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A review of Phillips-type right-tailed unit root **bubble** detection tests

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Abstract

Recent developments on the right-tailed unit root tests of Phillips et al., which are used to date stamp the origination and collapse dates of asset price bubbles, have generated considerable interest. This paper provides a review for both empirical researchers that adopt these new econometric tools to detect the presence of asset price bubbles, and theoretical papers that extend these testing procedures. This paper also uses the psymonitor package in R to demonstrate the practical use of such tests using an example based on data for British Railway Mania of the 1840s.

KEYWORDS

bubbles, generalized sup ADF test, right-tailed unit root

1 INTRODUCTION

Price bubbles are not new despite growing recent interest following the Global Financial Crisis. Dating back at least to the 17th century in the Netherlands, Tulipmania (1636-1637) is widely considered as the earliest example of a bubble. There the contract prices of tulip bulbs were ridiculously high in 1636 only to suddenly collapsed in 1637. Following the Dutch Tulipmania, we see the Mississippi Bubble in France and South Sea Bubble in England, during 1719-1720, with these three events becoming some of the most famous stock price bubbles. The twentieth century had its bubble episodes including but not limited to the German stock price bubble in 1927, the Wall Street Crash of 1929, Japanese asset price bubble of the 1980s to 1990s, the Dotcom bubble and the

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more recent US housing bubble during the 2000s. Bitcoin has also been characterized as a more recent example of a bubble, where its price reached an all-time high of close to US\$ 20,000 in December 2017.

Prior to the work of Phillips et al. (2015a, PSY) traditional bubble detection tests were typically based on ex post analysis, see Gürkaynak (2008) for a review. To address a need for real-time monitoring and surveillance of asset prices, a series of papers by Phillips et al. (2011, PWY), Phillips and Yu (2011), Phillips et al. (2015a, PSY), and Phillips and Shi (2020) develop novel right-tailed unit root tests for market exuberance by identifying the originating and collapse dates of asset price bubbles. These right-tailed unit root tests have been widely employed as early warning systems for bubble-like behavior in a wide variety of financial markets. Moreover, the PSY procedure has also been adopted by the Federal Reserve Bank of Dallas to monitor potential exuberance in international housing markets.

This paper provides a review of theoretical papers that extend the PWY or PSY procedures, and empirical studies that adopt these new bubble detection tools. The theoretical papers mainly focus on developing new date stamp strategies or proposing a new approach to address heteroskedasticity or comparisons of the asymptotic properties of existing bubble detection approaches. The empirical papers cited consider a range of applications and are not limited to stock markets and housing markets. This paper also highlights the usefulness of applying the PSY procedure with a new bootstrap method applied via the R package.

The paper is organized as follows. Section 2 presents a brief description of the econometric methods of PWY and PSY for those not familiar with the approaches. Section 3 surveys both theoretical and empirical papers based upon the right-tailed unit root tests in the literature. Section 4 shows how to use the PSY procedure via the R psymonitor package with an application to the British Railway share price index of the 1840s. Section 5 concludes.

2 | METHOD

This section provides a brief description of the econometric methods of PWY and PSY.

2.1 | The PWY approach

A highlight of the PWY approach is the ability to capture the periodically collapsing bubbles of Evans (1991). Extensive simulation studies show that the PWY approach is especially effective in detecting a single bubble episode. For this testing procedure, a sup Augmented Dickey–Fuller (SADF) method is utilized in testing for the presence of explosive behavior and such a testing procedure is implemented as follows. For each time series x_t , we apply the ADF test for a unit root against the alternative of an explosive root (right-tailed). The following autoregressive specification for x_t is estimated by least squares:

$$x_t = \mu_x + \delta x_{t-1} + \sum_{j=1}^{J} \phi_j \Delta x_{t-j} + \varepsilon_{x,t}, \qquad \varepsilon_{x,t} \sim \text{NID}(0, \sigma_x^2), \tag{1}$$

for some given value of the lag parameter J, where NID denotes independent and normally distributed. The null hypothesis of this test is H_0 : $\delta = 1$ and the alternative hypothesis is H_1 : $\delta > 1$.



The above equation is estimated repeatedly using subsets of the sample data incremented by one additional observation at each pass in the forward recursive regression. Thus the SADF test is constructed by repeatedly estimating the ADF test. Let r_w be the window size of the regression. The window size $r_w(r_w = r_2 - r_1)$ expands from r_0 to 1, where r_0 is the smallest sample window width fraction and 1 is the largest window fraction (the full sample). The starting point r_1 is fixed at 0, and the end point of each sample (r_2) equals r_w and changes from r_0 to 1. The ADF statistic for a sample that runs from 0 to r_2 is, therefore, denoted by $ADF_0^{r_2}$. The SADF statistic is defined as the sup value of the ADF statistic sequence:

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2}.$$

The SADF test statistic cannot locate the origination and collapse dates of a bubble. To identify the origin and the collapse dates, we can compare the recursive test statistic ADF_r against the relevant right-tailed critical values. If r_e is the origination date and r_f is the collapse date, we can construct estimates of these dates as follows:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : ADF_{r_2} > cv_{r_2}^{adf} \right\}, \tag{2}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 : ADF_{r_2} < cv_{r_2}^{adf} \right\}. \tag{3}$$

The ADF statistic and its corresponding critical value are used for dating the origination and termination dates of a bubble.

2.2 | The PSY approach

The PSY approach extends the work of PWY by allowing for more flexible window widths in the recursive regressions on which the test procedures are used. The PSY test has shown great power in detecting the presence of multiple bubbles. The martingale null with an asymptotic drift is specified as

$$H_0: y_t = dT^{-\eta} + y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \text{NID}(0, \sigma^2), \tag{4}$$

where d is a constant, the localizing coefficient η is greater than 1/2 and T is the sample size. The alternative hypothesis is a mildly explosive process:

$$H_1: y_t = \delta_T y_{t-1} + \varepsilon_t, \tag{5}$$

where $\delta_T = 1 + cT^{-\theta}$ with c > 0 and $\theta \in (0, 1)$. The following regression model is estimated:

$$\Delta y_t = \hat{\alpha} + \hat{\beta} y_{t-1} + \sum_{i=1}^k \hat{\gamma}_i \Delta y_{t-i} + \hat{\varepsilon}_t, \tag{6}$$

where $\hat{\alpha}$ is an intercept and k is optimum lag length.

The generalized sup ADF (GSADF) test relies on repeated estimation of the ADF test regression model on subsamples of the data in a recursive fashion. The window size r_w expands from r_0 to 1, where r_0 is the minimum window size. The end point r_2 varies from r_0 to 1 and the starting point



 r_1 varies from 0 to $r_2 - r_0$. The GSADF statistic is the largest ADF statistic over the range of r_1 and r_2 :

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} ADF_{r_1}^{r_2}.$$

The GSADF test is used for assessing explosive behavior for the entire sample period, however, it does not provide the origination and termination dates of identified bubble episodes. In order to provide a real-time monitoring of market exuberance, we will use the backward SADF (BSADF) test. The BSADF statistic is defined as the sup value of the ADF statistic sequence:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}.$$

The BSADF statistic and its corresponding critical value are used for dating the origination and termination dates of a bubble as follows:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) > cv_{r_2}^{\beta_T} \right\}, \tag{7}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < cv_{r_0}^{\beta_T} \right\}. \tag{8}$$

This approach, therefore, has greater power in the detection of multiple bubbles. Phillips et al. (2015b) also derive the asymptotic distribution of this test statistic under the null. The minimum window size r_0 needs to be large enough to allow initial estimation but not too large to miss an early bubble episode. As recommended in PSY, the minimum window size r_0 is set equal to 0.01 + $1.8/\sqrt{T}$.

3 | LITERATURE REVIEW

This section is divided into two parts covering (i) theoretical papers and (ii) empirical papers related to the right-tailed unit root tests in the literature.

3.1 | Theoretical papers

Turning now to theoretical developments since the publication of the seminal papers of PWY and PSY, we find that there exist many attempts to develop and extend these right-tailed unit root bubble detection tests.

Gutierrez (2011) develop a bootstrap version of PWY tests to test for asset price bubbles. The bootstrap procedure is used to generate a large number of simulated values of the test statistics proposed in PWY tests. Gutierrez (2011) applies this bootstrap version of PWY tests to the Nasdaq stock price index and Case–Shiller house price index and finds clear evidence of explosive behavior. Homm & Breitung (2012) propose several tests in detecting rational bubbles and investigate their power properties. The proposed Chow-type test and the modified version of Busetti and Taylor (2004) procedure are found to have higher power than the PWY test. The proposed Chow-type, the modified version of Busetti and Taylor (2004) and PWY tests find the presence of bubbles in the United States, United Kingdom, and Spanish house price indices. Pavlidis et al. (2017) also



develop two novel methods based on the PSY procedure and the rolling Fama regression for bubble detection that use spot and forward (or futures) prices.

In a number of papers, David Harvey and his coauthors contribute to various aspects of the PWY and PSY approach. Harvey et al. (2015) compare the local asymptotic and finite sample power of the PWY and a Chow-type test of Homm & Breitung (2012). The power of the two tests can differ substantially depending on the location of the explosive regime, and whether such a regime ends in collapse. Harvey et al. (2016) show that the presence of heteroskedasticity can affect the performance of the PWY test and they develop wild bootstrap versions of the PWY tests for explosive bubbles. When the PWY test is implemented in the presence of time-varying volatility, there is a possibility of spurious identification of a bubble. They show that the use of critical values that are robust to the presence of nonstationary unconditional volatility can lead to much less clear evidence of bubble behavior. Applications of the new bootstrap tests to crude oil, precious metals, and nonferrous metals identify fewer bubbles than the standard PWY tests. Harvey et al. (2017) develop a new approach that utilizes model-based minimum sum of squared residuals estimators combined with Bayesian information criterion model selection to improve the estimated start and end dates of a single bubble period for both PWY and PSY approaches. The procedure developed by Harvey et al. (2017) allows a wide range of different types of explosive episodes and this BIC-based procedure provides much improved power in accuracy in comparison to PSY procedure.

Astill et al. (2017) look at the issue of detecting explosive behavior by the popular PSY procedure when an explosive episode is both ongoing at the end of the sample and of finite length. It is wellknown from the literature that the PSY approach is particularly well-suited to detect an end-of sample bubble. However, a potential drawback of asymptotic validity of PSY is that it assumes that the length of the bubble regime is a fraction of the total sample size. A more practical assumption could be made based on one of a finite length end-of-sample bubble regime with possibly only a few bubble observations. They suggest a new testing strategy by applying the end-of-sample instability testing approach of Andrews (2003) and Andrews and Kim (2006). The new test statistics are robust to serial correlation and conditional heteroskedasticity. Their proposed approach shows power gains in detecting an ongoing end-of-sample bubble. They apply the new procedure to the S&P 500 price-dividend ratio for the period January 1871 to December 2010. Empirical results show that newly proposed procedure identify the origination of several bubbles in a number of months earlier of the dates compared with the PSY procedure. In Astill et al. (2018), they develop two procedures that use sequential computation of subsample-based test statistics from a training period of data for detecting asset price bubbles in real time monitoring. One procedure is based on comparing the real-time monitoring period statistics with the maximum statistic over the training period, and the other compares the number of consecutive exceedances of a threshold value in the monitoring and training periods, where the threshold value is obtained from the training period. A bubble is detected if either of the procedures reject the null hypothesis of no bubble.

Harvey et al. (2019) propose a weighted least squares-based variant of the PWY test for explosive behavior in financial data in the presence of time-varying volatility. In order to capture the relative power advantages of the OLS- and WLS-based PWY tests across different volatility patterns, Harvey et al. (2019) suggest a union of rejections procedure. This procedure involves a nonparametric kernel-based volatility function estimator for computation of the WLS-based statistic, together with the use of a wild bootstrap procedure applied jointly to a wild bootstrap version PWY test of Harvey et al. (2016), and a wild bootstrap version of the WLS-based test. The new testing procedure is robust to a wide range of time-varying volatility specifications. The WLS-based test can provide improved power in detecting explosive behavior compared to PWY procedures when applied to the FTSE and S&P 500 data. Harvey et al. (2020) develop a sign-based variant of the PSY test for

explosive behavior in financial time series. Compared with the PSY test of Phillips et al. (2015a), the newly proposed sign-based test does not require bootstrap-type methods to control size in the presence of time-varying volatility. Harvey et al. (2020) also develop a union of rejections procedure that combines the sign-based and original PSY tests and utilize the wild bootstrap method to control size. They apply the proposed sign-based PSY tests to daily Bitcoin price data between September 1, 2017 and January 28, 2018 and present strong evidence of bubbles. Harvey et al. (2020) propose a two-step procedure for dating multiple explosive regimes by extending the work of Harvey et al. (2017). The first step is to apply recursive unit root tests to identify a "date window" for the origination and collapse dates of an explosive episode. The second step is to apply a model-based BIC approach to precisely estimate the regime change points within each date window. The approach of Harvey et al. (2020) offers the advantages in allowing the multiple explosive regimes. Harvey et al. (2020) apply the two-step procedure to 22 international housing markets from 1975Q1 to 2018Q2. Among 20 of these international housing markets, the estimated start and end dates are earlier than the corresponding PSY date estimates.

Peter Phillips and Shuping Shi also contribute significantly in developing new methods of bubble testing. Phillips et al. (2014) provide empirical guidelines for the practical implementation of PWY tests. They suggest to use an empirical model that does not include a linear deterministic trend in the regression but has a fitted intercept in testing for explosive behavior. Phillips and Shi (2018) investigate the asymptotics and behavioral characteristics of the PSY approach under alternative collapse scenarios. They also propose a "reverse regression" implementation strategy for detecting bubble implosion and estimating the origination and termination dates of bubble implosion. The new framework uses a mildly integrated process that can capture a variety of forms of reversion in a financial market. Phillips and Shi (2019) introduce a new model for detecting financial market crises and collapses. The model involves an asymmetrically distributed input process that is particularly suited to capturing abrupt market falls. Phillips and Shi (2019) also show that the PSY approach can detect abrupt market falls, providing analytic solutions as evidence of this capability that have arisen in some prior studies. Phillips and Shi (2020) implements the PSY testing procedure that incorporates a new bootstrap method that simultaneously address heteroskedasticity and multiplicity (the probability of making false positive identifications increases with the number of hypotheses tested) in detecting bubbles.

Whitehouse (2019) consider a GLS version of the PWY test procedure and evaluate the power of original OLS-based PWY and GLS-based PWY approaches in detecting explosive behavior. Whitehouse (2019) shows that the GLS version of the PWY test offers superior power over the original PWY approach when a large proportion of the data is explosive. Whitehouse (2019) also develops a union of rejections procedure for both OLS- and GLS-based approaches to account for the initial condition and lengths of the explosive regime. Kurozumi (2020) investigates the asymptotic properties of the CUSUM and ADF-type-based tests and provide some general guidelines for practitioners to use. The CUSUM monitoring test is found to be suited to identify an early and short-range bubble, whereas the ADF-type test is suitable for detecting a middle-to-late bubble. Kurozumi (2021) further investigates the asymptotic properties of detecting the date of a bubble for both CUSUM and ADF-type based tests. The CUSUM-type procedure tends to identify a bubble sooner than the ADF-type procedure. Lui (2019) extend the work of PWY via a new test and dating algorithm for detecting explosive behavior when errors are strongly dependent. A heteroskedasticity autocorrelation robust (HAR) test statistic is used to improve the estimation accuracy in the bubble origination and termination dates. The new test does not suffer from the size problem. Pedersen and Schütte (2020) propose sieve bootstrap versions of PWY and PSY tests to control size distortions under serially correlated innovations. They apply the sieve bootstrap PSY tests



to 18 OECD housing markets and find weaker evidence of housing bubbles compared to existing studies in the prior literature.

Hafner (2020) extends the PWY approach of Phillips et al. (2011) to test for the existence of bubbles in cryptocurrency markets by allowing volatility to be time-varying. As discussed in Harvey et al. (2016), the PWY procedure could be biased in the presence of nonstationary volatility, leading to spurious indications of explosive behavior. It is expected that cryptocurrencies exhibit nonstationary volatilities. Harvey et al. (2016) also develop a wild bootstrap procedure to correct the size of the PWY test and the new procedure achieves good power by considering deterministic volatility functions only. It is possible to account for a possible volatility clustering due to stochastic effects, such as GARCH or stochastic volatility. Hafner (2020) extend the wild bootstrap approach of Harvey et al. (2016) by combining deterministic and stochastic components using the Spline-GARCH model of Engle and Rangel (2008). The proposed testing procedure identify the presence of bubbles in eight out of 11 cryptocurrencies.

Astill et al. (2021) extend the CUSUM-based procedure of Homm & Breitung (2012) for detecting explosive behavior in financial time series to allow for time-varying volatility. The CUSUM-based procedure assumes that shocks are unconditionally homoskedastic, which can be infeasible for many financial time series with time-varying volatility. Astill et al. (2021) modify the standard variance estimate in the CUSUM statistics by a nonparametric kernel-based variance estimate with an aim to reduce the false positive identification of bubbles. Astill et al. (2021) apply the modified version of CUSUM procedure to the daily Bitcoin-GBP price between January 1, 2017 and November 30, 2017 and identify explosive behavior in the sample period. Monschang and Wilfling (2021) explore the robustness of the PWY and PSY test and heteroscedasticity-adjusted variants tests of Harvey et al. (2016) and Harvey et al. (2020). Monschang and Wilfling (2021) show that the PSY procedure outperforms the PSY sign-based strategy of Harvey et al. (2020) in terms of bubble-date estimation accuracy.

3.2 | Empirical papers

A number of studies have adopted the PWY and PSY bubble detection methods to consider explosive behavior in a wide range of applications. Turning first to applications in agricultural and commodity markets, Gilbertet al. (2010) apply the PWY procedure to commodity futures prices for the years 2000–2009 and present evidence of explosive behavior in the nickel, copper, and crude oil series. In early applications of their work, Phillips and Yu (2011) find no explosive periods in coffee, cotton, cocoa, sugar, and feeder cattle cash prices using their PWY procedure. Gutierrez (2013) apply the PWY procedure to investigate the existence of bubbles in the daily CBOT future prices of wheat, corn, soya beans prices during 2007–2008 using a sieve bootstrap method to simulate critical values. They report evidence of explosiveness in the futures prices of wheat, corn, and rough rice. Etienne et al. (2014) adopt the PSY procedure to explore the presence of bubbles in 12 daily US agricultural commodities futures prices for the period 1970–2011. They apply the wild bootstrap procedure of Gonçalves and Kilian (2004) to take account of conditional heteroskedasticity. Their results suggest that bubbles in agricultural futures markets are short-lived. Etienne et al. (2015) also present evidence of short-lived bubbles in the corn, soybean, and wheat futures prices using the same PSY testing procedure along with wild bootstrap critical values.

The housing market has attracted a number of tests for bubbles. Yiu et al. (2013) test for explosive behavior in three Hong Kong residential property market segments (e.g., the overall market, the mass segment, and the luxury segment) between March 1993 and March 2011. They find

several bubbles in all three segments. Jiang et al. (2015) identify episodes of bubbles in the Singapore real estate market between 2006Q4 and 2008Q1. Their findings suggest that the cooling measures, implemented by the Singapore government during 2009-2013 to control house price inflation, were effective. Shi et al. (2016) examine the house price-rent ratios for housing bubbles in Australia major capital cities for the period December 1995 to January 2016. Their results suggest a nationwide occurrence of speculative behaviour in all capital cities in the 2000s. Turning to New Zealand, Greenaway-McGrevy and Phillips (2016) investigate the evidence of bubbles in the New Zealand regional housing markets for the period 1993Q1-2014Q4, and propose a timevarying regression approach to measure potential bubble migration across markets. They find evidence of a New Zealand-wide housing bubble over the 2003-2008 period and that the housing bubble was transmitted from Auckland to the other regional centers. Pavlidis et al. (2016) propose a panel setting for the PSY procedure to examine explosive behavior in housing markets for a large set of 22 countries, and finds strong evidence of explosive behavior between the early 2000s and 2006/2007. Engsted et al. (2016) examine house price bubbles using price-rent ratios in 18 OECD countries from 1970 to 2013. Their results show that 16 of the 18 markets exhibit bubble-like behavior. Also using rent-price rations, Shi (2017) examines the existence of bubbles in price-to-rent ratios of the US national and 21 regional housing markets during 1978–2015. She also proposes a new method for real-time monitoring of exuberance in housing markets by taking into account macroeconomic conditions for example, interest rates, per-capita income, employment, and population growth. After controlling for housing market fundamentals, the PSY approach identifies a bubble episode in the early-to-mid 2000s at the national level and two bubble episodes (e.g., late 1980s and early-to-mid 2000s) at the regional level. Gomez-Gonzalez et al. (2018) explore the existence and international transmission of housing market bubbles in 20 OECD countries from 1970 to 2015. They first apply the PSY approach to a price-rent ratio to test for explosive behavior in each of the 20 housing markets. They then use a time-invariant approach of Phillips et al. (2011) and a time-varying approach of Greenaway-McGrevy and Phillips (2016) to investigate the existence of international bubble transmission from the United States or the United Kingdom to other OECD countries. They identify five episodes of transmission from the US housing bubble. Hu and Oxley (2018b) investigate the presence of bubbles in the US housing market at the State level for the period January 1975 to December 2014. They present empirical evidence to show a housing bubble that originates in the early 2000s and collapses in the mid-2000s in more than 20 States and the District of Columbia concluding that the bubbles of the 2000s were more widespread than the 1980s.

Perhaps unsurprisingly, stock markets have attracted considerable interest from those seeking to identify bubble-like or exuberant behavior. One of the original applications used by Phillips et al. (2011) to demonstrate their new theoretical approach provided evidence to support the dotcom bubble (or NASDAQ bubble) using their PWY procedure. Phillips et al. (2015a) use the S&P 500 data, to show the existence of multiple historical bubbles including the post long-depression period, the great crash episode, the postwar boom in 1954, Black Monday in October 1987, and the dot-com bubble. Escobari et al. (2017) apply the original ADF test-based PWY and PSY procedures to test for bubbles in Latin American equity markets including Argentina, Brazil, Chile, Colombia, Mexico, and Peru. By way of comparison, they also apply tests similar to the PWY and PSY recursive procedures based on Phillips–Perron unit root test to the same stock markets. Both the ADF and Phillips–Perron-based PWY and PSY procedures illustrate similar date-stamping outcomes. Hu and Oxley (2018c) apply the PSY approach to investigate the famous South Sea Bubble in England and the Mississippi Bubble in France during 1719–1720, which are widely regarded as the most famous episodes of bubbles. They confirm explosive behavior in the South Sea



Company and several other British companies. Their full set of empirical results suggest that, during this period, the British share market was generally much more speculative than once thought, as the South Sea Company was not the only one experiencing exuberance in its share price. Hu and Oxley (2018a) also investigate the Japanese asset price bubble of the 1980–90s in Japan. During that time, Japan experienced the most intense episode of growth in asset prices (e.g., art, antiques, golf course memberships, land, real estate, stocks) with prices reaching historical highs. For the first time, they provide robust econometric-based empirical evidence about the timing of Japan's stock market and housing market bubbles and that the bubble migrated from the stock market to the housing market using the time-varying regression approach of Greenaway-McGrevy and Phillips (2016). Deng et al. (2017) found evidence of bubbles in Shanghai stock market Beijing housing market using daily data. They also present evidence of bubble migration from the stock market to the housing market in 2009.

Energy markets have also seen the PWY and PSY approaches used to consider whether evidence of bubble like behavior exists. Bohl et al. (2013) report explosive behavior in German renewable energy stocks during the mid-2000s using the PWY procedure. Gronwald (2016) and Caspi et al. (2018) provide evidence of explosive behavior in oil prices using the PWY and PSY approaches, respectively. Sharma and Escobari (2018) find strong evidence of explosive behavior in a wide range of energy indices/prices including three energy sector indices (crude oil, heating oil, and natural gas) and five energy sector spot prices (West Texas Intermediate (WTI), Brent, heating oil, natural gas, and jet fuel). In addition, they test for explosive behavior in the implied convenience yields constructed using the relevant futures prices for energy sector spot prices, and identify explosive behavior in the WTI, Brent, heating oil, and natural gas spot prices.

In addition to these particular markets, the PWY and PSY approaches have been applied to a range of other markets. Bettendorf and Chen (2013) finds no bubbles in the Sterling-dollar exchange rate using the PWY approach. Cheung et al. (2015) present evidence of bubbles in the MT. GOX Bitcoin prices for the period 2011-2013 using the PSY procedure. They also conclude that the bursting of these bubbles is in line with some major events that occurred in the bitcoin market. Kräussl et al. (2016) apply the PWY approach to detect explosive behaviors in six major art market segments (e.g., "Impressionist and Modern," "Post-war and Contemporary," "American," "Latin American," "19th Century European," and "Old Masters") using a unique dataset for the years 1970-2014. First, the presence of a bubble is confirmed in all six markets. Second, there is strong evidence of a speculative bubble that started in late 2010 and is still in the mania phase for four of the six markets including the "Impressionist and Modern," "Post-war and Contemporary," "American," and "Old Masters" art market segments. Figuerola-Ferretti et al. (2015) find explosive behavior in the cash and 3-month futures prices for six metals; copper, nickel, lead, zinc, and tin. Similarly, Figuerola-Ferretti and McCrorie (2016) also present significant evidence of short-lived bubbles in the spot and futures prices of gold, silver, platinum, and palladium. Finally, Corbet et al. (2018) utilize the PSY approach to examine the explosive behavior in a price-fundamental ratio for two of the largest cryptocurrencies-Bitcoin and Ethereum. They select three measures to represent the underlying fundamentals including mining difficulty, hashrate, and liquidity. They conclude that there are periods of clear bubble behavior, especially for Bitcoin.

4 | AN EMPIRICAL APPLICATION

A useful add-on package for the statistical software (R Development Core Team, 2020) is psymonitor. The main purpose of psymonitor is to implement the popular real-time monitoring

strategy proposed by Phillips et al. (2015a, 2015b) that incorporates the new bootstrap method of Phillips and Shi (2020). In this section, we will demonstrate the use of functions from this package using data from the well-known British railway mania of the 1840s. Section 4.1 provides the historical background and a brief history of British railways share prices in the 1840s, while Section 4.2 shows the R codes to perform the bubble detection test of Phillips et al. (2015a), and reports and discusses the corresponding results.

4.1 | Background and data

The British Railway Mania of the mid-1840s is one of the earliest and well-known examples of financial bubbles in British history. The Economist describes such an episode as "arguably the greatest bubble in history" (December 18, 2008). The Railway Mania resulted in a boom of railway construction and a subsequent financial panic. Jackman (1916, p.585) argued that during 1844–1846, Parliament sanctioned Bills for the construction of 8470 miles of railways, which was three times the mileage already constructed. The mania is accompanied by a speculative run-up and sudden collapse of railway shares. Railway shares soared during the early period, however, the sudden bursting of the share price bubbles ruined many investors and led to difficulties for many railway companies. According to Chancellor (1999), railway shares had fallen from their peak by over 85% by 1850.

Gayer et al. (1953) created a monthly index of total British share prices to represent the total price movement of shares quoted on the London Stock Exchange from 1811 to 1850. There are eight subgroups included in the total index including canals, docks, waterworks, insurance companies, gas-light and coke companies, mines, railways and banks. The subindexes are constructed from averages of actual prices weighted by the number of shares outstanding, and the total share price index is simply a weighted average of the subindex numbers. The total index of share prices is constructed in two forms: one including mine share prices, the other excluding mine share prices. For our particular interest, we select the subindex of prices of railway shares comprises fourteen companies from May 1827 and December 1850 (June 1840 = 100). A list of the 14 railways companies is presented in Table 1.

We also obtain the annual British retail price index (RPI) between 1827 and 1850 from www. measuringworth.com. To deflate the subindex of railways share prices into real values, we use the method of Dagum and Cholette (2006) to interpolate the annual RPI to a monthly series. We then renormalize the monthly RPI to June 1840 at 1. The real railway index is then calculated as the nominal railway index deflated by the monthly RPI, where the real railway index in June 1840 is set to 100. Finally, we convert the real railway index into natural logarithms before analysis. A time series plot of the real share price index for railways in natural logarithm is provided in Figure 1. As can be seen in Figure 1, there is a rising trend of railway share prices between 1827 and 1845, which can be attributed to the prospects of the new transportation by investors (Gayer et al., 1953).

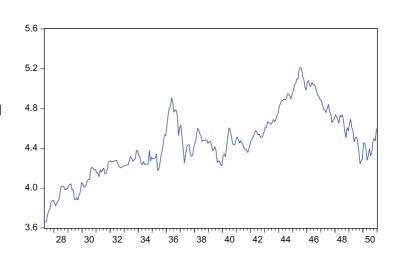
4.2 | Empirical analysis and discussion

The following R code is used for detecting explosive behavior in the real share price index for railways. We install the *psymonitor* package and import the data to R.



Railway companies	The period included in the index
Liverpool and Manchester	May 1827 to Mar 1836
Stockton and Darlington	Jan 1828 to Dec 1834
Cheltenham	May 1827 to Mar 1836
Forest of Dean	May 1827 to Mar 1832
London and Birmingham	Jun 1833 to Aug 1846
Great Western	Sep 1835 to Dec 1850
London and Greenwich	Jan 1835 to Dec 1850
Bristol and Exeter	Mar 1836 to Dec 1850
Manchester and Leeds	Nov 1836 to Jul 1847
North Eastern	Jan 1838 to Jul 1847
London and Southwestern	Nov 1839 to Dec 1850
Midland Counties	Jan 1841 to Dec 1850
Edinburgh and Glasgow	Jan 1842 to Dec 1850
Great North of England	Jan 1841 to Dec 1850

FIGURE 1 Real share prices for railways between May 1827 and December 1850 (June 1840 = 100) presented in natural logarithms [Colour figure can be viewed at wileyonlinelibrary.com]



```
library(psymonitor)
rail<- read.csv("railway.csv", header = TRUE)
date<- as.Date(rail[,1], format ="%d/%m/%Y")
y<-rail[,2]</pre>
```

We use a fixed lag order of 0 in this analysis. The minimum window size r_0 is equal to 0.01 + $1.8\sqrt{T}$, where T is the number of observations. The empirical size is controlled by a 2-year period. Bootstrap critical values are obtained using 2000 replications.

The PSY statistic sequence and the its maximum value (the GSADF statistics) along with the corresponding bootstrapped critical values can be calculated as follows:

```
bsadf<- PSY(y,swindow0, IC, adflag) # estimate PSY statistic sequence
max(bsadf)

# The GSADF statistics

# [1] 2.146163

quantilesBsadf<- cvPSYwmboot(y,swindow0, IC,adflag,Tb, nboot, nCore)

# BSADF bootstrap critical value sequence at the 90, 95 and 99 percent level.

# 90% 0.8203336

# 95% 1.2615360

# 99% 1.9974178
```

From the above code, the null hypothesis of no explosive behavior in the railway share index is rejected at the 1% level (2.1462>1.9974). The identified origination and termination dates can be shown as the following:

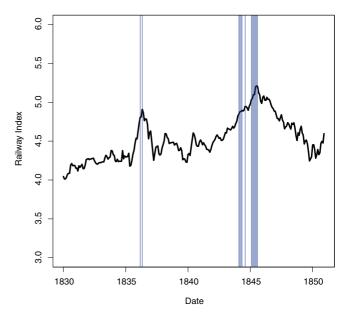
```
date<- date[swindow0:obs]
quantile95<-quantilesBsadf%*%matrix(1,nrow=1,ncol=dim)
ind95<-(bsadf>t(quantile95[2,]))*1

OT <-locate(ind95,date)
BCdates<-disp(OT,obs)
print(BCdates)

# start end
# 1 1836-03-01 1836-03-01
# 2 1836-05-01 1836-05-01
# 3 1844-02-01 1844-05-01
# 4 1844-08-01 1844-08-01
# 5 1845-02-01 1845-08-01</pre>
```



FIGURE 2 Date-stamping strategies for the British railway index between May 1827 and December 1850. The *solid line* is the the real railways index and the *shaded areas* are the periods where the PSY statistics exceed its 95% bootstrapped critical values [Colour figure can be viewed at wileyonlinelibrary.com]



The following codes for generating the date-stamping strategies in Figure 2 are:

As shown in Figure 2, we can identify two episodes, where the first episode is between 1836M03 and 1836M05 and the second is from 1843M02 to 1845M08 (with some small breaks). These two episodes coincide very well with the development of innovations in the British railway industry during the nineteenth century. The first exuberant episode in 1836 corresponds to the railway boom of 1836 and the second episode is often referred to as the great Railway Mania of the mid-1840s. We, therefore, have solid historical evidence from which to interpret the empirical findings as evidence of explosive behavior in the railways index.

The episode identified in 1836 in Figure 2 corresponds to a period of a speculative boom in rail-ways. Considerable interest in railway investment was stimulated by the successful launching of the Liverpool & Manchester and other steam railways (Lewin, 1914). The Liverpool & Manchester was the first railway to be built, and it was opened on September 15, 1830, making it one of the outstanding years in British history. The income enabled the Liverpool & Manchester to pay dividends of 10% (Porter, 1851). The financial success of the Liverpool & Manchester not only reacted to the prices of railway shares but also attracted promoters with new projects (Gayer et al., 1953).

As a result of the boom in 1836, there were 2000 miles of railway in operation by 1842 compared to only 200 miles in operation in 1833. However, Matthews (1954) argued that the railway mania of 1836 was a promotion boom, not a construction boom, as railway investment reached its peak in 1839/1840 rather than in 1836 (see, Mitchell (1964), Kenwood (1965) and Hawke and Reed (1969)).

The development during the first phase of the railway enthusiasm in the early 1830s was not a bubble but the railway boom during the second phase from later 1835 was a mania/bubble (Kindleberger & O'Keefe, 2005; Matthews, 1954). Confidence engendered by the prosperous state of trade and the relatively easy conditions in the market seemed to contribute to the share speculation in the mid-1830s (Kenwood, 1965; Matthews, 1954).

Gayer et al. (1953) argued that the formation of various railways led to the boom in the share market in 1835–1836, where his subindex of railway shares more than doubled between May 1835 and May 1836, rising from 60.2 to 129.4. In his book, the amount authorized by Parliament in 1836 for railway construction (nearly £23 million) exceeded the total sum authorized during 1826–1835 (£19 million). In 1837, £13.5 million were authorized by Parliament, however, it contrasts with only £2 million in 1838. Kenwood (1965) also concluded that a growing speculative interest in railway building was simulated by large-scale railway construction and the rising railway shares in the early 1830s.

The Railway Mania of the mid-1840s was reflected in the number of new Acts for railways. The construction of a railway required Parliament to pass an Act granting the right and privileges to the company at a considerable cost (Matthews, 1954). Based on statistics from Porter (1851, p.327), the number of Acts passed in England were 120, 270, and 190 in each year 1845–1847 while only 48 Acts were adopted in 1844. According to Simmons (1978, p.42), 330 Railway Acts were passed to establish new companies or extend existing lines in England and Wales during the period 1845–1847 (e.g., 79 Acts in 1845, 154 Acts in 1846 and 97 Acts in 1847). Even if these numbers of Acts authorized for railways from Simmons (1978) are different from those discussed in Porter (1851), the general conclusion of a boom in railway construction during 1845–1847 still holds. However, only 54 Acts were passed during the early railway boom of 1835–1836 based on the statistics from Porter (1851). It is evident that The Railway Mania in the mid-1840s is much larger in scale than the rail boom in 1835/1836 as the number of Acts authorized for railways during the great Railway Mania is far more than the number of Acts authorized during the 1835–1836 boom.

The Railway Mania was also reflected in the amount of capital authorized and miles of railway authorized by Parliament. In 1844, only £20 million authorized for the construction of 805 miles of track. The amount authorized by Parliament was £60 million for the construction of 2700 miles in 1845, £132 million for the construction 4538 miles in 1846 and £40 million for the construction 1354 miles in 1847, respectively. It is clear that the year 1846 was one of the most remarkable years in the history of British railways, where the Railway Mania of the mid-1840s was a construction boom due to promotional and speculative activity (Kenwood, 1965).

5 | CONCLUSION

Econometric methods for identification explosive behavior in financial time series have been the focus of much recent research. Since the seminal work of Phillips et al. (2011) and Phillips et al. (2015a), these real-time bubble detection methods are now becoming standard tools in detecting the presence of bubble-like behavior. These popular Phillips-type right-tailed unit root tests have been widely applied to investigate the existence of bubbles in a variety of applications. This paper covers a wide range of empirical papers that adopt the PWY or PSY testing procedure and reports on some attempts to extend these testing procedures in several ways. It presents a short review of empirical applications.

As Harvey et al. (2016) point out, the presence of heteroskedasticity affects the performance of the PWY test, leading to size distortions in testing. The PSY procedure also faces same issues in



the presence of heteroskedasticity. This paper illustrates the usage of a newly developed R package psymonitor to implement the PSY approach of Phillips et al. (2015a) along with a new bootstrap procedure of Phillips and Shi (2020) to deal with the potential heteroskedasticity and multiplicity issues in testing.

We apply the PSY approach with the new bootstrap procedure to the famous British Railway Mania of the 1840s. The test results provide evidence of explosive behavior in share prices of railways in 1835/1836 and 1846, which are related to the railway boom in 1836 and the more prominent Railway Mania in the mid-1840s, respectively. The railway speculation of 1836 has been a prelude to the great Railway Mania of the 1840s. This is the first empirical study to investigate railway share prices during the well-documented Railway Mania using the PSY approach. The empirical application provides new insights into the most documented bubble episodes in history as the findings from this paper will be of great interest to economic historians who are interested in the rage of speculation in historical share prices during the Railway Mania.

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ENDNOTE

¹The initial condition is the deviation of the first observation of the sample away from the deterministics of the process.

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