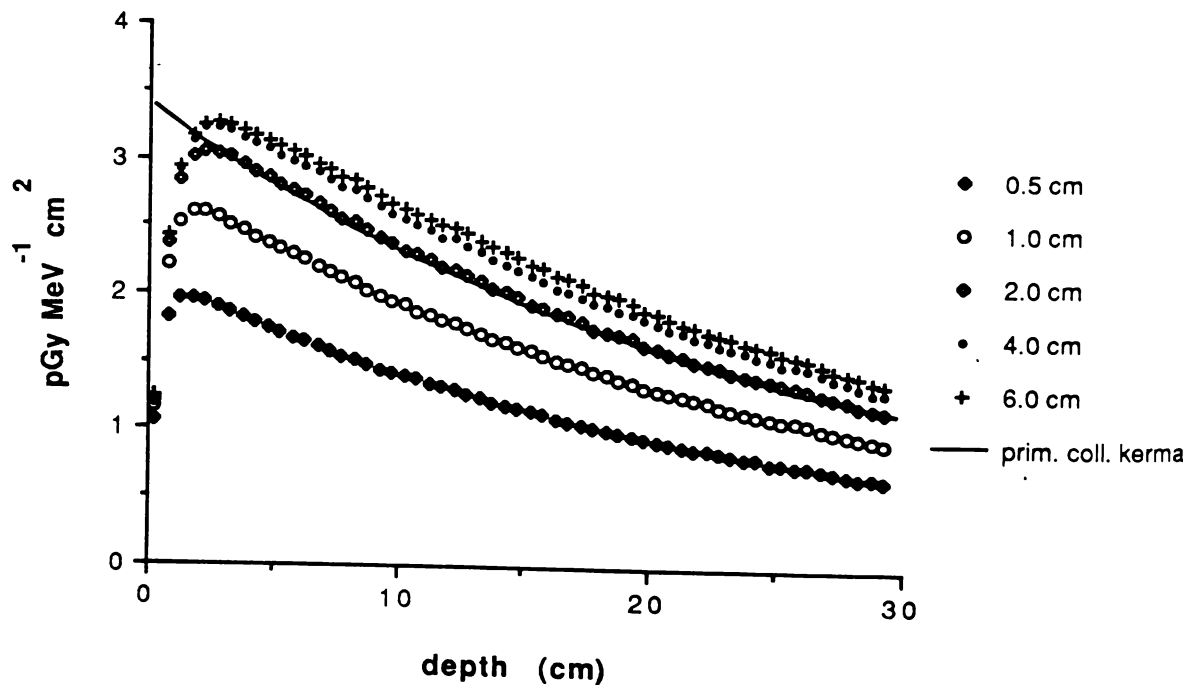


Figure 2.18. Total dose obtained by summing the primary and scattered doses. Note that the increase in dose from the 4.0 to 6.0 cm beams is entirely due to the scattered dose component. Primary collision kerma is included as a reference for comparison between the primary and total dose.

2.4.3. The Lateral Electron Equilibrium Factor

A quantity is proposed here with which to specify the amount by which the primary dose at central axis is reduced due to lateral electron disequilibrium. This quantity is the *lateral electron equilibrium factor* EEF_{lat} . If the primary dose at depth z for a beam diameter d is $D_p(z, d)$ then

$$EEF_{lat}(z, d) = D_p(z, d)/D_p(z, d_{eq}), \quad (2-23)$$

where $D_p(z, d_{eq})$ is the primary dose at depth z for a beam diameter d_{eq} large enough for equilibrium to be established. From figure 2.16, it can be seen that a suitable value of d_{eq} is 4.0 cm. The lateral electron equilibrium factor versus depth, for beam diameters of 0.5, 1.0, and 2.0 cm, is shown in figure 2.19.

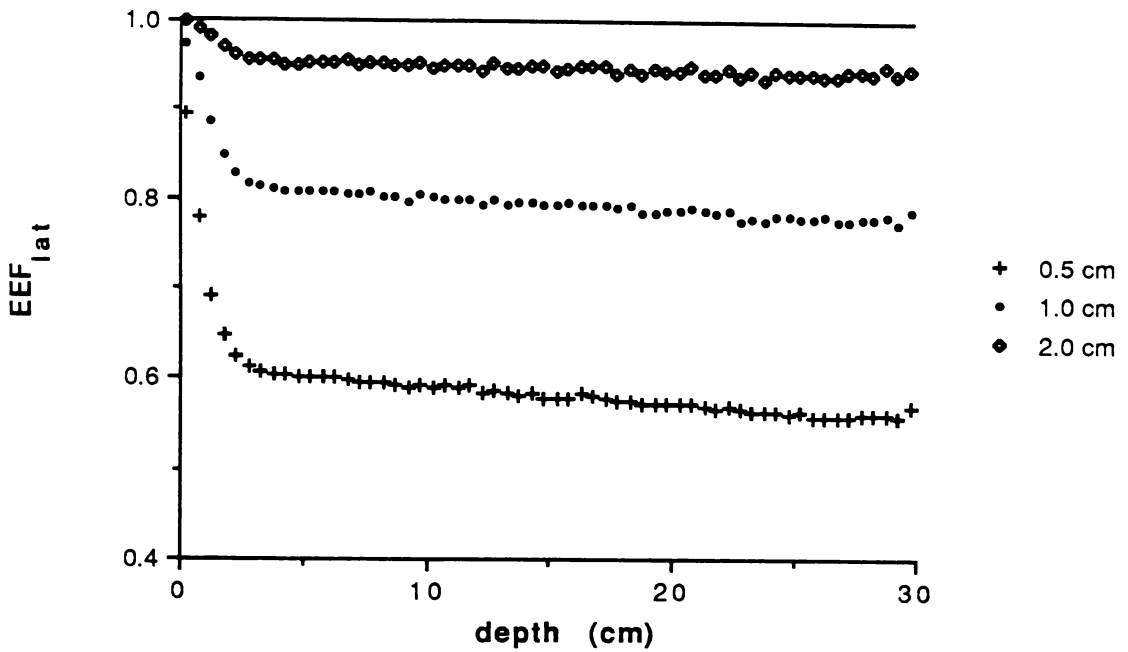


Figure 2.19. Lateral electron equilibrium factor as a function of depth for the 0.5, 1.0, and 2.0 cm beams.

After the initial sharp decrease with depth, the value of EEF_{lat} is approximately 0.6, 0.8, and 0.95 for the 0.5, 1.0, and 2.0 cm diameter beams respectively. Therefore, for a unit density medium, lateral electron disequilibrium causes a reduction in primary dose at central axis of approximately 40%, 20%, and 5% for these beam diameters. The lateral electron equilibrium factor as a function of beam diameter, at a depth of 5.25 cm, is shown in figure 2.20.

The initial decrease in EEF_{lat} with depth seen in figure 2.19, is because the contribution to the dose at central axis due to primary photon interactions away from the central axis region initially increases with depth. This is explained in terms of primary convolution energy deposition kernels (primary EDKs) in chapter 3. Note that there is a slight decrease

in EEF_{lat} with depth past this region. This is due to an increase in secondary electron range with the depth, which occurs because of *beam hardening* — this is also discussed in terms of primary EDKs in chapter 3.

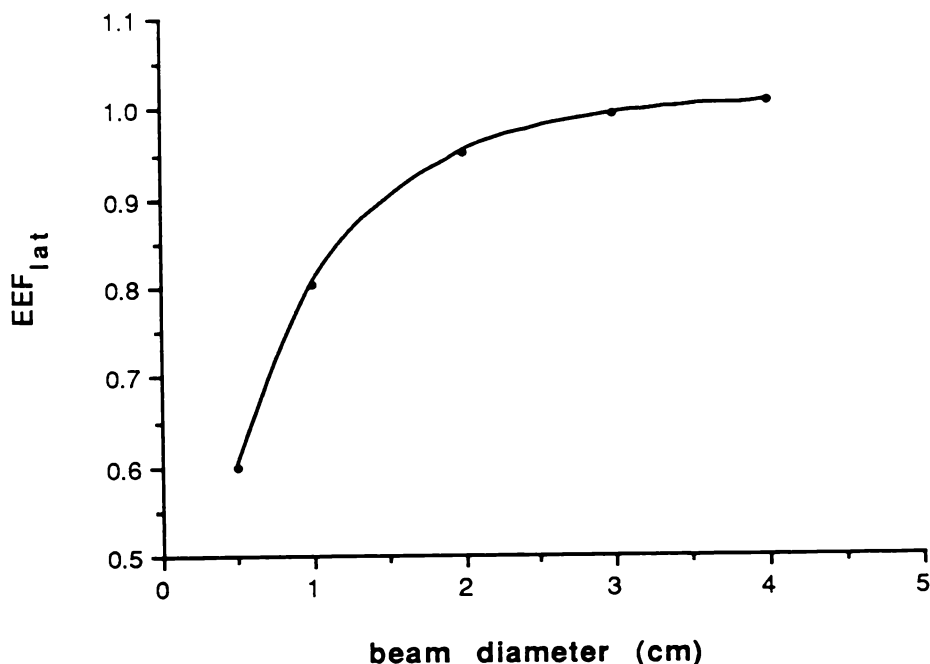


Figure 2.20. Lateral electron equilibrium factor at 5.25 cm deep for a range of beam diameters. As can be seen in figure 2.19, the increase is initially rapid before levelling off as equilibrium is approached.

2.4.4. Disequilibrium in a Medium Containing Lung

As the density of the irradiated medium is decreased, the secondary electron range increases in inverse proportion to the density. Therefore, for a particular beam diameter, the degree of lateral electron disequilibrium increases with a decrease in density. Also, the diameter at which a particular degree of disequilibrium exists increases as density decreases. Because of the inverse relationship of density and electron range, and by O'Connor's theorem (O'Connor, 1957), the central axis dose curves plotted in figures 2.16-2.18 apply to any homogeneous medium of similar composition to water, but with the units of depth and beam radius replaced by equivalent depth and radius in water. For example, for a medium of density $\rho = 0.25 \text{ g cm}^{-3}$, the equivalent beam diameters represented by the curves in figures 2.16-2.18 correspond to physical beam diameters of 2.0, 4.0, 8.0, 16.0, and 24.0 cm, and the maximum depth of 30 cm in water corresponds to a depth of 120 cm in the low density medium.

Beams of these radii incident on a medium consisting of a 20 cm thick 'lung' slab of density $\rho = 0.25 \text{ g cm}^{-3}$ sandwiched between two 5.0 cm thick unit density 'water' slabs were simulated. Primary dose at central axis for this configuration is shown in figure 2.21. In

the central region of the lung, equilibrium is established for a beam diameter of 16.0 cm — this is expected since this diameter in the lung corresponds to 4.0 cm diameter in a water medium. In the water regions away from the lung interfaces, disequilibrium exists for the 2.0 cm diameter beam but is established for the 4.0 cm beam as in figure 2.16.

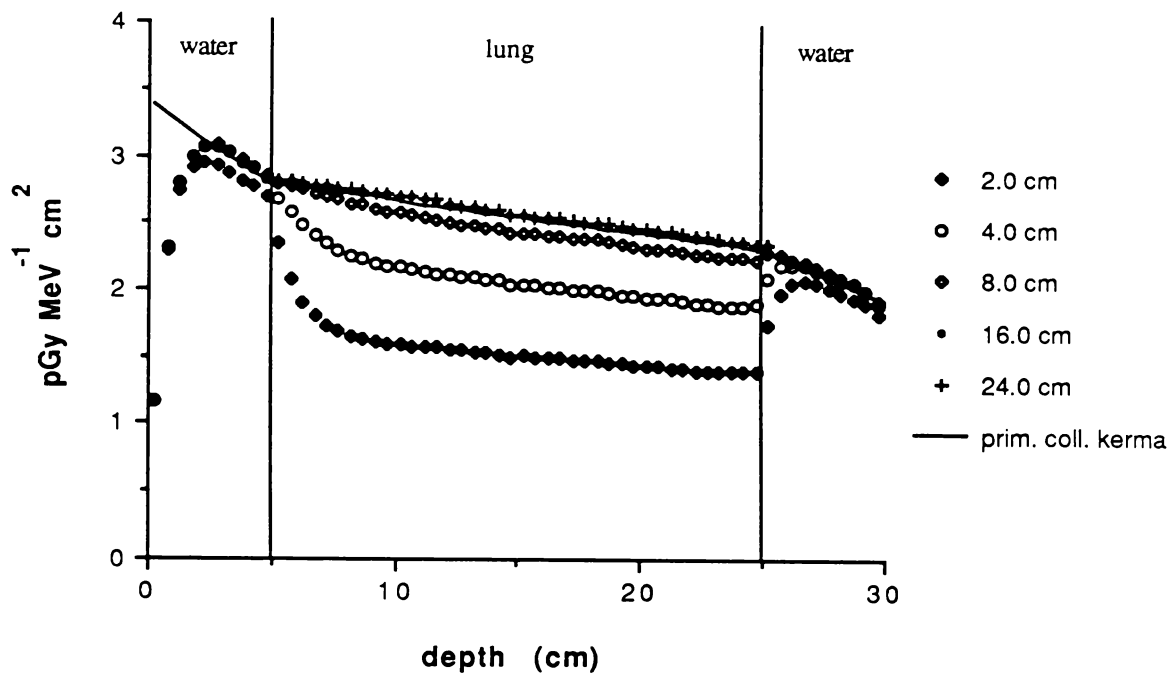


Figure 2.21. Primary dose at central axis for parallel beams with diameters of 2.0, 4.0, 8.0, 16.0, and 24.0 cm. A 20 cm ‘lung’ region of density 0.25 g cm^{-3} is sandwiched between two 5 cm thick ‘water’ regions. These diameters in the lung, are radiologically equivalent to the diameters in water in figures 2.16-2.18. Primary collision kerma is also shown.

Also shown in figure 2.21 is the collision kerma. This quantity was obtained using (2-19), except that the depth z is replaced by the radiological depth. As with the homogeneous water medium, the primary dose in equilibrium conditions is very close the collision kerma throughout the medium. Note that since the linear attenuation coefficient μ is proportional to density and the secondary electron range is inversely proportional to density, the product $\mu \bar{z}$ in (2-22) is independent of density. Therefore, the ratio of collision kerma to primary dose should be the same in the water and in the lung.

The scattered dose for the lung configuration is shown in figure 2.22. As with the homogeneous water medium, scattered dose continues to increase with beam diameter for all diameters shown. Note there is an increase in scattered dose for all diameters on entering the second water region. This is due to the increase in the frequency of scattered photon interactions as density increases.

Total dose for the lung configuration is shown in figure 2.23. Note that the increase in dose from the 4.0 cm to 16.0 cm beams in lung is mainly due to an increase in primary dose

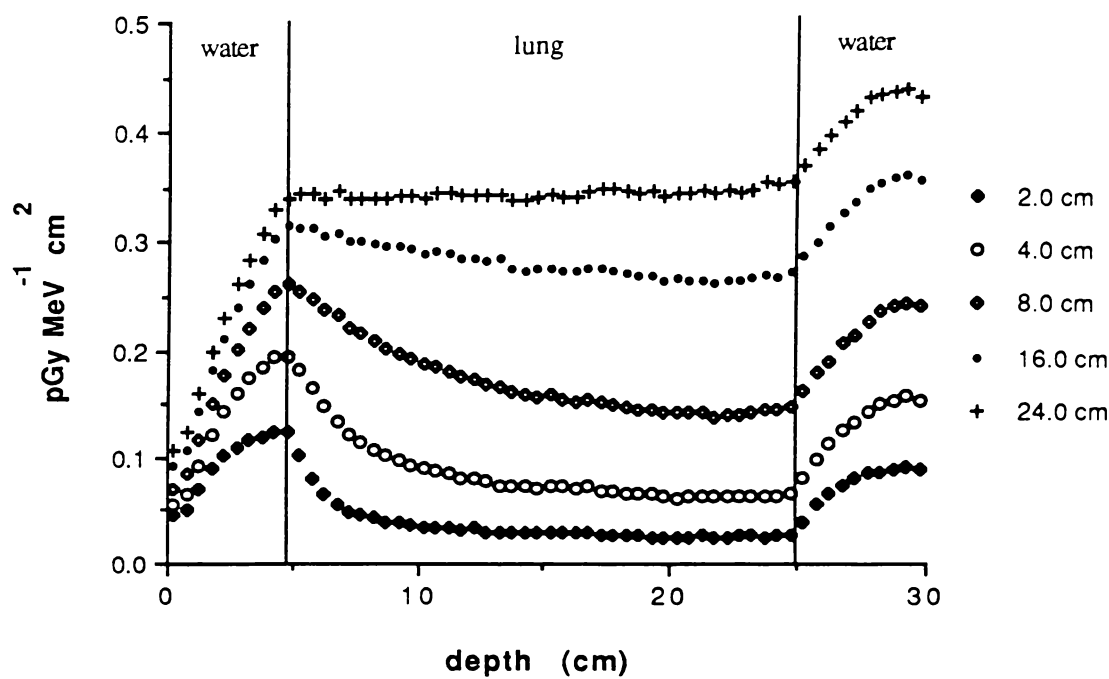


Figure 2.22. Scattered dose for the lung configuration. Due to the long range of scattered photons, dose increases with diameter for all diameters shown.

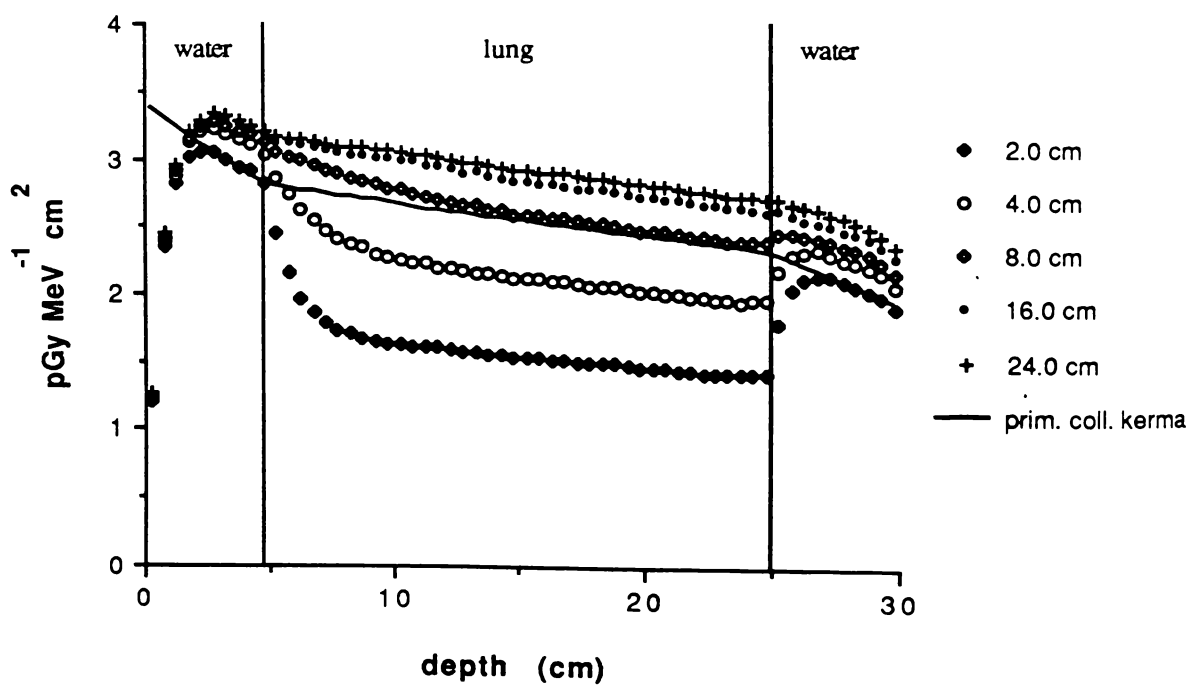


Figure 2.23. Total dose obtained by summing the primary and scattered doses for the lung configuration. Primary collision kerma is included as a reference for comparison between the primary and total dose.

whereas in the water region this increase is entirely due to an increased scattered dose. The collision kerma is included in the graph as a reference for comparison between the magnitudes of the primary and total dose.

A plot of the lateral electron equilibrium factor for the lung configuration is shown in figure 2.24. From this graph, it can be seen that in the water region near to the water→lung interface, disequilibrium exists for the 4.0 cm beam. This is because although equilibrium with respect to electrons originating in the water is established, a further increase in beam diameter would cause an increased dose due to those electrons originating in the lung. In the lung past the interface, disequilibrium increases as the primary dose due to secondary electrons originating in the lung becomes a larger proportion of the total primary dose. This increase continues until the distance past the interface becomes greater than the maximum distance of penetration into the lung by electrons originating in the water, at which point the degree of disequilibrium is only influenced by secondary electrons originating in the lung region. In the lung region near the lung→water interface, where electrons originating in the water beyond the interface contribute to the dose, disequilibrium decreases. Past the interface, as the dose contribution from the lung decreases, disequilibrium decreases until equilibrium is again established for the 4.0 cm beam.

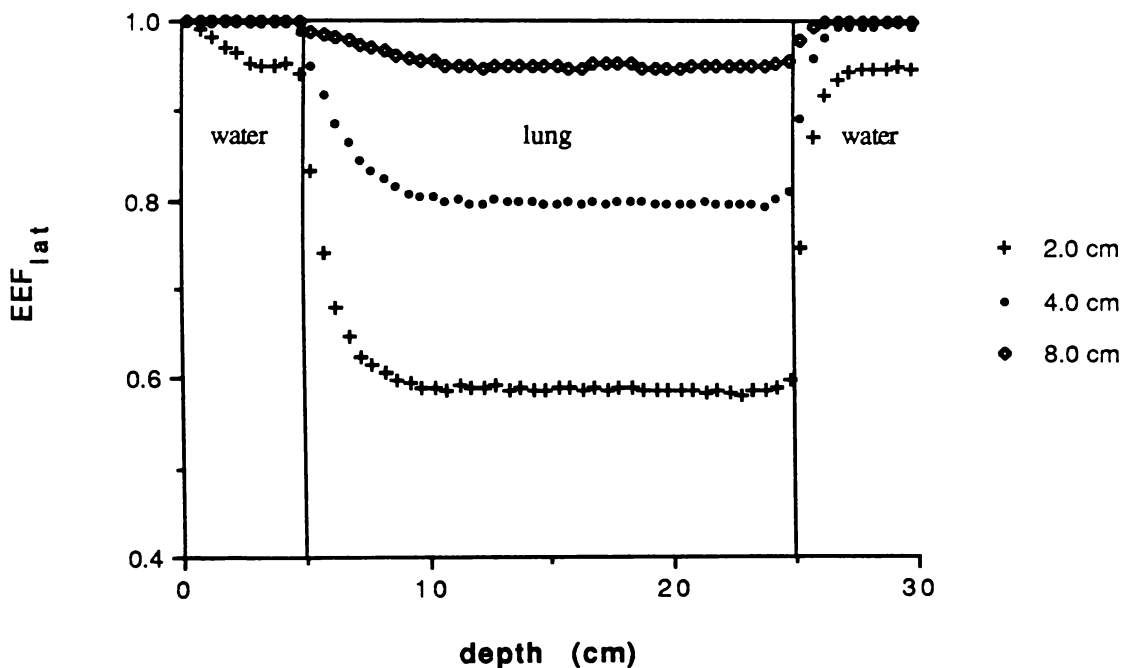


Figure 2.24. Lateral electron equilibrium factor for the lung configuration. The factor reduces in water as the lung interfaces are approached due to the increase in dose contribution from secondary electrons originating in lung. In lung, as the water interfaces are approached, disequilibrium decreases as the dose contribution from secondary electrons originating in water increases.

Investigations of the reduction in dose due to lateral electron disequilibrium in lung were carried out by Kornelson and Young (1982). In that work, the dose due to a 10 MV beam

in low density media, for a range of square field sizes was measured. Media of relative electron density 0.162, 0.332, and 1.0 were used in the investigation. From the results, a loss factor was obtained. This factor represents the 'loss' in total dose due to lateral electron disequilibrium and reduction in scattered photons, as a function of (equivalent circular beam radius) $\times$ (relative electron density) (Young and Kornelson, 1983). This loss factor is the central axis dose at d_{max} for the low density material relative to the dose in water at d_{max} for a reference field size and can be used to determining the dose rate for low density media.

Dose in a 'lung phantom' containing a slab of low density material has also been measured by Rice *et al* (1988). From these measurements, correction factors which convert the dose in water at a certain depth for a certain field size to the dose in the lung phantom have been obtained — a correction factor of less than 1 means that the dose at that depth is less than that in water. With an increase in the degree of disequilibrium in the lung, there is a decrease in the value of the correction factor. Both 4 MV and 15 MV beams were used to irradiate phantoms containing lung of density 0.015, 0.18, and 0.31 g cm⁻³. The correction factors in the lung were found to decrease as (i) beam energy increases, (ii) lung density decreases, and (iii) field size decreases.

The problems which lateral electron disequilibrium in lung can cause in dose calculations are discussed in chapter 4.

2.5. Comparison with Measured Radiosurgery Beam Data

Small circular beams such as those investigated here are employed in the treatment of brain tumours using stereotactic radiosurgery. Treatment is carried out by first rigidly fixing the patient so that the tumour is at the isocentre of the linear accelerator. During treatment, the gantry is rotated through several different arcs, each time with the tumour at the isocentre.

Radiosurgery is carried out at the Department of Radiation Oncology, Prince of Wales Hospital, Sydney. At that hospital, specially constructed collimators which attach to the linear accelerator treatment head have been engineered to define the small circular beams (Amies *et al*, 1991). During treatment, the primary collimators (photon beam jaws) in the treatment head are set to define a 4.0 cm square field. Measured central axis depth dose curves, as well as dose rates at 2.0 cm deep, were obtained for 10MV beams with diameters of 1.0, 2.0, and 3.0 cm. The SSD was 100 cm. Dosimetry was performed using the IAEA protocol (Andreo *et al*, 1987), with a 0.125 cm³ PTW ionization chamber. Output factors relative to a 10.0 cm square beam (defined by the primary collimators) were obtained by division of the dose rates at 2.0 cm deep for each beam diameter by the dose rate for the 10.0 cm square beam at this depth. Results were kindly forwarded to the author for comparison with Monte Carlo results. The measured output factors at 2.0 cm deep are given in table 2.5.

beam diameter (cm)	output factor
1.0	0.76
2.0	0.88
3.0	0.91

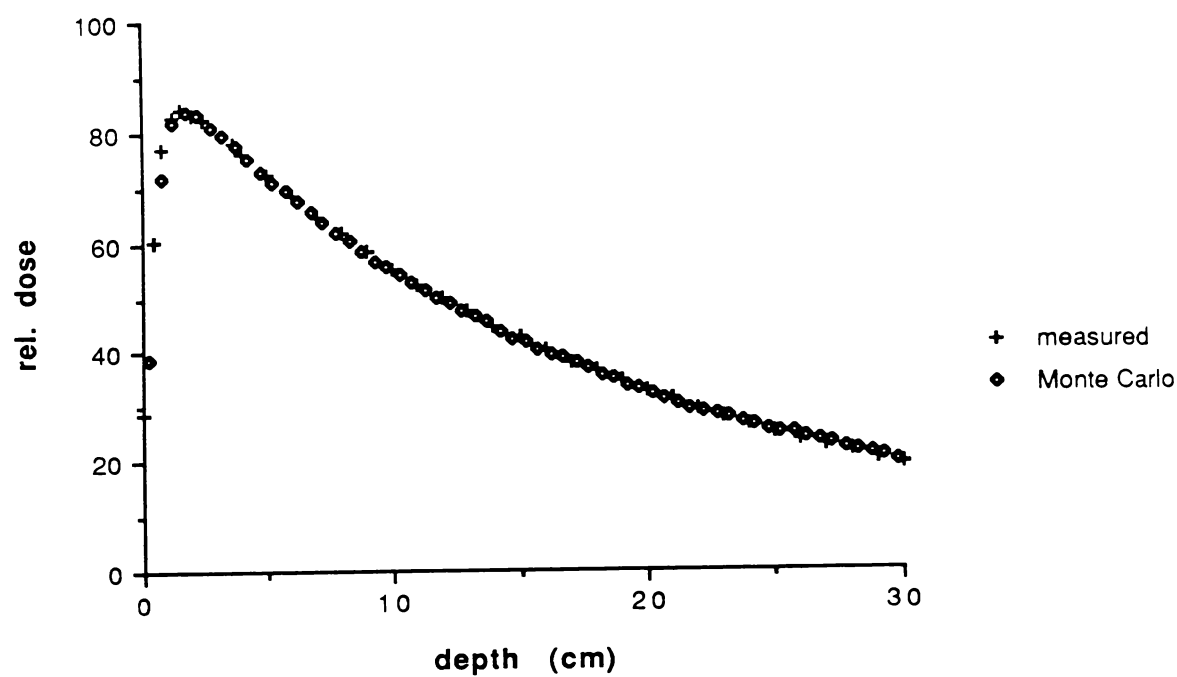
Table 2.5. Measured output factors at 100 cm SSD for circular 10MV beams with diameters of 1.0, 2.0, and 3.0 cm.

For comparison with Monte Carlo results, the measured dose for each beam diameter was normalised so that the dose at 2.0 cm deep for the 3.0 cm beam was 100%. Central axis dose calculated using the reciprocity technique, with the divergence correction discussed in section 2.3.4, was normalised to 100% at 2.25 cm deep for the 3.0 cm diameter beam. For a rigorous comparison with the measured data the Monte Carlo data should be normalised at 2.0 cm deep, however the centre of a voxel layer falls at 2.25 cm deep and the dose changes very little from 2.0 to 2.25 cm deep.

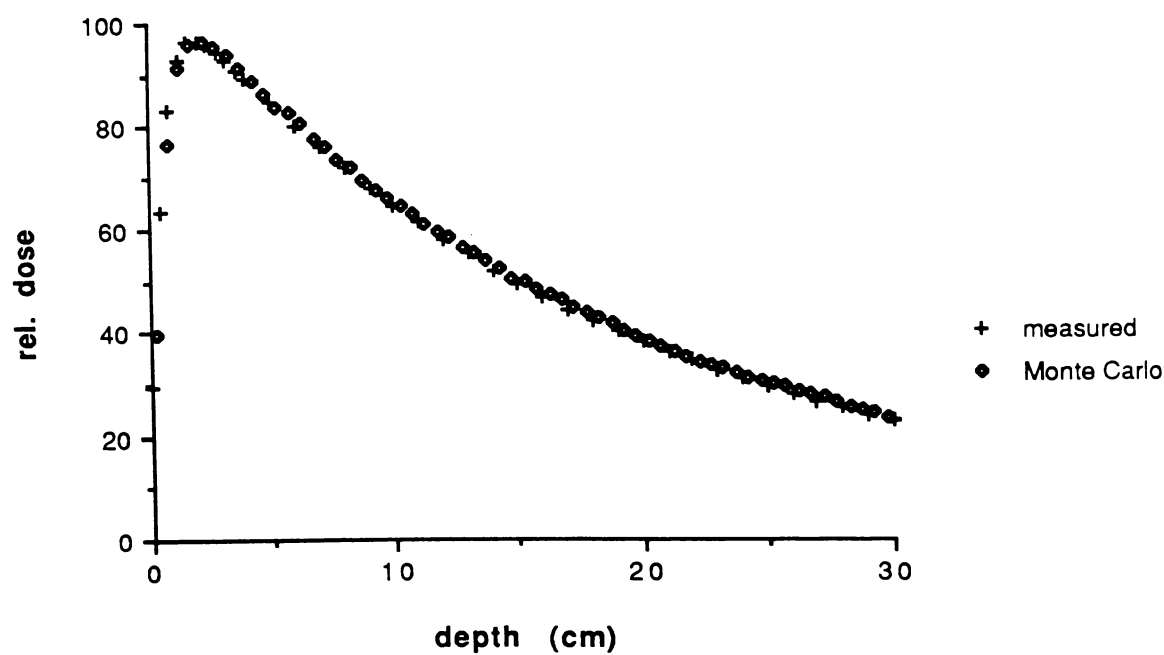
Comparison of measured and Monte Carlo generated relative depth dose for 1.0 and 2.0 cm diameter beams is shown in figure 2.25(a) and 2.25(b) respectively. The magnitudes of the dose relative to the 3.0 cm beam at the normalisation depth as obtained by each method is in excellent agreement for all diameters. This indicates that the variation of dose with beam diameter is accounted for accurately by the variation in lateral electron disequilibrium.

The rate of falloff of the depth dose curves is also in excellent agreement indicating the photon spectrum used for the Monte Carlo simulations is an accurate representation of that which exists for the radiosurgery beams. Since the same photon spectrum also gives depth dose results in good agreement with measured data for larger fields without a radiosurgical collimator attached (Metcalf *et al*, 1989), it appears that the spectrum is not altered significantly by the radiosurgery beam collimation system. Also, this good agreement indicates that the small beams are modelled adequately by a monodirectional beam — at least for the calculation of central axis depth dose.

In figure 2.26, the measured depth dose for a 10.0 cm square beam is compared with that predicted by a Monte Carlo simulation for a 10.0 cm diameter beam. As with the smaller beams, the dose is normalised to the maximum dose for a 3.0 cm diameter beam. The dose relative to a 3.0 cm diameter beam is seen to be significantly higher for the measured data. This phenomenon is further illustrated in figure 2.27, where the relative dose at 10.0 cm deep (measured) and 10.25 cm deep (Monte Carlo) is plotted for each field size. The relative dose predicted by Monte Carlo for the 10.0 cm diameter beam is 5% lower than that obtained from measurement for the 10.0 cm square beam. This indicates that for the small circular beams, the measured reduction in dose at central axis compared with the 10.0 cm square beam is greater than can be accounted for by lateral electron disequilibrium and a reduced scattered dose component. The reduction in dose must also be due to a



(a)



(b)

Figure 2.25. Measured and Monte Carlo generated central axis dose for radiosurgery beams with diameters of (a) 1.0 cm and (b) 2.0 cm. Measured dose is normalised so that the dose at 2.0 cm deep for a 3.0 cm diameter beam is 100%. The Monte Carlo dose is normalised to 100% in a 3.0 cm beam at 2.25 cm deep.

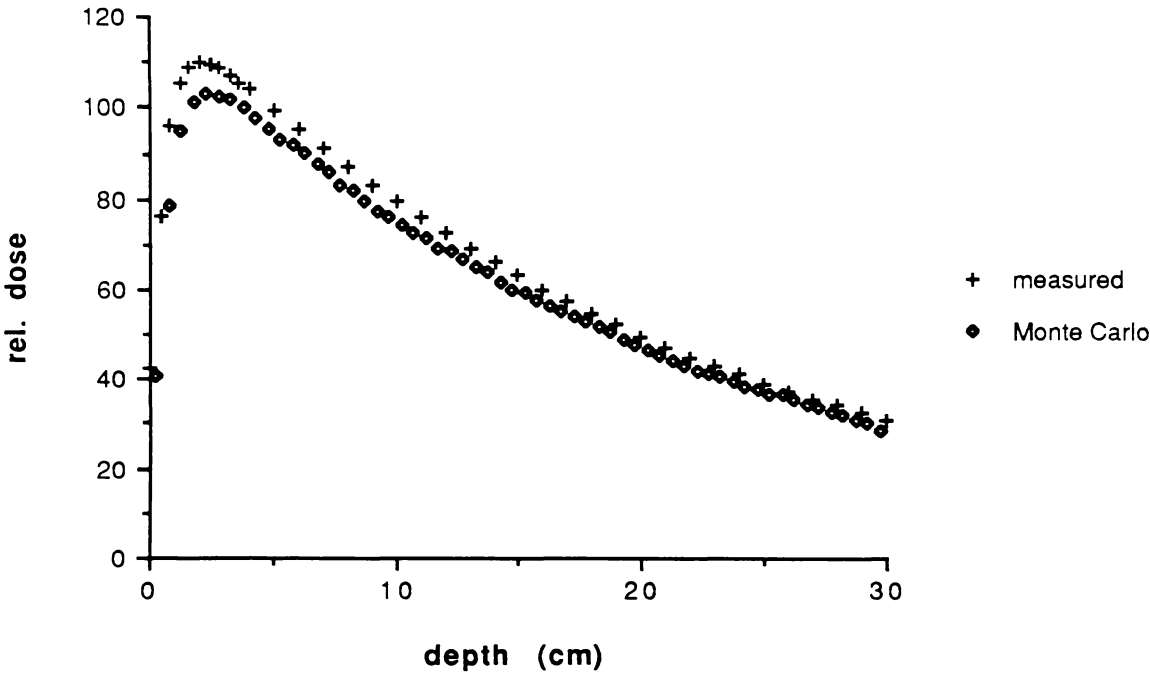


Figure 2.26. Measured dose for a 10.0 cm square beam normalised to 100% at 2.0 cm deep in a 3.0 cm diameter beam, and Monte Carlo generated dose for a 10.0 cm diameter beam normalised to 100% for a 3.0 cm beam at 2.25 cm deep.

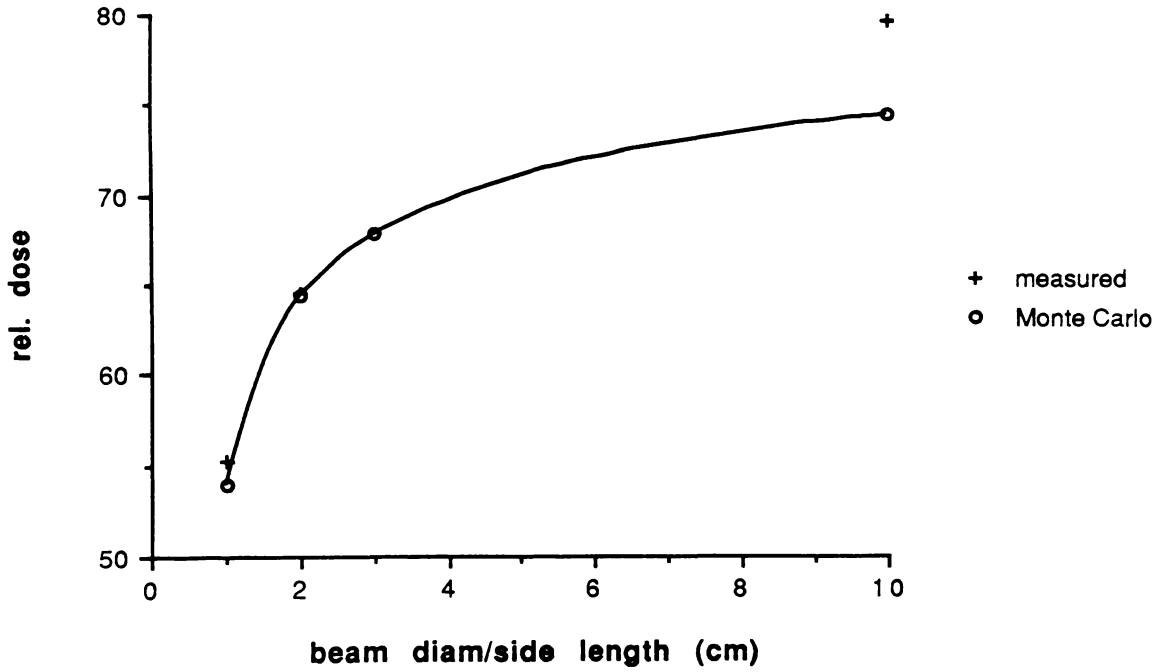


Figure 2.27. Measured and Monte Carlo generated dose at 10.25 cm deep for beam diameters of 1.0, 2.0, and 3.0 cm, as well as measured dose for a 10 cm square beam and Monte Carlo generated dose for a 10 cm diameter beam. Dose is normalised in the same way as in figures 2.25 and 2.26.

reduced primary photon fluence. Since the primary collimators are set to define a 4.0 cm square beam at the SAD for the radiosurgery beams, this reduced primary photon fluence from that which exists for the 10.0 cm beam is expected. This is due to the well known increase in photon scatter from the primary collimators as field size increases (Johns and Cunningham, 1983).

2.6. Summary and Discussion

When calculating central axis dose for a beam of circular cross section using a Monte Carlo simulation, a large increase in efficiency is obtained by employing the reciprocity technique, instead of a full simulation. With the reciprocity technique, the diameter of the incident beam is equal to the diameter of the central axis detection region which would be used in an equivalent full simulation. For a particular beam diameter, the efficiency gain increases as the diameter of the central axis detection region used in the full simulation decreases. This is particularly important for small beams where the dose profile is sharply peaked, since in this case a very narrow detection region is required in a full simulation to give a true representation of central axis dose. The efficiency gain was also found to increase slightly with a decrease in the thickness of the detection voxels. Using the reciprocity technique, the central axis dose for several beam diameters can be obtained in the same simulation — this further enhances the usefulness of the technique.

A modification has been made to the reciprocity technique, which allows central axis dose for small divergent beams to be calculated accurately. This generalisation of the technique accounts for the dependence on beam diameter of the degree of lateral electronic disequilibrium.

Lateral electron disequilibrium has a significant effect on the central axis dose for small beam cross sections. For the 10 MV beam, disequilibrium exists for beams with diameters of less than approximately 4 cm when the irradiated medium is of unit density. The beam radius required for equilibrium to exist corresponds to the maximum range of secondary electrons in a direction perpendicular to the incident beam. For lung of density 0.25 g cm^{-3} the minimum diameter for equilibrium is increased to approximately 16 cm.

Using the modification to the reciprocity technique to account for beam divergence, Monte Carlo results which agree closely with measured data for radiosurgery beams were obtained. This agreement is for both the relative output associated with each beam diameter, and the shape of the depth dose curves.

Chapter 3

Polyenergetic Convolution Energy Deposition Kernels

3.1. Introduction

The convolution method of photon beam dose calculation is becoming an increasingly practical alternative to traditional methods, because of the increasing availability of powerful computer hardware as well as continuing developments in the convolution algorithm. Dose calculations using the convolution method are possible for arbitrary beam geometries, and if the method is generalised to a superposition, calculations can be performed for a medium containing inhomogeneities (Mackie *et al*, 1985; Mohan *et al*, 1986; Ahnesjö, 1989). For beams of rectangular cross section, it is normal to represent the irradiated medium and the calculated dose distribution by three-dimensional cartesian arrays (Mackie *et al*, 1985).

In this chapter, the convolution method is used to calculate the central axis dose for small, non-divergent 10 MV beams of circular cross section, such as those investigated in chapter 2. A cylindrical array, identical to that used for the Monte Carlo simulations in chapter 2, is used to store the calculated dose. This array contains 60 layers of thickness $\Delta Z = 0.5$ cm, and 12 radial shells of thickness $\Delta R = 0.25$ cm (see figure 2.9). As with the Monte Carlo calculations, dose for multiple beam diameters may be determined in a single run of the convolution program. With the calculation array used, central axis depth dose distributions can be obtained for beams with diameters ranging from 0.5 to 6.0 cm, in increments of 0.5 cm.

As the photon beam penetrates the medium, each primary photon interaction ‘removes’ the energy of that photon from the primary beam. The energy removed due to a unit mass of the medium is represented by the term μ (see section 1.13.4). This removed energy is then present as the kinetic energy of secondary electrons and scattered photons, and is subsequently deposited in a *fractional energy distribution* about the primary photon interaction site. This distribution is represented by the energy deposition kernel (EDK). EDKs are generated in a Monte Carlo simulation by forcing primary photons to interact at a point, and recording the distribution of energy deposited about this point. If the primary photons used to form the EDK have a spectrum of energies, then the EDK is called a *polyenergetic* EDK. If a single photon energy is used, a *monoenergetic* EDK is produced. The process of EDK generation is discussed in section 3.3.2. In this work, separate primary and scattered EDKs are used, where the separation of these components is performed using the procedure outlined in section 2.4.2. The EDKs are represented by cylindrical arrays

containing voxels with the same dimensions as the dose calculation array. Since only central axis dose is being calculated, the reciprocity theorem may be applied.

In the convolution algorithm, the terma and the EDKs are *convolved*, where the EDKs are the convolution kernels, to obtain the dose distribution. Convolution using polyenergetic EDKs is not ideal because of the effects of *beam hardening* — the increase in mean primary photon energy with depth. The validity of using polyenergetic EDKs in the convolution calculation, is investigated here by comparing results with Monte Carlo calculations.

Primary EDKs are also used in this chapter to predict the lateral electron equilibrium factor (EEF_{lat}) discussed in section 2.4.3. Values predicted using polyenergetic EDKs formed from two different primary photon spectra are compared with the values found from Monte Carlo generated depth dose curves in section 2.4.3.

3.2. Beam Hardening

Since the linear attenuation coefficient μ decreases with increasing photon energy (Johns and Cunningham, 1983), photons at the low end of the energy spectrum are lost out of the primary beam more rapidly than those with a higher energy. This means that the high energy components in the spectrum become increasingly significant with depth — the beam *hardens*. If the primary photon spectrum at the medium surface has N components, each containing a relative fluence $F_n(0)$ then the relative fluence in component n at a depth z is obtained from

$$F_n(z) = \frac{F_n(0)e^{\mu_n z}}{\sum_{m=1}^N F_m(0)e^{\mu_m z}}. \tag{3-1}$$

As depth increases, values of $F_n(z)$ corresponding to the high energy components in the spectrum become higher in comparison to values of F_n corresponding the lower energy components. This causes a rise in the mean primary photon energy with depth. The primary photon spectra at depths of 0, 20, and 40 cm, calculated by applying (3-1) to the spectrum in figure 1.1 are shown in table 3.1.

	Energy (MeV)									
	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
$F_n(0cm)$	0.17	0.22	0.20	0.13	0.09	0.07	0.05	0.03	0.02	0.01
$F_n(20cm)$	0.06	0.18	0.21	0.16	0.12	0.10	0.07	0.05	0.03	0.01
$F_n(40cm)$	0.02	0.12	0.20	0.17	0.14	0.13	0.10	0.07	0.04	0.02

Table 3.1. Primary photon spectra for depths of 0, 20 and 40 cm in water for the Varian Clinac 18/10 10 MV photon beam. The 0 cm spectrum is from figure 1.1, and the 20 and 40 cm spectra are calculated using (3-1)

3.3. Convolution for Circular Beams

3.3.1. Terma Calculation

The terma at depth z for primary photons in the n th energy interval is given by

$$T_n(z) = F_n(0)e^{-\mu_n z} E_n \left(\frac{\mu}{\rho} \right)_n \quad (3-2)$$

where E_n is the mean energy in the n th interval. The linear attenuation and mass attenuation coefficients μ_n and $(\mu/\rho)_n$ are equal in this case since the density is unity. Values for μ_n can be obtained from tables (Johns and Cunningham, 1983). The terma at depth z is obtained by summing the terma for each energy component:

$$T(z) = \sum_{n=1}^N T_n(z). \quad (3-3)$$

Since the beam is assumed to be uniform in cross section, and the medium is homogeneous, the terma is a function of depth only. The terma is calculated and stored at discrete depths corresponding to the mid-point of each layer in the dose calculation array.

3.3.2. Cylindrical Energy Deposition Kernels

The EDK arrays used in this work were generated using the EGS4 Monte Carlo code RTPCYL (Murray, 1990), which contains special features for producing cylindrical EDKs. Each EDK array has 64 layers with indices running from -4 to 59 and has 12 radial shells with indices from 0 to 11. In the formation of an EDK, the origin of the primary photons is defined to be at a point in the axis of the EDK array, at the centre of the layer with depth index 0. Before each photon is transported, its mean free path is set to zero so that it interacts at this origin point. The origin point is near the ‘top’ of the EDK arrays since very little energy is deposited ‘upstream’ of the interaction site (Mackie *et al*, 1988).

The value stored in each voxel is the fraction of the energy imparted by primary photons interacting at the origin point, which is deposited in that voxel. This *fractional energy* is stored in the primary or the scattered EDK depending on whether the secondary electron depositing the energy is tagged ‘primary’ or ‘scattered’, as described in section 2.4.2. Values in the primary and scattered EDKs are denoted by $H_p(l, i)$ and $H_s(l, i)$ respectively, where l is the layer index in the array and i is the radial shell index. The sum of the primary and scattered EDK values is denoted by the *total* EDK $H(l, i)$.

For monoenergetic incident photons, values in the EDK can be determined by simply recording the energy deposited in each voxel, and dividing by the total energy of the incident primary photons. If the total energy imparted by primary photons at the interaction site is E_{tot} and the primary energy deposited in voxel (l, i) is $E_p(l, i)$, then the primary EDK value for voxel (l, i) is

$$H_p(l, i) = \frac{E_p(l, i)}{E_{tot}}. \quad (3-4)$$

For M monoenergetic primary photons of energy E , E_{tot} is given by ME . The primary energy deposited in voxel (l, i) is given by

$$E_p(l, i) = \sum_{m=1}^M \epsilon_p(l, i)_m \quad (3-5)$$

where $\epsilon_p(l, i)_m$ is the primary energy deposited in voxel (l, i) due to photon m . Similarly, the scattered EDK value for voxel (l, i) is

$$H_s(l, i) = \frac{E_s(l, i)}{E_{tot}} \quad (3-6)$$

where

$$E_s(l, i) = \sum_{m=1}^M \epsilon_s(l, i)_m. \quad (3-7)$$

For a polyenergetic beam however, it is not correct to simply force the interactions of M photons, and to form the EDKs by the same method as for a monoenergetic beam. This is the case for the following reason. In the primary beam, photons of different energies have different attenuation coefficients μ (the cause of beam hardening), and thus have different probabilities of interacting — this is accounted for in the terma calculation. This must also be accounted for in forming the EDKs, so that the fractional energy distribution in the EDKs is representative of that which would exist if the primary photons interacted with their true probabilities.

A conceptually simple way to obtain the correct EDK for a certain primary photon energy spectrum, is to direct a large number of photons (which have an energy distribution determined by the spectrum) toward a small volume representing the EDK origin and to discard any scattered photons or secondary electrons which originate from primary photon interaction at points which are not within that volume. The fractional energy distribution due to primary photons which interact in the small volume could then be recorded. In this way, the dependence on energy of the likelihood of a primary photon interaction is accounted for. This method however is a ‘waste’ of photons since very few incident photons would actually interact in the small volume.

A more efficient method of creating the correct EDKs is to force the interactions, but account for the variation in μ by weighting both the energy imparted in the interaction (the primary photon energy), and the energy deposited due to that interaction, by μ . Using this method, if there are M primary photons interacting then the total energy imparted E_{tot} is given by

$$E_{tot} = \sum_{m=1}^M E_m \mu_m. \quad (3-8)$$

Similarly, the primary energy deposited in voxel (l, i) is given by

$$E_p(l, i) = \sum_{m=1}^M \epsilon_p(l, i)_m \mu_m \quad (3-9)$$

and the scattered energy deposited in voxel (l, i) is

$$E_s(l, i) = \sum_{m=1}^M \epsilon_s(l, i)_{m\mu_m} \quad (3-10)$$

The values of the polyenergetic primary and scattered EDKs for voxel (l, i) are then obtained using (3-4) and (3-6) respectively. The summation over m in these equations, can be written as a summation over the energy components in the spectrum, and for each energy interval, as summation over the number of photons within that interval. In this way, if the relative fluence of component n is F_n , (3-8) can be written

$$\begin{aligned} E_{tot} &= \sum_{n=1}^N \sum_{m=1}^{F_n M} E_n \mu_n \\ &= M \sum_{n=1}^N F_n E_n \mu_n \end{aligned} \quad (3-11)$$

where M is the total number of primary photons as in (3-8). Similarly, if $\epsilon_p(l, i)_{n,m}$ is the primary energy deposited in voxel (l, i) due to photon m in energy interval n , then the primary energy deposited in voxel (l, i) is given by

$$\begin{aligned} E_p(l, i) &= \sum_{n=1}^N \sum_{m=1}^{F_n M} \epsilon_p(l, i)_{n,m} \mu_n \\ &= \sum_{n=1}^N F_n \mu_n \sum_{m=1}^M \epsilon_p(l, i)_{n,m}. \end{aligned} \quad (3-12)$$

Now for M monoenergetic photons of energy E_n , $E_{tot} = M E_n$, and using (3-5), the primary EDK for component n is

$$H_{p,n}(l, i) = \frac{\sum_{m=1}^M \epsilon_p(l, i)_{n,m}}{M E_n}. \quad (3-13)$$

Substituting (3-11) and (3-12) into (3-4), and using (3-13), gives the polyenergetic primary EDK:

$$H_p(l, i) = \frac{\sum_{n=1}^N F_n \mu_n E_n H_{p,n}(l, i)}{\sum_{n=1}^N F_n \mu_n E_n}. \quad (3-14)$$

From this equation, it is apparent that $H_p(l, i)$ is a weighted sum of the monoenergetic primary EDKs $H_{p,n}(l, i)$. These weights are simply the term for component n without any attenuation — this can be seen by setting z to zero in (3-2). The polyenergetic primary kernel for any spectrum defined by the relative fluences F_n is therefore given in terms of the unattenuated term components $T_n(0)$ for that spectrum:

$$H_p(l, i) = \frac{\sum_{n=1}^N T_n(0) H_{p,n}(l, i)}{\sum_{n=1}^N T_n(0)}. \quad (3-15)$$

Similarly, the scattered polyenergetic EDK is given by a weighted sum of the monoenergetic scattered EDKs.

$$H_s(l, i) = \frac{\sum_{n=1}^N T_n(0) H_{s,n}(l, i)}{\sum_{n=1}^N T_n(0)} \quad (3-16)$$

Note that for a polyenergetic 'surface spectrum' EDK, $T_n(0)$ in (3-15) and (3-16) is given by (3-2) with z set to zero.

3.3.3. Convolution using the Reciprocity Technique

The dose on the central axis of a non-divergent circular beam at layer k in the array, due to primary photon interactions on layer k' , may be determined by applying the reciprocity theorem (Bielajew and Rogers, 1988) to the convolution method. By this theorem, if the energy deposited in a unit mass on the central axis of the beam on layer k , due to a uniform term T within the beam limits on layer k' , is E , then the energy deposited within the beam limits on layer k due to a uniform term T in a unit mass on the central axis on layer k' , is also E . This is illustrated in figure 3.1, where the energy deposited in volume A (of unit mass), is equal to the energy deposited in volume B.

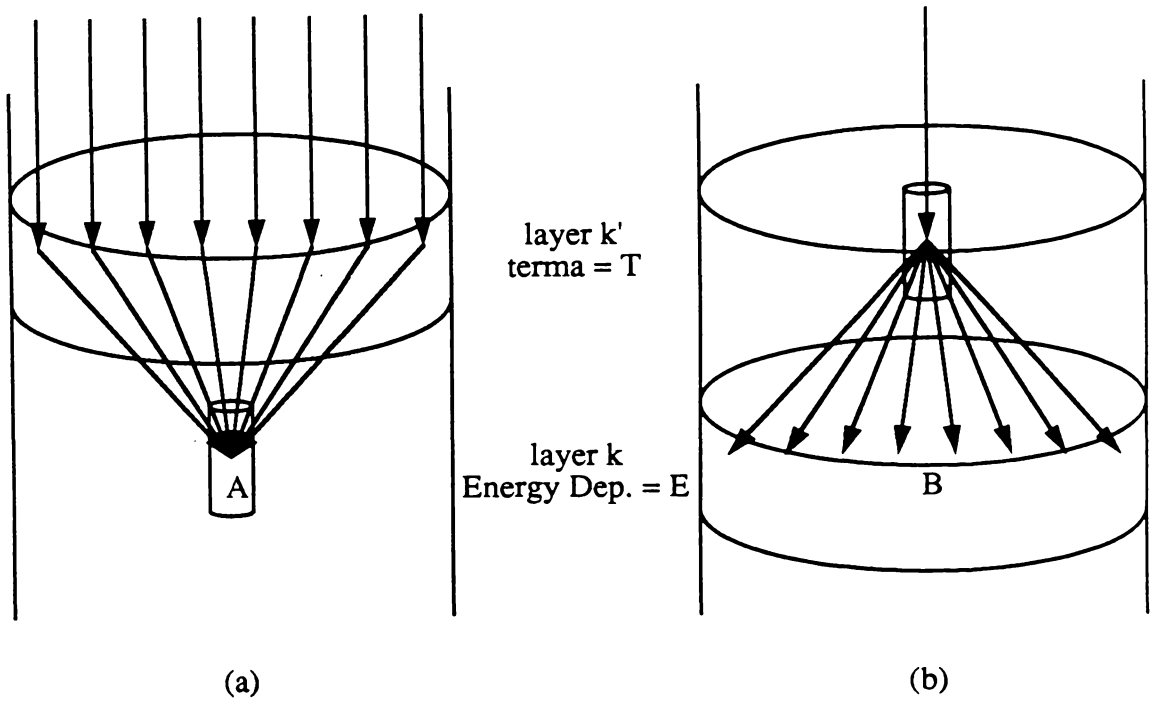


Figure 3.1 Reciprocity theorem applied to convolution dose calculation for a non-divergent circular beam. In (a), an energy E is deposited in a unit mass (region A), on the beam central axis on layer k , due to a uniform term T on layer k' . When the reciprocity theorem is applied as in (b), a pencil beam giving a term T in a unit mass on the axis on layer k' causes an energy E to be deposited on layer k (region B).

Using the reciprocity theorem, the dose at central axis on layer k due to primary photon interactions on layer k' for a beam of radius $I\Delta R$ is given by

$$D(k, k', 2I\Delta R) = \sum_{i=0}^{I-1} T(k')H(k - k', i) \quad (3-17)$$

where $T(k')$ is the term at the depth corresponding to the centre of layer k' . Note that the dose is given as a function of the beam diameter $2I\Delta R$ to be consistent with the results

in chapter 2. It is a simple process to determine the central axis dose on layer k due to primary photon interactions on layer k' for beams of multiple radii since (3-17) can be written as a recursive function of the dose due to a beam one shell thickness less in radius;

$$\begin{aligned} D(k, k', 2I\Delta R) &= \sum_{i=0}^{I-2} T(k')H(k - k', i) + T(k')H(k - k', I - 1) \\ &= D(k, k', 2(I - 1)\Delta R) + T(k')H(k - k', I - 1). \end{aligned} \quad (3-18)$$

The dose due to primary interactions on layer k' for beams with increments in radius of one shell thickness, can thus be stored in successive voxel shells. If this procedure is repeated for each layer k , the dose on the entire central axis due to interactions in layer k' can be calculated.

If there are K layers in the irradiated medium, a complete convolution can be performed by repeating this procedure for all k' from 0 up to $K - 1$. However, because of beam hardening, this repetition is not strictly valid unless for each k' , a unique EDK formed for the primary photon spectrum on that layer is used. Of importance to this work is the fact that the proportion of primary photon energy that is transferred to secondary electrons increases with photon energy, with a corresponding drop in the proportion of primary energy retained by scattered photons. The fraction of the energy removed from a monoenergetic primary beam per unit pathlength that is subsequently deposited in the medium is given by the ratio μ_{ab}/μ corresponding to the beam energy. A plot of this ratio versus energy is shown in figure 3.2.

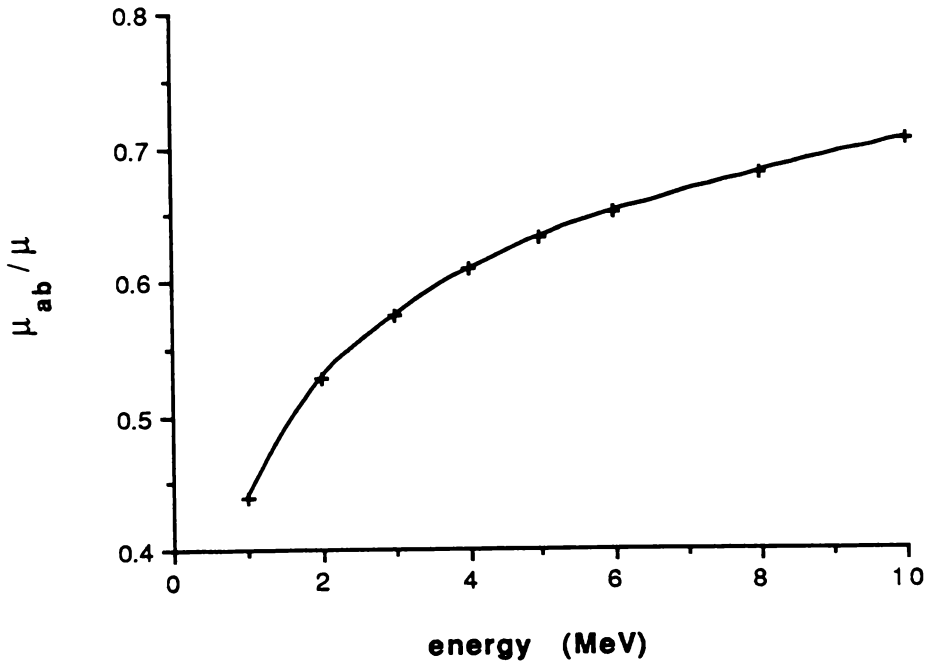


Figure 3.2 Ratio of the energy absorption coefficient for water μ_{ab} to the linear attenuation coefficient for water μ , as a function of photon energy.

Strictly then, primary and scattered EDKs should be created for each depth in the terma array. If this was done, the fractional energy contained within the primary EDK would increase with depth and the fractional energy contained in the scattered EDK would decrease. Furthermore, only some of the scattered photon energy is deposited within the extent of the scattered EDK, so as more of the total energy removed from the primary beam is transferred to secondary electrons the total fractional energy in the EDKs will increase. Also, as the primary photon spectrum hardens, the range of secondary electrons originating at the primary interaction site increases — the extent of non-zero values in the primary EDK should therefore increase with depth.

Beam hardening is fully accounted for if the dose calculation is a sum of separate convolutions carried out for each energy component in the spectrum (Mohan *et al*, 1986). With this method, the terma for each energy component is convolved with EDKs formed for that energy. This means that as the primary photon energy spectrum hardens with depth, EDKs formed for the higher energies in the spectrum become more significant since the terma for the higher energies becomes more significant. Using this method, the central axis dose on layer k of a medium represented by K voxel layers, for a beam of radius $I\Delta R$, is given by

$$D(k, 2I\Delta R) = \sum_{n=1}^N \sum_{k'=0}^{K-1} T_n(k') \sum_{i=0}^{I-1} H_n(k - k', i) \quad (3-19)$$

where T_n is defined in (3-2) and H_n is the total EDK for monoenergetic primary photons whose energy is the mean energy in the n th interval. With the EDK separated into primary and scattered components, (3-19) becomes

$$D(k, 2I\Delta R) = \sum_{n=1}^N \sum_{k'=0}^{K-1} T_n(k') \sum_{i=0}^{I-1} [H_{p,n}(k - k', i) + H_{s,n}(k - k', i)] \quad (3-20)$$

where $H_{p,n}$ and $H_{s,n}$ are the primary and scattered EDKs for energy component n .

The approach of summing over the energy components has been shown to be accurate for a variety of beam spectra (Mohan *et al*, 1986). The drawback with this method is the large amount of computation required in performing a separate convolution for each energy component. An alternative approach which uses primary and scattered EDKs produced for a single polyenergetic spectrum, but accounts for beam hardening with depth by correcting the EDKs, is proposed here.

3.3.4. Correcting for Beam Hardening

A convolution equation for this alternative approach is formulated by modifying (3-20) so that the sum over the energy components is not required. Single polyenergetic primary and scattered EDKs formed for the spectrum which exists at the surface of the medium are used. Primary and scattered EDKs for the surface photon spectrum are denoted by $H_{p,0}$ and $H_{s,0}$ respectively, where the subscript 0 indicates that the EDKs were generated for a 0 cm deep spectrum.

Expressions for the polyenergetic EDKs, $H_{p,0}$ and $H_{s,0}$, in terms of monoenergetic EDKs for each energy component are required. If the term at the surface obtained using (3-2) and (3-3) is denoted by $T(0)$ then using (3-15) and (3-16) the primary polyenergetic EDK is given by

$$H_{p,0}(k - k', i) = \frac{1}{T(0)} \sum_{n=1}^N T_n(0) H_{p,n}(k - k', i) \quad (3-21)$$

and

$$H_{s,0}(k - k', i) = \frac{1}{T(0)} \sum_{n=1}^N T_n(0) H_{s,n}(k - k', i) \quad (3-22)$$

These expressions are similar to that used by Ahnesjö (1989), who uses the primary photon energy fluence at the surface instead of the term to weight the monoenergetic EDKs. This approach however does not account for the variation in μ with energy whereas the method proposed here does.

It should be emphasised that the weighted sum of monoenergetic EDKs is *not* used in this work to generate polyenergetic EDKs. The reason for expressing the polyenergetic EDKs in this way is so that a beam hardening correction can be formulated. Such a correction is now suggested.

When (3-21) and (3-22) are combined with (3-20), the following expression is obtained with some manipulation:

$$D(k, 2I\Delta R) = \sum_{k'=0}^{K-1} T(k') \sum_{i=0}^{I-1} \left[H_{p,0}(k - k', i) \left(\frac{\frac{1}{T(k')} \sum_{n=1}^N T_n(k') H_{p,n}(k - k', i)}{\frac{1}{T(0)} \sum_{n=1}^N T_n(0) H_{p,n}(k - k', i)} \right) \right. \\ \left. + H_{s,0}(k - k', i) \left(\frac{\frac{1}{T(k')} \sum_{n=1}^N T_n(k') H_{s,n}(k - k', i)}{\frac{1}{T(0)} \sum_{n=1}^N T_n(0) H_{s,n}(k - k', i)} \right) \right]. \quad (3-23)$$

This expression requires far more computation than (3-20), and hence is not a sensible alternative in this form. However, if the terms in braces are simplified so that they are a function of the primary interaction layer k' only, they can be used to make a correction to the EDKs for each value of k' . The first of these terms will be referred to as term 1 and the second as term 2. The summation in the numerator of term 1 is the energy deposited in voxel (k, i) due to secondary electrons originating from primary photon interactions in a unit mass of material on the central axis on layer k' . This energy is thus a fraction of the primary collision kerma on layer k' , $K_c(k')$. This fraction is a function of the depth k' and the longitudinal and radial distances $k - k'$ and i , and is denoted by $\alpha_{k'}(k - k', i)$. The numerator of term 1 is then simplified to the form $\alpha_{k'}(k - k', i) K_c(k') / T(k')$. Similarly, the summation in the denominator is a fraction $\alpha_0(k - k', i)$ of the primary collision kerma at the surface $K_c(0)$, so the denominator is simplified to $\alpha_0(k - k', i) K_c(0) / T(0)$. If the approximation is now made that the fraction of the primary collision kerma deposited at coordinates $(k - k', i)$ relative to the interaction site is equal for primary photons with the energy spectrum which exists on layer k' and at the surface, then $\alpha_{k'}(k - k', i)$ and

$\alpha_0(k - k', i)$ are equal and cancel each other out. The numerator and denominator in term 1 then become simple ratios of the primary collision kerma to the terma, at layer k' and at the surface respectively.

In term 2, the summation in the numerator is a fraction $\beta_{k'}(k - k', i)$ of the scattered photon energy originating in a unit mass on layer k' . The scattered photon energy per unit mass is the difference in the terma and the primary kerma, $T(k') - K(k')$, so the numerator becomes $\beta_{k'}(k - k', i)(1 - K(k')/T(k'))$. Similarly, the denominator becomes $\beta_0(k - k', i)(1 - K(0)/T(0))$. Assuming that $\beta_{k'}(k - k', i)$ and $\beta_0(k - k', i)$ are approximately equal and may cancel, terms 1 and 2 can be written as the primary and scattered correction factors

$$C_p(k') = \frac{K_c(k')/T(k')}{K_c(0)/T(0)} \quad (3-24)$$

$$C_s(k') = \frac{1 - K(k')/T(k')}{1 - K(0)/T(0)}. \quad (3-25)$$

The correction to the primary EDK given above appears to be a logical choice since both the numerator and denominator are ratios of the primary energy deposited to the total energy removed from the beam. Since the correction factor is a ratio of this fraction at the actual depth of primary interaction to the fraction at the depth (0 cm) for which the EDK was formed, the effect of beam hardening on the proportion of primary photon energy transferred to secondary electrons should be effectively accounted for. Similarly, the scattered EDK correction factor is reasonable because it is the ratio of the scattered photon energy originating from the the interactions of primary photons with the energy spectra which exists at depth k' and at the surface respectively.

Note that because of the assumptions which allow the cancellations in terms 1 and 2, this correction does not account for the increase in secondary electron range as the primary beam hardens — only the increase in the proportion of primary photon energy removed from the beam that is subsequently deposited in the medium is corrected for.

In replacing terms 1 and 2 by the primary and scattered correction factors respectively, (3-23) becomes

$$D(k, 2I\Delta R) = \sum_{k'=0}^{K-1} T(k') \sum_{i=0}^{I-1} [H_{p,0}(k - k', i)C_p(k') + H_{s,0}(k - k', i)C_s(k')]. \quad (3-26)$$

A simple test of the validity of this method is to compare EDKs produced for a surface primary photon spectrum and a spectrum at a chosen depth z . The ratio of the fractional energy contained in the primary EDKs produced for the spectra at a depth z and at the surface, should be approximately equal to the primary correction factor for depth z . Similarly, the ratio for the scattered EDKs should approximately equal the scattered correction factor.

Primary and scattered EDKs were produced for the three spectra given in table 3.1. The upper part of the primary EDKs for the 0 cm and 20 cm deep spectra are shown in figure 3.3, where the axis of each EDK is at the left edge of the array shown, and each cell represents an annular voxel. The primary interaction site is indicated by a cross. Cells left blank indicate that the fractional energy in that voxel determined by the simulation is zero to three decimal places. For each simulation, 1.2 million incident photons were used and a CPU time of approximately 7 hours on a VAX 6000-430 was required. Simulation times increase slightly with hardening of the primary photon spectrum since secondary electrons transported away from the interaction site have increased energy and hence undergo more transport steps before stopping.

To obtain the primary correction factor given by and (3-24), the primary collision kerma at the depth of primary photon interaction is required. The primary collision kerma at depth z is given by

$$K_c(z) = \sum_{n=1}^N F_n(0) e^{-\mu_n z} E_n \left(\frac{\mu_{ab}}{\rho} \right)_n, \quad (3-27)$$

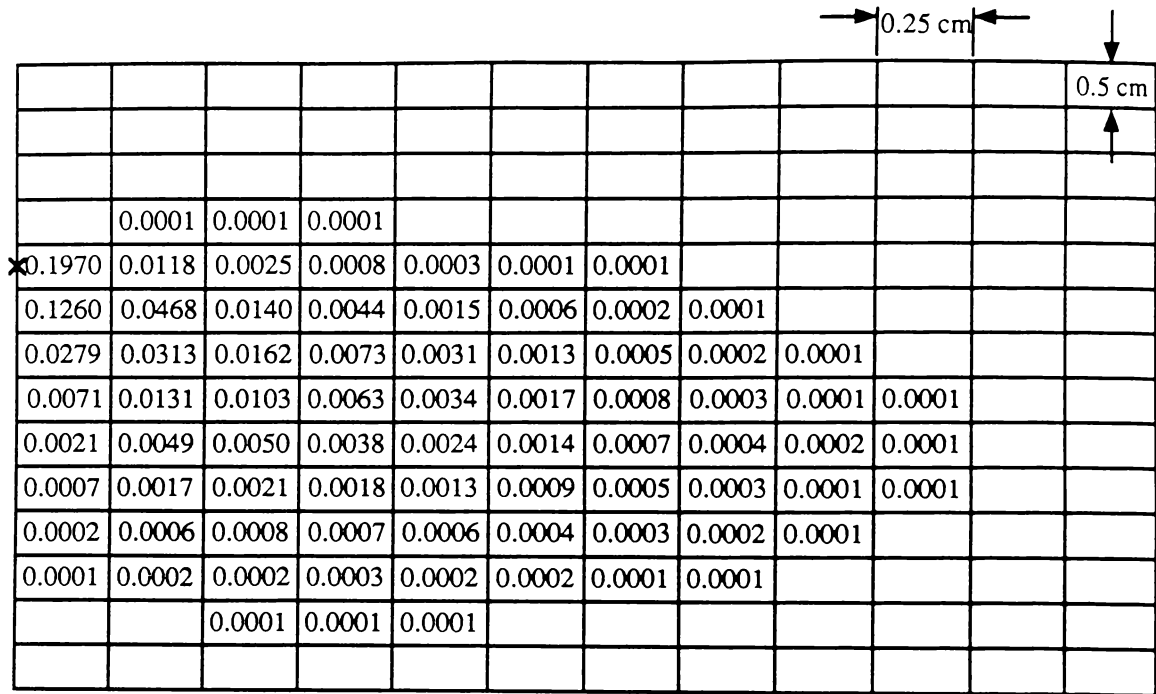
where $(\mu_{ab}/\rho)_n$ is the mass energy absorption coefficient for the n th component. Values for $(\mu_{ab}/\rho)_n$ can be obtained using tables (Johns and Cunningham, 1983). Since the fraction of secondary electron energy converted to bremsstrahlung is very small in this case (see sections 1.3.4 and 1.3.5), the primary kerma is approximately equal to the primary collision kerma. Therefore, the primary collision kerma is also used in the scattered correction factor.

By substituting calculated values for the terma and primary collision kerma into (3-24) and (3-25), the primary and scattered correction factors at 20 and 40 cm deep are found to be:

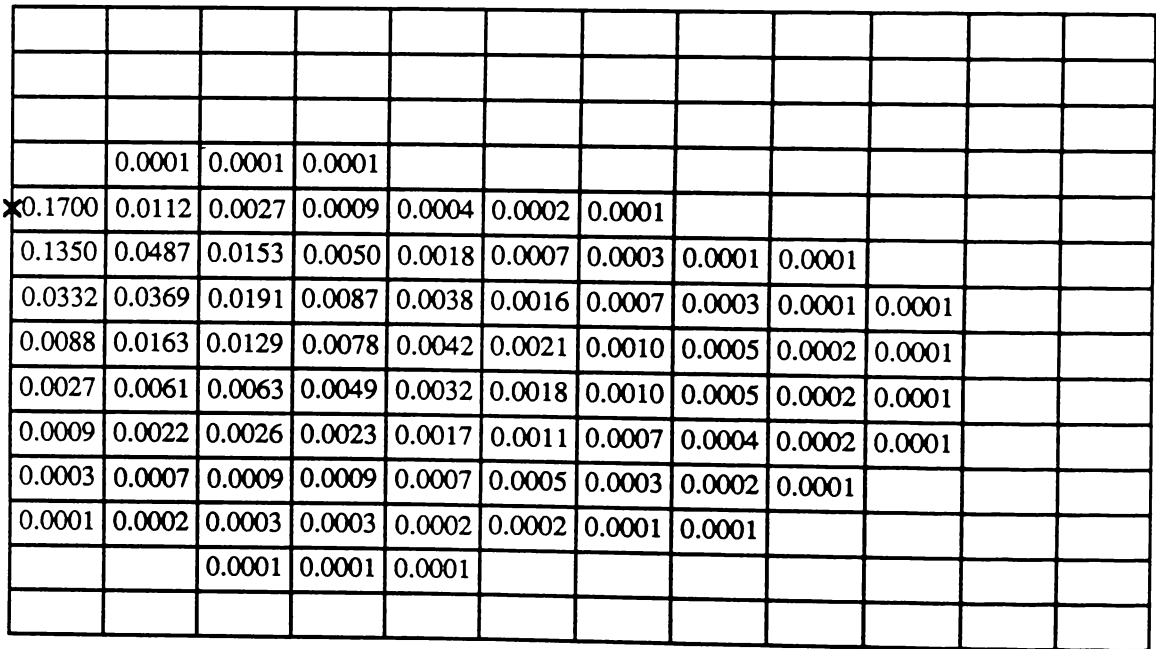
$$\begin{aligned} C_p(20\text{cm}) &= 1.046 & C_p(40\text{cm}) &= 1.072 \\ C_s(20\text{cm}) &= 0.940 & C_s(40\text{cm}) &= 0.903 \end{aligned}$$

Total fractional energy deposited within each primary and scattered EDK is given in table 3.2. Also given is the fractional energy in the 20 cm and 40 cm EDKs obtained by multiplying the fractional energy in the 0 cm EDKs by the correction factors given above. The sum of the primary and scattered components is given also. Note that the fractional energy deposited in a primary EDK is approximately 0.6 and the fractional energy contained within the scattered EDKs is approximately 0.08. This indicates that the total fractional energy deposited within these arrays is only about 0.7 — a considerable proportion (0.3) of the primary photon energy imparted, is carried out of the array boundaries by scattered photons.

From the results in table 3.2, it can be seen that the fractional energy contained in the 20 and 40 cm EDKs varies considerably from that contained in the 0 cm EDK. Note that as expected, the fractional energy in the primary EDKs increases with the depth at which the primary spectrum is obtained, whereas the fractional energy in the scattered EDKs decreases. The difference in fractional energy between the 0 cm EDKs and the 20 and 40



(a) 0 cm Primary EDK fractional energy contained = 0.574



(b) 20 cm Primary EDK fractional energy contained = 0.597

Figure 3.3 Top 14 layers of the primary EDKs for (a) surface and (b) 20 cm deep primary photon spectra. The axis of an EDK is at the left edge of the array shown, and each cell represents an annular voxel. The primary interaction site is indicated by a cross. Cells left blank indicate that the fractional energy in that voxel determined by the simulation is zero to three decimal places. For each simulation, 1.2 million incident photons were used and a CPU time of approximately 7 hours on a VAX 6000-430 was required. Fractional energy deposited within the EDK is greater for the 20 cm primary photon spectrum.

	Primary Spectrum Depth (cm)				
	0	20		40	
	direct	direct	0 corr.	direct	0 corr.
Primary	0.5740	0.5970	0.6001	0.6134	0.6153
Scattered	0.0843	0.0806	0.0792	0.0775	0.0761
Total	0.6583	0.6776	0.6793	0.6909	0.6914

Table 3.2. Fractional energy contained in 64×12 voxel EDKs. Values for the 0, 20, and 40 cm EDKs were obtained by direct simulation (direct). Values for the 20 and 40 cm EDKs were also obtained by correcting the 0 cm EDKs with the correction factors given above (0 corr.).

cm EDKs as a percentage of that in the 20 and 40 cm EDKs is given in table 3.3. Also given is the percentage difference if the 0 cm EDKs are corrected. The fractional energy in the corrected EDKs is seen to be very close to that in the 20 and 40 cm EDKS.

	Primary Spectrum Depth (cm)			
	20		40	
	0	0 corr.	0	0 corr.
Primary	-3.9	+0.5	-6.4	+0.3
Scattered	+4.6	-1.7	+8.8	-1.8
Total	-2.8	+0.3	-4.7	+0.1

Table 3.3 Percentage difference in fractional energy contained in 0 cm and corrected 0 cm EDKs, to that contained in 20 and 40 cm EDKs. Results show a much smaller difference for the corrected 0 cm EDK in all cases.

The results in tables 3.2 and 3.3 give the fractional energies contained within EDKs of 3.0 cm radius (12× 0.25 cm thick radial shells). These results indicate that the corrections are valid for beams of this radius since the beam radius is the radius in the EDKs out to which the fractional energy is summed in a convolution (see (3-17) for example). The validity of the corrections for beams of smaller radius than this, is indicated in plots of the fractional energy contained within a radius less than the outer radius of each shell in the EDKs, both for the direct Monte Carlo generation of 20 and 40 cm EDKs and for the corrected 0 cm EDKs. In figure 3.4, the fractional primary energy within shells of a radius greater than about 1.5 cm calculated from the 0 cm primary EDK is seen to agree well with the direct simulation for both spectral depths. As the shell radius becomes smaller, the fractional primary energy within the shell for the EDKs generated with the harder spectra, becomes smaller relative to that for the surface spectrum EDK. In fact, within the smallest radius (0.25 cm) the fractional energy is larger for the surface spectrum. This is because the secondary electrons set in motion by the surface spectrum photons deposit a

larger proportion of their energy within a short distance (also see figure 3.3). The primary correction factor is thus invalid for very small beams, but could be modified to cope with this situation by making the correction a function of beam diameter.

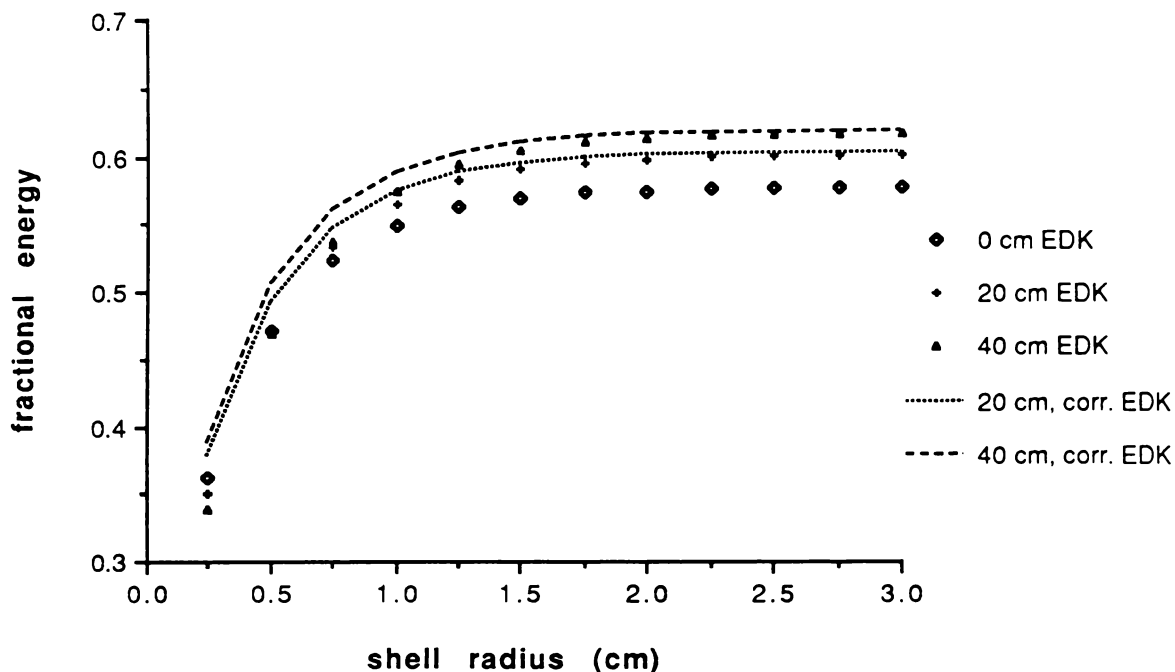


Figure 3.4 Fractional energy deposited within radial shells in the primary EDKs produced for 0, 20 and 40 cm deep primary photon spectra (see table 3.1). Fractional energy increases with spectral depth due to the increase in the proportion of primary photon energy transferred to secondary electrons. Fractional energy obtained by multiplying the 0 cm data by the primary correction factors is also shown.

The fractional scattered energy increases almost linearly with radius as seen in figure 3.5. The steady increase is because the number of scattered photons interacting in each shell is approximately equal. The plot shows that the correction to the scattered EDKs is effective. Since the fractional energy within the radial shells in the scattered EDKs increases steadily with radius, the scattered component gains in significance as beam diameter increases.

Figure 3.6 shows the total fractional energy, which is a sum of the energies plotted in figures 3.4 and 3.5. The initial increase in fractional energy within each radial shell is characteristic of the primary curve (figure 3.4), while the steady increase in fractional energy with radius is due to the scattered component. The corrected fractional energy curves give a good match to the 20 and 40 cm EDK points for a radius greater than about 1.5 cm.

3.3.5. Dose Calculation Results

Convolution calculations were performed using both invariant and corrected polyenergetic EDKs. Depth dose curves were also produced directly using a Monte Carlo simulation — the reciprocity technique was used to reduce the computation time and to allow the

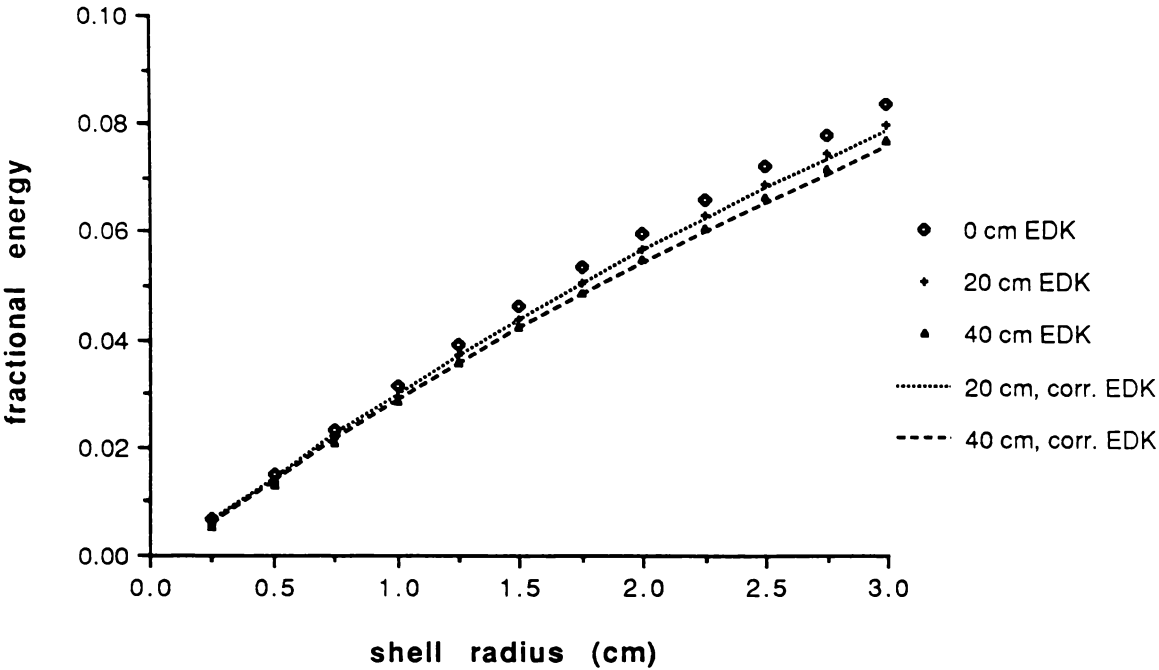


Figure 3.5 Fractional energy deposited within radial shells in the scattered EDKs produced for 0, 20 and 40 cm deep primary photon spectra (see table 3.1). Fractional energy decreases with spectral depth due to the decrease in the proportion of primary photon energy retained by scattered photons. Fractional energy obtained by multiplying the 0 cm data by the scattered correction factors is also shown.

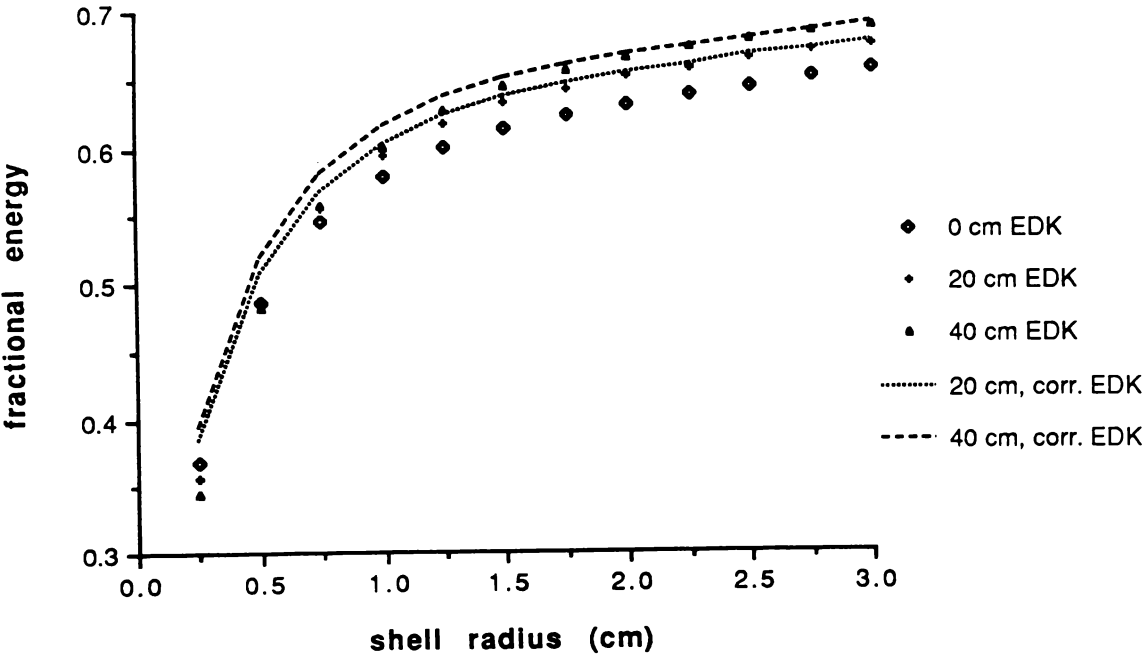


Figure 3.6 Sum of the fractional energy deposited within radial shells in the primary and scattered EDKs produced for 0, 20 and 40 cm deep primary photon spectra (see table 3.1). Fractional energy increases with spectral depth since the increase in the proportion of primary photon energy transferred to secondary electrons is more significant than the decrease in the proportion retained by scattered photons. A summation of the corrected curves in figures 3.4 and 3.5 is also shown.

calculation of central axis dose for beams of different radii. The simulation was performed using 7.2 million incident photons with energies selected from the surface photon spectrum. CPU time on a VAX 6000-430 was approximately 28 hours. Primary and scattered dose was recorded separately so that these components could be compared with those predicted by the convolution algorithm.

For the convolution calculations with uncorrected EDKs, (3-26) is used with the correction factors set to unity. Both the convolution and Monte Carlo results have units of dose per incident energy fluence ($\text{pGy MeV}^{-1} \text{ cm}^2$). These results are compared directly — no normalisation is performed. In figure 3.7, the primary, scattered and total dose for a 4.0 cm diameter beam, obtained by convolution using uncorrected 0 cm deep EDKs is compared with the Monte Carlo result. Near the surface, the primary dose obtained by convolution is seen to agree closely with the Monte Carlo result, but drops below the Monte Carlo result from approximately 5 cm deep. This behaviour is expected since the fractional energy contained in a 0 cm deep primary EDK is correct for primary interactions at the surface, but becomes increasingly lower than the correct value with the depth of primary interactions. Note that the fractional difference in the Monte Carlo and convolution curves increases with depth. The scattered doses appear to agree closely at all depths, although it is significant that the convolution dose is slightly higher. This is due to the fractional energy in the 0 cm scattered EDK becoming increasingly higher than the correct value as the depth of primary interactions increases.

If EDKs produced for a 20 cm deep spectrum are used in the convolution, the primary dose at shallow depths is higher than the Monte Carlo dose as seen in figure 3.8. This is due to excess fractional energy in the primary EDK for primary interactions at depths less than 20 cm. Between 20 and 25 cm deep the convolution curve crosses the Monte Carlo curve. This is as expected, since the fractional energy in the primary EDK is correct for primary interactions at 20 cm deep. The scattered convolution dose is slightly lower than the Monte Carlo dose near the surface since the fractional energy in the scattered EDK is lower than if the scattered EDK had been generated for a 0 cm spectrum. For depths greater than 20 cm, the scattered convolution dose is slightly higher than the Monte Carlo dose, also as expected.

To produce the convolution results in figure 3.9, EDKs generated for the primary photon spectrum at 40 cm deep were used. The primary convolution dose is now higher than the Monte Carlo dose for the entire depth range of 0 to 30 cm, and the scattered convolution dose is significantly less than the Monte Carlo dose at shallow depths.

Note that each of the three primary convolution curves fall off more sharply with depth than the Monte Carlo results. This means that if the convolution and Monte Carlo curves were normalised to the dose at d_{max} , then in all cases, the convolution dose would drop below the Monte Carlo dose for depths greater than d_{max} . It is interesting to note that the ratio of total dose in the voxel layer centred at 2.75 cm deep to the dose in the layer

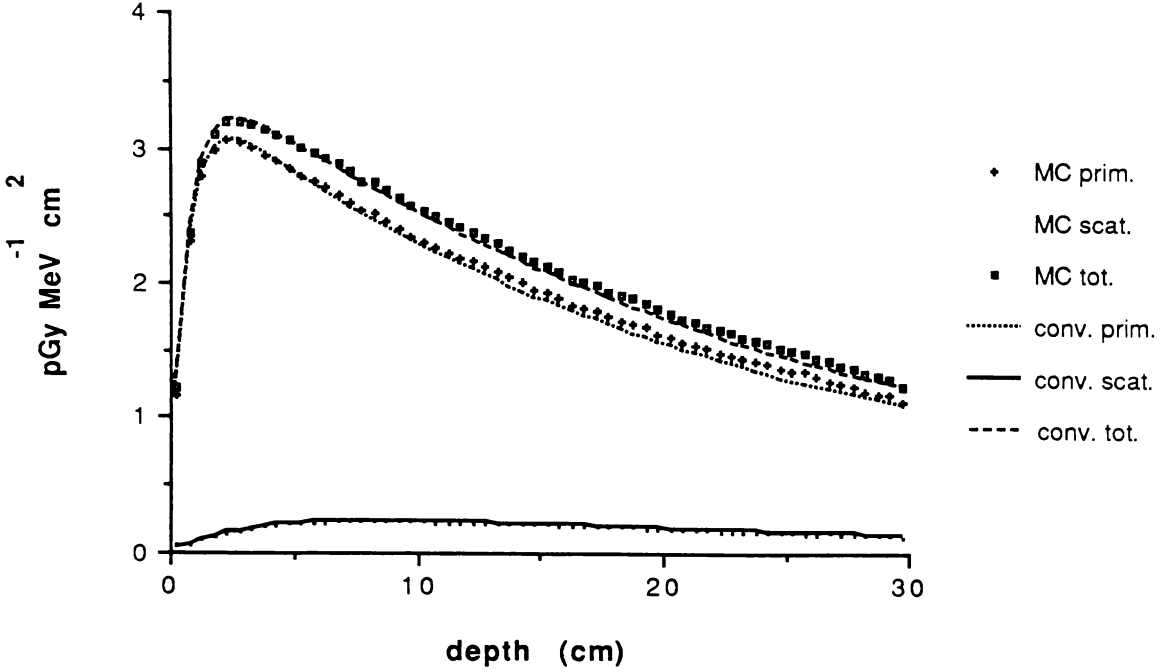


Figure 3.7 Primary, scattered and total dose per incident energy fluence versus depth for a 4.0 cm diameter beam. Convolution results were produced using 0 cm spectrum EDKs. The primary convolution curve drops below the Monte Carlo results as depth increases because the fractional energy in the primary EDK becomes too low as the primary beam hardens with depth. The scattered convolution curve is slightly higher than the Monte Carlo results because the fractional energy in the scattered EDK becomes too high with depth. The total dose behaves very similarly to the primary dose.

centred at 29.75 cm deep, is 2.63, 2.63, and 2.62 for convolutions performed with the 0, 20, and 40 cm deep EDKs respectively, while the ratio for the Monte Carlo result is 2.57. The difference of less than 0.5% in this ratio when different EDKs are used, means that if depth dose results were normalised to 100% at d_{max} , it would make very little difference which EDK was used. This invariance in the normalised depth dose curve is consistent with results produced by Metcalfe *et al* (1990).

From these graphs, it is apparent that separate EDKs formed for each depth increment k' in the convolution would produce results in good agreement with Monte Carlo, since the convolution curve coincides with the Monte Carlo result at the depth for which the EDK was formed. Although this approach is impractical since a very large number of EDKs would be required, a small number of EDKs produced for larger depth increments could be stored and the correct EDK value could be obtained by interpolation. An alternative is to use the correction to the primary and scattered 0 cm EDKs proposed here. Results using (3–26) are shown in figure 3.10, where primary, scattered, and total dose is seen to be in close agreement with Monte Carlo results. Normalisation of the results to 100% at d_{max} would not significantly alter the appearance of the curves since the maximum dose is very similar in the convolution and Monte Carlo results.

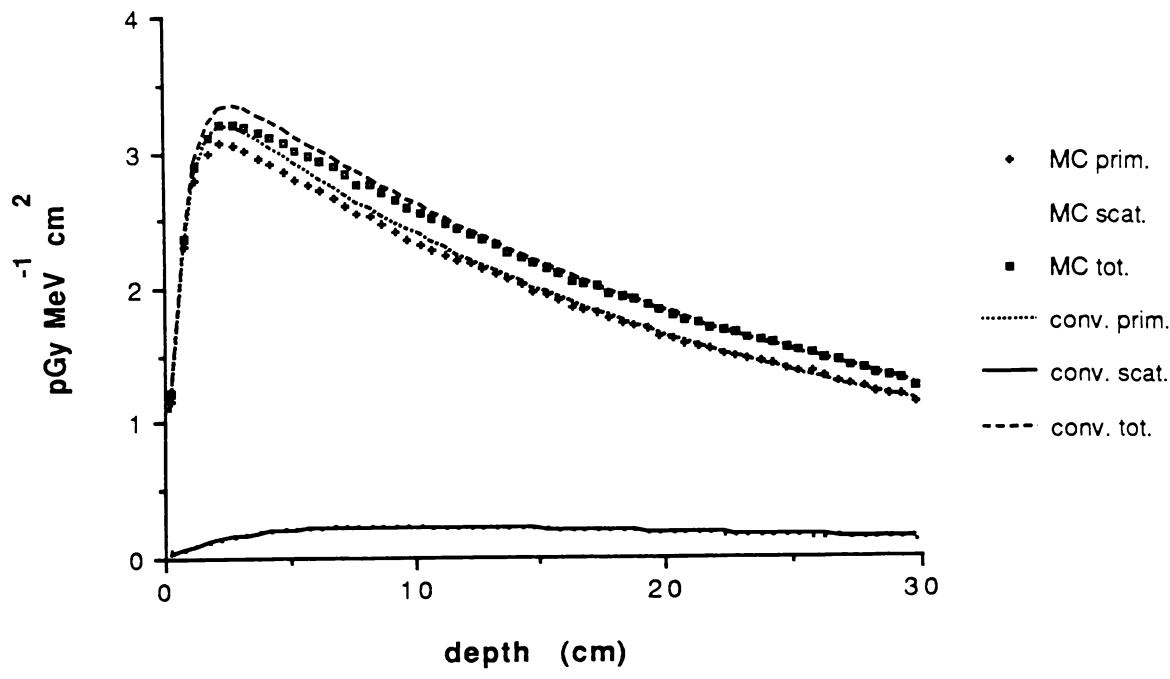


Figure 3.8 Primary, scattered and total dose per incident energy fluence versus depth for a 4.0 cm diameter beam. Convolution results were produced using 20 cm spectrum EDKs. The primary convolution curve is above the Monte Carlo results near the surface because the fractional energy in the primary EDK is too high for interaction depths of less than 20 cm. The scattered convolution curve is slightly below the Monte Carlo result near the surface because the fractional scattered energy is too low for interactions at depths less than 20 cm.

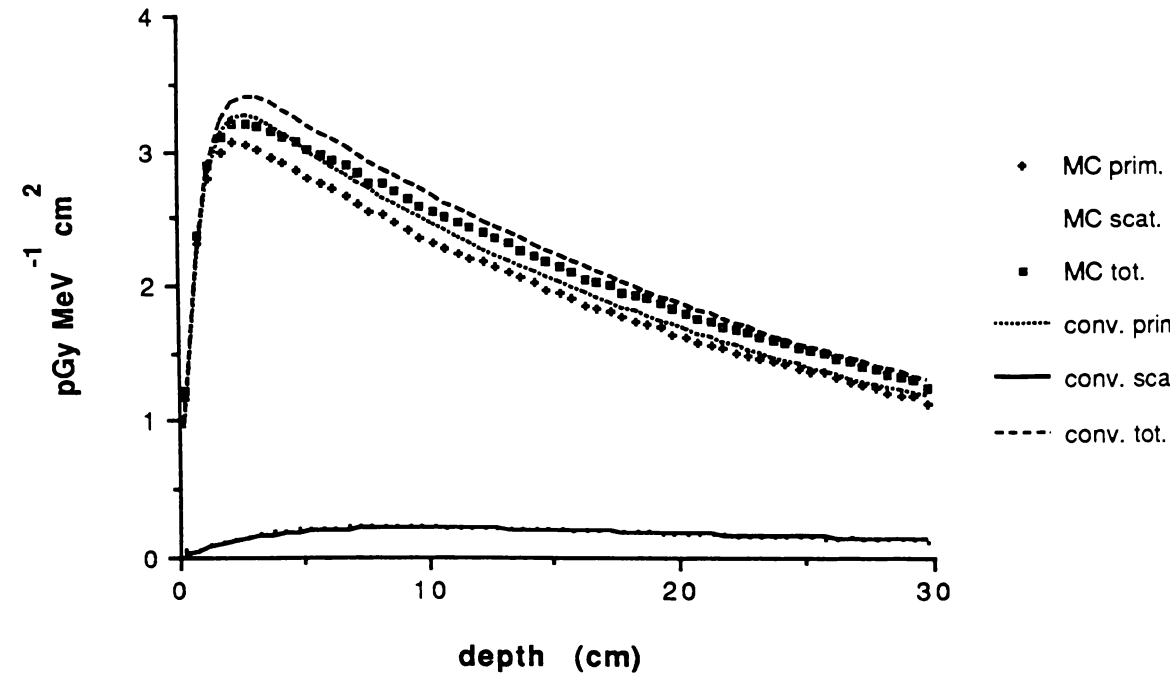


Figure 3.9 Primary, scattered and total dose per incident energy fluence versus depth for a 4.0 cm diameter beam. Convolution results were produced using 40 cm spectrum EDKs. The primary convolution curve is above the Monte Carlo results for the whole depth range because the fractional energy in the primary EDK is too high for interaction depths of less than 40 cm. The scattered convolution curve is significantly below the Monte Carlo result near the surface.

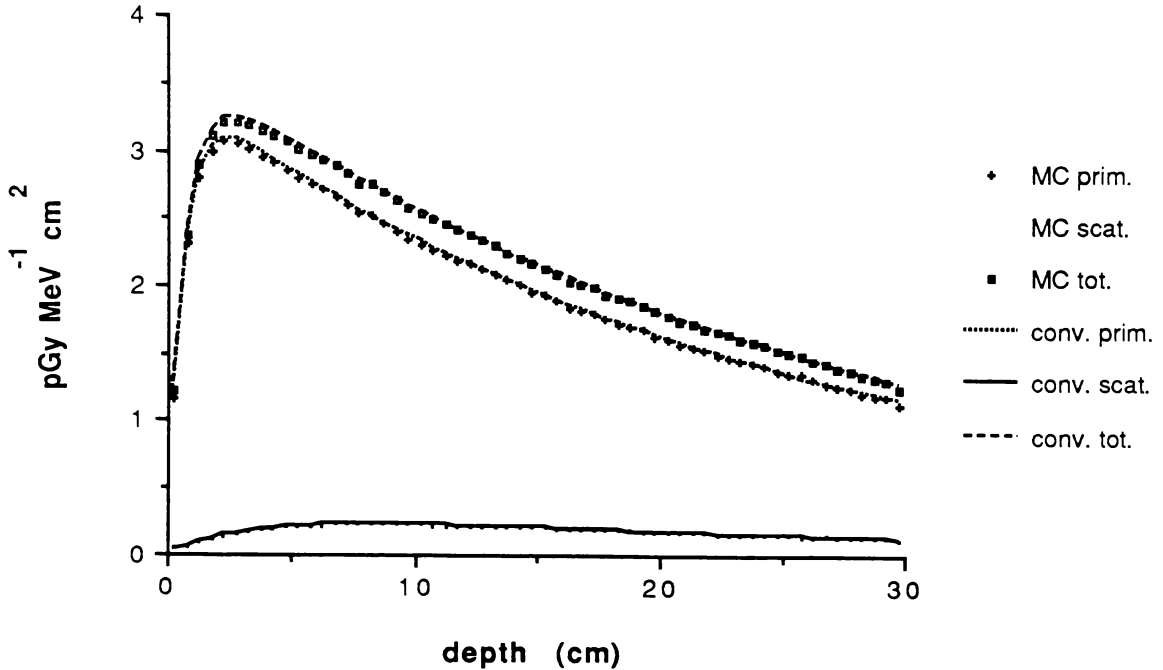


Figure 3.10 Primary, scattered and total dose per incident energy fluence versus depth for a 4.0 cm diameter beam. Convolution results were produced using corrected 0 cm spectrum EDKs. The primary and scattered convolution curves are both in good agreement with the Monte Carlo results.

Depth dose results for a 2.0 cm diameter beam, without the beam hardening correction, are shown in figure 3.11. Surface spectrum EDKs were used for the convolution. By comparison with figure 3.7, it is apparent that the difference in the convolution and Monte Carlo results is not as great as for the 4.0 cm beam. This is consistent with the results in figures 3.4, 3.5, and 3.6, where the difference in the fractional energy contained within a radius of 1.0 cm in the EDKs between the 0, 20, and 40 cm EDKs, is not as large as the differences in energy contained within a radius of 4.0 cm. Note that the falloff in the primary and total convolution depth dose curves is still slightly more rapid than that of the Monte Carlo curves.

In figure 3.12, convolution dose with the beam hardening correction is compared with Monte Carlo results for the 2.0 cm beam. The correction does not cause a large change in the convolution curves, although the primary and total dose is seen to be increased slightly too much by the correction. This is consistent with figure 3.4, where the corrected energy in the 0 cm primary EDK is greater than that in the 20 and 40 cm EDKs for a radius of 1.0 cm. The important result however, is that the slope of dose falloff with depth of the corrected convolution total dose curve, matches that of the Monte Carlo results better than that of the uncorrected curve in figure 3.11.

An alternative method for correcting the dose calculated using polyenergetic kernels has been proposed by Ahnesjö (1989), where a beam hardening factor is applied at the end of

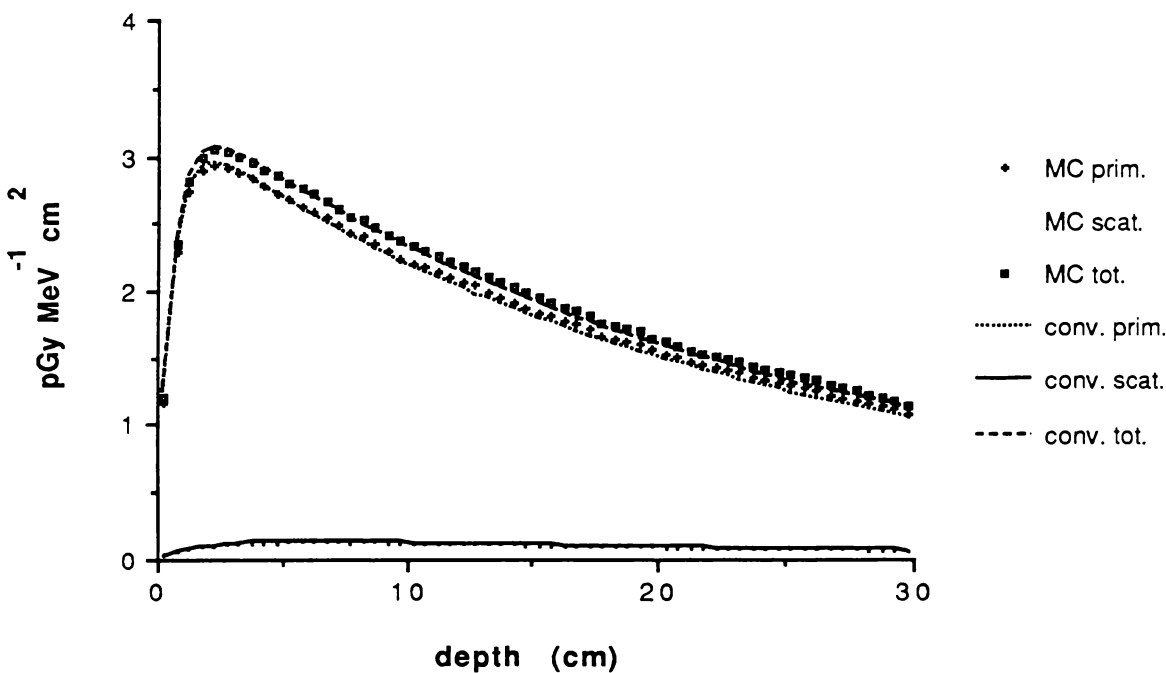


Figure 3.11 Same as figure 3.7 but for a 2.0 cm diameter beam. Note that the scattered component is smaller than for the 4.0 cm beam.

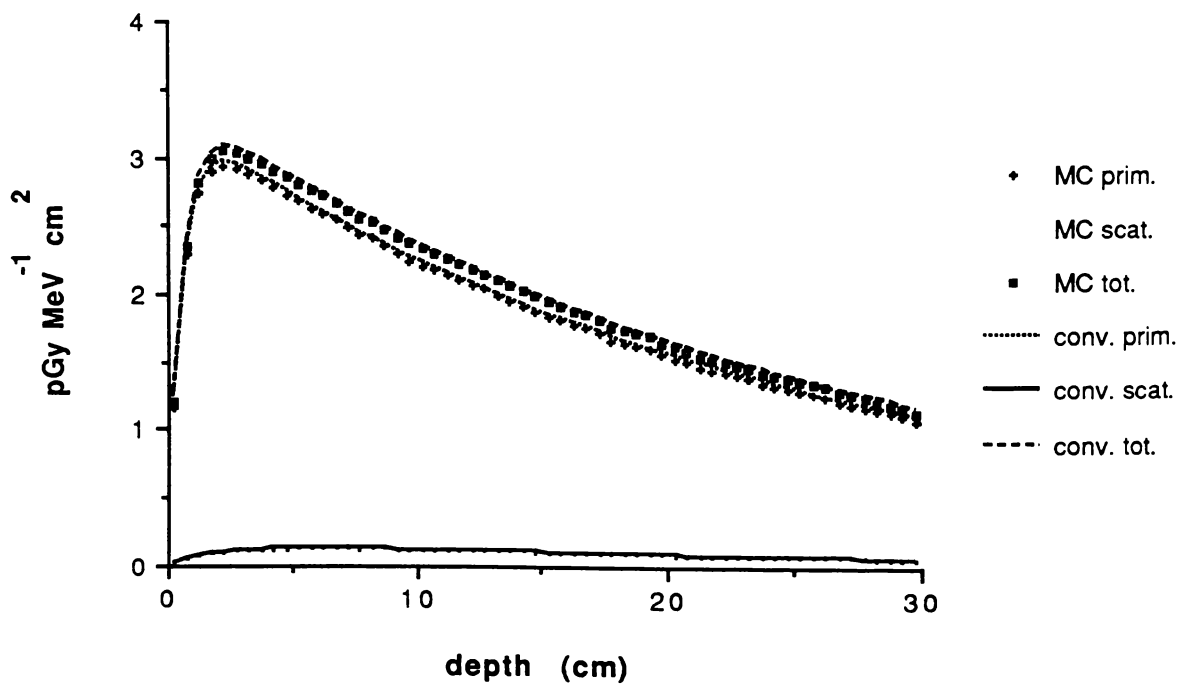


Figure 3.12 Same as figure 3.10 but for a 2.0 cm diameter beam. The fractional energy in the primary EDK is slightly overcorrected, giving a primary dose that is slightly too high, however the slope of the depth dose curve agrees well with the Monte Carlo results.

the convolution calculation. For a certain point, this factor is the ratio of the dose calculated using a summation of convolutions for each energy component to the dose calculated using polyenergetic EDKs.

3.4. Using Primary EDKs to Predict EEF_{lat}

Since the primary central axis dose for a range of beam diameters can be obtained from a convolution calculation, the lateral electron equilibrium factor, EEF_{lat} , defined in section 2.4.3, can be obtained using convolution. In this section, an expression for EEF_{lat} in terms of a primary polyenergetic EDK is obtained. The dependence of the predicted values of EEF_{lat} on the spectrum for which the EDK was generated is also investigated.

Beam diameters for which convolution dose can be obtained includes those which are multiples of twice the radial thickness ΔR of the EDK array voxels. Using the primary part of (3-26), with the correction factor C_p set to unity, the primary dose on layer k for a beam of diameter $2I\Delta R$ is given by

$$D_p(k, 2I\Delta R) = \sum_{k'=0}^{K-1} T(k') \sum_{i=0}^{I-1} H_p(k - k', i). \quad (3-28)$$

If the minimum beam diameter required for lateral electron equilibrium is d_{eq} then the minimum beam radius in terms of discrete radial voxel thicknesses is I_{eq} where $2I_{eq}\Delta R \geq d_{eq}$. Since lateral electron equilibrium is established for a diameter of 4.0 cm (see figure 2.16), $2I_{eq} \times 0.25$ must be at least 4.0 cm — therefore, $I_{eq} = 8$. The lateral electron equilibrium factor at voxel layer k , for a beam of diameter $2I\Delta R$, is given by

$$EEF_{lat}(k, 2I\Delta R) = \frac{\sum_{k'=0}^{K-1} T(k') \sum_{i=0}^{I-1} H_p(k - k', i)}{\sum_{k'=0}^{K-1} T(k') \sum_{i=0}^{I_{eq}-1} H_p(k - k', i)}. \quad (3-29)$$

Now if the approximation is made that $T(k')$ is constant over the range of k' for which $H_p(k - k')$ is non-zero, the term cancels and can be omitted from (3-29). Therefore, (3-29) may be simplified so that EEF_{lat} is approximated by:

$$EEF_{lat}(k, 2I\Delta R) = \frac{\sum_{k'=0}^{K-1} \sum_{i=0}^{I-1} H_p(k - k', i)}{\sum_{k'=0}^{K-1} \sum_{i=0}^{I_{eq}-1} H_p(k - k', i)}. \quad (3-30)$$

The numerator of (3-30) represents a sum of the values in the primary EDK which are within I voxel thicknesses of the EDK axis and between the voxel layers with indices $l = k - (K - 1)$ and $l = k$. In the denominator, the sum over l is the same as in the numerator but I_{eq} radial layers are included. Note that the minimum voxel layer index $(k - k')$ which can be included in the summation is -4 (see section 3.3.2).

The primary EDK generated using the surface primary photon spectrum is shown in figure 3.13, where a sum of the fractional energy in each radial shell as well the total fractional energy in the EDK is included. Note that approximately 57% of the total primary photon

energy removed in primary interactions is deposited by secondary electrons (also see table 3.2). It can be seen that less than 0.2% of the total fractional energy deposited lies in voxels with radial indices greater than $I_{eq} - 1 = 7$. The process of obtaining the lateral electron equilibrium factor at a depth of 2.75 cm (centre of voxel layer 5) for a beam of diameter 2.0 cm, ($EEF_{lat}(5, 2.0)$), using (3-30) is depicted in figure 3.13. The numerator in (3-30) is a sum over the values within the shaded region. Although $I_{eq} = 8$, the summation in the denominator can equally include values of i up to the maximum radial index in the array (11 in this case) — therefore the denominator is depicted in figure 3.13 as the sum of all values in the EDK above the bold line. The value of $EEF_{lat}(5, 2.0)$ obtained using this method is 0.828.

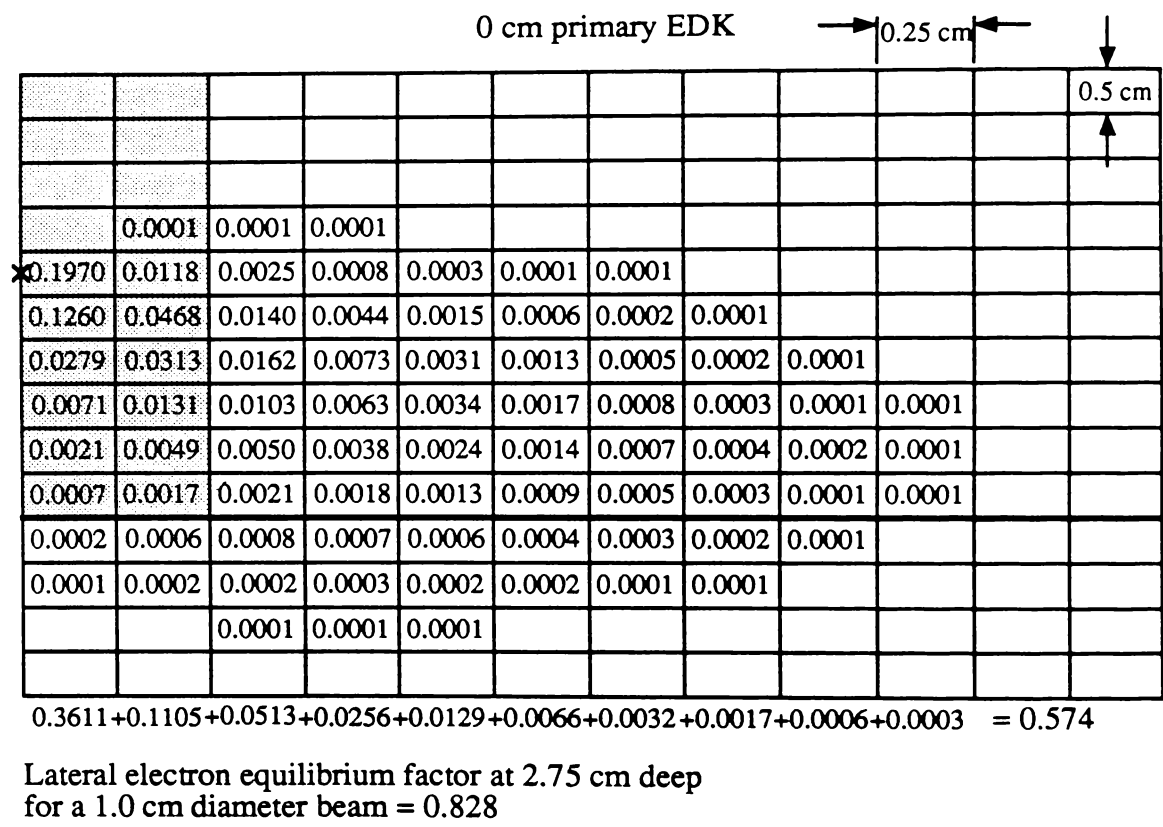


Figure 3.13. Primary energy deposition kernel formed for the primary photon spectrum which exists at the surface. Primary photons are incident along the axis of a cylindrical array of voxels, and are forced to interact at the point indicated by a cross. Energy deposited by secondary electrons in each voxel as a fraction of the total incident photon energy imparted is recorded. The variation in interaction probability with photon energy is accounted for. 1.2 million incident photons were simulated which took nine hours on a VAX 6000-430. The lateral electron equilibrium factor at 2.75 cm deep for a 1.0 cm diameter beam in water, is given by the ratio of the fractional energy contained within the shaded box to that above the bold line.

Values obtained using this method are compared here with the ‘true’ values of EEF_{lat} obtained in section 2.4.3. A comparison of the true and predicted values of $EEF_{lat}(k, 2I\Delta R)$ for $2I\Delta R = 0.5, 1.0$, and 2.0 cm is shown in figure 3.14. Values of EEF_{lat} are predicted

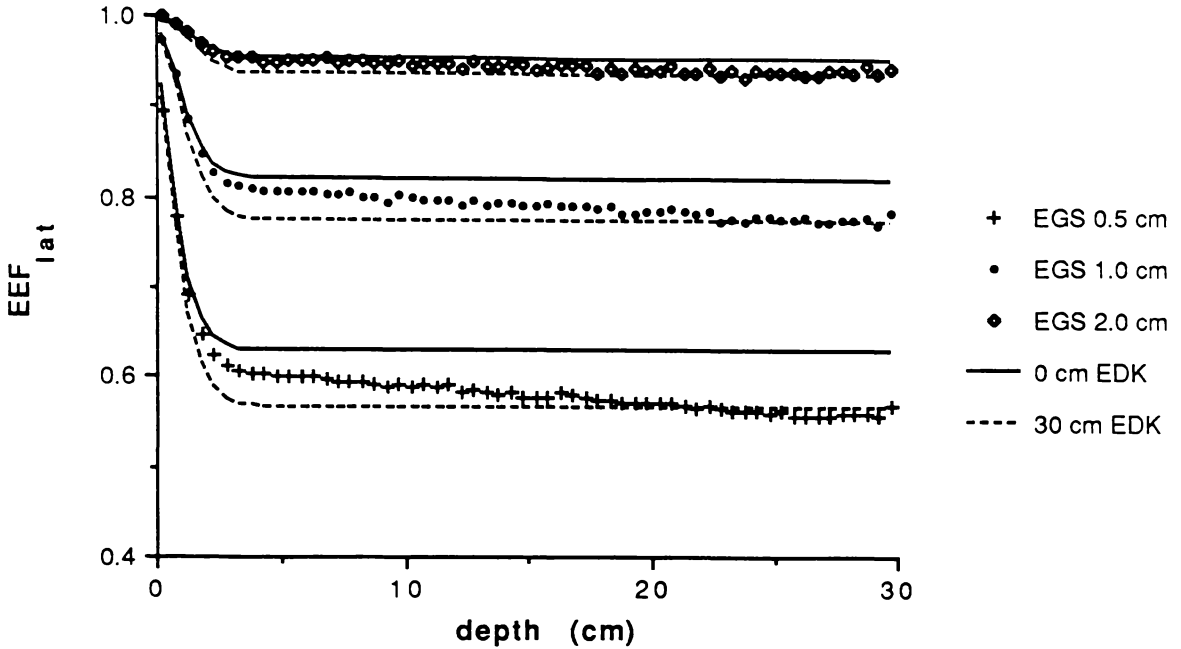


Figure 3.14. Lateral electron equilibrium factor as a function of depth for the 0.5, 1.0, and 2.0 cm diameter beams. True values obtained by division of the dose for each diameter by the dose for the 4.0 cm beam are shown by the sloping dashed lines. Values calculated using (3-30) are also shown, where the predictions made using a surface spectrum primary EDK are shown by the solid curves, and the predictions made using a hardened 30 cm deep EDK are shown by the dashed curves.

using primary EDKs formed for the spectra which exist at 0 and 30 cm deep in water.

Since the lateral extent of the non-zero values in the EDK shown in figure 3.13 initially increases with distance downstream from the primary interaction site, the ratio of the numerator to the denominator in (3-30) will decrease with increasing k . This is evident in the initial decrease in EEF_{lat} with depth seen in figure 3.14. The values EEF_{lat} predicted using the EDKs are constant for a depth greater than approximately 3 cm. This is because both the numerator and denominator in (3-30) are invariant for when k is greater than the longitudinal range of the secondary electrons.

Because the lateral range of the secondary electrons produced by the 30 cm deep primary photon spectrum is slightly greater than that of those produced by the surface spectrum, the numerator in equation (3-30) is a smaller proportion of the denominator for the 30 cm spectrum EDK than for the surface spectrum EDK. This is the reason for the difference in the values of EEF_{lat} predicted by the two different EDKs. The true value of EEF_{lat} near the surface and at approximately 30 cm deep, should be approximated by the values predicted using the surface spectrum EDK, and 30 cm deep EDK respectively, since these EDKs are representative of the energy deposited by secondary electrons originating at these depths. The true value is seen to make the transition between the two predicted values as depth increases, for each beam diameter.

The decrease in EEF_{lat} due to beam hardening with depth is further illustrated in figure 3.15. True values of EEF_{lat} at 5.25 and 25.25 cm deep are plotted for a range of diameters. The values are again seen to be lower at the greater depth — this is due to the increased lateral range of secondary electrons due to beam hardening.

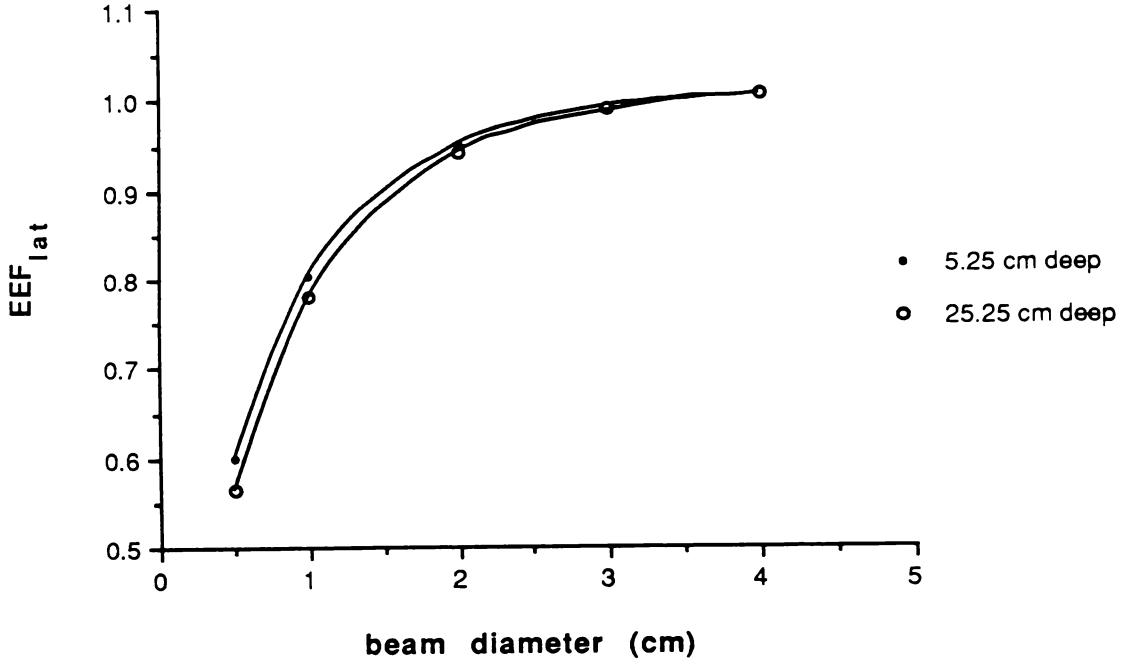


Figure 3.15. True lateral electron equilibrium factors at 5.25 and 25.25 cm deep. Values are slightly lower at the greater depth because the lateral displacement of secondary electrons arising from primary photon interactions increases as the primary photon beam hardens.

It has been shown here that as the primary beam hardens with depth, and the lateral range of secondary electrons increases, the degree of lateral electron disequilibrium increases for a particular beam diameter. This effect should ideally be corrected for in using polyenergetic EDKs for convolution calculations where the beam is small enough for lateral electron disequilibrium to occur. Such a correction has not been attempted here. The correction formulated in section 3.3.4 is for the increased proportion of primary photon energy transferred to secondary electrons with depth — not for the increase in secondary electron range with depth.

3.5. Summary and Discussion

In calculating the dose distribution for a polyenergetic photon beam using the convolution method, consideration should be given to the hardening of the primary photon beam with depth. As the beam hardens, increasing energy is transferred to secondary electrons by the primary photons. Most importantly, the proportion of primary photon energy transferred to secondary electrons, and then deposited in the medium, increases as the primary beam hardens. In forming primary and scattered EDKs using the primary photon spectra which

exist at different depths, it has been found that the fractional energy contained within the primary and scattered EDKs, increases and decreases respectively, as the spectrum hardens.

If polyenergetic EDKs are used in the convolution calculation, then ideally, unique EDKs should be used for different depths of primary photon interaction. This however requires that a large number of EDKs be generated. A technique has been developed which uses polyenergetic primary and scattered EDKs formed for the photon spectrum at the surface, and corrects the EDKs with depth by accounting for the change in the fraction of primary photon energy transferred to, and subsequently deposited by, secondary electrons. It has been shown that using this correction method, accuracy is improved over that achieved when polyenergetic primary and scattered EDKs are used without a depth correction.

This correction is particularly suited to convolution dose calculations for small beams, where the dose due to secondary electrons originating from primary photon interactions is much larger than the dose due to scattered photons. In this case, it is important to accurately calculate the separate primary and scattered convolution doses. With an increase in beam diameter, the error in the total dose calculation when uncorrected 0 cm EDKs are used will decrease. This is because the primary dose component, which becomes increasingly underestimated as depth increases, decreases in significance as beam diameter increases, and the scattered component, which is overestimated, gains in significance. As diameter increases therefore, the errors in the two components will increasingly cancel each other out when the components are added.

It should be emphasised that as found by Metcalfe *et al* (1990), it is the terma which has the greatest effect on the shape of the depth curve. Correct terma calculation is dependent on the use of a correct primary photon spectrum, and, for a divergent beam, correct inverse square correction with depth. Therefore, before corrections to the EDKs are worthwhile, it must be ensured that the terma is calculated correctly.

In section 3.4, a method for predicting the lateral electron equilibrium factor, EEF_{lat} , using a primary EDK was discussed. As the primary beam hardens with depth, the lateral secondary electron range increases. This causes the true value of EEF_{lat} to decrease with depth — the degree of lateral electron disequilibrium increases. This increase in disequilibrium is not predicted by a single EDK. It was found that an EDK generated using a surface primary photon spectrum gives values of EEF_{lat} which agree well with the true values near the surface, and that values predicted using an EDK generated using a 30 cm deep spectrum agree well with the true values at 30 cm deep.

Chapter 4

Superposition Dose Calculation in Lung

4.1. Introduction

In the irradiation of a water medium with 10 MV photons, a reduction in dose on the central axis due to lateral electron disequilibrium is shown in chapter 2 to exist for circular beams with diameters up to approximately 4 cm. Also, disequilibrium is shown to exist in lung of density 0.25 g cm^{-3} for beams with diameters less than approximately 16 cm. The investigations of Kornelson and Young (1982; 1983) and of Rice *et al* (1988) on the subject of lateral electron disequilibrium in lung are also discussed.

An outline of some of the photon beam dose calculation algorithms currently in use with commercial treatment planning systems is given in chapter 1. The Equivalent Tissue-air Ratio (ETAR) method is the most sophisticated of these, and is computationally intensive compared to the simpler algorithms. For high energy beams however, scatter function models such as this are not accurate in some situations. A particular situation where scatter function models are inaccurate, is in the irradiation of a medium containing a low density region such as lung, when the field is small enough for lateral electron disequilibrium to exist in the lung (Mackie *et al*, 1985; Tang *et al*, 1986; Woo *et al*, 1990; Hoban *et al*, 1990). Scatter function models such as ETAR make the assumption that electronic equilibrium exists at all calculation points, giving rise to an overestimate in the dose in lung in situations of lateral electron disequilibrium. In this chapter, the reasons why scatter function models are inaccurate in disequilibrium situations are briefly discussed.

Unlike the scatter function models, the superposition method *does not* assume the existence of electronic equilibrium. In this chapter, the theory of the superposition algorithm is discussed, with particular emphasis on the scaling of the energy deposition kernel (EDK) to account for inhomogeneities. A superposition program was written and applied to the problem of dose calculation in lung. Results generated by this algorithm are presented and compared with Monte Carlo predictions. All results are for a Varian Clinac 18/10, 10 MV photon beam.

The superposition algorithm described in this chapter is a simpler, preliminary version of that which has been incorporated into the GRATIS treatment planning system. That system is described in chapter 5.

This chapter is based on the paper "Superposition dose calculation in lung for 10 MV photons", *Aust. Phys. Eng. Sci. Med.* (1990) 13(2) 81-92.

4.2. The Effect of Disequilibrium on Scatter Function Model Results

A common feature of the currently used algorithms outlined in chapter 1, is the assumption that electronic equilibrium exists at all calculation points — the conditions required for electronic equilibrium are discussed in section 1.12. This assumption is the underlying cause of the overestimate in lung dose. The reasons why this assumption causes this error are explained in this section.

Scatter function models such as ETAR use tables of zero-area TARs and SARs to determine the primary and scattered dose components respectively. The error in the dose estimate stems from the use of the zero-area TAR to characterise the primary dose component (Woo *et al*, 1990). When a TAR versus beam diameter graph is extrapolated to find the zero-area TAR, the value which is in fact obtained is the TAR in the absence of dose due to scattered radiation — the equilibrium primary dose is still included. Hence, the zero-area TAR represents the primary dose component which would exist under equilibrium conditions.

This can be seen by referring to figure 4.1, where a point P on the beam axis, at which the dose is to be calculated, is depicted for two different situations. The radii of the beams shown are equivalent radii such as that used in the ETAR algorithm. In the diagrams, the darkly shaded regions represent the volume from which primary photon interactions give rise to a primary dose at P . The lightly shaded regions represent the volume from which primary photon interactions give rise to a scattered dose at P . In figure 4.1(a), the equivalent radius is large enough for lateral electron equilibrium to exist on the axis, since the beam edges are further from P than the lateral electron range. Also shown in this figure is an illustration of a plot of the TAR versus beam diameter such as that used to determine the zero-area TAR and the SAR. Measured TARs are indicated by the crosses. A curve passing through the measured points and extrapolated to zero diameter is also shown — the value of this curve at zero diameter is the zero-area TAR. The difference between the TAR and the zero-area TAR is the SAR. As shown by Woo *et al* (1990), the primary component should be represented by a curve which coincides with the zero-area TAR for beam radii larger than that required for equilibrium, but which falls off sharply for smaller radii. Such a curve is shown in the graph. Also on the TAR graph are bars indicating the correct ratios which should be used to find the primary and scattered dose components. Since lateral electron equilibrium exists, the zero-area TAR is the correct value of the primary dose ratio.

In figure 4.1(b), lateral electron equilibrium does not exist at P . In the corresponding TAR graph, the true value of the primary dose ratio is shown to be less than the zero-area TAR. It is for effective beam diameters less than that required for lateral electron equilibrium, that an overestimate in the primary dose ratio occurs. This problem has been addressed by Woo *et al* (1990), who have devised a method which uses Monte Carlo generated primary and scattered dose components to obtain the correct primary dose ratio for small beam diameters. The separation of primary and scatter dose using tissue-air ratios has also been

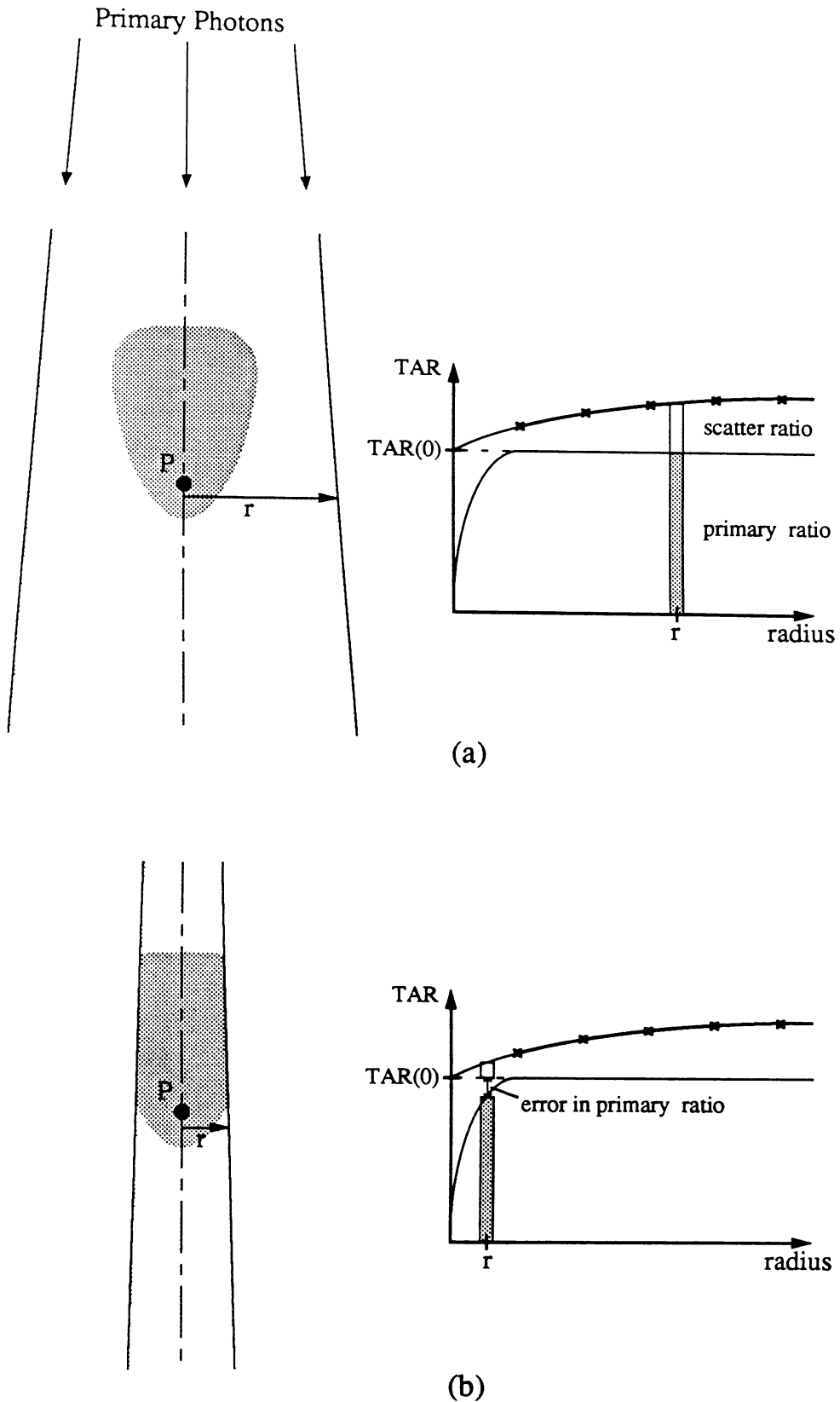


Figure 4.1. Calculation of central axis dose using a scatter function model. The darkly shaded regions represent the volume from which primary photon interactions give rise to a primary dose at P . The lightly shaded regions represent the volume from which primary photon interactions give rise to a scattered dose at P . In (a), lateral electron equilibrium exists at P , therefore the zero area TAR gives the correct primary dose. In (b), lateral electron equilibrium does not exist at P , therefore the zero area TAR gives an overestimate in primary dose.

questioned by Mohan and Chui (1985), who conclude that the non-existence of electronic equilibrium will cause errors near beam boundaries and inhomogeneities.

Experimental work was carried out here to further investigate the problem of disequilibrium, using a 'lung' phantom comprised of a 4 cm thick polystyrene ($\rho = 1.04 \text{ g/cm}^3$) slab on either side of 8 cm of cork ($\rho = 0.20 \text{ g/cm}^3$) representing the lung (figure 4.2). The cork is in 0.6 cm sheets so dose measurements are able to be made in the cork with a Baldwin Farmer ionization chamber placed in a hole between two sheets. Depth dose measurements are made in the lung by moving the two sheets up and down in the stack while the SSD remains constant. There is also a hole in which to insert a Baldwin Farmer ionization chamber in the top polystyrene block at d_{max} (2.1 cm deep).

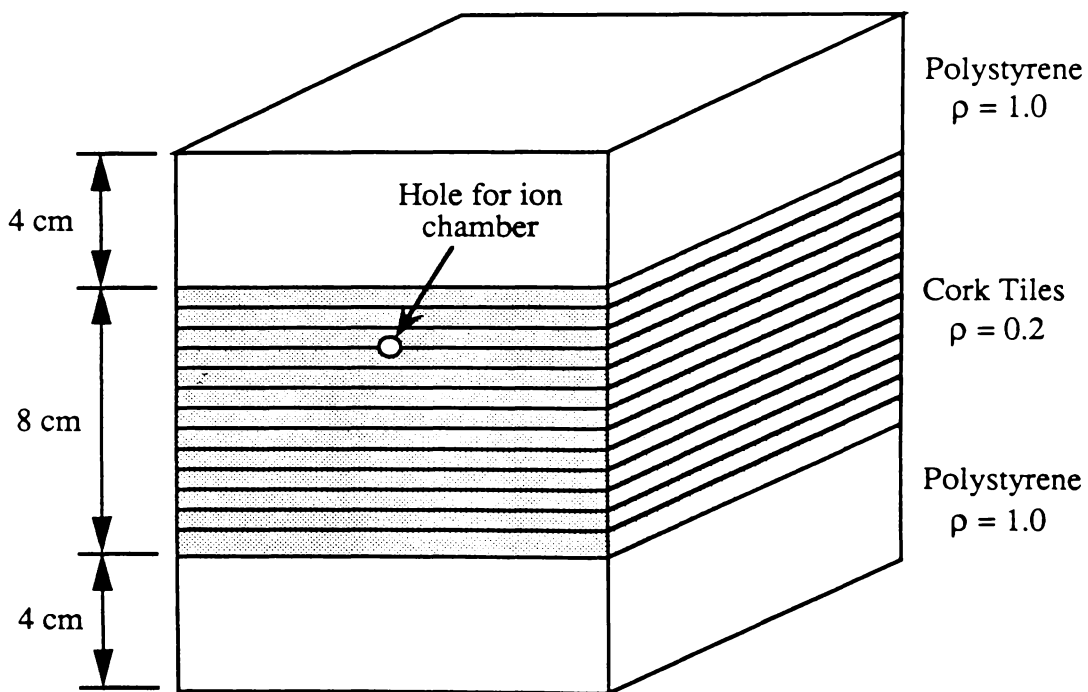


Figure 4.2 Lung phantom used for obtaining measured dose. Cork tiles of density 0.2 g cm^{-3} are sandwiched between two polystyrene blocks. Ionization chamber and film measurements can be made in the cork portion. There is also a hole in which to insert a Baldwin Farmer ionization chamber in the top polystyrene block at d_{max} (2.1 cm deep).

Measurements were made for 5×5 and 10×10 cm fields at 100 cm SSD and compared with the results obtained using the ETAR method. The ETAR results were obtained using the Theratronics Theraplan treatment system at Waikato Hospital. As illustrated in figure 4.3(a), the dose in lung for the 5×5 cm field is overestimated by ETAR. For a 10×10 cm field however, ETAR gives adequate depth dose results as shown in figure 4.3(b), since the

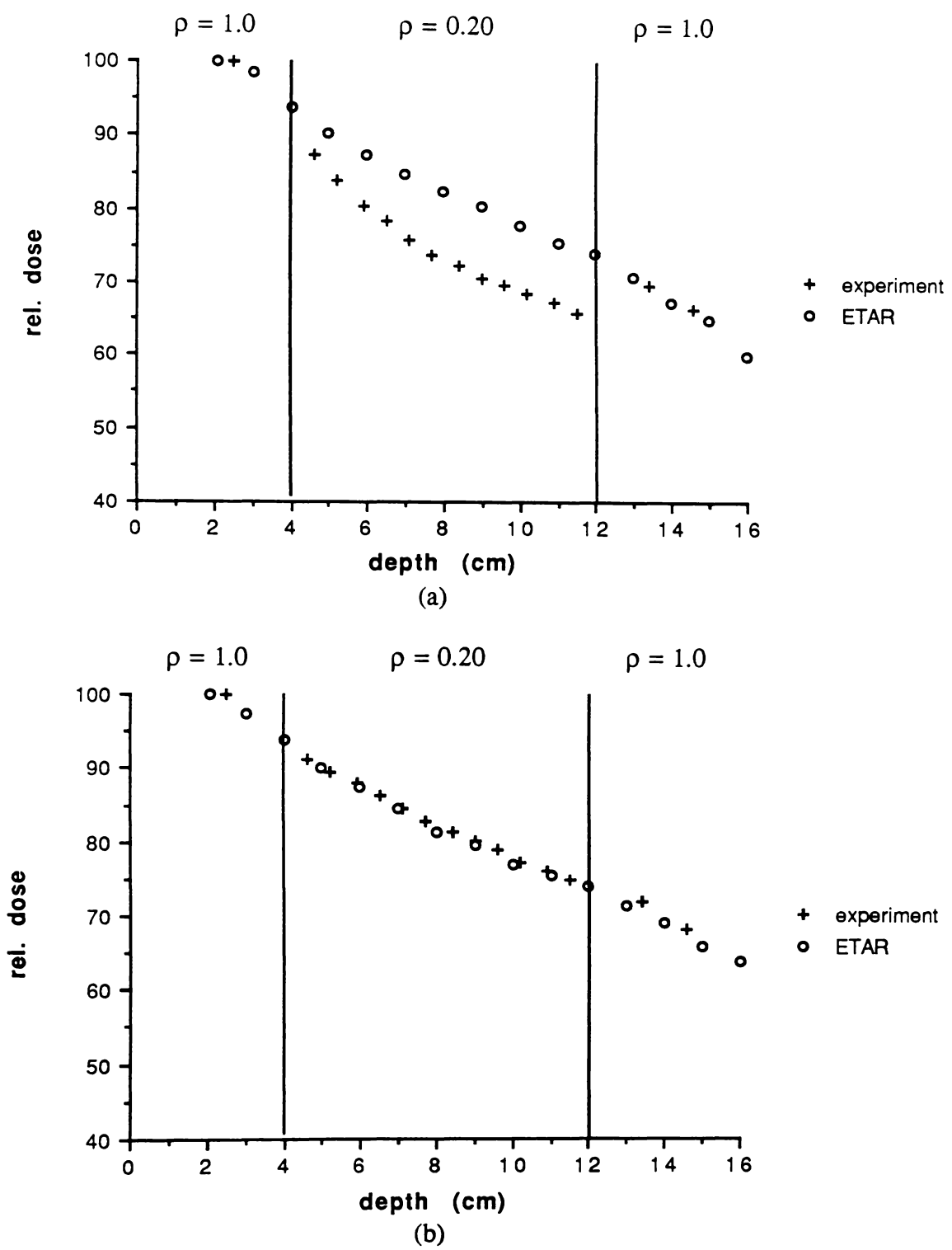


Figure 4.3. Comparison of measured depth dose results for (a) 5x5 and (b) 10x10 cm fields with ETAR. The ETAR algorithm assumes that electronic equilibrium exists in the lung, hence the dose is overestimated for the 5x5 cm field.

degree of lateral electron disequilibrium is reduced.

Associated with the over-prediction of dose in lung for the 5×5 cm field is that the buildup region in the second polystyrene slab (12-14 cm depth) caused by the re-establishment of lateral electronic equilibrium is not predicted. This is a potentially serious error since the proximal surface and underlying tissue of the interlung region receives a lower dose than predicted which may cause a local recurrence of tumour cells.

For any field size, lateral electron disequilibrium exists at points closer to the edge of the field than the lateral electron range. It is this phenomenon which is largely responsible for the shape of the beam penumbra (Mohan and Chui, 1985). The distance from the field edge at which disequilibrium occurs is increased by a decrease in density. It is therefore expected that near the field edges in the lung portion of the phantom, that the penumbra will be broader than in the water regions. This is not predicted by scatter function models since the penumbral shape generated is based on the observed shape in water and no correction to this shape is made for density variations. A dose profile in the lung for a 10×10 cm beam was obtained using film (Kodak XTL-2) sandwiched between two cork slabs. The film was at 8.2 cm deep which corresponds approximately to mid-lung. Results are compared with that predicted by ETAR in figure 4.4. It can be seen that the penumbra predicted by ETAR is narrower than that obtained by experiment.

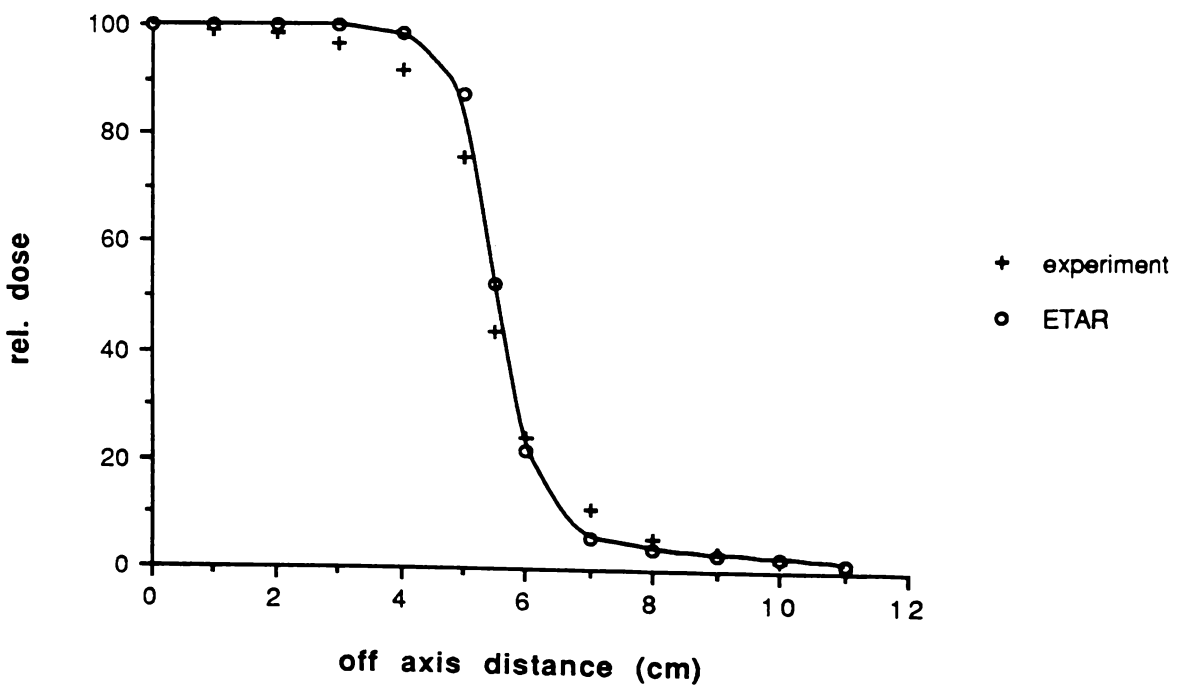


Figure 4.4 Comparison of film (Kodak XTL-2) profile with ETAR results for a 10×10 cm field at mid-lung. The broad penumbra due to lateral electron disequilibrium away from central axis is not predicted.

The inaccuracy of dose calculation in lung using scatter function models demonstrates the

need for a more rigorous approach.

A method of accounting for lateral electron disequilibrium in lung has been suggested by Young and Kornelson (1983), which uses a *loss factor* to model the loss of lateral electron equilibrium in the lung. This method takes into account the distance of the calculation point from the nearest ‘upstream’ boundary with tissue. The effect on the total dose due to the initial increase in the degree of disequilibrium in the lung (see figure 2.23) is therefore predicted. Similar methods have been proposed by van de Geijn (1987) and Petti *et al* (1987). Another possible solution to the problem of overestimating the primary dose component in scatter function models is to use the lateral electron equilibrium factor EEF_{lat} discussed in section 2.4.3. This factor may be used to modify the zero-area TAR so that it gives a more realistic value in situations of electronic disequilibrium. For example, in the ETAR method, the zero-area TAR for the radiological depth d_p' , $TAR(d_p', 0)$ could be replaced by $TAR(d_p', 0)EEF_{lat}(d_p', 2\bar{r})$ where $\bar{r}$ is the effective beam radius at the calculation point P (see section 1.13.3). With this method, EEF_{lat} is a correction factor for the zero-area TAR, and hence for the primary dose component. The value of the correction factor would initially decrease past a water-lung interface due to the decrease in $\bar{r}$, giving a decrease in dose. The use of more realistic tissue-air ratio data for small beams, as suggested by Woo *et al* (1990), has been shown to give accurate results in lung when applied to the ETAR method.

Errors arising from the assumption of electronic equilibrium are avoided by the superposition method, since the dose contribution due to primary photon interactions in all volume elements in the irradiated medium is considered explicitly. The remainder of this chapter is concerned with the development of a superposition algorithm which accurately calculates the dose distribution in lung.

4.3. Superposition in Inhomogeneous Media

In chapter 3, a convolution algorithm for dose calculation on the central axis of a non-divergent circular beam was discussed. This was a very specialised application of the convolution method. More general convolution methods are able to calculate the dose distribution throughout a three-dimensional region, for a beam of arbitrary shape. For a homogeneous medium, the dose at a point denoted by the vector $\mathbf{r}$, is given by a convolution of the terma and the energy deposition kernel:

$$D(\mathbf{r}) = \int_{\mathbf{r}'} T(\mathbf{r}') H(\Delta\mathbf{r}) d^3\mathbf{r}', \quad (4-1)$$

where $T(\mathbf{r}')$ is the terma at $\mathbf{r}'$ and $H(\Delta\mathbf{r})$ is the EDK value corresponding to the vector $\Delta\mathbf{r} = \mathbf{r} - \mathbf{r}'$. $H(\Delta\mathbf{r})$ is the fraction of the energy imparted by primary photon interactions at $\mathbf{r}'$ that is deposited in a unit volume at $\mathbf{r}$, and represents the sum of the primary and scattered EDK values. In this expression, H is dependent only on the vector between $\mathbf{r}'$ and $\mathbf{r}$ — it is therefore referred to as an *invariant kernel* and (4-1) is a true convolution equation. The integration over $\mathbf{r}'$ includes the entire irradiated volume. In this case, since

the kernel is invariant, the calculation may be done by *Fourier convolution* (Murray *et al*, 1989). Fourier convolution greatly reduces the calculation time (Field and Battista, 1987), but is not strictly applicable to inhomogeneous media because of the requirement of an invariant convolution kernel. Fourier techniques for inhomogeneous media have however been investigated (Boyer, 1984).

When the medium is inhomogeneous, an accurate distribution cannot be obtained unless H is also dependent on $\mathbf{r}'$. The kernel is then *non-invariant*, and the convolution becomes a superposition. In the inhomogeneous case,

$$D(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})} \int_{\mathbf{r}'} T(\mathbf{r}') \rho(\mathbf{r}') H(\Delta\mathbf{r}, \mathbf{r}') d^3\mathbf{r}'. \quad (4-2)$$

In this equation, the density $\rho(\mathbf{r}')$ inside the integral is used to convert energy imparted per unit mass at $\mathbf{r}'$ (the terma) to the energy imparted per unit volume. The division by the density at $\mathbf{r}$ outside the integral, converts the result from energy deposited per unit volume at $\mathbf{r}$, to energy deposited per unit mass (the dose).

4.3.1. Inhomogeneous Terma Calculation

In section 3.3.1, the calculation of terma for a non-divergent beam incident on a homogeneous medium was covered. For inhomogeneous media, the terma at each point depends on the radiological depth at that point. The radiological depth at a point is found from the distribution of electron densities in the medium along a ray from the beam source to that point (see section 1.11). Also, for a divergent beam, an inverse-square term must be included in the terma calculation. For an N component primary photon energy spectrum, the terma at $\mathbf{r}'$ is given by

$$T(\mathbf{r}') = \left(\frac{r_{ref}}{|\mathbf{r}'|} \right)^2 \sum_{n=1}^N F_n(0) e^{-\frac{\mu}{\rho}_n d'(\mathbf{r}')} \left(\frac{\mu}{\rho} \right)_n E_n \quad (4-3)$$

where $|\mathbf{r}'|$ is the distance from the virtual source to $\mathbf{r}'$ and $d'(\mathbf{r}')$ is the radiological depth at $\mathbf{r}'$. The quantity r_{ref} is an arbitrary reference distance.

In this superposition algorithm, the density distribution in the irradiated medium is represented by a 3-D cartesian array of voxels, where the voxels are cubic with side length 0.5 cm. This array contains 32 layers with indices $k = 0$ to 31. To represent the lung phantom, voxels in layers 0-7 and 24-31 are assigned the value 1.0 and voxels in layers 8-23 are assigned the value 0.2. An element in the density array with indices (i, j, k) is denoted by $\rho(i, j, k)$. An array with the same structure is used to contain the terma. The lung phantom consists of slabs, so, apart from the slight angulation of the beam away from the central axis, the radiological depth is a function of k index only. The terma is therefore also a function of k index only, except that the field edges must be accounted for. On each layer of the terma array, the radiological depth at the centre of the layer is determined from the density array. The central axis terma for that layer is then

$$T(k) = \left(\frac{SSD}{SSD + d(k)} \right)^2 \sum_{n=1}^N F_n(0) e^{-\left(\frac{\mu}{\rho}\right)_n d'(k)} \left(\frac{\mu}{\rho} \right)_n E_n \quad (4-4)$$

where $d(k)$ and $d'(k)$ are the physical depth and radiological depth at the centre of layer k respectively. The inverse square term uses the SSD as the reference distance — note that the greater the SSD, the less is the reduction in terma on successive layers due to inverse-square falloff. The position of the field limits at each depth are determined, and the terma at voxel (i, j, k) , $T(i, j, k)$, is assigned the value $T(k)$ if the voxel is within the field limits. If a fraction f of a voxel is within the field limits, then a terma $fT(k)$ is assigned to that voxel in the terma array.

4.3.2. Cartesian Energy Deposition Kernels

The EDKs used in this algorithm are 3-D cartesian arrays with the same voxel dimensions (cubic with 0.5 cm sides) as those in the density and terma arrays. Primary and scattered EDKs were generated using the EGS4 Monte Carlo user code RTPCART (Murray, 1990).

In the Monte Carlo simulation, two arrays of dimensions $19 \times 19 \times 72$ were used to form the primary and scattered EDKs. The i , j , and k indices in the arrays run from 0-18, 0-18, and 0-71 respectively. Primary photons with the energy spectrum in figure 1.1, with initial momentum in the direction of increasing k index, were forced to interact throughout the region corresponding to voxel (9,9,8). Note that this ‘interaction voxel’ is on the axis of the array. There are 8 voxel layers ‘upstream’ of the layer containing the interaction voxel and 64 layers ‘downstream’ (including the interaction voxel) in which the record the fractional energy deposited. The variation in the probability of interaction with energy is accounted for by weighting the energy imparted and deposited by the mass attenuation coefficient as described in section 3.3.2. Since the fractional energy distribution in both the primary and scattered EDKs should be cylindrically symmetric about the axis of the arrays, only one quadrant of the arrays is required. Arrays of dimensions $10 \times 10 \times 72$, representing one quadrant, were created using a weighted average of the fractional energy contained in corresponding voxels in all four quadrants (Murray *et al*, 1989). This averaging process means that the uncertainty in the fractional energy assigned to voxels in one quadrant is less than that which would exist if the other three quadrants were simply discarded. The index of the interaction voxel in these arrays is (0,0,8).

For the density scaling method explained shortly, EDKs must be created for a range of densities. A primary and a scattered EDK is therefore generated for media with the same composition as water, but with densities relative to water of 0.2, 0.4, 0.6, 0.8, and 1.0. A separate simulation is required for each density. For each simulation, 250000 incident photons were used, and the PRESTA algorithm was used for electron step control (Bielajew and Rogers, 1987).

The EGS4 generated EDKs were output to a text file for storage. When read into the superposition program, any subset (including the interaction voxel) may be stored in arrays. Since the lung phantom consists of only 32 voxel layers, EDKs with dimensions $10 \times 10 \times 34$ are used, where the interaction voxel is on the third layer (indices (0,0,2)).

The relative significance of the primary and scattered EDKs is illustrated by summing the energy deposited in a 2-D horizontal ‘slice’ of each array at each layer. Since the EDKs are represented in only one quadrant, this summation must weight the energy per voxel appropriately to obtain the total energy deposited in all four quadrants.

The fractional energy per layer for the unit density primary and scattered EDKs relative to the energy imparted is plotted in figure 4.5(a) where Δz is the distance in cm from the interaction voxel to the centre of each layer. Layers which are ‘upstream’ of the layer containing the interaction voxel are not shown. The reason why there is less fractional energy deposited in the layer containing the interaction voxel (layer 0) than in the second layer (layer 1) in the primary EDK, is because incident photons do not on average interact until they have travelled half way through the interaction voxel and the majority of secondary electrons travel in the forward direction. Secondary electrons originating at the centre of the interaction voxel and travelling at least 0.75 cm in the forward direction will deposit approximately twice the amount of energy in layer 1 as in layer 0. It can be seen from the primary EDK data in figure 4.5(a) that the fractional primary energy on each layer drops off continuously past layer 1 due to less and less electrons penetrating to each successive layer. Also, the maximum longitudinal range of the secondary electrons can be seen to be approximately 3 cm. The relative percentage of fractional energy deposited in each layer due to the primary and scattered components is plotted in figure 4.5(b). Note that the scattered EDK does not dominate until $\Delta z > 3$ cm — the maximum secondary electron range.

For the remainder of the work in this chapter, the EDK is not separated into primary and scattered components — a single summed EDK for each density is used. Also note that the beam hardening correction discussed in chapter 3 is not used as it is only applicable when the components are separated: also, the maximum radiological depth in the lung phantom is only 9.6 cm, so beam hardening would not be a significant cause of error in the results.

4.3.3. Scaling Energy Deposition Kernels

There are an infinite number of possible density distributions surrounding a primary photon interaction site. This means that the formation of a unique EDK for each $\mathbf{r}'$ in (4-2) is impossible for practical calculations. A method therefore needs to be devised to utilise a finite number of available EDKs to obtain a suitable value of $H(\Delta\mathbf{r}, \mathbf{r}')$ for all $\mathbf{r}'$. The scaling procedure implemented in this work is based on the density scaling method proposed by Mackie *et al* (1985).

Consider an inhomogeneous medium in which there is an interaction point $\mathbf{r}'$ from which secondary electrons and scattered photons originate. For media of similar elemental composition, the rate at which electrons lose energy and the number of photon interactions per unit pathlength is proportional to density. Therefore, if the bold assumption is made that secondary electrons spread out in straight lines from the interaction site, then the fluence of both secondary electrons and scattered photons reaching a volume dV at $\mathbf{r}' + \Delta\mathbf{r}$ is

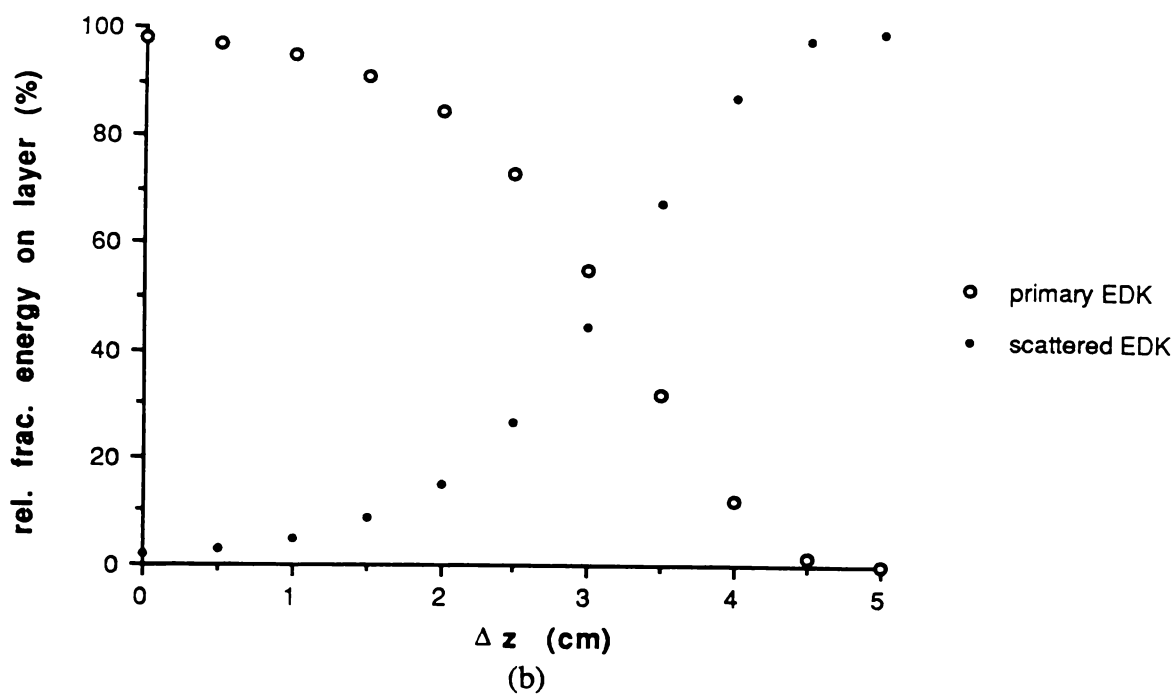
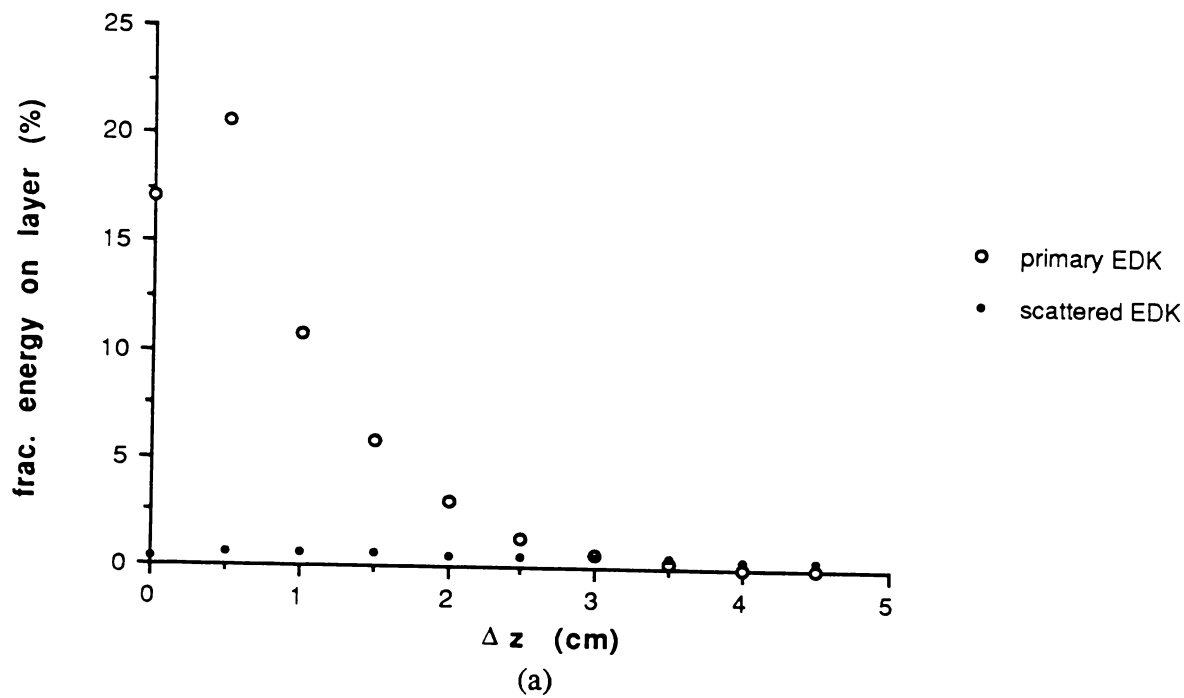


Figure 4.5. (a) Fractional energy deposited per layer for primary and scattered unit density EDKs. Δz is the distance in cm from the centre of the layer containing the interaction voxel to the centre of each layer. (b) Relative percentage of the fractional energy deposited per layer contained in the primary and scattered unit density EDKs.

dependent only on the average electron density along a ray between $\mathbf{r}'$ and $\mathbf{r}' + \Delta\mathbf{r}$. The average density is a function of $\Delta\mathbf{r}$ and $\mathbf{r}'$ and is given by

$$\rho_{ave} = \frac{1}{\Delta r} \int_{\mathbf{r}'}^{\mathbf{r}'+\Delta\mathbf{r}} \rho(r'') d\mathbf{r}'' \quad (4-5)$$

Also, the energy deposited in dV is related to the energy deposited in the same volume if the medium was homogeneous of density ρ_{ave} by

$$E(\Delta\mathbf{r}, \mathbf{r}', dV) = E(\rho_{ave}, \Delta\mathbf{r}, dV) \frac{\rho(\mathbf{r}' + \Delta\mathbf{r})}{\rho_{ave}}, \quad (4-6)$$

since the energy deposited per unit pathlength is proportional to density. In terms of the energy per unit volume H , (4-6) is written

$$H(\Delta\mathbf{r}, \mathbf{r}') = H(\rho_{ave}, \Delta\mathbf{r}) \frac{\rho(\mathbf{r}' + \Delta\mathbf{r})}{\rho_{ave}} \quad (4-7)$$

which can be substituted into (4-2). This density scaling method is illustrated in figure 4.6. In figure 4.6(a), the density varies between the interaction site $\mathbf{r}'$ and a unit volume at $\mathbf{r} = \mathbf{r}' + \Delta\mathbf{r}$. The average density between these two points is ρ_{ave} and the density at $\mathbf{r}$ is $\rho(\mathbf{r})$. In figure 4.6(b), the medium is homogeneous of density ρ_{ave} . On substituting (4-7) into (4-2), the expression

$$D(\mathbf{r}) = \int_{\mathbf{r}'} T(\mathbf{r}') \rho(\mathbf{r}') \frac{1}{\rho_{ave}} H(\rho_{ave}, \Delta\mathbf{r}) d^3\mathbf{r}' \quad (4-8)$$

is obtained. Note that the density at the calculation point $\mathbf{r} = \mathbf{r}' + \Delta\mathbf{r}$ cancels, making the dose calculation independent of the density at the calculation point.

Implementation of this method requires that $H(\rho_{ave}, \Delta\mathbf{r})$ be obtainable for all possible values of ρ_{ave} . It is for this purpose that EDKs are produced for a range of densities. For this scaling method to be strictly valid, secondary electrons must travel in straight paths. Electrons are in fact highly scattered by atomic electrons and nuclei, which causes some error in the scaling method (as discussed in section 4.4). The significance of electron scattering in the transport of secondary electrons is now discussed.

4.3.4. Secondary Electron Scattering

In spreading out from the primary interaction site, multiple Coulomb scattering continuously deflects the secondary electrons away from their direction of motion (Andreo, 1985). Consider a group of secondary electrons set in motion from an interaction site at the origin of a cartesian coordinate system, which initially travel in the same direction as the primary photon beam. These electrons form a parallel beam in the z direction. As the electrons traverse the medium, the beam becomes spread out with a lateral displacement distribution which is approximately Gaussian (Hogstrom *et al*, 1981). The standard deviation of the distribution is found using the mass angular scattering power $d\bar{\theta}^2/\rho dz$. If the beam

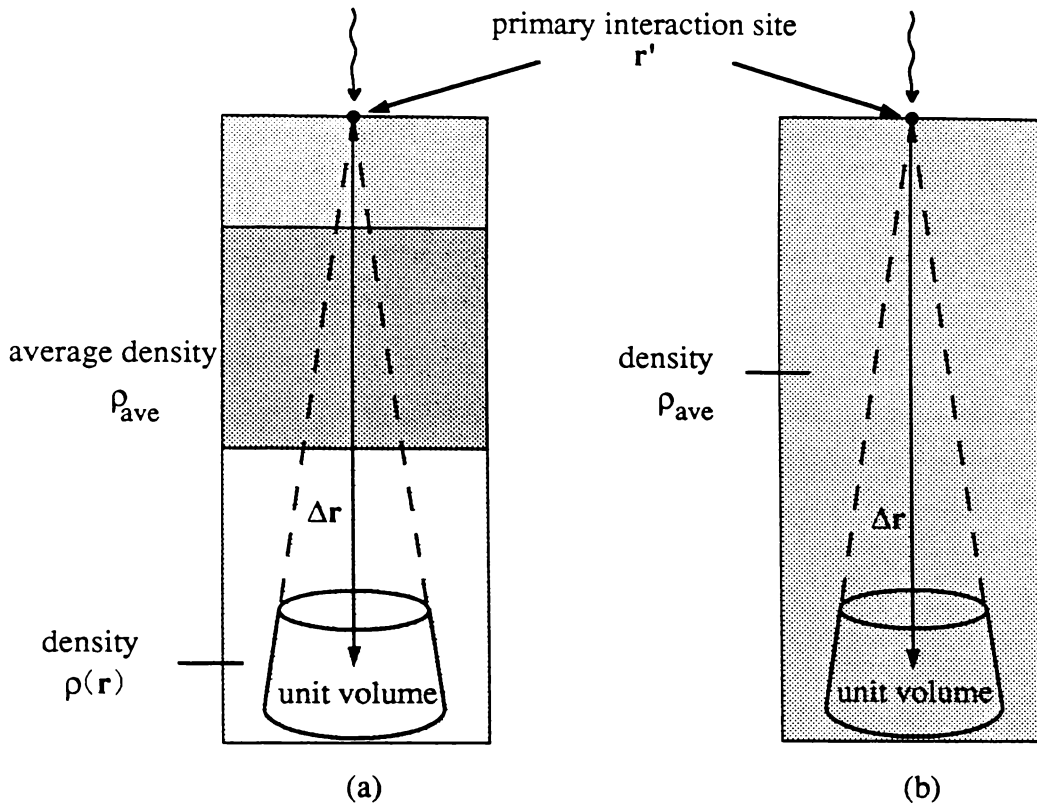


Figure 4.6 Illustration of the density scaling method. In (a), the medium is inhomogeneous with an average density ρ_{ave} between r' and r . In (b), the medium is homogeneous of density ρ_{ave} .

traverses a distance Δz in the z direction in a medium of varying density then the standard deviation of the beam is given by

$$\sigma_0(\Delta z) = \sqrt{\frac{1}{2} \int_0^{\Delta z} \frac{d\bar{\theta}^2}{\rho dz} \rho(z') (\Delta z - z')^2 dz'}. \quad (4-9)$$

For the scaling method to be valid, the lateral electron distribution at Δz must be the same as if the density between the origin and Δz was equal to the average density ρ_{ave} between the origin and Δz . This requires that the standard deviation be the same in both cases. The standard deviation for a density ρ_{ave} is

$$\sigma_1(\Delta z) = \sqrt{\frac{1}{2} \rho_{ave} \int_0^{\Delta z} \frac{d\bar{\theta}^2}{\rho dz} (\Delta z - z')^2 dz'}. \quad (4-10)$$

In general $\sigma_0 \neq \sigma_1$ since σ_0 is dependent on the density distribution between the origin and Δz — not just on the average density along this path. The consequences of this inequality are discussed in section 4.4.

The significance of scattering can be seen by calculating the standard deviation of a narrow beam of electrons after traversing a medium of known thickness. The mass angular scattering power is a decreasing function of energy given for a unit density medium by Werner

et al (1982) as

$$\frac{d\bar{\theta}^2}{\rho dz}(E) = 4.52E^{-1.78} \quad (4-11)$$

where E is the electron energy in MeV. If the approximation is made that the electrons in the narrow beam have a constant energy of 2 MeV, then from (4-10) the standard deviation in water is

$$\begin{aligned} \sigma(\Delta z) &= \sqrt{\frac{\frac{d\bar{\theta}^2}{\rho dz} \Delta z^3}{6}} \\ &= 0.47 \Delta z^{3/2} \end{aligned} \quad (4-12)$$

which gives a standard deviation at $\Delta z = 1.0$ cm of $\sigma = 0.47$ cm. This indicates that scattering is a significant factor in determining the spatial distribution of secondary electrons about the primary interaction site, and that the assumption of straight line motion is not strictly valid.

4.3.5. Dose Calculation Procedure

The superposition integral ((4-2) or (4-8)) implies that the dose at a single deposition point $\mathbf{r}$ is calculated due to all interaction points $\mathbf{r}'$ in the irradiated medium before the dose at another point is calculated. The superposition calculation is then referred to as being carried out using the *deposition point of view* (Mackie *et al*, 1985).

An alternative is to calculate the increment in dose at all deposition points $\mathbf{r}$ due to a single interaction point $\mathbf{r}'$ before proceeding to calculate the dose due to another interaction point. This method is referred to as the *interaction point of view*. Both methods have advantages, but the interaction point of view is used here since this method is more convenient for the density scaling method used in this algorithm. The algorithm for calculating the dose distribution due to an *interaction voxel* in the terma array with indices (i, j, k) proceeds as follows. Using (4-8), The dose contribution to a *deposition voxel* at $(i + \Delta i, j + \Delta j, k + \Delta k)$ is given by

$$D(i + \Delta i, j + \Delta j, k + \Delta k; i, j, k) = T(i, j, k) \rho(i, j, k) \frac{1}{\rho_{ave}} H(\rho_{ave}, \Delta i, \Delta j, \Delta k). \quad (4-13)$$

The first factor which must be calculated before (4-13) can be evaluated is ρ_{ave} . In the slab inhomogeneity case, ρ_{ave} is independent of Δi and Δj , so Δk is stepped through and ρ_{ave} is calculated for that Δk before the dose contribution to each voxel on layer $k + \Delta k$ is calculated. The average density is found by a simple ray trace between the centre of the interaction level to the centre of the deposition level:

$$\rho_{ave} = \frac{\left(\sum_{l=0}^{\Delta k} w_l \rho(k + l) \right)}{\sum_{l=0}^{\Delta k} w_l} \quad (4-14)$$

where the weighting factors w_0 and $w_{\Delta k}$ are normally set to 0.5 because on average, the increment in radiological pathlength for secondary particles due to these voxels is half

the radiological thickness of the voxel. For the intervening voxels, the weighting is unity (w_1 to $w_{\Delta k-1} = 1.0$). Of course, if $\Delta k < 0$ the summation is from $l = \Delta k$ to $l = 0$. A more sophisticated ray trace which is applicable to non-slab inhomogeneities has been implemented on the GRATIS treatment planning system and is described in chapter 5.

Once ρ_{ave} is found, $H(\rho_{ave}, \Delta i, \Delta j, \Delta k)$ is determined by interpolation using the set of EDKs generated for densities of 0.2, 0.4, 0.6, 0.8, and 1.0 g cm⁻³. Linear interpolation is used, although a second or third order difference algorithm may achieve slightly greater accuracy but at considerable cost to calculation speed.

Note that the the division by ρ_{ave} in (4-13) could be avoided if the Monte Carlo generated EDK values were divided by the density for which the EDK was formed on reading the data into the arrays. This however causes a problem with the interpolation of the EDKs, as for some EDK voxel indices, the distance from the interaction voxel is such that the fractional energy is much higher for a particular density EDK than the value in the voxel with the same indices in the EDK with the next highest density. When the fractional energy in these voxels is divided by density, the difference in the values in two EDKs becomes even larger. This makes interpolation between densities uncertain since the differences in values are large. This occurs for example if a voxel in the $\rho = 0.2$ EDK is within the secondary electron range, but the same voxel in the $\rho = 0.4$ EDK is not. For this reason, the fractional energy contained in each voxel is not divided by the density for which the EDK was formed at the time of reading in the data.

4.3.6. Extension to Bone Inhomogeneities

In deriving (4-8), the assumption was made that the medium was entirely water-like in composition (constant effective atomic number). For such material, both the mass attenuation coefficient and mass stopping power are approximately constant (Johns and Cunningham, 1983). The linear attenuation coefficient and linear stopping power can therefore be obtained by multiplying the value for water by the mass density. This simple relationship is used in calculating the terma, energy imparted per unit volume, and average density between the interaction and deposition sites.

For bone, both the mass attenuation coefficient and mass stopping power are less than that for water. The reason for this is that bone has a lower hydrogen content than water, and therefore contains less electrons per unit mass (see section 1.10). For the photon energies of concern here, Compton scattering is the major cause of attenuation for both water and bone. Since the Compton attenuation coefficient is proportional to electron density (Johns and Cunningham, 1983), the linear attenuation coefficient for bone is closely approximated by multiplying the mass attenuation coefficient for water by the electron density relative to water:

$$\mu_{bone} \approx \left(\frac{\mu}{\rho} \right)_{water} \rho_e^w(bone). \quad (4-15)$$

This relationship is verified by comparing the linear attenuation coefficient for bone calcu-

lated using (4-15) to the true value for bone. In figure 4.7, the linear attenuation coefficient for water is plotted, along with the calculated and true values for bone of mass density 1.65 g cm^{-3} . This bone is that for which attenuation coefficient data is given in tables in Johns and Cunningham (1983) and has an electron density relative to water of $\rho_e^w = 1.57$. The calculated and true values can be seen to agree very closely. It is therefore reasonable to replace $\rho(r')$ in (4-8) by $\rho_e^w(r')$. Also, the attenuation of scattered photons can be scaled by electron density. For higher energies, where pair production is significant, the procedure described above becomes invalid since the pair production attenuation coefficient is not proportional to electron density. This has been found to be the case for 20 MeV photons (Mohan *et al*, 1986). It should be remembered however that this energy is much higher than the mean energy of any currently used treatment beam. It can be seen from figure 4.7 that there is only a slight difference in the true and predicted values of the linear attenuation coefficient at 10 MeV – the true value is slightly higher due to pair production.

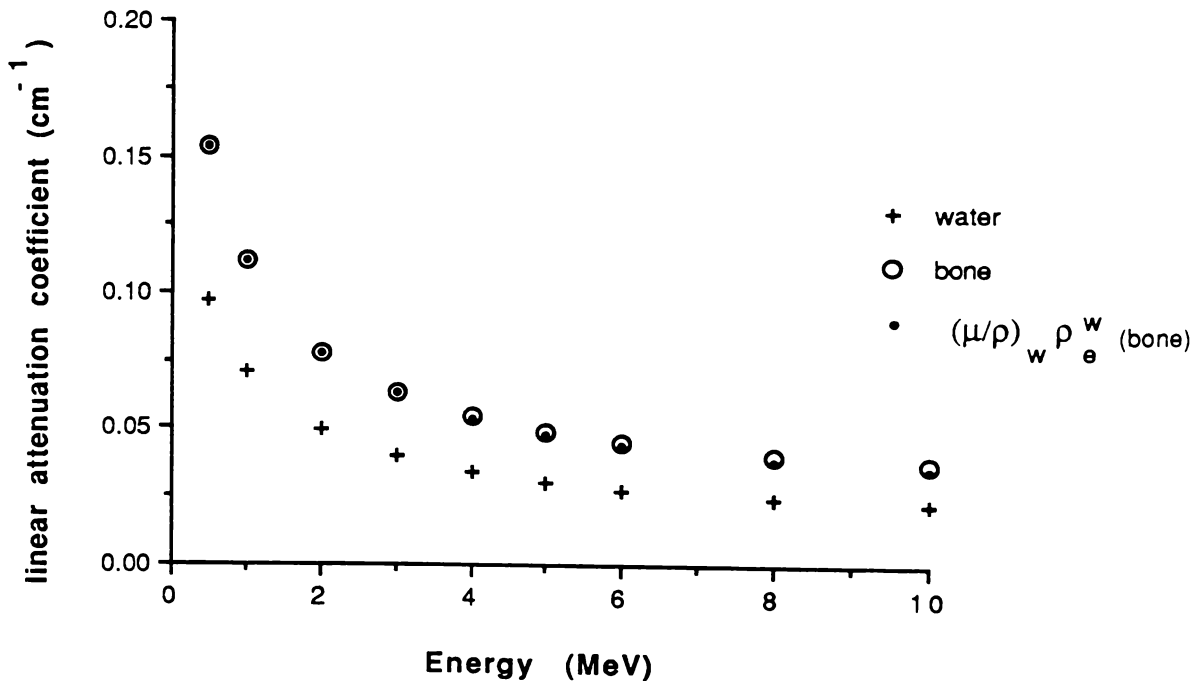


Figure 4.7 Comparison of the actual linear attenuation coefficient in bone of mass density 1.65 g cm^{-3} and the value predicted by multiplying the mass attenuation coefficient for water by the relative electron density of the bone (1.57). Mass attenuation coefficient for water is also shown.

Similarly to the linear attenuation coefficient, the linear collision stopping power is approximately proportional to electron density for these energies (Klevenhagen, 1985). Therefore, the slowing down of secondary electrons, and consequently the energy deposited per unit pathlength, is approximately proportional to electron density. However, since the scattering power is dependent on atomic number rather than electron density (Klevenhagen, 1985), it is not strictly valid to use electron density to scale the primary EDK (as pointed out by Mackie *et al* (1985)). The average effective atomic number could be found along with

the average electron density ρ_{ave} , and the interpolation performed between primary EDKs formed for media of different atomic number as well as different density.

Neglecting the dependence of electron scattering on atomic number, (4-8) can be rewritten with the density replaced by the electron density:

$$D(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})} \int_{\mathbf{r}'} T(\mathbf{r}') \rho_e^w(\mathbf{r}') \frac{1}{\rho_{ave}} H(\rho_{ave}, \Delta\mathbf{r}) \rho_e^w(\mathbf{r}' + \Delta\mathbf{r}) d^3r'. \quad (4-16)$$

where ρ_{ave} is now the average electron density between $\mathbf{r}'$ and $\mathbf{r}$. Note that in this expression, there is a division by mass density outside the integral (to convert from energy to dose) as in (4-2). This mass density does not in general cancel with the electron density inside the integral as was the case in deriving (4-8). If however, the dose in a small volume of water is required at $\mathbf{r}$, then $\rho_e^w(\mathbf{r}' + \Delta\mathbf{r}) = \rho(\mathbf{r}) = 1.0$ and the density terms cancel as before. The dose at $\mathbf{r}$ is then

$$D(\mathbf{r}) = \int_{\mathbf{r}'} T(\mathbf{r}') \rho_e^w(\mathbf{r}') \frac{1}{\rho_{ave}} H(\rho_{ave}, \Delta\mathbf{r}) d^3r'. \quad (4-17)$$

In the superposition algorithm, the dose contribution to voxel $(i + \Delta i, j + \Delta j, k + \Delta k)$ due to term in voxel (i, j, k) is now given by (4-13) with the density $\rho(i, j, k)$ replaced by $\rho_e^w(i, j, k)$.

If the actual dose to bone is required, rather than the dose to a small mass of water in the bone, the dose can be first calculated using (4-17) then converted to dose to bone using a multiplication factor. This factor is simply the ratio of the relative electron density to the mass density:

$$f(\mathbf{r}) = \frac{\rho_e^w(\mathbf{r})}{\rho(\mathbf{r})}. \quad (4-18)$$

If $\mathbf{r}$ is within bone of mass density 1.65 g cm^{-3} , this factor is $1.57/1.65 = 0.95$. Note that this is the ratio of the number of electrons per gram in bone to that in water ($3.19 \times 10^{23}/3.34 \times 10^{23}$).

4.4. Lung Dose Calculation Results

An illustration of how the dose distribution is related to primary photon interactions is given by separating the depth dose distributions arising from interactions in the three regions of the phantom. This is shown in figure 4.8 for a $10 \times 10 \text{ cm}$ field. The three dose components are generated by performing three separate superposition calculations, with the range of interaction voxel levels restricted to $k = 0 - 7$ (region 1), $k = 8 - 23$ (region 2) and $k = 24 - 31$ (region 3) respectively. Note the large contribution to dose throughout the lung from primary interactions in the first water region. Also note that the buildup region in the curve which is due to interactions in lung, exceeds the thickness of the lung. Both these observations indicate that the longitudinal secondary electron range in the lung exceeds the thickness of the lung. As well as the long electron range in the direction of the beam, secondary electrons have a significant lateral range, which may cause lateral electron

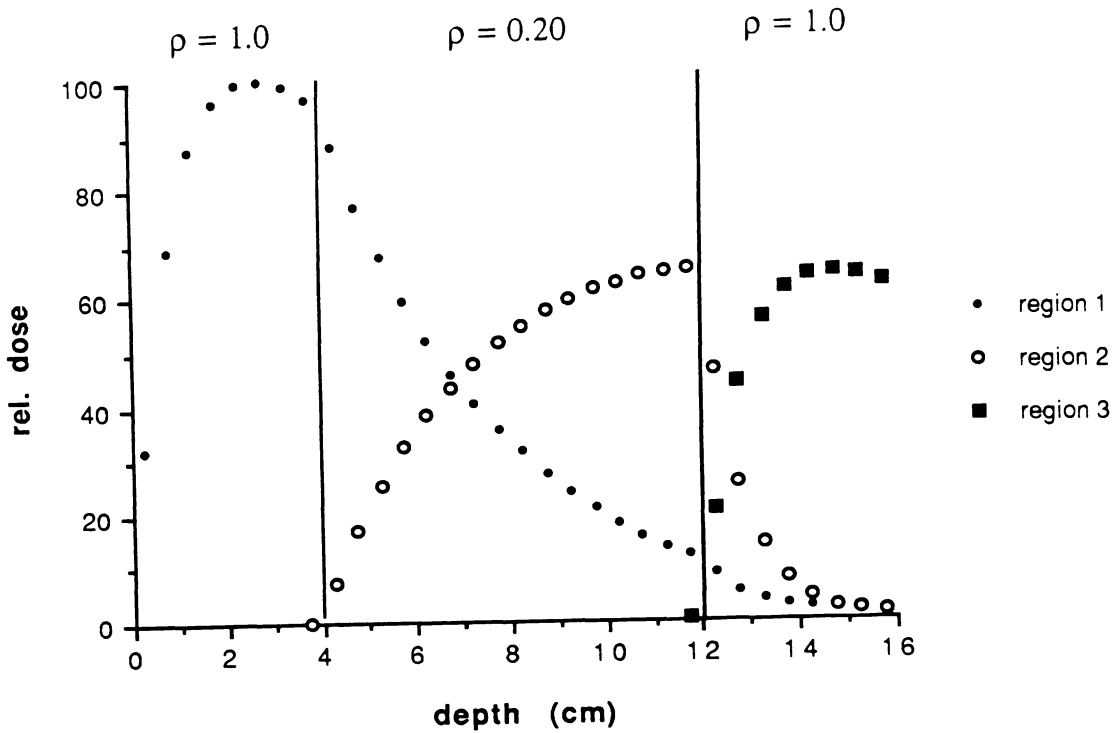


Figure 4.8 Superposition depth dose result for a 10×10 cm field, where the terma is non-zero in only one region at a time. A large dose contribution in the lung due to the top water region (region 1) is apparent. Also note the long buildup region due to TERMA in the lung (region 2). The sum of these three curves gives the total dose.

disequilibrium to occur at central axis. This is clearly a case where the range of secondary electrons cannot be ignored and the assumption of local energy deposition made by scatter function models is invalid.

In figure 4.8, dose is normalised to that on the layer containing the maximum dose. This layer ($k = 4$) is centred at a depth of 2.25 cm. The experimentally observed d_{max} is at 2.1 cm, so the layer predicted to contain the maximum dose is correct.

Experimental depth dose measurements made with a 0.6 cm^3 Baldwin Farmer ionization chamber in the lung phantom for a 5×5 cm field, were compared with EGS4 Monte Carlo simulations using 8.2 million incident photons with the energy spectrum in figure 1.1. Results are shown to be in good agreement in figure 4.9. Because the Monte Carlo results include more data points, these results are used for comparison with superposition.

Superposition depth dose results for a 5×5 cm field for a lung density of 0.20 g cm^{-3} are compared with Monte Carlo results in figure 4.10. The superposition dose is seen to be significantly higher than the Monte Carlo generated dose in the lung region. This suggests that the superposition algorithm, using the density scaling method discussed in section 4.3.3, still does not predict the correct amount of lateral electron disequilibrium in the lung. The agreement is still much better than that obtained using the ETAR and dSAR methods however (see figure 4.3(a)).

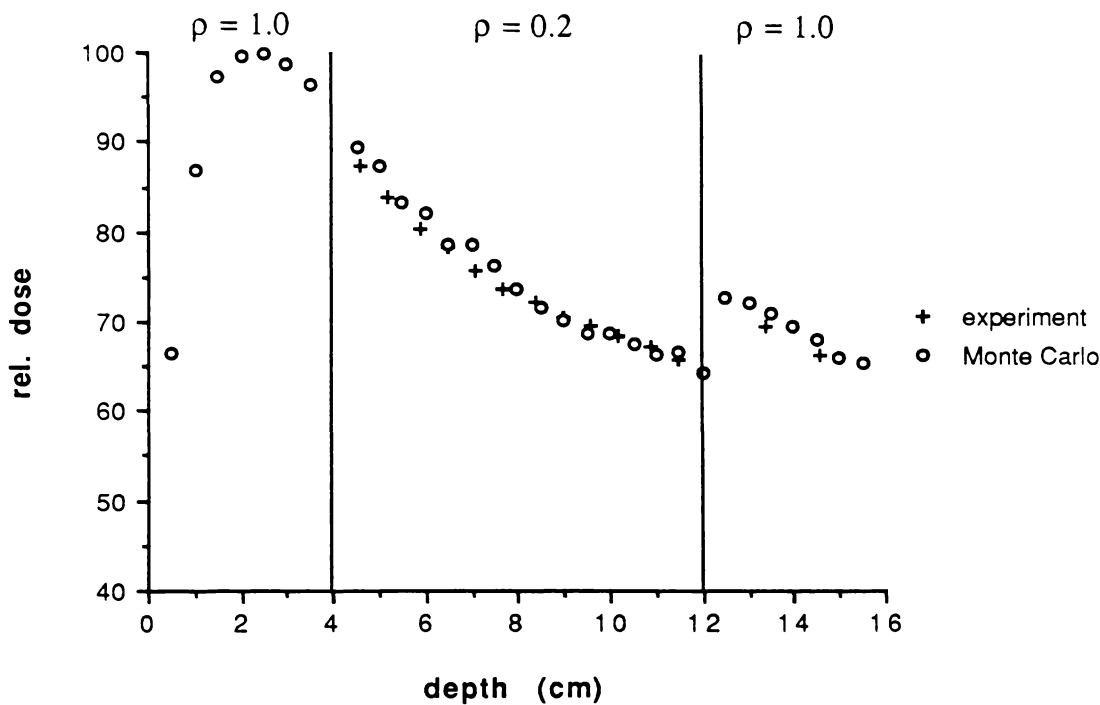


Figure 4.9 Depth dose measurements made with a 0.6 cm³ Baldwin Farmer ionization chamber, compared with EGS4 Monte Carlo results for a 5×5 cm field. 8.2 million incident photons, with the surface energy spectrum in figure 1.1, were used for the Monte Carlo simulation.

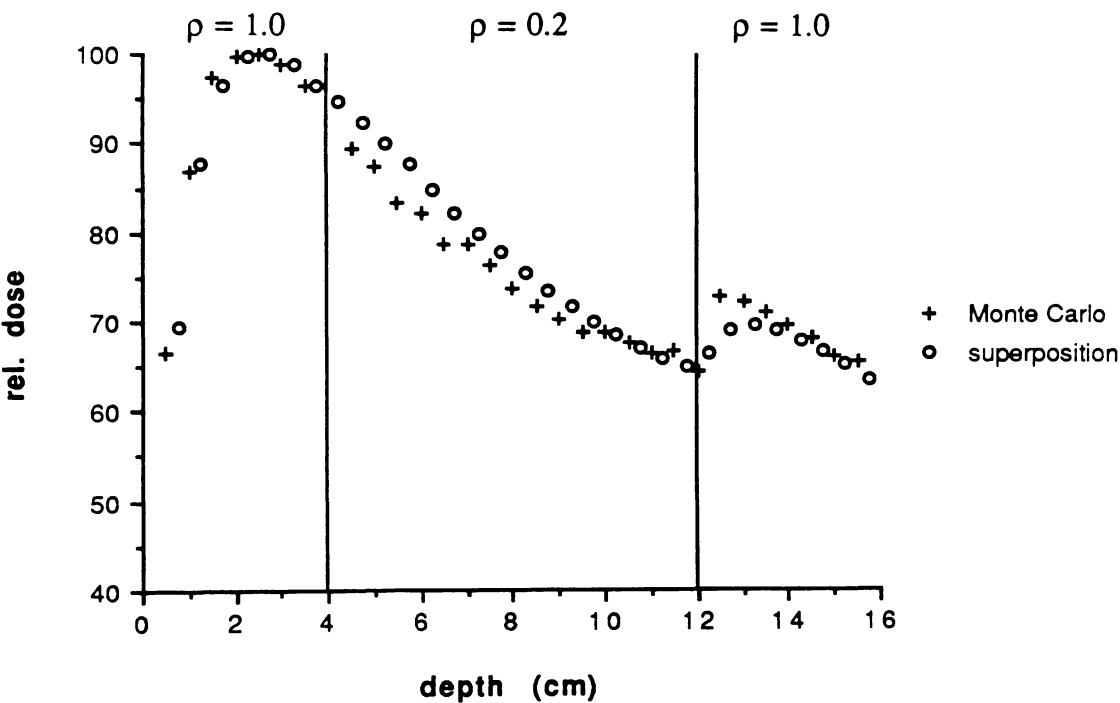
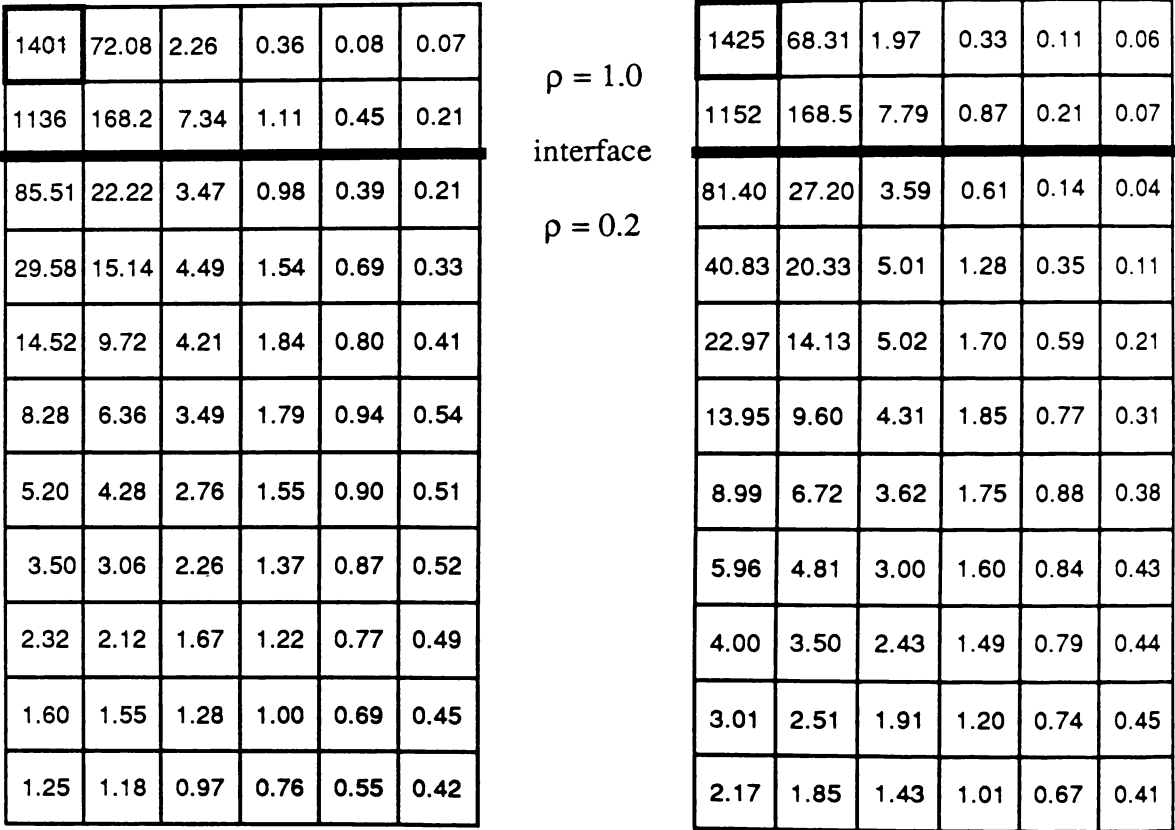


Figure 4.10 Superposition depth dose curve for a 5×5 cm field compared with Monte Carlo results. The disagreement is due to the assumption made by the superposition algorithm that secondary electrons spread out in straight lines from the primary interaction site.

This overestimate in lung dose can be explained by using the EDK scaling method to generate an EDK containing a water-lung interface, and comparing the result to an EDK produced using a Monte Carlo simulation. A portion of the slice at $j = 0$ in each of these EDKs is shown in figure 4.11. The values are the fractional energy multiplied by 10^4 . The superposition generated EDK is formed by assigning a unit term to the interaction voxel and calculating the dose to the surrounding voxels. The dose in each voxel is then multiplied by the density of that voxel to convert from dose to fractional energy. Inspection shows that the energy in voxels near the axis in the superposition array is in general higher than in the Monte Carlo generated EDK, and becomes slightly lower away from the axis.



(a) Monte Carlo generated EDK

(b) Superposition generated EDK

Figure 4.11 Total EDKs formed using (a) Monte Carlo and (b) superposition with a lung-water interface as shown. The interaction voxel is indicated by the bold box. Fractional energy multiplied by 10^4 is shown in a slice containing the interaction voxel. The superposition EDK gives an energy distribution which is more ‘forward peaked’ than the Monte Carlo result.

In figure 4.12, the total fractional energy (expressed as a percentage of the energy released in the interaction voxel) deposited in voxel layers 0-31 of the interface-EDKs is compared. Fractional energy in all four quadrants of a layer is computed by summing the energy in voxels in one quadrant with an appropriate weighting so that the summation includes energy deposited in the other three quadrants. It can be seen that the agreement in total energy per layer is better near the interface than further away. The sum of the fractional energy deposited in all 32 layers is $60.73 \pm 0.1\%$ and 61.67% for the Monte Carlo and super-

position EDKs respectively: the total energy deposited using superposition is within 2% of that predicted by Monte Carlo. The conclusion from this comparison is that the energy distribution calculated using superposition is more forward peaked than the Monte Carlo distribution, although total energy deposited within the region is almost the same.

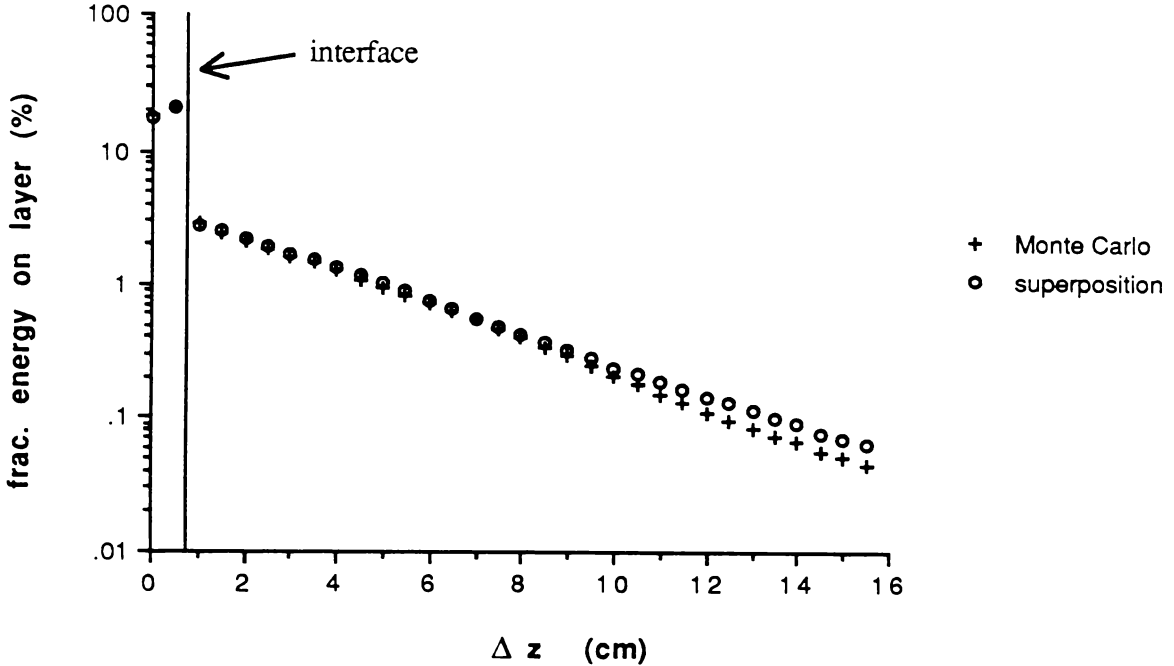


Figure 4.12. Fractional energy deposited per layer in EDKs containing a water-lung interface between layers 1 and 2. Results obtained from EDKs generated by direct Monte Carlo simulation and by the EDK scaling method used in the superposition calculation are shown. The horizontal coordinate Δz is the distance in cm from the centre of the interaction voxel to the centre of each layer. The sum of the fractional energy deposited in all 32 layers is $60.73 \pm 0.1\%$ and 61.67% for the Monte Carlo and superposition EDKs respectively.

The differences in the superposition Monte Carlo results in figures 4.11 and 4.12 are due to the invalid assumption made by the EDK scaling method that secondary electrons spread out in straight lines from the primary interaction site. The transport of electrons away from the interaction site is influenced significantly by scattering as shown in section 4.3.4. Because scattering is important, the inequality of (4-9) and (4-10) is also important. It can be shown from these equations that when the interaction voxel is in a higher density layer than the average density between the interaction and deposition voxels, the standard deviation is greater at the deposition voxel layer than if the medium were homogeneous of density ρ_{ave} . This is expected since although the number of scattering events depends on ρ_{ave} , it is the scattering events near the interaction voxel which are most significant in determining the standard deviation at the deposition voxel layer. This theory is verified by calculating the standard deviation of the fractional energy distribution on each layer of the EDKs in figure 4.13. The standard deviation on layer Δk in the EDKs can be found from the shape of the EDK profile across the array on layer k . The data used is in a row of the

EDK with j index equal to 0. If the EDK value at $(i, 0, k)$ is $E(i, 0, k)$ then

$$\sigma(k) = \sqrt{\frac{\sum_{i=1}^9 2 \cdot E(i, 0, k) \cdot i^2}{E(0, 0, k) + \sum_{i=1}^9 2 \cdot E(i, 0, k)}} \quad (4-19)$$

The plot of σ versus Δz in figure 4.13 illustrates that the standard deviation of the Monte Carlo generated EDK increases more rapidly than that of the superposition EDK past the interface, ie. the Monte Carlo EDK is 'broader' than the superposition EDK. The horizontal coordinate Δz is the distance in cm from the centre of the interaction voxel to the centre of each layer. If separate Monte Carlo generated EDKs were produced for each interaction voxel, the calculated dose in the lung due to the first water region would be lower than that predicted using the scaling method since a greater degree of lateral electron disequilibrium would be predicted by the broader EDKs.

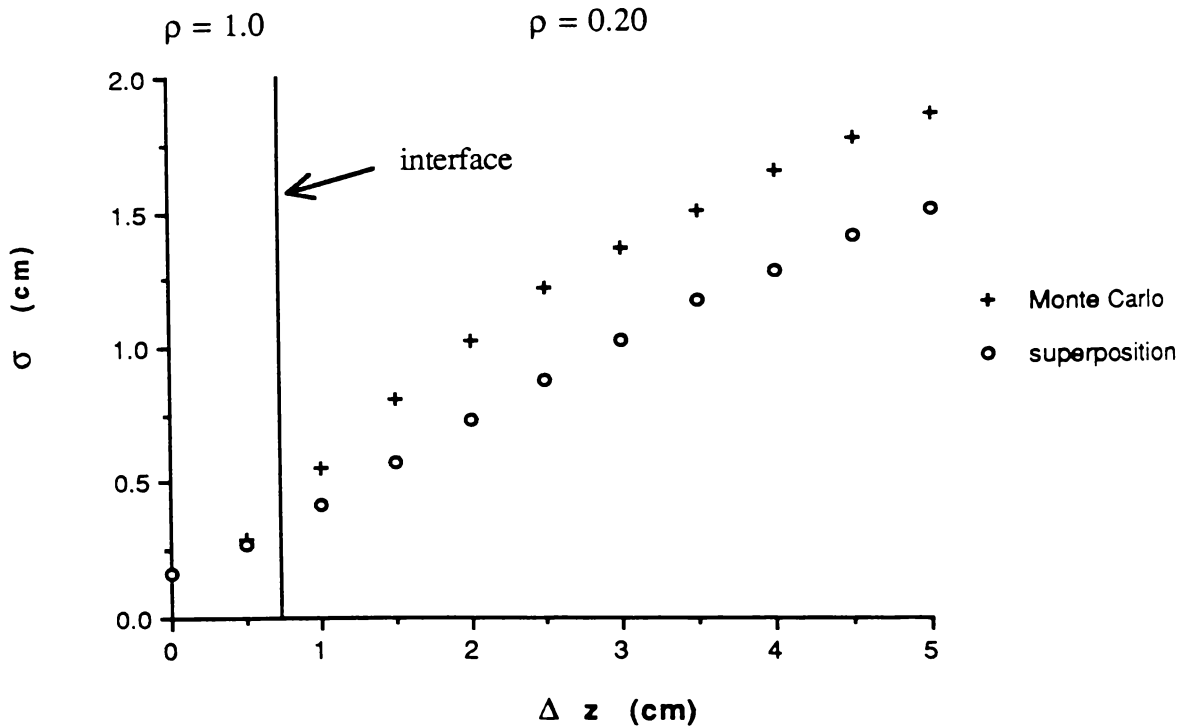


Figure 4.13 Standard deviation of the Monte Carlo and superposition generated EDKs containing a lung-water interface. The Monte Carlo result gives a more spread out distribution past the interface than superposition. This is because electron scattering is incorrectly accounted for.

A similar result to that in figure 4.10 was obtained by Woo and Cunningham (1990) who used superposition to calculate the central axis dose in a lung phantom for a 2.82 cm radius, 6 MV photon beam. A Monte Carlo generated depth dose curve for a 2.82 cm radius beam was compared with the superposition results obtained using density scaling. The superposition calculation gave a significant overestimate in dose in the lung region. These researchers also used the density scaling method to predict the dose beyond an air gap due to primary photon interactions in a thin 'source' region in a water slab before the air gap. The dose was either underestimated or overestimated depending on the position

of the source region with respect to the proximal surface of the air gap. It was concluded that because of scattering, the density scaling method does not predict the correct lateral spread of secondary electrons beyond the air gap — hence predicting an incorrect degree of lateral electron disequilibrium.

The error in the density scaling method can be corrected for to some extent by altering the weighting w_0 of the interaction voxel density in calculating ρ_{ave} in (4-14) (Mackie, 1985). If w_0 is made larger than 0.50, ρ_{ave} becomes closer to the interaction voxel density. When the interaction voxel is in the water before the water-lung interface, this has the effect of increasing ρ_{ave} between the interaction voxel and a deposition voxel in the lung beyond the interface. Since the value at a particular index in the EDKs generally decreases with an increase in the EDK density, and since there is a division by ρ_{ave} in (4-13), the increase in ρ_{ave} means that the dose delivered to the deposition voxel due to that interaction voxel decreases. For a lung-interaction water-deposition situation, ρ_{ave} is decreased by the increase in interaction voxel density weighting, with a consequent increase in dose contribution to the deposition voxel.

This correction partially accounts for the fact that voxels close to the interaction voxel have the largest influence on electron spread. The value of w_0 may be set differently for different lateral positions Δi and Δj in the EDK. Since the required effect of this modification for the water-lung interface is to flatten the EDK profile past the interface (see figure 4.11), it is best to set w_0 highest for voxels on the EDK axis ($\Delta i = 0$, $\Delta j = 0$), so that the calculated value of ρ_{ave} is greatest for these voxels. The most suitable values for w_0 can be found by intelligent use of the trial and error method. It was found here that values of $w_0 = 1.0$ for when Δi and Δj are both zero, and $w_0 = 0.8$ for all other Δi and Δj give good results for the 5×5 cm field, as seen in figure 4.14. Note that the depth dose curve drops significantly immediately after the water-lung interface due to this modification. This is expected since for interaction voxels in the water above the interface, the increase in ρ_{ave} due to increased interaction voxel weighting is larger than further away from the interface.

Note that this modification to the method of determining ρ_{ave} is strictly only applicable when determining primary EDK values since it is secondary electron scattering that is being accounted for. However, since the scattered EDK values are much less significant than the primary values, the slight modification to the scattered EDK values will not significantly affect the total dose distribution. Furthermore, the effect of the modified weighting of the interaction voxel decreases with distance from that voxel since more voxel layers are included in the average density calculation: therefore for layers far enough away from the interaction voxel layer for the scattered component to be significant, the effect of the modified weighting will be minimal.

The use of voxel weights to overcome a fundamental flaw in the transport of electrons must be recognised as a limitation. The trial and error method of determining the weight is likely to lead to some dose artefacts when the model is applied more generally. Formulating

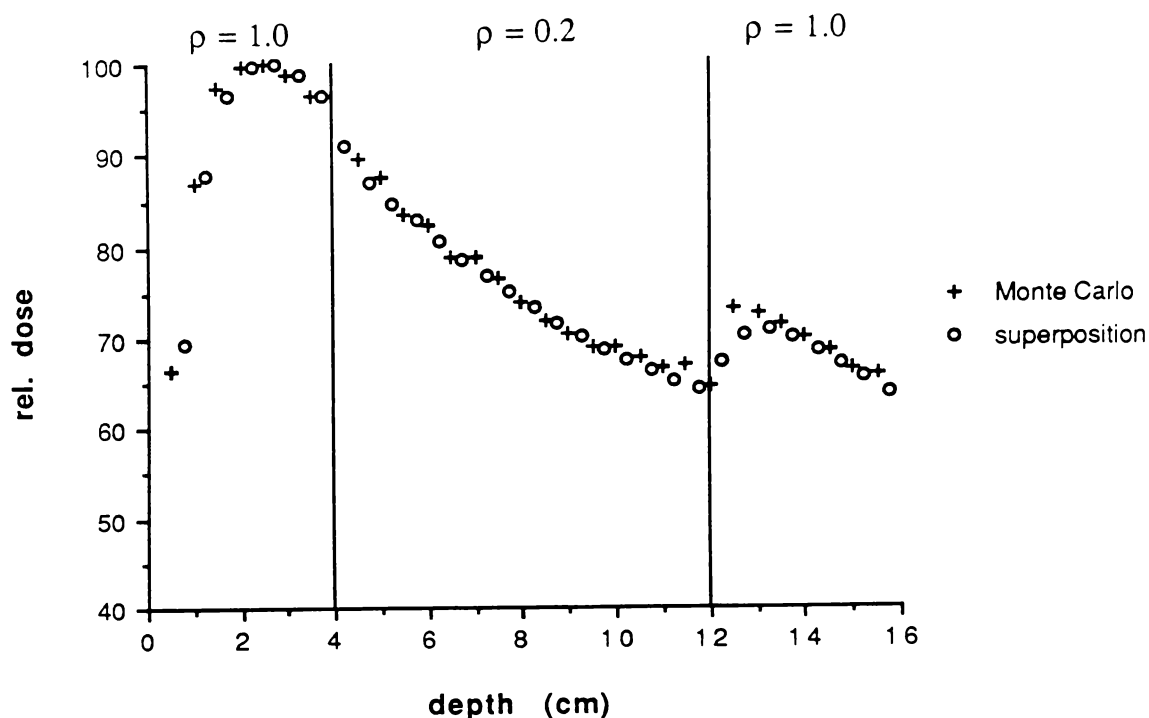


Figure 4.14 Comparison of Monte Carlo and superposition depth dose curves for a 5×5 cm field when a modification is made to the ray tracing method to find the density between the interaction and deposition voxels. An increased density weighting is given to the interaction voxel since this voxel is the most significant in the electron scattering process.

a scaling theorem which rigorously accounts for electron scattering requires that electron transport be modelled accurately. The scattering power is strongly energy dependent which means that since the electron spectrum is polyenergetic and electrons are decreasing in energy with distance from the interaction site, scattering is difficult to quantify analytically. As well as this, the initial directions of the secondary electrons have a wide distribution. A more rigorous method of correcting for non-idealities caused by electron scatter is required, but it must also be remembered that more complex correction methods are also more time consuming to compute.

A lung density of 0.20 g cm^{-3} is at the low end of the mean density found from CT scans (van Dyke *et al*, 1982). Since lateral electron disequilibrium increases as lung density decreases, the dose in 0.20 g cm^{-3} material is more difficult to predict than that in higher density media. Monte Carlo results for a 5×5 cm field with a lung density of 0.30 g cm^{-3} are compared with superposition in figure 4.15. Superposition results are given with and without the modification to w_0 . In comparing these results to those in figure 4.10, it is apparent that superposition without the modification to the average density calculation becomes more accurate as lung density increases. Note that the dose in lung actually increases with density, even though primary photon fluence is reduced. This emphasises the importance of a loss of lateral electronic equilibrium on the dose. Also note that there is now a less pronounced buildup region in the second water region — this is because the loss of lateral electronic equilibrium in the lung is less extreme than with the lower density.

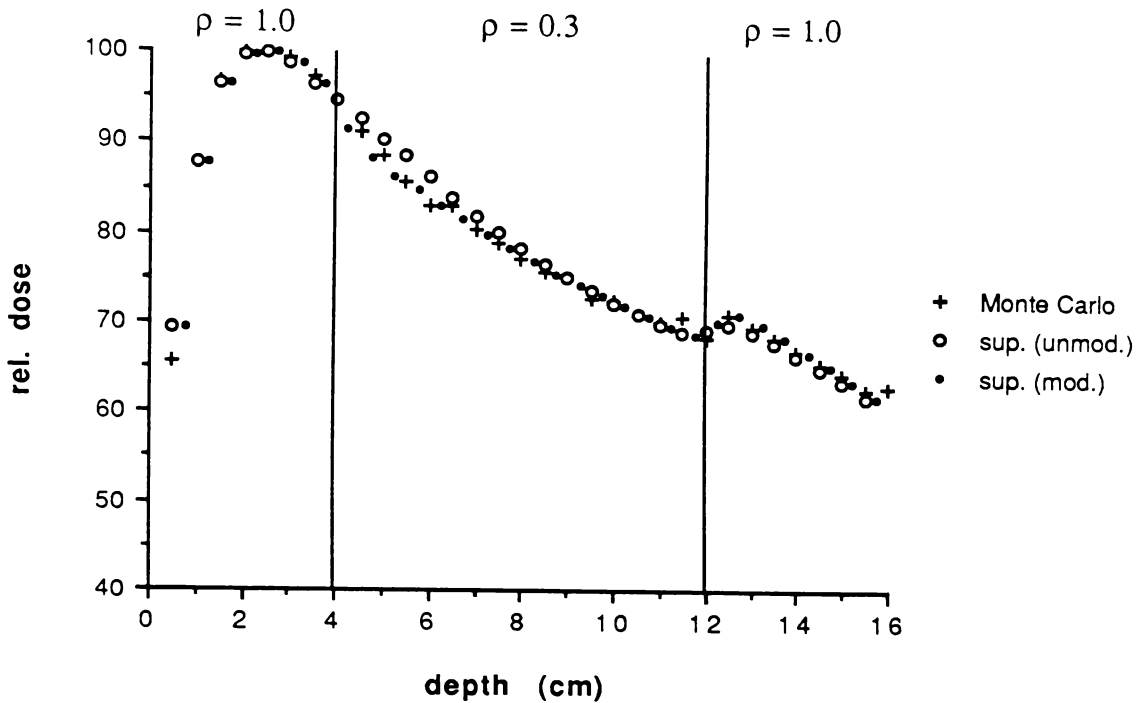


Figure 4.15 Monte Carlo and superposition depth dose curves for a 5×5 cm field when the lung density is 0.30 g cm^{-3} . Curves are shown with and without the modification to the ray trace.

An experimental dose profile obtained using film at mid-lung (8.2 cm deep) is compared with superposition in figure 4.16, where the superposition profile is at the level of voxels centred at 8.25 cm deep. The difference in field width between these depths due to beam divergence for a 100 cm SSD is negligible (0.15 mm). Note that the superposition profile shown was generated using a standard interaction voxel density weighting of 0.5. The modification required for the depth dose curves does not significantly alter the profile shape. The result shows a much closer agreement with experiment than does the ETAR result (see figure 4.4). This improvement is due to the penumbral broadening caused by lateral electron disequilibrium being accounted for. Note that the half width of the field projected to this depth is 5.4 cm which corresponds closely to the 50% dose level on both profiles. Profile results using superposition may be further improved if such factors as finite source size, scatter of primary photons off the collimator assembly, and angular distribution of primary photons are accounted for in the superposition algorithm (Mohan *et al.*, 1986).

4.5. Summary and Future Work

It has been demonstrated that the superposition method is superior to scatter function models in calculating dose in a lung phantom for small fields. The increased accuracy in this situation is due to the assumption of electronic equilibrium being eliminated.

Due to the importance of electron transport in the dose deposition process, it is found that the density scaling method based on the assumption of rectilinear motion of the secondary electrons and scattered photons is not ideal in the case of the secondary electrons. Electrons do not spread out in straight lines from the primary interaction site as do the scattered

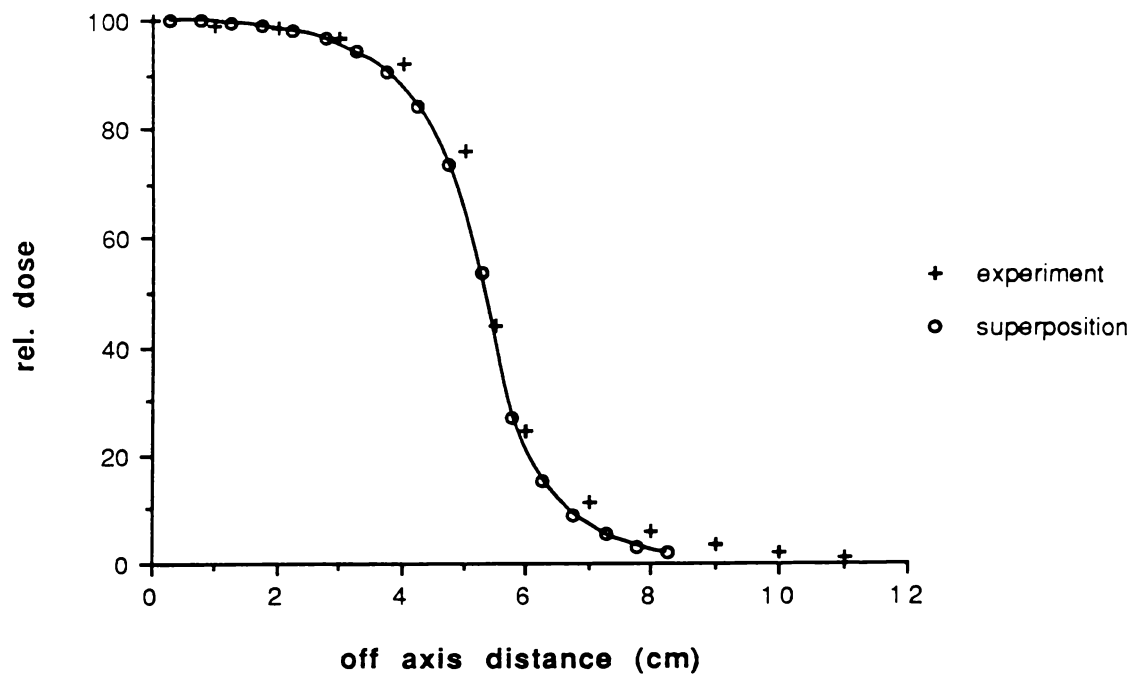


Figure 4.16 Comparison of film (Kodak XTL-2) profile with superposition results for a 10×10 cm field at mid-lung. The broad penumbra due to lateral electron disequilibrium away from central axis is predicted by superposition.

photons. This causes an inaccuracy in the scaling of the EDK. A rigorous scaling theorem for secondary electrons requires that multiple Coulomb scattering be taken into account. This has yet to be done, but a modification to the ray tracing method used in the scaling process achieves adequate results for the lung dose calculations performed here.

The superposition method has been implemented on the GRATIS system, where more sophisticated methods of creating electron density, radiological depth, and terma arrays are used. The electron density array is obtained from CT scan information. Also, separate primary and scattered cartesian EDKs with a resolution selected within the program are created from EGS4 generated spherical polar EDKs. This system is described in detail in chapter 5.

Chapter 5

The Superposition Algorithm in the GRATIS System

5.1. Introduction

The Superposition algorithm has been incorporated into the third release of the GRATIS Radiotherapy Treatment Design System, developed by George W. Sherouse and co-workers at the University of North Carolina (Sherouse, 1990). This is a Unix-based system, written in the C programming language, in which a number of X-Windows tools are provided for the user interface. The system is installed on a Sun Sparcstation IPC equipped with a 1280×1024 resolution colour monitor. The most computationally intensive part of the superposition algorithm may be performed using a network of transputers (Murray *et al*, 1991; Murray, 1991).

Features of the GRATIS system which have been left unaltered include a *virtual simulator* for beam selection and placement, a spreadsheet for adjustment of relative beam weights and calculation of treatment time, a dose grid selection tool, and an interactive display of isodose lines overlaid on outlines of anatomical structures. Apart from the installation of the transputer network, the modifications made to the system have been to provide the superposition method as a photon beam dose calculation option as well as to incorporate a 3-D pencil beam algorithm for electron beam treatment planning. The pencil beam algorithm and its implementation are described in chapters 6 and 7.

The programs used by the system are run using C-shell *scripts*. These are command files used to guide the user through the steps involved in the planning process. The dose calculation option used depends on which script is executed to control the calculation process. If the script `extbm` (external beam) is executed, the existing scatter integration program `c_photon` is used, whereas if `s_extbm` is executed, the superposition program `s_photon` is used. Furthermore, if `transbm` is executed, the superposition calculation is performed using up to 16 transputers. Dose distributions can be generated using superposition for the Varian Clinac 18/10, 10 MV beam as well as the Varian Clinac 600c, 4 MV beam. Currently, the data required to run `c_photon` for these beams has not been installed.

The account on the Sun in which the system resides, contains the following two subdirectories: (i) `gratis/` which contains C source code, patient directories, and data files required for dose calculation (in text format), and (ii) `GRATIS/` which contains the scripts, executable code, and data files in binary format. All source code and data which is specific to the superposition program, is stored in subdirectories of the `gratis/src/s_photon/` directory.

When the system is installed, source code is compiled and the resulting object code copied into `GRATIS/bin/`. As well as the source code for the virtual simulator, dose calculations, isodose display etc., there is code for building the binary data files required for dose calculation due to particular beam types. After compilation and installation into `GRATIS/bin/`, the code for building these data files is executed automatically, resulting in the text-format data files being read from subdirectories of `gratis/`, and written out in binary format to a file in `GRATIS/phys_dat/`. The data contained in these binary files is referred to as *physical data*. The procedure for installing the GRATIS system is detailed by Murray (1991).

This chapter concentrates on the incorporation of superposition into the system as well as on the procedures making up the superposition algorithm. An outline of the main components of the GRATIS system is given first, along with a brief description of the virtual simulation process.

5.2. The Virtual Simulation System

The virtual simulator is invoked by typing `simulate patient_dir`, where `simulate` is the script which controls the virtual simulation process, and `patient_dir` is the patient directory in `gratis/` where the data required for the treatment simulation of a particular patient resides. This directory may also contain the output from previous simulations and dose calculations involving that patient.

The required data includes a CT image in a special format created by the operation of a utility program `build_image` on a set of CT scans, and a set of anatomical structures (called *anastructs*) which are created from the image using `imex` and `build_anastruct`. The image file `plan_im` contains header information for the set of scans, and the CT data for each scan. Each *anastruct* consists of a set of 2-D *contours* representing a particular structure such as the skin or the lungs, and is stored in subdirectory `a/` of `patient_dir`. Each contour is a closed loop outlining a structure as seen in a transverse slice of the body. If these *anastructs* do not exist when `simulate` is run, they are created before the actual simulation begins.

Also in the patient directory are further subdirectories: `b/` for the beam specifications created by the virtual simulator; `g/` for the 3-D arrays of calculated doses corresponding to each beam; and `v/` for the list of point doses corresponding to each beam — points may be specified at which a dose calculation is required. There is also the file `grid` which contains the description of the dose grid geometry created by the grid selection tool, and `pt` which contains the coordinates at which point doses are required. Note that `grid` and `pt` are common to all beams, but a separate dose grid and point dose list exists for each beam.

When `simulate` is run, it begins by moving `b/`, `g/`, and `v/` so that they are subdirectories of another directory `frameN/`, where `N` is a number which is incremented by 1 each time this process is performed. New directories `b/`, `g/`, and `v/` are then created. The current `b/`, `g/`, and `v/` directories may easily be replaced by any of these stored ‘frames’ if a previous

setup needs to be recalled.

If patient anastructs exist in **a/** when **simulate** is run, the X-based code **xvsim** is executed. This program takes several command line arguments which specify input and output files. Included in the list of arguments are:

-a anastruct

At least one anastruct is required, with one labelled 'skin' being essential. The anastruct files are **a/skin**, **a/lung** etc., which are specified as input by including **-a a/skin -a a/lung** on the command line.

-i image

An image is not essential, since the anatomy is described adequately by the anastruct contours, and no patient electron density information is required at this stage. An image is usually specified to make the display more interesting, but increases the time spent loading data before simulation can begin. The image file is **plan.im**.

-p point_file

As mentioned in the introduction, points may be specified at which a dose calculation is required. A list specifying the coordinates and labels of these points is written to the point description file **point_file** on exit from **xvsim**. As well as points specified during the current simulate session, previously defined points read in using **-P** (which are not deleted) are included in the point list. The point description file is named **pt**.

-P old_point_file

Previously defined points are read in from **old_point_file** which is the current version of **pt**.

-b old_beam_file

The beams displayed when **xvsim** is started up are read from beam files in the **b/** directory in the most recently created frame. These beams are specified by including **-b frameN/b/aa -b frameN/b/ab** etc. on the command line, where **frameN** is the most recent frame and **aa**, **ab** etc. are the beam files.

Once the X-Windows display panels appear, the user can begin selecting beams, beam orientations etc. Beam placement is performed with the use of orthogonal wire-frame 3-D patient structures created from the anastructs, along with gantry and treatment couch controls. A detailed description of the features of **xvsim** is given in the manual supplied with the GRATIS software (Sherouse, 1990). Only a brief description of the important features is given here.

Calculations can only be performed for those beams for which a full set of physical data files exist. A menu allows selection of treatment units whose *unit description* is included in the unit file in **GRATIS/phys_dat/**. If a beam whose unit description exists is selected, but some other data file for that beam is missing, the program will exit when attempting to open

the non-existent file. Currently, the required data exists for beams labelled Clinac10MV (Clinac 18/10 10 MV) and Clinac4MV (Clinac 600c 4 MV). A treatment beam should be selected before further beam manipulation is performed.

The beam should now be specified as having an SAD (source axis distance) or SSD (source skin distance) setup. For both SAD and SSD setups, the isocentre is indicated on the patient views. If the setup is SAD, the isocentre position is set by dragging the patient in any of the three orthogonal views, which causes a corresponding shift in the treatment couch. Note that the couch can also be moved vertically, laterally, and longitudinally, but it is easier to position the patient directly. The isocentre can also be automatically positioned at the centre of a selected patient anastruct. Once the isocentre is in the desired position, it can be locked to prevent further movement while setting up the beams. The orientation of the beam is then set using the tools for gantry and couch rotation and/or patient rotation in any of the orthogonal views. Again, rotation of the patient causes a corresponding rotation of the gantry and table. If the setup is SSD, the patient is dragged and rotated so that the beam axis and orientation with respect to the patient surface is as desired. The SSD is known from the intersection of the beam axis with the skin anastruct, and is continuously displayed (for the SAD setup as well). The SSD is set to the desired value using a slider which is moved using the mouse — as the slider is moved, the patient moves accordingly.

Once the beam position and orientation with respect to the patient is selected, the user selects the field dimensions, beam modifiers etc. The beam is then named and saved, and another beam can be selected. Note that beams of a different type can be selected for the same plan.

Also displayed are ‘postage stamp’ size images corresponding to each slice in the CT scan. Overlaid on each image are the corresponding anastruct contours. If the image was not read into *xvsim*, only the contours are shown. The beams which have been specified are also shown on these images. Any of these images can be selected to appear in a 512×512 pixel image which includes a grey-scale windowing tool. This selected scan appears highlighted in the 3-D patient views. Dose calculation points are specified on this large image and then named for identification. As well as user specified points, an additional point is added per beam by default. This point is at the isocentre for SAD beams and is at the depth of maximum dose in water for SSD beams. The depth of dose maximum in water, d_{max} , is included in the unit description read in by *xvsim*.

During the simulation, any of the existing beams can be deleted. Also, any existing beam can be selected and modified. On exit from the program, beam description files are written to *b/* for all old beams which were not deleted plus new beams which were specified. Dose calculation points can also be deleted at any stage. On exit from the program, the *point_file* *pt* is written, which contains a description of any old points which were not deleted, plus new points, plus the one point per existing beam added by default. Also existing after *simulate* has been executed is an empty dose grid directory *g/* and an empty point dose

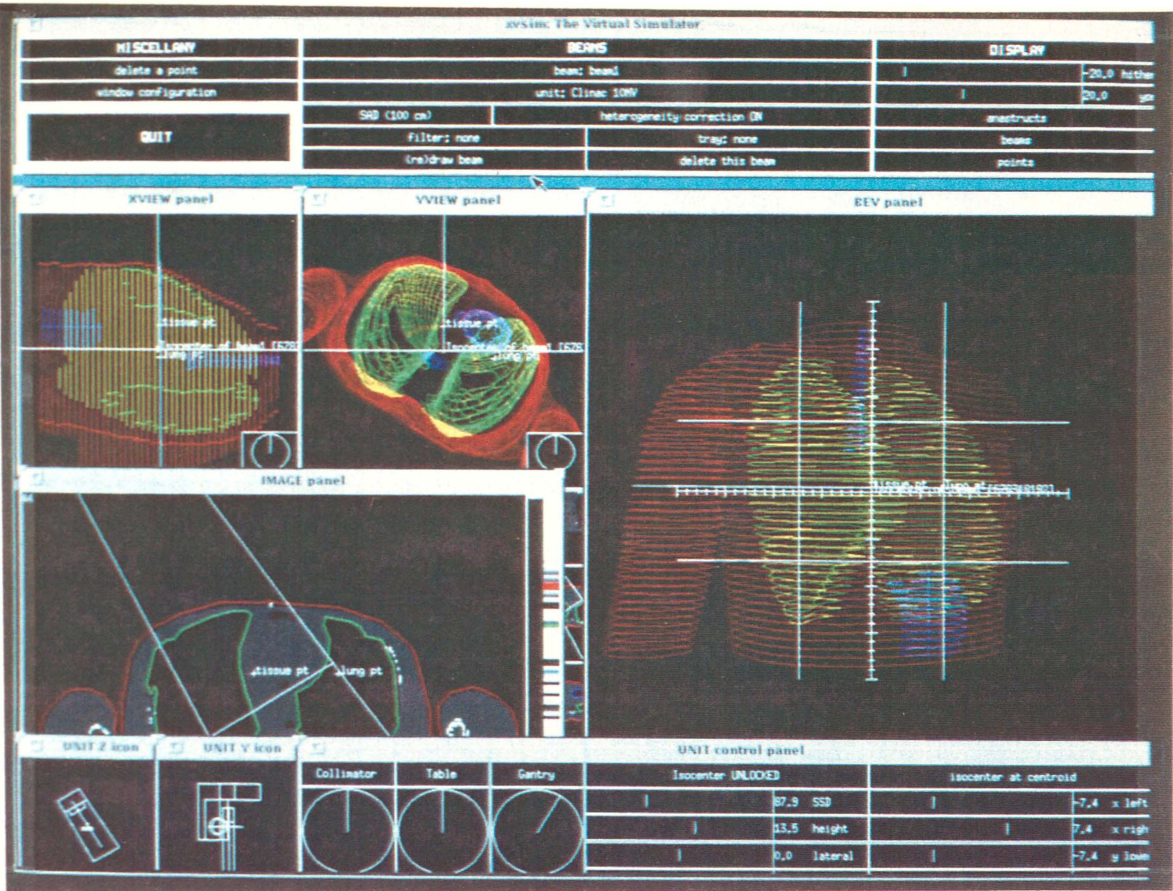


Figure 5.1 Virtual simulator display provided by `xvsim`. A 15×15 cm, 10 MV beam has been specified.

list directory `v/`.

Shown in figure 5.1 is the `xvsim` display. The three orthogonal wire-frame views of the patient can be seen, with the beam’s eye view in the large window on the right. A 15×15 cm, 10 MV beam has been specified which can be seen in the beam’s eye view window as well as on the CT scan in the other large window. Points in the lung and in tissue where dose is to be calculated are shown on the CT scan. The isocentre of the beam is indicated by the cross-hairs in each of the wire-frame images. Note that the beam appears to remain fixed in each of these views — the isocentre and beam orientation are adjusted by moving the patient anastructs. The gantry can be seen to be rotated in the windows at bottom left. Obscured behind the front windows are the ‘postage stamp’ CT scan images — some of these are visible between the beam’s eye view and large CT image windows.

5.3. Overview of the Dose Calculation System

If the Superposition dose calculation option is used, the script `sxtbm` takes over from `simulate` and controls the rest of the planning process. The main steps in `sxtbm` are outlined here before a more detailed description of the dose calculation program `s_photon` is given.

Similar to `simulate`, `sxtbm` is invoked by typing `sxtbm patient_dir`. The main pro-

cesses which the script guides the user through are as follows.

(i) calculating point doses

For each specified beam, a *point dose list* is calculated using `s_photon` for each of the calculation points specified in `xvsim`, where the coordinates of these points are stored in the point description file `pt`. Once calculated for a particular beam, the point dose list is written to a file in `v/` with the same name as the corresponding beam file in `b/`. Note that the point dose files in `v/` have the same structure as `pt`, except that in addition to a description of the point positions, they contain the dose at each point due to the corresponding beam.

(ii) beam weighting and treatment time calculation

The point doses are used to calculate the treatment time (in monitor units (MU)) required for each beam. This calculation is performed by the spreadsheet program `tele_spread`. First, the point dose lists are read and the dose at each point due to each beam is summed. This summed dose is displayed on the spreadsheet, along with the percentage contribution from each beam. The user then sets the dose at a normalisation point (eg. the centre of the tumour) to a prescribed value, and can vary the relative weights of each beam to minimise the dose to other points. Also included in the spreadsheet is a beam normalisation factor, a plan normalisation factor, and the number of treatment fractions — all of which are particular to an individual beam. The total number of monitor units required per beam is the product of the beam weight, the beam normalisation factor, the plan normalisation factor, and the number of fractions. When the beam weights and dose to the normalisation point have been specified, a hardcopy of the spreadsheet is obtained which includes the required number of monitor units for that beam, for the entire treatment. The number of monitor units per beam determined in the spreadsheet is also the factor by which the values in the dose grid for the corresponding beam are multiplied before isodose lines are displayed. When the values in each dose grid are multiplied by their respective factors and summed, the isodose level at the normalisation point will be that specified in the spreadsheet. Therefore, before exiting the spreadsheet, the dose at the normalisation point should be set to the value desired on the isodose display (eg. 100%) — this will modify the factors displayed on the spreadsheet accordingly. A list of these factors for each beam is written to the file `wt`, which is later read into the program `sum_grids`. The dose grids do not exist at this stage if `simulate` has just been run. Otherwise, the spreadsheet can be used to interactively alter the beam weights and optimise the point doses and isodose distribution.

(iii) placing the dose grid

The program `xplace_grid` is used to define a 3-D grid in which to store the calculated dose. This tool allows a grid to be created with appropriate dimensions and resolution for the beam setup. Included in the display are three orthogonal views of the patient anastructs with the beam outlines overlaid. A grid is overlaid on the anastructs and adjusted to an

appropriate size to cover the region in which dose will be delivered. The anastructs are dragged so that the grid is positioned correctly. There is also a set of ‘thermometers’ for adjusting the grid resolution. When the grid is satisfactory, the program is quit and the grid description file, `grid`, is written. The `xplace_grid` display is shown in figure 5.2. A grid has been selected which will contain the calculated doses for the beam specified in `xvsim` (see figure 5.1). The number of points in the grid in the x, y, and z dimensions is 31, 26, and 7 respectively as indicated by the thermometers. The x, y, and z axes are directed outward from the page in the X view, Y view, and Z view windows indicated, respectively. The spacings of points in each direction in the grid are 1.01, 1.01, and 1.04 cm respectively.

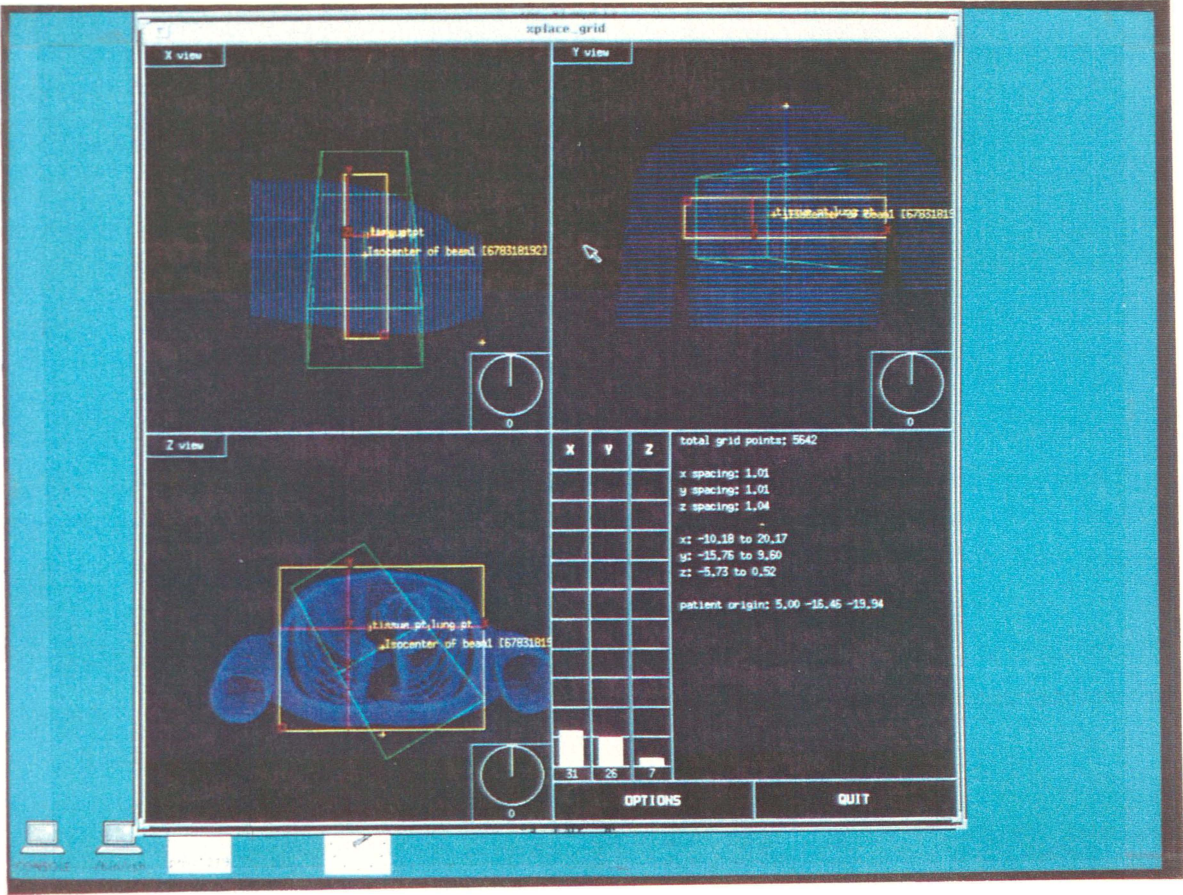


Figure 5.2 Placing of a dose grid using `xplace_grid`, in which to contain calculated doses for the beam specified in `xvsim`.

(iv) calculating dose grids

For each specified beam, the dose is calculated using `s_photon` for each point in the grid specified in `xplace_grid`. Once the dose at each point in the grid has been calculated for a particular beam, the grid is written to a file in `g/` with the same name as the corresponding beam file in `b/`. Note that the dose grid files in `v/` have the same structure as `grid`, except that in addition to a description of the grid, they contain the dose at each point in the grid due to the corresponding beam.

(v) *displaying isodose lines*

Before isodose lines are displayed, the dose grids must be summed. This function is performed by the program `sum_grids`, which uses the beam weights stored in `wt` to do a weighted sum of the dose grids in `g/`. The summed grid is output to a file, `sum`, and then read into the isodose display program `xplandisp`. In this program, three orthogonal views of the patient anastructs are displayed, along with a 'cut plane' which slices the anastructs. In the main view window, the anatomy and isodoses coinciding with the cut plane are displayed. In each view, the cut plane remains stationary and the patient is dragged and rotated to change the display. When the patient is in the default position, the main view window shows a transverse section of the patient. The isodose levels displayed may be altered at any time. The beam files are normally read in, allowing display of the beam outlines. Also, if the point description file `pt` is read in, the calculation points can be displayed with their associated labels. On quitting `xplandisp`, the plan file `plan` is written which can be used to obtain a hard copy of the plan on an HP plotter. The `xplandisp` display is shown in figure 5.3. Dose calculated using superposition is displayed in a slice of the grid specified in `xplace_grid`. This slice is in the plane indicated by the top of the blue 'T' shape in the two smaller windows. The white dots are points in the grid where the calculated dose is zero.

Coordinate systems used in the calculation

There are several cartesian coordinate systems defined in the GRATIS system which are utilised by the superposition algorithm. Included in these are the patient system, the beam system, and the grid system. The patient system is fixed with respect to the treatment couch and the beam system is orientated with the current beam. There is a grid coordinate system specific to each data structure containing a 3-D array used in the program. For converting between these systems, 4×4 transformation matrices exist. For example, for each beam specified in `xvsim`, there are transformation matrices which convert the beam coordinates of a point to patient coordinates, and vice-versa. Such transformations are discussed by Siddon (1981). The beam and patient coordinate systems are depicted in figure 5.4, where the x , y , and z directions in the beam and patient systems are denoted by B_x , B_y , B_z , and P_x , P_y , P_z respectively. The origin of the patient coordinate system is such that the centreline of the surface of the treatment couch coincides with the z axis and the x - y plane corresponds to the plane of the first CT scan. Note that the direction of increasing z in the patient system, is in the direction away from the gantry. The patient coordinate system is *right-handed*. The beam coordinate origin is located at the virtual source, with the direction of increasing z in the direction of the beam. The beam coordinate system is *left-handed*, as is the convention for viewing coordinate transformations in computer graphics.

5.4. The Superposition Program `s_photon`

Source files which are specific to the superposition program reside in the directory

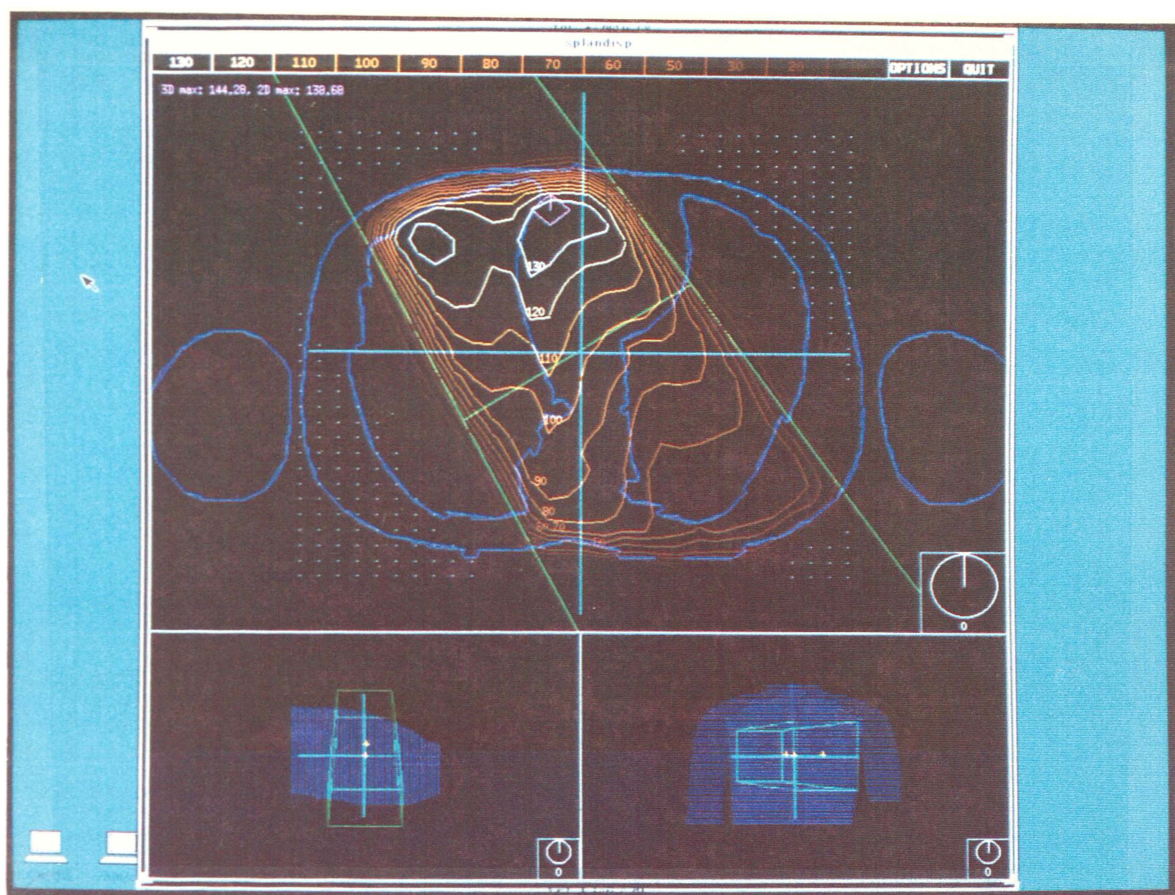


Figure 5.3 Isodose display provided by `xplandisp` for the specified beam. Dose has been calculated using `s_photon`.

`gratis/src/s_photon/c/`. These files are compiled, then linked both with each other and with required object files in other directories. This procedure is performed by a Unix `makefile`, written specifically for this task. The executable code produced, `s_photon`, is copied to `GRATIS/bin/`, also using the `makefile`. All programs and functions written or modified for use in superposition have names prefixed with `s_`; those without this prefix already existed.

5.4.1. Input of Requested Data

When `s_photon` is called for a particular beam, either to calculate point doses or dose throughout a grid, several command line arguments are supplied. These arguments include file names for input and output, along with other parameters, and are read by `s_read_request()`, the first function called by the main program `s_main`. This function is a modified version of `read_request`, which is used in the `c_photon` algorithm. In `s_read_request()` each argument is read in succession, and in the case of the file names, stored as a string which is used to identify the file to be opened. Each of these arguments is now specified along with a brief description of the functions which `s_read_request()` performs.

-b beam_file_name

Each time `s_photon` is run, dose calculations for a particular beam are performed. This

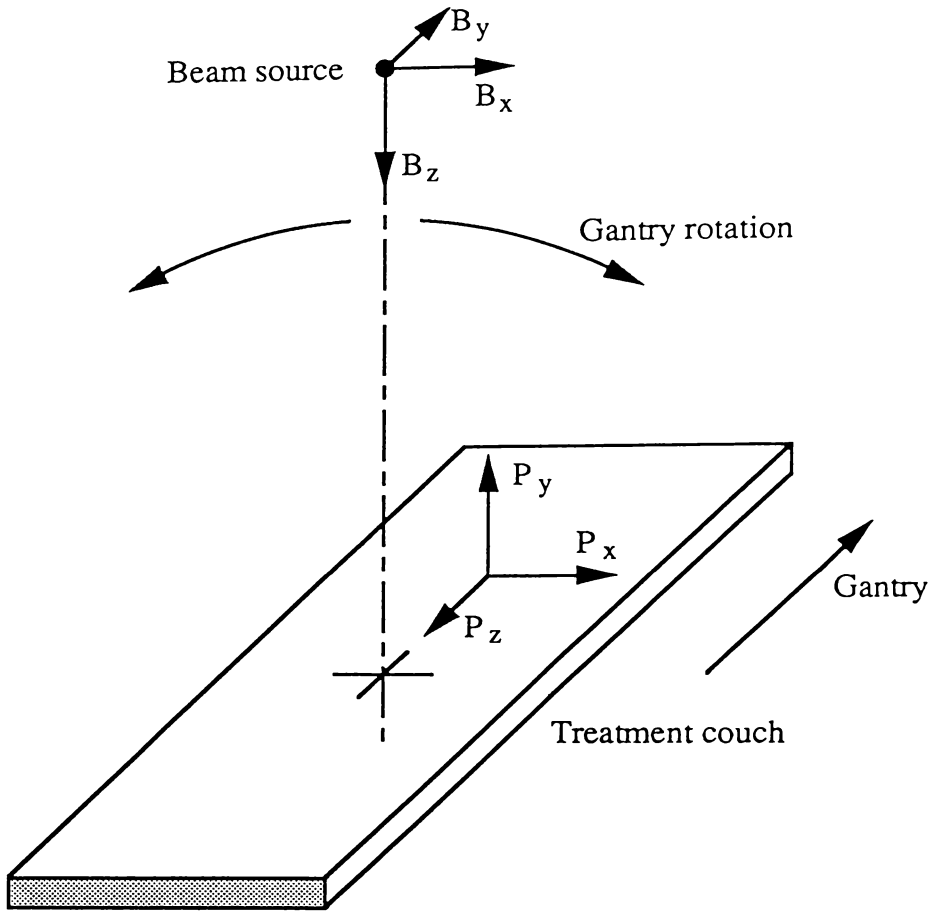


Figure 5.4 Axis orientations in the beam and patient coordinate systems. The x , y , and z directions in the beam and patient systems are denoted by B_x , B_y , B_z , and P_x , P_y , P_z respectively.

beam is referred to as the *current beam*. The beam description file corresponding to this beam is named `beam_file_name` — this is a name such as `b/aa`.

`-a anastruct_file_name`

The anastruct filename read is `a/skin`. This is the only anastruct required by `s_photon`.

`-g grid_desc_file_name`

The grid description filename `grid` is read. Note that this argument is only supplied if a dose grid is to be calculated.

`-o out_grid_file_name`

The name of the file which will contain the dose grid corresponding to the current beam is read. For example, if the current beam is `b/aa` then the dose grid filename is `g/aa`. Note that this argument is only supplied if a dose grid is to be calculated.

`-p point_desc_file_name`

The point description filename `pt` is read. Note that this argument is only supplied if point doses are to be calculated.

`-o out_point_file_name`

The name of the file which will contain the point dose list corresponding to the current

beam is read. For example, if the current beam is `b/aa` then the point dose filename is `v/aa`. Note that this argument is only supplied if point doses are to be calculated.

`-i image_file_name`

The filename of the CT image is read — this file is `plan.im`.

`-S scaling`

In the superposition program, there are three options of inhomogeneity scaling type which can be used. The parameter `scaling` sets the variable `scaling_type` to `NO_SCALING`, `Z_SCALING`, or `RAY_TRACE`, which are the three options in increasing order of complexity.

`-T processor`

This parameter specifies whether the dose calculation is to be performed using the Sun processor or using the transputer network. The `processor` parameter sets the variable `processor_type` to `SUN` or `TRANSPUTER`.

`-N processor_number`

The number of processors to be used if `processor_type` is `TRANSPUTER`.

`-U beam_unit_id`

To relate the result of the calculation to absolute dose, a table of dose data calculated using superposition for a range of square field sizes is required. The data in this table is referred to as *superposition data*. The parameter `beam_unit_id` is the unit identification number of the beam for which the table is to be produced. This parameter is supplied only when the system is being set up, or when data needs to be produced for a new beam. If this parameter is supplied, no other parameters are included on the command line. To signify that this type of calculation is to be performed, the 'flag' `making_sup_data` is set to true.

This concludes the list of possible command line arguments. The rest of `s_read_request()` is concerned with reading files and producing the required data structures as described below. Note that of the procedures described below, only that concerned with creating EDK data is applicable if `making_sup_data` is true.

Input of beam data

Unless `making_sup_data` is true, the beam file `beam_file_name` containing data for the current beam is opened and read into `request.beam`. The variable `request` is a large data structure containing several levels of sub-structures which are used to store data read from the requested files. The data read into `request.beam` includes parameters specified in `xvsim` such as: the name which the user has given the beam in `xvsim` (stored as `request.beam.name`); the number identifying the beam type (stored as `request.beam.unit_id`); the field limits defined by the primary collimators (beam jaws) at the SAD (stored in the structure `request.beam.jaw`); patient coordinates of the beam axis at the SAD (stored as the structure `request.beam.position`); gantry, treatment couch, and collimator angles (stored as `request.beam.gantry_angle`, `re-`

`quest.beam.table_angle`, and `request.beam.coll_angle` respectively); transformation matrices for converting between beam and patient coordinates and vice-versa (stored as the matrices `request.beam.T_beam_to_pat` `request.beam.T_pat_to_beam`); identification number of the tray used to support any beam defining blocks specified (stored as `request.beam.tray_id`); number of beam filters specified in `xvsim` (stored as `request.beam.filter_count`); identification number of each filter (stored as the array `request.beam.filter_id`). The parameters listed above are all determined in `xvsim` and read into `request.beam` via the beam description file. Other beam parameters are assigned in `s_read_request()` or are read from files specified by `request.beam.unit_id` (referred to as the unit id.). For example, the beam outline defined by the jaws at the SAD (stored as the structure `request.beam.beam_outline`) is obtained from the beam limits contained in `request.beam.jaw`. The beam outline is stored as a `CONTOUR` structure, which consists of a list of vertex coordinates. The beam outline can also be specified in `xvsim` using blocks: in this case the beam outline is read from the beam file. Measured dose rate information for the current beam is read into `request.beam.time_calc` from a file in `GRATIS/phys_dat/` specified by the unit id. This dose rate information consists of a measured dose at a calibration point and a set of *collimator output factors* for a range of square field sizes as explained in section 5.4.3.

Block tray data

If a block tray was specified in `xvsim`, data concerning the tray is read into `request.tray` from a file in `GRATIS/phys_dat/` using the identification number `request.beam.tray_id`. This data includes a factor for the reduction in primary photon fluence caused by the tray.

Beam filter data

If any beam filters were specified in `xvsim` (`request.beam.filter_count > 0`), data concerning those filters is read into `request.filter` from a file in `GRATIS/phys_dat/`, using the identification numbers in `request.beam.filter_id`. When this data is read in, a 2-D map (`request.filter_map`) of the reduction in primary photon fluence at the SAD caused by these filters is created

Beam flatness data

A map of beam intensity at the SAD as a function of off axis distance is required by the superposition program. This data is read from a file in `GRATIS/phys_dat/` specified by the unit id. The structure used to store the flatness data is `request.SAR_table` — in the `c_photon` algorithm, scatter-air ratio data is required and is stored along with the flatness data. This structure is also used here to assist in preserving the existing program structure as much as possible. SAR data has not been collected for the 10 MV and 4 MV beams.

Anastruct contour data

The anastruct data file `anastruct_file_name` is read into `request.patient_anastruct`. It is verified that the anastruct *label* is 'skin', since this is anastruct required for patient outline definition.

Output dose grid description

If a grid description filename was specified on the command line, the output dose grid description is read into the GRID structure `g.grid3`. A GRID is a much-used type of data structure which contains a description of a 3-D array of floating point numbers, as well as the array itself. The array description is referred to as the 'header' section of the grid structure. Contained in the header are: the grid coordinates of the origin of the array; the spacing and number of points in each dimension of the array; the transformation matrices to convert from grid to patient and patient to grid coordinates; and the maximum and minimum values contained in the array. The array is declared as a pointer to a 1-D array of floating point numbers, however the elements in the array are accessed as though they are in a 3-D array using the macro `GRID_VALUE`. When the grid description has been read, the required amount of memory is allocated to the array. The output grid file is opened and the pointer to this file assigned to `g.out_fdes`. The flag `request.have_grid` is set to true.

Point dose list description

If a point description filename was specified on the command line, the point description file is read into the `CALC_POINTS` structure `p.points`. This structure contains an array of point positions in patient coordinates and an array to contain the calculated dose at each point. When the point description has been read, an amount of memory which depends on the number of point doses which must be contained, is allocated for the array. The output point file is opened and the pointer to this file assigned to `p.out_fdes`. The flag `request.have_points` is set to true.

CT scan data

The `CGRID` structure `density_grid` is filled from the CT scan file using `s_plan_to_cgrid()`. This function is based on the existing routine `plan_to_grid`. A `CGRID` structure is identical to a GRID structure except that the elements of the array are characters instead of floating point numbers. The array in this grid is defined in patient coordinates. In `s_plan_to_cgrid()`, header information is read from the image file and used to assign values to the header variables in the `CGRID` and CT scans are read into the array. For 3-D planning, a set of scans which cover the irradiated region is required. Since each scan has 512×512 resolution, the memory required to store a single scan in 4-byte floating point representation is 1 Megabyte. The amount of memory occupied to store the required scans in this format, as well as the time required to read the data from file and store it, is unacceptably large. For this reason, `s_plan_to_cgrid()` stores the scans, with reduced resolution, as a array of characters (1 byte) instead of floating point numbers. Depending on the number of scans in the image file and on a set maximum number of scans which are allowed to be read, only every second or third scan may be read. When the scans are read, the CT numbers are first converted into floating point electron densities, then converted into an appropriate character representation. CT numbers are converted to electron density using

the method described in section 1.10, where to differentiate between bone and soft tissue, the factor R_γ is set to 1.0 if $N_{CT} < 200$ and 0.465 if $N_{CT} \geq 200$ (Metcalf and Beckham, 1988). Also, the resolution of each scan is reduced on reading it into the array by stepping through the scan four voxels at a time in both the x and y directions. This means that the memory occupied by a single scan is reduced to 16 kilobytes, enabling a sufficiently large number of scans to be stored, and reducing the time required to read the data to a few seconds.

Superposition data

The superposition data required for absolute dose determination is read from a file in `GRATIS/phys_dat/` specified by the unit id., using `s_read_sup_dat()`. This block of data includes a table of unnormalised superposition doses at a calibration point for a range of square field sizes. The calibration point and field sizes are the same as those specified for the calibrated dose rate and collimator output factors in `request.beam.time_calc`. The superposition data has been previously generated using a special ‘calibration’ run of `s_photon` when the only command line option was `-U beam_unit_id` — this procedure is detailed in section 5.4.8. In this block of data is a header containing the beam unit id., source to calibration point distance, calibration depth, and number of field sizes for which superposition data has been created. There is also an array of field sizes and an array of superposition doses corresponding to each field size. This data is stored in the structure `sup_dose_pars`.

Energy deposition kernel data

Primary and scattered EDKs, stored as cartesian arrays, are created for the current beam using `s_read_kernels()`. The file in `GRATIS/phys_dat/` from which the EDK data is obtained is specified by the unit id. The EDK data is stored in a structure of type `EDKS`, named `kernel`. The process of creating EDKs is detailed in section 5.4.2.

5.4.2. The Energy Deposition Kernels

5.4.2.1. Structure of the EDKs

The EDKs are all stored in the variable `kernel` which is a structure of type `EDKS`. This structure contains a header section as well as two arrays containing structures of type `EDK`, where each EDK structure contains a cartesian EDK array. The EDK structures in one of the arrays contain the primary EDK data and the EDK structures in the other array contain the scattered EDK data. Each array contains an EDK structure corresponding to relative electron densities of 0.25, 0.50, 0.75, 1.00, and 1.25. In each EDK structure, as well as the EDK array there is a header section containing the dimensions of the EDK array and the k-index in the array which contains the EDK origin. The important quantities contained in the header section of the `EDKS` structure are: the spectral energies used to create the EDKs; the spectral weight of each energy component; the mass attenuation coefficient and mass energy absorption coefficient of each energy component; the primary collision kerma and terma at the surface of a water medium at 100 cm SSD; the dimensions of the EDK

voxels; and the relative electron densities to which the stored EDKs correspond. Note that the primary collision kerma and the terma at the surface are included so that the beam hardening correction described in chapter 3 can be applied. Similar to the array in a GRID structure, elements in the EDK arrays are accessed using the macros `PRIMARY_EDK_VALUE` and `SCATTERED_EDK_VALUE`. These macros require the name of the EDKS structure, the density index of the EDK structure to be accessed, and the indices (i, j, k) of the required array location. The EDK arrays are always orientated with the current beam — the i, j , and k indices correspond to the x, y , and z directions in beam coordinates respectively. The process of creating the EDKs stored in `kernel` is now described.

5.4.2.2. Input of EDK Data

The values in the EDK arrays are assigned using a stored set of primary and scattered spherical polar EDKs (PEDKs), in the routine `s_read_kernels()`. The PEDKs are produced by an EGS4 Monte Carlo simulation using the user code RTPEDK (Murray, 1991). These arrays contain a fractional energy distribution in ring shaped voxels defined by the radial distance from the primary photon interaction site and the angular displacement from the primary photon direction (Mackie *et al*, 1988; Metcalfe *et al*, 1990).

The binary data file which `s_read_kernels()` uses to create the kernel structure contains all the header information for `kernel` except for the voxel dimensions, as well as the primary and scattered PEDK data for each density. This file has been previously created using the auxiliary program `s_make_kernels`, which reads the kernel header information from a text file pertaining to the current beam, and also reads a text file of Monte Carlo generated PEDK data. In `s_make_kernel`, this information is stored temporarily in the appropriate structures, and then written out to a binary file in `GRATIS/phys_dat/`. The order in which the binary data is stored is (i) EDK header information, (ii) primary PEDK data for each electron density, and (iii) scattered PEDK data for each electron density.

When the binary data file is read in `s_read_kernels()`, the header information is stored in `kernel` then the PEDK data is read into a PEDKS structure, referred to as the *polar kernel*. The polar kernel, named `polar_kernel`, contains the unit id. corresponding to the current beam type, an array containing a primary PEDK for each electron density and an array containing a scattered PEDK for each electron density. The PEDKs are stored as PEDK structures which each contain the dimensions of the PEDK and an array containing the PEDK values. The array elements are accessed using the macros `PRIMARY_POLAR_EDK_VALUE` and `SCATTERED_POLAR_EDK_VALUE`. Note that the PEDKs are 2-D arrays where elements are accessed by supplying the macros with the electron density index of the PEDK, and the array indices (i, j) corresponding to the radial and angular indices of the required element.

5.4.2.3. Setting the EDK Dimensions

The first step in creating the EDKs is to decide on the EDK voxel sizes and on the EDK dimensions. The voxel sizes depend on whether the calculation is to be performed for point doses (`request.have_points` is true), an output dose grid (`request.have_grid` is

true), or to generate superposition data (`making_sup_data` is true). If a point doses list or superposition data is required, the dimensions are set to be 0.5 cm in each direction. If an output dose grid is required, the voxel dimensions are set to be the cube root of the product of grid spacings. The reason for this choice of voxel size is as follows. The array in which the dose is to be calculated is contained in a `GRID` structure, defined in beam coordinates, referred to as the *beam dose grid*. The voxels in this *beam dose array* are later set to be of the same dimensions as those in the EDKs. The time required for a superposition calculation increases as the voxel size of these arrays decreases, so it is convenient for the user to have some control over these sizes. If the EDK voxel dimensions are linked to the point spacings in the output dose grid, then since these point spacings are set interactively in `xplace_grid`, the user can conveniently control the calculation speed. Of course, the greater the grid spacing the lower the spatial resolution of the dose distribution. Since the beam dose array can be orientated in any direction relative to the output dose grid, it was decided to make the EDK voxels, and hence the beam dose array voxels, cubic with side lengths equal to the cube root of the output dose grid spacings.

The number of voxels on each axis of the EDK arrays are referred to as the array dimensions. The absolute lengths spanned by an array on each axis are referred to as the array extents. The *lateral* dimensions or extent of the array pertains to the directions perpendicular to the beam axis — the x and y axes. The *longitudinal* dimension or extent pertains to the beam coordinate z axis. Each EDK contained in the kernel can have unique dimensions. This is so that the dimensions can be varied to account for the greater extent of the scattered fractional energy as well as the increasing secondary electron range with decreasing electron density. The voxel in each EDK where the origin of the fractional energy distribution is to be, is on the axis of the array at a set number of voxels from the top of the array — this number is only a small fraction of the z dimension of the array since most energy is deposited ‘downstream’ of the origin. For all the EDKs the origin voxel currently defined to be on the fifth layer of the arrays. This origin voxel is given the coordinates (0,0,0).

For the primary EDKs, the dimensions are set to values based on the maximum lateral and longitudinal ranges of the secondary electrons. These ranges are calculated using the approximate maximum secondary electron energy, the mass stopping power of electrons in water, and the relative electron density for which the EDK is to be created. The maximum secondary electron energy is approximated by the maximum primary photon energy in the spectrum (the spectrum is stored in the kernel). This is reasonable, since as stated in section 1.3.1.3, the maximum energy transferred to a secondary electron in a Compton collision by a 10.0 MeV electron is 9.75 MeV. The value assigned to the lateral range is half the total range of the maximum energy secondary electrons. If the maximum photon energy in the spectrum is E_{max} , the mass stopping power is S , and the electron density associated with primary EDK i is ρ_i , then the lateral range for primary EDK i is approximated by

$$r_{lat}(i) = \frac{E_{max}}{2\rho_i S} \quad (5-1)$$

The mass stopping power in water is approximately $2 \text{ MeV cm}^2 \text{ g}^{-1}$, so, for the 10 MV beam, the lateral range for unit density is assigned the value $10/4 = 2.5 \text{ cm}$. This is a suitable value since in section 2.4.2, it was found that the maximum lateral range in water is approximately 2 cm. The lateral EDK dimensions are set so that in both directions, the voxel width multiplied by the lateral dimension is greater than or equal to the lateral range.

The value assigned to the longitudinal range of the secondary electrons is the total range of the maximum energy secondary electrons:

$$r_{long}(i) = \frac{E_{max}}{\rho_i S}. \quad (5-2)$$

The z dimension of the primary EDK is then set so that the voxel width multiplied by the number of voxel layers from the origin voxel to the end of the EDK, is greater than or equal to the longitudinal range.

The dimensions of the scattered EDKs are set to values which depend on the field width, and if an output dose grid is to be created, on the extent of this grid. These dimensions are the same for each density. The lateral extent of the EDKs in each direction should be at least as great as the field width in that direction so that scattered photons originating near a field edge are allowed to contribute to the dose on the opposite side the field. The lateral EDK dimensions in each direction are therefore set so that the voxel width multiplied by the lateral dimension is greater than or equal to the field width at the SAD in that direction. The width of the field is obtained from field edge specifications in `request.beam.jaw`. If either lateral scattered EDK dimension found in this manner exceeds a set maximum value, then that dimension is set to the maximum value — this is a safeguard against unreasonably large EDK arrays. If superposition data is to be created, in which case no beam has been defined, the lateral dimensions of the scattered EDKs are set to their maximum allowed values.

If an output dose grid is required, the z dimension of the scattered EDK is set so that the voxel width multiplied by the number of voxel layers from the layer containing the origin voxel to the layer with the maximum *k*-index, is greater than or equal to the maximum extent of the output dose grid in any direction. As with the lateral dimensions, the z dimension is not allowed to exceed a maximum allowed value. If point doses or superposition data are required, the z dimension of the scattered EDKs is set to this maximum value.

5.4.2.4. Creating EDKs using PEDKs

The procedure for creating the EDKs from the PEDKs is performed first for the primary EDKs then for the scattered EDKs, looping through each electron density in succession. The PEDK structures containing the PEDKs each have a header section containing the number of radial shells in the array, the number of angular divisions, the thickness of the radial shells, and the increment in each angular division. These parameters are listed in the EGS4 text file from which the data was obtained, and are unique to each PEDK. The primary

PEDK arrays all have radial shell thicknesses of $\Delta r = 0.2$ cm, and angular increments of $\Delta\theta = 10$ degrees (18 angular increments). The primary PEDKs for each density each have a large enough extent to contain all the fractional energy deposited by secondary electrons set in motion by primary photons. For the relative electron densities stored (0.25, 0.50, 0.75, 1.00, and 1.25), the number of radial shells is 120 (24.0 cm extent), 60 (12.0 cm), 40 (8.0 cm), 30 (6.0 cm) and 24 (4.8 cm). Note that these extents are inversely proportional to density. The scattered PEDK arrays all have radial shell thicknesses of $\Delta r = 0.5$ cm, and angular increments of $\Delta\theta = 10$ degrees (18 angular increments). All scattered PEDKs have 60 radial shells (30.0 cm extent). The radial shells in the scattered PEDKs are thicker than those in the primary PEDKs because the fractional energy distribution varies less sharply. A cartesian EDK is created from the corresponding PEDK by the following process.

The radial shells in the PEDK are looped through, and the radius to the centre of the shell m , r_m , is calculated. For each radial shell, the 18 angular increments are looped through and the angle at the centre of increment n , θ_n , is calculated. The simplifying assumption is now made that all energy in PEDK voxel m, n is contained in the infinitely thin ring defined by r_m and θ_n . The origin of the PEDK is considered to be at the centre of the origin voxel in the EDK. Each thin ring in the PEDK then lies within a corresponding voxel layer with index k_{mn} in the EDK. This layer is found from the distance of the thin ring from the PEDK origin in the z direction, given by

$$z_{mn} = r_m \cos \theta_n. \quad (5-3)$$

A test is first performed to check that z_{mn} is within the EDK array for the current density, then the corresponding voxel level k_{mn} is calculated. The fractional energy contained in voxel (m, n) must now be assigned to the appropriate voxels in the EDK on level k_{mn} . The radius of the thin ring is required — this is given by

$$r_{mn} = r_m \sin \theta_n. \quad (5-4)$$

This geometry is depicted in figure 5.5(a).

The ring is now considered to be divided into 100 segments, each representing an increment in azimuthal angle of $\Delta\phi = 0.063$ radians. Each of these segments contains 1/100th of the fractional energy contained in the ring. For each of these segments, the lateral displacement in the x and y directions from the axis of the ring is calculated. If the mean azimuthal angle corresponding to step l is ϕ_l , then the x and y displacements are given by the expressions

$$x_{mnl} = r_{mn} \cos \phi_l \quad (5-5)$$

and

$$y_{mnl} = r_{mn} \sin \phi_l. \quad (5-6)$$

A test is performed to check that x_{mnl} and y_{mnl} are within the EDK array for the current density, then the corresponding lateral voxel indices i_{mnl} and j_{mnl} are calculated. The

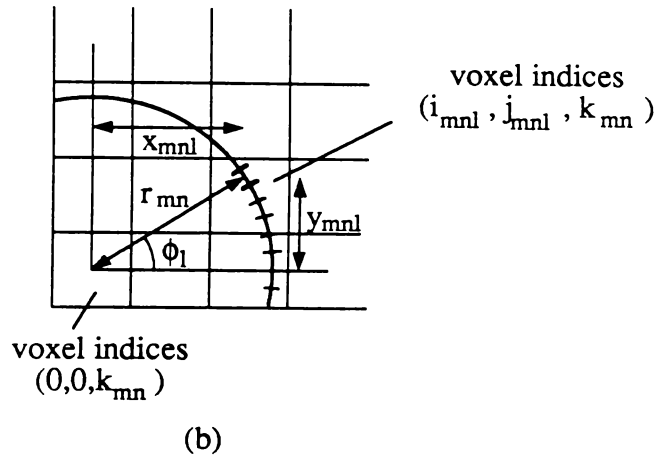
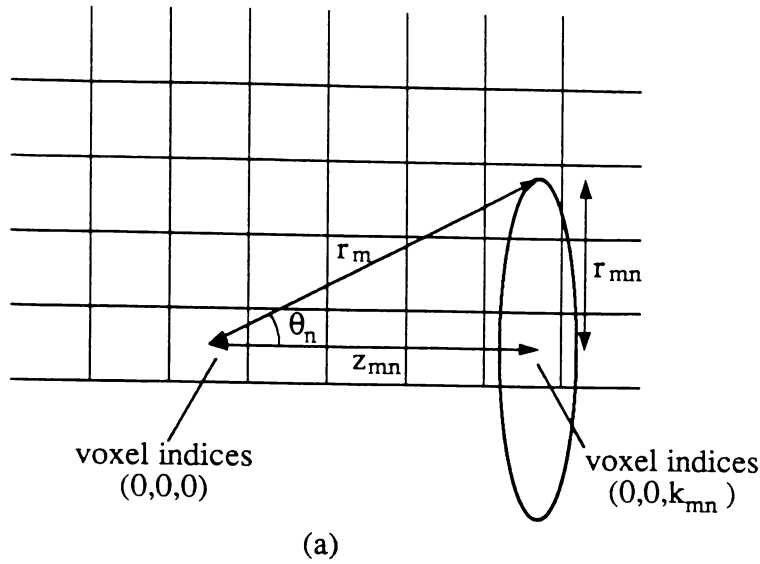


Figure 5.5 Assigning fractional energy to a cartesian EDK using a spherical polar EDK (PEDK). In (a), a voxel in the PEDK is represented by a ring at a radial distance r_m from the primary photon interaction site, and angular displacement θ_n from the direction of the primary photons. The PEDK is overlaid on the EDK so that the primary interaction site coincides with the origin voxel in the EDK. As shown in (b), the energy contained in the ring is divided between voxels in the EDK which it intersects.

fractional energy associated with the segment is then assigned to voxel $(i_{mnl}, j_{mnl}, k_{mn})$ in the cartesian EDK. This geometry is shown in figure 5.5(b) for the same ring as in figure 5.5(a).

5.4.3. The Reference Dose Rate

The structure `request.beam.time_calc` contains the calibrated dose rate in cGy MU^{-1} for the current beam type. This is the measured dose in water for a 10×10 cm field at d_{max} for an SSD of 100 cm. Also contained in the structure is the depth in water and distance from the virtual source of the calibration point at which this dose rate was measured. For

example, the Varian Clinac 18/10 10 MV beam at Waikato Hospital has a calibrated dose rate of 0.92 cGy MU^{-1} at d_{max} (2.1 cm deep), for an SSD of 100 cm. If point doses or an output dose grid are required, the dose rate at the calibration point for the current field size is required. This is obtained by multiplying the calibrated dose rate by a collimator output factor, normalised to 1.0 for a $10 \times 10 \text{ cm}$ field, corresponding to the equivalent square field size of the current beam.

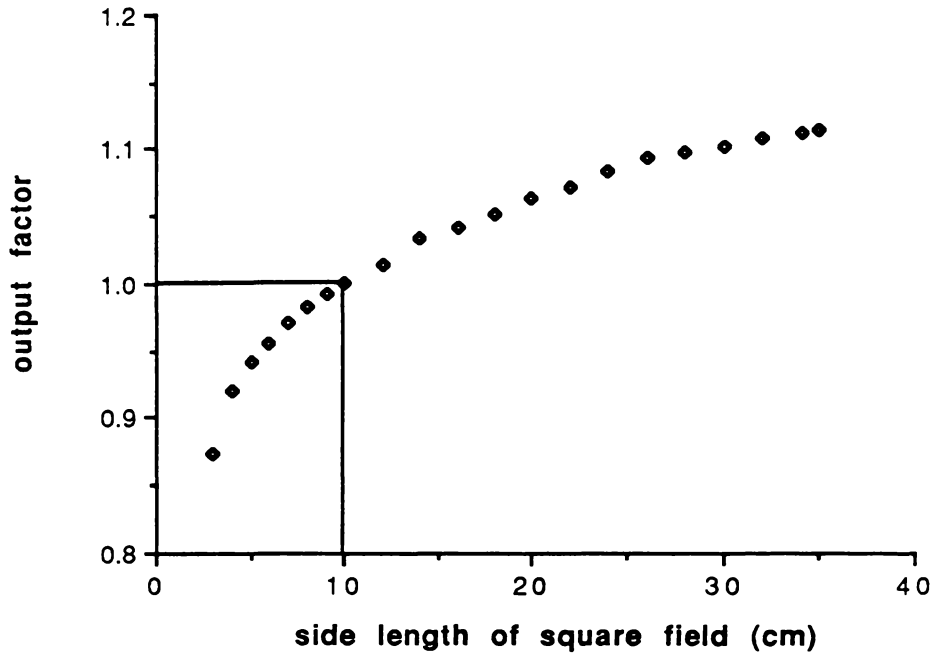


Figure 5.6 Plot of collimator output factor for the Waikato Hospital Varian Clinac 18/10, 10 MV beam. Values are normalised to 1.0 for a $10 \times 10 \text{ cm}$ field.

The equivalent square field is defined to be that which has a perimeter to area ratio equal to that for the rectangular field defined by the beam jaws — this definition is that which existed on the GRATIS system. The output factor is obtained by interpolation using a table contained in `request.beam.time.calc`. A plot of the collimator output factor for fields ranging from 3 cm to 35 cm square, for the 10 MV beam, is shown in figure 5.6. If a block tray was specified, the product of calibrated dose rate and output factor is multiplied by the corresponding tray factor, which for example is 0.98 for a polycarbonate tray in the 10 MV beam.

The product of calibrated dose rate, collimator output factor, and tray factor is stored as a global multiplication factor to be used after the superposition calculation has been performed.

5.4.4. The Electron Density Grid

To perform the superposition calculation, a GRID structure containing an array of electron densities defined in beam coordinates is required. This is formed using the patient-

coordinate electron density grid obtained from CT data in `s_plan_to_cgrid()`. This ‘source’ grid will be referred to as the *density grid* to distinguish it from the *electron density grid* being created. As with the EDK arrays, the array in the electron density grid (referred to as the electron density array) must represent an array of voxels rather than an array of points as is the case with the array in the density grid (referred to as the density array). The dimensions of the voxels in the electron density array are set to be the same as those in the EDK arrays. The process of creating the electron density array for when an output dose grid or point doses are required is now explained.

The density grid structure is defined in its own grid coordinate system, and contains transformation matrices which convert from grid to patient coordinates and vice-versa. These are in fact identity matrices since the density grid is defined in the patient coordinate system. The electron density grid to be created must be defined in the beam coordinate system, hence its grid-to-patient and patient-to-grid transformations are copied from the beam-to-patient and patient-to-beam transformations in `request.beam`. In order to create the electron density array from the density array, transformation matrices between the two grid coordinate systems are required. The density to electron density transformation matrices are obtained by concatenating the density grid’s grid-to-patient matrix and the electron density grid’s patient-to-grid matrix, using the function `cat_mat()`. The electron density to density transformation is obtained by finding the inverse of the density to electron density transformation, using the function `invert_mat()`. Points are transformed between coordinate systems using `transform_point()`. Each voxel in the electron density array has indices (i, j, k) , where those with a k index of zero are those in the layer of the array which is closest to the beam source. The direction of increasing k index in the electron density array is referred to as the z direction, and the directions of increasing i and j are the x and y directions respectively. Note that the beam axis is on the z axis of the beam coordinate system.

The limits of the electron density array form a rectangular prism whose ‘top’ surface is perpendicular to the beam axis, and whose boundaries totally enclose the points in the density array. This situation can be visualised as a box within a box, where the inner box (the density array) is sitting on the treatment couch, and the outer box (the electron density array) is aligned with the beam. The dimensions of the electron density array are found by transforming the eight corner points of the density array, and testing each transformed point to determine the minima and maxima on each of the electron density array axes. The minima found in this way define the beam coordinates of the electron density array origin. Note that this origin is defined by the minimum absolute coordinates of the region enclosed by the array. The *origin voxel* is that which has a corner at the array origin. The electron density array dimensions are set so that the extent of the array in each direction is at least as great as the distance from the minimum to the maximum points in that direction found from the transformed corner points. For example, if the minimum and maximum in the

x direction are X_{min} and X_{max} respectively, and the voxel size in the x direction is ΔX , then the x dimension of the electron density array is $(X_{max} - X_{min})/\Delta X$ rounded up to the next highest integer. A 2-D representation of this geometry is shown in figure 5.7 (for the beam coordinate x-z plane).

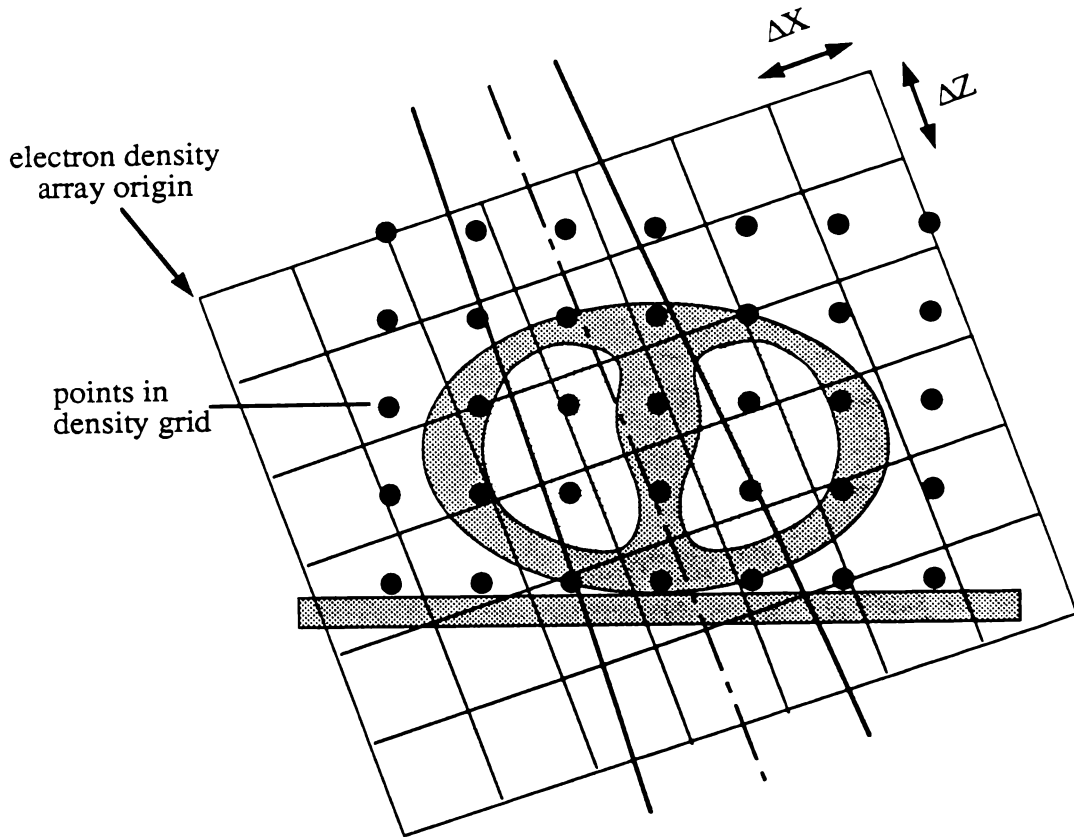


Figure 5.7 The electron density array (voxels) overlaid on the density array (points). The extent of the electron density array in each direction is large enough to enclose the density array. The electron density array has voxel dimensions ΔX , ΔY , ΔZ equal to those in the EDKs.

Once the dimensions have been determined, the electron density array is filled by looping through each voxel and obtaining the value of the electron density from the corresponding position in the density array by interpolation. First, the position of the centre of the origin voxel in the electron density array is transformed to density grid coordinates. Note that the centre of the origin voxel has coordinates which are half a voxel width greater in each direction than those of the array origin. This is the start point for the loop. The orthogonal vectors representing an increment of one voxel width in each direction in the electron density array are also transformed to density grid coordinates. The positions in density grid coordinates of the electron density voxel centres are stepped through using the transformed start point and vectors. At each position, the value of the electron density is found by interpolation using the function `s_ctrinterp()`. Those points which lie outside the bounds of the density array are assigned an electron density of zero.

If `making_sup_data` is true, a simple unit density electron density array is created which is used to calculate the superposition data. Voxel dimensions are set to be the same as those in the EDKs (0.5 cm sides in this case). The z coordinate of the electron density array origin is set so that the ‘top’ surface of the array coincides with the SSD at which the calibrated dose rate was measured. The number of voxel layers is then set so that the number of layers with a k -index greater than the layer containing the calibration point, is at least as great as the number of layers in the EDK arrays with a k -index less than zero. This ensures that fractional energy contained in the EDKs which is upstream of the primary interaction site is allowed to contribute to the dose at the calibration point. The beam axis coincides with a column of voxels at the centre of the electron density array, and the lateral array dimensions are defined to be large enough so that the greatest square field size for which superposition data is required, fits within the array limits. Once the dimensions and voxel sizes are set, the elements of the array are all assigned an electron density of 1.0 so that a water medium is represented.

5.4.5. The Inside_contour Grid

The superposition algorithm needs to know whether each voxel in the electron density array is inside or outside the set of contours comprising the skin anastruct. This information is used to decide whether to include the energy imparted at a particular voxel in the terma array, and whether to assign dose to a particular voxel in the beam dose array. A CGRID structure called the *inside_contour* grid is created and assigned the same header-values as the electron density grid. The *inside_contour* array consists of characters with values of either 0 or 1, where 0 signifies that the centre of the corresponding voxel in the electron density array is outside the patient anastruct and 1 signifies that it is inside.

Once this array has been created, those voxels in the electron density array whose corresponding voxel in the *inside_contour* array contains a 0, have their electron density value set to zero. This ensures that in creating the radiological depth array, the electron density of the CT couch or any other external structure is not taken into account. The *inside_contour* array is also used as a flag to indicate whether to include the energy imparted by primary photons in a particular voxel in the dose calculation, and whether to assign dose to a particular voxel in the beam dose array. The values in the *inside_contour* array are determined as follows.

The longitudinal position (patient z coordinate) of each of the anastruct contours contained in `request.patient_anastruct` is known. Also known, is the index of the CT scan with which each contour is associated. There may be more than one contour (for example, a torso and two arm contours) associated with each scan — these contours all have the same longitudinal position. Each contour lies in the x - y plane of the patient coordinate system. The position of the centre of each voxel in the electron density array is transformed to patient coordinates, and compared with the longitudinal position of each contour. The indices of the set of contours associated with the scan whose longitudinal position is closest to

that of the transformed point are recorded. The (x,y) coordinates of the transformed point are then tested using `s_inout()`, for being inside one of the contours whose indices were recorded. If the point is inside a contour, the corresponding element in the `inside_contour` array is set to 1, otherwise it is set to 0. If superposition data is to be produced, each element in the `inside_contour` array is simply set to 1, since the entire electron density array represents a water medium. The function `s_inout()` uses the coordinates of the vertices of a specified contour to determine whether a given point with coordinates (x,y) is within that contour. An imaginary line is drawn from (x,y) to (∞,y) , and the vertices stepped through to determine how many times the contour crosses the line. If the line is crossed an even number of times, the given point is outside the contour and if the line is crossed an odd number of times, the point is inside the contour.

5.4.6. The Radiological Depth Grid

Another grid containing a voxel array whose structure is identical to the electron density array is counteracted in which to store the radiological depth at each voxel in the electron density array. The radiological depth array is formed using the function `s_raytrace`, which uses Siddon's ray-tracing algorithm (Siddon, 1985). To reduce computation time, the range of voxels in the electron density array at which radiological depth is calculated, is restricted to include only those which are within the beam outline defined by the jaws at the SAD or less than a distance of 3 cm outside the beam outline. The minimum and maximum i and j indices which define this range are referred to as the i,j limits. Before proceeding to calculate the radiological depth, the value -1.0 is assigned to each element in the array to signify that a radiological depth has not yet been assigned to that voxel. The details of the raytracing algorithm will not be explained here, but an outline of the method used to ensure that radiological depth is assigned to each voxel is given.

For each voxel on the top layer (the layer with a k index of 0) of the electron density array, an imaginary line which originates at the virtual source and passes through the centre of that 'target' voxel is drawn. The end point of the line is defined to be far beyond the extent of the electron density array. The start and end points of the line are then passed to `s_raytrace()`. In this function, the electron density array is treated as three sets of orthogonal planes which are overlaid to form the voxel boundaries. Points on the line which cross any of these planes are found and assigned a parameter to define the relative position of the point on the line. This parameter is a value between 0.0 and 1.0, where 0.0 signifies the start of the line and 1.0 signifies the end of the line. The list of these 'crossing' values is sorted into ascending numerical order, so that each successive value represents the next crossing of a plane made by the line: the first value representing the crossing of the top surface of the array and the last value representing the exit of the line from the array. The list of values is stepped through and the position in the array corresponding to each crossing value is calculated using the coordinates of the array origin and voxel dimensions. The pathlength between each successive pair of crossings is then obtained, along with

the indices of the voxel which the line segment between that pair of crossings is within. The electron density stored in that voxel is used to calculate the increment in radiological pathlength between that pair of crossings, along with the total radiological pathlength to the midpoint between the crossings. The radiological pathlength to the midpoint is assigned to the corresponding voxel in the radiological depth array *only if* the physical pathlength of the line segment through that voxel is at least as great as the thickness of the voxel in the z direction — this is intended to ensure that the radiological depth assigned to a voxel is that calculated for a line which passes through the top and bottom surfaces of the voxel. After all voxels on the top layer of the electron density array have been targeted, values will have been assigned to all voxels on the top layer of the radiological depth array since rays have passed through the top and bottom surfaces of all of these voxels. Beneath the top layer, some voxels will have been assigned a radiological depth in `s_raytrace()` and some will not. Those voxels which have not had a value assigned, will still have the value -1.0. Voxels in the second layer of the electron density array, whose corresponding voxel in the radiological depth array has not been assigned a positive value are now targeted in the same way as the top level voxels. This ensures that all voxels in the second layer of the radiological depth array are assigned a radiological depth. This procedure of targeting unassigned voxels continues for all voxel layers. For each successive layer, the number of calls to `s_raytrace()` decreases as the number of unassigned voxels decreases, hence the computation time per layer decreases with layer index.

5.4.7. The Terma Grid

As with the radiological depth grid, the terma grid has the same structure as the electron density grid. The elements of the terma array are the terma values at the centre of the corresponding voxel in the electron density array.

Before the terma at each voxel is calculated, the function `mk_fluence_map()` is used to create a map of the relative beam intensity (referred to as fluence), in a plane perpendicular to the beam, at the SAD. This map is 2-D cartesian array of points, which are spaced 0.5 cm apart in both directions. In `mk_fluence_map()`, if the map point is outside the beam outline, the fluence value at that point is set to zero. If the map point is inside the beam outline, the fluence is found from the flatness value corresponding to the radial distance from the central axis, which is stored in the flatness table in `request.SAR.table`. The curve representing the flatness table for the 10 MV beam is plotted in figure 5.8. These values were obtained from a measured dose profile at 10.0 cm deep in water for a 30×30 cm beam at 100 cm SSD. This profile was obtained using a 0.1 cm³ PTW ionization chamber in a Therados RFA-3 plotting tank, and is along the x axis of the beam (across the treatment couch). Values in the profile are normalised to 1.0 at $x=0$. Since the flatness table must represent the fluence at the SAD (100 cm), and the profile was measured at 10.0 cm deep, the off axis distance of the points in the profile were first scaled by the factor $SAD/(SSD+10.0) = (100.0/110.0)$. The flatness values are made equal to the profile values for off axis distances up to that at

which the profile drops off in the penumbral region. For larger off-axis distances (up to 20 cm), the flatness value is set to the maximum value of the profile. The flatness value at central axis is normalised to 1.0. A profile was measured along the y axis as a check of the symmetry of the beam. Since the x and y profiles are very similar, it is considered that the flatness as a function of x is representative of the flatness as a function of radial distance from the axis (as is required for the flatness table). It is also assumed that the dose profile at 10 cm deep is representative of the primary photon fluence distribution at this depth — at least away from the penumbral region. The beam can be seen to be very uniform from figure 5.8 (the maximum value in the flatness table is 1.03) so any errors in creating the table will have a small effect on the result, especially for small beams where only flatness values at small off-axis distances are used.

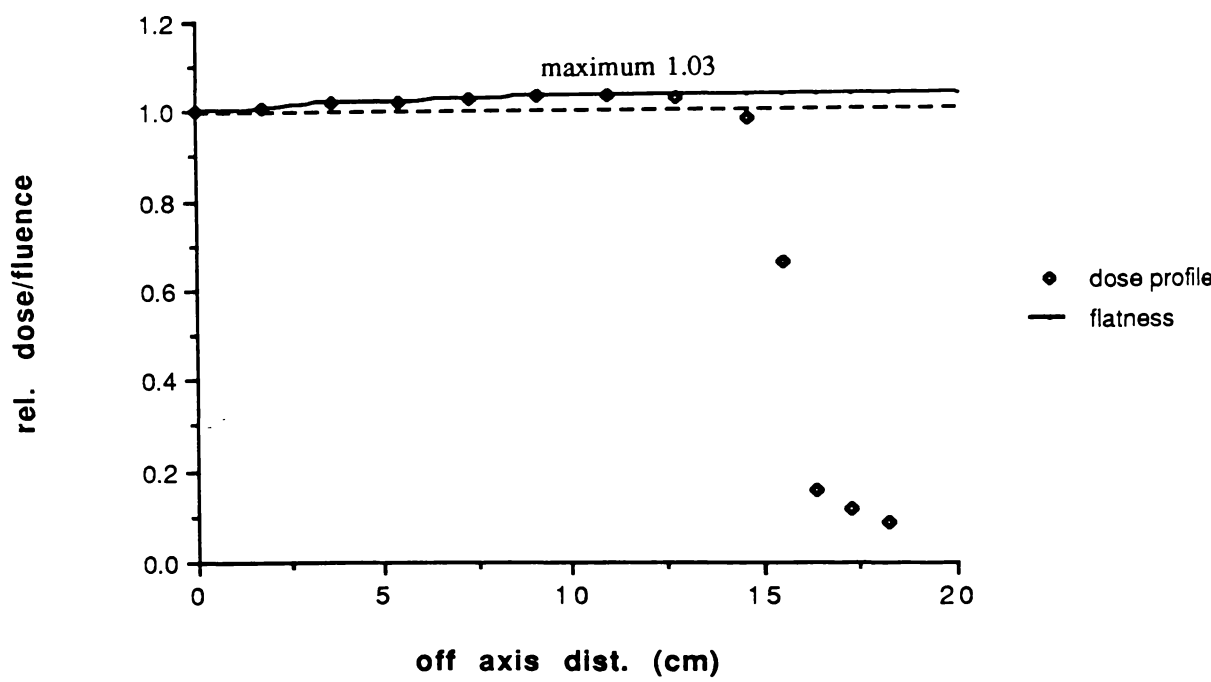


Figure 5.8 Plot of values in the flatness table for the 10 MV beam. The table was obtained from a profile along the x axis of a 30×30 cm beam, which is also shown.

If any beam filters were defined in `xvsim`, each value in the fluence map is multiplied by the corresponding filter factor, which represents the fraction of the primary photon fluence that is transmitted through the filter. The filter factors are stored in a map with the same structure as the fluence map.

The `terma` at each voxel within the i, j limits in the `terma` array is now calculated as follows. First, the relative intensity of the portion of the beam directed toward the voxel must be found from the fluence map. The voxel may be partially within the beam outline, hence the fluence at that voxel is taken to be the average at each of the four top corners of the voxel. In finding the fluence at each corner point, the distance of the point from the

beam axis in the x and y directions is scaled using a beam divergence factor to give the corresponding lateral position at the SAD. The fluence at the scaled lateral position of the corner point is then found by interpolation from the fluence map. Note that this fluence is the relative unattenuated beam intensity, as distinct from the primary fluence which exists after attenuation by the medium. If the relative fluence found as described above for voxel (i, j, k) in the terma array is denoted by $\Phi_{rel}(i, j, k)$, then the terma at (i, j, k) is calculated by summing the terma for each energy component in the primary photon spectrum using the expression

$$T(i, j, k) = \Phi_{rel}(i, j, k) \left(\frac{SAD}{spd} \right)^2 \sum_{n=0}^N F_n(0) e^{-(\mu/\rho)_n d'(i, j, k)} (\mu/\rho)_n E_n. \quad (5-7)$$

In this equation, spd is the distance from the virtual source to the centre of voxel (i, j, k) , $F_n(0)$ is the relative fluence of component n at the surface of the medium, $(\mu/\rho)_n$ is the mass attenuation coefficient for water of energy component n , $d'(i, j, k)$ is the radiological depth at voxel (i, j, k) , and E_n is the mean energy of component n . $F_n(0)$, $(\mu/\rho)_n$, and E_n are contained in tables in the header section of the EDKS structure `kernel`. The primary collision kerma, which is required for the beam hardening correction discussed in chapter 3, is also calculated in this way, except that the mass attenuation coefficient is replaced by the mass energy absorption coefficient $(\mu_{ab}/\rho)_n$, which is also contained in the kernel:

$$K_c(i, j, k) = \Phi_{rel}(i, j, k) \left(\frac{SAD}{spd} \right)^2 \sum_{n=0}^N F_n e^{-(\mu/\rho)_n d'(i, j, k)} (\mu_{ab}/\rho)_n E_n. \quad (5-8)$$

Once the electron density, `inside_contour`, terma, and collision kerma grids have been created, the dose calculation stage can begin. One of three types of calculation is to be performed: to create superposition dose data (if `making_sup_data` is true), to calculate point doses (if `request.have_points` is true), or to calculate an output dose grid (if `request.have_grid` is true). Each of these calculation types will be covered in turn.

5.4.8. Superposition Dose Data

5.4.8.1. Using Superposition Dose Data

In order for the unnormalised doses calculated by the superposition program to be converted to absolute dose, the unnormalised superposition dose at the point in a water phantom where the calibrated dose rate was measured is required. In fact, a superposition dose at the calibration point is required for each square field size for which the measured output factors were obtained. A table of superposition doses is constructed for each square field size contained in the output factor table, and stored in `sup_dose_pars`. This means that for a beam with an equivalent square field size f , the absolute dose at any point P can be found from the absolute dose at the calibration point P_{cal} and the ratio of the superposition dose at P to the stored superposition dose at P_{cal} . Note that P_{cal} is at d_{max} in water for the current beam type, is on the central axis of the equivalent square field, and the SSD is

that for which the calibrated dose rate was measured (100 cm). If the absolute dose at the calibration point is $D_{abs}(P_{cal})_f$, and the stored superposition dose at the calibration point is $D_{sup}(P_{cal})_f$, then the absolute dose at P is given by

$$D_{abs}(P) = D_{abs}(P_{cal})_f \times \frac{D_{sup}(P)}{D_{sup}(P_{cal})_f}, \quad (5-9)$$

where $D_{sup}(P)$ is the raw superposition dose at point P . The absolute dose at the calibration point is in fact given by the global multiplication factor, which is the product of the calibrated dose rate, collimator output factor, and tray factor. The ratio in (5-9) can be thought of as the factor which converts from the absolute dose at the calibration point, to the absolute dose at the calculation point. The conditions under which $D_{sup}(P_{cal})_f$ are obtained are shown in figure 5.9(a). The field is square with side length f , P_{cal} is at d_{max} , and the SSD is 100 cm. In figure 5.9(b), a calculation point P is shown in a medium irradiated by a beam of equivalent square field size f .

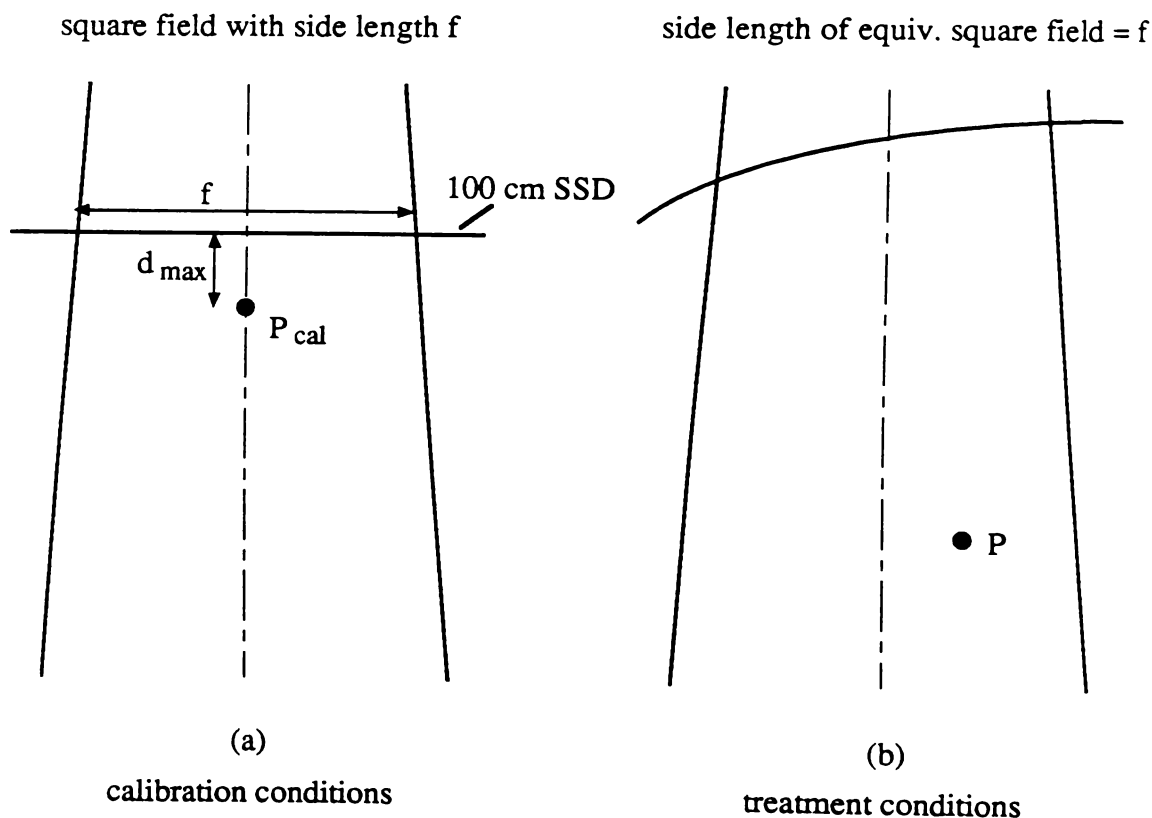


Figure 5.9 The use of superposition data to obtain the absolute dose at a calibration point P . In (a), the calibration point P_{cal} for a square field with side length f is shown. This point is at d_{max} and the SSD is 100 cm. In (b), a calculation point P in a medium irradiated by a beam with equivalent square field size f is shown. The relationship between the absolute dose at P_{cal} and P is given by (5-9).

For fields defined by blocks, this expression still applies, where the equivalent square is determined from the jaw positions, and the absolute dose for the equivalent square (the

first factor) is converted to that for the blocked field by the ratio of superposition dose for the blocked field to that for the equivalent square field (the second factor). Beam filters are accounted for by the ratio of superposition dose for the filtered beam to that for the unfiltered equivalent square field.

For fields smaller than the smallest stored in the output factor and superposition dose tables, the values stored for the smallest field in the tables are used. The reduction in dose for very small fields is accounted for by the ratio in (5–9) being less than unity.

Note that for a calculation point at the central axis of a square beam with side length f , at the same distance from the beam source and at the same depth as the calibration point ($P = P_{cal}$), then the ratio in (5–9) is unity and $D_{abs}(P) = D_{abs}(P_{cal})_f$ as must be the case.

5.4.8.2. Calculating Superposition Dose Data

The depth in water and distance from the virtual source corresponding to the calibrated dose rate stored in `request.beam.time_calc` is copied to the structure `sup_dose_pars`. Next, the voxel in the electron density array which contains the calibration point is located. This is referred to as the calibration voxel. The k (depth) index of this voxel is found from the calibration depth and the voxel side length, and the i, j indices are set to be those at the centre of the array. Since the voxels are small (0.5 cm side lengths) and the dose varies slowly with depth in the region of the calibration point, the approximation is made that the superposition dose at the calibration point can be represented by the dose in this ‘calibration’ voxel.

Superposition dose is now calculated at the calibration voxel for a range of square fields. These fields are defined by restricting the i, j indices of the ‘interaction’ voxels in the `terma` array. The dimensions of these beams do not increase with depth, but the `terma` used in the calculation has the correct inverse square dependence. Since the calibration depth is small, and the correct inverse square falloff is used, this non-divergence should not significantly affect the results. Square fields are defined with increments in side length of two voxels. The smallest field is one voxel wide, the next is three voxels wide (covering nine voxels) etc. The largest field has a side length which is at least as great as the maximum square field size in the collimator output factor and superposition dose tables. For example, if the maximum field size required is 35 cm and the voxel side length is 0.5 cm, then the largest field for which dose is to be calculated will have a side length of 71 voxels. The superposition dose at the calibration voxel is calculated for each field size, using the function `s_calc_points()`. This function, which will be explained shortly, calculates and returns the dose at a specified voxel due to the `terma` in interaction voxels within a specified range of i, j indices. In this way, fields with side lengths of 1, 3, 5, 7, etc... voxels can be defined. Note that these side lengths correspond to 0.5, 1.5, 2.5, 3.5, etc... cm. The superposition dose calculated for these field sizes is stored in an array. Superposition dose data is required for square fields with the side lengths stored in the superposition dose table in `sup_dose_pars`.

The field size in units of voxel widths is calculated for each field size in the table, and the corresponding dose value is obtained from the array of calculated doses by interpolation using `v_interp()`.

When the structure `sup_dose_pars` is complete, it is written to a file in `GRATIS/phys_dat/` using `s_write_sup_dat()`. The name of this file identifies it with the unit id. of the beam for which the data was created.

5.4.9. Calculating Point Doses

The patient coordinates of each point specified in `xvsim` at which a point dose is to be calculated, are stored in a list in the structure `p.points`. Before calculating the dose at a point, its coordinates are transformed to beam coordinates. The indices of the ‘deposition’ voxel in the electron density array containing the point are then determined using the beam coordinates of the origin of the array, and the voxel dimensions. This voxel should always be within the skin `anastruct` since dose calculation points should not have been specified outside the patient: it is ensured that this is the case using the `inside_contour` array. If the voxel is inside the `anastruct` (the `inside_contour` array value corresponding to that voxel is 1), the dose at that voxel is calculated using `s_calc_points()`, otherwise the dose is set to zero and the user is alerted that the point is outside the patient. As with calculating superposition data, the dose in this voxel is considered to be a good representation of the dose at the calculation point since the voxel size is small (0.5 cm side length) and the dose gradient is normally small also. In this case, the range of i, j indices over which the interaction voxels are limited in `s_calc_points()` includes those within the i, j limits. By imposing these limits, interaction voxels which are well outside the beam are not considered.

In `s_calc_points()` the dose at a specified voxel is calculated due to `terma` in interaction voxels within specified limits. Parameters passed to this function include: the indices in the electron density array of the voxel at which the dose is required, (i, j, k) ; the inhomogeneity scaling type to be used in the calculation; pointers to the `terma`, primary collision kerma, and electron density grids; a pointer to the kernel; and the i, j limits. In this function, the primary and scattered dose components are calculated separately and added together to give the value which the function returns. The procedure for each component is as follows.

First, the range of interaction voxel indices in the `terma` array, due to which dose contributions are to be considered is determined. This range is based on the i, j limits, but is further constrained by the dimensions of the EDK arrays. In the case of the primary EDKs, the EDK with the greatest dimensions is that which has been created for the lowest electron density. The range of `terma` array indices included in the primary dose calculation is therefore determined using the dimensions of the lowest density EDK. Note that the scattered EDK arrays all have the same dimensions. The dose contributions are summed by looping through this `terma` voxel range layer by layer. A two dimensional representation of the process of calculating the dose at a deposition voxel due to a layer of interaction voxels in the `terma` array is depicted in figure 5.10. The indices passed to `s_calc_points()`

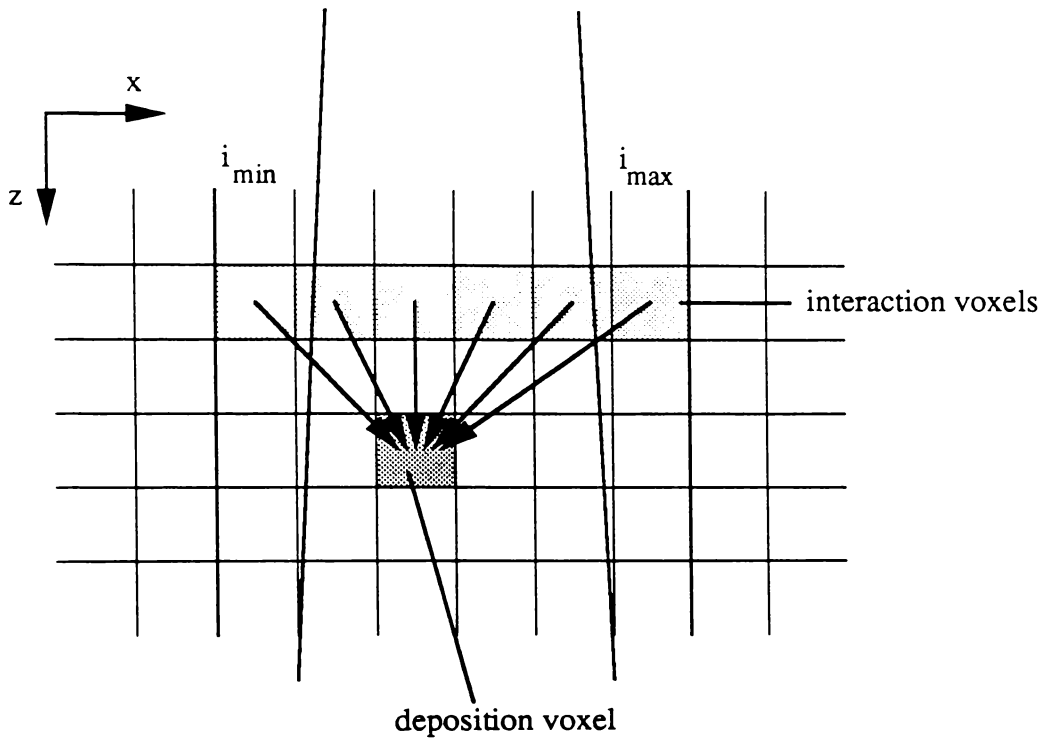


Figure 5.10 Dose contributions to a deposition voxel containing a calculation point, due to a layer of interaction voxels. The voxels in the x direction of the terma array between which contributions are considered are shown.

which limit the range of interaction voxels in the x direction of the terma array are shown.

For each interaction voxel, which has indices (l, m, n) , the terma is obtained from the corresponding voxel in the terma array. If the beam hardening correction discussed in chapter 3 is to be used, the terma is modified using the primary collision kerma at the same voxel, and the surface terma and kerma stored in `kernel`.

5.4.9.1. Average Density Calculation

If the specified scaling type is `Z_SCALING` or `RAY_TRACE`, the average density between the interaction and deposition voxels is determined. This is done using `s_z_trace()` or `s_full_trace()` for the two scaling types respectively, where the procedure performed by `s_z_trace()` is the same as that described in section 4.3.5. These ray tracing algorithms are outlined below.

The parameters which `s_z_trace()` takes are: the interaction voxel coordinates (l, m, n) , the k index of the deposition voxel, the relative weight of the interaction voxel, and a pointer to the electron density grid. In this function, the average density is determined between the centre of the interaction voxel and the centre of the voxel on the layer containing the deposition voxel, which has the same i, j indices as the interaction voxel. This average

density is found by stepping through each voxel in this column, from the interaction voxel layer to the deposition voxel layer, and performing a weighted sum of the electron density in each voxel. If the electron density at a voxel with coordinates (i, j, k) is $\rho_e^w(i, j, k)$, and if $k \geq n + 2$, then the average density is

$$\rho_{ave}(l, m, n; k) = \frac{int_wt \rho_e^w(l, m, n) + \sum_{p=n+1}^{k-1} \rho_e^w(l, m, p) + 0.5 \rho_e^w(l, m, k)}{int_wt + (|k - n| - 2) + 0.5} \quad (5-10)$$

where *int_wt* is the interaction voxel weighting. Note that *int_wt* is always set to 0.5 if the scattered dose component is being calculated. If the primary component is being calculated, *int_wt* may be set to a slightly higher value to help account for electron scattering, as discussed in section 4.4. If $n - 1 \leq k \leq n + 1$, the middle terms in both the numerator and the denominator are omitted, and if $k \leq n - 2$ the sum in the numerator is from $n - 1$ to $k + 1$. Note that if $k = n$, the same point (at the centre of the interaction voxel) is sampled twice.

The parameters which `s_full_trace()` takes are: the interaction voxel coordinates (l, m, n) , the deposition voxel coordinates (i, j, k) , the relative weight of the interaction voxel, and a pointer to the electron density grid. In this function, the average density is determined between the centre of the interaction voxel and the centre of the deposition voxel (the end voxels). The number of points sampled in the electron density array between the centres of the two end voxels, is set to be equal to the number of voxel layers between the layers containing the end voxels. This means that the number of sample points is independent of the lateral coordinates of the interaction voxel. As sample points are stepped through, the coordinates of the voxel containing that point are determined. If the i, j coordinates of the voxel containing the sample point on layer p are $(q(p), r(p))$, then, if $k \geq n + 2$, the average density is

$$\rho_{ave}(l, m, n; i, j, k) = \frac{int_wt \rho_e^w(l, m, n) + \sum_{p=n+1}^{k-1} \rho_e^w(q(p), r(p), p) + 0.5 \rho_e^w(i, j, k)}{int_wt + (|k - n| - 2) + 0.5} \quad (5-11)$$

In situations where $k < n + 2$, the same rules apply as in `s_z_trace()`. Note that if $k = n$, points at the centres of the interaction and deposition voxels are sampled.

A two dimensional representation of both ray tracing methods is illustrated in figure 5.11. Points which would be sampled if the scaling type was `Z_SCALING` are those on the vertical line and the points on the diagonal line between the centres of the interaction and deposition voxels are those which would be sampled if the scaling type was `RAY_TRACE`. The latter method is more rigorous since points are sampled on a direct path between the two end voxels. Note that the two methods will give the same results if the medium contains only slab inhomogeneities.

Once the average density has been found, the EDK value with indices corresponding to the interaction-deposition voxel pair is found by interpolation of the EDK values stored for the

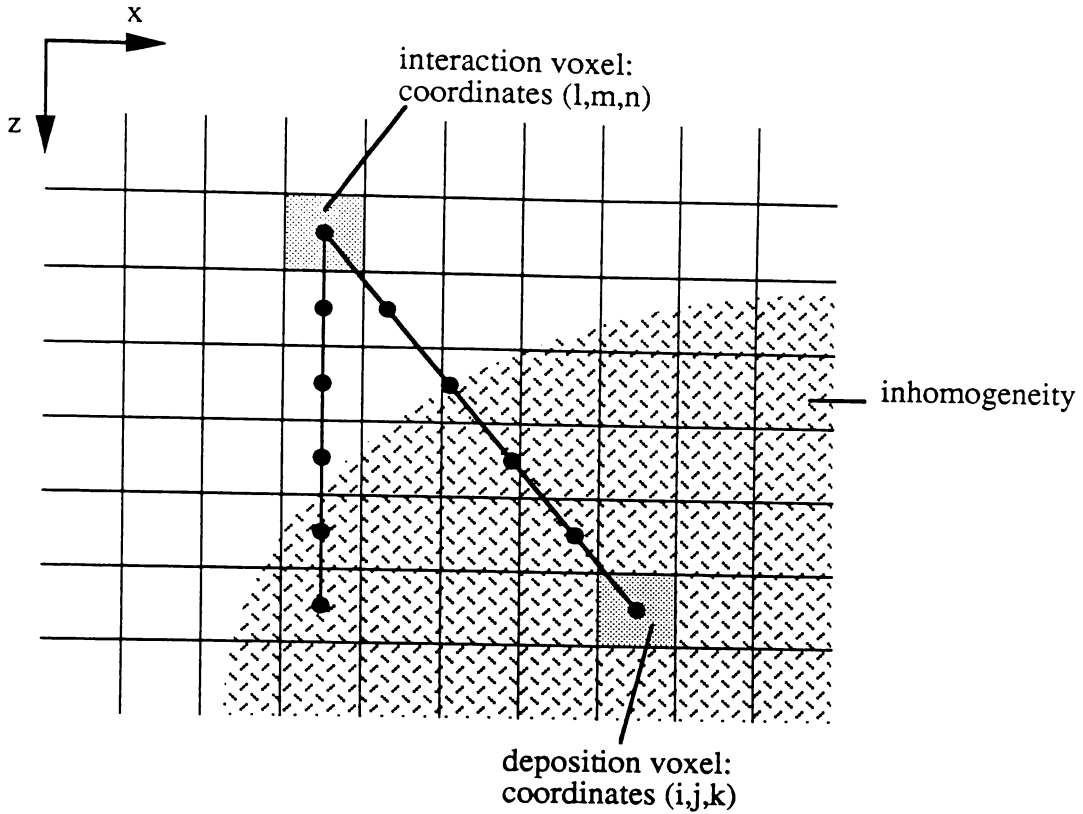


Figure 5.11 Ray tracing methods used to find the average density between the interaction and deposition voxels. If the scaling type is `Z_SCALING`, points are sampled in the column of voxels with the same i, j indices as the interaction voxel — these sample points are those indicated on the vertical line. If the scaling type is `RAY_TRACE`, points are sampled on a line between the interaction and deposition voxel centres, where one point is sampled per voxel layer. This is indicated by the points on the diagonal line.

different electron densities. If the primary dose component is being calculated, before the interpolation is performed a check is made to ensure that EDK values with the required indices exist for both densities used in the interpolation. If the EDK value calculated in this way for the interaction-deposition voxel pair is $EDK(\rho_{ave}; i-l, j-m, k-n)$, then the dose at the deposition voxel is incremented using the expression

$$dose(i, j, k) = dose(i, j, k) + terma(l, m, n) \times \rho_e^w(l, m, n) \times EDK(\rho_{ave}; i-l, j-m, k-n) / \rho_{ave}(l, m, n; i, j, k). \quad (5-12)$$

If the scaling type is `NO_SCALING`, no inhomogeneity correction to the EDKs is used and the expression for the dose increment is simplified to

$$dose(i, j, k) = dose(i, j, k) + terma(l, m, n) \times EDK(1.0; i-l, j-m, k-n) \quad (5-13)$$

where $EDK(1.0; i-l, j-m, k-n;)$ is the value stored for a unit density EDK. Note that $dose(i, j, k)$ is not contained in array — it is just a variable representing the primary or scattered dose at (i, j, k) .

Once this procedure has been performed for both dose components, the sum of the primary and scattered dose is returned to the main program. This value, **sup_dose**, is an unnormalised superposition dose which must be converted to an absolute dose using the superposition dose data. The superposition dose at the calibration point, **sup_output**, corresponding to the equivalent field size of the current beam is obtained by interpolation from the table in **sup_dose_pars**. The conversion from unnormalised superposition dose to absolute dose is now performed using the global multiplication factor, **global_factor**, and **sup_output**. Using (5-9), the absolute dose at the calculation point, **abs_dose**, is given by

$$\text{abs_dose} = \text{global_factor} \times \frac{\text{sup_dose}}{\text{sup_output}}. \quad (5-14)$$

This absolute dose is assigned to the entry in the point dose list in **p.points** which corresponds to the current calculation point.

When the dose at all points in the list has been calculated, **p.points** is written to a file in the **patient_dir/v/** directory using **write_points()**.

5.4.10. Creating the Output Dose Grid

If an output dose grid is required, the dose is first calculated throughout the beam dose array. The dose at each voxel in the beam dose array, due to terma voxels within the i, j limits, is calculated using **s_calc_dose()**. Each time this function is called, the values contained in the beam dose array voxels are incremented by the contribution from the specified interaction voxel.

Before **s_calc_dose()** is called for an interaction voxel, it is checked that the voxel is within the patient skin anastruct using the **inside_contour** array. This is to avoid calculating the dose due to voxels at which the electron density has been set to zero.

In **s_calc_dose()** the dose throughout the beam dose array is calculated due to the terma in a specified interaction voxel. Parameters passed to this function include: the indices in the terma array of the interaction voxel, (i, j, k) ; the inhomogeneity scaling type to be used in the calculation; pointers to the terma, primary collision kerma, electron density, and beam dose grids; and a pointer to the kernel. Elements of the dose array are incremented first for the primary dose component, then the scattered component. The procedure for each component is as follows.

First, the range of deposition voxel indices in the dose array at which dose increments are to be considered is determined. This range is based on the limiting i, j indices supplied, and on the dimensions of the EDKs in the same way as in **s_calc_points**, except that the limits now pertain to deposition voxels instead of interaction voxels. The dose contribution to each voxel in the dose array within this range of indices is calculated layer by layer.

Before the loop through the deposition voxels begins, the terma at the interaction voxel is obtained from the corresponding voxel in the terma array. If the beam hardening correction

discussed in chapter 3 is to be used, the term is modified using the primary collision kerma at the same voxel, and the surface term and kerma stored in the kernel. A two dimensional representation of the process of calculating the increment in dose on a layer of deposition voxels in the beam dose array due to an interaction voxel in the term array is depicted in figure 5.12.

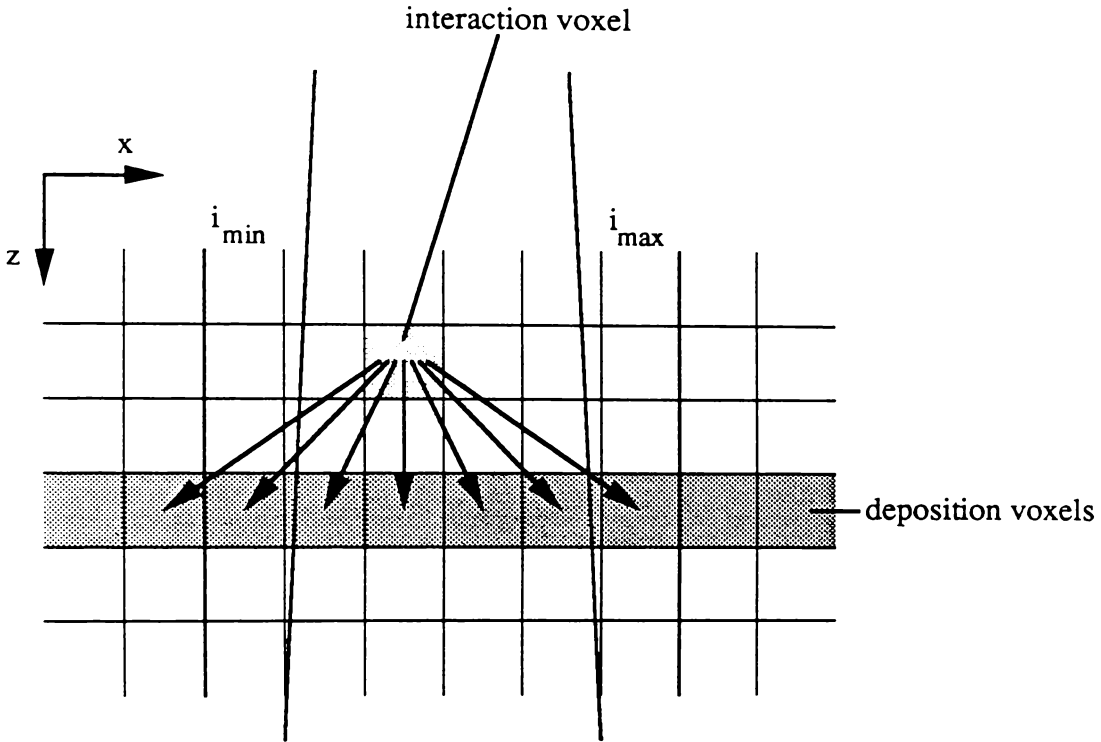


Figure 5.12 Dose contributions to a layer of deposition voxels, due to an interaction voxel in the term array. The voxel limits in the x direction between which term is considered are shown.

In the loop, if the scaling type is `Z_SCALING` the average density is calculated once for each layer of deposition voxels using `s.z.trace()`, whereas if the scaling type is `RAY_TRACE` the average density is calculated for each deposition voxel using `s.full.trace()`. Note that the interaction voxel now has indices (i, j, k) and the deposition voxel has indices (l, m, n) , hence the order in which these sets of indices are passed to `s.z.trace()` and `s.full.trace()` is the opposite from the point dose calculation case. As in `s.calc.points`, once the average density has been found, the EDK value corresponding to the interaction and deposition voxel pair is found by interpolation. If the EDK value calculated in this way for the interaction-deposition voxel pair is $EDK(\rho_{ave}, l - i, m - j, n - k)$, then the dose at the deposition voxel is incremented using the expression

$$dose(l, m, n) = dose(l, m, n) + term(i, j, k) \times \rho_e^w(l, m, n) \quad (5-15)$$

$$\times EDK(\rho_{ave}; i - l, j - m, k - n) / \rho_{ave}(l, m, n; i, j, k)$$

If no inhomogeneity scaling is used, the expression for the dose increment is simplified to

$$dose(l, m, n) = dose(l, m, n) + terma(i, j, k) \times EDK(1.0; l - i, m - j, n - k) \quad (5-16)$$

where $EDK(1.0; l - i, m - j, n - k)$ is the EDK value stored for unit density. Note that $dose(l, m, n)$ is now an element of the dose array representing the dose at voxel (l, m, n) , whereas in (5-15) and (5-13) $dose(i, j, k)$ is a value representing the dose at voxel (i, j, k) which is returned to the main program.

The computation time required for the calculation is far greater if the scaling type is `RAY_TRACE` than if it is `Z_SCALING`, since the `s_full_trace()` is called for each deposition voxel (l, m, n) . For example, on the Sun (without using transputers), the computation times required for a 15×15 cm beam when voxels of side length 1.0 cm are used, are approximately: 5 minutes with the scaling type set to `NO_SCALING`; 7 minutes using `Z_SCALING`; and 3 hours using `RAY_TRACE`. It is the result of this computation using `Z_SCALING` which is shown in figure 5.3. Obviously a great deal of optimisation is required before the full ray trace option is practical for routine use — a strategy such as the ‘collapsed cone’ approach of Ahnesjö *et al* (1989) is worth investigating. An investigation of the computation times using transputers for this configuration has been carried out by Murray (1991).

Once the loop through the interaction voxels is complete, each voxel in the beam dose array is tested for being inside the patient anastruct. If it is, the superposition dose contained in that voxel is converted to absolute dose using (5-14) in the same way as the point doses, otherwise the dose in that voxel is set to zero.

The output dose grid `g.grid3` must now be filled using the values stored in the beam dose array. This is performed by looping through each point in the output dose grid and obtaining the dose value from the corresponding position in the beam dose array by interpolation.

First, the coordinate transformation between the output dose grid and the beam dose grid is obtained. The coordinates of the origin point in the output dose grid are transformed to beam dose grid coordinates. It is important to note that the value associated with a particular voxel in the beam dose array, is stored at a point whose coordinates are the minimum x, y, and z coordinates of the region occupied by that voxel. Therefore, the transformed coordinates of the output dose grid origin point are decremented by half a voxel width in each direction. The orthogonal vectors representing the output grid point spacings are also transformed to beam dose grid coordinates. The transformed positions of the output grid points are stepped through using the transformed start point and vectors. At each position, the value of the dose is found by trilinear interpolation using the function `s_trinterp()`. Those points in the output grid which lie outside the bounds of the beam dose array are assigned a dose of zero.

The output dose grid, `g.grid3`, is then written to a file in the `patient_dir/g/` directory using `write_grid()`.

5.5. Future Improvements and Projects

There are a number of improvements which could be made to the system, and projects which the system could provide. From a software point of view, the most important future task is to speed up the ray-tracing method used for calculating the average density between the interaction and deposition voxels. One approach to this would be to sum the fractional energy in groups of voxels in the EDK arrays before beginning the superposition, so that the average density calculation would be performed for fewer deposition voxels. This may be a reasonable approach for the scattered dose component only, since the values in the scattered EDKs vary more slowly than those in the primary EDKs — also the scattered component is less important. Another approach is to calculate the contribution from an interaction voxel along radial lines originating at the voxel. EDKs stored in spherical polar arrays would be more suitable for this approach. The main advantage with this approach is that the average density could be calculated recursively for successive points along each line — this speeds up the calculation because a complete ray-trace is not required between each interaction and deposition voxel as is the case with the present method.

At this stage the accuracy of the system has not been investigated, except for the comparisons with experiment and Monte Carlo made for the simpler algorithm described in chapter 4. For slab inhomogeneities such as in the lung phantom, the full ray-tracing method would give the same results as those produced using the simpler z-scaling method. Investigations should be carried out to determine the errors which arise from using z-scaling in situations where interfaces between different material are not perpendicular to the beam. This would possibly involve the construction of another lung phantom with cork-polystyrene interfaces. A Monte Carlo simulation could also be set up with the same geometry as the phantom — this is straightforward using RTPCART. Also, a simple tool could be included in the GRATIS system to construct an electron density array with geometry resembling that of the phantom. As well as testing the validity of the z-scaling method, the validity of any modifications to the full ray-trace method could then be investigated.

Presently, transputers are used to perform the actual superposition procedure only. Other components of the algorithm, such as the ray trace to calculate radiological depth, could also be performed using the transputer network. Shortly, transputers will be available with approximately 10 times the computing power of the presently used T800 chips. These T9000 transputers could be easily installed in place of the T800s. The large increase in speed obtained would increase the practicality of developing a more sophisticated algorithm.

Chapter 6

2-D and 3-D Electron Pencil Beam Algorithms

6.1. Introduction

Radiotherapy treatment using high energy electrons has become increasingly available in recent years due to the large number of dual mode linear accelerators – such as the Varian Clinac 18/10 – installed in radiotherapy centres. Electrons, rather than photons are the modality of choice for a number of treatment situations because of the ability to limit the region to which dose is delivered. Tumours on the skin or at shallow depths are particularly suited to electron beam therapy since the uniformity of dose is high to a reasonably well defined depth, after which the dose rapidly falls off as the electrons run out of energy and stop (Klevenhagen, 1985).

The widespread use of electron beam therapy has seen a need for accurate electron beam treatment planning systems. With the availability of increasingly powerful computers on which to base treatment planning systems, it is now possible to implement algorithms which are able to realistically model the transport of electrons in the irradiated medium. The most realistic calculations are those done by Monte Carlo modelling where energy deposition due to individual incident particles is calculated (Jenkins *et al*, 1988). Due to the detailed treatment of each particle history, along with the very large number of incident particles required to produce a dose distribution with an acceptably low statistical uncertainty, this method is not suitable for routine use. Monte Carlo modelling is however a very important area of research which will inevitably gain in significance as available computing power increases.

Currently, the most sophisticated electron beam algorithms implemented on commercial treatment planning systems are based on the *Pencil Beam* method (Hogstrom *et al*, 1981). Using this method, broad beams are built up of narrow ‘pencil’ beams whose contributions to the dose throughout the medium are added. The superposition of measured narrow beam dose distributions to form a broad beam dose distribution was shown to be valid experimentally by Lillicrap *et al* (1975). In pencil beam algorithms, each pencil beam is considered mathematically to consist of an infinite number of ‘point’ beams, which are superimposed to form the pencil beam.

In figure 6.1, the tracks followed by 20 electrons in very narrow 12 MeV beam (radius 0.01 cm) incident on a water medium are illustrated. The tracks were generated with an EGS4 Monte Carlo simulation where the coordinates of each electron step are recorded using the

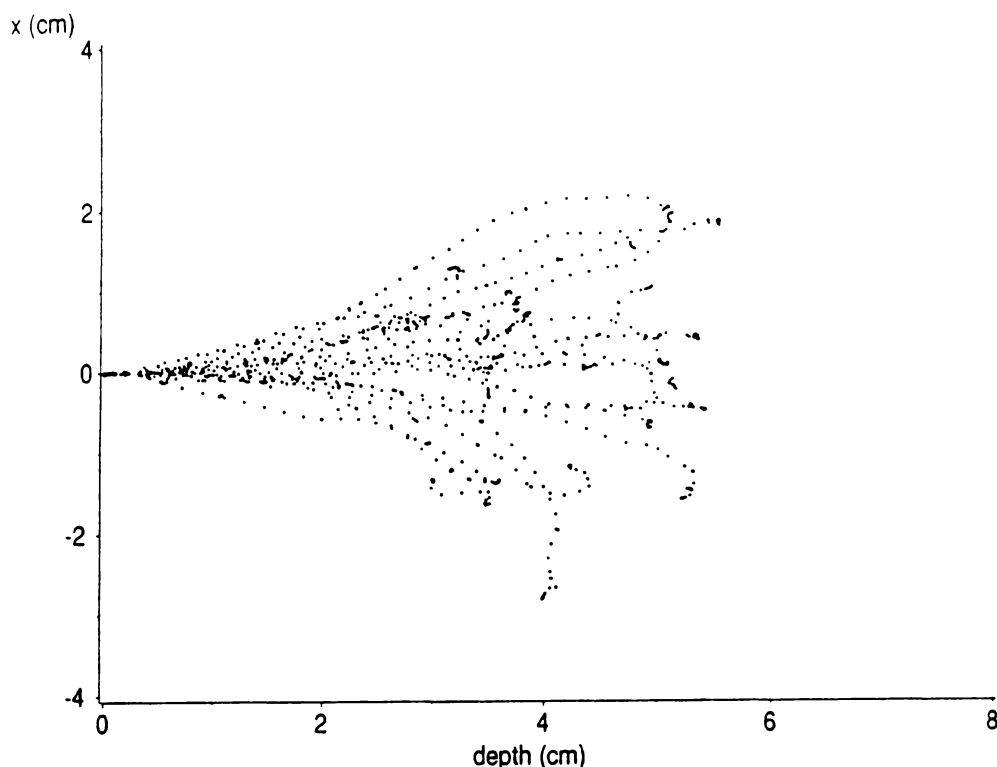


Figure 6.1. Tracks generated by an EGS4 Monte Carlo simulation of 20 monoenergetic 12 MeV electrons in a circular beam of radius 0.01 cm, incident on a water medium.

DOTPLOT option in the user code RTPCYL (Murray, 1990). Although Monte Carlo generated pencil beams are not used in the pencil beam model described here, the tracks in figure 6.1 give a useful visualisation of the behaviour of electrons in a point beam.

As with the superposition method, a 3-D pencil beam model has been incorporated into the GRATIS radiotherapy treatment planning system and a network of transputers can be used to reduce the computation time. An outline of that algorithm is given in chapter 7. Prior to installing the Pencil Beam algorithm on this system, 2-D and 3-D dimensional Pencil Beam algorithms were implemented on a PC based system, which also used transputers. In this chapter, these preliminary 2-D and 3-D methods are discussed and used to calculate the dose distribution in a water medium containing an air cavity. In particular, the accuracy of the algorithms in predicting the magnitude of the 'hot spot' beneath the cavity is investigated. Calculated results for a 12 MeV beam are compared with those obtained by measurement. The differences in the 3-D method described here and that implemented on the GRATIS system are mainly technical: the theory covered in this chapter applies equally to the algorithm described in chapter 7.

6.2. Outline of the 2-D and 3-D Pencil Beam Methods

A brief explanation of the geometry of the pencil beam methods used for this work, and

the differences in the 2-D and 3-D methods, is given here before the theory is discussed.

The electron density distribution of the irradiated medium is stored in a cartesian voxel array with dimensions $I \times J \times K$. The indices i, j, k (each beginning at 0), denote an element of the array. The directions of increasing i, j and k indices will be referred to as the x, y and z directions respectively. The central axis of the incident broad beam is perpendicular to the x - y plane. The calculated dose distribution is stored in another array, called the calculation array, also of dimensions $I \times J \times K$.

In the 2-D method, the pencil beams into which the broad beam is divided are defined by I 'strips' at the surface of the irradiated medium. These strips are one voxel wide in the x direction and extend the width of the beam in the y direction. The pencil beam incident on the strip of voxels at the surface with index i is referred to as pencil beam i . For each pencil beam, the dose contribution to voxels on each successive layer (k index) in the array is determined. With the 2-D method the medium is considered to be invariant in the y direction. The electron density variation in the x - z plane used for the calculation is that at a chosen value of j . This j -value is chosen to be that which coincides with the x - z plane in which the dose distribution is required. The dose distribution is calculated for this j -value only. Note that only a two dimensional electron density array and calculation array are required, but since the 2-D and 3-D algorithms are implemented in the same program, three dimensional arrays are used.

In the 3-D method, the broad beam is divided into $I \times J$ pencil beams of square cross section, each of which are incident on a voxel at $k = 0$. A pencil beam incident on voxel $(i, j, 0)$ is referred to as pencil beam i, j . The electron density variation in the y direction is now considered since the dose contribution from each pencil beam is calculated using an electron density distribution unique to that beam. Using the 3-D method, the dose distribution throughout the entire calculation array is determined, enabling a 'slice' of the distribution at each j -value to be examined.

The dose contribution due to pencil beams incident on the medium is calculated using the *Fermi-Eyges* theory of Multiple Coulomb Scattering (MCS) (Eyges, 1948; Andreo, 1985). Using this theory, the electron fluence distribution of the point beams comprising the pencil beams is calculated. An alternative to using MCS theory is to use the theory of *age diffusion* (Kawachi, 1975; Ayyanger *et al*, 1980).

6.3. The Point Beam Distribution

In traversing a medium, electrons are deflected from their paths by interactions with atomic nuclei and orbital electrons (Andreo, 1985). In the energy range applicable to therapeutic electron beams, deflections are predominantly due to the interaction between the electric fields of the incident electrons and the atomic nuclei. Since this process involves a very large number of small and independent deviations, the net deviation of an electron after having traversed a certain thickness of material may be predicted statistically. This statistical

distribution may be determined using the Fermi-Eyges theory.

According to the Fermi-Eyges theory, an electron incident in the z direction at the origin of a cartesian coordinate system, which has penetrated a depth z in a medium, has a probability $P(x, \theta_x; z)$ of having x component of lateral displacement x , and x component of angular displacement θ_x given by (Klevenhagen, 1985)

$$\frac{\partial P(x, \theta_x; z)}{\partial z} = -\theta_x \frac{\partial P}{\partial x} + \frac{1}{4} \frac{d\bar{\theta}^2}{dz} \frac{\partial^2 P}{\partial \theta_x^2}. \quad (6-1)$$

The *linear scattering power* $d\bar{\theta}^2/dz$ is defined as the mean square angular deviation of an electron in traversing a unit path length (ICRU 21, 1972), and is a function of medium composition and electron energy. Equation (6-1) was solved by Eyges (1948), yielding the expression

$$P(x, \theta_x; z) = \frac{1}{2\pi \sqrt{A_0 A_2 - A_1^2}} e^{(-A_0 x^2 - 2A_1 x \theta_x + A_2 \theta_x^2)/2(A_0 A_2 - A_1^2)}, \quad (6-2)$$

where A_0 , A_1 , and A_2 are called *scattering moments*. The expression for the n th scattering moment is (Hogstrom *et al*, 1981)

$$A_n(z) = \frac{1}{2} \int_0^z \frac{d\bar{\theta}^2}{dz'} (z - z')^n dz'. \quad (6-3)$$

Note that $A_0(z)$ is the mean square x component of angular deviation in traversing a path length z .

By integrating the Eyges solution over all θ_x , the *location probability density* $P_L(x; z)$ is obtained:

$$\begin{aligned} P_L(x; z) &= \int_{-\infty}^{+\infty} P(x, \theta_x; z) d\theta_x \\ &= \frac{1}{\sqrt{2\pi A_2}} e^{-x^2/2A_2}. \end{aligned} \quad (6-4)$$

This expression gives the probability that an electron at depth z has x component of lateral displacement x . By comparison with a normal distribution function whose mean is $\bar{x}$ and standard deviation is σ_x , it is obvious that $P_L(x; z)$ is a normal distribution where $\bar{x} = 0$ and $\sigma_x = \sqrt{A_2}$. The location probability density for point beam can then be written as (Jette, 1984a; Klevenhagen 1985)

$$P_L(x; z) = \frac{1}{\sqrt{2\pi} \sigma_x} e^{-x^2/2\sigma_x^2}. \quad (6-5)$$

Another quantity of interest is the probability $P_A(\theta_x; x, z)$ that an electron with x component of lateral displacement x at depth z , has x component of angular displacement θ_x . This quantity is given by

$$P_A(\theta_x; x, z) = P(x, \theta_x; z)/P_L(x; z) \quad (6-6)$$

$$= \frac{1}{\sqrt{2\pi(A_0 - A_1^2/A_2)}} e^{-(\theta_x - A_1 x/A_2)^2/2(A_0 - A_1^2/A_2)}.$$

This is another normal distribution with mean $\bar{\theta}_x(x) = A_1 x/A_2$ and standard deviation $\sigma_{\theta_x} = \sqrt{A_0 - A_1^2/A_2}$. From these expressions it can be seen that the mean angular displacement is proportional to x and the standard deviation is independent of x . Note that $\sigma_{\theta_x} < \sqrt{A_0}$, because σ_{θ_x} pertains to a particular x and not to all electrons at depth z .

There is of course an identical distribution to $P_L(x; z)$ for the lateral displacement in the y direction, $P_L(y; z)$. If the standard deviation in the x or y direction is denoted by σ_{MCS} (since the displacement is due to multiple Coulomb scattering) then the probability that an electron at depth z has been displaced by (x, y) is given by

$$\begin{aligned} P_L(x, y; z) &= P_L(x; z)P_L(y; z) \\ &= \frac{1}{\sqrt{2\pi}\sigma_{MCS}} e^{-x^2/2\sigma_{MCS}^2} \\ &\times \frac{1}{\sqrt{2\pi}\sigma_{MCS}} e^{-y^2/2\sigma_{MCS}^2}. \end{aligned} \quad (6-7)$$

If many electrons are incident in a point beam at the origin then provided no electrons are lost from the beam, the value of $P_L(x, y; z)$ is proportional to the *planar fluence* Φ_p at (x, y, z) with respect to the x - y plane (Nahum, 1985). A good approximation to the dose is the *total fluence* Φ (Nahum and Brahme, 1985), so by approximating the total fluence by the planar fluence, an approximate dose distribution for the point beam is obtained. Since the electrons have been deflected, they are not travelling in a direction normal to the x - y plane. Φ_p is therefore slightly less than Φ , so a small error is inherent in using $P_L(x, y; z)$ to approximate Φ .

Since $\sigma_{MCS} = \sigma_x = \sqrt{A_2}$, then using (6-3), σ_{MCS} is calculated using the expression (Hogstrom *et al*, 1981)

$$\sigma_{MCS}(z) = \sqrt{\frac{1}{2} \int_0^z \frac{d\bar{\theta}^2}{dz}(z')(z - z')^2 dz'}. \quad (6-8)$$

For media of similar composition, the linear scattering power is proportional to mass density, and can be expressed in terms of the *mass scattering power* and the mass density by,

$$\frac{d\bar{\theta}^2}{dz}(z') = \frac{d\bar{\theta}^2}{\rho dz}(z')\rho(z'). \quad (6-9)$$

The mass scattering power decreases with electron energy and is therefore dependent on depth. From a plot of mass scattering power versus energy, it has been found that the expression

$$\frac{d\bar{\theta}^2}{\rho dz} = AE^{-B} \quad (6-10)$$

can be used to determine the mass scattering power where E is the energy in MeV (Werner *et al*, 1982). For a water-like medium, $A = 4.52$ and $B = 1.78$. For bone $A = 5.58$ and

$B = 1.78$. Note that the mass scattering power of bone is higher than for water, although the energy dependence is the same. The determination of electron energy as a function of depth is outlined in section 6.4.

For a homogeneous medium of mass density ρ , if the energy dependence (and hence the dependence on z) of the scattering power is ignored, a simple expression for σ_{MCS} may be obtained. If the mass scattering power is set at a constant value T , then the standard deviation is given by

$$\begin{aligned}\sigma_{MCS}(z) &= \sqrt{\frac{1}{2} \int_0^z T \rho (z - z')^2 dz'} \\ &= \sqrt{\frac{T \rho z^3}{6}}.\end{aligned}\tag{6-11}$$

If an approximate value of T is known this is a useful expression for a quick estimate of σ_{MCS} .

6.4. The 3-D Calculation Procedure

Although the 3-D method is more calculation intensive than the 2-D method, it is best described first since it is more ideal and hence more straightforward conceptually.

The relative dose due to a point beam incident in the z direction at the origin of a cartesian coordinate system is approximated by the product of the location probability density $P_L(x, y, z)$ and a depth factor $g(z)$:

$$D(x, y, z) = P_L(x, z) P_L(y, z) g(z).\tag{6-12}$$

The depth factor is obtained from a measured depth dose curve as explained shortly.

In the 3-D algorithm described here, the dose is calculated due to $I \times J$ pencil beams incident normally on each of the voxels at the top of the calculation array. The slight angulation of the beam due to divergence from the virtual source is not considered, but the increase in field width with depth is accounted for (see section 6.4.4). Using (6-12), the dose contribution at voxel (i', j', k') due to pencil beam i, j is given by

$$\begin{aligned}D(i', j', k'; i, j) &= S(i, j, k') \int_{i\Delta i}^{(i+1)\Delta i} P_L(x_{i'} - x'; i, j, k') dx' \\ &\quad \times \int_{j\Delta j}^{(j+1)\Delta j} P_L(y_{j'} - y'; i, j, k') dy' \\ &\quad \times g(i, j, k')\end{aligned}\tag{6-13}$$

where the central point of voxel (i', j', k') has position $(x_{i'}, y_{j'}, z_{k'})$. The weighting factor $S(i, j, k')$ accounts for the falloff in beam intensity away from the central axis and is known as the *beam-drift factor* (Hogstrom, 1985). The integrals involving P_L in this equation are called the x and y scatter factors respectively. Note that P_L depends on i and j since it is

the medium in this column of voxels which determines the standard deviation of the point beams. Substituting the expression for P_L (equation (6-5)) into (6-13) gives

$$\begin{aligned}
 D(i', j', k'; i, j) &= S(i, j, k') \\
 &\times \int_{i\Delta i}^{(i+1)\Delta i} \frac{1}{\sqrt{2\pi}\sigma_{MCS}(i, j, k')} e^{-(x_{i'} - x')^2 / 2\sigma_{MCS}^2(i, j, k')} dx' \\
 &\times \int_{j\Delta j}^{(j+1)\Delta j} \frac{1}{\sqrt{2\pi}\sigma_{MCS}(i, j, k')} e^{-(y_{j'} - y')^2 / 2\sigma_{MCS}^2(i, j, k')} dy' \\
 &\times g(i, j, k')
 \end{aligned} \tag{6-14}$$

The integrals in this equation are referred to as the *x scatter factor* and *y scatter factor* respectively.

6.4.1. The Depth Factor

For each pencil beam i, j , the depth factor $g(i, j, k')$ must be determined for each voxel level k' . The depth factor is defined as the central axis percentage depth dose in a layered medium whose density varies as in voxel column i, j . The depth factor should be for a field size large enough so that lateral electron equilibrium exists at all depths (Mc Parland *et al*, 1988). The decrease in dose at central axis due to the lateral electron disequilibrium which occurs for small fields is then accounted for by the model. Note that in this case lateral electron disequilibrium is caused by a lack of primary electrons — not secondary electrons as is the case with photon beams. The depth factor is found using the value of a measured percentage depth dose curve in water for a 20×20 cm field, at the effective depth in water at the centre of voxel i, j, k' , $z_{eff}(i, j, k')$. The effective depth in water of a point P in an inhomogeneous medium is the depth at which the incident electrons have lost the same energy as at P (Klevenhagen, 1985). Percentage depth dose curves for the Varian Clinac 18/10 6, 9, 12, 15, and 18 MeV beams are shown in figure 6.2.

The effective depth may be determined by a discrete summation over depth intervals of thickness Δz . The effective depth after M intervals is given by

$$z_{eff}(M) = \sum_{m=1}^M \Delta z_{eff}(m) \tag{6-15}$$

where $\Delta z_{eff}(m)$ is the increment in z_{eff} for interval m . For a medium thickness Δz at interval m , the equivalent thickness in water for electrons crossing this interval, is that thickness in water over which electrons lose the same energy. This is found using the ratio of the total stopping power in the medium at interval m to that in water, $S_{H_2O}^{med}(m)$. The expression for $z_{eff}(M)$ can then be written in the form

$$z_{eff}(M) = \sum_{m=1}^M \Delta z S_{H_2O}^{med}(m). \tag{6-16}$$

A good approximation to the ratio of stopping powers in interval m , is the electron density relative to water in that interval $\rho_e^w(m)$ (Klevenhagen, 1985). This is due to the fact that in

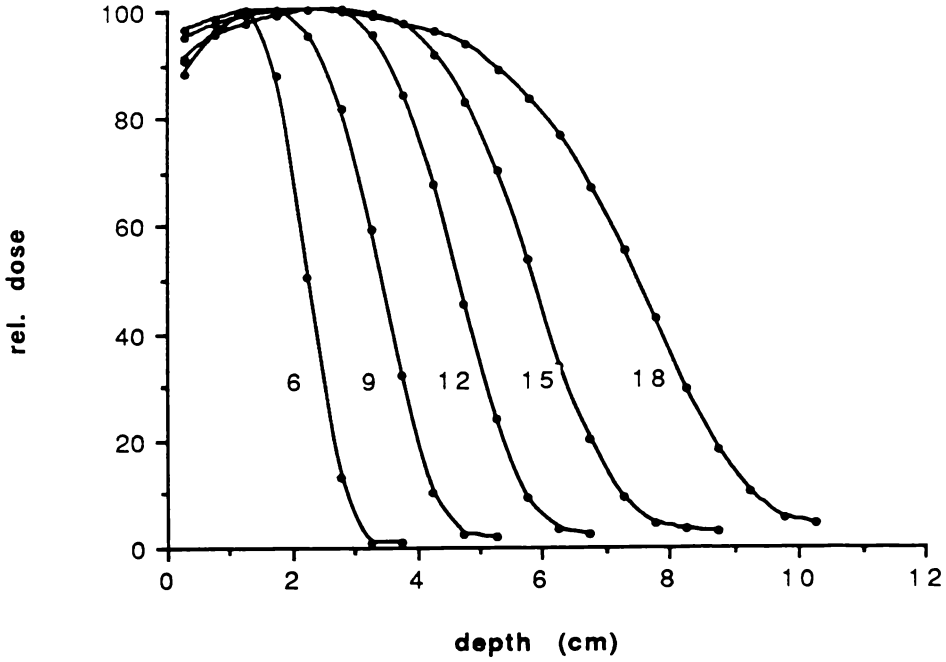


Figure 6.2 Percentage depth dose curves for the Varian Clinac 18/10 6, 9, 12, 15, and 18 MeV beams.

the electron energy range of interest, energy loss is primarily due to collisions with atomic electrons. The energy lost per unit path-length is then proportional to the number of electrons per unit volume. This relationship only becomes inaccurate when the medium has a high effective atomic number and the energy is high enough for radiative energy loss to be significant. Effective depth can therefore be expressed in terms of a summation of ρ_e^w for each interval m :

$$z_{eff}(M) = \sum_{m=1}^M \Delta z \rho_e^w(m). \quad (6-17)$$

This can be written as a recursive function of the effective depth at the end of the previous interval:

$$z_{eff}(M) = z_{eff}(M-1) + \Delta z \rho_e^w(M). \quad (6-18)$$

If each step length Δz is half a voxel thickness, then for each pencil beam, (6-18) can be used to calculate the effective depth at the centre and at the boundaries of each voxel layer. The effective depth at the centre of voxel $(i, j, 0)$ is $z_{eff}(i, j, 0) = \Delta z \rho_e^w(i, j, 0)$, and the effective depth at the centre of voxels for which $k' > 0$ is

$$z_{eff}(i, j, k') = z_{eff}(i, j, k' - 1) + \Delta z (\rho_e^w(i, j, k' - 1) + \rho_e^w(i, j, k')). \quad (6-19)$$

The effective depth at the voxel boundaries is also easily obtained using this two-step process.

An inverse-square correction factor must be applied to this value to account for the fact that the distance from the virtual source to the centre of voxel (i, j, k') , is in general different

than the distance from the virtual source to the depth of measurement $z_{eff}(i, j, k')$. In applying this correction, consideration must be given to the effective position of the virtual source, which for electron beams may not coincide with the nominal position used to define the SSD (Hogstrom *et al*, 1981). The effective source position of the Waikato Hospital's Varian Clinac 18/10, has been determined from the measured dose at a range of SSDs (Hoban, 1988). For all energies, it was found that the effective position of the source is approximately 15 cm 'downstream' from the nominal position of the source. This effective position is referred to as the *virtual source position* (*vsp*).

The value of the measured depth dose curve at $z_{eff}(i, j, k')$ in water is denoted by $g_{H_2O}(z_{eff}(i, j, k'))$. The depth factor for voxel (i, j, k') , including the inverse square correction, is then given by

$$g(i, j, k') = g_{H_2O}(z_{eff}(i, j, k')) \left(\frac{100 - vsp + z_{eff}(i, j, k')}{SSD - vsp + z_{k'}} \right)^2 \quad (6-20)$$

where SSD corresponds to the distance from the nominal source position to the surface of the electron density array.

The bremsstrahlung component in the depth dose curve is subtracted from the measured dose to obtain g_{H_2O} . Bremsstrahlung dose is assumed to have an inverse square dependence on distance from the virtual source. When the dose distribution has been calculated, the bremsstrahlung component is added, taking into account beam divergence and inverse square falloff. Although the dose due to bremsstrahlung comprises only 2-4% of the total dose at shallow depths, this component is responsible for the dose deposited at depths greater than the electron range. The dose at depths greater than the electron range is especially important when multiple fields are used, since the dose at the intersection of the fields can become significant.

6.4.2. Energy Loss with Depth

To evaluate σ_{MCS} , the mean electron energy as a function of depth is required since the scattering power is energy dependent as in (6-10). The mean energy at depth z , $\bar{E}(z)$, can be found from the mean energy at the surface $\bar{E}(0)$, the effective depth $z_{eff}(z)$, and the practical range in water for that beam energy R_p from (Harder, 1965)

$$\bar{E}(z) = \bar{E}(0)(1 - z_{eff}(z)/R_p). \quad (6-21)$$

The practical range is the depth at which the dose due to incident electrons drops to zero. The mean energy in MeV of the beam at the surface is found from the depth at which the dose has dropped to 50% of its maximum value. If this depth is denoted by R_{50} , then the mean energy at the surface is given by the expression (ICRU 21, 1972)

$$\bar{E}(0) = 2.33R_{50}. \quad (6-22)$$

Mean surface energy and practical range is stored for each energy at which the linear accelerator operates (see table 6.1 for appropriate values for the Varian Clinac 18/10 at Waikato Hospital).

Nom. energy (MeV)	$\bar{E}(0)$ (MeV)	R_p (cm)
6	5.4	2.9
9	7.9	4.4
12	10.7	5.8
15	13.7	7.3
18	17.5	9.5

Table 6.1. Mean energy at the surface $\bar{E}(0)$ and practical range R_p for each nominal electron energy of the Varian Clinac 18/10. R_p was obtained from ionization chamber measurements. $\bar{E}(0)$ was obtained from (6-22) where R_{50} was obtained from measurements.

6.4.3. The Scatter Factors

Before the scatter factors in (6-14) can be evaluated, $\sigma_{MCS}(i, j, k')$ must be determined. The linear scattering power in (6-8) is the product of the mass scattering power and the mass density of the medium, where the mass scattering power is different for bone and for soft tissue. So that the correct values of A and B are used in (6-10) to calculate the mass scattering power for a particular depth interval, it must be determined whether the medium is bone or soft tissue in that interval. Also, the electron density in that interval must be converted to mass density. To determine whether the medium at a depth interval is bone or soft tissue, the electron density is compared with a threshold value above which the medium is considered to be bone. This threshold is set at 1.10. For bone, the electron density is converted to mass density by multiplying by 1.02, and for soft tissue the mass density is equal to the electron density. The factor of 1.02 used for bone is based on work by Metcalfe (1990), where it was found that the ratio of mass density to electron density for a bone analog material and for human bone is 1.16/1.14 and 1.12/1.10 respectively. Note however that for denser bone such as that of mass density 1.65 in tables in Johns and Cunningham (1983), the ratio of mass density to electron density is higher than this. As stated in section 4.3.6, the electron density of this bone is 1.57, making the ratio of mass density to electron density equal to 1.05. The lower value (1.02) is used here however since bone of density 1.1-1.2 is more common than bone of density 1.6-1.7.

The standard deviation σ_{MCS} is determined by evaluating (6-8) using the method of Hogstrom *et al* (1981). Using this method, the scattering moments A_0 , A_1 , and A_2 are calculated recursively in steps of half a voxel thickness. The mean energy at the beginning of each step is required. This is calculated using (6-6) where z_{eff} is the effective depth at the beginning of the step. If the mass scattering power and mass density corresponding to the M th increment in depth in voxel column i, j are $d\bar{\theta}^2/\rho dz(i, j, M)$ and $\rho(i, j, M)$ respectively and the depth increments are of size Δz , then the scattering moments after increment M are given by

$$A_0(i, j, M) = A_0(i, j, M-1) + \frac{1}{2} \Delta z \frac{d\bar{\theta}^2}{\rho dz}(i, j, M) \rho(i, j, M); \quad (6-23)$$

$$A_1(i, j, M) = A_1(i, j, M-1) + \Delta z A_0(i, j, M-1) + \frac{1}{4} \Delta z^2 \frac{d\bar{\theta}^2}{\rho dz}(i, j, M) \rho(i, j, M); \quad (6-24)$$

$$A_2(i, j, M) = A_2(i, j, M-1) + 2\Delta z A_1(i, j, M-1) + \Delta z^2 A_0(i, j, M-1) + \frac{1}{6} \Delta z^3 \frac{d\bar{\theta}^2}{\rho dz}(i, j, M) \rho(i, j, M). \quad (6-25)$$

Prior to beginning this recursive procedure, the values of $A_0(i, j, 0)$, $A_1(i, j, 0)$, and $A_2(i, j, 0)$ are set to zero. Note that two iterations of these equations are performed for each increment in the voxel layer index k' . The value of $\sigma_{MCS}(i, j, k')$ is simply equal to the square root of $A_2(i, j, M)$ where M corresponds to the centre of voxel layer k' — note that σ_{MCS} can also be found on the voxel boundaries.

In calculating the standard deviation σ_{MCS} , the Fermi-Eyges theory assumes that the number of electrons in a point beam remains constant with depth. In fact, some electrons are lost out of the beam at shallower depths than others. This is mainly due to *energy loss straggling* — the energy lost per unit pathlength is not identical for each electron (even if all electrons have the same initial energy) (Berger and Wang, 1988; Sandison and Papiez, 1990). Because of straggling, the depth at which electrons run out of energy varies. Also, some electrons undergo large angle scattering events at shallow depths, causing them not to penetrate as far as other electrons.

The effect of this loss of electrons on the standard deviation was investigated using EGS4. A monoenergetic 10.7 MeV electron beam of radius 0.01 cm incident on a water medium was simulated using RTPCYL. This energy was chosen since it is the mean surface energy of the 12 MeV beam (see table 6.1). The beam is incident in the z direction at the origin of a cylindrical coordinate system. Energy deposition is recorded in annular voxels of radial index i and depth index k with radial and depth dimensions Δr and Δz respectively. The mean radius of voxels with radial index i is r_i . A dose distribution $D(i, k)$ is obtained by dividing the energy in each voxel by the voxel volume. This simulation was performed with $\Delta z = 0.5$ cm and $\Delta r = 0.1$ cm, and the standard deviation $\sigma(k)$ of the dose profile at depth index k determined using

$$\sigma(k) = \sqrt{\frac{\sum_i D(i, k) r_i^2}{\sum_i D(i, k)}}. \quad (6-26)$$

In figure 6.3, the standard deviation found using both Monte Carlo and Fermi-Eyges theory (using (6-8)) is compared. It can be seen that the Monte Carlo curve reaches a maximum of 1.14 cm, while the Fermi-Eyges curve continues to increase. The maximum occurs because electrons which are still in the beam near the depth of maximum penetration, tend to be those which have not been displaced by a large distance from the pencil beam axis.

So that the standard deviation does not increase unrealistically in the pencil beam model, a maximum is imposed on σ_{MCS} . For a homogeneous water medium, this maximum value can

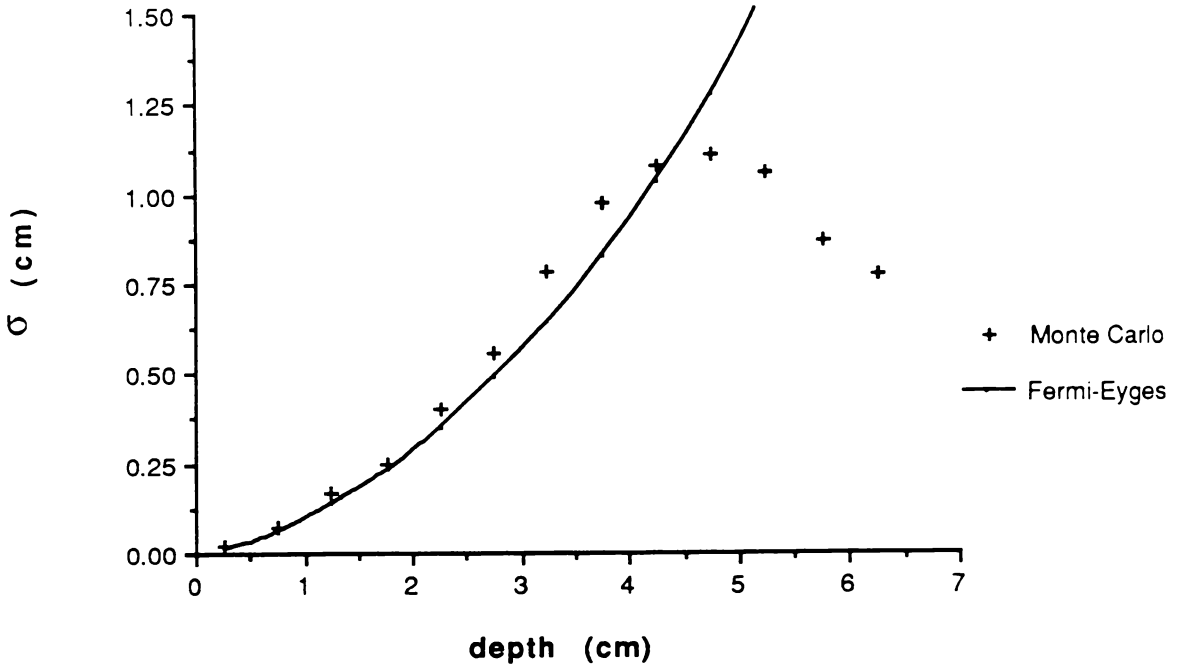


Figure 6.3. Maximum pencil beam standard deviation. The standard deviation calculated using Fermi-Eyges theory is not allowed to exceed the maximum calculated using an Monte Carlo simulation. Shown is a comparison of Monte Carlo and Fermi-Eyges calculated standard deviations for monoenergetic 10.7 MeV electrons.

be made equal to the maximum standard deviation found using a Monte Carlo simulation as described above. However for a medium of lower density for example, the maximum standard deviation should be higher since the electrons travel further before stopping. It can be shown using (6-8) and (6-9) that at a depth z in a homogeneous medium of water-like composition, where the effective depth is z_{eff} , the standard deviation is z/z_{eff} times the standard deviation at depth z_{eff} in water. For example, in a medium of density $\rho = 0.5 \text{ gcm}^{-3}$, the standard deviation at depth z is twice the standard deviation at depth $z_{eff} = z/2$ in water. Therefore, a reasonable value for the maximum standard deviation in the $\rho = 0.5 \text{ gcm}^{-3}$ medium would be twice that in water. To determine a value for the maximum allowed standard deviation at each voxel level in the pencil beam calculation, the mean density relative to water $\bar{\rho}(i, j, k')$ between the surface and depth $z_{k'}$ is found. The standard deviation σ_{max} which must not be exceeded at that level is then determined from

$$\sigma_{max}(i, j, k') = \frac{\sigma_{max, H_2O}}{\bar{\rho}(i, j, k')} \quad (6-27)$$

where σ_{max, H_2O} is the maximum standard deviation in water. The standard deviation is then set to the minimum of $\sigma_{MCS}(i, j, k')$ and $\sigma_{max}(i, j, k')$. Values of σ_{max, H_2O} found

for each electron energy are given in table 6.2. Note that the maximum found by using $\bar{\rho}(i, j, k')$ in (6-27) is strictly only valid for homogeneous media since the standard deviation at a particular depth is dependent on the density distribution, rather than on the mean density, between the surface and that depth.

Nom. energy (MeV)	σ_{max, H_2O} (cm)
6	0.68
9	0.93
12	1.14
15	1.33
18	1.57

Table 6.2. Maximum standard deviation in water for each energy found using EGS4 simulations. Energy was recorded in a cylindrical voxel array and the standard deviation on each layer of the array was found using (6-26).

More sophisticated methods of calculating σ_{MCS} , which account for the reduction in electron fluence, have been developed (Mc Parland *et al*, 1988; Bruinvis *et al*, 1989).

Once σ_{MCS} has been found, the scatter factors in (6-14) can be evaluated. These integrals may be written in terms of the error function, which is defined as

$$erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t'^2} dt'. \quad (6-28)$$

In this form, (6-14) becomes

$$\begin{aligned} D(i', j', k'; i, j) &= S(i, j, k') \\ &\times \frac{1}{2} \left[erf \left(\frac{(i+1)\Delta i - x_{i'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) - erf \left(\frac{i\Delta i - x_{i'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) \right] \\ &\times \frac{1}{2} \left[erf \left(\frac{(j+1)\Delta j - y_{j'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) - erf \left(\frac{j\Delta j - y_{j'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) \right] \\ &\times g(i, j, k'). \end{aligned} \quad (6-29)$$

6.4.4. The Beam Drift Factor

In this model, the pencil beams are considered mathematically to originate at the top surface of the calculation array. In reality, before reaching the surface, electrons in the beam pass through the scattering foil, beam monitors, air etc. in the linear accelerator treatment head (Hogstrom *et al*, 1981). Due to scattering, this introduces an angular spread into the beam. This angular spread gives rise to lateral disequilibrium near the edges of the beam, causing the electron fluence at the beam limits defined by the collimation system (the geometric limits), to drop off less sharply. The beam-drift factor $S(i, j, k')$ takes account of this by weighting each pencil beam i, j at the depth of calculation $z_{k'}$. In electron beam therapy, the

collimator closest to the medium surface is generally an electron applicator (Klevenhagen, 1985). For the Varian Clinac 18/10 there is an air gap between the applicator face and the surface of 5 cm for an SSD of 100 cm. Across the applicator face, the beam is modelled as having a constant intensity with an x component of angular standard deviation σ_{θ_x} . When projected to the depth of calculation $z_{k'}$, this gives rise to a lateral standard deviation σ_{air} in the medium of (Hogstrom *et al*, 1981).

$$\sigma_{air}(z_{k'}) = (z_{k'} + L_0)\sigma_{\theta_x} \tag{6-30}$$

where L_0 is the air gap between the collimator face and the medium surface.

Although the air gap is *physically* 5 cm for an SSD of 100 cm, the *effective* air gap may be found from experimental ‘in air’ dose profiles measured at a range of source detector distances (SDDs) (Hogstrom *et al*, 1981). The angular standard deviation may also be found from these measurements. This analysis assumes that the beam penumbra in the profile curves is due entirely to the projected angular standard deviation. The penumbral width is quantified by the difference in off-axis distance of the points of 10% and 90% dose, denoted by $\Delta_{10} - \Delta_{90}$. First, a plot is made of $\Delta_{10} - \Delta_{90}$ versus SDD. It can be shown that the plot should be linear, with slope $2.56\sigma_{\theta_x}$. Also, since the penumbral width is assumed to be due entirely to the projected angular standard deviation, $\Delta_{10} - \Delta_{90}$ should be zero at the collimator face, therefore the intercept with the SDD axis corresponds to the effective position of the collimator face. The air gap is then the difference between the SSD and the intercept with the SDD axis. This was performed for SDDs ranging from 100 cm to 120 cm for each energy (Hoban, 1988). Results using this method are shown in table 6.3, where L_0 is with respect to an SSD of 100 cm. The air gap for any SSD may be found by adding (SSD - 100.0) to the value of L_0 in the table. Note that σ_{θ_x} decreases with energy. This is expected since the scattering power decreases with energy as in (6-10).

Nom. energy (MeV)	L_0 (cm)	σ_{θ_x} (rad)
6	5.7	0.063
9	7.2	0.051
12	5.5	0.039
15	6.3	0.032
18	7.1	0.027

Table 6.3. Effective air gap L_0 between the collimator face and the surface (100 cm SSD) and angular standard deviation σ_{θ_x} at the collimator face for each nominal electron energy of the Varian Clinac 18/10.

The effective air gap for an SSD of 100 cm differs from the physical value (5 cm) and also varies with energy. This may be due to the assumption of a flat field with constant angular

standard deviation at the electron applicator face being slightly incorrect. The intensity of pencil beam i, j at depth $z_{k'}$ due to the standard deviation σ_{air} is given by the beam drift factor

$$S(i, j, k') = \int_{X_0(k')}^{X_1(k')} \frac{1}{\sqrt{2\pi}\sigma_{air}(k')} e^{-(x_i - x')^2 / 2\sigma_{air}^2(k')} dx' \quad (6-31)$$

$$\times \int_{Y_0(k')}^{Y_1(k')} \frac{1}{\sqrt{2\pi}\sigma_{air}(k')} e^{-(y_j - y')^2 / 2\sigma_{air}^2(k')} dy'$$

where the limits of integration are the beam limits at depth $z_{k'}$. If the beam axis coincides with the z axis, and the minimum and maximum x, y coordinates of the beam at 100 cm from the nominal position of the virtual source are $X_0(0), Y_0(0)$ and $X_1(0), Y_1(0)$, then the beam limits at depth $z_{k'}$ are

$$X_0(k') = X_0(0) \left(\frac{SSD - vsp + z_{k'}}{100 - vsp} \right) \quad (6-32)$$

$$Y_0(k') = Y_0(0) \left(\frac{SSD - vsp + z_{k'}}{100 - vsp} \right) \quad (6-33)$$

$$X_1(k') = X_1(0) \left(\frac{SSD - vsp + z_{k'}}{100 - vsp} \right) \quad (6-34)$$

$$Y_1(k') = Y_1(0) \left(\frac{SSD - vsp + z_{k'}}{100 - vsp} \right) \quad (6-35)$$

Note that conceptually the beam drift factor is a weighting of the pencil beams at the surface, although it is dependent on the depth $z_{k'}$. This weighting is illustrated in figure 6.4, where the beam intensity increases with an increasing shade of grey.

If the integrals in (6-31) are written in terms of the error function, the beam-drift factor becomes

$$S(i, j, k') = \frac{1}{2} \left[erf \left(\frac{X_1(k') - x_i}{\sqrt{2}\sigma_{air}(k')} \right) - erf \left(\frac{X_0(k') - x_i}{\sqrt{2}\sigma_{air}(k')} \right) \right] \quad (6-36)$$

$$\times \frac{1}{2} \left[erf \left(\frac{Y_1(k') - y_j}{\sqrt{2}\sigma_{air}(k')} \right) - erf \left(\frac{Y_0(k') - y_j}{\sqrt{2}\sigma_{air}(k')} \right) \right].$$

The two terms in this equation are known as the x beam-drift and y beam-drift factors respectively.

6.4.5. Assigning Dose Due to a Pencil Beam

Once the beam-drift factors, scatter factors, and depth term are known, the dose contribution to voxels on layer k' due to pencil beam i, j can be assigned. Substituting (6-20) and

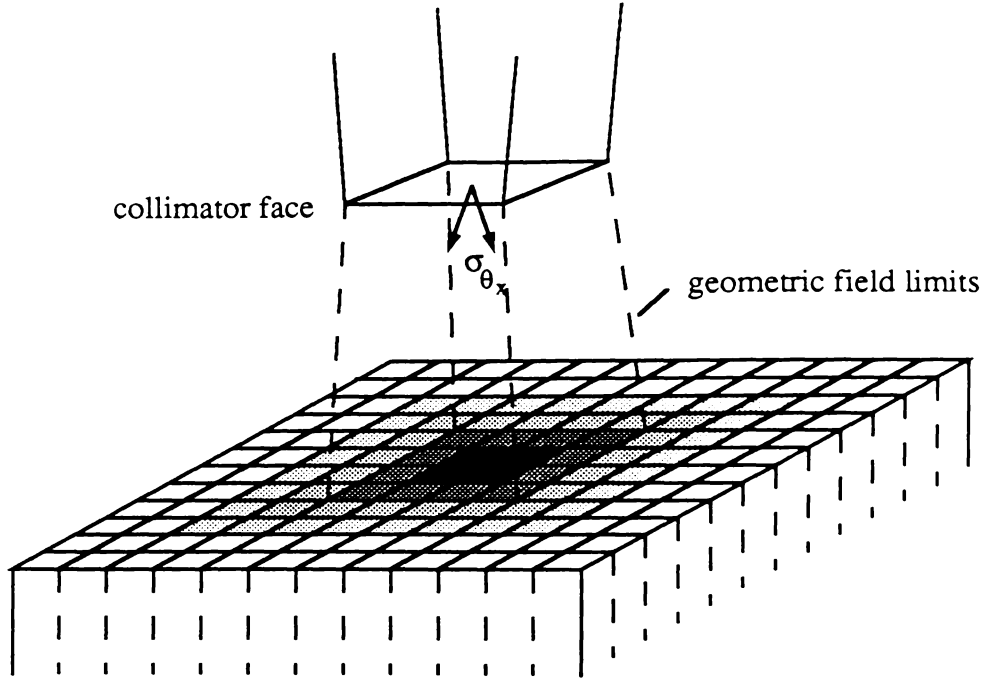


Figure 6.4. Weighting of pencil beams at the surface for the 3-D calculation due to the angular spread of electrons at the electron applicator face. The weight is a function of the pencil beam index i, j and decreases with distance from the beam axis as indicated by the decreasing shade of grey.

(6-36) into (6-29) then gives the final expression for the 3-D calculation:

$$\begin{aligned}
 D(i', j', k'; i, j) = & \frac{1}{2} \left[\operatorname{erf} \left(\frac{X_1(k') - x_i}{\sqrt{2}\sigma_{\text{air}}(k')} \right) - \operatorname{erf} \left(\frac{X_0(k') - x_i}{\sqrt{2}\sigma_{\text{air}}(k')} \right) \right] \\
 & \times \frac{1}{2} \left[\operatorname{erf} \left(\frac{Y_1(k') - y_j}{\sqrt{2}\sigma_{\text{air}}(k')} \right) - \operatorname{erf} \left(\frac{Y_0(k') - y_j}{\sqrt{2}\sigma_{\text{air}}(k')} \right) \right] \\
 & \times \frac{1}{2} \left[\operatorname{erf} \left(\frac{(i+1)\Delta i - x_{i'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) - \operatorname{erf} \left(\frac{i\Delta i - x_{i'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) \right] \\
 & \times \frac{1}{2} \left[\operatorname{erf} \left(\frac{(j+1)\Delta j - y_{j'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) - \operatorname{erf} \left(\frac{j\Delta j - y_{j'}}{\sqrt{2}\sigma_{MCS}(i, j, k')} \right) \right] \\
 & \times g_{H_2O}(z_{eff}(i, j, k')) \left(\frac{100 - vsp + z_{eff}(i, j, k')}{SSD - vsp + z_{k'}} \right)^2.
 \end{aligned} \tag{6-37}$$

The error function terms in this equation are evaluated using a look up table, where values of $\operatorname{erf}(t)$ are stored for $t = 0.00$ to 3.00 in steps of 0.01 .

The value of $D(i', j', k'; i, j)$ decreases with the radial distance from i, j to i', j' . When this distance becomes larger than about $3\sigma_{MCS}(i, j, k')$, the dose contribution drops to zero. For this reason, the loop through i', j' is restricted so that $|i' - i| \leq 4\sigma_{MCS}(i, j, k')$ and $|j' - j| \leq 4\sigma_{MCS}(i, j, k')$. If the dose distribution in a single slice in the x - z plane is

required, the loop may be restricted to only one j' , thus reducing the computation time. The calculation is performed for each successive depth index k' until the effective depth $z_{eff}(i, j, k')$ exceeds the practical range R_p . When this depth is reached the dose on an additional two voxel layers is calculated since the electron contribution to the dose has a small 'tail' past R_p . The dose due to bremsstrahlung is added at the end of the calculation as already described.

6.5. The 2-D Calculation Procedure

The 2-D calculation method is derived from the more ideal 3-D method using appropriate assumptions and approximations.

In the 2-D method, a new pencil beam standard deviation σ_{tot} is used. This is a combination of the standard deviation due to MCS and that due to initial angular deviation at the applicator face, and for pencil beam i, j , is given by

$$\sigma_{tot}(i, j, k') = \sqrt{\sigma_{air}^2(k') + \sigma_{MCS}^2(i, j, k')}. \quad (6-38)$$

If $\sigma_{tot}(i, j, k')$ is used in the scatter factors instead of $\sigma_{MCS}(i, j, k')$, the beam-drift factors can be incorporated into the scatter factors as in the original Hogstrom algorithm (Hogstrom *et al*, 1981). If this is done in the 3-D calculation, the pencil beams i, j over which the dose contribution is determined are restricted to those within the geometric beam limits. This variation is not used in the 3-D method implemented here since it reduces the ability of the algorithm to account for inhomogeneities close to the surface (Hogstrom, 1985).

The 2-D calculation method which is implemented here uses the total standard deviation in the y direction but retains the x beam drift factor. The broad beam is now divided into 'strip' pencil beams, and the dose contribution due to each of these is determined. The strips can be visualised as having a length $Y_1(k') - Y_0(k')$, which increases with depth due to beam divergence. The beam-drift factor now consists of the x beam-drift term only. In figure 6.5, the weighting of the strips at the surface due to the beam drift factor is illustrated.

The standard deviation σ_{MCS} and effective depth z_{eff} are now functions of the strip index i and the calculation layer k' . There is no functional dependence on j since the medium is considered to be invariant with j . The j -index J_0 at which the dose distribution is to be determined, is the j -value from which the density is obtained.

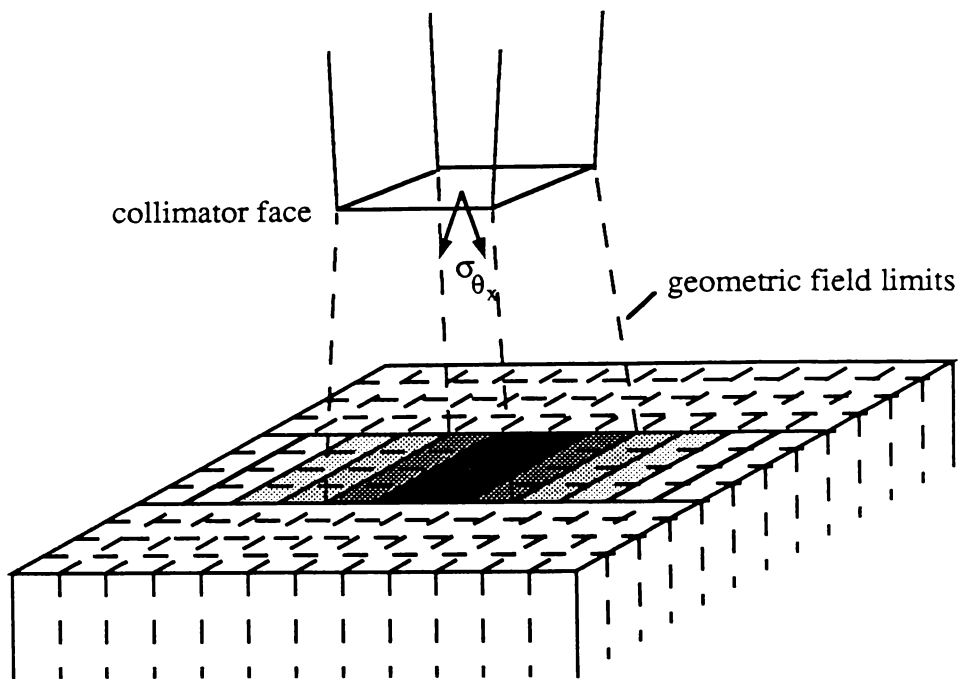


Figure 6.5. Weighting of pencil beams at the surface for the 2-D calculation. The weight is a function of the pencil beam index i and decreases with distance from the beam axis as indicated by the decreasing shade of grey.

The dose at i', J_0, k' due to a strip with index i is given by

$$\begin{aligned}
 D(i', J_0, k'; i) = & \frac{1}{2} \left[\operatorname{erf} \left(\frac{X_1(k') - x_i}{\sqrt{2}\sigma_{\text{air}}(k')} \right) - \operatorname{erf} \left(\frac{X_0(k') - x_i}{\sqrt{2}\sigma_{\text{air}}(k')} \right) \right] \\
 & \times \frac{1}{2} \left[\operatorname{erf} \left(\frac{(i+1)\Delta i - x_{i'}}{\sqrt{2}\sigma_{\text{MCS}}(i, J_0, k')} \right) - \operatorname{erf} \left(\frac{i\Delta i - x_{i'}}{\sqrt{2}\sigma_{\text{MCS}}(i, J_0, k')} \right) \right] \\
 & \times \frac{1}{2} \left[\operatorname{erf} \left(\frac{Y_1(k') - y_{J_0}}{\sqrt{2}\sigma_{\text{tot}}(i, J_0, k')} \right) - \operatorname{erf} \left(\frac{Y_0(k') - y_{J_0}}{\sqrt{2}\sigma_{\text{tot}}(i, J_0, k')} \right) \right] \\
 & \times g_{H_2O}(z_{\text{eff}}(i, J_0, k')) \left(\frac{100 - vsp + z_{\text{eff}}(i, J_0, k')}{\text{SSD} - vsp + z_{k'}} \right)^2.
 \end{aligned} \tag{6-39}$$

As with the 3-D method, the loop over the voxel indices i' at which the dose contribution is calculated is restricted so that $|i' - i| \leq 4\sigma_{\text{MCS}}(i, J_0, k')$.

6.6. Dose Calculation Results

In electron beam irradiation some inhomogeneities give rise to regions of unusually high and low dose known as 'hot spots' and 'cold spots' respectively (Jette, 1984b). The accurate prediction of the position and intensity of these regions is difficult because it requires a realistic treatment of the scattering process. The type of inhomogeneity which causes the

most dramatic hot and cold spots is the air cavity. Since electron beam treatment of the head and neck region (where such cavities exist) is common, an algorithm which accurately accounts for cavities is essential.

The dose distribution in a water medium containing a cubic air cavity of side length 2.0 cm was calculated using the 2-D and 3-D pencil beam methods described here. The voxels used in the calculation are cubic with side length 0.5 cm. For a 10×10 cm 12 MeV beam, the array dimensions used are $I=J=35$ and $K=20$. The cavity extends in the z direction from $k=1$ to $k=4$. In the x - y plane, the centre of the cavity is made to coincide with the centre of voxel $i = j = 17$. Voxels which are partially included by the cavity are given mean densities according to the proportion of the voxel included in the cavity.

Calculated results are compared with experimental results obtained using a polystyrene walled cavity immersed in water in a Therados RFA-3 plotting tank. The walls of this cavity are 0.3 cm thick. The cavity was positioned at the centre of the 10×10 cm beam with its top surface at a depth of 0.5 cm. Dose profiles at 4.0 cm deep were obtained using the setup illustrated in figure 6.6, where the detector is a waterproof 0.1 cm^3 PTW ionization chamber. All calculated and experimental results are for the 12 MeV beam. This energy was chosen since it is in the middle of the energy range for the Varian Clinac 18/10. A central axis percentage depth dose curve was first obtained without the cavity in place. The depth of maximum dose (d_{max}) in water was found to be 2.0 cm. All results are percentage doses relative to the dose at d_{max} in water.

A depth dose curve beneath the cavity was recorded, beginning at 4.0 cm deep (1.5 cm below the cavity). Points from the curve are plotted in figure 6.7(a). The measured dose at 4.0 cm deep is 122%. Note that this depth is 1.5 cm below the bottom surface of the cavity, and that beneath the cavity, the effective depth at 4.0 cm deep is 2.0 g cm^{-3} . Since $d_{max} = 2.0$ cm, dose calculation methods which use a simple effective depth algorithm where electron scattering is ignored (Klevenhagen, 1985), would predict a dose beneath the cavity at this depth of approximately 100%. The high dose beneath the cavity is due to a lateral electron disequilibrium situation which is explained below.

In reaching a depth of 4.0 cm, electrons incident directly above the cavity pass through less water and hence undergo less scattering than electrons incident adjacent to the cavity. Consequently, the number of electrons incident adjacent to the cavity which are scattered into the region below the cavity is greater than the number of electrons incident above the cavity which end up outside this region.

Included in figure 6.7(a) is the measured depth dose curve in water. If scattering was not significant, the curve with the cavity in place should be displaced from the falloff part of the water depth dose curve by the thickness of the cavity. The displacement is in fact about half this amount as can be seen by comparison with the 2.0 cm long horizontal line, indicating that electrons incident adjacent to the cavity contribute significantly to the dose

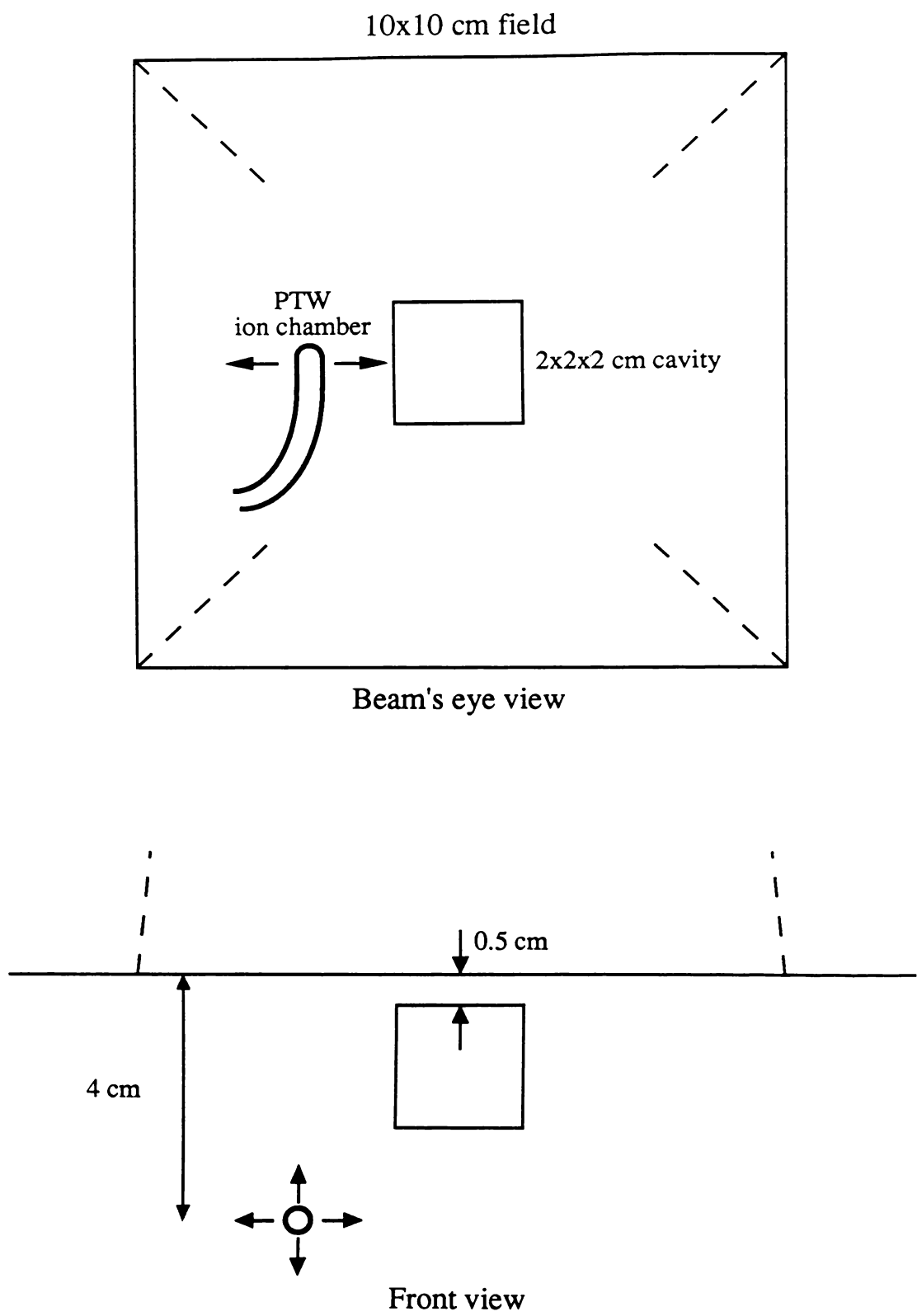


Figure 6.6. Cubic cavity (2 cm sides) in a water medium used to test the accuracy of the pencil beam method. Measured dose profiles and depth dose curves beneath the cavity are obtained with a waterproof PTW ionization chamber with cavity volume 0.1 cm^3 .

beneath the cavity.

Also shown in figure 6.7(a) is the curve calculated by the 2-D pencil beam method. A dose of 114% is predicted by the model at 4.0 cm deep. Note that 4.0 cm deep corresponds to the boundary between two voxel layers. Dose can be calculated on these boundaries as well as at the centre of the layers, if the increments in depth Δz used in calculating effective depth and standard deviation are set to half a voxel width (see sections 6.4.1 and 6.4.3).

The predicted dose enhancement beneath the cavity is due to a larger standard deviation σ_{MCS} being calculated for pencil beam strips outside the cavity boundaries than for pencil beams incident above the cavity. The dose at 4.0 cm deep predicted by the 2-D method is lower by 8% than that found by experiment. An underestimate in the dose is expected due to the assumption made by the 2-D method that the cavity extends infinitely in the y direction. This assumption means that the in-scattering to the region under the cavity is underestimated since the cavity is only considered to be bounded by water on two sides. Also note that the displacement of the calculated curve from the water depth dose curve is slightly greater than the displacement of the experimental curve.

In figure 6.7(b), the depth dose curve calculated by the 3-D method is shown. The relative dose at 4.0cm deep is now predicted to be 127%. The increase in dose predicted by the 3-D method over that predicted by the 2-D method, is because in-scattering from all 4 sides of the cavity is accounted for by the 3-D method. An important result is that the displacement of the calculated curve from the water depth dose curve is in much better agreement with experiment than when the 2-D method is used, meaning that the position of isodose lines will be more accurate.

Dose profiles were recorded at 4.0 cm deep by passing the detector across the beam in the x direction at the centre of the field in the y direction. These results are compared with the 2-D and 3-D pencil beam calculations in figures 6.8(a) and 6.8(b) respectively. The underestimate and overestimate in dose by the 2-D and 3-D methods respectively, directly beneath the cavity, is again illustrated. Curves calculated using both the 2-D and 3-D methods, are seen to closely match the experimental curve in the high gradient falloff regions near the edges of the cavity. In the region just outside the extent of the cavity, the 2-D calculation predicts a marked dip in the profile. This is due to electrons incident adjacent to the cavity being scattered into the region beneath the cavity and not being replaced by equal numbers of electrons which were incident directly above the cavity. Profiles obtained by experiment and using the 3-D calculation, do not show such a pronounced dip in this region. The reason for the exaggerated dip predicted by the 2-D method is that the electron fluence in this region does not have a contribution from the water near the back and front walls of the cavity.

An illustration of the dose distribution around the cavity as calculated using the 3-D method, is given by the isodose chart in figure 6.9. This was produced by exporting the

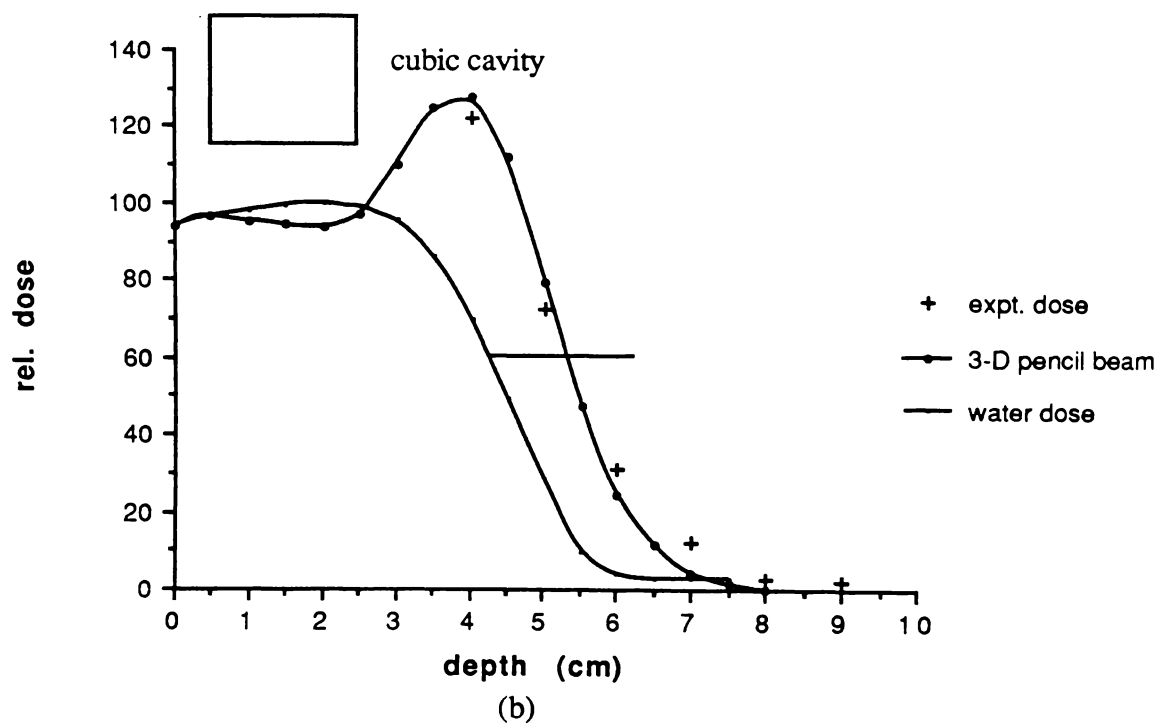
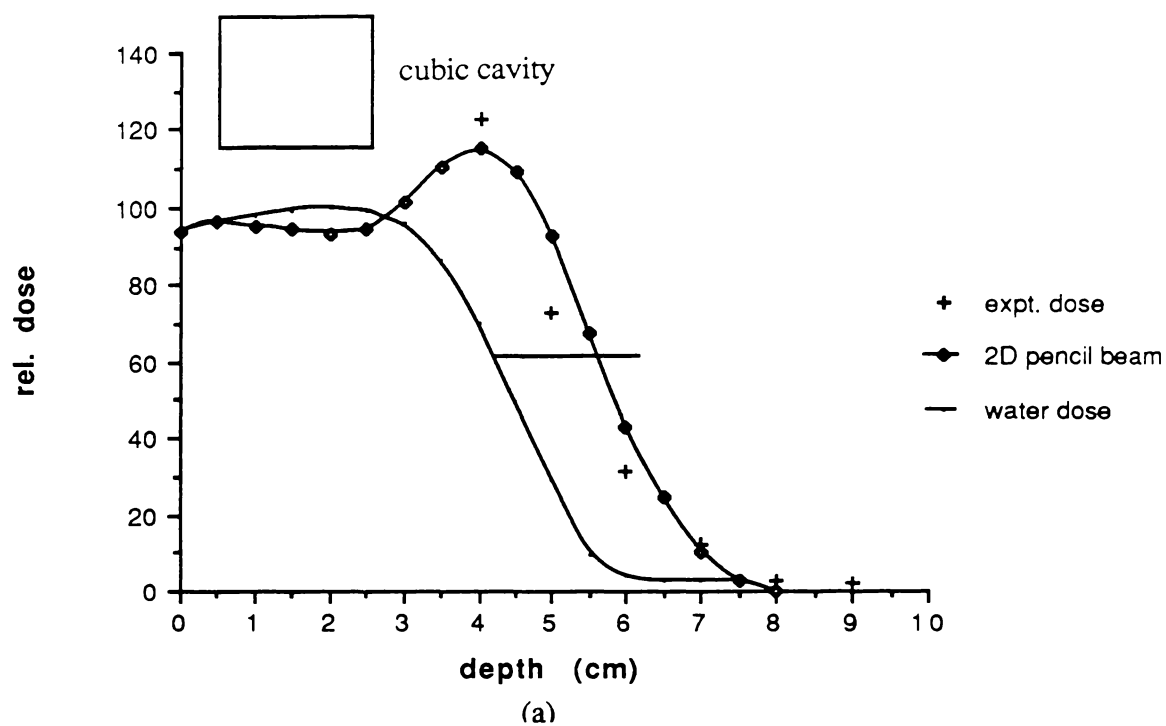


Figure 6.7. Comparison of measured depth dose with (a) 2-D pencil beam results, and (b) 3-D pencil beam results. Measured dose at 4.0 cm deep is 122%, compared with 114% and 127% predicted by the 2-D and 3-D methods respectively.

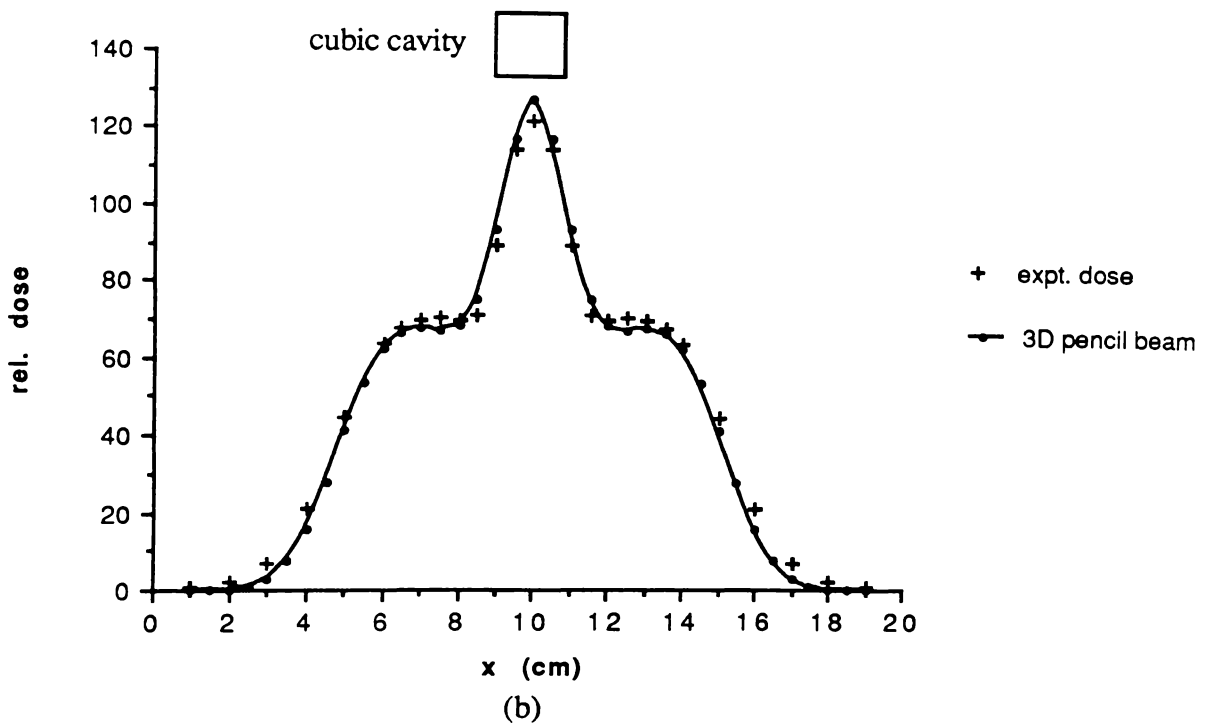
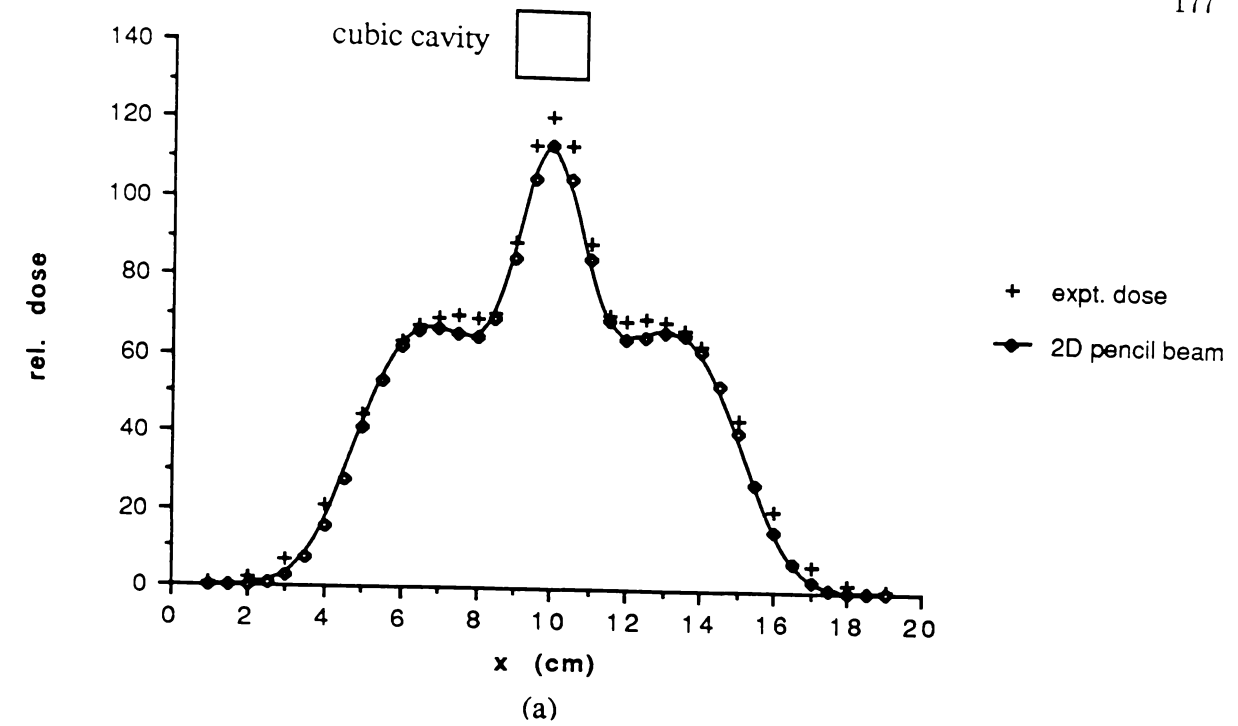


Figure 6.8. Profiles at a depth of 4.0 cm comparing the measured result with (a) 2-D pencil beam calculation, and (b) 3-D pencil beam calculation. In (a), the dose peak beneath the cavity is underestimated, and the dip in the profile due to disequilibrium just outside the lateral extent of the cavity is overestimated by the 2-D calculation. In (b), the dose peak under the cavity and the dose outside the lateral extent of the cavity are both in better agreement with experiment than is the case with the 2-D method.

data from a slice of the dose calculation array to a program written to generate isodose lines. In this case the dose was calculated at the centre of each voxel instead of at the voxel boundaries. The 127% dose at 4.0 cm deep beneath the cavity is within the innermost loop on the isodose plot.

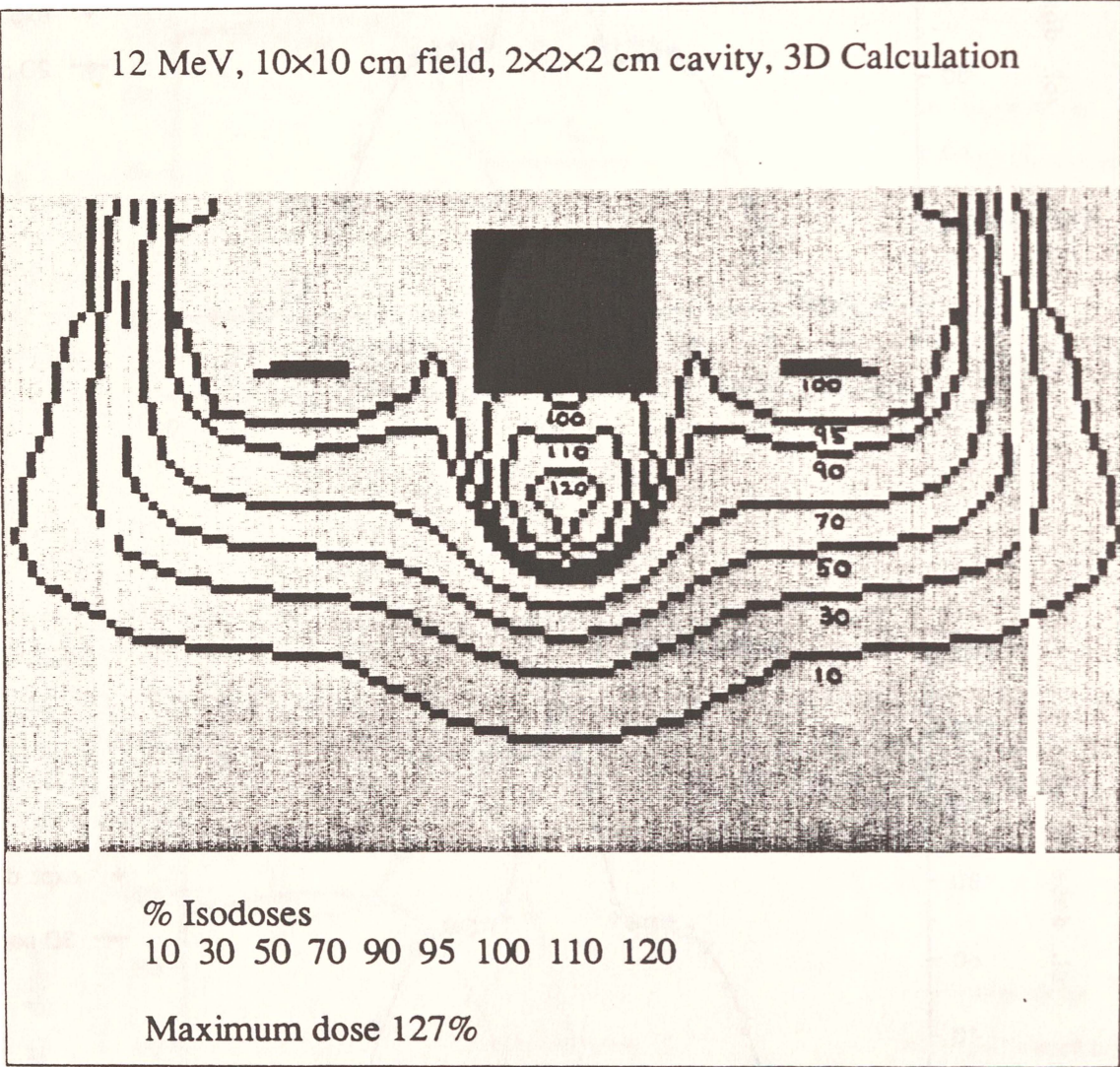


Figure 6.9. Isodose plot of the dose distribution in a plane at the centre of the cavity as predicted by the 3-D pencil beam calculation. The isodose plot is superimposed on a density distribution. The high dose (hot spot) beneath the cavity is within the innermost closed isodose line beneath the cavity.

The 3-D method has been shown to be superior to the 2-D method in calculating the dose distribution beneath the cubic cavity. However, the 2-D method should be accurate if the length of the cavity was greater than the beam width in the y direction. A polystyrene walled cavity with dimensions 2.0×2.0×12.0 cm was constructed and a profile recorded as for the cubic cavity. The comparison in figure 6.10 of the experimental profile with the

profile generated using the 2-D method shows good agreement. The measured dose at 4.0 cm deep is 111%, compared with 114% predicted by the model. Note that the dip in the experimental profile outside the edges of the cavity is now more pronounced than with the cubic cavity and agrees well with the 2-D method.

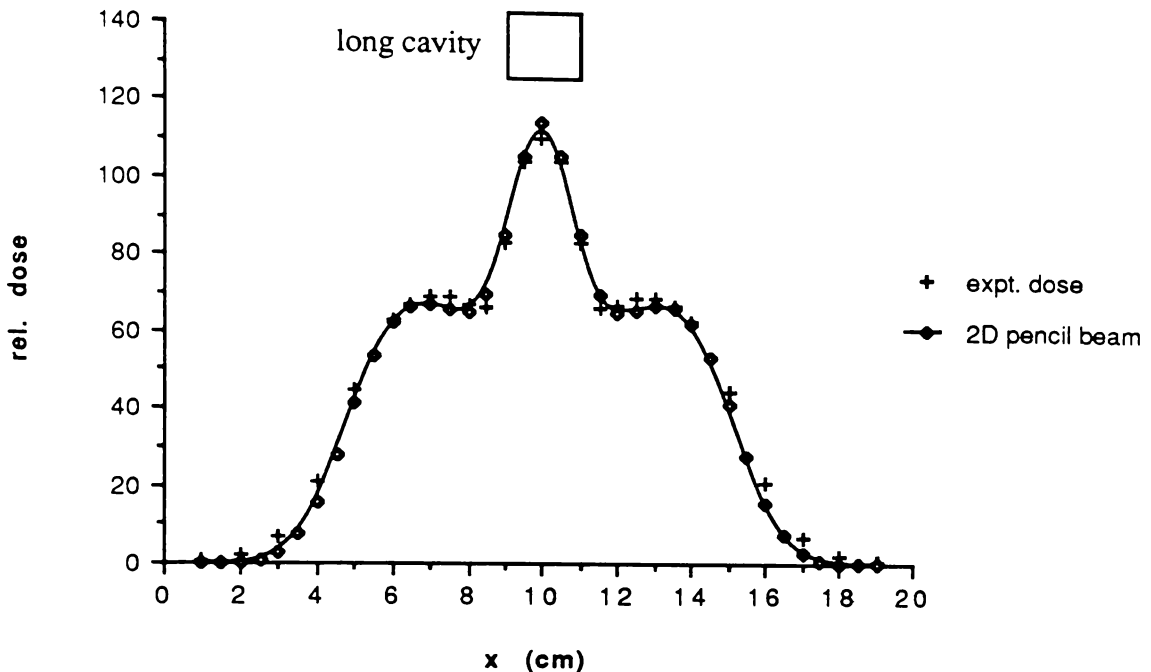


Figure 6.10. Profiles at 4.0 cm deep with a $2 \times 2 \times 12$ cm cavity (which extends across the width of the beam in the y direction), comparing the measured result with the 2-D pencil beam calculation. The measured dose at a depth of 4.0 cm beneath the cavity is now 111%, compared with 114% predicted by the 2-D pencil beam calculation. The dip in the profile predicted by the calculation is now seen in the measured result.

As a comparison with an existing commercial system, a dose profile beneath the cubic cavity at 4.0 cm deep was generated using the Theratronics Theraplan system installed at Waikato Hospital. The calculation algorithm which this system uses is a 2-D pencil beam method based on age diffusion theory. This system does not successfully calculate the dose profile as is seen in figure 6.11. The dose beneath the cavity is greatly underestimated, probably due to the failure of the method to accurately calculate the difference in spread of pencil beams which pass through the cavity and those which don't.

6.7. Summary and Future Work

In comparing the 2-D and 3-D pencil beam methods implemented here, it has been shown that the 3-D method is the more accurate in calculating the dose distribution beneath a small cavity. This is because the 3-D calculation accounts for the large increase in electron fluence beneath the cubic cavity.

Similar results to these were obtained by Starkschall *et al* (1991) who compared the results of 2-D and 3-D algorithms used to calculate the dose distribution in a unit density medium

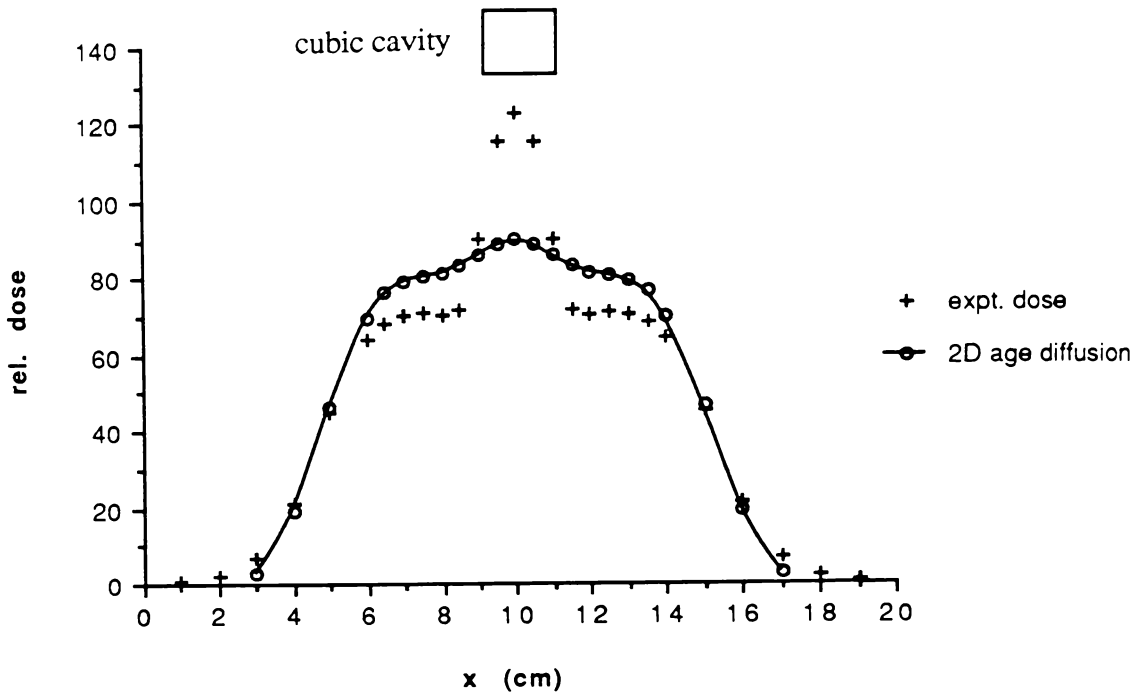


Figure 6.11. Profiles at a depth of 4.0 cm comparing the measured result with a pencil beam calculation which uses the age diffusion method. The age diffusion result was obtained from the Theratronics Theraplan system installed at Waikato Hospital. The calculated result does not agree well with the measured profile, probably due to an inaccurate calculation of the standard deviation (or the equivalent parameter $(k\tau)^{1/2}$).

containing a small 'pit'. The 3-D algorithm predicted a hot spot beneath the pit which was 20% higher than that predicted by the 2-D algorithm. Investigations have been carried out by Mah *et al* (1989), where the 2-D and 3-D Hogstrom algorithms were used to calculate the dose distribution in a water medium containing small pieces of bone and aluminium. The 3-D method was shown to give superior results when compared with experiment. In the case of inhomogeneities such as this, which are of higher density than water, there is a cold spot beneath the inhomogeneity.

The assumption made in calculating the dose contribution due to each pencil beam that inhomogeneities exist in layers has the potential to cause significant errors, particularly when inhomogeneities are long in the z direction but have a small cross section in the x - y plane (Brahme, 1985; Lax *et al*, 1986). For example, if a pencil beam is incident above a thin vertical air cavity, the depth factor and scatter factors are calculated as though there is a layer of air through which electrons in the beam travel. In fact, electrons are scattered by the air above the cavity into oblique directions so that they may not even enter the cavity, or if they do they will soon exit the cavity through the sides. A closely related problem can occur when electrons are incident adjacent to a cavity (of any size) and are scattered into the cavity. The unimpeded transport of the electrons through the cavity is not accounted for since the electrons are considered to be travelling entirely through water. Solutions to these problems have been proposed — in particular the *pencil beam redefinition algorithm*

of Shiu and Hogstrom (1990). With this new method, the pencil beams are propagated over small depth increments and redefined after each increment.

Another area where present pencil beam methods (2-D and 3-D) are not ideal is in predicting the dose distribution resulting from the irradiation of high atomic number materials such as bone. The mass scattering power of an element is proportional to $Z(Z + 1)/A$ where Z is the atomic number and A is the atomic mass (Klevenhagen, 1985). High atomic number materials therefore have a high mass scattering power. The mass stopping power on other hand, is approximately proportional to Z/A . This means that at a point in bone at depth z , which has an effective depth z_{eff} in water, the angular spread of the beam is larger than the angular spread at z_{eff} in water. Since a larger angular spread causes an increase in total electron fluence, the depth factor obtained from the water depth dose curve will be underestimated (Nahum and Brahme, 1985).

The algorithm described here is applicable to rectangular fields only. Since irregular fields can be defined by inserting a cutout in the electron applicator, an algorithm which copes with irregular fields would be useful. A 3-D algorithm which divides an arbitrarily shaped field into rectangles using quadtree subdivision has been developed by Kooy and Rashid (1989). Each rectangle is treated as a sub-beam in the pencil beam calculation.

Chapter 7

The 3-D Pencil Beam Algorithm in the GRATIS System

7.1. Introduction

A three dimensional Electron Pencil Beam algorithm has been incorporated into the GRATIS system. Although this system has been designed for photon beam planning, the user interface and planning procedure provide an adequate environment for basic electron planning. The pencil beam model implemented on the system is based on the 3-D algorithm discussed in chapter 6, although some technical innovations have been made to improve the compatibility of the algorithm with the GRATIS system. The theoretical aspects of the pencil beam algorithm are covered in chapter 6.

The structure of the software written and data used to implement the pencil beam algorithm has been kept as close as possible to that written for the superposition algorithm. All of the X-Windows user interface tools used in the photon beam planning system are left unchanged.

The `simulate` script which controls the virtual simulation process is used in the same form as in the photon beam case. The script `pextbm` is used to control the electron beam dose calculation process, in the same way as `sextbm` is used for the photon beam case. The script calls the pencil beam program `p_electron`, instead of `s_photon`, to calculate point doses and a dose grid. Apart from the calculation programs called, `pextbm` is identical to `sextbm`. As with the superposition program, up to 16 transputers may be used for the calculation intensive parts of the pencil beam program (Murray, 1991). Calculation using transputers is controlled by the script `transpbm`.

For a particular patient, the patient directory and its subdirectories, as well as the file names contained in these directories, are the same as those used for photon beam simulation. All source code and physical data which is specific to the pencil beam program, is stored in subdirectories of the `gratis/src/p_electron/` directory.

A brief description of the operation of the system, and the pencil beam algorithm is given in this chapter. Only those aspects of the system which are unique to the electron beam case are described in detail.

7.2. Virtual Simulation

As for photon beam planning, the virtual simulator is invoked by typing `simulate patient_dir`. The electron beam types listed in the treatment unit menu presently include

the Varian Clinac 18/10 – 6, 9, 12, 15, and 18 MeV beams. When `xvsim` is executed, the beams specified in the previous simulation are displayed — these may be electron or photon beams. The type of each existing beam is contained in the beam list which can be examined. Beams may be changed from one type to another or deleted as desired.

Since the simulator is designed for photon beams, the field dimensions are defined using the beam jaw settings. The rectangular field defined in this way simulates a field of the same dimensions defined by an electron applicator. Currently, circular or irregular fields defined by applicator inserts cannot be specified. The SSD and orientation of the beam is set in the same manner as for photon beams. The beam would normally be specified as being of SSD rather than SAD type, so that a point dose at d_{max} is calculated. For the Varian Clinac 18/10, the electron applicator physically imposes a minimum value on the SSD of 95 cm, hence the SSD should not be set to less than this value.

7.3. The Dose Calculation System

The script `pextbm` controls the rest of the planning process. First, a point dose list for each beam is obtained using `p_electron`. The dose at each of these points is then used by the spreadsheet program `tele_spread` to determine the treatment time for each beam, and to set the weighting factor for each beam to be used in summing the doses before displaying isodose lines.

The output dose grid is specified using `xplace_grid`, where the position and extent of the grid is decided using the displayed beam edges. This grid can have a smaller extent than is the case for photon beams, because of the restricted depth of penetration of the electron beam. Also, because the dose varies more rapidly than that due to a photon beam, the spacings of points in the grid should be smaller to allow a more detailed distribution to be displayed.

Once the dose grid has been specified, the dose at each grid point is calculated using `p_electron` for each beam. The grids for each beam are then summed using the weights determined in `telespread` to obtain the total dose at each grid point. An isodose distribution is then displayed using `xplandisp`. In figure 7.1, a dose distribution in the Rando phantom due to a 12 MeV electron beam is shown.

7.4. The Pencil Beam Program `p_electron`

Source files which are specific to the pencil beam program reside in the directory `gratis/src/p_electron/c/`. These files are compiled, then linked with each other and with required object files in the `gratis/src/s_photon/c/` directory and other directories. The executable code `p_electron` is copied to `GRATIS/bin/` where it resides along with other executable code. All code which is specifically concerned with the pencil beam algorithm is named using the prefix `p_`.

7.4.1. Input of Requested Data

The function `p_read_request()` is called to read the command line arguments. These

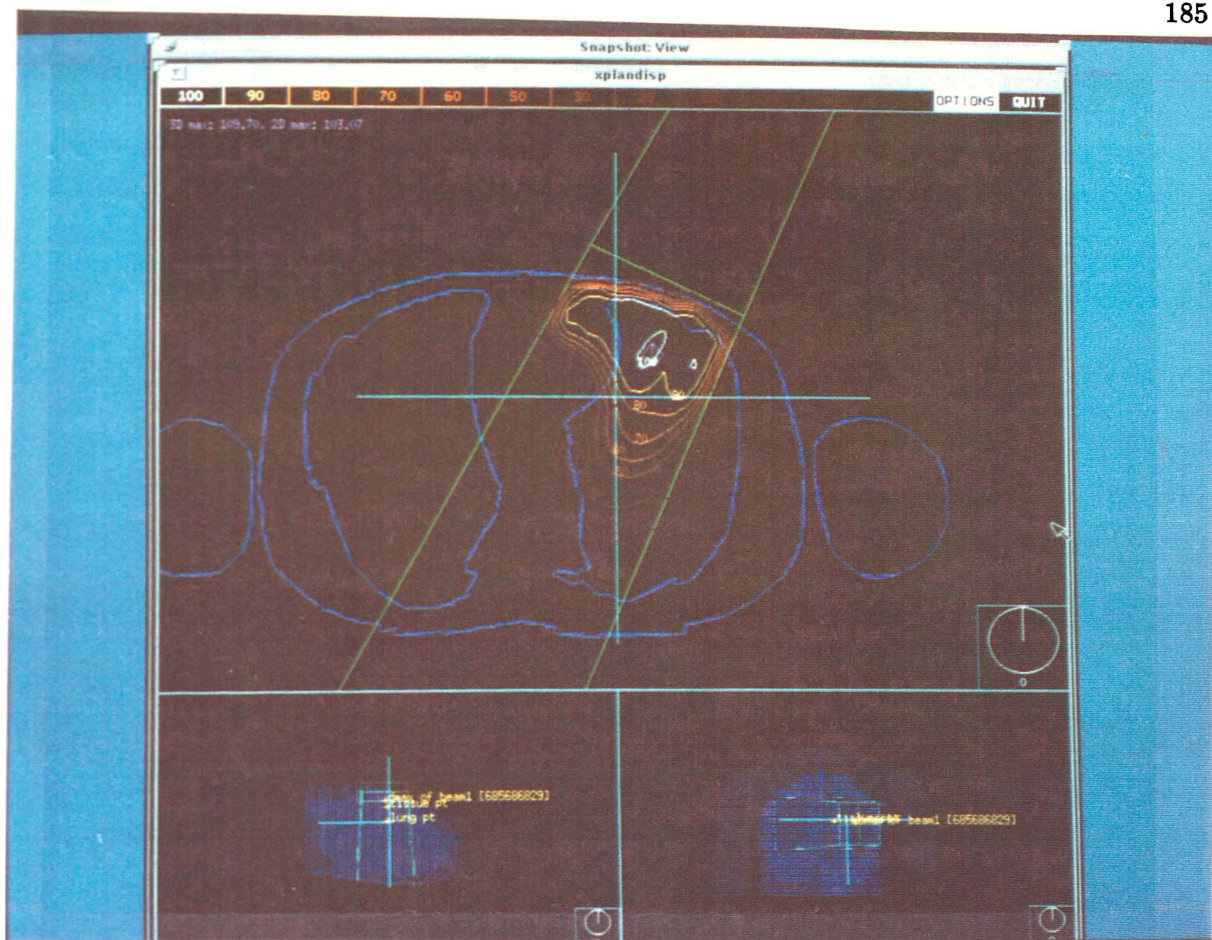


Figure 7.1 12 MeV electron beam dose distribution calculated using `p_electron` and displayed using `xplandisp`. Field size is 10×10 cm. Dose calculation points are spaced 0.5 cm apart in each direction.

arguments are the same as those passed to the superposition program and read by `s_read_request()` (see section 5.4.1), except that `-S` scaling and `-U` beam_unit_id are not required.

When the beam description file is read, the unit id. of the requested beam is checked to ensure that it is an electron beam — for photon beams the unit id. is less than 100, and for electron it is greater than 100. If a photon beam has been requested, the program exits with an appropriate message and continues with the next beam.

Procedures common to both `s_read_request()` and `p_read_request()` which are carried out include: defining the beam outline using the beam jaw settings; reading the dose rate file; reading the anastruct file; reading the dose grid description or point dose description file, and filling the density grid from the CT data file. Output factors are included in the dose rate data — these pertain to a range of square fields defined by different electron applicators. The CT image file is converted into a density grid with the same resolution as that used for the superposition calculation, using `s_plan_to_cgrid()`.

A table of error function data is required by `p_electron`. This table is produced by the function `p_create_erf_table()`, where values of $\text{erf}(x)$ are stored for $x = 0.00$ to 3.00 in

steps of 0.01.

The main block of data required for the pencil beam calculation is that which is read into the structure `elec_beam_pars` using `p_read_beam_data()`. This data structure contains

- (i) `elec_beam_pars.surface_energy` — mean surface energy of the beam.
- (ii) `elec_beam_pars.prac_range` — practical range of the beam in water.
- (iii) `elec_beam_pars.air_gap` — effective air gap between the electron applicator face and a plane at the SAD.
- (iv) `elec_beam_pars.sig_theta` — angular standard deviation of the beam at the electron applicator face.
- (v) `elec_beam_pars.max_sigma` — maximum lateral standard deviation of the point beams.
- (vi) `elec_beam_pars.surface_brem` — surface dose due to bremsstrahlung at 100 cm SSD as a fraction of the total dose at d_{max} .
- (v) `elec_beam_pars.SSD` — SSD at which the measured depth dose data was obtained.
- vi) `elec_beam_pars.vsp` — effective virtual source with respect to the nominal position.
- (vii) `elec_beam_pars.depth_incs` — depth increment between each depth dose value.
- (viii) `elec_beam_pars.n_depths` — number of depth dose values.
- (ix) `elec_beam_pars.depth_dose[]` — an array of depth dose values normalised to 1.0 at d_{max} , measured at 100 cm SSD.

7.4.2. The Reference Dose Rate

The calibrated dose rate contained in `request.beam.time_calc`, is the dose rate in cGy MU^{-1} for a 10×10 cm field, at d_{max} in water at an SSD of 100 cm. For example, the dose rate for the Clinac 18/10 12 MeV beam is 0.99 cGy MU^{-1} at a depth of 2.0 cm in water. Output factors are stored for square field sizes of 6, 10, 15, 20, and 25 cm. For the 12 MeV beam, the output factors for these field sizes are 0.96, 1.00, 1.00, 1.03 and 1.01 respectively. The equivalent square field corresponding to the current rectangular field is found by equating the beam perimeter to area ratios, as in `s_photon`. The reference dose rate is then found by multiplying the calibrated dose rate for the 10×10 cm field by the output factor for the current equivalent square field as in `s_photon`. This product is stored as a global multiplication factor which is used to convert the relative dose calculated using the pencil beam algorithm to an absolute dose.

7.4.3. The Electron Density Grid

An array of electron densities is created from the density grid using coordinate transformations, in the same way as in the superposition program (see section 5.4.4). The electron

density array voxels are cubic with side lengths which are set to 0.5 cm if a point dose list is required, or to the cube root of the output dose grid spacings if a dose grid is required. As with the EDK voxel dimensions in `s_photon` (see section 5.4.2.3) the cube root of the output dose grid spacings is used for the electron density array voxels so that the user has some control over the speed of the algorithm.

7.4.4. The Inside_contour Grid

As in `s_photon`, an `inside_contour` grid with the same dimensions as the electron density grid is created. This is a character array which signifies whether each voxel in the electron density grid is inside or outside the patient skin anastruct. The array is created using the contours stored in `request.patient_anastruct` and the function `s_inout()`, as explained in section 5.4.5.

If a voxel is inside the `anastruct`, the `inside_contour` value is set to 1, otherwise it is set to 0. Once this array has been created, those voxels in the electron density array whose corresponding voxel in the `inside_contour` array contains a 0, have their electron density value set to zero. This ensures that the effective depth array which is to be created next, does not take the electron density of the CT couch or any other external structures into account. The `inside_contour` array is also used as a flag to indicate whether to assign dose to a particular voxel in the dose array.

7.4.5. The Effective Depth and Standard Deviation Grids

In the majority of pencil beam algorithms, the effective depth z_{eff} and standard deviation σ_{MCS} are calculated for points on ‘fan’ lines diverging from the virtual source (Hogstrom *et al*, 1981). In `p_electron`, both quantities are stored in voxel arrays with the same structure as the electron density array. The value in each voxel represents the respective quantities at the centre of that voxel. Note that in the 3-D algorithm described in chapter 6, the standard deviation was calculated at the time of the actual dose calculation, whereas in `p_electron`, it is calculated and stored in an array before the dose calculation begins.

For electron beams, the increment in effective depth with an increment in path length is usually calculated using the ratio of the total stopping power of the medium at the current position to the total stopping power in water, where this ratio is taken to be energy independent (Hogstrom *et al*, 1981). As discussed in section 6.4.1, the stopping power ratio is closely approximated by the electron density relative to water. Therefore, the effective depth is calculated using the electron density array with the same ray-tracing method used in calculating the radiological depth in the superposition program (see section 5.4.6). This method of calculating effective depth is slightly more rigorous than that described in section 6.4.1, since the divergence of the beam, and the consequent obliqueness of the rays with respect to the electron density array voxels is now accounted for.

The standard deviation is calculated using a ray-tracing method adapted from that used to calculate the effective depth. As with the effective depth calculation, beam divergence is

now accounted for. In practice, the effective depth and standard deviation are calculated simultaneously, using the same function `p_ray_sdev()`. This procedure is now described.

7.4.5.1. Calculating Effective Depth and Standard Deviation

The function `p_ray_sdev()` used to calculate both the effective depth and standard deviation is largely the same as the function `s_raytrace()` used in the superposition program (see section 5.4.6), except that when effective depth is assigned to a voxel, the standard deviation is assigned also. To reduce computation time, the range of voxels in the electron density array at which effective depth and standard deviation is calculated is restricted to include only those which are within a distance of 3 cm outside the beam outline. The minimum and maximum i and j indices which define this range are referred to as the i, j limits. Prior to calculating the effective depth and standard deviation, each element in the effective depth array is assigned the value -1.0. This is used to indicate whether the effective depth and standard deviation has have been calculated for a voxel with particular indices. Ray lines are constructed for each ‘target’ voxel in the manner described in section 5.4.6 for `s_raytrace()`. The method used to assign effective depth and standard deviation to voxels which lie on a ray-line originating at the virtual source position and passing through a target voxel is as follows.

Before the ray is traced, the electron energy is set to the mean energy at the surface, `elec_beam_pars.surface_energy`, and the scattering moments A_0 , A_1 , and A_2 are all set to zero (see section 6.4.3). When the plane-crossing values for the ray-line have been calculated (see section 5.4.6), these values are stepped through in ascending numerical order. Between each successive crossing value is a line segment contained within a particular voxel in the electron density array. The indices of that voxel, increment in effective depth from the beginning to the midpoint of the line segment, and the total effective depth to the midpoint of the line segment are determined. The effective depth at the midpoint is assigned to the corresponding voxel in the effective depth array if the line segment is longer than the voxel width, as in `s_raytrace()`.

Next, the standard deviation at the midpoint of the line segment is determined. This is calculated using recursion relations based on those in section 6.4.3, where the increment Δz is half the length of the current line segment. In determining the mass scattering power, the medium is considered to be bone if the electron density is above 1.10, otherwise it is considered to be soft tissue. The mass density is obtained from the electron density by multiplying by 1.02 for bone and unity for soft tissue (see section 6.4.3). The standard deviation at the midpoint of the line segment, σ_{MCS} , is then found from the value of A_2 at that point ($\sigma_{MCS} = \sqrt{A_2}$). The value of A_2 at the midpoint is only calculated if $\sqrt{A_2}$ at the beginning of the segment is less than `elec_beam_pars.max_sigma`, otherwise A_2 is left at its value at the beginning of the segment. As with the effective depth, the standard deviation at the midpoint of the line segment is only assigned to the corresponding voxel in the standard deviation array, if the line segment is longer than the voxel width. The

electron energy at the midpoint of the line segment is determined using the effective depth at that point, surface energy and practical range as in section 6.4.2. This is then used to calculate the scattering moments at the end of the line segment, so that the standard deviation at the midpoint of the next segment can be obtained. This process is depicted in figure 7.2, for a ray-line targeting a voxel on the top layer of the electron density array. The effective depth z_{eff} , and the standard deviation σ_{MCS} at a point $p2$ are assigned to the elements in the effective depth and standard deviation arrays which correspond to the voxel containing $p2$. As indicated, the values of both these quantities are calculated recursively, using the calculated values at the previous crossing point on the ray-line ($p1$). Similarly, the values at $p3$ are calculated using the values at $p2$.

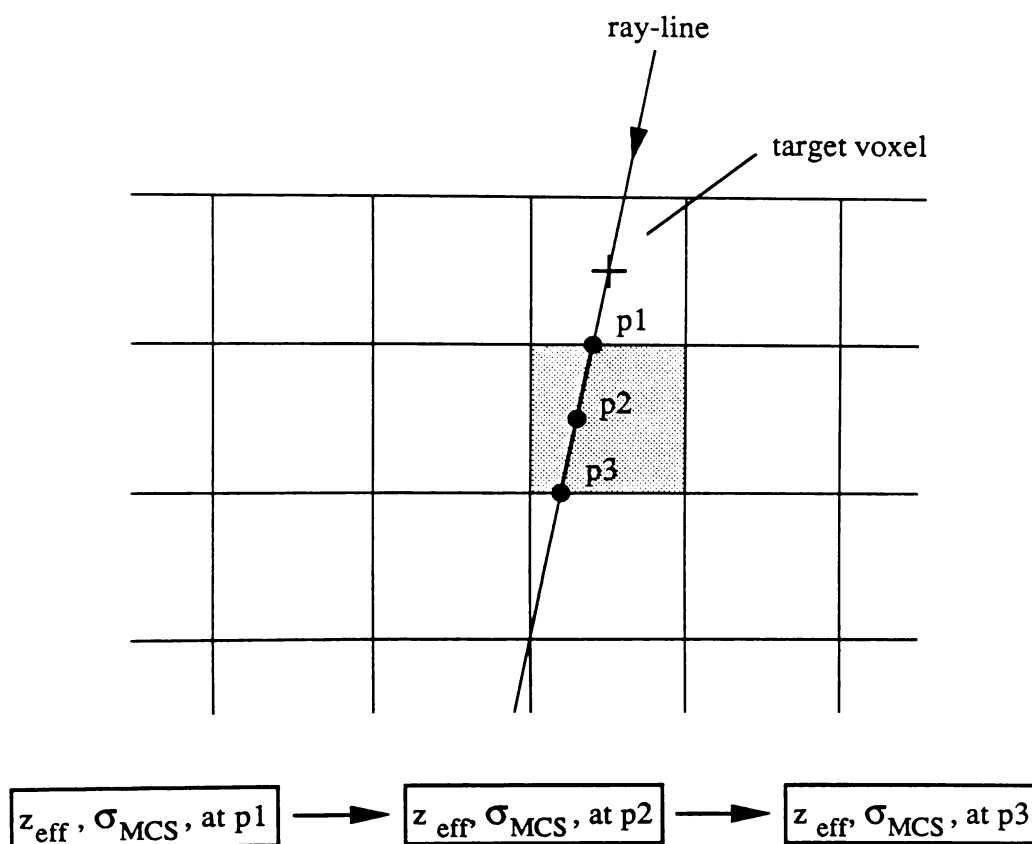


Figure 7.2 Process of calculating effective depth and standard deviation using `p_ray_sdev()`. Shown is a ray-line targeting a voxel in the top layer of the electron density array. The values of z_{eff} and σ_{MCS} at the midpoint $p2$ of the line segment through the shaded voxel, are calculated recursively using the values at $p1$.

When all crossing points on a ray line have been stepped through, effective depth and standard deviation have been assigned to the target voxel for that ray, and to some other voxels intersected by the ray. Voxels which have not been assigned these quantities, are indicated by the value -1.0 in the effective depth array — these voxels are targeted in subsequent calls to `p_ray_sdev()`. When this process is complete, effective depth and standard deviation arrays exist, which have a large enough lateral extent to encompass the

beam. Note that since the electron density array values have been set to zero for voxels which lie outside the patient, both effective depth and standard deviation will be zero in these voxels. To avoid a divide-by-zero error when the program is run (see section 6.4.3), a standard deviation of zero is set to a very small value (0.001). Note that the effective depth values continue to increase with depth in the array, whereas the standard deviation values do not exceed the maximum associated with the current beam, (`elec_beam_pars.max_sigma`).

7.4.6. Creating the Output Dose Grid

If `request.have_grid` is true, a beam dose grid is created. This structure contains a voxel array with the same dimensions as the electron density, `inside_contour`, effective depth, and standard deviation arrays. The dose at each voxel in the beam dose array is calculated by summing the contributions from each of the set of pencil beams which make up the broad beam.

There is conceptually one pencil beam directed along each column of voxels in the electron density array with i, j indices within the i, j limits. Each pencil beam has dimensions defined by the voxel width — in this case the beams are of square cross section, since the voxels in the electron density array are cubic. The values in a column of voxels in the effective depth and standard deviation arrays with indices i, j , are used to calculate the dose contribution due to the pencil beam defined by that column. Note however, that although the pencil beams are parallel and incident normally on the electron density array from the point of view of assigning dose due to each beam, the standard deviation and effective depth associated with a pencil beam at a particular depth are calculated as though the pencil beam had diverged from the virtual source. This is the major difference from the 3-D method described in chapter 6, where these quantities are calculated as though the beams are parallel. The effect of this difference should be minimal as beam divergence only causes a slight angulation in the pencil beam axes. For example, if the axis of a pencil beam is located 5 cm from the beam central axis at an SSD of 100 cm, then the axis of that pencil beam will intersect a plane at 5 cm deep, at an off-axis distance of approximately 5.25 cm. In particular, the results presented in chapter 6 would not be altered by using the ray tracing method described here, since the off-axis distances of pencil beams contributing to the dose beneath the cavity are very small: ray-lines targeting a voxel near the axis in the first layer would remain within that voxel column throughout all the layers in the array.

The contribution to elements in the beam dose array due to a particular pencil beam is calculated using `p_calc_dose()`. In this function, layers of the beam dose array are stepped through and the dose contribution to that layer due to the current pencil beam is calculated. First, the x and y beam drift factors at the current layer are determined using the field limits, and σ_{air} (the lateral standard deviation due to initial angular spread at the applicator face), as in section 6.4.4. The field limits are found from the beam coordinates of the current layer and of the virtual source position, and from the field limits at the SAD. σ_{air} is found using the angular standard deviation of electrons in the beam at the applicator

face, and the distance between the applicator face and the centre of the current layer as explained in section 6.4.4.

The depth dose value is obtained from the depth dose table `elec_beam_pars.depth_dose[]` using `p_central_axis_dose()`, where the depth at which the value is obtained from the table is the effective depth at the voxel where the axis of the current pencil beam intersects the current layer. This voxel is referred to as the *current voxel*. An inverse square factor is then applied to this value to account for the difference in the distance from the virtual source of the point at which the depth dose value was measured, and the distance from the virtual source of the current voxel (see section 6.4.1).

From the standard deviation grid, the standard deviation σ_{MCS} at the current voxel is obtained. The dose contribution on the current layer from the current pencil beam is calculated at ‘deposition’ voxels which are within a distance of approximately $4\sigma_{MCS}$ from the current voxel. The scatter factors and the dose contribution are then calculated for each voxel on the current layer which is within this range as described in section 6.4.5.

A pencil beam incident on a column of voxels in the dose array is shown in figure 7.3. Dose contributions are made to deposition voxels within the range indicated by the light shading. The current voxel – at which the depth dose, beam drift, and σ_{MCS} values are calculated – is indicated by the darkly shaded voxel.

When the contribution to the beam dose array due to each pencil beam has been calculated, the values in the array are converted to an absolute dose using the global multiplication factor (the product of calibrated dose rate and output factor). If a voxel is outside the patient anastruct, the dose in that voxel is set to zero. The output dose grid is then filled using the beam dose array by the method described in section 5.4.9.2.

7.4.7. Calculating Point Doses

If `request.have_points` is true, the dose at each point in the point dose list is calculated. The first step in calculating the dose to a point is to convert the patient coordinates of the point to beam coordinates. The voxel in the electron density array which contains the point is then located. After ensuring that this calculation voxel is inside the patient skin anastruct, the dose within it is calculated using `p_calc_points()`. In this function, the dose due to each pencil beam within the i, j limits is summed.

In calculating the contribution from a pencil beam, the standard deviation σ_{MCS} of the pencil beam on the layer containing the calculation voxel is obtained from the standard deviation array. If the lateral distance between that pencil beam and the calculation voxel is less than $4\sigma_{MCS}$, the dose contribution from that pencil beam is calculated. This involves determining the beam drift factors, the depth factor, and the scatter factors.

In figure 7.4, the calculation voxel is indicated by the lightly shaded region. The dose contribution to this voxel due to pencil beams in columns within a range of $4\sigma_{MCS}$ are

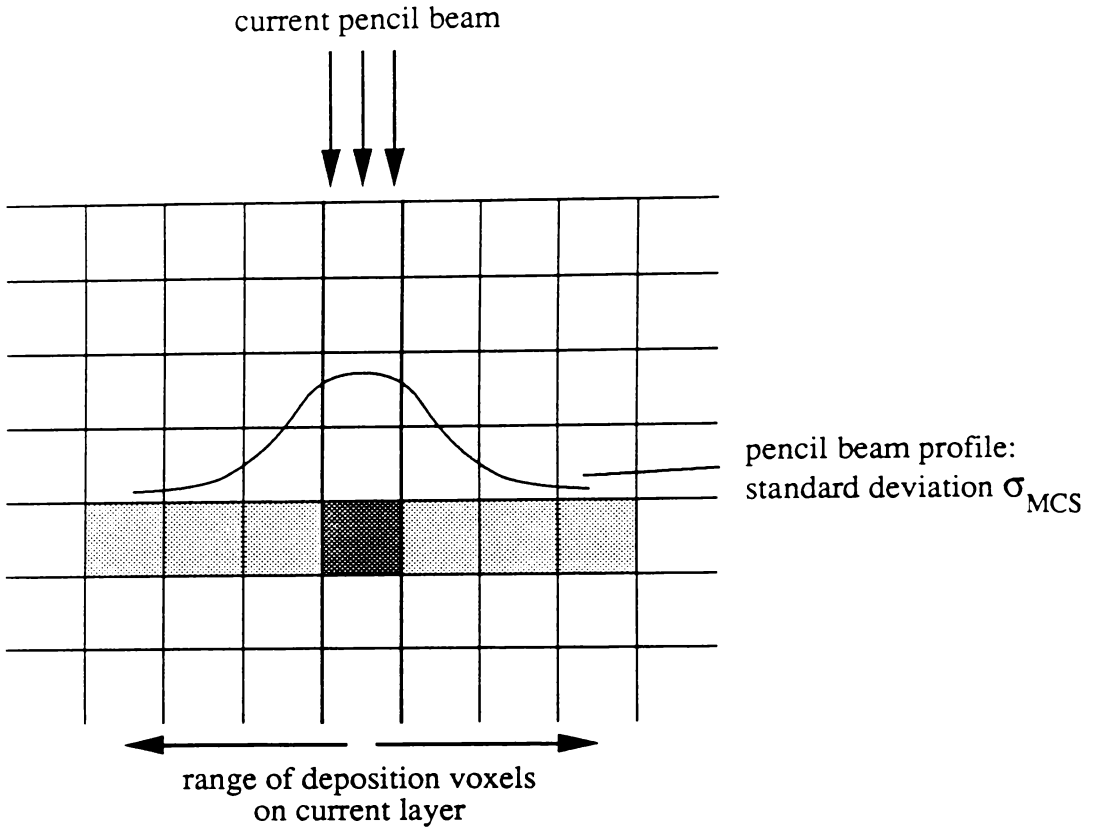


Figure 7.3 In `p_calc_dose()`, the dose contribution to voxels on successive layers of the beam dose array are calculated due to each pencil beam. Shown is a pencil beam with standard deviation σ_{MCS} at the layer indicated by the shaded voxels. The darkly shaded voxel is the current voxel at which the beam drift factor, depth dose factor, and standard deviation are calculated. The lightly shaded voxels are deposition voxels at which the dose contribution due to the pencil beam is calculated.

calculated. A pencil beam within this range is indicated. The depth dose, beam drift, and σ_{MCS} values used, pertain to the darkly shaded voxel.

The dose value returned by `p_calc_points()` is converted to absolute dose using the global multiplication factor. This value is then assigned to the appropriate element of the dose array in the point dose structure `p.points`.

7.5. Future Improvements and Projects

In order to make this a more useful electron beam planning system, the user interface should be modified to allow certain electron beam specific parameters to be specified. For example, the virtual simulator does not allow the specification of irregular fields defined by cutouts in the electron applicator. Also, the position of the electron applicator face is not indicated in `xvsim` — this should be shown, or at least the movement of the treatment couch should be restricted to the physical limits imposed by the applicator.

This algorithm assumes that the incident electron fluence is uniform across the beam. Measured dose profile data could be used to weight the incident pencil beams. In some

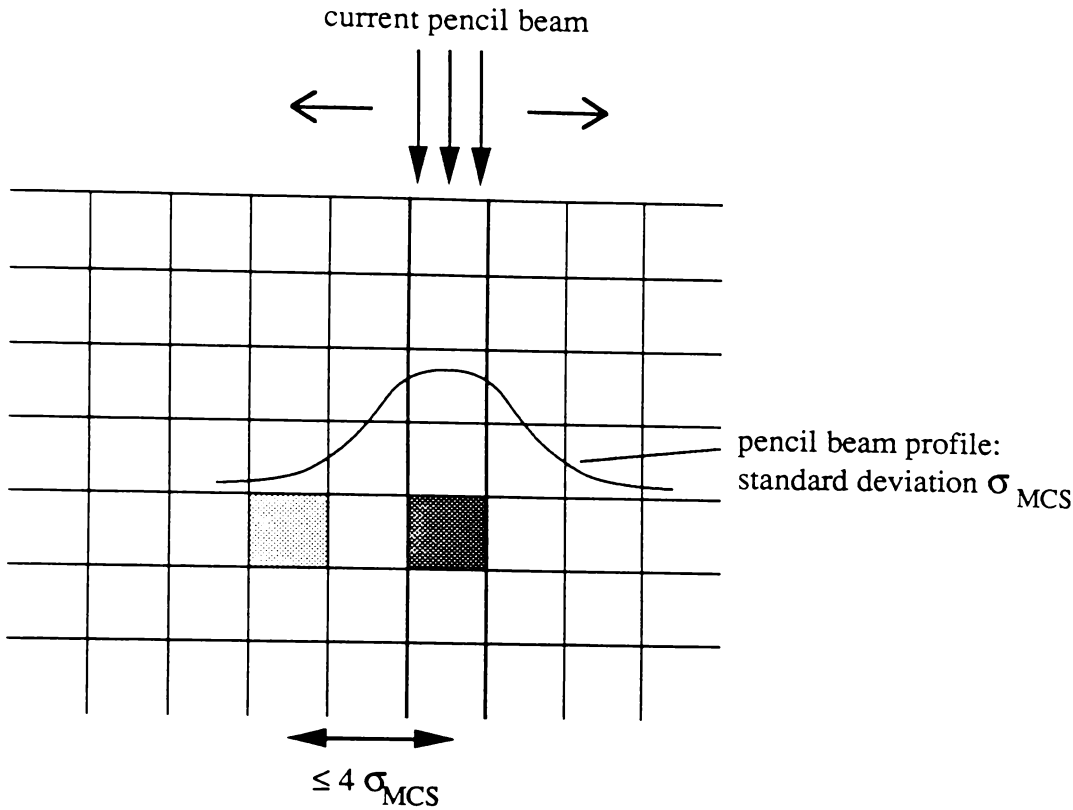


Figure 7.4 In `p_calc_points()` the dose contribution to a calculation voxel, indicated by the lightly shaded voxel, is determined for each pencil beam. Shown is a pencil beam with standard deviation σ_{MCS} at the layer containing the calculation voxel. The beam drift factor, depth dose factor, and standard deviation are calculated for the darkly shaded voxel.

implementations of the Hogstrom algorithm, correction factors obtained using measured dose profiles are used to improve the agreement between calculated and measured dose at large off-axis distances. (Hogstrom, 1985). Results obtained using the algorithm described here should be compared with measured dose profile data, and if necessary, a correction to the calculated dose should be included.

As with the photon beam planning system, a simple software tool to construct an inhomogeneous medium in which to perform dose calculations would be useful. Along with this, it would be useful to be able to specify vertical and horizontal lines in the medium along which depth dose or dose profile data is obtained — this data could then be compared with experimental results from an equivalent phantom.

Chapter 8

Discussion and Future Developments

8.1. Small Circular Photon Beams

The incidence of lateral electron disequilibrium for small, circular, 10 MV photon beams is investigated using the EGS4 Monte Carlo system in chapter 2. Depth dose curves generated using the reciprocity technique indicate that disequilibrium occurs for beam diameters of less than approximately 4 cm. For a beam diameter of 0.5 cm, the central axis primary dose is approximately 0.6 times that which exists under equilibrium conditions. A slight modification to the reciprocity technique is proposed which accounts for the variation in lateral electron disequilibrium with depth which occurs for divergent beams. Results produced using this technique agree closely with experimental results obtained from radiosurgery beams.

Polyenergetic primary and scattered energy deposition kernels (EDKs) are used to calculate central axis dose using the convolution method in chapter 3. The inaccuracy of this method due to the neglect of the effects of beam hardening on the EDKs is investigated. When EDKs generated using the primary photon spectrum which exists at the medium surface are used, the primary dose is increasingly underestimated with depth: this is due to the increase in primary photon energy transferred to secondary electrons as the beam hardens. There is a corresponding overestimate in the scattered dose when surface spectrum EDKs are used. Correction factors which modify the EDK values to account for beam hardening with depth are proposed. Convolution depth dose curves are in good agreement with Monte Carlo results when these correction factors are used.

8.2. The Superposition Algorithm

In situations of lateral electron disequilibrium, the dose in a low density medium such as lung can be overestimated by scatter function models such as ETAR. As discussed in chapter 4, this is due to the use of zero-area tissue-air ratios in determining the primary dose component — a procedure which is only valid in situations of electronic equilibrium.

The superposition algorithm does not assume the existence of electronic equilibrium, hence, in disequilibrium situations, the dose is more accurately calculated by superposition than by ETAR. The dose in lung for a 5×5 cm, 10 MV beam is slightly overestimated by the superposition method when the EDK values are found using a standard density scaling technique. In particular, the dose in the lung due to primary photon interactions in the overlying water medium is overestimated. This is due to the assumption that secondary

electrons spread out in straight lines from the primary interaction site: an assumption that is not strictly valid because of electron scattering. An increase in the weighting of the interaction voxel density when determining the average density between the interaction and deposition voxels is found to improve results.

The superposition algorithm has been incorporated into the GRATIS radiotherapy treatment planning system, as described in chapter 5. This is a Unix-based system running on a Sun workstation, where the user interface is provided by a number of X-windows tools. The computationally intensive part of the superposition algorithm is performed by a network of T800 transputers. The scaling of EDKs to account for inhomogeneities can be performed using either a 'vertical' ray-trace between the interaction voxel and each layer of deposition voxels (z-scaling), or a ray-trace between the interaction voxel and each deposition voxel (full ray-trace). The latter is more general but requires far more computation. Further work is required to determine whether the increased accuracy provided by the full ray-trace method is worth the large increase in computing time. The full ray-trace method could be optimised by calculating the average density recursively at points along ray-lines emanating from the centre of the interaction voxel. This would require considerable alteration of the existing code, but such a modification is warranted if the full ray-trace method proves superior to the z-scaling method. Hopefully, the T800 transputers currently in use in the system will in future be replaced by more powerful T9000 transputers — this will allow more sophisticated scaling algorithms to be implemented while retaining practical computation times. In order to test the accuracy of the dose calculation algorithms, a software tool to produce 3-D electron density arrays which replicate physical test phantoms would be useful. The results of the superposition algorithm could then be easily compared with measured results. Along with this tool on the GRATIS system, a similar tool to produce electron density arrays for EGS4 Monte Carlo simulations on the VAX system would be useful — this could be used to provide further verification of the superposition algorithm.

8.3. The Electron Pencil Beam Algorithm

The principles of 2-D and 3-D electron pencil beam algorithms, which use the Fermi-Eyges multiple scattering theory, are discussed in chapter 6. The 3-D method is shown to be superior to the 2-D method in calculating to dose in a water medium containing a small cubic air cavity. In particular, the enhancement in dose (hot spot) beneath the cavity is underestimated by the 2-D method but is predicted more accurately by the 3-D method. The increased accuracy of the 3-D method is due to the cavity being treated as being cubic rather than being treated as extending across the whole width of the field in the direction perpendicular to the calculation plane. Both methods predict the dose enhancement more accurately than the 2-D age diffusion method used by the Theratronics Theraplan system.

The 3-D pencil beam algorithm has been incorporated into the GRATIS system, as described in chapter 7. Calculations can currently be performed for the 6, 9, 12, 15, and 18 MeV beams produced by the Varian Clinac 18/10. The X-windows user interface is

currently identical to that used for the photon beam system. Presently, rectangular fields can be defined using the jaw settings in the same way as for photon beams. Further work could be carried out to allow the specification of different electron applicators, as well as applicator inserts to define circular or irregular fields. For fields other than those of rectangular shape, a more sophisticated method of defining the pencil beams such as a quadtree subdivision would be required. As with the photon system, a tool for defining a 3-D electron density array would be useful for the comparison of results with experiment.

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