Exploring the transition into Year 3 of Year 2 students who use counting on to solve mathematics problems

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Abstract

This research project examined how five Year 2 students, at stage 4 on the Number Framework (counting on), experienced mathematics as they transitioned into a Year 3 and 4 classroom. It investigated the support structures put in place to shift students from counting on to part-whole thinking, as part of the Numeracy Development Projects (NDP) approach to teaching mathematics. An additional transition of two teachers into Year 3 and 4 (one up from Year 2 and one down from Year 5 and 6) provided evidence of teacher transition experiences when shifting teaching levels. The setting, role of the teacher, and external influences were examined.

This research was a qualitative investigation framed within a case study approach. The main source of data was classroom observations and semi-structured interviews. The teachers' interviews focused on their approach to teaching and learning, attitude, student ability, assessment, and knowledge of the mathematics curriculum from Level 1 to Level 2. The combination of classroom observation and student interviews demonstrated the current level students were operating at and any signs of shift in their knowledge, as well as attitude towards mathematical learning.

The thesis illustrates how classroom practices and teaching approaches encouraged students to count on *instead* of shifting into part-whole thinking. The findings highlight possible barriers, student experience, the importance of teacher knowledge and understanding, and the impact of teaching practices that support and undermine the shift. The findings also show that teachers are still following the NDP material very closely, without a full understanding of the pedagogy of number knowledge which can bridge Level 1 to Level 2 of the New Zealand Curriculum.

The findings also indicate that the NDP teaching model is not being fully incorporated into classroom teaching, with a decrease of manipulatives used over the transition, a limited use of visualisation through diagrams and pictures, and students experiencing abstract representations without a full understanding of their meaning. The findings also show that the current reform in mathematics is only operating at a surface level. Teacher practices reflected an instrumental, procedurally-based approach to the teaching and learning of mathematics.

The evidence contained within this thesis points to the link between knowledge and strategy not being made explicit, with limited experiences of exploring relationships between numbers and quantity. It considers a critical aspect of student understanding is to develop a full understanding of number relationships through the concept of subitising, part-whole relationships, and more-and-less relationships. Continuing Professional Learning and Development is needed for teachers to develop a deeper understanding of these relationships and how they support student shift from 'counting on' to part-whole thinking.

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Chapter One: Introduction

1.1 Introduction

To support students' progress in mathematics, teachers need to recognise and build on prior learning experiences using evidence from a range of sources to continually develop mathematical understandings, skills, language, and confidence (Ministry of Education, 2012a). In the current climate of New Zealand teachers having to report on students' achievement against National Standards, increasing pressure has been placed on schools and teachers to have students working at the expected level within each school year. This raises questions about how schools are effectively managing the learning of mathematics and sharing information across the levels to achieve these expectations. This means building on students' prior knowledge and moving them forward in their learning, showing evidence of progress and achievement. It also means exploring the teacher's understanding of what precedes and follows each level and how key mathematical ideas are developed. It is also important to explore what support is put in place to help students transition their prior knowledge and develop it within a new classroom environment.

The transition of mathematical knowledge from one learning environment to the next is a complex issue and not as straightforward as reapplying knowledge in a different context. Changing classroom environments, different teaching pedagogies, and students' developing capability can all contribute to the mathematical experiences encountered when one moves from one year level to the next. Within the New Zealand curriculum, there are significant shifts in mathematical thinking as students move from one curriculum level to the next. The shift from Level 1 to Level 2 is a substantial step, as students who have previously been encouraged to use counting to solve problems now need to replace this strategy with grouping numbers in different ways. At the same time the majority of these students are shifting out of the junior syndicate and moving into the middle syndicate of a primary school, where approaches to teaching and learning may be very different.

An important question is about whether teachers in fact build on prior learning or instead take a fresh-start approach, valuing new assessments over the previous year's judgments (Bicknell, Burgess, & Hunter, 2010). If teachers value new assessments over the prior years, relying on assessment over information sharing, then when certain students dip in achievement from one year to the next, as reported by Anderson, Jacobs, Schramm, and Splittgerber (2000), does their learning regress as they cover material they have already experienced? In Young-Loveridge's (2010) review of mathematics reform over the last decade, she discusses the idea that prolonged exposure to counting to solve problems may limit students' progress in using more complex strategies. This is particularly important in Year 3 when the emphasis should be on partitioning and grouping numbers instead of counting. This raises questions about how teachers go about teaching in order to produce this shift in thinking.

ERO concluded in its 2012 report on transition between Year 9 and 10 that less than 10% of schools were highly effective at using achievement information from the previous year, and that poor transitions impacted on students' well being and future achievements (West, Sweeting, & Young, 2010). The work of Davies, Walker and Walshaw (2008) and Peters (2010) identify similar issues within the structural provision, assessment and information sharing from early childhood to primary. Therefore, if transition practices from one sector to the next are mostly ineffective, how coherent are the transition practices within one sector?

There is limited research on how effective transition is within one sector or how the shift from junior to middle school impacts on prior learning. By identifying what is valued and what is an effective transition for students, a smoother pathway could be promoted through the development of mathematical thinking and teaching practice. The purpose of this research was to explore these themes and highlight what is happening with the transition in mathematics from Year 2 to Year 3, as students who 'count on' to solve problems are expected to learn about part-whole strategies.

The primary research question guiding this thesis was:

> What support do "counting on" students receive in mathematics as they transition from Year 2 to Year 3?

1.2 Researcher Positioning

The researcher's interest in this topic came about when working as a numeracy adviser on a Ministry of Education professional development contract during 2007 - 2011. As part of this contract, the researcher worked in a number of schools participating in professional development in teaching mathematics. Within this period transition became an issue in her local community. In response, one of the local primary school principals organised a series of transition workshops for local early childhood centres and schools. As part of this initiative, the researcher conducted workshops examining what mathematics looked like in both sectors, shared conversations focused on building relationships, and examined the differences and similarities between each respective curriculum.

The interest in transition and students' experiences across levels deepened in 2012 when the researcher took up a part-time position in a school in which she had previously been a numeracy advisor. Observing changes in the students over the three-year period and different teachers' perspectives of student ability in mathematics sparked an interest in what occurs as students move from one year level to the next. The researcher observed changes in teaching approaches, the use of equipment and the development of key mathematical concepts. Teachers appeared to have a fresh-start approach to the beginning of each year.

The limited research on this topic to date motivated the researcher to investigate the issue of transition and the development of mathematical thinking as students move from one classroom to the next across two year levels within a primary school setting. The researcher observed one particular group of students identified as 'advanced counters' stage 4 on the New Zealand Number Framework, investigating in detail student mathematical experiences as they moved from a Year 2 to a Year 3 classroom and the role of the teacher within this process.

The following chapter provides background information from the literature on the complexities of transitioning from one learning environment to another and the changing mathematics as students move from Level 1 to Level 2 of the curriculum. Further, information on the pedagogical approaches of teachers framed within the structure of the NDP teaching material and the factors that support a student to successful transition from one year to the next are also discussed. The third chapter provides the underlying methodology of the data collection and describes how the study was conducted. Data that were analysed and compared from year to year, is presented in Chapter 4. Chapter 5 discusses the meaning of the results and links them to the literature. It is in this chapter that the researcher focuses on the classroom setting, student learning and mathematical content, the teachers' practices, and the external influences on transition. The conclusions from the research and the way that the results of this thesis add to the field of knowledge in this area, the significance of the research for teaching mathematics at Year 2 and 3 are the focus of the final chapter.

Chapter Two: Literature Review

2.1 Introduction

This chapter provides an overview of the relevant literature and a framework for this thesis. The review is divided into eight sections. The first section, 2.1 defines the term transition and investigates the theoretical approaches to educational transition. Section 2.2 describes the mathematical structure in the New Zealand Curriculum and developing concepts in number, focusing on part-whole relationships and place value. This is extended in section 2.3, with a description and elaboration of the mastery of basic number concepts. In section 2.4, modes of representation are examined and include the role of manipulatives and visualisation. The Numeracy Development Project (NDP) resources that are used by teachers to support numeracy understanding are examined in section 2.5. Issues related to transitioning classrooms and the role of the teacher are explored in sections 2.6 and 2.7, respectively. The final section (2.8), reviews assessment and its role in student transition.

2.2 Transition

The term ‘transition’ in educational terms is defined as the process of moving from one setting to another (Dunlop & Fabian, 2007). Beach (2003) describes it as a developmental change between an individual and one or more social activities. It can describe the changes a student experiences within and across a school setting, and includes a student moving from one classroom to another together with a change of teacher (Dunlop & Fabian, 2007). In much of the literature, the term transition is used interchangeably with transfer. However, transfer is generally used in the context of a move from one school to another or one phase to another (Demetriou, Goalen, & Rudduck, 2000; Dunlop & Fabian, 2007).

Transition in practice will likely involve a change of culture and status. It may entail leaving something behind that has constructed an identity (Gennep, 1960). This could also mean leaving the ‘comfort zone’ and encountering the unknown (Dunlop & Fabian, 2007). Most literature on transition has focused on the “entrance and exit” years of schooling, whereas relatively little attention has been given to sustaining progress across the years in between school transfer (i.e., within the same school) (Demetriou et al., 2000, p. 425).

Transition refers to not only a change of situation but also a change in relation to the individual internally. The concept of consequential transition refers to the situation when individuals reflect on, struggle with, and create a shift in their sense of self or social position (Beach, 2003). Two primary types of consequential transition outlined by Beach (2003) are lateral transition and mediational transition.

Lateral transition refers to an individual moving between two historically related activities in a single direction (i.e., students moving classes such as from Year 2 to Year 3). Mediational transition aligns itself broadly with Vygotsky's (1978) concept of a zone of proximal development. Teachers mediate an activity that embodies a particular notion of developmental progress for the students. Examples include students who are operating at a counting stage, then participate in activities that encourage students to operate at a higher stage. The mediation bridges the transition between the two operating systems of counting to part-whole thinking.

In this thesis, transition not only refers to a student's physical shift from one environment to the next, but also to the context of a student's developmental shift in thinking. Within this thesis, transition is considered a multilayered concept and is interrelated when examining continuity as well as the change of knowledge across different contexts.

2.2.1 Theoretical approaches to Transition

It is well documented that transition from one level to the next is not just a one-off event, but is influenced by a range of factors that can impact on students well after the initial adjustments and has long-term consequences for learning and achievement (Peters, 2010). By identifying the underlying theoretical ideas around transition, we can make sense of different students' experiences as they move from one level to the next. This raises a question about what contributes to successful adjustments to new teaching environments and what might challenge a transition of skills and abilities.

In this review of literature, dominant theoretical ideas are examined in relation to the transition of mathematical thinking across different classroom environments.

These include a ‘deficit model of assessment’, a socio-cultural view of ‘scaffolding the process’, and an ecological systems theory (Peters, 2003).

2.2.2 Deficit model of assessment

The deficit model of assessment is typically reflected at the beginning of the year in a mathematics classroom, and focuses on what a student lacks. It means using new assessments to gather information on what skills students do or do not have. The objective is to ‘fill the gaps’ before students can progress (Peters, 2003, p. 15). With this type of approach, a student’s prior knowledge and skills can sometimes be overlooked. This approach can create a level of apprehension and anxiety for a new student who may suddenly feel unprepared or lacking in ability to cope within a new mathematics environment. When comparing early childhood mathematical experiences and the early years of primary, Peters (2003) identified that primary schools tended to focus on the need to build on knowledge and fill gaps before students were considered ready to solve problems. Preschool mathematical experiences have been contextualised, thus creating possibilities for problem solving, whereas primary school mathematics learning can be isolated and knowledge driven (Aubrey, 1993; Perry & Dockett, 2004; Peters, 2003; Sherley, Clark, & Higgins, 2008). This approach focuses on the notion of student ‘readiness’ and places importance on “remediating skills deficits or other deficiencies inherent in the child” (Schulting, Malone, & Dodge, 2005, p. 2). The limitation of this theory is the lack of consideration for contextual factors and that successful transition does not solely sit within the child but is socially evolved (Peters, 2003).

2.2.3 Socio-cultural theory

A Vygotskian view considers that what students can do with the assistance of others is more indicative of their mental capabilities than what they can do alone. Vygotsky (1978) theorised that the developmental process lags behind the learning process, resulting in a zone of proximal development, described as:

> ‘the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers’ (Vygotsky, 1978, p. 6)

Good learning takes what students have initially mastered and provides opportunities for subsequent development by the student experiencing a variety of more complex tasks that shifts a student's thinking. Vygotsky (1978) states that testing students only determines the actual developmental level, whereas exploring what a student is able to do working alongside more capable others is more indicative of their mental capabilities, and takes into account not only the development that has matured but also those processes that are currently in the state of formation.

Learning occurs when the student is challenged to cross over into their zone of proximal development (Brostrom, 2007). With the use of materials that stimulate and the involvement of peer and/or adult interaction, knowledge is created and extended. Through the formal and informal instruction of more knowledgeable others, learning is socially constructed (Muijs, 2011). Engestrom (1987) promoted this 'learning by expanding' to develop higher levels of mental ability.

A number of researchers have discussed the problematic relationship between learning and development, and whether learning is the same as development or at least is the accumulation of items within learning that equate to developmental change (Baltes, Reese, & Nesselroade, 1977). Brown's (1982) research into learning, remembering, and understanding identifies a shift in development occurring once a concept has become context-free and can be applied in a range of situations flexibly. The "isolated skills can be connected together, extended and generalised" (Brown, Bransford, Ferrara, & Campione, 1982, p. 18).

For teachers to access the zone of proximal development and place students in the appropriate instructional group, they need to use diagnostic procedures grounded within an explanatory understanding of the child's current state of development (Chaiklin, 2003). In New Zealand classrooms teachers are encouraged to use NDP resources, which are aligned with The Number Framework (Ministry of Education, 2007a). This Framework helps teachers to identify a student's current state of development in number. The Number Framework is divided into two sections: knowledge and strategy. The knowledge part of the Framework outlines the stage of student conceptual development in number knowledge; it determines what knowledge is fluent and what key items of knowledge students need to learn in order to apply particular strategies. The strategy framework outlines the mental processes a student uses to estimate answers and solve number problems (Ministry of Education, 2007a). Teachers who can identify a student's current development by using diagnostic tasks or interview questions can gain a full understanding of the child's developmental stage in number and support the construction of new concepts through a process of 'scaffolding' (Bruner, 1986).

Sociocultural theory does not take into account the student's own biological disposition towards mathematics and how this interacts with the environment, which is multilayered within a complex system of relationships (Schulting et al., 2005). Bronfenbrenner's (1979) ecological systems theory explores the different levels of the environment and how it is influenced and influences the developing person.

2.2.4 Ecological systems theory

The ecological systems theory not only acknowledges the relationships within the current environment but also looks outside this to expectations and events in the larger society. To understand transition fully in relation to this thesis, it is important to look broadly both at the interactions between current and previous mathematics environments and at the wider context in which the transition takes place. Bronfenbrenner's model "recognises that the child is embedded within a group of interacting systems" (Schulting et al., 2005, p. 3) identified as the micro, meso, exo, and macrosystems (Bronfenbrenner, 1979). The microsystem is the environment the student currently operates in, the activities, roles, and relationships a student experiences. The mesosystem is the network of relationships between the microsystems. The exosystem refers to the systems outside the student's control but influences and is influenced by what happens in the microsystem. The macrosystem refers to the overriding beliefs, values, and ideologies within the context of Aotearoa New Zealand's culture and the current government policies that include National Standards, the current curriculum, the type of funding the school receives for curriculum development, as well as the learning culture within the school. (Bronfenbrenner, 1979, 1992; Peters, 2003; Vogler, Crivello, & Woodhead, 2008)

Bronfenbrenner's (1979) hypothesis 42 states that: "upon entering a new setting, the person's development is enhanced to the extent that valid information, advice, and experience relevant to one setting are made available, on a continuing basis, to the other" (p. 217). This links to the idea of continuity and communication between two environments. To succeed in mathematics from one level to the next, students need to understand the expectations, the developing curriculum, and see similarities in pedagogy from one environment to the next (Vogler et al., 2008).

As students transition from one setting to the next, they need to learn what their role is within this new setting before they can focus on the content. Communication between settings is important to support the transition as the greater the difference between the two environments, the less likely it is that the transition will be a smooth one (Peters, 2003). Understanding how a student's identity is shaped within different environments and how students construct a sense of self within mathematics is essential when examining how to foster positive dispositions towards mathematics (Peters & Rameka, 2010).

A study that examines both the act of transition across a year group and specifically within a curriculum area lends itself to a range of theoretical approaches. So far in this chapter, the researcher has considered transitional theories that sit alongside the students' developing mathematical understandings. The first is the deficit model of assessment where at the beginning of the year the focus is on 'gap filling' and teaching is informed by current assessments. The sociocultural transition refers to a process of scaffolding, which is both social and developmental as students make shifts in their mathematical thinking. This raises questions about how the peer group, use of manipulatives, and teacher scaffolding used within the zone of proximal development help shift students' mental processes from using counting as their main strategy on to using grouping numbers in different ways. Finally, the ecological transition is important, as the microsystem changes through a changing classroom, teacher, and peer group, and information may be shared and links made from one setting to the next (Bronfenbrenner, 1979).

2.3 Mathematics Structure in the Primary School

Within the framework of the New Zealand Curriculum (NZC), (Ministry of Education, 2007b), schools are encouraged to design a responsive curriculum specific to their community so that students find each stage prepares and connects them to the next stage. Within the "learning pathways' (p. 41), transition is encouraged through linking the learning and maintaining continuity throughout a student's journey.

The underling philosophy of the mathematics curriculum is for students to develop:

> 'the ability to think creatively, critically, strategically, and logically….. to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge.' (Ministry of Education, 2007b, p. 26)

Achievement objectives within each level build on and develop ideas from the previous level. In the learning area of mathematics, the shift from Level 1 to Level 2 happens for most children when they move from Year 2 to Year 3. The weighting given to the strands in mathematics change as students move up the curriculum levels. Initially, the number strand is given priority and is allocated 60 to 80 percent of the teaching time in Level 1 and 2 (Ministry of Education, 2009), as it is considered the most important at this early stage in young children's mathematical development. The achievement objectives for number knowledge and strategies are outlined in Table 1.

Knowledge and strategies are seen as interconnected, as knowledge provides the foundation for strategies and the use of strategies lead to the creation of new knowledge. The combination of both number knowledge and mental strategies develops a strong number sense that will lead to "the ability and inclination to use mathematics effectively – at home, at work, and in the community" (Ministry of Education, 2001, p. 1).

The mathematics curriculum levels are based on a number framework which was first developed from Steffe's psychological model (Steffe, 1992), and incorporates the stages of development children move through when solving number problems (Bobis et al., 2005). Level 1 focuses on counting-based strategies where students are building addition and subtraction through counting by ones, initially counting from one, then progressing to counting on and counting back, once students understand the cardinality of a set. The cardinal principle is the understanding that the number of objects within a set always remains the same so they can now count on or back from the cardinal number of one of the sets to solve problems. Essentially they see and use numbers as a collection of ‘ones’, but they also know that a set of objects can be represented by a single count (Ministry of Education, n.d.).

Table 1: New Zealand Curriculum - Level One and Two in Number

Mathematics Achievement Objectives

Level 1	**Level 2**
In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:	
Number strategies • use a range of counting, grouping, and equal-sharing strategies with whole numbers and fractions Number knowledge • know the forward and backward counting sequences of whole numbers to 100 • know groupings with five, within ten, and with ten	Number strategies • use simple additive strategies with whole numbers and fractions Number knowledge • know forward and backward counting sequences with whole numbers to at least 1000 • know the basic addition and subtraction facts • know how many ones, tens, and hundreds are in whole numbers to at least 1000 • know simple fractions in everyday use

The shift from counting-based to part-whole strategies happens as students move from Level 1 to Level 2 in the mathematics curriculum. A shift in thinking to a different and more sophisticated way of manipulating numbers enables students to solve more complex problems (Thomas & Ward, 2001; Young-Loveridge, 2001). Instead of counting, students move from seeing numbers as a collection of ‘ones’ to treating numbers simultaneously as a whole, which can be partitioned and recombined in different ways to solve addition and subtraction problems. This is called part-whole thinking (Ministry of Education, 2007a).

2.3.1 Part-whole relationships

Understanding and using part-whole relationships among numbers is first introduced to students within the Level 1 number knowledge achievement objective, where students learn groupings with five, within ten, and with ten. This objective explores the concept that numbers are made up of different parts which when put together make up a whole number; e.g., knowing that 3 and 2 make 5. Recalling basic addition and subtraction facts and using these number facts to solve problems is the beginning of shifting the students from solving problems by counting to partitioning and recombining numbers in different ways. The combination of understanding the "additive composition of numbers" and being able to retrieve number facts becomes essential as students need to know that numbers can be constructed in many ways; e.g., 6 can be made from 3 and 3, 4 and 2 or 5 and 1 (Young-Loveridge, 2001 p. 73). At Level 2, children are expected to be at 'early additive' and experience the simplest level of part-whole thinking, where they have to split and join numbers with only one or two splits. An example of this at Year 3, is that a student is able to solve a problem like 6 + 7 through 'number fact retrieval' (Young-Loveridge, 2001, p. 72) and the knowledge that 7 can be split into 6 and 1, they use the combination of number knowledge of doubles and splitting to work out that 6 + 7 is 6 + 6 + 1 = 13.

If children do not understand part-whole relationships, they will have a great deal of difficulty with addition, subtraction, and other mathematical problems (Baroody, 2000). An incomplete understanding of part-whole relationships is one of the main reason students perform badly on missing-addend word problems, such as 4 + _ = 9 (Riley, Greeno, & Heller, 1983) and relating this equation to the principle of inverse operations (Losq, 2005). In Fischer's (1990) research, the focus groups that received an intensive programme based around part-whole relationships showed a greater understanding of place value than those students who received the normal mathematical curriculum programme. Research supports a strong conceptual understanding of part-whole as the foundations to more advanced concepts, such as place value (Baroody, 2004).

2.3.2 Place-value understanding

As students move through stages of development in number, their number sense is continually evolving. By the age of seven or eight as students begin to experience Level 2 of the mathematics curriculum, their cognitive development has reached the point where making sense of distinct quantitative dimensions such as tens and ones is possible (Griffin, 2004). Students are now able to begin to understand the complex structure that makes up place value, which consists of the following four mathematical properties.

> 1. *Additive property*. The quantity represented by the whole numeral is the sum of the values represented by the individual digits.
> 2. *Positional property.* The quantities represented by the individual digits are determined by the positions that they hold in the whole numeral.
> 3. *Base-ten property*. The values of the positions increase in powers of ten from right to left.
> 4. *Multiplicative property.* The value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position. (Ross, 2002, p. 419)

In Level 1 of the mathematics curriculum document, place value begins in the same place as part-whole relationships in the objective grouping in fives, within tens, and with tens. Students must have a clear understanding of what makes up 10 and the different representations of 10 before they can move into Level 2, exploring how many ones, tens, and hundreds are in whole numbers.

To begin with, students have a unitary concept of numbers where number words and digits have no meaning on their own (Young-Loveridge, 1999a). They see the number word 25 as the whole quantity only. As the students shift from the unitary concept of 25 to a ten-structured concept they begin to understand that the number 25 can be partitioned into units of tens and ones (20 plus 5). At this stage, the only way students can ascertain how many tens are in the number is by counting and keeping track of the number of counts (10, 20, so there are two tens). From this stage, students' thinking shifts to a multi-unit understanding where units of tens and ones can be counted separately and can be traded and exchanged (10 ones for one 10 or one 10 for 10 ones) (Young-Loveridge, 1999). Students need to move through unitary, ten-structured, and multi-unit concepts with two-digit numbers as they progress through Level 1 and Level 2 of the NZC, with Level 2 extending multi-unit concepts into the hundreds, tens and ones.

Multi-unit understanding develops a student's awareness of a "positional base-ten structure" (Fuson & Briars, 1990, p. 180). Children have to construct named-value and positional base-ten conceptual structures for the number words for two- and three-digit numbers at Level 2. The difficulty with this is that two-digit numbers involve a two-step process. Students have to interpret the number position and value with no number-word clues. This is relatively easy for numbers in the hundreds and thousands and beyond, as the spoken number words indicate their value (e.g., three thousand seven hundred, the thousand gives the value of the three and the hundreds the value of the seven). With two-digit numbers, children have to interpret sixty-four as six tens and four ones. Fuson and Briars (1990) argue that this lack of consistency in the English language creates a barrier for children interpreting the named-value meanings of two-digit numbers.

The process of developing place-value understanding begins with treating a collection as a whole (unitary concept), 'a collection of ones', and then develops as a system that is built on the interaction of grouping collections into units of 10, 100, etc, requiring a significant cognitive reorientation (Thomas & Mulligan, 1998). There remains a body of research indicating that students who have difficulties with this shift in thinking are limited in their progress and future achievement in mathematics (Young-Loveridge, 2001).

Even though two-digit numbers do not have a named value aspect, what they do have is partitionable structure. For example, the number 28 separates into 'two discrete parts: a multiple of ten and a single digit number' (Thompson, 1998, p. 5); that is, 20 and 8. This partitionable aspect of the English counting-word system supports children to add and subtract in quantities, rather than add and subtract digits (Thompson, 1998), and this in turn supports partitioning and combining numbers within the number strategy objective and part-whole thinking.

2.4 Mastering Basic Number Combinations

Research has shown that children typically progress through three phases when mastering the basic number combination of single-digit addition and subtraction; that is, basic facts.

> 'Phase 1: Counting strategies – using object counting (e.g., with blocks, fingers, marks) or verbal counting to determine the answer

> Phase 2: Reasoning strategies – using known information (e.g., known facts and relationships) to logically determine (deduce) the answer of an unknown combination
> Phase 3: Mastery – efficient (fast and accurate) production of answers'
> (Baroody, 2006, p. 22)

The National Research Council (NRC) in the United States of America (USA) has concluded that the efficient, appropriate, and flexible application of combining single- and multi-digit numbers is an essential skill when developing mathematical proficiency (Baroody, 2006). Mathematical proficiency has been defined by the NRC as the combination of conceptual understanding, procedural fluency, strategic and adaptive mathematical thinking, and a productive disposition (Baroody, 2011).

Teachers view the learning of basic facts in different ways. The first view that is still widespread among teachers, principals, and parents today is the belief of "conventional wisdom" (Baroody, 2006, p. 24); i.e., that mastery grows through the memorisation of individual facts. This view considers phase 1 and phase 2 are not necessary and that basic facts do not need conceptual understanding or developmental readiness; they merely require practice through rote learned procedures, using flash card drills and timed tests (Baroody, Bajwa, & Eiland, 2009). Alternatively, the number sense view considers that phase 1 and 2 play an integral and essential role in achieving phase 3. When learning any large body of factual information, it is far easier to recall it if it is linked together in a meaningful way (Baroody, 2006). The mastery of basic facts comes from the student discovering patterns and relationships that interconnect the basic combinations. Garza-Kling (2011) defines this as the fluency approach, where mathematical strategies are used to derive unknown facts by effectively using knowledge that the student already has. She argues that quick recall of facts may display a good memory but may not illustrate a deep understanding of mathematics. This approach also encourages advanced mathematical thinking as students learn to effectively partition and recombine numbers.

By focusing on the patterns that arise from the basic facts to 20, the task of learning basic number combinations can be simplified. By understanding relational knowledge and the commutative property, the combinations that need to be learned are halved. Recent research indicates that by understanding this concept, the two combinations are stored in a person's memory as a single representation (Baroody, 2006). A student quickly learns the meaning of "+0" and most children learn the idea that facts with +1 or -1 and + 2 or -2 are related closely to the counting sequence (Garza-Kling, 2011). Research has shown that students memorise doubles reasonably quickly (Kilpatrick, Swafford, & Findell, 2001). Once students have their doubles memorised, they can use them to derive any near-double fact (Garza-Kling, 2011). Another set of combinations learned and heavily focused on in a number of countries (Kilpatrick et al., 2001; Steinberg, 1985) is the combinations of 10, such as 6 + 4 and 7 + 3, etc. When students can recall these facts, they can apply 'make 10' strategies when faced with facts that are near 10. If students can master the doubles and 'make 10' combinations, they can derive nearly every other fact.

When approaching the teaching and learning of these basic facts, it may be tempting to rote memorise the key elements mentioned previously. In contrast, research recommends engaging in interactive opportunities to notice patterns and relationships through a discovery-learning approach (Baroody, Eiland, Purpura, & Reid, 2012). Bruner (1961) defined discovery learning as the process in which students are guided to explore some mathematical idea in order to discover a formula, procedure, or some mathematical fact which the teacher has in mind (Yeo, 2007). Alfieri, Brooks, Aldrich, and Tenenbaum's (2011) meta-analysis of 164 studies revealed that guided discovery learning was the most effective form of instruction for number knowledge. Cognitive scientists agree that knowledge that is organised, connected, and structured is more powerful than simply memorisation as it is easier to access, retrieve, and apply flexibly (Kilpatrick et al., 2001). Where materials are arranged in a way that students can notice patterns and relationships together with questioning to scaffold the learner to take notice and discover mathematical regularity, learning is more powerful. Once patterns and relationships are recognised, students test the regularity and define where it can be generalised. Facilitating students' adaptive reasoning through discussion of the advantages and disadvantages of the different addition procedures will improve their understanding of addition processes (Kilpatrick et al., 2001).

Number relationships goes beyond counting and refers to the students' ability to represent a quantity in multiple, flexible ways. When exploring number combinations, students need to explore and understand number through subitising, part-whole relationships, and more-less relationships (Jung, Hartman, Smith, & Wallace, 2013). Research has shown students that develop strategies learn and maintain basic facts more effectively than those who rote learn (Neill, 2008). Through meaningful memorisation, facts become automatic in the long-term memory, freeing up the working memory for other aspects of mathematics (Neill, 2008). The following section explores how patterns, visualisation, and manipulatives support the learner to recognise number structure.

Subitising refers to the process of instantaneously recognising the number of items in any spatial structure without counting (Bobis, 2008). Subitising has long been recognised as an important skill for developing number sense (Clements, 1999). It is well known that even very young children are capable of instantaneously recognising numbers of objects up to four (Bobis, 2008; Jung et al., 2013). This is categorised as perceptual subitising, where the number of objects are recognised without using any other mathematical process (Clements, 1999). For most children, perceptual subitising occurs quite naturally. Once the quantity moves beyond five, subitising shifts to conceptual. Conceptual subitising is the ability to partition a spatial arrangement into its composite parts and at the same time recognise the whole (Bobis, 2008; Clements, 1999). This requires practise through showing students quick images where they can retain a mental picture of the image but do not have the time to count each item. Students learn to break the image up into parts. This promotes children's understanding of part-whole relationships through the composing and decomposing of numbers, exploring different number combinations, and place value.

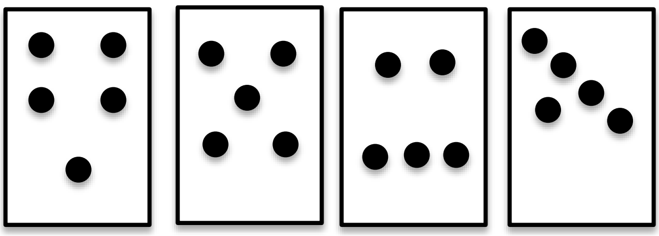


Figure 1: Different combinations lead to different decompositions of the number

Regular practise with dot patterns helps students not only recognise quantities without counting, but also recognise single-digit combinations building up their basic facts knowledge (Garza-Kling, 2011). Mulligan and Mitchelmore's (2009) research found that four- to eight-year-olds' awareness of mathematical patterns and structure was critical in the development of mathematical thinking. Effective mathematical reasoning involved the ability to notice patterns and structure, both in real-world situations and symbolic objects (Mulligan, 2013). Students who recognised and understood the underlying structure of one mathematical concept were able to transfer this knowledge to other concepts and learn to abstract and generalise.

Hunting (2003) found that students' ability to change from counting individual items to identifying the structure of a group was fundamental to the development of their number knowledge. Building a strong link between spatial structure contained in different patterns; that is, finger patterns and subitising activities and number knowledge, supported this development (Van Nes & de Lange, 2007). Mulligan (2013) advocates the need to teach mathematics as patterns, relationships and generalisations, rather than disconnected concepts and skills.

Research has shown that teachers in the early years do not provide students with enough subitising experiences (Clements & Sarama, 2014). When these did naturally occur in the classrooms, teachers undermined the experience by asking children to count and check after instantly recognising a pattern. The teacher's action of reinforcing counting over subitising unintentionally reinforced counting over pattern recognition, limiting student development of advanced counting and number sense (Clements & Sarama, 2014). Subitising is considered a more fundamental tool than counting for learning the cardinal value of numbers (Jung et al., 2013).

2.5 Modes of Representation

A representation is defined as concrete manipulatives, images/diagrams, or symbols, that symbolise or represent something else (Gagatsis & Elia, 2004). In this thesis, manipulatives refers to concrete materials and equipment used to teach mathematical concepts. As part of the NDP resources, a strategy teaching model encouraged teachers to move through phases of using concrete manipulatives, imaging and number properties (Ministry of Education, 2008a). The concrete-images-abstraction instructional sequence is recognised as an effective sequence for teaching a variety of mathematical skills and processes (Flores, 2010). First, concrete manipulatives are used to promote conceptual understanding. The teacher demonstrates the mathematical skill and/or process using manipulatives. Then the teacher guides students to use the manipulatives providing prompts and cues. Students then use manipulatives independently to demonstrate the skill and/or process (Flores, 2010). In the representation phase, which NDP defined as imaging, manipulatives are either shielded (Ministry of Education, 2008a) or replaced by diagrams or pictures. The final phase replaces imaging and the use of pictures and/or diagrams with the abstract representation of numbers. This process scaffolds the learner in developing mathematical understanding from the concrete to abstract. Folding back through previous phases is critical for students to connect mathematical abstraction with the actions of the concrete manipulatives (Ministry of Education, 2008a).

Sowell's (1989) meta-analysis of 60 studies on the effectiveness of teaching with concrete manipulatives confirmed increased achievement and positive attitudes in students with long-term use by effective teachers. Using a variety of manipulatives (Yetkin, 2003) and recording them through different representations, such as diagrams, pictures and symbols (Muijs, 2011), supported children to solve a range of problems in different contexts. Unfortunately researchers have found that students rarely have opportunities to use diagrams and/or pictures in mathematics class (Presmeg, 1986; Wheatley, 1991; Van Garderen, 2006). Additionally, the poor use of concrete manipulatives confused, distracted, and restricted learning if the student did not have the mathematical understanding to connect the manipulatives to the relevant concept (Boulton-Lewis & Halford, 1992).

The value of a concrete representation is that it mirrors the structure of the concept and the student should be able to use the structure to construct a mental model of the concept (Boulton-Lewis & Halford, 1992). Mathematical concepts do not inherently lie in manipulatives. Children construct number relationships by actively engaging with the manipulative in a variety of situations (Jung, 2011). Unfortunately, Baroody’s et al. (2012) research has found that some teachers use direct instruction to tell students what the manipulatives show and consider that this guarantees student learning. However, this may only produce routine expertise in which the strategy can only be applied in one situation, inhibiting transferable and adaptive knowledge.

Manipulatives should support the ability of a student to visualise the structure of a number. The most basic manipulative exists in the students' hands – their fingers are already grouped in fives. Finger-pattern activities help recognise the 'five and' quinary structure as well as other combinations (Whitenack, 2002).

The human eye cannot discern more than five objects without counting, unless those objects are set within an information rich context (Losq, 2005). Tens frames provide one context composed of a five-by-two grid in which counters or dots are placed. By exploring the features that make up the structure within the tens frame, students can be taught all number relationships, as the following example (Figure 2) shows within the combinations that total to 6.

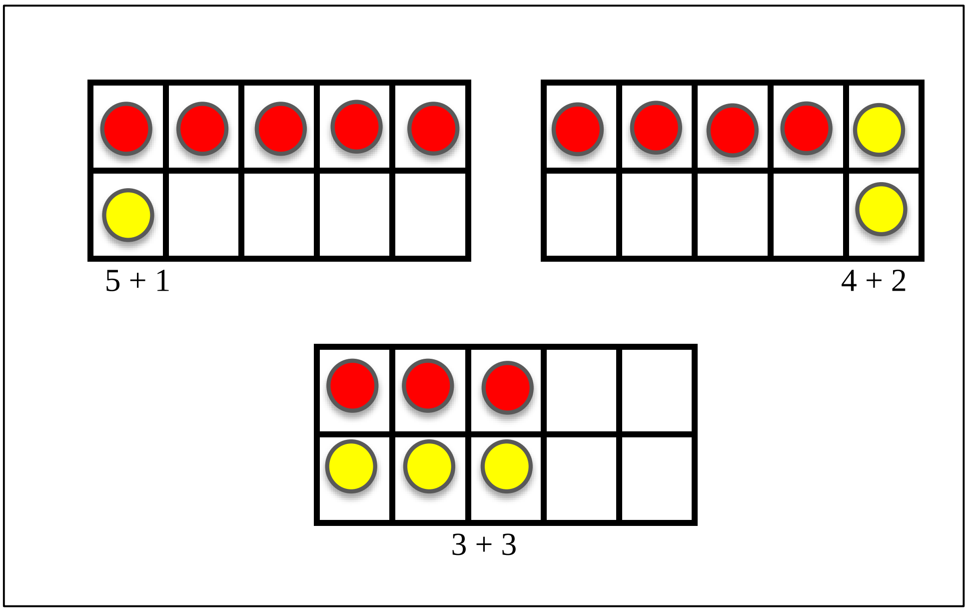


Figure 2: Ten-frames are effective in teaching number relationships, as this example of combinations that total 6

Activities that encourage students to investigate number composition and decomposition support rich connections that achieve fluency with their basic facts as well as becoming flexible, creative and strategic mathematicians (Garza-Kling, 2011).

When using manipulatives to support place value, it is important to use tools that support the concept and do not increase the processing load for the student. To avoid this, Boulton-Lewis and Halford (1992) recommend that the students know the material well. In their research around place value and cognitive loading, they felt exposing children in junior classes to a wide range of materials was effective when constructing a mental model of numbers. However, when it came to place value, they encouraged the use of a particular representation for sets of tens and units as they found students with regular experiences with one representation were more efficient at internalising the mathematical concept (Boulton-Lewis & Halford, 1992).

Teaching place value begins with the most concrete representation of ones and tens in which the students use sticks and bundle them into groups of ten. This encourages students to see a set of ten two ways; that is, one group of ten or one group of ten ones. Hugh's (in press) advocates that the part-whole connection is difficult for students to grasp and takes time to learn. Later on, base-ten value blocks can be introduced once the student has internalised the concept of tens and ones and understands that one object can stand for more than one countable unit (Losq, 2005). Unfortunately, many students are introduced to this model too early and do not fully understand this abstract representation (Losq, 2005).

Overall, research encourages students to spend time developing an understanding of mathematical concepts through the exploration of concrete manipulatives and visualisations. Through the exploration of structure and pattern, mathematical understanding becomes embedded. Research has shown that teachers do not use visual aids and concrete manipulatives enough (Westwood, 2006).

2.6 Numeracy Project Material

The NDP and support resources were implemented across New Zealand schools in early 2000 as a result of recommendations from the 1997 Mathematics and Science Taskforce (Higgins, 2003a). The taskforce highlighted a number of priorities in relation to improving mathematics performance in New Zealand, including the need to develop the pedagogical knowledge of teachers, improve quality teaching and teacher confidence, and provide resources and professional development to support mathematics teaching and learning (Ministry of Education, 1999).

The NDP resources used to support the teaching of the NZC achievement objectives (Table 1) in number included, *Book 1: The Number Framework, Book 4: Teaching Number Knowledge*, *Book 5: Teaching Addition, Subtraction, and Place Value*, and the nzmaths website. Young-Loveridge's (2010) review of mathematics education reform in New Zealand found that these components and the number framework required teachers to have a good understanding of mathematics as well as being very familiar with each resource. Research has also shown that many teachers closely followed a sequence in resource books, which may indicate low levels of confidence in their mathematics teaching (Young-Loveridge, 2010).

The following section provides an explanation and discussion of these resources, the recent update of Book 5, Book 4, and the nzmaths planning sheets, and how the key ideas link to the NZC.

The Level 1 achievement objective for number strategies outlined in Table 1 states that a student:

- use a range of counting, grouping, and equal-sharing strategies with whole numbers and fractions

Book 5 guides teachers through the development of student's mathematical thinking by presenting within each stage a set of key ideas that directly link to the Framework and NZC (Ministry of Education, 2012b). Each key idea is supported by a series of relevant learning experiences. Book 5 emphasises the link between place value and the different operations by incorporating place value thinking in a number of activities in each section by using the strategy 'with ten' or 'through ten' (Ministry of Education, 2012b p. 5). At the end of Level 1, students are advanced counters who have successfully mastered the following key ideas:

1. Numbers can be added by counting on from the largest number in increments of one.
2. Numbers can be subtracted by counting back from the largest number in increments of one.
3. Objects can be counted by creating bundles of ten.
4. Groups of ten can be added and subtracted by using simple addition facts.
5. Addition is commutative, so the order of the numbers can be rearranged to make counting on easier (p. 29).

At this point it is crucial that a student's place value knowledge is extended in preparation for part-whole thinking and students have a firm understanding of key ideas 3 and 4 (Ministry of Education, 2012b). Once secure in these concepts students are ready to move into developing early additive part-whole thinking. Again the Book 5 resource focuses on teaching strategy through place value. Initially part-whole thinking starts with doubles or fives strategies, then it becomes more sophisticated using with and through tens strategies (Ministry of Education, 2012b). The key ideas in this section are summarised below.

1. Addition and subtraction problems can be solved by part–whole strategies instead of counting.
2. Numbers can be partitioned and recombined to make a ten to solve an addition or subtraction problem. 9 + 5 = 9 + (1 + 4) = (9 + 1) + 4 = 14
3. Basic facts are essential when partitioning and recombining numbers (p. 37).

These key ideas, the learning progression and references are outlined within the nzmaths planning sheets, which also document key knowledge items that students need to learn to broaden their strategies. These two sections, strategies and knowledge, run parallel on the planning sheets reflecting the two main sections of the Framework. The NDP support resources separate strategy and knowledge with *Book 4: Teaching Number Knowledge* and *Book 5: Teaching Addition, Subtraction, and Place Value*. The Framework emphasises that it is important for students to make progress in both sections (Ministry of Education, 2007a). Johnston, Ward, and Thomas (2010) found that the practising of strategies supported the development of knowledge more so than vice versa. Students with a large body of knowledge may not necessarily be able to effectively use these facts to solve number problems or increase their strategy ability. In contrast, those students who were able to use strategies competently to solve number problems were also able to use these strategies to form new knowledge items.

The updated Book 5 has given teachers a more concise, user-friendly document. At the beginning of each key idea, key knowledge items are listed that students must know to work successfully in each learning experience. Book 4 lists a number of activities to build and maintain key knowledge if modified appropriately to meet the needs of the students (Ministry of Education, 2008b)

Researchers have raised concerns about the use of NDP support resources. These include the view that the separation of knowledge and strategies may encourage teachers to disconnect the development of each area, teaching them in isolation (Wall, 2004). Another concern is about strategies being emphasised at the expense of knowledge (Neill, 2008; Scouller, 2009). Subitising is briefly mentioned in Book 4 but is not encouraged as an alternative to counting, to determine the quantity of a small collections of objects (Young-Loveridge, 2010). Teachers who lack the confidence and understanding may use the support material as text books rather than see them as a source of ideas and strategies in which a framework of lessons can be formed (Scouller, 2009). Cobb (2012) noted that this may have resulted from the way the NDP has been presented as a ready-to-teach programme that if followed would ensure that most of the NZC achievement objectives in number and algebra were taught.

As part of the NDP, teachers were encouraged to "move their classroom practices away from an exclusive focus on computational procedures towards a focus on understanding mathematical ideas, relationships and concepts" (Anthony & Hunter, 2005, p. 26). This could change the learning from a process of knowledge acquisition to classroom mathematics communities where all students make sense and take ownership of the learning. NDP resources provided teachers with ready-made modes of teaching through the provision of problems linked to real-world experiences, examples of useful manipulatives, the structure of collaborative group work, open-ended questioning, and a range of possible student strategies and solutions. However, this does not ensure students' deep connections and developed understandings of mathematics, as teachers often adopt surface features of a reform programme (Fraivillig, Murphy, & Fuson, 1999).

2.7 Transition across numeracy classrooms

The process of transitioning learning to solve problems outside the initial context in which the concept is introduced involves looking at an idea or procedure many times in different ways, on different levels, and through different examples.

Failure to carry over prior learning can lead to more rigid patterns of thinking and behaviour (Haskell, 2001). Peters and Rameka's (2010) work in early childhood mathematics identifies that "learning is not just an accumulation of ideas and understandings, but a dynamic process of continuous germination, cultivation and pruning" (p. 12). As students make the transition from a junior classroom where counting is the main strategy, the question becomes about how they redefine and create a new way of solving problems.

Research has shown that young children construct their own knowledge and invent their own strategies in everyday situations (Aubrey, 1993). Aubrey suggests that young children need activities that offer problem-solving skills that extend and show purpose for knowledge. It is also important that their learning environment has an openness to be naturally inventive when solving problems and is not undermined by the teacher's single acceptable response (Aubrey, 1993).

As students shift from one learning environment to the next, they have to adapt to different roles, rules, and relationships (Bronfenbrenner, 1979). This includes different teaching philosophies, a shift in the curriculum content and the delivery. Belcher's (2006) research into the perceptions of new entrant children as they transition into a primary school numeracy classroom advocated the importance of sharing information across the sector. Communication across the settings and planned activities that link prior learning can help students to transition and adjust to their new environment.

Elements that make a transition difficult are outlined in Anderson and colleagues' (2000) research as institutional and social discontinuities. These discontinuities include changing class sizes, increased monitoring, more rigorous academic standards, the use of ability grouping, and teacher expectations. Social changes within the classroom makeup, relationships with peers and teachers, and a sense of belonging with more of an emphasis on behaviour, ability and competition can have detrimental efforts (Anderson et al., 2000).

In Bulkeley and Fabian's (2006) study of wellbeing and belonging during early educational transitions, importance is placed on levelling out change through continuity of experience, by having similarities in teaching approaches, an environment that promoted a sense of belonging and emotional well-being, and the teacher who is able to identify relevant, meaningful activities that created learning opportunities. Anthony and Walshaw (2008) advocate that an essential element of classroom communities is that students need to feel a sense of belonging before they are able to fully engage in mathematics.

Classroom communities make up the ‘cornerstone’ for building student mathematical identity and competence (Anthony & Walshaw, 2008, p. 196). The following section explores how teachers build on and develop student mathematical skills and abilities within the classroom community, as well as how scaffolding and grouping are used to support the transition of thinking.

2.8 Teachers

Teacher beliefs about mathematics have a significant impact on the teaching and learning that occurs in the classroom setting (Grootenboer & Ingram, 2008). The teacher has the most significant influence over student learning and this far outweighs the school itself at determining how a student will perform (Muijs, 2011). An effective teacher creates opportunities for students to think and experience ‘collaborative mathematical explorations’ (Anthony & Walshaw, 2008).

When examining the characteristics of effective pedagogical practices that support progress and achievement for students, the following areas are affected directly by the teacher. The first key area of the teacher’s mathematical knowledge is subject matter knowledge and pedagogical content knowledge (Shulman, 1987). Additionally, the teacher’s knowledge of the student as a learner (Hill, Rowan, & Ball, 2005) and their understanding of the curriculum (Chick, 2007) are important. Other characteristics include how a teacher facilitates the classroom community, and the tasks and activities students are set to enhance their learning and develop their mathematical thinking (Anthony & Walshaw, 2008; Bicknell & Hunter, 2009).

Subject matter (content) knowledge is defined as the mathematical expertise of the teacher. Teacher knowledge has significant impact on student learning of mathematics (Hill et al., 2005) with “low levels of content knowledge and the resulting lack of confidence about mathematics [limiting] teachers’ ability to maximise opportunities for engaging children" (Anthony & Walshaw, 2007, p. 45). The professional implication of having a strong mathematical knowledge is the potential to strengthen children's mathematical understanding and design activities that cater for a diverse range of learners (Anthony & Walshaw, 2007). However, only having a strong content knowledge may not improve student outcomes, as shown in Mewborn's (2001) research of pre-service teachers, as many participants were able to solve complex mathematical problems but unable to explain the concepts.

The Education Review Office (2006) report on the quality of mathematics teaching in Years 4 and 8 found that 78% of teachers had sound subject knowledge and 75% used effective strategies to engage the learners in mathematics. Effective teaching requires a strong pedagogical content knowledge. This includes the ability to mediate, facilitate, and support student thinking and reasoning.

2.8.1 Teaching and Learning

The separation of knowledge and strategy in the NDP teaching resources has been done for pedagogical reasons (Hughes, 2002). Hughes states that teaching knowledge and the development of strategically-based thinking warrant very different teaching models. Rowe (2007) agrees that a balanced approach is needed, combining direct-instruction (teacher-directed approach) for basic knowledge and skills with a student-centred (constructivist) approach when engaging in strategy development.

The NDP resource has limited material on how to go about teaching knowledge, as it is considered to be within the teacher's current models of teaching (Hughes, 2002). Historically, school mathematics teaching predominantly followed one of two approaches. The first, a **skills approach,** focused on procedurally-based teaching, with teachers transmitting information through direct instruction and practice, focusing on accuracy and routine expertise (Baroody, 2003). Mathematical knowledge was seen as simply a collection of facts, rules, formulas and procedures that students would master with repetitive practice.

The second instructional approach, labelled the **conceptual approach,** focused on meaningful memorisation, teaching skills, and concepts through a combination of procedural and conceptual content (Baroody, 2003). If students were shown why a procedure works, this in turn was expected to develop understanding and mastery of skills.

With the implementation of the NDP, recent mathematics education reform has encouraged teachers to focus on communication, interaction, and understanding of deeper mathematical ideas (Anthony & Hunter, 2005) This socio-constructivist learning perspective links to Piagetian and Vygotskian notions of cognitive development. The most radical of these, the **problem-solving approach,** focuses on the process of mathematical inquiry (Baroody, 2003). Students develop ways of thinking by searching for patterns in order to solve problems. The role of the teacher is to participate in the inquiry, at times pushing the process along, but it is expected that the students direct the inquiry with the focus being on reasoning, conjecturing, representing, and communicating. At times, content is accidentally discovered as students develop more mature ways of thinking. The assumption with this approach is that students have acquired sufficient prior knowledge and skills to engage effectively and productively to generate new learning (Westwood, 2006).

The final instructional approach, the **investigative approach,** utilises elements from the previous three approaches (Baroody, 2003). The development of mathematical thinking is seen as a network of skills and concepts, with the combination of meaningful memorisation. The teacher uses a process of inquiry in which students actively construct understanding. The difference between this approach and the problem-solving approach is that the teacher is more actively involved. The teacher mediates, guides, and prompts. Procedural learning is utilised in meaningful ways to support concepts and develop mathematical thinking. Some researchers interpret the investigative approach the same as guided discovery learning, but Yeo (2007) disagrees. He defines the differences as follows: with discovery-learning, students are given the opportunity to discover the solution that the teacher has already devised, whereas in an investigation the tasks are more open ended, unexpected discoveries are encouraged, and there are a range of possible solutions (Yeo, 2007).

A student's level of cognitive engagement ultimately determines what is learned from one episode of mathematical instruction. The ways and extent to which a teacher supports a student's thinking and reasoning may increase or decrease the level of cognitive demand. One of the factors that reduces cognitive engagement takes the form of *teacher lust* (Stein, Schwan Smith, Henningsen, & Silver, 2000). Tyminski (2010) has defined two types of teacher lust: enacted and experienced. Enacted teacher lust is an observable teacher action in which a teacher, unaware of their actions, removes an opportunity for students to think or engage in mathematics for themselves. By presenting their own conceptions and understanding, the teacher directly influences student thought, shutting down other avenues of thinking. Experienced teacher lust is an impulse by the teacher to act as stated above, consciously aware upon reflection or within the episode, of their influence on the possible outcome. Examples of enacted teacher lust include imposing mathematical structure or knowledge, directing and/or limiting student strategies, or telling information (Tyminski, 2010) in a manner that reduces the cognitive level of the task (Stein et al., 2000). An example of this is the decline into procedures without connection to meaning. Instead of students being given the opportunity to engage deeply and meaningfully with the mathematics, they end up using a more procedural, mechanistic, and/or shallow approach to the task (Pesek & Kirshner, 2000; Stein et al., 2000).

One feature prior to the implementation of the Number Framework and NDP resources was the reliance on an algorithmic approach to solving number problems. Students were taught that mathematics was about following certain procedures correctly with the main objective being to obtain the correct answer. NDP provided teachers with The Number Framework and resources to shift teachers' instruction away from procedural to conceptual, moving students through stages of development with an effective model to teach strategic thinking in number (Ministry of Education, 2008a). Researchers have raised concerns that even though the resources support a conceptual approach, strategies can also be taught in the same rigid way as an algorithm, if accurately followed step by step, guarantee a correct answer, side-stepping why and how the procedure works (Scouller, 2009).

There is a place for algorithms, which is clearly stated in NDP Book 1. That:

> Students should not be exposed to standard written algorithms until they use part-whole strategies. Premature exposure to working forms restricts students' ability and desire to use mental strategies. This inhibits their development of number sense (Ministry of Education, 2007a, p. 14).

The nature of mathematical knowledge and understanding is best described by Skemp (1976) as either instrumental or relational. Instrumental understanding, in a mathematical situation consists of being able to recognise a task as one of a particular kind for which one already knows a set process. Relational understanding is a more adaptable form of understanding in which mathematical problems are solved by relating elements within and outside the problem to appropriate schema (Carroll, 1994). In relational learning, the right answer indicates that the student understands the appropriate relationships between concepts, skills, and the problem situation. In instrumental learning, the right answer is the goal and the set process or rule is a means to get there.

The place of algorithms within instrumental and relational understanding is debated in skills versus understanding. Algorithms maybe considered harmful as they encourage students to give up their own thinking and "they 'unteach' place value, thereby preventing children from developing number sense" (Kamii & Dominick, 1998, p. 135). Alternatively instead of banishing algorithms completely from primary school classrooms, incorporating understanding within the teaching of algorithms is seen by some mathematicians as a very efficient process to acquire mathematical skills and understanding (Wu, 1999).

Skemp (1976) outlined the advantages and disadvantages of an instrumental approach to teaching, which focuses on learning facts and rules, and the relational approach, which focuses on strategies. He believes that the development of strong conceptual understanding comes from the recognition of the interdependence of the two approaches. Knowing how one is interrelated to the other "enables one to remember them as part of a connected whole, which is easier" (Skemp, 1976, p. 23). There is a common misconception that the demand for precision and fluency in the execution of basic facts undermines the acquisition of conceptual understanding. "The truth is that in mathematics, skills and understanding are completely intertwined" (Wu, 1999, p. 1). With "knowledge being the material on which the strategy operates" (Scouller, 2009, p. 6).

Primary teachers' subject knowledge is mediated by powerful feelings rooted in their own experiences of mathematics (Bibby, 1999). Research has shown that teachers' understanding of mathematics impacts the ways in which knowledge is used professionally in the classroom. Product-centred beliefs reflected in the teaching of procedural or algorithmic processes reflect the belief that there is one right method that is more right than all the others (Bibby, 2002).

There is a large body of research on affective issues in mathematics education. Many teachers experience strong negative feelings towards mathematics linked to internal constraints of anxiety and shame based on their feelings of inadequacy and limited understanding (Bibby, 2002). Additionally, the nature of the current teaching environment with time restraints, a crowded and constantly changing curriculum, teachers' work increasingly being scrutinised, and the practical aspects of the daily workload means that many teachers find it difficult to develop beyond the reassuringly straightforward product and/or absolutist certainties of procedural and/or algorithmic teaching (Bibby, 2002).

Evidence clearly indicates the need for teachers to have a greater understanding of mathematics and ways to support their students' mathematical development (Young-Loveridge, 2010). Students need to experience deeper more connected learning not only through explaining their strategies, but also having the ability to critique and justify them (Young-Loveridge, 2010). For teachers to make a substantial change in the way they approach mathematics, time is needed for them to develop a more sophisticated mathematical and pedagogical content knowledge base. The ability to elicit and use students' mathematical thinking is a complex and time-consuming process, which requires a lot of patience, practice, and the ability to effectively question, mediate, and prompt (Anthony & Hunter, 2005).

2.8.2 Scaffolding

Scaffolding, is a construct first used by Bruner (1986) to describe a form of assistance that supports student learning provided by the teacher or peer to bridge a gap in current knowledge to gain understanding. One way of forming new knowledge is through scaffolding learners when they are in what Vygotsky (1978) termed the zone of proximal development (Muijs, 2011). Scaffolding, in this thesis, is taken to mean one-to-one discussions between teachers and students, peer conversations, and the use of materials to support the learning. Cheeseman (2009) has described the connections between teacher and student as key to gaining insight into student understanding. “These moments offer rich opportunities for teachers to scaffold that particular student understanding through careful questioning, specific explanations, or by making links to situations, representations or manipulatives that resonated with that student” (Ferguson & McDonough, 2010, p. 178). The support of making connections to student prior knowledge, to develop a better understanding of the mathematical concept, is being able to utilise ‘teachable moments’ as they occur naturally throughout the session (Muir, 2008). A teachable moment refers to “a teacher’s simultaneous act in response to a student’s answer, comment or suggestion and is utilised to either address a possible misconception or to enhance conceptual understanding” (Muir, 2008, p. 362).

Building on student misconceptions is another important step in developing student reasoning and problem-solving skills (Eggleton & Moldavan, 2001). A necessary step in effective teaching is the art of questioning and letting the student explain how they came to that answer, whether it is right or wrong as this exposes student misconceptions and understandings (Muijs, 2011).

For teachers to be effective at scaffolding, they first have to access the point at which the student’s understanding is, and then support the construction of new knowledge by providing quality structure and feedback to develop cognitive growth (Anthony & Walshaw, 2008). To access students’ understanding, questioning is used to scaffold the learners from accessing what they know, to supporting the forming of new knowledge. Two types of questioning often identified are “funneling” and “focusing” (Wood, 1998, p. 167). The funneling scaffolding technique uses closed leading questions in which the questioning sequence directly guides students to a particular answer (Franke et al., 2009). The teacher dominates the conversation, using leading questions to guide the students’ thinking when presenting them with a new and challenging concept. In McGuire and Kinzie's (2013) study, when analysing place value instruction, leading questions were thought to be essential in the early years of schooling when building mathematical understanding and vocabulary.

The other questioning technique of ‘focusing’ identified by Wood (1998) is advocated by several researchers (e.g. Franke et al., 2009; Herbel-Eisenmann & Breyfogle, 2005). Focusing requires the teacher to listen to the student’s responses using a sequence of questions that probe the student to provide a full explanation and uncover detail of the student’s thinking. A teacher’s questions are based on student thinking rather than how the teacher would solve it. Research into teacher questioning as a tool to develop student thinking found that teachers used open-ended questions to gather students initial responses to solving a problem, but teachers found it difficult to follow up the students explanation to gather details on their strategy and connect this to other student strategies (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998). Anthony and Hunter (2005) agreed, adding that teachers struggle to extend to higher levels of mathematical thinking.

2.8.3 Grouping

This section briefly considers some of the positives and negatives for grouping students by ability. One of the significant changes facilitators noted with the implementation of the NDP was the implementation of small-group work in mathematics (Higgins, 2003b). In a recent Education Review Office (2013) report on mathematics in Years 4 to 8, most schools continued to use ability grouping within or across classrooms.

Grouping students for instruction by strategy stages makes it easier for the teacher to pose problems that are broadly in the students ‘zone of proximal development’. The aim of the grouping may be to teach a new type of strategy or key knowledge that is the foundation for the development of strategies (Ministry of Education, 2008b). The benefits of ability grouping for teachers are that they are able to streamline their planning and organisation, tracking student progress more closely within the classroom setting. Anthony and Walshaw (2008) found that the use of both individual and group talk helped to support and enhance student cognitive thinking. Higgins (2003b) found that the change to questioning and instructional grouping provided more opportunities for students to explain their problem solving strategies.

According to Wall (2004), the drawback of ability grouping is that learning becomes almost prescriptive, working through certain steps to gain the next stage.

Teachers in the United Kingdom (UK) tend to employ a more restricted range of teaching approaches, which impacts on the student in profound and largely negative ways (Boaler, Wiliam, & Brown, 2000; Nunes, Bryant, Sylva, & Barros, 2009). The exclusive use of ability grouping can limit students' expectations of themselves and negatively influence a student's self-confidence (Nunes et al., 2009). Grouping based on strategy stage may be limiting for knowledge development, as seldom do these two aspects correlate neatly (Wall, 2004). Professional learning encouraged teachers to identify, group, and teach students according to their strategy stage (Cobb, 2012). The teaching of knowledge was encouraged through whole-class warm ups and activities (Wall, 2004). Belcher (2006) has questioned the relevance of teaching whole-class items of knowledge, as students were passively engaged with no strong link between the forming of new knowledge, relevant context, and manipulatives. Specific targeted knowledge is needed to support students who lack the depth to apply certain strategies as the numbers become more difficult. Wall (2004) questions whether this specific, targeted knowledge development happens within the structure of ability grouping by strategy.

2.9 Assessment

Assessment information is one of the key sources of information shared from one classroom to the next. Successful transitions are based on teachers using assessment information to build on students' mathematical understandings and develop continuity from one setting to the next.

Effective assessment is a key component to quality teaching and learning and plays a significant role in informing teachers of the appropriate steps to improve student outcomes (Ministry of Education, 2011). Education reform in the NZC in the early 1990s signalled a change in the approach to assessment (Young-Loveridge, 2010). Formative and diagnostic assessment became a valid component for gathering information about the learner's needs and abilities, with an emphasis on contextualising assessment that demonstrated the "ability and inclination to use mathematics effectively – at home, at work, and in the community" (Ministry of Education, 2009, p. 6). The implementation of *Mathematics Standards for Year 1 – 8* built on the NDP (Ministry of Education, 2009). The purpose of the Standards is to improve mathematics education in New Zealand by aligning mathematics programming and assessments against the NZC When assessing students against the Standards, teachers are asked to make an overall teacher judgement (OTJ) based on multiple sources of information. A range of approaches is necessary in order to compile a comprehensive picture of the areas of progress, areas requiring attention, and what a student's unique pattern of progress looks like. Using a range of approaches also allows the student to participate throughout the assessment process, building their assessment capability. Because of this, to assess a student in relation to National Standards, teachers need to bring together a range of evidence in order to form an OTJ.

The diagnostic interview, Junior Assessment of Mathematics (JAM) and Global Strategy Stage Assessment (GloSS) are considered quality tools, which incorporate both 'formative' and 'summative' purposes. They are designed to give information about the knowledge and mental strategies of the student as well as make a judgment about the student's learning to date (Ministry of Education, 2008). JAM focuses on the first three years of school. It provides tasks that allow teachers to access student achievement in relation to Level 1 and 2 of the NZC (Ministry of Education, 2007b), and the Mathematics Standards for Year 1 – 3 (Ministry of Education, 2013). Formative assessment can be used to unpack a student's approach to problems and underlying misunderstandings.

The key to any assessment is the purpose behind it. If it has positive undertones of individual student support and improvement then it will be successful. However, when assessment is used to examine ability, competition, and comparison with others, it can diminish motivation for learning and increase feelings of negativity and anxiety (Black & Wiliam, 2001). Bicknell and Hunter's (2008) research of Year 6 students transitioning into Year 7 found 'fresh start' approaches meant reassessment before teachers were willing to form mathematics groups and begin the teaching of numeracy. Assessment tools like Progressive Achievement Tests (PAT), JAM, GloSS, and Individual Knowledge Assessment of Number (IKAN) were used but only as summative tools with their formative potential underutilised. In ERO's recent report on mathematics they suggest that teachers need to move beyond using achievement information mostly for grouping students and put more emphasis on inquiring into the effectiveness of their teaching strategies in terms of what works and what does not (Education Review Office, 2013).

Research into teachers' existing ideas around assessment, recording, reporting, and testing was that at times they were viewed as extra tasks (Swann & Brown, 1997). Specific features from these assessments were add-ons, as additional tasks within the classroom programme. Further, there is no evidence that these tasks are being integrated into the teachers' framework when thinking about their classroom teaching (Swann & Brown, 1997). It was intended that JAM would make up part of the evidence to moderate a teacher's OTJ (Mitchell & Poskitt, 2010). In Scotland as part of research into assessment in education, Hayward (2007) found that attainment targets dominated thinking in schools and classrooms. National tests intended to support or challenge the teacher's professional judgement instead replaced it.

In New Zealand, OTJs were intended to shift teacher focus from summative assessments to more formative processes, through triangulating data that incorporates situation learning and social interactions within the context of solving problems (Mitchell & Poskitt, 2010). Teachers as professionals, are capable of making appropriate judgements and are situated in the best position to gather information on a student in a range of contexts and assessment opportunities (Cumming, Wyatt-Smith, Elkins, & Neville, 2006). When making consistent judgements, a teacher must possess the understanding to assess the quality of a student response to a task in relation to the concept and link this to the Standard (Sadler, 1987). In the UK, the reliability of teacher judgements was low as the application of OTJs was problematic unless all teachers had a common understanding of the Standards, clear criteria, exemplars, and a process of moderation to ensure consistent judgements (Harlen, 2007). In New Zealand, in 2010 teachers had mixed understandings of the National Standards and applied them in different ways, and there was minimal experiences of moderation processes to keep judgements consistent (Poskitt & Mitchell, 2012).

Without effective moderation systems, teacher judgments may be unreliable and inconsistent from one teacher to another (Sadler, 1987). Some of the variables that can affect teacher judgements include:

- Inconsistences within one teacher over time
- The order effects (carrying over positive or negative impressions from one appraisal to the next)
- The halo effect (students personality interfering with the judgement of the student's achievement)
- Inconsistency among teachers with some teachers being too lenient and others being too severe (Sadler, 1987)

The concern with the implementation of National Standards is that there will be a shift in focus away from improving quality of mathematics teaching to an overemphasis on assessment comparisons between students, teachers, and schools (Young-Loveridge, 2010).

Research clearly shows positive outcomes for the learner when using detailed and quality assessment, which focuses on the student's thinking processes and next steps, rather than a measure at the end point. Assessment is a powerful tool that can either optimise or inhibit learning (Absolum, Flockton, Hattie, Hipkins, & Reid, 2009). When connecting student prior preschool experiences with the school setting, teachers who used more developmental approaches to learning created a more seamless transfer of knowledge with a child-responsive environment (Davies, 2009). Quality assessment practices are there to benefit the student's learning, and with effective, continuing support from the teacher, students are less likely to be discouraged and disengage (Absolum et al., 2009).

This chapter has reviewed the relevant literature identifying the different theoretical approaches to educational transition. The development of mathematical knowledge across Levels 1 and 2 of the NZC has been examined, including the key elements that support student shift from 'counting on' to part-whole thinking. The different teaching approaches, external influences, and assessment practices that impact on the teacher's role to mediate developmental progress have been outlined. Chapter three describes the methodology and research design used for this thesis.

Chapter 3: Methodology and Methods

3.1 Introduction

This chapter outlines the methodology and research design for this thesis. In Section 3.2, the interpretative methodology is framed within a case-study approach. Section 3.3 describes the method, including the context of the study, the role of the researcher, and the selection process for the participants. The methods of data collection are described in section 3.4. Section 3.5 presents ethical considerations, and issues relating to trustworthiness are considered in Section 3.6. The final section (3.7) is a description of the data analysis process, and the seven main themes and related sub-themes that emerged from the process and framed the data analysis.

3.2 Interpretative Methodology

The basis of this research was a comparison study examining the mathematical experiences a group of students have as they transition from Year 2 to Year 3. In researching the questions, both qualitative and quantitative methods were chosen in order to explore how students who were classified as being at Stage 4 Counting On in Year 2 experienced mathematical learning in Year 3. Additionally, the study explored the two teachers' perceptions on their transitions as they shifted levels, one from teaching juniors, and the other from teaching seniors, to both teaching Year 3 and 4 students.

The underlying research paradigm was an interpretative methodology, which used a range of qualitative methods from multiple perspectives. This explored a range of possible relationships, causes and effects that illustrated the complexities of knowledge acquisition. The strength of qualitative research is through the triangulation of data and the use of detailed descriptions of the setting, participants, and interactions bringing to light the evidence of how new knowledge is constructed. Good evidence is seen as embedded within "the context of fluid social interactions" (Tolich & Davidson, 2011, p. 33).

The purpose of this research was to translate the setting, situation, and group of participants in such a way that others reading the report would be able to make connections and relate it to their situation, in accordance with the ideas of Mutch

(2005). Case study is specific to the environment in which it is set and bounded within that particular complex system (Burns, 2000). The strength of a case-study approach is its ability to investigate and report on the complex and dynamic interactions within reality, providing a rich description of events. Blending this description with a detailed analysis that links themes to events to discover how children's competencies in mathematics develop over time.

Case-study research provides little scientific evidence for generalisation (Burns, 2000). Case studies are generalisable to theoretical propositions, not statistical populations. The researcher's goal was to expand theories, not to undertake statistical generalisation. When it comes to generalisation, researchers make no statements but try to help readers analyse the evidence and draw conclusions relevant to their personal situation (Burns, 2000).

The researcher is the primary instrument for data collection and interpretation. Qualitative researchers are "non interventionists" (Stake, 1995, p. 44) who attempt to blend into the surroundings to capture what would naturally occur if they were not there. They try to observe the ordinary and do this long enough to understand what the ordinary means within that particular case. As the research unfolds, researchers are guided by their own knowledge, instincts, abilities, and judgements. The notion of sensitivity is demanded within case-study research. The researcher must be sensitive to the setting, the people, and non-verbal behaviours.

Researchers deliberately or instinctively make role choices at the outset of the research process as noted by Bicknell (2009). Change can occur while researchers are engaged in the project, as they learn and develop their understanding of the process. As researchers consider their role in the research and level of participation in the research environment they need to strike a balance between participation and distancing. There is a range of choices the researcher makes throughout the research process, one of the first being to select the sample that will make up the case.

3.2.1 Case study

A case study is an in-depth exploration of a bounded system. Case studies focus on circumstantial uniqueness in which all evidence is “embedded in historical, social, political, personal and other contexts and interpretations” (Burns, 2000, p. 474). Stake (1995) divides case-study research into two main forms - intrinsic or instrumental (Hamilton & Corbett-Whittier, 2013). An intrinsic case study attempts to capture the case in its entirety. Instrumental case-study research concentrates on a key focus or concern. Case studies can be differentiated into three categories whether they are predominantly descriptive, interpretative, or evaluative (Cohen, Manion, & Morrison, 2007; Merriam, 1988).

Effective case-study research uses a range of sources to triangulate data, improving on the reliability and validity of the information gathered (Burns, 2000). In a case study, maintaining a chain of evidence is essential to verify conclusions through tracing back to the original source through the citation of specific observations, documents, and interviews.

Observations offer the researcher an opportunity to gather data from a naturally occurring social situation. An unstructured observation does not have predetermined categories, instead the researcher goes into the situation and observes what is taking place before deciding on its significance in relation to the research question (Cohen et al., 2007). Taylor and Bogdan (1998) describes the importance of “becoming an unobtrusive part of the scene, people the participants take for granted” (p. 45). The researcher cannot completely remove themselves from influencing the environment, but they can minimise their impact by not teaching, offering advice, providing assistance, speaking, or answering questions (Mertler, 2012). The more the researcher functions as a participant in the environment the more risk they have of losing their “eye of objectivity” (Mertler, 2012, p. 93).

The video camera allows a researcher to capture the multimodal nature of the classroom, including the physical and verbal interactions, body language as well as facial expressions. It provides a more permanent visual and audio record of what occurs, allowing a researcher to hone in on micro-details and preserving the ‘thick data’ that illustrates the dynamics of complex interactions, subtle interchanges, and sometimes the unexpected (Holm, 2008; Otrel-Cass, Cowie, & Maguire, 2010).

Interviews give participants the opportunity to share their views and discuss their interpretations of the world they live in (Cohen et al., 2007). Interviews are structured conversations between participants and researchers in which the researcher poses questions and gathers data from verbal and non-verbal responses. Rubin (2005) defined qualitative interviewing as responsive interviewing, where the researcher's aim is to gain a depth of knowledge rather than breadth. Questions are asked, followed up, and expanded on depending on the interviewee's responses. Rating scales within the research combine the opportunity for a flexible response with the ability to determine frequencies, correlations and other forms of quantitative analysis (Cohen et al., 2007, p. 327). The interviewer should consider the following characteristics throughout the interviewing process (Rubin, 2005): The style and approach which makes the interviewee feel most comfortable, the details of the research context and the researcher's ability to use appropriate follow-up questions to obtain depth, detail and clarity.

Documents obtained can be personal, official or gathered from an electronic medium. Certain documents such as planning and classroom artefacts provide evidence of what has happened prior to the observations, enabling links to be made with past events. The difficulty with documents is that they can be biased and selective as the author may only select certain information presenting an incomplete record. When reviewing documents, a researcher needs to consider the context in which the document was written, who the author was, and how reliable and valid the information was (Cohen et al., 2007).

The research question guiding this case study was: What support do "counting on" students receive in mathematics as they transition from Year 2 to Year 3? The present study fits the criteria of interpretive case-study as it was built around the teaching and learning of one group of students and the implementation of the mathematics curriculum. In this case, the bounded system is all the participants selected, who originated from one mathematics ability-based group within a Year 2 classroom. The researcher examined the teaching approach used at each year level and has described, compared, interpreted, and evaluated the observed events. This research is an interpretive case study as it contains rich descriptions and is used to develop categories based on common concepts to support and/or challenge theories. Data are gathered for the purpose of interpreting or theorising about the phenomenon.

3.3 Methods

3.3.1 Setting

This study took place in a decile 3, urban primary school (Year 0–6) in a provincial New Zealand town over two time periods. The first observation and interviewing period was a 2-week block in Term 4, 2013. The second 2-week block was in Term 1, 2014. The research was limited to one school and participants from one classroom to keep the study to a small scale so that the data obtained was manageable and detailed enough to determine informed findings.

3.3.2 Participants

The first step in the process of recruiting participants was the selection of a Year 2 teacher (Teacher A). This teacher was nominated by the Principal after careful consideration to ensure she would be willing and confident to share her classroom practices within the context of a detailed cross examination of a case study.

Once Teacher A agreed to participate, purposive sampling was used, based on her judgement of students who she felt best fitted the category of working at stage 4 "counting on to solve problems" (Ministry of Education, 2007a). These selected students were given information letters (Appendix F) and consent forms (Appendix G), and invited to participate. Consent was gained for Jack, Levi, Tamati, Aroha, Jessica, and Melanie (pseudonyms).

In the second phase of the study, circumstances dictated that Teacher A shift year groups (from Year 2 to Years 3 and 4) and the parents of four of the six students requested that their children be placed in her class. With this change in school structure, the study altered slightly from its original focus. The researcher decided to remain with the four students (Jack, Levi, Tamati, and Jessica) who had chosen to stay with Teacher A for Year 3. One student (Aroha) left the school and the final student (Melanie) shifted into a Year 3 and 4 classroom in which the teacher (Teacher B) was also new to that level, having had prior teaching experience at the more senior levels (Years 5 and 6). Alongside the student transition experiences, teacher transition experiences became relevant and were also included as part of the focus of the thesis. Teachers' shifting practices as they moved year groups and their developing knowledge of Level 2 of the mathematics curriculum were also examined.

3.3.3 Procedure

The collection of the data from the participants occurred at two distinct time points, one being Term 4 at the end of Year 2, and the second near the end of Term 1 in Year 3. The first set of observations and interviews were completed near the end of November 2013, during the fifth and sixth week of the final mathematics unit on number for the year. The rationale behind this timing was that this was the month the school management system wanted the final assessments for the year recorded. The Year 2 teacher (Teacher A) was identifying each student's stage of development. The students themselves had also experienced in-class organisation for a whole year and had an awareness of the way mathematics was organised and their ability within this setting.

The second set of observations and interviews was in 2014, at the end of March. This was during the students' seventh and eighth week at school in Year 3. The rationale behind this timing was that the students had had the opportunity to settle into their new class and be near the completion of their first mathematics unit on number for the year.

Within this study the multiple sources of data included interviewing, observations, assessment documents, and classroom artefacts. After each interview and observation, recordings were reviewed and written up in full while it was fresh in the researcher's memory. This ensured not only a more accurate recount of the dialogue between participants, but also the physical interaction, descriptions of the setting, activities and the researcher's initial reactions, feelings and interpretations.

3.3.3.1 Observations

The researcher's method of observation within this study was a participant observer. Within participant observation there are degrees of involvement within the environment. In this case the researcher chose to remain primarily an observer removed from the activity and simply observing and recording the interactions of the group.

The video camera allowed the researcher to capture how students reacted to each lesson, and how they went about solving problems, as well as what they said. The researcher used an iPad to video record all the teaching sessions. This enabled the researcher to capture the students' and teacher's movements as she followed the interactions between participants. The iPad was chosen because the students were familiar with this tool, as it was a piece of equipment that was already integrated into the everyday classroom environment. This hand held device was discretely used by the researcher on the perimeter of the teaching circle away from the focal point of the lesson. All images captured were dictated by the direction the researcher pointed the camera and this was determined by how the teacher directed the lesson and by alternating between pairs of students when they were set independent tasks.

3.3.3.2 Interviews

Within this case study, a semi-structured interview based on pre-determined questions initiated the interview (Appendix A and B). The researcher had the option to use follow-up questions or additional probing questions depending on participant responses to explore areas of interest relevant to the research. During and after each interview the researcher acknowledged and reflected on how their responses to the interviewee's answers may have influenced follow-up questions. The researcher also used member checking as a way to interpret the interviewees' responses through repeating and clarifying what was said; each interviewee was given the opportunity to validate their answers.

The researcher constructed two semi-structured interviews. One set of questions was designed for the teacher (Appendix B) and the second set of questions for the students (Appendix A). The initial interview was conducted after the first phase of observations while the students were in Year 2. The second interview was completed at the end of the second phase of observations the following year. Both the teacher and student interviews had predetermined questions that remained the same for each participant for cross-case analysis.

As part of the interview questions, questionnaire items were included. These questions were useful as they provided structure and numerical data that could be easily gathered and compared. These closed questions prescribed a range of responses in which the participant had to select one answer. This enabled comparisons to be made across the sample group and across the two time points. The researcher brought together a scale of measurement with opinion in the form of a Likert scale.

Within the student interview, there were three questions in which the answer used a Likert scale (see Appendix A). Two questions used a 5-point scale. The first of these questions asked how challenging the students perceived their maths, 1 being ‘too hard’ and 5 being ‘too easy’. As the scale used 5-points, the students were given the option of a mid-point of ‘just right’. The attitude question on how students felt about maths was also on a 5-point scale using smiley (1) to sad faces (5) to describe feelings towards maths. A neutral face was used for the midpoint. The other Likert scale question was based around assessment and how the students felt they were going in maths. A 4-point scale was used removing the midpoint so that the students had to select either a positive or negative position.

3.3.3.3 Documents and classroom artefacts

The range of documents that were investigated within this case study included the teacher’s planning, teaching resources, and classroom artefacts. The teacher’s planning and resources were guided by the NDP resources, particularly *Book 5: Teaching addition, subtraction, and place value* (Ministry of Education, 2012b) and the addition and subtraction planning sheets taken from the nzmaths website. JAM (Ministry of Education, 2013) was the main assessment document used to assess each student in relation to Level 1 and 2 of the New Zealand Curriculum and the Mathematics Standards for Years 1 – 3.

Classroom artefacts included anything that was written or visual pertaining to the study. Within this study, examples of schoolwork, the teacher-modelling book, performance-based assessments, and the follow-up games and activities contributed to the data collection.

3.4 Ethical Considerations

This research was conducted following the ethical guidelines laid out by the Ethical Conduct in Human Researcher and Related Activities Regulations, 2008, from the University of Waikato. Full ethical approval for this study was given from the Faculty of Education Ethics Committee. Informed consent was gained from the Principal and Board of Trustees to carry out this research within the selected school (Appendix C). All participants were given a clear description of what the research involved and how it would be reported. Permission was also sort from the parents/guardians of the children involved. The participants took letters of consent home to discuss with the parents/guardians (Appendix F and G).

The principle ethical issues in this case study were confidentiality, informed consent, and the value of the time given by participants. The issues of confidentiality were addressed by participants' identities being kept confidential, with the researcher having the only access to primary data collected (video-tapes and interview notes) and then by assigning pseudonyms to each participant during the writing process.

An introductory meeting was held within the target class to explain the research process and what their involvement would entail. Only students who had both parental and personal permission forms participated. The students were also encouraged to raise questions at this meeting and throughout the study rather than being held to a single binding permission statement, to try and resolve the issue of gaining a child's full comprehension of what it meant to be a research participant. Caregivers and the researcher independently explained the research process. Permission slips were used incorporating child-friendly language and graphics. Time was also allocated throughout the research to explain the process and answer questions as they occurred. A further safeguard was that students were advised (and reminded) that they had the right to withdraw their participation at any time (until data collection was completed).

Another ethical issue was the value of time for the participants. This issue was addressed by involving students who would normally have teaching on this topic, so the teaching sessions were not an extra to their planned programme. The two student interview sessions were scheduled at a time that was convenient to the teacher so as to minimise disruption. The teacher interview was scheduled at a time that was most convenient for each teacher involved, such as a before-school time slot.

3.5 Trustworthiness

Within case study research, reliability and validity is replaced with the notion of trustworthiness (Guba, Schwandt, & Lincoln, 2007). To address whether this research was a true representation of events, four key aspects were addressed: credibility, transferability, dependability and confirmability (Guba & Lincoln, 1989).

To validate the findings and create an accurate and credible picture of the events, the following criteria addressed issues of credibility. Persistent observations (Guba et al., 2007) over time contributed to gaining a true picture of what occurred in each mathematics classroom. Repeated observations removed the 'one-off performance,' giving a more accurate picture of what happened within each setting. Reflexive researchers not only record their role but consider their impact on the research site, participants, and data collected (Cohen et al., 2007).

The triangulation of data by using two or more methods of data collection verified aspects of student and teacher behaviour (Burns, 2000; Cohen et al., 2007). This occurred by gathering perspectives from each individual within the environment, cross checking with different types of data (e.g., observations, interviews) and data collection sources (documents and classroom artefacts) (Creswell, 2008). The researcher examined a range of data identifying evidence to support different themes establishing a chain of evidence that linked parts together (Burns, 2000).

Transferability was achieved when 'thick descriptive data' were used to describe the setting and events. This made the findings translatable, so that the reader can make comparisons to other situations.

In this case study, the researcher immersed herself within the context, establishing a rapport with both students and teachers. To remain objective and meet the criterion of an unobtrusive observer, the researcher clearly identified her role in the environment, thus enabling the reader to see the researcher's position. This needs to be considered with two of the students being prior students in the researcher's Year 1 classroom and both teachers having received professional support from the researcher in the years leading up to the study. The conclusions the researcher reached were based on the depth of experience the researcher had in the field of primary mathematics and qualified by the prior experience of being a mathematics lead teacher and numeracy advisor. They are the interpretations of events from one perspective and the researcher's viewpoint only. Highlighting the researcher's experiences and background can shape interpretations. The author's conclusions are only tentative and inconclusive and may open up more questions for the reader to consider.

3.6 Data Analysis

3.6.1 Organisation of data

With the research question as a focal point, key issues and reoccurring events were identified and organised into categories (Bogdan & Biklen, 2007). Video and audio recordings presented the researcher with a large amount of data. The video recordings provided data of what happened in the classroom observations. The audio recordings gathered data of participants' perspectives, beliefs, and values of mathematics within the interviews. Initially the researcher viewed and transcribed all information that was captured. In the second round of data collection, the researcher became more selective, transcribing events that related directly to the categories that had been identified after the first round of coding.

Each transcript was reread and checked against the original footage to correct errors, check for accuracy, and clarify points – adding details where needed. The advantage of the researcher transcribing all the records was that she was able to review the videos as many times as needed to ensure that each event was as accurate as possible. She was also able to add insights alongside events, interpreting episodes as she went, and identifying themes as they emerged. Photos of activities, modelling book information, and assessment documentation were attached to each appropriate transcript.

3.6.2 Analysis of data

A thematic approach is appropriate for open-ended research. As themes and/or concepts appear, they become the conceptual framework in which the researcher manages and organise data (Mutch, 2005). Using the research questions and critical literature review, the researcher developed a thematic framework (Menter, Elliot, Hulme, Lewin, & Lowden, 2011), which evolved as information was gathered and transcripts were reviewed and coded. Transcribing the video and audio recordings gave the researcher an opportunity to zoom in on micro-details, capturing subtle interchanges between the students and the materials they used to solve problems, and adding layers of complexity within the coding (Otrel-Cass et al., 2010).

Qualitative analysing software can assist the researcher with the managing, sorting, indexing, arranging, and rearranging of data (Menter et al., 2011). *Dedoose* software (Sociocultural Research Consultants, LLC, Manhattan Beach, CA, USA) helped organise material developing systems for categorising information. The framework for analysis evolved as the researcher gained a deeper understanding of the data and how the themes and concepts were related to each other. The advantage of using software to analyse the data was that the researcher was more flexible and adaptive as key themes appeared, and a more precise system of coding was applied. Another advantage of using *Dedoose* software was that it made the analysis as systemic and as transparent as possible, which when shown to others, provided a clear 'trail of evidence' increasing the rigour and trustworthiness of the study.

The researcher transcribed three in-class video-recordings of mathematics for the targeted students in week 5 and 6 of Term 4, 2013. Although the recordings were close to and facing the participants it was difficult to capture all the students within the one lens shot. Similarly, when they broke into pairs to solve problems, it was difficult to capture all student calculations at the same time. Consequently, the researcher alternated between pairs and groups of students, capturing their ways of working on particular problems. At times, the whole group was not captured so comparison across the group was made when problems of a similar nature were presented, rather than the exact same mathematics problem.

Similarly, when transcribing the three in-class video recordings the following year in week 7 and 8 of Term 1, 2014, at times only one set of students were captured processing the problem. It was a lot easier because the group was split into four students in one class and one student in another. The single student had a complete record of each session and the 4 students had a fuller version of events. Because the initial coding was completed before the second set of observations, the researcher targeted areas of interest relevant to the data that had already been collected.

The researcher also transcribed the digital audio-recordings of the initial and final interviews of the students and the two interviews from Teacher A and B. Transcripts were checked for accuracy and compared with the original recording. Teacher transcripts were checked by the teacher confirming the details by a signature on each page of the transcript.

Photos of group modelling books were taken to gather information on learning outcomes, sample problems, and teacher annotations. The modelling book also contained examples of how particular students had solved problems. Photos were also used to record examples of students' ways of working on particular problems.

The researcher used a thematic, inductive approach, which allowed categories, themes, and patterns to emerge from the video and audio data. Initially coding started in a simple way, by identifying big themes. As the process of analysis developed, big themes were added, and some themes became main themes with sub-themes attached, adding further layers of detail. As each transcript was coded, sections of text were coded based on whether the event fitted the description attached to each code. The initial ten themes were narrowed down to seven key themes (parent codes) and their sub-themes (child codes), and these are described in Table 2.

Each transcript that was put into *Dedoose* was connected to a descriptor field, which attached a participant to the document. A particular document may have been entered a number of times as different participants had to be attached separately. The descriptor field included information about the participant's ID, age, gender, and quantitative information of his/her JAM results from 2013. Each document was tagged whether it was phase 1 (November data) or phase 2 (March data). Then each document was coded according to the themes outlined in Table 2.

As part of the semi-structured interviews (Appendix A), the students were asked three Likert-scale questions (as outlined above). Of these three questions, two were integrated within the qualitative themes. The following rubrics linked these questions to evidence in the classroom-observation transcripts. Through a rating scale built into the coding structure of mathematical content, *Dedoose* was used to expose relationships between the two forms of data. The following table shows the combination of a Likert-scale question and the rating scale of 1 – 5 used to code mathematical content observed in the classroom.

Table 2: Coding for analysis for observations

Id	Parent Code	Depth	Child code	Description
1	**Transition models**	0		
	1i	1	**Ecological Model**	Expectations and events in the larger society.
	1ii	1	**Zone of Proximal development**	Refer to Challenge Rubric
	1iii	1	**Deficit model of assessment**	New assessments gathered in term 1 to gain information on what skills students do or do not have.
2	**Assessment**	0		Assessment data gathered on each student's achievement.
3	**Mathematical Content (T)**	0		Mathematical content connected to the **Teacher**
	3i	1	**Number Knowledge (T)**	Mathematical content that is linked to number knowledge areas: number id, sequencing, place value, fractions and basic facts
	3ii	1	**Strategies (T)**	Addition and subtraction strategies
4	**Mathematical content (S)**	0		Mathematical content connected to the **Student**
	4i	1	**Strategies**	Evidence of a strategy being used to solve addition / subtraction questions
	4ii	1	**Knowledge of number**	Key items of knowledge that students either already know or need to learn.
	4ii(a)	2	**Basic facts**	Subgroup of Number Knowledge
	4ii(b)	2	**Place Value**	Subgroup of Number Knowledge
5	**Modes of representation**	0		Representations used to clarify mathematical content within a lesson
	5i	1	**Diagrams**	Visualisations
	5ii	1	**Symbols**	Characters or symbols
	5iii	1	**Manipulatives**	Concrete Materials
6	**Questioning**	0		What types of questions are used?
	6i	1	**Focusing**	Sequence of questions that probe the student to provide a full explanation and uncover detail of the student's thinking
	6ii	1	**Funneling**	This scaffolding technique uses closed leading questions in which the questioning sequence directly guides students to a particular answer.
7	**Attitude**	0		Attitude towards mathematics from the perspective of the teacher as well as the student.

Table 3: Student mathematical content on a Likert scale and rating scale connecting an interview question with observations

Interview Question **Challenge** – How challenging did you find maths this year? Students selected one of the following within the semi-structured interview				
Too hard	Hard	Just right	Easy	Too easy
Theme: Mathematical Content (S) Each piece of text coded under the theme: mathematical content (S) had a rating scale attached. A number between 1–5 was assigned based on the criteria outlined below.				
1	2	3	4	5
Student is unable to solve or complete the task/problem and has no idea how to go about it.	Answers take a long time, and at times the student becomes confused or loses track of the process or reverts to a primitive strategy to solve the problem.	Answers are challenging but with materials or time, answers are calculated.	Answers are easily calculated	Answers are instant

An additional rating scale was added to the coding structure for manipulatives within modes of representation. The following scale (Table 4) was drawn from internationally accepted standards adopted by the National Council of Teachers of Mathematics (NCTM). The rubric is cumulative, with each column building on the previous column. Data were taken from classroom observations and a number assigned for how effectively manipulatives were used in each mathematics session.

Using a code rating scale along with certain codes opened up a variety of additional analyses. Giving a ‘value’ tag to each qualitative response in the student mathematical content and manipulatives themes divided coding into different categories and indicated average ‘value’ for certain situations and individuals creating an overall picture. This was linked to visualisation showing underlying patterns and trends of what was happening in the data.

Table 4: A rubric for the teacher and student use of manipulatives

Manipulatives	1	2	3	4
Students will understand and use manipulatives appropriately.	Manipulatives may not be visible in the classroom or accessible to teachers or students. The teacher does not model use of manipulatives and students do not use manipulatives. Students use manipulatives inappropriately.	Manipulatives are visible in the classroom, but not readily accessible to students. The teacher models use of manipulatives and directs student use of manipulatives. Students imitate use of manipulatives without reflection, exploration, or connection to symbols, pictures, and explanations.	The teacher models the Decision-making process for choosing appropriate manipulatives to give meaning to abstract concepts. The teacher actively engages students in using manipulatives to construct and give meaning to new concepts. Students independently select appropriate manipulatives to make connections from symbols, pictures, and explanations to concepts in order to problem solve and represent their understanding of mathematics.	The teacher scaffolds students' understanding so they become less reliant on manipulatives. Students can demonstrate their knowledge of abstract relationships using symbols, pictures and explanations, but are no longer dependent on manipulatives. Students have internalised use of manipulatives and can describe how manipulatives were used to develop an understanding of mathematical concepts.

Adapted from the Georgia Department of Education (Bermuda Ministry of Education, 2010).

3.7 Summary

This chapter has described the methodology and research design used in this thesis. Data collection methods have been described and discussed, together with the ethical considerations and issues relating to the trustworthiness of the data. The organisation and processes used to analyse the data was described in the final section. The categories defined by the key themes identified in the data analysis, frame the structure for Chapters 4 and 5.

Chapter Four: Results

This chapter reports on the analysis and findings from two different data sets. The first set, qualitative data, came from classroom observations and open-ended questions in the semi-structured interviews (Appendix A & B). The second set, quantitative data, came from student assessments, Likert-scale questions, and code-rating scales. The combination helped answer the following question:

> What support does a ‘counting on’ student receive in mathematics as they transition from Year 2 to Year 3?

The results pertaining to this question were divided into six sections. The first two sections (“Mathematics in Year 2” and “Mathematics in Year 3”) examine mathematical content and classroom practices. The third section examines evidence of how the teachers in the study assessed mathematical thinking. The fourth and fifth sections report on the modes of representation and questioning that teachers used to support, develop, and extend mathematical thinking in Years 2 and 3. The final section reports on both student and teacher attitudes towards mathematics and transition.

4.1 Mathematics in Year 2

The five case study students were selected by their Year 2 teacher based on the criterion that they were advanced counters ‘stage 4;’ that is, counting on to solve problems. Within this small mathematics group, there was still a range of ability and the students fell into two sub-groups. One student was above the criterion already solving problems by using part-whole thinking (stage 5), whereas the other four students were predominantly ‘counting on’ to solve problems.

At the end of Term 4 in the three classroom observations, the mathematical content of instruction was focused on beginning to shift the students from the advanced counting stage (stage 4) to the early additive stage (stage 5). The key idea in the first two observed lessons was that “groups of tens can be added and subtracted by using simple known addition facts” (Ministry of Education, 2012b, p. 35). Students used bundles of sticks and basic facts knowledge to join groups of ten in problems like:

Aroha had 64 lollipops. Melanie gave her 20 more. How many does she have now?

The third lesson focused on the key idea that "numbers can be rearranged and combined to make ten" (Ministry of Education, 2012b, p. 40). An example of the problems posed was:

Levi catches 6 fish, Tamati catches 7 fish and Jessica catches 4 fish. How many fish did they catch altogether?

Figure 3 shows the mathematical content coding activity and rating scale completed in 2013. This figure explains the case study student's ability to solve problems when dealing with mathematical content. Mathematical content was rated on a scale from 1 to 5, depending on student responses, as outlined in Table 5. A rating scale of 3 was given when content was considered to fall within the student's zone of proximal development.

Table 5: Descriptions of rating scale criteria

Rating Scale	Description
1	Student is unable to solve or complete the task/problem and has no idea how to go about it.
2	Answers take a long time, and at times the student becomes confused or loses track of the process or reverts to a primitive strategy to solve the problem.
3	Answers are challenging but with materials or time, answers are calculated.
4	Answers are easily calculated
5	Answers are instant

In category 2 on Table 5, primitive strategies had to be clearly defined when coding each excerpt. A primitive strategy was defined as a strategy that the lesson did not intend the student to use, and was classified as being at a lower stage on the Number Framework. 'Counting on' was accepted as the student's main strategy when solving problems independently and when the lesson was focused on advanced counting. When the lesson focus was on shifting the student from advanced counting to early additive, if the student reverted back to 'counting on', this was classified as a primitive strategy.

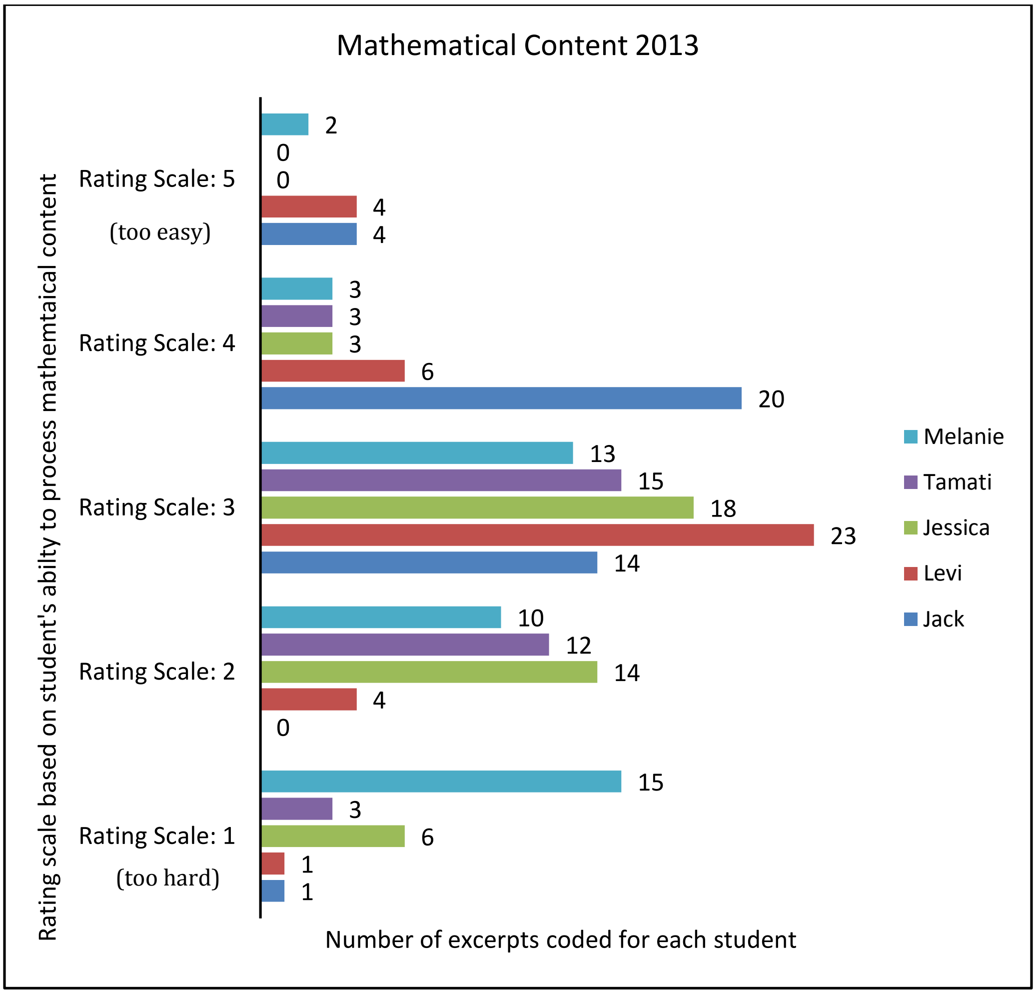


Figure 3: Student ability to solve mathematical content in classroom observations in 2013

4.1.1 Transitioning into part-whole thinking (Year 2)

The most able student, Jack, predominantly found the mathematical content 'relatively easy' (4) or 'just right' (3), solving a number of problems using part-whole strategies. In the first lesson, Jack, with the help of manipulatives, partitioned 37 into 20, 10 and 7 and used simple addition facts to solve the following problem: 40 + 37.

> Teacher A & students: 10, 20, 30, 40 [Teacher A points to each bundle as they count] So we've got our 40 haven't we [points to 40 in the equation] now we need to find 37…
> Teacher A: 10,
> Chorus: 20, 30
> Tamati: 7
> Student: (Places down another bundle)
> Teacher A: We don't want a bundle, we want 7.
> Teacher A: 1, 2 3, 4, 5, 6, 7 Let's see if she's got it. She's got her 40 and her 37 [pointing to each bundle] Can you count with me? [pointing to each bundle]
> Chorus: 10, 20, 30, 40, 50, 60, 70, 80 ….79

Jack: 77…
Jack: I had 40 in my head and I added 20 more then I added 10 more
Teacher A: What's 40 + 20 more first of all?
Jack: 50, oh no, 60
Teacher A: Then what did you do?
Jack: Added 10 more and then I added 7 more
(Teacher A records 40 + 20 = 60)
+ 10 =
Teacher A: What's 10 more first of all….60 + 10 more?
Jack: 70
Teacher A: and then you added
Jack: 7 more
Teacher A: and what did it equal?
Jack: 77

In the third lesson, Jack looked for known combinations focusing on his doubles knowledge to solve problems that were originally designed to have the students combine two numbers to 'make 10'. The following problems were examples of this:

Question 1
Teacher A: We had 7 paua, then we went and got 4 more paua, and Jessica caught 3 more paua.
(7 + 4 + 3 =)
Jack: I knew what 4 + 3 was, that equals 7 and then 7 + 7 was 14

Question 2
Teacher A: We have 6 + 7 + 4
Jack: I remembered what 6+6 was, then I added one more, and it equalled 13, and I added 4 more and it equalled 17.

In the second example, Jack partitioned 7 to access the double. These three examples indicate strong part-whole thinking. Jack reverted back to 'counting on' in the independent activity, Quick 10. One of the activities the majority of the class had to complete each mathematics session was to answer a list of basic facts questions (Quick 10) displayed on the whiteboard. The questions below are 8 of the 10 questions his group had to complete.

Wed 6th Nov	Mon 4th Nov
1. 5 + 5 =	1. 10 – 7 =
2. 8 + 3 =	2. 8 – 8 =
3. 6 + 3 =	3. 19 – 4 =
4. 3 + 4 =	4. 18 – 2 =
5. 10 + 2 =	5. 12 – 5 =
6. 7 + 3 =	6. 17 – 5 =
7. 7 + 5 =	7. 13 – 4 =
8. 5 + 3 =	8. 13 – 5 =

Figure 4: Quick 10 - daily basic facts

Jack was able to instantly answer 5 + 5 and 10 + 2, but for all the other questions, he ‘counted on’ using his fingers. Two days previously, the students had been given the subtraction quick 10. Jack used his fingers and ‘counted back’ to solve all of these questions.

In Term 4, 2013, Jack was observed using part-whole thinking when working with the teacher on addition strategies. Independently, he reverted back to ‘counting on’ and ‘counting back’ when presented with a list of basic facts. In this situation the use of video-stimulated interviews would have prompted the student to clarify why he was able to part-whole in the teacher’s lesson but reverted back to ‘counting on’ and ‘back’ when working independently. This was not possible under the framework of this study.

4.1.2 Counting (Year 2)

With the same lessons, the other four students in the group, Levi, Jessica, Tamati, and Melanie, predominantly used the ‘counting on’ strategy, both in the teaching sessions and independently. The difference between Levi and the other three students was in the process used to ‘count on’. Jessica, Tamati and Melanie all touched their heads before ‘counting on’ with their fingers. This was a technique the teacher encouraged in class to help students ‘count on’ and ‘count back’. The tap of the head represented the students putting the first number in their heads, then from that point they either ‘counted on’ or ‘counted back’ to find the answer. Levi and Jack were the only two members of the group that did not tap their heads to ‘count on’. It was a small difference but appeared to be an extra step to accessing the answer for Jessica, Tamati, and Melanie. Below is an example of this:

Teacher A: 6 + 7 + 4
Tamati: [Touches his head and counts on fingers]
Levi: [Mouths number and counts using fingers]
Jack: [Looks up and mouths numbers and counts without fingers]
Researcher: [unable to tell from video footage which number they started from]
Teacher A: Ok Jessica you had your hand up straight away. What did you do?
Jessica: We have to put the 6 + 7 first
Teacher A: Mm so what’s 6 + 7
Melanie: [touches head and counts 7 fingers]
Jessica: [gets distracted and does two attempts using fingers both times]
Teacher A: What’s 6 + 7
Jessica: … 13
Teacher A: Mm then what did you do?
Jessica: Put 4
Teacher A: Ok what did that come to?
Jessica: [touch head and count 4 fingers] … 17

Another difference between Levi and Jessica, Tamati and Melanie, was the number of mathematical content items scaled in the 4 and 5 zones (i.e., too easy). Levi was able to recall accurately more basic facts answers as indicated on Figure 3 rating scale 4 and 5. Although he was not transferring a lot of this knowledge into the addition strategy lesson at this time, he was able to find the missing addend that completed each equation to 10 very quickly at the beginning of the lesson.

Teacher A: What’s another one, 7 +…
Jessica: 4, 2
Levi: 3
Teacher A: 3 awesome, 8+…
Jack: 2
Melanie: 2
Levi: 2
Teacher A: 9 +…
Jack/Levi: 1

He also was the first to notice patterns in the group in the adding tens lesson.

Teacher A: Put your ten there Tamati [Tamati places it down] Where’s your ten
Jack: [Jack places it in front of himself]. Put it next to it [pointing to Tamati’s bundle]
Levi: 10, 20
Teacher A: Oh well done, Levi. What did he just do?
Melanie: He went 10, 20
Teacher A: So he is counting in his?
Melanie: 10s

Levi, who was more independent, could readily access mathematical knowledge and notice patterns and structure. His ‘counting on’ was very accurate and quick and he was able to access some basic addition facts easily. When examining the mathematical content, the majority of the excerpts fell into the ‘just right’ category.

The mathematical content Jessica, Tamati and Melanie worked on in class was split between the category of ‘just right’ and the category 1 and 2 in which either it was too hard or took too long to calculate. In category 2 Jessica, Tamati and Melanie became confused, losing track and having to start over. At times, the teacher or a more able peer heavily scaffolded them. Another example of category 2 was content that should have been calculated easily but was processed through a primitive strategy. An example of category 2 was part of a question Jessica should have easily recalled instantly as part of her ‘one more’ and basic facts knowledge without the use of fingers.

> 9 + 7 + 1
> Jessica: 9 + 7
> Student: [points to 9 + 1]
> Jessica: 9 + 1 [pointing to the 9 and 1] what is it [touches her head and holds up one thumb] 10 [touches her head…pauses]
> Student: [Records 10 + 7 and puts 17 without calculating]
> Researcher: [Unable to tell if Jessica knew the combination 10 + 7 instantly due to her partner recording it before she had a chance to respond or calculate]

This basic number knowledge of knowing that the combination 9 and 1 make 10 was limiting her ability to achieve the objective of the lesson, which was to rearrange and combine numbers to make 10. She lacked the knowledge of the different parts that make up 10, and without guidance would have worked through the question from left to right through a process of counting and storing numbers in her head.

All three students lacked fluency in their basic facts, using their fingers to solve all addition problems, including basic combinations during the teacher’s strategy-teaching session. Figure 3 reflected low numbers of questions with instant or easily calculated answers. Melanie deviated slightly with two responses in the 5-scale category, due to having the opportunity to answer two very easy basic facts questions, 3 + 0 and 6 + 0 which she responded to instantly.

4.1.3 Year 2 Teacher's navigation of the learning

To develop the students' knowledge of adding tens, Teacher A began with the students building bundles of 10. Using this material and the students' knowledge of skip counting, the teacher started off with the simple question: how would you combine 39 and 20? The first step was to place 3 bundles of 10 and 9 sticks in one area for 39, and in a separate area, 2 bundles of 10. The following example shows how the joining of 2 numbers was introduced:

> Teacher A: Shall we count them again? [points at each bundle as students count]
> Chorus: 10, 20, 30, 39, 40, 50
> Teacher A: That's a bit awkward isn't it? How else could we work it? What could we put altogether to make it easier to count?
> Levi: 10, 20, 30…
> Teacher A: Oh Levi I think you just said it. Tell us again.
> Levi: 10, 20, 30, 40, 50…
> Teacher: What did you just do…you joined all the…
> Levi: 10s together
> Teacher A: Could you do that for us? Can you move them over so they are all together? [Levi rearranges the 10s bundles so they are together]
> Teacher A: Do you think that will be easier now to count?
> Chorus: Yes
> Teacher A: Shall we give it a go? [Pointing to each bundle as students count]
> Teacher A & Students: 10, 20, 30, 40, 50
> Tamati: 39
> Jack/Levi: 59
> Chorus: 39 & 59 [mixed responses from the group]

After completing a number of examples in the same way, Teacher A challenged the students to try and complete a question without the bundles of sticks.

> Teacher A: What's another way you could work it out without sticks?
> [Silence]
> Teacher A: Not yet. Ok. What about if I went like this
> 40 + 37 = [draws an arrow 40 + 37 =]
> Levi: 4+3
> Students: Oh
> Teacher A: What's 4+3?
> Students: 7
> Teacher A: 4+3=7 [Records in modelling book]
> Teacher A: What else do we have to do?
> Student: Put a 3 at the end
> Teacher A: Do we want a 3 at the end? When we have already done 4+3… What do we need at the end?
> Jack: a 7
> Tamati: a 0
> Teacher A: What about this one [points from the 0 to the 7]
> Students: 77
> Teacher A: How did we get the 7?
> What did you add together?

[Pause]
Students: Put 7 in your head
Teacher A: Have a look at this [draws an arrow 40+37]
What's 0+7 [records 0+7=7]
So it's 77. Lets try a faster way. I want to see if you can get it straight away without using sticks.

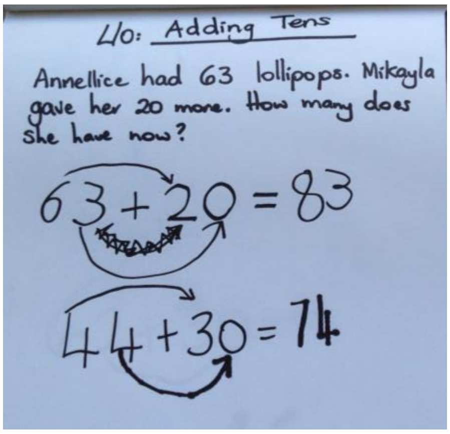


Figure 5: Modelling book examples of the arrow procedure

The rest of the session was focused on completing a number of problems using this new system of drawing arrows to the tens, and arrows to the ones, and recalling basic single-digit number combinations. The following lesson revisited adding tens with the focus being on accurately completing the arrow procedure.

The final lesson shifted into the beginning of using simple additive strategies with the session on how to rearrange and combine numbers to make ten. At the start of the lesson, the teacher reviewed the students' basic facts knowledge focusing on combinations that make ten. She used no manipulatives within this lesson even though this was their first experience of the content. In Book 5, counters and tens frames were recommended for this lesson. Instead the teacher used question prompts to help the students. "Are there two things there that might add up to 10?" Once the students located the pair of numbers that combined to make 10, arrows were drawn to highlight this fact.

Teacher A: [records 5 + 6 + 5 = 5 + 5 + 6 = 10 + 6 = 16]

In the follow-up interview, Teacher A was asked how she prepared the students for the following year.

Teacher A: Just generally trying to make sure they are up to the level that I would hope they would be at going into Year 3.

Researcher: Do you know what that is?
Teacher A: I'd like to see them all sort of part-whole by the end of the year, but its not quite there. Obviously I've got a lot of kids that are below and some who are just making it and some who are a tiny bit higher.
Researcher: So is this beginning of part-whole or …
Teacher: Yes just beginning part whole. Just getting them as far as I can at the end of the year ready for next year. You have limitations as to how far you can push them.

Teacher A's main goal at the end of the year was to expose the students to part-whole thinking in preparation for the following year's mathematical content.

4.2 Mathematics in Year 3

The following year, the Year 2 teacher shifted year groups and was teaching one of the three Year 3 and 4 classes. Of the five case-study students, four shifted into her classroom. This section explores her approach to the mathematical content and compares it to the Year 3 and 4 teacher who taught the one other case-study student. The next section reports on the learning experiences each student had in each classroom and the strategies and knowledge they used.

4.2.1 Teacher A navigating the learning

Teacher A started teaching her number unit three weeks into Term 1, after a settling period where a unit of statistics was taught. The four students that moved through with Teacher A into Year 3 and 4 were Jack, Levi, Jessica, and Tamati. The four students were members of the so-called 'triangles' maths group, which was the second to top group in the class with 12 students. Jack was also in the top group and potentially could have had two teaching sessions per mathematics lesson.

At the beginning of the number unit, the students revisited lessons in the 'counting all' to 'advanced counting' section of Book 5. When the researcher observed the lessons in Week 7 and 8, the triangles group with all four students were exploring the key idea that "groups of ten can be added and subtracted by simple known addition facts" (Ministry of Education, 2012b, p. 34). This was the same lesson objective the researcher had observed in Term 4 the previous year.

The top group, with Jack, had shifted back into the beginning of Level 2 and was beginning to explore simple additive strategies. The first lesson explored the key idea that "basic fact knowledge can be used to add and subtract tens" (Ministry of

Education, 2012b, p. 38). The top group's next two lessons investigated the key idea that numbers can be rearranged and combined to make ten. The focus was to 'make 10' by finding the pair of numbers that add up to ten, i.e., [6 + 4] + 7 = 17. Again this was a lesson that the researcher had observed the previous year.

Teacher A approached the learning in a similar manner to the observed lessons of the previous year. In the triangles group with all four case-study students, there had been a change in the type of equipment used to model groups of ten. In exploring, adding and subtracting groups of ten, beans and canisters had replaced bundles of sticks. The teacher explained this change when asked in the follow-up interview: "They were enjoying the canisters more than the sticks. I started off with the sticks nah…the canisters were much more fun." At the beginning of each session the teacher reminded the students that if they wanted equipment, they could use the bean canisters. Levi and Tamati opted for this support. The following subtraction problem 25 – 10 demonstrated the students using beans and canisters to solve the problem.

> Tamati: [grabs 2 canisters]
> Levi: [grabs 5 beans]
> Teacher A: You've got 2 canisters and 5 little beans [Teacher A records picture that represents canister and beans] then what did you do?
> Levi & Tamati: We took 1 away
> Teacher A: Is it 1 or is it 10?
> Levi & Tamati & Student: 10
> Teacher A: You took away the 10 [writes 25 – 10 and draws 1 canister]
> Teacher A: Then what did you have left?
> Levi & Tamati & Student: 15
> Teacher A: [records 15 by the equation and draws – one canister] Take away one of those and you ended up with 1 of those and 5 little ones [draws the canister and beans for the answer] Awesome…ok

Through these sessions the teacher recorded in the modelling book arrows linking the tens and the ones. She also challenged students in the final session to solve the problems without the bean canisters.

> 45 + 21
> Teacher A: Is there another way you two could have worked it out? Without materials you think…Levi can you work it out again with the arrows? I just want to see if you can do arrows. Have a go…
> Levi: [Hesitates]
> Teacher A: You are going to add your tens…show me your tens.
> Levi: [unsure]
> Teacher A: Tens are your 4 and your …
> Levi: 2

Teacher A: [to researcher] He's not quite at that stage yet
Levi: [draws an arrow from the 4 to the 2]
Teacher A: What's 4 + 2?
Levi: 6
Teacher A: write it down
Levi: [records 6]
Teacher A: Then add your ones
Levi: [hesitates again not willing to record an arrow]
Teacher A: Your ones are your 5 + your…5 + _ [pointing to the modelling book]
Levi: 1
Teacher A: is
Levi: 6
Teacher A: [to researcher] I just want to see if he can do it or if he still needs materials
Levi: [records arrows and writes 6]

4.2.2 Part-whole thinking

What had Jack retained from last year and what strategies was he using this year to solve problems? In the triangle group, Jack continued to be the most capable student in the group, finding problems relatively easy to solve.

Teacher A: Jack how did you work it out?
Jack: I went 25 – 10 is and I just knew the answer already
Teacher A: Looks like you are all understanding that, can you all grab a whiteboard each and then we will do a speed one…Ready for a speed one…are you ready
Teacher A: 46 apples… I'm taking away 20
Josh: [Very quickly records the equation on his board and the answer 46 – 20 = 26]

In the top group, Jack used a range of strategies to solve problems. His preferred strategy of doubling from last year was only used once, with a variety of strategies used when processing the problems from left to right.

1) 6 + 3 + 4 =
Jack: 6 + 3 is 9 + 1 is 10 + 3 is 13 (partitioned)
2) 7 + 4 + 3
Jack: 7 + 4 [counted on 4] = 11 + 3 [counted on 3] = 14
3) 8 + 5 + 2
Jack: 5 + 5 is 10 + 3 is 13 + 2 is 15 (doubling)
4) 9 + 5 + 1
Jack: [splits the 5 into 1 and 4 and allocates the 1 to the 9] 9 + 1 is 10 and 4 + 1 = 15 (partitioning)

From this evidence, it is clear that Jack was beginning to master early additive thinking confidently using a mixture of part-whole strategies. In Jack's 2014 mathematical content knowledge data, his average remained high at 3.64, with the majority of coding been allocated to the 3 to 5 zone of 'just right' to 'easy'. There was no substantial change in his coding from Year 2 to Year 3.

4.2.3 Counting (Year 3)

The previous year, one of the differences in the group of students was that Levi did not touch his head to 'count on'. In Year 3, Jessica and Tamati still used their fingers to solve all number combinations and the head-tap technique was still prominent. Tamati and Levi preferred to use equipment to help solve the adding-tens problems. Levi could accurately use the arrow procedure, but both Jessica and Tamati were inconsistent. When they recorded this procedure inaccurately they either reversed the digits or drew the arrow to the wrong digit.

> Teacher A: Show me…45 + 21
> Tamati: [records equation on the whiteboard and draws arrows incorrectly]
> Teacher A: We've got a little bit confused with our arrows haven't we…ok.

Evidence indicates that both Tamati and Jessica still do not have a clear understanding of place value and the difference between the tens and ones.

Figure 6: Tamati records the arrows incorrectly

Teacher A's attempt at stopping the finger counting is described below, but she also identifies classroom practices that undermine it.

> Teacher A: Mainly when they are in their groups we'll do a couple of fast ones within the group and stop using the fingers…Fingers behind your back, sit on your fingers…
> Teacher A: But the minute you put that test in front of them, out come the fingers again.

This was highlighted when the teacher played a basic facts head-to-head competition as part of a class warm-up. This was a very popular warm up that was requested by the students to play. Students are allocated into two lines; the front person of each line competes. The teacher calls out a basic-fact question. Students who are too slow or give an incorrect answer return to their seat, while the winner remains to compete against the next person in the line. Most students relied on their fingers to solve these quick-fire questions. Levi was very quick at 'counting on' with his fingers and won against a number of the leading mathematicians in the classroom. On the day he was observed, only one student could beat him.

In this situation, winning took priority and most students openly reverted to 'counting on' using their fingers. Levi, Jessica, and Tamati used fingers openly in this competitive structure, but in an independent follow-up activity after a group lesson, both Levi and Jessica hid their fingers under the table, as they calculated their answers to the problems on a worksheet. In the warm-up competition, fingers were used openly to solve problems but when working on independent worksheets, fingers were hidden from view.

When comparing the data of mathematical content on the rating scale of 1 (too hard) to 5 (too easy) across both time points, Jessica showed no evidence of shift with the average 2.44 across the two time points. Tamati dropped slightly from an average of 2.55 to 2.09, with an increase of coding in category 1. Levi's average also dropped from 3.21 to 2.83, Year 2 and Year 3 respectively. In Levi's data the numbers were similar for category 1 and category 2, but there was less evidence of category 3 (just right) in the Year 3 coding causing the drop.

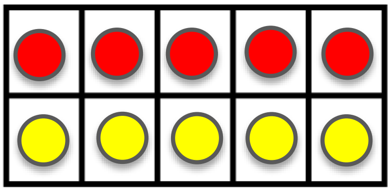
4.2.4 Teacher B navigation of the learning

Teacher B's approach to the beginning of the Year 3 period was very different from that of Teacher A. Teacher B launched straight into teaching place value and basic facts at the start of the year. She explained: "my focus points…place value and basic facts to 20 for 2/3 of the class because I have found that if you do not understand place value, maths makes no sense." She also explained her reasoning for a basic-facts focus: "You don't want to be caught up trying to figure things - that's what basic facts is - you just want to know it and use it". Teacher B also commented on her lack of experience with this particular age group: "I don't know this age group - the only strength I can fall on is what I think they need to know for the next stage up".

The three lessons observed focused on basic-facts knowledge and instant recall. The warm up was a flash-card drill, practising combinations up to 5. The main teaching point of the first two lessons was using doubles and adding 1 to find unknown facts:

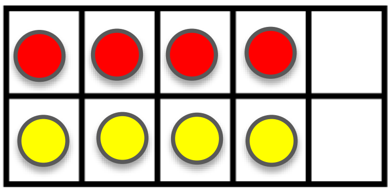
> Teacher B: [recorded 2 + 2 = 4 on a whiteboard] You all know this answer? If I know this, can I then know the answer to this?
> 2 + 2 = 4
> 2 + 3 =
> [Teacher B asks Melanie how she worked it out]
> Melanie: I put 2 in my head [touches her head] and I added on 3.
> Teacher B: Can you use doubles?
> Melanie: [shakes her head]

Most of the group struggled to make the connection between the doubles they knew, and adding 1 to find unknown facts. In the final lesson, Teacher B focused on doubles they knew and doubles they did not, using ten frames and counters to support the lesson. Teacher B asked the following questions using the doubles structure in the tens frames to highlight the teaching point. Students recorded the equation that matched the diagram shown.



Teacher B: Here comes the next one [shows 5 red and 5 yellow]
Melanie: [Records 5 + 5 = 10 on her whiteboard]

Figure 7: Tens frame showing double 5



Teacher B: Rub it out, here comes the next one and here we go [4 red and 4 yellow]
Melanie: [Records 4 + 4 = 8 on her whiteboard]

Figure 8: Tens frame showing double 4

4.2.5 Changing classrooms

The coding activity and rating scale remained very similar for the four students that kept the same teacher. The notable change was Melanie's data in Figure 8.

Melanie continued to ‘count on’ to solve problems and considered that her way of putting a number in her head was slightly different: “I do it a different way. I put the big number in my head and then I count.”

In Figure 9, Melanie either knew the answer instantly (5), or used her fingers (3), but predominantly she rated in the 1 and 2 category where she either did not know the answers, or in a number of cases, used fingers which depending on the situation, was categorised as a primitive strategy. When it came to calculating unknown facts, she reverted back to ‘counting on’ by tapping her head and using her fingers. This was classified as a primitive strategy, as the intent of the lesson was for the student to derive unknown facts from known facts.

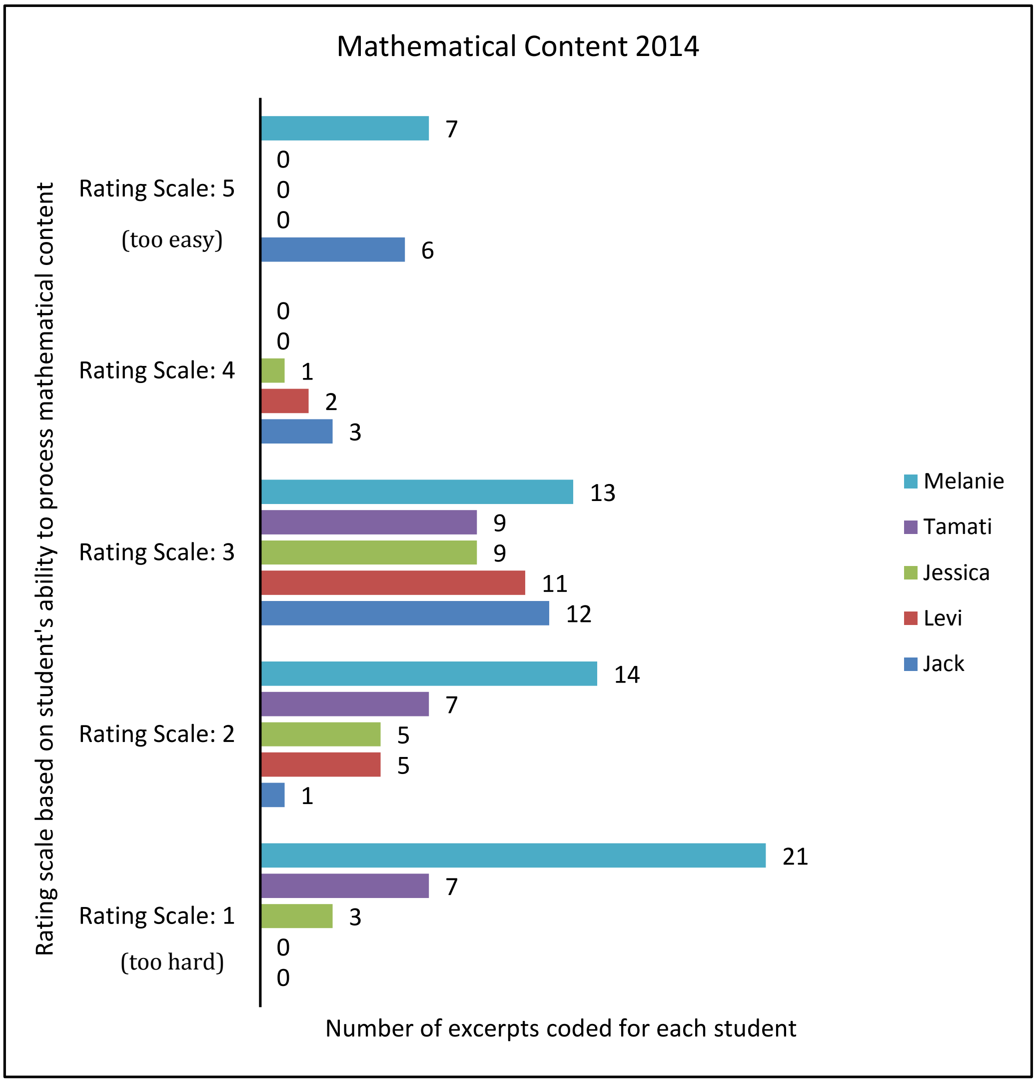


Figure 9: Student ability to solve mathematical content in classroom observations in 2014 (see Table 5 for description of rating scale criteria p. 57)

4.3 Assessment

In this section, key assessment items in the end of year assessment as well as assessment practices that influenced the beginning of the following year are reported.

4.3.1 End of Year Assessment

For Teacher A, a true indication of whether a student's knowledge and strategies had shifted, depended on how they responded in the Junior Mathematics Assessment. Key items from JAM, made up part of the warm ups in Term 4. Two of these items were counting back in 2s and counting from 80 to 130. The following excerpt is taken from the teacher's interview after reviewing Jessica's assessment data:

> Teacher A: And she can't skip count backwards either.
> Researcher: It is quite difficult and they looked like they were struggling to skip count backwards in 2s in class.
> Teacher A: But it is a big one they need to learn for this [holds up JAM document]

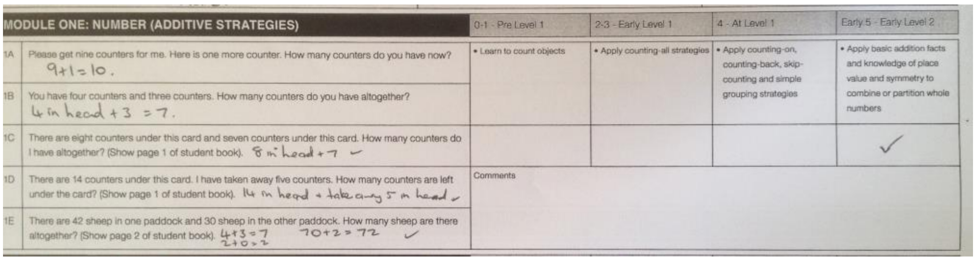
In the strategy lesson, "groups of ten can be added and subtracted by simple known addition facts" (Ministry of Education, 2012b, p. 34), the arrow procedure taught was evident in task 1E in the JAM. When questioned on this item in Levi's assessment review, Teacher A's response was as follows:

> Teacher A: Levi's awesome…He is able to do the sheep in the paddock boom, boom, 4 + 3 is 7, 2 + 0 is 2, so he's done his little arrows over and worked it out. I could see him using his finger and bouncing it across to the other one and going that way. So that was quite cool.
> Researcher: Does the decision say 4 tens and 3 tens, use language like that?
> Teacher A: No there was, but that was even higher, that was going straight onto the GLoSS. If they are able to say, I don't have it right in front of me. This was a 5 and then if they were able to say 4 tens +… I can't remember what it was
> Researcher: Three tens
> Teacher A: It's slightly higher
> Researcher: It's actually higher?
> Teacher A: Yes

This was the teacher's interpretation of the Teacher Guide Task 1E decision section, which states:

> If the student counts on in tens (42, 52, 62, 72), note that they are able to use increments of ten. Rate them at the **highest stage** demonstrated in **tasks 1C–1D** and **proceed to module two**.
> If the student uses additive strategies (40 + 30 = 70, 70 + 2 = 72), rate them as **early stage 5** and **proceed to module two**.
> If the student shows quick recall of 42 + 30 = 72, or says 4 + 3 = 7 so 70 + 2 = 72, go to the addition and subtraction tasks in **GloSS** to determine their strategy stage. (Ministry of Education, 2013, p. 4)

Teacher A's response identified tens and ones as being higher than recalling basic facts. JAM clarified that recalling tens and ones was lower and rated at early stage 5. The decisions section of the teacher guide advised the administrator of the test to rate them at early 5 if they used additive strategies. The 'counting on' strategy that Levi had used in the previous task, 1D was overlooked, and he was rated at Early Additive in the Addition strategies after successfully completing the arrow procedure for Task 1E in module one of JAM (Figure 10).



	MODULE ONE: NUMBER (ADDITIVE STRATEGIES)	0-1 - Pre Level 1	2-3 - Early Level 1	4 - At Level 1	Early 5 - Early Level 2
1A	Please get nine counters for me. Here is one more counter. How many counters do you have now? 9+1=10.	• Learn to count objects	• Apply counting-all strategies	• Apply counting-on, counting-back, skip-counting and simple grouping strategies	• Apply basic addition facts and knowledge of place value and symmetry to combine or partition whole numbers
1B	You have four counters and three counters. How many counters do you have altogether? 4 in head + 3 = 7.				
1C	There are eight counters under this card and seven counters under this card. How many counters do I have altogether? (Show page 1 of student book). 8 in head + 7 ✓				✓
1D	There are 14 counters under this card. I have taken away five counters. How many counters are left under the card? (Show page 1 of student book). 14 in head + take away 5 in head ✓	Comments			
1E	There are 42 sheep in one paddock and 30 sheep in the other paddock. How many sheep are there altogether? (Show page 2 of student book). 4+3=7 2+0=2 70+2=72 ✓				

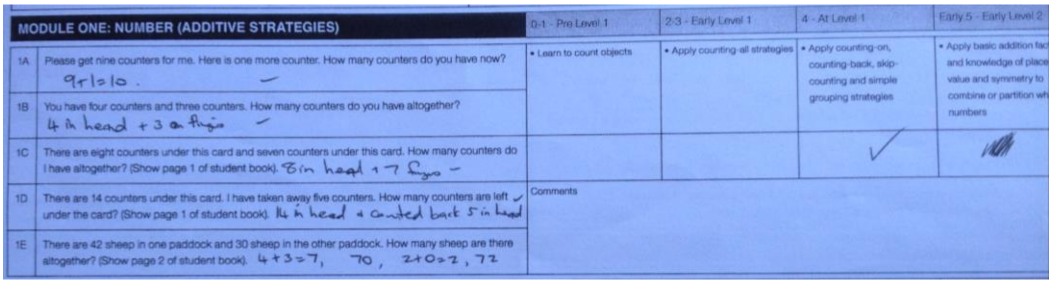
Figure 10: Levi's JAM assessment for additive strategies

4.3.2 External Expectations and Influences on End of Year Assessment

To supplement the JAM assessment, Teacher A had to complete a checklist of skills. This checklist was made up of all the items of knowledge identified in *Book 1: The Number Framework* (Ministry of Education, 2007a) and the strategies each student needed within each stage. The only items missing from this checklist were items listed under the heading: Written Recording. Teacher A explained: “I have one of these; it is beautifully filled out. Syndicate Leader says we have to do one of these [shows checklist of skills]; every child has one.”

In addition, there was researcher influence on some of the decisions Teacher A made at the end of the year.

> Teacher A: Ok Tamati … He used fingers for a couple of them. I was a bit disappointed with him because I know he can do it…but he was able to do this one
> Teacher A: mmm I was tossing up whether to put him on a [stage] 4 or [stage] 5 for that. Even though he got that which makes him a 5. He is using his fingers. I was like, argh, is he really a 5 if he is using his fingers?
> Researcher: I wouldn’t class him a 5 unless he does doubles… Oh no I’m giving you my opinion unless they are doing something in here [Researcher points to question 1C and 1D]. It’s nice they can do this [Researcher points to question 1E].
> Teacher A: But it’s not consistent yet
> Researcher: They need to be doing…see that’s why I’m interested in Jack because he is able to use doubles or tidy tens to solve this one then I say, yeah, definitely they are in here (point to early stage 5)
> Teacher A: Yip so I’ll bring him back to a stage 4



MODULE ONE: NUMBER (ADDITIVE STRATEGIES)		0-1 - Pre Level 1	2-3 - Early Level 1	4 - At Level 1	Early 5 - Early Level 2
1A	Please get nine counters for me. Here is one more counter. How many counters do you have now? 9+1=10.	• Learn to count objects	• Apply counting-all strategies	• Apply counting-on, counting-back, skip-counting and simple grouping strategies	• Apply basic addition fac and knowledge of place value and symmetry to combine or partition wh numbers
1B	You have four counters and three counters. How many counters do you have altogether? 4 in head + 3 on fingers				
1C	There are eight counters under this card and seven counters under this card. How many counters do I have altogether? (Show page 1 of student book). 8 in head + 7 fingers			✓	
1D	There are 14 counters under this card. I have taken away five counters. How many counters are left under the card? (Show page 1 of student book). 14 in head + counted back 5 in head	Comments			
1E	There are 42 sheep in one paddock and 30 sheep in the other paddock. How many sheep are there altogether? (Show page 2 of student book). 4+3=7, 70, 2+0=2, 72				

Figure 11: Tamati's JAM assessment for additive strategies

Teacher A was juggling between selecting stage 4 and early stage 5 for Tamati's stage in additive strategies. Above she refers to Task 1E influencing an early stage 5 selection, as Tamati was able to complete the task using the arrow procedure taught the previous week. Tasks 1B, 1C and 1D were all completed using fingers, which identified him as stage 4.

The final classroom activity that was influenced by assessment was the teaching of multi-choice questions at the end of Year 2. This was the most notable thing Teacher A did to prepare Year 2 students for the transition into Year 3 mathematics. The following excerpt gives her reasons for this teaching content.

> Teacher A: We have a look at multiple choice questions, which I will be doing with my Year 2s coming up within the next few weeks. Mainly because they have a PAT coming up next year and none of them have been aware of or have even seen a multi-choice question. So it just prepares them a little bit for how that would look.

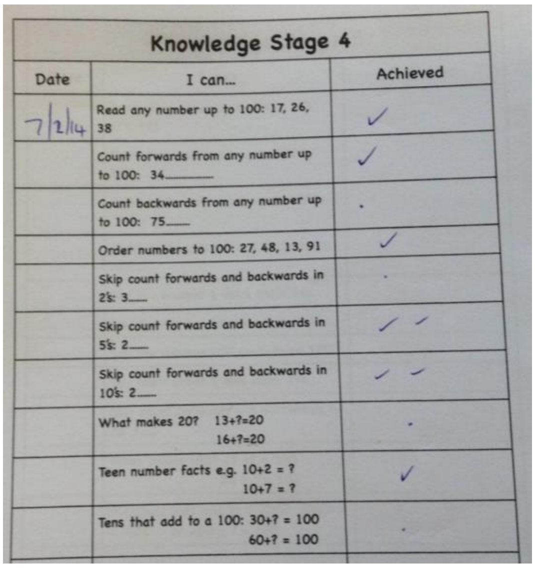
The end of year assessment was based on the JAM and how the teacher interpreted each assessment task. Additional assessments included classroom observations, a checklist of Number Framework knowledge and strategy items, and multi-choice questions similar to the Year 3 Progressive Achievement Tests (PAT).

4.3.3 Beginning of Year Assessment

The assessment information that was passed on with the student into their Year 3 classroom was an overall mathematics level. The end of year JAM information was also included in each student's file. Both Teacher A and Teacher B used the overall level to place students in mathematics groups at the beginning of the year. The JAM and other assessment data on the school management system had just been given to the teachers during the observation week (Week 8, Term 1). This meant that the only information to inform placement at the beginning of the year was the OTJ. Within the first two weeks, to supplement the overall level, both teachers had completed a quick knowledge assessment to validate student placement and identify gaps in their knowledge.

> Teacher A: I've got that little IKAN sheet to give me an idea.
> Researcher: A checklist.
> Teacher A: Yip a checklist, just so I can see if there is anything/gaps that I might be able to plug as a group or as individuals. I know they're not ideal, but that gave me a bit of an idea of where they were at the start of the year.
> Researcher: How did you gather this data (pointing to checklist)
> Teacher A: Just orally and getting the materials out in front of them.
> Researcher: So you sat down one-on-one with them?
> Teacher A: Yes
> Researcher: So was it almost like doing a little JAM?
> Teacher A: Yip very similar actually, I didn't want to go through the whole JAM test, so I wanted a quick snapshot of whether they were still at the same level and most of them were bang on.
> Researcher: So this is just based on their knowledge?
> Teacher A: Yeah
> Researcher: And how did you assess their strategy?
> Teacher A: Just looking at what they were doing in their groups. This was just a quick snapshot so that I knew they were in the right group.

Jessica's stage 4 knowledge assessment (Figure 12) identified gaps in her knowledge, which were targeted by Teacher A as part of the mathematics programme at the beginning of the year.



Knowledge Stage 4

Date	I can...	Achieved
7/2/14	Read any number up to 100: 17, 26, 38	✓
	Count forwards from any number up to 100: 34____	✓
	Count backwards from any number up to 100: 75____	.
	Order numbers to 100: 27, 48, 13, 91	✓
	Skip count forwards and backwards in 2's: 3____	.
	Skip count forwards and backwards in 5's: 2____	✓ ✓
	Skip count forwards and backwards in 10's: 2____	✓ ✓
	What makes 20? 13+?=20 16+?=20	.
	Teen number facts e.g. 10+2 = ? 10+7 = ?	✓
	Tens that add to a 100: 30+? = 100 60+? = 100	.

Figure 12: Jessica's IKAN assessment beginning of 2014

4.3.4 Routine expertise

Of the five case-study students, Melanie and Jessica were able to attain stage 3 in the basic facts section of JAM, recalling facts to 5, doubles to 10 and groupings within 10. In this quick-fire assessment where students must respond within 3 seconds, the students displayed a good knowledge of early Level 1. Transferring this same knowledge into a different context proved to be difficult for both students.

> Teacher A: What are we doing Jessica?
> Jessica: Put the 6 and the 2
> Teacher A: [records an arrow 63+20=] What are we doing with them?
> Jessica: Adding them
> Teacher A: Ok what is 6+2?
> Jessica: [Touches head and counts 2 fingers]

Tamati attained stage 4 in module eight, recalling all facts to 10, doubles to 20, and corresponding halves and teens facts. The teacher commented on the teaching process used to help Tamati achieve this level.

> Teacher A: Mmm, I was quite surprised with that one with him actually; he knows his doubles really, really well. I did a prize for my class, all my Year 2s, to get their doubles to 20, and he's remembered them all, thank goodness.

Again there was evidence of knowledge not transferring over into a different context.

> Teacher A: I have two mussels, along came…Melanie she got 6, and then Aroha got 8. How many have we got now?
> Teacher A recorded: [2 + 6 + 8]
> Tamati: [When working out 8 + 2 Tamati touched his head and counted 2 on]

Knowledge items were taught in isolation, using memorisation, rote learned procedures, and speed tests. Competition was encouraged, as the teacher saw it as a way to motivate individuals. Melanie, Jessica, and Tamati were able to instantly recall a basic-facts question in isolation, but struggled to retrieve the facts in different situations and apply them flexibly.

4.4 Modes of Representation

Teachers used a variety of representations to support the mathematical learning. These have been separated into three categories. The first category is symbols, the second images and diagrams, and the third, manipulatives. The following graph shows teacher use of these different types of representations.

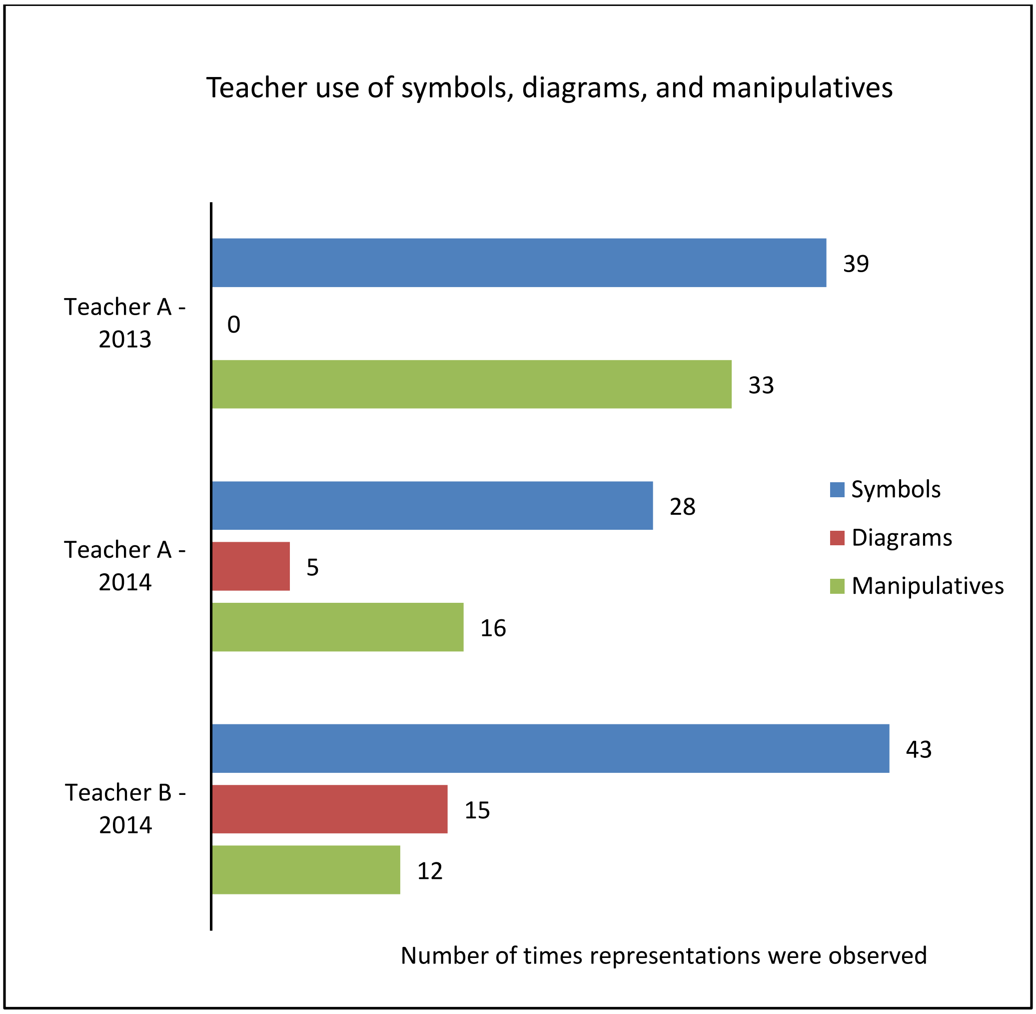


Figure 13: Coding activity of the teachers use of symbols, diagrams, and manipulatives in classroom observations

4.4.1 Symbols

Within this study, numbers (digits), mathematical symbols (i.e., + -), brackets, arrows, and equations for word problems, were all coded as symbols. This representation when compared with diagrams and manipulatives was used the most by both teachers. There were a number of occasions when it was the only representation used in the learning process. The following example is part of a teaching episode where symbols were the only representation used.

> Teacher B: [recorded 2 + 2 = 4] you all know this answer? If I know this, can I then know the answer to this?
> Teacher B: [recorded 2 + 3 =]
> Teacher B: [asked Melanie how she worked it out]
> Melanie: I put 2 in my head [touches her head] and I added on 3.
> Teacher B: Can you use doubles?
> Melanie: [shakes her head]

> Teacher B: Ok let me ask her another one…ok, lets look at this one [Records 5 + 6 = on the whiteboard] Can I use my doubles? Like double 5 and then work it out?
> [Melanie works it out on her fingers while teacher focuses on another student]
> Student: 5 + 5 is 10 and it's just one more.

Symbols were also used to replace concrete manipulatives in the lesson: "groups of ten can be added and subtracted by simple known addition facts" (Ministry of Education, 2012b, p. 35). Teacher A encouraged students to replace the canisters and the bundles of sticks with the arrow procedure. "Is there another way you two could have worked it out? Without materials you think…Levi can you work it out again with the arrows?"

There was evidence of symbol confusion when arrows were drawn to the wrong digits. While Tamati was trying to solve the problem 54 – 20, he was observed taking 2 fingers away from 4 fingers, and recording an arrow from the 5 to the 0 with his final answer as 52.

There was also evidence of student overgeneralisation with the arrow procedure. After a week of the arrow procedure used to combine the tens and the ones, an arrow was again used to link two numbers for which the sum equalled 10. Two students overgeneralised and used the arrow procedure to link the numbers that were at the beginning and end of each equation.

> Partner: Jessica bought 9 jellyfish, along came Aroha and she bought 1 more and she gave her 3 more. [Records 9 + 1 + 3 then passes pen to Jessica to work it out]
>
> Jessica: [drew an arrow 9 + 1 + 3]
> Partner: [pointed to the 1] that's 10 [but then went 9, touched her head and counted on using 3 fingers to 12]
> Partner: That's 12
> Jessica: [recorded 12 + 1 = 13]

In observed lessons, symbols were one of the key representations used by teachers and students.

4.4.2 Diagrams

There were limited examples of diagrams used as a form of representation during the nine classroom observations. The only diagram or image observed in Teacher A's room was an illustration of canisters used to demonstrate one of the student's explanations when subtracting groups of ten.

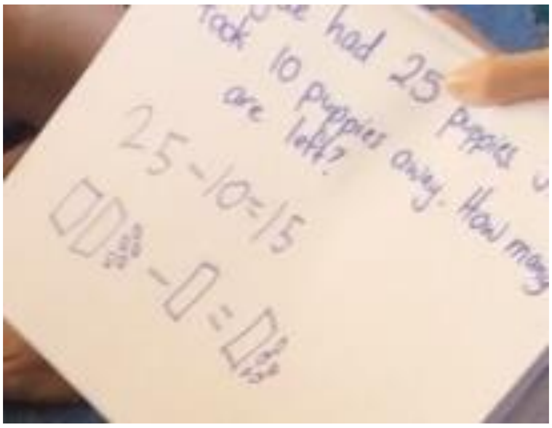
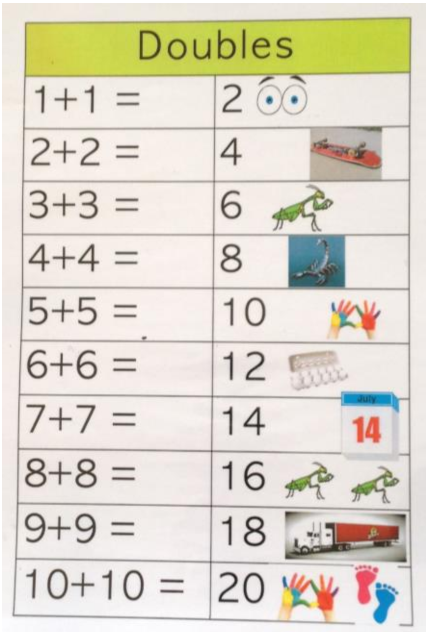


Figure 14: Teacher A’s modelling book and the use of a picture representation.

In Teacher B’s room, two types of images were used to support the development of student doubles knowledge. The first in Figure 15 was a poster with different picture representations of doubles. Teacher B discussed each picture and its link to the double fact it represented. She also noted the mistake she made for the double fact: 8 + 8, commenting that she needed to replace the two praying mantises with scorpions.



Doubles	
1+1 =	2
2+2 =	4
3+3 =	6
4+4 =	8
5+5 =	10
6+6 =	12
7+7 =	14
8+8 =	16
9+9 =	18
10+10 =	20

Figure 15: Doubles poster used in Teacher B's knowledge lesson

This poster was effective for the images that students readily associated with the double such as the pair of eyes and the two hands, as they were able to recall these doubles instantly when the picture was shown. For the images they were not so familiar with, students struggled to recall the fact. Over the three sessions where the poster was used, there was no substantial change in Melanie's doubles knowledge.

The second image that Teacher B used in combination with manipulatives was the tens frame model. Using the tens frame and magnets, Teacher B built different double combinations. At the beginning of this teaching episode, Teacher B highlighted that doubles have partners and the model mirrored their partnership (Figure 16).

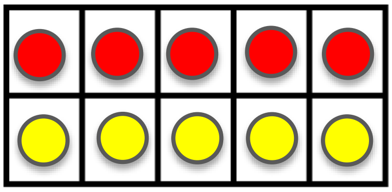
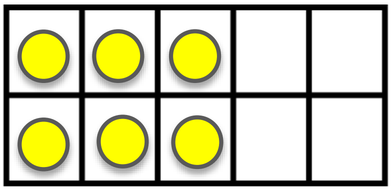


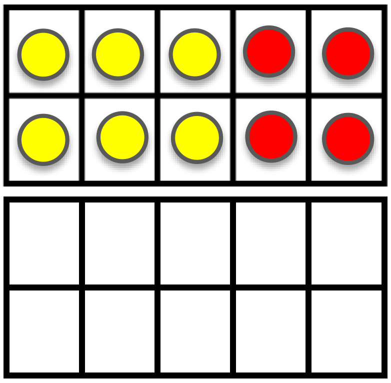
Figure 16: Tens frame showing double 5

The structure of the representation changed once the double was extended beyond the first tens frame. Instead of continuing with the double structure, Teacher B changed to the combinations that 'make 10' model.



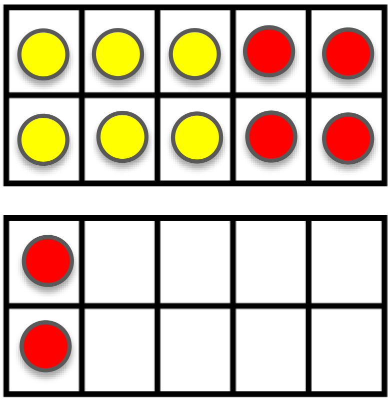
Teacher B: How many more do I add to make a double? Double 6 how many more do I add?
Student (a): You add another 3

Figure 17: Tens frame used to represent 6



Teacher B: No doubles another…6 more…[add 4 red magnets] 1, 2, 3, 4, Oh my goodness where do I go now?

Figure 18: Two tens frames used to represent '10 and'



Student: The bottom one
Teacher B: [Adds 2 more…]

Figure 19: Tens frames used to represent 12

This change moved the lesson focus away from the doubles structure, creating inconsistent modelling across the lesson as a whole.

4.4.3 Manipulatives

Manipulatives are concrete materials and equipment used to mirror the structure of the mathematical concept being taught. Teacher A's use of manipulatives in 2013 closely matched the use of symbols, as a mode of representation (Figure 13), whereas her 2014 data showed a substantial decrease. In comparison Teacher B's use of manipulatives was coded 12 times, and only 10 of these were within classroom observations. Six coded excerpts were from the tens frame/magnet lesson described in the diagram section (4.4.2). The other four incidents were when Teacher B used jellybeans to demonstrate the change when one is added to a double.

Teacher B: [hands out 10 jellybeans to each student] Make them into 2 equal groups.
Melanie: Splits the 10 jellybeans in 3 groups
Teacher B: [corrects her and counts out 5 and 5…needed to correct a number of students] How many is 5 + 5?
Students: 10
Teacher B: 10, now if 5 + 5 is 10, what is 5 + 6?
Melanie: [touches her head and holds up 6 fingers and counts on]
Teacher B: [hands out 1 jelly bean per student] 5 + 6, 5 + 6, 5 + 6 what's the answer Melanie?
Melanie: 11
Teacher B: How did you do it?
Melanie: [no response]
Teacher B: If 5 + 5 is 10 …
Melanie: [goes to use her fingers]
Teacher B: no fingers

Melanie: [no response]
Teacher B: [shows her the jellybeans] If 5 + 5 is 10…[pauses and waits] what's 5 + 6.
Melanie: [touches head and counts fingers]

Teacher A used manipulatives in the form of bundles of sticks and beans and canisters to represent tens and ones, but they were removed as soon as possible because of the following reasoning:

Researcher: Because you are coming to the end of the year, do you think you are using equipment less or more?
Teacher A: Definitely less
Researcher: Why's that?
Teacher A: Just because they are able to internalise ideas a little bit more and also I'm aware that they've got JAM coming up and they've got PAT next year. I don't want then using equipment because obviously they can't use it in those tests. Especially with JAM, I want them to actually be starting to use their fingers or in their head. Because that's what's expected… If I keep putting equipment out in front of them, they will be expecting to use it all the time, and when it's JAM and they have to go 14 – 5, they will not be able to use it, so I do draw it away.
Researcher: Is it a draw away at a certain time?
Teacher A: No it's a draw away when I'm aware they can do it. I don't want them to completely rely on it. I see them starting to use their fingers then it's drawn away.
Researcher: Is most of their maths based around their fingers right now?
Teacher A: Yip. I'm quite heavily reliant on them getting off that equipment as fast as possible and getting onto their fingers…Once they've been on their fingers for a little while, we start encouraging them to take their fingers away. Working in their head. So I'm push, push, pushing them as much as I can to get it internally rather than externally.

Teacher A introduced new mathematical concepts with concrete manipulatives, then removed them as quickly as she could to encourage them to use their fingers or internalise the concept. Teacher B started off with no manipulatives but as the lessons progress she increased their use but only under her direct control.

The following graph illustrates the number of times the use of manipulatives was observed in the six classroom observations in 2014. The rating scale indicates the effective use of the concrete manipulatives when they were used, based on the criteria set in Table 4. (From rating scale 1 where manipulatives are not being used or used inappropriately to rating scale 4 where manipulatives have been used effectively and are now been withdrawn as students make connections between the concrete and abstract representations.)

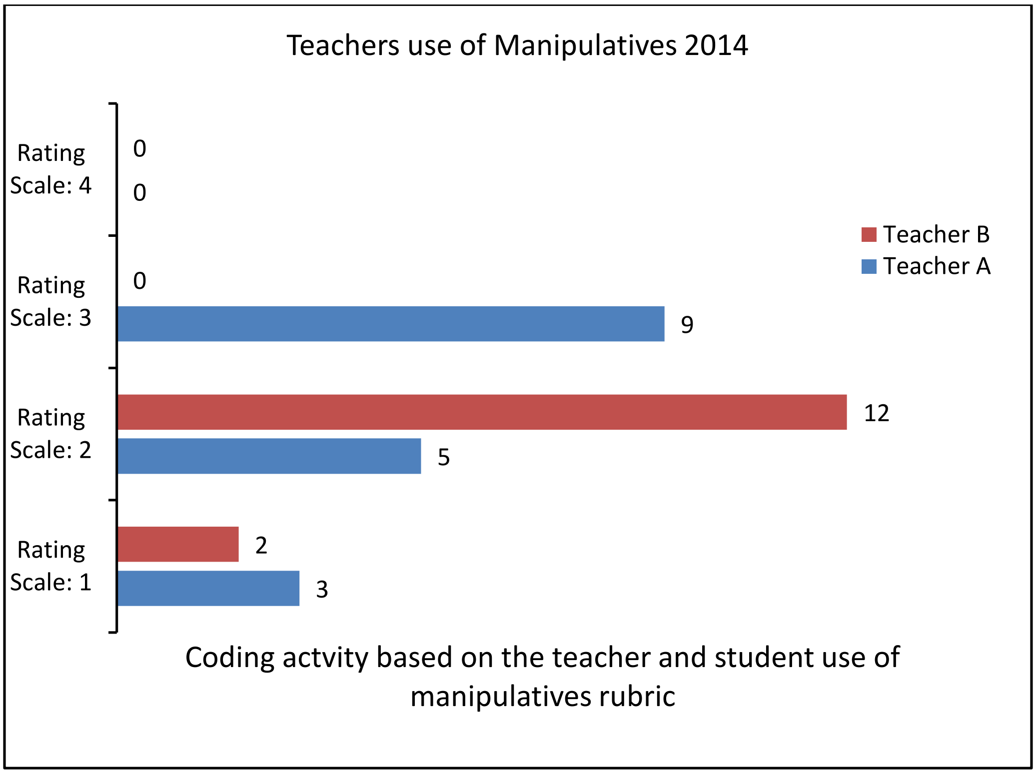


Figure 20: Appropriate use of manipulatives

Rating scale 1 was assigned to sections of the lesson where the teacher did not use manipulatives to model the mathematical concept. In the lesson: "Make ten (working with ten)" (Ministry of Education, 2012b, p. 40), Teacher A missed an opportunity to use tens frames and counters to model the structure and combinations that make 10. The lesson in Book 5 states that this structure helps students visualise numbers that make 10 and develop the understanding that the order of the numbers in an addition equation does not affect the final outcome, e.g., 5 + 8 + 5 = 5 + 5 + 8 = 10 + 8 = 18.

No manipulatives were used in Teacher B's first lesson on doubles + 1. The lesson was structured around teacher instruction and equations on the whiteboard.

> Doubles + 1
> Teacher B: [recorded 2 + 2 = 4] You all know this answer? If I know this can I then know the answer to this?
> Teacher B: [recorded 2 + 3 =]

Rating scale 2 was assigned when the teacher used manipulatives to model a mathematical concept and when students were directed by the teacher to use manipulatives to explain their thinking.

Teacher A: Can you get me out 5 canisters
Student: [Counts as student gets them out] 10, 20, 30, 40, 50
Students: [join in with the skip counting]
Tamati: 4 [leaning over and pointing to the container of single beans]
Teacher A: We've got 50?
Student: 4
Teacher A: Has he got 54?
Students: Yes
Teacher A: Are you sure? 10, 20, 30, 40, 50..4 How many ran away?
Students: 20
Teacher A: Can you get them to run away?
[Student leans forward and takes 2 canisters and puts them into the ice cream container]
Teacher A: How many have we got left?
Students: 34
Teacher A: Tell your partner how many have you got left.
Students: 34 [whispered]
Teacher A: Let's check 10, 20, 30…4 [points to each canister and staggered the count when saying 34 as she points to the canisters and the ones]

Rating scale 3 was assigned to specific occasions when students independently selected appropriate manipulatives to make connections from the problems posed and were able to use manipulatives as part of their explanation when reporting back their thinking.

Rating scale 4 criterion would have been assigned if teachers scaffolded students understanding so that they became less reliant on manipulatives. Teacher A removed the bundle of sticks and canisters of beans in the lesson: "groups of tens can be added and subtracted by using simple known addition facts" (Ministry of Education, 2012b, p. 34). Her reasoning was to encourage them to internalise and become less reliant on the manipulatives. On the occasions observed in class that equipment was removed, the arrow procedure and fingers replaced them so there was no evidence of internalising the concepts.

Students' use of manipulatives over the two time points is displayed in Figure 21. The data showed a change in Jessica's use of manipulatives over the two time points. In November 2013, she used equipment when directed and was heavily reliant on fingers. In the observations in March 2014, she elected not to use concrete manipulatives, favouring the arrow procedure instead. So the coding activity dropped, with only excerpts with finger counting recorded. She was still reliant on fingers on a number of occasions and when calculating basic number combinations.

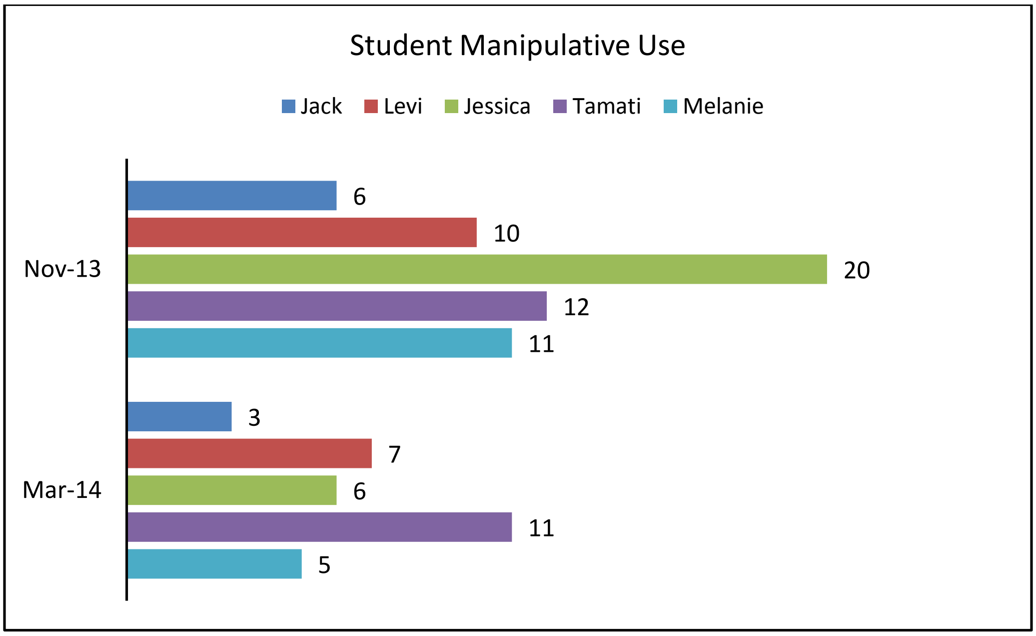


Figure 21: The use of manipulatives by the students at the two data collection points

Jack attempted most problems in his head, opting for the manipulatives at the beginning of the session, but quickly putting them to the side to calculate answers internally.

Levi and Tamati preferred using manipulatives when given the option. When manipulatives were removed, Levi was able to replace them with the arrow procedure. Tamati needed to use fingers alongside the arrow procedure to calculate answers.

Melanie's use of concrete manipulatives dropped due to a different classroom environment and lesson content. The focus was on recall and retrieving basic facts without concrete manipulatives or fingers.

4.5 Questioning

Both teachers used different types of questions to support, develop, and extend mathematical thinking in Year 2 and Year 3. Each question a teacher asked was coded according to one of two categories: funneling and focusing.

Funneling questions were defined as a closed leading question in which a particular answer was expected, or the questioning sequence directly guided the student to a specific strategy. The simplest form of funneling was a closed question used to recall basic facts. Students were expected to instantly recall the answer in a quick-fire situation:

> Teacher A: What adds up to 10? What's 5 plus [shows 5 on one hand]
> Teacher A: What's another one 7+…
> Teacher A: 3 awesome, 8+…
> Teacher A: 9+…
> Teacher A: 10+…

A funneling-question sequence used leading questions to guide students to solve a problem in a particular way. This sequence was used when both teachers were teaching one type of strategy. The following example is an illustration of funneling questions used to lead students to a doubles-plus-one strategy:

> Teacher B: [gets out jellybeans … 2 blue and 2 yellow] I got 2 blue ones and 2 yellow ones. How many is that?
> Students: Four
> Teacher B: Fantastic…did this change? [Pointing to the blue beans]… no. So I still got my 2 blue ones. [adds a yellow jellybean] How many yellows do I have now?
> Student: Three
> Teacher B: 3? How many did I add on?
> Students: 5, 3, 2, 1
> Teacher B: There's 2, there's 1, I added 1, 1 then my answer would be one more.
> Students: Yes
> Teacher B: I used my doubles to add on one. So if 2 + 2 is 4, then 2 + 3 is 5, it's just one more

In the next example, Teacher A posed funneling questions to lead students to use a certain strategy when solving two-digit addition.

> Teacher A: And we were going 76 – 40 pizzas. What was the first thing we had to do?
> Student: 7 – 4
> Teacher A: What was 7 – 4?
> Jessica: 3
> Teacher A: Well done then what did we have to take away?
> Students: 7, 6, 4
> Teacher A: [points to the numbers in the equation]
> Student: 4 and 0
> Teacher A: Because that's our tens and our ones. Wasn't it? Shall we get the canisters out?
> Students: [Mixed response] no/yes

Figure 22 shows the number of questions coded under each category and the differences between the two teachers.

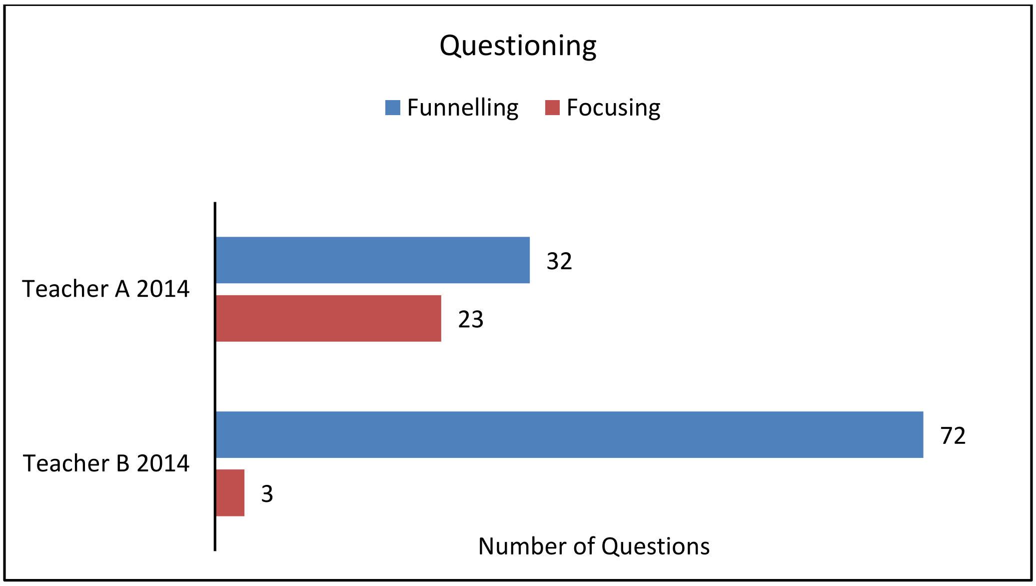


Figure 22: The two types of questions used by the teachers

The main emphasis in Teacher B's lessons was number knowledge and basic fact recall. A large number of the funneling questions asked during the observations were closed questions with a single numeric answer. When Teacher B focuses on the student strategies to solve unknown facts, she tried to funnel them away from using their fingers to instant recall.

Focusing questions are open-ended questions used to uncover details of a student's thinking. Teacher A used this questioning technique predominantly with Jack to uncover his thinking. For the example: 7 + 4 + 3 = , Jack explained: "I knew what 4 + 3 was, that equals 7, and then 7 + 7 was 14".

Both teachers predominantly used funneling questions when accessing student number knowledge. Focusing questions were used to uncover student strategies for solving equations.

4.6 Attitudes

The final section investigated student and teacher attitudes towards mathematics before and after the transition. Figure 23 shows the students' responses to the Likert scale question in the semi-structured interview at the two different time points.

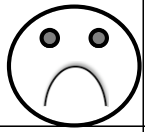
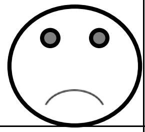
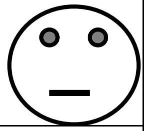
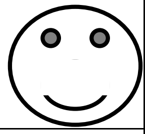
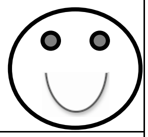
	Interview Question: How do you feel about maths this year? Students selected one of the following within the semi-structured interview				
					
	Really negative	Not so good	O.K.	Good	Great
2013			Melanie		Jack Levi Jessica Tamati
2014				Melanie Levi Tamati	Jack Jessica

Figure 23: Students' attitude towards mathematics recorded on a Likert scale over the two time points

4.6.1 Student attitudes

Each student was asked the following two questions:

- (2013) How do you feel about maths?
- (2014) How do you feel about maths compared to last year?

Jessica and Jack remained the same, Levi and Tamati dropped down one level, and Melanie went up one level.

Both Jessica and Jack selected the smiley face (great) when asked how they felt about mathematics at the two different time points. Jack's reason in 2013 was limited to the response 'good' and he was unable to elaborate on this. In 2014, he described himself as really happy about mathematics because it was harder. Jessica's reason in 2013 was that she was really happy about mathematics, but she did not elaborate. The following year, Jessica qualified her selection because she had access to a piece of equipment she had not had the year before. Jessica explained: "because it's fun using the whiteboards".

Levi and Tamati's attitude dropped one level over the transition from Year 2 to Year 3. Both selected the very smiley face (great) in 2013, then in 2014, opted for the smiley face (good). Levi's reasons were based around his performance. In 2013, he qualified his selection with the following response: "Because I'm good at doing maths… doing the right answers".

In 2014, he qualified his lower choice by saying maths had become harder. Tamati had similar reasons for the one level drop. In 2013, he felt happy about maths, and in 2014 when he was comparing how he felt about mathematics, his reasoning was that he liked mathematics more the year before, as it was easier. Earlier in the interview, Tamati had remarked that mathematics had become harder, and one of the reasons was: “we have to do heaps of stuff, we have to do arrows, to do the correct arrows to get the correct answer, and that’s hard.”

The last student, Melanie, shifted up one level over the transition. In 2013, she selected the neutral smiley face (O.K.), the lowest of the whole group. She remarked on how she was feeling nervous about maths, and there was a lot of uncertainty around what maths would be like the following year.

> Melanie: It’s so different, because our class is going to get busted down, because there is going to be no juniors next year. They’re all going to be middles.
> Researcher: It is going to be different. Are you looking forward to that? What do you think it will be like being in the middles?
> Melanie: Different. Different class.
> Researcher: What do you think middle maths will be like?
> Melanie: Hard. Super hard.

The following year, Melanie’s attitude had become more positive and she selected the smiley face (good), and commented that she felt proud when she could complete the maths work successfully by herself.

4.6.2 Teacher attitudes

In this section, Teacher A’s attitude is examined as she transitioned from teaching Year 1 and 2 to teaching Year 3 and 4, and the major changes in her attitude towards teaching mathematics over the two time points.

In 2013 Teacher A was very positive about teaching mathematics.

> Teacher A: It’s good. I quite enjoy it. It gives you a lot of scope in teaching maths. You can have lots of fun, you can get really down to the nitty gritty, you’ve got lots of talking, lots of hands-on stuff. It’s really varied. You can go outside, you can do things out there with maths. Maths is just everything throughout the whole curriculum.

She identified her strengths as her enthusiasm, lots of discussion and student talk, being well planned, and having an awareness of any changes within the students and the group, the ability to be flexible, and adapt and change planning when needed. In addition, she felt that students responded positively to some element of competiveness in the classroom environment.

When reinterviewed the following year, there was a dramatic change in Teacher A's levels of confidence:

> Teacher A: Confidence is a little bit low at the moment.
> Researcher: Why's that?
> Teacher A: Just not really knowing what I'm doing, just feeling my way, especially with the higher levels. It's all just looking at the book, going by the lessons, and not really knowing what the hell I'm doing, but winging it at the moment.
> Researcher: So strengths in your programme?
> Teacher: Not yet.
> Researcher: None…nothing?
> Teacher: Not yet, I am zero confidence this year.
> Researcher: Oh no!
> Teacher: Yeah…we will get there, it's just getting to the end of the term sitting down and thinking, ok, what went well and what didn't…I am changing it all the time, it's just trying to figure out the levels. The levels I've done before no problem, but the higher levels, I'm still feeling my way.

One of the weaknesses identified by Teacher A in her first interview was her understanding of the new support material - the updated version of *Book 5: Teaching Addition, Subtraction, and Place Value* released at the end of 2012.

> Phase 1 (November 2013)
> Teacher A: Still getting my head around the new NZ Numeracy Projects Book 5. Understanding the material. The new NZ maths book (Book 5) is terrible, it doesn't match up with the planning sheets. The pages are all out and you're trying to find your way round the new book. I've actually gone back to the old ones again, cause it drives me insane.

Planning sheets to complement this resource were put online in March 2013. From the teacher's comments, it appears that she may have been still working from old planning sheets, as the updated sheets match the resource. Over the January break, Teacher A had the opportunity to sit down and read the resource in detail and had recently downloaded the updated planning sheets.

Another weakness identified by Teacher A was her limited understanding of place-value development. She suggested the limited professional learning in the area of place value and her own incomplete knowledge as reasons for the lack of confidence and understanding.

> Phase 1 (November 2013)
> Teacher A: You can add that to one of my weaknesses, I'm still trying to get my head around that a little bit. We do use sticks, the ice block sticks, and we have

> used it quite a bit this year. I do know place value is a weakness of mine, just trying to figure out what equipment to use and what will help them even more…
> Yeah and I think that is quite a big puddle that I'm in at the moment where I'm really not sure what to do there and how to move them on using place-value equipment…

As Teacher A shifted into Year 3 and 4, she became heavily reliant on the resources to guide her teaching.

> Teacher A: These are all brand new lessons I haven't looked at before, so it's really going through that maths planning sheet on nzmaths looking through the pink books and just trying to work my way through it. Kind of flying blind a bit…

An additional reason Teacher A felt lost going into Year 3 and 4 was not only the lack of experience at that level, but also the lack of support. A newly appointed principal had restructured all the syndicates at the beginning of 2014, so there had been some major changes across the school as a whole. In the Year 3 and 4 syndicate, Teacher A replaced Teacher C, who was shifted into the junior syndicate.

> Teacher A: It's a relatively new syndicate, if we had Teacher C staying here, which I was really expecting to go into the middle syndicate with. That's why I asked to come in here, because I had Teacher C, I thought yip great, I can plug Teacher C all day, everyday and figure out what the heck to do, but then she went poof and I went hang on, there is no huge experience in the middle school now to tap into.
> Researcher: There's just not the experience?
> Teacher A: There's not the help or the experience there.

On top of Teacher A feeling the lack of support, first-term pressures, and a hectic timetable, meant that by week 8, she had taught four weeks of number.

> Teacher A: It's just been so frantic…time [pressures] just getting the balance of the day, trying to get through 4 groups where normally I'm only doing 3 or 2 groups a day. And I'm trying to work out what will work in this class.
> Researcher: So you would say the beginning of the year is quite hectic, there's lots going on, lots of external things going on that seem to crowd classroom teaching, is that what I'm hearing?
> Teacher: Yip, yip…you try and make time for all your core subjects. We've got an assembly at 10am on a Monday morning for half an hour, and that cuts straight through your numeracy and literacy time. Things crowding it all the time…

4.7 Summary

The strategy students used predominantly in the classroom observations, was the strategy they relied on throughout the transition into the next year. The content and placement of students in both classrooms was influenced by assessment practices. Manipulatives were used initially to support student thinking, but there was a substantial decrease in use in one classroom. In the comparison classroom over the course of the observations, there was an increased use of manipulatives, but only under the direct control of the teacher. The results identified the types of questions both teachers used within each classroom context. Classroom interaction, attitudes, and external influences impacted on both teacher and student transition. These results are discussed in the following chapter, including the connections students made between current knowledge and evolving knowledge, and the teacher's roles in the process of forming new knowledge.

Chapter Five: Discussion

5.1 Introduction

This chapter examines the evidence presented in the previous chapter and answers the question:

> What support do ‘counting on’ students receive in mathematics as they transition from Year 2 to Year 3?

This thesis highlights some of the difficulties and barriers Year 2 ‘counting on’ students have when attempting to shift into part-whole thinking. It also challenges aspects of assessment and teacher practice that may inhibit this progress. These findings are discussed and linked to the knowledge base reviewed in Chapter 2.

The first section (5.2) examines the influence of: the setting, classroom environment, and grouping. The next section (5.3) summaries the student learning and the mathematical content. 5.4 explores teacher practices, knowledge, and resources. The final section discusses the external influences on transition.

5.2 Setting

How the students experienced the learning environment, the organisation, structure, and grouping, was largely dependent on the routines, teacher pedagogical knowledge, and their underlying philosophy about how to teach mathematics. The initial classroom setting (Year 2) followed the guidelines recommended by NDP professional learning in that Teacher A had four ability-based groups (Cobb, 2012) . The case-study students were all in one group and had been selected by the teacher who felt confident they would meet the criterion of ‘counting on’ to solve problems. The teacher was beginning to teach early additive part-whole lessons during the observations, in preparation for the following year. She followed the planning sheets very closely, and when the sheets did not match the updated resources, she went back to the old version, a finding that indicated a prescriptive type step-by-step following of this resource (Boaler et al., 2000; Scouller, 2009).

Again as recommended by NDP professional learning (Cobb, 2012), Teacher A had a set routine of rotating through the groups to teach each group specific strategies (Ministry of Education, 2008a). She began each maths session with a class warm up (Ministry of Education, 2008a), and at this point in the year there was a major emphasis on certain tasks from the JAM assessment. The classroom environment was very settled, reflecting the calm, positive yet structured nature of Teacher A's approach to learning. When students were not with the teacher, there was a clear expectation of being quiet and self-directed learners. Each day, differentiated lists of basic facts (quick 10) were put up on the whiteboard for students to work on independently. Once this was completed, students were able to select a task of their choosing, with the guidance that it should improve their maths.

As Teacher A and Teacher B negotiated the change from teaching Year 1 and 2 and Year 5 and 6, respectively, both transferred their previous practices into the new environment. Both teachers grouped by ability, as they had done the previous year, and each group member was identified and listed. All students were aware that their grouping was based on ability, and the beginning year data was used to validate each student's placement, confirming ERO's (2013) report that achievement information was mainly used for grouping students. Lesson content was focused on Level 1 of the curriculum, revisiting learning intentions at an advanced counting stage.

Elements within each classroom environment encouraged 'counting' over 'grouping' numbers, reinforcing certain students to continue to count instead of developing more sophisticated strategies. In the Year 2 classroom, the quick 10 independent activities reinforced counting on. It was intended to help students build a bank of number knowledge, but a drill-like conventional approach (Baroody et al., 2009; Baroody, 2006) where speed to complete the task was emphasised, it inadvertently undermined this process and reinforced 'counting on'. The following year, Teacher A used a head-to-head competition to practise basic facts. Again this activity reinforced 'counting on' as a quick and effective strategy. In comparison, Teacher B was encouraging students to just know basic combinations using flash card drills and rote learning. Teacher B expected students to know items of knowledge. Limited experiences of subitising, part-whole relationships, and more-and-less relationships restricted students' concept of number and quantity (Jung et al., 2013). The consequence was that students struggled to recognise different number combinations and reverted back to 'counting on', as this was the only strategy they had to access the answer. There was evidence that students were overgeneralising when presented with flash cards with the addend missing (i.e., 2 + ? = 5), on a number of occasions, students added both numbers together. This displayed an incomplete understanding of part-whole relationships (Riley et al., 1983) and may also indicate a lack of knowledge of inverse operations (Losq, 2005); i.e., in knowing within the structure of this equation to work backwards from the known elements (the whole and one of the addends) to the unknown.

Both classroom environments 'unintentionally' reinforced counting over more sophisticated strategies (Young-Loveridge, 2010). After reviewing each teacher's beginning of year planning, it became clear that both had revisited advanced counting key ideas. This confirmed that both teachers had regressed at the beginning of the year, revisiting Level 1 material the students had already been taught the year before, consistent with previous findings of Anderson and colleagues (Anderson et al., 2000).

5.3 Student Learning and Mathematical Content

Students' experiences of mathematics are now examined, particularly in relation to the acquisition of number knowledge and problem solving strategies. The skills they transitioned across into the new environment, their developing attitudes, conceptual understanding and procedural fluency.

Jack was selected because his main strategy the previous term had been 'counting on'. All evidence indicated that Jack had already begun the shift into part-whole thinking. He confidently used his knowledge of doubles to partition and recombine numbers and his basic addition knowledge to combine tens. Yet when working independently, he reverted back to 'counting on' with the basic facts, quick 10.

His mathematics capabilities exceeded any of his fellow group members when exposed to tasks that mediated part-whole thinking. Through the teacher mediating an activity that embodied a higher order of thinking, Jack was able to identify the features that supported him to use part-whole thinking, replacing counting with a more sophisticated strategy and therefore created a developmental shift (Beach, 2003). He instinctively formed his own part-whole strategy reflecting a crossover into the zone of proximal development (Vygotsky, 1978). He was able to show this capacity without manipulatives or images, indicating an understanding of the number properties that made up the number problem. However, there was no evidence of him transferring this skill and connecting it to accessing basic facts. If Jack had been provided opportunities to link this doubles knowledge to deriving basic facts, this could have led to him not only using derived fact strategies in a group teaching session, but also applying part-whole thinking in a range of situations (Brown et al., 1982).

As Jack transitioned through to the Year 3 and 4 classroom, his use of part-whole thinking appeared to regress. Teacher A noted that he was the only one in the group who she thought had lost some knowledge over the holiday break. The initial part of the first observation in 2014 confirmed that Jack appeared to have regressed, but this dip in achievement is not unusual (Anderson et al., 2000). The strong doubles strategy he had used the previous year was only observed once. When observed in week 7 and 8, Teacher A had placed him in two groups, as she felt he was straddling between the two (Wall, 2004). Jack appeared to find the work in the lower group relatively easy, putting aside the manipulatives very quickly to calculate the answers internally (i.e., in his head). In the top group, he used a range of strategies including ‘counting on,’ but he also used ‘making 10’ and doubles, and was able to adapt his thinking after interacting with members of his group. It was evident that he had the level of understanding and the ability to explore more sophisticated strategies, so the placement in the top group seemed ideal for his zone of proximal development. If Jack had been given the opportunity to not only explain his strategy but also critique and justify it, he may have become more flexible and shifted away from his tendency to work systematically from left to right through an equation (Young-Loveridge, 2010).

The support Jack received in mathematics as he transitioned from Year 2 to Year 3 was through grouping and interaction with more capable peers. He was enjoying maths more as he was finding it more challenging and his attitude remained positive throughout the transition. He was proud of the fact he was in two groups with opportunities to shine in the lower group, and higher levels of cognitive growth in the top group fostering a positive disposition towards mathematics

(Peters & Rameka, 2010). Jack instinctively moved into part-whole thinking through the ability to use a range of part-whole strategies at the introduction of early additive material.

Levi's main strategy at the beginning of the observations was 'counting on,' and he alternated from 'counting on' in his head to 'counting on' using his fingers. When working with manipulatives in the group teaching sessions, Levi was the first on a number of occasions to notice a pattern or underlying structure within the symbolic objects (Mulligan, 2013). Levi's ability to recognise and understand the underlying structure in a number of situations demonstrated his ability to successfully transfer knowledge from one concept to a range of situations. This confirmed that the awareness of pattern and structure is critical in the development of mathematical thinking, as proposed by Mulligan and Mitchelmore (2009). It was noticeable that he picked up concepts quickly and was able to explain and justify his thinking using modelling with the manipulatives. In the initial observation, when he had been shown the arrow procedure to solve 2-digit addition problems, Levi accurately completed the addition problem using place value in the JAM.

Transitioning through to Year 3, Levi remained with the same teacher and he was still predominantly using 'counting on' as his main strategy in Week 7 and 8, Term 1. He had become very quick at accessing basic facts through 'counting on' as the head-to-head basic fact competition reinforced. In group work and independent work he was trying to use them less or hide them from view; this may indicate a small shift acknowledging this as a primitive strategy to accessing answers.

When revisiting 2-digit addition, Levi had forgotten the arrow procedure he had been shown the year before, reverting back to the manipulatives. Levi had made sense of this process using the bean canisters and was able to clearly communicate, verify, and validate his processes. He was hesitant to shift from this way of working to the teacher's arrow procedure, but was able to work through it systematically and accurately. Teacher A felt he was about to shift and during all the observed sessions, he was one of the students that picked up the concepts quickly and confidently, correcting others and leading peer discussions.

Levi was able to use a part-whole strategy, but needed more opportunities to explore and link manipulatives to part-whole thinking. An additional challenge was shifting him off using his fingers as a quick way to access answers as he thrived on competition, which seemed to reinforce the use of fingers. Quick images may have been a way to challenge him and build on his ability to notice patterns and structure, linking the skill of partitioning small numbers to part-whole thinking (Garza-Kling, 2011).

Jessica and Tamati were the last two students to transition through with Teacher A, and both students relied heavily on their fingers in observations in Term 4 of Year 2. Both these students used the technique of storing the first number in their head by tapping their head before ‘counting on’. They continued this strategy in Term 1 the following year. This was an extra step in processing and indicated that these students may not have a full understanding of cardinality. There were higher incidences of them losing track of the count and not transferring basic number knowledge into the group teaching session. It appeared that number knowledge and strategy were separated and clear links between the two had not been made explicit by the teacher (Wall, 2004). Through the teacher’s action of promoting ‘counting on’ by a head tap, students had not made the connection between basic fact recall and number relationships such as “one more” or “one less” (Jung et al., 2013). The focus on creating a large body of knowledge without explicit connections to number relationships, has been suggested to disconnect the interrelationship between knowledge and strategy (Johnston et al., 2010). As both Jessica and Tamati were able to complete a range of basic-facts questions but reverted back to fingers when asked to use knowledge to solve 2-digit addition. These students learned knowledge through rote-learnt procedures and speed tests, but were unable to connect this knowledge to another context or help support strategy development, these findings were consistent with observations of others (Baroody, 2006).

When it came to strategy teaching, Tamati commented that mathematics had become harder and mentioned the arrows as one of the reasons. It appeared that he had not made sense of the arrow procedure used to identify and link addition facts in 2-digit addition. He saw it as the teacher’s system that he had to make sense of. The teacher’s system of using arrows to link the two tens digits and two ones digits was very similar to the process used to complete a vertical written algorithm, in which Kamii and Dominick (1998) criticised “undoes” students number sense. The teacher had not ensured that students understood the quantity values of the digits (e.g., 30 and 40, or 3 tens and 4 tens) in her discussion and the students reporting back when describing the procedure. Students need to construct their own strategies to make sense and build understanding (Aubrey, 1993), and this may have been undermined by the teacher’s approach to push them to abstract representation that overlooked the quantity value of the digits in the tens position. Both Jessica and Tamati were inconsistent when applying the arrow procedure with digit reversal and arrows drawn incorrectly. This showed a lack of understanding of the tens and ones in each number problem. They both needed to fold back to the manipulatives to unpack this misconception and develop a clearer understanding of what each digit represented (Flores, 2010; Ministry of Education, 2008a).

Melanie, who was the only student in the case study to transition to a different classroom teacher, predominantly used a ‘counting on’ strategy in Year 2. In Year 3 she continued to use this strategy in the new classroom setting. She felt that the way she put the number in her head was different from other members of her group who were ‘counting on’. Melanie struggled to make links between facts she knew and unknown facts, returning to what she trusted worked. Even when she was prompted not to use her fingers, she struggled to see an alternative strategy. Teacher B did not make an explicit connection between number relationships of “one more than” and “one less than”, working from the students’ knowledge base (Jung et al., 2013). By making this connection explicit, she may have helped Melanie shift from ‘counting on’ to part-whole thinking.

Melanie had had a lot of experiences with rote learning, with special mention from Teacher B of the additional work she had done at home. The Term 1 focus on knowledge acquisition showed very little improvement from the first observation to the last. Evidence showed that her mathematical content was based around her either knowing the answer or having to use her fingers to access the answer. There was no evidence of her using other strategies to solve number problems or to form new knowledge items. There was no evidence of shift from Term 4 to Term 1, even with the intense focus on basic facts and place value for eight weeks in Term 1, confirming Johnston, Ward, and Thomas's (2010) findings that building up a body of knowledge may not necessarily be used to solve number problems.

In Term 4, Melanie had been very apprehensive about the shift into a new classroom, as she referred to it as 'our class is going to get busted down'. She also voiced her concern about how hard the mathematics would be. After the transition, Melanie had become more positive, acknowledging her ability to solve number problems more independently. She had a clearer understanding of what she needed to learn. Because of the emphasis of rote learned procedures and speed tests, Melanie struggled to retain basic facts in her long-term memory. Melanie had a number of her doubles memorised, and it was evident that these were easy to recall (Kilpatrick et al., 2001). She needed to work on the rest up to 20, then develop the near-doubles facts (Garza-Kling, 2011). Teacher B was attempting this approach, but Melanie was not given time or appropriate manipulatives to explore this concept in-depth.

Levi, Tamati, Jessica and Melanie continued to use 'counting on' as their main strategy throughout the transition. Jack was the only student to show evidence of progress. He displayed the ability to use a range of strategies within the early additive part-whole stage in Term 1, going beyond the doubles strategy he had used the previous year. There was evidence of student conceptual understanding being compromised by procedural learning and a lack of fluency in their basic-facts knowledge. Attitudes towards mathematics fluctuated depending on the student and the situation. Overall, the transition of student thinking was influenced by the teacher, her mathematical knowledge, and teaching approaches.

5.4 Teacher Knowledge and Resources

The teachers' pedagogical expertise combined with their mathematical knowledge was critical in influencing what stage of the framework the students were operating in, and whether they were beginning to transition to part-whole thinking. Bruner (1986) used the term scaffolding to describe the guidance and interactions given by a teacher when working in the learner's zone of proximal development (Vygotsky, 1978). Teaching approaches differed when working in the knowledge and strategy domains of the Number Framework. Several researchers have identified guided discovery learning as the most effective way to support student learning when acquiring knowledge (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; Baroody, Eiland, Purpura, & Reid, 2012; Bruner, 1961). The NDP resources encouraged a process of mathematical inquiry using either the problem solving or an investigative approach to learning.

5.4.1 Teaching approaches

Teacher A was very reliant on the NDP resources to provide guidance for how to approach the learning. She systematically worked through the material step-by-step ensuring everything was covered, a consequence described by Scouller (2009) of the NDP. Each strategy lesson began with a problem linked to a real-world experience. The teacher made a point of selecting problem contexts that students were familiar with and incorporated the student's names in the problem. The use of manipulatives and students working in pairs or small groups provided opportunities of collaborative group work. A range of possible student strategies and solutions were accepted and recorded.

Student strategies for the 2-digit addition lesson were replaced by a rote procedure, as the teacher demonstrated how to use arrows to combine the tens and the ones, moving away from an investigative open-ended inquiry into a procedurally-based operation. Teacher A's approach was consistent with the findings that some teachers adopt the surface features of current reform programmes (Fraivillig et al., 1999). In wanting to have the students access the right answer, the teacher turned the 2-digit addition problem into a mechanical and shallow approach (Stein et al., 2000). This is an example of moving away from conceptual understanding and replacing it with a procedural approach with instructional understanding (Skemp, 1976).

The concern with this procedure is that it was undermining place-value understanding, as the students did not refer to the digits in the tens as 3 tens plus 4 tens. Place-value language vanished very quickly from both the teacher and students as they both identified the numbers as 3 + 4 =. This approach focused on a product-centred belief that this was the most efficient process (Bibby, 2002). The teacher took ownership of the strategy as described by Tamati 'we have to do arrows, to do the correct arrows to get the correct answer and that's hard.' Telling students exactly what to do and controlling the direction of student thought illustrates that Teacher A gave into the temptation of *teacher lust* (Tyminski, 2010).

Both teachers' approach to knowledge acquisition aligned itself with conventional wisdom and the skills approach (Baroody, 2006). This was evident in Teacher A's approach to acquiring knowledge with the use of speed tests, drill practices, classroom competitions, and rote-learned procedures for reciting number sequences. For certain students, this may have been an effective way to motivate them, but for Jessica, Tamati and Melanie, these facts learnt in isolation only developed routine expertise with no evidence of transfer or adaptable knowledge (Baroody et al., 2012).

Teacher A's use of questions to scaffold the learning alternated between funneling and focusing. When Teacher A had a definite path she wanted the students to follow, the majority of her scaffolding was funneling-type questions (Wood, 1998). With the use of manipulatives and arrows, she led the students in a certain direction. The following year when Teacher A started working with mathematical content she was unfamiliar with, the predominant scaffolding was focusing questioning. She presented problems, recorded responses of different strategies but because of her uncertainty of what to do next, she left the students there. This reflected the limitations the teacher had, in that she lacked the content knowledge to support and push the problem-solving process along (Baroody, 2003). This is consistent with the work of several researchers (e.g., Anthony & Hunter, 2005; Baroody, 2003; Franke et al., 1998) showing that teachers find it difficult to follow up student explanations, connecting them with other student strategies and extending them to higher levels of mathematical thinking.

Teacher A missed opportunities to support the transition into part-whole thinking. This may have been due to a lack of confidence and understanding of the higher stages of the Number Framework. Some of the key strengths Teacher A identified in Year 2 was her ability to be well planned, knowledgeable of changes, and the skill to change and adapt when needed. She struggled to replicate these attributes the following year, possibly because of the lack of knowledge of Level 2 content.

Classroom practices reinforced counting and routine expertise, which may have also limited student progress into the higher levels of the Number Framework.

In comparison, Teacher B was very confident of her mathematical knowledge and she approached Year 3 and 4 very differently from Teacher A. She focused on building student knowledge of basic facts and place value. As Teacher B had transitioned from the higher levels of the primary school, she went through a process of constructing her own pedagogical knowledge of this particular year group. During the three observations, she worked through the teaching model in reverse order, starting with abstract number properties then simplifying these and incorporating more manipulatives and visualisations by the third observation. By the final lesson, Teacher B located the students' actual development level, accessing the doubles they knew and the doubles they needed to learn.

At the beginning of each group lesson, Teacher B used conventional wisdom and a skills approach to practise basic facts to five and doubles through drill, rote learning, and partner games (Baroody, 2006). These all focused on building that body of knowledge and creating knowledge in the long-term memory for instant recall. Teacher B did not incorporate different representations of quantities, limiting the students' development of number relationships. To support this development, Teacher B needed to consider part-whole relationships and develop understanding of more-than and less-than relationships as well as subitising (see Jung et al., 2013).

In the final session, Teacher B incorporated a guided-discovery learning approach, using tens frames and magnets to model doubles patterns. At the beginning of the session, she modelled the doubles structure in a single tens frame. As she crossed over to a double-tens frame she missed the link to use the double structure to cross over, replacing the doubles structure, which was based on quinary structure '5 and,' with the place-value 'make 10' structure. She missed a teachable moment to build the link from doubles students knew to doubles they did not. As Melanie discovered at the end of the session, she knew all her doubles in the single tens frame 1 + 1, 2 + 2, 3 + 3, 4 + 4 and 5 + 5, but struggled to access the doubles in the double frame. What initially started out as a guided discovery lesson became a heavily scaffolded session, with funneling questions and direct-teacher instruction with a focus on skill acquisition. Teacher B dominated the conversation, leading students away from ‘counting on’ and presenting them with “one more than” connection. In the early years of developing place-value understanding, it is considered necessary for teachers to heavily scaffold and support the learning (McGuire & Kinzie, 2013). To shift students off ‘counting on’ Teacher B had also heavily scaffolded to encourage students to use one part-whole strategy, limiting their experience of a range of other strategies.

Students were given limited opportunity to discover or explore ways to derive new facts from known facts. This was a major shortcoming on the part of the teachers and the essence of part-whole thinking and the understanding of relationships among numbers (Baroody, 2000). However, structure and pattern was incorporated into the knowledge to support student understanding of number properties. Once Teacher B had accessed their development level, she did not give them opportunities to work in their zone of proximal development, as she felt she had spent enough time on this area. She planned to put a poster up on the wall and thought that the students just had to learn it, as she needed to move on. This may have been because Teacher B felt the time restraints and pressure to cover other aspects of the mathematics curriculum (Bibby, 2002). If she had incorporated both the doubles structure, as well as the quinary ‘5 and’ and the place-value ‘make 10’ structure and let the students decide the best model to access unknown doubles, she would have fully incorporated guided-discovery learning, which lets the students build their own links and is considered more effective for long-term memory and retrieval mechanisms (Bruner, 1961). It is thought to be through identifying number patterns and noticing regularities that knowledge becomes connected and structured, making it easier for the student to access, retrieve and apply flexibly (Kilpatrick et al., 2001).

Both teachers had elements of a socio-constructivist approach to teaching with collaborative work, using questioning and a range of manipulatives to support transition. One of the limitations in their practice was the ability to recognise where students were operating currently, and what their capabilities were when presented with higher levels of thinking. At times the ability to anticipate and respond to particular student needs was also an issue. Teacher A needed to fold back to knowledge they had not acquired and build these skills before the students could successfully engage effectively and productively in using known facts to combine 2-digit addition problems, as suggested by Westwood (2006). The approach to constructing number knowledge needed to change to examine pattern and structure and develop a ten-structured concept, focusing on partitioning numbers into units of tens and ones (Young-Loveridge, 1999b).

Teacher B needed a more strategic approach to build knowledge items as the skills approach did not change student knowledge base (Johnston et al., 2010). Both teachers had a tendency to take mathematical concepts and break them down into a step-by-step progression (Scouller, 2009), similar to an algorithimic approach which may only encourage instrumental understanding (Skemp, 1976). Evidence indicates that explicit links between knowledge and strategy were not being made. Awareness of this issue and developing students' understanding of number relationships would help teachers shift students from 'counting on' to part-whole thinking.

5.4.2 Representations

Repesentations decreased over time in Teacher A's classroom, and in Teacher B's classroom they were used sparingly over the observation period. Teacher A used manipulatives to support place value, intially using bundles of sticks in Term 4, then replacing this representation with canisters and beans. She commented that the students preferred the canisters and beans and this was evident with more observations of independent work. The students knew this manipulative well and had a firm understanding of how this model connected to the ten-structured concept (Boulton-Lewis & Halford, 1992; Young-Loveridge, 1999b). When this manipulative was removed, it did increase the processing load for all the students as they had not yet fully internalised the concept. The introduction of the arrow procedure was too early and this abtract representation was not fully understood (Losq, 2005). The students needed more time to make the connection between the digits and their placement to develop positional property understanding (Hughes, in press; Ross, 2002).

Teacher A had made the assumption that students would identify numbers that 'make 10' in her part-whole lessons without manipulatives, confirming Westwood's (2006) finding that visual aids and concrete manipuatives are not used enough. With the aid of tens frames and counters, students would have had the opportunity to discover for themselves and independently identify the structural properties of the problem, creating a deeper learning experience (Anthony & Hunter, 2005). The combination of the manipulatives and their basic-facts knowledge would have encouraged students to investigate number composition and decomposition (Clements, 1999; Garza-Kling, 2011; Young-Loveridge, 2001). This opportunity could have provided students with rich connections, developing fluency (Garza-Kling, 2011) so that 'make 10' knowledge could become context free and adaptive in a range of situations (Brown et al., 1982).

Teacher B worked through the teaching model in reverse beginning with abstract representation: '2 + 2 is 4 then 2 + 3 must be?' When students were unable to access the + 1 strategy, the second lesson incorporated beans in two colours (two blue and two yellow), then adding another yellow. The students still struggled to distinguish that the teacher had added one more only, seeing the whole, two blue and three yellow. With the teaching example lacking structure, the students struggled to identify the change and reverted to 'counting on'. In the final lesson, Teacher B incorporated the two important elements of both pattern and structure (Mulligan, 2013). This appeared to be a lot clearer for the students to identify the doubles pattern, as the magnets were set within an information rich context; i.e., the tens frame (Losq, 2005). It was a lot easier for the students to explore the features of the double structure and investigate number composition.

Teacher B did not give the students any opportunitiy to use the manipulatives independently to demonstrate the skill or process (Flores, 2010). She used shielding as an imaging process and the tens-frame structure was the only visualisation used. Teacher A gave students opportunities to demonstrate their thinking with the manipulatives and for one example there was a picture drawn to represent the canisters. However, opportunities to use diagrams were rare and under-utilised (Presmeg, 1986; Wheatley, 1991; Van Garderen, 2006).

5.5 External Influences

Assessment played a significant part in the teaching programme at the end of Term 4 and the beginning of Term 1, as knowledge items were incorporated into the class programme in an attempt to fill gaps before the upcoming assessment. NDP resources (Ministry of Education, 2008a) recommend targeting knowledge as hot spots and class warm up, but in practice this was knowledge acquisition where there were limited opportunities for the students to make sense of and take ownership of the learning (i.e., students were passively engaged (Belcher, 2006)).

At the beginning of the year, both teachers reassessed students as there was limited information passed on with the student at the beginning of the year, as has been commonly found by researchers (e.g., Bicknell & Hunter, 2009). Teacher A used it as a checking system to ensure the accuracy of transition information. Teacher B used this information to target knowledge areas the students lacked and to group students. This approach reflected a deficit model of assessment approach in its entirety, as Teacher B had not progressed into the higher strategies with a focus on building knowledge and filling gaps (Peters, 2003). This approach was isolated and knowledge driven (Aubrey, 1993; Perry & Dockett, 2004; Peters, 2003; Sherley et al., 2008). Teacher B referred to getting them ready for the senior levels of the school through "remediating skills deficits" (Schulting, Malone, & Dodge, 2005, p. 2).

In an ecological examination of the expectations and events happening in the wider school community, a range of factors influenced both teachers. The isolation of knowledge and itemising may have been influenced by JAM, but also a checklist generated by the Syndicate Leader to ensure that all items in each stage were checked off. Assessment-driven teaching was evident in Teacher A's classroom, with the statement 'they need to learn for this (holds up JAM).' This gave the impression that particular tasks were seen as an 'add-on' (Swann & Brown, 1997) and assessment driven. National Standards dominated teachers thinking with school wide assessment, such as the preparation of Year 2s for the following year's PAT (Hayward, 2007).

The school infrastructure had a major impact on both teachers, with the Year 3 and 4 syndicate being completely changed. Teacher B had not been expecting the shift as placement had been imposed without consultation. Teacher A had requested a shift, but had expected more experience and support, however these were removed. Both teachers were feeling like 'fish out of water', experiencing their own personal transition into a new setting. Bronfenbrenner (1979) has stated that a “person’s development is enhanced to the extent that valid information, advice, and experience relevant to one setting (is) made available, on a continuing basis” (p. 217). He does state in terms of transition from one environment to the next that communication needs to continue from the previous environment, but in this case the support into the new environment had been removed. This in turn had created a level of uncertainty within both teachers.

Teacher A voiced her uncertainty in the interview and after the final observation asked for support with the higher stages of The Number Framework. A feeling of anxiety and uncertainty was portrayed as she commented ‘confidence is a little bit low at the moment’. It appeared that with the lower stages of the Number Framework, to overcome her anxiety around mathematics, Teacher A had become familiar with the content and then approached parts of the learning in a procedural, structured, almost algorithmic manner, as also noted by Bibby (2002). To support the teachers’ transition, the school needed to consider in their restructuring how they would provide the necessary advice and guidance to support teacher shift.

To make consistent judgements, teachers need to understand the quality of the students’ responses in relation to the tasks and link this to the Standard (Sadler, 1987). Both teachers studied here could identify students that were ‘counting on’, but there was evidence that assessment overruled what was happening in the classroom every day (Hayward, 2007). Teacher A rated the students higher if they could complete the task ‘using place value for addition problems’ in JAM by replicating the arrow procedure. This indicated a misinterpretation of the assessment task and the appropriate responses students should make at this point in their development.

OTJs for National Standards require judgements to be made on what students are doing independently, most of the time (Ministry of Education, 2009). The opposite occurred for Jack who was showing strong evidence of part-whole thinking in both the assessment and classroom observations, yet the teacher kept him in the same group the following year, which showed inconsistent judgement due to an ‘order effect’ where the teacher was carrying over impressions from the previous assessments (Sadler, 1987). Without moderation processes in place, Teacher A's inconsistent judgement on JAM had not been picked up, consistent with Poskitt and Mitchell's (2012) conclusions that minimal moderation processes were in place currently.

As students transitioned from Year 2 to Year 3, classroom practices and teaching approaches reinforced 'counting on'. When supporting the students to shift from advanced counting to part-whole thinking, procedural learning and routine expertise was evident. An additional layer of complexity was the teachers' own transition and the external influences, bringing together a multilayered ecological system where change is examined on a number of levels as mathematical knowledge transitions across into a Year 3 context. A possibility for future research is to investigate teacher transition into this particular level and compare it with a teacher who is currently teaching Year 3 and 4.

Chapter Six: Limitations and Implications

6.1 Introduction

Finally, it is important to consider the research structure and the limitations (6.2) that may have impacted on the trustworthiness of the data. Further questions and research ideas that arose from this thesis are noted in addition to addressing the implications that are drawn from the research.

Case-study research methodology investigates a bounded system completely unique within its own setting. The combination of this with the personal views and knowledge of the researcher led to informed decisions throughout the research (Stake, 1995). These decisions influenced processes such as posing questions and data analysis. The researcher provided sufficient detail about the research study for it to be replicated, providing 'thick data' that illustrated the dynamics of the complex interactions and subtle interchanges that occurred throughout the study (Holm, 2008; Otrel-Cass et al., 2010). The case-study approach allowed the investigation to be holistic in nature, gaining meaning from events set in real life via the use of classroom observations and semi-structured interviews. It is acknowledged that it is not possible to make generalisations from such a small number of participants and in one particular school setting. However, the amount of detailed data provided should help the reader to understand the findings and conclusions, and potentially make meaningful associations to their own personal and professional situation.

With the researcher's prior role of teaching Year 1 in the Junior Syndicate of that school the year before meant that the likelihood of some of students having been taught by her was high. Of the six participants selected, Levi and Jack fell into this category. This was not viewed as a conflict of interest but as an advantage as she already had a relationship and rapport with these students (Cohen et al., 2007). It was an easy introduction as most Year 2 students in the class already knew the researcher and the likelihood of students sharing authentic knowledge and understanding was higher due to the prior relationship built on experience, respect, and trust (Smith, 2011).

To validate the findings and create an accurate and credible picture of the events, the researcher triangulated by using a range of data. Through the following methods, the researcher was able to establish a chain of evidence that linked different events and types of data together (Burns, 2000). These parts included time triangulation utilising data that were collected from the same group at different points in the time sequence, in this case at the end of 2013 and the beginning of 2014. A combination of two levels of triangulation was used, including the individual level through student and teacher interviews, and the interactive level of the teachers and students operating in a group during mathematics time. Methodological triangulation within this research used the same method at different times and in different settings (Cohen et al., 2007). Finally, member checking was used after each interview question. The researcher validated the student and teacher responses by repeating back the key points and having the interviewee confirm that this was a true representation of their thoughts. The teachers also reviewed the transcripts and signed each copy validating their interviews. All these methods added together to increase the credibility and trustworthiness of this study.

Another strength of this study was the use of an iPad to explore the multimodal nature of the classroom. The researcher was able to review and revisit events over and over, to hone in on micro-details capturing the true nature of how students approached each mathematical situation.

There are weaknesses and limitations that need to be considered in a case study approach. These are the lack of control over extraneous variables that mean it is difficult to determine cause-and-effect relationships. The potential for bias was high with only one researcher, and the inability to cross check data may mean that data is selective, personal, and subjective (Cohen et al., 2007).

6.2 Limitations

Although informative and worthwhile in and of itself, the results of this thesis were limited by the participant selection process. Specifically, because the selection process was limited to one class and, due to time restraints, the potential range of participants was limited. It could have been a more effective study if the researcher had been able to observe groups of ‘counting on’ students across a number of Year 2 classes. This could have increased the potential to discover new and different examples of how the construction of knowledge and strategies in junior classes affects the processes of transitioning into part-whole thinking. Another limitation was the fact that most of the students transitioned into one classroom remained with the same teacher. This was an event that was unexpected and narrowed the transition experiences of four of the case-study students, limiting the data gathered on ‘change of teacher’ experiences.

Another limitation was that only selected parts of each lesson for each student were recorded, limiting a complete picture of the whole experience. Data were collected through the researcher’s lens (Otrel-Cass et al., 2010), recording what she considered significant or relevant in the moment. From an overall impression of each lesson, the researcher felt she had gathered enough relevant data for each student. During the analysis phase there appeared an inconsistency in her opinion with Levi. As noted, once data were analysed his mathematical content activity for the ‘just right’ category dropped in the second phase. Upon reflection, the researcher reviewed the footage gathered and realised that she had focused more on Tamati and Jessica as they had clearly revealed how they accessed the answers, whereas most of Levi’s work was completed internally (i.e., solved in his head), so it was hard to identify what processes he was using as an outside observer.

Another limitation was Teacher B, who treated the observations as ‘one off performances’ (Guba et al., 2007). Each observation followed on from the previous lesson even though the observations were spread over a two-week period. The researcher got the impression that the lessons were not following the teacher’s normal programme. In saying that, the data collected were rich enough to gain an understanding of where the students were operating, the teacher’s understanding of the Framework, and the processes she went through to access knowledge and scaffold the learning. Additionally, due to the lessons being sequenced, the researcher was able to observe modifications of lesson content, which provided evidence of teacher reflection and her ability to adapt learning intentions.

At times, the researcher struggled to be a complete observer. In moments of weakness she gave in to *teacher lust* instead of remaining completely neutral throughout the interviews. Specifically, on three separate occasions she provided her opinion, and this was identified in the results. The remarks reflected her professional ties to the school and to the teacher involved. When the teacher was uncertain, the researcher at times found it hard not to reassure or clarify items, as this had been the researcher's role in the past. To avoid this in the future, the researcher needs to consider selecting a school where she has not had such an in-depth involvement, or remain reflective in her role and actions inside the research. On the other hand, this relationship with the school provided the researcher with real access to teachers and students as keen participants in the study.

6.3 Implications

Despite the limitations discussed above, this thesis highlights that there is a need to look more closely at the teaching approaches teachers use when teaching number knowledge and strategies in the junior levels of a primary school. These include the development of number sense and ways to initiate part-whole thinking in Year 1, 2, and 3 to support transition into early additive strategies. There is also a need for teachers in Year 3 and 4 to understand that time is needed to develop relational understanding of the key concepts in Level 2.

This thesis highlighted that the use of drills and speed tests to build number knowledge through rote learning were mostly ineffective at creating flexible and fluent knowledge. This research supports the need to provide teachers with tools to develop a number sense approach, developing their understanding of ways to incorporate pattern and structure. The researcher recommends junior classes experience alternative ways to 'counting,' putting an emphasis on partitioning small numbers through conceptual subitising-type activities (Young-Loveridge, 2010). Additionally, students need to fully understand number relationships connecting number and quantity through part-whole relationships and more-and-less relationships.

Teachers could benefit from an in-depth inquiry into the development of place value. Professional learning could focus on helping teachers examine place value within each stage, the effective use of manipulatives, and the connections from one level to the next.

There appears to be an urgency to remove manipulatives from the learning, as teachers worry students will become too reliant on them. Teachers need to be aware that removing the manipulatives too soon may lead to a student regressing or replacing them with fingers, so the manipulatives have just changed form and there has been no shift in the student's thinking. The push to move students to an abstract representation can mean certain mathematical understanding becomes instrumental and procedurally based. When teachers introduce a procedure such as tapping the head, it is essential they connect this action with conceptual understanding. The danger of combining a rote skill (tapping the head) with conceptual understanding is that it interferes with meaningful learning and can lead to more rigid patterns of thinking and behaviour (Haskell, 2001). Indeed, this was highlighted by the lack of flexibility when solving problems and limited ability to construct new understandings with Tamati, Jessica and Melanie, as they continued to refer back to a fixed procedure. In the opinion of the researcher, which have been informed by Pesek and Kirshner (2000), and based on the findings of this thesis, the place for procedures is after students have clearly demonstrated their understanding of the relationship between the abstract representation and the manipulatives, and when students can recognise the efficiency of the procedure while also understanding how and why it works. Students then need to test why the procedure works and the places it works to avoid overgeneralisation.

There remains a misunderstanding of the second stage of the teaching model, when manipulatives are replaced with representations such as imaging, diagrams and pictures. The use of the term 'imaging' in the NDP material has been problematic, as some teachers consider this term refers to students completing problems in their head typically connected with Stage 3 of the Number Framework. Instead this is an aspect of progression, as part of the Strategy Teaching model where students transition from manipulatives into a representational phase where they use pictures, diagrams or imaging to link to the abstract (Flores, 2010).

The findings of this thesis highlight the influence that assessment tasks have on classroom practices and the need for school-wide moderation to keep consistency and accuracy. Student management systems need to pick up anomalies in achievement data from one year to the next and investigate inconsistencies as part of their self-review process on teacher capability. Teachers need to use assessment more formatively to inform classroom practices and incorporate tasks so they are meaningful and linked to strategic thinking.

In summary, this research has presented some of the possible barriers students experience when transitioning from Year 2 to Year 3, and from advanced counting to early additive part-whole thinking. The importance of teacher knowledge and understanding, and the impact of teaching practices that support or undermine shift, reinforces concerns that children are encouraged to count too long with limited opportunities to explore and discover structure and patterns within numbers.

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Appendices

Appendix A: Student Interviews

Phase One

<u>Introduction</u>

- What are some of the favourite things you like to do at school?

<u>Mathematics Class</u>

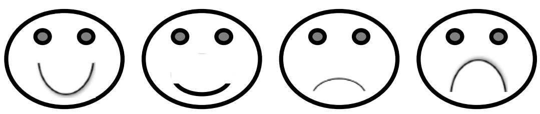
1. Tell me about your maths class (prompt re: grouping, teaching approach, activities, homework, maths books – how you record your work)
2. Mark on the scale below, how challenging did you find maths this year?

Too Easy Just right Too hard

3. What do you enjoy most about maths?
4. Are there any parts of maths that you do not enjoy? Why not?
5. Do you prefer working by yourself or with others?
6. What group are you in for maths? Why do you think you are in that group?

<u>Assessment</u>

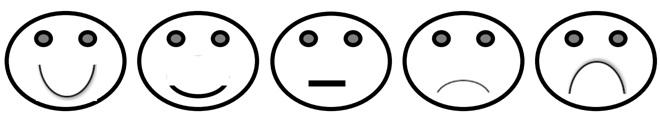
7. How well do you think you are doing in maths?



8. What do you need to learn next in maths?

<u>Attitude</u>

9. How do you feel about maths this year?



The Future

10. What do you think it will be like in year 3 in maths?

Phase Two

Transition

1. What is different about this year's maths compared to last year?
2. Are you using anything you learnt last year to help you this year in maths?

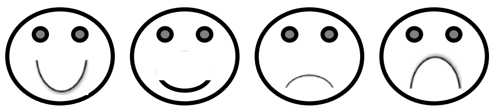
Maths Class

3. Tell me about your maths class (prompt re: grouping, teaching approach, activities, homework, maths books – how you record your work)
4. Mark on the scale below, how challenging do you find maths this year?

Too Easy Just right Too hard

Progress

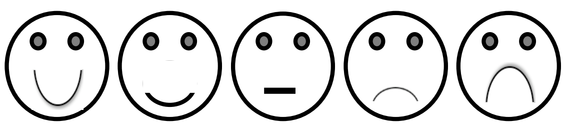
5. How well do you think you are doing in maths?



6. What do you need to learn next in maths?

Attitude

7. How do you feel about maths this year compared to last year?



Reflection

8. I'm sure the children in Room __ (last years room) now would like to know what it is like in year 3 maths. What would you tell them?

Appendix B: Teacher Interview

Teacher Year 2

Semi-structured interview

Phase One

<u>Background</u>

1. How long have you been teaching?

2. How do you feel about teaching mathematics?

<u>Programme</u>

3. What do you believe are the key features of your mathematics programme?

4. What have been the strengths/ weaknesses (if any) of the programme?

5. How do you measure the students' mathematical progress?

6. What consideration (if any) do you give to their attitude towards mathematics?

7. What information do you pass on to the next class?

8. Do you prepare the students for this transition? If so, in what way?

9. What can you tell me about *(name of student)* in maths?

10. What do you think is the next learning steps for *(name of student)?*

Teacher Year 3 semi-structured interview

Phase Two

Background

1. How long have you been teaching?

2. How do you feel about teaching mathematics?

Programme

3. What do you believe are the key features of your mathematics programme?

4. What have been the strengths/ weaknesses (if any) of the programme?

5. How do you measure the students' mathematical progress?

6. What consideration (if any) do you give to their attitude towards mathematics?

7. What academic information was made available to you about the students(s)?

8. How much was this information used to inform your planning?

9. What other factors inform your planning? (school long-term plans, individual profiles, other assessments - beginning year P.A.T) etc

10. How did you group your students at the beginning of the addition/subtraction program and have you had to make any changes. If so why?

11. What can you tell me about *(name of student)* in maths?

12. What do you think is the next learning steps for *(name of student)*

Appendix C: Information sheet: Principal and Board of Trustees

Department of
Mathematics, Science &
Technology Education
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Hamilton, New Zealand

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THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

Date

Dear (Principal) (BOT)

As part of the work towards a post-graduate qualification in education, I would like to undertake a small research project in your school in 2013-14. The focus of the project is to document the changes children make in their mathematical thinking and the experiences in mathematics they have as they move from year 2 to year 3.

I am looking to involve children who have been identified by their teacher as at "Counting on to solve problems (stage 4)" in Year 2 in one Year 2 classroom. This is likely to be one group of 6 - 10 students. I would like to observe these children in mathematics time for three lessons. I would then like to follow up this observation with an interview with each child and the classroom teacher. The interview times will be set in coordination with their classroom teacher so as to minimize disruption to the normal class program. The interviews are likely to take 20 minutes for each student and 40 minutes for the teacher. I have a two-week time frame in mind of November 4th – 14th.

In Term 1 2014, I would like to re-interview each child, their new classroom teacher and observe them in three math sessions in their Year 3 classroom. These days will be decided in consultation with the school and each classroom teacher.

Participation in this research project is entirely voluntary and throughout the interview participants may choose not to answer a question or to stop the interview - this will be respected. The participants can withdraw by email or by contacting me by phone up till the point when data analysis begins.

The interviews and teaching sessions will be video or audio-taped with the teachers' and children's consent. The participants' names will not be used in the final research report and everything they tell me will remain confidential. I will be the only person to have access to the audio-tapes. The school and class will also not be identified in any report that is written as a result of the research. I will provide you with a copy of the research report upon completion.

In agreeing to this research in the first year of study I would value your recommendation of a possible teacher who may be interested in having their children involved. The following year I would like to approach the Year 3 teachers in classes which the students have been placed. Once I have these teachers' permission, a separate letter of information will be sent to all teachers and to all students' caregivers. If you have any questions or require further information, please feel free to call me or email me.

Finally, I would like to thank you for considering this research and look forward to hearing from you.

Yours sincerely,

Jo Matthews

Appendix D: Information sheet: Year 2 Teacher and Year 3 & 4 Teacher

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THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

Date

Dear (Teacher) Year 2

As part of the work towards a Masters degree in education, I want to undertake a small research project in your school. The focus of the project is to document the changes children make as they move from year 2 into year 3 in mathematics, specifically their thinking when approaching addition and subtraction problems in Numeracy. I will also be examining their transition from one year-level to the next, looking at different teaching approaches in mathematics and the sharing of information about student learning in mathematics. I will then write a thesis based upon what happens and compare this to the current research literature. Information from the thesis may also be used to write articles for publication and/or presentation at conferences.

I am planning to work with a group of children in one class at year 2 and follow each student through to their new Year 3 classroom. I will be selecting children according to the following criteria and protocols:

Criteria

The children that fall into the category "Counting on to solve problems (stage 4)"

The children who agree to be part of the research with caregiver approval.

Protocols

I will observe three mathematics lessons at times that are convenient for you during the time period of November 4th – 14th. I would also like to interview you for around 40 minutes to discuss your teaching in mathematics.

I would interview the children individually at some mutually convenient time where disruption to your teaching program will be minimal.

The interviews and teaching sessions will be video or audio-taped (with all participants' permission) but no participant's real name will be used in the research and no identification will be made of either the class or the school.

At any time in the process you may contact me using the phone or email contacts as given below if you require more information.

If there are any problems you can contact my supervisors – Associate Professor Jenny Young-Loveridge: on …….. or email on………; or Dr Brenda Bicknell: on …….., or email on………..

Finally, I would like to thank you for considering this research and look forward to working along side you.

Yours sincerely,

Jo Matthews

Appendix E: Teacher's consent form

<u>Declaration of Consent</u>

I consent to participate in Jo Matthew's research assignment relating to the mathematical transition of year 2 students into year 3.

I have read and understood the information provided to me concerning the research project and what will be required of me as a participant in the project.

I understand that the information I provide to the researcher will be treated as confidential and that no findings that could identify either me, or my school, will be published.

I understand that my participation in the project is voluntary and that I may withdraw from the project at any time up until the point when data analysis begins.

Name: ……………………………………………..
Date:………………….

Signature: ………………………………………………………..

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THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

Date

Dear (Teacher) Year 3

As part of the work towards a Masters degree in education, I want to undertake a small research project in your school. The focus of the project is to document the changes children make as they move from year 2 into year 3 in mathematics, specifically their thinking when approaching addition and subtraction problems in Numeracy. I will also be examining their transition itself from one-year level to the next looking at different teaching approaches and the sharing of information about student learning in mathematics. I will then write a thesis based upon what happens and compare this to the current research literature. Information from the thesis may also be used to write articles for publication and/or presentation at conferences.

I am planning to work with a group of children who were identified as counting to solve problems in their year 2 class. Each student will then be followed through to their new Year 3 classroom. The children would have been selected according to the following criteria and protocols:

Criteria

The children were identified and observed in their previous year at school as "Counting on to solve problems (stage 4)".

The children agreed to be part of the research with caregiver approval.

Protocols

I will observe three mathematics lessons at times that are convenient for you during the time period of March 3rd – 14th I would also like to interview you for around 40 minutes to discuss your teaching in mathematics.

I would interview the children individually at some mutually convenient time where disruption to your teaching program will be minimal.

The interviews and teaching sessions will be video or audio-taped (with all participants permission) but no participants' real name will be used in the research and no identification will be made of either the class or the school.

At any time in the process you may contact me using the phone or email contacts as given below if you require more information.

If there are any problems you can contact my supervisors – Associate Professor Jenny Young-Loveridge: on …… or email on …… ; or Dr Brenda Bicknell: on …… , or email on ……..

Finally, I would like to thank you for considering this research and look forward to working along side you.

Yours sincerely,

Jo Matthews

Appendix F: Information sheet and consent form: Parents/Caregivers

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Hamilton, New Zealand

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www.waikato.ac.nz

THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

Date

To the Parents/caregivers of ....

Kia ora,

My name is Jo Matthews. I worked at *(school)* last year in Room 13 and previous to that I was a Numeracy Adviser at the University of Waikato. This year I would like to conduct a small research project at *(school).*

I want to document changes children make as they move from year 2 to year 3, specifically their thinking when approaching addition and subtraction problems in Numeracy. I will also be examining their transition from one year-level to the next and investigating different teaching approaches in mathematics. I will then write a thesis that will go towards gaining a Masters qualification in education at the University of Waikato.

The classroom work has four parts. The first involves observing a small group of children in mathematics sessions in year 2. Then I will be interviewing each child. Subsequently I will observe each student in their year 3 class the following year and re-interview. I will then write a report based upon what happens which will be compared with what other people have written and so form a thesis. Information in the report may be used to write articles for publication and/or presentations at conferences.

Your child's name will not be used in the report, in any other publications or used verbally at any conference. In the writing process I will assign pseudonyms to each child so that their identity is unknown. Everything that is said during the sessions and interviews will remain confidential. When completed, a copy of the report will be available at the school.

In order to accurately capture what the children will be saying, I will video-tape the interviews and teaching sessions as well as take notes. These tapes and notes will not be available to anyone else but kept securely by myself.

Can you please talk with your child about what is involved with the research and see if they are happy to take part. If you also agree with their involvement, then please complete the consent form below and return it to school by Friday of next week. It is important that the form is returned, as I cannot work with any children without consent from home.

I will also be explaining the process to the children in class and asking them to complete a personal consent form. Even if your child agrees to take part, they can withdraw at any time by contacting me.

If you have any questions or require further information before making a decision, please contact me through the email address or phone number listed above. If at any time during the research you have concerns, please feel free to contact (*school*) on ….. or my supervisor, Associate Professor Jenny Young-Loveridge ……………...

Yours sincerely,

Jo Matthews

Parent/Caregiver Consent Return Slip

I agree to (child's name) .... taking part in the research work as described in the letter I have received. I understand that video-tapes of interviews and teaching sessions will be made with my child's permission. I realize that all information will be kept private and that my child's name will not appear on any documents published from this project. I understand that my child's participation in the project is voluntary and that they can withdraw from the project at any time.

Name: .... Date: ……………

Signed: ………………………………….

Appendix G: Student consent form

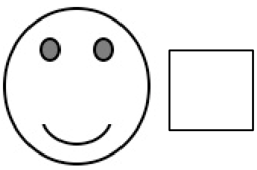
Student's consent form – Student Copy

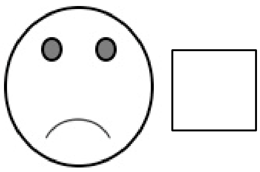
The project Jo Matthews wants to do on maths has been explained to me. If I have any questions, I can ask my teacher or Jo.

I am happy to be part of the project, have my voice recorded and to talk to Jo, so I have ticked the happy face.

Or

I don't want to take part so I have ticked the sad face.





If I want to, I can change my mind and not take part. I can tell my teacher or Jo.

Signed: .... Date ....