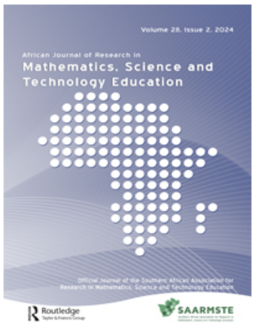

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Decolonising Content Narratives in Mathematics and Science Education: The Case of Reinventing Length Measurement

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In South Africa and in many other parts of the world, decolonising the curriculum has become a valued goal, while frameworks that would systematically support the decolonising project through *instructional design* are not broadly available. In this conceptual paper we bring readers to consider one framework for instructional design, the theory of Realistic Mathematics Education, and discuss how it can aid in decolonising education in primary years. We exemplify our conceptual position through an instructional design for teaching length measurement—a key grounding practice in both early years mathematics and science throughout the world. We propose that the resulting decolonised narratives are not only useful in marginalised contexts of countries with a history of colonisation. Due to their positioning of mathematics as a human endeavour, these narratives are capable of generating meaningful, equitable engagement with mathematics for diverse student groups in a variety of educational settings.

Keywords: *Decolonising content; equitable learning opportunities; Realistic Mathematics Education; number; measurement*

Introduction

In South Africa and in many other parts of the world, decolonising the curriculum has become a valued goal (Fataar, 2018; Miller et al., 2023) for a number of good reasons. Bishop in his 1994 article 'Cultural conflicts in mathematics education' noted that since the 1980s, in line with the aims of increasing mathematics accessibility for all learners, the field has witnessed 'increasing questioning of the relevance of ex-colonial models of education in developing countries' (p. 15). Yet despite increasing questioning and awareness, teacher and learner experiences in mathematics and science classrooms in many parts of the world are still largely shaped by curricular coloniality. The four authors' interest in this topic emerged from working in such classrooms and curricular contexts in Mexico, South Africa and Australia, and from the difficulties we encountered when seeking ways to empower students and teachers in these contexts with respect to mathematics.

We associate coloniality in the mathematics and science education curricular narratives with a power imbalance that results when contemporary disciplinary concepts and ideas are imposed on students with little regard for cultivation of students' meaningful personal connections to the ideas and forms of reasoning that we wish them to explore (see Sfard, 2003, 2015, who attends to meaningful personal connections through the notion of significance of what is to be learned, to the learner). We find it important to note that this view of coloniality in the curriculum, and the related project of decolonising it—that is, the work of removing the imposition—still operate within the ontology of the Western Galilean tradition, within which natural causes are viewed as providing explanations of causality of observed events (Radford, 2021). We acknowledge that the school institutions in the increasingly globalised world pursue knowledge creation within this specific ontology, and, more-or-less intentionally, displace the alternative ontologies in the process (see D'Ambrosio, 1985). We refrain here from taking a moral position on whether an advance of this ontology through global education itself is 'good' or 'bad' for the impacted parties. Instead, we accept this advance as an immediate context within which mathematics and science education take place in classrooms in Southern Africa and beyond today. The changes we advocate for and aim to facilitate within this context are those targeting the removal of negative experiences of forceful imposition and compromised identities and futures, that often result from what we view as curriculum-driven coloniality practices in mathematics and science education (Hoadley & Gallant, 2019). Recognising that the severity of negative impact is disproportionately high for groups that are already marginalised in other ways, we bring attention to the coloniality aspects that reside in curricula and play out daily in science and mathematics classrooms. By curricular and instructional changes, we seek to generate conditions for more democratic, inclusive and equitable education capable of 'emancipating' diverse individuals alienated from mathematics (see Frankenstein, 1983; Freire, 1970).

It is central to this debate that most contemporary disciplinary narratives in science and mathematics present the disciplinary ideas decoupled from the universal human needs that drove their exploration (Freudenthal, 1971; Otte, 2007). Instead, the ideas to be learned are selected and justified because they present important and currently relevant disciplinary innovations, which we need the students to master. These innovations are often presented as completed products or systems (Freudenthal, 1971) often attributed to European creation (Ernest, 2010). Combined with the lack of cultivation of meaningful personal connections to (or purposes for) learning, a curriculum constructed in this way would be prone to alienating many students. Ernest (2010) argues that this ultimately dehumanises and degrades the human spirit:

> through its training in rule following and often unquestioning obedience to imperatives, mathematics inculcates a way of seeing and being that helps degrade the human spirit. It focuses on objects rather than on people, feeling, empathy, caring, and being. It supports the spreadsheet metaphor that values everything in terms of its bottom line. It has been recruited into the postmodern Western project of consumerism, with its emphasis on having rather than being, lacking rather than becoming. (p. 81)

Take as an example, the scenario in which students might be expected to learn about length measurement through being introduced to standard measurement units and tools, including a metre stick. This may follow an initial period of measuring lengths with informal units, such as body parts. However—like in the case of South African curriculum (Department of Basic Education, 2011)—students usually do not get to learn about a chain of local innovations that connected meaningfully what they may see as crude body part measurement practices and sophisticated modern measuring tools. Curricula do not include problems that resulted in the creation of graduated measuring tools, or in the specific ways that numbers came to be used on those tools. More typically, students are provided with a demonstration, and a set of procedural rules, for how a ready-made metre stick should be correctly used to measure lengths (e.g. align the edge with a zero, read out the number at the last notch on the stick). In some cases, the recent history of the standardisation of metric units might be told to the students, positioning these as an innovation, which signified the beginning of a learning-worthy measurement practice. This kind of framing of mathematics—as something discovered by exceptional 'others' to be learnt by children in schools—positions students as receivers of mathematical discoveries, many of which can at best be '"parachuted" onto the learner from above' (Sfard, 2003: 367). A decolonised alternative, we propose, positions students as active in experiencing the need for and in inventing mathematics, and thus as being mathematical, just like all humans across geographic and historical contexts.

Indeed, as we point out elsewhere (Millán Gasca & Vale, 2021), a paper about the origin of measurement systems in South Africa, published as recently as 2011 in a technical journal for specialists in geo-informatics and surveying, claimed that '[t]he earliest units of land measurement in South Africa were derived from the units that were used by the Dutch pioneers in the 17th century' (Zakiewicz, 2011: 29). This suggestion sharply contrasts with the significant documented examples of measures emanating from Africa that predate the arrival of Dutch by millennia (e.g. Niangoran-Bouah, 1963). Indeed, multiple measurement practices have emerged across all cultures, driven by a pragmatic human need to understand and organise their environment. The fact that the need for international standardisation of measurement units emerged in history is itself a testament to the existence of varied local measurement practices and innovations, which at some point came to present as an obstacle to human cooperation. This suggests that when length measurement is first introduced in schools, it could be taught as all peoples' heritage, with subsequent important evolutions, including standardisation.

Mathematics is a pan-cultural activity (Bishop, 1994). Yet the disciplinary ideas deemed to be worth learning in primary school are pre-selected on the basis of alignment with current disciplinary end points (e.g. university mathematics of structures), not on the basis of their relevance to and learnability by the primary student. These ideas are introduced in curricula often at a considerable level of abstraction, and only later explained as being 'applicable' to practical everyday problems. When we focus on teaching the current, robust, abstracted systems of thought, the ideas in these have long been decoupled from the problems and phenomena that historically gave rise to them (cf. Otte, 2007). It is not surprising that teachers today routinely skip the difficult task of re-creating for (and with) students *the need for* mathematical ideas in the curriculum, given that this curricular focus is absent. Unfortunately, in these ways, we neglect to cultivate students' meaningful personal connection to mathematics. When students are told what to learn, without experiencing *a need for* the development of what they are learning, they can be denied the opportunity for sense making and identification with the relevance of the content. Their socio-historical experiences (cf. funds of knowledge; Moll, 1997) and identities are largely positioned as irrelevant to the learning process.

In this respect we suggest that there is the need for instructional design that transforms pedagogical and instructional approaches from their traditions of coloniality imposition and learner alienation towards traditions of inclusion in mathematics conceived as a universal human activity that enhances our ways of engaging in the world. We also propose that mathematics in which students' personal connections to it are cultivated from the start will better prepare them for their experiences and explorations in the school sciences.

Theoretical Framing and Positioning

In this conceptual paper, we work at the intersection of mathematics and science education. We draw on the distinctions of theoretical traditions that have been brought to bear on the issue of increasing access to mathematics and science for those historically and continually marginalised from central participation (cf. Wenger, 1998) in the activities of these disciplines. We first introduce two categorisations of approaches towards equitable learning opportunities, specifically:

- curriculum multicultural science education and instructional multicultural science education approaches (Southerland, 2000); and
- the cultural alignment orientation and the cultural participation orientation to mathematics teaching (Hodge & Cobb, 2016).

Like the authors above who distinguish between these different approaches, we position our work (within which 'the case of measurement' emerges) with the instructional multicultural science education approach and the cultural participation orientation.

Southerland (2000) writes about epistemic universalism in science education and notes two dominant approaches to engaging with multiculturalism in the field: *curriculum* multicultural science education (CMSE) and *instructional* multicultural science education (IMSE). The former looks to reconstitute or redefine conceptions of science taught at school so as to confer equal status to culturally local ways of understanding the world and those of the disciplinary science. Southerland cautions that this approach is prone to leaving students with unreconciled alternative perspectives unsuitable for forming a strong basis of further science studies. In contrast, in the IMSE approach, teachers and those designing learning sequences consider and build in the worldviews of students, positioning these historically as strengths, and pay particular attention to these when worldviews differ from those promoted in the disciplinary conception of science. This approach foregrounds the importance of effective teaching that is attuned and respectful to the diverse student worldviews, while cultivating appreciation of and participation in science as a practice, in which ideas and perspectives are developed in response to a need for understanding the world. Students learn that in doing science, differences between perspectives are explored and reconciled, resulting in disciplinary 'knowledge' that continues to evolve.

In mathematics education, Hodge and Cobb (2016) note a similar duality arguing that there are primarily two views of culture in the work of those focused on teaching for equity. They refer to these views as the *cultural alignment orientation* and the *cultural participation orientation* and argue that the vast majority of mathematics education studies on equity are conceptualised in the cultural alignment orientation. In this orientation, the problem of equity is considered in terms of finding ways to align classroom mathematics practices with out-of-school practices of the students and their communities. In this respect deep knowledge of students' out-of-school practices and resources as well as research into how to leverage these for learning opportunities are key. Hodge and Cobb point out that this orientation becomes challenging as classrooms become less homogeneous, because reflecting the local cultural and linguistic diversity of all students becomes increasingly difficult for the teacher. In contrast, the classroom participation orientation moves away from assuming a need for a match between in-school and out-of-school practices and instead considers how pedagogy and instruction must be adjusted so that all students participate substantially in developing disciplinary ideas in the classroom. The students' multiple backgrounds are considered among resources for planning instruction, rather than treated as something that must be replicated in the classroom for student engagement.

As Hodge and Cobb (2016) note, the two orientations they outlined – and, we add, the two scientific approaches outlined by Southerland – are grounded in contrasting views of culture. The former notions (i.e. CMSE and cultural alignment orientation) consider culture as 'a way of life characteristic of a bounded community' (p. 3) while the latter (i.e. IMSE and cultural participation orientation) view culture as

> a network of local hybrid practices that people jointly constitute as they negotiate the places in specific settings such as the mathematics classroom … Students seem to develop ways of participating in or resisting classroom activities by drawing on a range of resources that include practices of the home communities, broader discourses, popular culture, and images in the media. (p. 4)

This latter notion of culture connects with Bourdieu's (1990) notion of habitus and Moll's (1997) notion of funds of knowledge, and provides a productive orientation to instructional design for multicultural classroom settings, where creating access to powerful mathematical and scientific practices and ideas and to scientific career pathways is a goal. As we endeavour to illustrate in this paper, this goal is pursued by the cultivation of learner agency in relation to their participation in and co-constitution of classroom mathematical and scientific practices. We find this view of culture in particular relevant to the decolonising project, given that it actively pursues the removal of curricular imposition characteristic of coloniality and replaces it with content creation as emergent from the shared classroom experiences.

It is important to note, however, that adoption of the perspective of culture as a network of local hybrid practices does not negate the value of rich insights acquired within the CMSE and cultural alignment traditions, in particular those within the Southern African context. The research conducted within these traditions highlights the value of understanding alternative conceptions of knowledge. For example, Ngcoza and colleagues (Ngcoza, 2019; Seehawer et al., 2021) provide powerful examples of critical engagement on the re-appropriation of 'heritage practices' into the science curriculum and ways in which these can be aligned to key disciplinary practices. Similarly the work of Southern African mathematics education researchers working in the field of ethnomathematics have provided a range of rich examples of 'heritage practices' that can be drawn on in the mathematics curriculum. For recent examples see Machaba and Dhlamini (2021) and Mosimege and Egara (2023). These researchers identify powerful opportunities for building bridges between local and historical practices across communities on the one hand and developing disciplinary practices on the other hand.

The CMSE and cultural alignment orientation frequently underpin research in fields of ethnomathematics and indigenous knowledge systems in mathematics and science. These have a rich history of contributions to the decolonisation of science and mathematics curricula in Southern Africa (see for example Abonyi et al., 2014; Gerdes, 1988a, b; Laridon et al., 2005), Latin/South America (Abreu & Carraher, 1988; Cortina, 2013; D'Ambrosio, 1985; Nunes, 1992) and beyond. Paulus Gerdes (1988a) is one of the most well-known ethnomathematicians who brought attention to the need to challenge existing 'transplantation' of mathematics curricula across cultural and national contexts on the basis that: 'the mathematics curriculum has to be "inbedded" into the cultural environment of the pupils' (p. 35). He provided a range of examples of using African artefacts to guide students to discover various mathematical theorems and concepts through interacting with them, including discovering the Pythagorean theorem (1988a) or noticing arithmetic progressions and various geometric properties through the drawing tradition of the Angolan Tchokwe people (1988b).

We recognise these important contributions that draw on anthropological, historical and socio-psychological approaches and provide rich understanding of mathematical activity and practices across different social groups (Bishop, 1994) as sources that can usefully enrich curricula. Our focus in this paper however is on decolonising education through *instructional design* as the key means of supporting how teachers cultivate students' participation in classroom disciplinary practices (Visnovska & Cortina, 2022). Based on our work with a range of learners who have been marginalised from mathematics and science participation, we propose that focusing on instructional design for equitable and increasingly central participation in the practices of these disciplines is a useful and under-represented approach to decolonising the mathematics and science curricula. We note that frameworks that would systematically support the decolonising project through instructional design are not broadly available and have been less considered in relation to the project of decolonisation and working towards equity.

Foregrounding equitable participation connects with the 'content learning and identity construction' framework of Varelas et al. (2012), who work to strengthen African American students' mathematics and science learning in the United States. Varelas and colleagues argue that equitable access for African American students to mathematics and scientific disciplinary practices requires a framework that positions school learning as being at the intersection of content learning (of expected and promoted disciplinary ideas) and identity construction. They construct the content learning and identity construction framework as a tool for 'understanding how Black students negotiate participation in, and come to see themselves as doers of, science and mathematics in their school classrooms' (p. 319).

We agree that enabling positive identity work is key in supporting education of marginalised and underprivileged groups. However, such identity work might remain difficult if curricula are organised by coloniality narratives, which impose complex ideas on students without giving them opportunities to experience the need for these, and which focus on giving knowledge that students must have rather than on student ways of being and becoming (see Ernest 2010 quote above). Like Ernest, we point out that such narratives tend to subdue humanity even of educational 'winners' and are thus counterproductive to the project of democratic education at large (cf. Biesta, 2022). This is why we address the curriculum decolonising project through pursuing the following question: how can curricula be organised so that all students become 'doers' of science and mathematics in their classrooms, positioned as capable contributors to science and mathematics endeavours from the outset?

In the next section, we illustrate how adopting the Realistic Mathematics Education (RME) theoretical orientation in instructional design directs us to attend to (1) who we are asking the learners to become, and (2) how and why their becoming doers of mathematics/science in particular ways could be desirable for them. We describe how RME provides a tool for disrupting the coloniality curricular narratives of mathematics and replaces these with instructional narratives (cf. Hodge & Cobb, 2016; Southerland, 2000), in which the discipline emerges both historically, and in the classroom, through purposeful collaborative human activity (Freudenthal, 1973).

RME as a Design Theory for Decolonialising Curriculum

The RME instructional design theory (Cobb et al., 2008; Gravemeijer, 1994) is rooted in Freudenthal's (1973) interpretation of mathematics as a human activity. In Freudenthal's view, mathematics is highly relevant to many human endeavours, and thus it should be experienced by students as such first-hand. This is a notable departure from coloniality narratives built (more or less overtly) on claims like the one noted above, that the earliest use of units of land measurement in South Africa came from the Dutch. Narratives which start by presenting the end-point mathematical innovation (e.g. specific form of standardised unit, or metric system) prescribe what students *should* see as relevant, and lack a commitment to presenting students with experiences in which relevance of content *to them* would become obvious. Realistic Mathematics Education holds such commitment to be of high importance because lack of relevance of taught mathematics to the student has been identified as one of the main reasons for student alienation from school mathematics (e.g. Boaler, 2000; Ezeife, 2003; Wright, 2017). Realistic Mathematics Education, in simplified terms, makes the connection between content learning and identity construction (Varelas et al., 2012) central to instructional design.

When it comes to why mathematics was developed and how, RME holds that humans developed (and develop) mathematics *accumulatively*, in response to pragmatic or mathematical *needs*. They progressively organise (i.e. *mathematise*) their world and the mathematical insights and relationships developed previously. If learners are to engage with mathematical practice in ways consistent with this view, the sequencing of what is to be learned becomes a central concern for designers. By and large, learners should learn mathematics (1) for which they experience a need first and (2) which can be, in principle, developed by reorganising their existing mathematical and personal insights and experiences (Freudenthal, 1971; Sfard, 2003). These guiding principles for the nature of classroom mathematics cohere with Freire's (1970) notion that 'the only person who really learns is s/he who … re-invents that learning' (p. 101).

The two guiding principles relate closely to the three central RME tenets (Cobb et al., 2008), the first of which posits that learners encounter the material they are to mathematise as being *experientially real* for them. Problem situations and tools that are experientially real are those with which learners can immediately engage in an activity that is both *personally meaningful* to them and *mathematical* (Cobb et al., 1997). In psychological terms, these were described as problems and tools with which learners can readily become *imagistically involved* (Thompson, 1996). These descriptions share that the designed problem situations need to (1) elicit learners' personal and habitual responses of involvement, curiosity, and desire to proceed with untangling the situation, and (2) be well within the learners' immediate and habitual reach with respect to the mathematical tools needed for the initial untangling, even if at a basic mathematical level. The broader aim of this design tenet is to ensure that everyone in a classroom has a way to participate in the mathematical activity, experience mathematical competence and contribute to the discussions, in which new mathematical ideas are co-developed. In designing for this aspect of learners' engagement, we came to value the stories of the challenges that humankind faced historically, which portray how and why mathematical and scientific practices came to be, or how and why they had to be reorganised (e.g. Sfard, 2015).

The second principle (i.e. creating the necessity of learning by reorganising past insights) points towards the importance of appropriate sequencing of mathematics for education purposes. This issue is specifically attended to in two remaining central tenets of RME. Realistic Mathematics Education specifies that the starting points of classroom engagement should be justifiable in terms of the potential end points of the learning sequence (Cobb et al., 2008). In other words, RME prioritises the powerful mathematics practices that students are to develop over time. It insists that we arrive at such practices mathematically, that is, with mathematical coherence and integrity. This means that designers need to be selective in their choices so as to avoid building the foundations of students' mathematics on notions, tools and metaphors that would reveal themselves as didactical obstacles (cf. Brousseau, 1997), that is, something that halts rather than enables advances in subsequent mathematising. Unfortunately, current curricula are replete with notions that later act as didactical obstacles (Cortina et al., 2014a).

The RME tenet of curriculum allowing, over time, for the emergence of powerful mathematics practices could, on the surface, be read as devaluing the incorporation of indigenous insights and perspectives in school education. To refute such interpretation, we lean on the final, third central tenet of RME, which positions indigenous perspectives as a legitimate contributor to classroom mathematics. This tenet brings attention to the means of supporting student participation in *increasingly sophisticated* mathematical practices. This reminds us that modern mathematical ideas did not occur to mathematicians in their present form, but were, instead, a result of progressive mathematising over millennia. Indeed, much modern academic mathematics would not make sense to ancient Egyptian pyramid designers and scholars who (like primary school students) did not concern themselves with the projects of axiomatisation of mathematics or a set-theory approach to it (Phillips, 2014). Current students' mathematics education journeys, too, require intermediate mathematical tools and practices, which allow them to respond, with relative independence, to their immediate mathematical needs. However, as we are not raising ancient Egyptian scholars, the tools and practices we use in education need to be capable of being progressively refined and restructured in the classroom to acquire their contemporary shape once the purposes for the evolution of mathematics itself become apparent to students.

The overt attention to such classroom practices reminds us that choices about mathematics tools and representations were historically always based on their users' understanding of strengths and weaknesses of various available alternatives. From the perspective of instructional design, classroom invention and re-invention of mathematical practices also need to include the deliberations of the relative power and utility of different problem-solving approaches, tools and representations. The rich body of work on mathematics practices of different indigenous and culturally bounded groups is a valuable source of mathematical ideas from different eras and cultures that were part of the historical mathematics evolutions, and have the potential to contribute to well-designed classroom deliberations.

To summarise, in our experience, RME nudges instructional designers to transcend the view of culture as a set of practices of historically and geographically bounded groups, while at the same time keeping designers attuned to and respectful of utility of specific cultural tools and practices to advancing classroom mathematics and science.

Our aim in the remainder of this paper is to provide a vivid example (at the intersection of mathematics and science curricula) of an instructional sequence that is guided by RME theory and to highlight how this kind of design contributes to the project of decolonising curricula. We illustrate how the design commits to equipping teachers for supporting students' learning of mathematics and science by (1) generating experiences of a need for measurement first and (2) supporting students' reorganisation of their existing insights and experiences of measurement. Our example illustrates how RME theory directs educators to bypass existing coloniality narratives because they are not adequate for teaching mathematics well. We also illustrate how the RME approach to design includes indigenous mathematics and science in our classrooms by making them meaningful from students' perspectives and legitimate with respect to the mathematics to be developed. We note that there were much softer boundaries between mathematics and science in the early days of these disciplines. As such, initial school experiences grounded in curiosity about our world would inevitably, and helpfully, share the overlap.

The instructional sequence (Cortina et al., 2014b) starts with the classroom re-invention of possible ways in which various peoples, including Mayan or African people, may have engaged with measurement ideas based on their daily needs. We show how encountering the early practices of measurement of continuous quantities not only engages students on personal and identity levels but opens a powerful way for supporting the learning of number where all numbers (in particular whole numbers and fractions) co-exist as mathematical objects of the same kind.

Exploring Length Measurement: A Coherent Path to Fractions

In classical mathematics, number was a result of quantification of magnitude (Otte, 2007). In other words, measurement of discrete and continuous magnitudes, besides having a practical purpose of allowing quantitative comparison of various aspects of the world (e.g. length, numerosity), also gave rise to an initial, mathematically coherent notion of number, which included whole numbers and fractions (Chambris & Visnovska, 2022). In contrast, the mathematics instruction typically made available to students today does not enable many of them to adequately develop number sense (Graven et al., 2013; Venkat & Mathews, 2019) and students rarely come to understand fractions as a coherent extension of our numeration system (Park et al., 2013; Watanabe, 2002; Wu, 2011). To counter this trend, Cortina et al. (2014b) turned to the historical practice of length measurement to support students in developing sense of whole numbers and fractions as mathematical tools that help to quantify lengths. This allowed for extending the meaning of number from whole numbers towards rational numbers in a mathematically coherent way. It further provided students with opportunities to experience a need for the key aspects of measurement (e.g. precision, accuracy, unit), and introduce measurement practices as being inherently part of students' heritage.

To experience measurement as an invention that can give rise to numbers, students needed to encounter practices that could have existed *before* various measurement tools with number scales emerged. This provided a mathematical reason for situating the teaching narrative in the indigenous history of measurement practices. Working with students in a few Mexican classrooms, the sequence was originally developed around a narrative about a group of ancient Mayans, who needed to measure lengths as part of their craft making. The storytelling narrative takes the students on 'the journey of experience' (Phillips, 2012: 142), where they face various challenges of a mathematical and scientific nature alongside the main protagonists in the story, a wise elder of the group and her pre-teen daughter. Alongside the story characters, students develop a need to respond to each challenge, then evaluate the proposed responses, often giving rise to new, subsequent challenges. The storyteller motivates the classroom to the actions of mathematising and problem solving, as well as to the understandings of humanity 'via the cultivation of sympathetic imagination that storytelling fosters' (Phillips, 2012: 142). In this way, the story becomes a means of presenting the listeners with the purpose and need for mathematical and scientific actions in ways that foster students' understanding of common humanity, inventiveness, and the power of human imagination and thought. This is why the identity of the indigenous group in the story is easily adaptable to the desirable local context, or even to the context of the shared roots of human existence on Earth.

The instructional sequence starts with guiding students to recognise a need for a standardised unit when measuring length. Students are first asked to measure objects in the classroom using parts of their bodies (e.g. hands), like the indigenous group did. They first learn to measure with attention to precision by encountering examples of measurement failures (e.g. measurement with gaps or overlaps), and then come to experience and recognise that while measuring with body parts is practical, in some instances it can become problematic. To support this recognition, a story is told, in which one indigenous daughter took a measure for a clay pot, ordered by a villager, as 'three hands tall'. When her mother made the pot, using her own hand to measure 'three hands tall', the pot was the wrong size. Once students recognise how this and similar situations make measurement with body parts problematic, the standard unit of measurement (the stick) is introduced as an innovation that allowed the protagonists to make consistent length measurements (of one, two, three, … sticks long) by different people and at different times.

As a next step, the students are supported to become mindful of the limitations of solely using the stick for measuring lengths. This time, upon measuring everyone's height with the stick, the students are told they are the same height: five-sticks-and-a-bit tall. Students typically vehemently disagree with such a conclusion and suggest that a more accurate measurement tool is needed. At this point, they learn that the indigenous elders solved the accuracy problem by introducing additional, smaller length measures, *smalls*, which they created to fulfil the following pattern: 'A small of two is a rod of such a length that, when we measure the stick with it, the stick measures exactly two. A small of three is a rod of such a length that, when we measure the stick with it, the stick measures exactly three' (see Figure 1), and so on. Mathematically, each small represents the respective unit fraction of the length of the stick (e.g. a small of three is one-third as long as the stick).

Figure 1. A small of three rod (dark) is of such a length that three iterations of the rod cover the same length as the white stick (i.e. the reference unit)

Students are then given time to produce their own smalls, by making one rod for each small, iterating the rod alongside the stick, and adjusting its length until each rod fulfils the specified condition (see Figure 2). While progressing through trial and error, students are asked to predict whether the next small (e.g. small of four) will be longer or shorter than the previous one (e.g. the small of three) and why. As a result, two ideas that are often difficult—or even missing—in initial fraction instruction remain at the fore, as the meaning of fraction as a number is being supported. These are, first, the awareness of the size of a unit fraction relative to the size of the reference unit and, second, the inverse order relation of unit fractions (i.e. that 1/3 is longer than 1/4 even though 3 is less than 4).

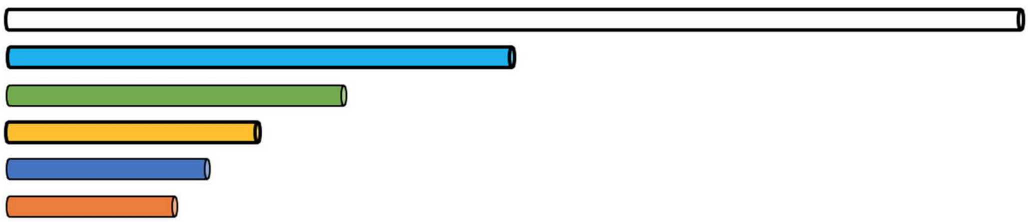

Figure 2. The measuring stick (reference unit) and measuring rods (subunits) of lengths 1/2, 1/3, 1/4, 1/5 and 1/6

Unpacking the RME-informed Design

We now discuss the shared sequence of activities from the perspective of the problem at hand, that of how we see RME contributing to the decolonising of curriculum via instructional design. The RME tenet that classroom activities be experientially real to students is addressed by support for students' identification with the protagonists of the story and by bringing students to consider the problems that these protagonists face as being legitimate and worth solving. The Mayans are the traditional owners of the land, where our early collaborating classrooms were located, and the direct ancestors of most of the students in those classrooms. However, given that length measurement is a universal activity, the initial narrative is highly adaptable to adequately portray an achievement of other civilisations and indigenous groups. We have used adaptations of the sequence with the protagonist group being the ancient Toltecs, when used in classrooms in central Mexico, and indigenous South Africans (see Figure 3), when used in South African classrooms (Vale & Graven, 2018).[1] However, we also used the South African version of the story successfully in Italy (Millán Gasca & Vale, 2021) and Slovakia (Višňovská & Slabý, 2019) where the invention of length measurement was introduced as one of the practices of the shared roots of humanity.

[1]The URL link https://www.ru.ac.za/sanc/mathsatschool/miclegr4-7/ offers the storybook (Vale et al., 2019) in English as well as translated into several additional languages (Afrikaans, Indonesian, Slovak).

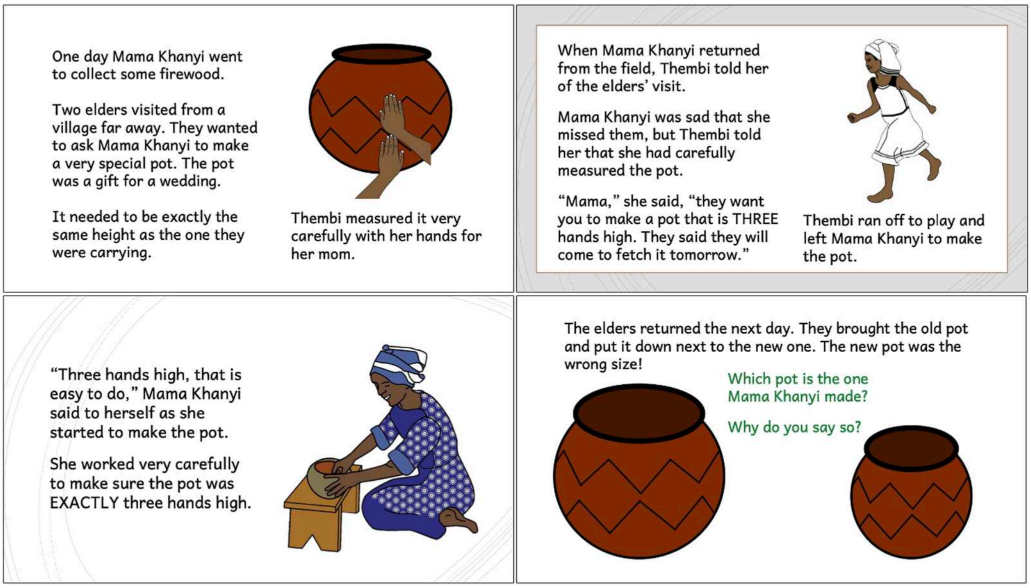

Figure 3. The collage of four consecutive pages from *Mama Khanyi and the Pots* storybook

In making the narrative adaptations, the overarching aim is not teaching historical truths about how today's mathematics and science came to be. The aim is instead to introduce engaging, plausible narratives of what could have happened, which are informed by history, but also – and primarily – by our understanding of how mathematics and science practices can be gradually re-invented in the classroom. Following RME, our aim is to create opportunities for all students to become imagistically involved with the ideas and inventions that give rise to mathematics and science and have ownership of such ideas. Creating the virtual world of stories (Gee, 2007) in which new mathematics and science practices are needed allows such opportunities to be facilitated.

The narrative plays a central role in the mathematics teaching. It is not merely used to entice the students into paying attention, or to 'hook' them in the classroom activities at the start of the lesson by appealing to their cultural heritage. Instead, the narrative is offered as a resource for the teacher to orient students to regard the presented problems as worthy of being investigated. In other words, it aims to help the teacher to establish the *need* for new ideas in the classroom, such as establishing the need for a standardised measurement unit, by allowing students to experience situations where measurement with body parts is problematic.

In addition, the sequencing within the narrative must present mathematics or science as a collection of tools and ideas that can be co-developed during investigations. In other words, the narrative also supplies the teacher with the types of problems that *can be*, in principle, advanced by reorganising students' prior understandings and insights at each step of the journey. The narrative thus avoids ideas 'being "parachuted" onto the learner from above' (Sfard, 2003: 367) in the name of progressing the learning. For example, finding a new way to measure must first intuitively make sense to students (e.g. when smaller units are needed to allow for better accuracy) before it is introduced, by the students or the teacher. We do not expect that students would spontaneously propose making the *smalls*. Yet it is important that they can independently evaluate whether the smalls provide the indigenous elder in the story narrative with a reasonable response to the problem she is facing.

In this way, the problems that students are to face in the classroom are considered within a longer-term narrative that provides the scope for advancing coherently towards the determined end point of mathematics and science practices, without introducing contradictions, incoherencies and didactical obstacles along the way.

Concluding Remarks

In this paper we have proposed an approach for decolonising the curriculum through instructional design. We have used RME instructional design theory as an example of an approach to instructional design which we have argued and shown to be well suited to supporting the decolonising project. Hoping to speak to both science and mathematics educators and researchers reading this journal, we have illustrated this position through an introductory sequence to length measurement that was both mathematically and scientifically relevant to student learning of both measurement and number.

Our measurement sequence foregrounded the use of storytelling and an extended, scientifically and mathematically coherent narrative, adaptable to various contexts in which the sequence was used. We attribute the success of the sequence (as attested to by research and teacher use across a range of contexts) to the decolonised aspects of the design—specifically, bringing students to:

(1) experience the problem as being of high relevance for peoples who did not yet have measurement tools; and
(2) appreciate the ingenuity of the different ways in which both the peoples around the world, and students in the classroom themselves, solved the relevant problems.

As a result, the activity design proactively contributed to building the view of students as capable doers of mathematics through supporting their active participation in proposing the ways in which various measurement challenges could be addressed. The decolonised narratives were not only useful in marginalised contexts of third world countries like South Africa or Mexico, where the example sequence was initially developed and tested. The measurement sequence was of considerable interest to students and teachers in diverse classrooms in different cultural contexts in Italy, France, Slovakia and Australia, because—we conjecture—it succeeded in recruiting student creativity and de-mystified the process by which mathematical ideas get to take shape.

We have worked with the concept of the universality of foundational ideas in mathematics and science, and highlighted the possibilities this offers for enabling learners to view themselves as part of that universality. By centring the universal ideas in what the learners share, or in the histories of those who need to be empowered, the teachers working with the sequences of this kind can demonstrate that who their learners are legitimately places them as participants in the disciplinary practice. Aligning with the cultural participation orientation (Hodge & Cobb, 2016), teaching can thus emphasise both learners' humanity and their identities as effective doers of mathematics and science.

While RME is a theory and design framework tailored to mathematics teaching and learning, we have attempted to illustrate that it has relevance to the foundational areas of science. We propose that there is likely much to be gained from exploring parallel instructional design strategies for science education, as a means of designing for 'emancipating' learners who are currently excluded from successful science participation in their classrooms. This would align with Southerland's (2000) notion of approaching multicultural science education through instructional design.

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