UNDERSTANDING DECIMAL NUMBERS:
AN INVESTIGATION OF STUDENT MEANING CONSTRUCTIONS
IN INTERMEDIATE AND HIGH SCHOOL

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ABSTRACT

This study grew out of informal observations of the poor achievement of 13 year old New Zealand students with decimal numbers. These observations were confirmed by the results of large-scale quantitative tests with 11 to 14 year olds both in New Zealand and overseas.

The research design combined quantitative and qualitative approaches. Initially a decimal numbers survey was administered to a sample of 11 to 14 year olds (n = 102). Next a series of stimulus cards was constructed based on the survey results and the researcher's observations. These cards formed the basis of interview protocols for a longitudinal study of 11 to 14 year olds (n = 28) over two years. Additionally, the individual interviews investigated the students' views of learning mathematics and mathematics teaching, including their affective responses to decimal numbers and mathematics.

The research was influenced by the constructivist view of learning in which the emphasis is on the individual actively constructing his/her own interpretations from incoming stimuli. In particular, use was made of the generative learning model to account for students' constructions of meaning. Students employed these constructions of meaning, or mini-theories, to describe and explain their ideas when solving problems with decimal numbers.

The findings gave further support to a constructivist view of learning mathematics, and pointed to the strong influence of affective variables in mathematics learning. It was considered that the generative learning model, with its strong cognitive emphasis, may be criticized for its inattention to attitudinal factors. As well, (and contrary to generative learning model postulates) the research suggested that learning was often of a fragmented and situation-specific nature and that students held incorrect mini-theories without testing them against a number of existing ideas.

The implications of the findings for teaching and learning, the curriculum, assessment procedures, and in-service and pre-service teacher education are discussed in the final chapter.

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CHAPTER ONE

THE RESEARCH PROBLEM AND RELATED BACKGROUND

The present study arose from informal observations of the poor achievement of New Zealand children with decimal numbers. These observations were reinforced by results of the 1981 International Educational Achievement Survey (Department of Education, 1982). The survey revealed that a large Third Form sample (n = 5177) of 13 year olds had an average facility level of only 39 per cent for test items involving computation with decimal numbers (see Appendix A).

This finding seemed to contradict the optimism of mathematics curriculum developers that the introduction in 1974 of metric measurement into New Zealand would make learning about decimal numbers an easier task (N.Z. Department of Education, 1974 a:4). Moreover, the survey results could be seen as supporting the argument of the Back to Basics movement that, on average, New Zealand school children lacked facility with the fundamentals of mathematics; further, that the introduction of 'new maths' curricula since the late 1960s had failed to serve the learning needs of many members of the school populace. The fact that surveys had uncovered similar low achievement patterns in other western countries (e.g. the United States - see Carpenter *et al*, 1984:488), reinforced the present investigator's concern.

One way of investigating this situation would be to conduct further cross-sectional surveys of achievement, to establish correlational patterns between mathematics achievement levels and a range of teacher, student, and teaching method variables, and then set up an experimental study aimed at improving knowledge, understanding and skill with decimal numbers. Rather than employing these quantitative research methods, however, the present study focussed on the observation of individual children and explored their ideas and beliefs through the use of the interview procedure.

This orientation was influenced by two relatively recent developments in educational research. The first is the constructivist view of children's learning in which the emphasis is on the individual actively constructing his/her own interpretations of information from incoming stimuli. The second is the move away from quantitative research methodologies to observational research using in-depth studies of relatively small sample.

Constructivist psychology has its roots in the work of Jean Piaget, but in recent years it has been restated and refined in theories such as the generative learning model. The essence of the generative learning model is that the learner does not passively accumulate information; rather, s/he selects and attends to the information received and actively constructs her/his own meanings from this in-coming data. Generative learning theory emphasizes the importance of generating links to relevant aspects of information held in memory (Osborne, 1985; Osborne and Wittrock, 1983, 1985; Wittrock, 1974a, 1980). The links that are generated by the learner are critical for the meanings that are constructed.

Research in science education has shown that learners frequently generate meanings that are different from those hoped for by the teacher, and by the writers of curricula and textbooks (Bell, 1981; Driver, 1984; Gunstone and White, 1981; Osborne, 1985; Osborne and Wittrock, 1983, 1985; Tasker, 1980, 1981). Another finding has been that where children's ideas are changed by science teaching, these changes are often quite different from those intended (Osborne, 1981; Osborne and Cosgrove, 1983). It is significant that the above studies have adopted naturalistic research methods, participant observation, interviews with individual learners, and action research in classrooms.

In mathematics, the generation by learners of incorrect strategies has been well documented (Ashlock, 1976; Brown and Van Lehn, 1982; Brownell, 1949; Reisman, 1972), and has led to comments on the diverse nature of children's errors (Davis, 1979, 1982a) as well as to empirical studies of the 'bugs' themselves (Van Lehn, 1982, 1983). Again, naturalistic methodologies have often been employed as researchers have focussed on individual and group conceptions of aspects of mathematics. Paralleling the research in science education, research in mathematics education has moved in recent years to greater use of the clinical interview technique, participant observation, and qualitative accounts of learners' behaviour.

Purposes of the Study

The 1981 I.E.A. survey showed that a substantial proportion of New Zealand 13 year olds lacked facility with decimal numbers. The researcher's interest, however, was in what might lie behind the children's difficulties and errors. While a survey could confirm that computational problems and conceptual difficulties existed, it was considered that the crucial question concerned how children constructed these inappropriate responses.

A small number of overseas studies using both quantitative and qualitative research methodologies have focussed on student misconceptions and error patterns in the area of decimal numbers (e.g. Bell *et al*, 1981; Brown, 1981a; Hiebert and Wearne, 1983, 1984; Wearne and Hiebert, 1984). In general, these studies have supported the finding from various surveys that many students demonstrate inadequate knowledge and understandings when working with decimals.

The present study was designed to tap into the mathematical thinking of individual students about decimal numbers. The study used diagnostic instruments, as well as a series of detailed individual interviews, in an attempt to uncover (a) how a sample of 11 to 14 year olds constructed their mathematical ideas about decimal numbers, and (b) what the basis might be for any mathematical errors that this sample exhibited.

A cross-sectional survey of achievement offers a "one-shot" assessment of different age groups at the same time. It was hoped that the longitudinal monitoring of a child's performance in decimal numbers in the present study would provide data on any changes in knowledge, understanding and skill over time. Such data might not only help educationalists understand how children learn to think mathematically, but it might assist them to make a more meaningful and comprehensive analysis of the results of surveys such as the I.E.A.

In addition, it was hoped that the present study would prove suggestive in the pedagogical sense. In other words, because the study looked at the learners' ideas constructed over a period of schooling, then these ideas should have relevance for teaching about decimal numbers.

The overall purpose of the present study, then, was to explore how learners actively constructed their concepts of decimal numbers, how they constructed computational procedures using decimal numbers, and how they viewed lessons about decimal numbers. This focus on the learner rested on the belief that children build upon their previous views of the world in acquiring new information, skills and attitudes rather than learning *de novo*.

Limitations of the Study

The research reported in this study concerns itself with only one topic within the broader mathematics curriculum. The naturalistic research design precluded a study of a wider range of subject matter, which would have demanded the attention of a research team.

The naturalistic orientation of the research methodology also limited the numbers of subjects in the research sample. Twenty-eight 11 and 12 year olds were studied for two years. During this time the research sample transversed the primary/secondary school interface and experienced systematic schooling in the curriculum area of decimal numbers. For logistical reasons a larger sample size and a wider age range would have proved unmanageable for investigation by a single researcher.

Organization of the Report

Chapter Two discusses the theoretical background to the research, with particular reference to constructivist and generative learning theory. The implications of these theories for research design are then discussed with reference to qualitative research, in particular longitudinal study and use of the interview as an evaluative tool.

Chapter Three reviews the findings from quantitative and qualitative studies in mathematics education. A critical analysis is made of concerns in mathematics education, of research studies focussing on children's errors or 'bugs', and of studies on children's knowledge and beliefs about decimal numbers.

Chapter Four describes the research method. The chapter presents the research questions, discusses the quantitative and qualitative approaches that were used (including the rationale for the longitudinal research design), details piloting procedures for the interview format, describes the research sample, and describes the interview protocol.

Chapter Five presents the results of the quantitative survey of students' knowledge of decimal numbers. The results are analysed, and areas for the naturalistic follow-up research are identified.

Chapters Six, Seven and Eight present and discuss the findings of the longitudinal studies with the main research sample, including an analysis of the individual and group constructions of meaning that the qualitative research methodology helped reveal. Attitudes of the students towards mathematics are also presented.

Chapter Nine summarizes the research findings, draws significant conclusions, and outlines some major educational implications of the study.

CHAPTER TWO

THEORETICAL BACKGROUND AND RESEARCH IMPLICATIONS

As indicated in Chapter One, the present study attempted an analysis of how and why students arrive at inappropriate responses to decimals-related problems. The researcher adopted the constructivist viewpoint in cognitive psychology that, in learning, students build upon their previous views of the world rather than attempting to construct ideas *de novo*. The present chapter examines constructivist theory to justify its application in this research. Because the constructivist position is best supported by a qualitative research methodology, the chapter examines also the general nature of qualitative approaches to educational research, including discussion of longitudinal-type designs and use of the interview as an evaluative instrument.

The Constructivist View of Learning

Many people have held a 'commonsense' view of learning as simply absorbing knowledge. As Driver (1984:3) noted, this might be reflected in everyday phrases such as *"I could not take it in"*. This view of the learner as a passive recipient can be contrasted with current perspectives on learning which suggest that learning is an active process.

Jean Piaget (1929) first brought attention to the idea that learning is an internal, active process. He considered that all knowledge was constructed by the individual as s/he interacted with the world and tried to make sense of it. In contrast to the absorption model, Piaget believed that knowledge was acquired, not by the internalization of some outside given meaning, but by construction from within of appropriate representations and interpretations of incoming stimuli.

Over the last two decades there has been an increasing tendency for teachers to cite Piagetian psychology - in particular, the developmental stages for the emergency of various epistemological constructions in mathematics - as the rationale for their curriculum and teaching approach. Yet Piaget was essentially a philosopher interested in the nature of knowledge, and not so concerned with the psychology of learning and classroom applications of his theory (Flavell, 1977; McNally, 1972). However, two other constructivist theorists have attempted to relate constructivist ideas to a learning model: (1) George Kelly in the field of personality development, and (2) Merle Wittrock in the area of learning theory.

Kelly's Constructive Alternativism

Kelly (1955, 1963) used the term constructive alternativism to summarize his view that people understood themselves and their surroundings, and anticipated future eventualities, by constructing tentative models of the world. This relates to Kelly's metaphor of 'man-the-scientist'. Such models could be tested for their predictive efficiency, and were subject to both revision and replacement. A concomitant of this theory of personal constructions of reality was rejection of an absolutist view of truth.

From Kelly's perspective, then, personal constructs are erected by people to account for phenomena, to predict events, and to evaluate previous forecasts once the events have occurred. These constructs are hierarchically arranged and possess what Kelly called a 'focus of convenience', by which he meant that a construct refers to those things to which individuals would find its application most useful.

Change to personal constructs is seen by Kelly as an evolutionary process in which groups of constructs are differentiated into independently organized substructures or mini-theories which, in turn, may be integrated into cross-referencing systems, or into higher levels of abstraction. This enables a person to draw upon more, as well as more sophisticated, relevant information.

A most important theme throughout Kelly's theory is that people are unique in their constructions of reality (Kelly's 'Individuality Corollary'). According to Kelly, two people cannot be presumed to have the same ideas even when they have ostensibly experienced the same set of events. Because constructs differ in their focus, range, permeability, position within a hierarchical framework, and linkage with one another, this would affect the content of a person's construct system.

Kelly also describes the relevance of phenomenology to his personal construct theory. Phenomenology emphasizes the fact that the outlook of the individual person is itself a real phenomenon. In psychology this has led to the study of individual differences, and the psychologist, claims Kelly, should not assume that a different viewpoint lacks substance of its own: "...the psychologist should not necessarily infer that what one person thinks has to be like what another would think in the same circumstances, nor can he accurately infer what one person thinks from what is publicly believed to be true" (Kelly, 1963:41). Kelly also describes the orientations of some psychologists (e.g. Allport) in distinguishing between nomothetic and idiographic classifications, and how, if individual descriptions are to have meaning, they must relate to principles of human behaviour.

The educational implications of Kelly's personal construct theory centre upon the importance of teacher recognition of, and sensitivity to, the personalized alternative frameworks that students possess and apply during learning. As Pope and Keen (1985:33) suggest: "When applied to an educational context, this constructivist view of knowledge lends support to teachers who are concerned with the investigation of students' views, who seek to incorporate these viewpoints within the teaching-learning dialogue, and who see the importance of encouraging students to reflect on, and make known, their construction of some aspect of reality."

Kelly believed that other people, too, are important for the validation or invalidation of a person's constructs. Thus, a person might possess a construct about the importance of authority. If this construct were paramount, then conceivably a student having this view would see the <u>teacher's</u> knowledge as being the most powerful and thus to be the most valued (c.f. Young, 1971). This might discourage a learner from experimenting with his/her personal constructs, thus limiting knowledge to the teacher's dicta.

Although Kelly recognized that there was an infinite number of possible ways for a person to construct meaning about some aspect of reality, some constructions would inevitably be inappropriate. Thus learners would need to be guided to construct 'better theories'. Crucial, here, was the need for the learner to see any new information ('theory') as intelligible, plausible, and useful (Hewson, 1981).

From a teaching-learning perspective this suggests that teachers should discover students' frameworks, recognise any emotional significance of these to the learner, appreciate that some mini-theories might be deeply entrenched and highly resistant to change, and devise learning activities that would help learners to evaluate the efficacy of their mini-theories.

Kelly's theory was to precede a shift in educational psychology towards cognitivism and renewed interest in the information-processing strategies of learners (White, 1983). This paradigmatic shift was perhaps a reflection of the growing concern with the details of learning, with how learners perceive and interpret their worlds.

To summarize, Kelly presented a potentially useful theory for educators, but his formulation of this theory was so complex and burdened by specialized terminology that most classroom practitioners would find it difficult to understand and apply (c f. Osborne and Wittrock, 1985:63). Kelly's theory of personal constructs is essentially a theory about personality, Kelly's main concern being with therapeutic practices related to satisfactory personality functioning (see, for example, Kelly's 1953 handbook for psychotherapists dealing with disturbed clients). However, Kelly's work represents a new thrust in cognitive psychology, namely, interest in the information-processing strategies of learners, and the idiosyncratic nature of this for each learner. A complementary contribution comes from the work of Merle Wittrock who looks at the way in which meaning is actively constructed from the interaction of existing knowledge and incoming stimuli.

Wittrock's Generative Learning Model

Wittrock (1974a, 1980), in explaining the generative learning model, presents the fundamental premise that people tend to generate perceptions and meanings consistent with their prior learning. In particular, learning with understanding is a process of generating semantic and idiosyncratic associations between stimuli before the learner, and information from memory store. Wittrock, then, indicates his reliance on a constructivist model of learning, and parallels the general orientation of Kelly's work. However, the generative model of learning, according to Wittrock (1974a), is unlike earlier cognitive models of learning because of its emphasis on concrete, prior experience and abstract verbal abilities "transferring to and being used to construct specific associations to stimuli" (Wittrock, 1974a:89).

Wittrock (1974a) describes three small studies that he undertook based on the learning model. The first of these, with 364 Fifth and

Sixth Grade students, provided support for the idea that when students are asked to generate a sentence that summarizes a passage of reading material and are given word organizers associated with this passage, they perform significantly higher than the control groups on retention and comprehension scores. The two other studies which required learners to generate hierarchies for recalling words, and trained school children to solve simple concept identification problems, provide support for Wittrock's generative learning model in that it is shown that the relation between the learner's background and the stimuli are crucially important.

Wittrock considers that learning with understanding involves generating and transferring meaning from a number of sources - one's background, attitudes, abilities and experiences. This indicates Wittrock's acknowledgement of the importance of the individual person's construction of reality and how these constructions vary from one person to another.

In summarizing research on the brain and learning, Wittrock (1980) indicates that although the brain often responds reflexively to incoming stimulation, it is much more than a 'tabula rasa' that passively stores incoming information. The 104 studies cited by Wittrock indicate that "the brain often selects the information to which it will attend, and constructs models of reality from the incoming stimulation" (Wittrock, 1980:397).

Wittrock claims that the encoding of information in memory is an active, constructive process (c f. Kelly, Piaget) that involves interaction between information processing strategies and stored memory, as well as sensory data from the environment. The construction of meaning is not viewed as a simple process but involves "expectations, intentions, voluntary attention, previous learning, and strategies for processing information" (Wittrock, 1980:397).

According to Wittrock, research on learning and development will only be fruitful if we understand the effects of prior learning. Thus, in teaching and learning we should first find out the developmental differences between students, and then plan instructional sequences accordingly.

Wittrock gives primacy to the learner. He states that those who study learning and individual differences should attempt to discover and understand learners' information-processing systems. Wittrock (1980) claims that traditional correlational studies are of little use; rather, educators should study how learners transform teaching and learning sequences into useful information. This suggestion implies that classroom instruction will have different meanings for different learners depending upon their information processing strategies and their fund of past experiences.

Whereas Kelly's theory of personal constructs is comprehensive and detailed, Wittrock's generative learning model is almost too broad. Apart from the model's fundamental premise that people construct perception and meanings for themselves, Wittrock fails to describe any other major postulates of the theory. Although three minor studies have been described (Wittrock, 1974a), as well as related research studies on cognition and the brain (Wittrock, 1980), elaboration of the generative learning model *per se* is absent from Wittrock's writings.

In the field of science education, Osborne and Wittrock (1983, 1985) have attempted to redress these imbalances, and expand upon and articulate the generative learning model. In an attempt to make Kelly's ideas more communicable to readers, Claxton (In press a, in press b) modifies and extends Kelly's work. Both sets of writers attempt to apply the theories to the classroom setting by making suggestions for classroom interaction, using illustrative material from research. These newer developments are discussed in the next section.

Constructivism Applied

Wittrock (1974a, 1980) had outlined the generative learning model in terms of its basic principles. It was not until he collaborated with Osborne, however, that the model was more fully explained (Osborne, 1985; Osborne and Wittrock, 1983, 1985).

Although the generative learning model incorporates ideas from information-processing theory (Larkin and Rainard, 1984; Newell and Simon, 1972), as noted earlier it received its major input from constructivists such as Piaget (1929), Kelly (1955) and Wittrock (1974a). From these sources, Osborne and Wittrock accepted wholly the principle that learning with understanding requires effort to actively generate links between incoming stimuli and store information. Diagrammatically this has been represented as follows (FIGURE 1):

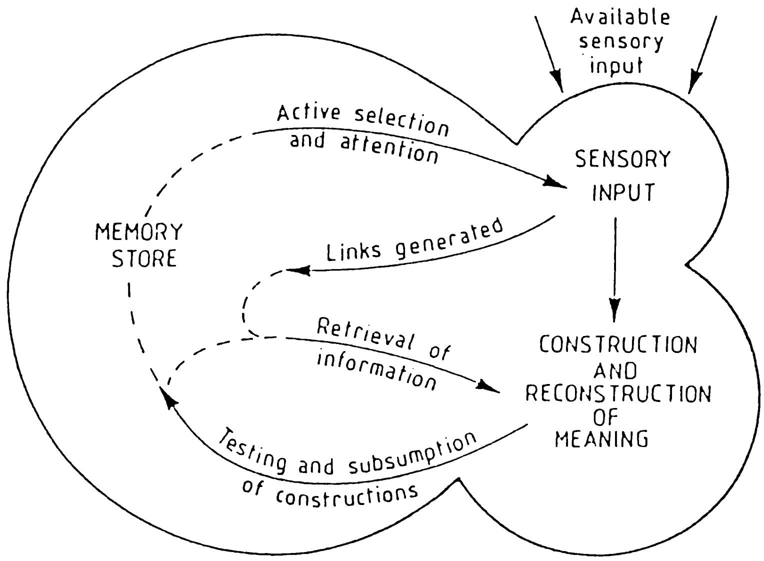


FIGURE 1 : Representation of the Generative Learning Model (from Osborne and Wittrock, 1985)

Osborne and Wittrock hold that a person's existing ideas influence which stimuli are selected and attended to, and what meaning is given to those stimuli: "What people learn begins with existing ideas rather than with the stimuli." (Osborne, 1985:10)

Osborne and Wittrock (1985) have listed the major postulates of the generative learning theory as follows:

1. The learner's existing ideas influence what use is made of the senses, and in this way the brain can be said to actively select sensory input.
2. The learner's existing ideas will influence what sensory input is attended to and what is ignored.
3. The input selected or attended to by the learner, of itself, has no inherent meaning.
4. The learner generates links between the input selected and attended to and parts of memory store.
5. The learner uses the links generated and the sensory input to actively construct meaning.
6. The learner may test the constructed meaning against other aspects of memory store and against meanings constructed as a result of other sensory input.
7. The learner may subsume constructions into memory store.

8. The need to generate links and to actively construct, test out and subsume meanings requires individuals to accept a major responsibility for their own learning.

Osborne and Wittrock (1985) accept that any model of human learning is an oversimplification of reality, and that further elaboration of the generative learning model will be necessary. Some empirical studies have used the generative model as their theoretical underpinning. Bell, (1984), for example, researched the fields of reading comprehension and conceptual change in science education, and focussed on the role of existing knowledge in both processes. Bell's main finding was that existing knowledge was used not only to construct a meaning, but also to evaluate it. To illustrate: (i) existing knowledge (e.g. the everyday concept of animal) was used both to construct and evaluate a meaning for the text that students encountered; (ii) the extent to which the perceived text information was used in evaluation varied according to the student's view of the acceptability, authority or 'correctness' of the constructed text meaning. Learning in science, Bell claimed, involved not only the construction of the intended meaning, but accepting it as well.

Other research by Baird (1984), and Baird and White (1984), has argued that improvements in learning, "depend on a fundamental shift from teacher to student in responsibility for, and control over, learning" (Baird and White, 1984:2). In other words there is concern with metacognition (the knowledge, monitoring and control over one's own thinking and learning), a concept that has a direct link with Osborne and Wittrock's Postulate 8. above, namely, that learners should assume responsibility for their own learning.

Baird and White's study used high school students' biology lessons as a basis for a project that ran through four phases: Exploratory, awareness, participation, and responsibility-control. Data were gathered from 15 sources. The students (n = 64) were given a question-asking check-list (e.g. 'What is the topic?'; 'What do I know about the topic?' etc.), evaluation notebook and card, and a techniques notebook to assist them to increase control over their learning. Baird and White concluded that students did gain greater control over their own learning (based on researcher observations and students' questionnaire responses), but that the teacher remained in control of activities and direction within the lessons. As well, the official syllabus and long-established teaching procedures militated against change (see White, 1984, for alternative proposals for schooling). The important finding, though, was that materials that encouraged students to think about and evaluate their own learning were successful. Given a chance, students could become more responsible for their own learning.

The generative learning model has described also the frequency with which children's ideas have not been influenced, or have been influenced in unanticipated ways, by classroom experiences (Biddulph, 1985; Carr, 1984; Tasker, 1980, 1981).

Osborne and Wittrock (1985) have suggested that the problem for teachers might be that learners frequently generate links to existing ideas which might be considered irrelevant by the teacher. Learners construct their own perceptions and ideas about the purposes of lessons, linkages to former lessons, the appropriateness of activities, and so on (as Baird and White (1984) described). These perceptions might be very different from those intended by the teacher.

Osborne and Wittrock (1985) have described the implications for the teaching and learning of science of the constructivist tradition and the generative learning model. They identified these as:

1. Selection: Initially, students need the opportunity to explore new situations for themselves.

2. Attention: The teacher should influence the learner's voluntary control over attention if he/she is to ensure that students attend to specific aspects of the learning experience.

3. Sensory Input: Teachers should reflect constantly on the fact that the sensory input selected and attended to by students has, in and of itself, no inherent meaning.

4. Generating Links: Teachers might revise earlier lessons, exemplify how the topic could be related to students' prior experiences, explain fully and repetitively, and provide a large number of exemplars.

5. Constructing Meaning: This involves much more than a single-path or single-loop process.

6. Evaluation of Constructions: Teachers need to ensure this occurs by making a range of models, experiences, demonstrations, worked examples and analogies available to the students.

7. Subsumption: Instruction should encourage learners to generate firm links between constructed meanings and a variety of appropriate aspects of the knowledge structures in memory.

8. Teachers could help students make better sense of their world by providing opportunities for them to consider and contemplate, to reflect back, to encourage the idea that learning is dependent on their own actions, and to ensure that effort is rewarded.

In addition to these teaching principles, Osborne and Wittrock have considered the implications of generative learning theory for curriculum writers, assessment practices, and future research in science education. However, whereas these two researchers have concentrated generally on the processes by which learners construct their views of the world, the nature of such constructions has been explored to a greater extent by writers such as Claxton (In press a; in press b).

Claxton's Concept of the Mini-Theory in Science Learning

Focussing on science education, Claxton (In press a; in press b) has attempted to refine, yet make more communicable, Kelly's theory of personal constructs. At the same time, Claxton seems to have incorporated in his work similar thinking to the generative learning model of Osborne and Wittrock.

Claxton's analysis of science learning rests upon a set of basic assumptions as follows:

1. Children know a lot of science.

2. Children are scientists. Not only do they know how the world works, but they possess theories that underline and produce these ideas.

3. Children are inconsistent scientists. Children possess not one grand theory, but a number of mini-theories. Some of these are inconsistent with each other.

4. Mini-theories have a focus and a range of convenience. Theories became expanded to situations other than the original domain, and this developed domain is called the 'range of convenience' of a mini-theory.

5. All knowledge and skill possesses a range of convenience. All cognitive mechanisms and representations are context-specific.

6. When existing theories are inadequate we learn. In other words, the most powerful stimulus for the development of a theory is that theory's failure.

7. We vary in our personal commitment to our mini-theories. Some points of view are easily given up; others might be very resistant to change.

In explaining his theory, Claxton used a a number of metaphors. For example, he described a developing set of mini-theories as being like a large number of amoebae which are attempting to find their 'right place' on a flat board that represents one's experience. Using this analogy, he points out that some domains of experience might be covered by no mini-theory ('gaps'), while others might be covered by more than one mini-theory ('overlap').

Claxton considers Kelly's constructs to be less general than mini-theories. However, he admits that much of what Kelly said could be applied to mini-theories. Claxton's acronym SPADE covers the essential points of the mini-theory viewpoint: There is a Situation (S) to which mini-theories are applied, this application consisting of Prediction (P) and Action (A): "A theory tells us what is likely to happen next, given what is happening now" (Claxton, In press a); 'SPA' often operate in an unconscious, unarticulated, spontaneous, and intuitive fashion. Once the child has developed language, however, Descriptions (D) and Explanations (E) usually feature.

Claxton also used the metaphor of a street light to describe the boundaries of mini-theories. Just as street lights illuminate a patch of ground and fade into shadow at the edges, or merge with the next light ('overlap'), so too one set of expectations might sharpen up while the other fades away.

How stable are mini-theories? Claxton has argued that it depends: if situations shift quickly, then so, too, will mini-theories. Overall, Claxton feels that a particular mini-theory lasts as long as it is successful.

Claxton has proposed that learning involves some change to SPADE: "Next time we may be inclined to expect something different (P), behave differently (A), or describe and account for what is going on in a different way (D and E)" (Claxton, In press a). Learning might occur when a mini-theory produces a successful prediction, action, and so on.

The failure of a mini-theory might be attributable to over-extension, under-extension, or conflict. Where failure occurs through over-extension of a theory, then we might advance another mini-theory into that area. On the other hand, the mini-theory might be modified. The most interesting case though, according to Claxton, is where conflicting mini-theories exist, as this is a problem often encountered by science teachers. For example, students may have different ideas about dynamics simultaneously, based on information from the school and out-of-school setting (Claxton, In press b).

Claxton isolated three types of science - 'gut', 'lay', and 'school'. 'Gut and 'lay' science can overlap, and as a result of this might be laminated or integrated. Laminated mini-theories are those that overlap but are not connected; integrated mini-theories are those where disparities and inconsistencies have been ironed out, and the two mini-theories are fully in accord.

Claxton has discussed the problems of overlap, conflict and resistance with regard to learning science. In doing this he drew not only on cognitive, but also social, aspects of the learner's world. For example, in cases of no learning, people may opt not to give up a cherished 'gut' or 'lay' mini-theory if that might place the person in conflict with his/her society, family and friends.

Claxton has suggested that linking school science and out-of-school science might be more difficult than teachers imagine. For the integration of the two to occur, eight conditions must be fulfilled. These are the need for: the two theories to be laminated; the theories to be simultaneously active; conflict to exist; the conflict to be perceived or acknowledged by the learner; the learner to feel the need or the benefit of resolving the conflict; appropriate learning strategies to be engaged; defensive strategies to be avoided; and a satisfactory resolution of the conflict to be found.

In the broader context, Claxton has categorized two aims for teachers: the teaching of symbolic science (traditional scientific knowledge and techniques), and amplifying competence (enhancing students' everyday competence). Claxton notes the disparities between these styles of teaching, and the complexities and difficulties associated with each: "Acquiring knowledge is a different activity from developing one's competence, and learners pursuing one or the other require very different kinds of assistance" (Claxton, In press a). Claxton concluded that teachers needed to offer students of science an approach that embodied considerable tact, understanding and respect.

Comment

Although Claxton's theory aims to provide a general model of learning, he has concentrated to date very much on the science education field. Moreover, while he has referred to anecdotal evidence and to various researchers in his theory-building, he has not provided empirical data to support his contentions. It seemed appropriate in the present study, therefore, to investigate the applicability of Claxton's conceptualization of mini-theories to the learning of mathematics - in this case, decimal numbers.

The generative learning model of Osborne and Wittrock presented a second theoretical position for empirical investigation in the present study. Not only might Osborne and Wittrock's postulates of learning bear study (see Pages 12, 13), but a study of mathematics learning might reveal interesting findings in the affective domain, an area neglected by Osborne and Wittrock, and to a lesser extent Claxton. Thus the present research explored attitudes towards mathematics in an attempt to investigate their importance in the learning of mathematics. In turn, it was hoped that this would provide evaluative data on the validity of the generative learning model.

This model also tends to present a sequential picture of learning. But learning might well be more of a piecemeal nature, that is, it might consist of a series of 'packages' that are situation specific (c f. Claxton, In press b). And these 'packages' might be more tentative and experimental than the generative model suggests. It was hoped that the present study of mathematics learning might illuminate this theoretical issue.

Implications of Constructivist Theory for Research Method

Educational research that is underpinned by constructivist theoretical thinking necessarily leans towards a methodology that will help the researcher get beneath surface behaviour to the phenomenological perspective of learners. As Katterns (1982) has put it, the aim is "to get between the ears and behind the behaviour". This section examines this qualitative approach to educational research, discusses its two features of longitudinal study and the use of interviews, and considers the advantages of combining the qualitative approach with a quantitative one in which learner performance is subjected to some kind of statistical analysis. Because the focus of the present study was on mathematics learning, points of principle are illustrated as often as possible by reference to research in that curriculum area.

Quantitative and Qualitative Research

Surveys of a quantitative nature in broad areas of the mathematics curriculum provide what is called a nomothetic assessment. The advantages are held to be: the sampling of large numbers of student behaviours to assess current status; providing data that can be used to compare the performance of various groups; the identification of major trends; and identifying for teachers problem areas that might be addressed in the future. Although some claim that large scale testing does not enjoy the prestige it once had, pressure has persisted to continue with this form of testing and measurement. Ebel, and Carpenter *et al* have summarized these sentiments in the United States:

> "...external tests of pupil achievement have strong support in the public that ultimately determines educational policy in this country. That support has already been expressed by the legislatures of two thirds of the states who have mandated some form of external testing of achievement...the fundamental logic of including assessments of achievement as an essential component in any purposeful instruction program is sure to prevail ultimately over the kind of specious arguments we have been hearing against the use of tests of achievement."
>
> (Ebel, 1980:16)

> "Recently the achievement of students in American schools has become a matter of national concern, particularly achievement in the sciences and mathematics. One of the best measures of achievement is the results of the National Assessment of Educational Progress (NAEP) (1983)...Over 45,000 students participated in the most recent mathematics assessment... the results provide an accurate sampling of the performance of elementary and secondary students over a broad range of objectives."
>
> (Carpenter *et al*, 1984:485)

Some researchers, however, advocate the use of qualitative approaches in educational research. Some of the earliest examples of this came from Piaget (1929, 1941). Since that time, qualitative studies of children's learning in mathematics have revealed much more information on how individual students go about constructing meaning from the data that they confront. In general, these studies have been loosely structured around a few key concepts or questions that the investigator has wanted to explore with the individual. Indeed many of the recent studies with young children could be linked directly to Piaget's earlier ideas:

> "What now began to be carried out in increasing numbers were studies which extended his (Piaget's) work, either by finding quite new ways of looking at the topics he had studied, or by using variants of the tasks which he used."
>
> (Donaldson *et al*, 1983:7)

In other disciplines such as sociology, a balance has been attempted between macro and micro studies - earlier positivistic studies such as those of Durkheim (1951) and Coleman (1961) have been replaced by smaller scale studies that examine the views that people as individuals attach to objects and actions as they continuously interact in group situations (Berger and Luckman, 1967). As well, participant observation techniques have necessarily involved small samples in a variety of settings (e.g. Becker, 1953).

In between these two stances, writers have advocated, on occasions, the use of both quantitative and qualitative approaches when studying children's learning. This has been suggested as a means of overcoming the problem of sampling a student population large enough so that the findings can have generality, and yet small enough so that the views of individual students can be recorded and analyzed. Brueckner (1935) mentioned this problem many years ago when discussing diagnostic procedures in arithmetic:

> "Survey and analytical tests help the teacher to locate the source of difficulty. Test results do not give adequate data concerning the pupils' methods of work, nor do they reveal the nature of errors made. There are four general methods that can be used to analyze errors and faulty methods of work: (1) observation of the pupil at work, (2) analysis of written work, (3) analysis of oral responses, and (4) interviews."
>
> (Brueckner, 1935:291)

The survey reported by Hart (1981) used both quantitative and qualitative methods, as have other studies (e.g. Krutetski, 1976). The Hart research, the Concepts in Secondary Mathematics and Science (CSMS) programme, tested approximately 10,000 children in all, but used interviews with about 30 children from appropriate age ranges to provide validating information for the written tests and data on the students' errors:

> "In order to present a picture of children's mathematical understanding which was representative of the English child population, it was necessary to use written tests.
>
> ...Items were written which were free from technical words and these were tried on interview with children and then replaced or revised. The interviews were used for a second purpose, that of finding the methods used and errors made by children when confronted with a mathematical problem."
>
> (Hart, 1981: 1)

Likewise, Brown's (1981a) study, which was part of the CSMS programme, used both written surveys and interviews with individual students. The interviews, although based on the written tests, did have some flexibility built into them:

> "In the first set of interviews most children were asked to tackle all the questions to give as much information as possible. However because there was a large number of items and a great variety of difficulty, the interviewer was allowed especially in later interviews to exercise some selectivity where it seemed appropriate and omit questions which on the basis of the child's answers to prior questions would be expected to be too easy or too difficult."
>
> (Brown, 1981a:220)

Taking its lead from research like that described above, the present study used both quantitative and qualitative approaches. In order to sample a broad domain of objectives in the curriculum topic Decimal Numbers, a written test was administered to a representative group of 11-14 year olds to assess their knowledge and understanding of decimal numbers. From the results of this test certain aspects of the students' knowledge and understanding of decimal numbers seemed to warrant closer study. It was decided that the best way this could be done was through the use of individual interviews with a sample size of manageable numbers. In order to probe more deeply than was possible with a 'one-shot' interview, the subjects were interviewed four times over two years. In other words, the present study used a survey to ascertain some trends from quantitative data, then employed interviews over a period of time that yielded qualitative data. Aspects of both nomothetic and idiographic assessment were thus employed.

Longitudinal Studies

Fox (1969) claimed that short-term longitudinal surveys were the single, most frequently employed research approach (by 'short-term' longitudinal studies Fox meant projects that ran for up to one school year). 'Genuine' long-term longitudinal studies, however, have been relatively uncommon. Terman and Oden's (1947) twenty year study of genius, and Bayley's (1955) twelve-year research into the relationship between intelligence and aging, are considered to be classical studies of the genuinely longitudinal type.

The problems of longitudinal studies have been well documented (Johnson, 1977; Wiersma, 1975) and include:

1. Attempting to answer research questions involving long-term objectives by means of study periods that are too short.
2. Vulnerability to Hawthorne Effect, particularly if comparisons are being made within a control/experimental group paradigm.
3. Subject attrition and/or difficulty in maintaining subject contact: in the case of attrition, remaining subjects might well be those who are more co-operative, more persistent, less assertive, and more stable, thus constituting a bias in the research sample.
4. The possibility of subject 'test-wiseness' as the result of too frequent exposure to data collection instruments.

5. Unintentional research sensitisation affecting performance e.g. parental reactions to their preschoolers' language usage affected by knowledge of a researcher's purposes.

6. Variable selection problems: for example, initial research questions and measures may or may not be pertinent at subsequent trials.

7. Difficulty in ensuring a representative research sample: for example, subjects willing to commit themselves to a longitudinal research programme might well be those who are more concerned, better educated, more curious and more open-minded, thus biasing the research sample.

8. Practical problems of economics, loss of research personnel, keeping track of subjects and maintaining systematic data collection methods.

Despite these difficulties, many claim that longitudinal studies make a special contribution to research. Vasta (1979:64), for example, states that such research "offers the psychologist a potentially valuable way to focus on changes in behaviour and to identify the determinants of such change. In addition, it permits the study of the long-term cumulative effects of these variables and their interaction with one another." Again, Fox (1969:439-440) points out that "...there is no substitute for truly long-term longitudinal studies...it avoids the assumption of comparability of different groups by using the same respondents at every data-collection interval."

One variation of the longitudinal approach has been the combined longitudinal/cross sectional design. In this research design several groups of subjects at different ages are studied over a period of time. For example, a research project of this nature might begin with subjects aged 10, 12, 14 and 16 years of age. If this sample were studied for two years, then there would be two years of data on each individual subject, as well as comparative data for the four age levels at the commencement of the study - and after two years, data for ages 10 years to 17 years.

The combined longitudinal/cross-sectional design, therefore, attempts to retain the best features of each design. Data can be built up about individuals over a period of time, yet some of the difficulties of long-term longitudinal studies are obviated.

In the present study the longitudinal research format was used. In addition, one feature of cross-sectional research methodology was incorporated in that two class levels comprised the sample under study. (This is discussed in Chapter Four.) The major feature of the research design, however, was its longitudinal dimension. Thomas (1980) has summarized much of the argument for the longitudinal design:

> "Where a research problem in cognitive development requires detailed qualitative monitoring of individual differences then the likelihood is that a longitudinal design would be appropriate. If the research problem was interested in the differences between earlier and later status for a particular variable then again a longitudinal design would be not only suitable but necessary."
>
> (Thomas, 1980:3)

The Interview as an Evaluative Tool

Just as longitudinal study lends itself to research that seeks to identify changes in how learners construct meaning, so too the individual interview appears to be a valuable tool for understanding how such meaning construction occurs and changes. This section reviews the individual interview as a diagnostic and evaluative tool in educational research with particular reference to mathematics learning.

As reviewed by Ginsburg *et al* (1983) and Hunting (1983), interviewing to probe student learning is a much-used but difficult tool to master. Cohen and Manion (1980) contrasted three conceptualizations of the interview in research. One view is that the interview has the potential for pure information transfer. Thus, if the interviewer does his/her job well, and if the respondent is sincere and well-motivated, then accurate information might well be obtained.

A second conception of the interview is that of a transaction model. In this view, bias is taken into account, and this can be recognized and controlled: "The interview is best understood in terms of a theory of motivation which recognises a range of non-rational factors governing human behaviour, like emotions, unconscious needs and interpersonal influences" (Cohen and Manion, 1980:250). However, these potential obstacles, it is believed, can be controlled or removed in some way.

A third conception of the interview describes it as an encounter, necessarily sharing many of the features of everyday life. Thus five unavoidable features of the interview situation arise: (i) interviews differ one from the other because of mutual trust, social distance and the interviewer's control; (ii) respondents may become defensive if questioning is too deep; (iii) both interviewer and interviewee are bound to hold back something of what they could state; (iv) meanings clear to one person may be relatively opaque to another; (v) interviews cannot ever be rationally controlled. Proponents of this third conceptualization of the interview consider that no matter how conscientiously an interviewer tries to be objective and systematic, "the constraints of everyday life will be part of whatever interpersonal transactions he initiates" (Cohen and Manion, 1980:251).

In order to obviate problems like these above, several writers have suggested strategies for interviewing. Bell and Osborne (1981), for example, have listed 15 "Do's and "Don'ts" for interviewers (see APPENDIX B). Schoen (1979) and Lankford (1974) have compiled suggestions for conducting interviews in the context of mathematics. In New Zealand the mathematics textbook School Mathematics 2 (Department of Education, 1984), has a number of interview formats that teachers might employ for evaluating specific objectives, such as children's ability to perform subtraction with decomposition.

All of these suggestions and proposals have been aimed at improving the validity and reliability of the interview. However, a dilemma has remained, in that measures taken to increase reliability (e.g. greater control, structure) have tended to decrease validity. Thus, the more interviewers have become rational, calculating and detached, the less interviewees have perceived the situation as a friendly transaction. Thus, more calculated and controlled responses might be elicited.

Some researchers have combined interviewing with a quantitative orientation in their research. Hart (1981), for example, has outlined the Concepts in Secondary Mathematics and Science (CSMS) research programme in which written tests administered to 10,000 children were supplemented with interviewing of 30 children to improve the written test items themselves, and to investigate the methods used and errors made by the students. Brown (1981a), as part of the CSMS study, used a similar research methodology. Other research described in Chapter Three (Bell *et al*, 1981; Hiebert and Wearne, 1983, 1984) employed similar methodologies.

Interview sequences have often been quoted by researchers to illustrate certain response patterns. Davis (1971), in discussing some of the issues in the content of 'New Maths', quoted the following sequence to show the original methods students can generate to solve problems. The extract also illustrates the type of information an interview can elicit:

"Teacher (says)	(writes)
Now, we can't take 8 from 4, so we'll have to regroup.	*64* *-28*
Kye a 3rd grade boy (interrupting)	
...oh, yes, you can! Four minus eight is negative four	*64* *-28* *- 4*
...and 20 from 60 is 40	*64* *-28* *- 4* *40*
...and 40 and negative four are thirty-six...	*64* *-28* *- 4* *40* *36*
...so the answer is 36"	

(Davis, 1971:36,37)

As Davis pointed out, 'Kye' was a bright seven year old who had a helpful background to assist him in developing advanced mathematical concepts.

Peck and Jencks (1973) also noted the usefulness of interviews for probing the understanding of students who got high scores on traditional written tests in mathematics, and provided illustrative examples of Sixth Graders' explanations for reducing fractions. They concluded that interviewing "allows the teacher to infer a depth of understanding not discernible by paper and pencil tests" and that "many testing programs could profitably be supplemented by asking children to verify their responses in the world of reality" (Peck and Jencks, 1973:56).

The use of interviews to elicit mathematical understandings from students has covered many aspects of the mathematics curriculum. Brown (1981), Brown and Kuchemann (1976, 1977), Jencks *et al* (1980), Katterns and Carr (1986), Kent (1979), Krutetskii (1976), and Lankford (1972, 1974) have focussed on the computational processes and understandings that children employ to solve problems involving some of the basic operations. Carr and Katterns (1984) looked at children's use of an often-used teaching aid in mathematics - the number line - and used individual interviews to go beyond data obtained from written tests.

In the area of problem-solving (using traditional textbook-type problems), Moser and Carpenter (1982) employed individual interviews with 100 Grade 1-3 youngsters in order to find out how they attacked problems. Their work revealed, to the surprise of some, that school beginners coped successfully with solving word problems in mathematics. Students' ability to generalize beyond the context of written test items to a much broader variety of contexts was probed by Alderman *et al*, (1979). In an exhaustive examination of 'Henry's' mathematical behaviour, Davis (1975) described and analysed the responses of one student and made suggestions aimed at producing conceptual thinking in all students. Lawler (1982) used observation and unstructured interview procedures to investigate 'Robby's' experiences with LOGO (Papert, 1980) and how these influenced 'Robby's' later problem solving. Krutetski's (1976) work involved an intensive study of the problem-solving processes used by primary and high school pupils of different ability levels in Russia.

Davis (1982b) looked at the heuristics used by an 18 year old student in solving problems in calculus at the university level - this study was based on three individual interviews with the particular student. At the tertiary level also, Knight's (1982) research was based on individual interviews with mathematically incompetent students. Finally, Ginsburg (1982) outlined a cross-cultural study that attempted to compare how African and American children solved problems, individual interviews being used as an integral part of the study.

Generally, then, the interview has been more frequently used in evaluating learning in mathematics in the last 15 years. On the disadvantages side, conducting interviews is time-consuming, requires sensitivity on the part of the interviewer, and may lack reliability and validity if certain procedures are not built in. However, research using the interview technique has provided much detail and insight into students' thinking and information-processing strategies in mathematics.

The work of Krutetskii (1976) is a good example of the detailed information that can be revealed by qualitative methods. For this reason, it was decided to incorporate interviewing procedures in the present study, but in a context of longitudinal as well as some cross-sectional survey research.

Finally, it must be remembered that whatever protocol method is used in interviewing, self-reports from learners can never reveal all the causal or processing factors of interest. However, "no source of data is ever complete, or can provide answers to all our questions" (Ginsburg *et al*, 1983:30): interviews may provide detail that pencil-and-paper tests may not.

Summary and Comment

This chapter has described the constructivists' theoretical position. The development of the tradition was noted, and difficulties in articulating and communicating the theoretical postulates outlined.

Later constructivist theories (Claxton, In press a, in press b; Osborne and Wittrock, 1983, 1985) have been described, and a critique of these theorists offered: (i) the generative learning model suggests a sequential and general theory of learning that might not pay sufficient attention to the situation - specific nature of learning; (ii) the generative learning model ignores the influence of affective factors on learning; (iii) Claxton's (In press a, in press b) notion of mini-theories has not been applied to learning in mathematics; and (iv) the theory of mini-theories describes and accounts for learning, but at present empirical research is required to assess its predictive power such as might be revealed through a longitudinal-type study.

A rationale has been provided for using a qualitative approach in the present study. In this regard a critical review was provided concerning the longitudinal research design and use of the interview as an evaluative research tool.

The next chapter reviews research related to the subject of the present study, namely, learning in mathematics, with special reference to students' constructed ideas about decimal numbers.

CHAPTER THREE

REVIEW OF RELATED RESEARCH IN MATHEMATICS LEARNING

As in many domains of research into human cognition, the number of research studies in mathematics education has burgeoned in the last decade. This chapter surveys major reasons for the increasing interest in and concern about mathematics education, and reviews the research literature on aspects of children's mathematical thinking and performance that were of special interest in the present study.

The chapter is organized in four sections as follows:

1. Concerns in Mathematics Education.
2. Mathematical Errors: Diagnosis
   - Errors or 'Bugs'
   - 'Maths Anxiety'
3. Mathematical Achievement in Decimal Numbers
   - Comprehensive Survey Studies
   - Studies with a Qualitative Component
4. Summary and Conclusions

Concerns in Mathematics Education

Towards the end of the 1950s dissatisfaction with the teaching of mathematics became apparent in some overseas countries. As changes in approaches to the teaching of mathematics were being considered, an event of significant importance occurred - the U.S.S.R. launched the first satellite. The effect this phenomenon had on the rethinking of school curricula in mathematics and science, particularly in the United States of America, has been well documented (Hayden and Rudolph, 1984; Howson *et al*, 1981). By the 1960s, new programmes in mathematics were appearing in schools and became known collectively as 'New Maths'.

Generally 'New Maths' attempted to emphasise the structure of the subject through themes that spiralled throughout the curriculum (e.g. sets, number theory, logic), and through the properties of the number system (e.g. commutativity, associativity). The hope was that such an approach would contribute to a deeper comprehension of the nature of operations and problem solving. As well, children were introduced to certain topics in mathematics (e.g. set theory) that often required a considerable level of formalism at an earlier age. The 'New Maths' programmes were supported by textbooks which made full use of logical lay-outs and illustrations that were designed to motivate students.

'New Maths' reached New Zealand around the late 1960s. Reform was instigated initially in mathematics for junior classes through the trialling and introduction of a two-year programme developed by Eric Lenz of the University of Canterbury and later modified by J.J. Lee (1965). This New Zealand scheme drew heavily on the research of Jean Piaget. For the middle and upper section of primary schools in New Zealand 'New Maths' was introduced by textbooks imported from the United States which were issued to all primary schools from 1967 (Duncan *et al*, 1967).

It did not take long for the 'New Maths' to become the subject of considerable debate. Textbooks generally emphasised set theory (e.g. Duncan *et al*, 1967; Graham, 1970; Schminke *et al*, 1973), and this aspect alone generated controversy (Kline, 1973). In particular, concern was expressed that children were being asked to learn mathematics they could not understand, and attempts were made to clarify the directions of modern mathematics (Davis, 1974a, 1974b). As Tafton put it:

> "Questions were eventually raised regarding the appropriateness of certain topics and approaches for young children. Thought was given to how ideas might be treated in a manner more congruent with the thinking of children. ...findings from developmental psychology did not appear to support the level of formalism sometimes found in the reform programs."
>
> (Tafton, 1975, p.26)

As reaction hardened in many quarters, the public at large joined in the debate over standards in mathematics learning (Suydam and Reys, 1978). Along with the issue of public accountability - and perhaps because of it - came the 'Back-to-Basics' movement (Brown, 1979; Lindquist, 1980). The public wanted skills-orientated mathematics programmes, and national reports and surveys alluded to these sentiments

(Carpenter *et al*, 1980; Cockcroft, 1982). In New Zealand this concern was reflected in the objectives stated in both the mathematics syllabuses for Infants to Standard Four, and Forms One to Four (N.Z. Department of Education, 1976, 1985) which stressed the importance of computational skills.

Paralleling the public's belief that children could no longer compute as well as their parents could in their day, doubts were expressed by researchers about the level of students' mathematical learning (Doe, 1979; Farnsworth, 1976; Geddes, 1980; Kline, 1973; Pike, 1976; Tafton, 1975). In addition surveys of students' attainment revealed the specific kinds of weaknesses that students possessed (Booth, 1982; Brown, 1981a; Carpenter *et al*, 1980; Carr and Katterns, 1984; Clements, 1982; Hart, 1981; Otterburn and Nicholson, 1976). Other studies that made a more detailed exploration of a smaller number of students' mathematical concepts also unearthed the large number of errors that children constructed (Alderman *et al*, 1979; Booth, 1982; Brown, 1981a, 1981b, 1981c; Brown and Kuchmann, 1976, 1977; Brown and Van Lehn, 1982; Carr, 1983; Clement, 1979; Davis, 1975; Davis *et al*, 1978; Davis *et al*, 1979; Davis and McKnight, 1979; Erlwanger, 1973, 1975; Katterns and Carr, 1986; Knight, 1982; Peck and Jencks, 1979, Van Lehn, 1982.

One disturbing finding of some national assessments of mathematical achievement in the United States had been that the longer students remained at school, the greater their decline in performance when compared with earlier years:

> "Mathematics scores of students seeking college (university) admission peaked in the early 1960s and has declined steadily since. Elementary and secondary school data from standardized tests and national assessment indicate steady or slightly increasing performance among the youngest students, but continued to decline at junior high and senior high school levels.
>
> However the pattern of decline has changed over the years. Through the 1960s, the greatest decline occurred in the area of computational skills; during the 1970s, comprehension and problem solving have declined."
>
> (Fey and Sonnabend, 1982:158)

The trend Fey and Sonnabend reported is incongruous with the emphasis of modern mathematics programmes on developing in students a deeper understanding of the subject, and the heuristics for problem-solving. But the trend has persisted (Hayden and Rudolph, 1984).

Mathematical Errors: Diagnosis

Some researchers have investigated the problem of why students in mathematics have had difficulties. Early studies investigated this problem through considering the contents of units of instruction, analysing teaching methods and teaching materials, and developing written diagnostic tests. These early attempts to analyse children's mathematical difficulties focussed on appropriate remedial teaching strategies, rather than an in-depth investigation of the children's errors (Brueckner, 1935:299). In more recent times, the focus has shifted to the specific difficulties of the learner.

This section describes approaches to studying students' errors, and reviews the special impact on mathematical performance of anxiety factors.

Errors or 'Bugs'

Some studies have focussed on the diagnosis of the specific errors made by mathematics students, and have attempted to trace the etiology of such errors.

One approach has been the integration of diagnostic and remedial teaching stragegies. Ashlock (1976), identified 38 computational errors often made by children, analysed each error type, and provided guidelines for teachers on how to correct error patterns. Reisman (1972) offered a 'diagnostic teaching model' which involved identifying the child's present cognitive level, and analyzing the task to be learned into components (e.g. in order for a child to do 'long' division, she/he must first be able to multiply and subtract). Reisman also outlined the applications of norm-referenced and criterion-referenced tests, and provided an example of a diagnostic screening instrument: the Reisman Sequential Assessment Mathematics Inventory. As well, she suggested how attitudes might be tapped, described common errors, and listed instructional suggestions for remediating errors.

A different approach for analyzing errors has come from Brown and Van Lehn (1982), and Van Lehn (1982, 1983). Using a computer programme known as DEBUGGY, student errors were analyzed in terms of 'bugs' and 'slips'. 'Bugs' refer to mistaken beliefs that are consistent and stable. 'Slips' are taken to be 'performance phenomena' and, as such, are unstable over time. Brown and Van Lehn (1982) proposed a generative theory of 'bugs'. The authors explained their theory thus:

> "The theory is motivated by the belief that when a student has unsuccessfully applied a procedure to a given problem, he or she will attempt a repair. Suppose he or she is missing a fragment (subprocedure) of some correct procedural skill, either because he or she never learned the subprocedure or maybe forgot it. Because the missing fragment must have had a purpose attempting to follow the improverished procedure vigorously will often lead to an impasse."
>
> (Brown and Van Lehn, 1982:122)

These writers described an 'impasse' as a step that could not be carried out by the student. Some students confronted with '0 - N = " face an 'impasse': "...he or she will often be inventive, invoking problem-solving skills in an attempt to repair the impasse and continuing to execute the procedure, albeit in a potentially erroneous way. We believe that many bugs can best be explained as patches derived from repairing a procedure that has encountered an impasse while solving a particular problem." (Brown and Van Lehn, 1982:123).

These repairs must first pass the 'critics' possessed by the student (examples of 'critics' are: "Don't leave a blank in the middle of an answer"; "Subtraction is not commutative, so don't reverse the column"). Brown and Van Lehn (1982) claimed that repairs which pass the screening of the 'critics', became part of the student's stable behaviour. As a corollary of this, knowledge of a student's impasses could lead to prediction of likely 'bugs'.

As the theory developed, Van Lehn (1982) has suggested that 'bugs' might not be as stable as was initially suggested. In reporting several empirical studies, Van Lehn concentrated on 925 students who were learning subtraction:

> "...many students' bugs appear to be manufactured as the test begins and held consistently for the duration of the test, only to be dismissed and evaporate from the memory as soon as the test is over and they have served their purpose, namely to get the student through the test...there is no reason for the student to retain the strategy after the test is over; it or another like it can be generated again next time."
>
> (Van Lehn, 1982:46,47)

In an extension of earlier work on errors (e.g. Ashlock, 1976; Brownell, 1949), Brown and Van Lehn (1982) and Van Lehn (1982, 1983) focussed on precise, formal notation of students' systematic errors. Van Lehn (1982) provides a 'Bug Glossary' of 144 'bugs' in subtraction and suggests how these might help student teachers understand children's error types.

Brown and Van Lehn examined many thousands of scripts in order to develop their repair theory and the associated generative theory of 'bugs'. Other studies have used a wider range of tests across several topics within mathematics. The National Assessment of Education Progress (NAEP) in America, the CSMS and Assessment of Performance Unit (APU) in Britain, the International Association for the Evaluation of Educational Achievement (IEA) in several countries, have all employed survey techniques based on written tests with thousands of students. Results from these empirical surveys that are relevant to the present study will be referred to later in this chapter.

Theory-based models have been used by some writers to explore the relevance of abstract conceptual schemas to the learning of mathematics (e.g. Skemp, 1971). As well, 'director systems' or the way people direct their actions (Skemp, 1982) have led to different categories of theories (Skemp, 1982). Attempts to provide empirical support for such theories have used interviews and teaching experiments, and have sometimes focussed on mathematically incompetent students (Buxton, 1981; Knight, 1982). Knight's (1982) study, for example, used Skemp's (1979) model of intellectual behaviour, and argued that the absence of frameworks, or schema, explained both the affective reaction of students having problems in mathematics and the development of specific areas of knowledge. Indeed, according to Knight (1982), the affective area was not sufficiently considered by teachers and curriculum writers. He concluded that teachers should "be sensitive to signals in the affective domain which indicate developing problems in the cognitive area." (Knight, 1982:152).

Maths Anxiety

In recent years affective aspects in the learning of mathematics have been studied in greater detail (Buxton, 1981; Tobias, 1978). 'Maths anxiety' has been defined as "involving feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and/or learning situations." (Reyes, 1980:169). Thus 'maths anxiety' has been offered as one explanation of low achievement in mathematics.

Brush (1980), through a longitudinal study, investigated the attitudes and behaviours towards mathematics of 1500 Sixth, Ninth, and Twelfth Grade students in the United States. The three-year study was questionnaire based, but used interviews with a sample of 45 students over the last two years of the study in order to gain more detailed information about the reasons for subject choice and attitudes towards mathematics. Brush found that her subjects felt mathematics became more difficult with time, that they preferred teachers of mathematics who allowed them to ask questions freely, and that they enjoyed challenging (but not too difficult) material. Female students were more prone than males to 'maths anxiety'.

Brush also reported that the topic 'Fractions, Decimals, Percents' was especially liked by 21 per cent of her sample, and disliked by 24 per cent. This topic was also the most frequently mentioned for both the 'Especially Liked' and 'Disliked' categories - a phenomenon Brush did not consider significant, judging from her failure to mention it in her discussion.

Other researchers have pointed to the relationship between confidence levels in mathematics and achievement in the subject. For example, Fennema and Sherman (1978) found that gender-related achievement levels in favour of male students were accompanied by higher confidence levels in male students. In addition, these researchers found that gender-related differences in confidence were present even when no differences in mathematics achievement were found. In summarizing much of the research into what might be termed "mathematics self concept", Reyes, (1984) wrote:

> "Considerable research about confidence in learning mathematics indicates the importance of this variable in relation to student achievement, gender-related differences, election of optional mathematical courses, and classroom processes."
>
> (Reyes, 1984:563)

Some researchers have seen 'maths anxiety' as a special problem of female students (e.g. Tobias, 1978), though Fennema and Carpenter (1981) have noted that data from NAEP in America has not substantiated sex-related differences in mathematics abilities for the nine and 13 year old groups, and has shown superiority for males over females only at the 17 year old level. Other researchers (e.g. Walden and Walkerine, (1982) have commented that standard explanations of sex-related achievement differences in mathematics are unsatisfactory, and a more complex model than the linear cause and effect model is needed. For instance, it might be possible that the expectations of society need to be taken into account as well, especially with regard to sex-role stereotyping.

However, basic research questions in this whole field remain unanswered. For example, what are the long-term effects of efforts to reduce 'maths anxiety' through programmes such as those offered by Tobias (1978).

An additional concern has been that studies in this area have presented correlational data only, and have not established causal links between mathematics anxiety and low achievement. This might suggest that a more qualitatively-oriented research methodology may be required.

Interest in attitudes towards mathematics has led to the development of scales such as the Mathematics Attitude Scale (Aiken and Dreger, 1961) and the Mathematics Anxiety Rating Scale (Suinn *et al*, 1972). As well, the semantic differential technique (Osgood, Suci and Tannenbaum, 1957) has been adapted by researchers to tap into student attitudes towards mathematics. In New Zealand, Clark (1977) used the semantic differential method to probe the attitudes of 12 to 14 year old students (n = 263) towards mathematics. In general, her subjects held very favourable attitudes. In the numbering scale of one to seven, with seven representing the most positive attitudes, categories such as 'Useful later in life', 'Good for you', 'Useful now', 'Good', and 'Interesting', all received mean scores greater than five. As Clark noted, however, the results of her pencil-and-paper survey might have been influenced by factors such as desirability, acquiescence, and the halo effect. Another New Zealand study (Knight, 1982) also described the importance of affective factors in the learning of mathematics, this time with 26 mathematically incompetent tertiary students. At the conclusion of his study, Knight claimed:

> "The tragedy is that many students, not understanding their failure, lose self-esteem and a vicious interaction of affective and cognitive reactions create a state in which any constructive approach to mathematics learning seems impossible."
>
> (Knight, 1982:151)

To summarise, the research literature strongly suggests a link between affective variables and the learning of mathematics. Confidence levels, in particular, have been shown to be positively correlated with achievement in mathematics, and negatively correlated with levels of 'maths anxiety'. Gender-related differences in mathematics achievement have not been established unequivocally, but some writers have linked the alleged low achievement of female students in mathematics to aspects of socialization (Tobias, 1978). Survey data in the field of 'maths anxiety' has sometimes been supplemented with information from interviews with individual students as researchers have tried to probe underlying beliefs and conceptions.

Mathematical Achievement in Decimal Numbers

As a result of a shift from imperial to metric measurement, the learning of decimals has become a significant part of the New Zealand mathematics curriculum for primary and high school students. Indeed, curriculum developers believed that the introduction of metric measurements, expressed through decimal numbers, would make for easier learning by mathematics students.

This section reviews two kinds of research that have focussed on the mathematical achievement of students in decimal numbers. The first group of studies are of the comprehensive survey type, while the second group derived qualitative kinds of data from the use of in-depth interviewing of mathematics learners. There has been a paucity of research of this latter kind on the mathematical topic of decimal numbers. The present reviewer located only four researchers working along qualitative lines. For this reason, it was decided to review these four studies in greater depth than those examined in the survey research group.

Comprehensive Survey Studies

The topic of decimal numbers has received more, as well as earlier, emphasis in most school mathematics programmes today. Perhaps because of this the subject matter has been assessed by various large-scale surveys. Four such studies are now described.

1. The National Assessment of Educational Progress (NAEP) (1978 and 1982). In America NAEP has traditionally investigated several aspects of student learning with very large student samples. Relevant results for the present study of the 1978 and 1982 assessments were: i) Most nine year olds interpreted decimal numbers as whole numbers (thus the decimal number 37 thousandths was seen as 37000 - and 40 per cent of 13 year olds also chose this answer); (ii) placing decimal numbers on a number line was difficult, as too was naming a given point on a number line - only one third of 13 year olds being able to perform this task; (iii) selecting decimal number equivalents for common fractions was poorly performed, the most common error for 'one fifth' being .5; (iv) computation with decimal numbers was difficult, most problems occurring with thousandths, when the operation was division, or when decimals were expressed in different units such as 1.5 and 0.23; (v) estimation and approximation skills were poor, only 21 and 37 per cent of 13 and 17 year olds respectively being able to estimate the answer nearest to 3.04 x 5.3.

Grossman (1983) commented that the use of zero as a place holder seemed to cause problems with decimal fractions, and that the topic should be introduced earlier in American schools. She also expressed surprise at the levels of difficulty demonstrated by students.

2. International Association for the Evaluation of Educational Achievement (IEA) - Mathematics Study (1981-83). This study was conducted in 23 countries from 1981 to 1983. The New Zealand sample consisted of 5177 Third Formers (13 year olds) and a similar number of Seventh Formers (17 year olds). There were 15 items from the core and sub-tests on decimal numbers. Relevant results for the present study for the 13 year old group were: (i) Selecting decimal number equivalents from verbal descriptions, and from mixed numbers (e.g. only 24 per cent chose the correct decimal number for $7 \frac{3}{20}$); (ii) computation with decimal numbers was difficult in all operations; (iii) and estimation

and approximation skills were poor (e.g. only 46 per cent of the sample could round off to the nearest centimetre).

3. Assessment of Performance Unit (APU). The APU monitors several aspects of students' learning in Britain. Foxman (1985) reports on nationwide tests over five years involving 150,000 students aged 11 and 15 years.

Relevant results for the present study from APU findings were: (i) Generally most students had serious misconceptions about numbers that included decimals; (ii) the most common error was a belief that 'longer' decimal numbers were smaller (thus 0.3753 was seen as smaller than 0.625, 0.25, 0.125, or 0.5); and (iii) the second most common error was to ignore the decimal point (thus 0.5 was seen as the smallest fraction from those in (ii) above).

In commenting on the 'largest is smallest' error, Foxman (1985) noted students were perhaps confused with common fractions where the larger the denominator the smaller the fraction, or with whole numbers where more digits indicated a larger number. In conclusion, Foxman (1985) recorded that a U.K. Board of Education publication in 1912 reported the 'longer is smallest' confusion. Obviously students have been constructing this error for some time.

4. Concepts in Secondary Mathematics and Science (CSMS) (1981). Relevant results from this study are reported under Brown (1981a) later in this chapter.

5. Joyce's (1984) Study in New Zealand. A very recent but smaller survey of mathematics achievement was made by Joyce (1984). In a personal communication with the writer, Joyce reported a survey of New Zealand 13 year olds' achievement in 'basic skill' areas, including decimal numbers. Over 200 students were tested. Difficulties with decimal numbers were: (i) Confusion over place value; (ii) ordering according to size; (iii) computational errors particularly with subtraction and division; and (iv) assigning decimal numbers to designated points on a number line.

To summarize, the surveys discussed above revealed certain common difficulties. These were: (i) Computational problems with decimal numbers in all of the basic operations; (ii) difficulties with estimating and approximating when 'rounding off', or as the result of an operation; (iii) errors in selecting decimal fraction equivalents for common fractions, and vice versa; and (iv) confusions with whole number conventions (e.g. the 'longer is smaller' error).

Overall, these survey studies (NAEP, IEA, CSMS, APU) have shown the same patterns for difficulties with decimals among students from different countries. However, only a few surveys (e.g. CSMS, APU) have employed individual interviews to probe error patterns more deeply. Indeed, Foxman (1985) reports that interviews were used to trace the genesis of the 'longer is smaller' error (above) only after mathematics educators could not explain the anomoly by examining the quantitative results.

Studies with a Qualitative Component

Research reported earlier in this chapter explores students' difficulties in mathematics and decimal numbers, and information on some of the causes of these difficulties can be inferred from the responses. However, this survey research method needs to be supplemented with detailed interviews if students' conceptual understanding and problem-solving strategies are to be explored fully. This latter research orientation is termed qualitative because it is concerned with the nature of responses from learners. The four research studies reviewed below all have a significant qualitative component.

1. Brown's (1981a) Research on Place Value and Decimal Numbers

Brown's sample consisted of 950 Year One to Four students (aged 12 to 15 years) in British comprehensive, mixed comprehensive, grammar and independent schools. Subsequently she conducted 39 individual interviews from 1976 to 1979. The interview sample comprised seven Year One, 15 Year Two, 11 Year Three, and 6 Year Four students. Interviews were based on the written survey test paper.

Brown's (1981a) main findings may be summarized as follows:

a) Approximately 60 per cent of Year Ones (n = 170) understood the basic ideas connected with the use of numbers expressed to the first decimal place (e.g. reading a scale marked in tenths; estimating a length to within an error of one tenth of a unit).

b) Numbers after the decimal point were treated as whole numbers. Only 25 per cent of Year Ones and 35 per cent of Year Fours could apply decimal place value principles consistently.

c) Addition was easier to perform than subtraction (with decimal numbers). 'Carrying' was an easier process than 'borrowing' for students.

d) Students used some 'rule' or principle when multiplying or dividing by powers of ten. Lower achievers often used the 'add-a-nought' rule without realizing this was inappropriate.

e) Problems with estimating and approximating were attributable to two misconceptions. First, very few students knew that it was <u>not</u> a universal rule that 'multiplication makes it higher and division makes it smaller'. Second, half the Year One students thought it impossible to divide a number by another larger than itself.

f) Problem-solving involving decimal numbers was very difficult. Some students believed decimals had limited applications and were "merely a game played in mathematics lessons" (Brown, 1981a:342). Selecting an operation to solve a word problem was more difficult with decimals than whole numbers.

Some of Brown's test items that are relevant to the present research are presented below. The facility levels come from approximately 200 students tested from each year.

Estimation and Approximation

				Facility level (%)			
				Year 1	Year 2	Year 3	Year 4
18(a)	Ring the number nearest in size to:						
	(i)	182	100/82/180/150/ 200/190	87	90	93	87
	(ii)	2.9	3/30/2/20/0/1	50	69	74	79
	(iii)	0.18	0.1/10/0.2/20/0/ 1/2	44	48	61	59
18(b)	Ring the number nearest in size to:						
	(i)	2.9 x 7	.002/.02/.2/ 20/200/2000	44	44	57	62
	(ii)	0.29 x 7.1	.002/.02/.2/ 2/20/200/2000	15	20	25	31

With Item 18(a)(ii), wrong responses ranged across all of the alternatives given. When asked to reconsider their answers carefully, incorrect respondents (in the sample interviewed) either stayed with their original answer, or changed but could not say why, or changed for an incorrect reason (e.g. Fi moved from 2 to 0 "because it's lower"). Thus, "...even some of the children who were fairly average in their understanding in terms of their final level were not able to appreciate an approximate order of magnitude of an apparently simple one-place decimal" (Brown, 1981a:286).

With Item Number 18(a)(iii), the proportion giving 20 as the answer was 30/25/18/17 per cent for Year One to Four students respectively. Brown considered that this might have been caused by the similar representations of '18' and '20' - there was no decimal point in either. Interviews showed that "...children seemed to reach answers very quickly and those who were asked to justify them tended to give tautological responses" (Brown, 1981a:286, 287).

In the items requiring approximation, very few pupils 'rounded up'. With Item 18(b)(i), unexpected strategies were adopted:

> "Julie, Billy, Danny and Richard all arrived at the correct answer of 20, but going via 'two sevens are fourteen'. However, both Fung Mei and David rounded up the seven explaining respectively, 'cos say that 7 was 10, then that would be 29, but it's less than that, so 20', and 'it'd be about 20 - 'cos it's timesing by 7 and 7 is near 10 so...'."
>
> (Brown, 1981a:287)

With Item 18(b)(ii), Brown noted that many pupils had just guessed, this being admitted during the interviews.

> "Wendy ringed 20 and when asked why simply said '200 is too big - I don't know why, it just looks it'."
>
> (Brown, 1981a:287)

The study revealed, too, that the working forms of algorithms suggested answering strategies:

> "Some of the children were so obsessed by algorithms that they were unable to understand the idea of approximation at all. Frances insisted on working out 18(b)(i), but lost the decimal point in doing so..."
>
> (Brown, 1981a:288)

Division with powers of ten. Where students were asked to divide by a power of ten (in this case by 100, and by 20) Brown found that if the answer required was a decimal number the facility level diminished. The results for two such items were:

			Facility level (%)			
			Year 1	Year 2	Year 3	Year 4
14(c)	Divide by one hundred					
	(i)	1600	52	57	69	60
	(ii)	3.7	24	27	34	41
14(d)	Divide by twenty					
	(i)	24	9	13	28	34
	(ii)	16	7	11	25	36

Children who had earlier been able to multiply by powers of ten by using an internalised rule such as moving the decimal point (or moving the digits around the decimal point), were able to use similar heuristics with the division process. But, as the facility levels indicate for Item 14(c)(ii), these 12 to 15 year olds had difficulty with division examples where the divisor was larger than the dividend, and with the example involving decimal numbers, Brown recorded some inconsistencies with her interviewed subjects:

> "Danny, Francis and Juli refused to divide 26 by 200 on the grounds that it was impossible, although they had all earlier managed to successfully divide decimals by a hundred. Jane B. and Lisa reverted to a 'remainder' strategy, although when asked to reconsider her answer of 0.16 to 14(d)(ii) (divide 16 by 20) Lisa did write 0.7 explaining '20 into 16 doesn't go, then add another 0... 20 into 160 goes 7...no, 20, 40, 60, 80, 100, 120, 140, 160... no, it's 8' and corrected her answer to 0.8."
>
> (Brown, 1981a:298)

Brown speculated that only a group of around 10 per cent of Year One students (12 year olds) rising to 35 per cent of Year Four students (15 year olds) accepted that division of a smaller number by a larger number was possible, and that the result could be expressed as a decimal. On the whole, though, Brown found that the students did behave consistently. Ideas learned in multiplication did transfer to division although, as pointed out earlier, facility levels for items like 3.7 divided by 100, and 16 divided by 20, were low.

When multiplying with decimal numbers, compared with dividing by decimal numbers, Brown found that almost all children considered that multiplication would give a bigger answer. As Tracey put it, "'..when you're timesing you get a bigger answer than when you share.'" (Brown, 1981a:299)

Most children confidently believed that multiplication enlarged and division decreased, irrespective of the numbers involved. Only between 13 and 18 per cent of Year One students recognized the effect of the multiplier or divisor being less than one.

Problem-solving and decimal numbers. In problems with decimal numbers, facility levels were again low. Some examples follow:

				Facility levels (%)
"19.	Ring the calculation you would need to do to find the answer:			
(a)	A table is 92.3 centimetres long. About how many inches is this? (1 inch is about 2.54 centimetres)	2.54 + 92.3 2.54 ÷ 92.3 2.54 - 92.3	92.3 ÷ 2.54 92.3 - 2.54 92.3 x 2.54	27/35/44/45
(d)	The cost of the 6.44 gallons of petrol was 4.86. What would the price of one gallon be?	6.44 + 4.86 6.44 ÷ 4.86 6.44 - 4.86	4.86 ÷ 6.44 4.86 - 6.44 4.86 x 6.44	19/23/30/28 "

(Brown, 1981a:307)

All word problems were difficult for students. In many cases the children could not start to solve the problem or guessed the answer. 19(d) involving the 'sharing' model of division was found to be a little easier than 19(a) which uses 'grouping'. Brown suggests that sharing seemed to be the best established model in terms of stories that the children constructed.

Items involving decimals were significantly more difficult than those involving natural numbers; a significant proportion of children could only relate number operations to real situations when using whole numbers. Brown suggested this difficulty stemmed from children's difficulties in extending the physical actions with whole number problems to situations where fractional numbers were used. Brown found little evidence of children using approximation to help them solve problems and choose the correct operation - only two of her interviewees used this strategy.

Another interesting finding was inversions to division items involving decimal numbers. Thus 34, 39, 43 and 40 per cent of Year One, Two, Three and Four students respectively chose $6.44 \div 4.86$ as the answer to Item 19(d), when the reverse was the correct response.

Comment:

Some limitations in Brown's research appear to be:

a) Interviews were used to validate the written test, and were not the major focus of her research.

b) The interviewer and subject worked through the written test paper which necessarily constrained interview content.

c) Individual students had significantly different interviews about the students' prior responses. This suggested under-standardization.

d) Full interview protocols comprised 70 questions (though individual interviews were shorter) which might be too long and result in a cursory exploration of many questions.

e) Details of the interview sample (age, score, level of understanding in the written test) were minimal.

f) Brown's aim was to investigate the formation of six levels of understanding in decimals (the orientation of the whole

CSMS study (Hart, 1981)) from wide-scale written tests. Checking was provided through 30 interviews. Thus, Brown's major concern was in identifying the six levels of understanding rather than in probing students' underlying ideas and mini-theories (Claxton, In press a).

g) The study was not longitudinal so the stability of errors described with time could not be explored.

2. Erlwanger's Studies of Children's Conceptions of Mathematics (1973, 1975)

Erlwanger's sample consisted of nine students from Grades Four, Five and Six levels (aged 10 to 12 years). Students were observed in their classrooms, discussions with pupils and staff were held, and each of the nine students was interviewed from four to eight times with an interval of one to two weeks between interviews. Interviews averaged 30 minutes duration.

Erlwanger's (1973, 1975) main findings may be summarized as follows with illustrated material coming from the case study of Benny:

a) A number of interesting errors showed confusions with 'rules' that students developed from their individualized mathematics programmes. For example, Benny thought there were different rules for different problems, i.e. rules were situation specific.

b) Above average students, such as Benny, were particularly adept at inventing procedures. For example, Benny thought the common fraction equivalent for 0.5 had to be $\frac{3}{2}$ or $\frac{2}{3}$, "as long as it comes out with the answer five" (Erlwanger, 1973:9).

c) Benny used whole number strategies when computing with decimal numbers. For example, he computed: $4 + 1.6 = 2.0$; $7.48 - 7 = 7.41$; $8 \times .4 = 3.2$; $.2 \times .3 \times .1 = .006$.

Erlwanger's research has relevance for the present study in terms of the students' responses, and heuristically. Some of this detail is now discussed.

Erlwanger (1975) adopted the view that in the course of learning mathematics the student developed his/her own conceptual system of interrelated ideas, beliefs, emotions and views concerning both mathematics and the learning of the subject. Such ideas directed and controlled mathematical behaviour in terms of what was learned, and what was understood.

Since learning and thinking were psychological rather than mathematical processes, Erlwanger argued, that the child's understanding and thinking are conceptual in nature. He explained: "...while mathematical principles and relationships may lead to inferences about specific aspects of a child's external behaviour, such inferences need not be the same as the ideas, beliefs and views underlying his work" (Erlwanger, 1975:167).

Erlwanger stated that research into children's conceptions of mathematics should be conducted through flexible explorations of children in the classroom setting, as well as by interviews and discussion which probe aspects of children's work. Erlwanger cited sequences from Brownell and Watson (1936) and Lankford (1972) and noted the lack of depth in these extracts. By contrast Peck and Jencks' (1973) interview sequences demonstrated the value of flexible, exploratory interviews, revealing "that apparently successful children may have developed a different understanding of aspects of mathematics; and it also suggests the direction for further discussions with the child" (Erlwanger, 1975:171).

Erlwanger's (1973, 1975) results come from his case studies. Benny, in particular, is often quoted, and some of his errors were noted a little earlier. The following sequences show: (i) Benny's use of whole number strategies with decimals, and (ii) his conception of rules in mathematics (E = Erlwanger: B = Benny).

i) "*E:* *Like, what would you get if you add .3 + .4?*
B: *That would be...oh seven (07).... .07.*
E: *How did you decide where to put the point?*
B: *Because there's two points; at the front of the 4 and the front of the 3. So you have to have two numbers after the decimal, because...you know...two decimals. Now like if I had .44, .44 (i.e. .44 + .44) I have to have four numbers after the decimal (i.e. .0088.* "

(Erlwanger, 1975:205)

ii) "*E:* *Let's take your first example, where you said 2 + .3 = .5. 2 is a whole number. What happens to it when you add it to a decimal?*
B: *It becomes a decimal.*

E: You mean it happens just like that?
B: No! Mm...I would really like to know what happens. You know what I'll do today? I'll go down to the library...I am going to look up fractions, and I am going to find out who did the rules, and how they are kept."

(Erlwanger, 1973:19)

An important aspect of Erlwanger's work that has attracted comment by Davis (1979), and Davis and McKnight (1979) concerns the types of errors made by Benny and his classmates. Many misconceptions were at the level of 'super' (or 'executive') procedures, rather than 'mini-procedures'. The students could add, multiply and subtract correctly (all 'mini-procedures'), but could not grasp which over-riding principles, decisions, and understandings were needed ('executive-procedures'). Davis (1979) has suggested that this was caused by the instructional programme. In terms of modern mathematics instructional programmes (which have stressed learning with understanding for the past three decades), this interpretation of Erlwanger's results is disquieting since the errors reflect a <u>lack</u> of the very understanding that modern mathematics programmes claim to stress.

<u>Comment</u>:

Erlwanger's (1973, 1975) studies followed the clinical interviewing procedures used by Piaget (1929, 1941). Thus Erlwanger engaged in relatively free conversations with the students with direction being provided by key questions. Notwithstanding the advantages of this approach for understanding students' mathematical ideas, Erlwanger's research has a number of limitations:

a) There was a tendency during interview sequences to positively reinforce correct responses from students. Bell and Osborne (1981) and Codd (1981) advise against this practice because it might encourage interviewees to follow particular 'paths'.

b) Some interview sequences suggested that further probing was needed - incongruous, considering Erlwanger's stated aim and his criticism of Brownell and Watson (1936) and Lankford (1972) to discover students' underlying ideas.

c) Erlwanger's sample was exposed to one teaching programme - programmed learning texts - so that his findings might not generalize to students encountering alternative and more commonly-employed teaching methods.

c) Erlwanger's research is primarily descriptive and lacks a theoretical base.

e) Erlwanger's research is not genuinely longitudinal, since field-work lasted only a few months.

f) The research focussed on student ideas about operations on, and recording conventions with, decimal numbers. But it did not explore problem-solving aspects.

3. Bell *et al*'s Study of Problem-Solving with Decimal Numbers

The sample consisted of 20 pairs of students (aged 12-16 years). Written tests and audio-taped or video-taped interviews were used. The students were given decimal-related problems to solve and asked to explain their methods, using prompts such as 'rephrase the problem in your own words'.

Bell *et al*'s main findings may be summarized as follows:

a) Understanding of decimal place value was poor, e.g. 1.071b was read as "one pound seven ounces".

b) Students thought that multiplication inevitably made 'things' larger. For example, 1.17 x 8.6 was easily solved, but 1.20 x 0.22 was difficult, with students believing that division was needed to reduce 1.20 (c f. Brown, 1981a).

c) Problems with division. Students assumed that the smaller number must always be divided into the larger. A second confusion was that a ÷ b, was $a\overline{)b}$ (c f. Brown, 1981a).

d) Some students were unhappy about multiplying two numbers with differing units, e.g. 1.07 x 0.86 was not performed because the first number represented weight, the second money.

e) Links through terms such as 'go into' proved helpful. The authors believed that a more deliberate supplying of these cues would assist students.

f) Where easier numbers were used in problems, students were more successful in constructing answers (c f. Brown 1981a). However, they found it difficult to choose easier numbers themselves, and their solutions were often intuitive and unclear.

g) Estimation was seen as a powerful way to discover an error, but gave the students no help in finding the error's origin, nor in correcting it.

h) Diagrams constructed by students to help solve problems were unsuccessful. Many students used diagrams for decoration, others simply to compare size, and only a few to indicate a possible algorithm. Diagrams presented by the interviewers, on the other hand, were found useful because they (i) removed words to give students an independent uncluttered view of the problem; (ii) enabled pupils to estimate solutions; and (iii) frequently led to a possible strategy for solving a problem.

The final phase of the research (Bell *et al*, 1981) involved one of the research team teaching the class 11 lessons, each of 70 minutes. Eight activities within these lessons were directed at improving the understanding of decimal-place value, and the effect of multiplication and division on the size of a number. (Teaching strategies involved teacher-explanations, pupil worksheets, calculator-based games, and making a peer's homework activity.)

Students' understanding of decimal place value improved significantly, there were more modest improvements in other problem areas, and the error that "multiplication always makes larger" persisted (Brown, 1981a).

Comment

A number of limitations in Bell *et al*'s study can be noted:

a) The study was essentially a teaching experiment using a diagnostic model (c.f. Ashlock, 1976; Reisman, 1972). Individual students' underlying constructions of meaning were not explored during the interviews.

b) Interviews were used as validation procedures for the written diagnostic test (c f. Brown, 1981a), not as primary instruments.

c) The genesis of particular errors was not discussed.

d) Interview formats were prescribed by given problems and associated number operations. Consequently, interviewees did not generate their own algorithms.

e) The teaching model was traditional: test, teach, re-test. This model did not allow students to construct their own questions as starting points for a lesson (c f. Biddulph and Osborne, 1984), take greater responsibility for their own learning (c f. Baird and White, 1984), nor did it break new ground heuristically.

f) Facility levels with parallel CSMS items (Brown, 1981a) provided confirmatory evidence, but new knowledge was absent.

In summary, Bell *et al*'s research lacked the flexibility of Erlwanger's study, and the breadth of information gained from Brown's investigation. A wider perspective for students' conceptions of decimal numbers was provided by the Hiebert and Wearne (1983, 1984) and Wearne and Hiebert (1984) studies which are reviewed in the next section.

4. Students' Conceptions of Decimal Numbers: Hiebert and Wearne (1983, 1984); Wearne and Hiebert (1984)

Hiebert and Wearne administered a written test to 115 Fifth Graders, 256 Seventh Graders, and 212 Ninth Graders. Subsequently structured interviews were held with 25 students from Grades One, Five, Seven and Nine. Interviews were held in the months of December, March and April.

Hiebert and Wearne's (1983, 1984) main findings may be summarized as follows:

a) Students had difficulties writing decimal numbers when based on a pictorial display using place value blocks as the stimuli.

b) Deficiencies in whole number knowledge were apparent with Fifth Graders. For example, the item 'In changing the 5 in 351 to 7, how much bigger does the number get?' was answered correctly by only a third of the Fifth Graders (n = 53). Other conventional pencil-and-paper items revealed adequate but not strong performance. For example, an item asking students to identify the place value of 3 in 645.37, was correctly answered by 48 and 74 per cent of Seventh and Ninth Graders respectively.

c) Confusions with whole number strategies were evident (c f. Brown, 1981a; Erlwanger, 1973, 1975). For example, the most common error for 'write a number ten times as big as 437.56' was to add a zero to the end (c f. Grossman, 1983). Many Fifth Graders ignored the decimal point (c f. Foxman, 1985).

d) Decimals as rational numbers were better understood in some cases by the younger students. For example, a greater proportion of Fifth Graders than Seventh or Ninth Graders could identify 0.4 of a region.

e) Writing decimal number equivalents for common fractions was difficult for Fifth Graders (c f. NAEP, IEA results).

f) Writing decimal numbers between two given numbers was easy for '1.56 and 1.72', but troublesome for '0.2 and 0.3'.

Many of Hiebert and Wearne's (1983, 1984) and Wearne and Hiebert's (1984) findings were similar to those given in the survey and interview studies reported earlier in this chapter. Some general points made by Hiebert and Wearne are now discussed.

In some cases the added knowledge possessed by older students seemed to produce confusions. Thus, knowledge about hundredths was weaker in Seventh and Ninth Graders than with Fifth Graders. The writers commented: "...it remains a disconcerting fact that instruction introduces some errors that would not otherwise appear" (Hiebert and Wearne, 1983:21).

Hiebert and Wearne hypothesized that most students did not come to decimal numbers through common fractions, and that they didn't give meaning to decimal number symbols based on their knowledge and understanding of common fractions. Rather, the authors felt that the linking between symbols and understanding came much later, after students had developed some high level skills in manipulating both common fraction and decimal number symbols.

Hiebert and Wearne distinguished between 'decimal form' and 'decimal understanding'. The former referred to rule components that could be thought of as the 'syntax' of the system. The researchers believed that students in their study had created few links between decimal form and decimal understanding.

In discussing schooling, Hiebert and Wearne felt that instruction was more successful in teaching the symbolic form of decimal numbers than understanding. Very few of the errors in their study violated the conventions of standard decimal notation. Thus student perceptions of decimal numbers seemed to be influenced more by their knowledge of form, than by their understanding. Students viewed a decimal number as a new way of writing a number. And accompanying this new form came a set of rules that guided the use of the relevant symbols (c f. Erlwanger, 1975). The authors also remarked that responses elicited during standardized interview tasks revealed some fundamental understandings of the decimal system of notation, but that this tended to be over-ridden by the students' own knowledge of form.

According to Hiebert and Wearne, the implications for schooling were:

i) that instructions must be designed to assist the linking of form and understanding;

ii) that learning would be increased by helping students to make connections between form and understanding from the outset; and

iii) that one possible strategy might be to take advantage of students' inventive powers, and have them (the students) "create an informal, transitional symbol system that has meaning for them".

(Hiebert and Wearne, 1983:27)

In subsequent writings Wearne and Hiebert (1984) have noted the meaningless manner in which students work with decimal numbers, the over-generalization of concepts about whole numbers to decimal numbers, and the failure of students to realise that inappropriate answers are inappropriate.

Three levels (or 'sites') of errors were identified. First, many students do not know what the symbols mean; second, many students do not know why a computational procedure works in a particular instance; third, many students fail to realize that their answers are unreasonable. The writers provided illustrative errors at each 'site', and Hiebert (1984) has suggested teaching strategies that might promote the linking of symbols with meaning. Additionally they state "...we believe that, in many cases, the critical instructional problem is not one of teaching additional information, but rather one of helping students connect pieces of information that they possess" (Hiebert and Wearne, 1984:6). The relevance of the generative learning model is brought to mind.

Comment:

The research of Hiebert and Wearne (1983, 1984) and Wearne and Hiebert (1984) yielded data similar to Brown (1981a), Erlwanger (1973, 1975) and Bell *et al* (1981). Hiebert and Wearne concentrated on the linkage that students make, or fail to make, between form (symbols) and understanding. As these researchers point out, the distinction is not new, and has been discussed previously by Skemp (1971).

In considering their work, the following critical points can be made:

a) The research was essentially cross-sectional in design. Results were presented in grade levels, and a genuine longitudinal study might have provided more valuable data.

b) Progress and conceptualizations for individual students were not provided in the reports. Data reporting generally followed a nomothetic pattern, and thus lacked qualitative detail.

c) The discussion of links between 'form' and 'understanding' parallels Skemp's (1971) 'habit learning' and 'intelligent learning'. Bloom's (1956) 'knowledge' and 'comprehension' levels of thinking were also similar. Perhaps Hiebert and Wearne (1983) have invented terminology unnecessarily.

d) The three 'levels' or 'sites' of errors identified cannot be thought of as representing the full set of potential 'sites'. For example: At which 'site' would a failure to transfer knowledge to the real world be found? Where would affective variables be situated?

e) Hiebert and Wearne's (1983) pedagogical suggestions might increase student confusions. The writers suggested that students should invent idiosyncratic informal, transitional symbol systems to link 'form' and understanding. Research in science and mathematics learning has shown the difficulty in shifting students from incorrect conceptualizations (Bell, 1984; Freyberg and Osborne, 1981; Osborne *et al*, 1982; Van Lehn, 1982, 1983).

Summary and Conclusions

This chapter has reviewed concerns in mathematics education in general, and research into students' knowledge and understanding of decimal numbers in particular. The trends in the results of survey research have been noted, and four studies with qualitative research components have been described and evaluated. These four studies (Brown, Erlwanger, Bell *et al*, Hiebert and Wearne) used quantitative and/or qualitative methods to identify student difficulties and confusions with decimal numbers. The four studies revealed a lack of articulation with learning theory. Thus, although Brown's (1981a) work was related to the CSMS research programme which had Piagetian overtones, Brown herself failed to explain student thinking on the basis of any clear theoretical standpoint. Likewise, Erlwanger, Bell *et al*, and Hiebert and Wearne reported their findings largely at the descriptive level without discussion in terms of a model of human learning.

These four studies had varying purposes. Brown and Hiebert and Wearne were concerned with diagnosis and levels of understanding. Bell *et al* explored teaching strategies, while Erlwanger focussed on the ideas that individual learners constructed. This variability meant the studies had differing degrees of relevance and suggestiveness for the present researcher's study.

Together, however, the studies provided comprehensive background on student conceptions of decimal numbers, and added to the data stemming from purely quantitative studies (e.g. NAEP; IEA; CSMS; APU).

In the present study it was decided to adopt theoretical positions from constructivist psychology and the generative learning model, giving primacy to the learner's existing ideas and the individual's construction of meaning. The methodological orientation of the present study is described in the next chapter and integrates quantitative and qualitative research designs as exemplified by Brown (1981a). However, unlike Brown's research, interviews with individual students over a considerable time period were undertaken. Methodologically this called for the present study to investigate and identify students' ideas on a large scale (the quantitative component), to explore these ideas in general detail by means of interviewing (the qualitative component), and most importantly, to conduct these interviews so that opportunity was provided for students to reveal the nature of their thinking on mathematical ideas - in short, the manner in which they constructed their mini-theories.

CHAPTER FOUR

THE RESEARCH DESIGN

Concerns expressed in the Introduction, together with clear indications in the research literature of the difficulties experienced by students in dealing with decimal numbers, determined the focus of the present study. The main purposes of the research were to explore what might lie behind student difficulties with decimal numbers and to monitor student conceptualizations over a period of time.

Recent research on students' mathematical understandings have shown the potency of qualitative assessment devices (Brown and Kucheman, 1976, 1977; Bryant and Kopytynska, 1983; Davis *et al*, 1978; Ginsburg, 1975; Knight, 1982; Krutetskii, 1976) and the usefulness of longitudinal studies of a quantitative nature (Hart, 1981) and the qualitative orientation (Erlwanger, 1973, 1975). In the present study, it was decided to combine the quantitative and qualitative approaches.

As an initial strategy, it was hoped that error patterns in student difficulties would be revealed by a written survey instrument used with a representative sample of students. Then, data from this survey (the quantitative study) might well suggest areas to be probed in a subsequent series of individual interviews over time with a further but smaller sample (the qualitative study).

Research Questions

The literature review indicated both general and specific areas of concern about students' knowledge and understanding of decimal numbers. Some studies were suggestive in terms of decimal-related content that should be addressed (Brown, 1981a; Carpenter *et al*, 1984; Hart, 1981; Hiebert and Wearne, 1983). Other studies were suggestive heuristically (Erlwanger, 1973, 1975; Knight, 1982), while others, by their interpretations of results,indicated a need for further research (Hiebert and Wearne, 1984; Wearne and Hiebert, 1984).

The present study investigated students' conceptions of decimal numbers, it being clear from the research literature that basic misunderstandings lay at the bottom of most incorrect answers to decimal-based computations and problems. These misunderstandings were to be interpreted by reference to the theoretical notions of mini-theories (Claxton, In press a, in press b) and the generative learning model (Osborne and Wittrock, 1983, 1985).

It was decided to focus on learners from age 11 to age 14, this period being one when schools give much emphasis to decimalisation. The transition from primary (elementary) to secondary (high school) schooling usually occurs during this time, and this primary/secondary interface has been an area of concern in the past (Cockcroft, 1982; Cornelius, 1982; Walton, 1979). Cornelius, for example, has stated that many secondary teachers are unfamiliar with the approach to mathematics which their pupils experienced at the primary stage. Likewise, Cockcroft has claimed that many primary teachers do not know the kind of mathematics which students undertake during their first term at secondary school.

A survey and a longitudinal study that spanned the primary/secondary schooling interface might thus provide data on the specific success and difficulties with decimalisation that students experience, and provide useful information for both primary and secondary mathematics teachers.

The present study investigated the following themes and related research questions:

<u>Theme A : Problems in Dealing with Decimals</u>

i) What difficulties do 11-14 year old learners encounter when using decimal numbers in the contexts of estimation and approximation, handling basic mathematical operations and problem solving?

ii) What types of errors are made when decimal numbers are written from a verbal stimulus?

<u>Theme B : Meanings for Decimal Numbers</u>

i) What meanings do students construct from the information presented by teachers, textbooks, peers and other sources?

ii) How persistent are students' constructed meanings over a period of time?

iii) Can these meanings be explained by the generative learning model, including the generation of mini-theories (Claxton, In press a, in press b).

Theme C : Affectivity and Decimals

i) How prevalent and significant are affective variables in learning about decimal numbers?

ii) What problems, if any, do students see in making the shift from primary to high school insofar as mathematics learning is concerned? What do students view as good mathematics teaching? What perceptions do students have of each other's views on mathematics teaching and learning?

Theme D : Implications for Schooling

i) What are the implications for teachers of the errors that students generate when dealing with decimal numbers?

ii) How do students relate their school knowledge of decimals to the outside world?

Research Design

The research design comprised three phases: (i) An initial survey to map student strengths and difficulties in handling decimal numbers and decimal fractions; (ii) the production of stimulus cards for interview purposes; and (iii) the main study with a small sample of students. The main study would attempt to identify, over a two-year period, students' successes and failures with decimal operations, the reasons behind these, and the students' attitudes towards mathematics in general. The research design could thus be described as a cross-sectional and longitudinal one.

The Research Schedule

The sequencing of the different phases of the research schedule are outlined below:

Date		
Date : 1980	Term III (September)	The Initial Survey 11-14 year olds (n = 102)
1981	Term I (February)	Piloting of Stimulus Cards Series One (n = 39)
	Term II	Longitudinal Sample chosen
	Term III	Interview I: 11 & 12 year olds (n = 30)
1982	Term I (February)	
	Term II	
	Term III	
		Interview II: 12 & 13 year olds (n = 28)
1983	Term I (February)	
	Term II	
	Term III	Interviews III & IV: 13 & 14 year olds (n = 28)

Phase 1: The Initial Survey

The subjects here were 102 students aged 11-14 years randomly selected from one intermediate and one high school to be representative of the normal population in terms of academic achievement, sex and ethnicity. FIGURE 2 shows the nature of the survey sample.

SCHOOL	Students								TOTAL
	Form I		Form II		Form III		Form IV		
	Male	Female	Male	Female	Male	Female	Male	Female	
Intermediate (urban)	16	16	13	10	-	-	-	-	55
High School (urban)	-	-	-	-	10	14	11	12	47
									102

FIGURE 2 : Research Sample: Initial Survey

The main source for the initial survey was a written test instrument designed by Brown (1981a) for 12-15 year olds in the United Kingdom. This test was modified so that it was content-valid for New Zealand students, and then checked against the New Zealand Department of Education's Mathematics Syllabus, Form I-IV (1972), and by consultation with the Heads of Mathematics in the research schools.

Students taking the written test were supervised by their usual mathematics teacher. Time limits were not imposed and, where necessary, help was given by the teacher with the reading of particular items. Most students completed the test in approximately 30 minutes, with a few taking as long as 50 minutes.

A copy of the written test used in the initial survey may be found in APPENDIX C. The instrument covered the following areas within the topic of Decimal Numbers.

Question 1: Naming the place value for digits in whole numbers (three items).

Question 2: Naming the place value for digits in decimal numbers (four items).

Question 3: Adding ten, one tenth, one hundredth, and subtracting one hundred (12 items).

Question 4: Identifying and naming points on a number line that uses decimal numbers (five items).

Question 5: Adding one more to the whole number "6399" (one item).

Question 6: Naming a number two less than seventeen thousand (one item)

Question 7: The significance of zero at the end of a number using decimal numbers (one item).

Question 8: Comparing decimal numbers and naming the larger (five items).

Question 9: Writing decimal numbers (four items).

Question 10: Writing numbers between two given numbers (five items).

Question 11: Adding decimal numbers (two items).

Question 12: Multiplying and dividing with decimal numbers (eight items).

Question 13: Estimation, including identifying the number nearest to an answer estimated from algorithms involving decimal numbers (six items).

Question 14: Problem solving using traditional 'textbook' problems with decimal numbers and identifying the process required to achieve an answer (five items).

Chapter Five provides a detailed report on the responses to this initial survey. Since the findings were the basis for the content of the interviews in the longitudinal study that followed (see Phases 2 and 3 below) we note here that particular difficulties for students were as follows:

i) Writing decimal numbers.
ii) Division yielding a decimal number less than one as the quotient.
iii) Estimation and approximation.
iv) Problem solving involving decimal numbers.
v) Locating decimal numbers on a number line.

Phase 2: Development of Stimulus Cards (Series One) for Interviews I to III

Stimulus cards were designed that reflected the student difficulties identified above. Additional cards involved the comparison of numbers containing identical digits but with differently sited decimal points (e.g. 123 and 12.3). These additional cards resulted from exploratory work with 11-14 year olds prior to the present research which revealed student confusion about how many times one number was larger or smaller than another (which also explores the ideas of estimation and approximation).

Three draft versions of the Series One Stimulus Cards were developed and trialled during late 1980 and the first three months of 1981. Interviews for this pilot work were conducted by the researcher in an urban high school and two urban intermediate schools. A total of 39 students was used, none of these students becoming members of the main research sample.

The trials resulted in very little modification to the mathematical content of the original stimulus cards, although card lay-out and interview patterns were changed to ensure that students clearly understood the content of each card and could construct their own responses. The latter was especially important because the individual conceptualisations of students were to be the focus of the main research study.

To illustrate: (i) For Cards K to N (see FIGURE 3), the partial working solutions were hidden from students, only the verbal problem being visible. Once a student had suggested (or had failed to suggest) strategies for solving a problem, the algorithms were revealed by the researcher and used to elicit further responses. (ii) The semantic content of cards was checked to ensure that it was correctly interpreted by students. Thus in Cards A to F (see FIGURE 3) the words "is nearest in size to?" replaced the original phrasing "nearest to" in Cards A to F because students read the latter wording to mean "nearest in physical proximity to", rather than the intended number relationship. (iii) The content of some cards was extended in order to explore student conceptions of more difficult quantitative expressions. Thus Stimulus Card J (see FIGURE 3) originally contained three decimal numbers but was extended to include the two verbal stimuli "seven hundredths" and "twenty hundredths" to cover the concept of hundredths, and the idea of a hundredths fractional number being reducible to tenths (twenty hundredths).

The final version of the Stimulus Cards (Series One) had the following content focusses:

i) Estimation and approximation.
ii) Estimation and approximation involving computation.
iii) Division, including examples where the divisor was larger than the dividend.
iv) Writing decimal numbers.
v) Problem solving involving decimal numbers.
vi) Comparisons with numbers including decimal numbers.

Phase 3: The Main Study

The Subjects

Once the final version of the stimulus cards had been decided, 30 students were chosen to form the basis for the main research project. The sample comprised 15 students from a Form One class and 15 students from a Form Two class in a Hamilton intermediate school. The rationale for this cross-grade selection was that a wider age spread could be studied over

Area	Card No.	Format of Card	Source
Estimation	A	186 is nearest in size to? 100/80/180/200/150/190	Brown (1981a)
	B	1.2 is nearest in size to? 1/10/0.2/12/0/2/0.12	Brown (1981a)
	C	0.8 is nearest in size to? 1.8/0.08/8/1/0	Brown (1981a)
Estimation after computation	D	1.9 x 5 is nearest in size to? 0.01/0.10/1.0/10/20/100	Brown (1981a)
	E	0.19 x 5 is nearest in size to? .01/0.10/1.0/10/20/100	Brown (1981a)
	F	0.19 x 0.5 is nearest in size to? 0.01/0.10/1.0/10/20/100	Carr
Division by 10 and its multiples	G	Divide by 10: 1000 100 1500 15	Carr
	H	Divide by 100: 1000 100 1500 15	Carr
	I	Divide by 20: 200 100 10 25 15	Carr
Writing decimal fractions	J	Write as decimals: three tenths seventeen hundredths fifteen hundredths seventeen tenths twenty hundredths	Brown (1981a) Carr
Problems involving decimal fractions	K	The price of potatoes is $1.50 for each kilogram. What is the cost of 0.5kg of potatoes? ---------------fold--------------- 1.50 + 0.5 0.5 + 1.50 1.50 + 0.5 0.5 - 1.50 1.50 - 0.5 0.5 x 1.50	Brown (1981a)
	L	The price of mince is $2.67 for each kilogram. What is the cost of a packet containing 0.58kg of mince? ---------------fold--------------- 2.67 + 0.58 0.58 + 2.67 2.67 + 0.58 0.58 - 2.67 2.67 - 0.58 0.58 x 2.67	Brown (1981a)
	M	The cost of 2.5 litres of soft drink is $0.75c. What would the price of <u>one</u> litre be? ---------------fold--------------- 2.5 + 0.75 0.75 + 2.5 2.5 + 0.75 0.75 - 2.5 2.5 - 0.75 0.75 x 2.5	Carr
	N	The cost of 6.44 litres of petrol was $3.58. What would the price of <u>one</u> litre be? ---------------fold--------------- 6.44 + 3.58 3.58 + 6.44 6.44 + 3.58 3.58 - 6.44 6.44 - 3.58 3.58 x 6.44	Brown (1981a)
Comparisons with numbers containing decimal fractions	O Q	0.4 4 0.80 80	Carr
	P	123 12.3	Carr
	R	249 2.49	Carr

FIGURE 3 : Stimulus Cards (Series One) Used in the Individual Interviews I-III on Decimal Numbers

the two year research study, than would have been possible if a sample from one class (grade) level were investigated for two years.

To select the sample, one Form One and one Form Two teacher were chosen at random from the staff of the intermediate school participating in the study. Each teacher's class was unstreamed - that is, it comprised a heterogeneous group ranging in general ability from 'very able' to 'slower learning' students. Both classes were administered two standardized group tests: the PAT Mathematics (NZCER, 1972) and TOSCA, a Test of Scholastic Ability (NZCER, 1981). The Primary Form A of TOSCA was used with subjects under 12.5 years and the Secondary Form A was used with subjects over 12.5 years. Data derived from these two norm-referenced instruments enabled the researcher to randomly choose 15 subjects from each of the Form One and Form Two classes such that a spread of general and mathematical ability was evident in each research group (see FIGURE 4).

Potential subjects were asked if they were willing to take part in a project that would last for two years, and would involve four 'conversations' over two years on a one-to-one basis with the researcher. The purpose of the study was described to the subjects as being to find out how ordinary New Zealanders of their age thought about some aspects of mathematics. At this stage none of the 30 students declined to take part.

The Interviews

Chapter Two and Chapter Three referred to the importance of clinical interviewing in research on mathematics learning. Clinical interviewing was a major focus of the present research. Indeed, because of the longitudinal nature of the present study and its focus on individuals' constructions of meaning, more than one interview was scheduled. Furthermore, the qualitative data from the interview series would be compared with the results from the pencil-and-paper survey of Phase 1 of the research. Subjects were interviewed on an individual basis four times over a two-year period. FIGURE 5 shows the time scale of these interviews.

Name (Gender retained) (pseudonyms)	Age (March 1981)	Sex	Race	TOSCA Percent-ile Rank	PAT Mathematics Percentile (age) Rank	PAT Mathematics Percentile (class) Rank
Form I (1981)						
John	11.5	M	European	83	97	99
Andrea	11	F	"	98	95	96
Trish	11	F	"	90	83	81
Don	11	M	"	42	66	63
Mary	10.5	F	"	95	63	49
Anne	11.5	F	Maori	53	62	69
James	11	M	European	59	55	52
Peter	12	M	"	53	54	81
Delwyn	11.5	F	"	49	52	60
Percy	12	M	"	51	43	60
Neville	11	M	"	9	24	17
Janette	11.5	F	"	24	17	17
Ronald	11	M	"	31	13	10
Dorothy	12	F	Maori	1	2	10
Rita	12	F	"	43	1	10
Form II (1981)						
Garth	12	M	European	79	95	96
Petra	12	F	"	53	87	85
Suella	12	F	"	75	76	76
Charles	11.5	M	"	79	75	64
Dwight	11.5	M	"	64	70	58
Trudy	12	F	"	53	58	54
Danny	12.5	M	"	75	54	61
Rex	11.5	M	"	46	54	40
Stu	12	M	"	72	49	43
Jackie	12.5	F	"	44	33	37
Andy	12.5	M	"	24	19	20
Doug	12.5	M	Maori	6	16	16
Stella	12	F	European	8	11	4
Wally	13	M	"	23	9	24
Jude	13	F	"	31	9	20

FIGURE 4 : The Research Sample

Interview I	:	Term III, 1981
Interview II	:	Term III, 1982; Term I, 1983
Interview III		Term III, 1983
Interview IV		Term III, 1983

FIGURE 5 : Schedule of Interviews

The purpose of the first three interviews was to explore students' responses to the Series One Stimulus Cards. Each interview was conducted according to the following procedure:

i) An introduction reminded the subject of the purpose of the study, emphasizing that the session was not a test, and that what was required were 'ideas' about some aspects of maths. The confidentiality of responses was stressed.

ii) The eighteen stimulus cards of Series One were presented to the subject in an unhurried fashion.

iii) Perceptions of decimal numbers in relation to other topics in mathematics were gauged by the use of topic cards which students were asked to rate on a like - neutral - dislike basis (see the section on 'Procedures for Measuring Attitudes Towards Mathematics and Decimal Numbers' for details).

iv) The subjects were asked to name the columns on either side of a decimal point, to label these, and to explain their responses.

v) The interviewer then explored student ideas that seemed to warrant further probing.

vi) The interview concluded with the interviewer thanking the subject for his/her help.

The fourth interview focussed on students' constructions of meanings about decimal numbers. Students' attitudes about mathematics in general, and decimals in particular were also explored. Although Interviews II and IV were both scheduled in Term 3, 1983, a one week gap was allowed between the two sessions (except in the case of students who were interviewed out of Hamilton). The format for the fourth interview was as follows:

i) Ten alternative Stimulus Cards (Series Two) based on those used in the first three interviews, were used to probe a subject's understanding in more depth if data from Interview III were insufficient.

ii) A version of the semantic differential test (Osgood, Suci and Tannenbaum, 1957) was administered, and the responses that the subject had given were discussed. It is important to note that the small size of the student sample (n = 28) precluded systematic processing of responses by factor analysis. In the circumstances, it was decided to merely describe the response patterns that occurred to the 12 sets of polar adjectives.

iii) An exploratory discussion on the primary/secondary schooling interface was held, focussing on the subjects' ideas about how mathematics differed at the two institutions.

iv) The subjects were asked to give advice to teachers of mathematics, using the following questions: *"I want you to give me some advice for teachers to help them teach maths better to people your age. What sorts of things would you tell them? What should teachers do to make maths easier to learn for people your age?"* The responses of the subject were explored in an unstructured fashion.

v) The opinion that the subjects' peers held towards mathematics was sought through flexible questioning based on a scenario given by the interviewer. *"I want you to imagine you're with a group of your friends at lunchtime. The bell rings to signal the start of afternoon classes. One of your friends says what have we got now? Someone else says, maths. What do people say when they hear this? What comments might be said, if any?"*

vi) Opinions were sought from students on the role, if any, of social disruption in the classroom with regard to the learning of maths. These views were elicited through informal questioning based on a scenario given by the interviewer: *"Some students your age talk about other students who play around in class. Is this a problem for you in the sense that it interferes with learning maths?"*

vii) Where necessary a check was made on the 'name the columns' activity from the previous interview. The student was asked to explain his/her responses.

The Conduct of the Interviews

By arrangement with the Principal and Head of Mathematics in each of the schools used in the study, it was possible to schedule 30-40 minute interviews with each of the subjects.

A special interview room was made available. This was set up with an audio tape-recorder, table and chairs for the researcher and subject, and pen and paper for the recording of student responses.

To help establish an emotionally supportive and non-threatening atmosphere which would encourage students to respond frankly and easily the following special measures were taken:

1. Audio taping: The subject was asked at the outset if the interview could be audio-taped. The recorder was placed on the table next to the interviewer and subject - no attempts were made to hide the recorder from the subject.

2. Establishing rapport: Time was taken before each interview started to outline the purpose of the study, and to emphasize to each student that this was not a 'test'. Students were told that anonymity would be preserved. It was explained that any comments used in the subsequent results would not be assigned to them by name.

   The purpose of the study was explained as being a project aimed at finding out how students of their age thought about 'some things in mathematics'.

   Because of the longitudinal nature of the research exercise, care was taken to explain to subjects that they had been chosen randomly and represented 'ordinary' students of their age. It was thought important to stress this factor in order to counter any feelings that the subjects might construct once the study was underway and they were revisited by the researcher.

3. Presentation of the cards: As the Stimulus Cards (Series One and Two) were presented during the interviews the researcher adopted a non-judgemental attitude towards subjects' responses. Use was not made of positive comments that might often be used by teachers such as *"Good"*, *"Yes"*, *"Right"*, etc. Rather, attempts were made to elicit the subjects' views and ideas by the use of neutral, non-judgemental verbal cues such as *"Uh-huh"*. When probing students' responses, the wording was in the form of; *"Why?"*, *"Why do you think*

that?", and when necessary, *"I don't understand that - could you explain it another way for me?"* For illustrative purposes, APPENDIX D provides a transcript of a complete interview.

4. Fatigue factor: Care was taken not to rush the subjects in their responding. Each interview was of 30-40 minutes duration. Fatigue did not appear to be a factor in any of the interviews.

All Stimulus Cards (Series One and Two) were read to the subject. The cards were presented one at a time. They were printed on ten by fifteen centimetre white lined card. Additional information to that printed on the cards was given in an informal oral manner with fifteen of the eighteen cards. This additional information is described below (see inside back cover for copies of the Stimulus Cards concerned):

i) Cards D, E, F: Subjects were told they did not have to work out the calculation. Rather, an estimation of a reasonable answer was required, and this should be matched to an alternative from those listed at the bottom of the card.

ii) Cards G, H, I: Subjects were reminded where necessary of the appropriate working form. For example, 'divide by 100: 15' would be '$100\overline{)15}$ ', not '$15\overline{)100}$ '.

iii) Card J: Subjects were reminded of the form of decimal numbers by referring to earlier cards in the series.

iv) Cards K, L, M, N: The algorithms were hidden from view by being folded back out of sight of the subject. These were only unfolded if the subjects were unable to generate a calculation from the mixed numerals in the problem.

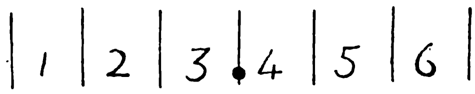
v) Cards O, P, Q, R: The subjects were asked to read the numerals presented. Then they were asked if one number was larger than the other number, or if they were of the same size. Depending upon the responses, subjects were asked how many times one was larger (or smaller) than the other, and the rationale for this.

Place Value Nomenclature

As indicated above, at the conclusion of Interviews I and III, subjects were asked to name the columns in terms of place value about a decimal point. This activity was designed to reveal growth in student understanding (if any) about the place values in decimal numbers.

Student ideas about place value nomenclature were probed as follows. First, the researcher drew columns in front of the subject...

...and next wrote in digits in each place



This numeral was read to the subject, as "one hundred and twenty-three, point four, five, six." Subjects were then asked to write the names of the place value columns, commencing with the whole number one hundred and twenty-three.

Once this was completed the subject-generated nomenclature was discussed and reasons for the responses were elicited.

Procedures for Measuring Attitude Towards Mathematics and Decimal Numbers

i) As mentioned above, Interview IV included an administration of a version of the semantic differential test (Osgood, Suci and Tannenbaum, 1957). The test included 12 contrastive adjective pairs (see APPENDIX E) and sought responses to the two concepts 'mathematics' and 'decimals'. Subjects were given unlimited time to complete the exercise and their responses were probed by the interviewer.

ii) Mathematical attitudes were also examined by means of a "card sort" by which students expressed their like or dislike for mathematical topics. These topics, worded in the kind of mathematical language that students would typically use, were as follows: take-away sums, adding sums, multiplication sums, times tables, fractions, decimals, sets, geometry, division sums.

Subjects were asked to place the topic cards at positions on the continuum that represented their opinions (cards could be placed on top of each other if necessary). It was explained that a point mid-way between "Things I don't like in maths' and "Things I like in maths' would represent a neutral stance.

No time limit was imposed but reasons for the positions taken were sought from the subjects.

iii) Advice to teachers. As discussed earlier this was elicited during Interview IV. The aim of this activity was to have students suggest strategies that teachers should adopt that would "help students your age" learn mathematics.

iv) Peer opinions of mathematics. These also were elicited during Interview IV. The aim of this discussion was to sample the opinions that 11 to 14 year olds held towards the subject of mathematics when recalling the comments that their peers might make.

v) The primary/high school interface and mathematics. Opinions here were also gathered during Interview IV. The aim was to collect student perceptions of the way mathematics was taught in primary (intermediate) schools compared with high schools.

Stimulus Cards (Series Two) for Interview IV

This alternative set of ten stimulus cards was constructed for use during Interview IV. The cards were based on the Series One set, but differed with regard to the calculations required and included a new problem requiring the use of decimal numbers in the context of money.

Series Two cards were used to further probe ideas (particularly errors) elicited during Interview III. These alternative cards were required for Interview IV so that memory from Interview III did not contaminate responses. As well, it was hoped that the Series Two cards would prevent the occurrence of any response set from students.

Copies of the cards themselves are in FIGURE 6 (over page). Not all Series Two cards were used with every student. Rather, Series Two cards were used to explore misconceptions, especially, shown during Interview III.

As with Series One set of stimulus cards, each card was read to the subject. Additional information to that printed on the card was given as follows:

i) Stimulus Cards (Series Two) A, B: Subjects were told they did not have to work out the calculation - rather an estimation of a reasonable answer was required, and then this should be matched to an alternative from those listed at the bottom of the card.

ii) Stimulus Card (Series Two) C: Subjects were reminded, where necessary, of the appropriate working form. For example, 'divide 10:50' would be '50) 10 ', not '10) 50 '.

iii) Stimulus Card (Series Two) D: Subjects were reminded of the form of decimal fractions by referring to earlier cards.

iv) Stimulus Cards (Series Two) E, F, G, H: Instructions for these cards were as for Cards O, P, Q, R of Series One (the original set).

v) Stimulus Cards (Series Two) I, J: Subjects were encouraged to write working forms for each problem if they thought this were needed.

Area	Stimulus Card No. (Series Two)	Format of Card		Source
Estimation after computation	A	0.3.4 x 3 is nearest in size to? 0.01/0.10/1.0/10/30/100		Brown (1981a)
	B	0.34 x 0.3 is nearest in size to? 0.01/0.10/1.0/30/100		Brown (1981a) Carr
Division by a multiple of ten	C	Divide by 50 200 100 50 10 60 5		Carr
Writing decimal numbers	D	Write as decimals two tenths nine hundredths twenty-five hundredths eleven tenths		Brown (1981a) Carr
Comparisons with involving decimal numbers	E	0.6	6	Carr
	F	432.0	43.2	Carr
	G	0.10	10	Carr
	H	548	54.8	Carr
Problems involving decimal numbers	I	A person sees that ice-cream is "on special" this week - a two litre pack for $1.00. At this price/rate how much would one litre cost?		Carr
	J	Next week, ice-cream is "on special" again - this time a 2.50 litre pack for $1.25. At this price/rate how much would one litre cost?		Carr

FIGURE 6 : Stimulus Cards (Series Two) Used in Individual Interview IV on Decimal Numbers

CHAPTER FIVE

RESULTS: THE QUANTITATIVE SURVEY

The approach adopted for integrating both quantitative and qualitative aspects of research methodology was discussed in the previous chapter. The first step in this study was to generate a pencil-and-paper test on decimal numbers, and to administer this to a representative sample of 11 to 14 year olds.

The survey was based upon Brown's (1981a) test, but modified to ensure that the survey was content valid for New Zealand students. The test was administered to a sample 11 to 14 year olds (n = 102) in Forms One to Four as described in Chapter Four.

Results and Discussion

The facility level for each item in the survey instrument is presented in TABLE 1 and a summary of the results is shown in TABLE 2 . The chapter then records in more detail the error types for each item and, where appropriate, comparisons are made with Brown's (1981a) results.

With two exceptions, the researcher focussed on items in the survey that a majority of students failed to answer correctly, i.e. questions that had a facility level of around 50 per cent or lower. These questions might indicate areas for more detailed investigation of children's knowledge and understanding of decimal numbers in the subsequent longitudinal investigations.

The mean for the survey was 34.69 out of 64 with a standard deviation of 12.85. Older students generally scored better than younger students, means for Forms One, Two, Three and Four being 28.84, 32.91, 35.08 and 44.22 respectively.

The variability of the scores as shown by the high standard deviation, is apparent from TABLE 3. The range for each grade/class level was 30 to 40 raw score points, indicating clearly that this was not a mastery exercise for any class level within the overall sample.

The individual items that caused student difficulty are now considered in detail.

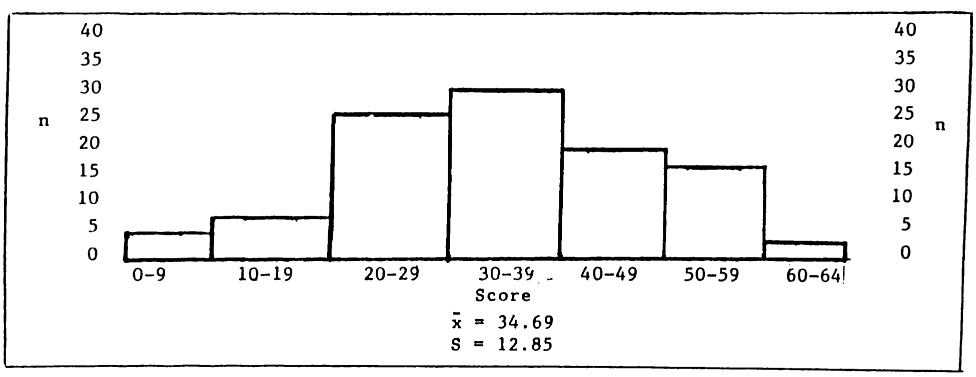
TABLE 1 : Results of the Survey: Decimal Numbers

Item Number	Form I (n = 32)	Form II (n = 23)	Form III (n = 24)	Form IV (n = 23)	Facility Level %
1a	28	19	22	17	86
b	16	15	15	12	58
2a	18	19	14	19	70
b	11	15	17	18	61
c	9	14	14	13	50
3a	24	12	24	19	79
b	6	5	6	11	28
c	25	12	21	20	78
d	22	10	19	17	68
e	9	11	15	13	48
f	5	5	12	17	39
g	28	16	23	20	87
h	17	11	16	17	61
i	11	11	18	19	59
j	15	12	15	17	59
k	21	13	19	22	75
l	21	10	15	20	66
4a	29	21	23	21	94
b	22	14	23	19	78
c	14	6	14	17	51
d	8	5	12	16	41
e	7	7	10	10	34
f	20	16	17	17	70
5	25	20	19	23	87
6	18	16	17	22	73
7a	13	14	20	21	68
b	12	9	19	20	60
8a	15	12	14	14	55
b	10	7	11	11	39
c	28	17	22	22	89
c	17	14	19	19	69
e	20	11	21	20	72
9a	12	16	15	16	59
b	4	9	4	10	27
c	4	8	6	8	26
d	6	7	4	7	24
10a	25	21	22	21	89
b	22	21	21	20	84
c	9	9	12	17	47
d	8	6	10	17	41
e	6	5	3	7	21
11a	31	19	21	23	93
b	22	17	11	17	67
c	23	15	15	15	68
d	11	12	7	13	41
12a	25	22	20	21	88
b	14	12	6	16	48
c	17	12	12	16	57
d	7	8	5	14	34
e	19	19	14	18	70
f	1	7	2	11	21
g	11	7	2	12	32
h	0	1	1	9	11
13a	27	22	23	22	94
b	12	16	12	20	60
c	11	14	14	16	55
d	8	11	7	19	45
e	1	2	2	7	12
f	2	6	3	7	18
14a	24	19	15	22	80
b	16	9	9	19	53
c	7	4	2	7	23
d	7	3	2	6	18
e	15	12	3	15	47

TABLE 2 : Summary from the Survey: Decimal Numbers

Topic	Item Number	Facility Level %	Major Errors
Name tens of thousands column	1(b)	58	'hundreds of thousands', 'hundreds'
Name thousandths column	2(c)	50	'thousands', 'ones', 'tenths'
Add 'ten' to '0.15'	3(b)	28	'0.25', '1.5', '1.50', 0.150'
Add one tenth to '4.254'	3(e)	48	'5.265', no answer attempted, '42.54'
Add one tenth to '2.9'	3(f)	39	'2.10', no answer attempted, '3.9'
Identify '14.65' on a number line	4(d)	41	'14.6½', '14.6', '14.7'
Identify '3.2' on a number line	4(c)	34	'3.1', no answer attempted, '0.1'
0.75 or 0.8 larger?	8(a)	39	
Why larger?	8(b)	39	no answer attempted, '75 larger than 8', '0.75 <u>is</u> a larger fraction than 0.8'
Write eleven thousandths as a decimal	9(b)	27	'.0011', '000.11', no answer attempted
Write eleven tenths as a decimal	9(c)	26	'0.11', no answer attempted, '0.011'
Convert four tenths to hundredths	9(d)	24	'four', no answer attempted, '0.4'
Name a number between 0.4 and 0.5	10(c)	47	no answer attempted, '0.4½', '0.4'
Name a number between 0.41 and 0.42	10(d)	41	no answer attempted, '0.41½', '0.41'
Numbers between '0.41' and '0.42'	10(e)	21	'10', no answer attempted, '9', '1', 'none'
23.12 + 54.7 =	11(d)	41	'77.19', '28.59', '17.65', no answer attempted
5.13 x 10 =	12(b)	48	no answer attempted, '5.130', '50.13'
2.3 x 100 =	12(d)	34	'no' (students indicated no answer possible), '23.00', no answer attempted
3.7 ÷ 100 =	12(f)	21	'no' (students indicated no answer possible), no answer attempted, '0.37'
24 ÷ 20 =	12(g)	32	no answer attempted, '1.4', 'no', (students indicated no answer possible)
16 ÷ 20 =	12(h)	11	'no' (students indicated no answer possible), no answer attempted, '0.16'
Number nearest to 2.9 x 7	13(d)	45	'200', no answer attempted, '0.002'
Number nearest to 0.29 x 7.1	13(c)	12	'0.002', '200', '20', no answer attempted
Number nearest to 59 ÷ 190	13(f)	18	'3', no answer attempted, '0.03', '0.003'
Problem solving: algorithm needed - 2.57 x 0.55	14(c)	23	'2.67 + 0.58', no answer attempted, '2.67 - 0.58'
Problem solving: algorithm 3.58 + 6.44	14(d)	18	'6.44 + 3.58', no answer attempted, '6.44 - 3.58'
Problem solving: algorithm needed - 11.8 x 8.37	14(e)	47	'11.8 + 8.37', no answer attempted, '8.37 + 11.8'

TABLE 3 : Results from the Survey (Grouped Data)



1. <u>Naming the Place Value of Digits: Items 1(b) and 2(c)</u>

Item 1(b)	
521400 ↑	The 2 stands for 2...............

Facility Level: 58%   Number of errors generated: 11

Error	Frequency
hundreds of thousands	(nine)*
thousands	(nine)*
hundreds	(eight)*
tens	(three)
tens of hundredths	(three)
tenths	(two)
tenths of thousands	(two)

The following errors occurred only once: 'millions', 'units', ten thousandths', and 'tens of thousandths'.

The three most common errors for Item 1(b)* account for two-thirds of the total errors and indicate a lack of knowledge about basic place value in the group of thousands, or imprecise knowledge of a particular column's label ('thousands', rather than 'tens of thousands'). Moreover, the range of errors shows that students will construct a diverse range of incorrect responses when confronted with a question that probes an area about which they are uncertain. Form I students generated the largest number of errors.

Item 2(c)	
0.412 ↑	The 2 stands for 2...............

Facility Level: 50%   Number of errors generated: 9

Error	Frequency
thousands	(sixteen)
ones	(eleven)
tenths	(eight)
tens	(four)
hundredths	(four)
oneths	(three)
hundreds	(two)

The following errors occurred only once: 'hundredths of hundredths' and 'ten hundredths'.

The other error types reveal confusion with whole number nomenclature ('thousands', 'hundreds', 'tens', 'ones'). An interesting error made by three students was in naming the 'thousandths' column as 'oneths', indicating again how whole number terminology seems to influence student ideas about decimal numbers.

The largest proportion of errors came from the Form I (11 year old) group, but all grade levels exhibited a similar range of error types.

2. Addition with Decimal Numbers: Items 3(b), 3(e), 3(f)

Item 3(b)
Add ten
0.15 ....

Facility Level: 28%   Numbers of errors generated: 16

Error	Frequency
'0.25'	(forty four)
'1.5'	(six)
'1.50'	(four)
'0.150'	(four
'25'	(three)
'0.015'	(two)

The following errors occurred only once: '10.5', '105', '1.25', '01.5', '15', '0.510', '2.5', '30', '020', and '0.1510'.

The responses show that students had a clear preference for the error '0.25'. Here, the error seems to have been generated by adopting a whole-number framework and disregarding the decimal point: fifteen plus ten does equal twenty-five.

Item 3(e)

Add one tenth

4.254 ....

Facility Level: 48%   Number of errors generated: 14

Error	Frequency
4.264	(nineteen)
no answer attempted	(eleven)
42.54	(eight)
4.255	(three)
4.254	(two)
4.0254	(two)

The following errors occurred only once: '4.274', '4.259', '42.64', '4.6794', '4.2540', '4.154', '5.254', and '4.260'. The non-response category accounts for a significant proportion of the errors. The most common error '4.264' can be obtained if the decimal point is ignored, and ten is added to '4254'. The error '42.54' (eight instances) would be the correct response if students had been asked to multiply by ten - perhaps students were applying the inappropriate strategy of shifting the decimal point.

Item 3(f)

Add one tenth

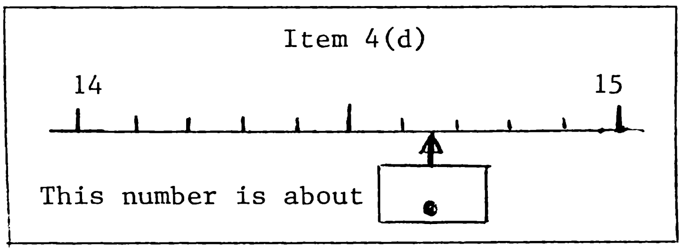
2.9 ....

Facility Level: 39%   Number of errors generated: 17

Error	Frequency
'2.10'	(thirteen)
no answer attempted	(thirteen)
'3.9'	(ten)
'29'	(six)
'2.19'	(three)
'2.09'	(three)
'2.19'	(two)
'2.9'	(two)
'2.91'	(two)
'0.39'	(two)

The following errors occurred only once: '2.29', '2.01', '2.14', '3.19', '2.11', and '2.8'. Several features of these errors are worthy of note. First, the number of errors generated by the 11 to 14 year old students is considerable. Second, the nil response was equal to the most commonly occurring error, suggesting that the item was either not understood by the students, or that it probed an unfamiliar area of mathematics. Third, the two most common errors ('2.10' and '3.9') continue the theme evident in earlier items - the power of beliefs about whole numbers influencing any ideas that the students possessed about decimal numbers.

3. <u>Assigning Decimal Numbers to Designated Points on a Number Line: Items 4(a), 4(e)</u>



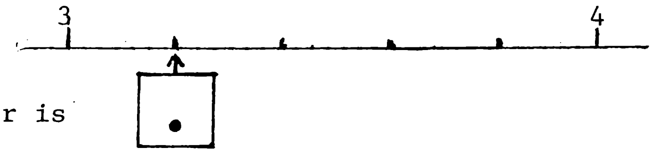
Facility Level: 41%   Number of errors generated: 25

Error	Frequency
'14.6½'	(twenty)
'14.6'	(seven)
'14.7'	(six)
'6½'	(three)
'14.5'	(three)
no answer attempted	(three)
'6.5'	(two)

The following errors occurred only once: '1.3', '8$\frac{3}{2}$', '14.8', '0.5', '14.165', '20.5', '0.4', '14.15', '14.7½', '2.1', '14.1$\frac{1}{5}$', '$\frac{3}{4}$', '14.13', '14.96', '18.15', '14.55', '14.06', and '6.14'. The question itself is perhaps poorly phrased as the word "about" can be variously interpreted.

If "about" meant that '14.6' and '14.7' were included, then the facility level rises to 58%. This might be a fairer indication of Item 4(d)'s difficulty.

Item 4(e)



This number is

Facility Level: 34% Number of errors generated: 12

Errors	Frequency
'3.1'	(forty nine)
'no response'	(six)
'0.1'	(three)

The following errors occurred only once: '4.4', '$\frac{1}{3}$', '4.0', '3.0', '6.0', '0.4', '.12', '2.0', and '3.25'. These errors show a lack of understanding when assigning a decimal number to a particular point on a number line. The responses are similar to those observed by Carr and Katterns (1984), i.e. a symbolic mode of responding seems to dominate the students' thinking. Carr and Katterns (1984) found a significant proportion of their 9 and 13 year olds could not perform simple addition and subtraction when using a number line model. In Item 4(e) the most common error is '3.1', which suggests that students understood the initial space to be '0.1' without first counting the number of spaces in order to determine the appropriate size of each space. Thus they are responding to the symbols they meet, rather than to the underlying principles of a number line (Rathmell, 1980).

4. Understanding 'Size' of Decimal Numbers: Items 8(a), 8(b)

Items 8(a) and 8(b)

(a) Ring the bigger of the two numbers:
0.75 or 0.8

(b) Why is it bigger?

....

Facility Level: 39% Number of errors generated: 13

The errors were categorized as follows:

a) nil response, or wrote 'don't know' (twenty five instances)

b) responses claiming 0.75 was larger than 0.8 because 75 is greater than eight, e.g.
"You can tell by its numbers - they are greater than eight."
(Tanya, aged 13)

"0.75" is bigger because it is 75, not 8." (Craig, aged 14)
"75 is bigger than 8. 8 is not more than 75." (Royce, aged 11)
"It (0.75) also has more units than the other." (Glen, aged 14)
(nine instances)

c) Responses claiming 0.75 was larger than 0.8 because 0.75 contained more digits, e.g.
"Because it has got more numbers." (Kim, aged 11)
"Because it has got one digit more after the decimal point." (Mark, aged 11)
"It (0.75) is bigger because it has two digits and the other (0.8) has only one." (Trudy, aged 12)
(six instances)

d) Responses claiming that 0.75 was a larger decimal number than 0.8, (self evident) e.g.
"In decimals, .75 is one 7th of something, .8 is one 8th" (Ian, aged 13)
"Because 5 (in 0.75) is 5 hundredths (sic) which makes things bigger than 8 tenths." (Donna, aged 12)
"Because 0.75 shows the 5 is a hundredth and the 7 a tenth. In 0.8 there is no hundredth, which is larger than tenths." (Trudy, aged 12)
"Because it has 7 tenths and 5 hundredths and the other one only has 8 tenths." (Michael, aged 12)
"It (0.75) is also in the hundreds column (sic)." (Michele, aged 13)
"Because .75 is bigger than .8." (Daryl, aged 14)
(eight instances)

e) Responses giving reasons for 0.8 being larger than 0.75 (or vice versa) that were unrelated to the problem, e.g. (0.8 larger than 0.75)
"Because the number after the decimal doesn't necessarily have to have a zero on the end of it." (Karen, aged 13)
(three instances)

Other incorrect explanations included a claim that the two decimal numbers were the same size: *"It's the same really, because the number behind the decimal is nothing."* (Teresa, aged 13), and that the more digits there were in a fractional number the smaller it was; *"Because the more numbers in a fraction, the smaller the fraction gets."* (Andrew, aged 11).

In this case, Andrew's 'rule' has helped him choose the correct response (0.8 being larger than 0.75), but it is not difficult to imagine examples where this would not be the case. For example, Andrew's 'rule' would have him choose '0.95' as a smaller number than '0.1', because '0.95' has more digits.

Another instance of a student choosing the correct alternative, yet revealing an underlying misconception was Mark (aged 13). Mark wrote that '0.8' was larger than '0.75' *"because this one (0.8) is a whole number."* Mark's reasons for believing that '0.8' is a whole number cannot be detected from his statement. A follow-up interview would be needed to explore his beliefs about decimal numbers and their relationship to whole numbers. The responses of both Andrew and Mark are representative of the phenomenon where students may give correct answers in a pencil and paper test (when asked to choose an alternative), in spite of possessing incorrect underlying misconceptions concerning the knowledge being assessed.

The question must be raised about the meaning of the word "bigger" in Item 8(a). For some students this could have meant bigger related to the number of digits: 0.75 has more digits than 0.8. Hence, size here could be ambiguous in a way not yet encountered in this survey. The errors under (c) above show that, for some children at least, this was the case.

5. Writing Decimal Numbers for Given Verbal Stimuli: Items 9(b), 9(c)

The next two items explored writing decimal numbers given verbal stimuli. Earlier, Item 9(a) asked students to write the decimal number 'three hundredths' (facility level: 59%)[1]. The next item's (Item 9(b)) results were:

Item 9(b)
How would you write as decimals:
eleven thousandths..............

Facility Level: 27% Number of errors generated: 13

Errors	Frequency
.0011	(thirty)
000.11	(fourteen)
no answer attempted	(eight)
0.11	(five)
11.000	(four)
.00011	(three)
11000	(three)
0.11000	(two)
$\frac{11}{1000}$	(three)

Errors that occurred only once were: '000.3', '11.1000', '1199', and '1.100'. However a diversity of errors is evident. The

[1] This was perhaps lower than might be expected since students were given place value nomenclature as part of the practice questions at the commencement of the survey (see APPENDIX C).

two most common errors ('.0011' and '000.11') represent 56 percent of the total errors. The most common error, '.0011', would seem to come from the idea that in order to write 'eleven thousandths', it is necessary to move the 'eleven' one place to the right, rather than to utilize the adjacent column to the left . Students who responded in this fashion were using strategies for writing decimal numbers that presented a type of 'mirror imaging' to the nomenclature of whole numbers (see Carr, 1983; Freyberg and Osborne, 1981).

Item 9(c)
How would you write as decimals:
eleven tenths..................

Facility Level: 26% Number of errors generated: 12

Errors	Frequency
'0.11'	(fifty three)
no answer attempted	(seven)
'0.011'	(five)
'11'	(four)
'11.0'	(two)
'11.10'	(two)
'$\frac{11}{10}$'	(two)

The following errors occurred only once: '0011', '1.0', '$1\frac{1}{10}$', '0.8', and 'eleven tenths'. The rèsults show a clear preference for the error '0.11', which demonstrates that these students do not understand that 'eleven tenths' is greater than one whole and, as in Item 9(b), are adopting strategies inappropriate for writing decimal numbers.

6. <u>Generating Decimal Numbers from a Given Stimuli: Items 9(d), 10(c), 10(d), 10(e)</u>

The next four items asked students to generate equivalent fractions from a given decimal, and to write decimal numbers between two given fractions.

Item 9(d)
Four tenths is the same as
.... hundredths

Facility Level: 24%   Number of errors generated: 17

Error	Frequency
'four'	(twenty one)
no answer attempted	(twenty)
'0.4'	(ten)
'two'	(six)
'zero'	(four)
'eight'	(three)
'0.04'	(three)
'one'	(two)

The errors that occurred only once were: 'eleven', 'five', 'thousand', 'four hundred', '4.8', '¼', '40 thousandths', '0.2', and '0.11'. In a pencil-and-paper survey such as this, the underlying reasons for writing the response 'four' can only be guessed at. Although this particular error may be symptomatic of a misconception, it could also result from a student not understanding the question. Significant, as well, is the number of students who failed to write <u>any</u> answer for this particular item. It cannot be said whether the 'no answer attempted' error was caused by a failure to understand the question *per se*, or the mathematical concepts involved (c.f. Donaldson, 1978). The large number of errors that were generated points to the creative nature of children's thinking in mathematics (c.f. Carr, 1984). The third most common error, '0.4', is the correct decimal number in numeral form for 'four tenths', but is incorrect in the context of this question. Error types were distributed evenly through the 11, 12 and 13 year old samples, while the 14 year olds made fewer errors in terms of 'four' and 'no answer'. Twelve different errors were generated by the 11 year olds, six by the 12s, six by the 13s, and ten by the 14 year olds.

Item 10(c)
Write down any number between 0.4 and 0.5..................

Facility Level: 47%   Number of errors generated: 20

Error	Frequency
no answer attempted	(sixteen)
'0.4½'	(eleven)
'0.4'	(five)
'no answer possible'	(three)
'0.04'	(two)
'04.1'	(two)
'0.40'	(two)
'0.3'	(two)

Errors that occurred only once were: '1 tenth', '.05', '$0.4\frac{3}{4}$', '0.05', '1.4', '4.5', '1', '10', '4.7', '0.6', '0.00', and '04.2'. Apart from the 'no answer' category, the most common error, '0.4½' reveals a confusion in the conventions for writing decimal numbers - in this case, an inappropriate combination of the conventions for writing common and decimal fractions. Most errors were generated by the 11 year old group within the sample (24 errors), with a gradual decline in the frequency of errors through to the 14 year old group (6 errors). The diversity of errors is worthy of note again - 20 different errors in all from a sample of 102 students.

Item 10(d)
Write down any number between 0.41 and 0.42..................

Facility Level: 41%   Number of errors generated: 19

Error	Frequency
no answer attempted	(nineteen)
'0.41½'	(twelve)
'0.41'	(five)
'0.42'	(three)
'no answer possible'	(three)
'41.2'	(two)
'$0.41\frac{3}{4}$'	(two)
'0.041'	(two)

Errors that occurred only once were: '1 hundredth', '1.141', '.45', '41.5', '41.6', '41.1', 0.041½', '0.00', '0.47', '41.0', and '0.425'.

Again the 'no answer attempted' category was common. This could not be explained by students accidentally over-looking the question, since in most 'no answer attempted' cases, the questions above and below item 10(d) were attempted. The second most prevalent error, '0.41½', reveals confusion of decimal and common fraction nomenclature (c.f. Item 10(c)). Here, students were constructing answers that revealed understanding of the question, but their existing ideas about decimal numbers were not sufficiently developed to allow them to construct a mathematically correct response.

These two error types, 'no answer' and '0.41½', account for half the errors manifested. The next most common error, '0.41', was the first decimal number given in the item, and was made just five times. The 22 different errors were distributed among the sample of 11 to 14 year age groups, as 26, 16, 14 and 6 errors respectively

Item 10(e)
How many different numbers could you write down which lie between 0.41 and 0.42?
....

Facility Level: 21% Number of errors generated: 13

Error	Frequency
'10'	(eighteen)
no answer attempted	(sixteen)
'9'	(fifteen)
'1'	(ten)
'none'	(nine)
'finite'	(eight)
'8'	(three)
'2'	(three)
'3'	(two)

The following errors occurred only once: 'not many', '5', '41½', and '4'. Without supporting qualitative data, the underlying student rationales cannot be gleaned. This item produced 81 errors distributed among the 11, 12, 13 and 14 year olds as 25, 19, 21 and 16 errors respectively. Obviously this question was difficult for all age groups.

The concept of infinity explored in this item, is introduced to 8 year olds in the New Zealand school system through the examples of finite and infinite sets. Item 19(c) combines the concepts of infinity with decimal numbers, a task beyond most students in the present sample.

7. <u>Operations with Decimal Numbers, or with Numbers that Give a Decimal Number as the Answer: Items 11(d), 12(b), 12(f), 12(g), 12(h)</u>

Item 11(d)
23.12 + 54.7 = ...

Facility Level: 41% Number of errors generated: 26

Errors	Frequency
'77.19'	(twelve)
'28.59'	(eight)
'17.65'	(seven)
no answer attempted	(seven)
'176.5'	(four)

There were 21 errors that occurred only once. These varied from '77.29' and '79.21' (which are reasonable approximations of the correct answer) to '1765.0' and '0.1765' (which show little evidence of approximation and estimation). Forty-seven students used the working space provided on the test script. The most common error, '77.19', is the result of students adding the decimal fractions in 23.12 and 54.7 as '12' plus '7' - as in whole number strategies. The second most common error, '28.59', warrants comment. This error is possible if the decimal numbers 23.12 and 54.7 are incorrectly aligned in place value:

```
23.12
 5 4.7
28.59    (Grant, aged 11 years)
```

Similarly:

```
23.12
 5 4.7
2.8 59   (Teresa, aged 13 years)
```

Other errors such as '17.65' and '176.5' appear to have been generated through using the operation of subtraction rather than addition, and again applying inappropriate whole number strategies. For example, 2312 - 547 gives 1765, which then requires the insertion of a decimal point to give '17.65' or '176.5'.

The previous example in the survey was a subtraction example, and this might have caused a 'mental set' that predisposed some students to attempt subtraction in spite of the algorithm reading '23.12 + 54.7 = ...'. There was a large number of individual errors and it is impossible to discover all of the strategies that students used to obtain these answers, although in the case of the student who obtained '2235', it would seem that 2312 was taken from 547 - at least as far as it would go (the researcher's working):

$$\begin{array}{r} 547 \\ \underline{-2312} \\ 2235 \end{array} \text{ (sic)}$$

The error in the thousands column is similar to that categorized by Van Lehn (1982) as '0 - N = N/EXCEPT/AFTER/BORROW', although Van Lehn did not allow students to generate their own working forms. His analysis was restricted to the responses to a prescribed subtraction survey.

Item 11(d) demonstrates several other points: (i) the creative nature of children's thinking in mathematics in terms of the range and number of errors they can construct (c.f. Carr, 1984); (ii) the difficulties students have when given working forms that have the potential for place value confusions; and (iii) the high incidence of students who mis-read questions in a pencil-and-paper survey - at least 14 students appear to have performed subtraction rather than the required addition operation.

An example of this latter error was:

$$\begin{array}{r} 2'3.'12 \\ \underline{'5'4.\ 70} \\ -4\ 6.\ 42 \end{array} \quad \text{(Karen, aged 14 years)}$$

Karen's use of a negative number results from persisting with subtraction, an error Van Lehn (1982) does not, in fact, list. It is interesting that the algorithm in this case is generated by the respondent.

Kay was correct, but not in the context of the question.

$$\begin{array}{r} 54.70 \\ \underline{-23.12} \\ 31.58 \end{array} \quad \text{(Kay, aged 13 years)}$$

Item 12(b)
Multiply by ten
5.13 ....

Facility Level: 48%   Number of errors generated: 11

Errors	Frequency
no answer attempted	(twenty one)
'5.130'	(fifteen)
'50.13'	(eight)
'.5130'	(three)
'5.23'	(three)

The following errors occurred only once: '10', '5130', '10.43', '513', '5103', and 'yes'. Although only 12 different errors were made (c f. Items 4(d) and 11(d) where over 20 errors were made) the task is much easier, involving application of a simple rule viz., to multiply by ten, shift the decimal point one place. Errors were allocated to the 11, 12, 13 and 14 year old age groups as 18, 12, 18 and 8 errors respectively.

Item 12(d)
Multiply by one hundred
2.3 ....

Facility Level: 34%   Number of errors generated: 18

Errors	Frequency
'no' (subjects indicated no answer possible)	(thirteen)
'23.00'	(eleven)
no answer attempted	(ten)
'2.30'	(nine)
'2.300'	(seven)
'200.3'	(five)
'.2300'	(three)
'20.3'	(three)
'0.023'	(two)

The following errors occurred only once: '30', 'yes', '2.00', '2.3 hundred', '2.03', '320.0', '3.3', '02.3', and '2.31'.

Errors were made by 11, 12, 13 and 14 year olds at the rate of 24, 18, 20 and 10 errors respectively. The errors '2.30' and '2.300' were not combined. The writer felt it wiser to record these as separate errors since the error '2.300' suggests that learners were inappropriately adding two zeros to the number in order to 'multiply by one hundred'. The position of the decimal point in the various error types is worth noting for it ranges from before the digits ('0.023', '0.2300'), to central locations ('2.30') to locations near the end of the number ('30', '320.0', '200.3').

Item 12(f)
Divide by one hundred
3.7 ....

Facility Level: 21% Number of errors generated: 10

Errors	Frequency
'no' (subjects indicated that no answer was possible)	(forty seven)
no answer attempted	(seventeen)
'0.37'	(six)
'0.0037'	(two)
'3.7'	(two)

Idiosyncratic errors recorded were '37', 'Yes', '30', '297r11', and '106,007,960'. Fewer error types were generated by the respondents, a clear majority opting for the 'no answer possible' category. Because students were not required to justify their responses, discussion can only be speculative. However, the belief that when a divisor is larger than the dividend then no answer is possible, is also noted by Brown (1981a) when she comments:

> "....only a group of around perhaps 10 percent of first year children rising to 35 percent of fourth years accept that division of a smaller number by a larger number is possible, and that the result can be expressed as a decimal."
>
> (Brown, 1981a:311)

The second most common response was 'no answer attempted' (space for answer left blank). These two most common errors ('no answer possible' and 'no answer attempted') account for 80 per cent of all errors, suggesting that students had difficulty constructing <u>any</u> response at all to the item.

Item 12(g)
Divide by twenty
24 ....

Facility Level: 32% Number of errors generated: 19

Errors	Frequency
no answer attempted	(sixteen)
'1.4'	(thirteen)
'no' (subjects indicated that there was no answer possible)	(eight)
'4'	(seven)
'2r4'	(four)
'yes'	(four)
'1'	(four)
'2.4'	(three)
'480'	(two)
'0.24'	(two)

The following errors occurred only once: '2400', '24', '4.8', '2', '204', '12', '1.5', '14r', and '201.4^{r}'. Errors were spread through the 11, 12, 13 and 14 year old age groups as 21, 18, 22 and 12 errors respectively.

The 'no answer attempted' category accounted for a considerable proportion of the total errors (22 per cent). The second most commonly occurring error, '1.4', results from the misconception that remainders to quotients are necessarily decimal fractions - in this case they are 'twentieths'. The diversity of errors again warrants comment. Many seem to be the result of attempted operations with whole numbers, e.g. the error '4' (seven instances) is the missing addend in 24 - 20 = . Division with whole numbers is related to the subtraction operation, so students who made this error attempted a process that would result in a decrement.

From her research, Brown (1981a) comments on the error 'no' (no answer possible), and conjectures that this may express the student's inability to cope with the remainder, which, if it were turned into a decimal, would again require division by a larger number. The more usual response of Brown's (1981a) sample, and the present survey sample as well, was to give '1 remainder 4' or '1.4' as the answer. This obviated the need for further division on the part of the students.

Item 12(h)
Divide by twenty
16 ....

Facility Level: 11%   Number of errors generated: 15

Errors	Frequency
'no' (subjects indicated that no answer was possible)	(fifty)
no answer attempted	(nineteen)
'0.16'	(six)
'16'	(two)
'0.4'	(two)
'yes'	(two)
'negative 4'	(two)

The following errors occurred only once: '18r', '1600', '320', '3.2', '1 + 6r', '11', '0.45', and '$\frac{20}{16}$'. This item was the most difficult for students in the survey, demonstrating a clear belief in many students' minds that no answer was possible, as was also the case with Item 12(f). Brown (1981a) reports that in her sample (which had facility levels of 7, 11, 25 and 36 per cent for Year 1 to 4 students) children seemed to behave consistently with this item, <u>if</u> they had grasped the idea in the context of multiplication. This they were able to transfer to division. Brown's (1981a) research also revealed that a large number of children believed it was not possible to divide by a larger number - 50, 47, 43 and 23 per cent respectively of 11, 12, 13 and 14 year olds. Follow-up interviews showed that: "In most cases (children) gave an answer such as 'it doesn't go'....or 'you can't'.... or '16 is lower'....". (Brown, 1981a:308,309)

The other common response, 'no answer given', accounted for 23 per cent of the errors in the present survey. The number of error types (15) is about average for this survey, and the two most common error types account for 75 per cent of all errors - errors were most common in the 11 year old age group (33 per cent) and least common with the 14 year olds (16 per cent). Some of the more uncommon errors show attempts to multiply ('320', '3.2'), to shift the decimal point ('0.16'), and to subtract ('-4'). This latter type of error will be commented on in Chapters Seven and Eight (see pp. 131, 137, 170).

8. Estimation and Approximation: Items 13(d), 13(e), 13(f)

Item 13(d)
Ring the number you think is
NEAREST IN SIZE to the answer
(do not work out the sum)
2.9 x 7   .002/102/.2/2/20/200/ 2000

Facility Level: 45%

Because this item was in the multiple choice format, errors were confined to the given alternatives.

Errors	Frequency
'200'	(twenty two)
'.002'	(five)
'.02'	(seven)
'.2'	(four)
'2'	(four)
'2000'	(four)
no answer attempted	(eleven)

Errors were most common with the youngest group of students, there being 23, 12, 15 and 6 errors amongst the 11, 12, 13 and 14 year olds respectively. '200' was by far the most common error; '29 x 7' (i.e. disregarding the decimal point) is nearest to '200'.

A question arises about the appropriateness of the wording of this question, in that 'in size' may be ambiguous. This point is discussed in Chapter Four and relates to the next two items as well.

Item 13(e)
Ring the number you think is
NEAREST IN SIZE to the answer
(do not work out the sum)
0.39 x 7.1   .002/.02/.2/20/200/ 2000

Facility Level: 12%

Because the item was in the multiple choice format, errors were confined to the given alternatives.

Errors	Frequency
'.002'	(eighteen)
'200'	(eighteen)
'20'	(fourteen)
no answer attempted	(fourteen)
'.02'	(ten)
'.2'	(nine)
'2000'	(eight)

Errors were most frequent with the 11 year olds (31), dropping progressively to 17 errors for the 14 year olds. The two most frequently chosen alternatives, '.002' and '200' are almost at opposite ends of the given continuum (.002 - 2000). The reasons behind student choices were not explored in this survey, but use of a 'rule' such as "When multiplying with decimals, add up the number of places in the factors and transfer this to the product" may influence the frequency with which '.002' was chosen - the decimal places in the factors '0.29' and '7.1' total three.

Item 13(f)

Ring the number you think is NEAREST IN SIZE to the answer (do not work out the sum)

59 ÷ 190 .003/.03/.3/3/30/300/3000

Facility Level: 18%

Because the item was in the multiple choice format, errors were confined to the given alternatives.

Errors	Frequency
'3'	(twenty three)
no answer attempted	(fifteen)
'.03'	(thirteen)
'.003'	(thirteen)
'30'	(nine)
'300'	(seven)
'3000'	(five)

Response patterns were not as even as in the previous item, the two most common errors accounting for almost half the total errors. The distractor '3' (chosen in 23 instances) is the answer obtained if the working form is reversed from that given, i.e. '190 ÷ 59' instead of

'59 ÷ 190'.

The results for the three items on estimation and approximation can be compared with Brown's (1981a) data from 12 to 15 year olds:

	Brown (1981a) Facility Levels	The Present Study Facility Levels
Item 13(d)	44% - 62%	45%
Item 13(e)	15% - 31%	12%
Item 13(f)	10% - 22%	18%

Brown (1981a) notes that many of the children interviewed on these items from her sample (17 subjects) admitted they had 'just guessed' with Items 13(d) and 13(e), although their guesses in many cases had some basis. For example:

> "With (Item 13(e)) she was more precise; having first ringed 0.2 as an approximation for .29 x 7.1 she changed to 2; 'it was too small - it's more than a whole one - it has to have wholes - it just looks it'."
>
> (Brown, 1981a:287)

9. Problem Solving: Items 14(c), 14(d), 14(e)

The next three items explore problem solving in mathematics.

Item 14(c)		
Ring the calculation you would need to do to find the answer:		
The price of mince is $2.67	2.67 + 0.58	0.58 ÷ 2.67
for each kilogram. What is	2.67 ÷ 0.58	0.58 - 2.67
the cost of a packet containing	2.67 - 0.58	0.58 x 2.67
0.58kg of mince?		

Facility Level: 23%

Because the item was in the multiple choice format, errors were confined to the given alternatives.

Errors	Frequency
'2.67 ÷ 0.58'	(forty five)
no answer attempted	(eleven)
'2.67 - 0.58'	(eight)
'2.67 + 0.58'	(seven)
'0.58 ÷ 2.67'	(five)
'0.58 - 2.67'	(three)

The number of errors made by the 11, 12, 13 and 14 year old groups was 24, 18, 23 and 14 respectively. All groups showed a clear preference for the alternative '2.67 ÷ 0.58'. The two most common errors account for more than half the total errors. Respondents chose the alternative where a smaller number was divided into a larger. This is perhaps consistent with the results from earlier items where students believed a larger number could not be divided into a smaller one, (Items 12(f), 12(h), 13(f)).

Item 14(d)		
Ring the calculation you would need to do to find the answer:		
The cost of 6.44 litres of	6.44 + 3.58	3.58 ÷ 6.44
petrol was $3.58. What	6.44 ÷ 3.58	3.58 - 6.44
would the price of one litre	6.44 - 3.58	3.58 x 6.44
be?		

Facility Level: 18%

Because the item was in the multiple choice format, errors are confined to the given alternatives.

Errors	Frequency
'6.44 ÷ 3.58'	(forty eight)
no answer attempted	(eleven)
'6.44 - 3.58'	(nine)
'3.58 - 6.44'	(seven)
'3.58 x 6.44'	(five)
'6.44 + 3.58'	(four)

The numbers of errors from the 11, 12, 13 and 14 year old age groups was 26. 24, 22 and 16 respectively for this item.

Again, the two most common errors account for over half of the total errors, and the most common incorrect choice is a division operation. In this instance, division is required to obtain the correct answer, but the needed order of the divisor and dividends is the opposite to that most often chosen above.

Many students chose the correct operation, but the unwillingness to operate with a divisor larger than a dividend is once again manifested (c.f. Items 12(f), 12(h), and 13(f)).

Item 14(e)		
Ring the calculation you would need to do to find the answer:		
My car can go 11.8 kilometres on each litre of petrol. How many kilometres can I expect to travel on 8.37 litres?	11.8 + 8.37 11.8 ÷ 8.37 11.8 - 8.37	8.37 ÷ 11.8 8.37 - 11.8 8.37 x 11.8

Facility Level: 47%

Because this item was of the multiple choice format, errors are confined to the given alternatives.

Errors	Frequency
'11.8 ÷ 8.37'	(seventeen)
no answer attempted	(thirteen)
'8.37 ÷ 11.8'	(nine)
'11.8 - 8.37'	(eight)
'11.8 + 8.37'	(five)
'8.37 - 11.8'	(five)

The 11, 12, 13 and 14 year old groups generated 16, 9, 21 and 8 errors respectively. The high number of errors in the 13 year old sample (third formers) included seven instances of 'no answer', but is still noteworthy. The 13 year olds chose the full range of incorrect answers, following the overall pattern of responding most frequently (and incorrectly) to the division alternatives.

The 13 year olds' performance on Item 14(e) could be a chance or sampling phenomenon, and/or an artifact of non-systematic error.

Identical items are not available from Brown's (1981a) survey for the problem solving items discussed above, since her items contained the British decimal currency forms ('pounds', 'pence') and used 'gallons' and 'miles' in measurement (Item 14(e)).

Brown (1981a) also notes the increased difficulty of problem-solving with decimal numbers compared with whole numbers.

Summary and Comment

The findings from the quantitative survey of the ability of a sample of 11 to 14 year olds (n = 102) in working with decimal numbers may be summarized as follows:

1. Range of errors. The mean for the survey was 34.69 (possible = 64) with a standard deviation of 12.85. The range of scores for each grade/class level was 30-40 raw score points. There was a tendency for the number of errors generated to be less for older than younger students. Facility levels ranged from 11 to 94 per cent. Twenty four items out of 64 yielded facility levels below 50 per cent.

2. Areas of most difficulty. The items that caused greatest difficulty were: (i) name the place value of digits; (ii) addition with decimal numbers; (iii) assigning decimal numbers to designated points on a number line; (iv) understanding the size of decimal numbers; (v) writing decimal numbers; (vi) generating decimal numbers from verbal stimulus; (vii) multiplication with decimal numbers; (viii) estimation and approximation with decimal numbers; (ix) problem-solving with decimals; (x) division with divisors larger than dividends.

3. Number of Error types revealed. As well as students constructing a number of errors at all levels, the number of error types generated overall for some items was large. For example, Item 12(d) - multiply 2.3 by 100 - yielded 18 different errors; Item 12(g) - 24 ÷ 20 - produced 20 different errors; Item 11(d) - 23.12 + 54.7 - produced 26 different errors.

The patterns indicated above may be interpreted by reference to the thinking of constructivist theorists (see Chapter Two). For example, the fact that students appeared to generate 'creative' answers when questions were difficult might be surprising to a teacher, but would support the views of constructivists that individuals construct knowledge for themselves from the available stimuli (Kelly, 1955; Piaget, 1929; Wittrock, 1974a, 1980). The large range of errors children constructed in the present survey is accounted for by the generative learning model, in that the students' ideas in this case were constructed by generating links between the stimuli and their stored information. Generative learning theory recognizes the crucial influence of existing ideas on what is attended to and selected (Osborne, 1985; Osborne and Wittrock, 1983, 1985; Wittrock, 1974a). All students in this survey had existing ideas on decimal numbers through school and out-of-school experiences. According to the generative learning model these existing ideas might influence the construction and development of new ideas, new ideas that were subsumed into memory store even though they were incorrect mathematical beliefs. The diversity of errors generated by this sample of 11 to 14 year olds (n = 102) is evidence of the individual constructions of meaning that students possess and supports Claxton's (In press a, in press b) notion of mini-theories. Students move from one theory to another as they attempt to construct meaning. Stimuli are linked to existing relevant knowledge in memory. These links are formed in diverse ways. Eventually a meaning is constructed, even though it may yield an incorrect response.

> "The (generative learning) model rests on the assumption that while individuals vary in terms of what they learn in a given situation, these variations are primarily determined by differences in existing ideas, in learners' memories...."
>
> (Osborne, 1985:11)

An individual's existing ideas influence all learning; the generative model emphasizes the importance of students' present ideas in the construction of meaning, and we shall see evidence of this in the next three chapters.

The present quantitative survey lacked the power to investigate the students' strategies that led to the generation of a particular error. The data obtained were thus limited in analytical, diagnostic and predictive power. In this sense the quantitative data presented in this chapter contrasts with the data obtained from the longitudinal qualitatively-oriented facet of this research. We now turn to the longitudinal study.

CHAPTER SIX

RESULTS AND DISCUSSION: INDIVIDUAL INTERVIEWS

THE HIGH COMPETENCY GROUP

As described in Chapter Four, the findings of the initial survey in the present study were followed by a series of four individual interviews with a sample of 14 Form One and 14 Form Two students over a period of two years. The purpose of these interviews was to monitor and explore students' cognitive constructions about decimal numbers, taking particular note of any error patterns, and probing to ensure that an accurate picture was being gained of the student perspective. During the two-year period of this research two subjects withdrew from the original group of 15 Form One and 15 Form Two students; the attrition level was therefore 6.6 per cent over the two-year period.

Findings from the interview series are reported and discussed in this and the next two chapters. The present chapter focusses on the conceptions and attitudes of those students in the interview sample whose response patterns indicated high competency with decimals. Their general behavioural features are compared with those revealed by the initial survey. A detailed case study of one student is also provided in order to illuminate further the thinking and attitudes typical of the more competent student group as a whole.

In Chapter Seven the same pattern of presentation is used with reference to students in the interview sample who could be classified as having average competency. Chapter Eight focusses on students with a low competency level in working with decimal numbers.

To assist the reader, a separate copy of all stimulus cards used in the interviews is available inside the back cover.

The High Competency Group

The High Competency Group comprised five students (pseudonyms are used throughout).

TABLE 4 : The High Competency Group

Name	Sex	Class level (1981)	TOSCA Percentile (Rank)	TOSCA Stanine	PAT (Maths) Percentile Rank (Class)	PAT (Maths) Percentile Rank (Age)	Interview Dates: Interview I	Interview II	Interview III	Interview IV
Trish	F	F I	90 (Primary Form A)	8	81	83	18. 8.81	26.10.82	4.11.83	4.11.83
John	M	F I	83 (Primary Form A)	7	99	97	18. 8.81	13.12.82	22. 9.83	5.10.83
Charles	M	F II	79 (Secondary Form A)	7	64	75	4.11.81	7. 4.83	11.11.83	14.11.83
Andrea	F	F I	98 (Primary Form A)	9	96	95	14.12.81	15.11.82	11.11.83	11.11.83
Garth	M	F II	79 (Secondary Form A)	7	96	95	15.12.81	25. 1.83	26.10.83	31.10.83

M = Male
F = Female

Interview Response Patterns

Most of the students in this group had little difficulty with the questions posed by the stimulus cards, the majority of errors in the early interviews being corrected by the conclusion of the research project. Three of the group, Charles, Garth, and Andrea, were competent from the outset in most areas explored by the interviews. The others in the group, John and Trish, developed correct strategies to eliminate most of their errors as the two years progressed.

1. Estimation and Approximation: Unlike the students in the survey sample (Chapter Five), the High Competency Group's strength in estimation and approximation was noticeable from Interview I. The exception to this was Trish who developed competence in this area over the two years. The strategy most often used was to perform mental computations and then choose an alternative nearest to the answer they had obtained. John's response is typical:

John: Interview I: Stimulus Card D (Series One)

I: *(Reads Card D to S)*
S: *Ten.*
I: *Why do you say that?*
S: *I think one point nine times five is nine point five. That's point five away from ten. And no (other) number is closer.*

2. Positioning the Decimal Point: Another characteristic of these learners was that they knew where the decimal point 'belonged' - where it best made sense. Some students used a rule based on the position of a decimal point in a product after multiplication has been performed. For example, when asked why she put the decimal point between nine and five Andrea (Interview I) replied, *"Because I took it out to multiply, and then I just put it back in the same place again."* With reference to the executive/mini-procedures distinction (Davis, 1979), Andrea was mathematically correct at both levels. She selected the correct alternative from those provided as a result of her computation (mini-procedure), and evaluated where the decimal point should be placed (executive-procedure) in a precise manner. On the whole these learners readily generated links between the input selected from the stimulus card, and their existing ideas and operational ability with decimal numbers.

Charles and Trish were exceptions to the above facility with estimation and approximation. These students were tentative in their reasoning with Stimulus Cards D, E and F (Series One). Charles claimed that 1.95 x 5 was nearest to 10 in Card D after a mental computation, *"Well, five ones is five. And nine times five... so it's point forty-five, but I'm not sure. Then it's closest to ten."* Next, he experienced difficulty in rounding off to the nearest whole number, but eventually remembered a rule that he correctly applied to tens:*"...if you've got say, 55, and you have to work out which is the nearest in size to 50 and 60, you go up to 60 and not down to 50."* When asked why this was the case Charles replied, *"I don't know why. It's what we've been taught."* His response to the next card in this topic, Card E, showed that he computed .19 x 5 correctly, and knew that this was nearest in size to 1.0. This suggests his earlier incorrect computation of 1.9 x 5 for Card D (Series One) was a 'slip' rather than a stable error or 'bug' (Van Lehn, 1982).

Charles' incorrect answer to 1.9 x 5 of 5.45 (Card D, Series One) still led him to the correct alternative. The reasons behind this fortuitously correct response could only be gleaned from the individual interview. With a pencil-and-paper survey (c f. Chapter Five) the teacher would be unaware of Charles' underlying faulty computation that managed to lead him to the correct alternative from those given. Further examples of this phenomenon are discussed in Chapters Seven and Eight.

3. Division and Decimal Numbers: The next series of cards probed students' ability to divide with whole numbers, particularly where this gave a quotient that was a decimal. The survey results had shown that the item, 16 ÷ 20, was the most difficult in the whole test (see Chapter Five). In Interview I most students in the High Competency Group constructed correct answers, but these were not always based upon computational strategies. Rather, this group tended to use broad understandings, the result of strong links with existing ideas about fractional numbers in general. The following sequence with Andrea is illustrative of this characteristic.

Andrea: Interview I: Stimulus Card I (Series One)

I: *What about 10 divided by 20?*
S: *Point five.*
I: *How did you get that?*
S: *...it's point five, 'cause you can't do it one point. Because ten is less than twenty...*
I: *Say you had to do it on a piece of paper. Just write it out...how would you go about that?*
S: *(writes* $20\overline{)\,10\,}$ *with* 00 *written above)*
I'd go twenty into one, is zero. Twenty into ten is zero...(pause 15 seconds)...Then, if I hadn't realized it at once, I'd realize that ten was half of twenty.

This ability to generate links with existing knowledge*("I'd realize that ten is half of twenty")* characterized this group's responses. John put it more succinctly than Andrea when he responded to the operation 10 ÷ 20 by saying, *"Half. Point five."*

This ability to generate links with existing knowledge in order to construct meaning resulted in some of the High Competency Group drawing on other aspects of memory store to help them in providing an answer. Garth revealed a novel (but mathematically correct) method for computing 15 ÷ 20 involving the generation of links with ideas about common fractions, division, times tables, and decimal equivalents for common fractions.

Garth: Interview I: Stimulus Card I (Series One)

I: *15 divided by 20?*
S: *Point seven five.*
I: *How did you get that?*
S: *It's three quarters of twenty and fifteen...*
Five is a quarter of 20, 'cause it goes in four times.
I: *How did you work out in your head the answer to that?*
S: *Well, five is a quarter of twenty, and so it's three fives.*
I: *What is three fives?*
S: *Fifteen. So that's three quarters. Three quarters is point seven five.*

Not all of the High Competency Group understood as clearly as Garth, but even where tentative links were made with existing ideas, the answers provided were usually correct, and were rarely bizarre. Charles, for instance, solved the problem 10 ÷ 20 by using a conventional algorithm by adding a zero to 10, and then performing the division operation. His explanation for this was, *"You can't really stick 20 into 10, so you put 20 into 100."* He then placed a decimal point before the five with the explanation, *"You point the point there or something...yeah, I think it works out something like that."* His casual response was nevertheless based upon intelligible mathematical thinking - intelligible both to himself and the researcher.

4. Writing Decimal Numbers: The writing of decimal numbers proved troublesome. Although this group responded more correctly than did the overall survey sample, only two of the five (John, Garth) gave completely correct responses in Interview I. TABLE 5 shows the error patterns for the group.

TABLE 5 : Write decimal numbers: Interview I
(Stimulus Card J, Series One)

	The High Competency Group				
	three tenths	seven hundredths	fifteen hundredths	seventeen tenths	twenty hundredths
Trish	.3	.70	.15	.17	.20
John	.3	.07	.15	1.7	.2
Charles	.3	.07	.015	.17	.020
Andrea	.3	.07	.015	.17	.02
Garth	.3	.07	.15	1.7	.20 or 2

The most frequent errors were with writing seventeen tenths, fifteen hundredths, and twenty hundredths, a pattern matching the survey with 11-14 year olds (see Chapter Five).

Andrea wrote seventeen tenths as .17 and explained her response as follows, *"Well, if it was just ones it would be one point seven. But seeing it's tenths it's point seventeen. And that's the number of decimals."* As happened with students in the Average Competency and Low Competency Groups (see Chapters Seven and Eight), learners in the present group failed to realize that seventeen tenths was greater than one. From the data given to them, and from the form in which it was presented to them on the stimulus card, they were unable to construct an appropriate meaning.

John and Garth could correctly write seventeen tenths from the outset. Garth's explanation for his answer 1.7 was simply, *"Seventeen tenths is more than one."* John faced a small dilemma when writing the decimal number twenty hundredths in that he was unsure about the need for the last zero. He finally wrote 0.20 or 0.2 and explained, *"...it doesn't actually have the last zero on the end, I don't think."* Asked why this was the case he responded, *"Because twenty hundredths is the same as two tenths. Twenty divided by ten is two, and a hundred divided by ten is ten. So it's two tenths."* This is a novel method, demonstrating John's facility with manipulating numbers.

5. <u>Problem Solving</u>: Problem solving with decimal numbers was difficult, on average, for the survey sample (Chapter Four) and apart from Garth, was difficult also for the High Competency Group. With Stimulus Card K (Series One), for example, most of the group chose division as the operation required to obtain an answer. When asked why he chose 1.50 ÷ 0.5, John replied, *"Because...it's got one fifty and five. That's half of it, so it's dividing it in half. And that's half of it so that gets the answer."* The idea that multiplication with a decimal number is equivalent to division with a whole number was difficult for this group, but some were able to generate a correct response to the problem without using an operation with the given decimal numbers. Thus, for Card K (Series One) Charles explained, *"Point five is half, so half a kilo...No...(mumbles)...oh, seventy-five cents, 'cause it's half."* Rather than using the conventions of a mathematical operation, Charles relied on his existing ideas about common fraction equivalents for decimals *("Point five is half")*.

He constructed meaning from the crux of the word problem, and generated an answer from the relevant information. He was not distracted by generating tentative links to knowledge about how to write the operation, where to put the decimal point, or which operation to perform. Andrea adopted this approach as well.

Andrea: Interview I: Stimulus Card K (Series One)

I: *(reads card to S)*
S: *Seeing point five is half of whatever the whole is, I'd make it two into a dollar fifty...and that would make it seventy-five cents.*

(writes: $2\overline{)150}$ with quotient $75c$ *)*
I: *O.K. (unfolds algorithms) Now down the bottom here are some working forms. Which one of those would give you the answer do you think?*
S: *(reads) 0.5 ÷ 1.50.*
I: *Why have you chosen that one?*
S: *Because it's the same as I did except I just made it into two, 'cause two wholes make a half...(later)...*

(writes: $0.5\overline{)1.50}$ *)*
...I wouldn't do it like that probably, I'd just make it (0.5) into a two.

Andrea's explanation raises several points. First, like Charles, she generated a correct answer without having to work through a formal algorithm. Second, when forced to choose an algorithm she responded incorrectly, relying on an existing idea that division inevitably gave a quotient smaller than the divisor. Third, the requirement by the interviewer for Andrea to use a formal algorithm interfered with her attempts to solve the problem and made her uncomfortable. In the classroom context a similar situation might easily apply. In this context, the mathematically correct algorithm was not helpful. Rather, her informal and intuitive attempts to construct meaning were more productive. The low facility levels apparent with these items in the initial survey, may also be attributed to this factor.

Stimulus Cards K to N (Series One) also pointed to the different constructions of reality of the researcher and the student. The researcher believed these cards were exploring students' ability to solve problems with decimal numbers but, in the case of Andrea, the algorithms supplied interfered with the construction of a correct response to the problem. There was a mismatch between the existing ideas of the researcher and the student.

6. Understanding the Size of Numbers: The final set of cards probed understandings about the size of numbers with and without decimal points (e.g. 0.4 and 4), exercises that had not been part of the initial survey. The High Competency Group found this task relatively easy, although there was some tentativeness. Thus John's response to Card Q (Series One) was correct, but when asked to explain he replied, *"I'm not sure. But I think so...because we haven't done these before, you know, and I'm not sure how to do it."* However, he confirmed his earlier response, and could not be shifted by the interviewer to an alternative (and incorrect) answer. Garth's response was more typical of the group.

Garth: Interview I: Stimulus Card O (Series One)

S: *(reads card)*
I: *Which is the larger number there or are they the same size?*
S: *Four. (is larger)*
I: *How many times larger is it, than point four?*
S: *Ten.*
I: *Exactly ten?*
S: *Mm. (assent)*
I: *Are you sure?*
S: *Yes...'cause if you multiply point four by ten you move the decimal point one to the right.*

Charles explained that 123 was ten times larger than 12.3 (Card P, Series One) by working it out, *"Ten twelves is a hundred and twenty. Ten (times) point three equals three. So you add them together and get a hundred and twenty-three."*

7. Developmental Trends: Over the two years of the research project members of the High Competency Group repeated their pattern of largely correct responses. They also progressed in the areas that they had earlier found difficult. Thus Andrea, who wrote seventeen tenths as .17 in Interview I correctly wrote 1.7 in Interview II and a year later in Interview III. Her rationale for this during Interview II was, *"Ten tenths is one whole. Seventeen tenths is over one whole."*

With estimation and approximation, the group held its pattern of correct responding over all four interviews. Garth, during Interview I, for example, quickly realized that 1.9 x 5 was nearest in size to 10 because, *"One point nine times five is, uh, nine point five."* Thirteen months later in Interview II he responded to the same question with, *"One point nine is almost two. Two times five is ten."* Another nine months later in Interview III, Garth responded that 1.9 x 5 was nearest in size to 10 because, *"One point nine is about two. And five times two is ten."*

Likewise, with division that would yield a decimal fraction, the group as a whole consistently displayed correct responding. The students' existing mini-theories (Claxton, In press a, in press b) were adequate, and thus there was no need to generate alternative ideas. Compared to the general picture in the survey sample these above average learners seldom responded that no answer was possible.

Problem-solving (Stimulus Cards K-N, Series One) was initially difficult for most of the High Competency Group (as with the survey group described in Chapter Four). However, progress was evident in that four, seven, and fourteen correct responses were given in Interviews I, II and III respectively (out of a possible 20). But variations within the group were evident e.g. Garth mastered the problem solving tasks by Interview II, whereas other students made gradual progress over the two years.

With Stimulus Card K (Series One) most errors continued to show confusion about the meaning of division with a decimal number. During Interview II, John, for example, said that he would divide 1.50 by 0.5 because, *"You'd split it in half...'cause that's dividing it in half. You're only taking half the potatoes."* A year later in Interview III, he suggested the same method of obtaining an answer (1.50 ÷ 0.5) with the rationale, *"The price is $1.50 so you're only buying half of it... so you'd divide it in half."* John's own construction of a suitable method is correct in that one would have to halve $1.50 to obtain an answer. Earlier in Interview III on this same topic he noted that, *"In this case, seeing it's 0.5...I might just, um, divide it by two."* But when asked to identify an algorithm that would yield the answer he could successfully generate intuitively, he chose incorrectly. Some errors, therefore, seemed to be an artifact of the question format rather than the students' conceptualizations. John, above, realized that half of $1.50 was required - *"divide it by two"* - but failed to comprehend that dividing by 0.5 would not "divide it in half". This construction of meaning was very similar to Andrea's interview sequence earlier in this chapter.

A more eloquent explanation for a solution to Stimulus Card K (Series One) came from Garth.

Garth: Interview III: Stimulus Card K (Series One)

I: *(reads card to S)*
S: *Divide...$1.50 divided by 0.5 'cause it's half a kg of potatoes, so you have to divide the price by 2... Oh! I meant to multiply.*
I: *Why?*
S: *'Cause it's the same as dividing by two.*
I: *How can multiplying be the same as dividing?*
S: *Point five is a reciprocal of two.*
I: *How do you know that?*
S: *I've been taught it.*

With Stimulus Card L (Series One) Garth correctly suggested that multiplication with a decimal number would obtain the answer, *"Because when you multiply by point five eight you get point five eight of the answer."* When asked why he should divide by 0.58 Garth answered, *"'Cause that would make it bigger - it's (0.58) less than one."* Assigning place value nomenclature to decimal number columns was a simple task for this group. All students responded correctly at Interview I and held these responses to Interview III.

8. Attitude towards Mathematics: In the affective domain, the High Competency Group displayed very positive attitudes towards mathematics. The pencil-and-paper semantic differential test (Osgood, Suci and Tannenbaum, 1957) in APPENDIX E showed that these students saw mathematics as very good, and very valuable. In discussing his responses John said, *"Maths is a valuable course - whatever you do you'll need it."* Garth noted that, *"You need it (mathematics) for money and things and have it for jobs."* Andrea commented that mathematics was valuable, *"'Cause you need it all through life. It's sort of good in the way it's valuable."* Charles suggested, *"You use it (mathematics) later in life ...nearly everything is related to maths."*

Decimals were also viewed positively, but not as positively as the broader 'mathematics'. Garth and John saw decimals as extremely good and valuable, although John did comment that decimals was a rather boring topic. He explained further by saying, *"Well, not boring, just less exciting than, say, going on an orienteering course or something like that."* Andrea expressed her opinion of decimals (which was less positive than that of Garth and John) by noting their value and the ease with which she could master them, but also their over-riding dullness as a topic. *"They're just boring,"* she commented. Trish expressed neutral feelings towards decimals but saw them as slightly valuable: *"Mainly just...things like money and things like that."* Charles, though, expressed a negative attitude towards decimals, and saw them as extremely unfair: *"It's unfair to me because I'm not too good at them,"* he explained. Charles' comment was rather surprising considering his competence at decimals. He also remarked on their lack of use for the outside world, a contrast with his earlier view about mathematics.

Most of the High Competency Group saw mathematics as more complicated at high school than at primary school. Most also felt that they covered more topics at the high school level.

At the conclusion of Interview III the students were asked to sort nine cards with mathematical topics written on them into a 'like'/ 'dislike' continuum (see Chapter Four, Page 70). TABLE 6 shows the results for four of the group. (Trish's responses are presented later in this chapter.)

TABLE 6 : Responses to Sorting Task, Interview III: The High Competency Group

	Andrea	Garth	Charles	Jason
Like Most:	s	g	g	g
	g	d	+	d
	xt	fr	-	x
	+	↑	x	+
Neutral:	-		d	-
	÷	the rest	fr	s
	x		÷	x
	fr		s	÷
Dislike Most:	d		x	fr

Key: ÷ (division sums)
x (multiplication sums)
- (take-away sums)
+ (adding sums)
s (sets)
xt (times tables)
g (geometry)
fr (fractions)
d (decimals)
} (equal ranking)

The topic of decimals received a low ranking only from Andrea, who was unable to explain her response. Garth commented, *"I just like most of them,"* and placed the cards towards the 'like most' position. Jason explained his placement for decimals with, *"Well, decimals are reasonably easy to work."* This was the rationale adopted by these students. Topics they found easy were placed towards the 'like most' end of the scale and the converse held as well.

In the 'Advice to teachers' segment of Interview IV, the High Competency Group varied in their responses. Andrea felt there was nothing teachers could do to change things, while Charles and Trish recommended that teachers explain more clearly, take more time, and make sure "we've got it". Garth and John were reluctant to give any advice. Garth, for example, wouldn't generate any advice to teachers because, *"It depends on the (individual) people...just depends."*

The scenario that explored peer opinions of mathematics (see Chapter Four) revealed that most of the High Competency Group thought their peers held negative opinions towards mathematics. Andrea claimed, with regard to her friends, *"They growl, complain...How boring it is."* John remarked on the lack of success that his peers experienced when they said to him, *"Oh, I couldn't do my maths test. Couldn't do my homework ...,"* and the depressing effect this had on their view of mathematics.

Social disruption in the class was seen as a hindrance to learning mathematics by only one of the High Competency Group, Garth, who considered the noisy nature of the classroom was a problem for him. Others did not comment on social problems in the classroom. Andrea mentioned ability streaming in her school, in the sense that she was a member of an elitist group, the top stream class.

Andrea: Interview IV: Social Disruption in the classroom and peer opinions of mathematics

I: *(scenario for peer opinions, see Chapter Four)*
S: *They growl, complain.*
I: *What sorts of things do they say?*
S: *How boring it is.*
I: *(later) (exploring opinions of all third formers)*
S: *I don't really know (their opinions) 'cause I'm in the brightest maths class, and some of those kids probably don't like it. I don't know about the 'dumber' classes.*
I: *How many 'dumber' classes are there here?*
S: *Oh, they're not dumb, dumb, and that. Just dumber than the top stream. There's six third form classes. Sort of four in the middle and a top one and a bottom one.*
I: *Is it a good idea to do that?*
S: *In some ways...You don't have the kids holding...you don't sort-of have to hold other kids back.*

Stimulus Cards I and J (Series Two) presented problems with and without decimal numbers. Card I (Series Two) was answered correctly by all members of the group, but Card J (Series Two) was correctly answered by only John and Charles.

Garth's problems were illustrative of the difficulties that he, Andrea and Trish encountered when he complained, *"...the numbers confuse. That other one* (Stimulus Card I, Series Two) *was easy...This one's not got straight figures like one dollar."* Charles generated a correct answer by taking the *"half a litre off two litres, and the 25 cents off the dollar. One litre is 50 cents."* In this case he fortuitously obtained the correct response. For Card J (Series Two), therefore, the introduction of decimal numbers to a problem-solving context made the construction of a correct answer significantly more difficult, even for this High Competency Group. The links with the previous card (Stimulus Card I, Series Two) were not generally made. Only John performed the mathematical computation that was required.

The importance of real-life contexts for problems in mathematics was suggested in a sequence during Interview IV. During Interview III John had struggled to construct an appropriate meaning for the algorithm 15 ÷ 20, eventually saying it would be *"less than one"* and subsequently *'0.7 something...0.7"*. When this same card (Stimulus Card I, Series One) was re-explored in Interview IV, John was having difficulty still in constructing meaning. Eventually the researcher suggested that the numbers be put in some practical context. Initially John did not understand, but once he was able to generate links with existing knowledge he replied, *"15 apples shared out to 20 people...And they get less than one apple each."* When asked how much each person would receive he answered, *"Three quarters."* John readily constructed an appropriate response when the question was reformulated, a similar phenomenon to Andrea's response (Interview I, Stimulus Card K, Series One). John's sequence is interesting, as well, for demonstrating how individuals who accept a major responsibility for their learning necessarily need to generate links and actively construct, and test out meanings. John constructed the simple problem-solving response himself, tested the meaning to assess its plausibility (c f. Hewson, 1981), and eventually produced an answer that was satisfactory to his way of thinking about the problem. To use Brown and Van Lehn's (1982) explanation, John's Repair strategy had passed the Critic, and had now become a stable part of his behaviour.

Summary and Comment

The responses of the five members of the High Competency Group may, in summary, be characterized as listed below.

1. These learners were generally competent in coming to a reasonable answer. Their strength with estimation and approximation was apparent from the start, and was held and consolidated as the two years progressed. Thus, in the 'executive/mini-procedures' distinction (Davis, 1979), these students were competent at both levels.

2. A facility with generating links to appropriate knowledge in their memory store was noticeable. To assist them when working with decimal numbers, they actively generated links with their knowledge about common fractions and whole numbers (c f. Osborne and Wittrock, 1985).

3. Progress was made in earlier areas of student difficulty. In writing decimal numbers and problem solving, for example, this group of learners showed evidence of constructing more correct meanings over the two years. Progress in these two areas was in marked contrast to the picture, on average, in the large sample used in the initial survey in the study.

4. Where there remained errors at the conclusion of study, these errors may have been more an artifact of the questions *per se*, than the students' thinking. For example, although almost a third of the problem-solving cards (Stimulus Cards K-N, Series One) were responded to incorrectly at the conclusion of the study, interview sequences suggested that these students could generate correct answers if they were allowed to work independently of the associated (and given) formal working forms. Interview extracts with John and Andrea illustrated this clearly. Given a chance to construct meaning independently, these competent students did so.

5. Positive attitudes were generally shown towards mathematics, and the group found the topic of decimals relatively easy (except for Charles). On the other hand, this group considered that their peers did not find mathematics so easy to learn; nor did they enjoy the subject at school.

6. The group showed flexibility and ease in manipulating numbers, and a desire to dispense with existing theories if they were inadequate

(c.f. Claxton, In press a, in press b). Andrea's change from writing seventeen tenths as .17 in Interview I, to 1.7 in Interview II exemplifies this. (Further examples are given in the case study on Trish in the next section.)

7. These students normally possessed and chose the appropriate mathematics strategy (mini-theory) for tasks related to decimal numbers. As Claxton (In press a) says, "Probably the majority of a young person's learning is to do with choosing the right tool for the right job - when to use a conceptual screwdriver and when to use a hammer." The High Competency Group was able to estimate by rounding-off decimal numbers correctly, to choose appropriate and relevant common fraction equivalents, and to choose the appropriate reason for one number being larger than another. In short, they generally chose the right tool.

8. Many of the interview sequences revealed an apparent confidence in the High Competency Group. Responses were often taciturn, precise and assured. The following sequence showed this:

   Garth: Interview IV: Attitude towards Decimal Numbers

   I: *(discussing Garth's response to Semantic Differential). Decimals are pretty clear to you?*
   S: *Yep. I understand them.*
   I: *You're quite good at Maths?*
   S: *Yes.*
   I: *Do you think you're good at Maths?*
   S: *Yep.*
   I: *How do you know you're good at Maths?*
   S: *I'm in the top stream in maths classes.*
   I: *Are you! Top stream...What about School Certificate next year, Garth? Do you think you'll pass that all right?*
   S: *Yep.*

9. Finally, where problems were encountered, the translation of the situation into an every-day context assisted the learners to generate a correct response. John's impasse with 15 ÷ 20, and his subsequent resolution of the problem, was an example of this phenomenon: *"If it was 20 (divided by 20) it would be one each...so anything less than 20 will be less than one...zero point seven something..."*

Trish

A Case Study from the High Competency Group

Trish was interviewed four times over a period of 26 months (see TABLE 7). Her standardized test results in mathematics (PAT) suggested that she was in the top 20 per cent of students in her age and class levels.

TABLE 7 : Personal Data for Trish

Interview I 18 August, 1981	Age at Interview: 11 years 5 months
Interview II 26 October, 1982	Standardized Test Results (1981) *TOSCA P.R. 90
Interview III 4 November, 1983	*TOSCA Stanine 8 PAT (Maths) P.R. (class) 81
Interview IV 4 November, 1983	PAT (Maths) P.R. (age) 83 *(Primary Form A)

Interview I:

The initial set of stimulus cards probed students' ability to estimate and approximate. Trish quickly encountered problems.

Trish: Interview I: Stimulus Card B (Series One)

I: *(reads card to S)*
S: *Point one two.*
I: *Why?*
S: *Um...'cause that's really ten and a two (.12)...and... sort of that's (1.2) a kind of a ten and that's a two.*
I: *One point two?*
S: *Mm (assent)*

She indicated uncertainty with the next four cards. Comments such as, *"I don't really know', "haven't done these',* and *"we've never done these things before...with the times"* were scattered through her responses. Generally she attempted to generate links with existing ideas about the multiplication operation, *"I times one point nine by five and I got ninety-five."* Her computation (or 'mini-procedure') was correct; her understanding or evaluative move to position the decimal point ('executive procedure') was incorrect.

Dividing by ten caused no problems for Trish (Stimulus Card G, Series One) as exemplified by her response to 100 ÷ 10. She responded, *"Well, if you divide it, ten goes into ten once, and you just put the zeros."* And with 15 ÷ 10 she replied, *"Well, divide ten into fifteen goes once, and there's five remainder and fives half of ten, so it's one and a half."* Stimulus Card H (Series One) involved division with 100, and again Trish coped successfully. When solving 1500 ÷ 100 she answered, *"I did a hundred into a hundred and fifty goes once with fifty remainder. Fifty with zero makes five hundred. And a hundred into five hundred goes five."* The solution to 15 ÷ 100 did cause her difficulties (c f. Item 12(f) in Chapter Four), however, but she realized an answer was possible, *"...might be able to divide it right down into... little numbers."* Here she thought the answer would be expressed by "little numbers", but her existing mini-theory (Claxton, In press a, in press b) was inadequate in its range of convenience to provide an answer to the particular problem 15 ÷ 100. Stimulus Card I (Series One) was answered correctly, but she relied on her existing ideas about common fractions to answer 15 ÷ 20 and 10 ÷ 20 by responding, *"three quarters"* and *"half"* respectively. Even so, this was an improvement on the survey sample (Chapter Four), most of whom thought no answer was possible.

The writing of decimal numbers proved more difficult for Trish. When asked why she wrote .17 for seventeen tenths, she responded, *"Well, there's ten tens in one...ten tens in ten...and there's more than that so I put the seven next to it in the hundreds column."* Although her exact reasoning was unclear, it would appear that she was using a strategy linked to whole number counting such as changing columns when they became overloaded. The construction of meaning in this area, though, proved to be troublesome for Trish over the two year period. Her error reflected the pattern of many 11-14 year olds discussed in Chapter Five.

The next set of cards (Stimulus Cards K-N, Series One) contained problems involving decimal numbers. As with Charles' response discussed earlier in this chapter, Trish generated a strategy that would yield a correct answer to Stimulus Card K (Series One), *"...point five equals half. If you divide one point fifty by half you'll get the right answer."* However, this led her to choose the incorrect working form 0.5 ÷ 1.50. Left to her own reasoning she was capable of generating a correct meaning. Her mini-theory here was over-extended (Claxton, In press a, in press b) in the sense that she applied her theory to a new situation (the working forms) and it let her down.

The next card (Stimulus Card L, Series One) was answered correctly after some thought, but eventually she decided, *"...two point six seven...for each kilogram. You want point five eight kilograms of mince. You'll have to times it."* Card M (Series One) provided an example of a learner testing various strategies, and rejecting them one-by-one. With this card Trish started with times, then moved to plus, and finally settled with take-away. To use Claxton's (In press a) explanation, Trish was tentatively advancing different mini-theories into the same area, *"to see if it works any better"*.

The final problem-solving card, Card N (Series One), revealed that Trish could construct meaning from the word problem in terms of realizing that division was the needed operation. However the algorithm chosen was again incorrect (c f. Stimulus Card K, Series One, above), and from her rationale it can be seen that an inappropriate link had been made with existing ideas.

Trish: Interview I: Stimulus Card N (Series One)

I: *(reads card)*
S: *Six point four four divided by three point five eight.*
I: *How did you work that out?*
S: *I just realized it was one litre...you divide it because that's six point four four litres and you only want one. If you divide it you'll get your answer.*
I: *And so how would you divide it? Which one into which one?*
S: *Six point four four...oh...what do you mean?*
I: *Which one into which one? (unfolds card) You see, there are two dividing ones there (reads them to S). Which one will give you the answer Trish?*
S: *Six point four four divided by three point five eight.*
I: *Why did you choose that one?*
S: *'Cause you can't divide six point four four into three point five eight.*

Trish was correct, of course, if one considered concrete discrete objects that could not be partitioned, e.g. sheep, houses, cars etc. It was also the answer most often chosen in the quantitative survey. This immature conceptualization of division was evident with the Low Competency group as well, discussed in Chapter Eight.

Finally, Interview I probed Trish's understanding of the relationship between numbers with and without decimal points (e.g. 0.4 and 4). Trish responded incorrectly to all cards (Stimulus Cards O-R, Series

One). Her existing idea about the place value nomenclature of columns in decimal places (tens and hundreds) led her to describe the decimal number as the larger number in all cases. For example, she saw 0.4 as ten times larger than 4, because, *"The first column is tens...the first column after the decimal on the right."* Thus 0.4 represented 40 in her construction of meaning. This failure at the 'executive-procedure' level was consistently held for all numbers during this facet of Interview I.

Interview II:

The noticeable feature of Trish's responding during this interview was the progress she had made.

Illustrative of this positive trend were her responses to Stimulus Cards A-F (Series One) on estimation and approximation where four correct answers were given (previously one). In choosing which alternative was nearest in size to 1.9 x 5, Trish answered, *"Well, one point nine times five is nine point five. And if you get a five you round it up to the nearest one...ten."* In the two instances of incorrect responses in this interview, she was partially correct. With Card E (Series One) she correctly computed 0.19 x 5 as .95, but then chose a wrong alternative from those listed. This could well have been a 'slip' rather than a 'bug' (Van Lehn, 1982).

A second area of significant improvement was in problem-solving with decimal numbers, with three of the four cards correctly answered (Stimulus Cards K-N, Series One). Her response to Card M (with which she had explored several strategies in Interview I) was simple and clear.

Trish: Interview II: Stimulus Card M

I: *(reads card to S)*
S: *I would divide. (Why?) Because 75 cents is for 2½ litres of soft drink and I only want one litre. So I'd divide it to see how much one litre would cost...I'd divide point seven five by 2.5.*

(writes $2.5\overline{).75}$ *)*
I: *And why have you done that?*
S: *Because, um, it's 75 cents for 2½ litres. And I want one litre. So I have to divide to see how much one litre would cost.*

A third series of cards she mastered (in contrast with Interview I) were Stimulus Cards O-R (Series One) to do with comparing numbers.

During Interview II her explanation moved towards correct mathematical ideas. When discussing the size of 0.4 and 4 she noted, *"4 is greater. Point four is only four tenths, and four is a whole four."* With 123 and 12.3 (Stimulus Card P, Series One) she explained, *"Twelve point three is only 12. 123 is a whole 123."* Card R (Series One) posed some problems for her, but she eventually generated the working from 2.49 x 100, worked this through, and came to the conclusion that 249 was 100 times larger than 2.49.

The writing of decimal numbers continued to cause her problems. Seventeen tenths was still written as .17, and twenty hundredths as .020 (c f. Items 9(b) and 9(c) in the survey, Chapter Four). However, she knew the correct place value nomenclature for decimal numbers (an improvement over Interview I's responses). When asked how she learned place value terminology, Trish replied, *"I don't know. That was just the way we were shown."* Later she remembered learning this from *"the teacher and my dad...my dad is a maths teacher."*

Trish's background of existing ideas from memory store (Osborne and Wittrock, 1985; Wittrock, 1980) was apparent when the researcher discussed in an informal manner the notions that she had built up over several years. The discussion focussed on whole number place values, and the researcher asked why our number system used the names it did. Trish replied, *"'Cause, like those little things we used to play with - had all those spikes on them - the board like that (indicates flat surface) and we used to slip those little wee things on...(Researcher: Abacus?) Mm. Well, when we came up to nine it became one ten, so we put one on there, and so that became tens..."*

Trish expressed confidence in dealing with decimals, but she found the topic lacked interest. At one stage in Interview II she commented, *"I mean, I know how to do them (decimals), but I find them boring."* She then explained how nobody had ever outlined to her the meaning and use of decimal numbers.

In pursuing this comment, Interview II ended with a discussion on the use of decimal numbers outside school.

Trish: Interview II: Social Usefulness of Decimals

I: *Do you think we use decimals outside school? In the outside world? Say, shopping, buying things?*
S: *Not really. Only in things in building and that I think. Builders might use it...I'd say when they're working out the thickness of something.*

I: *What about when mum is in the supermarket? Would she ever use decimals there?*
S: *No, I don't think so. 'Cause things are usually in whole numbers, like seven dollars and four cents or something... 'cause you can't make a tenth of a cent!*
I: *So decimals wouldn't come into money?*
S: *...(shakes head)...Only with things like width and that.*

Clearly there existed a 'gap' (Claxton, In press a, in press b) in Trish's mini-theories concerning decimal numbers. She articulated one application of decimals for linear measurement, but apart from this no alternative uses were recalled. For Trish, decimals was very much something you studied at school.

Interview III:

The general characteristic of Interview III was that Trish consolidated upon the progress she recorded during Interview II. Correct mathematical ideas were retained, and a developing understanding of decimal numbers was evident. When challenged by the researcher, she articulated her reasons for the choice she had made:

Trish: Interview III: Stimulus Card C (Series One)

I: *(reads card to S)*
S: *One.*
I: *Why?*
S: *Zero point eight is over five...it's sort of like over one half...the closest is one.*
I: *Why isn't it closest to one point eight?*
S: *Because zero point eight isn't one whole thing. And one point eight is almost two.*

With the Stimulus Cards A-F (Series One) Trish progressed in her ability to estimate and approximate. In discussing Card E (Series One) she knew that .19 x 5 would *"be about one"*, and she could not be shifted from that view.

Two other areas that were mastered during Interview II - her ability to solve problems with decimal numbers, and to successfully compare numbers with and without decimal points - were again mastered in Interview III. However, with some cards there was hesitation in coming to the mathematically-correct response, and evidence of testing various mini-theories to eventually generate a link with an appropriate existing idea. Stimulus Card M (Series One) had been difficult for Trish in Interview I. By Interview II she quickly generated a correct response. But in

Interview III she initially chose take-away as the operation needed to get the answer, only switching to the correct division algorithm after fully exploring whether or not take-away was appropriate in this context, *"Divide it (75 cents) by two point five, 'cause that would give you an answer of one litre,"* she finally decided.

Writing of decimal numbers continued to be a problem area for Trish. Her Interview III responses were exactly the same as in Interview II, errors being made with fifteen hundredths (0.015), eleven tenths (0.11), and twenty hundredths (0.020). When asked to generate the names for the place-value columns in decimal numbers she did this successfully. When asked if she used the columns to help her write decimal numbers, Trish replied, *"Not really."* Her explanation for writing fifteen hundredths as .015 was, *"Well, I first put it (15) straight after the decimal point. Then I thought that was like 15 tens so it was 15 hundreds I needed. So I needed to move one more space to the right, so I put a zero in."* Likewise, Trish explained the writing of seventeen tenths as .17, *"Well it was tenths so I just put it straight after the decimal point."* At this point the researcher returned to fifteen hundredths and explored Trish's rationale in greater depth.

Trish: Interview III: Stimulus Card J (Series One)

S: (writes fifteen hundredths as .015)
I: *Fifteen hundredths, again. Why is the zero there?*
S: *'Cause if I'd put it straight after the decimal point it would have been like fifteen tenths.*
I: *So, in other words, what does the zero do?*
S: *Moved it over one, on to the right.*
I: *But if you've got a one and five there, the five will be in tenths of hundredths (to use her term for thousandths). Are you happy with that?*
S: *Well, I thought you read the first number first.*
I: *What do you mean?*
S: *Well, you go fifteen, and because the first number is in the hundredths column, that would be fifteen hundredths... It wouldn't be fifteen tenths of hundredths!*

This stable error, or 'bug' (Brown and Van Lehn, 1982; Van Lehn, 1982) had become part of Trish's repertoire of knowledge about decimal numbers. Until a more intelligible, plausible and fruitful idea was presented (Hewson, 1981), then Trish would conceivably continue with her present mini-theory. Her theory had links to existing ideas about decimal numbers column nomenclature, and with conventions for reading numerals. To use her words: *"You read the first number first."*

The final set of Stimulus Cards (Cards O-R, Series One) posed no problems for Trish. As with Interview II she was able to construct mathematically correct meanings in all cases. For example when comparing 249 with 2.49 Trish explained, *"Two hundred and forty-nine is larger. One hundred times larger (than 2.49)."* When asked if it were exactly one hundred times larger, Trish replied, *"Yeah - 'cause two point four nine times one hundred is twenty-four thousand nine hundred and then if you put the decimal point in where it's meant to go then you have two hundred and forty-nine."* Although her initial reasoning was not strictly correct (2.49 x 100 ≠ 24900), Trish was able to place the decimal point in the appropriate place and so show that 249 was one hundred times greater than 2.49.

Interview III incorporated a device for measuring attitude towards decimals. This sorting activity (see Chaper Four, Page 70) was also used in Interview II, and the results are presented in TABLE 8.

TABLE 8 : Trish: Responses to Sorting Task: Topics in Mathematics

	Interview II Responses 26.10.1982	Interview III Responses 4.11.1983	
Like Most:	xt	x	
	g	xt	
	x	÷	
	-	+	
	+	fr	} equal
Neutral:	s	d	
	÷	-	
Dislike Most:	fr	g	
	d	s	

Key: ÷ (division sums)
x (multiplication sums)
\- (take-away sums)
\+ (adding sums)
s (sets)
xt (times tables)
g (geometry)
fr (fractions)
d (decimals)
} (equal ranking)

The results revealed considerable differences between interviews for some topics (e.g. geometry) and consistent placings for others (e.g. times tables, multiplication sums). Trish said during Interview II that her high placement for times tables was, *"'Cause when I was in Standard Two I found I couldn't learn them, then the next day I knew them all and I've never forgotten them."* During Interview III she said she liked multiplication sums best of all, *"'Cause I learnt my times tables in Standard One and I've never ever forgotten them... and it's really easy for me."* Decimals improved its ranking (in Trish's perception). Interview II comments from Trish were, *"'Cause no-one's really told me what they really are. I mean I know how to do them, but I find them boring."* This opinion contrasted with her largely neutral stance towards the topic in Interview III.

Interview IV:

Interview IV was carried out later in the same day of Interview III. (Trish had shifted to a town 130 kilometres from Hamilton and it was necessary that both interviews be conducted on the same day.) Because the interviews used different Stimulus Cards, it was thought that memory would not contaminate responses in the second interview, nor was fatigue felt to be a limiting factor.

Stimulus Cards A-C (Series Two) were checking devices and confirmed Trish's earlier competence with estimation and approximation. Stimulus Card C (Series Two) showed her ability to perform division with a divisor larger than a dividend, *"Well, fifty can't go into ten, so I added a zero"* - by generating the correct algorithm. She then computed a correct response.

Stimulus Card D (Series Two) asked her to write decimal numbers. Again Trish responded incorrectly, writing .11 for eleven tenths, and .025 for twenty-five hundredths. When asked to explain her response for eleven tenths she replied, *"Well...you are in the tenths column so you just start your figures in the tenths column."* For Trish, there was no conflict in this rationale.

Stimulus Cards I and J (Series Two) presented word problems without (Card I) and with (Card J) decimal numbers. As with the High Competency Group in general, Trish found the latter problem more difficult because, *"You had a half that you had to get rid of...the two point five one you couldn't find half out straight away..."*

The remainder of Interview IV explored Trish's opinions and attitudes towards mathematics and the topic of decimals. Initially she was asked to consider the primary/high school interface. Primary school maths was seen as: *"We often did a lot of cards and sets, and things like that...but here (high school) you do a lot of percentages and decimals and fractions and everything like that."* She also indicated that she was taught better at high school; *"Because he's (the mathematics teacher) showing us more things, he's got more time."* About primary school maths, on the other hand, Trish said, *"We were put in three groups, then you'd be down there for about ten minutes and then she'd (the mathematics teacher) tell you to get on with the card. And you'd just pick the card up and do it."*

In discussing her semantic differential test result (Osgood, Suci and Tannenbaum, 1957), Trish thought maths was good because, *"I can understand it easy."* It was seen as valuable because you *"have to have maths"* for most jobs. Maths to her was clear, *"Yeah, well, I can understand things if I'm shown it once...I can sort of figure it out."*

Decimals were not so positively perceived by Trish, *"I don't like doing them."* Although seen as valuable, Trish's comment did not support this opinion, *"Well, for a job it's not as if you're going to be doing a lot and a lot of decimals."* Other responses for decimals were largely neutral, with a very slight preference for the labels 'sick', 'sad', and 'cruel'. Decimals were *"just a little bit (cruel). I don't mind it very much."*

Trish was next asked to give advice to teachers on how they might improve the teaching of mathematics. She gave five strong and clearly articulated pieces of advice:

(i) She considered that teachers should not stay on the same topic too long, *"or else it becomes boring"*; (ii) she advised teachers to explain and *"make sure they understand it, 'cause half the time they just say, 'oh, yeah, yeah, yeah' they know. But half the time they don't"*; (iii) Trish hinted at the need for a diagnostic model of teaching when she said, *"give them some problems to answer, and just see how long it takes them to do it...because if it takes them a really long time it's obvious that they don't really know what they're doing... they're just guessing"*; (iv) Trish wanted teachers to motivate learners to a greater extent. She explained this by saying, *"put a bit of variety in it...'cause it's quite good to do something like decimals and that and then...go on to geometry where you're drawing things with compasses...that's sort of interesting"*; (v) Trish advised against teachers explaining work to only part of the class, *"take them all at once because then everybody knows what's going on"*.

After listening to the scenario for eliciting peer opinions of mathematics (see Chapter Four, Page 67 Trish replied, *"Oh...aw... Gor..."* in imagining her friends' reaction. Next, she commented that *"it's really just the teacher...he's better now, but he used to keep us in."* Trish next explained that most students may say they don't like mathematics, but they may judge the subject on the tests associated with it. In her words, *"'Oh, I don't like maths'. They sort of judge it from one test."*

Trish described the role of the teacher in mathematics by stating how essential a teacher was. She felt that *"you can do maths anywhere. You can do it outside and that, but you can only do it if you know how to do it - you've got to have someone show you how to do it first.* Later she commented, *"...maths, you just can't go to a book and copy it out and sort of know what you're doing. Like in social studies if you read it, then you copy it out, you know what you're talking about. But in maths, you can't do that."*

During Interview II Trish had indicated that decimal numbers had limited application to the world outside school. This topic was explored in Interview IV. She was asked if decimals were used mainly in, or out, of school. Trish replied, *"Mainly in school."* When asked why, she continued, *"Well, it's not as if you're going into a shop and say I want three tenths worth of apples please, or something like that..- it's just in problems and things like that, to work out."* For Trish, decimals had remained very much a school pursuit.

Summary and Comment

Trish's responses over the 26 months that she was studied may be summarized as follows:

1. Trish progressed markedly over time in her ideas about decimal numbers. Initially incorrect answers and uncertainty were gradually replaced by correct constructions of meaning and a greater confidence. By the final interview she was saying, *"I can understand things if I'm shown it once."*

2. Trish appeared to accept responsibility for her own learning, an important characteristic for successful learning according to Osborne (1985). She grappled with ideas and coped. At various

stages she would comment, *"I can sort of figure it out"*, *"I know that..."*, *"I was thinking that..."*. When asked if social disruption in the classroom was a problem for her, Trish replied, *"Oh, not really. 'Cause if I'm concentrating I don't hear anything else around me."* During Interview IV she said, *"I just sort of remember what I did in Form One, and then sort of guess what it would be...making it up as I go along."*

3. Learning was seen by Trish as a process of active construction. Correct ideas were generally held on to once they were constructed. That is, she subsumed appropriate constructions into memory store. For example, her ability to estimate and approximate was assisted by links that were generated to existing ideas about 'rounding-off' numbers and common fraction equivalents for decimal fractions.

4. A stable 'bug' (Van Lehn, 1982) was evident in the topic of writing decimal fractions. Seventeen tenths, for example, was written as .17 right through the study. Trish's mini-theory for this response was obviously intelligible and plausible to her, and she did not recognize the potential conflict of this mini-theory with her other mini-theories about place value nomenclature, and the potency of the abacus (discussed in Interview III).

   During the last minutes of the final interview with Trish (Interview IV) the researcher revisited seventeen tenths. The common fraction version was written $\frac{17}{10}$, and the researcher explained how this meant, therefore, that seventeen tenths was 1.7. Trish watched and commented, *"Yeah...Mm...See, I've never been shown that before...because tens into seventeen you put one down and you've got seven left over."* It appeared that she was now ready to construct a correct mathematical meaning.

5. Mathematics was generally viewed positively by Trish, but decimals less so. Advice to teachers was a plea for more explanation, more variety in lessons, and attempts to really find out if students understood the material. Trish indicated the crucial importance of the teacher in the subject of mathematics.

6. A disquieting and consistently held view by Trish over the two years was that decimals had little application to the world outside school. Decimals were, in her view, *"Just in problems, and things like that, to work out."*

CHAPTER SEVEN

RESULTS AND DISCUSSION: INDIVIDUAL INTERVIEWS

THE AVERAGE COMPETENCY GROUP

This chapter focusses on the conceptions and attitudes of those students in the interview sample whose response patterns indicated average competency with decimals.

The Average Competency Group

The Average Competency Group comprised 14 students (pseudonyms are used throughout).

TABLE 9 : The Average Competency Group

Name	Sex	Class level (1981)	TOSCA Percentile (Rank)	TOSCA Stanine	PAT (Maths) Percentile Rank (Class)	PAT (Maths) Percentile Rank (Age)	Interview Dates: Interview I	Interview II	Interview III	Interview IV
Stu	M	F II	72	6	43	49	19. 8.81	12. 4.83	11.11.83	14.11.83
Percy	M	F I	51	5	60	43	10.12.81	4.11.82	14.11.83	17.11.83
Suella	F	F II	75	6	76	76	15.12.81	4. 3.83	27. 9.83	6.10.83
Delwyn	F	F I	49	5	60	52	18. 8.81	7.12.82	14.10.83	18.10.83
Trudy	F	F II	53	5	54	58	16. 9.81	23. 1.83	25.10.83	26.10.83
Rex	M	F II	46	5	46	40	15.12.81	21. 2.83	27. 9.83	4.10.83
Mary	F	F I	95	8	49	63	10.12.81	16.11.82	14.10.83	18.10.83
Don	M	F I	42	5	63	66	18. 8.81	15.11.82	5.10.83	6.10.83
Anne	F	F I	53	5	69	62	14.12.81	2.11.82	4.11.83	4.11.83
Peter	M	F I	53	5	81	54	16. 9.81	13.12.82	13.10.83	17.10.83
Danny	M	F II	75	6	61	54	15.12.81	22. 2.83	28.10.83	31.10.83
Dwight	M	F II	64	6	58	70	14.12.81	26. 1.83	1.11.83	2.11.83
James	M	F I	59	5	52	55	10.12.81	13.12.82	14.10.83	18.10.83
Petra	F	F II	53	5	85	57	14.12.81	26. 1.83	1.11.83	2.11.83

M = Male
F = Female

Interview Response Patterns

1. Estimation and Approximation: The initial pencil-and-paper survey (Chapter Five) found low facility levels amongst the 11-14 year olds (n = 102), (e.g. only 12 per cent of the survey group chose the correct alternative nearest in size to 0.29 x 7.1, the most common errors being .002 and 200).

The Average Competency Group found it difficult to estimate and approximate with decimals. None of the group correctly responded to all Stimulus Cards (A to F) in Interview I, and only one student, Peter, could do so by Interview III. Responses to Stimulus Card A (Series One) were correct in Interview I but the introduction of decimal numbers resulted in more errors. Stimulus Card B (Series One) requiring respondents to identify the number nearest in size to 1.2 (correct answer 1) produced a variety of responses. Trudy (Interview I) considered that 0.12 was nearest in size to 1.2 because, *"It's the decimal, it's different from an ordinary number. It wouldn't be just one."* Mary (Interview I) chose .2 as nearest in size to 1.2, but was unable to justify her choice. Don also chose .2 as nearest to 1.2, *"Well, that's the closest to point two, but it hasn't got the one in front of it...(later)...I'm not very good on those."* Percy, on the other hand chose 1 as closest in size to 1.2, *"Well, 'cause...it's only point two away from one, and that's the closest one...to one."* Suella gave evidence of "rounding off" numbers when she suggested (correctly) that 1.2 was nearest to 1, *"It's a whole number, and part of it...one point six would be nearer to two."*

For Cards E and F (Series One) only three of the 14 students in the group generated a mathematically-correct response. Existing ideas about multiplication with whole numbers were used extensively, but subsequent placing of the decimal point was either ignored or incorrectly performed. Don decided .19 x 5 (Stimulus Card E, Series One) was nearest in size to 100 because, *"Five times nineteen is...around a hundred or in the nineties...and it's nearest to it."* Generally, there was a shifting of responses as the students tried first one mini-theory (Claxton, In press a, in press b) then the next. Trudy (Card E, Series One) estimated 100, then .10, then .01 to be nearest in size to .19 x 5. Another common response with members of this group was to give up, to recognise that there was a 'gap' in their knowledge. Peter (Card E, Series One), responded, *"I don't know. I'd just times the fives and the nineteen. I don't know how to do this. But, ah, five nineteens, hang on...be ninety-five. And there'd be a point. I can't really remember how to do that."*

Peter had attempted to generate links with his existing knowledge, but finally he recognized that he was unable to construct anything meaningful from the particular problem. Anne's response represented many of the problems faced by these learners.

Ann: Interview I: Stimulus Card D (Series One)

I: *(reads card to S)*
S: *One hundred.*
I: *Why do you think that?*
S: *...point zero one.*
I: *Sorry?*
S: *That one (points to 0.01).*
I: *Why is it nearest to that?*
S: *'Cause that there (1.9 x 5) is nearest to a hundred.*
I: *Sorry, what do you mean?*
S: *If you times it, it will be near to a hundred, then you'll be just a small point off...*
(works 1.9
x5
9.5)
I: *So what's one point nine times five?*
S: *Ninety-five (then discusses placement of decimal point).*
I: *So one point nine times five will be nearest in size to which one of those down the bottom, do you think?*
S: *Either that one (100) or that one (0.01).*
I: *Would you like to say which one out of those two? Can you work it out?*
S: *A hundred.*
I: *Why?*
S: *See, there's nine point five, which is five away from a hundred.*

Anne's behaviour in this sequence was very much an exploration of mini-theories (Claxton, In press a, in press b), until she adopted one that did not trouble her. To the teacher of mathematics Anne's theory may be incongruous, but to her it was not. Anne's response was correct at the 'mini-procedures' level, (Davis, 1979) but wrong at the 'executive' or 'macro-procedures' level - computationally she was error-free, but her final evaluation and selection were incorrect.

2. Division and Decimal Numbers: Stimulus Cards, Cards G-I (Series One) probed the students' ability to divide, particularly when the divisor was larger than the dividend. In the quantitative assessment (Chapter Five) this task had proven one of the most difficult of the entire survey. The results from Interview I confirmed this. Most of the group responded correctly to Stimulus Card G (Series One) which involved division by ten with no dividends less than fifteen.

To the item 15 ÷ 10, for example, James replied, *"...you can only get one ten out of fifteen, and there's five left. So you, um, have got five remainder."* Delwyn explained in a similar fashion, *"Ten goes into fifteen once, and it doesn't go again...because you haven't got enough."*

Stimulus Cards H and I (Series One) were especially difficult to this Average Competency Group, particularly the two items 15 ÷ 100 and 15 ÷ 20. These two questions elicited a range of error types. For some students, the response was simply "You can't do it". Mary and Delwyn reversed the order of the divisor and dividend and achieved an answer. Delwyn explained her method as, *"...two fifteens are thirty... six fifteens are ninety...and there isn't enough left over to make a hundred."* Six students suggested that negative integers could be used to solve the problem of dividing a larger divisor into a smaller dividend. Percy thought you could not solve 15 ÷ 100, *"unless you go minus, minus eighty-five."* When asked what was meant by "minus eighty-five" he explained, *"Well, if you get to zero then go minus one, minus two, minus three..."* Suella was another who thought negative numbers could provide an answer to 15 ÷ 100.

Suella: Interview I: Stimulus Card H (Series One)

I: *What about fifteen divided by a hundred?*
S: *Ah, it would be negative something.*
I: *Negative something?*
S: *Well, there couldn't be an answer for it, 'cause it can't be divided by a hundred unless you went into negatives.*
I: *I see. Have you learned about negative numbers this year, have you?*
S: *Oh, we just did, a couple of days ago. We just did a little bit.*
I: *A couple of days ago...*
S: *Yeah...*
I: *So, what makes you think there would be a negative number then?*
S: *'Cause it's impossible to divide fifteen by a hundred. You'd have to go below zero.*

Suella's mini-theory was based on a recent classroom experience (according to her explanation) and might be described as an unintended curriculum outcome. However, according to the generative model of learning (Osborne, 1985; Osborne and Wittrock, 1983, 1985; Wittrock, 1974a), primacy is given to the learner's existing ideas. Thus Suella generated links to her existing ideas about negative integers.

3. Writing Decimal Numbers: The writing of decimal numbers proved difficult in the survey described in Chapter Four, with a facility level of 26 per cent only for the writing of eleven tenths. Only two students from the Average Competency Group, Suella and Peter, gave totally correct responses during Interview I.

TABLE 10 : Write Decimal Numbers: Interview I (Stimulus Card J, Series One)

	The Average Competency Group				
	three tenths	seven hundredths	fifteen hundredths	seventeen tenths	twenty hundredths
Stu	.3	.07	.015	.17	.020
Percy	3.0	7.00		17.0	20.00
Suella	.3	.07	.15	1.7	.20
Delwyn	.3	.07	.015	.17	.020
Trudy	3.0	7.00	15.00	17.0	20.00
Rex	3.0	.07	.015	.17	.020
Mary	.30	.700	.150	.17	.2000
Don	30.	700.	1500.	170	2000
Anne	3.01		no responses		
Peter	.3	.07	.15	1.7	.20
Danny	.3	.07	.015	.17	.020
Dwight	.3	.07	.015	.17	.020
James	3.0	7.00	15.00	17.0	20.00
Petra	3.0	.07	.015		

As with the survey (Chapter Five), a wide range of errors was made by this group. Probing during the interviews revealed that most students possessed stable mini-theories to explain their answers. Percy's explanation for seventeen tenths (he had written 17.0) was linked to his idea that the zero stood for tenths and so he "put one seven point zero." James explained his writing of twenty hundredths as 20.00 with, *"'Cause you put the twenty, and then you put the point, and two zeros for the hundredths."* Petra's responses were the result of links she generated with her place value nomenclature for decimal fractions: she named the columns after the decimal point as onesth (sic), tensth (sic), hundredths, and thousandsth (sic). When asked why she wrote .007 for seven hundredths, Petra responded, *"Because seven hundredths, it's in the* *hundredths column...the seven."* The responses then, were based on existing ideas about whole numbers, and existing ideas about column nomenclature. There was a strong element of a literal translation in many of the constructions e.g. James' response above and Don's explanation for twenty hundredths (he had written 2000): *"...well, it's two hundred and you add a zero on the end and it's twenty hundred."*

4. Problem Solving: Problem solving was difficult for the Average Competency Group during Interview I. Twelve problems (out of 56) were answered correctly, and no student solved more than two out of the four problems posed by Stimulus Cards K to N (Series One). Most errors resulted from students being unable to generate any responses; other errors resulted from guessing. Some students generated their own strategy to obtain a correct answer but, when asked to choose an appropriate algorithm, chose incorrectly. Dwight's sequence illustrated this phenomenon:

Dwight: Interview I: Stimulus Card K (Series One)

I: *(reads card) How would you go about getting an answer?*
S: *Zero point five is half a kilogram. So you probably halve the dollar fifty.*
I: *What would you get?*
S: *Writes "* $2\overline{)1.50}$ *.75c " seventy-five cents*
I: *(unfolds card) Which one of these (working forms) would you choose to give you the answer?*
S: *That one - one point 'oh' five divided by 'oh' point five.*
I: *Why?*
S: *Well, that would halve the answer. You'd get the right amount of potatoes you wanted.*

This sequence was similar to some discussed in Chapter Six. It appears that the presentation of working forms hindered rather than helped this student to construct a correct answer. Like some members of the High Competency Group, Dwight generated a correct response, but mistakenly chose division with a fractional number as the mathematical operation. Dwight's mini-theories were in conflict - one correctly suggested he should divide by two to halve an amount; the other mistakenly led him to divide with 0.5 in order to halve. He was able to hold both mini-theories concurrently, seemed satisfied with these conflicting explanations, and thus was not motivated to change his existing ideas (Claxton, In press a).

5. Size of Numbers: Stimulus Cards O to R (Series One) asked students to compare numbers with and without decimal points. Only two learners, Suella and Stu knew the 'rule' associated with moving the decimal point and correctly responded to all cards in this series. Suella appreciated that 249 was one hundred times larger than 2.49, *"Because to get a higher number you have to move the decimal point."*

Seven students failed to answer any of Cards O-R (Series One) correctly during Interview I, making a range of errors. Peter attempted to divide one number into another to obtain an answer (a plausible strategy), but set his working forms out incorrectly - for example when comparing 12.3 and 123 (Stimulus Card P, Series One) he wrote $12\overline{)123}$ which gave him the quotient 10.3. Petra, like others who could construct no appropriate meaning, commented, *"I don't get it, because of the decimal point"* when she pondered how many times 4 was larger than 0.4. Some students could see the relationship between the two numbers in terms of global size differentials, but were confused by the decimal fraction. Trudy was such a student.

Trudy: Interview I: Stimulus Card R (Series One)

S: *(reads numbers)*
I: *O.K. Which is the larger there?*
S: *Two hundred and forty-nine...Two is only two point four nine.*
I: *So how many times larger do you think that two hundred and forty-nine is than two point four nine?*
S: *About two hundred...no about a hundred...it (2.49) goes into two hundred about a hundred (times).*
I: *Is it exactly a hundred times?*
S: *No.*
I: *Why not?*
S: *'Cause point four nine is left over...remainder...just a hundred with forty-nine left over.*
I: *How could you work it out exactly?*
S: *On a piece of paper.*
I: *And what would you expect the number to come to?*
S: *A hundred and four?*

On the whole most of the Average Competency Group were clearly unsure of the number of times one number was greater than (or smaller than) another number with identical digits. Don's comments summed up most of this group's feelings when he muttered, *"I don't really know...I'm not very good on points...I don't know...um...no...I don't know."* These cards (O-R, Series One) probed areas for which many students appeared not to have generated mini-theories, suggesting that this topic had either not been covered by the students or that the students had not seen a need to make sense of this situation. Their textbook had presented the idea as follows:

> *"WHAT DO YOU THINK?*
>
> *Instead of saying, "Move the digits to the right to divide by a power of 10," some people say, "Move the decimal point to the left." Which rule do you prefer? Can you state a similar rule for multiplying by powers of 10?"*
>
> *(Duncan et al, 1978:129)*

FIGURE 6 : Extract from N.Z. Department of Education textbook for Form 2 students

Trudy, for example, had been through the above textbook extract as part of her school instruction in mathematics. It would appear that she was unable to construct meaning from the passage, or had built up a mini-theory that was not recalled in the context of Stimulus Cards O-R.

6. Developmental Trends: Interview II: The second interviews took place approximately one year after the first. Most of the 14 students in the Average Competency Group made some progress in the content areas explored in terms of correct responses and supporting explanations during individual interviews. One student (Trudy) appeared to have regressed at the time of the second interview, and three students made no progress (Stu, Percy, and Don). Of the ten students who made some progress, five made minimal gains. In short, less than a third of the Average Competency Group made what might be termed 'adequate' or 'reasonable' progress between Interviews I and II.

Mary was one student whose responses suggested a greater understanding of decimal numbers at the time of Interview II. When estimating the number closest to 0.8 (Stimulus Card C, Series One) she explained that point eight was closest to 1.0 because *"point eight goes point nine, then one point 'oh'"* (when counting). Peter made the most progress of any member of the group. In estimating 1.9 x 5 (Stimulus Card D, Series One) he quickly realized that 1.9 was nearly two, and rationalized *"five twos are ten...it's (1.9 x 5) only point five from ten."* Using his existing ideas about Stimulus Card D (Series One) Peter appreciated that for Stimulus Card E (Series One) 0.19 x 5 would be nearest to *"one, 'cause the one before was one point back."* Danny, another who made considerable progress to Interview II, was very strong in estimating and approximating. For Card D (Series One) he explained that 1.9 x 5 was closest to ten because, *"one point nine times ten is nineteen, and I halved it. That's about eight or nine. And that, ten, is the closest."* Finding that this strategy was successful (in his view), he applied it to Stimulus Card F (Series One) in approximating .19 x .5 - *"I did the same trick! Made that (.5) ten, then halved it. That's (10) got a decimal point and it stays there."*

Of the three students who made no discernible progress between Interviews I and II, Don's replies were characterized by *"I don't know... no idea...don't really know."* Anne, a student who made minimal progress commented for Cards B and C (Series One), *"I don't know...I haven't done that before...I don't know...it's just a guess."* Trudy, who appeared to know less about the content of Interview II than Interview I used a variety of mini-theories to help her solve Cards A-F (Series One). With Card C (Series One) she speculated that 0.8 was nearest in size to 1.8 because *"it's the same as that one (0.8) but only different."* Here Trudy seemed to operate with a mini-theory that emphasized the physical similarity of numerals (c.f. Erlwanger, 1973, and interview sequences with Benny). With Card E (Series One) Trudy believed that 0.19 x 5 was nearest to *"one hundred...it comes out about ninety-five...it's five off a hundred."* With this card her mini-theory focussed on an operation with whole numbers (19 x 5) which would give ninety-five. This mini-theory failed to acknowledge the positioning of a decimal point.

The next series of cards explored students' ideas about division. Again progress was variable, eight students making some progress, five remaining as they were and one regressing. Delwyn made the greatest progress in this area. Her responses were rather tentative, but she constructed an appropriate meaning for each question by generating links to her existing ideas about common fractions as she studied the problems. For example, in solving 15 ÷ 20 (Stimulus Card E, Series One), Delwyn realized that if fifteen objects were shared amongst twenty people then *"they would get three quarters each."* Anne, who also made progress in this area, commented with the same card, *"You'd have to go* *into fractions and stuff"* (to get an answer).

The six students who explained 15 ÷ 20 and 15 ÷ 100 by using negative integers repeated their ideas about negative numbers during Interview II. With Stimulus Card H (Series One) Rex argued that to solve 15 ÷ 100, *"you'd have to go into negatives, behind the positive numbers. You can't take one hundred away from fifteen!"* With the same problem James pondered *"it's below...unless it's minus so much... that one would be minus eighty-five."* Petra thought an answer to 15 ÷ 100 and 15 ÷ 20 would only be *"possible if it were negative."* All students gave some response to the items on Cards G-I (Series One). They considered that an answer was possible, even if they could not give one, in contrast to the survey (Chapter Five) where the most common response to 16 ÷ 20 was *"No"* (indicating that no answer was possible). In the survey (Chapter Five) there were also instances of negative integers being used with 16 ÷ 20: *"negative four"* was generated as an answer by two respondents.

The use of negative integers to solve a division problem that should yield a fractional number for the quotient was a stable 'bug' for the six members of the Average Competency Group. To use Claxton's (In press a, in press b) analysis the mini-theory brought into play when the divisor was larger than dividend, was 'nested' within other mini-theories to do with division. The fact that it persisted from Interview I to Interview II suggested that nothing had been learned in that time to change the students' views, and that the mini-theory lived on successfully in its own domain - "Basically we might say that a theory lasts as long as it is successful." (Claxton, In press a) Such a mini-theory was obviously not correct mathematically, but in the <u>learner's</u> view it was intelligible, plausible, and successful.

The writing of decimal numbers continued to be a problem.

TABLE 11 : Write Decimal Numbers: Interview II
(Stimulus Card J, Series One)

	The Average Competency Group				
	three tenths	seven hundredths	fifteen hundredths	seventeen tenths	twenty hundredths
Stu	.3	.07	.015	.17	.02
Percy	.3	.07	1.05	1.7	2.0
Suella	.3	.07	.15	1.7	.20
Delwyn	.3	.07	.015	.17	.020
Trudy	.03	.007	.0015	.17	.020
Rex	.3	.07	.015	.17	.020
Mary	.3	.07	.15	1.7	.20
Don				.71	
Anne		(no responses generated)			
Peter	.3	.07	.15	1.7	.20
Danny	.3	.07	.15	1.7	.20
Dwight	.3	.07	.15	.17	.020
James	.03	.007	.0015	.017	.0020
Petra	.3	.07	015	.17	.020

In comparison with the responses from Interview I, the following points can be made: (i) fewer errors were generated (e.g. three tenths and seven hundredths were correctly written by almost three quarters of the group compared to half the group in Interview I); (ii) fewer bizarre errors were evident, the students having a better 'feel' for a correct or reasonable answer; (iii) the range of errors diminished; (iv) more of the students could correctly answer the difficult items seventeen tenths, fifteen hundredths, and twenty hundredths; and (v) the students who made progress were those who had made minor errors during the previous interview. By contrast, those students experiencing serious difficulty during Interview I (Anne, Don, James and Trudy) performed poorest during Interview II. Anne, for example, generated only one response during both interviews.

An interesting 'bug' was generated by Don. For seventeen tenths he wrote .71. He reasoned, *"it works the other way (place value)...it goes tens, hundreds, thousands in decimals."* See also, findings quoted by Freyberg and Osborne (1981).

Problem solving with decimal numbers (Stimulus Cards K-N, Series One) had been a difficult topic during Interview I and in the survey (Chapter Five). In Interview II only 8 problems (out of 56) were answered correctly, which was less than in Interview I (12 answered correctly). Peter and James were the only students to give more correct responses. Peter could construct appropriate meaning for the problems. For Stimulus Card K (Series One) he realized that you would *"halve it,"* adding that the method of obtaining the answer was *"point five times one point five 'oh'...'cause it would have to be half the number."* James demonstrated that he understood the problems as well. With Stimulus Card M (Series One) he considered that the answer needed was, *"zero point seven five divided by two point five, 'cause that's dividing two and a half into seventy-five. That works out the price."*

The remaining twelve students in the Average Competency Group generated tentative links with existing knowledge about problem solving strategies, and decimal and/or common fractions. Often they guessed - Percy chose an operation because you *"had to"* or *"it works...that you... get your answer."* Rex adopted the subtraction strategy for Card L (Series One) because it *"looks best"*. These students could neither recall nor construct any mini-theories to explain their actions.

Other students tested ideas then abandoned them one-by-one. Don suggested first subtraction, then division, then reverted to subtraction with the explanation, *"Take away one litre...I don't know..."* Danny reached an 'impasse' (Brown and Van Lehn, 1982) with Card N (Series One), but then decided 6.44 ÷ 3.58 would get the price of one litre of petrol. However he remained unsure, - *"Wouldn't you really need to include the <u>one</u>? (one litre)...I can't see how that would give an answer..."* All four problems were equally difficult to the Average Competency Group. Patterns of errors did not generally exist, but for Stimulus Card N (Series One) six students chose the correct operation but reversed the order of divisor and dividend - the most common error (48 instances) for this same item in the survey (Chapter Five). The reasons given by the six Average Competency Group members lacked explanatory detail, and were akin to Mary's: *"'Cause you'll come up with the answer and that'll be the price of one litre."*

The final series of cards in Interview II, investigated relationships between numbers. Five students progressed, eight students remained static, and one student regressed. Peter (who progressed) was now far more assured than in Interview I. His responses (all correct) were precise and cryptic. In his construction of meaning, 80 was one hundred times larger than 0.80 because *"point eight 'oh' had two places."* He was able to generate strong links with existing ideas about the position of the decimal point, and the effect of shifting it in line with the decimal system of numeration. Danny, another to improve, noted that 249 was a hundred times larger than 2.49 because *"the decimal point is two times left...that means one hundred."* The responses of the eight students who made no progress in comparison with Interview I, fell into two categories. First, there were comments suggesting the task *per se* had little meaning for them - *"I'm not sure; I don't know..."* (Petra), *"I don't know that one"* (Don), and *"I don't know times larger"* (Anne). In Claxton's (In press a, in press b) terms, these students did not possess a mini-theory to fill the gap. Second, there were explanations that suggest the decimal fractions in 2.49, 12.3, 0.80, and .4 were the sources of confusion. Delwyn grappled with the numbers 123 and 12.3, and reasoned *"it (123) might be ten times larger...I think it is ten..."* She then worked out 12.3 x 10 to be 123.3. This then led her to say, *"Probably you'd have to times it (12.3) by nine point seven."* Nine point seven is point three less than ten, which Delwyn considered might solve the perplexity of the .3 in 123.3 (her construction). James considered 0.80 would need to be multiplied by 100½ to make it 80 but his reasoning was unclear. This confusion persisted in the following sequence.

James: Interview II: Stimulus Card R (Series One)

I: *Could you read those numbers to me?*
S: *Two hundred and forty-nine and two point four nine.*
I: *Which is the larger number there?*
S: *Two hundred and forty-nine.*
I: *How many times larger is it than 2.49?*
S: *Two hundred...'cause that one is two point four nine and the other is two hundred and forty-nine...that one (2.49) is two hundred and forty-seven less.*
I: *So when multiplying, I'd have to multiply two point four nine by what?*
S: *One hundred and twenty.*
I: *Why?*
S: *'Cause, if it (120) was times by two point four nine it should go up to two hundred and forty-nine.*
I: *How did you get a hundred and twenty? I'm sorry but I didn't quite understand...*
S: *'Cause it's (120) around about half of two hundred and forty-nine...and it's multiplied by two...*

James appeared to adopt a strategy that multiplied the 120 by the 2 of 2.49. The decimals .49 were largely ignored - for some of the interview they had been confusing for him, after all. For most of the Average Competency Group the simple rule of shifting the decimal point either left or right to manipulate with powers of ten was not part of their understandings about decimals.

7. Developmental Trends - Interviews III and IV: Interview III focussed on the same topics as Interview I, except for some probing into attitude by means of a sorting task. The first series of cards investigated estimation and approximation skills. By Interview III most of the Average Competency Group were coping satisfactorily with Cards A-F (Series One). Half the group had made progress over the time between Interviews II and III. Delwyn, when explaining why 0.19 x 5 was nearest in size to 1 said, *"Because I think when after you've multiplied it... you'd get a whole number, but that you wouldn't get it up to ten, which is the next closest...so it would be one point 'oh'."* Danny continued with the strategy he used in Interview II when estimating 1.9 x 5 (Card D, Series One), *"I went one point nine times ten, and halved it, roughly...and got ten."* Dwight exhibited confidence with the same card - *"One point nine is only point one away from two...two fives are ten."* James studied 0.19 x 5 (Card E, Series One) and estimated it nearest to 1.0 with the comment, *"Zero point one nine times five times five is point nine five, and that's closer to one...it's ninety-five hundredths...um, whereas the other ones, um, a hundred hundredths...so it's a bit closer...it's only five hundreds away."* Various mini-theories were brought into play, therefore, but for the students who mastered the estimation and approximation phase, their mini-theories were based on correct and appropriate mathematical reasoning.

Petra, Trudy, and Mary continued to have difficulty in this topic. Petra at Interview III was still basically unsure of decimals and had developed only inadequate mini-theories to fill the 'gap'. Comments like, *"I don't know,"* and *"I don't really know"* characterized some responding. At other times she could construct an answer - 0.19 x .5 was nearest to 100 she believed because, *"I don't take any notice of the decimal points...I can't do them"*. Ideas were based on multiplication with whole numbers. Trudy possessed similar ideas. Mary had a better grasp of the topic than Trudy or Petra, as indicated with Card C (Series One) when explaining the difference between 0.80 and 80 - *"Point eight* *is eight tenths or four fifths and that's almost one."* With Stimulus Card E (Series One) she again generated links to ideas about common fraction equivalents but could not construct a correct meaning for the "times five" part - *"point nineteen is...That would be nineteen over one hundred."* However she then thought multiplying this by five would give an answer of about ten, *"'cause you include the decimals in."*

The next topic proved more troublesome in Interview III. Division with divisors smaller than the dividend had been difficult for this group in the previous interviews. Of the fourteen students in the group, three students progressed, eight students made no appreciable gains, and three students regressed when compared with Interview II. (Overall, compared with Interview I, seven students progressed, six students made no progress, and one student regressed.) James was the most successful of the three students who progressed. By Interview III he had shifted to a more plausible mini-theory that relied on connections with the division process and common fractions. James was one of six students who used negative numbers to generate an answer to 15 ÷ 20 from the time of Interview I. In Interview III, James replied, *"You divide fifteen by twenty...fifteen is three quarters of twenty"* and responded 0.75. Likewise, his reasoning for 15 ÷ 100 was full, accurate, and intelligible. The students who made no discernible improvement between interviews continued a pattern of variable responses.

Percy: Interview III: Stimulus Card H (Series One)

I: *(reads 15 ÷ 100 to S)*
S: *Ah...you can't do that.*
I: *Why not?*
S: *Well, 'cause fifteen's lower than one hundred.*
I: *Is there any way you can do it?*
S: *um...maybe if you added a couple of zeros on...I don't really know. If you put another zero there (on 15)... even if you added a couple of zeros it wouldn't work.*
I: *Why not?*
S: *Well, if you add two zeros on to the fifteen you have to add two zeros on to the hundred...'cause you've got to keep it even both sides when you're adding, putting zeros on...*
I: *Is there any other way you could do it?*
S: *No, not really.*

Of the others who made little or no progress with 15 ÷ 20 and 15 ÷ 100, Trudy and Anne considered it *"can't fit"*, or *"doesn't go"*. Don, Stu and Danny knew that the answer would be a 'fraction' or 'point something', but could not generate a response beyond this; and Petra and Rex continued with the idea of negative integers, a 'bug' they had carried from the beginning of the research project.

Responses in this part of Interview III from three students (Suella, Delwyn and Mary) suggested (disturbingly), that they had regressed over time. Delwyn and Suella, during Interview II had readily solved 15 ÷ 20 by generating a common fraction equivalent (three quarters). During Interview III Suella, with 15 ÷ 100, confessed, *"I don't know. I just had to make it* (the answer) *up."* For 15 ÷ 20 she replied, *"Zero point one five."* When asked how she got this she answered, *"I dunno...just a guess...but I don't think it's right."*. Delwyn, who had made good progress in this topic to Interview II, attempted to answer 15 ÷ 20 by working through an algorithm - *"Twenty doesn't go into fifteen (adds a zero to fifteen)...and twenty goes into one hundred and fifty... seven and a half times...so it would be seven point five...yeah."* Here, Delwyn's mini-theory generated a correct answer to the algorithm 150 ÷ 20, but her 'executive-procedure' was at fault in selecting this as the appropriate working form. Computationally, she was correct - at the 'mini-procedures' level (Davis, 1979).

The next facet of Interview III investigated the students' generation of written decimal numbers.

TABLE 12 : Write Decimal Numbers: Interview III
(Stimulus Card J, Series One)

The Average Competency Group					
	three tenths	seven hundredths	fifteen hundredths	seventeen tenths	twenty hundredths
Stu	.3	.07	.015	.17	.02
Percy	.3	.07	.15	1.7	.20
Suella	.3	.07	.15	1.7	.20
Delwyn	.3	.07	.015	.17	.020
Trudy	.03	.007	.015	.17	.020
Rex	.3	.07	.15	1.7	.20
Mary	.3	.07	.15	1.7	.20
Don	.3	.07	.051	.71	.002
Anne	.3	.007	.015	1.7	.020
Peter	.3	.07	.15	1.7	.02
Danny	.3	.07	.15	1.7	.20
Dwight	.3	.007	.015	.17	.02
James	.3	.07	.15	1.7	.020
Petra	.03	.007	.015	(no response)	(no response)

In looking at changes over time, the following characteristics emerged with the Average Competency Group: (i) gradual progress was made by most students over the interviews, with 7 students (half the group) correctly constructing all fractional numbers (the corresponding numbers for Interviews I and II are 2 and 4 respectively; (ii) the errors that were still generated were plausible rather than bizarre; (iii) there were no instances of regression; (iv) half the group were still unable to write all decimal numbers correctly (students in this category generally constructed different errors in each interview - e.g. Petra, James, Dwight, Mary, Percy and Trudy - with Delwyn being the only student to repeat, exactly, the same errors for each of the interviews over the 26 months that she was interviewed); (v) those students who had constructed correct answers by Interview II held these for Interview III; and (vi) the most difficult number to write at Interview III was twenty thousandths - only six students correctly wrote this. Concerning point (vi), Dwight's reason for writing .02 was linked to his existing ideas about place value nomenclature. In explaining the decimal places as ones, tens, and hundreds, Dwight elaborated, *"It's just the reverse I think...I just thought it was the reverse (of whole numbers)...that's how I remember."* Later he continued with regard to the decimal point, *"It's just the turning point."* With a final attempt at writing 'twenty hundredths', Dwight fathomed, *"It, (.02) still doesn't look right...could be the same as two hundredths...I can't get a way to make it twenty hundredths."*

The next cards, on problem solving, continued to be very difficult indeed for the Average Competency Group. Only four of the fourteen students gave more correct answers. None of the group answered all four cards (Stimulus Cards K-N, Series One) correctly. Seven students could not respond to <u>any</u> of the four problems correctly. Characteristic responses from these students were (not unexpectedly), *"I'm not sure why"* (Suella), *"I don't know"* (Percy), *"I can't think of it"* (Peter), *"I don't know why - you just would"* (Danny) and *"I'm not really sure"* (Dwight). At other times attempts were made to construct meaning, and there was some evidence that appropriate links were being generated. As with Interview II, Stimulus Card K (Series One) seemed to make sense to many of the students who were eventually unable to construct or identify the appropriate algorithm. Percy noted that *"point five is a half - there's half a kg of potatoes, that's what you want to find"*, and then chose 1.50 ÷ 0.5 to get an answer. Delwyn, Suella, Rex, Mary, Don, Danny and Petra all gave similar rationales, and chose the same incorrect algorithm. In other words meaning was constructed from the problem *per se*, but not from the associated working forms at the bottom of the stimulus card. The categorization of student responses as incorrect may be rather unfair, but the seven students (half the Average Competency Group) who could not generate correct responses did not generally construct meaning from the wor d problem.

Trudy and Anne made the greatest gains in problem solving. Both these students deliberated on their responses for some time before deciding on the appropriate strategy - Trudy, with Stimulus Card N (Series One) for example, decided that 6.44 divided into $3.58 would give the price of one litre. Then she reversed the divisor and dividend. Next she reconsidered, *"Oh, no! It would be* <u>*that*</u> *way I had it (first) - 'cause you want to get the amount of one litre, instead of six point four four."* Anne, with Stimulus Card M (Series One), studied the problem and decided, *"Divide, 'cause you've got to find out how much one litre would cost... two point five divided into zero point seven five."* She then wrote the algorithm $2.5\overline{)\,0.75}$. With Stimulus Card N (Series One) the researcher attempted to make her change her response from division (correct) to multiplication (incorrect). She refused to shift her idea, *"You'd get the wrong answer! (if you multiplied). 'Cause you want to find out how much* <u>*one*</u> *costs, not a whole lot!"*

The final series of cards in Interview III probed the students' ideas about the relative size of numbers with and without the decimal point (Stimulus Card O-R, Series One). Between Interviews I and II progress had been evident, and this was again the case at Interview III. The students in the Average Competency Group (except Petra and Trudy) could construct appropriate mathematical meanings. Students sometimes used computational strategies to generate a response: Percy regarded 123 as larger than 12.3 because, *"Well, you times 12 by 10 and it will give you 120...and then 10 threes are 30. That will give you three in the ones."* Most of the group, however, used a rule related to moving the decimal point and the power of ten. Danny: *"The decimal point moved one place to the left just means that it's ten times smaller."* Peter (Stimulus Card O, Series One): *"If that (0.4) was timesed by ten you'd move it one decimal place...it would be four."* Petra struggled to say how many times 249 was larger than 2.49, eventually speculating it was 20 times larger because, *"You move the decimal point twice, to get rid of it, and then it would be 249."* It would seem that each movement of the decimal point was 'worth' 10 and that to move the decimal point twice was two lots of 10 i.e. 20. This was the first appearance of this 'bug' for Petra (Brown and Van Lehn, 1982).

The Stimulus Cards (Series Two) were used during Interview IV to check on difficulties that had surfaced (or reappeared) during Interview III. The first two cards (Series Two) explored estimation and approximation. Petra, again, was unable to construct meaning from the stimuli. Repeated questioning from the research confirmed that Petra's trouble came from a 'gap' in her knowledge - *"I've just forgotten how to do it...it's the decimal places..."* she commented during Interview IV. Trudy considered 0.34 x 0.3 would be nearest in size to 100 (in Interview III she believed 0.19 x 0.5 to be nearest to 100). When the researcher asked her to justify this she computed (with pencil and paper) the algorithm (see right) which she deemed nearest in size to 100. Asked if the decimal point in .102 made any difference, Trudy replied, *"I don't know, but I don't think so."* This 'bug' had been a consistent feature of her responding from Interview I.

$$\begin{array}{r} .34 \\ x\ \underline{.3} \\ \underline{.102} \end{array}$$

Students from this Average Competency Group who had experienced difficulties with writing decimal numbers during the previous three interviews experienced similar problems in Interview IV. Delwyn had named the columns for decimal fractions as *"ones, tens, hundreds"*, and wrote the number twenty-five hundredths as .0025. Next she changed the column nomenclature to *"tenths, hundredths, thousandths"* and rewrote twenty-five hundredths as .025 with a rationalization based on the column names. Trudy, another who had difficulties, exclaimed when Stimulus Card D (Series Two) was introduced, *"Is that the same as the other day's (interview)?...Oh, I can't do them! I can't remember..."*

Stimulus Cards I, J (Series Two) investigated problem solving with and without decimal numbers. Card I (Series Two) was answered correctly by all fourteen students in the Average Competency Group. Card J (Series Two) was answered correctly by less than half the group, as was the case with the High Competency Group. Most learners indicated that Stimulus Card J (Series Two) was more difficult because it contained decimal numbers. Danny explained, *"Card I was all nicely rounded off to ones and...you know, one point zero zero, and two point zero zero, and things like that...but this (Card J) is all around the thing with one point two five, and two point five and that...makes it more difficult to work out."* When discussing Card J (Series Two) Percy considered, *"It doesn't sort of* *go into one litre, with two point five litres."* For this group of students, the addition of decimal numbers to a problem that had earlier been easily solved with whole numbers (Stimulus Card I, Series Two) made it significantly more difficult to construct meaning. Assigning place value nomenclature to decimal fraction columns was correctly answered by half the group at both Interviews I and III. Mary was the only student to progress from incorrect ('ones, tens, hundreds') to correct labels by Interview III. Other students kept their stable mini-theories for the duration of the study. The most common error was to write 'ones, tens, hundreds' (Anne, Dwight, Petra, James, Delwyn, Trudy at Interviews I and III).

8. Attitudes Towards Mathematics: The final phase of Interview III explored the students' rankings of topics within mathematics, and their reasons for their choices. Students were asked to sort nine cards with topics written on them into a 'Like Most'/'Dislike Most' continuum (see Chapter Four, Page 70). TABLE 13contains the results (Stu's responses are listed later in this chapter).

TABLE 13 : The Average Competency Group: Responses to Sorting Task, Interview III

	Percy	Suella	Delwyn	Trudy	Mary	Anne	Peter	Danny	James	Petra
Like Most:	+	g	xt	-	g	x	s	g	g	x
	-	xt	x	x	xt	÷	÷	x	÷	xt
	g	x	÷	÷	x	xt	-	÷	fr	+
	x	+	g	÷	÷	+	d	-	-	÷
Neutral:	s	÷	+	fr	-	-	g	fr	x	s
	xt	-	d	xt	+	g	fr	xt	d	f
	fr	s	fr	s	s	fr	xt	d	X	-
	÷	d	-	d	f	s	÷	÷	÷	g
Dislike Most:	d	fr	s	g	d	d	x	s	s	d

Key: ÷ (division sums)
x (multiplication sums)
- (take-away sums)
+ (adding sums)
s (sets)
xt (times tables)
g (geometry)
fr (fractions)
d (decimals)
} (equal ranking)

Responses could not be collected from Dwight, Rex and Don. Obvious features from TABLE 13 are the popularity of geometry with many of the group, and the dislike of decimals, which was often relegated to the dislike most zone. The reasons for such a placement were quickly elicited. Percy: *"Decimals I hate the most...I can't do them really. I* *__can__* *do a few of them. They seem so complicated...all the points stuck every-*

where." Mary: *"I find it difficult and I don't like doing it...All the times (with decimals) and all those..."* Anne: *"Sometimes they (the decimals) get hard...not all the time."* Petra: *"I used to like doing them (decimals), but now I don't. I don't know how to do them...I can't think of any method."*

A large part of Interview IV probed the attitudes of this group towards the subject mathematics and the topic decimals. A version of the semantic differential technique (Osgood, Suci and Tannenbaum, 1957) was administered. Ratings for mathematics were generally more positive than those for decimals. Most students in the Average Competency Group recorded neutral opinions on the descriptive scales for mathematics. Only James and Danny expressed strong positive opinions - both saw the subject as extremely valuable, and very pleasant, clean, and good. Danny justified his extremely valuable rating for mathematics: *"Well, it's the main subject for most of the jobs around these days - you've got to get School 'C' Maths and 'U.E.' Maths for that..."* James thought too, that mathematics was valuable for future employment. Ten students indicated largely neutral feelings towards mathematics, although many of these included 'extremely valuable' as a departure from their neutral stance. Mary and Rex demonstrated negative opinions towards mathematics, viewing it as extremely distasteful (Mary) and very awful, hazy, unpleasant, and sad (Rex). Mary summed up the reasons for her responses: when asked why she had indicated unpleasant, distasteful - *"Oh, probably 'cause I don't like them and I'm not so good at them."* Decimals were perceived less favourably on the whole. Ten students indicated neutral feelings on all the given descriptive scales, while four students, Delwyn, Mary, Rex, and Percy, expressed negative opinions. Towards decimals Percy felt that, *"With decimals I never seem to remember where those little points go, and how to add them up."* Later he added, *"They (decimals) seem too complicated to remember."* Similar reasons were advanced by Delwyn, Mary, and Rex.

The next part of Interview IV asked students to compare the mathematics they had been taught at primary and high (secondary) school. Eight students in the Average Competency Group felt the main characteristic of high school mathematics was that it was harder than that at primary school. Danny: *"Maths we do here (at high school) is a lot more harder. It's got more detail to it, and that, you know - especially trigonometry, you know, sine, cosine, and that.....Here we get into the technical stuff - and there's heaps of it you've got to learn."* Delwyn and Rex felt the teachers' style at high school was different in that less support was given to students. Rex regarded this as a problem, *"They (the teachers) will go on to one subject, like percentages, and the teacher would put up a couple...but then she'll just put up about twenty and just say 'Do those..' ...put up ones different to the others I can't remember."* Percy considered more responsibility was given to students at high school, and James commented on the structured nature of the high school day.

The next segment of Interview IV elicited *"advice to teachers"* - advice students would give teachers to help learners with mathematics. All students listed several pieces of advice they would offer to teachers. Three related suggestions were most commonly given: for teachers to explain in greater depth; spend more time on topics (don't rush); and to work longer with individual students. These three recommendations accounted for 70 per cent of the suggestions given. Other recommendations were: set regular homework; desist from pushing or threatening; use less confusing textbooks; group students in smaller numbers; eliminate 'waffling' (by the teacher); and move more quickly from one topic to the next. Suella explained, *"Well, some teachers tend to like the topics that we find easy - 'cause they (the teacher) don't have to do much work. They spend a lot of time on them...it's boring."*

Advice to teachers was elicited through flexible interviewing formats. The following sequence illustrated this:

Danny: Interview IV: Advice to Teachers

S: *My teacher Mrs ____ was really good at making you learn it. She tells you about it. If she's giving you something, she'll show you how to do it, do heaps of examples on the board, and tell you to write it down in your textbook for future reference. ...once you've got the hang of that she'll give you about 10, 15 homework examples to do that night, to just keep them in your head.*

S: *(later) She's good at explaining things. And I like the way that she gives us ten questions at the beginning of the period.*

I: *Are these ten questions marked?*

S: *Yeah...just to see how we're going. That's her way of checking understanding...*

I: *What other things do teachers do that help you learn maths?*

S: *...just their style of teaching. They're ready to help you if you've got a few questions and that...if you don't get how to do something, just put your hand up and they'll come over. I find it easier to understand if the teacher's right there telling you...instead of telling the class.*

I: *Is there any advice for teachers about things they shouldn't do?*

S: *...once or twice this year Mrs ___ I think may have moved on* *to another subject a bit fast - without getting us all knowing how to do it. Mr ___ he wasn't really a good teacher at all...*

he was a good fellow but not a good teacher. He'd waffle away too much, and talk about other stuff and do too much stuff like that.

I: *Waffling on?*
S: *Yeah, yeah. He might go and tell us about his night at the pub or something like that...showing off kind of thing...*
I: *Did you enjoy that kind of waffling on?*
S: *Yes. I find I do, actually.*
I: *Did it help you learn maths?*
S: *No.*

The need for teachers to explain clearly in language that students understand emerged again and again. Percy's comment was similar to that of many others: *"He doesn't explain it. He knows how to do it, but most of the time we don't really understand what he is talking about."*

A wide range of opinion about social disruption in the classroom was recorded. James found peers who didn't work a disturbance: *"It does (interrupt) a bit 'cause the teacher always is trying to tell them off and that."* Trudy admitted, *"Me and my friends are always getting told off for talking."* Delwyn considered: *"With maths, here, someone's talking...the girl in front of me was talking to the girl next to me so I told them to shut up and they just kept on talking. And the teacher tells everyone to be quiet but a lot keep on talking... There's usually a few kids in the class that are stupid, you know."* Other students in the Average Competency Group did not see social disruption as affecting them. James thought it could be a problem for other classes, *"I'm second from highest (in class level) or something like that. All the bad ones are in the lower classes...they just get all crazy and that in class. They annoy other people."* Most students in this group knew of other classes where social disruption inhibited learning, but for themselves they did not view it as a significant concern.

When peer opinions of mathematics were gathered through the scenario outlined in Chapter Four, most of the Average Competency Group claimed that their peers disliked the subject. Three students thought their friends felt neither pleasure nor displeasure when confronted with a mathematics lesson next period, and Percy and Peter thought some of their peers enjoyed the subject. (Percy and Peter had made progress during the research project.)

Percy indicated the potency of peer pressure when he said, *"I think quite a few people like maths, but if their friends don't like it, they don't. They sort of want to fit in."* Peter: *"It really depends on* *what class you're in. From the middle (stream) down I don't think they like it. The top group usually likes it. I haven't heard them worry about maths that much."* Overall, the peer group perceived maths as an undesirable subject for next period, but most students agreed that it depended on the individual. This was perhaps best expressed by Danny:

Danny: Interview IV: Peer Opinions of Mathematics Scenario

I: (scenario for eliciting peer opinions)
S: John, he doesn't like maths. And when there's maths he goes along knowing it's maths - "what a bummer" type of thing. I've another mate, he's in my same class, he likes maths. He enjoys it, he understands it well. And there's another fellow, Richard, I think he likes it a bit. (later)
S: It depends if you're good at maths whether you like it or not...if you understand it you like it. But mostly, I've heard people complaining because they think that some subjects (topics) are too hard.

Summary and Comment

Characteristics of the responses of the 14 members of the Average Competency Group may be summarised as follows:

1. A major feature of this group was the range of progress. Some of the group made good progress (e.g. Anne, Peter, Dwight and James); most made modest progress (Stu, Percy, Delwyn, Trudy, Rex, Mary, Don and Danny); and two students made minimal or no progress (Petra and Suella). Suella, however, was reasonably competent at the start of the research project and remained at this level. Generally, progress in knowledge and understandings about decimal numbers was a slow and gradual process over the two years of the research project.

2. A difficult topic for this group was division with the divisor larger than the dividend (this had been the case with the survey sample, Chapter Five). Half of the students made no progress. Inappropriate mini-theories, such as using negative integers to solve 15 ÷ 20, were held by some students over a year or two-year period. Two students regressed in their understanding between Interviews II and III. The generation of the negative integers 'bug' (Brown and Van Lehn, 1982) was related to a teaching episode as explained by Suella on Page 131. This particular mini-theory demonstrated how mini-

theories may be quickly generated to fill a gap, and how these constructs may become stable over time.

3. Where students had difficulties it was generally because they generated links with inappropriate existing ideas about whole numbers (e.g. hundredths has three places), or, because they had never constructed any meaning from the stimuli presented. Many gaps (Claxton, In press a, in press b) remained for the duration of the research project. Students in this category were unwilling to advance any generated theory into the 'gap' to provide an answer, and characteristically responded, *"I don't know", "I'm not sure", "I can't think of it", and "I don't know why - you just would".*

4. The problem-solving exercise was the most difficult for this group but, as with the High Competency Group, the structured nature of the questions (Stimulus Cards K-N, Series One) may have precluded correct responding. The most common error for Stimulus Card K (Series One) was to choose the algorithm 1.50 ÷ 0.5, when half of 1.50 was needed. Claxton, (In press a, in press b) noted that which theory a child will choose may depend on apparently small details. A parallel was observed in the above, where constructed meaning from the word problem was evident, but the detail of choosing a correct algorithm *("I need half, so I must divide")* was incorrect.

5. Conflicting mini-theories (Claxton, In press a, in press b) were held simultaneously by many of the students in the Average Competency Group. For example, six of the fourteen students believed the place value nomenclature for decimal fractions read (to the right of the decimal point) - "Ones (or oneths), tens (or tenths), hundreds (or hundredths)." This idea was held through the two-year research study. In spite of holding these views, most of the six could write correct decimal fractions by Interview III. It would appear that the mini-theory concerning place-value column labels was confined to that particular task (i.e. generating names for columns). Thus the conflicting theories never confronted one another.

6. The topic of decimals was disliked by the group. Students indicated this was because they found the topic difficult. A version of the

Semantic Differential Technique showed that decimals was most often viewed neutrally or negatively by the group. Mathematics was also viewed neutrally or negatively, on the whole, but seen as valuable or extremely valuable by most students. Peer pressures to conform *("If their friends don't like it (maths), they don't")* were evident.

7. This group considered that teachers should take time to explain ideas in mathematics thoroughly, should spend longer on topics in the syllabus, and should provide (and be aware of the need for) individual tuition. A range of opinions emerged concerning the significance of social disruption in the classroom setting as a hindrance to learning, and most students thought high school maths was more difficult than mathematics at primary school.

8. These students knew their times tables, could add correctly, and do division (except if the divisor was larger than the dividend) - all 'mini-procedure' level skills, (Davis, 1979). Most errors were manifested at the 'executive-procedures' level. The interview sequence with Ann (Page 130) illustrated this phenomenon.

Stu

A Case Study from the Average Competency Group

Stu was interviewed four times over a period of 27 months (see TABLE 14). His standardized test results in mathematics indicated he was 'average' for students in his age and class level.

TABLE 14 : Personal Data for Stu

Interview I 19 September, 1981	Age at Interview: 12 years 11 months
Interview II 12 April, 1983	Standardized Test Results (1981) *TOSCA P.R. 72
Interview III 11 November, 1983	*TOSCA Stanine 6 PAT (Maths) P.R. (Class) 43
Interview IV 14 November, 1983	PAT (Maths) P.R. (age) 49 *(Secondary Form A)

Interview I:

The initial stimulus cards probed Stu's ideas about estimation and approximation. Stu was unable to construct a correct mathematical answer for the first Stimulus Card (Stimulus Card A, Series One), but this may have been a 'slip' rather than an 'error', (Van Lehn, 1982). This notion was supported by the fact that Stimulus Card B (Series One) was correctly answered.

By Stimulus Card E (Series One) Stu was experiencing difficulties. He correctly estimated 0.19 x 5 to be nearest in size to 1.0, but had computed 0.19 x 5 to be *"point fifty-five, which is pretty near one... it's over half way."* Stu's error here at the 'mini-procedures' level was continued into Stimulus Card F (Series One), where he correctly estimated 0.19 x 0.5 to be nearest in size to *"point ten"*. However, his reason for choosing 0.10 (or "point ten" to use his terminology) was based on his response to the previous card, and the realization that an extra decimal place was required. Interestingly, his errors in these instances were computational ('mini-procedures' level) while at the higher or 'macro' level his strategies were (fortuitously perhaps) correct.

These sequences showed how learners may provide the correct answer in a test-type situation (where the student was required to choose a response from a number of alternatives), but have used incorrect or incomplete strategies in arriving at the correct answer. If the above responses had been elicited from Stu in a busy classroom environment or in a pencil-and-paper test situation, then the teacher could have assumed that he had constructed an appropriate meaning (or possessed a mathematically-correct mini-theory (Claxton, In press a, in press b)).

The next three stimulus cards in Interview I explored Stu's processes when dividing, particularly where the required quotient was a decimal fraction. Stimulus Cards G and H (Series One) were answered correctly. For example, when solving 15 ÷ 100 Stu noted, *"a hundred doesn't go into fifteen, so it's point one five...it's fifteen over a hundred."* The generation of a link with existing knowledge about common proper fractions provided him with the opportunity to construct an appropriate response.

The third card in this series was more difficult for Stu.

Stu: Interview I: Stimulus Card I (Series One)

I: (reads 10 ÷ 20)
S: Two, I suppose...I think.
I: How did you get that?
S: I divided ten by twenty...I divided ten by twenty... tens into twenty.
I: Well, dividing by twenty - what does that mean? Ten into twenty? Or twenty into ten?
S: Twenty into ten.
I: And what was the answer?
S: Point two.
I: And how did you get that?
S: I divided twenty into ten. Two tens are twenty, so point two. Put a decimal point in.
(later)
I: (reads 15 ÷ 20)
S: Well, twenty into fifteen doesn't go.
I: So what do you do then?
S: Point one five.
I: How did you get that?
S: Put a decimal point in front if it (the 15)...I said if the divider (sic) is larger than the thing that you're dividing, put a decimal point in front of it.

However, when he was questioned further Stu reverted to his earlier idea of reversing the divisor and dividend.

(later)
I: What say I was dividing by thirty, and I had ten?
S: Well, you still put a decimal point in front of it 'cause it's still larger.
I: Will it be point one?
S: ...I mean tens into thirty is two, I mean three, three...point three.

Stu seemed to know that a decimal number less than one was needed, but he was unable to use the appropriate procedures for doing so. In the above extracts he moved from one mini-theory to a second theory and then back to his initial mini-theory, (Claxton, In press a, in press b). When dividing with a divisor larger than the dividend, Stu wanted to place a decimal point in front of the dividend, or to divide the dividend into the divisor (reversing the accepted mathematical convention). Both errors reflected a lack of understanding, and could be categorized at the 'executive-procedure' level, (Davis, 1979). Stu's responses were similar to those of the subsequent case study Ronald (Chapter Eight).

The next Stimulus Card (Stimulus Card J, Series One) looked at Stu's ability to write decimal numbers. TABLE 15 has results over the two years of study.

TABLE 15 : Stu: Write Decimal Numbers

	Interview I	Interview II	Interview III
three tenths	.3	.3	.3
seven hundredths	.07	.07	.07
fifteen hundredths	.015	.015	.015
seventeen tenths	.17	.17	.17
twenty hundredths	.020	.02	.02

Stu made little progress in this area over the two years, correctly answering just three tenths and seven hundredths at each interview. During Interview I Stu was unsure of why he had written .015 for fifteen hundredths. He explained, *"I don't know, I don't know fifteen hundredths - I just guessed."* Later in the same interview he elaborated, *"I just put zero one five - I really put fifteen and put a zero in front of it."* When asked why he had put a zero in front of the one five, Stu explained, *"Well, it's a place holder and it makes it hundreds instead of tens."* Seventeen tenths remained a mystery to him, *"I don't know how to do seventeen tenths yet...hundredths and tenths are only in one column and these (fifteen hundredths; seventeen tenths) are two column figures."* From Interview I through to the conclusion of the study, Stu correctly named the place value columns in decimal fractions as tenths, hundredths, and thousandths. The over-loading conventions of place value appeared beyond him. His explanations during Interviews II and III were more assured, but equally incorrect, as TABLE 15 revealed. This aspect of Stu's mathematical behaviour was explored in Interview IV.

Stu: Interview IV: Stimulus Card D (Series Two)

I: *(reads Stu Card D)*
S: *(writes:* *.2*
.09
.025
.11)
I: *How did you write twenty-five hundredths?*
S: *Well, I just wrote the twenty-five down, and so I missed out the tenths.*

I: And what's your reasoning for putting the zero down?
S: 'Cause it's hundredths, not tenths...
I: Are you sure about that?
S: Not really...I think it's right.

Stu, therefore, remained unsure of his written responses, and unaware of the need to shift the digits one place to the left because of the overloaded hundredths column. This 'bug' was stable for the duration of the research project.

The next series of cards explored Stu's ideas about problem-solving with decimal numbers in the contexts of money and measurement. Stu answered all Stimulus Cards K-N (Series One) incorrectly. His responses during Interview I revealed a mixture of strategies. For Stimulus Card K (Series One) he chose division as the operation needed to get an answer because, *"It just looked right, actually - that's why I chose it."* When asked again he replied, *"I don't know, it just looked right."* With Stimulus Card L (Series One) Stu did attempt to construct some meaning. After the researcher read Card L (Series One) to him, Stu paused for 32 seconds then answered, *"Two dollars sixty-seven plus point five eight."* When asked to explain this, Stu realized that addition would be inappropriate, and changed to division with the rationale, *"I don't know - it's just the first one (algorithm) that came to my eyes, actually, when I just looked."* With Stimulus Card M he attempted the subtraction operation with the offering, *"I'll work it out if you want me to."* Stu next generated the subtraction algorithm:

$$\text{"}\quad \begin{array}{r} .75 \\ \underline{2.5} \\ .60 \end{array} \quad\text{"}$$

In his explanation the .60 stood for sixty cents, an answer obtained *"by taking away twenty-five from seventy-five."* Stimulus Card N (Series One) produced a tentative answer from Stu, but with the realization that a lower "number" was needed than those in the problem. Generally, he was unable to construct appropriate mathematical strategies to solve the word problems, although he intimated that a certain operation was needed.

The final segment of Interview I investigated his ability to compare numbers with and without decimal points. Stu found this a relatively simple task. The only error he made was with Stimulus Card R (Series One) when he regarded 249 as 1000 times larger than 2.49. Generally he saw relationships quickly, and employed a mixture of correct 'mini-procedures' *("ten threes is (sic) thirty")* and 'executive-procedures' *("you put the decimal point in between the three and the 'oh' 'cause you want a hundred and twenty-three point zero").*

Interview II:

At the time of Interview II Stu was in the fourth form at an urban high school for boys. Generally his responses during Interview II were very similar to those elicited during Interview I.

The first series of cards explored estimation and approximation. As with Interview I, Stu had some difficulties with these cards. For example, he considered 0.8 was nearest in size to zero, because *"it's only eight away"* - a correct statement *per se*, but incorrect in the context of the problem (Stimulus Card C, Series One). Stimulus Card F (Series One) was also responded to wrongly. When estimating .19 x .5 Stu considered the answer would be closest to 1.0, from the alternatives given (the correct response was 0.10), and noted, *"it (.19 x .5) comes to .95...I don't know...(laughs)...nineteen times five and put a decimal point in front."* In contrast with Interview I, Stu responded incorrectly to Card F (Series One), but on that occasion his correct response was underpinned by incorrect 'mini-procedures' (Davis, 1979), as discussed earlier in this Chapter. As with Interview II a continuing lack of good estimation and approximation skills were apparent. His comment, *"nineteen times five and put a decimal point in front"* suggested a mini-theory that was only tenuously held.

While discussing Stimulus Cards A-F (Series One) Stu made a moaning sound indicating displeasure and commented, *"I'm not very good at this."* When asked if he disliked decimals, he answered, *"No, it's just that I can't times decimals without thinking a lot."*

Stu had no difficulty in constructing meaning from Cards G and H (Series One) on division. In giving the answer to 15 ÷ 100 as "point one five", he exlaimed, *"It's just come to me!"* He continued, *"There's two zeros in there (100) and there's two digits there (15), and a one in front of the two zeros which is a hundred. So I just put a point instead of the one."*

Stu was unable to articulate any rule or convention to explain his move, repeating, *"I just did it."* Stimulus Card I (Series One) was again troublesome for Stu, particularly 15 ÷ 20 but he was able to generate links with ideas that the answer must be a fractional number, *"'Cause it's (15) not over twenty."* Responses to these three cards were very similar to those noted in Interview I.

Problem-solving with decimal numbers was another source of concern for Stu. As with Interview I he constructed some meaning from some of the word problems, but was able to identify only one required operation that would yield a correct answer. At the commencement of the four problem solving cards (Stimulus Cards K-N, Series One) he complained, *"Hard test, hard test"*, and was unable to provide suitable reasons for his answers. With Card N, (Series One) Stu did choose the correct operation, division: *"then you find out what one is worth."* When pressed to explain his choice for the algorithm 3.58 ÷ 6.44 Stu could only respond, *"Well, it looks good."* The researcher next asked if there was an element of guessing with this response, and Stu replied, *"Yeah, there was, quite a bit...every single one was almost guessed."* He had made no progress in this facet of the interviews over the previous year.

The final segment of Interview II explored understandings of numbers with and without a decimal point. In Interview I Stu had found this task an easy one. During Interview II this trend continued, except for Stimulus Card O (Series One).

Stu: Interview II: Stimulus Card O (Series One)

I: *(reads card) Which is the larger number, or are they the same size?*
S: *Well, it depends on what scale you're using.*
I: *I just mean as numbers.*
S: *They're both the same.*
I: *Both the same size?*
S: *Yeah. They're both the same size. One's got a positive number and one's got a negative number.*
I: *Oh, I see. Which is the negative number?*
S: *It's got a decimal point...it's got a negative number.*
I: *Are negative numbers the same as decimals?*
S: *(draws number line +4 +3 +2 +1 0 -1 -2 -3 -4) Well, you see, you've got two apples over here (+2)... Negative of that is negative two, which means you've got less than...you've got under two.*
(later)
I: *But these two here, four and point four, are they the same size?*
S: *Yeah.*
I: *Why?*
S: *Well, they're the same size rationally, but they're not the same size if you're talking about objects. You see, if you're talking about objects, um, it could be four telephones...but it can't be negative four telephones, 'cause you can't have negative four telephones!*

I: *Would negative four be the same as point four?*
S: *Well, negative four and point four would be the same.*
I: *You used the words 'rational numbers'. What do you mean by that?*
S: *Oh! Just numbers. I just say 'rational' numbers because it sounds good!*

Stu's mini-theory in the case appeared to have very 'fuzzy' boundaries, (Claxton, In press a, in press b) and to be a mixture of ideas about positive and negative integers and fractional numbers. Stu had generated inappropriate links between these two ideas and had resolved the conflict by generating a mini-theory that incorporated elements of both mathematical ideas. Fractional numbers are less than whole numbers, and negative integers are, likewise, below positive integers (on, say, a thermometer scale).

Stu's lengthy and sinuous explanation incorporated the distinction between numerals and numbers. As well, he used the term rational in its everyday sense, rather than with its mathematical meaning. The interview sequence demonstrated Stu's construction of ideas, and his constant attempts to generate links with a wide range of existing knowledge. On this occasion a 'bug' rather than an acceptable mathematical response was developed. The sequence also showed how students will actively attempt to generate a response when asked to justify their ideas in the interview situation.

Interview III

The opening series of Stimulus Cards (A-F, Series One) revealed that Stu had made some progress in estimation and approximation between Interview II and III. When the reasons behind Stu's correct responses were explored he was tentative, *"Let me see, I'm not too sure about this one...I guessed it."* The remnants of a partly-remembered strategy surfaced during his explanation for Stimulus Card F (Series One), *"It's got three decimal places (.19 x .5) so I wouldn't think it* (the answer) *would be three decimal places that way* (to the right)*!"* Stu considered 0.19 x 0.5 would be nearest in size to 0.01.

The next cards explored division. As with the two previous interviews Stu knew the strategy to adopt, but could not explain the underlying reasons for the strategy. He correctly noted that 15 ÷ 100 was 0.15, *"I moved the decimal point two places...if you take off two zeros, or move the decimal place twice, it's the same thing."* Later he concluded, *"It's logical."* (This compares with his response during Interview II when he exclaimed, *"It's just come to me!"*) The researcher then asked him why it was logical, and was met with the response, *"I don't know"*. Stimulus Card I (Series One) had proved difficult for Stu during previous interviews, but during Interview III he grappled with the stimulus material, and eventually constructed correct answers. With 25 ÷ 20 he expanded 25 into 20 + 5 and divided each part by 2. Next he moved the decimal point "forward" one place to generate the answer 1.25. After this lengthy computation Stu admitted, *"I'm not too sure of this one... You see I'm not very good at mathematics at the moment..."* However he held on to his correct answer when this was explored in greater depth.

Stu's lack of success with Stimulus Cards K to N (Series One) on problem-solving continued. As with Interview II he chose division as the operation needed in all four cards, commenting, *"I don't know"*, *"It's really hard 'cause you have to explain how you think"*, *"It looks good"*. He was able to construct some meaning from the word problems in Cards K, M and N, but chose the wrong algorithm. Reference was made in Chapter Six to the nature of the word problems and the associated algorithms, where it was considered that left to their own devices the High Competency Group could have generated correct responses. This same phenomenon was evident with Stu. With Card K (Series One) he realized *"you divide that (1.50) by two"*. Left to solve the problem by his own means, Stu may well have correctly answered the majority of Stimulus Cards K-N (Series One). But when asked to choose an algorithm, however, the meaning he had earlier generated evaporated.

The final series of Stimulus Cards on comparing numbers used in Interview III (Cards O-R, Series One) were correctly answered by Stu. The 'bug' that Stu had generated in Interview II when comparing 0.4 and 4 was not present during Interview III. The mini-theory, (Claxton, In press a, in press b) he used to demonstrate why 0.4 and 4 were the same in the previous interview had been replaced: "Basically we might say that a theory lasts as long as it is successful." (Claxton, In press a). Stu had outgrown his earlier ideas.

Interview III incorporated a device for measuring attitude towards decimals. This sorting activity (see Chapter Four, Page 70) was also employed during Interview II. The results are presented in TABLE 16.

TABLE 16 : Stu: Responses to Sorting Task: Topics in Mathematics

	Interview II Responses	Interview III Responses
Like Most:	g	g
	d	+
	s	-
	-	s
Neutral:	x	fr
	+	÷
	fr	x
	÷	xt
Dislike Most:	xt	d

Key: ÷ (division sums)
x (multiplication sums)
\- (take-away sums)
\+ (adding sums)
s (sets)
xt (times tables)
g (geometry)
fr (fractions)
d (decimals)
} (equal ranking)

The results showed a considerable difference between interviews for the topic decimals, but most other topics received similar rankings in the two interviews. Geometry was Stu's most liked topic on each occasion. Decimals were positively viewed during Interview II, *"Decimals I find a bit hard, but they're quite good"*. A shift was evident at Interview III when they were 'most disliked'. Stu explained, *"I'm not very good at them. I don't like them much. Sometimes they're quite good, but other times they're boring. They're boring. They're hard to do. It's hard sort-of. Thinking's always hard..."*

From the outset Stu had correctly named the place value columns for decimal fractions. His generation of the correct nomenclature was probed during Interview III:

Stu: Interview III: Discussion on Place Value Nomenclature

S: *(writes -✓ tenths / hundredths / thousandths /)*
I: *How do you know that's the case?*
S: *Well, there's always a strange thing! I was told that they never had ones in that ('tenths') column. I asked one teacher who said he didn't know. It's just a strange thing about decimal points...*

I: *How do you mean?*
S: *It's hard to explain. This is tenths (points to 'tenths' column). I would have put 'ones' there because I thought it might be ones there...you're starting again almost.*
I: *But you don't do that?*
S: *Well, I did, except that I got told off.*
I: *Do you understand why it isn't 'ones' that first ('tenths') column?*
S: *No. We were never told that.*
I: *Did it make sense to you?*
S: *No. No sense at all.*

Stu remembered the appropriate terminology, but this held little meaning for him.

The final part of Interview III consisted of an informal discussion concerning the applications of decimal numbers. Stu saw decimals as a school-based topic, rather than one with wide application to the outside world. The only use he could see for decimals apart from in school was with money, *"Well, probably 'yes' in money. But not in anything else."*

Interview IV:

The stimulus cards associated with Interview IV were checking devices for responses elicited during previous interviews. Stimulus Card B (Series Two) proved difficult for Stu, as had similar cards in earlier interviews. When estimating the number nearest to 0.34 x 0.3 (Stimulus Card B, Series Two) Stu chose either 1.0 or 0.01, but not the correct alternative, 0.1. It could not be the latter, he reasoned, because, *"it's times...it's really times three with a point in front of it... point one is lower than point three."* At the conclusion of this Card, the researcher indicated that 0.1 was the correct answer, and explained to Stu why this was the case. Stu listened and remarked, *"I didn't think timesing here (with 0.3) would be like dividing!"* He had retained strong links with ideas about multiplication with whole numbers, and this had obviously interfered with his construction of appropriate meaning when multiplying with decimal numbers. His mini-theory for multiplication with whole numbers had been over-extended (Claxton, In press a, in press b).

Stimulus Cards I and J (Series Two) investigated Stu's problem-solving ability when decimal numbers were and were not present. Stimulus Card I (Series Two) was answered correctly and quickly, *"I divided by two."* Stimulus Card J (Series Two) was more difficult for Stu. He realized that division was required, and tried 2.50 and 1.25 as both divisor and dividend. Unable to compute an answer he first estimated $1.25 as a reasonable answer for one litre, than changed this to 80 cents. If he had had a calculator, he considered the strategy would be, *"two point five divided by one point two five."* The addition of decimal numbers to the word problem made it significantly more difficult for Stu to construct meaning.

The remainder of Interview IV was conducted in a flexible manner based on some key questions. The first discussion focussed on Stu's responses to the pencil-and-paper Semantic Differential Technique (Osgood, Suci and Tannenbaum, 1957). With mathematics, Stu had indicated mostly neutral positions on the descriptive scales except for "valuable", "good", "sad", and "unfair". He thought mathematics was valuable because, *"You use maths in most...for computing, you need maths and you use it..."* Maths was perceived as "slightly sad" because, *"Whenever I hear the word maths, or we've got maths next day, I feel down-hearted."* Maths was seen as slightly unfair because, *"They slap all this work on you, and they don't really explain to you what it does, or why you need it."* Decimals received neutral rankings on the twelve associated descriptive scales except for a "slightly valuable" perception.

Stu saw mathematics at the high school as different from that at primary school in two main ways. First, teachers did not explain so well at high school and did not involve themselves so much with the students. Second, high school mathematics was harder and more topics were covered at greater speed. *"I mean, one day you're in one thing and then you're on another,"* he commented.

The "Advice to teachers" segment of Interview IV elicited clear suggestions from Stu who offered six inter-related recommendations on the teaching of mathematics:

i) Explain

"I reckon they should explain things clearly, instead of using great huge eight syllable words that are very hard to explain."

ii) Use simpler language

"They've used quite big words. Like geometry. I mean, they just don't explain what geometry is. They don't explain what the word is, what it means, what it all is."

iii) Assist individual students

"When you put your hand up they have to always come. Yeah that's a good one, that one...to help people more."

iv) Vary lesson speed to suit individual needs

"Some people get things fairly simple, and others...I mean at intermediate school they don't have this trouble because they spend more time with kids who don't know much about it."

v) Judicious use of textbooks

"Well, they just make us copy stuff out of the book. And when you copy stuff out you don't exactly read. You just copy the words...that's a bit of a problem."

After listening to the scenario described in Chapter Four for eliciting peer opinions of mathematics, Stu replied, *"Well, some of them groan"* when imagining his friends' reaction. When asked what they might say, he continued, *"Oh no! Not maths! Probably that's what they would say...they have to sit down and think, and it's not very easy to think, for some people..."* Stu next outlined how unwilling students copied answers from the back of the textbook, and how they avoided detection by the teacher, *"One of the tricks is to make a few mistakes.. you just make a couple of deliberate mistakes, so it doesn't look like you're writing them (the answers) straight out of the book."*

The final informal discussion investigated Stu's developing knowledge in mathematics, in terms of his view of where and how he'd acquired knowledge in the subject:

Stu: Interview IV: Discussion on genesis of mathematical knowledge

S: Well, most of it I just made up myself. I mean when I came along and I found different things I just fitted in... except I fitted some things in the wrong place!
I: Have you?
S: Yeah.
I: How do you mean?
S: Well, 'to the power of' (writes 2^2)...two to the power of two... In intermediate school we just kind of ran over that 'cause they say that's what you're going to learn in high school. And when you're in high school they say you're meant to learn that in intermediate school. So you've got to make it up for yourself.
I: Is that a problem for you?
S: You've usually got enough background to make up things for yourself - depends on how much you've picked up, or how much attention you've paid.

Stu's reliance on existing ideas and his attempts to construct meaning independently (Osborne, 1985) were clear themes in the above sequence. The implications for teachers were also apparent, in the sense that students actively construct meaning, drawing on their fund of existing knowledge in the subject.

Summary and Comment

The general nature of Stu's responses over the 27 months that he was studied may be summarized as follows:

1. Stu progressed gradually over time in his ideas about decimal numbers. In some cases earlier 'bugs' were eliminated (e.g. negative integers and decimal fractions), while in other cases 'bugs' were stable for the duration of the study (e.g. writing the decimal numbers fourteen tenths, fifteen hundredths, and twenty hundreds).

2. Stu gave some evidence of attempting to construct meaning for his own purposes, of accepting responsibility for his own learning. With the problem-solving cards (Cards K-N, Series One: Cards I, J, Series Two) Stu independently constructed meanings to a greater extent as the research project progressed, but the associated mathematical conventions (algorithms) remained a 'gap' (Claxton, In press a, in press b) in his knowledge. In contrast with this orientation, he also appeared to accept imposed conventions and strategies that had no meaning for him: place-value nomenclature was an example of this aspect. Stu vacillated, therefore, between accepting responsibility for his learning, and uncritically accepting mathematical rules and conventions imposed by the teacher.

3. The existence of conflicting mini-theories in Stu's pool of mathematical knowledge became apparent as the research progressed. He accepted .7 as less than a whole number in the context of writing decimal fractions, but saw .4 as the same size as 4 in the context of comparing numbers.

4. A major source of Stu's errors come from inappropriate links that he had generated with existing ideas about whole numbers, and operations on whole numbers. For example, he failed to appreciate

that multiplication with a decimal fraction reduced the other factor. In this case his mini-theory concerning multiplication with whole numbers was over-extended (Claxton, In press a, in press b). When this was pointed out to him during Interview IV, Stu indicated that he was ready to change.

5. Stu made errors at both the 'mini-procedures' and 'executive-procedures' level (Davis, 1979). At times he computed incorrectly and this led to the construction of an incorrect answer. At other times he made wrong evaluative moves, and this led to other errors.

6. Mathematics and decimals were viewed neutrally by means of the semantic differential technique, but the value of both was recognized. The topic rating, though, showed that Stu disliked decimals. The opinions of peers about mathematics was thought to be low, and he thought this was a barrier to learning in the subject.

7. In the advice to teachers segment Stu articulated five recommendations. These were a need for teachers to explain mathematical concepts more clearly, to simplify the terminology, to help individual learners, to vary the lesson speed, and to make careful use of textbooks. Other implications for teachers surfaced at different times, and included the over-reliance teachers placed on previous coverage of a topic, the boring nature of mathematics and decimals, and the creative efforts of students to provide correct answers through undetected cheating. Strategies for making cheating difficult for teachers to detect were articulated. As well, Stu gave a clear indication of how he had constructed knowledge in mathematics: *"You've got to make it up for yourself,"* he explained. Compared with mathematics at primary school, Stu thought high school teachers did not explain as clearly, covered topics more quickly, and high school mathematics per se was more difficult.

CHAPTER EIGHT

RESULTS AND DISCUSSION: INDIVIDUAL INTERVIEWS

THE LOW COMPETENCY GROUP

This chapter focusses on the conceptions and attitudes of those students in the interview sample whose response patterns indicated low competency with decimal numbers.

As TABLE 17 shows, the Low Competency Group comprised nine students (pseudonyms are used throughout).

TABLE 17 : The Low Competency Group

Name	Sex	Class level (1981)	TOSCA Percentile (Rank)	TOSCA Stanine	PAT (Maths) Percentile Rank (Class)	PAT (Maths) Percentile Rank (Age)	Interview Dates: Interview I	Interview II	Interview III	Interview IV
Ronald	M	F I	31	4	10	13	10.12.81	4.11.82	13.10.83	17.10.83
Andy	M	F II	24	4	20	19	15.12.81	22. 2.83	27. 9.83	4.10.83
Neville	M	F I	9	2	17	24	10.12.81	5.11.82	13.10.83	17.10.83
Jude	F	F II	31	4	20	9	7.10.81	8.12.82	20.10.83	25.10.83
Rita	F	F I	43	5	10	1	10.12.81	4.11.82	20.10.83	1.11.83
Stella	F	F II	8	2	4	11	14.12.81	22. 2.83	28.10.83	31.10.83
Doug	M	F II	6	2	16	16	30.9.81	14.12.82	16.11.83	16.11.83
Wally	M	F II	23	4	24	9	28.10.81	20.12.82	1.11.83	2.11.83
Janette	F	F I	24	4	17	17	16. 9.81	2.11.82	28. 9.83	5.10.83

M = Male
F = Female

Interview Response Patterns

All students in this group had great difficulty with the questions posed by the stimulus cards. With few exceptions the errors recorded in early interviews were sustained to the conclusion of the research project.

1. <u>Estimation and Approximation</u>: The group found this topic very difficult. The survey sample as a whole (see Chapter Five) had shown a facility level of 45 per cent with 2.9 x 7, and only 12 per cent with 0.29 x 7.1. But the Low Competency Group encountered problems before multiplication was introduced. During Interview I, of the nine students, only one correctly estimated the number nearest in size to 1.2 (Stimulus Card B, Series One), and none estimated the number nearest in size to 0.8 (Stimulus Card C, Series One). Errors were often the result of guessing: *"I just took a guess; looked as if it was related"* (Jude). The reliance on a physical similarity between the decimal numbers meant that some thought 0.8 was nearest in size to .08: *"It's got the same numbers, but they're round the wrong way and that. It's (0.8) got point zero eight, and the top's (one) got zero point eight"* (Stella). Counting strategies with whole numbers were used to explain why 0.8 was nearest in size to 1.8: *"Well, when you count you go zero, one"* (Neville). Some students relied upon intuition: *"It just looks the correct one"* (Neville). *"'Cause zero comes before the one"* (Jack). Wally constructed an answer by moving the decimal point to the left, and then claimed that 0.8 was nearest in size to 0.08. With Cards B and C (Series One), therefore, the Low Competency Group generated links with existing ideas about counting, physical similarity, and moving *"the dot"* (Wally). None of these existing ideas helped answer the questions posed by the Stimulus Cards.

The Low Competency Group answered incorrectly all remaining stimulus cards exploring estimation and approximation (Stimulus Cards D, E, F, Series One). The interview transcripts for Interview I recorded comments such as, *"I don't know"* (Andy), *"Don't know that one"* (Neville, Doug), *"Um...that's hard!"* (Janette), *"I just don't know points and that"* (Wally), *"I wouldn't have a clue"* (Jude). Although Rita had a mini-theory of counting the total number of decimal points in the factors and applying this to the product, she then ignored the decimal point and used a whole number approximation. To illustrate: Rita estimated 0.19 x 5 to be nearest to 100 because 19 times five was *"ninety-five - it would have been point nine five."* At the 'mini-procedures' level (Davis, 1979) Rita was correct, but in terms of understanding ('executive-procedures') she was in error. Stella used a mini-theory based on the similarity of 0.19 and 0.10 when estimating an answer for both Cards E and F (Series One). She thought 0.19 x 5 and 0.19 x 0.5 were nearest in size to 0.10 because, *"The decimal point is in the right place.*

"They've both (0.19 and 0.10) got a point one beside it (the decimal point." Apart from these attempts to construct meaning, Stimulus Cards D, E and F (Series One) exposed 'gaps' in the Low Competency Group's knowledge about estimation with decimal numbers.

2. Division and Decimal Numbers: Stimulus Card G (Series One) investigated division by ten, and two of the nine students responded to this card correctly. Neville, in noting 1000 ÷ 10 was 100, explained *"Well, a hundred times ten is a thousand."* Stella answered Card G (Series One) correctly except for 15 ÷ 10, to which she commented, *"I don't know that one."* On the whole, Stimulus Cards H and I (Series One) which involved division yielding a decimal fraction, were beyond this group at Interview I, and remained a mystery to these students for the duration of the research project. Only Andy and Neville were able to construct an appropriate meaning for Stimulus Cards H and I (Series One). Their responses showed that they understood the consequences of division with a divisor larger than a dividend: *"It's a fractional number"* (Andy), *"Ten divided by twenty would be a half"* (Neville). With Stimulus Card H (Series One) Neville was unable to generalize his mini-theory to the more discrepant 15 ÷ 100: *"Fifteen divided by one hundred wouldn't work 'cause a hundred is miles bigger than fifteen...just wouldn't work!"* The most common response to Cards H and I was *"You can't do it"*. When asked why, the group replied with words to the effect that the divisor was larger than the dividend. Most students simply repeated, *"You can't do it"*. Wally used a mini-theory involving negative integers (c.f. the Average Competency Group in Chapter Seven). Initially he believed 15 ÷ 20 was equal to *"minus five"*. With 15 ÷ 100 he considered the answer might be *"Minus something...minus thirty?"* Parallel items to Stimulus Cards H and I (Series One) had been the most difficult in the initial survey (Chapter Five), and this difficulty was repeated with the Low Competency Group.

3. Writing Decimal Numbers. Interview I probed the writing of decimal numbers. TABLE 18 contains the results.

TABLE 18 : Write Decimal Numbers: Interview I
(Stimulus Card J, Series One)

	The Low Competency Group				
	three tenths	seven hundredths	fifteen hundredths	seventeen tenths	twenty hundredths
Ronald	3.10	7.00	15.00	17.00	20.00
Andy	.03	.700	.1500	.17	.2000
Neville	3.10	7.00	15.00	17.0	20.00
Jude	.30	.700	.1500	.170	.20,00
Rita	3.10	7.100	15.100	17.100	20.100
Stella	3.10	7.100	15.0	17.10	20.100
Doug	3.10	7.100ths	15.100	17.tenths	20.100ths
Wally	3.10	7.00	15.100	17.0	20.0
Janette	3.10	7.100	15.000	17.1	20.000

The pattern of errors was clear. The group appeared to generate links with whole number nomenclature, and construct a decimal number from this. For example, "three tens" as digits, would translate to 3 10s. With a decimal point added, it became 3.10, which these students saw as three tenths. The responses for other decimal numbers in TABLE 18 revealed similar constructions of meaning. In writing three tenths as 3.10: *"Well, there's three, and then you've got a decimal 'cause it's point...and then it's ten"* (Janette); in writing seven hundredths as 7.100, *"Seven point hundred...I don't know, I just wrote it down"* (Stella). Most students' rationales equated with the above but, when pressed for further ideas on why decimal numbers had been written in a certain manner, students in the group characteriscally responded, *"I don't know why"*, or gave confused answers that could not be followed. Neville, though, made strong links with whole numbers.

Neville: Interview I: Write "Seventeen tenths"

S: *Seventeen point 'oh' (writes 17.0)*
I: *And why is that seventeen tenths?*
S: *Because...seventeen times ten is a hundred and seventy.*
I: *Uh huh...*
S: *So I just put seventeen point 'oh'.*

I: *What made you put the decimal point there?*
S: *Because there's a hundred and seventy, so I put the one seven, and the point there.*
I: *What does that say as a decimal fraction?*
S: *Seventeen point 'oh'.*
I: *But what about this one here (on the card)? What is it?*
S: *(reads) Seventeen tenths.*
I: *Is that the same as seventeen point 'oh'?*
S: *Yeah.*

The level of correct responding to this card, Stimulus Card J (Series One), remained low for the duration of the research project. No student from the Low Competency Group constructed more than one correct response out of five during any of the interviews. Four students (Ronald, Rita, Doug, and Wally) held to their ideas based on phonetic similarities to whole number equivalents for Interviews I, II and III - there was no evidence of them restructuring their incorrect mini-theories. Janette shifted to a more correct mode of responding, but did not link this to her correct labels for decimal number place value columns. Janette was the only member of the group to develop correct decimal place value nomenclature (tenths, hundredths, thousandths) over the two years.

4. Problem-Solving (Stimulus Cards K to N, Series One): This caused difficulty for the Average Competency Group, concerned some of the High Competency Group (except for Garth), and many of the survey sample (Chapter Five) had found items in this area troublesome. But the Low Competency Group were unable to construct any correct responses here over the two years of the research project. Only Wally identified the correct algorithm for one card, but his underlying reasons were faulty. For example, in Interview II he correctly considered the algorithm 0.5 x 1.50 would solve the problem (Stimulus Card K, Series One), but when asked why he chose this working form he explained, "*Because five times one dollar fifty is the cost per litre.*" His mini-theory of treating decimal numbers and whole numbers as one and the same would eventually have led him to an incorrect answer, albeit that he might have modified his mini-theory in the light of its over-extension.

The remainder of the Low Competency Group gave various reasons for choosing the incorrect alternatives. Sometimes these were linked with ideas about the conventions of certain operations *per se*. For example, during Interview II, Stimulus Card N (Series One), 6.44 ÷ 3.58 because, "*You can't have the quotient bigger than the divisor*" (Janette); "*'Cause* *that number (3.58) is smaller than six dollars forty-four cents"* (Jude). On other occasions the choices were a matter of convenience. To Stimulus Card K (Series One) Doug thought 1.50 + 0.5 would give the answer, *"'Cause it's much easier (to do)."* At times, some students realized that a certain number had to be reduced to get the price of one litre, and division or subtraction was chosen. *"Six point four four minus three point five eight...to reduce it...you only want one litre. Minus it to reduce it"* (Neville). This student had been able to construct some meaning from the problem, but the choice of algorithm was incorrect in this context.

On the whole, the Low Competency Group usually guessed an answer and could give no reason for their guesses: *"I dunno - just a guess"* (Janette, Interview III); *"I just divided - don't know why"* (Andy, Interview I); *"I'm not sure"* (Neville, Interview II), and *"I just guessed"* (Rita, Interview III) were common responses in the problem-solving context.

5. Size of Numbers: The final series of Stimulus Cards (Cards O to R, Series One) investigated students' ideas about the relationship between numbers with and without decimal points. During Interview I, the larger of two numbers (e.g. 4 and 0.4) was correctly identified only one time out of three. For example, 0.4 was thought to be larger than 4 because: *"The decimal point makes it larger"* (Neville);*"'Cause a point might give it extra count"* (Rita). Similarly, some described 0.80 as larger than 80 (Stimulus Card Q, Series One): *"It's a decimal fraction, and eighty is just a number"* (Stella); *"That one's got a point and that one's got eighty"* (Doug). In other cases there was no response from the students, or comments were passed such as: *"I don't know, I just guessed"* (Andy); *"I haven't got any idea"* (Neville); *"I'm guessing"* (Janette). Those students who attempted to rationalize their answers, most commonly suggested that possession of a decimal point made that number greater, the opposite of the correct mathematical idea. In some cases students correctly identified the larger number, but in doing so revealed incorrect underlying constructions of meaning. Janette chose the larger number of 0.80 and 80 (Stimulus Card Q, Series One) but thought 0.80 was smaller, *"'Cause if it takes eight to reach the negative mark, and then it takes eighty...so it would be a hundred and sixty."* Janette's mini-theories concerning decimal numbers apparently led her to view 0.80 as a negative 80, and the difference as 160. Janette's

constructed meaning here corresponded to her response with Stimulus Cards H and I (Series One) when she also generated links with ideas about negative integers to answer 15 ÷ 100 and 10 ÷ 20. This association between decimal numbers and negative numbers was also discussed in Chapter Seven.

6. Developmental Trends: Over the two years of the research project the Low Competency Group repeated their pattern of largely incorrect responses. Their progress in terms of correct responses to the full set of Stimulus Cards (A to R, Series One) was as shown in TABLE 19.

TABLE 19 : Correct Response to Stimulus Cards A to R (Series One)

The Low Competency Group (nine students)	Interview I	Interview II	Interview III
Correct Responses (Possible = 162)	14 (8.6%)	28 (17.3%)	23 (14.2%)

The responses changed over time, but only in the sense that new, incorrect mini-theories were substituted for existing, incorrect mini-theories. Two exceptions to this pattern (Andy and Janette) made modest gains by Interview III in estimation and approximation, and the comparison of numbers. Even so, Andy's underlying ideas remained tentative. Thus he correctly estimated .19 x 5 to be nearest in size to 1.0: *"'Cause of the decimal point there (in 1.0), it's the biggest one with the decimals."* But when probed again, he admitted it was a guess. Likewise, Andy could correctly identify the larger number of a pair, but he was unable to say how many times one was larger than the other. By Interview III, he considered 123 was about ten times larger than 12.3 (Stimulus Card P, Series One): *"Pretty roughly it'd be um...ten (times),"* and 249 was about 120 times larger than 2.49. When asked why, Andy said, *"Well, one hundred and twenty times two is two hundred and forty."* Decimal numbers were still troublesome for Andy, but developing skill in approximation with whole numbers was evident.

Janette also progressed in these areas but was unsure of the answers she gave: *"It's just a guess"*, *"I dunno, just a guess"*, *"I suppose it's just closest, you know"*, and *"It just looks the closest"*. When comparing numbers during Interviews I and II, Janette considered decimal numbers to be negative integers. By the time of Interview III she identified the larger number through more conventional mathematical means. For example, when comparing 123 and 12.3, she noted that 123 was ten times larger than 12.3, *"'Cause it's got all the same digits apart from the decimal point."* She was beginning to construct a mini-theory based on the idea that digits moving around a decimal point would become ten times greater or smaller for each place that was shifted.

Stimulus Cards A to J (Series Two) were used during Interview IV to probe errors revealed in the three previous interviews. These cards confirmed the low performance of the group. Stimulus Card I (Series Two) was correctly answered by all students in the group, but no students correctly answered the associated card (Stimulus Card J, Series Two), which introduced decimal numbers. When asked to compare these two cards, Stella thought Stimulus Card J (Series Two) was the more difficult, *"'Cause it's got two point five, instead of just two...and it's got one twenty-five instead of a dollar."* Stella and Jude were the only students to construct any plausible meaning from Stimulus Card J (Series Two): *"Well, you're going to halve two point five and also the amount it costs...because that's two point five litres and we want to find out how much one litre will be"* (Jude); *"About sixty-three cents... a half of a dollar twenty-five...around sixty-two, sixty-three"* (Stella). Stella admitted using the same strategy she had used with the previous card (halving), but with Stimulus Card J (Series Two) this was inappropriate. She was correct at the 'mini-procedures' level in working out half of $1.25, but it was at the higher order 'executive-procedures' (Davis, 1979) that she failed.

Assigning place value nomenclature to decimal number columns remained a mystery to these students. Only Janette correctly labelled the columns at Interviews I and III. All others in this group responded incorrectly at both check-ups. The most common error was to continue whole number patterns, thus naming the decimal columns (from left to right) 'hundreds, tens, ones'.

7. Attitude Towards Mathematics: The pencil-and-paper semantic differential test (Osgood, Suci and Tannenbaum, 1957) showed that these students had varying attitudes towards mathematics. Rita and Wally saw mathematics as extremely "clean" and "valuable". In fact, six of the nine students in the group viewed the subject as either very closely or quite closely related to the "valuable" end of the descriptive scale: *"...'cause you learn more about it...you need to do maths and it's valuable for a job and that...it's boring!"* (Wally). When asked what was meant by this, Wally added, *"You've got to be clever to know how to do some of it."*

Doug, Andy, Stella, Jude and Neville indicated neutral positions on most continuums, with some exceptions. Stella considered it very unpleasant: *"I hate maths; it's boring' it's hard and that."* Neville saw maths as very "awful": *"I don't like maths...it's boring...decimals and fractions and..."* Janette recorded the most negative attitude towards maths, seeing the subject as very cruel, unpleasant, awful, and bad. She also saw it as quite sad and sick.

Janette: Interview IV: Discussing Semantic Differential Test

I: *You see maths as very cruel? Why?*
S: *It's just the working at it, 'cause I'm not very good at maths. I don't know about maths; can't do some of it.*
I: *You also have indicated maths is very awful.*
S: *Yeah...*
I: *What's awful about it?*
S: *The whole lot! The teacher, the way we have to work at it, the things we're on at the moment...*
I: *Well, what are you on at the moment?*
S: *Decimals! (laughs) ...decimals and fractions...*
I: *You don't like them?*
S: *Nah.*
I: *Why not?*
S: *'Cause for one thing I don't understand it. And another, if I don't understand it I don't enjoy it. If it was basic it would be really good, but I don't understand it.*
I: *What sorts of things don't you understand?*
S: *The whole lot! Everything!*

Janette explained that she had had trouble understanding decimals from her first encounters with the topic, and considered that greater individual attention to her in the classroom setting would have been of greater benefit than the use of concrete apparatus in helping her understanding. She could imagine little use for decimals and fractions in the world outside school, and had a low opinion of her ability in mathematics.

Decimals and mathematics were both viewed negatively by most of this group. Furthermore, most of the group thought mathematics was more difficult at the high school level: *"...well, primary maths was easier. That's from now when I look at it. But when I was at that stage it was quite hard for me, but when I'm in the sixth form I think it will seem easy and that"* (Stella).

At the conclusion of Interview III the students were asked to sort nine cards with topics written on them (from mathematics) into a 'like'/'dislike' continuum (see Chapter Four, Page 70). TABLE 20 has the results for eight of the group (Ronald's responses are presented later in this chapter.

TABLE 20 : The Low Competency Group: Responses to Sorting Task: Interview III

	Andy	Neville	Jude	Rita	Stella	Doug	Wally	Janette
Like Most	+	xt	-	÷	xt	+	xt	xt
	-	x	xt	-	+	x	x	x
	xt	+	÷	x	x	xt	+	+
	x	-	x	+	-	s	-	-
Neutral	d	÷	+	xt	fr	fr	s	s
	fr	fr	fr	s	s	d	÷	÷
	g	g	g	fr	d	-	g	g
	d	d	s	g	÷	g	fr	d
Dislike Most	s	s	d	d	g	÷	d	fr

Key:
÷ (division sums)
x (multiplication sums)
\- (take-away sums)
\+ (adding sums)
s (sets)
xt (times tables)
g (geometry)
fr (fractions)
d (decimals)
} (equal ranking)

It can be seen that, at best, decimals received a neutral ranking, and most often was placed near the dislike end of the continuum: *"Can't do them...can't do them and they're boring"* (Janette); *"They're* *complicated most of the time. It's just that it's hard to get the point...I just don't understand most of the time"* (Jude); *"I have trouble putting the decimal points in the right place...I always put them in the wrong place...I get frustrated 'cause I can't do it"* (Rita).

The next segment of Interview IV elicited "advice for teachers" - advice students would give teachers to help learners with mathematics. The Low Competency Group mentioned four areas that teachers should address:

i) Teachers should spend longer on each topic. *"Don't rush us so much, because we miss a lot of work rushing us. You have to get finished, and then if you don't finish you have to do it next day, and then if you get left behind you have to do the next day's work, as well as that day's work... Don't rush us and let us learn how to do it instead of rushing us, 'cause we want to know how to do it"* (Neville);

ii) Teachers should devote more time to individual students (Janette, Wally and Rita) and provide individual tuition when the need arose. Wally recognized this was partly due to class sizes: *"I think they should have more teachers to pupils, because only one teacher can't always get around to you when you want to know something."* Later, *"Sometimes you can't really learn in a bigger environment...you can't learn as fast if they've got to teach the whole class."*

iii) Stella and Neville considered teachers should explain more clearly to their students. Stella (whose first piece of advice to teachers was to cease teaching the subject entirely) added: *"They tell us to go and do it, and we don't have a clue what we're doing. They haven't explained it well enough."*

iv) There was a need for a simpler, more basic curriculum since many of the topics studied were beyond their scope and interest. Jude mentioned the need to study operations with money, and *"just work up to it 'cause it's complicated now, 'cause I don't understand half of it."*

The scenario that explored peer opinions of mathematics (see Chapter Four) revealed that most students in the group thought their peers held negative opinions towards mathematics, and no student thought their peers would express positive feelings. Jude's comments were typical:

Jude: Interview IV: Peer Opinions of Mathematics

I: *(scenario for eliciting peer opinions)*
S: *Um, "Oh, how exciting!" (said sarcastically)...It's not going to be! Most of them would want to wag that period. And they say, "Here comes another boring afternoon."*
I: *They don't think maths is interesting?*
S: *No.*
I: *Why not?*
S: *Oh, I guess they're finding it complicated.*
S: *(later) You either want to listen to maths or you don't. If it's something good which she's (the teacher) going on about then you like it. But if you walk in there and she starts saying, "All right, decimals...", well I'll just play up, or something like that.*
I: *Why?*
S: *'Cause I don't get it. We've been on those all the time and I don't get it, so I don't bother.*

Social disruption in the class was seen as a hindrance to learning mathematics by only two of the Low Competency Group, Doug and Wally. Doug considered *"kids get easily distracted"* during the mathematics lessons. It is possible that some of the group may not have recognized inappropriate behaviour as interfering with learning because they, themselves, contributed to the misbehaviour. Stella, for example, said peer misbehaviour wasn't a problem because *"I usually do it too... flinging paper around the room."*

Summary and Comment

In considering the responses of the nine members of the Low Competency Group, the following characteristics emerged:

1. These learners had great difficulty in all areas that were probed during the four interviews over two years. Most students made no progress at all in terms of generating a greater number of responses over time. Only two students, Andy and Janette, recorded progress in terms of selecting correct alternatives, but their answers were usually based on faulty underlying mini-theories.

2. Inappropriate links were frequently generated with existing ideas in memory store. Thus strategies for multiplying with whole

numbers were incorrectly used with decimal numbers; the writing of decimal numbers was wrongly based on whole number nomenclature, as was the assigning of place value nomenclature to decimal fraction columns; division was performed by employing negative integers; and numbers were thought to be closest in size to one another because they contained identical digits.

3. The interviews revealed a large number of gaps in the students' knowledge so that they frequently admitted to guessing, or simply replied that they *"didn't know"*. This remained unchanged with Stimulus Cards G to I (Series One), and Stimulus Cards K to N (Series One).

4. Some of the group constructed mini-theories that were precisely opposite in meaning to the correct mathematical idea, e.g. a number of students considered the number containing the decimal point (e.g. 0.80 and 80) to be larger simply because it possessed a decimal point.

5. A wide range of mini-theories was constructed by these students, both within a topic and over time. New but incorrect mini-theories were substituted for existing incorrect mini-theories. The students held little commitment to their existing ideas and changed these willingly. Errors were common at the 'executive-procedures' level (Davis, 1979) rather than the 'mini-procedures' level, so that students often performed simple calculations correctly but used these in inappropriate contexts.

6. Most of the group thought mathematics was a valuable subject. Some viewed the subject quite positively, others with distaste, but all members of the group considered that their peers disliked maths. Social disruption of mathematics lessons was not seen as interfering with learning. The topic of decimal numbers was rated as largely disliked when compared with other topics in mathematics.

7. The students thought that teachers should proceed more slowly, spend longer on each topic, work with individual students to a greater extent, explain ideas clearly, and provide a simpler curriculum in mathematics.

8. These students revealed a lack of confidence about and a fatalistic attitude towards their present and future performance in mathematics and decimal numbers. Stella's observations during Interview IV were typical of the group, when asked about her pending school examination in mathematics : *"I'll just fail it anyway, 'cause I did last year. Yeah, I'll probably fail my exam."*

9. Some of the questions were inappropriately worded for students in this group since the mathematical ideas were not accessible from the stimuli provided. This was particularly apparent with the Stimulus Cards K to N, (Series One), where responses indicated that students could construct little or no meaning from the written statement - let alone generate appropriate mathematical strategies for solving the problem.

10. Students correctly solved a simple problem with whole numbers (Stimulus Card I, Series Two), but could not transfer the same strategies to the new (and similar) problem that contained decimal numbers (Stimulus Card J, Series Two). This suggested that links with existing ideas were not generated because attention was diverted by the decimal numbers (c.f. Krutetskii, 1976).

Ronald

A Case Study from the
Low Competency Group

As shown in TABLE 21 Ronald was below-average in achievement and scholastic ability. At the time of his first interview he was a Form One student at an urban intermediate school. By his third and fourth interviews, Ronald was near to completing his Form Three year at an urban high school.

TABLE 21 : Personal Data for Ronald

Interview I 10 December, 1981	Age at Interview I: 11 years 10 months
Interview II 4 November, 1982	Standardized Test Results (1981) *TOSCA P.R. (Form A) 31
Interview III 13 October, 1983	*TOSCA Stanine (Form A) 4 PAT (Maths) P.R. (class) 10
Interview IV 17 October, 1983	PAT (Maths) P.R. (age) 13 *(Primary Form A)

Interview I:

From the outset Ronald's response patterns indicated confused thinking and negative attitudes concerning decimal numbers. His response to Stimulus Card B (Series One) is illustrative.

Ronald: Interview I: Stimulus Card B (Series One)

S: *(reading card) One point two is nearest in size to?*
I: *Which one is it nearest in size to, down below?*
S: *...ten.*
I: *Uh huh. Why do you think that?*
S: *Oh, it was guess actually. I didn't know that bit.*
I: *Which bit didn't you know?*
S: *The one point...the decimal.*
I: *You don't like those decimals?*
S: *No, not at all.*
I: *Why not?*
S: *I find them hard to work out...why you have to cut things in half, or like that. Cut one point two in...It confuses me actually. I'm terrible in class work.*
I: *What sorts of things confuse you?*
S: *Oh, it's the one point...you know, in two, because the point confuses me, 'cause it could be another number.*
I: *All right...Well, do you want to make a guess then? What do you think one point two could be nearest in size to?*
S: *...point one two; point twelve.*
I: *Uh huh. Point twelve...Why?*
S: *'Cause, um, I'm not quite sure.*
I: *Not quite sure?*
S: *...'cause that's about the only thing you can have there. I have heard that one point two is something like two fives, or something like that.*
I: *Sorry... One point two is something like...what?*

S: *Two fives.*
I: *Two fives?*
S: *I'm not quite sure...*
I: *How do you mean, like two fives?*
S: *Um... He said that "one" was five, or something... That's Bruce, my friend. It could be wrong.*
I: *That's what Bruce told you?*
S: *Mm...*
I: *I see... Right...*
S: *He's quite a mate of mine.*

This extract indicated the difficulty Ronald had in linking the problem with his existing knowledge. Not only did he offer *"It confuses me actually"*, but he had accepted uncritically the incorrect ideas of a close friend. Ronald's confusion, tentativeness and somewhat desultory thinking were also evident with Stimulus Card D (Series One). After a first response of *"Point ten, I think"*, he corrected himself, *"...'cause it hasn't got the number there I estimated it as."* Ronald's second response was *"One point forty-five...'cause nine fives are...forty-five.. but I could be wrong...I thought point ten, 'cause two fives are ten. Nine and one is ten. It is those decimal points that confuse me..."* Later, Ronald tried again: *"One point nine is nine...five and nine is... fourteen, so the twenty is closest to that..."*, and then changed his mind to *"One and nine (1.9) is ten...and five is fifteen."*

Confusions like these continued with the stimulus cards that probed estimation and approximation skills. Thus, in responding to Stimulus Card F (Series One), Ronald chose 1.0 as the answer, *"...'cause there's two decimal points, the one and the zero...'cause it has two decimal places...two decimals."* He explained, *"Well, there's not really two, but from there, from one to the point is two...there's two places... numerals...from the zero to the one is two...there...I think..."* It was clear that Ronald computed decimal places by using both the ones and tenths places as if they were places belonging to decimal fractions. He was confused between the digits in a decimal number and the number of places in a decimal fraction, and it was significant that Ronald referred to decimal places as decimal points.

Ronald also found it difficult to perform division when the divisor was larger than the dividend.

Ronald: Interview I: Stimulus Card H (Series One)

I: *What's 15 divided by 100?*
S: *15 divided by 100... (pause 5 seconds) ...I don't think you can do it.*
I: *Why not?*

S: *Well, first you have to have something you can put into it...and you can't divide anything out of it.*
I: *Is there any way that you know that you could do it? (15 ÷ 100)*
(later again)
I: *Is there any way you'd get an answer to that? (15 ÷ 100) That you know of?*
S: *No, not that I know of...only with adding a zero (to 15) but zero's not written on the piece of paper.*

This reasoning was repeated with the algorithm 15 ÷ 20: *"Twenty's too big for fifteen...you'd have to add five more...so you can't do it that I know of."*

The operation division has often been introduced to New Zealand elementary school children through the 'sharing' concept. This operation may be represented with concrete materials such as counters or place value blocks. In a physical representation it would be inappropriate to introduce the idea of sharing a number of objects where there was less than one per share (dividend smaller than divisor). Ronald obviously considered such a strategy impossible, in any case, and appeared to generate links with ideas about the physical representation of the operation: *"You can't get any hundreds out of fifteen."*

The next segment of Interview I probed Ronald's strategies for writing decimal numbers (this had been found difficult by many students in the initial survey).

Ronald: Interview I: Stimulus Card J (Series One)

(Ronald's responses: 3.10
7.00
.15,.00
.17.00
.20,.00)

I: *Right...read them to me, now three tenths...you've got?*
S: *Three point ten.*
I: *uh huh...why have you written that?*
S: *Three point ten...'cause three is only quite a small number so you wouldn't have to have any decimal points before it... so you could write it three point ten.*
I: *So that's three tenths is it?*
S: *Yeah.*
I: *Uh huh. What about seven hundredths?*
S: *Seven point zero zero.*
I: *Why have you written seven point zero zero?*
S: *Again, it's another small number...and so you put the point in there and hundreds is just two zeros.*
I: *Uh huh. And that's the decimal fraction for seven hundredths is it?*
S: *I hope so.*

His attempts to construct meaning were based on linkages to existing ideas about whole number nomenclature: *"Hundreds is just two zeros."*

With the remaining decimal numbers, Ronald was unable to respond correctly, although a first attempt to write fifteen hundredths (.15,00) was correct, except for the comma. However his response was based upon incorrect reasoning: *"Once you get to hundreds you put the point before the fifteen, or whatever the hundreds may be."*

At various times during the interview sequences Ronald showed an unwillingness to accept responsibility for his own learning. Earlier in this chapter his friend's influence was noted. At another time he commented his strategies were given to him by his teacher, but *"I don't want to accuse her (the teacher) if it's wrong."* Several interview sequences revealed that he believed learning in mathematics depended on input from outsiders: *"I was told somewhere along the line"* occurred several times. His own active internal construction of a set of knowledge and understandings was not given prominence. Osborne commented:

> "When students accept that they, rather than their teachers, their parents, other people, or other factors, are responsible for constructing the meanings that represent their success or failure in school, their learning is likely to increase."
>
> (Osborne, 1985:14)

Stimulus Cards K to N (Series One) investigated problem solving with decimal numbers. Ronald's responses were incorrect with errors at both the 'executive' and 'mini-procedures' level (Davis, 1979). He failed to generate appropriate meanings from the questions and he indicated that rules given to him by outsiders guided his thinking: *"I was told a couple of years back."*

The final set of Stimulus Cards (O to R, Series One) investigated Ronald's understanding of numbers with and without decimal points. He answered all cards wrongly. The interview sequences showed that he possessed a range of mini-theories and that he moved from one to the other. He commented that the decimal point *"doesn't really mean much"*; next, it *"doesn't mean anything, really"*; finally *"point, I think is taking the place of another number."* As a consequence of holding these views, Ronald considered 0.4 to be (consecutively) smaller than, equal to, and finally larger than 4.

Ronald named the place value columns for decimals, from right to left, as *"tens of hundreds", "tens of tens",* and *"ones".* This response had elements of whole number nomenclature within it. It would appear that he had generated links with his existing ideas of whole number places. His written form for fifteen hundredths had been *".15,.00"* because in his explanation *"hundreds is just two zeros".* Ronald, used zero as a place holder for both whole numbers and decimal fractions, but with decimal numbers the use was redundant.

During Interview I, Ronald made comments on decimals such as: *"I find them hard to work out", "It was a guess actually", "I think I've made a 'boo-boo'",* and *"It confuses me actually".* Given Ronald's lack of confidence with decimal numbers, it was not surprising that his achievement was so low.

Interview II:

The second interview took place 11 months after the first. At the time of Interview II Ronald was in a Form Two class at the same school (metropolitan intermediate) as the previous year. He was aged 12 years 9 months. His lack of knowledge and confidence about decimal numbers was immediately apparent: *"We did do them but I've forgotten them."* To Stimulus Card B (Series One) Ronald mumbled, *"It (the answer) just seems to be...you know...ah, I can't think..."*

Data from Interview I showed that Ronald held conflicting mini-theories regarding the decimal point, one being that the decimal point could be ignored when comparing, say, 0.4 and 4: *"It (the decimal point) doesn't really mean anything really."* Ronald, at the time of Interview II had not shifted his thinking. Thus, when comparing 0.8 and 8 he noted, *"Nothing point eight would be eight, I think..."* (Stimulus Card C, Series One). In other words he generated links with ideas about the digits, rather than the place value of the numbers (Hiebert and Wearne, 1984).

With Stimulus Cards G to I (Series One), Ronald responded similarly to Interview I. 15 ÷ 100 elicited, *"It won't fit!",* and 15 ÷ 20, *"I don't think so...you're five too short."* He appeared to have made no progress in this area over the previous year.

Some progress, however, was evident with writing decimal numbers (Stimulus Card J, Series One). Three correct responses were generated - three tenths, fifteen hundredths, and twenty hundredths - and links between decimals and with existing ideas about whole numbers were also evident, e.g. in explaining his construction of *".700"* for seven hundredths: *"I know it was a hundred - it's point seven hundred... 'cause also I've forgotten what the meaning of the point was."* This response was similar to that in Interview I.

Problem solving with decimal numbers remained too difficult. With Stimulus Card L (Series One) Ronald chose the correct operation, but he could give no mathematical reason for his correct response. His strategy was to eliminate the other alternatives with reasons such as *"fifty-eight plus two dollars sixty-seven is going to rip the shopkeeper off!"* He also believed that multiplying by 0.58 would increase the other factor, because he saw multiplication as a process that inevitably gave a product larger than either factor. Ronald constructed a correct response, but the links he generated to existing ideas were inappropriate.

The three other problem-solving cards proved troublesome for Ronald. He was unable to construct any correct meanings, and floundered from one mini-theory to another.

The next cards in Interview II asked Ronald to compare numbers with and without decimal points (Stimulus Cards O to R, Series One). Ronald responded that both numbers (e.g. 0.4 and 4) were the same size: *"I don't think the decimal point stands for any number."* In comparison with his responses to the same cards in Interview I, his ideas during Interview II were stable although some dissatisfaction with his answers was evident. At one stage, with regard to the meaning of the decimal point: *"I'd like to find out about that, but I don't know."* Later, *"The point's there for some reason, but I don't know the reason...I do know, but I've forgotten it,"* and, *"If I know what the point stands for I'd be able to..."* During Interview I Ronald thought that the decimal point stood for another number. During Interview II he thought that the decimal point had some unknown purpose. The difficulty he had in constructing meaning and generating links with his memory store was obvious.

In naming decimal places, Ronald named the first place after the decimal point as *"tenths of ones"*, but was unable to proceed. Interview I responses suggested a link with whole number nomenclature, but by Interview II even these links were not apparent.

Interview II probed Ronald's attitude towards the topic decimals. The ratings are given in TABLE 22 below.

TABLE 22 : Ronald: Responses to Sorting Task: Topics in Mathematics

Interview II : 4 November 1982

Like Most:	-
	+
	s
	g
Neutral:	÷
	x
	xt
	fr
Dislike Most:	d

Key:		
÷ (division sums)	xt (times tables)
x (multiplication sums)	g (geometry)
- (take-away sums)	fr (fractions)
+ (adding sums)	d (decimals)
s (sets)	} (equal ranking)

Ronald initially assigned division towards the dislike end, but then re-assigned the topic to a neutral position: *"I don't mind division, but I don't exactly love it."* When asked why he had placed decimals at the dislike end of the continuum he explained:

Ronald: Interview II: Sorting Task

S: *I don't understand decimals at the moment. I used to, and when I did do them I still couldn't understand them fully. I just don't like them.*
I: *What don't you understand about them?*
S: *Their uses and their meanings.*
I: *How do you learn about decimals? From the textbook? From Dad? From the teacher? From...*
S: *(interrupting) The teacher.*
I: *What sorts of things do you remember the teacher saying about decimals?*
S: *I've forgotten all of them!...*

Ronald's dislike for decimal numbers was related to his inability to understand the topic. The above sequence, like data from Interview I was characterized by feelings of doubt, and at one point Ronald contradicted himself. Ronald's claim that he did not understand the uses of decimals was investigated further:

Ronald: Interview II: Uses for Decimal Numbers

I: *In the outside world, do you use decimals...outside of school?*
S: *I don't. I haven't come across having to use decimals before.*
I: *Can you think of anybody who would use decimals outside of school?*
S: *No, not really.*
I: *Can you think of any uses that decimals might have?*
S: *They could have some, but I don't know them. They wouldn't be around if they had no use!*

Ronald was unable to generate links with ideas about money and measurement. His restricted view of decimals inhibited his ability to construct appropriate meaning. The generative learning model has suggested that if a number of links were generated (in the above extract, with say, ideas about decimal currency and metric measurements), then subsumption into memory store could more easily take place. Osborne wrote:

> "The greater the number of links generated to other aspects of memory store, and the greater the number of these links that reaffirm a useful meaning has been constructed, the more likely the idea will be remembered and made sense to the learner."
>
> (Osborne, 1985:14)

Interview III:

From the outset of this interview Ronald encountered problems: *"I still can't understand those things."* His strategies continued to be based on his existing ideas about whole numbers and decimal points were often ignored: with Stimulus Card E (Series One) Ronald estimated 0.19 x 5 to be about 100, because 19 times five would be almost 100.

Ronald persisted with the view that if the divisor were greater than the dividend, then the operation of division could not be performed. His mini-theory, stable for the two years of this study, remained (in his view) satisfactory, had not failed, and was not therefore inadequate. During Interview I, *"You can't get any hundreds out of fifteen"*, and later, *"You can't do it 'cause 15 is smaller than 100 and you need something like 100 or above before you can get 100 out of it."*

With reference to writing decimal numbers, all five of Ronald's answers were incorrect (in Interview II three answers were correct).

When asked how he constructed his answers, Ronald indicated that he had guessed,which was characteristic of his responses in all four interviews.

Stimulus Cards K to N (Series One) looked at problem solving with decimal numbers. Ronald could not construct any correct or reasonable answers, although his responses showed a realization that decimal numbers were less than whole numbers: *"Point five eight is less than one kilogram."* (In Interviews I and II Ronald had no meaning for the use of the decimal point: 0.4 and 4 had been the same size.)

Some progress was evident in Ronald's ability to compare numbers. He was consistently able to recognize the larger number out of, say 123 and 12.3, but unable to say how many times one number was larger than or smaller than the other (this idea had been introduced to Ronald in Form One). The textbook used in New Zealand schools states:

> "3. When multiplying a factor by 10, the decimal point is moved one place to the right.
> 4. When multiplying a factor by 100, the decimal point is moved two places to the right.
> 5. When multiplying a factor by 1000, the decimal point is moved three places to the right."
>
> (Duncan *et al.*, 1980:274)

and suggests that teachers should present this topic to students as:

$$" \quad 10 \times 27.3 = 10 \times \frac{273}{10} = 273$$

$$100 \times 2.73 = 100 \times \frac{273}{10} = 273$$

$$1000 \times 0.273 = 1000 \times \frac{273}{1000} = 273 \quad "$$

(Duncan *et al.*, 1980:275)

Interview III continued with the sorting exercise (described in Chapter Four). Ronald's ratings for topics within mathematics were as follows:

TABLE 23 : Ronald: Responses to Sorting Task: Topics in Mathematics

Interview III : 13 October 1983	
Like Most:	s
	+
Neutral:	
Dislike Most:	xt
	-
	÷
	x
	fr
	d

Key: ÷ (division sums) — xt (times tables)
x (multiplication sums) — g (geometry)
- (take-away sums) — fr (fractions)
+ (adding sums) — d (decimals)
s (sets) — } (equal ranking)

Ronald had placed most topics together under the dislike label, saying that he couldn't understand them, and, *"I don't really care much for maths at all."* During Interview II Ronald had claimed he could see little application for decimal numbers in the outside world. This topic was revisited, with consistent results:

Ronald: Interview III: Uses for Decimal Numbers

I: Do you think decimals are useful outside of school as well as inside school?
S: Well, I haven't come across them yet.
I: Outside school?
S: No! I haven't come across much maths out of school. (later) I haven't found much maths out of school anywhere really.

Ronald did not progress in terms of correct responses to the stimulus cards over the two years. The sole mathematically-correct idea he constructed occurred in the latter stages was that decimal fractions were smaller than whole numbers.

Interview IV:

Ronald could construct no correct meaning from the alternative stimulus cards used during the opening segment of Interview IV. In an attempt to assist his responding a calculator was introduced.

Ronald: Interview IV: Stimulus Card C (Series Two)

I: *And ten divided by fifty?*
S: *None...There's no fifties in ten.*
I: *Can you do it any way?*
S: *...um...*
(at this point a calculator was introduced by the interviewer)
I: *Could you do it on the calculator?*
S: *...no...too difficult.*
I: *(executes 10 ÷ 50 on calculator)*
S: *(reads display) Zero point two.*
I: *Do you believe that?*
S: *I'm not going to argue with a calculator. But* <u>*I*</u> *wouldn't be able to work it out.*

Ronald's latter comments restated his view that outside factors rather than his own active construction of meaning were primarily responsible for the development of his mathematical knowledge.

Stimulus Cards D and G (Series Two) illustrated Ronald's problems with writing decimal numbers and comparing numbers with and without a decimal point. With Stimulus Card J (Series Two) Ronald believed 2.5 litres meant half a litre, but he could not say <u>why</u> this was the case: *"I don't know why it is, but I do know it (2.5) is half a litre."* He reiterated this belief with Stimulus Card M (Series Two).

With the Semantic Differential Test Ronald indicated that decimals, to him, were *"unpleasant"*, *"hazy"*, and *"awful"*: *"...I don't like doing decimals or maths 'cause half of it I can't understand."* With mathematics, Ronald saw the subject as *"hazy"*, *"awful"*, and *"sick"*: *"Everytime I think of maths it makes me feel yuk...'cause I don't understand it."* Ronald was then asked if this negative opinion of mathematics had been held for some time.

Ronald: Interview IV: Opinion of Mathematics

I: *Have you often felt that about maths in the past?*
S: *Yes.*
I: *When?*
S: *About Standard One (7 year old level)*

I: *You've never enjoyed it?*
S: *No, 'cause I don't think I put enough work into it; but I never enjoyed it because it never seemed to appeal to me from the start...it just never appealed to me. And when I did try I just didn't get good marks anyway. So I gave up...trying.*
I: *Do you try much nowadays?*
S: *Yeah. I try and get some good marks. But...in my maths test I only get about 12 or 13 (marks). Everyone else gets 16s or 17s...it makes me feel really awful, really.*

Ronald was then asked to give advice to teachers of mathematics (see Chapter Four) and he spoke for five minutes 30 seconds on the subject, offering five related pieces of advice.

i) Ronald suggested teachers should be more tolerant of learners who were having difficulty: *Don't start yelling when you put up your hand and say, 'I don't understand'."*

ii) Ronald advised teachers to explain mathematical ideas to students more slowly and thoroughly: *"Go through it more thoroughly. Cover each thing thoroughly, and not just go on 'Page eight, blah, blah, blah. You ought to be able to do that'."*

iii) As a corollary of the above, Ronald suggested teachers should be less demanding of their students: *"They really demand untold of you,"* he complained.

iv) Ronald cautioned teachers on the use of textbooks in mathematics: *"They (textbooks) also don't explain things very well in some cases. They might give you examples on the top of the page, but they don't really give you enough to work from, to work out."*

v) Ronald thought teachers should put more variety into the teaching of mathematics. Most mathematics teaching was *"just straight out of the book. Just doing the good old textbook style and pen and paper. 'Do page blah, blah, blah...'. Whereas you could be outside doing something, still using maths."*

Ronald's constructions of reality in mathematics included a body of strongly-held ideas about the unpleasantness, difficulty, and irrelevance of much of what was taught. His advice to teachers came from years of failure in the subject: *"When I did try, I just didn't get good marks anyway."*

Summary and Content

The main features of Ronald's responses over the 24 months that he was studied may be summarized as:

1. Ronald did not appear to accept responsibility for his own learning in mathematics, considering that outside agencies (peers, teachers and the textbooks) were responsible for his learning. Ronald copied answers from the back of the textbook rather than attempting to construct meaning for himself from the textbook exercises. The generative learning model has suggested that learning would be likely to increase only when learners accepted that they "are primarily responsible for constructing the meanings that represent their success or failure in school" (Osborne, 1985:14). This might explain Ronald's lack of progress.

2. Ronald's negative attitude towards mathematics and decimal numbers was very apparent. He viewed mathematics as 'unpleasant', 'hazy', and anxiety-provoking, which doubtless contributed to his low achievement in the subject (c f. Brush, 1980; Knight, 1982; Reyes, 1984). *"I don't like doing decimals or maths 'cause half of it I can't understand,"* he commented.

3. Most of Ronald's errors were at the 'executive-procedure' (Davis, 1979) level. By contrast, many of his 'mini-procedures' were correct. It was a failure to understand and to evaluate that characterized his thinking.

4. Ronald was unable to generate links with existing ideas that provided relevant and supportive data. He moved from mini-theory to mini-theory in his attempts to construct meaning.

5. Ronald viewed decimal numbers as having limited application to the world outside school. During Interview II Ronald could not articulate uses for decimal numbers outside the school context. Likewise, in Interview IV he could not see the relationship between decimal currency and decimal numbers. In terms of Claxton's (In press a, in press b) mini-theories Ronald possessed separate theories that did not overlap and were confined to their own domains: decimal numbers belonged to the domain of school; money belonged to the world

outside school. In other words school maths was not related to his general knowledge of the world (c.f. White, 1984).

6. Ronald gave clear advice to teachers. He thought that learning in mathematics was inhibited by teachers who failed to discuss ideas with students, who expected too much, who moved too quickly through new mathematical content, who assumed students possessed ideas when they did not, and who were intolerant of slower learners like himself. In particular, he considered the assignment of textbook pages to be a worthless exercise. Much of Ronald's advice to teachers related to the introduction of new material.

CHAPTER NINE

SUMMARY, CONCLUSIONS AND IMPLICATIONS OF THE RESEARCH

The present research was designed to investigate students' thinking about decimal numbers. A large sample was surveyed, and a smaller group of 28 students was interviewed four times over a two-year period about: (i) their concepts of decimal numbers; (ii) their computational and problem-solving procedures with decimal numbers, and (iii) their views about the learning of decimal numbers and mathematics in general. These 28 interviewees were separated into above average, average, and below average competency groups

This final chapter summarises and draws conclusions from the research findings. The theoretical implications of the study are considered, and then some implications for teaching and learning outlined.

Summary of the Results

Details of both New Zealand and overseas quantitative studies in decimal numbers were given in Chapter Three. In particular, the 1982 IEA survey in New Zealand revealed that Third Formers had trouble estimating and approximating, as well as performing computations, with decimal numbers. Other research that has combined quantitative and qualitative methodologies (Bell *et al*, 1981; Brown, 1981a; Hiebert and Wearne, 1983, 1984) as well as qualitative research (Erlwanger, 1973, 1975) has reported concerns about the poor performance of students when working with decimal numbers.

Such research motivated the present study in the New Zealand setting. This research into how difficulties with decimal numbers developed, took its theoretical orientation from constructivist psychology (Kelly, 1955; Piaget, 1929; Wittrock, 1974a, 1974b, 1980), the generative learning model (Osborne and Wittrock, 1983, 1985) and related work (Claxton, In press a, in press b). Three ideas provide overlap and integration between these theories: (i) that existing ideas influence the stimuli attended to; (ii) that learners actively construct ideas by generating links between the input selected and attended to and parts of memory store; and (iii) that the mini-theories which learners construct vary in their stability, incidence of conflict, overlap, and correctness.

In the context of mathematics, the concept of mini-theories (Claxton, In press a, in press b) refers to the ideas that learners construct in order to make sense of mathematical situations. Mini-theories may apply to general understandings and explanations of principles (the 'executive-procedure' level) and/or to specific skills (the 'mini-procedures' level). An inappropriate mini-theory that persists over time may be termed a 'bug' (Brown and Van Lehn, 1982; Van Lehn, 1982, 1983).

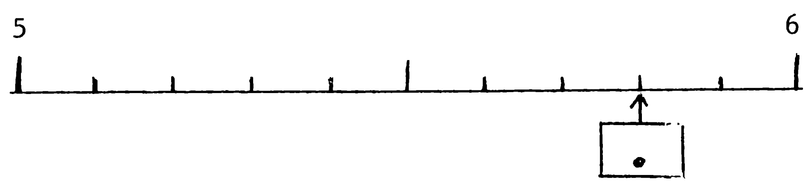
Main Findings from the Initial Survey

The initial survey with 11-14 year old students (n = 102) revealed the following strengths and weaknesses in working with decimal numbers.

Strengths

Items containing only whole numbers were frequently correctly answered. Few strengths were revealed when working with decimal numbers except for the items that follow.

1. Choosing an algorithm that would solve a simple word problem where one number was a whole number. e.g. A table is 92.3 centimetres long. How long would six tables this length be?
2. Assigning a decimal number to a designated point on an appropriately partitioned number line. e.g. This number is:

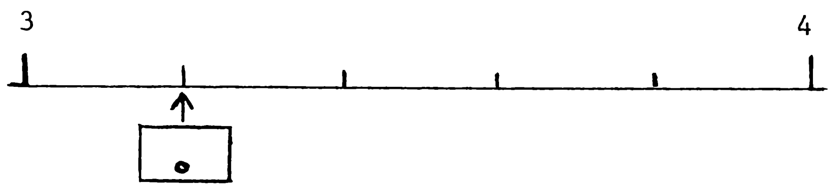


3. Adding a decimal fraction to a whole number e.g. 43.00 + .01 = ...

Weaknesses

1. Naming the place value of decimal number digits e.g. Naming the value of 2 in 0.412.
2. Assigning a decimal number to a designated point on a number line where the responses required a greater understanding of decimals

e.g. This number is:



3. Understanding the size of decimal numbers e.g. Identifying the larger of 0.75 and 0.8, and giving a mathematically-correct reason for the choice.

4. Writing decimal numbers from given verbal stimuli.

5. Writing decimal numbers between two given numbers e.g. Writing a decimal between 0.41 and 0.42.

6. Operations with decimal numbers, or with numbers that give a decimal as the answer e.g. Dividing 16 by 20.

7. Estimation and approximation involving decimal numbers e.g. Identifying the number that is nearest in size to 0.39 x 7.1.

8. Choosing an algorithm that would solve a word problem where the operation with decimal numbers was multiplication or division.

These results compare closely with those obtained by Brown (1981a). Generally, this sample of New Zealand students showed strengths and weaknesses in similar areas to their British peers (Brown, 1981a).

Characteristics and Developmental Trends of the Interview Groups

For the most part, performance patterns from the high, average, and below-average competency groups reflected the major findings from the initial survey. However, in-depth interviewing of individual students four times over a two-year period gave better insight into how these students arrived at their understandings and attitudes, as well as into any changes.

The High Competency Group (n = 5) showed strength in estimation and approximation skills, and quickly generated links with relevant aspects from memory store. They were confident and held positive attitudes towards mathematics and the topic decimal numbers. They could manipulate numbers easily, chose the appropriate strategies for resolving an impasse, selected fruitful mini-theories for suitable contexts, and were able to perceive applications for algorithms in every-day situations. Some of the group felt that teachers of mathematics should explain more clearly, take more time, and make sure students understood material before going on to new topics. Most of the group felt that their peers held negative opinions towards mathematics: *"They growl, complain.... How boring it is"* (Andrea).

Trish, the reported case study from the High Competency Group (see Chapter Six), possessed many of the above traits. She also accepted responsibility for her own learning, and she easily subsumed appropriate constructions into memory store. She indicated the importance of the teacher in mathematics: *"....maths, you just can't go to a book and copy it out and sort of know what you're doing."*

The Average Competency Group (n = 14) displayed much variability in progress. Over the two-year period four students in the group made good progress, but for most students progress was slow. Particular difficulties were: (i) division with a divisor larger than the dividend; (ii) writing decimal numbers, particularly seven hundredths and fifteen thousandths; and (iii) problem-solving with decimal numbers.

Problems arose when students generated links with inappropriate ideas about whole numbers. Conflicting mini-theories were held simultaneously by many learners, and these errors were often at the 'executive-procedure' level (Davis, 1979) even though the students knew such 'mini-procedure' skills as their times tables, being able to add correctly, and divide correctly (except if the divisor were larger than the dividend). Decimals as a topic was generally disliked by this group. They considered teachers should take time to explain more clearly, spend longer on syllabus topics, and provide individual tuition when needed. Most students felt that their peers disliked mathematics.

Stu, the case study from the Average Competency Group (see Chapter Seven) progressed gradually over two years. Some 'bugs' were eliminated from his mathematical behaviour, while others remained. He possessed conflicting mini-theories, and the major source of his errors came from inappropriate links with existing ideas about whole numbers and operations on whole numbers. Stu made errors at both the 'mini-procedures' and 'executive-procedures' levels, and vacillated between accepting responsibility for his own learning and uncritically accepting mathematical rules and conventions imposed by the teacher. When giving advice to teachers, Stu suggested that they should explain concepts more clearly, use simpler language, assist individual learners, vary lesson speed, and refrain from assigning textbook pages: *"Well, they just make* *us copy stuff out of the book,"* he complained.

The Low Competency Group (n = 9) had great difficulty in all topics and most students made no progress in understanding. Errors came from inappropriate links being generated with existing ideas, or simply from gaps in their knowledge. Some errors involved procedures very different from the correct mathematical idea. For example, most of the group thought that a number containing a decimal point was larger than one without (e.g. 0.80 and 80). On the whole these learners held little commitment to their mini-theories and took little responsibility for their learning. They had lucid advice for teachers: (i) explain ideas more clearly; (ii) proceed more slowly; (iii) provide individual tuition where needed; (iv) teach a simpler and more basic curriculum; and (v) use teaching methods other than textbook-based lessons. The group members had mixed feelings towards mathematics. It was interesting, however, that they considered that their peers disliked the subject. Generally the group revealed a lack of confidence and a fatalistic attitude towards their present and future performances in mathematics and decimal numbers.

Ronald, the reported case study from the Low Competency Group, possessed many of the above traits. Basic knowledge and understandings that he had failed to master during primary schooling were not learned at high school. He could see little application for decimal numbers to the world outside school. Ronald held strong views about the unpleasantness, difficulty, and irrelevance of much of what he was taught. During the final interview he admitted, *"Everytime I think of maths it makes me feel yuk...'cause I don't understand it."*

Discussion and Conclusions

1. The Use of Existing Knowledge in the Construction of Meaning

Students of all abilities generated links with their existing ideas when asked to construct answers to problems and to explain them. What distinguished the Low Competency and High Competency students was that inappropriate and appropriate links were generated respectively. Low Competency Group members tended to focus on perceptual similarities between numbers (e.g. 0.8 and .08), phonetic distractors (e.g. three tenths was written as 3.10 (sic)), and 'rules' to which they could imagine no exceptions (e.g. multiplication always makes larger; division always reduces). High Competency Group members generated links with fruitful and existing ideas about fractional and whole numbers. Although Low Competency students also generated links with existing ideas, their new mini-theories were as incorrect at the end of two years of interviewing as they were at the beginning.

In some cases the meaning intended by the researcher (through the Stimulus Cards) was not that constructed by the students (e.g. Stimulus Cards K-N, Series One). In turn, interviewees constructed answers based on these unintended interpretations (c f. Bell, 1984).

Students in the High Competency Group tended to accept responsibility for their learning. Their replies suggested that their learning was a result of active internal processes. For example, interview sequences with Trish (case study from the High Competency Group) indicated that she actively attempted to make sense of what the teacher said, of the textbook passages, of advice from her father (a teacher of mathematics) and her peers.

By contrast, students in the Low Competency Group indicated that they were not attempting to come to grips with the material presented, and that they lacked motivation. These learners relied to a greater extent on direct input from 'outsiders': their teachers, their friends, and passages from the mathematics textbook. When asked for the reasons for their responses, they quoted these outside sources. They admitted to frequent cheating, and several gaps existed in their knowledge: *"I just don't know points and that"* (Wally, Interview I); *"...I don't understand it"* (Janette, Interview IV); *"I wouldn't have a clue"* (Jude, Interview I).

All students could readily generate links to existing ideas, and construct mini-theories. Often, however, the existing ideas were of no help in the context of the particular problem, a point mentioned by Stu, the case study in the Average Competency Group.

Frequently unusual responses were constructed to problems the students found difficult. For example, some students solved 15 ÷ 100 and 15 ÷ 20 by using negative integers. This unintended curriculum outcome might surprise some teachers of mathematics, but those who have focussed on what students can do might consider this response less surprising (Carpenter and Moser, 1979; Carr, 1984; Donaldson *et al*, 1983; Hughes, 1983; Hughes and Grieve, 1983; Krutetski, 1976; Osborne and Wittrock, 1983, 1985).

The Linkages between 'In School' and 'Out-of-School' mathematics have received attention both overseas (Carpenter *et al*, 1984; Cockcroft, 1982) and in New Zealand (Department of Education, 1985). Suggestions have been made regarding the drawing together of school and out-of-school mathematics. The present study demonstrated how students hold mini-theories based on experiences at school and in the wider world, and how learners may make little linkage between the two domains (c f. Claxton's gut science/lay science/school science distinctions in Claxton, In press a, in press b). For example: (i) decimal currency in the outside was not seen as applicable to decimal numbers in the schooling context, and (ii) problems presented via stimulus cards in the school setting were not linked to real-life problems in the wider world. These findings support the view that much learning might be situation specific. Mini-theories appear to be developed for particular purposes and behavioural settings.

In addition, however, several students possessed conflicting mini-theories concerning decimal numbers. For example, Stu (case study from the Average Competency Group) saw 0.7 as less than 1 when writing decimal numbers, but saw 0.4 as equal to 4 when comparing the two numbers. These conflicting mini-theories tended to remain undiscovered and thus unresolved by the students concerned, a state Claxton (In press a) has described as 'laminated'. One student who did recognize the conflict, however, was Trish. During Interview IV she indicated that she was ready to write seventeen tenths in the correct mathematical form (the researcher had alerted her to her conflicting mini-theories).

For many students, conflicting mini-theories might remain a characteristic of their mathematical behaviour. Just as learners in science tolerate conflicting theories, ("I may be told that Newton's Laws are 'righter' than my own gut theories - but I know that my own gut theories work better." - Claxton: In press a), so, too, may different constructions be held simultaneously in mathematics.

The present research also investigated the stability of mini-theories over time. Although all students possessed both stable and unstable mini-theories, some mini-theories were held for the duration of the study. Ronald (case study in the Low Competency Group) omitted the decimal point in the product when multiplying with decimal numbers, and considered division impossible if the divisor were larger than the dividend. Other members of the Low Competency Group also held many incorrect mini-theories for the duration of the project.

Although less prevalent than with the Low Competency Group, members of the Average and High Competency Groups also constructed and held incorrect mini-theories for lengthy periods. For example: (i) Average Competency Group members used negative integers to solve 15 ÷ 20 over one and two-year periods; and (ii) Trish (case study from the High Competency Group) had constructed .17 in naming 17 tenths, and held this view for two years.

The more able students had a greater commitment to their particular mini-theories, unstable mini-theories being found mostly in the Low and Average Competency Groups. These latter students substituted new but equally incorrect mini-theories for existing incorrect mini-theories. To illustrate, in attempting to solve word problems containing decimal numbers (Stimulus Cards K-N, Series One), the Average Competency Group moved from one explanation to another. Again, Low Competency Group members had problems estimating and approximating, and constructed mini-theories to which they held little commitment. In contrast, members of the High Competency Group generally did not move rapidly from one construction to another.

For convenience, students in the present study were assigned to reasonably homogeneous groups (see Chapters Six, Seven, Eight). Within each group individual students progressed at considerably different rates. This variation was particularly evident within the Average Competency Group where some students made good progress, most modest gains, and two students made minimal or no progress. This variation might be considered surprising if an absorption model of learning were adopted. From a constructivist viewpoint, however, the variation is to be expected: individual learners construct their own meanings from the stimuli they encounter. Hence learners of similar ability will tend to progress and learn at different rates as they attend to different stimuli. The results of the present study support this view.

2. Affective Factors in Mathematics Learning

Research in this area was discussed in Chapter Three where the positive correlation between achievement in, and attitude towards, mathematics was noted. The problem for researchers has been to establish a causal link between the two, something that correlational studies have only suggested.

The present study indicated again that high achievers generally held positive attitudes towards mathematics. The Average Competency Group held neutral-to-negative opinions towards the subject, while the Low Competency Group expressed variable opinions towards mathematics.

Several students in the present research had established a cycle of negative opinions towards, and failure in, mathematics. The two variables seemed inextricably linked, but establishing a causal link between the two was not possible within the research design of this study. In spite of this, the importance and relevance of affectivity in learning mathematics was certainly suggested by the data collected.

Knight (1982:151) has commented: "The tragedy is that many students, not understanding their failure, lose self-esteem and a vicious interaction of affective and cognitive reactions create a state in which any constructive approach to mathematics learning seems impossible."

The consequences of continuing failure in learning about decimal numbers were exemplified in Chapter Eight and summarized in previous sections of the present chapter: Lack of progress; absence of understanding; a tendency to subscribe to wrong mini-theories; and movement from incorrect-to-incorrect mini-theories.

Failing students, in particular, had clear suggestions for teachers: (i) spend longer on each topic; (ii) provide tuition for individual students; (iii) explain concepts clearly; and (iv) teach a simpler and more basic curriculum. In their view, teachers were not adopting appropriate teaching methods, particularly with regard to textbook usage. Thus, many of the low achievers felt poor teaching was at least partly to blame for their failure.

Characteristically, many of the Low Competency Group generated links to unhelpful ideas in memory. This failure to make links to helpful and appropriate existing ideas was a major weakness with these students. Because the present research followed students who transversed the primary/high school interface, data was gathered on whether students would learn concepts about decimal numbers at high school which they had failed to learn at primary school. On the whole, the results suggested that topics not learned at primary school were not grasped at high school. In short, earlier failure was not remediated. This finding supports that of Hart (1981).

Most students, nevertheless, perceived mathematics as a valuable subject. Wally's (Low Competency Group) view was representative: *"You need to do maths and it's valuable for a job and that...it's* *boring!"*

3. The Generative Learning Model

This model of human learning provided a major theoretical underpinning for the present study. The fundamental notion that students tend to generate perceptions and meanings consistent with their prior learning was suggested by the responses that students gave in the interview sequences.

The constructivist tradition, to which the generative learning model belongs, takes the view that people strive to learn so that they can act on, predict, describe and explain their world. From a teaching perspective the model emphasises the provision of classroom experiences that students feel are genuinely helping their learning. This is said to be preferable to a 'carrot and stick' approach to teaching and learning (Pope and Keen, 1981). Osborne and Wittrock (1985) claim that students not only need to be successful in their own terms concerning their learning, but need to perceive this success as largely due to their own efforts. In the present study, high achievers appeared self-motivated in that their success, they believed, came as a result of their own actions. By contrast, low achievers felt that their failure was because of poor teaching, wrong advice from their peers, and incomprehensible textbooks.

As a result of the present study, a number of concerns emerged about the generative learning model. First, although the generative learning model recognizes the importance of motivation in learning, it does not really consider the possible pervasive effects of the affective component on learning. In particular, the model does not address the effect that attitude towards learning may have on the generation of links between incoming stimuli with existing knowledge. In short, the generative model is strongly cognitive in its emphasis and may be criticized for its inattention to attitudinal factors in learning (c f. Brush, 1980; Buxton, 1981; Fennemma and Sherman, 1978; Knight, 1982; Tobias, 1978).

In the present study Average and Low Competency Group members, in particular, held negative opinions of mathematics for the duration of the study. It was these students who: (i) uncritically generated links with inappropriate ideas; (ii) held little commitment to their ideas; (iii) had large gaps in their knowledge about decimal numbers; (iv) could see little or no linkage with decimal number applications in and out-of-schools; and (v) held negative opinions of the topic decimals and the subject mathematics.

Second, the generative learning model presents a rather sequential and logical image of learning; further, the suggestion is made that many key learnings can readily be applied to a number of different situations: "Whenever existing ideas have been found to apply successfully to a wide range of situations...it is likely that the pupil will tend not to restructure these ideas..." (Osborne and Wittrock, 1985:74). Data from the present study, however, indicate that learning is often of a fragmented and context-specific nature. Links with existing knowledge may or may not be generated often on the basis of what might be considered unexpected and inappropriate detail. Thus, students focussed on: (i) phonetic cues to write $\frac{3}{10}$ as 3.10; and (ii) negative integers to solve 15 ÷ 100. Learning may be more compartmentalized than the generative learning model suggests.

Third, Postulate (vi) of the generative learning model (Osborne and Wittrock, 1985) states that "the learner may test the constructed meaning against other aspects of memory store and against meanings constructed as a result of other sensory input" (Osborne and Wittrock, 1985:65). As a corollary of the situation specific-nature of learning, information from the present survey suggests that in many situations learners do <u>not</u> attempt to test their constructed meanings against other parts from memory. Rather, learners appear to make a link to a <u>single</u> aspect from memory, without subsequently testing this against other aspects. Thus, students in the present study consistently held ideas such as solving 15 ÷ 100 with negative integers, an idea that might have been rejected had it been tested against other aspects from memory about division and the division process (e.g. division as 'sharing'). Testing constructed meanings against existing ideas may not be as common as the generative learning model implies.

4. <u>Claxton's (In press a, in press b) Mini-Theories</u>

Wide use of the notion of mini-theores was made in the present research in the sense that students' ideas were labelled as mini-theories because they applied to a situation, predicted what might happen next, indicated actions to the learner, and were described and explained through articulation (by the learner). The existence of conflicting mini-theories was noted, and the stability of mini-theories over time described.

Overall, Claxton's theory of mini-theories to describe students' ideas in mathematics appears a useful and powerful strategy. That Claxton's proposals transfer so readily from science to mathematics, suggests his ideas have generality.

However, the 'theory of theories' (mini-theories) needs to operate at a deeper level than the merely descriptive. Where do mini-theories come from? Why do students hold on to them? These are concerns that might assist in the exploration of deeper structures that learners possess.

In some cases, data from the present study suggest that mini-theories are built up gradually, over many years. In other cases, they appear to be constructed over a very short period of time (perhaps a single mathematics lesson). Thus, in mathematics, mini-theories about the meaning of the decimal point may be constructed over a long period of time. In contrast, mini-theories about how to solve 15 ÷ 100 may come from a single mathematics lesson (c f. responses from the Average Competency Group). Therefore, mini-theories can come from ideas that have resided in memory store for some time, or from ideas that have been newly acquired.

In terms of deep structures, Brown and Van Lehn (1982) have suggested that 'bugs' may be acquired when impoverished repair rules are applied to an impasse. Claxton (In press a) has suggested that mini-theories come partly from direct experience and partly from intuition. Both writers view learning as an internal, constructivist process. The present study supports this view, and points to the web of knowledge and attitudes and beliefs that interact during the formation of a mini-theory.

The distinction is made in the present study between students' 'executive-procedures' and 'mini-procedures'. It is suggested that a large portion of students' difficulties is at the 'executive-procedures' level. That is, students could often compute successfully ('mini-procedure' skills) but failed to understand or evaluate appropriately ('executive-procedure' skills). Thus, in terms of Claxton's mini-theories, there seems to exist a hierarchy of mini-theories that each student possesses. Claxton (In press a) fails to make this distinction, but the present research suggests levels of mini-theories rather than the undifferentiated collection Claxton describes.

The stability of mini-theories is referred to earlier in this chapter. The question remains as to why students hold on to their theories. Claxton (In press a) has claimed that a mini-theory will last as long as it is successful. The present study found that some students held incorrect mini-theories for up to two years, yet these students were not reinforced for possessing these mini-theories in terms of success in the school context. This raises questions about the theory which states that mini-theories last if successful. The present research suggests that something other than being 'successful' is necessary for the retention of a mini-theory. Festinger's (1957) work is brought to mind whereby individuals will cling to ideas and rationalize their behaviour in order to justify some course of action. Again, other contexts need to be considered (c f. White, 1985), and in particular the affective domain may be relevant in explaining why students held their mini-theories. The present study suggests that mini-theories last if: (i) the student can articulate a description and explanation of the mini-theory; (ii) the student is comfortable with this explanation. Success *per se* may not be a necessary condition for the retention of a mini-theory.

Implications of the Study

1. Teaching and Learning

The generative learning model which underpinned much of this study, was influenced by the work of Jean Piaget. Although Piaget may be considered a constructivist, applications of his theory to mathematics teaching have focussed on 'ages and stages' for cognitive development rather than on his constructivist thesis (Bell, 1980; Copeland, 1979). Thus textbooks for teachers have tended to identify children's intellectual stages, describe learners' characteristics at each stage, and suggest appropriate curriculum content. This has persisted at the expense of considering constructivist ideas in the learning of mathematics.

The structured and axiomatic nature of mathematics has tended to discourage alternative methods of teaching the subject. Most recent innovations have centred around alternative textbook presentations (e.g. programmed texts), computer-based and computer-assisted learning (Papert, 1980), diagnostic teaching models (Ashlock, 1976; Bell *et al*, 1981; Reisman, 1972) and the appropriate use of teaching aids.

Although there has been renewed recent interest in children's errors or 'bugs', the constructivist model of learning has been subordinate to other explanations of learning in mathematics that are more akin to an absorption paradigm (see Romberg and Carpenter, In press). Generally there has been a failure to acknowledge the individual ideas that students construct as a result of instruction in mathematics.

Students experiencing difficulties in the present study have alluded to this orientation of teachers, when they pleaded for better explanations from teachers, more individual attention, and less use of textbooks. In other words, these learners were failing to 'absorb' the knowledge presented to them, and required assistance to construct appropriate meaning from the mathematics lessons. Teachers seemed largely unaware of this, if the students' perceptions are to be believed.

The present study suggests that the individual interview is a potent instrument for eliciting students' ideas. (Knight (1982) has argued that individual interviews should be part of the normal classroom evaluative programme. Teachers operating with an absorption model of learning may benefit, in particular, from interviewing children, and thus becoming more aware of students' mini-theories. Interview formats have been included in some textbooks (Department of Education, 1974b, 1984), but these tend to be structured in terms of questions that teachers should ask, and student responses that teachers should expect. The present study indicates that flexible interviews using some key questions combined with follow-up probing might provide richer and more useful data for teachers.

Another teaching and learning implication of the present study centres around the need for students to take responsibility for their learning. Baird (1984) notes the importance of this, and considers that there has been a lack of explicit teaching of pupils about learning and taking responsibility for their own learning. His research demonstrates that systematic attempts could produce some change in this area. Bell (1984) comments that learning may be enhanced if learners are aware that they need to attend to sensory data, and actively construct meaning. The present research shows how high achievers accept responsibility for their own learning, but it is the low achievers, in particular, who are failing in this orientation. Of course, it could be argued that students failing in a subject (or topic) <u>have</u> accepted responsibility for their learning. Perhaps they have decided, within themselves, to make little or no effort, to attend to alternative stimuli, to allow their strongly aversive reactions to dominate, or to engage in social disruption. This last option was, at least, mentioned by some students. The present study suggests, however, that positive gains in learning can best be made if teachers interact with students and explain concepts to them, provide individual help, spend longer on some topics, avoid textbook-based lessons, and generally help students feel that mathematics is for the students themselves, and not just knowledge to be transmitted by the teacher.

2. Curriculum

a) Content. In Chapter One the contradiction between low levels of performance with decimal numbers, and the optimism of curriculum developers who introduced decimal numbers via metric measurement, was noted. Generally, the present research supports the view that a considerable portion of the content in the present offical syllabuses is beyond the understanding of many 11 to 14 year olds, given present methods of teaching and current textbooks. Even relatively simple tasks such as writing decimal numbers had not been mastered by several students in the study. The question remains as to whether low achievement was caused by inappropriate content, poor teaching, negative attitudes, or other factors.

Certainly the sequencing of some topics seems to require attention. To illustrate, the requirement that Form Two students (12 year olds) solve division algorithms such as $0.375 \div 25$ is questionable, particularly in terms of the limited understanding of estimation and approximation that was apparent in the present study's student sample. The usefulness of such an exercise might be questioned, also, in view of the free availability of calculators.

Chapters Six, Seven, and Eight in this study noted that many students could not see the relevance of decimal numbers to the world outside school, a disturbing finding in view of the wide use of metric measurements and decimal currency in our society. Students holding the view that school and out-of-school mathematics are separate, may continue to view school mathematics as boring, impractical, and lacking in applications to real-life situations.

The present study suggests that developing links between existing knowledge about the world and what is taught in the mathematics lesson is a difficult task. Sections in textbooks about applications of mathematics to our world do not appear to be sufficient. This may be because the textbook extracts are not seen as relevant to the topic under study, or because they are, necessarily, providing vicarious experiences instead of first-hand opportunities. As well, teachers may: (i) assume that students can make the links themselves, or (ii) fail to recognize the need to draw together school and out-of-school mathematics. In a wider context, some writers have suggested that a more radical change to our schooling system might provide the answers (e.g. White, 1984).

b) Textbooks. Mathematics teaching in New Zealand schools has traditionally relied on the use of comprehensive textbooks that cover many of the syllabus objectives for a particular class level. These have been overseas (e.g. Duncan *et al*, 1978, 1980) as well as New Zealand publications (e.g. Department of Education, 1974b, 1984).

Reliance on textbook-based teaching sustains the commonly-held view that teaching mathematics involves the transmission of knowledge, and learning the passive reception of that knowledge. The central ethic of this model is one of teacher control and authority rather than learners' control of, and responsibility for, their own learning.

In the present study, students' comments on the use of the textbook in mathematics lessons varied, but Ronald (case study in the Low Competency Group) typifies the view of a number of students. He viewed mathematics teaching as, *"Straight out of the book. Just doing the good old textbook style and pen and paper. 'Do page blah, blah, blah...'"* (Ronald, Interview IV). Ronald, a slow learner, might be assigned the remedial examples below (a strategy suggested in the mathematics textbook currently in use in New Zealand Form One classes).

> "Remedial. Those students who have difficulty with these pages may be asked to complete a pattern of multiplications in which the decimal point has different locations.
>
> | | | |
> |---|---|---|
> | 26 x 483 | 2.6 x 483 | .26 x 483 |
> | 26 x 48.3 | 2.6 x 48.3 | .26 x 48.3 |
> | 26 x 4.83 | 2.6 x 4.83 | .26 x 4.83" |
>
> (Duncan *et al*, 1980:335 Teachers' Edition)

In view of Ronald's opinion of textbook examples, exercises such as the above are questionable. All students in the present longitudinal study spoke of the need for a different remedial strategy involving teacher explanations and individual tuition in mathematics lessons.

c) Assessment. The desirability of individual interviews for evaluation is mentioned earlier in this chapter. Quantitative and objective-referenced tests (e.g. IEA, NAEP, PAT) record only the number of correct responses to items, rather than informing teachers of how students work through problems in mathematics. Well-conducted interviews with students, on the other hand, can provide data on the processes students use to work algorithms and solve problems. Some textbooks in primary school mathematics (Department of Education, 1974b, 1984) provide interview formats, but these are structured in terms of questions that teachers should ask and student responses that teachers should expect. As noted earlier, more open and flexible interviewing might provide richer data for teachers.

d) In-Service and Pre-Service Teacher Education. The results of of this research suggest that there is an urgent need for teachers to become more aware of how children learn mathematics. In particular, teachers need to become aware of how children construct their ideas, and how these ideas may be very different from those presented by the teacher. For teachers who have operated with an absorption model of learning for many years, this will require a major re-orientation.

The present study points to the importance of affective factors in learning mathematics. Again, teachers need to develop strategies that will improve students' attitudes towards mathematics. 'Advice to teachers' provided by students in this study contains suggestions that might foster more positive attitudes towards mathematics.

As well, teachers need to be aware of the wider social contexts and characteristics of schools and classrooms, and attempt to elicit students' views of these contexts (White, 1985). To illustrate, in the present study Andrea (High Competency Group) viewed slower learners with some disdain: *"...I don't know about the 'dumber' classes,"* she commented. Again social disruption appeared to hinder learning in some cases *("I usually do it* (misbehave) *too...*

flinging paper around the room" Stella, Low Competency Group, Interview IV). In other words there are social and people features to classrooms.

The present research also suggests that teachers need to help students learn how to learn. Failing students are most at risk in this area. The strategies adopted by Baird (1984) may be helpful.

Indeed, a comprehensive 'package' incorporating constructivist notions of learning and sensitising teachers to students' constructions of meaning needs development. Ideally this would lead to teachers developing their own strategies that incorporate the above.

Further Research

1. Mathematics and the Generative Learning Model. The present study focussed on students' conceptions of decimal numbers and attempted to show that students' existing constructions influenced their learning. Given the broad nature of mathematics, research into other areas of the subject is required so that the generality of the theory might be assessed. If the generative learning model is to gain acceptance as a valid representation of human learning, then wider application will be needed to subjects other than science and mathematics.

2. Affective Factors and the Generative Learning Model. Although the influence of motivation on learning has been discussed with regard to the generative learning model (Osborne and Wittrock, 1983, 1985), the influence and effect of affective variables in the broadest sense have not been incorporated in the model. The present study suggests that attitude plays an important role in learning about decimal numbers and mathematics.

In science, Bell (1984) has discussed affective variables influencing the acceptance or rejection of concepts. Further research is needed in expanding the generative learning model to incorporate the affective domain by re-interpreting data that has formed the basis of the theory, and by conducting fresh empirical research.

3. Classroom-based Research. The qualitative aspects of the present study were conducted with individual students who were interviewed away from the classroom setting. Few genuinely classroom-based

studies have been carried out in mathematics education, and these might provide a better picture of how learning takes place c.f. suggestions by Romberg and Carpenter (In press). Research methodologies exist (e.g. ethnographic studies involving participant observation) that may provide data to help bridge the learning-teaching gap. Research questions might come from how constructivistic teaching methods can be incorporated in the mathematics classroom.

4. Developing Teaching and Learning Strategies to Link School and Out-of-School Mathematics. Major reports from overseas countries have noted the lack of articulation between mathematics in schools and in the wider world (e.g. Cockcroft, 1982). The present study revealed the inability of students to relate their knowledge about decimal numbers to situations in the world outside school. As discussed earlier in this chapter, facilitating this linkage will be no easy task. The problem is not new, but will require: (i) alternative teaching methods incorporating more than an occasional mention of mathematical applications in society; (ii) a view of learners that incorporates the idea that individuals construct their own, and differing, views of the world; and (iii) action-research based in the classroom setting.

Concluding Remarks

This study was a response to the investigator's observation that students were having difficulty with a mathematical topic (i.e. decimal numbers) that curriculum writers had assumed could be easily mastered. The investigator wanted to examine the appropriateness of a model of human learning that gave primacy to a person's existing ideas, and assumed that learning was an active, internal, constructivist process. As well, it was decided to describe students' conceptualizations through the use of Claxton's mini-theories (In press a, in press b), a notion that is embedded in the constructivistic tradition of learning theory.

This study with 11 to 14 year olds showed that students used their existing ideas to a great extent when constructing meaning. These constructed mini-theories were often situation-specific, and could be stable or unstable over time. Generally the less able learners moved from incorrect mini-theories to new equally incorrect mini-theories, more so than the more competent learners. Low Competency students were also less likely to accept responsibility for their learning.

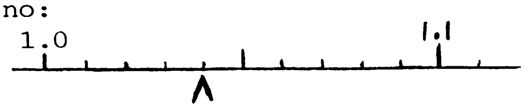
The introduction of constructivist psychology into mathematics education programmes at the pre-service and in-service teacher training levels would seem to be a priority. This might help teachers: (i) become more sensitive and sympathetic towards the students' views of the world; and (ii) start to develop strategies that might break the cycle of continual failure that is the norm for many learners in mathematics.

APPENDIX A : INTERNATIONAL EDUCATIONAL ACHIEVEMENT SURVEY (1981)

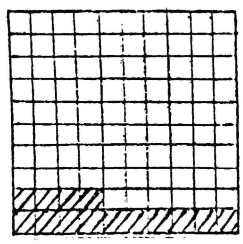
Preliminary Item Analyses of New Zealand Population A (13 year olds) Responses to Items Containing Decimal Numbers, (from Department of Education (1982).

(i) Core Test (n = 5177)

Item no:	CORRECT RESPONSE % CORRECT	COMMENT
1) 2 metres + 3 millimetres is equal to A 2.0003 metres B 2.003 metres C 2.03 metres D 2.3 metres E 5 metres	B 32	25% chose C
15) The value of 0.2131 x 0.2958 is approximately A 0.6 B 0.06 C 0.006 D 0.0006 E 0.00006	C 18	43% chose E
20) In a discus-throwing competition, the winning throw was 61.60 metres. The second place throw was 59.72 metres. How much longer was the winning throw than the second place throw? A 1.12 metres B 1.88 metres C 1.92 metres D 2.12 metres E 121.32 metres	B 62	
26) 0.40 x 6.38 is equal to A 0.2552 B 2.452 C 2.552 D 24.52 E 25.52	C 39	

APPENDIX A (contd)	Correct response % correct	Comment
32) $7\frac{3}{20}$ is equal to A 7.03 B 7.15 C 7.23 D 7.3 E 7.6	B 24	51% chose either A or C
(ii) Test A (n = 1297) Item no: 18)  The position on the scale indicated by the arrow is A 1.004 B 1.04 C 1.08 D 1.4 E 1.8	B 62	26% chose D (misread scale)
21) $0.004 \overline{)21.56}$ In the division above, the correct answer is A 0.614 B 6.14 C 61.4 D 614 E 6140	E 25	
(iii) Test B (n = 1319) Item no: 2) Which of the following is thirty-seven thousandths? A 37000 B 37 C 0.37 D 0.037 E 0.0037	D 21	46% chose A (misread thousandths) 30% chose E

APPENDIX A (contd)

	Correct response % correct	Comment
7) Rosemarie walked from Riverview to Bridgeport, which are 3.1 kilometres apart. During her walk she lost her watch, went back 1.7 kilometres to find it, and then continued in the original direction until she reached Bridgeport. How many kilometres had Rosemarie walked altogether when she arrived at Bridgeport? A 1.4 B 4.8 C 6.5 D 8.2 E None of these	C 36	39% chose B (3.1 + 1.7 = 4.8)
27) The large square has area 1 square unit. The area of the shaded part is  A 14 square units B 1.4 square units C 0.14 square units D 0.014 square units E 0.0014 square units	C 39	38% chose A (counted small squares)
31) The length of a box was measured and found to be 9 centimetres TO THE NEAREST CENTIMETRE. Which of these could have been the length of the box measured more accurately? A 10 cm B 9.9 cm C 9.62 cm D 9.6 cm E 8.6 cm	E 46	19% chose B

APPENDIX A (contd)	Correct Response % correct	Comment
(iv) Test C (n = 1303) Item no. 21) 323×10^5 is equal to A 0.0000323 B 3.23000 C 32 300 D 323 000 E 32 300 000	D 28	31% chose E (5 zeros)
30) The arithmetic mean (average) of: 1.50, 2.40, 3.75 is equal to A 2.40 B 2.55 C 3.75 D 7.65 E None of these	B 36	
(v) Test D (n = 1293) 3) A runner ran 3000 metres in exactly 8 minutes. What was his average speed in metres per second? A 3.75 B 6.25 C 16.0 D 37.5 E 62.5	B 22	39% chose A ($3000 \div 800 = 3.75$)
33) $\boxed{847.36}$ In the number in the box the digit 6 represents A $6 \times \frac{1}{100}$ B $6 \times \frac{1}{10}$ C 6×1 D 6×10 E 6×100	A 51	20% chose C

APPENDIX B : BELL AND OSBORNE'S (1981) SUGGESTIONS FOR INTERVIEWING CHILDREN

CHECKLIST FOR THE INTERVIEWER

1.	Try to clearly establish how and what the pupil thinks. Emphasize it is the pupil's ideas that are important and are being explored.	Do not give any indication to the pupil of your meaning(s) for the word or appear to judge the pupil's response in terms of your meaning(s).
2.	Provide a balance between open and closed questions and between simple and penetrating questions. In so doing, maintain and develop pupil confidence.	Do not ask leading questions. Do not ask the type of question where it is easy for the pupil to simply agree with whatever you say.
3.	Listen carefully to the pupil's responses and follow up points which are not clear.	Do not rush on, e.g. to the next card, before thinking about the pupil's last response.
4.	Where necessary to gain interviewer thinking thinking time, or for the clarity of the audio-record, repeat the pupil response.	Do not respond with a modified version of the pupil response; repeat exactly what was said.
5.	Give the pupil plenty of time to formulate a reply.	Do not rush but on the other hand do not exacerbate embarrassing silences.
6.	Where pupils express doubt and hesitation encourage them to share their thinking.	Do not allow pupils to think that this is a test situation and there is a right answer required.
7.	Be sensitive to possible misinterpretations of, or misunderstanding about, the initial question. Where appropriate explore this, and then clarify.	Do not make any assumptions about the way the pupil is thinking.
8.	Be sensitive to the unanticipated response response and explore it carefully and with sensitivity.	Do not ignore responses you don't understand. Rather follow them up until you do understand.
9.	Be sensitive to self-contradictory statements by the pupil.	Try not to forget earlier responses in the same interview.
10.	Be supportive of a pupil querying the question you have asked, and in this and other ways, develop an informal atmosphere.	Do not let the interview become an interrogation rather than a friendly chat.
11.	Read the question out loud to pupils.	Do not rely on pupils' reading ability.
12.	Where all efforts to develop pupil confidence fail, abort the interview.	Do not proceed with an interview where the pupil becomes irrevocably withdrawn.
13.	Verbally identify for the audio record, the pupil's name, age and each card as it is introduced into the discussion.	Do not return to earlier cards without verbal identification for the audio-record.
14.	Be sensitive to the possibility that pupils will give an answer simply to fill a silence.	Do not accept an answer without exploring the reasoning behind it.
15.	Appreciate that a card omitted will result in missing data.	Make no assumption about the way a pupil would respond to a particular card.

APPENDIX C : TEST FOR INITIAL SURVEY WITH 11-14 YEAR OLDS (n = 102)

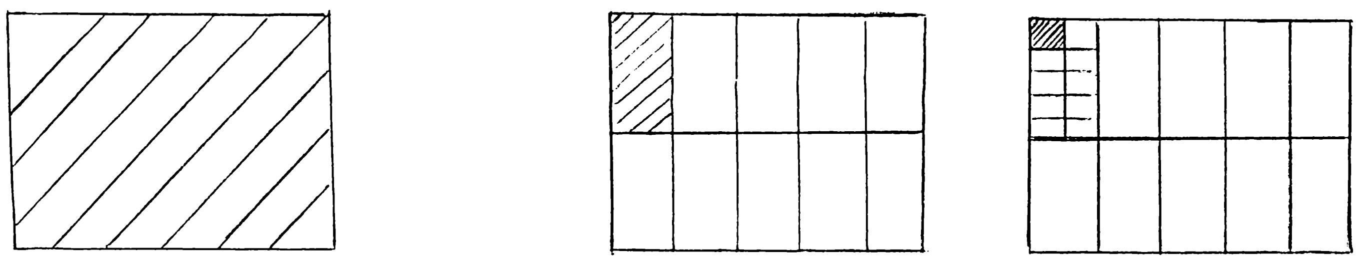
PLACE-VALUE AND DECIMALS

NAME........................ TODAY'S DATE........................

BOY OR GIRL........... CLASS................................

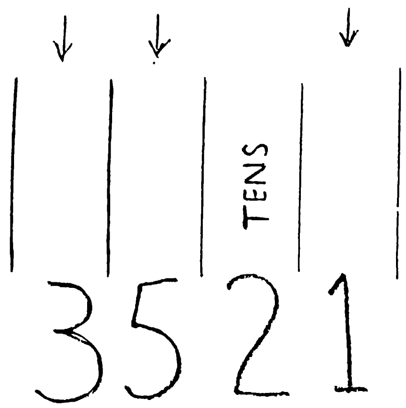
SCHOOL...................... DATE OF BIRTH... ....
day month year

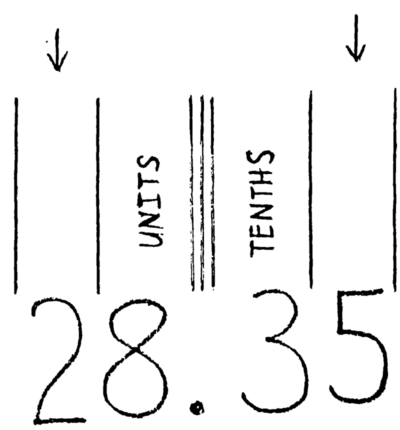
IF THIS IS ONE UNIT (1), then THIS IS ONE TENTH ($\frac{1}{10}$), and THIS IS ONE HUNDREDTH ($\frac{1}{100}$)



Trial Items

Fill in the empty columns:

(1) 

(2) 

Notes

These mean the same:

(a) 30 000 ↔ 30,000

(b) 4.21 ↔ 4·21

NOW TURN OVER

CHECK YOUR ANSWERS TO THE TRIAL ITEMS

THOUSANDS	HUNDREDS	TENS	UNITS (ONES)
3	5	2	1

TENS	UNITS	TENTHS	HUNDREDTHS
2	8.	3	5

NOW DO THESE: | Working

1) 5214 The 2 stands for 2 HUNDREDS

(a) 521 The 2 stands for 2 ....

(b) 521 400 The 2 stands for 2 ....

---

2) 0.126 The 2 stands for 2 HUNDREDTHS

(a) 0.2 The 2 stands for 2 ....

(b) 0.260 The 2 stands for 2 ....

(c) 0.412 The 2 stands for 2 ....

---

3) Add ten

(a) 3597 → ......

(b) 0.15 → ....

Add one hundred

(c) 21 534 → ......

(d) 19 930 → ......

Add one tenth

(e) 4.254 → ......

(f) 2.9 → ......

Take away one hundred

(g) 583 → ......

(h) 30 000 → ......

Add . 1

(i) 26.41 → ......

(j) 3.01 → ......

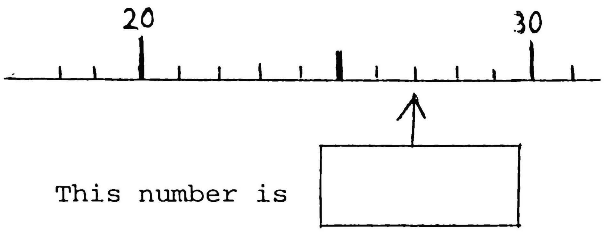
Add .01

(k) 43.00 → ......

(l) 18.69 → ......

Working

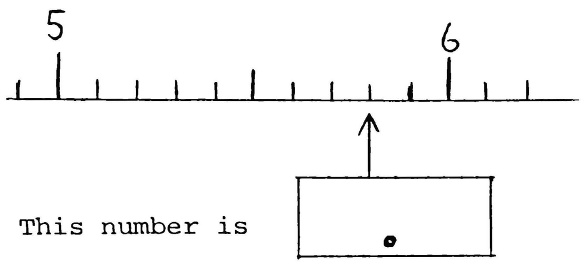
4)
(a)



This number is

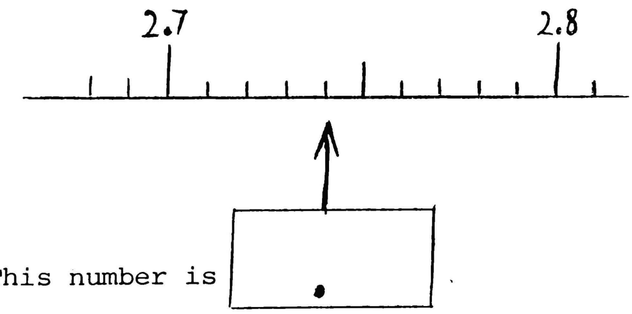
GIVE THE REST OF YOUR ANSWERS AS DECIMALS

(b)



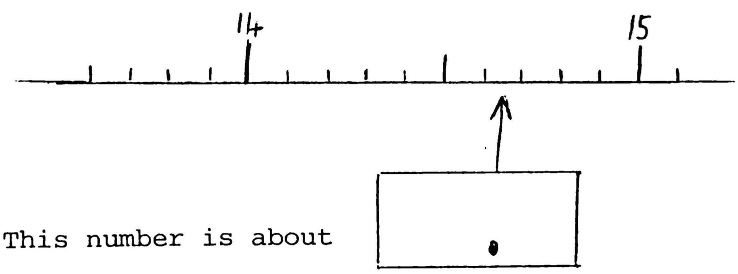
This number is

(c)



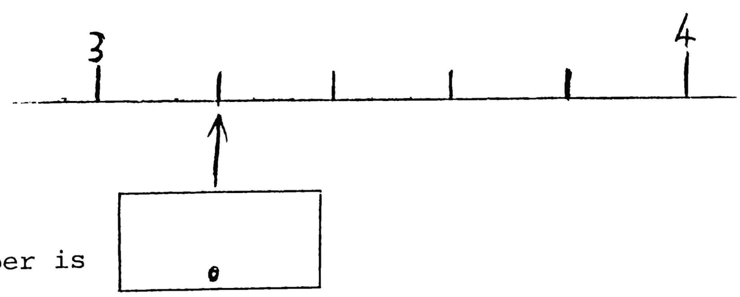
This number is

(d)



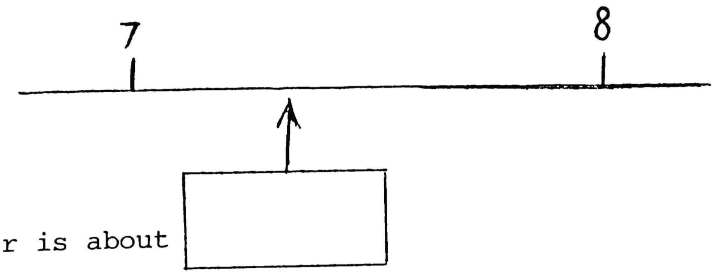
This number is about

(e)



This number is

(f)



This number is about

Working

5) This meter counts the people going into a football stand

0	6	3	9	9

After one more person has gone in, it will read:

---

6) The number that is 2 less than 17 000 is ....

---

7) For the answer to a maths question Dave got 4.9 and Lynn got 4.90

(a) Is there any difference between them? ....

(b) Why? ....

....

....

---

8)

(a) Ring the BIGGER of the two numbers: 0.75 or 0.8

(b) Why is it bigger? ....

....

....

For each pair, ring the BIGGER number:

(c) 20 100 or 20 095

(d) 7.55 or 7.5

(e) 4.06 or 4.5

---

9) Six tenths as a decimal is 0.6

(a) How would you write as decimals: three hundredths ......

(b) eleven thousandths ......

(c) eleven tenths ......

(d) Four tenths is the same as ....hundredths

Working

10) Write down any number between:

(a) 4000 and 5000 ....

(b) 4100 and 4200 ....

(c) 0.4 and 0.5 ....

(d) 0.41 and 0.42 ....

<u>How many</u> different numbers <u>could</u> you write down which lie between 0.41 and 0.42?...................

---

11) (a) <u>Add</u>

$$\begin{array}{r} 263 \\ +\ 97.8 \\ \hline \\ \hline \end{array}$$

(b) $\boxed{26.3 + 97.8}$ = ...

(c) <u>Subtract</u>

$$\begin{array}{r} 2312 \\ -\ 54.7 \\ \hline \\ \hline \end{array}$$

(d) $\boxed{23.12 + 54.7}$ = ...

---

12) $\boxed{\text{Multiply by ten}}$ $\boxed{\text{Multiply by one hundred}}$

WRITE <u>'NO'</u> IF YOU THINK THERE IS <u>NO ANSWER</u>

(a) 4 — ......

(b) 5.13— ......

(c) 317 — ......

(d) 2.3 — ......

$\boxed{\text{Divide by one hundred}}$ $\boxed{\text{Divide by twenty}}$

(e)1600 — ......

(f) 3.7 — ......

(g) 24 — ......

(h) 16 — ......

13) Ring the number NEAREST IN SIZE to:

(a) $\boxed{182}$ → 100 / 92 / 180 / 150 / 200 / 190

(b) $\boxed{2.9}$ → 3 / 30 / 2 / 20 / 0 / 1

(c) $\boxed{0.18}$ → 0.1 / 10 / 0.2 / 20 / 0 / 1 / 2

Ring the number you think is NEAREST IN SIZE to the <u>ANSWER</u>

(do <u>not</u> work out the sum):

(d) $\boxed{2.9 \times 7}$ → .002 / .02 / .2 / 2 / 20 / 200 / 2000

(e) $\boxed{0.29 \times 7.1}$ → .002 / .02 / .02 / 2 / 20 / 200 / 2000

(f) $\boxed{59 \div 190}$ → .003 / .03 / .3 / 3 / 30 / 300 / 3000

14) Ring the calculation you need to do to find the answer:

A. A table is 92.3 centimetres long. How long would 6 tables this length be?

92.3 + 6	6 ÷ 9.23
92.3 ÷ 6	6 - 92.3
92.3 - 6	6 x 92.3

B. My car tank is full after I put in 25.3 litres. The tank holds 34.68 litres. How much petrol was in it to start with?

25.3 + 34.68	34.68 ÷ 25.3
25.3 ÷ 34.68	34.68 - 25.3
25.3 - 34.68	34.68 x 25.3

C. The price of mince is $2.57 for each kilogram. What is the cost of a packet containing 0.58 kg of mince?

2.67 + 0.58	0.58 ÷ 2.67
2.67 ÷ 0.58	0.58 - 2.67
2.67 - 0.58	0.58 x 2.67

D. The cost of 6.44 litres of petrol was $3.58. What would be the price of <u>one</u> litre be?

6.44 + 3.58	3.58 ÷ 6.44
6.44 ÷ 3.58	3.58 - 6.44
6.44 - 3.58	3.58 x 6.44

E. My car can go 11.8 kilometres on each litre of petrol. How many kilometres can I expect to travel on 8.37 litres?

11.8 + 8.37	8.37 ÷ 11.8
11.8 ÷ 8.37	8.37 - 11.8
11.8 - 8.37	8.37 x 11.8

APPENDIX D (PART ONE) INTERVIEW SAMPLE FROM INTERVIEWS I - III

Subject: James Age at Interview: 13 yrs.7 months

Date: 14 October 1983

Interview Number: III

(I = Interviewer S = Subject)

Stimulus Cards used: Series One

I (Introductory comments, setting subject at ease, explaining what will take place)

Stimulus Card A

I. (Reading to S) 186 is nearest in size to: 100, 80, 180, 200, 150, 190?
S. 190
I. Why do you think that?
S. 'Cause six is above five which is ...um...it's above the half way point so it's closer to 190 ... you can round it off to that.

Stimulus Card B

I. (reading to S) 1.2 is nearest in size to: 1, 10, 0.2, 12, 0, 2, 0.12?
S. 2
I. Why do you think that?
S. 'Cause its one and a half and there's none of them. That one's 1½ but there's only one.

Stimulus Card C

I. Uhhuh (reading to S) 0.8 is nearest in size to: 1.8, 0.08, 8, 1, 0?
S. One ... 'cause it's above the ½ way point.
I. Yes. Any other reason?
S. 'Cause it's 8 tenths. The other is just one which is ten tenths.

Stimulus Card D

I. Uhhuh...(reading to S) 1.9 x 5 is nearest in size to: 0.01, 0.10, 1.0, 10, 20, 100?
S. Ten
I. Why?
S. 'Cause 20 in that is a bit high, and the other one is too low.
I. And how did you know that? That ten would be the best? What did you say in your head when you worked it out?
S. Well, 5 nines are 45, and five ones are five. That's nine point five.

Stimulus Card E

I. Uhhuh... 0.19 x 5 is nearest in size to 0.01, 0.10, 1.0, 10, 20, 100?
S. One point zero.
I. Why?
S. 'Cause zero point one nine times five is point nine five and that's closer to one ... it's 95 hundredths ... um, whereas the other one's, um, a hundred hundredths ... so it's a bit closer ... it's only 5 hundredths away.

Stimulus Card F

I. If you estimated 0.19 x 0.5 which one of these would it be nearest to: 0.01, 0.10, 1.0, 10, 20, 100?
S. Zero point ... um ... zero point ten I think..
I. Why?
S. 'Cause that's got two decimal places in it ... and it's ... zero point ... zero nine five.

I. Sorry I don't quite understand that James. Could you explain it again to me please?

S. It's hard to explain ... the answer's roughly zero point zero nine five, and that ... oh, zero point ten is a little bit higher than it, so it's closer ... no, zero point one, zero one I mean ... cause that's one hundredth, whereas the other one's zero nine fiveths of a hundred ...

I. Are you happy with that explanation?

S. Not really.

I. Why not?

S. Gosh, it's hard to explain!

I. So point to the one you think it's nearest to, again, James.

S. That one.

I. Zero point oh one?

S. Mm (assent)

I. (reading to S) Divide by 10, now. What's 1000 divided by ten?

Stimulus Card G

S. A hundred.

I. 100 divided by 10?

S. Ten.

I. 1500 divided by 10?

S. 150

I. 15 divided by ten?

S. One point five.

I. Very good. How did you get that so quickly James?

S. 'Cause 10 goes into 15 once and a ... one whole time and five is left over, which is half of ten, so you've one point five.

I. Well, what is point five the same as?

S. One and a half.

I. ... $1\frac{1}{2}$?

S. Oh, point five's the same as half.

I. (Reading to S) Divide by 100 now. What's 1000 divided by 100?

Stimulus Card H

S. Ten.

I. 100 divided by 100?

S. One.

I. 1500 divided by 100?

S. Fifteen.

I. 15 divided by 100?

S. Um ... ... point one five?

I. Sorry, what was that again?

S. Point one five.

I. How did you get that?

S. ...(works out on paper).... $100\overline{)15.00}^{\;1.5}$ ....

I. I see....

S. 'Cause, um, you start with 15 and then you put point there and you put some zeros, and a 100 doesn't divide into 15, so you put 2 zeros. And a 100 goes into 150 once, and five over ...oh,... fifty over ... yeah ... and there's a zero there makes it 500, and it goes into 500 five times.

I. I see, very good.

I. (reading to S) Divide by 20, now. What's 200 divided by 20?

Stimulus Card I

S. Ten.

I. 100 divided by 20?

S. Five.

I. 10 divided by 20?

S. Um ... ... point 5.

I. And... 25 divided by 20?
S. Um ... 1 and a ½.
I. And 15 divided by 20?
S. ... point one five, no, um, ... is it all right if I work it out?
I. Of course, ... you work it out....
S. ... (works out) ... point seven five.
I. How did you get that?
S. Um ... you divide 15 by 20 ... 15 is 3/4 of 20.
I. What did you say for 25 divided by 20, James?
S. One and a half.
I. 1½ Uh huh. How did you get that?
S. Well 20 goes into 25 once, and 5 left over, and it's ... should be 1 and a ¼.
I. Why do you think that?
S. 'Cause 5 goes into 20 four times ... it's ... (silence)
I. O.K. let's go on to the next card. Reads to S.) Write as decimals:

three tenths
seven hundredths
fifteen hundredths
seventeen tenths
twenty hundredths

Stimulus Card J

S. (writes) "
.3
.07
.15
1.7
.020 "

I. Let's have a look at the decimals you've written there. For fifteen hundredths, read what you've got there?
S. Point fifteen.
I. How did you get that?
s. Well 15 over 100 is ... um ... when you've got 15 over 100 ...is ... um ... there's two places taken up. And, like, if say it was 150 over 100 it's wrong 'cause it would be 150 over (inaudible) ... um ... it must be one decimal, ...
I. O.K. ... what about 17 tenths? read what you've written for that James?
S. (changes answer from '.17' to '1.7')
I. What have you got now?
S. One point seven.
I. How did you know to put that?
S. Well 17 over 10 is ... one and seven tenths.
I. O.K. (Reads problem to S). The price of potatoes is $1.50 for each kilogram. What is the cost of 0.5 kg. of potatoes? How would you get an answer for that?

Stimulus Card K

S. ... (pause) ... um ... take away ...
I. Why?
S. Because one point fifty minus zero point five, and then what you take away is the answer.
I. (unfolds card). So which one of these down the bottom would you choose to get an answer?
S. $1.50 - 0.5.
I. What's your reason for that?
S. So then you could have $1.50 and you take away 0.5 and that's ... um ... you're taking away a third of it ... and so you could ... you know what you've taken away ... and then you could just put that down for the answer ...

Stimulus Card L

I. O.K. Here's another problem. I'll read it to you from the card (Reads problem to S). The price of mince is $2.67 for each kilogram. What is the cost of a packet containing 0.58 kg. of mince? ... How would you get an answer?
S. Minus.
I. Minus. Uh huh. Which one would you take away from which one, and why would you do it?
S. Um... 0.58 ... no, the kilogram minus 0.58... and what you take away is the answer.
I. Uh huh (unfolds card) ... which one of these would you use to get an answer?
D. The $2.67 minus 0.58
I. Why?
S. 'Cause then you could, um, um ... no... divide I think...
I. ... Which one would you divide into which one?
S. The ... ah ... $2.67 into 58 .... 0.58 divided by 2 dollars 67
I. Why?
S. 'Cause then you end up with a percentage of what it is....
I. ... Mm ... you're not sure?
S. ... Mm (assent) ....

Stimulus Card M

I. O.K. then. Let's try another one James. (Reads problem to S). The cost of 2.5 litres of soft drink is $0.75c. What would the price of one litre be? Would you add, subtract, divide or multiply to get an answer?
S. ...Divide that number (2.5) into that one (0.75)?
I. 2.5 into 0.75?
S. Yeah....and then you'd, um, divide it by one.
I. ... O.K. (unfolds card). What one of these down the bottom, then, would you use to get an answer? In your way of thinking about it.
S. 2.5 divided by 0.75.
I. Why?
S. 'Cause then if you divide 2.5 by 7.5 you work out how much it is for each fifth of a litre, or something... and then you can justtimes that by 2.
I. Why do you say each fifth of a litre?
S. Um... Cause 2.5 litres into 75 would go 3 times ... and that ... oh ... (mutters) ...
I. ....You're not sure?? ... Well, which one do you think would be the best one ?
S. ... 2.5 divided by 0.75
I. O.K.... You want to stick to that?
S. Mm. (assent).
I. What would that look like as a working form?
S. (Writes $0.75\overline{)2.5}$)

Stimulus Card N

I. O.K....James (Reads problem to S). The cost of 6.44 litres of petrol was $3.58. What would the price of one litre be? ... What would be the best way to get an answer?
S. Divide.
I. Why?
S. ... 'Cause you'd work out what one litre of it would be.

I. And which one would you divide into which one?
S. \$6.44 divided by \$3.58 .... 6.44 litres I mean ...
(writes '$3.58\overline{)6.44}$ ')
I. Why?
S. Cause then you'd be able to work out what 1 litre was by dividing \$3.58 into 6 litres point four four ... um ... you're dividing what it cost into ... um ... how many litres you got ... and you end up with ... um ... the answer of what one litre would cost ... no, what 3 litres would cost ... (pause) ...
I. ... You're not sure? ...
S. ... Not really ...
I. ... O.K... James ... Look at these numbers Could you read them to me?

Stimulus Card O

S. Zero point four, and four.
I. Which is the larger number there? Or are they both the same size?
S. Four (is larger).
I. How many times larger is it than zero point four?
S. ...um... four tenths ...
I. What I mean is, what would I have to multiply 0.4 by to make it into 4?
S. Ten.
I. Exactly ten times?
S. Yeah, I think so.
I. Are you sure?
S. Yeah.
I. O.K. James. Could read these numbers to me?

Stimulus Card P

S. One hundred and twenty-three and twelve point three.
I. Which is the larger number there James?
S. A hundred and twenty-three.
I. How many times larger is it than twelve point three?
S. Ten.
I. Exactly ten?
S. Yeah.
I. How do you know that?
S. If you times twelve point three by ten it brings it up to a hundred and twenty-three.
I. Uh,huh. What about this one James? Could you read those to me?

Stimulus Card Q

S. Zero point eight 'oh', and eighty.
I. Uh huh. Which is the larger number there?
S. Eighty.
I. And, James, once again in your way of thinking about it, how many times larger do you think eighty is...?
S. Ten.
I. Exactly ten?
S. No, a hundred.
I. Oh! Exactly a hundred, James?
S. Yes, 'cause if you times eighty by a hundred it should come up to eighty ... it's got two decimal places after ... it's got decimal ...
I. If you times point eight eh?
S. Yeah...

Stimulus Card R

I. What about these ones James?
S. Two hundred and forty-nine and two point four nine.
I. Which is the larger number there?
S. Two hundred and forty nine.
I. And how many <u>times</u> larger is it than two point four nine?
S. A hundred. ...
I. James is it exactly a hundred times larger?
S. Yes.

Nomenclature

I. Are you sure of that?
S. Yeah.
I. ... O.K. ... (Explains decimal column naming exercise to S)
S. (writes
" oneth / tenth / hundredth / ")
I. James, how did you know to write that? Had you learned that from a book, or a teacher, or what?
S. Um, teacher I think.
I. Do you remember learning it?
S. Oh, not really. I just remember it,... the way it is.
I. James, did you use that idea (place value column names) when you wrote decimal fractions?
S. Um... kind of ...
I. In what way?
S. Um... well, I didn't think of those (the columns).
I. ... You didn't think of them?
S. No, I just think of the number and where the decimal point was, and that ...
I. All right ... O.K. ... Time's getting on ... What have you got now (next period)?
S. Just tech. drawing.
I. Just tech. drawing?
S. Mm.

Sorting Activity

I. Isn't that important?
S. Not with the teacher we've got ...
I. James, here's some things we do in Maths (explains card sorting activity). (later)
R. Read them to me in your order from 'like' to 'dislike'.
S. Geometry, adding sums, fractions, take away sums, times tables, decimals, multiplication sums, division sums, sets.
I. Why do you enjoy geometry so much?
S. 'Cause I do it in tech.drawing.
I. What parts do you enjoy about it?
S. Um...dividing circles, and all that.
I. Sets and division, what do you think of them? You've got them down the end there - you don't like them. Why is that?
S. Um, oh, not too bad... But the 'sets', um,... they're just a little bit harder than some other things.
I. James, decimals and times tables are sort-of in the middle there... why's that?
S. Oh, decimals isn't too bad.
I. Not too bad? Why do you think that?
S. Well, ... 'cause in maths we're doing algebra now, and that's got some decimals in it. It's quite good.
I. James, when you think about decimals outside of school, do you see them used much outside of school?

S. Um, just when you're working out money sums and that... And... oh, they're used quite a bit. Used in banks, and um...when you're writing out cheques and that.

I. Any other time you'd use it?

S. Mostly at school, probably.

I. Is it mostly a subject you do at school, decimals?

S. Um...not really. You see we're doing algebra most of the year. Just going on to different points of it.

I. But when you think about decimal fractions, are they most useful in school, or outside of school...Work in school or work outside school?

S. Um... Probably both ... when you get into the 3rd form you don't use them quite so much as you used to do ... or else you go into them a bit deeper ... Out of school they use them in supermarkets, banks, and all sorts ...

APPENDIX D (PART TWO) INTERVIEW SAMPLE FROM INTERVIEW IV

Subject: Delwyn Age at Inverview: 14 years 1 month

Date: 18 October 1983

Interview number: IV

(I = Interviewer S = Subject)

Stimulus Cards Used: Series Two

I. (Introductory comments)

Stimulus Card A

I. (Reads card to S) Zero point three four times three is nearest in size to: 0.01, 0.10, 1.0, 10, 30, 100
S. Um ... One point 'oh'
I. How did you get that Delwyn?
S. Oh, I just thought three thirty-fours... when they're being multiplied I think you'd get a bit over one ... but I don't think you would reach five ... That would be ten, so the closest would be one point 'oh'.
I. Why a bit over one?
S. Because 34 times three you'd get a whole ... about one and a half ... about one whole ...
I. Uh huh...The way you did that, you sort-of multiplied it did you?
S. Yeah. Uh huh.
I. Then?
S. Well I just thought three threes are nine, and three fours are twelve ... take them out ... a hundred and two or something, because it's a bit over one whole.

Stimulus Card B

I. What about this one? (Reads card to S) One point three four times zero point three is nearest in size to : 0.01, 0.10, 1.0, 10, 30, 100.
S. ...One point 'oh' again.
I. Why?
S. ...I'm having problems...yeah, one point 'oh'.
I. How did you get that Delwyn?
S. Well, I don't know if it would quite reach a whole, because it's kind of same as the other one, only except the point three instead of three, which is a whole ... so ... I multiplied that ... it would still be a pretty small ... would be quite a large number near ... (mutters inaudibly).
I. What say someone said, zero point three is about a third and times means of; so you could say that means one third of <u>that</u> - and that would be about - one third of point three would be about point one, so therefore that would be that one there, zero point one. Would that be a way you would think of working it out?
S. No.
I. Why not?
S. Because it's three times and it's going ... O.K., it's 'of' as well, but, of can be times as well, both the same thing, and that's point thirty-four, and so you're saying you end up with zero point ten, which is smaller

than what you started off with...

I. It's not possible?

S. No.

I. Why not?

S. Because ... you're timesing it - point three times point four ... I mean you could get it if you take away or divide three into thirty-four. Because you're timesing it you're, unless you had a negative or something, but it's just two positives...so, get a higher number - can't get a lower number than what you started with.

I. If you times it?

S. Mm.

I. (Reads card to S) Divide by fifty now. What's 50 divided by 50?

Stimulus Card C

S. One.

I. Ten divided by fifty?

S. None.

I. Is any answer possible?

S. Fifty doesn't go into ten. It would be point something.

I. Uh huh ... (Explains decimal column naming exercise to S).

S. (Writes "/ones/ tens/ hundreds/"

I. Why have you put that?

S. Well (pointing), I think that could be the ones column, the tens column, the hundreds column; except that <u>could</u> be the tens column, the hundreds column, the thousands column ....

I. You're not sure?

S. ... No ...

I. Why not?

S. 'Cause I forget!

I. O.K. Delwyn could you write these as decimals (Reads card to S)

Stimulus Card D

S. (Writes " .02
.009
.0025
.011 ")

Oh, I think I know now! ... Tenths, hundredths, thousandths! You don't get ones, tens - you don't have a 'ones' column.

I. Why not?

S. 'Cause ones are whole.

I. One's wouldn't be decimals?

S. Well, I don't know ... No, I'm not sure but I think they're tens, hundreds, and thousands. (Rewrites decimal fractions
".2
.09
.025
.11 ")

I. How did you get twenty-five hundredths?

S. Well, that's tenths (the first column), that's the hundredths (the second column), but you can't put five on top (of 2) ... so, 25 hundredths.

I. Uh huh ... Now (Reads card to S), a person sees that ice-cream is 'on special' this week - a two litre pack for $1.00. At this price, or at this rate, how much would one litre cost?

Stimulus card I

S. Um ... 50 cents
I. How did you get that?
S. Oh well, two litre is a dollar, and you only want one, so one's half of two, so then you'd halve the dollar - that's 50 cents!
I. (Reads card to S) Next week, ice cream is 'on special' again - this time a 2.50 litre pack for $1.25. At this price, or at this rate, how much would one litre cost?

Stimulus Card J

S. Aw... you'd say that ... two point five litre ... one litre would be one point two and a half - half that's one point two and a half ... no ... one point a half ... one point five ... so you'd say one point five into a dollar twenty-five ... and you'd move the point when you're dividing it ...

(Writes "1.5)1.25" then " 15)12.5")
...you'd say fifteen into twelve doesn't go, so you'd say fifteen into a hundred and twenty-five, and get the answer
I. What would it be?
S. Hm... one point fives divide, and you want to see how many times that goes into that. You only want one litre, and that's (2.5) two litres point five, so you'd have to halve the ones once and get one, then you'd halve the five and you'd get one point two and a half but you can't have that, so, ... think it would be point five of a half ... that's not quite right though.
I. Why not?
S. 'Cause the point 5 isn't half of point five... I don't know how to do that one.
I. Does having decimals make it more difficult??
S. No... it wasn't the decimals, it was that you see, that one (card I) find out a 2 litre pack and you only had to find a one litre pack and that was a hundred ... that's got a point in it too - was easy. But this one it's two point five, so it's a bit over two, so you can't say one... you just can't say one ... and halve it ... and halve the price 'cause that's a bit more.
I. (explains semantic differential exercise)

Semantic Differential

I. You see Mathematics as valuable. Why?
S. 'Cause you need maths for a lot of jobs... You use maths. every day, to do different things like, say you wanted to ... add 2 sandwiches, you know. 2 .. one, two. Maths all the time.
I. You see it as good. Why?
S. Because you use it a lot and if you didn't have maths I suppose you'd have something else, you need maths. kind of maths ... you use it every day, how many windows you want in a house, or something. How much money you need to pay something... you know. You use it all the time.
I. O.K. Now Decimals ... You see them as good, valuable. Why?
S. Because the same kind of thing. You use them quite a lot, like, if you want to buy something, and it's a dollar ten you need to know, you know. And if you only want a bit of something you need to know the decimal cost of things.
I. You see them also as slightly cruel, hazy?

S. ...Cause sometimes you don't understand them, or something, so, you know, they're a bit hazy.

I. Awful?

S. I don't think they're really nice to do. I don't think they're really awful though. But I don't really enjoy them that much. But, they're more towards awful than nice.

I. (Asks S. about differences between teaching mathematics at intermediate and high school.)

Intermediate/High School Differences

S. Oh, it's not really different. You do the same kind of things. Um, but except its ... its ... well ... since you're going into set classes and everything ... um ... the subject that you like, last year you didn't really pay ... didn't really worry about it ... like when you worry about maths or anything. This year you've got to swot a bit and ... exams coming up. It seems more important ... and a bit harder, a lot harder.

I. Is the <u>way</u> it's taught different at high school?

S. Yeah, a <u>lot</u> ... um, last year the teacher, because they're having you every day and they taught you everything, um, they were more personal sort of thing. They're more unpersonal when they teach now ... That's the main difference. And I think that last year the teacher would come around you know, and explain something often to you, you know, again, and that ... and they do that again this year but they're a bit more of a ... you know you put up your hand and, you say ... I don't know, it's just different.

I. The textbook?

S. Oh, you use the text book a lot more. Last year you used the textbook, but - like last year we were put into groups and would have a textbook and then you would have a discussion and then would work on the board and that sort of thing ... it's usually all from the board or all from the textbook ... it's more serious.

Advice to teachers

I. (Advice to teachers asked for from S)

S. Well, like, some things I really don't understand them, and then we go on to something else and I'm still not sure about that ... Like they're kind of ... you know, I suppose there are so many kids they can't go to everyone individually, and that ... They get the classes a bit smaller would be a bit better. And that before any test ... if you had a test that was a piece of paper you could take it home and study it and so that before the actual test for that so that you know what you have to brush up on and that ... like we had a pre-test the other day but it was just stuff off the board and the ones you got wrong - since you only had the answers down you couldn't really know what you did wrong, and that didn't really help you. He told us on the board but while you were busy marking everything else, and quickly skim through the thing. Quickly do the ... he does it really quick and you can't really

understand it well. It's O.K. overall.
I. Does the teacher go too quickly?
S. I don't know ... I suppose some people have no trouble with it. It just depends on the group.
I. Anything else?
S. I feel there should be smaller classes.
I. Why?
S. So that, um, like, in the big classes you don't have so much ... if you're not sure how to do something in ... like ... I had to make an appointment and see him at lunchtime and that ... last year ... I don't know, but we seemed to have more to do with the teacher, kind of thing, you know ...

Peer Opinions of Mathematics

I. (Scenario for eliciting peer opinions of mathematics)
S. It's not really an enjoyable subject. You need to learn and ... you try and ... it's probably one of the subjects I'd worry about the most. Science I'd worry about the most 'cause I don't u'stand science, but then maths would come second, definitely. You don't really enjoy yourself while you're doing it ... sort of thing ...

Class Disruption

I. ... Is there any 'playing up' in maths classes?
S. Yeah, definitely.
I. Does it worry you much?
S. It does because ... yeah, that's one thing I find too, with maths here, someone's talking ... the girl in front of me was talking to the girl next to me so I told them to shut up and they just kept on talking... you know ... and the teacher tells everyone to be quiet but (inaudible) a lot keep talking. The teacher doesn't really make sure that everyone's quiet.
I. Is that a problem?
S. Yeah, sometimes. There's usually a few kids in the class that are stupid, you know.
I. Is it more of a problem at secondary school? Kids mucking around in class?
S. No ... I think kids mucked around more in class as primary school ... like now you know that you need maths and if you don't get a good mark, you know, you won't get school 'C', and all the rest of it. Now you realize the importance of it. And so I think most people do knuckle down a bit more than at primary school.

Stimulus Card H

I. O.K. let's go to this. Could you read these numbers to me?
S. Five hundred and forty-eight. Fifty four point eight.
I. Which is the larger one there?
S. 548.
I. How many times larger is it than 54.8?
S. Ten times.
I. Exactly ten times larger?
S. Yeah.
I. How do you know that?
S. Um ... I think 10 fives ... oh no! 100 times larger ... yeah a hundred ...

I. How do you know that?

S. Because five hundreds is five hundred. Four hundreds ... oh no! ... um ...

I. You're not sure?

S. ... No ... um ... about ten times I think ... I'm not exactly sure.

I. (Checks on eleven tenths. S had written

" .011")

S. Eleven tenths, that's ... so it would be '1.1'
Oh! These are all wrong!
(Changes responses " .2
.09
2.05
1.1 ")

I. How did you get 2.05 for twenty-five hundredths?

S. 'Cause if you put 25 there (in decimal columns) it would be two hundreds and five thousandths, so it would be five hundreds and the two is two wholes.

I. (Shows S how to write twenty-five hundredths as ".25")
Twenty-five hundredths ... break it down into twenty hundredths and five hundredths. Another name for twenty hundredths is two tenths ...

S. Are you a maths teacher? After doing this sort of thing ...

I. Sorry?

S. Did you used to be a maths teacher?

I. Oh, I used to be! ...
Why do you think a lot of people your age get that wrong? See, a lot want to call that ones, tens, hundreds (the decimal place value columns).

S. Do they?

I. Yeah.

S. That's good to hear someone else does!

I. Oh you're not the only one ... about half of the form two and three do that. Why do you think that?

S. Um... because that's got ones, tens, and hundreds (whole nos.) and you immediately think that this side you'd have ones, tens, and hundreds, too. But you forget that they've changed the hundredths cause ... I remember being taught about the " t. h" and that ... um, last year. Now it comes back to me again ... I'll remember that now. Hopefully ...

APPENDIX E : VERSION OF SEMANTIC DIFFERENTIAL TEST USED IN INTERVIEW IV (Osgood, Suci and Tannenbaum, 1957)

"The purpose of this is to measure the meanings of certain words to people, by having them judge each word against a series of descriptive scales. In taking this test, please judge the words on what they mean to you. Each numbered item presents a WORD (such as SKY), and a scale (such as up - down). You are to rate the word on the 7 - point scale indicated.

If you felt that the word was very closely associated with one end of the scale, you might place your cross as follows :

SKY:

Up__X__:____:____:____:____:____:____Down

If you felt that the word was quite closely related to one side of the scale, you might check as follows:

HOUSE:

Straight____:__X__:____:____:____:____:____Crooked

If the word seemed only slightly related to one side as opposed to the other, you might cross as follows:

CLOUD:

Easy ____:____:__X__:____:____:____:____Difficult

If you considered the scale completely irrelevant, or both sides equally close, you would cross the middle space on the scale:

TREE

Happy____:____:____:__X__:____:____:____Sad

Sometimes you may feel as though you have had the same item before on the test. This will not be the case; every item is different from every other item. So do not look back and forth throughout the test. Also, do not try to remember how you marked similar items earlier in the test. Make each item a separate and independent judgment. Work at fairly high speed, without worrying or puzzling over the items for long periods. It is your first thoughts that we want.

Of course, some of the items will seem a bit strange to you. It is necessary, in the design of this test, to match every word with every scale at some place, and this is why some items seem irrelevant or strange, so give the best judgment you can and move along."

MATHS:

Good ____ : ____ : ____ : ____ : ____ : ____ : ____ Bad

MATHS:

Beautiful ____ : ____ : ____ : ____ : ____ : ____ : ____ Ugly

MATHS:

Clean ____ : ____ : ____ : ____ : ____ : ____ : ____ Dirty

MATHS:

Tasty ____ : ____ : ____ : ____ : ____ : ____ : ____ Distasteful

MATHS:

Valuable ____ : ____ : ____ : ____ : ____ : ____ : ____ Worthless

MATHS:

Kind ____ : ____ : ____ : ____ : ____ : ____ : ____ Cruel

MATHS:

Pleasant ____ : ____ : ____ : ____ : ____ : ____ : ____ Unpleasant

MATHS:

Happy ____ : ____ : ____ : ____ : ____ : ____ : ____ Sad

MATHS:

Clear ____ : ____ : ____ : ____ : ____ : ____ : ____ Hazy

MATHS:

Nice ____ : ____ : ____ : ____ : ____ : ____ : ____ Awful

MATHS:

Fair ____ : ____ : ____ : ____ : ____ : ____ : ____ Unfair

MATHS:

Healthy ____ : ____ : ____ : ____ : ____ : ____ : ____ Sick

DECIMALS:

Good ____ : ____ : ____ : ____ : ____ : ____ : ____ Bad

DECIMALS:

Beautiful ____ : ____ : ____ : ____ : ____ : ____ : ____ Ugly

DECIMALS:

Clean ____ : ____ : ____ : ____ : ____ : ____ : ____ Dirty

DECIMALS:

Tasty ____ : ____ : ____ : ____ : ____ : ____ : ____ Distasteful

DECIMALS:

Valuable ____ : ____ : ____ : ____ : ____ : ____ : ____ Worthless

DECIMALS:

Kind ____ : ____ : ____ : ____ : ____ : ____ : ____ Cruel

DECIMALS:

Pleasant ____ : ____ : ____ : ____ : ____ : ____ : ____ Unpleasant

DECIMALS:

Happy ____ : ____ : ____ : ____ : ____ : ____ : ____ Sad

DECIMALS:

Clear ____ : ____ : ____ : ____ : ____ : ____ : ____ Hazy.

DECIMALS:

Nice ____ : ____ : ____ : ____ : ____ : ____ : ____ Awful.

DECIMALS:

Fair ____ : ____ : ____ : ____ : ____ : ____ : ____ Unfair

DECIMALS:

Healthy ____ : ____ : ____ : ____ : ____ : ____ : ____ Sick.

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STIMULUS CARDS (SERIES ONE)

A

186 is nearest in size to?

100 / 80 / 180 / 200 / 150 / 190

B

1.2 is nearest in size to?

1 / 10 / 0.2 / 12 / 0 / 2 / 0.12

C

0.8 is nearest in size to?

1.8 / 0.08 / 8 / 1 / 0

D

1.9 x 5 is nearest in size to?

0.01 / 0.10 / 1.0 / 10 / 20 / 100

E

0.19 x 5 is nearest in size to?

0.01 / 0.10 / 1.0 / 10 / 20 / 100

F

0.19 x 0.5 is nearest in size to?

0.01 / 0.10 / 1.0 / 10 / 20 / 100

G

Divide by 10:-

1000
100
1500
15

H

Divide by 100:-

1000
100
1500
15

I

Divide by 20:-

200
100
10
25
15

J

Write as decimals

three tenths
seven hundredths
fifteen hundredths
seventeen hundredths
twenty hundredths

K

The price of potatoes is $1.50 for each kilogram. What is the cost of 0.5 kg of potatoes?

----------------(fold)-----------------

1.50 + 0.5	0.5 ÷ 1.50
1.50 ÷ 0.5	0.5 - 1.50
1.50 - 0.5	0.5 x 1.50

L

The price of mince is $2.67 for each kilogram. What is the cost of a packet containing 0.58 kg of mince?

-----------------(fold)---------------

2.67 + 0.58	0.58 ÷ 2.67
0.58 ÷ 2.67	0.58 - 2.67
2.67 - 0.58	0.58 x 2.67

M

The cost of 2.5 litres of soft drink is $0.75c. What would the price of <u>one</u> litre be?

----------------(fold)---------------

2.5 + 0.75	0.75 ÷ 2.5
2.5 ÷ 0.75	0.75 - 2.5
2.5 - 0.75	0.75 x 2.5

N

The cost of 6.44 litres of petrol was $3.58. What would the price of <u>one</u> litre be?

----------------(fold)----------------

6.44 + 3.58	3.58 ÷ 6.44
6.44 ÷ 3.58	3.58 - 6.44
6.44 - 3.58	3.58 x 6.44

O

0.4 4

P

123 12.3

Q

0.80 80

R

249 2.49

.../over

STIMULUS CARDS (SERIES TWO)

A	B	C
0.34 x 3 is nearest in size to? 0.01 / 0.10 / 1.0 / 10 / 30 / 100	0.34 x 0.3 is nearest in size to: 0.01 / 0.10 / 1.0 / 10 / 30 / 100	Divide by 50:- 200 100 50 10 60 5
D	**E**	**F**
Write as decimals two tenths nine hundredths twenty-five hundredths eleven tenths	0.6 6	432.0 43.2
G	**H**	**I**
0.10 10	548 54.8	A person sees that ice-cream is 'on special' this week - a 2 litre pack for $1.00. At this price/rate how much would one litre cost?
J		
Next week, ice-cream is 'on special' again - this time at a 2.50 litre pack for $1.25. At this price/rate how much would one litre cost?		.../over