Erratum: Lie theory and separation of variables. 3. The equation $f_{tt} - f_{ss} = \gamma^2 f$ [J. Math. Phys. 15, 1025 (1974)]

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(Received 11 April 1975)

PACS numbers: 01.85., 02.20.Q

Dr. Charles Boyer has kindly pointed out an error in the computation of the spectrum of the operator $L_B = \partial_{xx} + \gamma^2 e^{2x}$ on $L_2(R)$, which corresponds to the Bessel function basis of solutions for the Klein—Gordon equation. This error, which is the responsibility of the second author, consists in the assertion that the self-adjoint extensions $L_{B,\alpha}$ have only discrete spectrum $\lambda = (2n+\alpha)^2$ and an orthonormal basis of eigenfunctions

$$f_n^{B,\alpha}(x) = \sqrt{2(\alpha+2n)}\, J_{\alpha+2n}(\gamma e^x), \quad n = 0, 1, 2, \cdots,$$

where $0 < \alpha \leq 2$. In fact, as is shown on pp. 93—95 of the book *Eigenfunction Expansions. Part One.* (Oxford U.P., Oxford, 1962), 2nd ed., by E.C. Titchmarsh, these operators also have continuous spectrum.

Taking the case $\alpha = 2$ for simplicity, we find that the operator L_B has discrete spectrum $\lambda = 4(n+1)^2$, $n = 0, 1, \ldots$, and continuous spectrum $\lambda \leq 0$ with generalized eigenfunctions

$$\tilde{f}_\lambda^B(x) = [J_{i\sqrt{-\lambda}}(\gamma e^x) + J_{-i\sqrt{-\lambda}}(\gamma e^x)]/2[\sinh(\pi\sqrt{-\lambda})]^{1/2},$$
$$\langle \tilde{f}_\lambda^B, \tilde{f}_{\lambda'}^B \rangle = \delta(\lambda - \lambda').$$

Here, $\langle \cdot, \cdot \rangle$ is the usual $L_2(R)$ inner product. The functions $\{f_n^B, \tilde{f}_\lambda^B\}$ together form a complete set for $L_2(R)$.

The separable solutions of the Klein—Gordon equation corresponding to the continuum basis are

$$\tilde{F}_\lambda^B(s,t) = [\sinh(\pi\sqrt{-\lambda})]^{-1/2}\,[J_{i\sqrt{-\lambda}}(\gamma u) + J_{-i\sqrt{-\lambda}}(\gamma u)] \times K_{i\sqrt{-\lambda}}(-i\gamma v)$$

where

$s = (u^2 + u^2v^2 + v^2)/2uv, \quad t = (u^2 - u^2v^2 + v^2)/2uv, \quad v > u > 0.$

There are similar expressions for other regions of the (u,v) plane.

The error, while regrettable, in no way affects the principal methods and conclusions of this paper and of following papers in our series.

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