Digital Competencies of High School Mathematics Teachers in Pakistan

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ABSTRACT

The dynamic demands of the contemporary educational landscape, which includes a variety of digital tools and resources, require teachers to develop digital competencies. This need is particularly pertinent for mathematics teachers in developing countries like Pakistan, where there is a requirement to bridge the digital divide and equip teachers with the knowledge and skills to integrate digital resources into their teaching practices. Prior research has acknowledged the importance of integrating digital resources in education; however, there is limited research on the challenges and opportunities that high school mathematics teachers encounter when using digital resources in resource-constrained environments like Pakistan.

This study aimed to investigate the digital competencies of high school mathematics teachers in Pakistan, focusing on their knowledge, beliefs, practices, and experiences in integrating digital resources into mathematics teaching and learning. To achieve this aim, the study adapted the "Digital Competencies for Teaching Mathematics with Technology" (DCTMT) framework. Employing a mixed-methods explanatory sequential design, the study was divided into two phases for data collection. In Phase 1, quantitative data were collected through an online survey, which received a total of 306 responses. I analysed the survey data using descriptive (Mean, Mode, Median, and Standard Deviation) and inferential (intra- and inter-construct Spearman's rho (ρ) correlation tests) statistics. To further understand and explain important quantitative results, semi-structured interviews were conducted in Phase 2 with six (Phase 1) teachers. I analysed the qualitative data using pre-identified themes and sub-themes based on the constructs of the DCTMT framework.

The results of both phases indicated that teachers held firm beliefs about various aspects of mathematics education. Teachers expressed strong beliefs about pedagogical practices and held positive beliefs about the importance of digital resources for teaching and learning mathematics. Correlations were observed between beliefs, teachers' reported ability to create videos, knowledge of dynamic graphing software (DGS) and equation editors, and participation in professional training. The findings revealed that most teachers possessed some level of digital competency. Teachers predominantly relied on videos and graphics, indicating

potential bias and a gap in their professional development and training. They lacked access to (particularly at school) and possessed a weak knowledge of mathematics-specific digital resources like Dynamic Graphing Software (DGS). Other factors, such as lack of resources, inadequate formal training, and limited professional development opportunities, hindered their ability to integrate digital resources into their teaching and learning of mathematics. However, the results indicated that teachers could overcome some of these challenges through resilience and self-learning initiatives.

The study findings suggest that equipping teachers with appropriate resources and targeted professional development could enhance their digital competencies. This would consequently promote equitable and diverse use of digital resources in mathematics education. Within the confines of resource-constrained environments, these findings support for nurturing teachers' resilience and cultivating a self-learning attitude among them. Teachers who equip themselves with digital skills, receive training and have the confidence to integrate digital resources develop digital competencies, irrespective of resource constraints.

This thesis contributes to the understanding of high school mathematics teachers' digital competencies in Pakistan and indicates the opportunities and obstacles they encounter while using digital resources. This research inquiry provides empirical evidence for the modified DCTMT framework and reveals significant interconnections between the five constructs of the framework. The study reveals critical insights into the intricate relationship between beliefs, access to resources, and competencies in utilizing digital tools for mathematics education. This thesis establishes a compelling link between training, self-learning, teachers' resilience, and their digital competencies, highlighting the dynamic interplay that shapes mathematics teachers' digital competencies. This thesis offers implications for teachers' professional development, curriculum design, and policy formulation. It emphasizes the importance of supporting teachers with the resources and knowledge to integrate digital resources and enhance mathematics education in resource-constrained contexts.

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CHAPTER ONE

1. Introduction and context of the study

1.1. Introduction

Globally, teachers are increasingly using digital resources, ranging from online learning platforms to educational apps, to enhance student engagement and facilitate learning. Like other educators, mathematics teachers are challenged to rethink their teaching and learning approaches within this changing technology landscape. To meet these challenges, mathematics teachers need to enhance their digital competencies so that they can recognise and make use of the teaching and learning opportunities that digital resources can provide. Digital competencies will enable mathematics teachers to recognise and exploit the affordances provided by computational and representational tools, dynamic geometry environments, virtual manipulatives, and computer simulations of real-world phenomena. The need to acquire digital competencies is particularly relevant to mathematics teachers in developing countries like Pakistan, where bridging the digital divide is imperative. (Golden et al., 2023) define digital divide as a gap between teachers with access to information and communication technologies (ICTs) and the skills to use them, and teachers without such access and skills. Equipping teachers with the relevant knowledge and skills to integrate digital resources into their teaching practices is crucial in bridging the digital divide.

The recent COVID-19 pandemic further highlighted the importance of digital competencies, as schools around the world had to shut down and teachers were required to move to online teaching and learning. In the context of developing countries such as Pakistan, this transition was particularly challenging for numerous reasons including limited access to technology, skills, and digital resources. At the same time, the demands highlighted by the pandemic also presented an opportunity to accelerate efforts to bridge the digital divide and investigate how these changes impacted the use of digital technology in both online and face-to-face teaching and learning situations.

In this research inquiry, my goal was to contribute to the ongoing discourse about the importance of digital competencies in mathematics education and to offer recommendations for enhancing the digital competencies of mathematics teachers in Pakistan. The thesis starts with a narrative (Section 1.2) of how I developed a research interest in the field of digital competencies for teaching and learning mathematics. Following this, I present the general and specific context of the study (Section 1.3), encompassing an overview of the education system, culture, and state of mathematics teaching and learning in Pakistan. In Section 1.4, I highlight the challenges in mathematics education at the high school level in Pakistan and the factors affecting teachers and teaching quality within this context. Section 1.5 offers a concise overview of the evolving digital technology landscape in Pakistan, notably in the aftermath of the COVID-19 pandemic. In Section 1.6, the research focus and the guiding research question(s) are delineated. A synopsis of each of the chapters in the thesis is presented in Section 1.7.

1.2. Personal statement and research interest

As a mathematics teacher for nearly 15 years, I have learned that teaching is not just about imparting knowledge but also about facilitating understanding. My teaching career began at Bahria Foundation College, Karachi, where I initially believed that my strong mathematical knowledge and communication skills were sufficient for teaching. However, I soon realized that students needed more than just content knowledge; they required guidance in navigating the abstract concepts of mathematics.

Particular moments in the lessons highlighted the need to reimagine my role as a mathematics teacher. One example that stands out to me is when I was teaching Complex Numbers[1] . I noticed that several students were struggling to plot the imaginary part on the XY plane. Their questions revealed confusion and a desire to understand the practical applications of these concepts. They had questions like: "Where does the imaginary part of complex numbers exist on the XY-plane? If this number is 'imaginary', how can we plot it on the XY plane"? As an experienced mathematics teacher, I knew how to address these questions. I provided them with some real-world examples, such as the oscillation of a pendulum and a 2D graph of a complex plane. I also knew that 3D models could further help students to understand complex numbers better than traditional methods. However, to teach using such methods and resources, I needed both knowledge and access to digital resources to support my pedagogical imagination. I learned that the combination of knowledge, digital skills and access plays a pivotal role in actualizing my teaching innovations and enhancing the learning experiences of my students.

As I continued to develop as a mathematics teacher, I came to appreciate the importance of not just having strong mathematical knowledge, but also having the skills and resources to respond to students' concerns in a way that resonated with

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[1] Complex Numbers ($\mathbb{C}$) extend the concept of Real Numbers ($\mathbb{R}$). A complex number, denoted as z, takes the form $z = a + ib$, here a and b belongs to $\mathbb{R}$, and i is a solution to the equation $x^2 = -1$. The term "imaginary" is used because no $\mathbb{R}$ satisfies this equation. In the expression $a + ib$, a is referred to as the real part, and b as the imaginary part. Despite being historically labeled as "imaginary," complex numbers are real numbers and play a fundamental role in describing various real-world aspects in mathematics.

them. This included the use of digital resources for real-world examples and relevant representations to clear up confusion and help students grasp the concepts. Now, after witnessing the COVID-19 outbreak and living in a post pandemic world, I recognise that having digital competencies is becoming increasingly important for teachers. These competencies include being able to regularly use digital resources to create interactive and engaging learning experiences that help students understand mathematical concepts in both online and offline teaching situations.

1.2.1. Navigating digital competencies challenges in Pakistan: Insights from two projects

In 2013, my interest in digital competencies was further boosted when I became part of an exciting project called "Digital Education Streaming for Pakistan." I, along with others, initiated the project to develop digital resources for high school mathematics students and teachers in Pakistan. As the team leader, I co-authored and created digital content for mathematics education. Within this collaborative endeavour, I encountered diverse levels of teacher understanding in creating and using digital resources. These experiences and expectations were particularly unique given the context at that time, where the use of digital resources was not yet commonplace in Pakistan. This situation presented a challenge, compelling me to foster a shared understanding of using digital resources among teachers. For example, some teachers were more comfortable with simply using them (users), while others were more interested in designing and developing their own (designers). The variance in comfort levels among teachers was accompanied by distinct perspectives. Teachers who were only users of digital resources believed that digital resources might replace them, while at the other extreme, there were teachers thought that using technology would in itself improve student learning outcomes without any additional effort on their part.

A further opportunity to extend my knowledge and application of digital resources in education occurred when I worked for the Pakistan Navy School of Logistics (PNSL). I worked at PNSL from 2010 to 2019 and had the opportunity of designing an Operation Research and Business Mathematics curriculum with a global perspective. This task required integrating a diverse range of digital tools in teaching methods to accommodate the varying needs of curriculum and students from different backgrounds. For example, students from some countries such as China, were accustomed to using specific software or online platforms that are not commonly used in other countries. To meet these diverse needs, I realized that teachers must be skilled in a wide range of digital resources and be able to adapt their teaching styles accordingly. I learned that it is important for teachers to know how to embrace this diversity of digital resources to create a curriculum that meets the needs of students, resonating with their backgrounds and previous experiences with digital resources.

My teaching experiences informed me that the process of integrating digital resources into teaching is likely to be unique to each individual teacher. Factors, such as their level of digital competence, content knowledge, as well as their teaching style and preferences, may influence how they integrate digital resources into their teaching. Although studies (de Araujo et al., 2017; Reisoğlu, 2022; Trouche & Fan, 2018) have explored the use of digital resources in mathematics classrooms, they have often lacked a contextual understanding of developing countries like Pakistan. The challenges of teaching mathematics in Pakistan may involve unique factors like limited access, overcrowded classrooms, socio-cultural norms, and religious beliefs. As the literature on digital competencies continues to strengthen and expand, there is a growing need for multidimensional and contextual investigations in this area. Recognising these exigencies, I decided to focus my own research inquiry on the use of digital resources for mathematics teaching in Pakistan. Specifically, I aimed to explore teachers' beliefs, current practices, and potential challenges related to using digital resources and developing their digital competencies. The next section provides the context of the study that includes an overview of mathematics education in Pakistan.

1.3. The context of the study

The context of this study lies at the intersection of the rapidly evolving educational landscape and the ubiquitous presence of digital tools and resources. We live in the post-pandemic era of technological advancements led by Artificial Intelligence (AI). In this digital landscape, students are also skilled in using digital technologies, and technology is seamlessly integrated into various aspects of their lives. This pervasiveness of technology means that teachers worldwide require more time and a higher level of digital expertise to deal with the challenges arising in both face-to-face and online teaching and learning environments. In the pursuit of enhancing teachers' digital competencies and especially those of mathematics teachers, researchers need to evaluate the nature of these competencies in different contexts. One such context is a resource-constrained environment, highlighting the problem of the digital divide and analysing the fair distribution of access to mathematics education. In the context of technology integration in education, (Mabila et al., 2017, p. 15) defines resource-constrained environments as "environments that are characterized by limiting economic circumstances, inadequate infrastructure, and basic amenities." Teachers have less-than-favourable circumstance to implement technology within these environments with these constraints. They face challenges related to electrical power and network connectivity as well as economic conditions which are characteristic of low-income communities (Mabila et al., 2017).

Recognising the impact of resource-constrained contexts on technology-enhanced teaching and learning is pertinent in the context of Pakistan, the site of this study. Within Pakistan's educational landscape, integrating digital resources into mathematics education poses both local and universal opportunities and challenges. Challenges within Pakistan's education system are an important backdrop to implementing educational initiatives. Following its independence from British rule on 14 August 1947, Pakistan has been striving to address concerns surrounding the quality of education, a persistent issue evident in various national education policies. Pakistan is an Islamic country with deeply rooted cultural and religious legacies (Amirali & Halai, 2010). The country's educational landscape is characterized by the location of a significant proportion of schools in rural and underserved areas, catering for more than 60% of the population (UNICEF Pakistan, 2022). This rural distribution poses distinct challenges for mathematics teachers in accessing and employing digital resources for educational purposes. Consequently, the availability and accessibility of technology, along with teachers' digital competencies, plays a critical role in influencing the integration of digital resources in mathematics classrooms. Thus, understanding the opportunities and challenges experienced by mathematics teachers in using digital resources in resource-constrained settings becomes imperative. This study's relevance extends not only to the specific context of Pakistan but also to other developing countries confronting similar educational landscapes and technological limitations.

1.3.1. Education system in Pakistan

The education system of Pakistan consists of two levels: elementary (primary) and secondary (high). Out of 53.8 million students, 29.2 million students are enrolled in government institutions, whereas the remaining 24.6 million students attend private sector institutions (Qadeer et al., 2022). The total number of high school students across 41,312 schools is 6.07 million. Generally, Pakistani high school classrooms have high student-to-teacher ratios and are overcrowded (Qadeer et al., 2022).

1.3.1.1. Policy to Integrate ICT in Education

The constitution of Pakistan (Article 25A) mandates the state to provide free and compulsory education to all children aged 5-16 years as a fundamental right. In line with this, the National Assembly passed the Right to Free and Compulsory Education Bill in 2012 (Constitution of Pakistan, 2012). However, despite this legal framework, a significant number of school-aged children in Pakistan remain out of school (UNICEF Pakistan, 2022). Pakistan is also bound by its international obligations as a signatory to several organizations to fulfil its responsibilities of providing free and compulsory basic education to its citizens. Key international organizations actively promoting ICT in education include United Nations Educational, Scientific and Cultural Organization (UNESCO), United Nations Children's Fund (UNICEF), Islamic World Educational, Scientific and Cultural Organization (ISESCO) and international banks.

Two important statements highlighting inclusive, equitable and quality education are the Incheon Declaration on Education 2030 and the ICT Strategy Guangdong. These emphasize the use and importance of ICT under the Education for All (EFA) goals since 2000 and the education-related Millennium Development Goals (MDGs), focusing on lifelong learning for all through ICT integration. In addition, the Ministry of Federal Education and Professional Training (2017) highlights the importance of integrating ICTs in education across Pakistan. By 2025, the policy aims to prepare students for the digital world and equip teachers with the necessary skills to facilitate ICT-enabled learning experiences. The main pillars of ICT policy are providing access to ICT in schools; use of ICT to strengthen quality of teaching and enhance student learning; develop complementary approaches to ICT in education; build on best practices in existing ICT programmes; and develop the capacity of education departments. Integration of Computer and other digital devices like smart phones for teaching and learning is needed to prepare students in early grades to be ready for the digital revolution underway. It is important to note that the current policies do not specifically state the current or expected level(s) of digital competencies for teachers in Pakistan.

However, the implementation of these policies remained a challenge as the majority of them were only partially executed, largely due to political instability and a lack of sustained political commitment in Pakistan (Atique et al., 2024). Furthermore, the existing national education and ICT policies fail to adequately encourage the use of ICTs in education, let alone their meaningful integration into the teaching-learning process (Ministry of Federal Education and Professional Training, 2017). Consequently, the potential of ICTs to transform and improve educational practices remains largely untapped (Memon & Tunio, 2022).

Moreover, after the 18th constitutional amendment in Pakistan, the responsibility for education primarily shifted to the provincial governments (Siddiqui et al., 2021). Now "each province is proceeding according to its own choices in developmental fields that have already resulted in creating a vast gap among the provinces. The situation is further aggravated in absence of a common platform for policy dialogue" (Ministry of Federal Education and Professional Training, 2017, p. 90). Moreover, this devolution has resulted in lower levels of education, both at primary and secondary levels (Siddiqui et al., 2021). Especially in mathematics teaching and learning, as recent National Achievement Test (NAT) results for 2023, show a concerning decline in countrywide mathematics scores at the intermediate level (8th grade), with the average score slipping from 42.7 to 41.6 out of 100 during the 2023-24 academic year

(Pakistan Institute of Education, 2024). Moreover, based on international benchmarks of mathematics achievement (TIMSS – Trends in International Mathematics and Science Study), 73% of Pakistani high school students not reaching the low benchmark of 400/1000 in mathematics (National Education Assessment System, 2020).

1.3.1.2. Deteriorating Infrastructure and ICT Deficiency at Government schools

In Pakistan, the local infrastructure of most government schools is archaic and deteriorating. Severe limitations often characterise the government schools, such as a lack of basic facilities (Dundar et al., 2014; Memon & Tunio, 2022; Naviwala, 2019; Siddiqui et al., 2021), inadequately trained and absent teachers (called ghost teachers) (Jaffer, 2010), and a severe shortage of learning materials and books (Baron & Bend, 2023; Mahmood, 2011). The use of information and communication technologies (ICTs) in government schools is extremely low. The Pakistani government has limited funding for efforts to provide schools with basic ICT facilities to improve teaching and learning outcomes (Baron & Bend, 2023). Limited efforts which are only targeted towards urban government schools have been made to establish and update computer labs and instal multimedia devices. More substantial actions, such as providing ICT infrastructure and qualified teachers with digital skills, are required to meet global standards (Atique et al., 2024; GCI, 2018). In contrast, private schools are relatively well equipped with basic infrastructure and teaching staff. This is reflected in the higher average enrolment ratio of 180 students vs. 146 students per school in private vs. government schools (Shah et al., 2018). This disparity in resources and infrastructure between private and government schools has implications for the quality of education that students receive in government schools. Government schools' students face greater challenges in their academic development due to limited resources and inadequate facilities.

1.3.1.3. Teacher professional development and quality of mathematics education

Mathematics is a core subject taught in high schools in Pakistan. The National Curriculum governs the content for teaching mathematics (Ministry of Federal

Education & Professional Training, 2018). This curriculum is developed in consultation with all four provinces for each grade up to Grade 12 (Mahmood, 2011). The National Curriculum of Mathematics lists the content of the curriculum, outlines the learning objectives, and recommends teaching methods. Examination syllabuses and papers specify what kinds of things students must do to acquire the qualifications they need. Textbooks published by the examination boards, claim to provide the information students need and the learning activities that will enable them to meet the objectives of the curriculum (Mahmood, 2011; Mughal et al., 2021). All the teacher do is deliver the curriculum as it has been defined (Ali et al., 2023).

In Pakistan, "training" and "professional development" are often used interchangeably, as evidenced by the Ministry of Federal Education and Professional Training's nomenclature. However, in the context of educational technology and this thesis, these terms have the following meanings. Training refers to short-term, targeted programs designed to equip teachers with specific skills and knowledge, particularly related to the immediate implementation of technology in their classrooms. These programs focus on acquiring the technical competencies necessary for using digital tools and educational software. In contrast, "professional development" encompasses a broader, long-term process of continuous learning and growth for teachers. It includes training but extends beyond the acquisition of specific skills. Teacher training, therefore, can be seen as a step within the broader framework of professional development. Training provides the necessary foundation of technical skills (Wasserman & Migdal, 2019), whereas professional development promotes an ongoing commitment to educational excellence and adaptability in the face of technological advancements (Gegenfurtner et al., 2020).

1.4. Challenges in mathematics education: Factors affecting teachers and teaching quality.

Several challenges impact the teaching and learning environment of Pakistani mathematics classrooms. These challenges include teacher qualifications, student-to-teacher ratio, availability of pedagogical resources, teaching practices and feedback mechanisms.

1.4.1. Teacher qualifications and student-to-teacher ratios

Despite a substantial increase in new teacher hires since 2019, the persistent shortage of well-trained teachers continues to impact the quality of teaching and learning outcomes in Pakistan. Particularly at the middle (11 to 14 years old), high (15 to 17 years old), and higher secondary (16 to 18 years old) school levels, the scarcity of subject specialists remains a major obstacle to improving student learning outcomes (Rizwan & Taniguchi, 2022). Notably, recent recruitment efforts have failed to address the shortages of qualified mathematics teachers, leading to more pronounced demand-supply gaps, especially in rural areas.

While precise figures on the number of teachers capable of teaching high school mathematics are unavailable (UNESCO Institute for Statistics, 2020), the Pakistan Education Statistics for 2022 reveal a total of 180,553 teachers (including mathematics teachers) holding graduate or post-graduate qualifications. If this cohort possesses the necessary expertise to teach mathematics, a theoretical ratio of 39 students per teacher at the high school level can be calculated. However, this ratio is likely to be skewed due to contextual factors and constraints, such as not all teachers being qualified to teach mathematics and not all students studying the subject. Consequently, the actual ratio is likely to be higher, as evidenced by the prevalence of overcrowded classrooms in Pakistan.

1.4.2. Availability of pedagogical resources

Mathematics teaching in Pakistani high school mathematics classrooms revolves around three primary resources: a teacher, a textbook, and a blackboard. The textbook is the first resource that students get when they enter any class (Amirali &

Halai, 2021). It is noteworthy that most students use this single textbook for the rest of their academic year (Ministry of Federal Education & Professional Training, 2018, p. 54). The content of the mathematics textbook adheres to the content, curriculum guidelines, and standards set for teaching mathematics in the Pakistan National Curriculum. However, this curriculum content has been subject to criticism. Teachers, academic consultants, and politicians regularly highlight the fact that archaic and monotonous content is commonly found in the curriculum of various compulsory and elective subjects, including mathematics textbooks (Dundar et al., 2014; International Crisis Group, 2014). For instance, a report outlining the education reforms in Pakistan mentioned that the textbooks of government schools in Pakistan are "deeply flawed" with printing and content errors (International Crisis Group, 2014, p. 1).

Complementing the textbook is the conventional blackboard, used by teachers using chalk derived from sedimentary carbonate rock. This pedagogical tool has historically exerted a substantial influence, particularly within resource-constrained environments like Pakistan. Blackboards' large writing surface enables mathematics teachers to write, draw, explain, calculate a sequence of operations, and make corrections, which can engage multiple students simultaneously (Wylie, 2011). However, it is essential to note that the use of blackboards alone may not lead to learning. The end goal of blackboard use in the mathematics classroom has been memorization, achieved through students' imitation of the teacher's model (Wylie, 2011). It is imperative for teachers to possess both subject matter and pedagogical knowledge to present accurate models or sequences of operations, ensuring teaching and learning outcomes. However, the efficacy of mathematics education at this level remains a subject of contention in Pakistan, largely due to the teachers' inadequate content and pedagogical knowledge (Pakistan Institute of Education, 2024).

Technology is infrequently used in teaching because teachers believe it to be complex, expensive, and scarce (Dundar et al., 2014). Memorization and rote learning are especially prevalent in rural areas of Pakistan, where teachers often lack subject-matter knowledge due to inadequate professional development opportunities (Tayyaba, 2010). This leads to a lack of understanding and engagement of students, hindering their academic progress and resulting in early dropouts and children not attending school (Shuja et al., 2022). Currently, Pakistan has 22.8 million out-of-school children aged five to 16, the second-highest number in the world (UNICEF Pakistan, 2022). This means that 44% of children in Pakistan are not attending school (Baron & Bend, 2023). Despite the government's efforts to provide teachers with ongoing professional development and support, learning outcomes are not improving. The existing pedagogical approaches have resulted in low student achievement levels nationwide, with a significant proportion of school leavers failing to meet the minimum standards in mathematics required by the curriculum (Pakistan Institute of Education, 2024).

1.4.3. Teaching practices and feedback mechanisms

Mathematics teaching time in Pakistan varies from 40 minutes to one hour per lesson depending on the teacher. Typically, teachers spend their time in class writing and solving questions from the textbook on the blackboard. Unfortunately, government schoolteachers are often inadequately equipped with manuals or lesson plans, whereas most private schools do not allow teachers to teach without them (Alam et al., 2021). Moreover, due to high student-to-teacher ratio, it is common for teachers to ask students to memorize solutions and applications after presenting calculations on the blackboard. In government schools, half-yearly and final examination reports are the formal (official) feedback mechanism. Private schools have established monitoring and evaluation mechanisms, which rely on regular internal assessments of cognitive attainment, and examinations (Jaffar, 2010). During the COVID-19 pandemic, the schools were closed for almost eighteen months. All formal examinations and assessments were halted for the same time (Dawn, 2021). Most students progressed to the next levels based on their previous year's assessment results (Zubairi et al., 2022). This led to significant challenges for high school students, especially those in remote and underprivileged areas who already had limited quality face-to-face learning time and minimal access to technology (ASER Pakistan, 2020).

1.4.4. Student learning and the role of private coaching academies in Pakistan

Students spend most of their classroom time imitating teachers' solutions (answers, calculations) from the blackboard (Akhter et al., 2015). In this process, the chances of actual learning and understanding the concept may depend on the student's abilities for spontaneous reading and writing (Wylie, 2011). In most of the cases, due to insufficient time and high student-to-teacher ratios, students became passive and constrained about asking questions (Mughal et al., 2021). Students barely get the time to combine knowledge with understanding so that each informs the other (Beghetto & Sriraman, 2017). Consequently, students prefer home tuition or join private coaching academies (Alam et al., 2021). These academies operate as unregulated private businesses. They cater to the individual needs and financial abilities of students and their parents and are often favoured for their ability to improve students' grades. This trend is particularly prevalent in the middle class (group of people with moderate socioeconomic status), which comprises 38% of the population in Pakistan (N. Hassan et al., 2022).

In summary, high school mathematics education in Pakistan faces various challenges. They have limited opportunities, rely on rote learning and focus solely on textbook problems (Amirali & Halai, 2010, 2021), while disregarding real-world applications. Existing literature identifies several factors contributing to the poor state of mathematics education in Pakistan. These include the quality of teachers' education (Akhter et al., 2015; Halai, 2017; Shiraz & Qaisar, 2017), political hiring practices (Naviwala, 2019), the presence of ghost teachers[2] (Jaffer, 2010), insufficient teacher training (Amirali & Halai, 2021; Dundar et al., 2014; Naviwala, 2019), limited use of technology (Dundar et al., 2014), unappealing job profiles, and bribery tainting the hiring process (Naviwala, 2019). In many instances, teachers resort to direct teaching methods that prioritize textbook content delivery through rote learning and memorization (Amirali & Halai, 2021). According to (Skemp, 2006), memorization is a way to acquire proficiency in several mathematical techniques in the classrooms and it complements pedagogical resources such as textbooks.

[2]The term ghost teachers refer to those teachers who are on the payroll but do not show up for their job (Jaffer, 2010).

1.5. CHANGING DIGITAL LANDSCAPE AND LITERATURE ON DIGITAL COMPETENCIES OF PAKISTANI MATHEMATICS TEACHERS

Over the last five years in Pakistan, particularly following the COVID-19 pandemic, teachers have increasingly started using digital resources, particularly at urban schools. In 2022, UNICEF Pakistan published a post-pandemic digital learning landscape analysis. The report, Zubairi et al. (2022), which is based on 2366 teachers' interviews across rural and urban regions of the country, highlights significant shifts in using digital resources and the changing digital competencies of teachers. During the COVID-19 pandemic, teachers used digital platforms (social media, and video conferencing tools) for communication and teaching. The report, which is not subject-specific, indicates limited (lower) use of web resources and less frequent use of digital tools for student assessment. The report further mentioned variations in adoption and use of digital resources based on school type and location indicating a discrepancy in the level of digital competencies among teachers. For example, urban and private schoolteachers have higher levels of adoption of digital resources, implying a reasonable familiarity with technology and some level of digital competence in these settings. The report suggests prevalent use of smartphones, SMS, and messaging apps for communication among teachers showing their digital skills to engage with students and parents through these channels.

Moreover, in STEM (Science, Technology, Engineering and Mathematics) context, recent investigations by Ali et al. (2023), Aslam et al. (2022), and Makhdum et al. (2023) highlight the challenges encountered by teachers in integrating technology into their teaching practices. These challenges include limited access to technology, inadequate training, and insufficient support from educational institutions. Ali et al.'s (2023) survey of 566 STEM teachers in Karachi, Pakistan, emphasizes the need for teachers to enhance both pedagogical knowledge and the use of technology. The authors emphasize the importance of providing in-service teachers with access to professional development opportunities to augment their capabilities in classroom teaching and learning. However, the examination of PhD studies on mathematics education in Pakistan reveals a historical focus on student-centric investigations, as indicated by Higher Education Commission (HEC) of Pakistan's research repository. Using the search criteria “mathematics AND education,” the repository revealed only 24 dissertations (1960–2020) related to mathematics education. Among these, five used “technology” in their titles, with none specifically addressing teachers’ use of digital resources.

The above overview shows a gap in literature that highlights the use of digital resources by mathematics teachers in Pakistan. There is a need for research specifically focused on the digital competencies of mathematics teachers. Such research could investigate teachers' ability to integrate digital resources, the benefits, and challenges of using these resources, and teachers’ professional development needs in this area. With these considerations in mind, this study aims to address the lacunae in the available literature. In the next section, I present the aim and research questions that guided this study.

1.6. Research question

The aim of this study was to explore and understand the digital competencies of high school mathematics teachers in Pakistan, focusing on their beliefs, practices, and experiences in integrating digital resources into teaching and learning mathematics. This study highlights the challenges and opportunities within the context of a resource-constrained educational environment and recommends solutions for enhancing teachers' digital competencies. The overarching research question that guided this study was:

Overarching research question (RQ):

What are the digital competencies of high school mathematics teachers in Pakistan?

This question was explored through the following sub-questions (SQs):

SQ1: How do Pakistani teachers' beliefs about mathematics teaching influence their decisions to integrate digital resources in teaching and learning mathematics?

SQ2: What influences teachers to use digital resources well?

SQ3: How well do Pakistani teachers use digital resources to teach mathematics topics?

SQ4: What professional development opportunities do Pakistani teachers have that influences their digital competencies and approach to integrating digital resources?

SQ5: What are the barriers to and enablers for teachers using digital resources face-to-face and online?

Aligned with this focus, (belief, practices, and experiences of integrating digital resources), this study adapted and used the Digital Competencies for Teaching Mathematics with Technology (DCTMT) framework, as proposed by Tabach and Trgalová (2020). This theoretical framework served as a guide, offering a structured approach to examining mathematics teachers' digital competencies and their skills for using digital resources. Tabach and Trgalová's (2020) digital competencies' framework includes dimensions, encompassing personal orientation (beliefs, values, and preferences) (Schoenfeld, 2011), instrumental genesis (Haspekian, 2011; Thomas & Palmer, 2014; van Dijke-Droogers et al., 2021), specialized digital resources knowledge (SDRK), knowledge of digital resources and students (KDRS), knowledge of digital resources and teaching (KDRT), and knowledge of digital resources and curriculum (KDRC) (Tabach & Trgalová, 2019, 2020). These dimensions provided a systematic approach to examining digital competencies, offering categories and criteria for analysis. The framework also serves as a methodological tool for structuring the analysis of the data within its defined dimensions and aspects.

To answer the overarching research question, using the digital competencies framework, this study adopted a mixed-methods approach (comprising Phase 1 and Phase 2) with an explanatory sequential design. In Phase 1, I collected quantitative data using an online survey of high school mathematics teachers in Pakistan. The survey received 302 responses, however, after screening, 270 responses were suitable for analysis. Quantitative data from Phase 1 underwent descriptive and inferential statistical analysis to unveil patterns and trends. Once Phase 1 was completed, Phase 2 was initiated in which I collected qualitative data using semi-structured interviews with six high school mathematics teachers. The Phase 2 investigation was designed to gain a deeper understanding of teachers' experiences and perspectives, building upon the Phase 1 correlations and findings. Phase 2 employed thematic analysis using the pre-identified themes and sub-themes to explore the qualitative dimensions of teachers' digital competencies. For both phases, I collected the data during the COVID-19 pandemic and school lookdowns.

Triangulation was used as a validation technique for this mixed-methods research. Quantitative data from an online survey and qualitative data from semi-structured interviews were combined to gain a deeper understanding of the digital competencies of high school mathematics teachers. The aim was to look for patterns, similarities, and differences in teachers' responses as reflected in both sets of data. The chosen explanatory sequential design aligned well with the study's purpose. The integration of both quantitative and qualitative data collection methods enhanced the understanding of the digital competencies of high school mathematics teachers in Pakistan.

This research adhered to ethical guidelines and was approved by the University of Waikato Faculty of Education Ethics Committee on 06/04/2020 (Approval number: FEDU024/20, refer to Appendix H). Meeting these ethical requirements ensured participant confidentiality, informed consent, and privacy were maintained throughout the study.

1.7. Thesis overview

This thesis "digital competencies of high school mathematics teachers in Pakistan" is divided into seven chapters. The synopsis of each of the chapters in the thesis is as provided in the ensuing sections.

Chapter one introduces the thesis, presenting the personal statement and research interests of the author. The chapter sets the context of the study within the Pakistani education system, highlighting issues such as deteriorating infrastructure, ICT deficiencies in government schools, and the state of mathematics education. It delves into the challenges faced by mathematics teachers, including teacher qualifications, student-to-teacher ratios, the availability of pedagogical resources, teaching practices, and the role of private coaching academies. The changing digital landscape and relevant literature are explored, leading to the formulation of the research question and an overview of the thesis.

Chapter two provides a comprehensive review of the literature on digital competencies for mathematics teachers. It begins with an introduction and explores various aspects of digital competencies, including definitions, digital resources, historical overviews, theoretical frameworks, and evidence supporting the importance of these competencies. The review covers the complexity of digital competencies, historical developments from the 1980s to the post-pandemic era, and theoretical frameworks such as TPACK (Technological Pedagogical and Content Knowledge), MKT (Mathematical Knowledge of Teaching), PTK (Pedagogical Technology Knowledge), DAD (The Documentational Approach to Didactics), and DCTMT. Further, the literature review indicates that DCTMT serves as a guide, offering a structured approach to examining mathematics teachers' digital competencies and their skills for using digital resources. Then the literature review discusses the evidence on digital competencies for mathematics teachers, with a focus on problem-solving, dynamic graphing software, videos, interactive whiteboards, and game-based learning. Before presenting the summary of the literature review, the chapter considers the digital competencies of mathematics teachers in developing countries. Finally, this chapter concludes with a summary of the literature review.

Chapter three outlines the methodology employed in this study, aiming to elucidate the processes and approaches integral to the study. The chapter commences with the research paradigm, the mixed-methods approach, and the explanatory sequential design, which is employed, emphasizing its relevance in mathematics education. The chapter then delves into the quantitative Phase 1 of this study, elucidating the structure of the online survey instrument and the intricacies of the pilot study, addressing aspects such as feasibility, quality measurement, and response rate evaluation. Subsequent sections discuss participant recruitment, survey distribution, handling outliers and missing values, and the validation and reliability of the survey instrument. Phase 2 (qualitative) is introduced and encompasses the recruitment of participants for semi-structured interviews and the subsequent qualitative data analysis. Subsequently, I address methodological strategies such as triangulation and reflexivity and ethical considerations, including those related to the unique challenges posed by the COVID-19 pandemic. The chapter culminates in a summary, encapsulating the key methodological decisions and procedures that underpin the study's research design.

Chapter four presents the findings of Phase 1, focusing on quantitative data. It begins with an introduction and proceeds to discuss teachers' demographic information, their use of digital technologies, access to digital devices and the internet, and use of digital resources. Blocks of data related to general beliefs about mathematics teaching and learning, beliefs about digital resources, knowledge of digital resources and students, and knowledge of digital resources and teaching are explored. The chapter concludes with a description of overall correlations between these blocks and a summary of the findings.

Chapter Five delves into the qualitative findings of Phase 2 of this study. It covers general beliefs about mathematics teaching and learning, beliefs about digital resources and their use, digital pedagogical resource knowledge (DPRK), barriers and enablers in using digital resources, and changes in beliefs due to COVID-19. Finally, a summary of the chapter's key points is presented.

Chapter Six provides a detailed discussion of the findings from both the quantitative and qualitative phases. It explores teachers' beliefs about mathematics teaching and learning, their access to and use of digital resources, and their knowledge of digital resources in relation to students, teaching, and curriculum planning. The chapter explains correlations between different constructs of the digital competencies' framework, highlighting the interplay between various factors influencing digital competencies. A summary concludes the chapter.

Chapter Seven, the final chapter, serves as the conclusion of the thesis. It begins by reiterating the main contributions of the study, emphasizing insights into teachers' beliefs, the interplay between beliefs and digital competencies, and the link between training, self-learning, and resilience. The chapter discusses the implications of the findings for teachers, policymakers, and professional development providers. It acknowledges the limitations of the study, particularly those related to the COVID-19 pandemic, potential framework limitations, and personal biases. The conclusion ends with final thoughts on the overall importance of the research.

CHAPTER TWO

2. Literature review

2.1. Introduction

In this chapter, I examine literature on digital competencies in education and focus on studies related to mathematics education in Pakistan. I seek to identify the digital competencies that research studies have contended are necessary for high school mathematics teachers. Furthermore, the review examines the literature to report on how teachers across the world integrate digital resources in their teaching and learning of mathematics.

The literature review commences with an exploration of the concept of digital competencies, investigating what is regarded as constituting digital competence, and outlines the digital competencies which are seen to be necessary for mathematics education (Section 2.2).

In Section 2.3, I provide a historical overview of expectations of digital competencies for teachers. This overview includes early developments, evolution of ideas over decades, key initiatives, and major frameworks for examining the digital competencies of teachers. Further details about frameworks such as Technological Pedagogical and Content Knowledge (TPACK), Mathematical Knowledge of Teaching (MKT), Pedagogical Technology Knowledge (PTK), and Digital Competencies for Teaching Mathematics with Technology (DCTMT) and their use in mathematics education are presented in Section 2.4. This section also includes views on the role and influence of teachers' capabilities, competencies, beliefs, knowledge, and skills in using digital resources. At the end of this section, I assert that the DCTMT (Section 2.4.9) offers a structured approach to understanding the digital competencies required by mathematics teachers.

In Section 2.5, I present studies that highlight the role of digital competencies for mathematics teachers in teaching. Section 2.5 also elaborates on the importance and use of various types of digital resources, such as videos and dynamic graphing software (DGS). In Section 2.6, I present the obstacles teachers may encounter and the advantages of digital resources, particularly in the context of developing countries like Pakistan. Finally, in Section 2.7, I conclude by summarizing the key findings and highlighting the gaps in the literature.

2.2. DIGITAL COMPETENCIES

The following sub-sections draw on the literature to define the term digital competence and competencies. Following this, discussion focusses on the definitions found in several policy documents related to the use of digital resources in mathematics education.

2.2.1. Digital competence and digital competencies

In literature, the terms digital competence and "digital competencies are often used interchangeably (Skantz-Åberg et al., 2022). However, while related, they encompass distinct understandings and implications. Digital competence refers to the ability of a person to use digital technologies across work, education, or daily life (Reddy et al., 2020). Ferrari (2012) referred to digital competence as a broad set of skills, knowledge, and attitudes that enable individuals to use digital tools and resources. From this perspective, digital competence encompasses technical skills, information literacy, communication, collaboration, and problem-solving, and digital citizenship (Bozkurt & Uygan, 2020). In essence, digital competence represents the foundational, multifaceted expertise required to function in a digital world. In the educational context, digital competence may refer to the teachers' use of digital technologies within pedagogical settings (Heine et al., 2023). Just as teachers require various forms of professional knowledge, including content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK), they also need to develop digital competence to navigate the complexities of integrating technology into teaching (Guerreiro, 2017; G. Kaiser & König, 2019; Shulman, 1986). This competence extends beyond technical skills to encompass cognitive performance dispositions that are responsive to the demands of digitalization in schools (McFarlane, 2019; Selwyn, 2012).

In contrast, digital competencies refer to the specific abilities and proficiencies that contribute to an individual's overall digital competence (Reisoğlu, 2022). Digital competencies may include proficiency in using particular software applications (e.g., data analysis with Microsoft Excel, graphic design with Photoshop), being adept in the use of digital resources for specialized tasks (e.g., data visualization, content creation), and or in-depth familiarity with industry-specific digital best practices and tools (e.g., programming languages, cybersecurity protocols, digital marketing techniques). Importantly possessing digital competence does not imply mastery of all potential digital competencies, as these specialized abilities may vary depending on the context or domain (Reisoğlu, 2022). In summary, digital competence is the overarching, multidimensional capacity to operate in digital environments, whereas digital competencies are the skills and abilities that contribute to and enable that broader competence in the various contexts of an individual's home and working life. Developing digital competence involves the acquisition of the range of relevant digital competencies needed to achieve one's specific goals and requirements (Caena & Redecker, 2019; Reisoğlu, 2022).

Associated with digital competence is the availability of digital resources (DR). Ferrari (2012) considers digital competence as a "moving target" due to the ever-evolving nature of digital resources that an individual can engage with across their work, education, and life. Without an understanding of digital resources, a person cannot become digitally competent because digital competence involves the appropriate use of digital tools and resources. (Ferrari, 2012). Therefore, in the context of this research, it is important to understand what the term digital resources constitutes and what the digital resources for teaching and learning mathematics are, particularly at the high school level.

2.2.2. Digital resources (DR)

There is inconsistency in how the term 'digital resources' is defined in the research literature (Heine et al., 2023). The term, digital resources (DR) is frequently used synonymously with digital content materials (Gaffney, 2010) or digital curriculum resources (Pepin, Choppin, et al., 2017) that can be used in a variety of formats by teachers. For example, Intel Corporation (2011) provides a list of common components of DR referring to them as digital content. Intel's list includes multimedia components such as images, graphics, infographics, audios, videos, texts, animations, simulations, and interactive games. Intel's definition specifies components of DR; however, it is more focused on the digital content than the broader set of tools and applications encompassed by digital resources.

Another way of conceptualising digital resources is to describe them as online curriculum content. For example, Gaffney (2010), refers to digital resources as online curriculum content that can be used and customised by teachers in a variety of formats, including interactive multimedia resources, interactive assessment resources, and digital curriculum resources, which have been sourced from cultural and scientific institutions and private collections. Gaffney's conception of DR as digital curriculum resources resonates with the view of (Pepin, Choppin, et al., 2017). These authors describe digital curriculum resources as an elastic concept starting from one-off worksheets to a full range of curriculum programmes. Pepin, Choppin et al. (2017) suggest that the focus of the definition should be on digital resources in electronic formats that can organise and communicate curricular content for specific age groups, levels, grades, and topics. Other examples of digital resources that have been noted in the literature include e-books, online databases, web applications, social media platforms, and multimedia educational resources (Alberola-Mulet et al., 2021; Clark-Wilson et al., 2020).

There have been recent attempts to arrive at a comprehensive definition of digital resources. For example, Heine et al. (2023) conducted a systematic literature review to analyse the definitions of digital resources. Their review analyses the definitions of digital resources in 23 articles and examines and compares their facets in the context of digital competencies. Based on their review, (Heine et al., 2023) define digital resources in education as digital or technical tools that provide knowledge, innovation, and interaction. These tools can be accessed through hardware, such as computers, smartphones, laptops, and interactive whiteboards. (Heine et al., 2023) include hardware as a digital resource and argue that hardware is used as a presentation medium and correspondingly also serves as a source of knowledge. In their definition, the authors indicate that digital resources can be used for content delivery (including communication, connection, and exchanging feedback and information) and content creation (developing, modifying, and designing teaching materials). Heine et al. (2023) provide examples of digital resources such as emails, digital textbooks, social networks, platforms, blogs, clouds, websites, online information, software, apps, programmes, simulations, and multimedia content. In summary, (Heine et al., 2023) explain digital resources as tools that promote innovation, interaction, and knowledge in 21st-century schools. The definition is derived from technological, educational, and pedagogical perspectives. These perspectives consider digital resources as technical tools accessible through hardware, facilitating content delivery, creation, innovation, interaction, and knowledge enhancement in 21st-century educational environments. These perspectives collectively acknowledge digital resources as integral to teachers' professional digital competence, emphasizing their role in teaching and learning processes..

In the following section, I use the definitions discussed in Section 2.2.2 to establish a definition of digital resources in the context of teaching and learning mathematics.

2.2.3. Digital resources for teaching and learning mathematics.

There is no agreed definition of what constitutes DR for mathematics. However, based on the reviewed definitions, digital resources for mathematics may refer to the educational content that the mathematics community (teachers, students, institutions, developers, and others) design, develop, customise, use, and update, and that is available both online and offline, and may contain multimedia components as well as dynamic interactive features. The definition includes multimedia as numerous studies (e.g., Gaffney, 2010; Heine et al., 2023; Pepin, Choppin, et al., 2017; Tabach & Trgalová, 2020) consider them important in digital learning and as “particularly valuable in helping students acquire the initial mental imagery essential for conceptual understanding” (Miller, 2009, p. 396). In addition, dynamic graphing systems (DGS) such as GeoGebra, Cabri, and Geometers Sketchpad (GSP), dynamic geometry tools, algorithmic programming languages, spreadsheets, data loggers (motion detectors and GPS), computer algebra systems (CAS), online calculators, wikis, academic networking with students and experts are available to further enhance DR for mathematics education (Alabdulaziz, 2021; Dockendorff, 2020).

To ensure clarity and consistency in my thesis, I have adopted the definition of digital resources for mathematics teaching and learning as outlined in Section 2.2.3. Therefore, every time I use the term "digital resources," it will refer to the definition provided in Section 2.2.3. The next section presents digital competencies as defined in different frameworks.

2.2.4. Digital competencies in different frameworks

Educational policy documents in several countries emphasise the development of the technological knowledge and skills of teachers as one of the key targets of professional development programs (Kelentrić et al., 2017). Documents such as the Digital Literacy Framework (DigEuLit), DigComp 2.1: The Digital Competence Framework for Citizens (European Union), DigCompEdu (2017), Professional Digital Competence Framework for Teachers (Norway), Information Communication Technologies Framework for Schools in Ireland (NCCA), and UNESCO ICT Competency Framework for Teachers, emphasise the acquisition of digital competencies for teachers and students. The various frameworks that exist provide similar definitions of digital competence. For instance, (Martin & Grudziecki, 2006) define digital competence in the Digital Literacy Framework (DigEuLit Project) as follows.

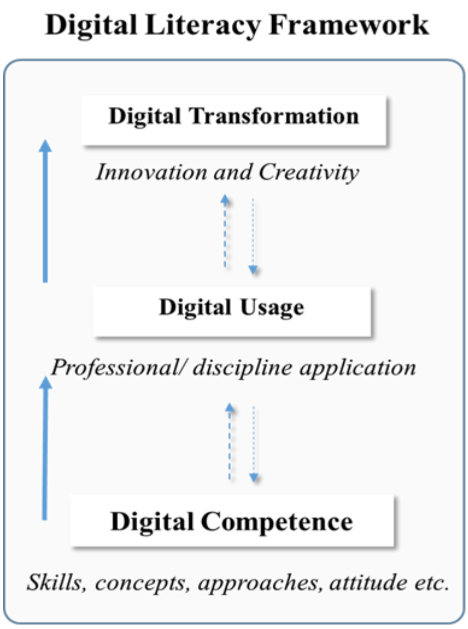
> Digital competence covers a wide range of topics, encompasses skill levels from basic visual recognition and manual skills to more critical, evaluative, and conceptual approaches, and also includes attitude and awareness (p. 255).

Martin and Grudziecki (2006) present digital literacy as a three-level framework as shown in Figure 1 (p. 30) (Martin & Grudziecki, 2006): digital competence, digital usage, and digital transformation. These authors consider digital competence as the foundational level of digital literacy. They define digital literacy as individuals' capacity to use digital tools to access, manage, integrate, assess, analyse, and synthesize digital resources. These skills enable them to generate new knowledge, craft media content, and engage in communication within specific life contexts, all with the goal of facilitating social engagement and capacity for reflective thinking on these actions.

Martin and Grudziecki (2006) organised an understanding of digital competence around thirteen processes: statement, identification, accession, evaluation, interpretation, organization, integration, analysis, synthesis, creation, communication, dissemination, and reflection. They explain that to solve a problem or a task, users can sequentially carry out these processes with digital tools in working with digital resources of any type. These authors argue that the regular application of digital competence within a specific professional realm is mandatory to reach the ultimate level of digital literacy i.e., digital transformation. Such a level allows user to create, innovate and "stimulate significant change within the professional or knowledge domain" (p. 259). However, it is recognised that every user does not need to attain digital transformation. The level of digital competence as shown in Figure 1 (p. 30) is seen to provide sufficient knowledge and skills to operate in a digital environment.

Figure 1

Digital Literacy framework by Martin and Grudziecki (2006)



Note. The three levels of the Digital Literacy Framework: Digital Competence, Digital Usage, Digital Transformation. From "DigEuLit: Concepts and tools for digital literacy development," by Martin and Grudziecki, 2006, *Innovation in teaching and learning in information and computer sciences, 5(4), p. 255.*

Martin and Grudziecki's (2006) concept of digital transformation aligns with (Verillon & Rabardel, 1995) theory of instrumental genesis. It emphasizes that teachers must acquire knowledge about digital tools and use them regularly for specific tasks, both professionally and personally. An illustrative example, offered by (Haspekian, 2011), involves teachers using a spreadsheet like Microsoft Excel for basic functions such as attendance recording. Haspekian (2011) explains that through continuous engagement, teachers gradually develop advanced skills, including automated grading and data organization for research. This process transforms the spreadsheet from a simple tool into a potent instrument for teaching and research, exemplifying instrumental genesis in the digital resource context. The idea of instrumental genesis introduced by Verillon and Rabardel (1995), underscores the evolution of a tool into a functional instrument as users continuously interact with it while executing specific tasks (Bozkurt & Uygan, 2020; Trouche, 2004).

Martin and Grudziecki's (2006) framework provides a foundation for understanding digital competencies. However, its broad scope and focus on a single linguistic context limit its application. Pangrazio et al. (2020) highlight this limitation, demonstrating how the concept of digital literacy translates differently across English, Spanish, and Scandinavian contexts. For example, the broader term digital literacy in English-speaking countries is often narrowed down to digital competence or digital Bildung in Spanish and Scandinavian contexts, reflecting a stronger emphasis on specific skills relevant to their educational systems and cultural practices.

This tension between the conceptual, practical, and political dimensions of digital literacy is further amplified by the rapid evolution of technology. Reddy et al. (2020) argue that Martin and Grudziecki's (2006) definition may need to adapt as new technologies emerge and transform how people interact with digital tools. While Martin and Grudziecki (2006) rightly emphasize the multidimensional nature of digital literacy, encompassing technical skills, cognitive abilities, and ethical considerations, a focus on digital competencies might be more practical. Digital competencies prioritize the application of these skills within specific contexts. By focusing on the tasks individuals need to perform in particular situations, the concept of competencies offers a more actionable approach to digital literacy. Ferrari (2012) instead of using digital literacy, prefers the term digital competence, understanding it as the set of knowledge, skills and attitudes needed today to be functional in a digital environment. Ferrari argue that “moving towards competences instead of literacies requires taking into account attitudes, which are often left aside in certification and assessment discourses, but which are so intertwined with knowledge and skills to be often difficult to isolate” (p. 19). Ferrari (2012) analysed 15 frameworks to identify the key components of digital competence in terms of knowledge, skills, and attitudes. Ferrari developed digital competence descriptors and proposed a roadmap for the possible use and revision of a digital competence framework and its descriptors for all levels of learners. Ferrari (2012) adjusted the existing definitions and defined digital competence as follows:

> Digital Competence is the set of knowledge, skills, attitudes (thus including abilities, strategies, values and awareness) that are required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge effectively, efficiently, appropriately, critically, creatively, autonomously, flexibly, ethically, reflectively for work, leisure, participation, learning, socialising, consuming, and empowerment (p. 3).

The above definition of digital competence provides a broad overview encompassing knowledge, skills, and attitudes; however, it does not explain overlap between competencies. For example, information literacy (ability to find, evaluate, and use information effectively and ethically) overlaps with digital media literacy and critical thinking (ability to assess the quality and relevance of information sources), making it challenging to clearly distinguish between them.

Another framework, the Digital Competence for Citizens (DigComp 2.1) (Carretero et al., 2019), defines five areas of individual digital competencies. The framework does not provide a precise definition. However, it explains that an individual is digitally competent when s/he can demonstrate skills in the following five areas: 1) information and data literacy 2) communication and collaboration 3) digital content creation 4) safety and 5) problem-solving skills. The framework provides exhaustive details for each area and its respective dimension pertinent to education, training, employment, and lifelong learning (Carretero et al., 2019).

Specifically designed for the digital capacity building of teachers, the DigComp developed a separate framework called DigCompEdu. This framework was introduced in 2017 by the European Commission to help teachers develop their digital competencies. Compared to other frameworks, DigCompEdu takes a holistic approach to the pedagogical knowledge and skills of teachers, including learning methods and techniques (Reisoğlu, 2022).

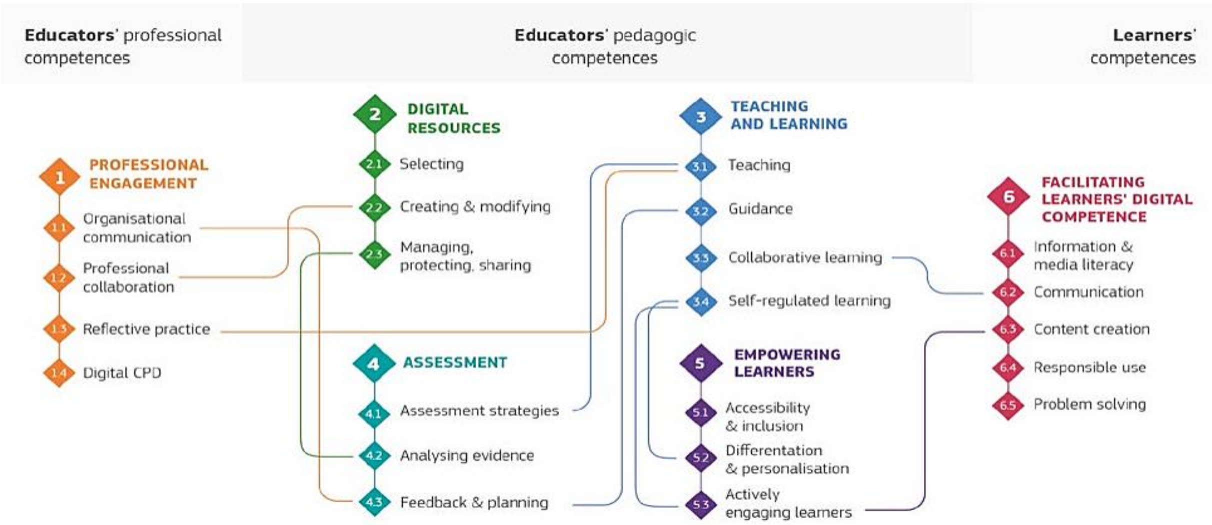
DigCompEdu defines the digital competence of teachers by proposing 22 elementary competencies organised in six areas: professional engagement, digital resources, teaching and learning, assessment, empowering learners, and facilitating learners' digital competence as shown in Figure 2 (p. 34) (Redecker, 2017). Each area has a series of competencies that "teachers must have in order to promote effective, inclusive and innovative learning strategies, using digital tools" (Redecker, 2017).

In DigCompEdu framework, Area 1 (professional engagement) is directed at teachers' use of digital technologies in professional interactions and communication with colleagues, learners, parents, and other interested parties. Furthermore, this communication through technology allows for individual professional development and collective and continuous innovation in an educational organisation (Cabero-Almenara et al., 2020). Area 2 (digital resources) looks at the competencies that teachers need to use, create, and share digital resources. Likewise, this area indicates that teachers must know how to use and administer digital content responsibly, respecting an author's rights and protecting personal data. Area 3 (teaching and learning) is dedicated to managing and orchestrating the use of digital technologies in teaching and learning. Cabero-Almenara et al. (2020) call this digital pedagogy i.e., knowing how to design, plan and implement the use of digital technologies in all the phases of the teaching process, and promoting student-centred approaches and methodologies. Area 4 (assessment) addresses the use of digital strategies to enhance assessment. Area 5 (empowering learners) focuses on the potential of digital technologies for learner-centred teaching and learning strategies adapted to the competence level, interests and learning needs of each student. Areas 6 (facilitating learners' digital competence) details the specific pedagogical competencies required to facilitate students' digital competence.

These six areas may jointly represent the competence level for teachers. According to DigCompEdu, teachers with different experience and contact with educational technology may fall in any one of the six levels (Novice level (A1), Explorer (A2), Integrator (B1), Expert (B2), Leader (C1) and Pioneer (C2)).

Figure 2

The DigCompEdu framework (2017)



Note. The Figure shows three main areas (professional, pedagogic and learners' competences) and their respective sub-areas of digital competencies for teachers. From "European Framework for the Digital Competence of Educators: DigCompEdu," *EUR 28775 EN. Publications Office of the European Union, Luxembourg, 2017, p. 8.*

Although DigCompEdu is a useful framework to capture and describe teacher-specific digital competencies, it is not without its drawbacks. Further contextual, subject-specific, and situational (such as COVID-19) adaptation in the frameworks are needed to meet changing requirements for skills and competencies. For example, during the COVID-19 pandemic, teachers were compelled to transition to online teaching quickly. Due to the urgency of the situation, some teachers may have bypassed certain competencies levels outlined in the framework and focused only on learning the tools needed for initiating online teaching, such as video conferencing via Zoom, Microsoft Teams, and Google Meet (Calder et al., 2021; Syahrin et al., 2023). These self-trained teachers may have progressed to different competence levels in online teaching environments, making it difficult to assign a competency level based on DigCompEdu.

Although the DigCompEdu framework provides general descriptions of digital competencies, it does not offer concrete examples or practical guidance for teachers to develop those competencies (Zhao et al., 2021). González et al. (2017) noted that the framework does not fully consider the needs and perspectives of learners and suggested that it could be expanded to include more learner-centred approaches. They further articulate that the framework could be strengthened by including more attention to ethical and social issues related to digital technologies.

Another framework aims to extend the scope of digital competencies. Kelentrić et al. (2017) in the Professional Digital Competence Framework for Teachers (Norway) define digital competence as including a wider range of 21st-century skills: creativity, critical thinking and problem solving, collaboration; communication; Information and Communication Technology (ICT) literacy; and social and/or cultural competencies [including citizenship]) as follows:

> The confident, critical, and creative use of ICT to achieve goals related to work, employability, learning, leisure, inclusion and/or participation in society. Digital competence is a transversal key competence, which enables the acquisition of other key competencies. It is related to many of the so-called 21st Century skills, which should be acquired by all citizens, to ensure their active participation in society and the economy (p. 12).

Kelentric et al. (2017) conceptualizes digital competence as a transversal key competence. Their concept is similar to the other definitions presented in this section, as it signifies digital competence as a skill set applicable across various domains of life and learning. That is, digital competence empowers individuals to use information and communication technologies (ICT) to achieve goals in work, education, leisure, and social participation (Ferrari, 2012; Ghomi & Redecker, 2019; Reisoğlu, 2022). The framework emphasizes the importance of digital competence in the teaching profession and provides a set of competences for teachers to develop. The framework is adaptable and flexible, allowing for customization and integration into existing teacher education programs (Nagel et al., 2023). However, it has been argued that it lacks specificity and places too much emphasis on technology and digital tools, and not enough on pedagogy and the specific educational context (Nagel et al., 2023; Starkey & Yates, 2022).

There is considerable similarity between the frameworks (Cabero-Almenara et al., 2020). The discussion of these frameworks highlights that digital competencies are more than just the ability to use software or operate digital devices. In general, these frameworks conceptualise digital competencies as the knowledge, skills, and attitudes required to make appropriate use digital technologies and tools for communication, information, and problem-solving purposes. The frameworks also recognise that digital competencies include the ability to navigate, analyse, and evaluate information, communicate using digital resources, and engage in responsible and ethical online practices (Redecker, 2017).

2.2.5. Digital Competencies for high school mathematics teachers

For the teaching of mathematics, digital competencies and skills may refer to some distinctive specific abilities. Studies (e.g., Dockendorff, 2020; Pepin, Choppin, et al., 2017; Pepin, Gueudet, et al., 2017) have shown that, to teach mathematical concepts, it is crucial for teachers to have knowledge and skills to use digital resources that enable dynamic and multiple representation of mathematical concepts. Teachers need to acquire competencies, a process that involves specific ways of dealing with and using the tools or resources, holding specific beliefs and values about them, and engaging in specific social interactions with them (Tabach & Trgalová, 2019). Over time teachers need to develop appropriate knowledge of particular digital tools, use them to perform tasks in certain ways and then continue to use them for both professional and personal purposes (Haspekian, 2011; Siedel & Stylianides, 2018; Trouche et al., 2020). Verillon and Rabardel (1995) refer to this process of appropriating tools (in our case, digital resources) as an instrumental approach. These authors introduced an important concept called instrumental genesis through which an available tool becomes a functional instrument for the user while performing a particular task over time (Bozkurt & Uygan, 2020; Trouche, 2004). Verillon and Rabardel (1995) further defined instrumental genesis as having two sub-processes. First, artifact-oriented instrumentalization that focuses on how the tool itself evolves or adapts through use. For example, a teacher might create custom templates or macros within a digital tool to improve its functionality for their specific needs. Second, subject-oriented instrumentation, which refers to the development of the user's knowledge and skills in using the tool. The teacher, in this case, develops a deeper understanding of how the tool can be used for teaching specific mathematics topic.

Tabach and Trgalová (2020) articulate that mathematics teachers' digital competencies may include their ability to select and integrate digital resources, adapt them to diverse learning needs, and orchestrate meaningful learning experiences. Teachers' knowledge of digital resources such as multimedia objects, games, animations, simulations, and software allows them to teach mathematical concepts by establishing their connections with real-world contexts (Karakoç & Alacacı, 2015). In essence, high school mathematics teachers' digital competencies can be understood as the skills, knowledge, and abilities that they need to integrate digital resources into their mathematics teaching at the high school level (Tabach & Trgalová, 2019, 2020). These competencies may include:

- Knowledge of technology tools and resources that can support mathematics instruction at the high school level, such as graphing calculators, dynamic graphing software (DGS), and mathematical modelling software (Albano & Dello Iacono, 2019; Dockendorff, 2020).
- Ability to design and implement technology-enhanced mathematics lessons that are aligned with high school curriculum standards and student learning objectives (Bozkurt & Ruthven, 2017; Bozkurt & Uygan, 2020).
- Familiarity with online and digital resources for high school mathematics, such as e-textbooks, simulations, and interactive activities.
- Ability to use technology to differentiate instruction and provide personalized learning experiences for high school students (Clark-Wilson et al., 2020).
- Understanding of how to integrate technology into assessment and evaluation of high school student learning, including the use of online assessments, grading software, and learning management systems (Pepin, Choppin, et al., 2017; Yerushalmy et al., 2017).
- Familiarity with data analysis and statistical software, including the ability to use technology tools to track high school students' progress and make data-driven instructional decisions (van Dijke-Droogers et al., 2021).
- Understanding of online safety, security, and ethical considerations related to the use of technology in high school mathematics instruction (Caena & Redecker, 2019; Ferrari, 2012; Ghomi & Redecker, 2019).

This study used the definition provided in this section for high school mathematics teachers' digital competencies. Understanding these competencies can serve as a roadmap for integrating technology into mathematics teaching and preparing students for success in a digital world.

2.2.6. The complexity of digital competencies in the educational context.

In schools, digitalization (using digital technologies to improve processes, workflows, and overall functioning) has created a complex school environment. Teachers need to rethink and transform traditional educational practices by integrating digital technologies (Pettersson, 2018). Schools need to develop strategies to support the digital competencies required for high-quality teaching and learning. Various studies have attempted to elaborate on the digital competencies needed for teachers, with a focus on pedagogical aspects as specific characteristics of digital competence (Ghomi & Redecker, 2019; Heine et al., 2023; Redecker, 2017; Starkey & Yates, 2022). Pettersson cites several studies, including From (2017), Vanderlinde and van Braak (2010), and Wastiau et al. (2013) that support embedding the pedagogical aspects of digital competence, not only for teachers, but also in the wider school organization.

Furthermore, Pettersson (2018) argues that despite a considerable amount of research in the field of digital competencies, there has been confusion among researchers, policymakers, and practitioners about the meaning of digital competence in an educational context. In addition, the terms digital pedagogy, ICT competence, digital literacy and pedagogical digital competence seem seldom to be well-defined and are often used interchangeably when describing the competencies needed for teachers (Zhao et al., 2021). Another complexity is that the policies related to developing digital competencies are formulated at multiple levels of the educational system, rather than at the school level. This can create a gap between the level of policy defining the development of digital competencies and the level of teachers acquiring the digital competences (Pattersson, 2018). Patterson articulates the idea that digital competencies can be seen as a school-level characteristic and schools can become digitally competent by structuring and organizing resources and institutional infrastructures. Zhao et al. (2021) argue that the current research on these aspects have applied quantitative methods, and that future research combining both qualitative and quantitative techniques could explore this aspect further to inform school leaders about how policies related to digital competence can be cemented in educational contexts.

The next section of this literature review presents the historical overview of digital competencies for mathematics teachers.

2.3. Historical overview of digital competencies for mathematics teachers

In the context of this thesis, it is important to understand how digital competencies have evolved and advanced over time and the challenges teachers have faced during this development. This exploration is crucial for contextualizing the current landscape of digital competencies among high school mathematics teachers in Pakistan.

The historical overview of digital competencies for mathematics teachers can be traced back to the integration of technology in education and the increasing demand for teachers to have the skills and knowledge to integrate technology in their teaching practices.

2.3.1. 1980s and 1990s: The emergence of educational technology

Technology integration in education began in 1960s with the introduction of computers in the classrooms. The 1980s and 1990s witnessed the widespread adoption of personal computers, digital devices, and digital resources, which enabled teachers to experiment with new ways of teaching and learning. These developments were part of e-Learning or electronic learning initiatives, when researchers began exploring the use of computers in education. One of the earliest forms of educational technology used during the 1980s was computer-assisted instruction (CAI) software. CAI allowed students to interact with computers and receive feedback on their work (Fletcher-Flinn & Gravatt, 1995). However, largely due to lack of access and issues related to teachers' professional development, the efficacy of CAI became the subject of debate during the 1980s and early 90s. Researchers such as Clark (1985) consistently failed to find significant learning differences across varying delivery mechanisms, including those using technology. Clark (1985) argued that wherever computers were used to deliver instruction (including the teaching of programming languages), any resulting change in student learning or performance could be attributed to the uncontrolled effects of different teaching methods, content, and novelty. However, other later studies found that CAI may provide better quality of instruction over other traditional approaches of instruction, such as the use of paper and pencil (Fletcher-Flinn & Gravatt, 1995).

A significant next phase was the development of digital content. The use of multimedia technologies such as CD-ROMs and video cassettes enabled the production of interactive and stimulating educational materials. Initially, multimedia learning was perceived as learning from words and pictures (Dunning et al., 2007). However, later it involved words (such as printed text presented on screen or spoken text presented via speakers) and pictures in the form of animation, video, drawings, photos, and other graphics presented on screen (Mayer, 2003). During the 1990s, the educational technology industry went from CBT (Computer-Based Training) and rudimentary synchronous learning applications to sophisticated e-Learning platforms that combined both CBT and synchronous learning options (Dunning et al., 2007).

During the 1980s and 1990s, several multimedia tools were developed and used for mathematics education. The earliest and most widely used tools were the *Cabri Geometry* (1994) and *Geometer's Sketchpad* (1995), developed in the mid-1980s (Sinclair et al., 2010). They were among the earliest computer-based tools that allowed students and teachers to manipulate geometric objects on the screen using a mouse, rather than having to draw them by hand (Scher, 2000). On the one hand, these developments provided dynamic, interactive opportunities for learning mathematical concepts. On the other hand, these innovations revealed a gap in teachers' understanding of the resources (Sierpinska et al., 1999).

A range of other notable multimedia tools for mathematics teaching was developed during the 1980s. These tools included: *The Logo Programming Language and Turtle Graphics*, which allowed students to write computer programs to draw geometric shapes; *The Math Blaster Series of Educational Games*, which used animation and graphics to make learning math fun; *The Math Workshop Software*, which provided students with interactive tutorials and drills on a variety of mathematical concepts; and *The MathSoft Mathcad Software*, which allowed students to create and solve mathematical problems using symbolic notation and graphing capabilities.

However, learning through multimedia technologies received a dramatic boost with the development of computer networks and the internet for educational purposes. The development of networked computers allowed for the creation of virtual classrooms and online learning communities, which provided teachers and students with access to a wealth of educational multimedia resources and allowed them to collaborate with their peers from anywhere in the world (Nordin et al., 2005).

The Internet paved the way for future developments in technology integration in education. However, the progress of technology integration in education was uneven across different parts of the world. Developing countries faced various challenges in adopting and using educational technologies (Dundar et al., 2014). In the 1980s and 1990s, many developing countries lacked access to the necessary infrastructure and resources to integrate technology effectively into their education systems (Dundar et al., 2014; UNESCO Institute for Statistics., 2006).

Despite the benefits of educational technology, there were concerns about its impact on teaching and learning. During the 1980s and 1990s, teachers faced several challenges related to using technology. One of the biggest challenges (which remains) was the lack of training and support provided to teachers in using these technologies (Ertmer & Ottenbreit-Leftwich, 2010). Many teachers did not have the skills and knowledge to integrate CAI and other technologies into their teaching, and the available professional development opportunities were often inadequate (Fletcher-Flinn & Gravatt, 1995). Another issue was the high cost of technology and software, which limited access to these resources for many schools and teachers, particularly in developing countries (Dundar et al., 2014). Furthermore, there were concerns about the quality of CAI software and other educational technologies. Some software programs were criticized for being too simplistic or not aligned with curriculum, while others were seen as lacking the personal touch and interaction that traditional classroom teaching offered (Fletcher-Flinn & Gravatt, 1995). Some teachers were worried that CAI and other technologies would make their jobs obsolete (Dunning et al., 2007) or that students would become too reliant on technology and lose essential social and interpersonal skills (Leidner & Jarvenpaa, 1995).

2.3.2. 2000s: Digital revolution and the emphasis on teachers' professional development

Throughout the 2000s, more and more studies (e.g., Bakia et al., 2007; Gaffney, 2010; Jasute & Dagiene, 2012; Means et al., 2009; Sinclair et al., 2010) highlighted the need for teachers to have digital competencies due to the expansion of the integration of technology in education. Studies primarily highlighted that teachers lacked the skills and knowledge necessary to use technology in their teaching and learning. This led to the development of various teacher training programs (e.g., Enhancing Education through technology - EETT) and initiatives designed to provide teachers with training and support. However, these initiatives were mainly established in developed countries in North America and Europe. For example, the International Society for Technology in Education (ISTE) developed the National Educational Technology Standards for Teachers (NETS-T) to develop teachers' digital competencies. The standards emphasized that teachers must use their understanding of the subject matter, teaching and learning, and technology to create opportunities that will stimulate student learning, creativity, and innovation in both physical and virtual learning environments (Banister & Vannatta Reinhart, 2012).

In relation to mathematics education, during the 2000s several initiatives were aimed at supporting mathematics teachers to integrate technology in teaching and learning. The most popular initiative was The National Council of Teachers of Mathematics (NCTM) Standards. In 2000, NCTM revised its mathematics standards to include a greater emphasis on the use of technology (Sinclair et al., 2010). NCTM stated that technology was an essential tool for mathematics education and called for teachers to integrate technology into their teaching t(Lee & Hollebrands, 2008)rning (Lee & Hollebrands, 2008). Later, further progress was made via Math Forum: an online community for mathematics teachers, (originally established in 1992) that gained significant popularity and influence during the 2000s. This forum provided teachers with access to a wide range of resources and tools, including software, lesson plans, and professional development opportunities (Renninger & Shumar, 2002). In the U.S., the National Science Foundation (NSF) also provided funding for several initiatives aimed at improving mathematics education, including the Math and Science Partnership Program. This program provided funding via partnerships between school districts and institutions of higher education to improve the quality of mathematics and science education.

Similar initiatives included: Singapore's *Masterplans for ICT in Education* (1997); *e-Japan Strategy* (2001); The European Union (EU) *eLearning Action Plan* (2001), US program *The Partnership for 21st Century Skills* (2002); Australian *Mathematics Online Interview* (MOI) project (2003) and the US *The National Math and Science*

Initiative (NMSI) (2007). All of these initiatives promoted the use of technology in teaching and learning, with a specific focus on mathematics education. The programs aspired to nurture 21st-century skills among students, emphasizing critical thinking and problem-solving. Collectively, these initiatives worked towards elevating the quality of mathematics and science education on a global scale through professional development, technology resources, and collaborative efforts.

Regarding pedagogy, the 2000s saw several initiatives and research contributions that highlighted ways to integrate technology into mathematics education. One notable framework is the Technological Pedagogical and Content Knowledge (TPACK) framework, introduced by (Mishra & Koehler, 2006). TPACK, a theoretical framework, highlights the interplay between technology, pedagogy, and content knowledge in the context of teaching and learning (Crompton, 2015). Since its inception, researchers have extensively employed TPACK in teacher education and professional development programmes. Other notable framework includes the SAMR model (Puentedura, 2006), which describes four levels of technology integration (substitution, augmentation, modification, and redefinition). The SAMR model was designed to help teachers to select, use, and evaluate technology in K-12 education (Hamilton et al., 2016). The European Commission also started a program in the 2000s called the Digital Literacy Project (DigEuLit), which was part of the eLearning Initiatives (discussed in Section 2.2.4). Specifically targeting mathematics education, Ball et al. (2008) developed a framework called The Mathematical Knowledge for Teaching (MKT), which focused on the mathematical knowledge needed for teaching. The MKT framework used and modified the seminal work of (Shulman, 1986)'s pedagogical content knowledge (PCK) in the context of mathematics teaching. Later, MKT was further modified and used by (Thomas & Palmer, 2014) and Tabach and Trgalova (2020) to explain the process of technology integration in the teaching and learning of mathematics.

2.3.3. 2010s: Mobile Learning and personalization

With the rise of mobile devices and the increasing use of digital technology in society, the need for teachers to have digital competencies became even more critical. This resulted in a growing body of research (Calder, 2015; Sawaya & Putnam, 2015; Thomas & Palmer, 2014) on digital competencies for teachers and the development of various frameworks and models. During the 2010s important development trends were mobile technologies, massive open online courses (MOOCs), digital libraries and designing learning objects, collaborative learning using digital technology, and blended and flip learning environments (Borba et al., 2016). Other initiatives similar to MOOCs, such as open educational resources (OERs) or openly licensed educational resources, also gained popularity to reduce costs and increase access to educational materials. OERs provided teachers new opportunities for the design and use of mathematics teaching resources (Trouche et al., 2018).

In the early 2010s, digital technologies started to offer more personalized experiences to meet the individual needs and preferences of teachers and learners. This gave rise to a range of new resources for teachers and students to create customized learning experiences. For example, online teaching and learning platforms were equipped with tools to monitor learners interaction with content and determine (at least in part) the nature of materials delivered subsequently (Kerr, 2016). Such platforms are also called adaptive learning platforms, i.e., platforms for online learning that use data analytics and artificial intelligence to personalize the learning experience (Kem, 2022). Although adaptive learning technology empowered teachers' creativity and ability to customize learning experiences for every student, anywhere, and at any time, the technology required that teachers have skills to use them (Kem, 2022). Nonetheless, the development of adaptive learning technology remains ongoing, and there is still no systematic understanding of adaptive learning practices and the factors contributing to successful implementation (Kem, 2022; Muñoz et al., 2022). Other solutions included Learning Management Systems (LMS), which were adopted by many educational institutions to provide personalized teaching and learning experiences. These platforms were equipped with tools that allow teachers to create custom-designed curricula tailored to each student's needs and to track their progress through the course (Muñoz et al., 2022).

During the 2010s, courtesy of mobile technologies, the creation of digital content (e.g., videos, graphics, and animations) began to be considered a part of every teacher's digital skillset. The easy access to handheld mobile (smart) devices equipped with cameras, digital applications and internet connectivity allowed users to create, upload, access, transform and share digital content worldwide (van Dijk & van Deursen, 2014). Teachers also faced the challenges of content creation and innovative curriculum design to teach using OERs (Rocha, 2018). Teachers needed to understand the distinction between the traditional classroom and an online classroom, which required a different set of skills. These developments required teachers to acquire knowledge of online discussions, understand feedback mechanisms, and develop the competence to create a sense of community in a virtual environment (Alabdulaziz, 2021; Syahrin et al., 2023). These abilities required further skills; for example, to facilitate online discussion, teachers need to know how to use discussion forums, online chat rooms, and video conferencing tools (Gandolfi & Kratcoski, 2020). Teachers also needed to know how to provide feedback using different media such as video, audio, and text (Lemay et al., 2021).

The rise of smartphones also generated new forms of communication through online resources sharing platforms (for example, Dropbox, web forums etc.), and social media platforms (such as Twitter, Facebook, YouTube, etc.). The responsive and collaborative environment offered by digital technologies gave Mathematics teachers an opportunity and spaces to create collective work. All these possibilities had an enormous impact on teachers' digital competencies in using, designing, and sharing educational resources (Rocha, 2018). At the same time, mobile technologies also faced widespread criticism as pedagogical tools in secondary and higher education (Herrington et al., 2009). There were widespread concerns about using the devices with students, based mainly on teachers' lack of understanding of the affordances offered by mobile technologies (Borba et al., 2016; Lefoe et al., 2009). Researchers such as Clark-Wilson et al. (2020) and Ruthven (2014) indicated that mobile technology played only a marginal role in many mathematics classrooms.

During the 2010s, digital technologies allowed mathematics teachers (at least in developed countries) to access (via the internet) a profusion of open and free open educational resources (OER). Due to variation in the quality and coherence of OER and teachers' difficulties in integrating them, several theoretical perspectives and frameworks emerged to investigate the interaction of teachers with these resources. For example, Pepin, Gueudet, et al. (2017) highlighted three theoretical perspectives on teacher interaction with digital resources as: (1) design-based approaches and didactical engineering proposed by Artigue (2011) and Gravemeijer and Cobb (2006); (2) the interpretation of and participation with resources e.g., Brown (2002) and Remillard (2005); and (3) the Documentational Approach (Gueudet et al. 2012;

Gueudet and Trouche 2009). Among them the Documentational Approach also known as DAD (the documentational approach to didactics) gained popularity and was used by several researchers (e.g., Gueudet et al., 2016; Rocha, 2018b; Trouche et al., 2018) especially in Europe.

DAD used an instrumental approach with the aim of understanding teachers' professional development by studying their interactions with the resources they used and designed in and for their teaching (Trouche et al., 2020). During this time, more studies on mathematics teachers' interaction with digital resources, used the concept of instrumental genesis. Furthermore, Haspekian (2011) modified the concept of instrumental genesis and introduced the concept of double instrumental genesis, which refers to the idea that both the construction and use of mathematical tools can influence the development of mathematical concepts and understanding.

Additional frameworks focused on how to integrate technology in mathematics teaching were designed during the 2010s. Thomas and Palmer (2014) introduced the framework called teachers' pedagogical technological knowledge (PTK). In 2019, Tabach and Trgalová (2019) built on the research done by Thomas et al, (2008) and Thomas and Palmer (2014) in designing the PTK framework. Their aim was to gain a deeper understanding of the knowledge and skills required by mathematics teachers when using digital resources in the classroom. Tabach and Trgalová (2020) examined different international standards describing mathematics teachers' digital competencies by using the PTK framework. They modified the PTK framework in the context of digital technology and introduced the DCTMT framework. However, the DCTMT framework lacks empirical evidence and needs testing within different contexts such as developing countries. Sections 2.4.6 and 2.4.9 of this literature review provide further details on these frameworks.

2.3.4. 2020s: Post-pandemic era and the age of Artificial Intelligence (AI)

The COVID-19 pandemic accelerated the adoption of technology in education, further emphasizing the need for teachers to have digital competencies to teach in a digital environment. During the COVID-19 pandemic many teachers became more interested in digital resources (Scully et al., 2021). Their beliefs and attitudes supported the transition to digital teaching and learning. and substantially increased the use of digital resources (Alberola-Mulet et al., 2021). However, the transition was not easy. For developing countries, such as Pakistan, the challenges were enormous (Jafri, 2022). The rapid transition was dependent on the teachers' access to digital resources, their digital competencies, and internet access for both teachers and students. Additionally, transition required changes in curriculum, content, teaching methods, and teaching skills (Clark-Wilson et al., 2020). Importantly, deeper changes were also required i.e., changes in teachers' beliefs towards using digital technologies. Both teachers and students responded quickly to the need for creative and critical engagement by opting for eLearning tools (Ferdig et al., 2020).

Mathematics teaching and learning responded and developed an efficient and effective design to integrate online teaching and learning environments during the time of crises. The unprecedented urgent demand reinforced pre-existing challenges related to the use of technology in mathematics education. These challenges include diverse theoretical perspectives, changing interactions from learner to teacher and teacher to learners, dynamic interrelations between technology and mathematics (S. Hegedus et al., 2016), and the digital competencies of teachers (Tabach & Trgalová, 2020). The primary concern was to find ways for, digital resources to produce knowledge of mathematics that was appropriate for particular contexts and tasks. The mathematics community including teachers, researchers and schools, responded quickly to this challenge, and proposed ideas for integrating a variety of online teaching and learning solutions into mathematics education. During COVID-19, teachers demonstrated digital competencies by (re)-discovering the educational uses of video conferencing, screencasting, breakout rooms, screen sharing, online meetings, and virtual collaboration via Zoom, Flipgrid, Microsoft Team, Google platforms, and many more (Calder et al., 2021). Teachers who employed these digital resources reported that they discovered innovative possibilities for the teaching and learning of mathematics. For example, (Morge, 2020) discovered that screencast and screen sharing could improve inquiry and problem-solving skills; breakout rooms could improve collaborative problem-solving skills (Carey et al., 2020; J. Green & Johnson-Whitt, 2020), and Google Slides could promote creativity and produce thoughtful learning outcomes for students (Nagle, 2020).

Furthermore, accessible technologies, especially Twitter, quickly enabled teachers around the world to come together for a common cause and build Communities of Practice (CoP) (Gandolfi & Kratcoski, 2020). Teachers who were familiar with these environments made use of their affordances such as through the use of targeted hashtags (e.g., #RemoteLearning and #RemoteTeaching) to facilitate seamless community conversations. Teachers shared resources, discussed challenges, and found solutions about online teaching and learning (D. Maher, 2020). (Ervin-Kassab, 2020) noted that Learning Management System (LMS) allowed co-planning and co-teaching activities. The LMS provided a space to hold a community of practice for the guidance and supervision of teachers.

The impact of the pandemic on teachers' use of digital resources was significant. Syahrin et al. (2023) used the digital competence framework to measure the set of digital competencies of teachers after the pandemic. The result of the study shows that most pre-service teachers scored Level 3 (Intermediate) in their self-assessment competency test score. The participants' digital competencies improved significantly as the result of online learning accelerated by the COVID-19 pandemic. The participants scored highest for digital content creation. Syahrin et al. (2023) consider that early career teachers are active and frequent users of Web 2.0 which allows users to generate content for other end users. Similarly, Alabdulaziz (2021) investigated the use of digital resources for mathematics education during the COVID-19 pandemic and found that 98% of participants believed that COVID-19 was a gateway for digital learning in mathematics education. The use of online education by schools also expanded greatly following the pandemic, resulting in various forms of software being used to facilitate communication between teachers and students mainly through mobile technologies. Alabdulaziz (2021) also noted the increased uses of digital libraries, designing of learning objects, use of MOOCs in mathematics, and computer algebra systems (CAS) such as Mathematical, Maple, MuPAD, MathCAD, Derive and Maxima.

Furthermore, teaching during the lockdown provided space for teachers to co-design activities with students (Coleman & MacDonald, 2020). Teachers who were familiar with resources like Flipgrid created "grids" to facilitate video discussions. Each grid works like a message board where teachers start the discussion with a posting prompt, and their students posted video responses that appeared in a tiled grid display (Goddard, 2020). Coleman and MacDonald (2020) suggest interactions like these increase teachers' and students' capacity to (re)-invent media, an important way to develop digital skills of instructional design, and augment digital competencies. (Hulon et al., 2020) observed that in-service teachers possess a wide range of technological knowledge and skills experienced fewer challenges during the lockdown. However, Hulon et al. (2020) found that many teachers who participated in the study "were not equipped with instructional design skills related to virtual learning" (p. 44). (Lindsay & Whalley, 2020) reported similar findings that during the pandemic, teachers with digital skills were more ready and confident to teach and learn online than those who did not have those skills. These teachers faced fewer issues in relation to using camera and microphone, integration of online resources, communicating via chat rooms, designing online instruction, and assessing the performance of students during the COVID-19 lockdown.

In this historical overview, I have reviewed how the required digital competencies for mathematics teachers have evolved during the last four decades. The reviewed literature emphasized the acquisition of skills that enable mathematics teachers to integrate digital resources in their teaching and to develop skills in pedagogy, collaboration, and teaching innovation, using digital resources. The historical overview also highlights that the continual emergence of new technologies required teachers to keep developing and extending their digital competencies. Teachers need to be adept in using digital resources, navigating the internet, and to understand emerging trends and technologies (Tabach & Trgalová, 2020). With the emergence of artificial intelligence (AI) a basic understanding of coding and programming languages, as well as the ability to solve technical problems, are becoming part of necessary digital skills. Teachers should be able to create, store, retrieve and share digital content (Ministry of Education NZ, 2017). They need to be familiar with cloud computing and its associated features and benefits. Mathematics increasingly required to use digital technology to access and analyse data, create models, identify trends, and make informed decisions. These competencies require understanding of data science principles, machine learning, and artificial intelligence (AI). Correspondingly, it is imperative to continually monitor mathematics teachers' development of digital competencies so that they be supported in maintaining their digital skills to remain functional in the digital environment.

The next section of this literature review will analyse major theoretical frameworks identified during the historical overview of digital competencies of mathematics teachers.

2.4. Theoretical frameworks of digital competencies for mathematics teachers

Theoretical frameworks are important research tools for structuring inquiry and advancing theory development across many fields. Frameworks provide a foundation of concepts, assumptions, values and practices to guide research (Partelow, 2023). The following sections discuss various relevant frameworks in mathematics education developed to understand teachers' capabilities and competencies. The section starts with the details of how researchers' have framed the aspects of developing technological capabilities of mathematics teachers in general by using TPACK and then specifically for mathematics by using MKT (Mathematical Knowledge for Teaching) and PTK (Pedagogical Technological Knowledge).

2.4.1. Teachers' technological capability

Technological capabilities have been described in a variety of ways. For example, the Irish National Council for Curriculum and Assessment (NCCA) framework emphasizes the application of foundational knowledge and skills through creative thinking and action (Gleeson, 2022). This framework incorporates competencies such as communication, design, problem-solving, and the ability to critically evaluate technological activities (Doyle et al., 2019). Teachers with technological capabilities, according to the NCCA, can apply competencies to solve practical challenges and participate in value-added decisions such as how to enhance collaboration, accessibility and personalize teaching, and learning experiences (Doyle et al., 2019; Gleeson, 2022). This implies that teachers' ability to apply knowledge and skills (competence) in various situations (capability) is crucial for solving real-world problems and making informed decisions.

Gaffney (2010) defined technological capabilities as a teacher's potential and facility in using digital technologies, involving the transfer of knowledge, skills, and values through technology. Technologically capable teachers can develop, analyse, and adapt digital resources for effective integration into teaching and learning. Gaffney (2010) highlighted the importance of intrinsic dimensions, such as teachers' beliefs and confidence, and extrinsic dimensions, including factors like availability, access, and institutional policies for developing teacher capabilities.

Both the NCCA and Gaffney (2010) stressed foundational knowledge and skills. However, the term 'digital competencies,' as discussed in Section 2.2, encompasses a specific array of skills and knowledge than the term capabilities. The term competencies also encompass beliefs and attitudes towards the use of technology. In the context of this study, focused on understanding mathematics teachers' knowledge and skills in using digital resources, the term digital competencies is deemed more fitting than capabilities. The upcoming section will delve into approaches for developing teachers' digital competencies.

2.4.2. Developing Teachers' Digital Competencies

Professional development has always played an important role in developing teachers' competencies. Several studies (e.g., Anabousy & Tabach, 2022; Beswick & Fraser, 2019; Moreno et al., 2020; Naidoo & Singh-Pillay, 2020; Reisoğlu, 2022; Schoenfeld, 2011) highlight the role of training programs and professional development for enhancing teachers' digital competencies and their ability to use technology for teaching and learning mathematics. The rationale for selecting studies was to focus on research that addresses teachers' professional development through technology integration mainly within the high school context. Several studies have investigated the impact of particular training initiatives on teacher's digital competencies. For example, Reisoğlu (2022) conducted a study in which teachers received collaborative and applied digital competencies training for creating interactive eBooks. Their findings revealed that the professional training enhanced teachers' knowledge and skills in various areas, such as professional engagement, using digital resources, organizing teaching activities, improving assessments, and empowering learners. Reisoğlu found that training allowed teachers to enhance organizational communication and cooperation using digital technologies while developing various digital content formats compatible with student requirements and curriculum objectives. Moreno et al. (2020) focused on the impact of perceived Teachers' Digital Competence (TDC) on the preparation of educational videos for the flipped classroom model. The study found that while teachers had an intermediate level of TDC and prepared satisfactory videos, there were deficiencies in the pedagogical and mathematics teaching components. The integration of technological, pedagogical, and mathematics teaching components was emphasized as crucial for developing high-quality teaching videos. They suggested professional development and teachers' training programs can assist teachers to integrate technology into the pedagogical processes of mathematics teaching.

The role of professional development in developing teachers' digital competencies has also been examined in other studies of mathematics teaching. (Naidoo & Singh-Pillay, 2020) explored the role of mathematics teachers' professional development within the context of the Fourth Industrial Revolution. Schwab (2016) defined the Fourth Industrial Revolution as a paradigm shift characterized by the fusion of advanced technologies, leading to transformative changes in how we live, work, and interact. Naidoo and Singh-Pillay (2020) emphasized the need for teachers to keep abreast of new educational trends and pedagogies to equip learners with 21st-century skills. Their findings provided insights into the experiences and needs of mathematics teachers in terms of blended teaching and learning, data handling, coding, and analytics in preparation for the Fourth Industrial Revolution. Similarly, Beswick and Fraser (2019) addressed the challenge of developing 21st-century learning skills in students through integrated STEM approaches. They emphasized the importance of teachers having expertise in STEM disciplines and possessing their own 21st-century competencies. Beswick and Fraser proposed a framework that enables novice teachers to consider what they need to know, find out, or think about as they plan, enact, and reflect on teaching, highlighting the complexity of knowledge required and the significance of teachers' own 21st-century skills.

Schoenfeld (2011) argued that among the most critical mathematics teachers' resources are personal and interpersonal skills and that these also develop through training and experience. Schoenfeld emphasized the importance of understanding the factors that shape mathematics teachers' behaviour, including their knowledge, resources, goals, and orientations towards technology. He argued that teachers' beliefs, values, and preferences interact with these factors to influence their decisions related to the use of technology. However, the interconnected nature of these elements suggests that their evolution is gradual and stimulated through professional training. Schoenfeld proposes using this understanding as a foundation for mathematics teachers' professional development, outlining how these factors can be addressed in a participatory professional development course for middle and high school mathematics teachers.

Communities of inquiry is another approach to professional development. Anabousy & Tabach (2022) investigated the impact of a Community of Inquiry (CoI) professional development program on teachers' beliefs about the value of technology. They found no significant difference in the rate of change of beliefs between senior and less senior teachers, indicating that all participants experienced change in their beliefs about technology's value. They suggested that teachers' practices with technology contributed to meaningful learning outcomes, which, in turn, enhanced their beliefs about technology. Teachers with a low level of previous technology integration demonstrated significant growth in their confidence in using technology and pedagogical knowledge related to technology integration. Anabousy and Tabach (2022) suggested that adhering to the inquiry cycle and participating in CoI can provide support for teachers, particularly those with less pedagogical and technological knowledge.

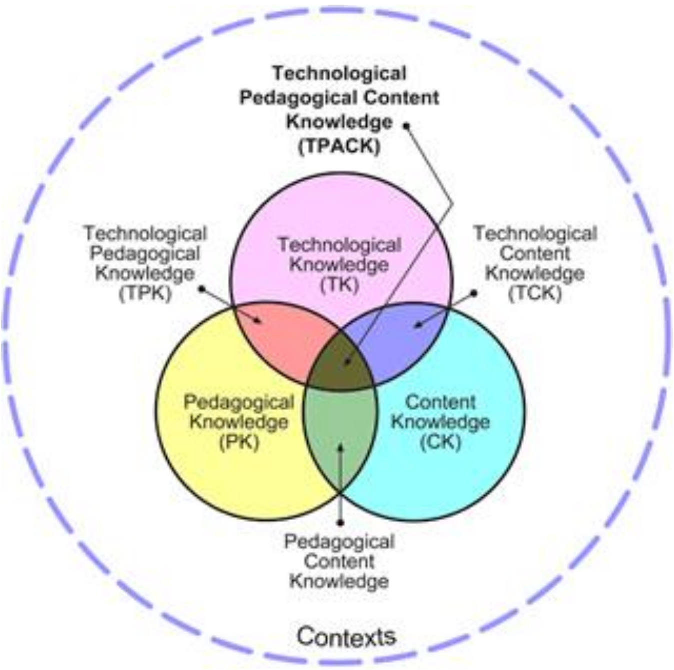
These studies highlight the role of training in developing teachers' digital competencies. However, in the context of high school teaching and learning the studies do not adequately explore the role of infrastructure, access to technology, and digital resources in shaping teachers' digital competencies. These factors are particularly critical in resource-constrained environments where limited technological infrastructure and the availability of digital resources can restrict the development of teachers' digital skills. Further there is a need to identify how training programs can be designed to address challenges such as limited infrastructure, restricted technology access, and a shortage of digital resources?

The following sections review the suitability of various frameworks proposed for the enhancement of teachers' digital skills. The initial framework under consideration is TPACK (Technological Pedagogical and Content Knowledge). This review aims to determine its efficacy as a model for both professional development and the evaluation of digital competencies among mathematics teachers.

2.4.3. Efficacy of TPACK as a framework for professional development of mathematics teachers

TPACK is a framework that built on and evolved from Shulman's 1986 concept of Pedagogical Content Knowledge (PCK), Informed by PCK, (Mishra & Koehler, 2006) propose a framework for developing teachers' capability and technological skills, and introduced an additional body of knowledge called technological knowledge (TK). They explained that there are three overlapping types of knowledge: content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK), as depicted in Figure 3. These knowledge types overlap, with the intersection of all three representing a special kind of teacher knowledge called "Technological Pedagogical Content Knowledge" (TPCK or TPACK). The TPACK model has other multiple intersections, such as the intersection of pedagogy with technology (TPK), content with technology (TCK), and pedagogy with content (PCK). According to Mishra and Koehler (2019), the outer dotted circle represents "context," which they emphasize is a crucial consideration in the development of teacher capability.

Figure 3
TPACK model



Note. Reproduced by permission of the publisher, © 2012 by http://tpack.org/.

Crompton (2015) suggested the TPACK framework as a model to guide teacher education programs in integrating mathematical content knowledge, pedagogical knowledge, and technological knowledge. Crompton found that pre-service teachers (PSTs) show an increase in their TPACK as they progress through teacher education programs that incorporate the TPACK framework by integrating technology skills with mathematical content and pedagogy. However, Crompton also mentioned two limitations of the model. Firstly, there is a need for more studies exploring the impact of pedagogical knowledge on linking content and technological knowledge. Secondly, beliefs about technology integration change as PSTs develop a better understanding of technology's affordances in mathematics education. Further research is, therefore, needed to establish the connection between TPACK development and teachers' beliefs.

The TPACK model may need adaption and modification to suit the requirements of particular contexts. This may be the case for mathematics education in Pakistan, the focus of this current study. For example, teachers' access to technology and resources in Pakistan may be limited (Aslam et al., 2022; Raza, 2016), which may affect the development of teachers' technological knowledge (TK) and the implementation of technology in the classroom (Koehler et al., 2013). The pedagogical approaches and content knowledge (PK and CK) may also differ according to the local culture and context (Mishra, 2019). Furthermore, there may be differences in the expectations and requirements for teachers in Pakistan, such as larger class sizes or a focus on rote learning (Amirali & Halai, 2010; Aslam et al., 2022). Considerations like these may impact how teachers use technology in their teaching and the emphasis placed on developing technological pedagogical content knowledge (TPACK) (Crompton, 2015). According to (Koehler et al., 2013), implementation of TPACK may involve providing additional training and support for teachers, identifying, and addressing barriers to the implementation of technology in the classroom, and recognizing the importance of cultural and contextual factors in shaping teacher knowledge and practice.

Moreover, the use of TPACK for empirical studies in mathematics education has been questioned. For example, Graham (2011) criticized the TPACK framework for relying on broad and undefined constructs, such as pedagogical content knowledge (PCK) and technological knowledge (TK). Graham argued that the TPACK framework lacked theoretical clarity, and that more in-depth attention was required from researchers to clarify the balance between the parsimony and complexity of its constructs and to provide precise definitions for each of its constructs. Crompton (2015) argues that a limited number of studies address the connection between technology, mathematics, TPACK, and teachers and that more evidence is required to determine the approaches that create TPACK development in teacher education programs and particularly how this development influences pre-service teachers' beliefs about technology integration.

Another important criticism that TPACK has received relates to its generalizability, as the framework is not mathematics-specific (Clark-Wilson et al., 2020; Koehler et al., 2007; Tabach & Trgalová, 2019). Mathematics has its own nuances of content knowledge, as exemplified by Ball et al. (2008) and Hill et al. (2008) in their frameworks of mathematical knowledge for teaching. This suggests the need for frameworks that are developed with mathematics teaching and learning in mind, such as Pedagogical Technological Knowledge (PTK) and Double Instrumental Genesis (Tabach & Trgalová, 2020; Thomas & Hong, 2013; Thomas & Palmer, 2014). These frameworks offer more specific and nuanced understandings of the intersection between pedagogy, content, and technology in mathematics education, which may better serve the needs of mathematics teachers and learners.

Several studies have highlighted that teachers with weak pedagogical and content knowledge in mathematics may struggle to make an appropriate link between pedagogy, content, and technology, even if they have a high degree of technological knowledge (Crompton, 2015; Niess, 2015). This may constrain their ability to identify "solid mathematical and didactical knowledge presented in a digital resource" (Pepin, Choppin, et al., 2017, p. 654). For example, when teachers use a digital game to teach numbers and coordinate geometry, they need more than just familiarity with the digital game. They require a foundation in both the mathematical content (coordinate geometry) covered by the digital resource (digital game) and the pedagogical approaches that enhance its effectiveness in the classroom. This emphasizes the importance of well-rounded expertise that combines mathematical proficiency with teaching strategies when integrating digital resources into the educational process (Pepin, Choppin, et al., 2017). It is therefore, important to understand how teachers' knowledge of teaching mathematics is defined and how this knowledge can facilitate the selection of digital resources and the development of digital competencies among teachers.

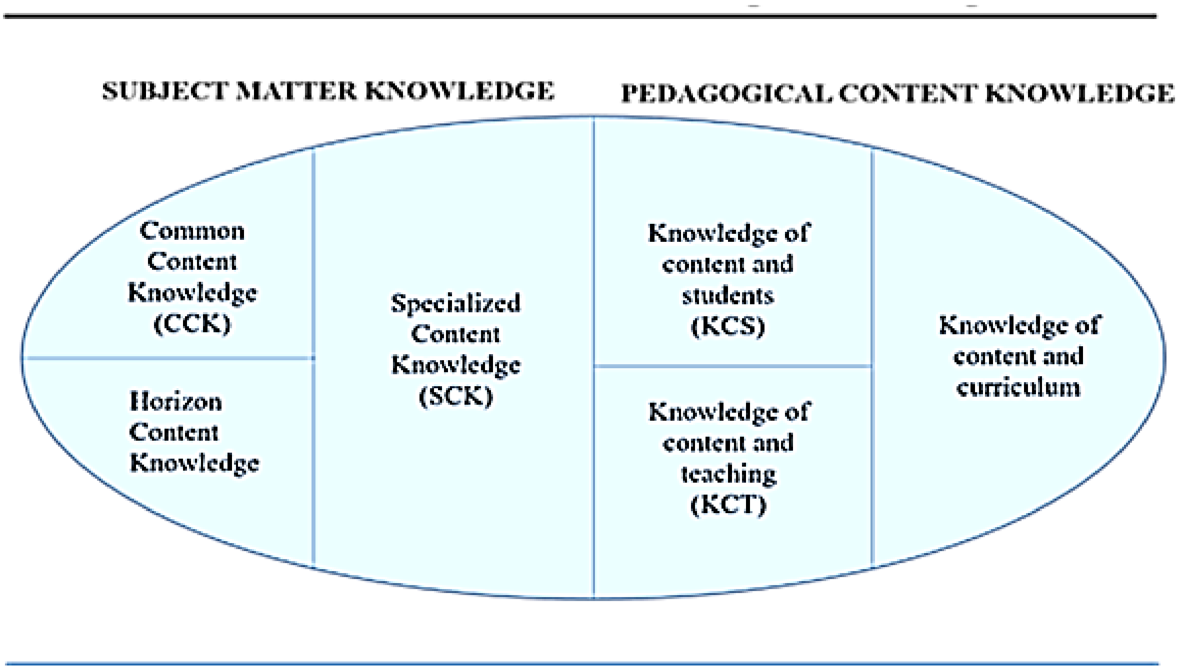
2.4.4. Mathematical knowledge for teaching (MKT)

Ball et al.'s (2008) Mathematics Knowledge for Teaching (MKT) framework is crucial for understanding the digital competencies of mathematics teachers. The MKT delineates the essential mathematical knowledge required for teaching, which is foundational for integrating digital tools in mathematics education. Ball at el. (2008) emphasized the importance of teachers having MKT, which is defined as "the mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). Their definition of MKT starts with teaching rather than teachers, which encompasses everything teachers need to do to support their students' learning. To develop this framework, Ball et al. (2008) analysed the tasks and problems that teachers face when teaching mathematics and identified three content areas of elementary mathematics. They used a matrix of these content areas and knowledge dimensions to develop a general test, measuring the mathematical knowledge for teaching (MKT) of elementary schoolteachers, based on item response theory.

Ball et al. (2008) used the concept of Pedagogical Content Knowledge (PCK) developed by (Shulman, 1986). They argued that PCK is the knowledge that teachers have about how to teach specific content knowledge, while MKT is the knowledge that teachers have about the mathematical content they are teaching and how to teach it. They proposed that MKT includes both subject matter knowledge (similar to content knowledge in PCK) and pedagogical content knowledge. They expanded on PCK by identifying four domains of mathematical knowledge needed for teaching: common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT) (as shown in Figure 4, p. 60). This shows a relationship between PCK and MKT in which PCK is a component of MKT. While PCK focuses on the intersection of content and pedagogy, MKT encompasses a broader range of mathematical knowledge that is necessary for teaching. Although PCK is essential for teaching, it is not sufficient on its own. Teachers also need a deep understanding of the mathematical content they are teaching and how it relates to their students' learning (Ball et al., 2008).

Figure 4

Domains of Mathematical Knowledge for Teaching



From "Content knowledge for teaching: What makes it special?" by Ball, D. L., Thames, M. H., & Phelps, G., 2008, *Journal of teacher education, 59(5), p. 403*

Figure 4 shows the correspondence between Ball et al's. map of the domain of content knowledge for teaching and two of Shulman's (1986) initial categories: subject matter knowledge and pedagogical content knowledge. Ball et al. define the terms as follows:

- CCK: The mathematical knowledge and skill used in settings other than teaching. Teachers need to know the material they teach; must be able to recognize expected wrong answers by students and inaccuracies in the textbook.
- HCK: Horizon Content Knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum.
- SCK: The mathematical knowledge and skills unique to teaching. The teacher must be able to identify patterns in students' errors or size up whether a

nonstandard approach would work in general. The teacher should demonstrate a unique mathematical understanding and reasoning.

- KCS: The knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing.
- KCT: This knowledge combines the knowledge about teaching and knowing about mathematics.

Ball et al. (2008) defined each construct (CCK, SCK, KCC, KCT) specifically in the mathematics context (as shown in Figure 4, p. 60). They proposed that the mathematical tasks of teaching constitute and revolve around teachers' knowledge and skills in finding, presenting, representing, linking, and selecting mathematical ideas for teaching. They emphasised that teachers' MKT is culturally specific or dependent on teaching styles. Ball et al. argue that cultural context is an important consideration when developing mathematical knowledge for teaching. Teachers need to adapt their teaching strategies to the context in which they are working. However, regardless of the cultural context, the explanation of mathematical ideas so that they make sense to students is central to teaching (Ball et al., 2008). This means that teachers must possess a deep understanding of the mathematical concepts they are teaching, as well as an understanding of how to present and represent those concepts in ways that are meaningful to their students (Ferrare & Hora, 2014).

Although the MKT framework provides a broad understanding of the knowledge and skills that teachers need to teach mathematics, it has received some criticism. Schoenfeld (2011) criticised MKT as it fails to consider the importance of teachers' beliefs. The importance of including beliefs in studies of teachers' knowledge has been emphasized, and some even argue for the equivalence of beliefs and knowledge (Beswick, 2012). According to Schoenfeld (2011), beliefs are part of teacher orientation and goals, they are part of affective aspects and inform "how and why teachers make the choices they make, as they teach" (Schoenfeld, 2011, p. 458). Schoenfeld (2011) emphasised the need to include teachers' beliefs to increase the validity of studies on teachers' knowledge of teaching mathematics, particularly when technology is involved. (Potari, 2012) argued that the MKT framework may be too complex and difficult to implement in practice, particularly in contexts where teacher education and professional development are limited.

The above criticism of MKT suggests that teachers' beliefs are important contributors to mathematics teachers' knowledge for teaching. Therefore, before discussing other frameworks, the section 2.4.5 will discuss mathematics teachers' beliefs particularly in the context of Pakistan and in relation to digital technology.

2.4.5. Importance of mathematics teachers' beliefs

Understanding teachers' beliefs has been a focus of attention in educational psychology (Ernest, 1989; Pajares, 1992; Schoenfeld, 2011). To understand why teachers do what they do and how they behave in response to different pedagogical situations, there is a need to understand teachers' systems of beliefs (Schoenfeld, 2011). Teachers' beliefs are influenced by their personal life experiences, socio-cultural and religious contexts. They inform teachers' understanding, judgments, evaluations, and justification of their teaching practices (Pajares, 1992).

Different studies have set out to identify the characteristics of mathematics teachers' beliefs. Drawing on the differing characteristics of teachers' beliefs articulated by Ernest (1989) and Pajares (1992), mathematics teachers' beliefs may be referred to as a psychological construct that encompasses the opinions, dispositions, (pre)conceptions, and philosophies held by mathematics teachers about the nature of mathematics, its teaching, and learning. These beliefs serve as cognitive frameworks that teachers use to make sense of and navigate their teaching practices, influencing how they approach teaching mathematics and interact with their students. For example, a teacher could develop a self-confirming bias about a particular teaching practice or give preference to a particular mathematics textbook because this is how they learned as a student. Such beliefs may inhibit their desire to changes teaching practices (Pajares, 1992) and are normally unaffected by new information (Karatas, 2014). Subsequently, these beliefs become a widely shared belief system(s) among most of the teachers, serving as the foundation for their thought processes, behaviour, and interactions with each other and their students (Pajares, 1992). However, according to Usó-Doménech and Nescolarde-Selva (2016) the term "belief system" can be highly confusing, as psychologists, political scientists and anthropologists use the term in different senses. Usó-Doménech and Nescolarde-Selva (2016) explained belief systems as a complex network of interrelated concepts, norms, and propositions that guide an individual's or a group's understanding of the world. These systems are dynamic and evolve with experience, reason, and context. They are not merely a collection of random beliefs but are organized in a manner that allows for coherence and functionality within a social or individual framework. Whereas Green (1971) described belief systems in a broad context as a complex, interconnected network of beliefs held by an individual, characterized by three key dimensions:

- Structure: Beliefs are organized in a hierarchy, with primary beliefs serving as foundations for derivative beliefs. Primary beliefs are those for which no further justification can be given, while derivative beliefs stem from these primary beliefs.
- Intensity: Beliefs vary in strength or centrality. More central beliefs are strongly held, highly connected to other beliefs, and resistant to change. Peripheral beliefs are less strongly held and more susceptible to modification.
- Clustering: Beliefs are often held in isolated clusters, which allows for the coexistence of potentially conflicting beliefs without apparent contradiction. These clusters can develop independently based on different contexts or experiences.

Green (1971) argues that belief systems can also include evidentially and non-evidentially held beliefs, with the latter being more resistant to change even in the face of contradictory evidence. Belief systems can further encompass working assumptions, conjectures, and hypotheses, which are tentatively held propositions that may eventually become more established beliefs or be discarded. Usó-Doménech and Nescolarde-Selva (2016) articulated the importance of understanding belief systems as they explain why individuals can hold seemingly contradictory beliefs, why some beliefs are more resistant to change than others, and how new ideas can be integrated into or rejected from an existing belief system.

However, not all beliefs in mathematics teachers' belief systems are equally important for teaching and learning mathematics. Only the central beliefs are important because they have more connections with other beliefs in a belief system (Rokeach, 1968). Central beliefs are most difficult to change (Ertmer & Ottenbreit-Leftwich, 2010). These beliefs need to be addressed and examined as they drive classroom actions and influence teacher change processes. (Richardson, 1996).

It is helpful to identify the features of central beliefs and what distinguishes them from more peripheral beliefs. Rokeach (1968) suggests that beliefs become central if we learn them during childhood and involve a direct encounter with the object of belief. For example, a belief that mathematics textbooks are important for teaching not only involve an object; it is further reinforced by the unanimous social consensus among all of one's reference people and groups (Jamieson-Proctor & Carmen, 2008). It has been argued that if beliefs are considered to be a collection of attitudes, then beliefs which play a critical role within a person's belief system and help to determine his or her behaviour are important (Rokeach, 1968). Rokeach (1968) argues that such beliefs are held within the core of our belief systems, and they act as central beliefs. Such beliefs may inhibit a teacher's desire to change teaching practices related to the belief (Pajares, 1992) and are normally unaffected by new information depending upon the reason for which they are held (Ernest, 1989). Therefore, it is essential to determine the relative importance or centrality of various beliefs held by mathematics teachers.

Rokeach (1968) defined centrality of beliefs in terms of connectedness. Rokeach suggests that the more a given belief communicates or has functional connections with other beliefs, the more we can consider it a central belief (p. 5). Nonetheless, central beliefs are a small set of beliefs that has more implications and consequences for other beliefs in the belief system. Rokeach proposed criteria for functional connectedness or functional communication. He argued that beliefs that are directly concerned with our existence and identity and that we share with others are important. These beliefs have more functional connections and consequences than others. Conversely, beliefs that are derived and are concerned with arbitrary matters have fewer functional connections and consequences for other beliefs. Although, Rokeach's theory of central beliefs has been influential in psychology and sociology, it has been criticised due to its oversimplification. Rokeach's theory suggests that individuals have a small set of central beliefs that guide their behaviour. However, other researchers (Festinger, 1962; T. F. Green, 1971; Pajares, 1992) argue that beliefs are complex and multifaceted and that individuals may hold conflicting or contradictory beliefs. Within a particular social context of teaching and according to the teacher's level of thought processes and reflection, beliefs could change (Ernest, 1989).

Influenced by the social context, teachers are likely to adopt the same teaching methods despite holding differing beliefs about mathematics (Pajares, 1992). For example, when teachers employ “curriculum materials in their classrooms, they may develop new mathematical and pedagogical beliefs and skills based on their design of lessons, conversations with students, use of technology, and so on” (Lloyd, 2002, p. 152). Yurekli et al. (2020) argued, however, that constraints in the educational environment, such as assessment methods and students' understanding of mathematics, can cause discrepancies between beliefs and practices.

2.4.5.1. The Influence of social context on teacher beliefs

Beliefs change when the social context of the teaching situation and the teacher’s level of thought processing and reflection change (Ernest, 1989). Influenced by the social context, teachers may adopt the prevailing teaching methods despite holding differing beliefs about mathematics. As such, witnessed during the pandemic, for example, the change in social context intensified the personal and professional uses of video conferencing, virtual collaboration, and social networks, and other digital resources. Studies have identified different examples of changes in teachers’ beliefs during the COVID-19 pandemic. For instance, Morge (2020) used screen-sharing during online video conferencing for problem-solving professional learning and found that it improved teachers’ confidence in using digital resources. Similarly, Carey et al. (2020) observed changes in teachers’ beliefs towards online teaching when they employed breakout rooms for their students collaborative problem-solving. This suggests transformation of beliefs can happen when changes in teachers’ practice produce outcomes consistent with their social context. With regard to using specific digital resources, (Antonietti et al., 2022) explored the relationship between teachers' digital competence and its influence on beliefs, their acceptance of technology, and their intention to use digital tools. They argued that teachers' familiarity with and consistent use of digital resources can lead to preferences beliefs favouring these resources. The gradual evolution of preferences is influenced by teachers' beliefs about the ease of use of technology, perceived utility, technology acceptance, and intention to use. Antonietti et al.'s findings provide empirical validation for the interplay between teachers' digital competence, beliefs, and technology acceptance, confirming the importance of teachers' interactions with and perceptions of digital resources in shaping their beliefs.

During the pandemic positive beliefs about digital resources and affordances increased teachers' confidence in digital technology and its potential to enhance outcomes for learners (Scully et al., 2021). Teachers were able to create innovative and supportive communities, collaborate for problem-solving, hold online conversations, and share resources thanks to the affordances of persistence, visibility, and searchability made available by accessible technologies (Carey et al., 2020). However, Christopoulos and Sprangers (2021) suggested that although the pandemic played a positive role in enhancing teachers' digital skills, it was still important to keep an eye on concerns and constraints as they affect teachers' beliefs negatively. For example, Reich et al. (2020) identified that many teachers struggled to motivate students during online sessions. Teachers who were unfamiliar with digital tools felt isolated and suffered a loss of self-efficacy and professional identity (Reich et al (2020). Also, less privileged teachers from developing or poor countries with limited or no access to digital resources, became victims of social and economic inequalities and were unable to teach during the pandemic (Ndambakuwa & Brand, 2020).

2.4.5.2. Pakistani mathematics teachers' beliefs and their social context

The Section 2.4.5.1 identified how contextual factors can influence teachers' beliefs about using technology in teaching. Contextual factors can also impact on teachers' thinking about teaching particular academic subjects. In the context of Pakistan, socio-religious experiences, school education, and pre-service training experiences shape teachers' beliefs about teaching mathematics (Amirali & Halai, 2010). Teachers hold traditional beliefs and regard mathematics as a constantly evolving discipline, where mathematicians continually revise their body of knowledge while everyone else consumes it (Amirali & Halai, 2010). Teachers in Pakistan use direct teaching methods with a strong emphasis on delivering textbook content (Amirali & Halai, 2021) and using memorization. The act of memorizing mathematical rules and formulas for solving specific problems, is a fundamental component of instrumental understanding, as described by Skemp (1976). Memorization is often regarded as a laborious and repetitive facet of the learning process, but, in Islam, the memorization of the Holy book (Quran) holds distinct values, encompassing moral, spiritual, and intellectual dimensions (Kabir, 2021). As a result in an Islamic country, instrumental understanding and traditional beliefs about mathematics teaching and learning could be preferred by teachers and students as opposed to other approaches to learning as aimed at developing relational (Skemp, 2006) or conceptual understanding (Yurekli et al., 2020).

Additionally, there are other contextual factors in Pakistan which shape mathematics teachers' beliefs. Technology is infrequently used in teaching because teachers believe it to be complex, expensive, and scarce (Dundar et al., 2014). Interestingly, most teachers, regardless of their professional role (government or private), gender, or teaching experience, hold identical beliefs (Amirali & Halai, 2021). In a comparative study of Pakistani government and private school mathematics teachers, Shiraz and Qaisar (2017) found that teachers' beliefs were not entirely aligned with their teaching practices. Factors such as classroom environment, resource availability, senior teachers, and career opportunities influence Pakistani teachers' teaching practices, including making decisions contrary to their beliefs (Shiraz & Qaisar, 2017). Christopoulos and Sprangers (2021) regard these factors as first-order barriers to successful technology integration. According to Christopoulos and Sprangers (2021), first-order barriers refer to external factors that are extrinsic to educators. These include aspects such as Internet access, sufficient bandwidth, and access to technology hardware. In contrast, second-order barriers are intrinsic to educators and involve their attitudes, beliefs, and practices related to technology integration. Specifically, second-order barriers can be influenced by personal attitudes, social contexts, cultural landscapes, and learned pedagogical practices. However, any of the first or second-order barriers, a piece of new knowledge, or a situation may influence teachers' use of technology for teaching (Christopoulos & Sprangers, 2021). As evidenced by the COVID-19 situation, many Pakistani teachers, regardless of their beliefs, employed digital resources for online teaching and learning (Alabdulaziz, 2021). Their beliefs and attitudes changed as they experienced the "ease of use" and recognized the "perceived usefulness" of digital resources in mathematics teaching (Scully et al., 2021).

The above discussion emphasizes the role of teachers' beliefs in mathematics teaching particularly with digital resources. In Section 2.4.4, we noted that the Mathematical Knowledge for Teaching (MKT) framework has been criticized for not considering teachers' beliefs. To address this limitation, we consider the Pedagogical Technology Knowledge (PTK) framework, which incorporates teachers' beliefs, personal and professional knowledge, and attitudes toward using digital resources in teaching mathematics (Thomas & Palmer, 2014). The PTK framework acknowledges the interrelatedness of teachers' knowledge and beliefs and their influence on pedagogical choices and practices. PTK is relevant to this study as it provides a theoretical foundation for understanding the role of teachers' beliefs and knowledge in mathematics teaching.

2.4.6. Pedagogical technology knowledge (PTK)

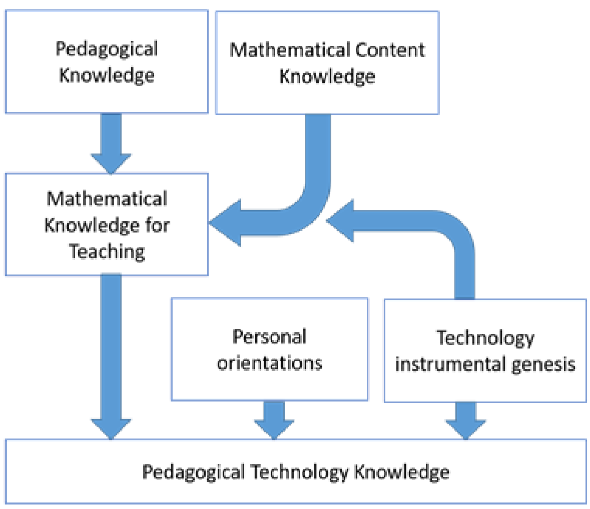
The introduction of technology demands that teachers are able to adopt a broad perspective about the implications of technology for the teaching and learning of mathematics (Thomas & Hong, 2005). Incorporating technology into mathematics teaching requires more than technological knowledge competencies. Teachers also need to develop pedagogical technology knowledge (PTK), that is, knowing how to teach mathematics with technology. Thomas and Hong (2013) indicate that teachers develop PTK when they advance through instrumentalisation and instrumentation of the tool and gain a personal appreciation of its role in learning mathematics.

Trouche, et al. (2018) defined instrumentation as a process focussed on the effects of resources on a teacher's work whereas instrumentalisation is a process focusing on the effects of a teacher on the resources she works on/with. Instrumentation and instrumentalisation are the two components of the concept of instrumental genesis (Guin & Trouche, 2002). Thomas and Hong (2013) argued that teachers need to work on both instrumentation and instrumentalization. Focusing on either alone may result in delays in the progress of instrumental genesis and hinder the overall effectiveness of using digital resources in mathematics education.

The theoretical basis for instrumental genesis was originally developed in educational psychology and cognitive ergonomics by Verillon and Rabardel (1995). Verillon and Rabardel emphasise that there is a difference between an artefact, a material object, and an instrument that is a psychological construct. They further explained that an instrument does not exist as an independent entity, and only becomes an instrument when the subject has been able to adapt it to him or herself and has integrated it into his or her activity (Verillon and Rabardel, 1995, p. 85). This construction is called instrumental genesis, which is a complex process, linked to the characteristics of the artefact (its potentialities and limitations) and to the activity of the subject, their knowledge, and their former working habits (Guin & Trouche, 2002).

Thomas and Hong (2005) explained PTK as including instrumental genesis, mathematical content knowledge (MCK), MKT and personal orientation. Figure 5 shows how these three teacher-related factors are combined to produce PTK. PTK uses Schoenfeld (2011) definition of personal orientation that emphasises teacher's beliefs and goals about the value of technology, and the nature of learning mathematical knowledge, the affordances and constraints these involve, and other affective aspects, such as confidence (Tabach & Trgalová, 2019, p. 190; Thomas & Hong, 2013). These foundations of PTK differentiates it from TPACK that articulates the relationship between PCK, TPK, and TCK. PTK takes account of strong mathematical content knowledge, teacher's personal orientation towards technology with specific predefined goals and the level of teachers' confidence in using technology (Thomas & Palmer, 2014). This suggests a teacher with strong PTK may be more capable of demonstrating and developing true knowledge of mathematical (digital) objects under study and may be able to embed mathematical conceptions and understanding rather than technology in the centre of classroom activity.

Figure 5
A model of the framework for PTK



Note. An outline of the construction of Pedagogical Technology Knowledge (PTK). From "Teacher integration of technology into mathematics learning," by M. Thomas and Y. Hong, 2013, *International Journal for Technology in Mathematics Education, 20(2), p. 70*

Studies have shown that the PTK framework is a useful tool for investigating teachers' use of technology in teaching mathematics. For example, Anabousy and Tabach (2022) used the PTK framework in a study of 42 mathematics middle schoolteachers to investigate how they use technology in their teaching practices. They found that the PTK framework facilitates understanding of the complex interplay between teachers' beliefs, knowledge, and practices in the context of teaching with technology. Thomas and Hong (2013) in their study of mathematics teachers' use of graphing calculators, employed PTK as an investigative lens and found that a lack of instrumentation and instrumentalization of digital tools (resources) could lead to a lack of confidence in teaching mathematics with technology, which further restricts the growth of PTK. However, Tabach and Trgalová (2020) argued that developing digital competencies can help teachers overcome this lack of confidence and integrate digital resources into their teaching practice. They explained that by building digital competencies, mathematics teachers can develop the technical and pedagogical expertise necessary to select, use, and design effective instruction using digital tools and resources. This can enhance the quality of mathematics teaching and support the growth of PTK, as teachers are equipped to integrate mathematics content knowledge and pedagogical knowledge with digital technologies to design and deliver teaching.

These are important arguments in the context of this study and understanding the knowledge and use of digital resources by mathematics teachers and their digital competencies. In addition to the above, there are several other approaches that investigate the knowledge and use of digital resources in mathematics education. For example, Tabach and Trgalová (2019, 2020) and Pepin, Choppin et al. (2017) discussed standards in the use of digital resources for mathematics education whereas van den Bogaart et al. (2019) have discussed the issues of co-design and development of digital content for mathematics. Trouche et al. (2018) propose a theoretical framework the documentational approach to didactics (DAD), as a tool for analysing the changes brought about by digitalization in the design and uses of mathematics teaching online resources. Given the complexity and multifaceted nature of digitalization in education, it is essential to examine various approaches to develop an understanding of how mathematics teachers' knowledge and use of digital resources are influenced and transformed. The next section explores these diverse approaches to provide a thorough investigation of the subject.

2.4.7. Other approaches to investigate teachers' knowledge and use of digital resources

Several studies have used the instrumental approach to investigate user activities with digital resources in a technology-rich environment. This approach is referred to as instrumental orchestration (Drijvers et al., 2010). In an instrumental approach, the knowledge and use of a technological tool includes a process of instrumental genesis, through which an entity or artefact becomes an instrument (Verillon & Rabardel, 1995). Instrumental orchestration emphasizes guiding students' instrumental genesis. Drijvers et al. (2010) differentiated instrumental orchestration into three components: a didactic configuration (arrangement of artefacts/tools), an exploitation mode (decisions to exploit a didactical configuration) and a didactical performance (how to perform in the chosen didactic configuration and exploitation mode). Drijvers et al. (2010) considered the instrumental orchestration framework as "a productive lens" through which researchers can examine teacher behaviours with a digital tool (p. 231). Another approach that includes instrumental genesis is the documentational approach to didactics which was first introduced by Gueudet and Trouche (2009) and has been developed further in joint work with Pepin (Gueudet et al., 2012).

The Documentational Approach to Didactics (DAD) is a theoretical framework that considers teachers' work beyond the classroom and over the long term (Trouche et al., 2018). DAD emphasizes the interplay between teachers' activity and their resource systems, which evolve over time and constitute a structured entity aligned with teachers' needs and professional knowledge. DAD also views teachers' professional development as the joint development of knowledge and competencies, which includes a mathematics teacher's design capacity. Furthermore, DAD recognizes the collective dimension of teachers' documentation work, as it takes place in a social context involving colleagues, students, and resources such as textbooks and digital materials (Trouche et al., 2018).

DAD is an evolving theoretical approach that adapts to internal and external changes, leading to the emergence of new concepts and theoretical appropriations (Trouche et al., 2020). For example, DAD introduced the notion of distance to analyse different steps of the documentational genesis, while the concept of community of practice was used to analyse teachers' collective documentation work. The relevance of new concepts, such as the community resource system or the community documentational genesis, is also being questioned. However, DAD has been originally conceptualised in French and further developed mainly in English (Gueudet et al., 2016), due to which it lacks strong empirical evidence that could support the generalizability of the framework in different educational cultures and language contexts (Shao et al., 2023). In 2020 HAL open science started a project to develop a DAD living multi-language glossary to address this challenge. The aim of the project was to ensure consistency and clarity in the use of key terms and concepts across different languages and contexts (Shao et al., 2023; Trouche et al., 2020).

Gueudet (2015) used a Documentational approach to investigate mathematics teachers' work when they solved a problem with students using (digital) resources. Gueudet argued that a teacher interacts with a set of resources – both old and new – to achieve certain pedagogical goals (e.g., problem-solving). She distinguishes "old resources" as resources that are already appropriated, whereas "new resources" are those which are often found on the Internet, selected, or designed by colleagues, or presented in in-service training sessions (Trouche et al., 2018, p. 5). The Documentational approach considers how multiple interactions of teachers with old and new resources results in a document. The Documentational approach and instrumental orchestration frameworks both grew out of the instrumental approach (Rabardel, 2002), therefore "they show significant similarities" (Pepin, Choppin, et al., 2017, p. 648).

In using DAD to examine digital competencies, the main challenge may be to understand the potential complexity of teachers' documentation work and resource systems. Digital resources and tools are rapidly evolving, and mathematics teachers need to continuously adapt and update their digital competencies to keep pace with these changes. This means that teachers' documentation work and resource systems may be diverse and multifaceted and may require a significant amount of time and effort to analyse and interpret. Another challenge is the need to ensure that the DAD framework is adapted appropriately to capture the specific digital competencies required in mathematics teaching. This may involve identifying and defining the key digital competencies relevant to mathematics teaching and assessing how these competencies are reflected in teachers' documentation work and resource systems. Overall, the need to establish clear criteria and standards for assessing digital competencies within the DAD framework would be a main challenge for researchers. This may involve developing appropriate measures or rubrics in different languages to assess teachers' digital competencies in mathematics education, as well as establishing benchmarks and standards for what constitutes effective and responsible use of digital tools and resources in their contexts and cultures.

2.4.8. Double instrumental genesis

Instrumental genesis, as previously highlighted in Section 2.4.6, is a concept in cultural studies and philosophy that refers to the process by which technology and tools are created and developed over time, and how they shape our understanding of the world. The concept emphasizes the idea that technology and culture are intertwined, and that the development of technology is not just a matter of scientific progress but also of cultural and historical practices (Haspekian, 2011). Double instrumental genesis is a framework proposed by Haspekian 2011) which incorporates two instrumental geneses (personal and professional) of teachers. The framework has its theoretical foundation in Rabardel’s (2002) concept of instrumental genesis. According to it, “teachers must first acquire basic skills to master the specific technology they intend to use and develop utilization schemes related to this technology (personal instrumental genesis). Teachers must also develop their understanding of how to support students’ mathematics learning in a digital environment (professional instrumental genesis) (Tabach & Trgalová, 2019, p. 188). While personal genesis may be seen as common to any teacher (although tool-specific), professional genesis is unique to mathematics teachers. Pepin, Gueudet et al. (2017) and Ruthven (2017) proposed that most studies on digital resources have considered digital content as a digital mathematical tool which can be examined under the lens of Rabardel's (2002) instrumental approach.

Concepts such as instrumental genesis and double instrumental genesis, have informed the modifications of MKT (Ball et al., 2008) and PTK (Thomas & Hong, 2005) by Tabach & Trgalová (2020). Tabach and Trgalová (2020) proposed a new framework called Digital Competencies for Teaching Mathematics with Technology (DCTMT) to provide more relevant constructs such as double instrumental genesis and digital knowledge of teaching mathematics to investigate knowledge and skills of mathematics teachers using DR. The next sections, discuss the modifications of the MKT and PTK frameworks by Tabach and Trgalová and explain how these modifications informed this study.

2.4.9. Digital Competencies for Teaching Mathematics with Technology (DCTMT) framework

Tabach and Trgalová (2019) extended the previous work on the PTK framework (Thomas et al., 2008; Thomas & Palmer, 2014) to understand the specific knowledge and skills needed by mathematics teachers to use digital resources in classrooms. They examined different international standards describing mathematics teachers' digital competencies by using the PTK framework. They consider using the PTK framework appropriate as the framework is specifically developed with mathematics teachers in mind. However, they suggested two modifications to the PTK. First, instead of "technology instrumental genesis", Tabach and Trgalova used a double instrumental genesis approach (Haspekian, 2011). Second, Tabach and Trgalová (2019) modified the original MKT framework by Ball et al. (2008) by adding the dimension of digital technology in four of the six domains of MKT (for MKT domains see Figure 4, p. 60). Their modification extends MKT to MDKT (Mathematical Digital Knowledge for Teaching) and defines each component of MDKT as follows:

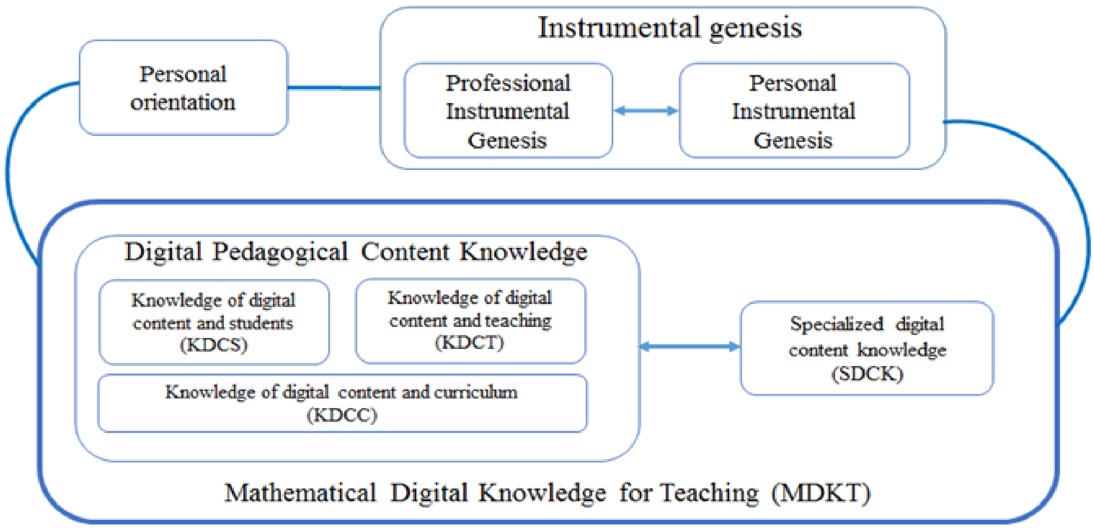
- Specialized Digital content knowledge (SDCK): Teachers specialized digital content knowledge (SDCK) with respect to the mathematics to be taught.
- Knowledge of Digital content and students (KDCS): Knowledge of content and students, which in a technological environment includes additional aspects that may be formulated as knowledge of digital content and students.

- Knowledge of Digital content and teaching (KDCT): Knowledge of content and teaching, which in a technological environment may be interpreted as knowledge of digital-content and teaching.
- Knowledge of Digital content and curriculum (KDCC): Knowledge of content and curriculum in a digital environment, e.g., knowledge of prescribed use of ICT.

These modification in the PTK i.e., introducing double instrumental genesis and adding a dimension of digital technology in MKT led to a new framework called "Digital Competencies for Teaching Mathematics with Technology" (DCTMT) as shown in Figure 6.

Figure 6

Digital competencies for teaching mathematics with technology (DCTMT)



Note. The original DCTMT framework was developed and used in a dialectical process of implementation and refinement. From "Teaching mathematics in the digital era: Standards and beyond," by M. Tabach and J. Trgalova, 2020, *p. 240.* Springer Nature Switzerland AG © 2020

The framework shown in Figure 6, comprises three domains: teachers' personal orientations (beliefs, values, and preferences), MDKT, and double instrumental genesis related to digital technology (Tabach & Trgalová, 2020). MDKT further comprises Specialized Digital Content Knowledge (SDCK) and Digital Pedagogical

Content Knowledge (DPCK). Tabach and Trgalová (2020) articulate SDCK as teacher's knowledge about integrating global content into the curriculum and operating various packages appropriate to mathematics, such as visualization, data analysis, role-play simulations, and online references. SDCK facilitates technology-enhanced mathematical experiences that foster creativity. Teachers with a high level of SDCK use ICT resources to support their own acquisition of subject matter and pedagogical knowledge. They understand how digital developments are changing and expanding the content of mathematics. DPCK, which comprises KDCS, KDCT, and KDCC defines teachers' ability to search in digital repositories for suitable resources, to select those resources best suited to their students' needs, to create new digital resources themselves or with their team members, to share their resources with their peers, as well as to evaluate the resources' efficiency and appropriateness with respect to the learning. The framework also acknowledges the role of teacher personal orientations and goals when using any specific technology or content for teaching (Tabach & Trgalová, 2020). Personal orientation and goals are related to the affective domain and teachers' beliefs regarding mathematics, teaching mathematics and digital technology. Other studies (e.g., Ruthven, 2014; Tabach & Trgalová, 2019; Thomas & Hong, 2013; Thomas & Palmer, 2014) have also mentioned the impact of teachers' orientations and goals on their confidence in using any digital resources for teaching. Schoenfeld (2011) explains that a positive orientation towards technology can serve as an important driving force for the integration of digital technologies.

2.4.9.1. The framework underpinning this study

Subsections 2.4.6, 2.4.7, 2.4.8 and 2.4.9 presented various framework and their relevance in the adoption of technology for mathematics education. The "Digital Competencies for Teaching Mathematics with Technology (DCTMT)" framework was selected for use in this research. The framework recognizes the complex interplay between mathematical digital knowledge for teaching (MDKT), instrumental genesis and personal orientation. The MDKT domains consider teachers specialized digital content knowledge (SDCK), Knowledge of digital content and students (KDCS), Knowledge of digital content and teaching (KDCT), and Knowledge of digital content and curriculum (KDCC). Among the four domains (constructs), Tabach & Trgalová (2020) consider that SDRK is linked to a teacher's personal instrumental genesis. The other three constructs KDCS, KDCT, and KDCC are linked to the professional instrumental genesis, the student instrumental genesis, and the genesis of learning mathematics with digital technology (Tabach & Trgalová, 2019, 2020). These domains are widely used and mentioned in the various international policy documents to define ICT-related digital competencies, knowledge, and skills of mathematics teachers. (Examples of these policy documents, include the EU DigComp framework and Australian national framework for professional standards for teaching).

Although the DCTMT framework captures important aspects of teachers and teaching in digital environments, the use of the term “content” may be confusing. Therefore, I consider modifying the term “content” to “resources” within the different constructs of the DCTMT. This modification is justified in the following section.

2.4.9.2. Modification of the DCTMT framework

The aim of this study is to understand the digital competencies of high school mathematics teachers in Pakistan. In this regard, I adopt the use of the term “digital resources” instead of “digital content” in the framework. This altered terminology indicates a conceptual transition to one that aligns better with the evolving nature and different aspect of digital tools (see Sections 2.2.2 and 2.2.3). The breadth of meaning of the term ”resources” is suggested in the literature. Ruthven (2014) claimed that the term “resource” has a multitude of everyday and educational meanings. Resources refer to any assets teachers might draw on to support any stage of their everyday teaching practice (Siedel & Stylianides, 2018). Furthermore, in this literature review, I have defined the concept of digital resources for teaching and learning mathematics (Section 2.2.2). Based on the definition, I elaborated on the digital competencies of high school mathematics teachers (Section 2.2.3). Considering these definitions and the context of this study, the change aligns with the scope of technology integration in mathematics education.

Moreover, it is argued that there are limitations associated with using the term "content" in the components of the framework. The term "content" may restrict understanding of the value of interactive and engaging digital resources, which differ from traditional knowledge sharing. The term may undermine recognition of teachers' creative potential in curating and creating materials and may shift the focus of the reader to the "what" rather than the "how" of teaching. Besides, the term "content" implies subject-specific information that needs to be conveyed. However, in the digital era, educational resources have evolved to include a diverse range of content, such as interactive simulations, multimedia objects, online databases, and collaborative platforms (Maciejewski, 2019). The term resources highlight that technology is not solely about content transmission, but also about employing various tools (Gadot & Tsybulsky, 2023).

The term “resources” also resonates with the framework's emphasis on teachers' abilities to search, select, create, and evaluate digital materials. The term includes the dynamic process of harnessing digital tools to support teaching objectives. The rationale which has been outlined demonstrates that the change in terminology involves more than just a linguistic adjustment. Teachers have a variety of roles, content, and tools in digital classrooms, collectively referred to as "digital resources" (Schoenfeld, 2011). These roles include curating content, facilitating learning, and providing personalized guidance to students (Gadot & Tsybulsky, 2023). A further argument is that the use of the term resources not only communicates the notion of greater flexibility, but also accords with the concepts of self-paced and self-directed learning for students (Reddy et al., 2020). These reasons justify the view that using "resources" instead of "content" would more aptly reflect digital technologies that can enhance teaching in several ways.

Moreover, during the pandemic, the traditional boundaries of the classroom blurred, and education systems worldwide found themselves in the realm of online and blended learning. The pandemic compelled teachers to adapt their pedagogical approaches to accommodate remote teaching. In this context, the term "content" might emphasize the one-way flow of information from teacher to student. It may neglect the interactive, collaborative, and creative aspects that resources brought to the forefront during the COVID-19 pandemic.

Particularly, in the context of mathematics teachers in Pakistan, the shift from content to resources allows for the capture of the broader aspects of digital competencies such as:

- Teachers’ use of collaborative, personalized platforms over reliance on fixed curriculum texts. Teachers can leverage social media, adaptive software, educational games, simulations etc. as per individual student needs.

- Knowledge about informal resources that teachers create to supplement formal (official) textbooks content. That is, how teachers integrate digital materials created by them that map better to the classroom context.
- Understanding of the role of the teacher in continually evaluating, modifying, remixing or adding to the materials accessed - casting them more as active curators rather than passive content consumers.

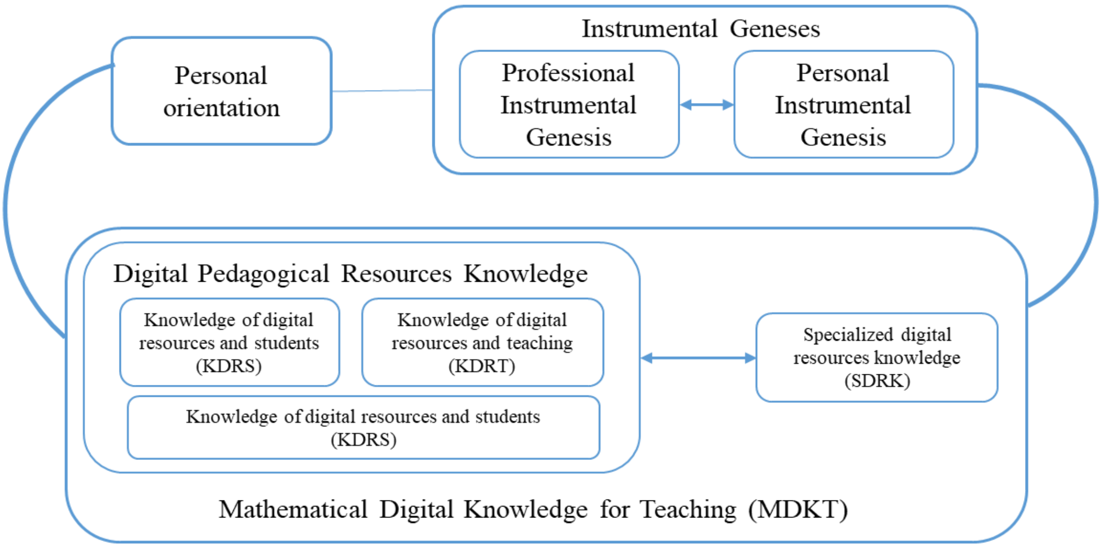
With this modification in mind, the components of the framework are as follows:

- Specialized Digital Resources knowledge (SDRK): Teachers specialized digital resources knowledge with respect to the mathematics to be taught.
- Knowledge of Digital Resources and students (KDRS): Knowledge of resources and students, which in a technological environment includes additional aspects that may be formulated as knowledge of digital resources and students.
- Knowledge of Digital Resources and teaching (KDRT): Knowledge of resources and teaching, which in a technological environment may be interpreted as knowledge of digital resources and teaching.
- Knowledge of Digital Resources and curriculum (KDRC): Knowledge of resources and curriculum in a digital environment, e.g., knowledge of prescribed use of ICT.

The study explored the digital competencies of mathematics teachers using the modified Digital Competencies for Teaching Mathematics with Technology (DCTMT) framework as shown in Figure 7.

Figure 7

Modified theoretical framework for this study.



Note. The modified theoretical framework shows the changes in the original framework (see Figure 6). The term "content" is replaced with "resources."

2.5. Evidence on digital competencies for mathematics teachers

The acquisition of digital competencies for mathematics teaching and learning has become important in recent years (Tabach & Trgalová, 2020). To use various digital resources, teachers need to possess the required digital competencies (Dockendorff, 2020; Gueudet, 2015; Pepin, Choppin, et al., 2017). This section examines studies on the digital competencies of mathematics teachers, including the skills and knowledge necessary for using digital resources in mathematics classrooms. It includes studies that highlight the importance of digital competencies for mathematics teachers in teaching and applying problem-solving and reasoning. The aim is to present research on digital competencies for mathematics teachers and identify areas for further investigation.

2.5.1. Problem Solving and teachers' digital competencies

People with good mathematical problem-solving skills can adapt quickly to situation. Problem-solving skills complement other attributes such as a goal-oriented approach, finding quick solutions, and making informed decisions. Starting in elementary school, these abilities need to be continually developed throughout the academic life of students (Szabo et al., 2020). The role of digital tools and resources in developing these skills is well documented in the literature. Examples are provided in the subsections that follows.

2.5.1.1. Digital competencies in promoting inventive-semiotic acts

Several methods have been used to teach mathematical problem-solving. These methods have focused on cognition, and paid less attention to affect or cognitive-affective interactions (Goldin, 1998). Goldin (1998) defined inventive-semiotic acts as a process in which children create new signs and assign them meaning (Goldin, 1998). Goldin's (1998) approach emphasizes the construction of students' own representational systems (internal and external) during the problem-solving process, involving inventive-semiotic acts. In this way, students can explore the logico-mathematical consequences of their inventions and learn to build their own (external and internal) mathematical representations. In relation to teachers, the ability to use signs, symbols and notations represents teachers' inventive semiotic acts. It is an important ability, which failure to acquire could introduce errors and produce inaccurate representations of mathematical content in digital format (Junqueira, 2006). Thus, teachers need to know how to creatively manipulate mathematical symbols and notations within a digital environment (Junqueira, 2006). They can employ digital tools (such as equation editors) to go beyond traditional pen-and-paper approaches and engage in a dynamic process of symbol manipulation, fostering new ways of representing mathematical concepts (Goldin, 1998). However, Junqueira (2006) also notes that manipulating signs, symbols and notation using equation editors can be difficult to learn, can be a distraction for students, and that they can lead to students becoming less engaged in the learning process.

With the advent of dynamic computer environments, like GeoGebra, the possibilities for displaying aspects of external representational structure explicitly have increased (Dockendorff, 2020). However, the education system in most parts of the world still devotes much of school mathematics explicitly to the manipulation of formal notational systems, with fewer efforts to change classroom practice to emphasize problem-solving strategies, visualization, pattern recognition, and other more conceptually-oriented techniques (Ruiz-López, 2018). This could be due to teachers needing to acquire digital skills to bring about the change. Digital technologies, particularly mobile ones, provide teachers with new opportunities to develop digital competencies that allow them to teach problem-solving more effectively (Carreira & Jacinto, 2019). Therefore, incorporating digital technologies into teaching mathematical problem-solving is essential for improving students' understanding and engagement in the subject.

2.5.1.2. Enhancing logical-mathematical intelligence through digital technologies

Mathematical problem-solving is an important component of Logical-mathematical intelligence (Dockendorff, 2020), which includes skills such as deductive reasoning, pattern detection, sequencing, logical thinking, and cause-effect relationship recognition, as well as knowledge of complex and abstract mathematical ideas. Dockendorff (2020) examined how digital resources (like GeoGebra) can enhance mathematical activities that are part of Logical-mathematical intelligence.

Dockendorff found that the use of Dynamic Geometry Software (DGS) in problem-solving can improve teachers' capacity to think in images and pictures, accurately visualize abstract concepts, and possess a keen sense of space, distance, and measurement. Furthermore, the use of learning technology by teachers can optimize both logical-mathematical and visual-spatial intelligences, complementing each other with improved learning outcomes. Therefore, integrating digital resources into mathematics education has the potential to enhance student engagement and understanding. However, Dockendorff considers that many teachers find it challenging to integrate digital resources in their teaching practice, leading to ineffective teaching of mathematical problem-solving. In addition to the technical, operational, and infrastructural difficulties, Dockendorff suggests that teachers who lack digital skills may experience additional challenges such as in making a fundamental shift in the role that teachers play (from instructor to facilitator/mediator) in teaching with digital resources.

2.5.1.3. Technology as a tool for developing problem-solving and critical thinking skills

Digital technologies can support teachers in developing problem-solving and critical thinking abilities in students (Barana et al., 2019). Digital technologies support multiple perspectives of reality, knowledge construction, and lifelong learning (Reddy et al., 2020). Barana et al. (2019) examined secondary school mathematics teachers' reflections on the role of Advanced Computing Environment (ACE) for problem posing and solving. The authors found that most teachers agreed on the importance of ACE as a method for posing and solving mathematical problems. Their findings indicate that ACE provides multiple options for interactively observing, representing, manipulating, generalizing, and extending reasoning about mathematical problem-solving strategies. Barana et al. reported that ACE diverts students' attention from calculators to more interesting activities, such as conjecturing, searching, and concluding. This can lead to "meaningful and motivated learning" that "promotes critical thinking and creativity" in students (p. 17). This shows that technology can optimize students' potential by promoting critical thinking, problem-solving, and computational skills essential in the 21st-century.

2.5.2. Orchestrating mathematical problem-solving in digital environments

Recent literature (e.g., Morge, 2020; Santos-Trigo, 2019) shows that digital tools including online communication platforms offer problem solvers a set of affordances, which they can use to orchestrate activities (such as explore, (re)-construct, explain, and communicate) during solo and collaborative mathematical problem-solving. The affordances offered by digital resources are critical in augmenting problem solvers' understanding of mathematical concepts beyond formal settings (Santos-Trigo, 2019). However, in intermittent and unpredictable online learning situations such as in COVID-19, teachers and students' reflections are necessary to examine the efficacy of digital tools and communication platforms (Hartshorne et al., 2020). Morge's (2020) study highlighted the role of digital competencies in mathematics teaching, particularly during collaborative problem-solving activities. Using a range of digital tools, such as encyclopaedias, online discussion walls, small chats, emoticons, and breakout rooms, teachers during COVID-19 facilitated meaningful conversations and engaged students in both small and large group discussions. The affordances of breakout rooms, for instance, allowed teachers to visit other groups synchronously, ask questions, explain reasoning, and critique the reasoning of others. This kind of technology-rich teaching configuration is essential for developing digital skills in teachers (Drijvers et al., 2010). While online teaching offers a lot of potential as a method for collaborative learning and problem-solving, Trenholm et al. (2016) cautioned about the difficulties preservice teachers could face due to a lack of knowledge. This includes accumulating and responding to students' feedback and helping students develop appropriate pedagogies before transitioning to fully online instructions. Therefore, it is crucial for teachers to acquire and develop digital competencies that can help them overcome these challenges and engage students in collaborative problem-solving activities.

Gueudet (2015) examined mathematics teachers' work with resources using an approach she called the documentational approach. Gueudet argues that a teacher interacts with a set of resources (both old and new) to achieve certain pedagogical goals. Gueudet designed a scenario for grade 9 students. Gueudet created graphics of a football-training field and placed it in GeoGebra. The students had to find the measure of the shooting angle for a player placed on Point P (point-to-shoot penalties) in the Goal represented as a line segment AB. The researcher, along with teachers, placed 11 hints using text boxes and some already drawn circles for the students to use in case of any difficulty. During the inquiry-based activity, students found several complications in finding the solution. For example, some vocabulary used in hints, "shooting angle", the position of already placed circles, the image used as a football field, and the written description of the problem were confusing for the students. The author had to change hints and find an appropriate football field image (via the Internet) to clear the students' confusion. Gueudet emphasizes that the teacher's digital competencies are key in determining how well they can select, combine, use, and modify digital resources in conjunction with other resources to create a stable organization of the teaching activity. The use of digital resources such as GeoGebra requires teachers to go through several stages of learning to optimize their use in the classroom.

Furthermore, Pepin, Choppin et al. (2017) conceptualized a structure to organize the digital Learning Space for solving problems in mathematics. Their organization starts with the Presentation Space, in which a digitally competent teacher can use multimedia resources such as videos, animation, and graphics to explain a concept (or a problem). Then there is the Problem Space, which includes the types of problems and the range of solution paths or responses for the problems the students work on. The Workspace, which refers to the set of tools and resources available to solve a problem, follows the Problem Space. Teachers can manage the Workspace with digital resources by providing access to a wide range of tools, including graphing calculators, spreadsheets, wikis, and academic networking. This application of digital competencies can help students develop creative and critical thinking skills. Finally, the Navigation Space refers to the potentially non-linear ways in which learners may progress through mathematical topics available via other sources. To manage these learning spaces, teachers' digital skills like searching, locating, installing, creating, evaluating and designing can facilitate the integration of digital resources. Teacher through these skills, can structure and design a digital learning environment and add further study materials to the space.

Pepin, Gueudet, et al. (2017) emphasised the importance of a mathematics teacher's design capacity when working with digital resources in a digital environment. They emphasized that digital resources offer incentives and increased opportunities for teachers' design work, but also require enhanced design expertise due to the changing nature of the requirements. They proposed ten questions to study teacher design processes and introduce three components for exploring teacher design capacity. They stressed the increasing role of digital resources in developing teacher design capacity, particularly in collaborative environments. Pepin, Choppin, et al. (2017) argue that the research on digital curriculum resources (DCR) (or simply digital resources) and their design is framed by socio-cultural theories and highlights the ongoing connection between design and use due to the unique nature and affordances of digital resources. They further mention the blurring of boundaries between pedagogy and assessment, as well as between summative and formative assessment techniques, influenced by the design of automated learning systems. The study emphasizes the expanded space of interaction facilitated by dynamic/interactive digital resources and its potential to support personalized learning. The authors asserted that digital resources provide opportunities for changing understanding of their design, use, quality, and the processes of teacher and student interactions. In all these processes, teachers must not ignore the importance of their digital competencies.

2.5.3. Teachers' digital competencies and online collaboration

Previous studies (e.g., Means et al., 2009; Molinillo et al., 2018) have pointed out that the participation, interaction, and meaningful work of peers are significant components of an online learning environment. Means et al. (2009) performed a meta-analysis of 200 studies and found that the studies in which online teaching was collaborative or instructor-directed, the effect sizes of learning gains were greater than those in which online learners operated independently. Means et al. (2009) highlight the importance of teachers having digital skills to facilitate online teaching and learning. Molinillo et al. (2018) also emphasized the importance of social presence, interactions between teacher and students, and emotional engagement in online collaborative learning. They empirically analysed the impact of social presence and interactions on active learning within the context of social web-based collaborative learning (SWBCL). Molinillo et al. (2018) found that social presence and interaction between teacher and students positively influenced "students' active learning, both directly and indirectly, through emotional engagement" (p. 41). These findings highlight the critical role of teachers' digital competencies in creating and facilitating collaborative learning environments in online settings.

As previously mentioned in Section 2.3.4, during the COVID-19 pandemic, teachers' digital competencies emerged as a critical factor in ensuring the success of online transition., Gandolfi and Kratcoski (2020) reported that teachers who possessed a knowledge of digital learning and social media platforms (such as Google Meet, Padlet, Facebook and Twitter) were able to transfer these skills to create an emergency Community of Practice (CoP) group. Teachers used the group to share common domains, practices, and social connections remotely. The authors guided teachers asynchronously on remote teaching strategies and provided training about non-content-specific tools (e.g., Book Creator). For synchronous CoP, they used a Facebook page, Twitter posts, and weekly emails. They authors found that it is important for school leaders seeking to implement aspects of remote teaching and learning into their educational programs to provide additional opportunities for teacher collaboration beyond the online courses. They must train teachers in the use of online tools that support online teaching and learning.

In another example, Gronseth et al. (2020) investigated the teaching strategies and technologies that facilitate online learning interactions and connections. They redesigned course activities that suited the online format. Gronseth et al. (2020) used synchronous and asynchronous digital technologies, such as mobile instant messaging (GroupMe), digital whiteboard (Padlet), and synchronous sessions via Zoom. Their findings suggest that a significant social link developed among students (165 pre-service teachers) and teachers through collaborative strategies and resources. They found that the promotion of connections and collaboration in online education involved the re-imagination and re-design of training activities with additional knowledge of technologies. An online learning environment requires the active and collaborative participation of learners to explore, develop awareness and understand what is referred to as the "Community of Inquiry" CoI (Garrison, 2009 cited in Gronseth et al. (2020). The study also found that connective technologies, including GroupMe, Padlet, and Zoom, have been instrumental in preserving social closeness despite social distances.

In summary the above studies illustrate that the digital competencies that encompass online communication, collaboration, and digital pedagogy are pivotal to navigate in a digital environment. These skills transcend traditional classroom boundaries, preparing teachers to navigate digital spaces and cultivate meaningful learning experiences for their students. As the role of digital resources continues to expand in mathematics education, these competencies will remain at the forefront of teaching in the digital age.

2.5.4. The knowledge of dynamic graphing software (DGS)

Dynamic graphing software, like GeoGebra, has become an integral component of modern mathematics education (Dockendorff, 2020). DGS's dynamic and interactive features empower teachers to bridge the gap between geometry and algebra and serves as a visual learning environment for students facilitating engaging and interactive mathematics education. GeoGebra can be classified as both a Computer Algebra System (CAS) and a Dynamic Geometry Software (DGS) (Zengin et al., 2012). This dual classification stems from its incorporation of symbolic and visualization features. These features encompass direct coding of equations and coordinates, as well as the capacity to define functions in algebraic terms (Dockendorff, 2020). GeoGebra supports a repertoire of concepts ranging from points and lines to more complex ones, such as conic sections. DGS's dynamic interplay between geometry and algebra not only support students' understanding but also nurtures their analytical thinking and problem-solving skills (Zengin et al., 2012).

The use of DGS has been extensively highlighted in the literature, as evidenced in studies such as those of Dockendorff (2020), Esguerra-Prieto et al. (2018), Maria et al. (2015), Pierce and Stacey (2010), Yerushalmy et al. (2017) and Zengin et al. (2012). Dockendorff (2020) used DGS for problem-solving and showed that affordances such as immediate feedback, interactive and multimedia experiences, and dynamic aspects can help both teachers and students stay motivated, track the progress of learning, and identify areas that need additional support. Such affordances also help students self-regulate and understand mathematics problems (Maciejewski, 2019).

Esguerra-Prieto et al. (2018) in a comparative study of MATLAB and GeoGebra showed that teaching complex numbers can be simplified by using graphical tools.

They used MATLAB and GeoGebra to create and perform several operations of complex numbers graphically such as addition, multiplication, division, subtraction, and conjugate. Esguerra-Prieto et al. created operations such as inverse and 3D representations using both software. They found that the dynamic representation of different operations provides teachers with better options, representations, and a new way to communicate the challenging task of teaching complex numbers. Similarly, Maria et al. (2015) demonstrated the use of GeoGebra to study complex numbers and complex functions. They produced multiple representations of complex numbers using 2D and 3D graphic windows in GeoGebra to solve and teach advanced mathematical concepts in complex equations.

Pierce and Stacey (2010) explained the efficacy of DGS beyond geometry. They suggested that DGS (they call it MAS - mathematical analysis software) provides improved speed in moving between representations such as functions and their graphs. Pierce and Stacey (2010) argue that DGS provides access to representations that would be challenging or "impossible in a pen-and-paper environment (e.g., 3D graphs of functions of two variables, dynamic diagrams), including displays which are easy to share" (p. 4). Zengin et al. (2012) in an experimental study of 51 students found that computer assisted instruction (using DGS) as a supplement to constructivist instruction is more effective than the constructivist teaching method. The findings were consistent with the other studies which found a positive impact of using DGS in enhancing students' learning and understanding.

In the context of Pakistan, Khalil (2016), in an experimental study design (post-test equivalent group), investigated the effect of a DGS (GeoGebra) on Grade-12 students' mathematical thinking and mathematical achievement in analytic geometry. Khalil used six aspects of mathematical thinking commonly found in literature such as generalization, analytical thinking, logical thinking, abstract thinking, problem solving and representation thinking. The author designed twenty-two lessons of GeoGebra, relevant to analytic geometry, for the experimental group and used the criterion-referenced test (post-test) to assess the students' responses. The study findings showed that there were statistically significant differences between the mathematical thinking and mathematical achievement scores of the two groups. In the result, the experimental group developed significantly more in both mathematical thinking and mathematical achievement except in mathematical achievement for high achievers. In multiple comparisons, the experimental group students performed significantly better than the control group. The only aspect that differed was problem-solving where the Mean score of the experimental group improved more than that of control group but did not reach statistical significance.

Yerushalmy et al. (2017) showed that DGS like GeoGebra can aid in the assessment of students' understanding of mathematical concepts. Yerushalmy et al. (2017) study explored the use of online interactive assessment content and its potential in minimising the need for teachers' interpretation of students' meaning making. By using a formative assessment platform called STEP, students were able to create sketches and submit their understanding of a mathematical problem. The researchers used one e-task to study the design principles of innovative assessment items, requiring students to construct confirming examples of a correct existential statement in a dynamic graphic environment (GeoGebra). The results of their study showed that digital technology not only enabled the collection of a different type and quality of data, but also allowed for easier measurement and quicker online feedback. The study concluded that GeoGebra could potentially change the discourse and norms of the classroom, encouraging discussion of solution spaces, heterogeneous response spaces, and the characteristics of examples and mathematical attributes.

These studies show that the use of DGS in mathematics education highlights the transformative power of digital resources within classrooms. This shift prompts a departure from conventional rote learning methods and movement towards an engaging and exploratory pedagogical paradigm. DGS and its potential also emphasizes the pivotal role of teachers, equipped with digital competencies, in order to exploit the rich pedagogical potential of dynamic graphing software like GeoGebra. A digitally competent mathematics teacher can revolutionise his or her mathematics teaching with DGS, creating an engaging and interactive learning experience for students.

2.5.5. Teachers' use of Videos

The omnipresence of videos on the internet is addictive in nature (Sari et al., 2020). Videos have a far greater reach than any other medium on the internet (Nacak et al., 2020). Various video streaming platforms, including YouTube, offer a wide range of educational videos designed to facilitate teaching and learning across different disciplines. Research has demonstrated the influence of videos not only on improving teaching skills (de Araujo et al., 2017; C. Maher et al., 2014; Nacak et al., 2020) but also on enhancing students' learning outcomes (Guo et al., 2014).

The use of videos in mathematics education has opened new possibilities for pre-service/in-service teachers and students, fostering opportunities for the development of digital competence. In particular, the use of videos has been shown to be a valuable tool for professional development of pre-service teachers (McDuffie et al., 2014). In an experimental study, Maher (2014) examined a total of 177 pre-service and in-service mathematics teachers to explore the impact of video usage on various aspects of teaching. The study found that videos help teachers to identify patterns of students' mathematical reasoning, enhance teacher engagement in mathematical discussions, improve their ability to construct mathematical arguments, and enhance their skills in critiquing the reasoning of others. They found that the videos have the potential to help teachers to recognize student reasoning, an important goal in the teaching and learning of mathematics.

Similarly, McDuffie et al. (2014) implemented a video analysis activity to support pre-service mathematics teachers ($n = 73$) in developing their ability to recognize equitable instructional (EI) practices. The authors define EI practices as those where every student has equal access to learning, is treated equitably by the learning community, and feels valued and supported by their instructor and peers. The pre-service teachers participated in the activity multiple times throughout the semester, using four different lenses (teaching, learning, task, and power and participation) to analyse teaching and learning in video clips. The authors found that, through analysing videos of children's learning, pre-service teachers demonstrated improved depth and expanded focus in identifying equitable teaching practices.

In another study related to the use of videos, Moreno et al. (2020) designed a study to assess the relationship between teachers' digital competencies among mathematics teachers and their ability to select and edit teaching videos for use within the Flipped Classroom Model (FCM). The descriptive univariate study involved 50 pre-service mathematics teachers and used a non-experimental approach. The study found that teachers with intermediate digital competencies can create acceptable teaching videos for mathematics education. The study also highlights deficiencies in the pedagogical and mathematics teaching components of the videos. Moreno et al. argues that combining images and explanations through video has been found to enhance students' understanding of the material, compared to relying solely on a textbook. By using videos as an instructional tool, students are provided with a visual representation of the content accompanied by explanations, which helps to connect and contextualize the information. The combination of visuals and explanations in videos facilitates a deeper understanding and retention of the material compared to relying solely on text-based resources.

Similarly, Beatty et al. (2019) analysed students' use of videos in a flipped classroom. They found that the use of videos could lead to improved learning outcomes when implemented appropriately. Their findings suggest that unique and important content in the videos can increase student engagement and persistence through watching the videos throughout the term. Teachers with the knowledge of how videos work can increase the likelihood of students watching each video. Teachers can adjust class activities or content can be timed based on students' video-viewing behaviour. This could allow teachers to align in-class activities with completed video assignments or timely introduction of new content in the online environment. They suggest that instead of pre-loading all videos at the beginning of the semester, instructors can release videos when the corresponding content is needed, creating anticipation and curiosity among students for each week's new video content.

Studies have also included examples of teachers who create their own teaching videos. De Araujo et al. (2017) discussed the case of a teacher who created and used videos as an alternative to the textbook in a mathematics classroom. The teacher adapted a method of teaching by scripting the video as per the content of the textbook and enriching it with her imagination and representations. The knowledge of creating videos solved her problem of teaching and attending to her students at the same time. The author found that using videos improved her students' understanding of mathematical concepts. Trouche et al. (2018) also articulated the value that videos add to the existing pedagogies and teaching approaches. They emphasized that videos are not only a critical component of alternative teaching approaches such as flip and blended learning, but they also provide chances for extending mathematics teachers' resource options. They found that teachers attempt to map the content of the textbook and its presentational style in the self-created videos, so the videos aligned with and replaced textbooks in some ways. This shows the role of teachers' competencies in appropriating resources for their own needs and converting them into a pedagogical instrument, thus initiating the process of instrumental genesis (Rocha, 2018; Trouche, 2004).

In the context of developing countries or resource-constrained environments, videos (particularly YouTube educational videos) can be considered as a tool for self-development, improving knowledge and problem-solving skills. For instance, Nugroho et al. (2019) conducted a quasi-experimental study in Indonesia to examine the impact of combining ethnomathematics, a culturally aligned learning approach, with YouTube learning media, on students' problem-solving abilities. The researchers employed a 2 × 2 experimental design and utilized a problem-solving ability test as the research instrument, administered both as a pretest and a post-test. The study revealed a significant relationship between the learning model factors, specifically YouTube media and the ethnomathematics approach, and the impact of mathematical material on students' problem-solving ability. Notably, they found that the student experimental group demonstrated higher problem-solving abilities compared to those who learned without this approach, after controlling for their initial abilities.

While there is strong evidence for the benefits of videos as an educational tool, there are also cautions and provisos. (Sari et al., 2020) investigated the use of YouTube Educative as an educational tool to improve knowledge, skills, and creativity in mathematics. The authors found that the appropriate use of YouTube videos can remove students' learning difficulties. However, they further argued that YouTube is not fully used as an educational resource in mathematics education. Students primarily use it for entertainment purposes. Similarly, Nacak et al., (2020) argued that the use of YouTube in education comes with its own set of disadvantages and considerations. They mentioned drawbacks including the potential for technology addiction, reduced eye contact and concentration, the inability to ask questions in real-time, the risk of accessing unreliable or unverified information, and students becoming isolated from social interactions. The authors suggest the need for caution and responsible usage when using YouTube videos as an educational tool. Jalaluddin

(2016) also discussed various disadvantages of YouTube in the classroom, such as privacy, the absence of external control, the potential for inappropriate content, and the lack of restrictions on comments. The author emphasizes the importance of establishing guidelines and promoting safe and appropriate usage of YouTube as an assistive resource for teaching and learning. The guidelines must consider safe and responsible use, selecting age-appropriate and reliable content, and ideological sensitivities as essential factors before integrating YouTube into classrooms.

2.5.6. Interactive whiteboard

(Mishra, 2014) emphasised that the basic idea behind teaching and learning in any discipline is that; there are ways of teaching and learning that are central to the way knowledge is represented in that discipline and that resources and techniques are intricately linked to them. However, the pace at which digital resources are changing has meant that our thinking about the pedagogy and content is also changing. Mishra suggests if the teacher changes any one element (content, pedagogy, and technology) in a classroom other aspect will also change. For instance, the introduction of Interactive White Boards (IWB) provides opportunities to display an "infinitely wide range of secondary" digital content when connected with the internet (Hennessy, 2011, p. 476 cited in (Saville et al., 2014). Teachers can use, draw, manipulate, create, display, and disseminate content through IWB, and can conduct assessments (Saville et al., 2014). Rather than focusing on traditional methods, teachers should be provided with opportunities to play and explore in a small micro-domain and take risks using alternate instructional (digital) resources. Institutions need to support and nurture these small micro-domains or creative classrooms instead of adding more courses related to technology in professional development programmes for teachers (Mishra, 2104). In this way, teachers develop new knowledge and skills that enable them to create, select and use technology-mediated new ways of representing the subject matter content knowledge, monitor student work and assess student learning using technology (Tabach & Trgalová, 2019).

2.5.7. Edutainment and game-based learning and teachers' digital competencies

Edutainment is a derived term that specifies a merger of education with entertainment (Aksakal, 2015), first coined by Walt Disney back in 1954. The concept gained momentum in mathematics education when approaches such as game-based learning and gamification were introduced for teaching and learning of mathematical concepts (Hwa, 2018). Nowadays, digital games have become an important digital resource for mathematics education (Gros, 2015). The concept flourished with the penetration of mobile technologies, provided a real impetus to the emerging themes in mathematics education of "learning across multiple contexts" and learning "through social and content interactions" (Crompton & Traxler, 2015). In the context of high school students, games allow them to progress at their own pace, providing opportunities for differentiated learning. High school students and adults have diverse learning needs and possess varied levels of digital skills. Game-based learning can accommodate these differences by tailoring challenges to individual proficiency levels.

There is a corpus of literature on edutainment as a tool to develop 21st Century skills in students. For instance, scholars such as Hwa (2018), (Lynch-Arroyo & Asing-Cashman, 2016) and Kolovou et al. (2013) have shown that digital games have a positive impact on learning, problem-solving, increasing scientific knowledge, and enhancing learning motivation. Edutainment can facilitate the progression of students' thought processes to levels of critical-creative thinkers (Lynch-Arroyo & Asing-Cashman, 2016). Games increase engagement and promote the transition from fixed mindsets to growth mindsets (Hwa, 2018). However, in the implementation of games, literature emphasises the importance of teachers' knowledge and in-depth understanding of different game categorization to successfully integrate educational methods and game design. Gros (2015) explained four categories of games as serious, epistemic, multiplayer, and social games. Gros (2015) emphasized that teachers with better digital skills could select appropriate games from the above categories to meet chosen learning goals. The extent of teachers' digital skills determines how they will use games either as a small unit to support teaching and learning (game-based learning) or to convert the "entire model of teaching to be a game or game-like" (gamification) (p. 39).

Along with its potential and possibilities, there are several challenges in implementing the concept of edutainment and game-based learning for mathematics education. These challenges include teachers' beliefs about using games as an educational phenomenon/tool, acquisition of mathematical knowledge from gameplay, difficulties faced by teachers while implementing games in classroom teaching, and selection of appropriate games for students who are learning at different levels (Gros, 2015). These challenges directly and indirectly relate to the teachers' digital competencies. Teachers with a low level of digital competencies may find it difficult to select appropriate games, design learning experiences, and evaluate student progress (Redecker, 2017). Digital competencies play a vital role in understanding a clear distinction between different gaming concepts used for learning, such as game-based learning or gamification, which may resolve some of the challenges mentioned in the literature.

Another challenge while using games for home-based learning is parental engagement in achieving desired learning goals (Kolovou et al., 2013). In online (home) learning, parents work conjointly with teachers to organize, structure, and operationalize the learning environment. The parents' role in situations such as special education and working with multiple students in different study groups/levels may require tools and strategies to create an effective home learning environment (McCarthy & Wolfe, 2020). This is particularly relevant to Pakistan, where the dynamics of resource availability within households play a crucial role. In settings where the number of users surpasses the available devices, the accessibility to digital resources becomes a critical concern. Another challenge could be students' and teachers' varying levels of digital competencies. While some students might readily adapt to game-based learning environments, others could struggle due to limited exposure to digital tools (Lowrie & Jorgensen, 2015). This discrepancy can create a disparity in learning outcomes, potentially leaving some students behind. Similarly, teachers' digital competencies play a pivotal role in using game-based learning. Teachers need not only to understand the educational potential of these games but also possess the skills to integrate them into the curriculum (Redecker, 2017).

2.6. Digital competencies of mathematics teachers in developing countries

Limited literature exists that discusses the digital competencies of high school mathematics teachers in developing countries. Existing studies focus on the challenges faced by schools, teachers, and students in this area. For instance, the World Bank report (Dundar et al., 2014) on student learning outcomes in South Asia reported that limited training opportunities, inadequate resources and funding, and a prioritisation of traditional teaching methods constrain the use of technology in schools. Moreover, their study found that teachers' access to and use of technology depended on the opportunities provided by their schools, which are often governed by policies designed by school leadership. The report highlighted a wider gap in technology adaptation between rural and urban teachers, particularly for the teachers in Sri Lanka, Bangladesh, India, and Pakistan (Dundar et al., 2014), further compounding these challenges. Umugiraneza et al. (2018) also highlighted these challenges in the South African mathematics education context. The study found that only a quarter of teachers had access to ICT for teaching mathematics, and the use of ICT is even lower in earlier grades.

Makhdum et al. (2023) conducted a study on the use of digital technology in mathematics classrooms with seventh grade students in Pakistan, focusing on the effectiveness of Desmos and GeoGebra in learning algebra. While the study primarily discussed the students' learning outcomes with the resources, it briefly highlighted the role of teachers' digital competencies. They found that the two participating teachers who had prior knowledge of using Desmos and GeoGebra were able to integrate the technology into their teaching practice, resulting in improved learning outcomes for their students. This finding highlights the importance of teachers having knowledge of and confidence in using digital tools to enhance student learning through technology. Another study in Pakistan (e.g., Khalil, 2016) investigated the effect of a dynamic geometry software (GeoGebra) on Grade 12 students' mathematical thinking and mathematical achievement in analytic geometry. However, the study did not consider the role of teachers' digital competencies in the effective use of GeoGebra in the classroom.

In the context of developing countries, many studies on mathematics education have been conducted under the umbrella of STEM (Science, Technology, Engineering, and Mathematics) education. For example, (Ali et al., 2023) used the STEM Pedagogical Content Knowledge (STEMPCK) Scale to measure teachers' STEM knowledge. The study found that teachers in Pakistan's largest city, Karachi, do not possess the necessary understanding to educate students about scientific concepts through the application of various digital resources. The authors suggest that the Pakistani in-service teachers need access to professional development opportunities to acquire a more in-depth understanding of STEM subjects. In another study, (Chowdhury et al., 2020) identified challenges in implementing STEM education in Bangladesh. The study argues that infrastructure and resources as well as professional development are the key factors for implementing STEM. In addition, inadequate scientific laboratory facilities, lack of financial resources, lack of training for trainers and large class size have been identified as the major challenges for the implementation of STEM.

STEM is a broad field that encompasses various aspects of science and mathematics education. However, it tends to focus more on science and technology-related topics and may give less attention to the specific context of mathematics education. In Pakistan, mathematics education may have unique challenges and opportunities that are different from other STEM fields, such as science or engineering. Thus, it is important to study the use of digital resources in mathematics classrooms specifically to understand the context of mathematics teachers in Pakistan. A more focused and contextualized approach to studying the use of digital resources in Pakistan can provide valuable insights into the ways in which digital resources can be integrated into mathematics classrooms to enhance student learning and teachers' digital competencies.

Section 2.6 explored how teachers' digital competencies allow the integration of several digital resources (DR) into their mathematics teaching and learning. Despite the many opportunities and benefits of using digital resources highlighted in that section, their use in developing countries has not been optimized. One likely reason is teachers' inadequate digital competencies. Inadequate technological infrastructure and resources, coupled with a lack of knowledge and skills to recognize the role of specific digital resources in transforming teaching and learning processes, may also contribute to this issue (Kalolo, 2019). It is likely that many teachers in Pakistan have not been introduced to or trained in using digital resources like e-textbooks and dynamic graphing software, such as GeoGebra. Since these teachers may not have been exposed to digital resources during their own education, they may not fully understand the potential use, complexity, and challenges involved in using digital resources for teaching and learning mathematics (Jafri, 2020).

2.7. SUMMARY OF LITERATURE REVIEW

This literature review has explored the importance of digital competencies for mathematics teachers, and the challenges faced by developing countries in integrating technology into their education systems. Digital competencies refer to the knowledge, skills, and attitudes necessary for using digital technologies for communication, information, and problem-solving purposes. The review examined theoretical perspectives and frameworks for investigating the interactions between digital resources and teachers, including the Mathematical Knowledge for Teaching (MKT), Pedagogical Technology Knowledge (PTK), Technological Pedagogical and Content Knowledge (TPACK), and Digital Competencies for Teaching Mathematics with Technology framework.

While the TPACK model has been criticized for not being mathematics-specific and not accounting for teachers' beliefs, other frameworks, such as the MKT and PTK focus on teachers' mathematical knowledge and how to teach mathematics with technology. The Digital Competencies for Teaching Mathematics with Technology framework (Tabach & Trgalová, 2020) was developed for examining the digital competencies essential in teaching mathematics with technology. However, an empirical foundation was absent from this theoretical framework within the literature. This study addresses this gap by adapting and using the framework (Section 2.4.9.1), offering an investigative lens that acknowledges the intricate relationships of mathematical digital knowledge for teaching (MDKT), instrumental genesis, and personal orientation, thus contributing much-needed empirical evidence to support its effectiveness. Moreover, this modified framework will also guide data collection and analysis for this study, ensuring a comprehensive exploration of the intricate connections among teachers' digital competencies in the context of mathematics education.

The review highlights the challenge of providing professional development to develop teachers' digital competencies. The review indicates that an ongoing and goal oriented professional development that includes collaboration, mentoring, and coaching is necessary. Providing access to high-quality digital resources and technologies is crucial, and developing digital competencies requires systemic changes at the school, district, and national levels. It also indicates that future research combining qualitative and quantitative techniques can explore this aspect further to inform stakeholders about how policies related to digital competence can be cemented in educational contexts (Zhao et al., 2021).

The review also identifies gaps in the literature, including the limited exploration and the perspectives of mathematics teachers in developing countries. In particular, the review noted the lack of research on digital competencies in the context of developing countries. There is a need for more comprehensive research on the challenges faced by mathematics teachers in developing countries in building digital competencies and integrating technology into their teaching practice. Further research in this area can provide valuable insights into how teachers from developing countries can adopt and use digital technologies in mathematics education to improve student learning outcomes.

CHAPTER THREE

3. METHODOLOGY

3.1. INTRODUCTION

In this chapter, I present the methods, design, and instruments that I used in this research study to understand the digital competencies of high school mathematics teachers in Pakistan. The chapter begins by providing an overview of the research paradigm (Section 3.2). The methodological approach and research design are then outlined, including a discussion on mixed-method research and its application in mathematics education research (Section 3.3). The explanation of the mixed-methods approach sets out the steps in each of the two distinct but sequential phases (Phase 1 and Phase 2). Section 3.4 focuses on Phase 1, the quantitative study, detailing the structure of the online survey instrument, the pilot study's methodology, and actions taken for the main study. This section also covers recruitment, distribution, and responses to the survey, discussing outliers and missing values. The validity and reliability of the online survey instruments are examined, followed by a description of the quantitative data analysis using both descriptive and inferential statistics. Section 3.5 describes the conduct of the qualitative phase of the study. This section covers the recruitment of teachers for semi-structured interviews and the subsequent qualitative data analysis. The overall approach allowed for the sequential use of both quantitative and qualitative methodologies, resulting in an overall mixed-methods analysis. The chapter further explains trustworthiness and rigor (Section 3.6), including triangulation of the findings and reflexivity. The chapter concludes with an account of ethical considerations (Section 3.7) and a chapter summary (Section 3.8).

3.2. Research paradigm

A research paradigm constitutes a fundamental belief system that shapes the researcher's approach to understanding research phenomena (Khatri, 2020). The research paradigm represents the researcher's worldview, perspective, school of thought, or a set of shared beliefs that guides the interpretation and meaning ascribed to research data (Guba & Lincoln, 1994). The paradigm is pivotal in research and influences the researcher's choices in terms of research design, methods, and data analysis (Guba & Lincoln, 1994; Khatri, 2020).

In this study, I adopted a pragmatic paradigm to understand the digital competencies of high school mathematics teachers. Pragmatism provides flexibility to use diverse approaches to understand the underlying phenomenon. Pragmatism bridges the gap between quantitative and qualitative approaches, as described by several scholars such as Creswell and Creswell (2018), Creswell and Plano Clark (2017), Biesta (2015), and Tashakkori and Teddlie (2021). Pragmatism does not adhere to a singular philosophical system or reality, allowing researchers to select methods, techniques, and procedures that best align with their needs and objectives (Creswell & Creswell, 2018).

Given the pragmatic paradigm's flexibility, the study employed both quantitative and qualitative methods to collect and analyse data. Pragmatism supports the use of abductive reasoning, which involves forming plausible explanations by connecting theory and data (Creswell and Creswell, 2018). This approach is particularly suitable for mixed methods research, where the inductive goals of qualitative research are based on the deductive results of quantitative research, and vice versa. Walton (2014) explains that abductive reasoning progresses from observations towards formulating the most plausible explanation, a process that is employed in this study to sequentially combine qualitative and quantitative methods.

Thus, the pragmatic paradigm not only informs but also justifies the use of a mixed methods approach, integrating diverse worldviews and assumptions. This ensures that the research findings are both contextual and generalizable (Creswell & Creswell, 2018; Tashakkori & Teddlie, 2021). By leveraging the strengths of both methodological approaches, the pragmatic paradigm provides a coherent rationale for the integration of varied data sources, aligning well with the study's aims and ensuring robust and versatile methodological choices.

3.3. METHODOLOGICAL APPROACH AND RESEARCH DESIGN

A methodological approach and research design are crucial components of a research study, guiding how the study is conducted and how data are collected and analysed. These aspects ensure that the research is conducted systematically, and in a way that aligns with the study's goals and objectives. This study used mixed-methods and followed an explanatory sequential design.

3.3.1. Mixed-method research

Mixed-method research is primarily used for empirical studies involving smaller sub-studies to answer specific questions that can be combined to answer the general research question of the project (Kelle & Buchholtz, 2015). Typically, these sub-studies collect either quantitative or qualitative data. The two sub-studies for this study were Phase 1 (quantitative) and Phase 2 (qualitative).

The rationale for employing a mixed-methods approach in this study was twofold. Firstly, quantitative, and qualitative methods have their specific limitations and strengths, such as providing a broad overview versus an in-depth investigation or exploring context versus researcher bias. Combining both methods helped offset their overlapping weaknesses (Biesta, 2015; Creswell & Creswell, 2018). Secondly, the aim of the overarching research question (RQ) and sub-questions (SQs) was to understand the digital competencies of mathematics teachers, as well as the relationships between the factors (constructs) involved. To meet these goals, different kinds of data were required; for instance, to measure the relationship between the constructs of modified DCTMT framework (Section 2.4.9.2, Figure 7, p. 80), required quantitative data. In contrast, qualitative data were needed to answer "why" questions, such as why teachers choose specific digital resources for teaching a particular mathematics topic and what they think about using specific digital resources. In addition, the authors of the framework consider that a PhD study is a good opportunity to apply both qualitative and quantitative approaches to explore mathematics teachers' digital competencies, making it an ideal approach for this PhD study (Michal Tabach, personal communication, January 24, 2020) (see Appendix G).

Moreover, mixed-methods research has been increasingly used in mathematics education studies. For example, (Hart et al., 2009) examined 710 research articles published in six prominent educational journals between 1995 and 2005 and found that 50% of the studies used qualitative methods only, 21% used quantitative methods only, and 29% used a combination of both methods. Similarly, (Kelle & Buchholtz, 2015) used mixed-methods research to restructure the teacher training program in mathematics. Kelle and Buchholtz suggested that mixed-methods research particularly suits, and is fruitful in, mathematics education as it overcomes shortcomings that arise specifically in mathematics educational research when using a single method.

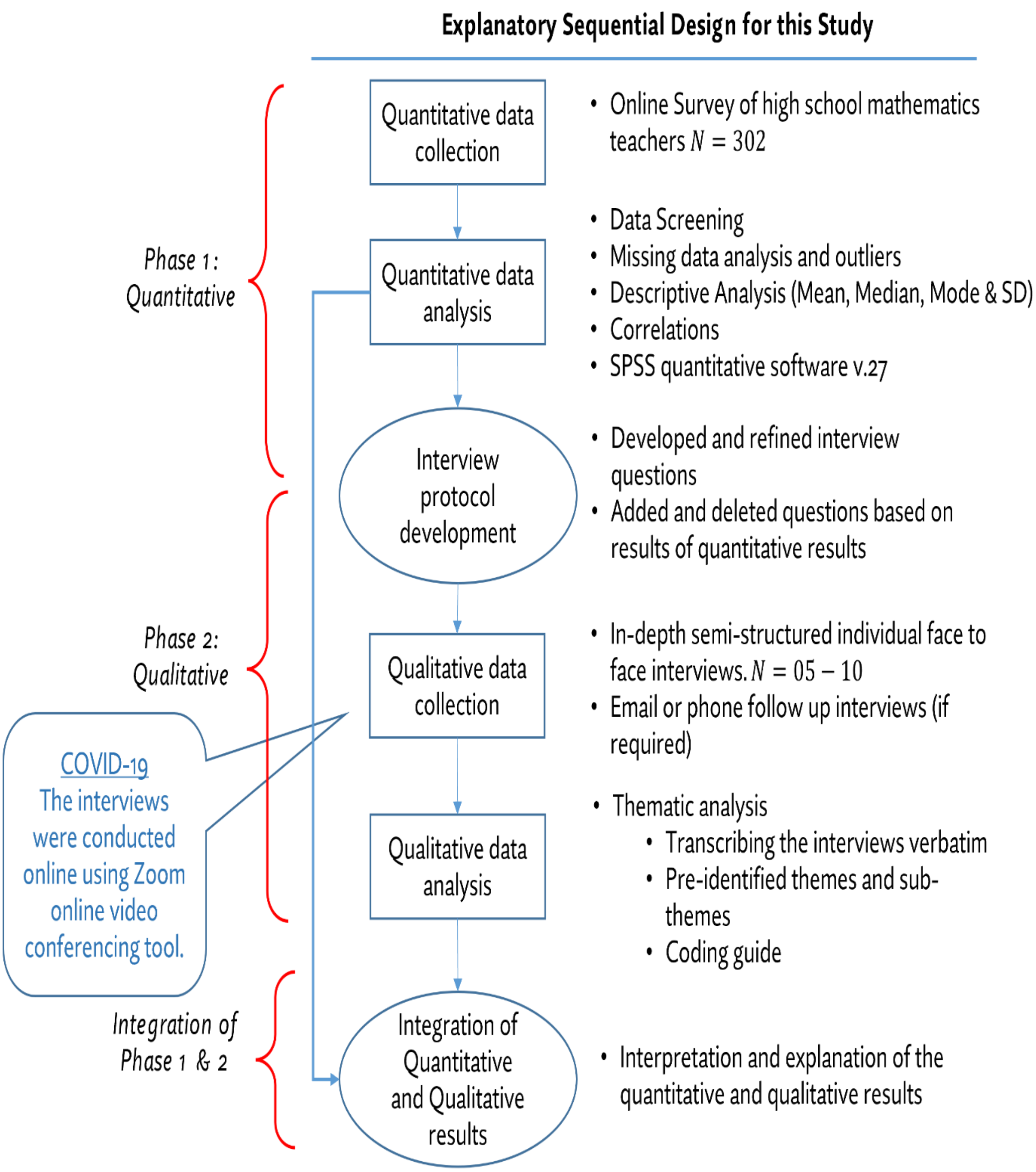
3.3.2. Explanatory sequential design

This study used an explanatory sequential design, which is a well-established mixed-methods research design. In this design, "the researcher begins by conducting a quantitative phase and follows up on specific results with a subsequent qualitative phase to help explain the quantitative results" (Creswell & Plano Clark, 2017, p. 77). In this study, I started with quantitative data collection using an online survey of mathematics teachers in Pakistan. The findings from the quantitative data were then used to inform qualitative data collection and analysis. Both data sets were analyzed separately, and the results were compared to determine potential connections and understand responses from one method to another. It was recognised that the data collection and analysis of one sub-study may have an impact on the other sub-study (Creswell & Creswell, 2018).

The connecting, comparing, and contrasting of the inferences that emerge from both quantitative and qualitative findings in this study helped to develop and extend understanding of high school mathematics teachers' digital competencies. It further allowed me to address the context, impediments, and challenges that directly or indirectly influenced the use of digital resources and the development of teachers' digital competencies in Pakistan. In this way, the research design sequentially shifted from quantitative to qualitative methods, positioning the qualitative method as one that complemented the quantitative method (Creswell & Creswell, 2018). Figure 8 provides an overview of the steps involved in implementing this mixed-methods explanatory sequential research design.

Figure 8

Explanatory sequential design for this study



3.4. PHASE 1: QUANTITATIVE STUDY

Phase 1 of this study (quantitative study) aimed to investigate the multidimensional nature of mathematics teachers' digital competencies, specifically the personal orientation of teachers, their instrumental genesis, and mathematical digital knowledge for teaching (MDKT). This focus was based on the modified DCTMT framework (see Section 2.4.9.2). The framework goes beyond knowledge and skills, and includes beliefs about digital technologies, making it a measure of digital competencies for mathematics teachers.

An online survey was considered an appropriate method for collecting data and investigating the various constructs of the modified framework. The original DCTMT framework (see Section 2.4.9) has previously been applied only to analyse policy documents on teachers' digital competencies. Tabach and Trgalová (2020) recommended its use in empirical studies to assess mathematics teachers' digital competencies. Online surveys are convenient research tools due to their timeliness, cost-effectiveness, and ease of administration (McInroy, 2016). Online surveys offer benefits, such as flexibility in recruitment and sampling, improved response rates, diverse design options, and automated data collection (McInroy, 2016). Another factor that particularly influenced the use of an online survey for this study was the COVID-19 pandemic, which imposed travel restrictions worldwide. According to Hillier and Harrison (2007), as cited in McInroy (2016), online surveys can help overcome geographical barriers, which was particularly relevant to this study. I was based in Hamilton, New Zealand, and was unable to travel to Pakistan due to the pandemic. Hence, the online survey method was a viable method for the recruitment of participants from different regions of Pakistan. The next section presents the account of the survey construction that involved a pilot study.

3.4.1. Structure of the online survey instrument

To represent various domains (constructs) of the framework (as shown in 2.4.9.2), I organised the online survey into six blocks. Each block represents a specific aspect of the digital competencies of mathematics teachers as shown in Table 1.

Table 1

Structure of the survey – Block number and title of each Block

Blocks	Caption of Each Block	Constructs	No. of items	Measurement Scale
Block 1	Demographic/Professional Information	Demographic	8	Multiple scales
Block 2	General Beliefs about Mathematics Teaching	Personal Orientation (P_O)	7	Likert Scale *Strongly Agree (05)* *To* *Strongly Disagree* (01)
Block 3	Beliefs about DR and their use in Mathematics Education (SDRK)	Personal Instrumental Genesis	5	
Block 4	Knowledge of DR and Students (KDRS)	Professional Instrumental Genesis	24	
Block 5	Knowledge of DR and Teaching (KDRT)			
Block 6	Knowledge of DR in curriculum planning & assessments (KDRC)			

Note. The table provides a structured layout with columns for blocks, captions (name), construct of the DCTMT framework, the number of items, and the measurement scale for each block.

Block 1 used multiple scales, and Likert scales were used for Block 2, 3, 4, 5 and 6. For example, Block 2 represents general beliefs, and it is related to the constructs "Personal Orientation (P_O)" of the DCTMT framework. There are seven items in this Block, measured on the five points Likert scale.

Table 1 shows the survey structure, offering an overview of the organization, content, and measurement scales employed in each block. Subsequent to the design of the online survey, the study progressed to the testing phase. The next section describes the pilot study which was conducted to validate the survey items.

3.4.2. Pilot study: Development and testing of the online survey

One of the important phases of a research project is a pilot study. (Z. A. Hassan et al., 2006) defined a pilot study as "a small study to test research protocols, data collection instruments, sample recruitment strategies, and other research techniques in preparation for a larger study" (p. 70). I conducted the pilot study to develop and identify potential problem areas, deficiencies, and strengths in my online survey instrument and protocols before implementing them for the main study. Two pre-existing surveys (Gencturk, 2012; Thomas & Hong, 2013) were adapted to create an online survey for the pilot study. Thomas and Hong (2013) used a survey instrument to explore mathematics teachers' attitudes towards graphing calculators (technology), a validated instrument used in several studies (Hong & Thomas, 2006; Thomas & Hong, 2005; Thomas & Palmer, 2014). For the pilot study, the word "graphing calculator" was replaced with "digital resources" (DR).

I also added nine items distributed in different Blocks aimed at assessing teachers' competencies in using several types of digital resources to solve high school level mathematics problems. For instance, items were added to Blocks 5 and 6 (see Table 2) to measure teachers' abilities in accessing, evaluating, organising, creating, and synthesising digital resources, including graphics, videos, equation editors, and dynamic graphing software (DGS) like GeoGebra. These competencies were assessed for both personal and professional use.

Table 2

Additional items related to videos, DGS and equation editors

Additional Items
I know how to write mathematical equations and expressions using an equation editor.
I can teach using dynamic graphing software (e.g., GeoGebra, Maple etc.).
I can solve geometry or trigonometry problems using dynamic graphing software.
I can use dynamic graphing software for calculus and algebra.
I want to improve my ability to teach mathematics with dynamic graphing software.
I can locate and search databases/websites containing digital resources.
I can download and install digital applications for teaching mathematics.

Moreover, as shown in Table 1, the survey for the pilot study was divided into six blocks. Except for Block 1, each block consisted of items agreement with which was measured on a five-point Likert scale ($5 = \mathit{Strongly\ Agree\ to}\ 1 = \mathit{Strongly\ Disagree}$). Block 1 was designed to gather general demographic and professional information from participants, using a range of categorical variables (survey items). A categorical variable is defined as having two or more categories, but there is no order to the categories (Agresti, 2010). These variables collected information about gender, current professional role (government, private, and online teacher), highest academic qualification, teaching qualification, teaching level (classes), and teaching experience (number of years). To ensure participants understood the meaning of terms used in the survey items, each block started with a brief description and definition of digital resources. The survey for the pilot study was developed and administered using Google Forms. An open-ended text input field was included at the end of each block for participants to provide feedback on the survey items.

3.4.2.1. Pilot study methodology

The methodology for the pilot study involved an analysis of four key aspects: research objectives, employed methods, obtained results, and necessary follow-up actions for the main study. I conducted the pilot study at the University of Waikato over three months from August 2020 to October 2020. The recruitment process employed online and in-person strategies to invite participants. The online recruitment approach involved sending survey links to 35 individuals, comprising 33 teachers and two proofreaders. This method yielded a response rate of 31%. Additionally, an in-person, paper-based workshop targeted seven pre-service and in-service mathematics teachers, achieving a response rate of 100%. The response rate for the pilot study was 36%, with the majority (87%) of participants held postgraduate qualifications, and 70% being certified teachers. The selection criteria for participants included mathematical content knowledge, experience in teaching with technology, and proficiency in editing and proofreading. 45% of the participants had familiarity with the Pakistani educational context. To evaluate the quality of the survey items, I followed the protocol proposed by (Z. A. Hassan et al., 2006), where participants were invited to provide feedback on item clarity, definition, and logical presentation. This feedback, gathered through text input fields at the end of each survey block, facilitated a comprehensive analysis and offered valuable insights for refining survey items. These findings highlighted the feasibility of survey protocols, the effectiveness of quality measurement strategies, and provided essential insights for optimizing the survey instruments in preparation for the main study.

3.4.2.2. Results and actions taken for the main study

I made several changes to survey items based on the results of the pilot study. These changes included rewording of item statements, adding new survey items, and deleting items from the survey (for summary of changes see Appendix F). The changes were aimed at improving the survey's validity and ensuring that it could measure the digital competencies of high school mathematics teachers.

In Block 1, participants provided feedback regarding access to different types of digital technologies, the Internet, and the terminology used for specific qualifications. Participants noted that due to variations in socio-economic status among rural and urban teachers in Pakistan, it would be important to know the types of digital devices teachers can access at school or at home. In response, I added three items (access to digital devices at home or work, frequently used digital resources, and availability of the Internet) to Block 1 to collect data regarding these factors. The two items "access to digital devices" and "frequently used digital resources" also included the text input option "Other" that collected information not otherwise listed.

Additionally, some participants raised concerns about the terminology used to collect professional information from teachers in Block 1. To address this, I modified the items related to academic and teaching qualifications to be more understandable in the Pakistani context. For example, I replaced the options of "Graduate" and "Postgraduate" with the abbreviations commonly used in Pakistan: "B.Sc./B.A." and "M.Sc./M.A.". Similarly, I replaced "Are you a qualified teacher?" with "Do you have a teaching qualification?" and provided the common terms used in Pakistan: "B. Ed" and "M. Ed".

For most participants, "dynamic graphing software" (DGS) was likely to be an unfamiliar term. For example, one participant asked, "What is dynamic graphing software? If possible, add the name of the software." Therefore, depending on the context the term dynamic graphing software was elaborated with "e.g., GeoGebra." Although the abbreviation DR was defined at the start of each survey Block, participants struggle to recall the meaning. Therefore, for the main study, the term digital resources (DR) were defined as "digital files/content that can be used to support teaching and learning of mathematics including multimedia objects (audio, videos, images, and graphics), simulations, digital games and dynamic graphing software."

For a summary of all changes, see Appendix F, which highlights changes that I made to the blocks of the online survey. For instance, a participant was unsure about the meaning of "practical activities" in the item "Mathematics teaching is to evaluate and share knowledge and ideas and to discuss a variety of practical activities and real-world contexts." For the main study, the term was deleted, and the item was reworded. Another participant pointed out that the word "diverts" in the item, "The use of DR to teach mathematics diverts the attention of students," was inappropriate. As a result, the item was reworded for the main study to read, "Digital resources distract the attention of students from doing calculations."

3.4.2.3. Addition of COVID-19 related items

The pilot study was conducted during the COVID-19 pandemic, which led to the closure of schools and the shift to online teaching. As a result, many participants suggested adding items related to COVID-19 and online teaching strategies. Given that video-conferencing, screen sharing, screencasting, and breakout rooms were widely used by teachers during this time (Ferdig et al., 2020), I considered this feedback important. Therefore, for the main study, I added three Likert scale items and one open ended question (total four items) related to the COVID-19 teaching and learning strategies in Block 5 (as shown in Table 3).

Table 3

Added Items related to COVID-19 and online teaching strategies

Block 5 Items related to COVID-19 teaching strategy and digital Tools.	I know how to use video conferencing applications (e.g., Zoom, Skype etc.).	Likert Scale Strongly Agree (05) To Strongly Disagree (01)
	I know how to do screen recording, share screens and resources during video conferencing.	
	If required, I can teach mathematics in a fully online environment	
	Please tell us briefly about your COVID-19 teaching strategy. What tools did you use (or using) for teaching mathematics during COVID-19?	Qualitative Item (descriptive)

Further, I moved the survey for the main study from Google Forms to Qualtrics, which offered advanced options for creating a logical flow of survey items. Qualtrics allows respondents to save their work and return to it later. It also enables survey questions to be integrated with other data sets. The use of Qualtrics improved the overall efficiency and the distribution of the online survey.

Overall, the findings and improvements of the pilot study provided essential suggestions for the main study survey. The final online survey is in Appendix I.

3.4.3. Recruitment of participants for the main study

Once the feedback was incorporated and the survey was updated, the recruitment of participants for the main study began. In Phase 1 of the study, high school mathematics teachers from Pakistan were targeted, with the aim of collecting data from 200 to 300 teachers. To ensure ethical considerations were upheld, all participants in the study were informed about the research purpose and provided with a consent form at the start of the online survey. Additionally, the study collected only non-sensitive data, and participant anonymity was maintained throughout the study. I also provided my email address at the end of the survey for those who wished to seek more information about the study and wanted to volunteer for Phase 2 semi-structured interviews (see Appendix I).

3.4.4. Distribution of the online survey

The online survey was distributed through email and social networking platforms such as Facebook, WhatsApp Groups, and LinkedIn. Social media platforms provide an information-rich opportunity to reach targeted populations that would otherwise be difficult to identify (Franz et al., 2019). The criteria for selection on LinkedIn was to search and invite members who had keywords "mathematics AND high school AND teachers AND Pakistan" in their profile description. Facebook and WhatsApp are dominant players in the social media landscape and have the most users in Pakistan. Therefore, in February 2020, I created a dedicated Facebook Page[3] to reach a country-wide target population and to get maximum responses. The page named "Mathematics Teachers Pakistan" drew 570 followers to the page (in 10 months)

[3] https://www.facebook.com/profile.php?id=100063918760904

before the online survey started. I released the survey on Facebook on 10 December 2020. Initially, I allowed the survey post to reach the audience organically. Organic reach is the number of people who see the content without paid distribution. Once the survey stopped receiving organic responses it was boosted (paid promotion) for two weeks. The Facebook results showed, the survey post reached 11,396 pre-defined audiences of which 489 were post engagements. The post engagements included reacting to the post (Liking), commenting on, or sharing the survey post and clicking on the survey link. Figure 9 shows how the Facebook displayed the online survey post in different feeds.

Figure 9

The online survey post display feeds on Facebook

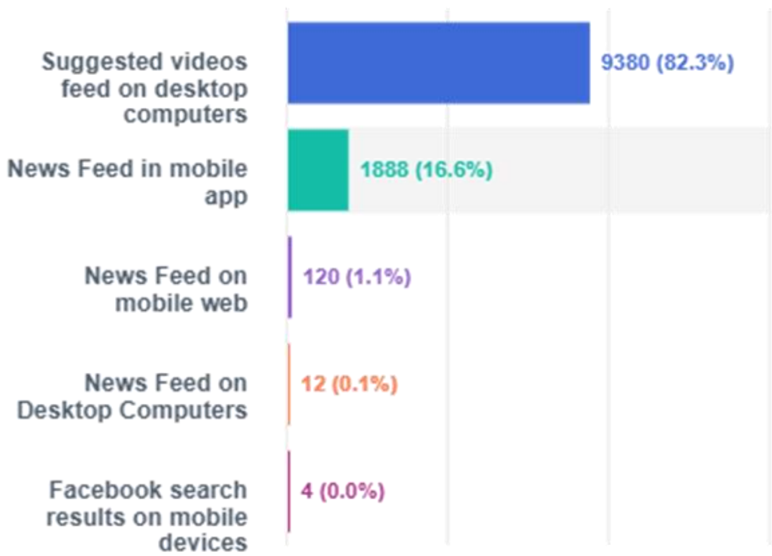
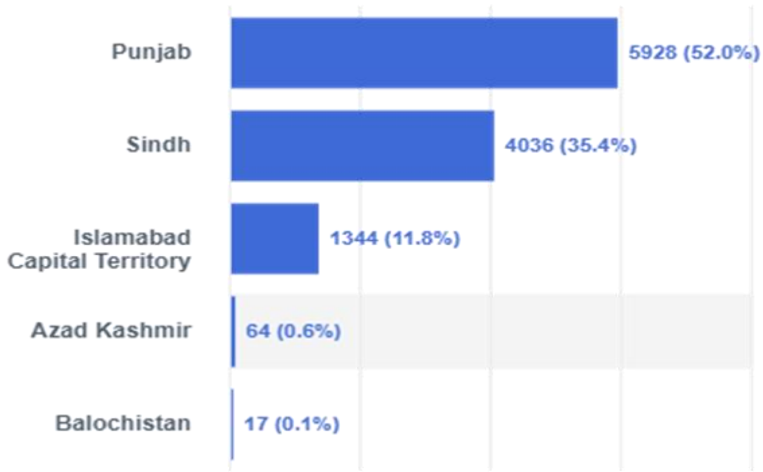


Figure 10 shows more than half (52%) of the Facebook audience were in Punjab, which is the largest province of Pakistan in terms of population. 35.4% were in Sindh and the remaining audiences were located across the rest of the country.

Figure 10

Location wise distribution of survey post on Facebook



3.4.5. Survey response

Across all social media platforms (Facebook page, LinkedIn, WhatsApp groups) and email, the survey received 306 responses in total. Responses were screened using the "completion rate" criterion i.e., responses with less than 10% completion rate were excluded. Following the screening process, I selected 270 cases for analysis.

3.4.5.1. Outliers in the online survey data

Once the screening of the responses was completed, I scanned the data for outliers to identify responses that appeared to deviate markedly from other observations in the sample. Identification of potential outliers was important as outliers can affect variance, and standard deviation of the data distribution (Gupta, 2002). I examined outliers using Mahalanobis Distance (MD), to determine whether a participant response belonged to a group or not. The test applied to 36 items through Linear Regression Analysis in SPSS. The results indicated the presence of outliers. To confirm, the MD values were compared with Chi-square distribution with the same degree of freedom (i.e., $df = 36$, total number of items/predictors). The Probabilities which belonged to six cases were significant ($p < 0.001$), confirming them as outliers. Further, to examine which items contained outliers, Boxplots and Kurtosis were obtained. The Boxplots and Kurtosis showed four items

($B5\_6, B5\_9, B5\_11, and\ B5\_12$) contained outliers at the lower end of the scale with values of 2 (Somewhat disagree).

Table 4

Summary of items with outliers

Item No.	Items statement	Missing values (n)	Kurtosis	Err
B5_6	I can use dynamic graphing software for calculus and algebra.	129	3.94	0.41
B5_9	I want to improve my ability to teach mathematics with dynamic graphing software (e.g., GeoGebra).	0	6.17	0.30
B5_11	I know how to screencast, share screen and resources during video conferencing.	19	8.78	0.31
B5_12	If required, I can teach mathematics in a fully online environment (e.g., School closure during COVID-19).	21	7.71	0.31

Note. Table 4 consists of five columns. The first two columns contain item number and statement. The third column lists the number of missing values (n) for each item. The fourth column Kurtosis and the fifth column shows Error calculated.

Table 4 shows high Kurtosis values for all four items $B5\_6, B5\_9, B5\_11$, and $B5\_12$, indicating that outliers were present in items belonging to Block 5. This Block pertained to teachers' knowledge of digital resources for teaching (KDRT) mathematics. In addition, these items had a higher proportion of missing values than other items in the survey. I retained the outliers, as they may provide valuable insights; further analysis of each block determined the potential impact of these outliers on the study's outcomes. In the following section, I elaborate on the presence of missing values and the measures taken to address them.

3.4.5.2. Missing values in the data set

The results of the search for outliers highlighted the presence of missing values. Therefore, I scanned the data to discover missing values' pattern, location, percentage, and possible reasons. Missing values potentially affect the validity, reliability, and generalizability of the results (Jakobsen et al., 2017). Missing data are usually accounted for by participants either not answering a question or submitting their response before they completed the survey. The SPSS 27 procedures identified patterns of missing data. In total 142 out of 270 cases (52.5%) were incomplete. The missing values were concentrated in block 5 that was designed to measure teachers' knowledge of digital resources and teaching (KDRT). Table 5 shows the list of items having missing values as more than 5%. Notably, items $B5\_5$ and $B5\_6$ had more than 45% of values missing. It shows that almost half of the participating teachers missed the items related to dynamic graphing software (DGS). However, the information did not tell me what caused the missing values.

Table 5

Summary of items with missing values greater than 5%

Item No	Item	Missing values	
		n	Percent %
B5_4	I can teach using dynamic graphing software (DGS)	25	9.3
B5_5	I can solve geometry or trigonometry problems using DGS	122	45.2
B5_6	I can use DGS for calculus and algebra.	129	47.8
B5_11	I know how to screencast (screen recording), share screen and resources during video conferencing.	19	7
B5_12	If required, I can teach mathematics in a fully online environment (e.g., School closure during COVID-19).	21	7.8

Note. Table 5 consists of four columns. The first two columns contain item number and statement. The third column lists the number (n) of missing values for each item and the fourth column presents percentages (%).

Rubin (1977) first explained that data could be missing for three reasons: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). According to Rubin (1977), when the missing values do not depend on either observed (data we have) or missing data, it is MCAR. Normally, five percent or less of the missing values is MCAR. Second, if the probability of missing values depends only on observed data, then the missing value is MAR. When the missing values depend on both the observed and the missing data, it is MNAR. There are some statistical tests that a researcher can perform to understand the reasons for missing values. I used separate-variance t-Tests for my study.

The test helps to identify items whose pattern of missing values may be influencing the other items (Zimmerman & Zumbo, 2009). I found that teachers who answered items related to digital graphs and equation-editors were most likely to have answered items on dynamic graphing software (DGS) and video conferencing. Further, I cross-tabulated demographic items (gender) versus items with missing values. The cross-tabulation showed that the female teachers were more likely to miss items related to DGS than male teachers.

These results indicated that instead of the data being missing completely at random (MCAR), there may be a relationship between missing values and observed data. In which case, the missing values could be MAR. To test this assumption, I conducted Little's MCAR (chi-square) test, in which an insignificant value shows that the missing data is MCAR (Howell, 2018). The test results were statistically significant ($Chi-square = 1282.818, DF = 646, Sig. = .000$), which rejected the assumption of MCAR. The results showed that there may be a relationship between the missing values and the observed data. However, it is difficult to say for certain. The missing values pattern could have a relationship with the observed data, not with the missing data. To conclude, the data may be missing at random (MAR) given teachers' demographic and professional background. Female teachers with less teaching experience, academic qualifications and information technology skills are likely to skip items related to dynamic graphing software.

When not addressed, high proportions of missing data can introduce bias in the analysis. Therefore, the next stage was imputation. In statistics, imputation is a procedure for entering a value for a specific data item where the response is missing or unusable (United Nations, 2000). There are many ways a researcher can impute missing values, including listwise or pairwise case deletion, *Mean* substitution, maximum likelihood (ML), and multiple imputation (MI).

The first three methods are preferred when missing values are MCAR (5% or less), but ML and MI techniques are preferred for MAR. I preferred MI over ML for my dataset because unlike other techniques, MI replaces missing values with a "set of plausible values which contain the natural variability and uncertainty of the right values" (Kang, 2013, p. 405). I also found that MI produced more accurate estimates than the other methods (Leite & Beretvas, 2010). This does not imply that the other methods are inferior, but that I found the MI results were more representative of my dataset.

The MI procedure was performed (via SPSS 27) by selecting the "automatic imputation method." All items were included in the imputation procedure. To obtain values within the range of the measurement scale ($1 = strongly\,disagree$, $5 = strongly\,disagree$) the imputation results were constrained ($range$; $1 = \min and\ 5 = max$) and rounded-off to 1 digit (integer values). SPSS filled in the missing values, created five different imputations (data sets), and pooled the data by combining the five imputations. All datasets were analysed using typical complete-case methods (linear regression). The method produced descriptive results, correlations, and model fit summary for all the data sets. Table 6 shows the Mean values of original versus pooled data.

Table 6

Comparison of Mean scores Original versus Pooled data

Item Code	Item Statement	Original		Pooled
		Mean	SD	Mean
B4_2	Students should not be allowed to use digital resources until they have mastered the concept, rule, or method.	3.76	1.25	3.76
B4_3	Digital resources allow students to creatively think about mathematics problems.	4.48	0.71	4.48
B5_1	I have received training for the use of technology and other digital resources.	3.25	1.41	3.25
B5_3	I know how to write math equations and expressions using an equation editor.	4.27	1.05	4.27
B5_4	I can teach using dynamic graphing software (e.g., GeoGebra).	3.69	1.25	3.65
B5_5	I can solve geometry or trigonometry problems using dynamic graphing software.	4.21	0.88	3.90
B5_6	I can use dynamic mathematics software for calculus and algebra.	4.48	0.75	4.12
B5_7	I teach mathematics better when my students use pen and paper for solving problems.	4.34	0.74	4.35
B5_11	I know how to screencast (screen recording), share screen and resources during video conferencing.	4.61	0.82	4.56
B5_12	If required, I can teach mathematics in a fully online environment (e.g., School closure).	4.67	0.62	4.64

Note. Table 6 contained five columns. The first two columns present item number and statement. The third and fourth columns represent Mean value and standard deviation (SD) of original data. The last column presents Mean value of items (pooled data) after MI procedure.

The rows in grey (item $B5\_4, B5\_5, B5\_6, B5\_11$ and $B5\_12$) represent items whose percentage of missing values were more than 5% (see Table 5).

After imputation, the scale-reliability score (value of Cronbach Alpha) of the survey items improved (see Section 3.4.6.2). However, the Mean scores of three items related to DGS slightly decreased after imputation. The change was expected due to a considerable proportion of missing values (47%) in items related to DGS. The drop did not impact the interpretation of the items' results. The Mean scores of the original data showed teachers somewhat agreed that they can use DGS for teaching geometry, calculus, and trigonometry, which can be interpreted as being the same for the complete dataset after MI. Therefore, the items with the missing values were filled in by imputed values using the Multiple Imputation (MI) method so that a complete set of data could be used for the analysis.

The next section presents reliability statistics for the online survey instrument.

3.4.6. Validity and reliability of online survey instruments

The assessment of any research instrument, including online surveys, is a critical step in ensuring the credibility and trustworthiness of the data collected. This section explains the validity and reliability of the online survey instrument used in this study. The evaluation of the survey's validity and reliability is essential to determine whether it accurately measures what it intends to and whether it produces consistent results over time (Taber, 2018). In this section, I present the approaches employed to ascertain the validity and reliability of the online survey instrument.

3.4.6.1. Validity of the instrument

For the online survey I modified and expanded "The Attitude Questionnaire" (Thomas & Hong, 2013) and used a few items from "Teacher Beliefs Survey" (Gencturk, 2012) to measure the digital competencies of high school mathematics teachers. Previous studies (Hong & Thomas, 2006; Thomas & Hong, 2005, 2013; Thomas & Palmer, 2014) have used The Attitude Survey and its validity is well established. These studies have used the survey to evaluate the pedagogical technology knowledge (PTK) of mathematics teachers. To further ensure content and construct validity, I made modifications to the instrument by including additional items associated with digital resources and their use in mathematics teaching and learning. These modifications included adding items related to videos, dynamic graphics software (DGS), online learning environment, database, and online assessment, which aimed to add components of digital resources and measure the beliefs and attitudes of mathematics teachers regarding their use. The five constructs of the framework (personal orientation, SDRK, KDRC, KDRS, and KDRT) were measured using a Likert scale of 1 to 5. In addition, the survey was piloted with 42 teachers (the majority of them were mathematics teachers) to further validate the items. The final survey instrument, therefore, consisted of close-ended questions on the Likert scale and one open-ended question. The open-ended question was placed to record participants' observation related to the survey. This provided a valid instrument that measured mathematics teachers' use of digital resources and reflected their digital competencies.

3.4.6.2. Reliability of scores on the survey instrument

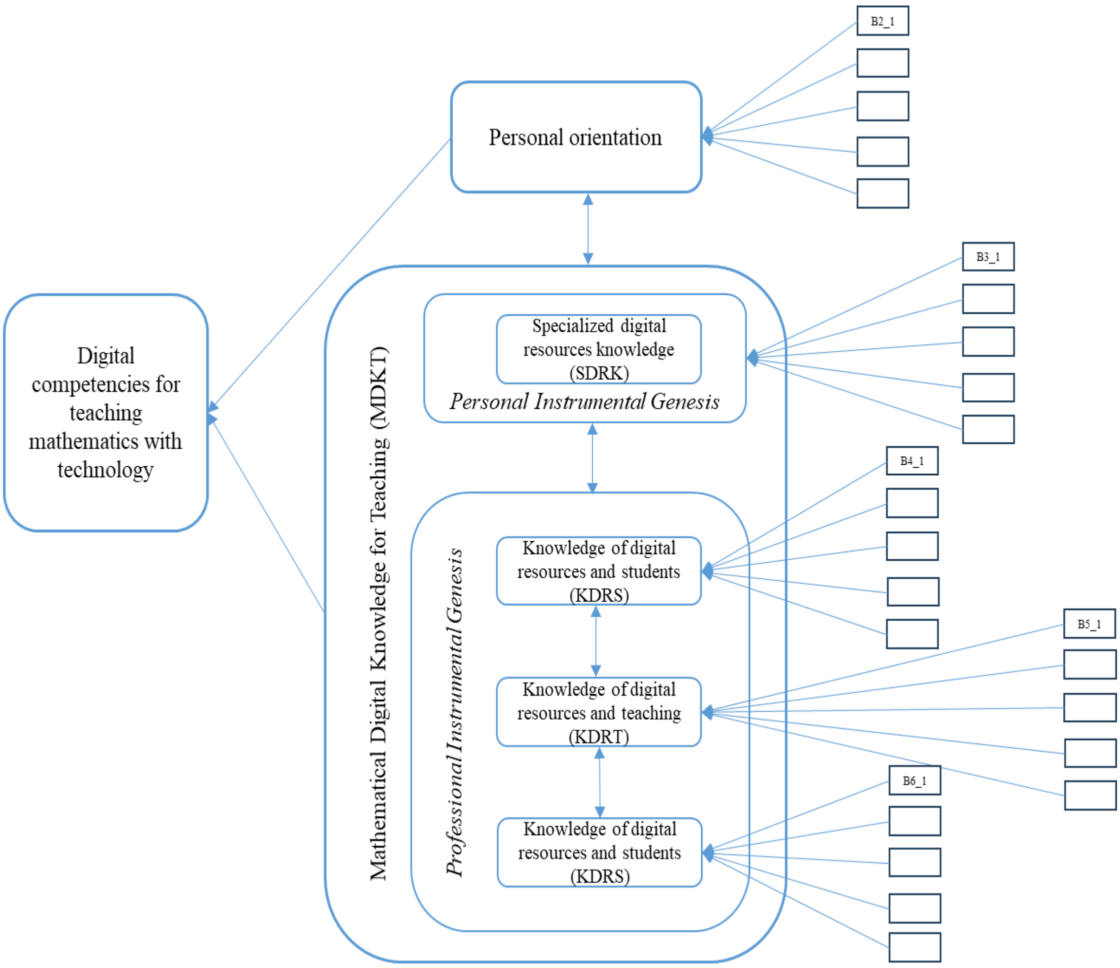
As shown in Section 3.4.1 (see Table 1, p. 108), the survey instrument contained five blocks measuring the constructs of the modified DCTMT framework. Altogether 36 items were distributed across the five blocks, which made the survey a multi-item instrument. The most important form of reliability for multi-item instruments is the instrument's internal consistency. It is the degree to which sets of items on an instrument behave in the same way (Taber, 2018). Internal consistency helps to understand whether the instrument scale items are assessing the same underlying construct or not, i.e., there exist acceptable intercorrelations between the survey items (Taber, 2018). I measured internal consistency using Cronbach's alpha (α), commonly used by social and educational scientists for scale reliability (Ghomi & Redecker, 2019; Taber, 2018; Touron et al., 2018). Alpha value ranges between 0 and 1, with optimal values ranging between 0.7 and 0.9. The value of Cronbach's Alpha for 36 survey items was $\alpha = 0.85$ suggesting that the items have high (good) internal consistency, and the scale is reliable.

3.4.7. Survey quantitative data analysis

The quantitative data were collected using items distributed among five constructs as shown in Figure 11. I analysed the collected data by using IBM SPSS Statistics 27 for Windows Software and used Microsoft Excel for illustrations and graphs. The SPSS is a professional software package widely used to do descriptive and inferential statistics and it was available through the University of Waikato IT services. Further, I had previous experiences of using SPSS to calculate descriptive and inferential statistics.

Figure 11

Proposed model for Phase 1 quantitative data analysis



3.4.7.1. Descriptive statistics

I used descriptive statistics to analyse the quantitative data collected via the online survey. Descriptive statistics were used to describe, present, summarise, and organise data through numerical calculations, graphs, and tables where necessary, including normal distribution, measures of central tendency (Mean, Mode, Median), and standard deviation. To examine relationships within the data that may not be readily apparent, I performed cross-tabulation (known as crosstab), a statistical tool used for categorical data. For example, I cross-tabulated the Block 1 item "access to digital devices" with "professional roles" to determine the percentage of government schoolteachers had access to various numbers of devices when compared to all other types of teachers. I also cross-tabulated the data for Internet access with professional roles to examine what percentage of different types of teachers had regular access to the internet.

3.4.7.2. Inferential statistics

I used inferential statistics to understand the complex mathematical calculations. Inferential statistics allow researchers to infer trends and make assumptions and predictions about a population based on a study of a sample taken from it (Kalish & Thevenow-Harrison, 2014). I used correlation, which is a bivariate analysis that measures the strength of association between two variables and the direction of the relationship (Mukaka, 2012). The correlation coefficient ranges between $+1$ and -1, with a value of ± 1 indicating a perfect degree of association between the two variables. The closer the value of the coefficient is to 0, the weaker the relationship between the variables. A positive coefficient suggests a positive relationship, while a negative coefficient suggests a negative relationship. I used Spearman's rho (ρ) correlation test. The Spearman's rho test measures the strength of association between non-parametric random variables (Schmid & Schmidt, 2007) and correlates unvalidated survey instruments or Likert-scale (ordinal) survey responses. The two assumptions for Spearman's rho were satisfied. First, the scores were measured using an ordinal scale[4]. Second, the relationships between items were monotonic, i.e., the value of one item increases, so does the value of the other item. The Bivariate Correlations procedure computed the pair-wise associations for items and displayed the results in a matrix (for example see Table 12, p. 145).

[4] A measurement scale that uses labels to classify cases into ordered classes, such as the Likert scale (strongly agree to strongly disagree.

Moreover, to understand the complex relationships of the modified DCTMT framework's constructs, I used correlations in two ways: Intra-construct and inter-construct correlations.

1. Intra-construct correlations were examined to explore the relationships between the different items within each individual construct. This provided insight into how the teachers' knowledge and skills that make up each competency area interacted with each other.

2. Inter-construct correlations were also computed to investigate relationships across the different overarching constructs. This highlighted how knowledge and skills in one construct (e.g., SDRK) may potentially influence or relate to another (e.g., KDRS).

3.5. PHASE 2 (QUALITATIVE)

Phase 2 of the study collected qualitative data using interviews. There are three common types of interviews: structured, semi-structured, and narrative. The primary difference between them is the amount of control the interviewer has over the encounter and the aim of the interview (Stuckey, 2013). Structured interviews are sequential and specified, whereas semi-structured interviews are oriented but flexible in order to be able to adapt to participants' responses. Narrative interviews are unstructured and usually start with a broad, open-ended question about a participant's experience, where the participant is rarely interrupted in telling their story (Stuckey, 2013).

For this study, I used semi-structured interviews (for sample interview questions, see Appendix E) to understand the digital competencies of high school mathematics teachers in Pakistan. A semi-structured interview is the type of interview which is most commonly used in qualitative research, and many studies illustrate its use in mathematics education research (Siedel & Stylianides, 2018; Thomas & Hong, 2013). The study followed a sequential design (as shown in Figure 8, p. 106), therefore, questions were added to understand quantitatively significant results, outliers and perplexing Phase 1 results. The semi-structured interviews facilitated collecting information regarding beliefs, preferences and perspectives in relation to using digital resources in mathematics teaching and learning. Along with the overarching research question (RQ), the following sub-questions guided the Phase 2 investigation.

SQ1: How do Pakistani teachers' beliefs about mathematics teaching influence their decisions to integrate digital resources in teaching and learning mathematics?

SQ2: What influences teachers to use digital resources well?

SQ3: How well do Pakistani teachers use digital resources to teach mathematics topics?

SQ4: What professional development opportunities do Pakistani teachers have that influenced their digital competencies and approach to integrating digital resources?

SQ5: What are the barriers and enablers to teachers using digital resources face-to-face and online?

3.5.1. Recruiting teachers for semi-structured interviews

Of 306 survey participants, 11 submitted their contact information and chose to volunteer for individual face-to-face online semi-structured interviews (up to one hour). I contacted these 11 teachers to further discuss the project details and request profiles. The aim was to identify 5 to 10 high school mathematics teachers with varied years of teaching experience from both government and private schools in Pakistan's rural and urban areas.

The primary focus was on recruiting specialist mathematics teachers with prior experience in using DR in their teaching. In the consent form, I clearly mentioned that the non-specialist mathematics teachers who have plans to use DR but are not currently using DR may not be considered (see the Participant Information Letter Appendix C). Based on the screening criteria, I selected eight teachers. Of these six replied and provided their consent, while the remaining two did not respond. The six teachers were assigned codes T1 to T6 (as shown in Table 7, p. 126). Due to COVID-19, the semi-structured interviews were conducted and recorded online using the Zoom video conferencing application.

Table 7

Profile of six teachers participated in semi-structured interviews

Code	Gender	Teaching Level	Teaching Qualification	Experience	Area	Government /Private
T1	Male	High School	No	5 yrs	Rural	Private
T2	Male	High School	No	20 yrs	Urban	Government
T3	Female	High School	Yes	4 yrs	Urban	Private
T4	Female	High School	Yes	8 yrs	Urban	Government
T5	Male	High School	Yes	13 yrs	Rural	Government
T6	Male	High School	No	4 yrs	Urban	Government

3.5.2. Qualitative data analysis

The qualitative data analysis began with transcribing the semi-structured interviews verbatim, followed by seeking participants' consent for analysis. I analyzed the semi-structured interview data using the pre-identified themes and sub-themes based on the multi-construct digital competencies framework. To ensure a clear understanding of pre-existing themes, a coding guide (Table 8) was created that included descriptions and definitions. This guide was used to identify relevant text segments during the coding process. Each relevant text segment was coded under the appropriate pre-existing theme, and the coded data were reviewed for accuracy. Table 8 shows the themes, sub-themes, and descriptions used as deductive codes for conducting a content analysis of semi-structured interview data. It is important to note that the same deductive codes were also used to design the survey items used in Phase 1. The Phase 2 semi-structured Interviews covered three main areas:

1. Teachers' general beliefs towards mathematics teaching, relevant to the use of digital resources.
2. Teachers' familiarity with theories about students' learning that relate to using digital resources. Teachers' ability to search, select, evaluate, integrate, create, download, and share digital resources best suited to their students' and peers' needs. Teachers' ability to evaluate the resources' efficiency and appropriateness in the context of mathematics education.
3. Understanding the benefits, enablers, and barriers in using of digital resources.

Table 8

Description of pre-identified themes and sub-themes to code interview data

Main Concept	Themes		Sub-themes	Description of themes and sub-theme to code interview data	
Digital Competencies	PO		Personal Orientation	Teachers' beliefs, values, emotions or attitudes relevant to ICT integration. General beliefs about pedagogical aspects such as memorization, signs, symbols ands notations in mathematics	
	MDKT	SDRK	SDRK Specialized digital resources knowledge	Knowledge of: how to integrate global content into curriculum; Operate various packages appropriate to their subject matter area, such as visualization, data analysis, role-play simulations, and online references; facilitate technology-enhanced mathematical experiences that foster creativity; Use ICT resources to support their own acquisition of subject matter and pedagogical knowledge. Understanding "how digital developments are changing and expanding the content of subjects"	
		Digital Pedagogical resources knowledge	KDRT Knowledge of DR and Teaching	•To structure the lesson so that different (teacher-led and learner-led) digital activities jointly re-enforce the learning objective. •To set up learning sessions, activities and interactions in a digital environment. •To structure and manage content, collaboration and interaction in a digital environment. •To consider how educator-led digital interventions—whether face-to-face or in a digital environment—can best support the learning objective. •To reflect on the effectiveness and appropriateness of the digital pedagogical strategies chosen and flexibly adjust methods and strategies.	Teachers' ability to search in digital repositories for suitable resources, to select those resources best suited to their students' needs, to create new digital resources themselves or with their team members, to share their resources with their peers, as well as to evaluate the resources' efficiency and appropriateness with respect to the learning
			KDRS Knowledge of DR and Students	•Teachers familiarity with theories about students' learning that take into account the use of ICT. Such familiarity will allow teachers to develop a detailed understanding of how young people learn, and in particular provide them with a clear understanding of the role of the teacher in leading this endeavour. •To set up learning activities in digital environments, having foreseen learners' needs for guidance and catering for them. •To assist learners in identifying areas for improvement and jointly develop learning plans to address these areas. Understanding "what pupils' digital skills entail and how they can be fostered in the subjects"	
			KDRC Knowledge of DR in curriculum planning & assessments	Teachers familiarity with the curriculum and assessment procedures in digital environments and at the same time must understand content and teaching in these environments.	
	Instrumental geneses		Personal instrumental genesis	•Teachers need to develop "confidence to embed ICT more into their practice. Include teachers' competencies: an ongoing process that refers to what teachers can do with technology for their own needs. How they use resources for communication and collaboration. •Teachers to develop their own digital skills. Expected to have already developed basic digital skills.	
			Professional instrumental genesis	•Include teachers' competencies for their students to benefit from ICT as an integral part of their learning. •Teachers must search for digital resources, select the appropriate resources based on pedagogical •and mathematical considerations, and create documents to be used by the students in class. •To plan for and implement digital devices and resources in the teaching process to enhance the effectiveness of teaching interventions. •To appropriately manage and orchestrate digital teaching strategies. •To experiment with and develop new formats and pedagogical methods for instruction. •Professional development must continue throughout teacher's professional life. •Teachers must develop their own professional digital competence during their initial teacher education, and later, during their teaching career.	

3.6. TRUSTWORTHINESS AND THE RIGOR OF THE STUDY

Trustworthiness in qualitative research and rigor in quantitative research are vital aspects ensuring the reliability and validity of collected data (Stahl & King, 2020). Trustworthiness pertains to the quality and credibility of qualitative data, emphasizing the need for confidence in the accuracy and authenticity of the information. Rigor, on the other hand, focuses on the accuracy and precision of quantitative data, ensuring that measurements and findings are robust and dependable (Wong & Cooper, 2016).

As presented in Section 3.4.6, for the quantitative phase, I ensured rigor through the utilization of reliability statistics (Wong & Cooper, 2016). Moreover, the online survey was adapted from a well-established instrument, which was piloted before using it for the main study. To ensure accuracy, I screened data, identified outliers (Section 3.4.5.1) and addressed the issue of missing values using established procedures (Section 3.4.5.2). I employed both descriptive and inferential statistics during the analysis, allowing for a thorough examination of the data and facilitating a nuanced understanding of the quantitative findings. To enhance trustworthiness in this study, I triangulated the overall findings and maintained reflexivity throughout the research process.

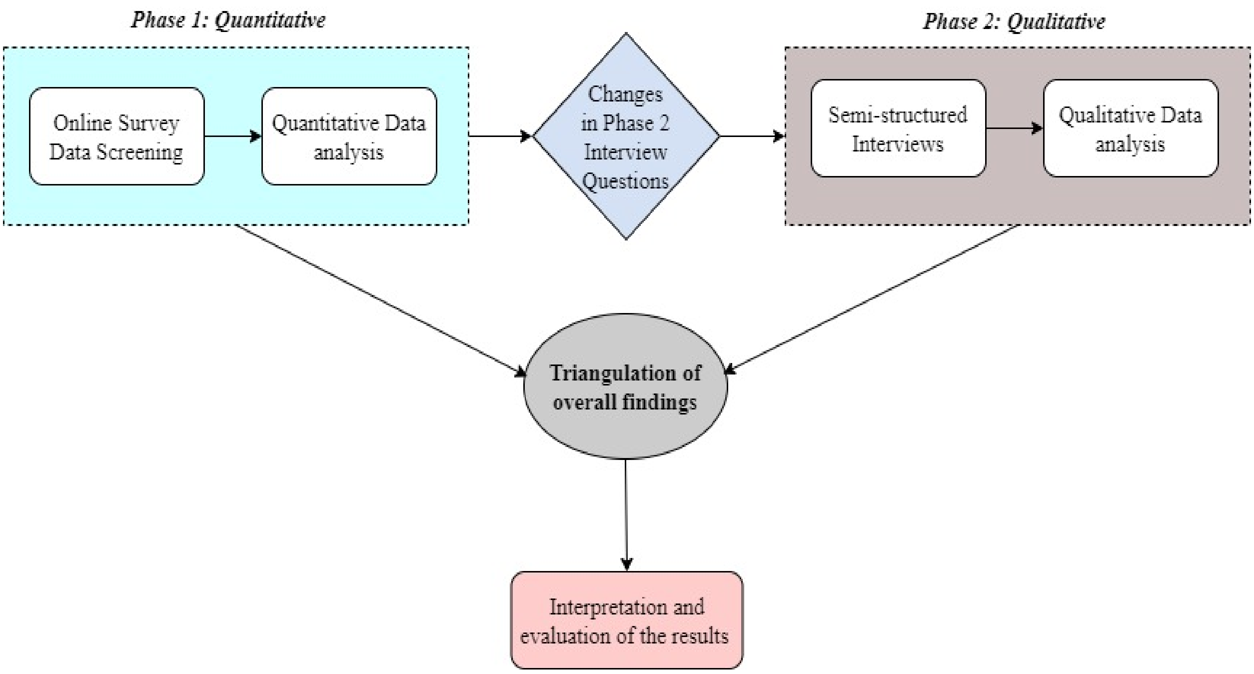
3.6.1. Triangulation of the overall findings

Triangulation, a strategy to enhance the validity and comprehensiveness of study findings, is a key methodological approach in mixed-methods research. (Mok & Clarke, 2015) broadly define triangulation as using different research methods in the study of the same phenomenon. In this explanatory sequential design, where I collected and analyzed quantitative data (Phase 1) followed by qualitative data (Phase 2), I applied triangulation, I compared the results from Phase 1 with the results of Phase 2. That is, I combined quantitative data from an online survey and qualitative data from semi-structured interviews to gain a deeper understanding and interpretation of the digital competencies of high school mathematics teachers. This combined approach contributed to the overall rigour and robustness of my mixed-methods study. Triangulation also provided a comprehensive perspective on the research phenomenon and enriched the interpretation of the research results.

Figure 12 shows how I used triangulation to corroborate and validate the overall findings.

Figure 12

Model of triangulation for interpretation of Phase 1 and Phase 2 data



Using the model in Figure 12, first, I examined the areas of convergence and divergence between my quantitative and qualitative data. The aim was to look for patterns, similarities, and differences in teachers' responses as reflected in both sets of data. This enabled the validation of my findings. For instance, I compared the Mean values from the quantitative data with the themes and narratives emerging from the qualitative responses. I looked for alignment or any contradictions to strengthen the credibility of my research findings. Second, I explored the data for anomalies or unexpected findings. For example, a few items in the survey reported high Mean values. I specifically looked for a disconnect between the high Mean values in the quantitative data and negative sentiments in the qualitative responses and screened the results with the support of the literature to gain a deeper understanding of the reasons behind this disparity. These steps enhanced the interpretation of the data and facilitated drawing a complete picture of teachers' digital competencies, highlighting both the commonalities and variations across different contexts and experiences.

3.6.2. Reflexivity

Reflexivity is a way of attending systematically to the context of knowledge construction, especially to the impact of the researcher and their positionality, at every step of the research process (Corlett & Mavin, 2018). To ensure rigorous research procedures, the researcher must acknowledge his epistemological identity, which refers to an individual's understanding of their own knowledge-related beliefs, values, and perspectives (S. J. Hegedus, 2010). It involves how a researcher perceives and defines knowledge, the nature of truth, and the methods of acquiring knowledge. Hegedus (2010) provides the following set of questions that offer a non-exhaustive account of generic lines of knowledge inquiry, which might be made:

- Can I realise my position in the process of research?
- Can I react to my position in this process?
- Have I answered the questions I have deemed necessary to be established?
- What is the nature of the knowledge I have established?
- How does my knowledge exist?
- Are my questions convincing?
- What are my questions?
- What are the assumptions made about this knowledge?
- Are my answers convincing? Do they use a suitable methodology? Have I assessed my data collection?
- How does my epistemological identity affect my new knowledge?
- Have I assessed the limitations of the new knowledge I am generating?

I took deliberate steps to manage and reduce any personal biases during the research process. I made sure to remain objective and impartial throughout the entire process. As acknowledged in Section 1.2, I had a background in mathematics education, digital resources, and curriculum development that could influence the research trajectory. This recognition prompted a transparent discussion with the supervisors about the potential impact on research questions, data analysis, and findings interpretation. These discussions were crucial in identifying and mitigating potential biases. To further ensure objectivity, my supervisors independently reviewed and validated 20% of the coding for the semi-structured interviews. Additionally, I have made conscious efforts to keep the principle of epistemological identity at the forefront of my research. I examined my biases, assumptions, and perspectives. Being reflexive helped me avoid letting my personal orientation affect the research process. For instance, this was important during the complex decision about changing the term “content” to “resources” for the mathematics teachers' digital competencies framework. Although my previous experience of digital content development and dealing with digital resources provided direction in making these decisions, I relied on the insights gleaned from the mathematics education literature. The diverse perspectives from literature (as reviewed in Sections 2.2 to 2.5) provided understanding of the implications of this change, ensuring that the modification resonated with both the literature and the teachers' needs and perceptions.

I addressed the issues of reflexivity by developing a reflexive diary (Nadin & Cassell, 2006). It was a simple A4 notebook, where I made regular entries during the research process. In these entries, I recorded methodological decisions, the impact and changes due to the unprecedented situation of COVID-19, the reasons for the changes, the logistical challenges of the study that arose due to COVID-19, and reflections on what was happening in terms of my values and interests. One notable instance was the unanticipated impact of the COVID-19 pandemic, which posed restrictions on my planned travel for data collection. As an alternative strategy, I resorted to the creation of a Facebook page aimed at engaging mathematics teachers in Pakistan. This approach, which was not originally part of my research plan, allowed for remote data collection.

Moreover, during the semi-structured interviews, I used this diary not only to note the biographical details of the participating teachers, but also to reflect on the interview experience, i.e., how well the interview was conducted and what changes I had to make in conducting the next interview. In summary, the reflexive diary helped me stay self-aware and transparent during the research. It showed that I followed a strict methodological process and adjusted to unexpected situations while analysing sensitive study data.

3.7. Ethical considerations

This research was approved by the University of Waikato Faculty of Education Ethics Committee on 06/04/2020. Approval number: FEDU024/20 (Appendix H).

3.7.1. Access to participants

Throughout my research journey, I adhered to ethical considerations to ensure the protection of participants' rights and well-being. Gaining access to participants involved navigating a complex approval process, securing consent from teachers and principals (wherever required), and aligning with both institutional and ethical standards (see Appendixes A and B). I implemented the informed consent procedures. All participating teachers (Phase 1 and Phase 2) received detailed information, including the study's purpose, potential benefits, and associated risks. During Phase 2, the participants had the right to decline or withdraw at any stage, with transparent arrangements made for them to receive information. In Both Phases of data collection, I communicated the use of information, specifying its application for the doctoral dissertation and potential publication in academic forums.

3.7.2. Cultural sensitivities and participant preferences in the Pakistani context

In Pakistan, educational research and surveys are relatively uncommon. Teachers often show reluctance to participate due to potential career implications. To tackle these challenges, I adopted a methodological approach designed to navigate them. In the Qualtrics platform, a deliberate and ethical approach was taken to prioritize participant privacy and data security. Specifically, email addresses and social media profiles were not collected as a mandatory field during the Phase 1 online survey. Instead, an optional field was provided, allowing participants the choice to share their email addresses or any contact information should they wish to participate in Phase 2 of the research. Teachers were assured that their responses would remain confidential, with identifiers (if any) removed from the collected data. This approach aimed to create a secure space for honest and uninhibited participation.

Additionally, I took into account cultural and social considerations when engaging with male and female mathematics teachers. In the context of Pakistan, issues related to female participants in research, particularly those pertaining to cultural and gender-specific considerations, are noteworthy. During my research, I encountered situations where female participants expressed hesitation or concerns related to engaging with male researchers. This hesitancy is often influenced by cultural norms and values that prioritize privacy and modesty, especially in interactions with individuals of the opposite gender.

One notable example involved a female participant during a Zoom meeting who communicated her preference to engage without enabling her video. This decision was rooted in cultural considerations, possibly linked to the practice of "purdah" (veil) where women may choose to limit visual exposure to unrelated males. In utmost respect for her preferences, I honoured this request, recognizing the importance of creating a comfortable and culturally sensitive environment for participants.

Language considerations aligned with the preference of participants. They were invited to use English, Urdu or a mix of both. The commitment to ethical principles extended to dispute resolution, where I proactively aimed to avoid conflicts and outlined a transparent process for resolution if needed. These comprehensive ethical considerations underscore my dedication to conducting research with integrity, transparency, and a profound respect for the individuals contributing to the study.

3.7.3. COVID-19 and research ethics

In Pakistan, educational research and surveys are relatively uncommon. Teachers often show reluctance to participate due to potential career implications. To tackle these challenges, I adopted a methodological approach designed to navigate them. Due to the COVID-19 travel restrictions and health issues, I conducted the survey and teachers' interviews online. The approach was recommended by the University of Waikato's Human Research Ethics Committee. I obtained the committee's approval before proceeding with data collection. I used Facebook and WhatsApp to share the online survey link. Teachers found it easy to participate in the survey due to the convenience and accessibility of social media platforms.

Ethical considerations remained at the forefront of my approach. I meticulously adhered to principles of transparency and credibility. The survey did not collect or record email addresses or social media profiles. I communicated my research affiliation and study objectives clearly in the online survey (see Appendix I).

Despite taking precautions, qualitative data collection remained a challenge due to COVID-19. Some interviewees experienced technical difficulties or were unable to participate due to their own COVID-19-related circumstances. To address these issues, I had to be flexible and reschedule interviews, as necessary. I made sure to modify my Participant Information Sheets and Consent Forms to accommodate the revised approach for data collection due to COVID-19, which required additional time and effort by those interviewed.

3.8. CHAPTER SUMMARY

In this Chapter, I presented the overall methodology employed to understand the digital competencies of high school mathematics teachers in Pakistan. I used mixed-methods research and divided the research process into two phases: Phase 1 and Phase 2. The methodology started with a quantitative phase, facilitated by an online survey. I adapted the survey from "The Attitude Questionnaire" and the "Teacher Beliefs Survey." With the addition of a few items, I piloted the survey with mathematics teachers. Based on feedback, the survey was further improved and then used for the main data collection. Using the survey, I measured five constructs of the modified digital competencies' framework. These included personal orientation, specialized digital resources knowledge (SDRK), knowledge of digital resources and curriculum (KDRC), knowledge of digital resources and students (KDRS), and knowledge of digital resources and teaching (KDRT). The survey included both Likert-scale and open-ended questions about the teachers' digital competencies. Across all social media platforms and email, the survey received 306 responses in total. Responses were screened using the "completion rate" criterion, i.e., responses with less than 10% completion rate were excluded. Following the screening process, I chose 270 cases for analysis. The screening also highlighted that the data contained outliers and missing values. Most missing values were concentrated in Block 5 and related to the use of DGS and video conferencing tools. To deal with the missing values, I performed the Multiple Imputation (MI) procedure via SPSS. Once the data was ready for analysis, I performed descriptive and inferential statistics. Overall, the survey helped assess the digital competencies of high school mathematics teachers.

The research then moved to the qualitative phase. In Phase 2, I collected data to understand the Phase 1 results in depth. Teachers ($n = 6$) were asked to explain quantitatively significant results, outliers, and perplexing Phase 1 results. The aim was to understand factors that underpin digital competencies within the context of mathematics education in Pakistan. Using the pre-identified themes and sub-themes of the modified digital competencies' framework, I analyzed the interview data. Then came the stage of triangulation, a methodological approach employed to augment the research findings' validity. Triangulation unveiled potential concurrences or disparities as I compared the statistical findings of the online surveys with teachers' qualitative responses.

Throughout the research, reflexivity remained a guiding principle. I consistently acknowledged my epistemological stance and maintained a reflexive diary. In this diary, I talked about my struggles with COVID-19 and how it affected my work. Crucially, it synthesized my position and the perspectives of the participants. Prior to commencing data collection, the study's design was reviewed and approved by the University's Ethical Committee, thus ensuring a robust and ethically sound research approach.

CHAPTER FOUR

4. Phase 1 Findings (Quantitative data)

4.1. Introduction

In this Chapter, I begin by presenting the results of Phase 1. I used the online survey "Digital Competencies of High School Mathematics Teachers in Pakistan" (See Appendix I) to collect the quantitative data that address the overarching RQ.

Overarching research question RQ:

What are the digital competencies of high school mathematics teachers in Pakistan?

As explained in Chapter three, this study uses the explanatory sequential design. I collected and analysed Phase 1 (quantitative) and Phase 2 (qualitative) data separately and sequentially. Phase 2 (qualitative data) results will be presented in Chapter 5. Both instruments, the online survey and semi-structured interviews, were expanded to include items related to online teaching and learning strategies during COVID-19 (see Section 3.4.2.3). This modification facilitated understanding of the extent to which teachers' beliefs and preferences about using digital resources changed because of the transition to online learning during COVID-19 lockdown.

As explained in Section 3.4.1, the online survey was divided into six blocks. The items (statements) in each block (2 to 6) of the online survey were connected to the dimensions (constructs) of the modified DCTMT framework (see Section 2.4.9.2, Figure 7, p. 80). The findings from each block are recorded in this chapter. The results from each block of items are presented in the subsections that follow, beginning with Block 1 that collected demographic data from the participants.

4.2. Block 1: Teachers' demographic information

As explained in Chapter 3 (see Section 3.4.2), a number of categories were used to identify the demographics and professional information of participating mathematics teachers. I calculated percentages to describe the composition of categorical items and highlight proportions within the sample according to teachers' demographic data. The results are presented in Table 9.

Table 9

Demographics items, their categories, and frequencies

Item	Categories	n	%
Gender	Male	129	48
	Female	141	52
Academic Qualification	Graduate	99	37
	Postgraduate	159	59
	Other	12	4
Teaching Qualification	Yes	137	51
	No	133	49
Teaching Classes/Level	Classes/Level 8-10	241	89
	Classes/Level 11	102	38
	Classes/Level 12	139	51
	Classes/Level Other	100	37
Teaching Experience	Less than 1 year	39	14
	1 – 3 years	78	29
	3 – 6 years	58	21
	6- 10 years	49	18
	more than 10 years	44	16
Professional Roles	Government School Teacher	93	34
	Private School Teacher	80	30
	Private Tutor	38	14
	Private Academies	18	7
	Online Teacher	33	12
	Other:	64	24

Note. Table 9 consists of four columns. The first two columns contain item names and their categories. The third column lists the frequencies (n) for each category and the fourth column presents percentages (%).

The first three items (rows) in the first column present the self-reported characteristics such as gender, academic and teaching qualification. The professional information such as teaching classes/levels, teaching experience, and professional roles are presented in the remaining part of the Table.

The results in Table 9 indicate a balanced representation of gender, with 52% of the teachers identified as female and 48% as male. Educational qualifications varied, with 59% holding postgraduate degrees, 36.7% being graduates, and 4.4% falling into the "other" category, encompassing individuals with advanced degrees like MPhil and PhDs. Furthermore, 50.7% of the teachers possessed a teaching qualification (e.g., B.Ed. or M.Ed.), and 85% reported having more than one year of teaching experience in mathematics across various levels.

Additionally, the cross-tabulation of items with each other shows that of the qualified teachers, the majority had postgraduate degrees (64.5%). A gender breakdown shows more female teachers had attained a teaching qualification (57%) than males (43%). Teachers (19.6%) who selected the option "other" for variable professional roles, specified that they also work as visiting faculty members at universities.

4.2.1. Use of digital technologies

Table 10 shows the results of items that examined teachers' access to digital devices, the Internet, and the use of digital resources for teaching and learning mathematics.

Table 10

Use of Digital devices and resources for mathematics teaching and learning

Variable	Categories	n	%
Access to Digital Devices	Desktop Computer	78	29
	Laptop	177	65
	Smartphone	137	50
	Interactive Whiteboard	77	29
	Digital Notebook/ Notepads	30	11
	Projectors	52	19
	Other	15	6
Use of Digital Resources	Videos	130	50
	Image & Graphics	134	51
	Digital Graphs	47	18
	Websites	100	38
	Animations	39	15
	Simulations	12	5
	e-textbooks	124	47.5
	Interactive Edu Games	38	15
	Dynamic graphing Software	24	9
	Other	35	13

Note. The table displays the distribution of responses for each category within the variables "Access to Digital Devices" and "Use of Digital Resources," providing counts (n) and percentages (%) for a comprehensive overview of digital technology use among the surveyed teachers.

For instance, in the "Access to Digital Devices" variable, the categories include "Desktop Computer," "Laptop," "Smartphone," "Interactive Whiteboard," "Digital Notebook/Notepads," "Projectors," and "Other." Each category specifies a distinct type or mode of digital technology or resource that the surveyed teachers might have access to or use. The second column provides a picture of the diverse digital tools and resources that teachers use in their teaching and learning mathematics and the number of teachers who use particular digital tools.

4.2.1.1. Access to digital devices and the Internet

Table 10 shows the prevalence of laptops and smartphones among the teachers, indicating their widespread adoption for educational purposes. Among the array of digital devices, 44.8% of teachers had access to only one digital device, whereas 54% had access to multiple devices. The option “other” identified devices not listed in the online survey, such as calculators and digital writing pads. Access to devices also varied by gender and professional role. For example, female teachers reported more access to smartphones (52%), IWB (61%), Digital notebooks/tablets (53.3%), Projectors (52%) and “other” (80%), whereas male teachers reported more access to Desktop Computers (54%) and Laptops (53%). The crosstabulation of the item “access to digital devices” with “professional roles” indicated government schoolteachers had access to the least number of devices when compared to all other types of teachers.

Most of the surveyed schoolteachers (74.5%) had Internet access most of the time during the day. Female teachers (53.2%) had marginally greater internet connectivity than male teachers (46.8%) during the day. Of the total number of government schoolteachers, 63.4% had access to the internet most of the time, 29% had limited access and 7.5% had no access to the Internet. Other mathematics teachers had more access to the internet than did government schoolteachers.

4.2.1.2. Use of Digital Resources

The second part of Table 10 (p. 141) shows that images, and graphics (51%), videos (50%), e-textbooks (47%), educational websites (38%), were the resources which were mostly used by high school mathematics teachers. Other digital resources such as digital graphs, animations and Interactive educational games were used by fewer than 20% of respondents. Few teachers had used dynamic graphing software (DGS) such as GeoGebra (9.2%) and simulations (4.6%). These findings indicate that teachers may either be unaware of these tools or have insufficient knowledge of them.

In gender terms, there appeared to be little difference between females and males in the use of digital resources. However, there were gender differences in the use of specific digital resources. Noteworthy differences existed for using digital graphs, dynamic graphing software (DGS) and interactive educational games. More male teachers used DGS and digital graphs than female teachers. By contrast, female teachers, used interactive educational games to teach mathematics more often than male teachers. Table 10 also shows that teachers mostly use videos, graphics, and websites.

Although Table 10 provides important information about Pakistani mathematics teachers' use of digital technologies, interpreting. this information is complicated by the unprecedented situation of COVID-19. On the surface, the data suggest that teachers regularly use digital technologies in classrooms for teaching and learning mathematics, which may or may not be the case during normal circumstances. However, I collected Phase 1 data during the COVID-19 lockdown when most teachers were forced to teach from home (online) and use digital technologies. Correspondingly, the responses may provide a distorted picture of the usage of digital resources in mathematics teaching.

In the following section, I present the findings of Block 2 that measured high school mathematics teachers' beliefs about mathematics teaching and learning.

4.3. BLOCK 2: BELIEFS ABOUT MATHEMATICS TEACHING AND LEARNING (P_O)

As discussed in Section 3.4.1 (see Table 1, p. 108), Block 2 ($B2$) items were designed to investigate teachers' personal orientation i.e., their beliefs and preferences about mathematics teaching. B2 contained seven items ($B2\_1$ *to* $B2\_7$), measured on a 5-point Likert scale (5 = strongly agree to 1 = strongly disagree). Table 11 shows the descriptive statistics for Block 2 items.

Table 11

Descriptive statistics for Block 2 items

Descriptive Statistics						
Item No.	Item Statement	N	Mean	Median	Mode	SD
B2_1	Learning mathematics means exploring problems to discover patterns and make generalisations.	270	4.71	5.00	5	0.46
B2_2	Mathematics teaching is to teach students how to create and assign meanings to signs, symbols, and notations.	270	4.24	4.00	5	0.62
B2_3	Mathematics teaching is to share knowledge and ideas, and to discuss a variety of real-world contexts.	270	4.51	5.00	5	0.51
B2_4	Mathematics lessons should be followed by a critical discussion with students.	270	4.50	5.00	5	0.61
B2_5	Mathematics curriculum textbooks are the best medium of instruction and source of knowledge.	270	4.07	4.00	4	0.64
B2_6	Mathematical knowledge is retained more easily if it is acquired using multiple representations.	270	4.59	5.00	5	0.63
B2_7	While learning mathematics, it is important to memorize rules, facts, and formulas.	270	4.73	5.00	5	0.55

Note. Each row in the table corresponds to a specific item, and the columns provide relevant statistics for each item, offering an overview of the distribution and central tendency of responses from the surveyed teachers. For example, the first column represents the identification code for each item in Block 2 and the corresponding row for $B2\_1$ presents item statement and its respective descriptive statistics.

Except for items $B2\_2$ and $B2\_5$, the Mean values ($4.5\ and\ above$) suggest that teachers on average strongly agreed with the item statements. In Block 2 Mode was 5, indicating teachers' robust beliefs about the pedagogical practices mentioned in the items, such as exploring problems to discover patterns and generalizing, conveying mathematical knowledge through multiple representations, critical debate with students, and sharing mathematical ideas. The highest Mean score (4.73) for item $B2\_7$ indicates that teachers preferred memorizing rules, facts, and formulas.

The Mean values for item $B2\_5$ ($Mean = 4.07, Mode = 4$) suggest that teachers somewhat agreed on textbooks as the best source of information for mathematics learning. This finding may signal a gradual shift away from textbook-centred teaching to other contemporary educational resources, such as digital resources. This possibility was investigated during the Phase 2 semi-structured interviews. The Mean scores for $B2\_2$ reflect Pakistani teachers believed that mathematics teaching is teaching students how to create and assign meanings to signs, symbols, and notations. The findings from the survey do not show the extent to which the beliefs with which teachers agreed translated into their pedagogical practices. Consequently, during Phase 2, it was considered important to investigate the correspondence (or otherwise) between beliefs and practices while using digital resources.

Further inquiry was undertaken to investigate the correlation between the different items in Block 2 of the online survey. Spearman's rho correlation test was used for this analysis. The two assumptions for Spearman's rho test were satisfied. First, the scores were measured using an ordinal scale - a measurement scale that uses labels to classify cases into ordered classes such as a Likert scale (strongly agree to strongly disagree). Second, the relationships between items were monotonic i.e., as the value of one item increases, so does the value of the other item. SPSS produced the following Spearman's rho correlation test results (as shown in Table 12).

Table 12

Results of Spearman rho correlation test for Block 2 items

Item No.	B2_1	B2_2	B2_3	B2_4	B2_5	B2_6	B2_7
B2_1	--						
B2_2	.433**						
B2_3	.223**	.207**					
B2_4	.208**	.195**	.210**				
B2_5	.215**	.337**	.271**	.199**			
B2_6	.125*	.132**	.273**	.166**	.302**		
B2_7	.138*	.336**	.158**	.132*	.331**	.168**	--

Note. The table displays a correlation matrix where each cell represents the correlation coefficient (ρ) between the corresponding items. The symbols $**$ and $*$ indicate the significance levels, with $**$ denoting significance ($p - values$) at the 0.01 level and $*$ denoting significance at the 0.05 level, both being two-tailed tests.

For most items, SPSS reported significant p-values ($p \leq 0.001$) for the Spearman rho test. These values suggest that items of B2 are monotonically correlated in the sample population. It also confirms the relationship between teachers' pedagogical practices and pedagogical beliefs in mathematics. For example, the significant positive correlation of $B2\_1$ with $B2\_2$ ($\rho = 0.433, p < 0.01$) suggests a relationship between learning mathematics through exploring patterns in mathematical problems and to generate new signs, symbols, and notations. Notably, item $B2\_2$ significantly correlated with all the other items of Block 2 indicating teachers' strong beliefs in the role and affordances offered by signs, symbols, and notations. It is an interesting finding that may reflect teachers' preferences for and understanding of signs, symbols, and notations. It may be interesting to know how these beliefs are transferred and translated in a dynamic technological environment and their role in the journey of attaining mathematical digital competencies. Additionally, a good positive relationship was found between the use of a mathematics textbook ($B2\_5$) and other items of Block 2. At the qualitative phase, further probes revealed more about these beliefs.

This section has presented the descriptive results and correlation among $B2$ items. Trends identified included teachers' strong agreement with pedagogical practices, emphasizing patterns, multiple representations, critical debate, and ideas sharing. A notable preference for memorization was indicated, but only moderate agreement on textbook importance was apparent. Interestingly, positive correlations between textbook usage and broader pedagogical beliefs underscore a coherence in mathematics education practices. The findings of Block 3 items are presented in the following section.

4.4. Block 3: Beliefs about digital resources and their use in mathematics education (SDRK)

Block 3 ($B3$) items were designed to investigate teachers' general beliefs about the use of digital resources for mathematics teaching. The beliefs about digital resources could reflect upon the teachers' digital competencies (RQ) and their orientation towards digital resources. Table 13 presents the descriptive statistics for five Block 3 items ($B3\_1$ *to* $B3\_5$).

Table 13

Descriptive statistics for Block 3 items

Item No.	Item Statement	N	Mean	Median	Mode	SD
B3_1	Digital resources play an important role in mathematics teaching.	270	4.68	5.00	5	0.53
B3_2	Digital resources make mathematics problems and tasks more engaging.	270	4.61	5.00	5	0.67
B3_3	I have lots of ideas about how I can make use of digital resources in the mathematics classroom.	270	4.27	4.00	5	0.84
B3_4	Digital resources help students in self-regulating, understanding, and solving mathematics problems.	270	4.54	5.00	5	0.67
B3_5	Digital resources distract the attention of students from doing calculations.	270	3.74	4.00	4	1.20

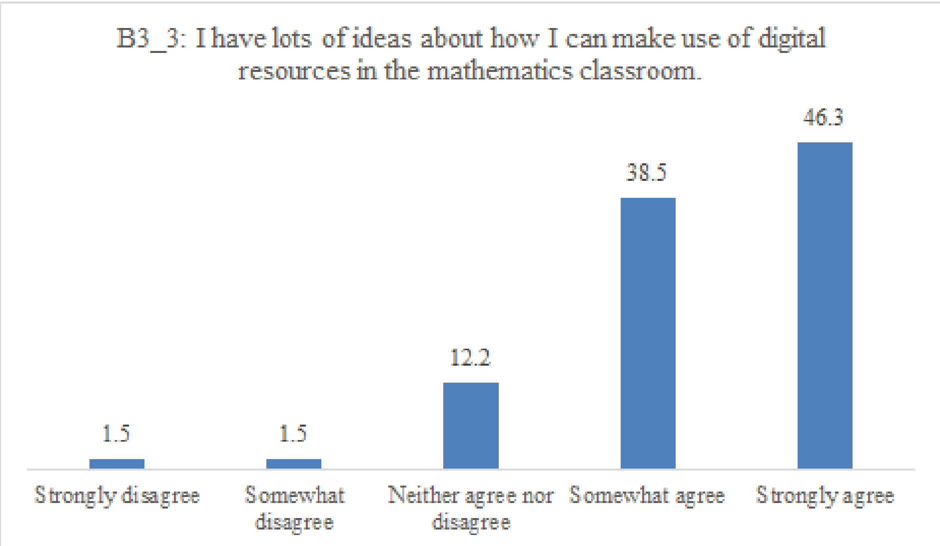
Note. Each row in the table corresponds to a specific item, and the columns provide relevant statistics for each item, offering an overview of the distribution and central tendency of responses from the surveyed teachers. For example, the first column represents the identification code for each item in the Block 3 and the corresponding row for $B3\_1$ presents the item statement and its respective descriptive statistics.

The three items ($B3\_1, B3\_2$ *and* $B3\_4$) with Mean values greater than 4.5 and $Mode = 5$ demonstrate teachers' strong belief in the role of digital resources in mathematics teaching. Teachers strongly agreed that digital resources help in engaging and self-regulating students in the classroom ($B3\_2$; $Mean = 4.61$).

However, for item $B3\_3$ teachers were split on whether they have lots of ideas about using digital resources in classrooms or not. More than half (53.7%) of the teachers' responses were in the range of somewhat agree to strongly disagree (see Figure 13). Teachers' slightly weak agreement with $B3\_3$ $(Mean = 4.2, Mode = 4)$ could be pointing towards infrequent use or maybe lack of confidence about their abilities for using digital resources in the classroom. This finding can be linked to the personal instrumental genesis through which individual teachers develop their personal understanding, ideas and mastery of tools or instruments in a specific context. However, the online survey did not collect data about factors that contribute to or impede the regular use of digital resources in classrooms. During Phase 2 interviews, such factors were investigated to understand the reasons.

Figure 13

Frequencies in percentage (%) of responses for Item B3_3



For item $B3\_5$ the Mean value (3.74) suggests teachers agreed that digital resources do not distract students' attention away from calculations. It is important to highlight that, to reduce acquiescence, the $B3\_5$ statement was negatively worded. Negatively worded items are designed to elicit responses that indicate disagreement or a lack of endorsement. In this case, the wording suggests that the use of digital resources is perceived as a potential source of distraction from students' engagement in mathematical calculations. In the context of this thesis, a lower Mean value (3.74) for item $B3\_5$ suggests that fewer teachers agreed with the statement, indicating that they do not perceive digital resources as causing distraction from students' calculations. The mention of "acquiescence bias" highlights the potential for respondents to agree with statements regardless of their true beliefs, and in this case, the negative wording of the item aims to mitigate this bias by encouraging respondents to express disagreement when appropriate.

To understand how the teachers' general beliefs about the use of digital resources for mathematics teaching are related, the analysis of Block 3 continues with Spearman's correlation test. The results are shown in Table 14.

Table 14

Results of Spearman's rho correlation test for Block 3 items

Item No.	B3_1	B3_2	B3_3	B3_4	B3_5
B3_1	--				
B3_2	.502**				
B3_3	.288**	.325**			
B3_4	.458**	.410**	.440**		
B3_5	0.104	-0.001	.157**	0.098	--

Note. The symbols $**$ and $*$ indicate the significance levels, with $**$ denoting significance ($p - values$) at the 0.01 level and $*$ denoting significance at the 0.05 level, both being two-tailed tests.

Except for the negatively worded item $B3\_5$ (Digital resources distract the attention of students from doing calculations), SPSS reported significant p-values for the Spearman rho test as being $p < 0.01$. The significant p-values for $B3\_1$ to $B3\_4$ shows strong evidence that these items were monotonically correlated in the sample. For example, the correlation between $B3\_1$ (digital resources' importance) and $B3\_2$ (enhancing engagement) is relatively strong and positive ($\rho = 0.502, p < 0.01$), suggesting that teachers who perceive digital resources as important may believe in their capacity to make mathematics problems and tasks more engaging. The positive correlations between $B3\_1$ and $B3\_4$ (digital resources aiding self-regulation) ($\rho = 0.458, p < 0.01$) and between $B3\_1$ and $B3\_3$ (possessing ideas about digital resource use) ($\rho = 0.288, p < 0.01$) indicate that teachers who recognize the importance of digital resources may tend to believe in their potential to enhance self-regulation and possess ideas for their use.

There was a positive correlation between $B3\_4$ and $B3\_3$ ($\rho = 0.440, p < 0.01$), indicating that teachers who believe in the potential of digital resources to help students self-regulate are also more likely to possess ideas about their classroom integration. Further, the correlation ($\rho = 0.325, p < 0.01$) between $B3\_2$ and $B3\_3$ shows a relationship between generating new ideas in mathematics classrooms and the use of digital resources to make mathematical problems more engaging and interesting. The correlation between $B3\_5$ (digital resources' potential to distract) and other items is weaker and not statistically significant.

This section presented the findings for Block 3 items. The correlation analysis reveals interesting patterns of association between teachers' beliefs about different aspects of digital resource integration in mathematics teaching. The strong positive correlations reflect that teachers' perceptions of digital resources' importance extend to their beliefs about engagement, self-regulation, and generating ideas for use. The next section presents the results of Block 4 items.

4.5. Block 4: Knowledge of digital resources and students (KDRS)

Block 4 ($B4$) items were designed to understand teachers' knowledge and use of digital resources in relation to students learning in a technological environment. The descriptive statistics for seven items ($B4\_1$ *to* $B4\_7$) of Block 4 are presented in Table 15.

Table 15

Descriptive statistics for Block 4 items

Item No	Item Statement	N	Mean	Median	Mode	SD
B4_1	Mathematics students need to know how to use digital resources for learning.	270	4.71	5.0	5	0.53
B4_2	Students should not be allowed to use digital resources until they have mastered the concept, rule, or method.	270	3.76	4.0	4	1.24
B4_3	Digital resources allow students to creatively think about mathematics problems.	270	4.48	5.0	5	0.70
B4_4	Dynamic mathematics software (DGS) (e.g., GeoGebra) makes students better problem solvers.	270	4.11	4.0	5	0.84
B4_5	Mathematics software removes some learning opportunities for students. (e.g., drawing graph using pen and paper)	270	4.07	4.0	4	0.96
B4_6	Students are more confident in maths when they use digital graphs.	270	4.19	4.0	4	0.74
B4_7	Videos about abstract mathematics concepts (e.g., complex numbers) improve student understanding.	270	4.52	5.0	5	0.54

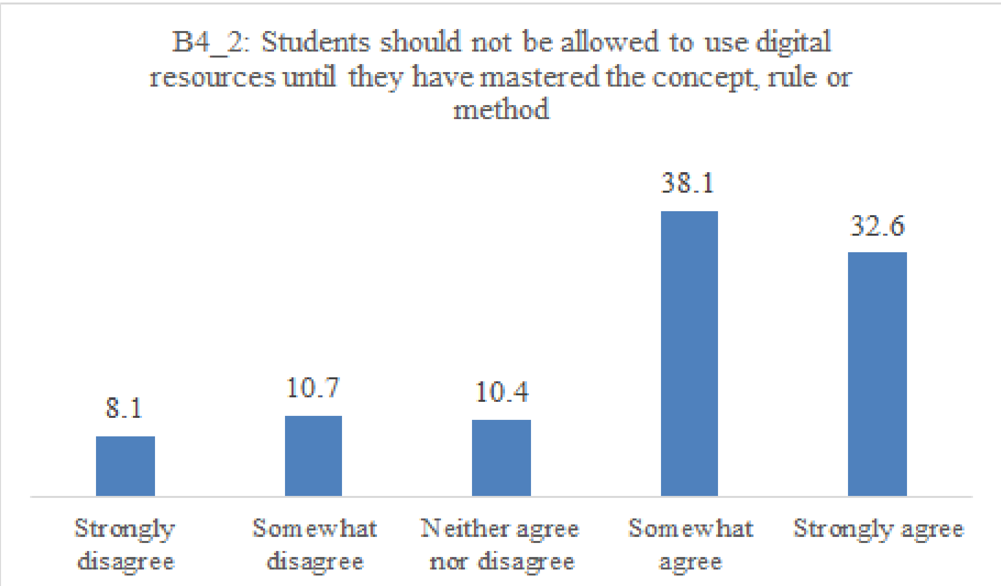
Note. Each row in the table corresponds to a specific item, and the columns provide relevant statistics for each item, offering an overview of the distribution and central tendency of responses from the surveyed teachers. For example, the first column represents the identification code for each item in the Block 4 and the corresponding row for $B4\_1$ presents item statement and its respective descriptive statistics.

The Mean values show only two items in B4 ($B4\_1$ *and* $B4\_7$) were strongly agreed upon while on the remaining five, teachers somewhat agreed. Teachers strongly agreed that mathematics students need to know how to use digital resources and manifested their confidence ($B4\_7$; $Mean = 4.5, Mode = 5$) in videos about abstract mathematics concepts to improve student understanding. Teachers expressed positive belief in item $B4\_3$ ($Mean = 4.48$) which suggests that digital resources can enhance creative thinking. However, teachers somewhat agreed on the impact of dynamic graphing software (DGS) such as GeoGebra on students' problem-solving abilities. Teachers were similarly split on whether mathematics software removes some learning opportunities such as the use of pen and paper. These are contrasting results when compared with Block 3 items where teachers indicated their strong belief in the role of digital resources, especially in solving mathematics problems and promoting students' self-regulation in the classroom. In Block 5 more items pertaining to DGS would help to determine whether the use of DGS is an issue or not.

The item B4_2 (students should not be allowed to use digital resources until they have mastered the concept, rule, or method) received the lowest Mean score ($Mean = 3.76$). Almost 70% responses were in the range of somewhat agree to strongly disagree (see Figure 14). Teachers' reservations (as reflected in $B4\_2$) suggest considerations about maintaining a balance between using digital resources and mastery of foundational concepts.

Figure 14

Frequencies in percentage (%) of responses for item B4_2



To understand how the items measuring teachers' knowledge and use of digital resources in relation to students learning in a technological environment are related, the analysis continues with Spearman's rho correlation test. The results are shown in Table 16.

Table 16

Spearman's rho correlation test for items of Block 4

Item No.	B4_1	B4_2	B4_3	B4_4	B4_5	B4_6	B4_7
B4_1	--						
B4_2	0.023						
B4_3	.336**	.139*					
B4_4	0.095	.181**	.135*				
B4_5	-0.008	.459**	0.061	.127*			
B4_6	0.084	0.079	.194**	.183**	.222**		
B4_7	0.115	0.047	.329**	.270**	-0.085	.164**	--

Note. The table displays a correlation matrix where each cell represents the correlation coefficient (ρ) between the corresponding items. The symbols $**$ and $*$ indicate the significance levels, with $**$ denoting significance ($p - values$) at the 0.01 level and $*$ denoting significance at the 0.05 level, both being two-tailed tests.

The results of the Spearman correlation test were both significant and insignificant. The item $B4\_1$ was significantly correlated with only one item ($B4\_3;\ \rho = 0.336, p < 0.01$) in the Block. This suggests that teachers who believe students should possess digital resource skills also tended to believe that digital resources contribute to enhancing creative thinking among students.

A relatively strong correlation ($\rho = 0.459, p < 0.01$) between items $B4\_2$ and $B4\_5$ suggests that teachers who believe in delaying the use of digital resources until mastery of concepts is achieved, are more likely to hold the belief that digital resources might remove certain learning opportunities for students. They may also consider that it may affect their ability to think creatively about mathematics problems ($\rho = 0.139, p < 0.05$). Comparing this correlation to Block 2 item $B2\_7$ (memorizing rules, facts, and formulas), which reflects a preference for memorization, both correlations align in pointing towards a cautious perspective regarding the integration of digital resources before a strong foundation is established. It is noteworthy to mention that Section 4.2.1.2 showed, apart from videos, less than 20% of teachers had access to other listed digital resources. Limited access to digital technologies and an associated lack of confidence could be a reason for emphasising traditional pedagogies. Teachers could be perceiving digital resources and pedagogies as two distinct things rather than mutually supported entities. Item $B4\_3$ (creative thinking) showed positive correlations with items $B4\_1$ (importance of digital resources), $B4\_4$ (DGS), $B4\_6$ (digital graphs), and $B4\_7$ (videos). These correlations suggest that teachers believed digital resources (DGS, digital graphs, and videos) allow students to think creatively about mathematics problems. Additionally, positive significant correlations $(\rho = 0.183, p < 0.01)$ were observed between DGS and digital graphs and $(\rho = 0.271, p < 0.01)$ between DGS and videos. These correlations are consistent with many other studies that also reported such relationships and the positive impact of dynamic graphing software and videos on students' problem-solving abilities. However, given contrasting results, it would be useful to further investigate these relationships. The next section presents the findings for Block 5 items.

4.6. Block 5: Knowledge of Digital Resources and Teaching (KDRT)

Block 5 ($B5$) items were designed to understand teachers' knowledge of digital resources in relation to teaching in a technological environment. Initially, $B5$ contained nine items, however, based on pilot study findings (see Section 3.4.2.3), four items related to teaching during COVID-19 (see Section 3.4.2.3) were added later. The results of the four items are presented separately in Section 4.6.1. In this section, only the descriptive statistics (see Table 17) and correlations among the nine items ($B5\_1$ to $B5\_9$) are presented.

Table 17

Descriptive statistics for items in Block 5

Item No	Item Statement	N	Mean	Median	Mode	SD
B5_1	I have received training for the use of technology and other digital resources.	270	3.27	3.00	5	1.41
B5_2	It is difficult to decide which digital resource to use to teach a particular mathematics topic.	270	4.16	4.00	4	0.77
B5_3	I know how to write math equations and expressions using an equation editor.	270	4.28	5.00	5	1.04
B5_4	I can teach using DGS (e.g., GeoGebra)	270	3.65	4.00	5	1.27
B5_5	I can solve geometry or trigonometry problems using DGS.	270	3.90	4.00	4	0.93
B5_6	I can use DGS for calculus and algebra.	270	4.12	4.00	5	1.03
B5_7	I teach mathematics better when my students use pen and paper for solving problems.	270	4.35	4.00	5	0.75
B5_8	I can create images and videos for teaching purposes.	270	4.41	5.00	5	0.88
B5_9	I want to improve my ability to teach mathematics with DGS	270	4.59	5.00	5	0.75

Note. Each row in the table corresponds to a specific item, and the columns provide relevant statistics for each item, offering an overview of the distribution and central tendency of responses from the surveyed teachers. For example, the first column represents the identification code for each item in the Block 5 and the corresponding row for $B5\_1$ presents item statement and its respective descriptive statistics.

The item $B5\_1$ which recorded the lowest Mean score 3.27 suggests that a significant number of participants had received some form of training for the use of technology and digital resources. Approximately 63% of teachers expressed varying degrees of uncertainty about their training. These responses ranged from strongly disagreed to neither agree nor disagreed. This suggests that a significant proportion of teachers may feel that they lack sufficient training or are not fully confident in their training for using digital resources in their teaching. Lack of technology training could be a reason for the high Mean ($B5\_7$; $mean = 4.36, mode = 5$) for teachers preferring pen-and-paper for solving mathematics problems. Further, inadequate training may also have contributed to their weak agreement with items related to dynamic graphing software. The finding allows us to understand the previous findings related to DGS (Section 4.5), where teachers' responses indicated confusion as to the extent to which DGS contributes to students' understanding of mathematical problems.

The result for $B5\_2$ indicates that, on average, teachers agree to a moderate extent ($Mean = 4.16$) with the statement that it is difficult to decide which digital resource to use when teaching a specific mathematics topic. This implies that teachers may face challenges in selecting appropriate digital resources given the range of options available. The relatively high standard deviation ($SD = 0.77$) suggests some variability in responses, indicating that some respondents may find it more challenging than others to choose the right digital resource for teaching mathematics topics.

The results about using the equation editor tool ($B5\_3$; $Mean = 4.38, Mode = 5$) and teachers' ability to develop graphics and videos for mathematics teaching ($B5\_8$; $Mean = 4.4, Mode = 5$) indicated teachers strongly agreed with having these important competencies. However, descriptive scores alone cannot determine a teacher's knowledge of equation editors and content creation tools, nor can it be determined whether teachers understand how to use them with other digital resources. Phase 2 was used to investigate these concerns more thoroughly. Additionally, teachers strongly agreed with item $B5\_9$ ($Mean = 4.59, Mode = 5$) which indicates teachers' desire to improve their ability to teach mathematics with DGS. The consolidation of two results (items $B5\_1$ and $B5\_9$) related to training suggests most mathematics teachers wanted to improve their abilities to teach mathematics with technology, especially skills related to the use of dynamic graphing software (DGS).

In Section 3.4.5.2, the high percentage of missing values were associated with items related to the use of DGS ($B5\_5$ & $B5\_6$) to solve mathematics problems. The gaps here are linked with the participants' demographic and professional backgrounds. The high percentage of missing values along with low Mean scores for items $B5\_4$, $B5\_5$ and $B5\_6$ suggests teachers' weak knowledge and limited use of dynamic graphing software (DGS) ($B5\_4$; $Mean = 3.65, Mode = 4$) in Pakistani mathematics classrooms. It appears, teachers with less or no experience of using DGS skipped items related to the use of DGS. However, this conclusion cannot be made with absolute certainty. In summary, the results of Block 5 items related to DGS, equation editor, videos, graphics, and teachers' training demonstrated Pakistani teachers' knowledge of digital content and mathematics teaching. The results suggest that other than for a handful of teachers, the specific tool-related digital competencies of a majority of survey respondents are moderate to weak.

To understand how the items on teachers' knowledge of digital resources and teaching are related, the analysis continues with Spearman's rho correlation test. The results of the tests are presented in Table 18.

Table 18

Results of Spearman's rho correlation test for items of B5

Item No.	B5_1	B5_2	B5_3	B5_4	B5_5	B5_6	B5_7	B5_8	B5_9
B5_2	.435**								
B5_3	.398**	.202**							
B5_4	.432**	.212**	.380**						
B5_5	.348*	.297**	.423**	0.450					
B5_6	0.215	0.089	.371**	0.275	.554**				
B5_7	.199**	.183**	0.009	0.115	0.108	0.018			
B5_8	.237**	.208**	.278**	.187**	.160*	0.060	.154*		
B5_9	0.107	.139*	.289**	.169**	.250*	0.186	0.023	.292**	--

Note. The table displays a correlation matrix where each cell represents the correlation coefficient (ρ) between the corresponding items. The symbols $**$ and $*$

indicate the significance levels, with $**$ denoting significance ($p - values$) at the 0.01 level and $*$ denoting significance at the 0.05 level, both being two-tailed tests.

The Spearman correlation test yielded both significant and insignificant results. $B5\_1$ (Received training) shows positive and significant correlations with various aspects of digital competencies ($B5\_2, B5\_3, B5\_4, B5\_5, B5\_6, B5\_8, and\ B5\_9$). This suggests that teachers who have received training for the use of technology and digital resources are more likely to feel confident in their abilities and have a desire to improve further. These correlations highlight the importance of professional development and training programs for enhancing teachers' digital competencies. $B5\_2$ (Difficulty in deciding on appropriate digital resources) is positively correlated with the ability to teach using DGS ($B5\_4, B5\_5, and\ B5\_6$) and the ability to create images/videos ($B5\_8$). This indicates teachers who struggle with choosing the appropriate digital resources for teaching specific mathematics topics may also face challenges in using dynamic geometry software (DGS) and creating videos.

$B5\_3$ (Ability to write math equations) exhibits positive correlations with the ability to teach using DGS ($B5\_4, B5\_5, and\ B5\_6$) and the ability to create images/videos ($B5\_8$). This suggests that teachers who are proficient in writing mathematics equations using an equation editor are more likely to be confident in their abilities to use dynamic geometry software and create visual content for teaching purposes. This finding pinpoints connections between different abilities, such as the ability to write mathematics equations being related to the ability to teach using DGS. In addition to the above-mentioned correlations, $B5\_4$ (Ability to teach using DGS) demonstrates positive correlations with the ability to solve geometry/trigonometry problems using DGS ($B5\_5$), the ability to use DGS for calculus/algebra ($B5\_6$), and the ability to create images/videos ($B5\_8$). This indicates that teachers who feel competent in teaching with dynamic geometry software are also more likely to excel in using it for specific mathematical domains and incorporating visual content into their teaching.

However, $B5\_7$ (Preference for pen and paper) shows a positive correlation with the ability to create images/videos ($B5\_8$), received training ($B5\_1$) and difficulty in deciding digital resources ($B5\_2$). This implies that teachers who prefer pen and paper for problem-solving also express an interest in creating visual content for teaching purposes. Interestingly $B5\_7$ showed no correlation with $B5\_9$ (desire to improve teaching with DGS). It highlights the potential value of combining traditional and digital approaches in mathematics education. $B5\_8$ (Ability to create images/videos) exhibits a positive correlation with the desire to improve teaching with DGS ($B5\_9$). This suggests that teachers who are proficient in creating visual content for teaching purposes also express a desire to enhance their teaching abilities specifically with dynamic geometry software. The correlation between $B5\_3$ and $B5\_9$ could indicate that teachers who possess the skills to write mathematics equations using an equation editor are more interested in improving their ability to teach mathematics using DGS.

These correlations collectively indicate that there are interconnections between different aspects of teachers' digital competencies and attitudes towards digital resources in mathematics education. They further suggest that training related to digital technologies may improve teachers' mathematical digital competencies. Many previous studies have also reported such results but not specifically related to the use of equation editors and DGS. The next section is a subsection of Block 5, which discusses the results of items related to video-conferencing and teaching during the COVID-19 pandemic.

4.6.1. COVID-19 teaching strategy and tools

Like much of the educational world, high school mathematics teachers in Pakistan responded to COVID-19 constraints by undertaking emergency remote teaching when they were forced to move from face-to-face classroom teaching to an online learning environment. To understand teachers' online teaching and learning experiences, four items ($B5\_10\ to\ B5\_13$) were added to Block 5 in which the following three items ($B5\_10\ to\ B5\_12$) were measured on a 5-pointLikert scale.

- $B5\_10;$ I know how to use video conferencing applications (e.g., Zoom, Skype etc.)
- $B5\_11;$ I know how to screencast, share screen and digital resources during video conferencing.
- $B5\_12;$ If required, I can teach mathematics in a fully online environment (e.g., School closure during COVID-19).

The fourth item ($B5\_13$)sought qualitative responses to the following prompts: "Please tell us about your COVID-19 teaching strategy. What tools did you use (or using) for teaching mathematics during COVID-19?" The qualitative items asked teachers to share their experiences and list resources that they used to support teaching and learning during COVID-19. The descriptive statistics of items $B5\_10$ to $B5\_12$ is presented in Table 19.

Table 19

Descriptive statistics for the three COVID-19 related items

Item No.	Items	Descriptive Statistics				
		N	Mean	Median	Mode	SD
B5_10	I know how to use video conferencing applications.	270	4.50	5.00	5	0.892
B5_11	I know how to screencast, share screens and resources during video conferencing.	270	4.56	5.00	5	0.864
B5_12	If required, I can teach mathematics in a fully online environment (e.g., School closure during COVID-19).	270	4.64	5.00	5	0.674

Note. Each row in the table corresponds to a specific item, and the columns provide relevant statistics for each item, offering an overview of the distribution and central tendency of responses from the surveyed teachers.

The most common value in the dataset for the three items was $Mode = 5$, which suggests that most teachers knew how to use video conferencing applications. Teachers strongly agreed that they can screencast and share digital resources during a video conferencing ($Mean = 4.56$) session and could teach in a fully online environment such as school closure during COVID-19 ($Mean = 4.64$).

The Spearman's rho correlation test was used to understand how the COVID-19 items were related. The results of the test are presented in Table 20.

Table 20

Results of Spearman's rho correlations test for three COVID-19 items

Item No.	B5_10	B5_11	B5_12
B5_10	--		
B5_11	.264**		
B5_12	.203**	.626**	--

Note. The table displays a correlation matrix where each cell represents the correlation coefficient (ρ) between the corresponding items. The symbols $**$ and $*$ indicate the significance levels, with $**$ denoting significance ($p - values$) at the 0.01 level and $*$ denoting significance at the 0.05 level, both being two-tailed tests.

The correlation among the items was found statistically significant. For example, the fairly high positive correlation between items $B5\_11$ and $B5\_12$ ($\rho = 0.63, p < 0.01$) suggests a relationship between teachers' knowledge of using video conferencing features and their ability to teach in an online teaching environment. The items did not offer scope, however, to probe their skill in using these tools, nor an explanation of how they used them. Therefore, it would be useful to know how different, difficult, or easy online teaching was compared to face-to-face teaching and what role they played while teaching mathematics online, whether as facilitators or teachers.

4.6.2. Results of qualitative item

In this sub-section, the results of the qualitative item ($B5\_13$) are presented. The objective was to gather information about pedagogical innovations, digital tools, and applications that teachers used during the pandemic. Of 270 overall respondents only 167 (62%) provided feedback for $B5\_13$. Teachers mentioned they used video conferencing solutions like Zoom (54.6%), Microsoft Teams (9.8%) and Google Meet (9.2%) to teach mathematics online. In synchronous online sessions, teachers delivered live lectures, shared PDFs, MS Word documents and PowerPoint presentations. About 8% used digital pens on online interactive writing applications (few used Microsoft Paint) to solve mathematics problems. According to a teacher, "it is very easy to convey mathematics concepts online using a digital pen and writing pad." 18.4% of responses were categorized as Miscellaneous (Misc). In this category, tools and applications employed by fewer than five teachers were placed. Teachers mentioned Skype, Desmos, Jamboard, Discord, DingTalk, Graph Solver, and Microsoft OneNote in this category. Crosstabulation of COVID-19 teaching tools with gender revealed more female teachers had relied on Google Meet, recorded videos, WhatsApp, Facebook, Whiteboard and Miscellaneous tools. Meanwhile, male teachers made more use of Microsoft Team, YouTube, Google Classroom, and Live Digital writing. There were not many differences between males and females for the use of Zoom, GeoGebra and PowerPoint.

A few teachers (12%) reported that because of the poor Internet connectivity, they only used asynchronous independent teaching via learning management systems (LMS) such as Google Classroom. Others who had faced Internet issues mentioned they recorded videos and uploaded them on YouTube and WhatsApp groups. For example, one male teacher mentioned, "Sometimes I tried to use Zoom but was unable to do so due to poor internet connectivity. Therefore, I recorded my lesson and uploaded it on YouTube."

Another teacher explained, "I recorded lectures on my laptop, I made the classroom in my home and gave lectures on the whiteboard using markers while the camera was on me." In addition to internet connectivity, some teachers raised other difficulties with online teaching experiences. One female teacher who faced difficulties reported:

> The online teaching experience was good, but it makes a teacher's life more hectic... a lot of work is required to uplift the eLearning landscape in Pakistan and to make better use of online teaching... Mathematics is difficult to understand by students in online sessions.

The issue of students' engagement in online classes was also mentioned by teachers. Absenteeism was the main problem, along with students' inattention in online classes and not performing more than the absolute minimum. One female teacher who used Skype mentioned, "it is difficult to deliver online lectures because I do not know who is active, who is present, whatever" These are common, yet important concerns Phase 2 interviews were used to probe teachers' thoughts about student engagement.

Six teachers in the data set indicated, beside digital resources, they were able to organize face-to-face teaching at schools where classes were allowed to be conducted only on alternate days (50% of students on each day). One male teacher mentioned:

> I followed traditional teaching methods such as discussion on topics from the textbook, solving questions on the whiteboard while maintaining social distancing, using sanitiser, face mask and washing hands with soap.

In a nutshell, it appears the quick transition to online learning for Pakistani teachers was challenging. However, teaching during COVID-19 provided opportunities to learn new tools, conduct online classes, bring innovation to lessons, design and develop digital resources otherwise not possible with their daily teaching routines. The next section presents the findings of Block 6.

4.7. Block 6: Knowledge of digital resources and curriculum (KDRC)

Block 6 (B6) items measured teachers' knowledge and use of digital resources concerning curriculum planning and assessment in a technological environment. Table 21 presents descriptive statistics for five items ($B6\_1$ *to* $B6\_5$) of $B6$.

Table 21

Descriptive statistics for items of Block 6

Item No.	Item statement	Descriptive statistics				
		N	Mean	Median	Mode	SD
B6_1	I know how to design and organize lessons that include digital resources.	270	4.24	4.00	4	0.607
B6_2	I can locate and search databases/websites containing digital resources.	270	4.26	4.00	4	0.604
B6_3	I can download and install digital applications for teaching mathematics.	270	4.54	5.00	5	0.571
B6_4	I know how to create and evaluate assessments using online tools (e.g., Google Forms).	270	4.22	4.50	5	0.633
B6_5	I recommend the use of digital resources for teaching and learning.	270	4.51	5.00	5	0.602

Note. Each row in the table corresponds to a specific item, and the columns provide relevant statistics for each item, offering an overview of the distribution and central tendency of responses from the surveyed teachers.

Teachers on average somewhat agreed ($B6\_1$; $Mean = 4.24, Mode = 4$) that they know how to design and organize lessons that include digital resources. This shows that teachers consider that they can design and organize lessons with digital resources. However, the teachers, on average, reported a relatively high level of agreement ($B6\_3$; $Mean = 4.54$) with $B6\_3$. This shows that teachers strongly believed in their competence to download and install digital applications for teaching mathematics. We can link this finding with the Mean value of item $B5\_2$ (Section 4.6, Table 17, p. 156), with which teachers moderately agreed. They find it difficult to decide which digital resources to use from the Internet to teach a particular mathematics topic. Seen together, these two findings address different aspects of digital resource usage among teachers.

For example, the ability to download and install applications ($B6\_3$) relates more to technical skills and familiarity with digital tools. Teachers might feel comfortable with the technical process itself. On the other hand, deciding which resources to use ($B5\_2$) involves evaluating content relevance, pedagogical suitability, and alignment with curriculum goals. This decision-making process might be more complex and involve considerations beyond technical proficiency. However, when we look at the $B6\_2$ results ($Mean = 4.26$), in which teachers expressed weak agreement about having the ability to access and search databases online, it validates the $B5\_2$ finding. It appears that the respondents feel more confident in their ability to download and install digital applications for teaching mathematics (Item $B6\_3$) compared to locating and searching databases/websites for digital resources (Item $B6\_2$). In summary, the confidence reported in downloading and installing digital applications ($B6\_3$) might pertain to a specific technical competence, whereas the difficulty reported in deciding which digital resources to use ($B5\_2$) could reflect the complexity of pedagogical decision-making in a digital environment. The apparent contradiction between these two findings could arise from the different aspects of digital resource utilization that the items address. The original interpretation could still hold to a certain extent, with the clarification that confidence in technical skills does not necessarily translate to ease in pedagogical decision-making. This insight adds depth to our understanding of how teachers navigate the integration of digital resources into their teaching practices.

About creating and evaluating assessments using online tools, teachers reported somewhat agreeing ($Mean = 4.22$). However, the Median value was 4.50, indicating a slightly higher agreement level among a subset of teachers. This suggests that teachers can create and evaluate assessments using online tools. However, it cannot be ascertained from the response how regularly they employed online tools for assessment. Item B6_5 results suggest that teachers would strongly recommend the use of digital resources for mathematics teaching and learning.

The Spearman's rho correlation test was conducted to understand the relationship between the items of Block 6. The results of the test are presented in Table 22.

Table 22

Results of Spearman's rho correlation test for Block 6

Item No.	B6_1	B6_2	B6_3	B6_4	B6_5
B6_1	--				
B6_2	.551**				
B6_3	.239**	.247**			
B6_4	.510**	.478**	.219**		
B6_5	.149*	.148*	.245**	0.085	--

Note. The table displays a correlation matrix where each cell represents the correlation coefficient (ρ) between the corresponding items. The symbols $**$ and $*$ indicate the significance levels, with $**$ denoting significance ($p - values$) at the $\mathbf{0.01}$ level and $*$ denoting significance at the $\mathbf{0.05}$ level, both being two-tailed tests.

The Spearman's rho correlation test yielded both significant and insignificant results. The item B6_1 reported significant positive correlations with all the other items in Block 6. The results suggest teachers with the ability to design/organize lessons with digital resources may also possess skills to search, download and install applications, and to create assessments online. The Spearman correlation also suggested that teachers who know how to download and install digital applications are more likely to recommend the use of digital resources for teaching and learning mathematics. A strong correlation $(\rho = 0.478, p < 0.01)$ was also found between $B6\_2$ and $B6\_4$ which suggests those who can search and locate databases or websites can also create and evaluate assessments using online tools (e.g., Google Forms).

There were few weak but positive correlations as well. For instance, there was a correlation between the ability to locate and search databases/websites containing digital resources ($B6\_2$) and the ability to download and install digital applications for teaching mathematics ($B6\_3$). This suggests that these competencies coexist to some extent. This could be attributed to their shared foundation in digital competencies and digital resource use. Both skills involve navigating online spaces and engaging with digital resources. Teachers who feel comfortable in online environments may be able to locate resources and also to successfully install digital applications. Similarly, there is a positive and weak correlation between the ability to download and install digital applications for teaching mathematics ($B6\_3$) and the ability to create and evaluate assessments using online tools ($B6\_4$). This suggests a modest relationship between these skills. The recommendation of using digital resources for teaching and learning ($B6\_5$) showed weak positive associations with all the other skills/items, indicating some relationship but at a lower level.

4.8. CORRELATION BETWEEN THE BLOCKS

As mentioned in Section 3.4.1, I organized the online survey into different blocks, each representing a specific aspect of the digital competencies of mathematics teachers. Table 23 presents the correlation between the Blocks and their corresponding constructs.

Table 23

Overall correlations between the Blocks (Constructs of the framework)

Block				P_O	SDRK	KDRS	KDRT	KDRC
Block 2			P_O	1				
Block 3	MDKT	Personal instrumental genesis	SDRK	0.371**	1			
Block 4		Professional instrumental genesis	KDRS	0.705**	0.819**	1		
Block 5			KDRT	0.628**	0.353**	0.845**	1	
Block 6			KDRC	0.298**	0.238**	0.534**	0.812**	1

Note. The table displays a correlation matrix where each cell represents the correlation coefficient (ρ) between the corresponding items. The symbols $**$ indicate the significance levels denoting significance $(p - values)$ at the 0.01 two-tailed tests.

Personal Orientation (P_O) shows weak to moderate positive correlations with other constructs of the framework. The strongest correlations were with KDRS at 0.705 and KDRT at 0.628. These correlations indicate relatively stronger connections between P_O and knowledge of digital resources in the context of students' learning and teaching mathematics. It suggests that that stronger beliefs about mathematics teaching may support or influence the use of various digital resources in teaching mathematics to students.

Specialized Digital Resource Knowledge (SDRK) correlated positively with personal orientation (P_O) at 0.371 and knowledge of digital resources and teaching (KDRT) at 0.353. This suggests that there may be a relationship between teachers' beliefs and their knowledge of specialized digital resources, but the strength of this relationship is not very strong. However, SDRK had a high correlation with KDRS at 0.819. Notably, KDRS displayed strong positive correlations with all other constructs: 0.705 with Personal Orientation (P_O), 0.819 with Specialized Digital Resources Knowledge (SDRK), 0.845 with Knowledge of Digital Resources and Teaching (KDRT), and 0.534 with Knowledge of Digital Resources in Curriculum Planning and Assessments (KDRC). This indicates that understanding how digital resources can enhance students' learning experiences is linked to the overall development of digital competencies.

The construct of Knowledge of Digital Resources and Teaching (KDRT) strongly positively correlated with Knowledge of Digital Resources (0.845) and Students (KDRS) and a relatively strong correlation of 0.628 with Knowledge of Digital Resources in Curriculum Planning and Assessments (KDRC). This suggests teachers' knowledge of digital resources and how to use them in teaching was associated with their teaching of students and planning curriculum and assessments using digital resources. Notably, Knowledge of Digital Resources in Curriculum Planning and Assessments (KDRC) exhibited moderate positive correlations with all other constructs. This shows the importance of KDRC in digital environments and developing digital competencies.

As mentioned in Section 2.4.8, personal instrumental genesis reflects teachers' beliefs about digital resources and their specialized digital resources knowledge in mathematics education. It pertains to the ways teachers perceive the role and impact of specialized digital resources in their teaching practices. Block 3 items contributed to this construct, and the items within this block provided insights about teachers' knowledge of specialized digital resources. The positive correlation of 0.371 between Block 3 (SDRK) and Block 2 (P_O) suggests that teachers' broader pedagogical beliefs are related to their perceptions of digital resource efficacy. The correlations between the belief in exploring mathematical problems to discover patterns ($B2\_1$) and recognizing the importance of digital resources in teaching ($B3\_1$) underscore the alignment between traditional pedagogical practices and the potential benefits of digital resources.

The construct of Professional Instrumental Genesis represents the combination of KDRS, KDRT, and KDRC. That is, it combines teachers' knowledge of digital resources and their integration with students’ (Block 4), teaching (Block 5), and curriculum planning (Block 6). Strong positive correlations between these blocks revealed interconnections in teachers' understanding of digital resource use across different professional dimensions. It further suggests the importance of these dimensions in developing digital competencies.

In conclusion, the correlations suggest varying degrees of interconnectedness between these constructs. For instance, knowledge of digital resources in relation to students (KDRS) appears to have strong associations with other constructs, including personal orientation and teaching-related knowledge. Conversely, the knowledge of digital resources and specialized aspects of teaching (SDRK) have weaker connections.

4.9. CHAPTER SUMMARY

The online survey focused on Pakistani mathematics teachers' digital competencies. The results highlighted teachers' access to, use of, beliefs about, and practices in using digital resources in mathematics teaching and learning. The findings showed that most teachers (74%) had internet access and knew how to use videos, e-textbooks, and websites for educational use. However, the use of DGS and simulations was less common among them.

In relation to beliefs, teachers held strong beliefs about pedagogical practices, such as pattern exploration, conveying mathematics knowledge through multiple representations, critical discussion, and idea sharing. They strongly believed in traditional approaches such as memorization and preferred use of signs, symbols, and notations. Teachers believed that digital resources aid in engaging and self-regulating students in the classroom. However, they had varied opinions regarding whether they possessed ideas about using digital resources in classrooms. Further in Block 3, teachers' responses revealed varying degrees of agreement on the role of digital resources in mathematics education. Significant correlations among Block 3 items revealed relationships between teachers' beliefs about digital resources' importance, their potential to enhance engagement and self-regulation, and the possession of ideas for using digital resources.

In Block 4, two items (digital resources are important and use of videos for students) received strong agreement, indicating that teachers consider digital resources important for students. Teachers believed that videos improve students' understanding about abstract mathematical concepts. However, for the remaining five items, teachers expressed moderate agreement. The Spearman correlation test results showed various correlations among these items, indicating associations between teachers' beliefs about digital resources and their perceived impact on teaching and learning.

In Block 5, teachers' responses revealed a range of beliefs related to digital resources in mathematics education. A significant number of teachers expressed uncertainty about their training, suggesting they may lack confidence in their ability to use digital resources. Teachers were not confident in using DGS and found it somewhat challenging to decide which digital resources to use when teaching specific mathematics topics. They faced difficulties in selecting the most suitable digital resources given the variety of options available online. Positive correlations among Block 5 items suggests that receiving training in technology is linked to teachers' confidence and their desire to improve digital competencies.

In Block 6, teachers reported varying degrees of self-perceived competencies related to using digital resources in teaching mathematics. They believed they could design and organize lessons with digital resources and were confident in their technical skills for downloading and installing applications. However, they expressed weaker agreement in their ability to access and search for digital resources online, suggesting challenges in this aspect. Teachers reported somewhat positive agreement with their ability to create and evaluate digital-based assessments and strongly recommended the use of digital resources for mathematics teaching and learning.

Overall, the digital competencies of Pakistani mathematics teachers exhibit variations that can be attributed to many factors. These differences in competencies could be due to a range of factors that needs to be investigated. The findings suggest that those who have received training tend to exhibit higher competencies. Notably, teachers' beliefs play a pivotal role; strong convictions about the efficacy of digital resources drive motivation to acquire digital competencies. Moreover, external circumstances, like the abrupt shift to online teaching during the COVID-19 pandemic, have also impacted. Challenges in selecting the most suitable digital resources for specific mathematics topics contribute to variations, reflecting the complexity of pedagogical decision-making in the digital realm. To further investigate these variations and understand the pedagogical context that influences teachers' digital competencies, semi-structured interviews (Phase 2) were conducted to gain deeper insights into the underlying reasons behind these findings.

CHAPTER FIVE

5. PHASE 2 FINDINGS (QUALITATIVE DAT)

5.1. INTRODUCTION

In this chapter I present the finding of Phase 2 of the study. I collected Phase 2 data using semi-structured interviews to explore the Phase 1 results in depth. Interviews with participating teachers ($n = 6$) were employed to probe quantitatively significant results, outliers, and perplexing Phase 1 results. Along with the overarching research question (RQ), the following sub questions (SQs) were considered important based on Phase 1 findings:

SQ1 - How do Pakistani teachers' beliefs about mathematics teaching influence their decisions to integrate digital resources in teaching and learning mathematics?

SQ2 - What influences teachers to use digital resources well?

SQ3 - How well do Pakistani teachers use digital resources to teach mathematics topics?

SQ4 - What professional development opportunities do Pakistani teachers have that influences their digital competencies and approach to integrating digital resources?

SQ5 -What are the barriers to and enablers for teachers using digital resources face-to-face and online?

These research questions also aligned with the corresponding domains of the modified DCTMT framework. This alignment ensured that the semi-structured interviews focused on and addressed the specific aspects of teachers' digital competencies. For instance, SQ1 was linked to Personal Orientation (P_O) and Personal Instrumental Genesis (SDRK) and provided details about Pakistani mathematics teachers' beliefs and their influence on integrating digital resources. SQ2–SQ5 were linked to SDRK and Professional Instrumental Genesis (KDRS, KDRT, and KDRC). During the interviews, further sub-questions related to professional instrumental genesis arose. One such question concerned teachers' familiarity with theories about students' learning that relate to the use of ICT. Another point of discussion related to teachers' ability to evaluate the efficacy of particular digital resources and their appropriateness in the context of mathematics education. An associated conversation concerned teachers' competence to search for, select, evaluate, create, share, organise, download, and modify digital resources. I analysed the interview data using the pre-identified themes and sub-themes (see Table 8, p. 127) to connect the qualitative data with the digital competencies' framework.

In Pakistan, due to COVID-19, schools remained closed for almost 18 months (from April 2020 to October 2021). All the Phase 2 participating teachers quickly transitioned from face-to-face to online teaching. Their responses would have been influenced by the necessity of coping with COVID-19 and moving to teaching online. Correspondingly, their responses may reflect some of the anxiety that they experienced due to the urgent pressure to teach online which they may not have done previously., Recognising this possibility, at the end of the individual interviews I asked to what extent their beliefs, values, and preferences about digital resources had changed because of the COVID-19 lockdown. I aimed to understand the impact of digital resources' affordances on their beliefs due to the pandemic. While responses to this question are linked to the "personal orientation" theme of the framework, they also link to other themes.

The following section presents findings on teachers' beliefs about mathematics teaching and their use of digital resources in teaching and learning mathematics.

5.2. General beliefs about mathematics teaching and learning

To understand teachers' general beliefs about mathematics teaching and learning (SQ1), I asked each teacher the following two questions:

- In your opinion, what is the main purpose of teaching Mathematics?
- Why are you teaching mathematics?

Teachers' responses demonstrated varied beliefs concerning teaching and learning mathematics. The majority perceived mathematics as a subject that connects and explains knowledge in other subjects such as physics, chemistry, and computer sciences. They believed that mathematical concepts act as tools that construct knowledge and facilitate understanding of real-life contexts. For example, T2, an experienced mathematics teacher, believes that:

> Mathematics provides lots of valuable contributions to other subjects. Mathematics is a tool. No matter what subjects you study, from the beginning to the advanced level, mathematics works as a tool. Without mathematics, it would be impossible to understand many other subjects... In real life, math is everywhere. Whether you want to do engineering or medicine, become a scientist, or do business, math is everywhere. It connects... I say that without mathematics, we are zero.

T2's response reflects a strong belief in the foundational importance of mathematics as a tool that connects and supports various subjects and real-life applications. T2 emphasised that mathematics is not an isolated subject but a universal language that bridges different disciplines and careers. T2's perspective aligned with the domain of "Personal Orientation" within the digital competencies' framework. The teacher's perception about mathematics as a crucial tool across different subjects and industries demonstrated a personal orientation in relation to the broader impact and significance of mathematics education. This perspective could influence how T2 approaches the integration of digital resources in teaching and learning mathematics. Furthermore, T2's emphasis on the ubiquity of mathematics in various domains of life resonates with the framework's domain of "Specialized Digital Resources Knowledge" (SDRK). His view that mathematics is essential for diverse fields and professions suggests an understanding of the potential of digital resources to enhance students' understanding of and engagement with mathematics across different contexts. Overall, T2's response represents relatively strong subject knowledge, which may play an important role in predisposing him positively towards integrating digital resources into teaching and learning. Other teachers, such as T1 and T6, held similar beliefs. For example, T1 said:

> Today is the age of mathematics... It is a fundamental subject. I want my students to progress, and mathematics helps in achieving this goal... In our school time, we were only taught abstract ideas. We were taught that this is a question and how to solve it. When we use digital resources, we physically look at what is happening or what is being said or how this is happening.

T1's perspective on the significance of mathematics aligns with the sentiment expressed by T2 and T6. T1's emphasis on mathematics as a tool for advancement echoes the view that mathematics provides the foundation for various fields and careers. However, T1's mention of the effect of digital resources on learning experiences is notable. T1 recognized the value of digital resources in providing visual and interactive learning opportunities. This recognition aligns with the digital competencies' framework's construct SDRK (i.e., beliefs about digital resources and their use in mathematics education). T1 believed in the potential of digital resources to offer a more tangible and engaging learning experience and promote deeper understanding.

T1's perspective can also be related to the "Knowledge of Digital Resources and Teaching" (KDRT) domain. The teacher's awareness of how digital resources can offer dynamic ways of interacting with abstract mathematical ideas suggests an understanding of how these resources can enhance teaching methods and student engagement. T1's remarks indicated a shift from traditional abstract teaching methods toward more visual and interactive approaches made possible through digital resources.

Moreover, the findings suggest that experienced teachers are more likely to exhibit strong beliefs (as in the case of T2 and T1) than are the less experienced or early career teachers. For example, when asked about the purposes of teaching Mathematics, a young private school female teacher (T3) explained the role of mathematics as follows:

> I am at an early stage of my teaching career. I think I am still figuring out the purpose of mathematics teaching. Mathematics is interesting... I have seen that those who are good at mathematics can go far on Instagram... They know how to use and manipulate the numbers on Insta, how to increase followers and when to post on Instagram...

T3's initial reflection on the purpose of mathematics teaching indicates a sense of exploration and curiosity. The teacher's statement about still figuring out the purpose of mathematics teaching aligns with the notion of teachers at the beginning of their careers actively seeking to understand the impact of their instructional practices. This perspective can be related to the "Personal Orientation" (P_O) construct in the digital competencies' framework, where T3's evolving beliefs about the role of mathematics teaching align with her position as a less experienced teacher who is still shaping her teaching identity. However, T3's observation of individuals skilled in mathematics finding success on social media platforms like Instagram introduces an interesting dynamic. This comment can be considered through the lens of digital competencies and the evolving nature of education in the digital age. It shows that T3 is aware of the practical applications of mathematical skills in various contexts, including social media. It also suggests T3's potential to find the real-world relevance of mathematics and capture students' interest and emphasise its broader applicability.

The findings also indicated that the context of teaching such as whether the teacher worked in a government or private school could also impact beliefs about mathematics teaching. Teachers from government schools considered mathematics a difficult subject. They believed that teaching and learning mathematics did not come naturally or intuitively. Their view was that it takes significant effort, time, and energy from both teachers and students. The following response of T5 reflects such beliefs:

> Most people do not prefer to teach mathematics here (rural government school) because it requires time and hard work. They have to use their brains quite a lot, which requires extra effort and struggle to convince students. It is a very tough job.

> Many teachers are teaching different subjects, but I think by teaching mathematics I can work more for the betterment of students. But we don't have good mathematics teachers here.

The above response shows that the teaching environment, especially in a government school setting, could impact beliefs about mathematics education. T5's response highlights the challenges of teaching mathematics in a resource-constrained rural context especially because of the complexity of the subject matter. He believed that teaching mathematics contributes to students' improvement and demonstrates a sense of responsibility about and dedication to their role as teachers. The context of a rural government school setting may have influenced T5's personal orientation towards mathematics teaching, due to the government's policy of hiring, training, and assigning teachers to remote (rural) locations. However, another government (urban) schoolteacher (T4) (who had also taught in a private school) had a different opinion about the difficulty of teaching in a government school. She replied:

> Previously I was teaching in a private school where it was easy to teach mathematics. But right now, I am working in a government school, it is very difficult to be a mathematics teacher or teach mathematics here. Mathematics is boring and a dry subject. Therefore, very few students like mathematics. Due to these reasons, I face difficulties in teaching mathematics at the government school.

T4 considers mathematics “boring” and a “dry” subject. I asked her why she was teaching mathematics if it was boring, she clarified:

> It is not boring for me but for students. They don’t like it; they have a presumption that it is boring and dry. Especially students in the government schools are not interested in mathematics.

Having taught in both private and government schools, T4 could compare the challenges and perceptions of students in both contexts. Her statement that mathematics is easy to teach in private schools but difficult in government schools supports the idea that the educational environment (or social context of teaching) influences teachers' experiences and beliefs. However, T4's clarification that mathematics is not boring for her suggests a personal enthusiasm for the subject.

This shows a contrast between T4's beliefs and students' perceptions. T4's beliefs align with the Personal Orientation (P_O) construct. Her beliefs about how students view mathematics align with the "Personal Instrumental Genesis," which shows how teachers' beliefs affect students' attitudes.

T4's perspective may provide insights into SQ1, which aims to understand how teachers' beliefs influence their decisions to integrate digital resources. T4's experiences highlight the need to address students' attitudes toward mathematics in government schools. This was explored further in interviews (Sections 5.3 and 5.4) by inquiring about any strategies or practices T4 has employed to make mathematics more engaging for students. In brief, T4 and T5's responses may have been based on their students' beliefs and the environment in which they teach mathematics. T4 and T5's positions on government school teaching, students, and difficulties in mathematics articulate issues of resource-constrained environments such as the availability of teaching resources, the environment, and the lack of professional training in government schools.

5.2.1. Beliefs about memorisation, and use of signs, symbols and notations

In Phase 1, teachers strongly agreed with memorising rules and formulas and the use of signs, symbols, notations. To further understand these results, I asked teachers the following question:

- What is your preferred method of teaching?
- How do you believe students best learn and retain information related to signs, rules, symbols, and formulas in mathematics?

All the teachers considered memorization to be an important part of mathematics teaching and learning. T2 explained.

> It is very important to understand the signs and formulas. But to use them or to give an exam, students have to memorise them. Though my emphasis is always on expounding the concept, our education system does not allow us to do it properly. You have to understand the final exam paper pattern, which is about 80% theoretical

> and 20% objective. The student has to memorise a little bit. If students do not memorise, how will they remember the concepts?

T4 emphasised the importance of memorisation as follows:

> We must ask students to memorise the signs, symbols, and notations. Without memorisation, how can they recognise them? For example, the first chapter in the grade 10 textbook is about SETS, which includes several signs such as union, intersection, proper set, etc. My students easily understand these signs. They understand and perform better in set notation form than in tabular form. Without memorising these signs, it would be difficult for them to learn such topics.

T1 and T3 expressed similar beliefs about memorisation. T6 and T2, in addition to memorization, also emphasised understanding and explaining the concepts. T6 replied, "formulas, rules, signs, and notations need to be explained rather than memorised. Students must learn how to implement or utilise them." All teachers preferred speed or rote memorisation of mathematical concepts. T2 explained the factors that drive the emphasis on memorization. He considered the Pakistani education system's examination patterns create a necessity for students to memorise certain elements. T2 acknowledged that conceptual understanding is also essential.

T4's perspective further highlighted the role of memorization in the pedagogical practices of Pakistani high school mathematics teachers. T4 believed that students can enhance recognition and understanding of mathematical signs and symbols through memorization. T4's example of teaching sets highlighted the utility of memorization in this context. We can relate these responses to the general beliefs about mathematics teaching (P_O) within the digital competencies' framework. It shows that traditional pedagogical approaches like rote memorization held a prominent place in the Pakistani educational system. T2 and T4's explanations highlight the interplay between students' success in examinations and the need for memorization, reflecting a contextual constraint that influences teachers' instructional decisions.

These responses provide valuable information in relation to SQ1, which explores how teachers' beliefs about mathematics teaching influence their integration of digital resources. The emphasis on memorization indicates a traditional perspective of teaching and assessment, which can impact the extent to which teachers see value in integrating digital resources. These responses offer insights into the potential challenges and opportunities for integrating digital resources in a context where memorization is emphasised. The combination of comments that students find mathematics boring and difficult with the emphasis on memorization, raises concerns about the nature of mathematics education in Pakistan. The association between memorization-centred teaching methods and negative student perceptions of mathematics highlights a potential disconnect between pedagogies and a meaningful engagement with the subject. The reliance on memorization without real-world applications or conceptual understanding might contribute to the perceived lack of relevance and interest in the subject.

These findings can be linked with multiple research questions and the digital competencies framework. They have a clear link with SQ1, which explores the influence of teachers' beliefs on integrating digital resources. The responses suggest that teachers (especially government schoolteachers) might not be addressing negative perceptions of mathematics through other pedagogical approaches like using digital resources. T4 and T5's comments also tie into SQ2 and SQ5. Memorization-focused pedagogy could be one of the factors that limit teachers' use of digital resources.

The findings also revealed that textbooks were the main resource for teaching mathematics. Teachers also considered textbook-focused pedagogies and examination patterns responsible for maintaining the legacy of repetition and memorisation in teaching practices. All teachers strongly agreed that textbooks were the most crucial resource, and they barely go beyond the content in textbooks. As T2 explained:

> This is because our students find it difficult to even study the textbook. They are not even interested in reading the textbook. As a result, we do not go beyond the textbook. The students get confused, so I only use the textbook... Our students' primary focus is to achieve good grades and gain admission to engineering and computer sciences. Their priority is to complete what is in the textbook rather than look for extra material. Teachers have adjusted their teaching accordingly.

Interestingly, all teachers used “we”, the plural form of the pronoun, instead of “I”, when they reported common pedagogical practises such as a focus on the textbook, memorization, and use of other resources for teaching. For example, T3 explained:

> The real problem is that we can only teach here with a textbook and whiteboard. We don't have any other tools here... The level at which we have learned from our own teachers, we are also teaching our students at the same level.

The teachers' use of the collective pronoun "we" rather than "I" when discussing pedagogical practices reflects a shared sense of constraint and conformity within their social context of teaching. This linguistic choice suggests that these practices might be driven by systemic factors, institutional norms, or peer influence, rather than solely by individual preferences. It speaks to a larger context where teachers might perceive that they have limited agency to deviate from established norms and practices. The observation of teachers adjusting their teaching methods to align with their own learning experiences further suggests the influence of tradition and prevailing norms. The responses to other themes in the next sections confirm the presence of peer influence that forces them to adjust (change) their existing beliefs or behaviour to fit in with others. This could be useful in the context of this research to understand changes in beliefs that would be classified as "conformity beliefs" due to peer pressure. The "level" mentioned by T3, referring to the teaching methods inherited from their own teachers, highlights a cycle of instructional continuity that could perpetuate less effective pedagogical approaches.

The identification of "conformity beliefs" due to peer pressure is a perspective that aligns with the broader theme of professional instrumental genesis. Teachers adapting their practices to fit within the existing norms and practices of their teaching community reflects a form of socialisation. It also suggests that the development of digital competencies requires addressing not only individual beliefs and skills, but also the broader cultural and social dynamics that shape teaching choices.

These insights contribute to understanding the factors that influence teachers' decisions to integrate digital resources (SQ2). The tension between traditional practices, student expectations, and the absence of awareness of the potential of digital resources could be linked with the SQ5. Moreover, the findings suggest the role of peer influence in shaping "conformity beliefs" highlights the mechanisms through which Pakistani teachers' digital competencies could develop and evolve within a professional community (SQ4).

Overall, the findings highlight the intricate web of influences that impact teaching practices and the potential for change in the context of digital competencies. It suggests mathematics teachers in Pakistan shared beliefs about mathematics teaching. Main teaching resources included textbooks, teachers, and blackboards. Their beliefs differed according to their working environment. The government schoolteachers showed more inclination towards traditional pedagogical approaches (e.g., rote memorisation). Whereas private schoolteachers were more likely to indicate affinity with more contemporary beliefs. Nevertheless, both appear to be influenced by peers, leading to conformity to others' beliefs. Their understanding of student-centred teaching approaches is limited, which could be attributed to "lack of professional development" and an ineffective evaluation system. The next section presents the findings related to beliefs about digital resources and their use in mathematics education.

5.3. Beliefs about digital resources and their use in mathematics education

This section reports on how the participating high school mathematics teachers perceived digital resources and their use in mathematics education. It was important to ascertain their views about digital developments which are changing and expanding the teaching of mathematics and investigate the impact of these changes on their beliefs about digital resources. Their responses here would help to shed light on their knowledge and integration of global content into the local mathematics curriculum, as well as their beliefs about various digital packages for visualisation, dynamicity, and data analysis. The following sub-questions guided the inquiry to understand teachers' attitudes towards digital resources, their perceived utility, and the extent to which they have been exposed to training in digital technologies:

- Do you use digital resources for teaching? (SQ3)
- What do you personally say about them? (beliefs) (SQ2)
- Are they useful for teaching mathematics? (SQ3 - SQ4)
- Did you receive any training for the use of digital technologies for teaching and learning mathematics? (SQ4)

In response to the first question, all six teachers agreed that digital resources play a vital role in mathematics teaching and learning. They believed that visualisation and digital graphical representation of mathematical concepts would play an important role in enhancing students' interest in mathematics. However, pedagogical practices and the availability of digital resources seemed also to influence their beliefs. For example, T3 explained:

> In our case, we can't use digital technology much. If we used audiovisual aid[s] in our school, it would definitely help students and teachers, but we can't use them. Because there is no system, there is no such thing (digital resources) right now in the school, we just use a whiteboard and two markers, black and blue... I can ask our principal for digital tools, but if we become more efficient, then the rest of us (peers) will not allow us to be more efficient. This is the problem.

To understand what she (T3) meant by "become more efficient" and what kind of change she expects, I asked her to further elaborate on the statement. She replied:

> Actually, digital learning is not so much appreciated here. I know I can use them if I want to… But we have to follow what the headteacher tells us to do or use. We have to follow her advice as the school made her headteacher for that purpose… Some (senior) teachers did not even know how to use the Internet and Zoom, and it was very difficult for them to figure out how to use them because they were not skilled. They also do not want to learn… So, during COVID, we took their classes… To bring about the change, I started my own (YouTube) channel. I thought, instead of teaching students as a routine, I needed to do something different. I am teaching them by making videos of mathematics topics. Students don't need to remember things; they have to learn instead of memorising them…

T3 understood the importance of digital resources (audiovisual aids) but lacked the confidence to embed them in school teaching for two reasons: lack of resources, and peer pressure or institutional constraints. Her mention of "just" having a whiteboard and markers reflects her perception of the limitations of available resources. She believed that initiating change might transgress the pedagogical practices of some senior teachers. She might be influenced by senior teachers, which is not surprising for an early career teacher. In Pakistani private schools, job security and other regulatory issues exist. However, when she clarified her views, T3 showed confidence about embedding digital resources into her personal practice. Her statement about becoming "more efficient" suggests she desired to introduce teaching methods through digital resources. This aspiration highlights potential for positive change in teaching practices. However, she also emphasised the challenge of navigating peer dynamics and traditional norms, which could generate resistance to such changes. This connects with the earlier observations of "conformity beliefs" where peer influence and institutional norms can limit the adoption of new practices.

T3's initiative of creating a YouTube channel to engage students with video-based learning was another example of how teachers can innovate despite challenges. Her emphasis on learning rather than memorization reflected that she wanted to move away from rote memorization towards more meaningful learning experiences. This transformational perspective also speaks to the perceived potential of digital resources to facilitate conceptual understanding and active engagement. It confirms the existence of already developed basic digital competencies and points towards personal instrumental genesis. Her position also aligned with the idea of professional instrumental genesis, where teachers' experiences, beliefs, and the social context of teaching influence their development as teachers.

T3's proactive approach to using YouTube indicated a form of self-directed professional development. She recognized the limitations of traditional teaching methods and wanted to use digital platforms to involve students in an engaging manner. In the context of the research questions, her response aligned with the themes related to SQ4, which investigated the professional development opportunities that influence teachers' digital competencies and their approach to integrating digital resources. T3's response also contributes to understanding teachers' beliefs about the use and challenges of integrating digital resources (SQ2 and SQ3).

Similarly, T5, a male teacher teaching in a rural government school for more than ten years mentioned similar issues as follows:

> We should learn the use of digital technologies. But neither do we have the facilities, nor do we do the type of teaching that requires digital tools. Especially when we talk about the government sector, there are no such things as digital resources. We only make limited use of WhatsApp to communicate with each other.

T5's response indicated two factors that limit the use of digital resources: the current teaching methods and the lack of resources. Although T5 later mentioned receiving training for blended learning, his intent to blend digital tools in the classroom seems vague. This suggests a gap between the training provided and its practical application. T5 lacked confidence in adapting new pedagogies and technologies due to the constraints of the teaching environment. His response explained his reluctance to change traditional pedagogical practices as stemming from the potential lack of approval or funding constraints and illustrated the challenges that teachers face in resource-constrained environments. T5's mention of the lack of facilities and the use of teaching methods that do not require digital tools highlights the existing barriers to digital resource integration. The limited use of WhatsApp highlighted the minimal use of digital platforms, even for basic purposes. Feedback from T5 supported the finding that Government teachers tend not to use digital resources. T5's response aligned with the themes related to SQ4, which explores the professional development opportunities that impact teachers' digital competencies and their approach to using digital resources. It also contributed to SQ5 by indicating the challenges faced by rural government schoolteachers.

T6, who demonstrated a good understanding of digital resources, mentioned similar factors as follows:

> I think digital resources are very helpful... Without a doubt, videos, graphics, and animation enhance students' learning abilities. When we make videos for students, it not only helps students but also allows us to see where we are making mistakes or how we can improve our teaching techniques. But unfortunately, in Pakistan, we don't have much use for these resources, and we also don't have ideas about their uses.

T6's acknowledgment of the benefits of digital resources, such as videos, graphics, and animations, in enhancing students' learning abilities, aligns with Phase 1 findings (see Sections 4.5 and 4.6). Teachers affirmed that these resources engage students and improve teaching outcomes. Moreover, his comments about limited use and ideas for using digital resources were also consistent with Phase 1 findings. Teachers agreed that they did not have many ideas about using digital resources (Section 4.6, Table 17, p. 156). I asked, why this was the case and why teachers in Pakistan did not have many ideas for using technology in mathematics teaching. He replied:

> There are numerous reasons. For example, during my pre-service training, we were told to use activity-based learning. They (trainers) did, however, also compel us to use low-cost or free means. Any teaching idea that involves cost is not encouraged. Policymakers are responsible for this. Why would teachers go the extra mile if they know the idea will be rejected because of resources? Look at the situation at our school computer lab, where Pentium 3 computers are still used to teach students... We don't have a single projector in our school. Whatever funds we get, they are enough to do a whitewash or replace broken classroom furniture.

The influences of limited technology infrastructure, policy decisions, and resource limitations were evident in T6's response. T6 believed that these constraints are responsible for teachers' limited ideas about using digital resources. At a personal level, T6 was familiar with the use of digital resources for teaching. He mentioned using them to teach himself, and communicate, collaborate, and share content with peers. This usage demonstrates already developed digital competencies, indicating the presence of personal instrumental genesis. However, he failed to develop professional instrumental genesis, the competencies that allow him to plan for and implement digital devices and resources for students' learning. Underdeveloped professional instrumental genesis is often caused by a lack of professional development. Therefore, T6's response directly contributes to SQ4 (professional development) and also contributes to SQ2 and SQ3.

In relation to professional development, I asked participating teachers what training they had received during their professional career for using digital resources to teach mathematics. This was related to SQ4 that examines any professional development opportunities teachers may have had during their teaching careers. The reasons for asking this question were twofold: first, to see whether the professional training (if any) helped teachers establish beliefs about the use of technology in mathematics education; and second, to understand whether training plays a part in developing digital competencies.

The responses showed that teachers had attended training related to general teaching practices and leadership. Three of the six teachers (T1, T4, and T6) had received only emergency training sessions for the use of technology during COVID-19. This included using video conferencing applications (Zoom and MS Teams). Before COVID-19, most teachers said they had little experience with digital resources and had never received any training for the use of technology or digital resources in mathematics education. This highlights the impact of the COVID-19 pandemic on their engagement with digital resources and technology and indicates the pivotal role that external factors and events can play in shaping teachers' digital competencies. The pandemic was a unique opportunity to learn how to use digital resources. For instance, T2 mentioned that:

> I had never used digital applications before COVID. I didn't even have a computer and I did not even know how to use them. But now I know how useful and comfortable they are... I've been taking online classes from home for almost two years now.

T2's experience is particularly noteworthy, as it highlights a transformation from limited exposure to digital applications to becoming adept at using them. He had two decades of teaching experience. Before COVID-19, he used traditional teaching pedagogies that required only a textbook and whiteboard. The pandemic compelled T2 to acquire new digital skills, indicating the potential for professional development to occur out of necessity and immediate demand. The use of digital resources during COVID-19 established and deepened his beliefs about them. He mentioned:

> As COVID is all over these days, you can't survive without digital things. Students are at least connected with education because of them. If not 100%, then at least 50% or 60% are learning... Today I took the Zoom class from home in the morning in which 50% of students were present. For those who were not present, I sent them the YouTube link of the recorded Zoom class. Digital resources are very useful for students who want to learn. Those who don't want to study don't even study in the face-to-face physical environment... I haven't used these things before COVID. This is the first time I've used a laptop (ThinkPad)... Since I am using a laptop, I can send links, write and solve maths problems. I can also share pictures and notes on ThinkPad. If a student asks a question, I can write, type and share it on the screen.

The above response shows a teacher's beliefs taking shape in the process of using digital resources. T2's experience with technology improved his agency and self-efficacy. His already developed subject-matter knowledge further expanded through using digital resources; technology deepened his pedagogical reservoir and gave him more confidence in teaching mathematics. His observation that students were able to continue learning through digital resources emphasises the accessibility and inclusivity that digital resources can offer, even in challenging circumstances. Moreover, the transition from traditional teaching methods to integrating digital tools, such as Zoom and recorded class links, reflects T2's willingness to adapt and evolve his teaching practices. This enabled him to deliver content remotely, interact and engage with peers and students, and remain flexible in his teaching approach. T2's narrative aligns with the broader theme of professional instrumental genesis and its role in shaping teachers' beliefs, practices, and digital competencies. However, during his teaching career to that point he had never received any professional development training specific to mathematics education, including the use of visualisation, dynamic graphing software, equation editors, and basic computational modelling. The same situation was evident in the responses of other teachers. For example, T5 another government schoolteacher mentioned:

> I have received training about blended learning but not on how to use technology to teach mathematics. There is no such training we have received that tells us how to convey mathematical ideas to students using technology. In teaching, we also don't do things that require technology. Our students also do not have access to technology.

T4, who had experience working in both government and private schools, indicated that "apart from MS Team, I did not attend any other training related to using digital resources." T6, a young mathematics teacher, mentioned, "I attended a pre-service teachers' training course at the time of joining. I have been teaching for four years now. I did not receive any training on how to teach mathematics using digital resources." The majority of the teachers reported that whatever knowledge they had about digital resources was mostly self-acquired using the information on the Internet. However, it appears that they were unable to apply it for students due to a lack of resources. T5, explained:

> For students' learning, we can include digital resources such as videos and images. But unfortunately, we don't have such facilities here, and even if I wanted to, I couldn't. For example, I can show videos using my laptop, but students do not have devices to watch themselves. Many teachers probably don't have digital devices. If a teacher wants to arrange them, they can do it personally, but it will be very expensive. I only focus on textbooks. I do not use digital resources in my planning and delivery of lectures.

It was noticed that teachers' self-learning is dependent on the availability of computer equipment and the cost associated with them, such as Internet packages. Most teachers mentioned that they can hardly afford a high-speed internet connection, online (paid) courses, and smart digital devices. T1, a teacher at a private school, stated that:

> Recently, I attended a workshop on MyMaths. The online application is available only via paid subscription. One of our colleagues showed us how the application works and how we could use it. But the institution did not provide any funds to buy the subscription.

A female private school teacher (T3) explained, "I would like to enhance my maths teaching using digital resources. But it all depends on how much money I have to spend on training and how much money I earn from my teaching." There appeared to be a trade-off between professional development and the cost associated with it. Neither teachers nor schools had the financial means to develop teachers' skills to address students' learning challenges using digital resources. It would be useful to examine the government's role in the professional development of teachers against teachers' expectations. The findings show teachers held positive beliefs and expected the school/government to take responsibility and provide resources to enhance the quality of teaching. As T6 explained:

> I believe teachers want to be trained... Since 2013, the government has been continuously hiring new teachers. They are all young and want to do good. They want to use digital resources. For me, the biggest hurdle is teaching theorems. At the moment we ask students to memorise theorems. Digital resources could help me explain and teach theorems easily.

In summary, this section suggests common and specific themes for both government and private schoolteachers. The findings suggest that most teachers' beliefs about digital resources were established through their use, not because of professional development. The responses indicate a gap in the professional development opportunities provided to mathematics teachers in relation to the integration of digital resources in their teaching practices. Teachers, regardless of their professional role (government or private), believe that "lack of resources and funding" are the primary reasons for not using digital resources. In particular, for government teachers, the "type/style of teaching" could be another factor that influences their beliefs. Private schoolteachers seem to work under peer influence and prefer local practices. COVID-19 appeared to have played a major role in influencing teachers' beliefs and increasing the use of digital technologies. However, students' use of digital resources was still limited due to financial issues. In terms of instrumental genesis, teachers had already developed personal instrumental genesis, i.e., they had self-learned how to use digital resources for personal learning. They had created their own workarounds for teaching and learning. However, professional instrumental genesis was underdeveloped. Teachers expected that the school/government should take responsibility as they are do not have the finances to invest in their own professional development. Therefore, it would be useful to discuss the effect of "professional development versus self-learning" on the use of digital resources. The next section presents findings related to pedagogical digital resources knowledge that include teachers' knowledge of students, teaching, curriculum, and assessment in a digital environment.

5.4. DIGITAL PEDAGOGICAL RESOURCES KNOWLEDGE (DPRK)

In this section, I inquired about teachers' digital pedagogical resources knowledge. This knowledge covers teachers' competencies to teach mathematics with digital resources (KDRT), the understanding of challenges relevant to students' learning and needs (KDRS), and the implementation of the curriculum, and assessment in a digital environment (KDRC). These competencies allow mathematics teachers to search in digital repositories for suitable resources and to select, create, share and evaluate those resources best suited to their students' and peers' needs. The focus was on collecting further information related to the research questions SQ3, SQ4 and SQ5 as follows:

SQ3: How well do Pakistani teachers use digital resources to teach mathematics topics?

SQ4: What professional development opportunities do Pakistani mathematics teachers have that influenced their digital competencies and approach to integrating digital resources?

SQ5: What are the barriers to and enablers for teachers using digital resources face-to-face and online?

I started this section of the interviews specifically with the following two sub-questions:

1. How do you plan your lessons? (SQ3)

2. Do you include digital resources in your lesson? (Yes/No) If yes, how do you choose them? If no, why? (SQ5)

For preparing lessons most of the teachers mentioned they followed learning objectives specified in the curriculum. For example, T2 starts the lesson preparation by asking himself key questions such as: "What background knowledge do I need to tell before the start of this lesson?" For T3 it was important to identify, "how much I want my students to learn?". All the teachers' planning revolved around the content of the mathematics textbook and using digital resources was not a prominent factor in their initial lesson planning process. As T2 explained:

> I start by studying the topic myself. Then I look, how to convey it to the students in the easiest way possible… I look at the textbook and how the textbook conveys the idea. What are the main points? Then I add my knowledge and understanding. Sometimes, I also search using Google to look for the main points and relevant material. I have to consolidate all the things in such a way that they can be completed in the available time duration. Time is an important factor. You cannot teach everything to students in 50 minutes related to the topic… Too much detail can sometimes become overburdened for them.

It is evident that T2 relies on two things: own subject matter knowledge and the textbook. There seemed to be no urgency to include digital resources. However, during COVID-19, his teaching took a different approach when he had to teach online. With the help of friends, he taught himself the use of digital technologies. This connects with the study's research questions explaining how teachers' use of digital resources has evolved and how they navigate challenges and changes in their teaching practices. T2 further explained:

> I haven't used these things before COVID. Initially, I was taking the classes on my phone via WhatsApp group call. I was facing a lot of difficulties because I had to prepare the lecture and explain it by taking pictures of my written notes. It was difficult to take an online class on mobile. The students were not satisfied.

T2 faced multiple challenges, including inadequate knowledge of appropriate digital tools for teaching online, a lack of training, and financial capacity to buy a laptop or desktop PC. However, he overcame these challenges when the Tuition Academy (where he teaches for extra income) provided him with a ThinkPad on monthly instalments. His online learning and teaching skills improved, and he discovered comfort and confidence in teaching during COVID-19. He explained:

> When I bought the ThinkPad, I learned the use of Microsoft Whiteboard and Zoom apps. If you are teaching maths, then digital whiteboards are very important. They contain graphs, animations, diagrams, etc. From there, I can pick any mathematical object or simple graphics I want to show. I can also draw any image I want using a

digital pen. If there is any good video, I send the link to the students… I took my ThinkPad to the college to practically show my colleagues how I was using it for online classes. Most of them do not know how to use such devices. They were taking classes from the college studio… (where) a camera is installed in front of the board and the session is going live on Zoom… When I trained a couple of teachers on the ThinkPad, they got excited. I showed them how to use a digital pen and solve any mathematics problems on the screen. Students also actively participate while at home… It is like students are with you, like a face-to-face class, and they can ask any question they want… and I can interact with them more…

T2's response demonstrated how he transitioned to online teaching and developed pedagogical digital resources knowledge. He was the most experienced teacher of the six. Maybe his strong subject matter knowledge helped him to quickly understand the affordances of digital resources that could support mathematics teaching. Being a senior teacher, he shared his experiences of using digital resources with other teachers (collaboration). He augmented his digital competencies by starting a YouTube channel (self-initiative). He mentioned, "I now record my lectures and arrange them on the channel so that students can access them at their own time." It shows T2's "self-learning" spree and the competencies that allow him to create content using a digital pen, structure his lessons and set up learning sessions, activities, and improve interactions with students in a digital environment. The main factors appeared to be motivation and acquiring a device. The ThinkPad (a device), and on-screen writing using a digital pen (affordances) and subject matter knowledge, helped him to apply face-to-face teaching methods in online teaching and learning sessions. T1, a private school teacher who use digital resources to teach geometry explained:

I use multimedia to teach conic sections. Students always find it difficult when I teach it on (white/black) board. I use videos to elaborate the concept. In classrooms, digital resources are fine but not completely online. Students do not have basic computer skills such as file attachment, signing into an account, or copying and pasting… During the online class, sometimes they get disconnected when the power goes out… Normally, 6 to 8 hours is loadshedding time here… The problem is that we use digital resources only when they are needed most. No one would allow us to use it otherwise.

T1's use of multimedia to teach conic sections showcases how digital resources can enhance the teaching of complex topics like geometry (SQ3). T1's experience indicates the potential of videos and multimedia to facilitate understanding and engagement among students (KDRS). However, T1's comments about students' lack of basic computer skills and the disruptions caused by power outages during online classes highlight the contextual challenges faced when integrating digital resources into teaching, particularly in a resource-constrained environment (SQ5).

T3, a female private school teacher, describes her planning and delivery of a lesson as follows:

> We have lesson plans which we make by hand (writing) and then we type on MS Word. For example, my last lecture was about variations that I gave to the 10th class. I taught a relationship between antecedent and consequent. First, I gave students background knowledge about the topic. I take 5 minutes to explain the importance of the topic. I am very fortunate to be teaching boys because they are extremely intelligent in maths. They can catch things very fast as compared to girls. For example, during teaching antecedent, it was hard for them to pronounce the word antecedent. But, as I repeated the word so many times, they were saying that word at the end of the lecture. In addition, I always try to link maths problems with physical (real-life context) entities so that they can relate and learn quickly.

T3 response, indicating that she prefers creating hand-written lesson plans and then digitising them using MS Word, reflects a blend of traditional and digital approaches to curriculum development. She followed the traditional style and emphasises explanation and repetition. Additionally, she maintains a belief (personal bias) about boys being better at maths, which might not help her teach girls. T3 seems unaware of social and cultural factors and other educational challenges faced by both genders (boys and girls) in Pakistan. Despite her belief, she mentioned earlier that "I was good at maths". However, her attention to the pace of instruction, pronunciation practice, and connecting mathematical concepts to real-life contexts highlights pedagogical considerations that aim to enhance student engagement and understanding. In this regard, I asked: Do you think that the inclusion of digital resources could help in connecting maths problems with real-life contexts? How would you search, access, and select digital resources for any mathematics topics and real-life connections? She replied:

> I never used any of these tools in my teaching. I did not have to do it. You know, we just have to teach using a whiteboard and markers. It is that simple. I am teaching students of the Federal Board in which teachers are not required to go deep into the topic they teach. They just have to stay within the (book) questions and teach students which formula needs to apply... I must say that we are not learning or teaching mathematics; we are just memorising and summarising things. We are not producing good students here. It is not difficult to learn these tools, but if I am good at them, others might not be happy with me. I need to be on par with other teachers. I don't want to be better than them.

T3 once again mentioned the peer pressure and textbook-focused pedagogy that restricted her from integrating digital resources into the lesson. This highlights the balance between her aspirations for innovation and the need to conform to the standard teaching practices. Her statement about not wanting to be "better" than other teachers suggest the potential presence of a norm that discourages teachers from deviating too far from established practices. This resonates with the notion of peer pressure and the impact of the social context of the teaching on teachers' choices and practices.

I further probed the reasons as to why other teachers might not be happy. I asked: "Are they digitally not skilled enough to teach?" She replied: "No, (apart from some seniors) my colleague teachers are very skilled, and they can do it all, if they want. It is easy to make a video for anyone now. But they do not want to use it." T3 acknowledges her colleagues' digital competencies, suggesting there may be other reasons for their reluctance to embrace digital resources. This reinforces the idea that various complex factors, including norms, social context of teaching, and personal preferences, might influence the use of digital resources. T3 was an early career female teacher who had attained higher qualification (MPhil) and may possess advanced digital skills. Therefore, despite her reply and reluctance. I asked her to tell me if she had to use digital resources, how she would go about selecting them. She replied:

> The first thing I will do is research. Honestly, I have never taught in such detail. Many things could be related to a topic... For example, for real numbers, it is quite easy to show students cartesian coordinates in the 3D plane or using PowerPoint animation. First, we can show an image of the XY plane. Then the X-axis should be animated,

> and all the points on the axis should come one after another as 0,1,2,3, ... Then we can show an animation of the y-axis. At this point, show them how it intersects the X-axis and how the point of intersection, called the "origin," is formed. It should be done step by step, so that it will embed as an image in students' minds, just like when they watch a cartoon from beginning to end.

Her detailed explanation of how she would incorporate digital resources demonstrated her thoughtful approach and understanding of the pedagogical benefits of digital resources. Her willingness to explore such strategies reflected her commitment to improving student learning experiences and conceptual understanding through innovative teaching methods. It aligned with the broader goal of connecting mathematics problems to real-life contexts (SQ2). Further, it reflected personal instrumental genesis i.e., personally what she can do with digital resources to enhance her teaching practices. To confirm this, I further inquired whether she would be able to modify or edit a video or image and whether she had done it before. She replied:

> Yes, if I want to do it, I can. I mostly use my smartphone to make videos. I can learn anything from YouTube. Recently, I bought a laptop, a webcam, and a Wacom Bamboo (a digital writing pad) to make videos for my YouTube channel. We can do wonders with videos. We can animate any concept, update them, put them online, etc... I can use them for lectures so that I do not have to repeat myself. Students can watch videos over and over again. If I had digital technology at school, it would be easy for me to teach. It saves time and could allow me to spend it on other things, such as how to conduct online exams, etc., or I can create one in my spare time.

T3 confirmed that she can modify and edit videos and images indicating a level of digital competencies and her confidence in using digital resources. Her use of a smartphone to create videos and her intention to use a laptop, webcam, and a digital writing pad for her YouTube channel emphasize her approach to integrate digital technologies into her teaching methods. T3's responses aligned with multiple domains of the framework, particularly in the context of digital pedagogical resources knowledge (DPRK) and its sub-domains (KDRT, KDRS and KDRC). T3's response confirmed that she had developed her own digital skills, which she was expected to acquire during her education. However, she made limited use of her digital skills for professional purposes due to peer pressure, limited training, access to resources. Nonetheless, during COVID-19, T3's direct interaction with learners improved and provided her with an opportunity to understand and respond to their needs. Directly gaining feedback from students is important not only for teaching and learning but also for lesson planning and boosting the active engagement of students. She mentioned:

> Before COVID we were not able to connect with the students directly. The teacher could never communicate directly. Because students' mobile numbers were in the office, we did not have any contact number of the students. But now every teacher is running a class WhatsApp group where students can directly interact with the teacher and each other.

T5 and T6 offered similar responses about planning lessons and integrating digital resources into teaching and learning. Both were rural government teachers with fewer facilities available at their respective schools. T5 explained how he planned and delivered his lesson:

> Let me give you an example from my recent lecture on real and complex numbers. I first introduce all the different classifications of numbers, such as real numbers, odd, even, prime and complex numbers. I talk about how different numbers behave, explain their tabular form, their arrangement, how to recognise and how to differentiate these numbers. After the explanation, I ask questions about the lecture to understand the students' understanding of the topic.

T5 followed a teaching approach that focused on explanation and recapitulation. Due to a lack of resources this could be seen as an effective way to teach mathematics concepts in a government school. I asked whether he included digital resources and he replied:

> No, I don't bring digital resources in my planning and delivering of lectures. My main focus is the textbook, which is the best to teach mathematics. We can combine digital resources such as videos and images, but unfortunately, we don't have such facilities here and even if I want to, I can't. We do not have Internet, Laptop, Projector, etc., even if we bring our own laptop, children cannot benefit from it. I can show videos using my laptop, but students do not have computers at home. Only 5% to 10% of children have access to digital technology. To be honest, even many teachers don't

> have digital resources. If a teacher wants to arrange it, they can do it personally, but we will not do it as it will be very expensive... Only we (teachers) use WhatsApp on smartphones. If we (teachers) want to share anything, like a book to read, we share it using WhatsApp. I have joined other teachers' communities on WhatsApp, but most of the time the group is silent.

T5's reliance on textbooks aligned with a curriculum-centred approach where traditional resources play a central role. His explanation reflects an approach determined by the available resources and practices within his social context of teaching mathematics (SQ2 and SQ3). T5's description of limited facilities and lack of Internet, laptops, and projectors at his school highlights the challenges he faces in integrating digital resources (SQ5). Somewhat differently, T6, a young mathematics teacher explained how he planned and delivered his lessons, and his comments show his awareness of the possibilities offered by digital resources in mathematics teaching:

> The most important factor is SLOs (student learning outcomes). We have to see how bringing in new ideas or content related to any topic impacts student learning outcomes. The second is learning objectives, which means the selected content is applicable and will help students in exams or not. We do not teach out of textbooks. These are the two things that I keep in my mind while selecting any (digital/non-digital) content for teaching... For example, 9th and 10th-grade mathematics theorems are the most difficult topics. Both teachers and students avoid theorems. Traditionally, the best way of learning theorems is through memorization. However, if we use digital resources, theorems can be taught very easily. For example, rather than showing a diagram printed in a textbook, bringing digital graphs and designing the theorem will be attractive for students. It will develop students' interests, and they might try to draw, create, and design theorems using digital resources.

T6 mentioned using SLOs (student learning outcomes) in planning lessons. By using SLOs, T6 is using "backward course design," which is a method of planning lessons by considering learning outcomes before choosing teaching methods and assessments. In other words, he starts planning with the end (i.e., outcomes to achieve) in mind and reverses the typical approach. Second, he understood the drawbacks of current teaching methods (memorization) in teaching theorems. Third, he showed a desire to use digital resources to explain theorems and but explained how he would like to teach them. He also followed textbook-focused pedagogies. Following on from T6's expressed desire to design content to teach theorems, I asked, "Did you ever design or create any digital content for teaching?" He replied:

> We did not make or use videos. During COVID-19, the government started a television channel for students, so we did not find a need to develop content. There are other reasons as well. In government schools, most students do not have smartphones, and those who do have them cannot afford regular Internet access. Besides, where I live, the Internet connection is bad, and we face regular issues with the speed of the Internet. And most students live in rural parts, and they come from the mountains to study. They hardly get a mobile signal there. The Internet is a far bigger problem for them. So, for us, it was of no use to make videos and other digital content as students would not be able to use it.

T6 acknowledged the potential of digital resources but also recognized the limitations imposed by students' access to technology outside of school. Overall, T6's responses demonstrated a forward-thinking attitude, whereby he saw the value of digital resources in teaching and learning, particularly for challenging topics. He also mentioned a television channel for students in Pakistan that could be a valuable alternative to learning for less privileged students who cannot afford expensive digital devices and the Internet. Therefore, I asked about the usefulness of the channel and the type of content broadcast on it. He replied:

> No... I was not able to watch the content. I tried a couple of times but most of the time the channel was not working. One teacher told me that the quality of content (on channel) is uneven. Some content is hard to understand, while some is better than the content children get in the government schools.

T4, a female teacher teaching in an urban government school located in a different province, explained her approach to teaching:

> We consider the lesson plan's learning objectives and the textbook content. On the Internet, I can find videos explaining a complete mathematics topic. We share these videos with students. It is not easy to find relevant videos and takes time. I normally use videos when I believe they will be useful to students or when I am teaching a

difficult mathematics topic. In most cases, digital resources are not part of my lesson planning.

T4 responded to my inquiry in the context of COVID-19 teaching and learning. Therefore, I asked whether she normally used videos in her teaching. She replied:

> No, I do not. Before COVID, it was very rare, not regularly. I used to take my students to the computer lab in my school to show them videos. But it depends on whether the lab is free or not. I teach on (black/white) boards and use mathematics textbooks.

To investigate T4's COVID-19 teaching strategies, I ask T4 to describe how she taught during the COVID-19 school closure. She replied:

> We used MS Team to create an online classroom in which different teachers teach their subjects at different times. We also trained our students to use the MS team. We coordinate with each other using WhatsApp and set a timetable so that every teacher can teach at a particular time without clashes. We also inform students via WhatsApp about the timetable... In the MS team, there are options to share content. During the lecture, we use these options to share images or videos. Mostly, we share YouTube video links... If we want to share something after the online class, then we use WhatsApp... We also trained our students to use the MS team.

T4's response shows that, with minimal training and support, the teachers quickly identified the affordances of digital tools and flexibly adjusted their teaching strategies. Additionally, it shows their ability to manoeuvre resources across multiple applications and devices to fit students' learning needs and their professional roles. They demonstrated an understanding of their role in leading COVID-19's transition to online teaching. To further understand T4's digital skills for teaching and learning, I asked about searching, creating, modifying, editing, and updating any digital resources. She replied:

> Yes, I make videos of myself solving the textbook questions using pen and paper. I record while explaining the solution so that the videos may contain both my audio and the solution to the question. The video is then shared with the group. I do not use a digital pen... I haven't modified any videos yet. But whenever I feel that something is missing in the video, I record an audio message related to that video and upload it

> on WhatsApp. For searching, yes, I like subscribing to channels where good videos are available. First, I do a random search on YouTube. Once I find a good video, I look for more videos from the same channel (if any). There are many YouTube channels run by private schoolteachers that contain very relevant and easy-to-understand videos. I am subscribed to three to five such YouTube channels. I prefer to use videos from the same channel because students get familiar with the style and presenter.

T4's choice of using YouTube videos and her subscription to relevant channels demonstrated her resourcefulness and ability to search for supplementary teaching materials that aligned with her pedagogical goals. She created her own teaching videos, where she recorded herself solving textbook questions while explaining the solutions. This hands-on approach indicated her digital competencies and her willingness to enhance her teaching resources independently (SQ3 and SQ4). Her recognition of the value of consistency in teaching style by using videos from the same channels indicated a strategic approach to ensuring a coherent learning experience for her students (KDRT). She bridged the gap between physical and online teaching by recording audio messages to complement videos. This highlighted her commitment to maintain communication with her students (KDRS).

However, the responses suggested a difference in digital competencies between urban and rural government schoolteachers of Pakistan. T2 and T6 (urban government teachers) could apply their digital skills in a variety of teaching functions, which is crucial for developing advanced digital competencies. T3 could identify, add on, and create resources that best meet her learning objectives, students, and teaching style. Such applications of digital skills were less noticeable in the responses of rural government schoolteachers' (T5 and T6) responses. T5 indicated a lack of access to technology and resources, making it challenging for him to integrate digital resources into teaching. T6, while recognizing the potential of digital resources, did not have the opportunity to receive training specific to integrating technology into mathematics education.

The teachers were not fully aware of the copyright and licensing issues related to digital resources. They also were less aware of the need to protect sensitive information while making and sharing videos online that contained students' names and visuals. For example, T2, the most experienced teacher, without editing the names or other information related to school and students, uploads recorded Zoom online classes on YouTube for public use. When I asked about data protection and copyright, he replied:

> Is this a data protection or copyright issue? I am aware of copyrights, we have a 40-minute class, so there is no need to buy copyrights. We try to organise classes within 35 to 40 minutes long so that there is no issue with buying a subscription. I keep my lectures on YouTube. My son is very interested in YouTube and also keeps adding animation and graphics to my videos.

T2's focus on the duration of the class and his son's involvement in adding animation and graphics suggested an approach to online content sharing, that did not take into account legal and ethical considerations. T3, a private school teacher, replied in relation to awareness of copyright issues: “I am not aware. And I don't need to be aware of it because we don't use digital resources here.” The rest of the teachers also replied in a similar way to questions about copyright and data protection issues. This indicates a general lack of attention to these matters across the board. This could be attributed to a lack of formal education or training on digital ethics, copyright regulations, and data privacy when it comes to creating and sharing educational content online.

The recapitulation of this section suggests several themes. The teachers alluded repeatedly to the COVID-19 situation and the changes arising in their teaching due to the pandemic. The pressures of COVID19 led teachers to teach themselves to use digital resources. Notably, teachers' use of digital resources (video-conferencing applications and videos) increased and their competencies in using digital applications improved significantly. However, it is difficult to predict whether this recourse to digital tools will continue in the future. Consistent with the findings of the previous section, teachers' financial capability affected their functioning in a digital environment and the development of digital competencies. Teachers thus relied on traditional approaches (direct lecture, textbook-focused) that might not include digital resources. Teachers' subject matter knowledge supported the rapid acquisition of digital skills and understanding of the affordances offered by digital resources for teaching. Apart from COVID online teaching, there was personal motivation to use digital resources, but professional use was limited. Urban government schoolteachers were more skilled at using digital resources than were rural teachers. The government focused on urban government schools and teachers (e.g., establishing digital studios), whereas rural schools were neglected. For the female teachers in the research peer pressure restricted the use of digital resources. The report on the findings continues in the following section with information about teachers' knowledge and use of mathematics specific digital resources.

5.4.1. Use of mathematics specific digital resources

Teaching mathematics with technology requires that teachers know and can use the appropriate digital resources for teaching mathematics topics. Digital competencies may include teachers' knowledge of tools for graphical representation, visualisation of mathematical concepts, use of equation editor, and dynamic graphing software (e.g., GeoGebra), and computational mathematical modelling using spreadsheets and simulations. I, therefore, asked each teacher the following questions:

- Do you ever use digital applications that are designed specifically for mathematics teaching? (SQ3)
- Why do you use these tools for mathematics teaching? What influences you to use such tools? (SQ2 and SQ3)
- If you have options, which digital resources would you prefer to use for mathematics teaching? (SQ3 - SQ4)

The majority of the teachers were aware of the equation editor available in MS Word for typing mathematical equations, symbols, and notations. Interestingly, despite using this feature, they never knew that it was called an "equation editor." The second most cited digital tool was spreadsheets which teachers used for administrative purposes, such as making timetables and preparing mark sheets. For example, T3 mentioned, "Yes, I have used spreadsheet but not for teaching only for timetable and scheduling". These examples illustrated teachers' personal instrumental genesis. Most teachers were unaware of dynamic graphing software. For example, T4 who is teaching in an urban government school candidly admitted: "No, I never heard about this software. Not even in my own student days." Only T1, a private school teacher mentioned that he used dynamic graphing software like GeoGebra in the classroom and he had "self-learned skills to use it for geometry, trigonometry and calculus." T6 mentioned that he had "used it for personal learning but has never used it in the classroom for teaching." I asked T1, what his main reason for using GeoGebra in the classroom would be and he replied:

> I prefer graphical visualisation such as in GeoGebra, Desmos and Digital Calculator… so that students can visualise the concept. The videos are fine too, I have never used a spreadsheet. I only use and share Khan Academy videos.

T4, a female government schoolteacher, conveyed her familiarity with spreadsheets and PowerPoint from her postgraduate studies. However, she revealed a reluctance among teachers to use new tools due to the potential burden of additional responsibilities. T4 articulated that expressing a desire to learn new tools might lead to an increased workload. Thus, teachers often refrain from voicing such aspirations to management and instead opt to conform to existing practices. This resonates with the theme of peer pressure and the influence of the social context of teaching. T4 explained:

> I used spreadsheets in my Masters (Postgraduate) to solve mathematical problems. Or sometimes we used PowerPoint for presentations. But it was a long time ago, so I hardly remember what I did in those days. But the scenario in which I am working, If I wish to introduce innovations in my teaching or want to do something outside of the box, it means that I should be ready for loads of extra work. To be very honest with you, if I tell management that I know how to use these tools or that I want to learn to use these tools, then I will be burdened with all the work related to that tool. Many teachers do not want to learn new tools; therefore, whoever is interested should be ready for extra work. For this reason, we do not express such wishes to management and try to follow what is happening and being told.

The responses show that most teachers were unaware of how to use specific digital resources for mathematics. For administrative needs, teachers use software such as Microsoft Word and Excel, possibly because pre-service and in-service training exposed them to these tools for administrative purposes only. Teachers stated that they had never received any training in digital tools for mathematics teaching and learning. The additional reason for the female teachers appeared to be peer pressure and conformity with local pedagogical practices. The most used digital resource appeared to be videos. All the teachers mentioned the use of videos for different purposes, with the main source of finding relevant videos was YouTube. The next section presents the teachers knowledge and use of digital resources for examination and assessment.

5.4.2. Use of digital resources for examination and assessment

Teachers' knowledge of digital resources for conducting examinations and assessments is an important aspect of their teaching in a digital environment. This competency is a part of teachers' pedagogical digital resources knowledge (PDRK). To understand their use of digital skills in assessment and examination I asked the following question:

- How much do you rely on digital resources to prepare and conduct tests, exams, and assessments?

The majority of teachers mentioned that in regular face-to-face classroom settings, all types of assessments are paper-based. Teachers first prepare a question paper which is mostly handwritten. However, a few teachers use MS Word. On a specific date, students assemble in the examination hall (or classroom) to take the examination on answer sheets. The teacher assesses answers using predetermined marking criteria to award grades or numbers. In the case of government schools, teachers send the compiled results to their respective Education Boards, whereas in private high schools, final exams are organised and conducted by the Education Board. Teachers only conduct half-yearly and quarterly exams. A government teacher (T6) explained as follows:

> We do not use digital means to conduct exams. To produce question papers, we usually use hard copies. We use physical means to communicate with the Provincial Education Board, such as hard copies sent by mail (post). There is a policy that states how much content teachers need to complete every three months. The Board must send us question papers based on that policy. However, the Board does not follow its own policy. Mostly, teachers at the school prepare and conduct exams.

T5, a rural government schoolteacher responded in similar fashion:

> We have a traditional setup for exams; every two to three months, we conduct exams. But because of COVID, the school was closed, and no exams were conducted. You must have known that the government announced that all students would be promoted to the next level without exams. I am talking specifically about government schools. Though private schools had conducted online classes, they did not conduct exams because of government announcements.

The above responses demonstrated very limited use of digital resources for examinations. There seemed to be no sense of urgency about considering how digital resources could be used to improve existing examination strategies or build new assessment approaches. COVID-19 brought about a shift, prompting teachers to explore online assessment strategies. I, therefore, asked teachers to elaborate their COVID-19 examination strategy. T1, a private school teacher explained:

> These days (COVID), we conduct online exams. We create a paper in MS Word and share it on WhatsApp with our students. We give time to solve the questions (by hand) in their notebooks, take pictures, and send them to us. They create a PDF file of the pictures and send them to the WhatsApp group. Sometimes, when we are using Google Meet, they attach the PDF file there. We assess all the questions according to their weight. When we finish marking, we then upload the marks to the WhatsApp group. Students are also notified by SMS.

T2, a government schoolteacher who was teaching in a different city explained:

> You can call it an online exam. We sent them a test paper using the WhatsApp group and asked them to solve it. Once they solve it, they take a picture of the solution and send it to the same group. You know, in Pakistan, we do not have the resources to do advanced things on computers. We are not trained, and no one is concerned. We only create pictures of MCQs and test papers and ask them to email us the answers in the form of pictures or PDF files.

T3, a private school female teacher explained: "Students are sitting in their homes and they are writing online and giving the test. After completion, they send me the PDF of the exam answers. Not all students participate; only those who are present give the exam." I asked what happened to the rest of the students who are not present? She said:

> Not all students have access to the Internet or mobile phones. Only 50% to 60% attend online exams. The rest are those who (during COVID) went back to their villages, where they do not have access to the Internet. Once the school reopens, they can come and give the exam. Our school has a policy that students can send the papers from home, or they can come and submit them at school.

T4, a government school female teacher explained:

> Until now, I haven't conducted any online exams. The issue is that all students don't have personal devices, and even their parents do not have smartphones or computers. The last exams were held in classrooms, with half of the students showing up on alternate days.

The responses of T1, T2 and T3 showed a similar pattern of conducting exams during COVID-19. Regardless of their location (urban or rural), professional role (government or private), and teaching experience, they mostly followed the process in which teachers send an MS Word paper, students solve it in a notebook, take pictures, and send it to the WhatsApp group. It appeared that teachers found a way to map the face-to-face assessment to online using video conferencing. Students solved questions on answer sheets (notebooks) and then they submit a PDF file of images of answers. This process illustrates students' digital skills of organising and converting content into other digital formats. The teacher then checked the solution and shares the marks and their feedback on the same WhatsApp group. The feedback mechanism violated data privacy and protection because any student could access the work, marks, and feedback related to other students. This confirms the earlier statements by teachers that they are unaware of data protection issues.

The other important aspect of online assessment during COVID-19 was students' participation. The majority of teachers who conducted exams during COVID-19 mentioned that only 50% of students participated online. The main reason was access to digital technologies and the Internet at home. During the first year of COVID-19 due to government announcement exams were not conducted. However, during the second year exams were conducted physically with no online portion.

None of the teachers mentioned using online and remote assessment tools (e.g., Google Forms or Kahoot) for formative and summative assessments. When I specifically asked about such tools, T5 replied:

> I have seen assessment tools during training but have never used them for teaching. You know, during one or two weeks of training, everyone feels motivated and enthusiastic about the new things, but soon we come back to school and start working in an environment with no resources. Our enthusiasm starts to go away, and we start working according to our environment and sincerity. We do not have things available in schools that are taught in training.

Summarising this section, it is apparent that teachers did not use digital resources for examinations. Under the influence of external factors such as the COVID-19 pandemic they used WhatsApp showing teachers' adaptability and resilience to changing circumstances (SQ2 -SQ3). These conversations also highlight the digital divide as not every student had access to the digital resources at home to take online exams (SQ5). The majority of teachers were unfamiliar with online assessment tools, which may indicate insufficient training (SQ4) or poor motivation (SQ5). Teachers who knew about them do not use them because they did not have resources at school. The next section presents the information about barriers and enablers in using digital tools and resources.

5.5. BARRIERS AND ENABLERS IN THE USE OF DIGITAL RESOURCES

The last phase of the interviews focused on the enablers of and barriers to using digital resources. I asked teachers to indicate factors that:

- promote and assist the use/integration/implementation of digital resources in the teaching and learning of mathematics (enablers).
- restrict/impede or pose any challenges in the use/ integration/ implementation of digital resources (barriers).

Various teachers' responses indicated understanding of how digital resources can enhance mathematics learning. While discussing enablers, T1 explained:

> We use digital resources for demonstrating mathematical ideas graphically just like visualisation so that students can visualise the concept … Digital resources can address the diverse learning needs of the students. Each student can run a video or an animation at their own pace to understand the concepts… We can teach children more in less time. In a classroom different students have different levels of understanding, and they behave differently. But after visual representation they understand what that concept is all about.

T1 emphasised the use of digital resources in providing visual representations that enhanced students' understanding of complex mathematical concepts. He believes that these resources address the diverse learning needs of students by allowing them to engage with content at their own pace. T2 considered digital resources best for communication and collaboration during the COVID-19 pandemic. He explained:

> Today (COVID-19) students are at least connected with education because of digital technologies. Students are learning. If not 100%, then at least 50 or 60% are learning. If there was no Zoom App, the students would have lost two years… Now, we have WhatsApp groups to communicate timetables, other updates, and discuss our daily issues and share resources that could be important for other teachers.

T3, a private school teacher explained how digital applications facilitate learning:

> Students take an interest and think more when they see new ideas in videos, etc. While learning, students can think in multiple dimensions. Last time, I showed them associative and commutative properties in Venn diagrams. I taught the concept using PowerPoint animations (video). They learned it quickly and spontaneously gave me more examples of Venn diagrams. I can create digital content for teaching and my students, who have internet access and a computer, can also do the same.

T3's response demonstrated how digital applications can engage students and encourage multidimensional thinking. She used PowerPoint animations to teach concepts, promoting active learning and creativity among students. Other teachers gave similar responses, such as, T4, who considered digital resources a time saver, and T5 who thought that he could "expand his teaching using digital tools."

In terms of barriers and challenges to the use of digital resources, diverse opinions emerged which were linked to teachers' place of teaching and experience. The most commonly mentioned factors were peer pressure, financial constraints, poor motivation, training, the role of senior teachers, and the availability of resources For example, T1 (a private school teacher) believed that digital resources provide convenience, but they could hinder the students' learning experience, which is gained by doing mathematics manually.

> When using digital resources, we hardly do anything manually. We give input to the software and get the answer. If I am only using digital resources without teaching background concepts, i.e., how it happens and what is happening, they might even get the answer using digital applications, but they might not understand the concept and know how to do it manually.

T1 considered students' beliefs about mathematics to be one of the barriers to the use of digital resources:

> Some children understand mathematics concepts, and some do not. Here mathematics is considered something out of this world. The image of mathematics in our society, also reflected by students' attitude towards mathematics, is weird. They mostly avoid mathematics... At matriculation (high school) level they always ask why we learn mathematics. They don't know that mathematics is used everywhere.

For T2, parents' participation and awareness are the major challenges for motivating students to learn online. He explained:

> Unless parents are supportive or they know how to monitor their children using digital technology, it is difficult to motivate students. During online classes, I have seen some students simply open their laptops and fall asleep. When we inform their parents about it, they normally reply that our children keep their rooms closed during online classes, so we have no idea what they are doing inside.

He further described the challenges as follows:

> The other main challenge is the problem of Internet connectivity. Fast Internet access is expensive. In government schools, most children belong to middle and lower middle-class families. They do not have access to the internet, smartphones, or computers. Many students went back to their villages during COVID, where the issue of Internet connectivity is worse, so they are unable to study online. If the government could provide them with low-cost internet connections and devices, this problem could be solved. Second, the number of digital devices, middle-class households normally have one or two devices, but more users (children). Normally, all the children have classes at the same time. Some attend college, while others attend a university or a school...It happens that one child has a class, and the other is waiting to take the class. In the case of a single device, one has to compromise. I have a ThinkPad and a smartphone. Whenever my children need it for class, one takes a smartphone from me and to the other, I have to give the ThinkPad. The problem is, I have to take the classes as well.

T3 thinks internet connection and electricity are the main problems. She explained:

> The Internet facility is not available for all of us in the school and also for students. We have computers, but we cannot use them because there is often no light (electricity).

T4, a female government teacher also considered affordability of digital technology as the barrier in the implementation of digital resources at school. She explained:

> It is easy to teach online to the students at private schools. They can afford digital technology. However, in government schools, the majority of students cannot afford

> digital devices. Many students complain that they do not have access to the Internet at home. In such a situation, it is quite complicated. The government does not have the resources to provide internet access to each student. Therefore, it is difficult to teach online at the moment.

She (T4) further considered students' (in)appropriate use of technology as the biggest challenge, she explained:

> Parents need to get involved and take responsibility. If students get the chance to own a smartphone or a laptop, they will use it for online classes, but after that, they are using it for their own entertainment, such as watching movies, chatting unknown people on the Internet etc. It all depends on the student's choice of how they want to make use of digital technologies. Teachers can only engage them for online teaching sessions, not for the whole day. I see such students' attitudes as the biggest challenge. Other major challenges are financial, especially in the government sector, where very few students can afford a digital device.

T5, a government schoolteacher explained four main challenges including mathematics teachers' subject matter knowledge, lack of ideas, funds and training, and senior teachers who can train junior teachers. He explained:

> There are many challenges. Let me explain a few. First, nothing has been done to improve mathematics education. The kind of teachers needed for teaching mathematics are not available in this part of the country. Teachers who are teaching mathematics have nothing to do with mathematics. They do not have subject knowledge. Second, teachers do not try new ideas and things. Third, the role of the government is very important in facilitating teachers. The government does not provide funds for teacher development and resources. If we were fully equipped with training and resources, we would be performing better. And finally, we lack master trainers (senior teachers) who are capable of teaching mathematics with digital resources.

T6, a young government schoolteacher, indicated funding, Internet availability, and lack of professional development as the main challenges in the use of digital resources at government schools. He explained:

> Resources and funding are the main challenges. For example, to draw students' interest in geometry, I want to show images of geometrical objects. But we don't have projectors to show it in the class. The second challenge is internet availability and connection speed. In school, we have a dedicated computer for administrative tasks such as sending and receiving emails. They cannot be used for other purposes because their configuration can only handle one action at a time. The issue is worse for students who mostly belong to poor families with not much to spend on technology. Third, I believe teachers, especially young teachers, should be trained to use digital resources for teaching mathematics. They are motivated and can quickly learn digital skills.

The data reported in this section suggest that there were several enablers of and barriers to the use of digital resources in schools. The enablers were associated with affordances such as dynamic representation, flexibility, visualisation, flexible communication and collaboration, and the availability of a wealth of digital resources. Online learning embedded with communication and collaboration tools provided unique and customised and enhanced learning experiences for students. Teachers were motivated by their desires, such as to support teaching and learning, bring new ideas and expand teaching practices. The main barriers were traditional beliefs about mathematics teaching (e.g., manual solution and memorisation), parental participation and awareness, the Internet, funding, large families with too few devices, unaffordability, students' inappropriate use of DR and inappropriate online behaviour. Additionally, most of the teachers considered a lack of professional development to be the main challenge that hinders teachers' ability to use digital tools in the classroom. The next section is about changes in teachers' beliefs due to COVID-19 online teaching and learning.

5.6. Changes in beliefs due to COVID-19

At the end of the interviews, I asked each teacher how much their beliefs about digital resources had changed due to the COVID-19 lockdown. The majority of the teachers agreed that during the lockdown their teaching practises totally changed, and the use of digital technologies increased sharply. It appeared that this sudden increase and the transition to online learning and teaching significantly impacted teachers' beliefs (both negatively and positively) about digital resources. T1 (a private school teacher) who used digital technology during COVID-19 to teach his students explained as follows:

> Digital resources are great for giving students ideas about mathematics topics. After teaching during the lockdown, I believe online teaching and learning is too early for our students. Face-to-face teaching is good. We need to find ways to increase the use of digital resources in our classrooms so that students can visualise mathematical concepts.

T2, a government schoolteacher experienced a significant shift in beliefs about digital resources, as he explained:

> When COVID came, I used to think, how the world would go, how I would teach, how much children's time would be wasted, how children would move forward. But since I started using the digital resource, it seems that the world is not over. Your home could be your classroom. You can sit, read and learn... Honestly, before COVID, I was not aware of the potential of digital resources. I was always surprised to see some of my friends who were taking online classes at that time. But since I am using it myself, it is a wonderful experience. I used to travel a lot to give private tuition in the evening. Now I can give them using Zoom while sitting at home with my family.

However, T2's attitude and preference for using digital resources, appear to be constrained by the lack of resources at both sides of the educational spectrum. He explained:

> Some children are good at using digital technologies, and some are not. In Pakistan, not everyone has resources. Some have smartphones, some use laptops, and many do not have any. Children with no digital devices do not participate in online classes.

> I think about 40% of students know how to use digital technologies. The remaining 60% take (class/home) work from other children... Most households normally have one or two smart devices, but if there are more users, for example, four or five children, then there are problems. In my house, I have three school-going children and two smart devices. When I teach my students online using a laptop, my children also need devices to attend their online classes. So, you can understand what would happen. They fight for devices.

T3, a female private school teacher observed changes in her beliefs and also in those of her colleagues and the school principal. She commented:

> More than me, the beliefs of my colleagues and students have changed. Our principals have also realised that if it were not for smartphones and the Internet, we would not have been able to teach students. Now they think that it is good to use it for students. So that they can learn, this is a good source. Before COVID, some teachers did not even know how to use the Internet and Zoom, and it was very difficult for them to figure out how to use them because they were not skilled. Now they know. But the senior teachers do not want to learn, so we have to take their classes.

T5, a rural government schoolteacher, recognised digital resources and online learning as a way to self-learn many skills that the government and schools are unable to provide. He explained:

> My beliefs have changed quite a lot. I taught online for the first time during COVID. Now I know where to get information, how to use YouTube channels, and how to improve my skills online. Before, we used to avoid these online resources. But now, I mostly use them for learning and teaching mathematics. I also share links with my students.

T6, another rural government schoolteacher acknowledged that the use of technology had increased due to the pandemic, as he explained:

> COVID has brought the world's focus towards digitalization. Before, nobody was talking about digital resources in Pakistan, but now everyone is talking about Zoom and WhatsApp. Recently, many students told me that during COVID they were taking

> classes online and learning from YouTube. The use of digital resources has increased quite a lot.

T4 was the only teacher who did not experience changes in her beliefs about digital resources and online learning. She explained that:

> I still believe that students should not be allowed to use digital devices such as smartphones. It is hard to monitor the activities of (high school) students on smartphones. They are adults and know how to dodge their parents. Children from lower-middle-class families attend government schools. The parents work in manual labour or factories. They have no understanding of how to use or monitor digital technologies.

The responses show that the personal and professional use of digital resources transforms teachers' beliefs about them. COVID-19 triggered digital education, and teachers' beliefs changed with practice, ease of use, and awareness of the affordances offered by digital resources. This could be used to argue the critical role of instrumental genesis in shaping beliefs about digital resources. However, the lack of resources, unaffordable technology, and social and financial constraints, affected teachers' preferences and attitudes towards digital resources.

5.7. CHAPTER SUMMARY

The Phase 2 findings illustrate the complexities surrounding the digital competencies of mathematics teachers in Pakistan. The findings suggest that teachers share common ground when it comes to their beliefs about mathematics teaching, which traditionally has relied on textbooks, teachers, and blackboards. However, variations in these beliefs become apparent in the context of different working environments. The government schoolteachers were inclined to use more conventional pedagogical methods, like rote memorization, while their counterparts in private schools exhibited greater confidence in contemporary teaching practices. Nevertheless, both groups appear to be influenced by peer pressure, which fosters conformity to the beliefs of their colleagues.

A common thread emerged in teachers' understanding of student-centred teaching approaches, which can be attributed to a lack of professional development and an ineffective evaluation system. The findings also underscore the importance of beliefs being shaped through practical experience rather than formal training. This suggests a crucial gap in professional development opportunities for mathematics teachers concerning the integration of digital resources into their teaching practices. Both government and private schoolteachers expressed that a lack of resources and funding was a primary hindrance to using digital resources. The government teachers were further influenced by the type/style of teaching that was standard practice. Conversely, private school teachers often also conformed to local practices as a result of peer pressure.

The impact of COVID-19 was evident in the shaping of beliefs and increasing the use of digital resources, although financial constraints still limited students' access to digital resources. Teachers had developed personal instrumental genesis through self-education, but their professional instrumental genesis remained underdeveloped. Teachers expected schools or the government to take responsibility for their professional development. This brings to the forefront the discussion of the influence of professional development versus self-learning on the use of digital resources in education. The study also highlighted various enablers and barriers to the use of digital resources in schools. The main enablers included the affordances of digital resources, online learning with communication and collaboration tools, and teachers' motivations to support teaching and learning. Barriers encompassed traditional beliefs, involvement and engagement of parents in their children's online educational activities, Internet accessibility, funding, family size and device availability, affordability, students' behaviour, and a lack of professional development opportunities.

The findings further revealed that personal and professional use of digital resources had a transformative effect on most teachers' beliefs, particularly during the COVID-19 pandemic, with practice, ease of use, and awareness of the benefits of digital resources playing a pivotal role. However, external constraints, such as limited resources and financial challenges, continue to shape teachers' beliefs and preferences in relation to digital resources Overall, the Phase 2 findings indicated the dynamic interplay of factors that influenced teachers' digital competencies and beliefs, shedding light on the multifaceted nature of the educational landscape in Pakistan.

CHAPTER SIX

6. DISCUSSION OF FINDINGS

6.1. INTRODUCTION

This chapter discusses the Phase 1 and Phase 2 findings which were reported in the chapters 4 and 5 with reference to the overarching research question and sub-questions. These questions are recapitulated here for ease of reference in the ensuing discussion:

Overarching research question

What are the digital competencies of high school mathematics teachers in Pakistan?

Sub-questions

SQ1 - How do Pakistani teachers' beliefs about mathematics teaching influence their decisions to integrate digital resources in teaching and learning mathematics?

SQ2 - What influences teachers to use digital resources well?

SQ3 - How well do Pakistani teachers use digital resources to teach mathematics topics?

SQ4 - What professional development opportunities do Pakistani teachers have that influenced their digital competencies and approach to integrating digital resources?

SQ5 -What are the barriers and enablers to teachers using digital resources face-to-face and online?

In this chapter, I aim to connect my research findings with existing literature on mathematics education. It is crucial to acknowledge the integral role of research methodology and theoretical underpinning in shaping the study's framework and interpretation. Therefore, the discussion includes not only the findings in the context of mathematics education literature but also addresses the methodological approach and theoretical perspectives that underlie and enrich the interpretation of the research outcomes. To enhance clarity and coherence, I combine the findings from Phase 1 and Phase 2 of the research. Through this analysis, I aim to highlight whether these findings converge, diverge, or present contradictions. This process contributes to a holistic understanding of Pakistani high school mathematics teachers' digital competencies, thereby adding depth to the study's overall insights.

As mentioned in Chapter 3, I employed a mixed-methods approach involving quantitative and qualitative methods. I used a modified digital competencies framework (as shown in Section 2.4.9.2, Figure 7, p. 80) to analyse the results of both phases. The framework includes three constructs: personal orientation, instrumental genesis, and mathematical digital knowledge for teaching (MDKT). MDKT combines SDRK and DPRK (KDRS, KDRT, and KDRC). SDRK reflects teachers' personal instrumental genesis while DPRK reflects their professional instrumental genesis.

This chapter starts with Section 6.2, which discusses the Phase 1 and Phase 2 findings related to beliefs about mathematics teaching and learning (personal orientation). Section 6.3 discusses the beliefs about using digital resources in mathematics education and the factors that influence these beliefs, such as professional development, funding, and peer pressure. Section 6.4, discusses findings related to knowledge of digital resources and students. Section 6.5 discusses teacher knowledge of digital resources (such as dynamic graphing software) and the factors that influence their use, such as professional development and self-learning. The findings related to knowledge of digital resources, curriculum, and assessment are discussed in Section 6.6. Section 6.7 explicates the correlations between the constructs of the digital competencies' framework, and Section 6.8 presents the summary of the chapter. The following section discusses the findings about teachers' beliefs about mathematics teaching and learning.

6.2. BELIEFS ABOUT MATHEMATICS TEACHING AND LEARNING (P_O)

To address the overarching research question and SQ1, I examined Pakistani mathematics teachers' beliefs about mathematics teaching and learning. Beliefs are integral to the digital competencies of teaching mathematics within a technology framework (Tabach & Trgalová, 2020) and can be described as personal orientations (Schoenfeld, 2011). Data were collected in both phases of the study to explore the personal orientation of participating mathematics teachers.

In Phase 1, seven beliefs about mathematics teaching and learning were examined (Section 4.3). The findings (see Table 11, p. 144) showed that the teachers on average expressed strong agreement on two key beliefs: the importance of students memorizing rules and formulas ($Mean = 4.73$) and learning mathematics through problem exploration to discover patterns and make generalizations ($Mean = 4.71$). Additionally, a majority ($63\% - 67\%$) endorsed the use of real-world contexts, critical discussions, and multiple representations for teaching and learning mathematics. However, there was somewhat less agreement on the importance of textbooks ($Mean = 4.07$, 39% strongly agreed) and the need to create new signs, symbols, and notations (47% strongly agreed). There were positive correlations among all belief statements (see Table 12, p. 145), with a moderate correlations between exploring problems and teaching new signs, symbols, and notations ($\rho = .433, p < 0.01$). Furthermore, beliefs about critical discussion and multiple representations weakly correlated with other mathematics-related beliefs.

6.2.1. Memorization-centric approaches and instrumental understanding

One of the key findings is that memorization-centric approaches and instrumental understanding were important aspects of mathematics education in Pakistan. Memorization-centric approaches emphasize rote learning of facts, rules, and formulas (Schoenfeld, 2016; Skemp, 2020), while instrumental understanding involves the practical application of mathematical procedures without necessarily grasping the underlying concepts (Skemp, 2020). The evidence indicates strong teacher emphasis on memorization of rules and formulas ($Mean = 4.73$) and a focus on instrumental understanding, particularly in problem exploration ($Mean = 4.71$).

The prevalent emphasis on memorisation is in keeping with the literature (e.g., Säljö, 2010; Schoenfeld, 2016; Skemp, 2020) which shows that this is a well-established belief in mathematics education. This belief is rooted in the perception of mathematics as a set of fixed rules, which Skemp (2006) conceptualises as instrumental understanding. In resource-constrained environments like Pakistan, where educational resources, teacher training, and classroom materials are often limited, prioritizing instrumental understanding of mathematics can be practical. This approach, which emphasizes learning mathematical rules and procedures without understanding underlying concepts, allows for efficient teaching and quick skill acquisition. It may help students meet basic competency requirements and perform well in assessments (as stated by T2, T3 and T4), aligning with immediate educational goals and resource availability. However, scholars such as Yurekli et al. (2020) argue that it may limit students' deeper conceptual understanding.

Experienced teachers (T2, T4, T6) justified memorization for the learning of foundational concepts like Sets (see T4 comments, p. 181) and Theorems (see T6 comments, p. 192). For example, T2 believed that students can enhance recognition and understanding of mathematical signs and symbols use in Sets through memorization. Similarly, T4 contended (Section 5.2.1, p. 181) that his students perform better when they memorize signs and notations to learn Sets. However, the challenge arises when these teachers struggled to justify their adherence to memorization-based approaches, resonating with Skemp's (2006) argument about the limitations of instrumental understanding. Skemp (2006) argued that it is challenging to evaluate the effectiveness of this method due to the difficulty in measuring the impact of knowledge acquired through instrumental understanding. The focus on memorization, coupled with an inability to justify it, suggests potentially limited teaching goals (Yurekli et al., 2020).

Studies such as those of Murphy et al. (2021), and Schoenfeld (2011) advocate for clear teaching goals and use of alternative teaching methods. They argue that teachers need to think beyond their existing goals (Schoenfeld, 2011). However, Phase 2 data indicated a focus on more immediate outcomes that tends to shape teacher behaviours and approaches such as providing answers or preparing for exams (Section 5.2.1, see comments on p. 180). This is consistent with Gulistan et al. (2017), who found a correlation between Pakistani high schoolteachers' beliefs about their abilities to teach mathematics and the students' academic achievement. The current study, while not measuring students' academic achievement, notes teachers (such as T2, T5, T6) mention better results are attained by students who prioritize memorization. This suggest that an exam-focused culture of teaching and learning may limit creativity in the teaching and learning process.

An important factor that could play a role in teachers' adherence to memorization-based teaching strategies was their socio-religious affiliation. Amirali and Halai (2010) and (2021) found that school education, pre-service training, and socio-religious experiences shape high school mathematics teachers' beliefs about mathematics in Pakistan. Typically, memorization is seen as a repetitive learning method, but in Islam, Muslims deem it beneficial for moral, spiritual, and intellectual reasons (Kabir, 2021). Such influences may explain the teachers' connection with the practice of memorization as a core method of teaching in mathematics classrooms. It could be part of the reason for the weak relationship of "critical discussion after lessons" with other beliefs in the survey data, The cultural and religious emphasis on memorization might influence the prioritization of this method over other pedagogical approaches that involve more critical discourse. However, it Is important to consider the potential limitations associated with this approach (Schoenfeld, 2016; Skemp, 2020). While memorization based on socio-religious principles may have moral and spiritual benefits, it could limit the depth of understanding, as suggested by Säljö (2010). When students primarily memorize and imitate teachers' explanations, the knowledge acquired may be confined to the textbook's selected information, potentially restricting a broader and more comprehensive understanding of mathematical concepts (Säljö, 2010).

6.2.2. Nexus of textbooks, memorization, and problem-solving

Phase 1 data suggest an interconnected relationship among textbooks, memorization, and the application of problem-solving skills in teaching and learning mathematics. The findings indicated that teachers who believed in memorizing rules and formulas were also likely to have positive beliefs about students' ability to create and assign meaning to new signs, symbols, and notations ($\rho = .336, p < 0.01$) for problem-solving, as well as the importance of mathematics textbooks ($\rho = 0.331, p < 0.01$) (Section 4.3, Table 12, p. 145). Considering that high school mathematics textbooks involve a wide variety of signs, symbols, and notations along with rules and formulas, their relationship with a teacher focus on memorization may be crucial. First, the textbook emphasis prompts important questions such as whether textbooks are designed to prioritize memorization over alternative approaches and whether there are specific teaching methods associated with textbook use that promote a focus on memorization? Criticism of Pakistani school textbooks which has described them as "deeply flawed" (International Crisis Group, 2014, p. 1), highlights a need for reform concerning the content and methodologies within textbooks, which serve as foundational resources shaping students' learning experiences.

Second, the finding is interesting in the context of problem-solving, as it shows a link between memorization and creating new signs and symbols to assist the process of problem-solving. This is similar to the notion of an "inventive-semiotic act" (Goldin, 1998), a process that helps students understand mathematics better. Goldin (1998) argued that with the advent of dynamic computer environments, the possibilities for explicitly displaying aspects of external representational structure have increased. Dockendorff (2020) explained that this can occur with the help of GeoGebra and showed that digital resources like GeoGebra can support the process of creating new meaning using semiotics and visualization. Phase 2 data suggest that mathematics teaching in Pakistan revolves around manipulation of formal notational systems. Despite this, certain teachers (T1, T2, T3, and T4), particularly during the COVID-19 pandemic, mentioned modifying classroom practices to emphasize problem-solving strategies, visualization, pattern recognition, and other conceptually oriented techniques using digital resources (see comments on p. 195; 199; and p. 207). This shows that teachers like T1, T2, and T3 may have the capacity to bridge the gap between visualization-focused beliefs and practical implementation and promote a more dynamic mathematics learning environment aligned with problem-solving approaches using digital resources (Carreira & Jacinto, 2019).

6.2.3. Resource constraints and professional development navigate beliefs

Phase 2 findings highlight that lack of resources and professional development opportunities shape teachers' beliefs. This finding is consistent with Dundar et al's. (2014) findings. They reported similar challenges in educational technology adoption across developing countries, including Pakistan. The experiences shared by teachers in the Phase 2 interviews illustrates these challenges. For example, T6 mentioned that the pre-service training emphasized activity-based learning but discouraged using resources that required a cost (Section 5.3, p. 188). T6 believed that these constraints are responsible for teachers' limited ideas for using digital resources. Furthermore, T5's reference to educational practices (Section 5.2, p. 187), characterized by deeply ingrained legacies, societal norms, and limited resources aligns with the observations made by Amirali and Halai (2021). Other such as Ernest (1989) have also emphasized the influence of broader societal factors on teachers' beliefs.

Phase 2 findings suggest that resource constraints can have a direct bearing on teachers' beliefs and can influence the adoption of new pedagogical strategies and digital resources as exemplified by T5's reply that "neither do we have the facilities, nor do we do the type of teaching that requires digital tools" (Section 5.3, p. 187).

In a resource-constrained environment, it is rare for teachers to apply contemporary pedagogical approaches (Botha & Herselman, 2018). Due to constraints, teachers may not practice what they believe to be important (Yurekli et al., 2020). Findings from these studies suggest some of the practices of Phase 2 teachers (such as T3) did not reflect their beliefs about mathematics (Section 5.3, p. 198). Shiraz and Qaisar (2017) reported similar findings that Pakistani teachers teaching practices do not reflect their beliefs. Shiraz and Qaiser (2017) found that teachers routinely adhered to teaching practises that were socially acceptable, such as memorisation and repetition.

Schoenfeld (2011) argues that teachers act in accordance with what they perceive to be the most important things to do and the resources that they have at their disposal (Yurekli et al., 2020). Phase 2 data (Section 5.3 and 5.4) confirms Schoenfeld's (2011) and Yurekli et al.'s (2020) argument that these factors and resources influence teaching practices and teachers take pedagogical decisions that are contrary to their beliefs (Shiraz & Qaiser, 2017). However, the finding cannot be generalised due to the small sample size ($n = 6$) of Phase 2 teachers.

6.2.4. Dynamics of mathematics textbooks and teachers' beliefs in the digital age

The Phase 1 and Phase 2 data suggest an evolving relationship between mathematics textbooks and the beliefs held by teachers, particularly in the context of the digital age. Phase 1 findings indicated that the integration of digital resources may be influencing teachers' perspectives on mathematics textbooks. For example, only 39% of survey teachers strongly agreed, while 40% somewhat agreed that mathematics textbooks are the best source of information. This result was surprising given the historical value that mathematics teachers have placed on textbooks (de Araujo et al., 2017). It raises questions about whether this signals a shift away from textbook-centred teaching towards other contemporary educational resources, such as digital resources, or if it is influenced by historical criticisms of Pakistani school textbooks (International Crisis Group, 2014). Interpreting this finding is complicated by the fact that the data were collected during lockdown when teachers predominantly used digital technologies (Jafri, 2022) for online COVID-19 teaching and learning. The observed shift in perspectives could have been a temporary adaptation to the circumstances.

Phase 2 findings aligned with the existing literature, as all six teachers considered textbooks to be crucial for teaching mathematics. Previous studies, including those of Amirali and Halai (2021) and Mahmood (2011) have shown that in Pakistani mathematics classrooms textbooks have always played a key role in directing teaching approaches and are widely available to both teachers and students.

Before this study, little was known about how teachers' beliefs about textbooks are related to their other beliefs about mathematics teaching and learning (Lloyd, 2002). During Phase 1 data analysis, this study calculated that there were the significant correlations between beliefs about textbooks and the other six beliefs (see Table 12, p. 145). The use of a textbook had a positive correlation with memorizing, multiple representations, critical discussion with students, teaching real-world contexts, how to create and assign meanings to new signs, symbols, and notations, and exploring problems to discover patterns. The correlation was relatively moderate with the memorization of rules and formulas; the use of multiple representations; and the creation of new signs, symbols, and notations in mathematics teaching and learning. Phase 2 further confirmed the relationship as five out of six teachers considered that the exam paper pattern concentrates their focus solely on the textbook which means students have to practice and memorize the solutions (see T2 comments in Section 5.2.1, p. 180). It demonstrates, teacher's preference for memorisation over understanding. It confirms a connection between exams (students' achievements) with instrumental teaching and learning that necessitates memorizing rules and formulas.

6.2.5. Discrepancies in beliefs

A discrepant belief is one which is inconsistent with another belief held by an individual (Harmon-Jones & Mills, 2019). An example of such inconsistency was evident in the findings. More than 63% of teachers surveyed in phase one stated that they believed in in post-lesson critical discussion with students and using multiple representations in classrooms. The articulation of ideas involved in critical discussion (Brien, 2020), appeared to be somewhat inconsistent with the strongly held belief about the importance of memorisation. This seeming contradiction raises questions about the coherence of teachers' beliefs, aligning with Beswick's (2012) argument that theoretical consistency does not guarantee uniformity among individual teachers. Beswick emphasized that teachers adapt their beliefs based on specific teaching contexts, such as student dynamics and available resources. This aligns with the idea that teachers navigate their beliefs in response to the exigencies of their teaching environments.

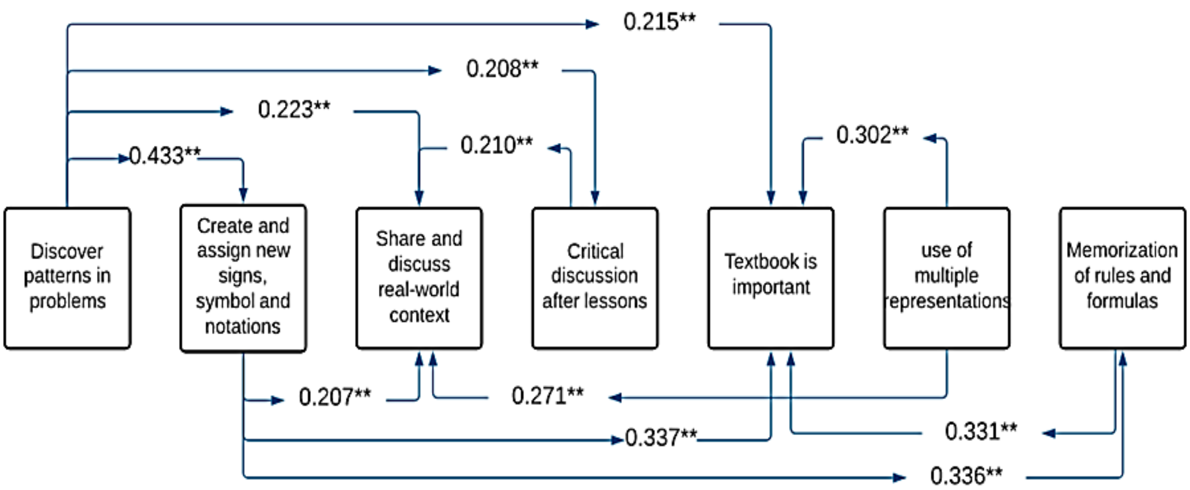
Similar inconsistencies among teachers' beliefs were also evident in phase 2 of the research. During Phase 2, all six teachers expressed contemporary beliefs and a desire for reforms in mathematics education. However, they simultaneously indicated that they preferred to conform with accepted practices in their school contexts. This conformity, particularly among early-career or pre-service teachers (see comments in Section 5.4, T3, p. 198; T4, p. 207), aligns with Ernest's (1989) notion that teachers, who are influenced by their social context, are more likely to adopt existing teaching methods. Despite their desire for improvement and integration of technology, teachers may struggle to develop new pedagogical beliefs, and persevere with traditional teaching methods. Other studies offer a more hopeful reading of discrepancies among teacher's beliefs and suggest that teachers may layer existing beliefs with ideas about change and be open to different approaches and resources. The presence of discrepancies among teachers' beliefs is an indicator of the complex relationship between beliefs and practices and the critical influence of context on the shaping of actual practices.

6.2.6. Summary of key beliefs about mathematics teaching and learning

Teachers' beliefs about memorization, signs, symbols and notation, and textbooks were strongly associated with other beliefs about mathematics teaching and learning. These beliefs in turn correlated relatively highly with beliefs about textbooks, which also correlated relatively highly with beliefs about memorizing rules and formulae (as shown in Figure 15, p. 231). As per the idea of Rokeach (1968) central beliefs, memorization, textbooks, and signs, symbols, and notation may be considered central beliefs of Pakistani mathematics teachers.

Figure 15

Graphical representation of the correlation coefficients among the seven observed beliefs in the online survey



Note. The Figure shows the values of correlation coefficients $\rho > 0.2$ *** shows the correlations is significant at the* 0.01 *level (2-tailed).*

Though memorization revealed fewer, albeit relatively higher correlations with other beliefs, it may have strong socio-religious belonging and acceptance. Rokeach (1968) argues that beliefs based on faith or religion are more central than others. This socio-religious context could influence the finding that memorization appeared to be one of the central beliefs of the mathematics teachers in this study.

The literature suggests that core beliefs like the conviction about the importance of memorisation are inflexible and hard to change because they have connections with related beliefs (Rokeach, 1968). This was reflected in T4's statements who believed that mathematics is boring and a dry subject and that very few students like mathematics (Section 5.2, p. 179). Such beliefs may inhibit teachers' desire to change teaching practices (Pajares, 1992) and are unaffected by new information (Karatas, 2014).

Overall, the discussion indicates the important role of the social context in teaching and learning mathematics. Phase 1 data suggest that these beliefs may be an outcome of limited professional development opportunities (Section 4.6, Table 17, p. 156). This was confirmed by Phase 2 semi-structured interviews (Section 5.3 and 5.4). Particularly, professional development opportunities are limited in rural areas where teachers face additional challenges in accessing resources and support (Yurekli et al., 2020). The next section discusses findings related to beliefs about using digital resources in mathematics teaching and learning.

6.3. Beliefs about using digital resources in mathematics teaching and learning and the influence of access to technology (SDRK)

In this section, I discuss the findings related to beliefs about using digital resources and how these beliefs are shaped by access to technology and use of it. The discussion centres on data collected to address the overall research question and, sub-questions 1 and 2. However, data for other research sub questions also contributes to the discussion. According to Galvis (2012), teachers who have limited access to technology may not have had the chance to develop positive beliefs about technology and may feel less confident in their digital skills. In other words, access to technology can both shape and be shaped by teachers' beliefs about technology and impact the development of instrumental genesis (Guin & Trouche, 2002). Section 6.3.1 discusses the findings about teachers' access to and use of digital resources (see Sections 4.2.1, Table 10, p. 141). Following this, Section 6.3.2 discusses the findings related to the beliefs about using digital resources.

6.3.1. Access to and use of digital resources

Phase 1 revealed that the majority of teachers reported having access to a range of online resources, such as images (51%), videos (50%), e-textbooks (47.5%), and educational websites (38%) (see Table 10, p. 141). However, only 18% of teachers knew how to use digital graphs, 9% dynamic graphing software, and 4% knew about using simulations. These are important results as different digital resources offer distinct affordances, thus presenting diverse avenues for enhancing the teaching and learning of mathematics (Dasgupta et al., 2019). For example, DGS (such as GeoGebra) facilitates the construction, modification, manipulation, and measurement of any geometrical figure (Dockendorff, 2020; Bozkurt & Uygan, 2020) and simulations offer an easy and open-ended way to contemplate the implications of a scenario (van Dijke-Droogers et al., 2021).

Phase 1 findings (see Table 10, p. 141) also revealed a mismatch between Pakistani teachers' access to and confidence about using digital resources. The data indicated limited access to mathematics-specific digital resources like DGS and simulation. Trouche and Fan (2018) showed that limited access to and use of such resources hinder teachers' development trajectory toward instrumental genesis and their ability to improve their teaching using these resources. In the online survey in this study a high percentage of teachers reported having access to graphics (51%) and videos (49%) compared to all the other digital resources. This finding is consistent with Antonietti et al. (2022) who argued that the potential development of beliefs towards specific digital resources is a natural consequence of the resources which teachers could interact with regularly. This preference can influence decisions about resource selection and allocation of teaching time. Consequently, teachers rely more on these resources while underutilizing others. Familiarity with and regular use of particular resources creates a natural tendency to rely on those resources.

Government schoolteachers reported the lower access to digital resources. Only 63.4% of government schoolteachers had regular Internet access. This limited access might not only contribute to disparities in using resources but also give rise to a distinctive set of beliefs specific to their constrained environment. This aligns with Beswick's (2012) emphasis on the impact of the teaching context on the formation and adaptation of teacher beliefs. The Phase 2 data further support this, indicating that government schoolteachers (as exemplified by T5, p. 181, and T6, p. 216), due to limited access, have developed strong beliefs in using traditional methods, viewing digital resources as inaccessible or irrelevant, and relying instead on conventional teaching practices.

6.3.2. Beliefs about using digital resources in mathematics education

The Phase 1 online survey included five belief statements about using digital resources in mathematics teaching and learning (Section 4.4, Table 13, p. 148). The results (as shown in Table 13, p. 148) suggested that most teachers held positive beliefs about the role of digital resources for teaching and learning mathematics. The main findings were that teachers strongly agreed ($Mean = 4.68$) digital resources are important for teaching mathematics. Teachers recognised that digital resources could not only help them in engaging students ($Mean = 4.61$), but also help students to self-regulate, understand, and solve mathematics problems ($Mean = 4.54$). However, teachers were split on whether they had lots of ideas about using digital resources in classrooms or not. More than half (54%) of the teachers' responses were in the range of "somewhat agree" to "strongly disagree" ($Mean = 4.27$). Further investigation revealed that of those who strongly agreed (the remaining 46%), only 2.9% were familiar with simulations, 4.4% with DGS and 8% with animation. These were significantly low numbers that indicate teachers may have limited knowledge of mathematics specific digital resources. During Phase 2 (Section 5.3), despite facing challenges, teachers also articulated positive beliefs about using digital resources for teaching and learning mathematics.

6.3.2.1. Interplay of beliefs, challenges, and social context of teaching

The findings discussed in Section 6.3.2 showed that teachers held positive beliefs about using digital resources but also faced challenges in integrating digital resources into teaching and learning of mathematics. On the one hand, teachers' positive beliefs indicate that they had the necessary knowledge, skills and confidence to integrate digital resources into their teaching practices and to demonstrate specialized digital resources knowledge (SDRK) as described by (Tabach & Trgalová, 2020). On the other hand, their SDRK contrasted with their limited access to digital resources, especially DGS and simulations (see Table 10, p. 141). Phase 2 data revealed that despite being aware of these resources, some teachers had not used them due to technological constraints at school. The gap between awareness and implementation may also reveal inadequate training. Phase 2 teachers reported limited exposure to relevant training beyond platforms like Microsoft Teams. This lack of training resonates with Christopoulos and Spranger's (2021) finding about the challenges teachers face in integrating technology, despite recognizing its advantages. The barriers identified in the findings of the current study included limited access, lack of training, and contextual factors like peer pressure correspond to the first-order barriers outlined by Christopoulos and Sprangers, which, in turn influence second-order barriers such as beliefs and preferences. Other researchers such as Beswick (2012) have helped to illuminate the gap between positive beliefs and practical implementation which was apparent in the findings in Phase 2 of the current research. Beswick's (2012) argument emphasizes that teachers, are likely to adopt different teaching methods depending on contextual factors. Consistent with this view, in Phase 2 findings, limited access to digital resources, inadequate training, and contextual factors contributed to the perpetuation of traditional teaching practices.

The Phase 2 data supports Ernest's (1989) argument that the social context reinforces resistance by creating a culture that resists change and constrains teachers from developing their digital competencies. For example, T3 and T4, both female teachers, believed that peer pressure of some senior colleagues the school influenced their reluctance to adopt digital resources and technological innovations (see T3's comment in Section 5.3, p. 198 and T4's on p. 207). As these were both early career teachers, their reluctance illustrates Hulme and Wood's (2022) insight that early career teachers are influenced by their peers and seniors and that senior leadership support is important in shaping attitudes to using technology in teaching. Support by senior leadership was not reported by Phase 2 respondents; instead, there was evidence (see T3's comments on p. 186 and T6's on p. 188) of discouragement that inhibited new ideas and constrained the innovative use of technology among teachers also reported by (Scully et al., 2021).

The importance of contextual influences and the trends in a school environment was illuminated by teachers' reports of their activities during COVID-19 when they were working from home and possibly less subject to peer pressure. According to the Phase 2 data, teachers felt more autonomous at home in relation to using technology and making changes in their practice.

6.3.2.2. Interplay of beliefs, training, and digital resources in teaching and learning of problem-solving

Phase 1 revealed positive correlations between beliefs about using digital resources, training, and problem-solving. For example, the belief that "digital resources play an important role in mathematics teaching" was significantly correlated ($\rho = 0.502, p < 0.01$) with the beliefs that "digital resources make mathematics tasks and problems engaging" and "digital resources help students to self-regulate and understand mathematics problems" and ($\rho = 0.458, p < 0.01$). These correlations suggested that when teachers consider using digital resources important for teaching, they recognised that their use could make problem-solving engaging, and, could improve students' understanding. These findings are important because, unlike previous studies (Ruiz-López, 2018), they provide quantitative evidence to support the relationship between teachers' beliefs about digital resources and their beliefs about the benefits of using digital resources in mathematics teaching. These findings are also consistent with the literature (Carreira & Jacinto, 2019; Dockendorff, 2020;

Maciejewski, 2019; Murphy et al., 2021) about the use of digital technologies in problem-solving.

6.3.2.3. Connection between teachers' beliefs and problem solving

The other important correlations were between beliefs about digital resources and their impact on student self-regulation, problem-solving abilities, and student engagement. In particular, the belief that "digital resources help students in self-regulation," strongly correlated with "digital resources are important" ($\rho = 0.458, p < 0.01$), "engage students in problem-solving" ($\rho = 0.410, p < 0.01$), and "teachers have ideas about using digital resources in mathematics classrooms" ($\rho = 0.440, p < 0.01$). The study found no significant correlation between self-regulation and digital resources distracting students' attention. These findings are consistent with previous literature on the role of digital resources in self-regulation and motivation among students (Dabbagh & Kitsantas, 2012; Kocdar et al., 2018).

There was a correlation between teachers' beliefs that digital resources can assist student self-regulation, and their own ideas about using digital resources. This correlation has not been frequently highlighted in prior studies, which have primarily focused on the impact of digital resources on students' self-regulation. In the context of digital competencies, this finding suggests that teachers' awareness of digital tools influences their thinking and prompts new ideas that can guide students in self-regulation, addressing gaps, and enhancing learning. Although this correlation connects teachers' ideas and student outcomes, the Mean values (for item $B3\_3$; Table 13, p. 148) showed a lack of consensus among teachers about having ideas for using digital resources. A larger sample size or post-COVID-19 data could potentially explain the extent to which teachers lack ideas about using digital resources to solve mathematical problems.

The next section of this chapter discusses the results concerning teachers' knowledge of digital resources in relation to students, an important construct of teachers' digital competencies.

6.4. Knowledge of digital resources and students (KDRS)

During Phases 1 and 2, the study examined teachers' understanding of their students' digital skills and the use of digital resources for problem-solving and learning mathematics. The findings are linked with KDRS, a major construct of the digital competencies' framework. The data collected for the overarching research question and SQ2 and SQ5 contributes to the discussion. However, data for other sub questions such as SQ1 and SQ4 also facilitate the interpretation and analysis. The Phase 1 survey included the seven items (Section 4.5, Table 15, p. 152) related to the knowledge of digital resources and students.

The survey results (see Table 15, p. 152) showed that most teachers (96%) considered the students' use of digital resources important for learning mathematics. Teachers strongly agreed that digital resources allow students to think creatively about mathematics problems. However, teachers only somewhat agreed on the impact of dynamic graphing software (DGS) such as GeoGebra on students' problem-solving abilities. They were similarly split on whether mathematics software removes some learning opportunities associated with traditional methods like using pen and paper. Further, Phase 2 data demonstrated teachers' desire for students to develop digital competence, as exemplified in T1's statement, "we need to find ways to increase the use of digital resources in our classrooms so that students can visualize mathematical concepts." This aligns with the DigCompEdu framework (2017), which considers teachers' adeptness in using digital resources crucial to nurturing students' active and creative engagement with subject matter. The framework emphasizes teachers' knowledge for using digital resources such as videos, to visualize and explain concepts in a motivating and engaging way (Ghomi & Redecker, 2019). These teachers' abilities are one of the core elements (e.g., KDRS) that define teachers' digital competencies. Phase 1 results reported such beliefs. Teachers strongly agreed ($Mean = 4.52$) that videos improve students' understanding of abstract mathematical concepts. This teachers' view resonates with numerous studies (e.g., Caena & Redecker, 2019; Redecker, 2017; Tabach & Trgalová, 2019, 2020) and frameworks like DigCompEdu that emphasize students use digital resources creatively and responsibly for information, communication, content creation, wellbeing and problem-solving.

6.4.1. Bias in using digital resources and its impact on digital competencies

The use of videos significantly and positively correlated with students' confidence when using digital graphs, DGS, and creative thinking about mathematics problems (see Table 16, p. 154). These correlations indicate that teachers tended to believe that using videos and using other digital resources like digital graphs ($\rho = 0.164,\ p < 0.01$) and DGS ($\rho = 0.270,\ p < 0.01$) were associated. These results provide quantitative evidence, which is often lacking in previous studies (e.g., de Araujo et al., 2017; Maher et al., 2014; McDuffie et al., 2014; Nacak et al., 2020; Sari et al., 2020) that have focussed primarily on learning outcomes and teaching practices. The results may hold significant implications for mathematics teaching. The findings in Table 10 (p. 141) showed that Pakistani teachers' perceptions of students' use of videos could be influenced by the fact that videos were reportedly the most readily available digital resource. Further, all six teachers in Phase 2 mentioned using videos, and four out of six mentioned knowledge of creating videos for teaching mathematics. Teachers mentioned creating, editing and uploading videos on WhatsApp groups and YouTube channels during COVID-19. T2, for example, mentioned that he recorded his lectures and provided them on the YouTube channel so that students could access them in their own time (p. 196). Government schoolteachers, including T2 had more interest in using and recommending videos whereas early career teachers were more interested in creating videos for their students.

Teachers' positive beliefs towards using videos in mathematics education can be considered from a number of perspectives. As Antoinetti et al (2022) have argued, the development of bias towards specific digital resources is a natural consequence of the resources teachers interact with regularly, and in this instance their knowledge of and ease of access to mathematics videos (particularly YouTube videos) may have influenced their opinions about using the resource. The prevalence of using videos suggests that the teachers believed they had potential for innovative and creative approaches to mathematics learning in resource-constrained contexts, such as in developing countries. The use of videos can be integrated into a range of other innovative teaching approaches such as flipped teaching Beatty et al. (2019). However, the Phase 2 findings did not show any evidence of teachers using videos as part of a deliberate component of other innovative pedagogies. In keeping with a recurring theme in the findings, this absence may be associated with inadequate training and professional development. It is also arguable that an over reliance on videos and the paucity of other digital resources could limit teaching initiatives and students' learning outcomes (Nacak et al., 2020).

The preference for videos may be perpetuating a cycle of limited resource allocation and professional development for teachers. With videos being the most accessible resource, Phase 2 teachers primarily focused on self-learning and familiarizing themselves with using video, as exemplified in statements made by Phase 2 teachers (see comments T1, p. 212; T3, p. 199; and T4, p. 203). This emphasis on videos may result in a neglect in acquiring skills needed to use other digital resources. (Drijvers et al., 2010) argued that a lack of diversity in teaching approaches, may hinder the development of teachers' instrumental genesis and limit their ability to leverage the full potential of digital resources in mathematics teaching.

6.4.2. Videos reshaping teachers' beliefs about other resources

The findings (as shown in Table 15, p. 152) provide insights into a complex interplay between preference for using a particular digital resource and traditional teaching resources, such as textbooks. The study found that teachers' perceptions of the role of videos in their students' learning experiences could be influencing their views on the relevance and importance of textbooks in mathematics education. This is supported by the relatively modest level of agreement regarding the importance of primary teaching resource mathematics textbooks ($Mean = 4.07$). These findings align with existing literature, including studies by de Araujo et al. (2017), Howard et al. (2018), and Moreno et al. (2020), which have highlighted the changing role of textbooks in education, a transformation significantly influenced by the growing prevalence of digital resources, particularly videos. De Araujo et al. (2017) argued that teachers no longer rely on single printed textbooks for teaching and learning mathematics, as teachers and students can both access videos about mathematics concepts online.

Phase 2 indicates that the main motivation for videos (such as for T3 and T4) appeared to be reusability of the digital content by students. In a resource-constrained environment, the reusability of digital content through videos can be seen as a potential solution to address limitations in teaching resources. Videos can relieve teachers from the need to repetitively deliver the same content, particularly in the context of high student-to-teacher ratio such as in Pakistan (UNESCO Institute for Statistics, 2020). During Phase 2 interviews, teachers admitted that use of videos could support teaching to large numbers of students and address learning inequalities (see T5's comment on p. 191 and T6's on p. 188).

The study found evidence of Pakistani teachers' knowledge of digital resources and students (KDRS). Teachers such as T1, T2, T3, and T4 mentioned using digital resources other than video to enhance students' mathematical understanding. For example, T3 confirmed using PowerPoint presentations (p. 198); T4, Microsoft Teams (p. 203); T2 mentioned the use of Zoom and digital writing using ThinkPad (p. 195); and T1 mentioned Desmos and digital calculators (p. 207). However, these instances occurred during the COVID-19 school closure and may not represent a widespread trend, particularly in a context in which there is limited professional development for teachers. Of course, not all Phase 2 teachers possessed the same KDRS. In resource-constrained environments, particularly in rural areas, where access to training and technological infrastructure may be limited, teachers face challenges in acquiring and applying digital skills (such as in the cases of T5 and T6) and may possess low KDRS.

6.4.3. Challenges in managing digital device use and ensuring student well-being

Studies such as Redecker (2017) and Ghomi and Redecker (2019) have highlighted the challenges associated with the integration of digital devices into education, particularly in managing their use and ensuring student well-being. Redecker (2017) specifically emphasized the importance of striking a balance between digital learning experiences and student well-being. The findings from Phase 2 showed that for many teachers it is challenging to align their practices with the recommendations in DigCompEdu 2.0 (Cabero-Almenara et al., 2020; Redecker, 2017) concerning safe and ethical digital practices. These recommendations focus on crucial aspects such as privacy protection, data security, and responsible social media use, all of which play a vital role in managing students' well-being online (Redecker, 2017).

Phase 2 teachers demonstrated limited awareness of these recommendation and strategies to safeguard students' digital identities and manage digital footprints. Some teachers seemed unaware of these requirements and their implications as documented in Section 5.4 (e.g., T2's comment; p. 205).

This lack of awareness around safeguarding students' digital identities may contribute to a lower level of KDRS, which is an important construct, as KDRS revealed strong correlations with other constructs in the framework (Section 4.8, Table 23, p. 169). These include perceptions that students may not be able to use digital devices wisely. For example, T4 believed in restricting students' use of digital devices, particularly smartphones (p. 215). Her concerns were grounded on the potential misuse of devices by adult (high school) students and the associated monitoring requirements (p. 219). She mentioned that it is very hard to monitor the activities of adult students on smartphones. Her sentiment was relevant for students whose parents' limited familiarity with digital technologies restricts their ability to guide and monitor their children's interactions with devices. T4's concern that many parents, particularly those engaged in manual labour or factory work, lack the understanding and skills to use or monitor digital technologies highlighted barrier to parental participations in their children's use of technology. This issue is consistent with findings from several studies (e.g., Kolovou et al., 2013; Lowrie & Jorgensen, 2015; McCarthy & Wolfe, 2020) in which parent engagement is considered important in achieving desired learning outcomes. According to Kolovou et al. (2013) many parents lack the digital literacy necessary to support their children, stressing the need for collaboration with teachers to ensure successful technology integration.

Phase 2 data suggests that teachers assumed that students possessed greater digital skills than did they themselves, highlighting a gap in both teachers' and students' digital competencies. T3's statement exemplifies this assumption, as students were seen as proficient in manipulating numbers on Instagram and understanding social media dynamics (Section 5.2, p. 178). These assumptions may mean that teachers may not guide students sufficiently around the use of digital tools, such as how to search, select, and download appropriate content from the internet, or make ethical use of devices as part of the mathematics curriculum. Ghomi and Redecker (2019) considered that it is important for teachers to guide students in crucial aspects such as evaluating digital content for appropriateness, understanding copyright licensing, and obtaining permissions (Ghomi & Redecker, 2019). DigCompEdu 2.0 recognized these skills as advanced digital competencies that play a crucial role in students' well-being in digital environments.

6.4.4. Impact of socioeconomic factors on teachers' perceptions of digital resource use

Teachers reported that that students from low-income families may face socioeconomic barriers when accessing digital resources. Particularly, government schoolteachers like T2 (p. 214), T4 (p. 214), T5 (p. 191), and T6 (p. 202) considered access to digital technologies for students of poor and low-income families the biggest obstacle to using digital resources. Pakistani teachers' knowledge of students' social contexts (parents' financial capacity and digital skills) and students' online behaviour may play a role in shaping their beliefs about students' learning with digital resources (Jafri, 2022).

Pakistan is suffering from its worst economic crisis and facing the risk of default due to its external debt obligations (Rana, 2023). This could further constrain parents' financial capacity to afford digital devices, which may drive teachers to develop beliefs associated with these evolving social contexts.

The next section discusses the results related to teachers' knowledge of digital resources for teaching and learning mathematics in digital environments.

6.5. KNOWLEDGE OF DIGITAL RESOURCES AND TEACHING (KDRT)

In this section, I discuss the results related to teachers' knowledge of teaching with digital resources (see Table 17, p. 156). The data collected to address the research question and SQ3, SQ4 and SQ5 informs this part of the discussion. These sub-questions focussed on Pakistani teachers' mathematical digital knowledge of teaching (MDKT - SDRK, KDRS, KDRT, and KDRC) and linked to their personal and professional instrumental genesis. Both Phases 1 and 2 involved questions about knowledge and use of specific mathematics-related digital resources (see Sections 4.6 and 5.4.1). The investigation focused on the use, level of training received and teachers' desire to learn and operate various digital resources appropriate for teaching and learning mathematics. These are important aspects of specialized digital resources knowledge (SDRK) and reflect on teachers' KDRT crucial construct of digital competencies.

Phase 1 included nine items (Section 4.6, Table 18, p. 158) related to KDRT. The results (as shown in Table 18, p. 158) indicate that a limited number of teachers (26%) had received some form of training for using technology and digital resources ($Mean = 3.27$). Approximately 63% of teachers expressed varying degrees of uncertainty (strongly disagreed to neither agree nor disagreed). This suggests that a significant proportion of teachers may have felt that they lack sufficient training for using digital resources in their teaching. Teachers also expressed difficulty in deciding which digital resource to use for teaching specific mathematics topics, as they showed a moderate agreement ($Mean = 4.16, SD = 0.77$) with the statement. Notably, teachers on average reported a high level of competence in writing mathematics equations and expressions using an equation editor ($Mean = 4.28$). Teachers reported varying levels of skills in teaching mathematics using Dynamic Graphing Software (DGS) across different mathematical domains; Mean scores ranged from 3.65 to 4.12. The teachers' continued to trust in traditional tools as is evident from their strong belief ($Mean = 4.35$) in the importance of students using pen and paper to solve mathematical problems. Teachers reported a positive inclination ($Mean = 4.41$) towards (creating) videos for teaching. Teachers acknowledged the potential benefits of specific digital resources in mathematics education and want to enhance their skills and knowledge of DGS. In summary, these Phase 1 findings suggest teachers' reliance on other digital resources other than videos is low, but they do want to improve their ability to teach using mathematics specific digital resources such as DGS.

6.5.1. Challenges in evaluating, integrating, and collaborating for digital competency

An important finding indicates that due to the abundance of digital resources, teachers experience difficulties evaluating the quality and appropriateness of particular digital resources and identifying ways to integrate digital resources into their teaching and student learning of mathematics. Around 85% of Phase 1 teachers strongly agreed or somewhat agreed with this statement, indicating widespread agreement among the teachers. Notably, there was relatively a high degree of standard deviation (0.77) among the responses (Section 4.6, Table 18, p. 158), suggesting that some teachers find it more challenging than others. The finding is consistent with the literature (Albano & Dello Iacono, 2019; Dockendorff, 2020; Gueudet, 2015; Levinsen & Sørensen, 2018; Pepin et al., 2017) that indicates limited training, and the proliferation of digital resources potentially impacts teachers' ability to use, evaluate, and integrate digital resources into their teaching. This result fits into the broader context of this research findings as it reveals a concern shared by the teachers. Evaluating this finding in conjunction with other findings and previous discussion, several key points emerge.

First, this finding confirmed teachers' desire to develop and improve digital competencies which was apparent in Phase 2 data. However, Phase 2 also revealed limited access to resources, lack of training, and contextual factors like peer pressure are barriers to implementing this aspiration (Section 5.4). Second, the interplay between beliefs and practices (Sections 6.3.2.2) indicates that beliefs could influence teaching practices and the integration of digital resources. Third, the findings discussed in Section 6.4 revealed a gap between teachers' awareness of the benefits of digital resources and their ability to implement them, which is a recurring theme. This gap is exacerbated by challenges such as limited access and inadequate training (Christopoulos & Sprangers, 2021). With these challenges, teachers find difficulties in selecting the right resources for teaching and learning mathematics topics (Pepin, Choppin, et al., 2017).

Teachers' challenges in this regard may also stem from the lack of a conducive environment for online collaboration and knowledge sharing. Phase 2 data indicates limited opportunities for teachers to engage in online collaboration and knowledge exchange, and so they cannot enjoy the benefits of participation in a community of practice. The literature on communities of practice (CoP) and collaborative platforms (Gandolfi & Kratcoski, 2020; Maher, 2020) has shown that accessible technologies, such as Twitter and Facebook facilitate the formation of communities of practice and improve collaboration among teachers particularly during the pandemic. However, Phase 2 suggest that teachers had limited success in using collaborative spaces. They mostly used WhatsApp groups as a collaborative environment with the limited goals of sharing ideas and discussing difficulties faced during the COVID-19 lockdown (see T5's comments, Section 5.3, p. 187) None of the teachers mentioned using other social media platforms like Facebook and 𝕏 (Twitter). Literature supports the notion that collaborative platforms, communities of practice, and communities of inquiry (CoI) provide valuable spaces for mathematics teachers to share insights and access resources (Anabousy & Tabach, 2022; Trouche et al., 2020). They can collectively solve problems, which promotes a culture of learning and improvement in the context of using digital resources in education (Calder et al., 2021).

6.5.2. A link between training and digital competencies

Phase 1 revealed correlations (see Table 18, p. 158) between "teachers receiving training" and teachers' knowledge of digital resources and teaching. For example, receiving training correlated with the use of equation editors ($\rho = 0.398, p < 0.01$), DGS ($\rho = 0.432, p < 0.01$), videos ($\rho = 0.273, p < 0.01$), and difficulties in finding resources online ($\rho = 0.435, p < 0.01$). This indicates that teachers who receive training for using digital resources are more likely to possess the ability to use the equation editor, DGS, create images and videos for teaching purposes, and may find less difficulty in making decisions about digital resources online. These results suggest that training related to digital technologies may improve mathematics teachers' digital competencies. This is not only in line with recent literature emphasizing the role of training in developing digital competencies but also conforms to the historical emphasis of scholars and practitioners on professional development, as reviewed in Section 2.3. For example, Anabousy and Tabach (2022), Clark-Wilson et al. (2020), Crompton (2015), Tabach and Trgalová (2020) and Trouche et al. (2018) strongly emphasized the importance of training in developing digital competencies.

Previous studies that have focused on the role of teachers' training did not specify training for the use of equation editors and DGS. Therefore, our findings provide evidence for the relationship between digital resources like DGS, videos, and equation editors with training. This is an important finding in the context of this research because it provides answers to the arguments already mentioned in relation to the lack of professional training in digital technology. Notably, Phase 1 of the study further demonstrates that the desire to improve ability (through professional development) to teach mathematics with DGS was also correlated with most other aspects; such as finding resources online, equation editors, knowledge of DGS, images and videos (Section 4.6, Table 18, p. 158). Together. these two results resonate with the existing literature on mathematics teaching and the relationship between training and enhancing teachers' digital competencies.

Teachers' preferences for particular digital tools may also be connected with training. For example, their tendency to prefer the use of videos was identified in Section 4.2.1.2 and discussed in Section 6.4.1). Although teachers reported limited training, programs may prioritize specific teaching resources and strategies. This argument is based on the observation that all Phase 2 teachers mentioned videos often during their responses. They showed preferences for teaching mathematics using methods introduced during pre-service teachers' training programs. For example, T6 attributed his teaching preference to his pre-service training, which primarily focused on activity-based learning rather than introducing him to a wide range of digital resources (p. 188). These indicators that training only focussed on certain areas is out of line with Sections 2.3.2, 2.4.1, and 2.4.2, which argues that teachers' training programs should provide teachers with access to a wide range of resources and tools, including software, lesson plans, and other professional development opportunities However, phase 2 findings do not substantiate the assumption that the emphasis on videos was linked to a concentration of training in this area as T1, T2, T3 and T4 indicated that accessibility was the main reason for the use of videos.

6.5.3. Equation editors: a tool enhancing teachers' digital competencies

. Phase 1 of the study found that 56% of the teachers had used equation editors. Four out of six teachers interviewed in Phase 2, had used an equation writing tool available in Microsoft Word for writing mathematics equations, symbols, and notation. However, these respondents did not know that it was called an equation editor. There is limited literature which discussed the use of equation editors. One study that has, is of Junqueira (2006) who compared two groups of student-teachers, found that the knowledge of equation editors helps mathematics teachers in setting mathematics tests and examinations for students and use different mathematics learning software. There appears no prior literature concerning the relationship between equation editors and the enhancement of teachers' digital competencies.

The teachers' knowledge of equation editors had positive significant correlations with DGS, technology-related training, and the teachers' ability to create videos. Knowledge of equation editors associated with the use of other mathematics-related digital resources and competencies. Equation editors serve as crucial tools for accurately representing mathematical expressions, symbols, and notations in a digital format (Junqueira, 2006). They enable teachers to go beyond traditional pen-and-paper approaches and engage in a dynamic process of symbol manipulation, fostering new ways of representing mathematical concepts (Goldin, 1998). The correlations between equation editors and other digital competencies highlight the interconnectedness of teachers' inventive semiotic acts. Equation editors not only insert different types of mathematical symbols, notations, or text, they can also introduce and create multiple mathematical equations, which include fractions, integrations, matrices, and mathematical symbols. An equation editor serves as a foundational tool that supports teachers in using a range of digital resources and technologies to facilitate mathematical teaching and learning. Without the knowledge and proficiency in using equation editors, mathematics teachers may struggle to convey mathematical concepts and use digital resources that rely on equation input tools.

From the perspective of instrumental genesis, the use of equation editors can be linked to both personal and professional instrumental genesis which is the process whereby teachers' appropriate digital tools and transform them into pedagogical instruments (Guin & Trouche, 1998; Tamborg, 2017). We can argue that mathematics teachers' mastery of equation editors is an important part of their digital competencies. Using equation editors, teachers can express signs, symbols, and notations in a digital format, appropriating the affordances of various digital resources, including DGS. Both teachers and students can engage in what Goldin (1998) has termed the "inventive semiotic act," (Section 2.5.1.1) a process where they can explore the logico-mathematical consequences of their creations (Goldin, 1998). This act of invention goes beyond symbol and notational manipulation; it facilitates a deeper understanding of mathematical concepts by enabling the construction of one's own external and internal mathematical representations (Goldin, 1998). This finding is an important contribution showing the link between mathematics specific digital resources (equation editors) and instrumental genesis, a vital construct of digital competencies.

While the evidence from phase 1 points to a widespread usage of the tool, there was no evidence of training in the use of equation editors and Phase 2 interviews revealed that teachers' knowledge of equation editors was self-acquired. Interestingly, neither in the teacher training programs attended by the Phase 2 teachers, nor during their own student years, were they taught how to use equation editors. There are different ways of reading this finding. Firstly, it aligns the recurring theme of inadequate professional development opportunities available to high school mathematics teachers in Pakistan. Second, this finding illustrates teachers' self-learning and self-development efforts. These personal initiatives and self-education have positive aspects for, as Mabila (2017) argues, self-development is crucial for success in resource-constrained environments. Mabila contended that teachers "take charge of their own self-development by identifying areas in which they need to develop and to seize opportunities that are available to them through various forums" (Mabila, 2017, p. 97). Similarly, Tabach and Trgalová (2019) noted teachers' self-development efforts and referred to teachers' ability to work independently on and demonstrate skills in using digital resources. They argued that self-development not only contributes to their professional profile but enhances their competencies and confidence. These purported benefits of self-education were apparent in the findings on teachers' skills with equation editors during both Phases of the current study. Findings showed how knowledge of fundamental tools was associated with teachers' confidence. As teachers feel confident about teaching with a specific tool or artefact, they may be on the path of self-developing professional instrumental genesis (Ruiz-López, 2018). The model of teacher's educating themselves in the use of particular tools can also be problematic as it may facilitate a shift of responsibility to individual teachers and may lead to learning skills that may be not relevant to their needs. Studies (Kleden, 2015) have shown that without proper guidance, it is likely that essential knowledge and pedagogical approaches might be overlooked.

6.5.4. Untapped potential of spreadsheets and the influence of teaching culture

In the Phase 2 interviews three of the six teachers (T3, T4, and T6) mentioned using spreadsheets, something which was not identified at all in Phase 1. It seemed worthwhile to investigate the uses teachers make of spreadsheets. Teachers generally used spreadsheets for administrative purposes (see Section 5.4.1, p. 206), such as making timetables and preparing mark sheets. This administrative use of spreadsheets may have been a norm in the school environments because at school level, the culture of using a particular resource (tool) for specific purposes plays an important role in its potential to become an instrument for teaching. As Trouche (2004) explained, artifacts (or digital resources) always carry a social element: they are products of social experience. Therefore, for any resource to become an instrument of teaching in Pakistani mathematics classrooms it requires social backing or peer support. There are indicators in the findings that this may be the case for spreadsheets. In particular, female teachers (T3 and T4) mentioned the use of spreadsheets for making timetables and showed interest in learning more ways to make timetables using spreadsheets. It is important to note that none of the teachers mentioned spreadsheets as a tool for teaching mathematical concepts to students. They appear to be unaware that spreadsheets are not only a tool but a set of various tools that could be used for doing several mathematical tasks and solving problems (Guin & Trouche, 2002; Society & 2011, 2011; Trouche, 2004). The issue here is not resource availability but a lack of knowledge about a resource's pedagogical potential.

6.5.5. The knowledge gap: Limited familiarity with dynamic graphing software (DGS)

The use of rich content platforms or applications such as Dynamic Geometry Software (DGS), where a teacher and students can create, share, and explain content (Pepin et al., 2017), was not evident in either Phases 1 or Phase 2. Although the Phase 1 results revealed significant correlations between teachers' self-perceived abilities to teach using DGS, the study found limited, or no evidence of DGS use in Pakistani mathematics classrooms. Even during the pilot study (Section 3.4.2), the majority of teachers asked what DGS were. Most teachers were thus unaware of DGS's potential and affordances that could enable them to access diverse ideas and help explore any mathematics concept in detail (Statti & Torres, 2020). This is a significant gap as the literature on DGS reviewed in Section 2.5.4 provides strong evidence that whether for teaching complex numbers (Esguerra-Prieto et al., 2018; Maria et al., 2015) or the dynamic representation of Euclidean geometry figures (Dockendorff, 2020), or problem-solving (Bozkurt & Ruthven, 2017; Khalil, 2016; Ruiz-López, 2018) DGS provides various options for calculation, manipulation and visualization of mathematical expressions.

Furthermore, Phase 2 data suggests that due to a lack of awareness and inaccessibility, it is unlikely that teachers would recommend DGS to students. Despite the positive correlation found between students' confidence in doing mathematics and using DGS for problem-solving that was reported in Phase 1, the descriptive results of Phase 1 indicated that teachers only somewhat agreed on the positive impact of DGS on students' learning and problem-solving abilities (see Table 15, p. 152). Teachers tended to believed that mathematics software (like DGS) can remove the learning opportunities that students can get from drawing graphs using pen and paper (see Table 15, p. 152). This perception may limit their willingness to value the potential for DGS provide access to representations that would be very challenging or "impossible in a pen-and-paper environment, e.g., 3-D graphs of functions of two variables, dynamic diagrams, including displays which are easy to share" (Pierce & Stacey, 2010, p. 4). However, when considering that only 9% of Phase 1 teachers and two out of six Phase 2 teachers knew about DGS, their views could be primarily based on their limited access to and awareness of DGS.

In Phase 1, the high percentage of missing values (48%) were also associated with items related to the use of DGS (Section 3.4.5.2, Table 5, p. 117) to solve mathematics problems. This finding is consistent with teachers' limited familiarity with DGS, suggesting that many (91% of the current study's respondents have weak or no knowledge of the tool. DGS appeared to be still largely unexplored in Pakistani mathematics classrooms.

The Phase 2 results about DGS suggest that teachers (particularly in government schools) have neither resources nor knowledge and experience to use this tool (see T4's comment in Section 5.4.1, p. 206). Phase 1 of the study found that almost 50% of the teachers preferred and recommended pen-and-paper for solving mathematics problems and believed students learn more when they draw graphs and do calculations using pen and paper. This could be linked to the recurring theme of preferring traditional approaches to teaching and learning mathematics. However, it could at least partially be a result of insufficient or inadequate professional development. The finding that more than 72% of the teachers showed a desire to learn more about using digital resources for teaching and learning mathematics further support the view that professional development in this area was lacking. The Phase 2 interviews confirmed these findings, all six teachers showed interest and expressed a wish to receive technology-related training specifically for the teaching and learning of mathematics.

However, the COVID-19 transition to online learning stimulated awareness about the rich learning opportunities offered by digital resources compared to pen-and-paper in developing their professional knowledge of teaching mathematics with technology (Ruiz-López, 2018). Their interest in whether or to what extent digital resources can contribute to students' understanding of mathematical problems was sparked by their unexpected exposure to online teaching and learning environments. This was supported by self-learning that allowed teachers to develop techniques for using digital resources (artifacts) as well as domain-specific conceptual understanding (van Dijke-Droogers et al., 2021).

6.5.6. Role of teachers' training and self-learning in digital competencies

One of the key findings of this research highlights the impact and possible role of self-learning (self-development) on teachers' digital competencies. Phases 1 and 2 reported that the teachers received little training related to technology. Only 26% of Phase 1 teachers mentioned receiving some training for using technology and digital resources and one out of the six Phase 2 teachers (T4) mentioned receiving such training (MS Team during COVID-19, p. 203). These are grim but important statistics, considering the role of technology during the COVID-19 and post-COVID-19 scenarios. They are also consistent with the literature that suggest Pakistani mathematics teachers receive limited professional development opportunities (Amirali & Halai, 2021; Awan et al., 2011; Halai, 2017; Malik, 2015; Mubeen et al., 2013). However, the results of the Phase 1 (Section 4.6.1 and 4.6.2) covering teaching strategies and struggles during the COVID-19 and Phase 2 interviews revealed that despite limited training and immediate official support, the majority of the teachers were able to transition to online teaching and learning. Teachers were able to use a wide variety of online tools, including video conferencing software. Further, 72% of Phase 1 teachers reported that if required they could teach in a fully technological (online) environment (Section 4.6.1, Table 19, 161). These findings show teachers' resilience, but also raise questions about the role of formal training in the context of teachers' adaptability and resilience during the COVID-19 pandemic. Without formal support and with limited technology-related training, teachers were able to successfully conduct online classes and create digital resources for their students during the COVID-19 pandemic.

The Phase 2 data revealed two possible explanations for teachers' engagement in self-learning and exploration of new digital tools. The first explanation is that teachers demonstrated a willingness to engage in self-directed learning and explore new technologies independently. Additionally, it is important to consider the factors that may influence teachers' engagement with informal professional development opportunities such as self-learning. The Phase 2 data suggest that some teachers may have taken on self-learning initiatives due to a lack of faith in the effectiveness or relevance of available professional development programs. The example of T6 who expressed distrust in teacher training programs by commenting on the lack of encouragement for cost-related reasons (p. 188) highlights concerns about the perceived quality and impact of traditional training initiatives. Second, being isolated from the social context of teaching, often imposes constraints that teachers embraced and started using various digital resources to support their online teaching (see Section 5.6). In contrast to teaching to fit with peer-pressures and with limited training within the school environment, online teaching provided them (T2, T3, T4) the freedom to improvise and find solutions.

It can be argued that although teachers have demonstrated resilience in the absence of formal training during the pandemic, such training could further support and strengthen their adaptability and resilience. As evidenced by the literature (Anabousy & Tabach, 2022; Beswick & Fraser, 2019; Moreno et al., 2020; Naidoo & Singh-Pillay, 2020), equipping teachers with relevant knowledge, skills, and beliefs, formal training can enhance their ability to navigate challenges, integrate technology into their teaching practices, and adapt to changing educational contexts.

However, considering the context of limited professional development opportunities and lack of trust in training, the findings suggest that teachers take the initiative for their own professional growth and acquire digital competencies through self-learning.

While the teachers' ability to train themselves in using technological tools testifies to their competence and motivation, self-education cannot be a substitute for appropriately designed professional development. Programs that are designed to meet the specific needs and goals of teachers. Such programs are comprehensive and structured and include. Formal professional development provides teachers with opportunities to engage in ongoing learning and to interact with peers and experts in the field. Such interactions allow for more collaborative and reflective learning (Ramírez-Montoya et al., 2021). A blended or hybrid approach to teacher training can be combined with opportunities for teachers to engage in self-learning and explore innovative digital resources. The post-Covid literature (e.g., Calder et al., 2021; D. Maher, 2020; Syahrin et al., 2023) suggests there is indeed potential for a blended (or hybrid) approach to teacher training which could enhance teachers' adaptability and resilience. Correspondingly, this study suggests that a blend of professional training with self-learning can have a positive impact on teachers' adaptability and resilience, particularly in the face of unforeseen circumstances such as the COVID-19 pandemic. The importance of professional development is noted in the literature (Section 2.4.2), as is the creative technological adaptations that teachers made during the lockdown, but there is no research that argues for the blend of self-learning with formal professional development.

6.5.7. Empowering through self-learning: The case of T2's digital competencies

The potential for self-learning in the use of digital resources for Mathematics teaching can be illustrated by a close focus on and discussion of the experiences of T2, the most experienced government schoolteacher out of six Phase 2 teachers, who developed his digital competencies during the school closure. Prior to this point in time, T2, typically follows traditional teaching approaches, which are a combination of his mathematical subject knowledge, pen-and paper (board and marker), and the textbook. However, in response to the school closure, T2, a non-tech enthusiast with no technology background or training, developed the capacity to teach online (see comments on p. 195 and 217). T2 was flexible and adaptive and overcame challenges by finding support (through peers and friends) and learned new tools by watching YouTube videos. His starting point was evidenced in his recall that his first encounter with a laptop occurred during the school closure. He said that he was so unfamiliar with the technology that he struggled to switch it off and had to seek assistance from a friend to figure out the process. Despite these initial challenges, T2's determination and self-motivation led him on a transformative self-learning journey. His online learning and teaching skills improved, and he discovered comfort and confidence in teaching during the COVID-19 lockdown. However, his preference for pen-and-paper remained intact. The most important part of his transition to online teaching and learning was using a digital pen. He enjoyed the affordances of digital pen and whiteboard. This newfound digital skill empowered T2 to deliver engaging and interactive online instructions, capture his students' attention and facilitate a deeper understanding of learners' mathematical concepts.

6.5.7.1. Mapping traditional teaching to the digital realm

The example of T2 illustrates a convergence of traditional methods with digital affordances. T2 found a way to map face-to-face teaching approaches symmetrically in a digital (online) environment. The digital pen became his marker and Microsoft

Whiteboard resonated with the physical whiteboard. His strong subject matter knowledge helped him to quickly understand the affordances of digital resources and support mathematics teaching (see Section 5.4, p. 195). Moreover, T2's commitment to sharing his experiences led him to find ways to share his developed content. He created a YouTube channel, where he records and organizes his lectures for students' convenience. These efforts show T2's self-learning spree and the competencies that allow him to create and share content using a digital pen, structure his lessons and set up learning sessions, activities, and improve interactions with students in a digital environment. Notably, the main factors appeared to be motivation, access to the device (ThinkPad), affordances of digital resources (on-screen writing using a digital pen) and subject matter knowledge, which helped him to map face-to-face teaching methods to online teaching and learning sessions. During COVID-19 lockdown, he enjoyed access to digital technology, made own decisions, which ultimately changed his beliefs about the role and potential of technology in teaching and learning mathematics.

T2's case serves as evidence supporting our argument that teachers' learning experiences outside of their social context of teaching can be a transformative opportunity to learn and develop new skills. T2's experience illustrates how expanding beyond the usual teaching environment and having fewer constraints can increase a teacher's knowledge and abilities (Yurekli et al., 2020). By engaging in a different teaching environment or exploring alternative approaches, teachers are exposed to new perspectives, challenges, and possibilities (Beswick, 2012; Beswick & Fraser, 2019). This exposure can challenge their existing beliefs, encourage reflective thinking, and prompt them to reconsider their instructional practices (Christopoulos & Sprangers, 2021; Jafri, 2022).

The next section discusses the results related to teachers' knowledge of digital resources for teaching and curriculum in digital environments.

6.6. Knowledge of digital resources and curriculum (KDRC)

This section discusses the results of Phases 1 and 2 related to the teachers' knowledge of digital resources and the role they can play in curriculum and assessment. Phase 1 included five items (see Section 4.7, Table 21, p. 165). The results (as presented in Table 21, p. 165) showed that teachers had low to moderate understanding of digital resources and their potential for use in curriculum and planning assessments. Fewer than half of the teachers (43%) consider they can design and organize lessons that include digital resources ($Mean = 4.24, Mode = 4$). The same percentage (43%) reported their ability to locate and search within databases/websites for digital resources that could be used as curriculum resources. These are relatively low numbers considering the data was collected during COVID-19, when most of the teachers were teaching online.

In terms of correlations (see Table 22, p. 167), the two skills (ability to design and organize and ability to locate and search for digital resources for curriculum design) showed a positive and relatively strong correlation ($\rho = 0.551, p > 0.01$). Moreover, competencies in designing and organizing lessons correlated with teachers' skills of downloading and installing digital resources ($\rho = 0.239, p < 0.01$), and skills to create and evaluate assessments using digital tools ($\rho = 0.510, p < 0.01$) (see Table 22, p. 167). The correlations also indicated that teachers who include digital resources in their curriculum were more likely to recommend digital resources for the teaching and learning of mathematics. Phase 2 revealed similar results for curriculum design and use of digital resources for assessments (Section 5.4.2). Teachers considered that they could search and locate content particularly videos for use in their lessons. However, none of the six teachers mentioned using digital assessment strategies, for student assessments or creating quizzes/tests. The next section will discuss these findings in detail.

6.6.1. The intersection of digital skills and curriculum design

In a digital landscape, the intersection of digital skills (search, download, install, evaluate and create) with strategic considerations in curriculum design, encompassing learning objectives, goals, outcomes, and teaching approaches, plays a pivotal role (Pepin, Choppin, et al., 2017). This study's findings suggest relatively strong connections between digital skills like designing and organizing lessons that include digital resources ($\rho = 0.551, p > 0.01$), locating and searching databases ($\rho = 0.510, p > 0.01$) and creating and evaluating assessments using online tools (e.g., Google Forms). Previous studies (e.g., Caena & Redecker, 2019; Redecker, 2017; Tabach & Trgalová, 2020) have considered these skills important for enhancing curriculum design. Caena and Redecker (2019) contended that these digital skills can extends teachers' ability to adeptly design and organize lessons, navigate databases and websites, locate relevant digital resources, and download and install digital applications for the curriculum. In mathematics education, Tabach and Trgalová (2020) and (Moreno et al., 2020) also stressed the importance of teachers' ability to search for, select, download, install, and integrate appropriate digital resources into the mathematics curriculum. Pepin, Choppin, et al. (2017) highlighted the interconnected nature of these digital skills, constituting a valuable addition to teachers' curriculum design abilities. However, despite the relatively strong correlation, Phase 1 reveals that only 43% of surveyed teachers considered that they have these skills. This suggests Phase 1 mathematics teachers may have limited understanding of how basic digital skills can be employed in curriculum design and to conduct digital assessments.

Four Phase 2 teachers (T1, T2, T3, and T4) showed evidence of digital skills for curriculum design and assessment. Moreno et al. (2020) considered these skills important particularly in teaching models like the flipped classroom, where teachers are tasked not only with content selection but also with its augmentation through thoughtful questions and reflections. An example of these competencies was apparent in the comments of T4 who drew on online sources to supplement her pedagogical and curriculum goals (see p. 203).

6.6.2. Teachers' use of digital tools for assessments

There was limited evidence of teachers' understanding and integration of digital curriculum resources for personalized and interactive learning and assessment. The findings, consistent with those of Zubairi et al. (2022), showed that Phase 1 teachers demonstrated only moderate awareness of developments in online assessment tools enabled by digital platforms like Google Forms ($Mean = 4.22$). Specifically, while authors such as Pepin, Choppin, et al. (2017) have discussed how digital resources can blur boundaries between pedagogy and assessment, as well as the shift from static print to dynamic/interactive digital curriculum resources, teachers in this study were not fully leveraging these possibilities. Pepin, Choppin, et al. (2017) argued that developments such as assessment in adaptive learning systems and using digital resources for formative assessment offers new dimensions in assessment form and function. These factors contribute to the expanding space of interaction associated with digital resources, enabling personalized learning and diverse forms of engagement with the materials (Yerushalmy et al., 2017). However, teachers without the digital skills may face difficulties in designing and selecting appropriate digital resources due to the ever-changing nature of these resources (Pepin, Choppin, et al., 2017).

Phase 2 findings further revealed limitations in teachers' abilities to use digital resources for assessment and addressing students' learning needs. None of the Phase 2 teachers named specific applications for creating or conducting assessments using digital resources. This aligns with Pepin, Choppin et al.'s (2017) argument that although digital resources provide expanded interactive spaces for personalized learning and engagement, teachers face difficulties in integrating these resources.

6.6.3. Improvisation of digital assessment methods during the pandemic

Although teachers did not generally use digital tools for assessment, Phase 2 teachers implemented improvised methods for digitizing assessments such as sharing handwritten tests via WhatsApp (see Section 5.4.2, p. 209). However, these approaches were aimed at replicating rather than transforming traditional practices. Although some success was achieved in the short term through these practices (Gandolfi & Kratcoski, 2020; Pócsová et al., 2021; Scully et al., 2021), the pandemic-induced solutions cannot be characterized as skilful, strategic digital integration. For example, Phase 2 revealed that teachers instructed students to photograph completed worksheets for submission via WhatsApp. This strategy represents a digitized duplication of paper-based tests rather than purposefully designed digital assessments as suggested by Pepin, Choppin et al. (2017) and (Yerushalmy et al., 2017). Moreover, the consistency of strategies across different Phase 2 teachers suggests that existing skill levels constrained innovative practices. (Pepin, Choppin, et al., 2017; Pepin, Gueudet, et al., 2017; Yerushalmy et al., 2017) all showed the potential of digital resources to enable more adaptive, interactive, and integrative assessments. Features like adaptive releases, embedded formative checks, and multidimensional analytics can enhance learning outcomes (Kem, 2022). However, this study found little evidence that teachers were capitalizing on these possibilities.

These limitations were more evident in the practices of rural government schoolteachers (i.e., T1, T5 and T6), which is consistent with the finding of Zubairi et al's. (2022) post-pandemic report. They also found limited use of online resources and less use of digital tools for student assessment in Pakistan, particularly among rural schoolteachers. However, their report was not subject specific. Nevertheless, other literature, as reviewed in Sections 2.3.4 shows that mathematics teachers in other countries during the COVID-19 pandemic made innovative uses of online resources for both teaching and assessment (Ferdig et al., 2020; McAndrew et al., 2021; Syahrin et al., 2023).

In brief, the emergency response to the pandemic evidenced teachers' aptitude in adopting technology when essential. It also revealed a lack of expertise in using digital applications tailored to unique need, particularly for assessment. Simply digitizing traditional methods represents an emergent (Hartshorne et al., 2020) but limited stage of digital competencies (Martin & Grudziecki, 2006). The findings align with the literature suggesting that teacher readiness still lags behind the possibilities that digital resources can provide (Dockendorff, 2020; Trouche et al., 2020).

6.7. EXPLICATING THE INTER-CONSTRUCT CORRELATIONS

As detailed in Section 2.4.9.2, the modified DCTMT framework (see Figure 7, p. 80) consists of five main constructs:

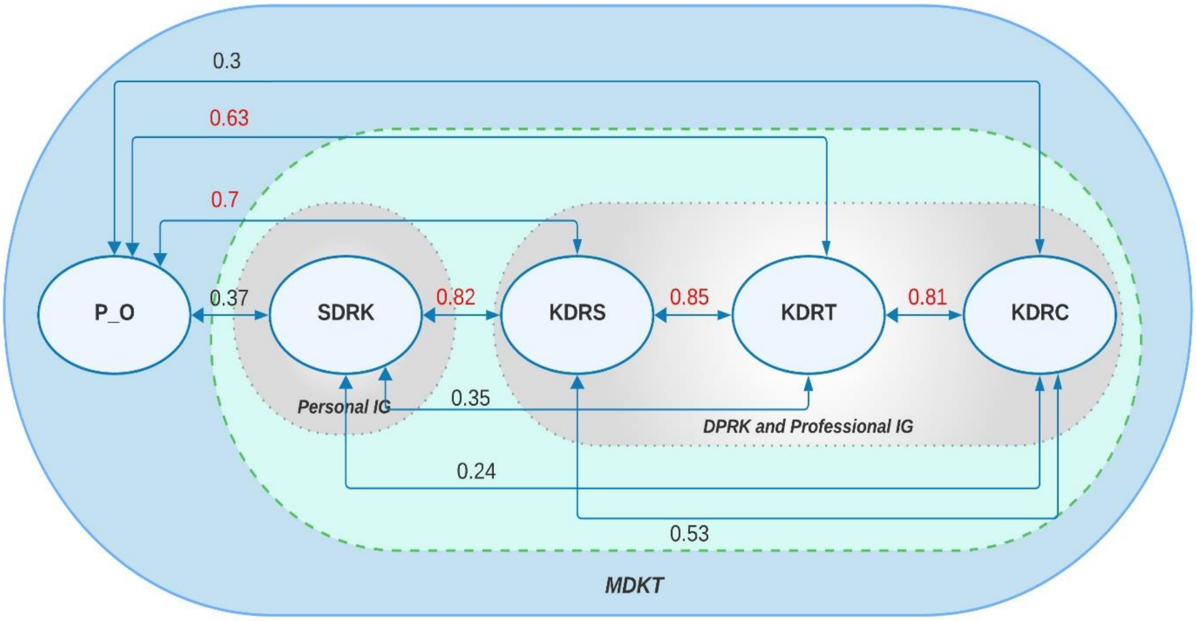
1. Personal orientation (P_O)
2. Specialized digital resources knowledge (SDRK)
3. Knowledge of digital resources and students (KDRS)
4. Knowledge of digital resources and teaching (KDRT) and
5. Knowledge of digital resources and curriculum (KDRC)

KDRS, KDRT, and KDRC, collectively contribute to teachers' professional instrumental genesis (IG) whereas SDRK is more linked with personal instrumental genesis (IG). Additionally, the constructs KDRS, KDRT, and KDRC constitute core components of Digital Pedagogical Resources Knowledge (DPRK). Along with SDRK, DPRK contributes to a teacher's overall Mathematical Digital Knowledge for Teaching (MDKT), which brings together the content, pedagogical, and technological capabilities required in digital environments.

Up to this point, the focus of Chapter Six has been on discussing what Phase 1 and Phase 2 findings reveal about high school mathematics teachers' digital competencies in the context of Pakistan and examining these findings in relation to the pertinent literature. Discussion has included explanation of intra-construct correlations, that is correlation between distinct items within each construct of the modified DCTMT framework. Transitioning into the current section, I discuss the relationships between (inter-construct) the five constructs of the modified DCTMT framework. I calculated inter-construct correlations to understand relationships across the overarching constructs and their sub-constructs (as shown in Table 23, p. 169). These correlations not only provided overall empirical evidence across the constructs, but also facilitate understanding of how knowledge and skills defined by one construct (e.g., SDRK) may relate to another (e.g., KDRT). Figure 16 depicts the inter-construct correlations of the modified DCTMT framework.

Figure 16

Inter-construct correlations of the modified DCTMT framework



Note. Personal IG stands for personal instrumental genesis, Professional IG for professional instrumental genesis. DPRK is Digital Pedagogical Resources Knowledge. Whereas P_O, SDRK, KDRS, KDRT, and KDRC are the five constructs of the modified DCTMT framework.

As shown in Figure 16, all five constructs positively correlated with each other. To explain this complex interplay between the constructs and sub-constructs, each of them are visually differentiated by distinct colours. Correlations exceeding 0.6 are emphasized in red to draw attention to the strongest associations in the framework and enable quick interpretation. The following sections explicate each meaningful correlation further.

6.7.1. Correlation between KDRS, KDRT and KDRC

The strong ($\rho > 0.6$) positive inter-construct correlations (see Figure 16, p. 262) between KDRS, KDRT, and KDRC highlight the interconnected nature of teachers' knowledge domains encompassing digital resources, students, teaching, and curriculum. This association emphasizes the importance of comprehensively approaching teachers' Digital Pedagogical Resources Knowledge (DPRK) and being aware of its correlation with KDRS, KDRT, and KDRC. Studies such as that of (Koehler et al., 2013) have emphasized the interconnectedness of teachers' professional knowledges and competencies. In digital pedagogy, (Mishra & Koehler, 2006) TPACK framework proposes that teaching with technology requires understanding of the interplay between technological knowledge, pedagogical knowledge, and content knowledge. The correlations between KDRS, KDRT and KDRC support this idea and emphasize that teachers integrate digital resources (technological knowledge) within their pedagogical and content knowledge (Clark-Wilson et al., 2020).

The strong correlation ($\rho = 0.85$) between KDRS and KDRT signifies that teachers attuned to digital affordances for enriching student learning are likely to be better equipped to align these resources with pedagogical strategies (Carreira & Jacinto, 2019; Dockendorff, 2020). Based on Rokeach's (1968) idea of centrality, KDRS and KDRT can be considered central components of mathematical digital knowledge of teaching (MDKT). The centrality of KDRS and KDRT, along with the strong connection of KDRT with KDRC, emphasize that understanding these knowledge domains can play an important part in understanding and developing mathematics teachers' digital competencies. This is consistent with the literature (Pepin, Gueudet, et al., 2017; Trouche et al., 2020), which suggests that teachers who integrate digital resources aligned with curriculum goals are likely to be well-positioned to select digital resources to align with and augment specific learning objectives. The Phase 2 teachers (T2, T3, and T4) demonstrated these connections between the KDRS, KDRT, KDRC in practice. They supplemented their teaching with customized videos targeting student needs during the COVID-19 pandemic (as exemplified in comments on p. 196, 199, 203). Their approach shows teachers' personal fluency with digital tools (SDRK) enabled them to effectively apply digital resources to aid their teaching (KDRT) in alignment with needs (KDRS) they perceived from analysing the curriculum (KDRC).

6.7.2. Factors influencing SDRK

Figure 16 (p. 262) shows relatively weak correlations between SDRK and three of the four constructs in the modified DCTMT framework. For example, SDRK weakly correlated with KDRC ($\rho = 0.24$), KDRT ($\rho = 0.35$), and P_O ($\rho = 0.37$). The only strong correlation of SDRK is with KDRS, which is well aligned with literature such as Drijvers et al. (2010) and Ruiz-López (2018). (Drijvers et al., 2010) considered that, in digital environments, mathematics teachers with adequate personal instrumental genesis facilitate the instrumental genesis of their students (students becoming competent with technology). As explained in Section 2.4.9, SDRK reflects teachers' personal instrumental genesis, whereby teachers use digital resources for their own subject matter and pedagogical knowledge acquisition (Tabach & Trgalová, 2020). This self-driven aspect of digital competence then enables a teacher to recognize more possibilities for learners and may facilitate introducing digital resources to students in pedagogically meaningful ways (Ruiz-López, 2018). Therefore, while constructs such as KDRT and KDRC focus outwardly on application to teaching and curriculum, SDRK emphasizes the inwards cultivation of digital competencies.

Another significantly weak correlation was between SDRK and personal orientation (P_O). ($\rho = 0.37$) which suggests that there may be other factors that contribute more strongly to an individual's level of SDRK. This study identified factors such as socio-religious context, perceptions, training, and availability of digital resources playing roles in shaping teachers' beliefs about using digital resources. These external factors likely moderate SDRK levels, irrespective of underlying beliefs. This contradicts research that highlights personal beliefs as determinants of technology adoption (Schenfeld, 2011), with perceptions of usefulness driving acceptance (Antonietti et al., 2022). The weak SDRK-P_O association suggests a possible inconsistency between conceptual models and the realities of resource-constrained educational environments such as Pakistan.

The findings suggest beliefs need to be viewed along with other drivers in predicting personal use of digital resources and understanding digital competencies. Other factors may contribute, especially in the context of resource-constrained environments, which may present additional barriers to realizing technological potential.

6.7.3. Explaining weaker KDRC correlations

The construct KDRC also demonstrated relatively weaker correlations with two other constructs. KDRC's correlation with P_O was 0.3, 0.24 with SDRK. Both P_O and SDRK reflect self-driven perceptions about mathematics and using digital resources. The only strong (red) correlation was with KDRT i.e., teachers' knowledge of digital resources and teaching. This critical connection aligns with findings from studies by Pepin, Choppin, et al. (2017) and Trouche et al. (2018), reinforcing the importance of curriculum alignment with teaching using digital resources. Nonetheless, other correlations were under the 0.6 threshold, suggesting potential additional factors driving KDRC development.

A possible reason behind these weaker correlations may stem from the inherent complexity of the curriculum itself. The curriculum encompasses diverse elements, including learning objectives, teaching strategies, and assessment methods. All are crucial for integrating digital resources into specific mathematics topics (de Araujo et al., 2017). However, as discussed in Section 6.5.1, The mathematics teachers in this study did not report that they integrated digital resources into the curriculum, and they conducted assessments using them. Teachers in both phases acknowledged a lack of ideas for curriculum design and the difficulty in choosing suitable digital resources from the abundance of options available online. Trouche et al. (2018) helps to explain these difficulties as studies have emphasized the need for teachers to have enhanced design expertise due to the changing nature of digital resources (Pepin, Choppin, et al., 2017). As is evident throughout this study, there are also contextual and situational pressures which influence curriculum choices.

Five of the Phase 2 teachers (T2, T3, T4, T5, and T6) focussed lessons on examination patterns rather than maximizing learning outcomes and students' needs (e.g., see T2's comment on p. 180). This leaves little room for perceiving the importance of digital resources in enhancing understanding of mathematical concepts. As Mishra (2019) discusses, skilled integration involves matching digital tools to specific educational contexts.

It is possible that teachers' knowledge of digital resources and how to integrate them into the curriculum is influenced by a wide range of factors beyond those measured by the framework. Other constructs in the framework, such as SDRK, KDRS, and KDRT may be more directly related to the use of digital resources in mathematics teaching and be better measures of digital competencies. Alternatively, KDRC and P_O may be indirect factors in the use of digital resources in mathematics teaching and may be influenced by other factors beyond those included in the framework. As (Partelow, 2023) discusses, sometimes it is hard to include all concepts (factors/constructs) in the framework.

6.7.4. The Interplay of personal and professional instrumental genesis

The correlations between the constructs also reflect association between personal and professional instrumental genesis. As previously mentioned, personal instrumental genesis can be referred to as the SDRK construct (Tabach & Trgalová, 2020). It involves constructing and adopting digital tools to become an "instrument for math work," a resource that enhances teacher's own mathematical understanding and competency (Haspekian, 2011, p. 2300). This is personally motivated facet of digital competencies centred on building knowledge and skills in using a variety of digital resources. In contrast, professional instrumental genesis entails taking those personalized mathematics learning instruments and evolving them further into "didactical instruments for math teaching" (Haspekian, 2011, p. 2300). This constitutes appropriating previous tools to serve pedagogical purposes in conveying and demonstrating mathematics concepts for others. Thus, professional instrumental genesis can be collectively represented by KDRS, KDRT and KDRC.

The relatively strong SDRK-KDRS correlation ($\rho = 0.82$) highlights that personal instrumental genesis is related to recognition of student affordances. Moreover, SDRK contributes to the wider set of constructs that constitute a teacher's overarching Digital Pedagogical Resources Knowledge (professional instrumental genesis) which enables the purposeful integration of technology into pedagogy (previously discussed in Section 6.7.1). The correlations suggest personal and professional instrumental genesis are strongly linked to each other and are not independent processes (Haspekian, 2011).

In conclusion, the correlations among the constructs demonstrate the complex structure of relationships within the components of the framework. This complexity highlights the need for and importance of a multifaceted approach in developing digital pedagogies in mathematics education. Standalone competencies can be productive but insufficient. Coupling personal knowledge and skills with student teaching and curriculum is essential for optimizing digital tools in practice, enabled by instrumental knowledge accrual across domains.

6.8. CHAPTER SUMMARY

In this chapter, I discussed both Phase 1 and Phase 2 findings. I focused on the relationships between teachers' beliefs about mathematics education and aspects of their digital competencies. The discussion highlighted the prevalence of memorization-centric teaching approaches and the nexus between instrumental understanding, socio-cultural influences, and religious affiliation. The discussion showed textbooks, memorization, and problem-solving practices shape Pakistani teachers' beliefs and there is an impact of resource constraints on how they navigate these beliefs. The examination suggested that thinking about the use of textbooks is evolving and changing and that there are also inconsistencies in teachers' beliefs. Findings showed that core beliefs about memorization, textbooks, signs and symbols and are likely central for many Pakistani mathematics teachers.

In addition, I discussed how beliefs about mathematics teaching and learning, access to and use of digital resources, knowledge of digital resources and students, knowledge of digital resources and teaching, and knowledge of curriculum and assessment are all interconnected. While most teachers acknowledged potential benefits of digital resources, limited personal experience, social context, and barriers hindered integration. A detailed discussion of the relationship between knowledge of videos, equation editors, professional development and digital competencies was also presented. The discussion related to other digital resources showed the untapped potential of DGS and spreadsheets and the influence of teaching culture that limited using these resources for professional purposes.

The findings revealed that the pandemic prompted teachers to improvise in their use of digital resources. Teachers with limited professional support and training shifted to online teaching and learning. This highlights the crucial role of self-learning in developing digital competencies. The COVID-19 pandemic ignited both resilient adaptation and assessment digitization rather than fundamental practice transformation, pointing to needs for sustained competency scaffolding. The discussion also confirmed a link between professional development and the development of digital competencies. Moreover, the divide between urban and rural teachers in terms of curriculum design capacity was also evident.

The digital competencies framework used in this study was useful for understanding mathematics teachers' digital competencies. Empirical evidence confirmed that the modified DCTMT framework provided a structured approach to examining the multifaceted aspects of teachers' knowledge and skills related to personal-professional use of digital resources. The framework's five key constructs (P_O, SDRK, KDRS, KDRT, and KDRC) allowed for a comprehensive analysis of teachers' digital competencies. They proved beneficial in highlighting the interplay between the constructs and their role in shaping teachers' digital competencies. However, a few constructs are found to be better predictors of digital competencies than others. Furthermore, the framework aligns with existing research on the interconnectedness of teachers' professional knowledge and competencies, specifically emphasizing the need for digital pedagogical resources knowledge (DPRK) in mathematics education. The findings further suggest that it might be useful to incorporate the recognition of contextual factors that influence digital competencies. Factors could include the availability of resources, socio-economic context, and access to training and professional development opportunities.

CHAPTER SEVEN

7. CONCLUSION

7.1. INTRODUCTION

In this PhD study, I examined the digital competencies of high school mathematics teachers in Pakistan. I wanted to understand what Pakistani high school teachers think about digital resources and how they integrate them into teaching and learning mathematics. Employing a mixed-methods approach, I undertook two distinct phases of data collection. Phase 1 involved an online survey to gather quantitative data and Phase 2 captured qualitative insights using in-depth semi-structured individual interviews with teachers. I used statistical analysis to find patterns and trends in the online survey data. For Phase 2, I analyzed and interpreted semi-structured interviews using predefined themes. This combination of methods revealed results that demonstrate the challenges Pakistani teachers encounter, available opportunities, and teacher viewpoints on integrating technology in their mathematics classrooms. The findings highlight the complexities of using digital resources in mathematics education, particularly in the context of a resource-constrained environment in a developing country.

In this final chapter, I reflect on the importance of this study and how it contributes to ongoing conversation and initiatives to improve mathematics education in the digital age. This chapter has six sections (7.1 to 7.6). Section 7.1 is introduction, The next section (Section 7.2) highlights the main scholarly contributions of this research. The implications of these contributions are presented in Section 7.3. In Section 7.4, I make suggestions for future research. The limitations of the research are presented in Section 7.5, and Section 7.6 provides concluding thoughts about the research process and findings.

7.2. MAIN CONTRIBUTIONS OF THE STUDY

This study makes valuable scholarly contributions by illuminating the digital competencies of high school mathematics teachers in Pakistan, a context marked by specific cultural, socio-economic, and educational characteristics. The research addresses crucial gaps in the literature by shifting the focus from students' use of digital resources to understanding teachers' digital competencies and their beliefs about using digital resources in their teaching. This shift is particularly important in the high school context, where teachers play a pivotal role in preparing students for tertiary education and everyday applications. Beyond its significance for Pakistan, this study contributes insights which can inform mathematics teacher training and professional development initiatives across the mathematics teaching and learning community. These contributions are set out as follows: Section 7.2.1 highlights the importance of modification and empirical evidence for the DCTMT framework; section 7.2.2 outlines the crucial aspects of mathematics teachers' beliefs about mathematics teaching and learning; the interplay between teachers' beliefs, access to digital resources, and competencies in using them for mathematics education is presented in Section 7.2.3; the challenges posed by training and the resilience exhibited by teachers, particularly during the COVID-19 pandemic are presented in Section 7.2.4 and 7.2.5.

7.2.1. Modification of and empirical evidence for the DCTMT framework

The foundation of this thesis rests on the Digital Competencies for Teaching Mathematics with Technology (DCTMT) framework (see Figure 6, p. 75) which was, identified through an extensive literature review. This framework serves as a structured guide, offering a comprehensive understanding of high school mathematics teachers' digital competencies. The five essential constructs in the framework (P_O, SDRK, KDRS, KDRT, and KDRC) shed light on the intricate interplay between these elements and elucidate their combined influence on teachers' digital competencies. All stages of data collection and analysis in this study align with the DCTMT framework. In relation to this framework, I have made two significant contributions.

First, based on my analysis and understanding of the current literature, I have made a modification to the framework. I have moved from the term "digital content" with its potentially restrictive meaning, to "digital resources." This adjustment enables the exploration of how teachers curate and create digital resources for specific purposes, including their competencies in utilizing collaborative and personalized platforms like social media. This adjustment enabled understanding of the informal resources teachers generate to complement traditional materials such as textbooks and highlight the teacher's role in navigating evolving digital content, especially in challenging scenarios like the COVID-19 pandemic.

Second, this study provides empirical evidence supporting the benefits of the DCTMT framework. Previously, the framework had been applied only to analyse policy documents on teachers' digital competencies. There were no studies that used the framework for collecting empirical evidence. Tabach and Trgalová (2020), the authors of the framework, recommended its use in empirical studies to assess mathematics teachers' digital competencies. The findings presented in this study have filled this empirical void and provided evidence by employing both quantitative and qualitative methods. The findings revealed that the constructs within the DCTMT framework are positively correlated. These correlations (see Table 23, p. 169), some weak and some strong, identify the intricate relationships between different components of digital competency for mathematics teaching. The correlations further reveal that teachers' competence in one area may signify proficiency in others. Thus, the empirical evidence from this study supports the idea that the digital competencies of mathematics teachers is a multi-construct phenomenon that requires a multifaceted approach for investigation.

7.2.2. Insight into the crucial aspects of teachers' beliefs about mathematics teaching and learning

With the help of the empirical evidence, I was able to understand different aspects related to the digital competencies of high school mathematics teachers in Pakistan. Firstly, my work has provided greater insight into crucial elements of teachers' beliefs about mathematics, and their perspectives on teaching and learning. These insights contribute to the ongoing inquiry into teachers' beliefs, values, and preferences about mathematics teaching and learning. I examined seven beliefs about mathematics teaching and learning and found that, despite challenges, teachers held beliefs across the multifaceted dimensions of mathematics teaching and learning. The identification of central beliefs, coupled with insights into practical pedagogical approaches, enriches understanding of how teachers conceptualize and implement their beliefs in the classroom. These beliefs are crucial perspectives when considering Pakistani teachers' uptake of digital resources, showing the potential to use digital resources. Some of the most significant insights are recapitulated in the next part of this conclusion.

The study showed the prevalence of teachers' belief in the importance of memorisation in mathematics learning. Repeated evidence of teachers' strong belief in memorizing rules, facts, and formulas, demonstrates that memorisation is a familiar and well-entrenched pedagogical practice among them. I found that socio-cultural, religious contexts, and other factors such as examination question patterns and the "type of teaching" they prompted were reasons for a focus on memorization (see Section 6.2.1). These findings provided deeper insights into the teachers' beliefs, which revealed their relationship with ideas about mathematics textbooks.

Limited evidence exist that examined the relationships between beliefs about the importance of mathematics textbooks and other beliefs about mathematics teaching and learning. This study revealed (see Table 12, p. 145) that teachers' reliance on textbooks and their instructional choices are interconnected. Notably, using Rokeach's (1968) concept of central beliefs, in this study, I identified beliefs about textbooks, memorization, and using signs, symbols, and notations as the central beliefs of high school mathematics teachers in Pakistan. The finding showed that these central beliefs have strong connections with other beliefs in teachers' belief systems.

Not only does this study shed light on the relationship between different teacher beliefs, but it also highlights the importance of context in shaping beliefs and practices. The study's focus on a resource-constrained environment provides context-specific knowledge, revealing how teachers navigate challenges while adhering to their beliefs.

7.2.3. Interplay between teachers' beliefs, access, and competencies

The empirical evidence from this study unravels the interplay between access to digital resources, teachers' beliefs, and their competencies, highlighting the dynamics shaping Pakistani mathematics teachers' knowledge and use of digital resources. Drawing on the data from both research phases, I elaborated on the various dimensions of teachers' beliefs, which encompassed the role of digital resources in teaching mathematics, promoting student engagement, fostering self-regulation, and addressing potential distractions. The study found that teachers hold positive beliefs about and have knowledge of using digital resources to enhance students' learning. This contributes to the existing literature on teachers' beliefs and provides an additional perspective on the effects of a resource-constrained environment. The findings highlighted a mismatch between mathematics teachers' beliefs and their actual access to and knowledge of mathematics-specific digital resources.

An important contribution of this study lies in the finding that despite knowledge of personal use of digital resources, teachers consider that they had the limited ability to make professional use of mathematics-specific digital resources in their teaching practices. It found that government schoolteachers, who had the least access to digital resources, face additional challenges due to limited infrastructure and internet access at schools. This predicament was exemplified by the comment of one teacher (T5) who said, "neither do we have the facilities, nor do we do the type of teaching… (here) there are no such things as digital resources". While generally teachers were constrained by poor resources and other contextual factors from experimenting with the use of digital resources, the study did indicate increase access to videos and graphics due to their easy access and availability. Teachers showed skills in searching, selecting, creating, and using videos for teaching and learning mathematics. Teachers may have acquired these skills through personal use, allowing them to use videos and graphics effectively in their profession. However, it was also suggested that this prevalence of specific resources such as videos could deter Pakistani teachers from using a range of other useful and innovative digital resources in mathematics education.

7.2.4. Link between training, self-learning, teachers' resilience and digital competencies

The empirical evidence provides a deeper understanding of the link between training, self-learning, and digital competencies. This finding is an important contribution as it reveals an interesting paradox among teachers concerning their interaction with digital resources. Although, teachers endorsed the importance of digital resources in mathematics teaching. However, they perceived themselves as inadequately trained and have limited access to these resources. This highlights a disconnect, between beliefs and realities on the ground. The teachers are placed in a situation where the application of what they saw as beneficial was constrained by barriers.

7.2.5. This coexistence of endorsement and inhibition lead teachers to use their own initiatives to introduce digital resources. But the continuity of such efforts needs to be complemented by formal professional development, mentoring, and a supportive infrastructure. For instance, the relatively high knowledge of equation editors highlights skill development through workarounds and self-learning. However, sole reliance on self-learning can be unsustainable. Influence of COVID-19 on beliefs and digital competencies

This study makes a notable contribution to the evolving landscape of COVID-19 teaching and learning literature. The findings (Sections 4.6.1 and 5.6) highlighted the impact of the pandemic on the beliefs and digital competencies of Pakistani mathematics teachers. The identification of the pivotal role played by the COVID-19 pandemic in changing teachers' beliefs about digital resources and enhancing the development of their digital competencies constitutes a novel insight in the Pakistani context. The observed increase in use of digital resources during the pandemic reflects a practical adaptation by teachers to the new educational landscape. Teachers, (such as T2, T3 and T4), engaged in self-learning, and devised innovative workarounds to teach mathematics digitally. The positive impact on teachers' confidence, stemming from the affordances and situational use of digital resources during the pandemic, signified an important shift in their digital competencies. Teachers' beliefs evolved based on firsthand experience, leading to new digital skills. This study thus contributes new knowledge by documenting how the unique circumstances of the COVID-19 pandemic served as a catalyst for changing beliefs and enhancing digital competencies among Pakistani mathematics teachers. The adaptive strategies such as assessment methods using WhatsApp employed by the teachers in response to unprecedented challenges, can be improved and used beyond the pandemic era.

7.3. IMPLICATIONS

This study's contributions have several implications for understanding teachers' beliefs, policy, practices, professional development, and mathematics teaching. In this section, I first present the implications highlighting the need for leveraging central beliefs (Section 7.3.1). The implications extend to policy considerations, emphasizing the importance of policies that support the development of teachers' digital competencies in mathematics education (Section 7.3.2). Then I present implications for teachers' professional development, highlighting the necessity of tailored programs (Section 7.3.3). Lastly, I highlight the implications for teachers and the need for them to navigate the evolving landscape of mathematics education in Section 7.3.4.

7.3.1. Leveraging central beliefs to enhance digital competencies and teaching practices

This study demonstrates the role and interconnected nature of teachers' beliefs within the digital competencies' framework. Based on these findings it is recommended that consideration of beliefs needs to play a role in the development of mathematics teachers' digital competencies. Teacher training programs could include modules that encourage teachers to reflect on their beliefs regarding digital resources. This reflective process can help professional development initiatives aimed at developing digital competencies. Curriculum developers can ensure that educational materials and resources align with teachers' beliefs and teaching methods. They can do this by incorporating teachers' feedback into the development process, ensuring that textbooks and resources address specific classroom needs and challenges. They may also encourage teachers to try different teaching approaches, in addition to their traditional instrumental pedagogy.

The findings related to beliefs further indicate that culturally sensitive interventions are essential, given that socio-cultural and religious contexts influence teaching practices in Pakistan. Teachers' central beliefs, including notions about textbooks, memorization, and using symbols and notations, offer an opportunity for teacher training endeavours. Findings reinforce the importance of context and the recognition that teachers' beliefs do not develop in isolation and that beliefs are not separate entities. Teacher training programs and ongoing professional development initiatives need to acknowledge teachers' central beliefs and design targeted interventions, such as belief-aligned workshops, reflective practice sessions, collaborative discussions on integrating technology with core pedagogical values, and mentorship programs, to enhance teaching approaches and use of digital resources in the classroom. Overall, the implications of the findings reside in the importance of understanding the interconnection between teachers' belief systems, digital competencies, and the potential for innovative mathematics teaching practices in a resource-constrained environment.

7.3.2. Implications for policy

This study finds that the mathematics teaching and learning environment in Pakistani schools faces a variety of challenges. These challenges include a shortage of qualified high school mathematics teachers, resources, poor infrastructure, and a reliance on traditional teaching methods. These findings have important implications for education policy, especially in resource-constrained environments such as Pakistan.

The findings implies that policy needs to focus on improving existing teachers' professional development setup, teaching methods, teaching resources and supportive infrastructure. It is argued that improvements in these areas could have a significant impact on the overall current mathematics education in Pakistan. By prioritizing and addressing key challenges such as teachers supply, resources, and technological infrastructure, policymakers can start laying the foundation for broader changes in the long run.

It is argued that policy needs to acknowledge and build on current practices. While there is room for improvement and integration of digital resources, it is essential to build on the strengths of traditional methods that provide value in certain contexts. Policy can promote blending instrumental and conceptual understanding with digital resources. The focus of policy can be on creating a balanced learning environment that equip teachers with the knowledge and skills, allow them to combine traditional teaching approaches, such as memorization and textbook-centred pedagogy with the knowledge of using digital resources. Policy can achieve this by designing and implementing professional development initiatives that encourages the use of mathematics-specific digital resources (like DGS) and blend them with local practices. This blend of approaches can enhance teachers' skills and capacity and may help them generate their own ideas to integrate technology into their existing curriculum and teaching practices.

Another policy consideration is promoting a growth mindset among teachers regarding technology integration. Our findings indicate that teachers have resilience and a self-learning attitude that could be encouraged to develop digital competencies. Massive Open Online Courses (MOOCs) like Coursera can facilitate this development. Access to these platforms can address limited professional development and promote inclusivity by offering free or affordable access to high-quality educational content. MOOCs provide courses from global institutions, enabling teachers to acquire and update skills relevant to teaching and learning mathematics at their own pace. This approach complements teachers' self-learning efforts in Pakistan, allowing them to engage, collaborate, and continuously learn from global academic leaders. MOOCs can enhance collaboration between teachers, minimizing barriers to peer interaction, and promoting an appreciation for diverse perspectives, such as those from early-career teachers. However, above all, policymakers need to address the digital divide and ensure equitable access to technology and internet connectivity, particularly in rural areas. It is important to bridge the gap in technology infrastructure and ensure that all teachers have equal opportunities to access digital resources.

7.3.3. Implications for teachers' professional development (PD)

The study findings have several important implications for the teachers' training and professional development (PD). Most teachers were dissatisfied with current PD efforts. They criticized PD programmes for teaching non-specific mathematics approaches and including limited technology applications. Teachers who were trained in the last five years (such as T6), noted various issues like teachers being forced by academic leaders to adopt low-cost or free methods for teaching and learning activity.

Based on these findings, a number of steps could be considered as follows.

- Incorporate Teachers' Feedback into PD Design: Teachers' feedback can reveal shortcomings in current PD programs, allowing for tailored and relevant initiatives.
- Engage Academic Leaders: Dialogue with academic leaders responsible for teacher training can bridge gaps between their expectations and teachers' needs. Interviews or surveys can help understand their perspectives and challenges.
- Include Digital Competencies in Pre-Service and In-Service Training: Addressing the discrepancy between advocated teaching approaches and constraints faced by pre-service teachers can prepare future teachers for modern digital classrooms.

The approaches including, incorporating teachers' feedback, engaging academic leaders, and adjusting pre-service training can enhance the digital competencies of mathematics teachers in Pakistan. Another finding suggests the need for PD focusing on personal instrumental genesis. Teachers' familiarity with digital resources often comes from self-learning, leading to a reliance on videos and graphics. PD should address this by introducing mathematics-specific digital tools, designing interactive digital lessons, and utilizing technology for formative assessment. Encouraging diverse use of digital resources can help overcome biases and enhance overall digital competencies.

The picture emerging from this study of significant self-education initiatives about digital tools was prompted in the first instance by the realities and challenges of the COVID-19 pandemic. Teachers mentioned their capacity for self-learning and resourcefulness in acquiring digital skills. Future PD programmes need to explore how self-learning initiatives can support teachers in developing their digital competencies. PD programmes can introduce them to relevant online tutorials, courses, encourage them to participate in webinars, access open educational resources, and actively engage with online communities and networks of teachers. These strategies can help teachers to stay motivated, maintain their well-being, and effectively navigate the demands of their profession.

In resource-constrained environments, blending self-learning with structured professional development (PD) allows teachers to learn at their own pace while benefiting from expert guidance and peer collaboration. Self-learning could offer flexible, accessible, and cost-effective training opportunities. Further research is needed to examine the effectiveness of these blended approaches in developing digital competencies. Education systems may acknowledge this form of professional growth, provide recognition, and support, and bridge the gap between self-learning and formal PD. PD initiatives may combine self-directed learning with structured institutional support, exploring their long-term impact and identifying strategies for teacher training.

7.3.4. Implications for teachers

The empirical evidence from this study suggests several implications for mathematics teachers. The findings suggest teachers have limited resources, inadequate PD opportunities, and work within their social context of teaching. But that teachers significantly contribute to shaping this social context. The findings reveal a tendency among teachers to prefer traditional methods and have limited opportunities to explore other pedagogical approaches, particularly those involving integrating technology into their teaching practices. This environment hinders the creation of opportunities for digital integration in educational settings. Teachers needs to take actions, decisions, or collective practices, to change and create a social context that provide more opportunities than constraints.

Constraints, such as peer pressure and traditional teaching methods within schools, may stem from the teachers themselves. For example, T3, an early career teacher, hesitated to embrace her potential and expertise in digital competencies. Despite having a sound understanding of technological tools, she mentioned a lack of confidence in applying these skills in her teaching practice. T3 hesitated because she thought she was more knowledgeable than her experienced colleagues. A limited mindset may stem from a desire to avoid possible conflict with peers. Self-imposed curbs can hinder teachers' willingness to try new approaches. From an institutional point of view, a supportive and encouraging culture of innovation that provides opportunities for collaborative learning and sharing of best practices can reduce the impact of peer pressure. From an individual standpoint, teachers can transform their own teaching approaches through self-learning and development as well as advocate for reformed organizational cultures that foster more pedagogical autonomy, collaborative support, and reduced peer pressure. Instead of waiting for systemic changes, equitable resource allocation, professional development opportunities, and supportive leadership, teachers can take steps towards growth and progress. To develop their digital competencies, teachers must leapfrog contextual constraints and create a self-empowering environment.

One way in which teachers can recognise and foster their own digital competencies is through becoming involved in academic research. During the data collection I faced resistance from teachers about engaging in this research. Research can allow teachers to stay abreast of the latest development and critically assess and apply evidence-based practices in their teaching. For teachers, it is important to explore, evaluate, and use knowledge from current research that align with curriculum goals and pedagogical approaches. Furthermore, research-based evidence can help teachers to persuade school leaders of the importance of and strategies for employing digital resources.

Overall, it is argued that teachers can undertake some of the initiatives for change themselves. They can engage in more experimentation, embrace risks, and draw insights from their experiences.

7.4. POTENTIAL AVENUES FOR FUTURE RESEARCH

This study's empirical evidence provides comprehensive understanding of the digital competencies and associated beliefs of mathematics teachers. The investigation also delineates potential avenues for future research. In this section, I present unexplored dimensions within the current study's scope and propose areas that merit deeper investigation. The section starts with the ideas for future research designed to examine the utility of mathematics-specific digital resources (Section 7.4.1). Following this, I discuss the feasibility of mixed-methods for the future research in Section 7.4.2. In the last Section 7.4.3, I explore the complexities of the digital competencies framework's constructs.

7.4.1. Research designed to examine the utility of mathematics-specific digital resources

A small number of participating teachers knew about mathematics-specific digital resources like DGS and its potential for teaching and learning mathematics. This finding has several implications for the future research.

Future research can employ comparative study design to compare the effectiveness, impact, or outcomes of using mathematics-specific digital resources with existing teaching methods or resources. In these studies, it would be important to assess the impact of access to resources and infrastructure on developing digital competencies in local contexts. These studies can compare several aspects of teachers and teaching. For instance, I found a gap in competencies of government and private schoolteachers as well as between rural and urban teachers. Future comparative studies can be conducted within the same educational context, where different groups of teachers are assigned to use either digital resources or traditional teaching methods. Alternatively, research could compare outcomes across different schools (government and private) that have varying levels of access to digital resources.

Another potential focus for comparative research could be examining the cultural and contextual factors influencing digital competencies in mathematics teaching across different countries or regions Research of this nature can highlight the variations in teachers' beliefs, attitudes, and practices regarding the use of mathematics-specific digital resources and inform the development of culturally responsive strategies for enhancing digital competencies in diverse educational settings.

7.4.2. Feasibility of mixed-methods approaches for future research

In this study, I used mixed-methods encompassing sequential use of a quantitative online survey followed by qualitative semi-structured interviews. This approach enabled both breadth and depth of analysis and provided insights into the complexity of digital competencies' constructs. Quantitative data provided a broad overview of the prevalence and patterns of digital competencies among teachers. Complementing this the qualitative data offered deeper insights into teachers' experiences, beliefs, perceptions, barriers, facilitators, as well as unique challenges that high school mathematics teachers face in using digital resources in Pakistan.

Linking the qualitative themes to pre-identified quantitative constructs also allowed for triangulation and cross-validation. For example, interview insights on resource limitations were mapped to survey results on technology access barriers and COVID-prompted belief changes were connected to measured teacher belief constructs. Overall, integrating two datasets and analytical lenses strengthened the reliability, credibility and confidence of the research inquiry. This study demonstrated the feasibility of using mixed-methods to examine issues in mathematics education such as digital competencies. It offered a comprehensive, triangulated understanding of the topic, explored contextual factors and individual experiences, including teachers' voices.

One major drawback that occurred in this research was that due to the COVID-19 lockdown, I was unable to undertake classroom observations. Teachers were at home and did not have access to their official documents or lesson plans. Future research can include classroom observations (including documents) as part of a mixed-methods approach to gain a deeper and richer understanding of how teachers integrate digital resources in teaching, learning and curriculum design. Through classroom observations, researchers can identify challenges and successes, and explore the specific strategies used by teachers to engage students and facilitate learning using technology. This rich qualitative data can complement the survey data used in this study. This would allow a comprehensive analysis of teachers' digital competencies and their impact on student learning.

Classroom observations could also facilitate incorporation of students' perspectives and experiences, crucial for understanding the impact of digital resources on their mathematics learning. Researchers can include student interviews, surveys, or focus groups to gather students' perceptions of how specific teacher digital skills contribute to their engagement, understanding, and problem-solving skills.

7.4.3. Future research exploring the constructs of the DCTMT framework.

Future research can address several key areas identified by the study's findings related to the constructs of the DCTMT framework (as shown in Figure 7, p. 80). Firstly, future research could explore the factors influencing the levels of SDRK, given its weak correlations with three constructs in the modified DCTMT framework. Investigating additional factors beyond beliefs, such as resource availability, access to training, and socio-cultural influences, and understanding how these factors moderate SDRK is essential for a comprehensive understanding.

Secondly, a deeper exploration of the relationship between beliefs, particularly personal orientation (PO), and digital competencies is warranted. The study suggests a potential inconsistency between conceptual models and the realities of resource-constrained educational environments. The study suggests that conceptual models linking beliefs to usage may have limitations in low-access environments or may overlook crucial variables. Future research can emphasize how teachers' beliefs align with or diverge from factors influencing their digital skills.

Thirdly, future research can examine the impact of contextual factors, including socio-religious context and resource constraints, on the framework constructs. This involves assessing how cultural and religious beliefs shape attitudes towards digital technology in education and evaluating the effects of limited resources on the effectiveness of technological PD initiatives. Inadequate training and PD opportunities, identified as potential barriers, require further exploration within the specific context of resource-constrained environments.

Fourthly, a view of the drivers influencing digital competencies is crucial. Future studies may investigate factors such as teachers' personal and professional beliefs about digital technology, the quality, relevance, and accessibility of training programs, and external contextual constraints like socio-economic conditions, institutional support, and infrastructure availability. This comprehensive understanding could illuminate the multifaceted nature of teachers' digital competencies.

7.5. LIMITATIONS

The study's findings should be interpreted with caution, considering limitations that impact their generalizability. This section addresses these limitations to present a nuanced understanding of the research. It begins by examining potential limitations associated with the Digital Competencies for Teaching Mathematics with Technology (DCTMT) framework (Section 7.5.1). Subsequently, the discussion extends to encompass limitations related to the broader context, including constraints and challenges faced, with specific attention to the unprecedented situation of COVID-19 (Section 7.5.2). Lastly, I highlight potential limitations associated with the personal biases inherent in my research that may have influenced certain aspects of the study (Section 7.5.3). This thorough exploration of limitations serves as a critical reflection to enhance the credibility of the study's findings.

7.5.1. Potential limitations associated with the framework

I modified and used DCTMT framework introduced by Tabach and Trgalova (2020). While the framework provided a structured approach, its specificity may have restricted the exploration of aspects not represented by the five constructs of the framework. This may have resulted in the potential oversight or under-representation of certain dimensions of teacher digital competencies. For instance, the framework may not have fully accounted for the influence of socio-religious and cultural beliefs on digital competencies for the Pakistan context.

Moreover, absence of directional arrows between framework constructs made inferring or empirically testing relationships/causality between competencies challenging. Arrows in a framework typically signify the directionality or causality between concepts, helping to clarify how one concept influences or affects another (Partelow, 2023).

One significant challenge faced during the study was the inflexibility of the framework in adapting to emerging trends, such as the COVID-19 teaching and learning. Frameworks are important research tools typically developed based on existing knowledge (Partelow, 2023). They are developed in well-resourced settings that assume consistent access and ideal conditions. However, during the pandemic technology and digital practices changed rapidly. The changes, including rapid shifts in teachers' digital competencies to transition to online teaching and learning and changing beliefs about technology use, were not anticipated during the development of the research instrument based on the framework's constructs. As a result, modifications had to be made to the survey instrument. I had to incorporate additional items to capture the COVID-19-related aspects. This adaptation allowed for an examination of teaching and learning strategies during the pandemic. However, it resulted in an increased length of the online survey, which may have potentially impacted teachers' engagement and response rates.

I also encountered challenges in finalizing the items for the construct KDRT (knowledge of digital resources and teaching). KDRT encompasses a wide range of teaching aspects in the context of digital technologies. These include, but are not limited to, familiarity with mathematics-specific digital resources, content curation abilities, digitized pedagogies, online feedback and support mechanism, digital citizenship and online safety and digital/online ethics. The multidimensionality of this construct may require additional items to fully capture the attributes associated with it. Alternatively, the construct may need to be further divided into sub-constructs that align with the field of mathematics education and highlight the integration of digital resources within the discipline. Examples of possible sub-constructs are provided here:

1. Mathematical Pedagogical Knowledge and Digital Resources (MPKDR): This sub-construct may focus on mathematics teachers' knowledge and skills related to the pedagogical aspects of using digital resources in mathematics education. It will cover the understanding of instructional strategies, the use of mathematics-specific digital resources, and knowledge of affordances offered by these resources for mathematical teaching and learning. The suggested name "Mathematical Pedagogical Knowledge and Digital Resources (MPKDR)" for the sub-construct aligns with mathematics education and the integration of technology. It may capture the essential elements related to mathematics teachers' knowledge and skills in using digital resources for mathematics teaching and learning.

2. Mathematical Digital Content Knowledge (MTCK): This sub-construct may emphasize mathematics teachers' knowledge and understanding of the mathematical content. It could involve the ability to design, create, modify, select, adapt, and present digital content that supports mathematical teaching, considering the specific context of mathematics education.

The existing set of items, although carefully developed, may not capture the entirety of teachers' knowledge of digital resources and teaching (KDRT).

7.5.2. COVID-19 pandemic

I started my PhD journey in November 2019, almost three months before the COVID-19 lockdown in New Zealand. The unprecedented challenges of COVID-19 pandemic introduced several limitations for my study. I intended to use mixed-methods to collect data using a survey, face-to-face interviews, and teacher documents to investigate the digital competencies of high school mathematics teachers in Pakistan. However, in March 2020, during New Zealand's first COVID-19 lockdown, I realised that my data collection efforts would be limited to being completed online. I created a Facebook page to reach a country-wide target population of mathematics teachers in Pakistan. While online methods allowed for wider reach and convenient data gathering, it also posed limitations. For instance, as most teachers were confined to their homes during the COVID-19 lockdown, they did not have access to their official documents and artifacts. Teachers' documents could have provided further context for and evidence of their digital competencies. Lockdown and travel restrictions also restricted in-person interactions and observations. Therefore, I had to rely on online surveys and interviews using the Zoom application.

7.5.3. Personal biases

Throughout the research process, I managed and mitigated potential personal biases. I ensured that my background in mathematics education, digital resources, and curriculum development would not influence the selection of research questions, data analysis, or interpretation of the findings. I scrutinized my biases, assumptions, and perspectives, particularly during critical decisions, such as modifying terminology in the mathematics teachers' digital competencies framework. Using the reflexive diary further ensured continuous self-awareness and transparency throughout my research journey. However, despite all the efforts, it is possible that my own biases and preconceived notions may have influenced the interpretation of the data.

7.6. CONCLUDING THOUGHTS

The findings of this research extend beyond the specific context of Pakistan, offering valuable insights for mathematics education globally. For example, the challenges revealed in the current study, such as inadequate training and resource constraints for digital integration, are common in educational systems, especially in developing and resource-constraint environments. Understanding the influence of socio-religious contexts, on teachers' beliefs and practices provides a framework for exploring similar dynamics in other countries. The blended approach for PD, which combines self-learning with structured in-service and pre-service training, offers a flexible model that balances autonomy with expert guidance, relevant to diverse educational settings. Insights into the impact of teachers' beliefs on their use of digital technology can inform the design of PD programs worldwide. Strategies to address resource constraints can also serve as inspiration for similar solutions in other contexts.

The modification of and empirical evidence for the DCTMT framework provided a comprehensive lens for examining teachers' digital competencies. However, it has limitations in not evaluating classroom practice. Future research could explore additional methodologies to gain a more comprehensive understanding of teachers' digital competencies.

The implications of this study's findings are far-reaching, extending to policy development, mathematics education research, and the design of teacher professional development programs. The resonance of these findings has been acknowledged through fruitful engagements in conferences and publications, contributing substantively to the ongoing scholarly discourse in mathematics education. These conferences and publications provided valuable platforms for engaging with the mathematics education community and being part of a scholarly conversation.

The four-year trajectory of this research has not been without its trials, and I extend my deepest gratitude for the unwavering support received along this arduous path. Two and a half years away from family, exacerbated by the challenges imposed by the COVID-19 pandemic, have underscored the resilience required in the pursuit of knowledge. Communicating with respondents amidst the pandemic and navigating the complexities of ensuring the survey reached the right audience presented formidable challenges, which were met with determination and perseverance.

Looking ahead, my commitment to advancing the educational landscape remains unwavering. I plan to continue research, collaboration, and investment in mathematics education. Particularly, I intend to delve into further research endeavours that build upon the foundations laid by this study. Future research endeavours will unlock layers of complexity surrounding teachers' digital competencies and their profound impact on beliefs, students' engagement, problem-solving prowess, and overall mastery of mathematics. This commitment extends to a broader horizon, where I aspire to shape educational policies that prioritize the integration of digital resources and the professional development of teachers, especially in resource-constrained environments.

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APPENDIX A: Letter of Invitation for Principal

The Principal,

ABC College, Karachi

Subject: Project Information Statement/Letter of Invitation

Dear Sir!

I am a PhD student at the University of Waikato, Hamilton New Zealand. I am conducting research on Mathematics Education. In this study, I am investigating the "Digital competencies of high school mathematics teachers in Pakistan." Should you agree for your school to participate in this research, I request to invite mathematics teachers to participate by either passing this information to them or giving me the permission to come and talk to them for half an hour (introduction). I need to talk to them as one group to explain the project. Teachers will be invited to participate in a one-to-one interview that will be approximately one hour in length. The interview will be audio recorded. The overall maximum time required for participants in this research will be two hours (Introduction ½ hour, interview one hour, second interview if required 15 minutes, review of transcript 15 minutes). These interviews will take place at a time when teachers are available (mutually agreed time) to talk within School timing. Their interview data is private to them and me only. At the end of the project, I can provide you with a summary of key findings that might help you to improve professional development programmes for your teachers.

The role of the school and participating teachers will be voluntary. Your mathematics teachers may wish to participate or refuse to participate in this research. Participants may withdraw from the study at any stage until they have approved and returned the first copy of their interview transcript(s). Anything that teachers say during the interview will remain confidential. The school and participants' names will not be used and individual teachers will not be identifiable in any written reports about the study. I will anonymise information by quoting pseudonyms for my thesis and publications. However, I cannot guarantee anonymity in full, but confidentiality will be rigorously pursued. The data collected from this study will be used for academic purposes (writing doctoral thesis, journal publications, and conference presentations). A summary of the key findings will be available to you at the end of the research.

I request your support of the research to allow me to engage with the participants in your school. If you have any queries or you require further clarification about the research project, please contact on the following:

Primary point of contact (Researcher)	**Secondary point of contact Supervisor**
Mairaj Jafri (mj115@students.waikato.ac.nz)	Dr Sashi Sharma (Senior Lecturer)
Phone: +64273187067, +923003913917	Sashi.sharma@waikato.ac.nz

This research has been approved by the University of Waikato Faculty of Education Ethics Committee on 06/04/2020. Approval number: FEDU024/20

APPENDIX B: School Principal Consent Form

I am happy for you to come into the school to recruit participants (mathematics teachers) for a doctoral research project titled as "Digital competencies of high school mathematics teachers in Pakistan." Anything that teachers say during the interview will remain confidential to the researcher. As a Principal, I will not see what participants said to the researcher. All information about school and teachers will be anonymized by quoting pseudonyms. However, you will provide a summary of the findings to school when they are available. I also understand that the data collected from this study will be used for a doctoral research report, within journal publications and conference presentations. The overall maximum time required for participants in this research will be two hours (Introduction ½ hour, interview one hour, second interview (if required) 15 minutes, review of transcript 15 minutes).

I have been given the guarantee that this research has been approved by the University of Waikato Faculty of Education Ethics Committee on 06/04/2020. Approval number: FEDU024/20

I may seek further information on the project from Mairaj Jafri on +64273187067, +923003913917 or at the following email address: mj115@students.waikato.ac.nz

_____________________ _______________________

Principal Signature

____

Date

This research has been approved by the University Of Waikato Faculty Of Education Ethics Committee on 06/04/2020. Approval number: FEDU024/20.

APPENDIX C: <u>INFORMATION SHEET FOR TEACHERS</u>

Thank you for your willingness to learn more about this research project. In this research, I am investigating Digital competencies of high school mathematics teachers in Pakistan. What they currently do? And what challenges and impediments might come in the way in using digital resources? I want to know their beliefs, values and perspectives towards using digital resources in the mathematics classroom.

I would like to invite you to participate in this study for an individual face-to-face interview that is expected to take an hour. The interview will be AUDIO recorded and will require a quiet place to conduct within school timing. If required, you may be asked to answer some follow-up questions via email or phone that can take 15-20 minutes of your time. You will be provided with a transcript of the interview for review, amendment, and approval to be returned within 07 days. It will take 15 - 20 minutes of your time to read the transcript. The overall maximum time required for participants in this research will be approximately two hours (Introduction ½ hour, interview one hour, second interview if required 15 minutes, review of transcript 15 minutes).

The data collected from this study will be used for academic purposes (writing doctoral thesis, journal publications, and conference presentations). Anything you share with me remains confidential. I will anonymise information by quoting pseudonyms for my thesis and publications. However, I cannot guarantee anonymity in full, but confidentiality will be rigorously pursued. Interviews data will be archived for at least five years as per the University Human Research Ethics Regulations. The audio recordings of the interviews will be kept in a secure location for the duration of the research process.

During the study, you have the right not to answer the whole or part of any particular question asked of you during the interview process. You will be given access to your individual transcript and a summary of the findings from the study when it is concluded. You can withdraw from the study up until you returned the first transcripts.

Contacting the Researcher

For further questions or any concerns about the project, please feel free to contact either me or my chief supervisor, Dr Sashi Sharma. If you want to contact someone not involved in the study, you can contact Professor Linda Mitchell from the University of Waikato. I also request to pass my information and contact details onto other mathematics teachers who you think can participate in the research project.

Please note, I am looking for 15-20 participants for the study. Mathematics teachers having knowledge of digital resources will be chosen for semi-structured interviews.

Researcher: Mairaj Jafri

Email: mj115@students.waikato.ac.nz

Phone: +64273187067, +923003913917

Chief Supervisor: Dr Sashi Sharma

Email: sashi.sharma@waikato.ac.nz

Associate Professor Linda Mitchell

Email: linda.mitchell@waikato.ac.nz

This research has been approved by the University Of Waikato Faculty Of Education Ethics Committee on 06/04/2020. Approval number: FEDU024/20.

APPENDIX D: INFORMED CONSENT FORM FOR TEACHERS

Project title: Digital competencies of high school mathematics teachers in Pakistan

I agree to participate in a doctoral research project led by Mairaj Jafri from the University of Waikato, New Zealand.

- My participation in this project is voluntary. Participation involves a face-to-face interview(s).
- The interview may last up to an hour. The total maximum time requirement for me in this research will be for two hours (introduction half an hour, one hour interview, if required second interview 15 minutes via email or phone, and review of transcript 15 minutes).
- I allow Mairaj Jafri to AUDIO record and take written notes during the interview.
- I do not have to answer any particular question, I do not have to answer all of them. I do not have to have a second interview. But I do need to check the transcript summary for accurate reflection and return it within one week time.
- I recognise that if I do not return my copy, the researcher will view it as my willingness to use it in his study. I have the right to withdraw up until I reviewed and approved the first transcript. However, you wish to retain data already collected.
- Mairaj Jafri will also share the approved findings of the research with me.
- All information shared with Mairaj Jafri by me will remain confidential.
- My name will not appear, and you will anonymise information by quoting pseudonyms for thesis and other publications.
- For any further details Dr Sashi Sharma, the chief supervisor of Mairaj Jafri can be contacted via email: sashi.sharma@waikato.ac.nz

I am fully aware of everything that is expected of me during this research. I have had all my questions answered to my satisfaction, and I volunteer to take part in this research. I am also issued a copy of this consent form co-signed by Mairaj Jafri. I understand that the data collected from my participation will be used for academic purposes (i.e. thesis, journal publication, conference presentations), and I consent for it to be used in that manner.

_________________________ _________________________

Participant Date

_________________________ _________________________

Mairaj Jafri Date

Email : mj115@students.waikato.ac.nz

Phone : +64273187067 ; +923003913917

This research has been approved by the University Of Waikato Faculty Of Education Ethics Committee on 06/04/2020. Approval number: FEDU024/20.

APPENDIX E: Interview Protocol

Introduction

I am Mairaj Jafri, a PhD candidate in Mathematics Education at the University of Waikato, New Zealand. My research focuses on the digital competencies of high school mathematics teachers in Pakistan. I've designed this interview to gather data on the use of resources by high school mathematics teachers. Your participation involves audio recording and note-taking during the interview. You have the option to skip any questions you prefer not to answer.

I want to assure you that your identity and responses will remain confidential. The collected data will be used for my doctoral dissertation and potentially for academic papers in journals and conference presentations. I kindly request that you refrain from sharing details discussed during the interview with other teachers or individuals. Your cooperation in this matter is greatly appreciated.

Personal/Preliminary Information

Date: ________________ Start Time: _____________ End Time: _____________

Level of Teaching ______________________ Years of Teaching _______________

Interview Questions

1. What is your preferred method of teaching?
2. How do you believe students best learn and retain information related to signs, rules, symbols, and formulas in mathematics?
3. Have you used digital resources in teaching mathematics in the past?
4. How do you use digital resources for teaching mathematics?
5. What are your beliefs, values and perspectives (orientation) towards digital technologies and using digital resources in the mathematics classroom?
6. What factors influence you to use digital resources in your planning and teaching mathematics? What factor(s) influence you to keep using digital resources?
7. While selecting or using digital resources what you are thinking?

8. Do you apply different types of digital resources while teaching mathematics or do you think it's better to stick with one? Like videos, e-textbooks, animations, graphing software, spreadsheets etc.
9. Did you attend any professional development programme for the use of digital technology? (Self-funded or funded by the institution) Do you feel a need to learn more about the use of digital technologies?
10. Have you ever designed and developed digital resources for teaching mathematics? If yes, please explain the process.
11. What challenges and impediments you might face in the use of digital resources?
12. Please give the main advantages or benefits you have found or feel to be true of using digital resources in mathematics lessons.

APPENDIX F: Changes Made to Survey Items Based on Pilot Study Feedback.

Item	Teachers' response(s) italic / New or Reworded item	Final decision
Dynamic graphing software (DGS)	Majority of the participants were unaware of the term. For final survey the term DGS was further explained as e.g., GeoGebra	Elaborated
Mathematics teaching is to evaluate and share knowledge and ideas, and to discuss a variety of practical activities and real-world contexts.	*Unsure about what is meant by "practical activities" in statement 2?* *Please define practical activities* *Real world context seems to be unnecessary.* *The statement is too long and ambiguous.* <u>Reworded Item</u> Mathematics teaching is to share knowledge and ideas, and to discuss a variety of real-world contexts.	Item reworded
Learning mathematics is memorizing a set of facts and rules.	*Facts! I think there are other things as well such as formulas.* *The Statement needs to be rephrased for better understanding.* <u>Reworded Item:</u> While learning mathematics, it is important to memorise rules, facts and formulas.	Item reworded

I can design lessons using DCR. I can organize DCR within the lessons I teach.	*I think these two items have the same meaning.* <u>Merged Item:</u> I know how to design and organize lessons that include digital resources.	Items merged
The use of digital resources to teach mathematics diverts the attention of students.	*Could Item 6 be reworded for further clarity? Does it mean "diverts their attention to something else" or "distracts - unsure?"* <u>Reworded Item</u> Digital resources distract the attention of students from doing calculation	Item reworded
Mathematics is easier if digital resources are used to solve problems.	<u>Reworded Item</u> Digital resources help students in understanding and solving mathematics problems.	Item reworded
I can communicate with students using online communication platforms.	*Here (in Pakistan) students are not allowed to use digital devices for various reasons, communication with them using online platforms seems unrealistic.*	Item deleted
Digital resources help in classroom management	*It is very early to measure such statements because the use of digital resources is at an initial stage in mathematics teaching in Pakistan*	Item deleted

APPENDIX G: Communication with the Authors of DCTMT Framework

Mairaj Jafri <mj115@students.waikato.ac.nz>

Interested to use MDKT framework

Michal tabach <tabachm@tauex.tau.ac.il> Fri, Jan 24, 2020 at 6:41 PM
To: Mairaj Jafri <mj115@students.waikato.ac.nz>
Cc: TRGALOVA JANA <jana.trgalova@univ-lyon1.fr>

Dear Mairaj,

Thank you for your interest in our work. Our, as I have developed it in collaboration with Jana Trgalova [cc to this e-mail]. In fact, we are still developing it together.

I think that a PhD study is a good opportunity to apply both qualitative and quantitative approaches.

I will be happy to follow your work and talk about it, and I am sure Jana is also willing to further hear about your work and discuss it.

Good luck

Michal [and Jana]

--

Michal Tabach, PhD

Associate Professor

Head, Department of Education in Mathematics, Science and Technology

School of Education

Tel-Aviv University

מיכל טבח

פרופ' חבר

ראש החוג לחינוך מתמטי, מדעי וטכנולוגי

בית ספר לחינוך

אוניברסיטת תל-אביב

אתר אישי

[Quoted text hidden]

APPENDIX H: Ethics Application Approval Letter

Te Kura Toi Tangata
Division of Education
The University of Waikato
Private Bag 3105
Hamilton, New Zealand, 3240

DivEd Ethics Committee

fedu.ethics@waikato.ac.nz
07 8384500 ext. 7870
www.waikato.ac.nz/education

THE UNIVERSITY OF
WAIKATO
Te Whare Wānanga o Waikato

6/4/2020

Dear Syed Mairaj Hussain Jafri

Division of Education Ethics Application Approved FEDU024/20

I am pleased to advise you that your ethics application for the project entitled "Investigating knowledge and skills of high school mathematics teachers who use digital content resources for teaching" was approved by Te Kura Toi Tangata Division of Education Ethics Committee on April 6th, 2020.

Please be aware that the Te Kura Toi Tangata Division of Education Ethics Committee must be advised (by memo) of any changes to the details recorded in your ethics application. Please send any such advice to fedu.ethics@waikato.ac.nz. You will receive a memo of approval once the change(s) has been considered.

Kind regards

Co-chair
Te Kura Toi Tangata Division of Education Ethics Committee

APPENDIX I: THE ONLINE SURVEY

THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

INTRODUCTION

Thank you for becoming part of this research project. In this survey, I want to know what you think about using digital resources (DR) (videos, images, dynamic software, digital games, animations, and mobile applications) in the teaching and learning of mathematics. I will collect anonymized data from this survey that will be used for academic purposes. If you have any questions about the project or your valuable participation, please feel free to contact me at the following email address: mj115@students.waikato.ac.nz. Please note: This research has been approved by the University of Waikato Faculty of Education Ethics Committee on April 6, 2020. Approval number: FEDU024/20. I also request that you share the link to the survey with other mathematics teachers who you think can participate in it.

SURVEY - Nov 10, 2020

CONSENT:
I have read and understood what the survey data are for and how they will be used. I understand that I do not have to submit this form. The data is anonymous and so I cannot retract it once submitted.

- ◯ Yes. I want to continue
- ◯ No. Thank you and close

General Demographic and Professional Information

General Demographic and Professional Information

Gender

- ◯ Male
- ◯ Female

Please select the options that describe your professional involvement.
Check all that apply.

- ☐ Government school teacher
- ☐ Private school teacher

- ☐ Private tutor
- ☐ Tutor teaching in private academies or coaching center
- ☐ Online teacher
- ☐ Other: __________

Please select your Academic qualification.

- ○ Graduate
- ○ Postgraduate
- ○ Other __________

Are you a qualified teacher? (Qualified teacher means you have done B.Ed or M.Ed)

- ○ Yes
- ○ No

Please select classes/grades/levels in which you are currently teaching mathematics.

- ☐ Level 8
- ☐ Level 9
- ☐ Level 10
- ☐ Level 11
- ☐ Level 12
- ☐ Other __________

How many years of teaching mathematics?

- ○ Less than 1 year
- ○ 1 - 3 years
- ○ 3 - 6 years
- ○ 6- 10 years
- ○ more than 10 years

What types of digital devices do you have access to at work (school) or home?

- ☐ Desktop Computer
- ☐ Laptop
- ☐ Smartphone
- ☐ Interactive Whiteboard

- [ ] Digital Notebook/ Notepads
- [ ] Projectors
- [ ] Other

Do you have regular access to the Internet?

- ○ Yes, most of the time
- ○ Sometime
- ○ No

General Beliefs about Mathematics Teaching

Your Beliefs about Mathematics Teaching.

This section of the survey consists of questions regarding your beliefs about mathematics teaching. Please choose any options that best describe your level of agreement with the statement.

Learning mathematics means exploring problems to discover patterns and make generalisations.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

Mathematics teaching is to teach students how to create and assign meanings to new signs, symbols, and notations.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

Mathematics teaching is to share knowledge and ideas, and to discuss a variety of real-world contexts.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree

- ○ Somewhat disagree
- ○ Strongly disagree

Mathematics lessons should be followed by a critical discussion with students.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

Mathematics curriculum textbook is the best medium of instruction and source of knowledge.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

Mathematical knowledge is retained more easily if it is acquired using multiple representations.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

While learning mathematics, it is important to memorize rules, facts and formulas.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

Beliefs about using DR in teaching and learning of mathematics.

Your Beliefs about Teaching Mathematics with Digital Resources.

This section of the survey consists of questions regarding your knowledge and skills for teaching mathematics with digital content resources. Please select options that best describe your level of agreement with the statement.

Please Note: Digital Resources (DR) means videos, images, graphics, digital graphs, info-graphics, simulations, animations and dynamic mathematics software that can be used for teaching and learning of mathematics.

What kind of digital resources do you use frequently in teaching mathematics? Please select from the following options. Check all that apply.

- ☐ Videos
- ☐ Graphics (e.g. digital images, charts, representations)
- ☐ Digital Graphs
- ☐ Websites
- ☐ Animations
- ☐ Simulations
- ☐ e-textbooks
- ☐ Interactive educational games
- ☐ Dynamic mathematics software (e.g. GeoGebra)
- ☐ Other

Digital resources play an important role in mathematics teaching.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

Digital resources make mathematics problems and tasks more engaging.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree
- ○ Strongly disagree

I have lots of ideas about how I can make use of digital resources in the mathematics classroom.

- ○ Strongly agree
- ○ Somewhat agree
- ○ Neither agree nor disagree
- ○ Somewhat disagree

○ Strongly disagree

Digital resources help students in self-regulating, understanding, and solving mathematics problems.

○ Strongly agree
○ Somewhat agree
○ Neither agree nor disagree
○ Somewhat disagree
○ Strongly disagree

Digital resources distracts the attention of students from doing calculation.

○ Strongly agree
○ Somewhat agree
○ Neither agree nor disagree
○ Somewhat disagree
○ Strongly disagree

Knowledge of digital content resources and students

Knowledge of Digital Resources and Students.

This section of the survey consists of questions regarding your knowledge of digital resources and students. Please select any options that best describe your level of agreement with the statement.

Please Note: Digital Resources (DR) means videos, images, graphics, digital graphs, info-graphics, simulations, animations and dynamic mathematics software that can be used for teaching and learning of mathematics.

Mathematics students need to know how to use digital resources for learning.

○ Strongly agree
○ Somewhat agree
○ Neither agree nor disagree
○ Somewhat disagree
○ Strongly disagree

Students should not be allowed to use digital resources until they have mastered the concept, rule or method.

○ Strongly agree
○ Somewhat agree

- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Digital resources allow students to creatively think about mathematics problems.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Dynamic mathematics software (e.g. GeoGebra) makes students better problem solvers.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Mathematics software removes some learning opportunities for students. (e.g. drawing graph using pen and paper)

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Students are more confident in maths when they use digital graphs.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Videos about abstract mathematics concepts (e.g. complex numbers) improve student understanding.

- Strongly agree
- Somewhat agree

- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Your understanding of DR and Teaching

Knowledge of Teaching Mathematics with Digital Resources

This section of the survey consists of questions regarding your knowledge and skills for teaching mathematics with digital resources. Please select any options that best describe your level of agreement with the statement.

Please Note: Digital Resources (DR) means videos, images, graphics, digital graphs, info-graphics, simulations, animations and dynamic mathematics software that can be used for teaching and learning of mathematics.

I have received training for the use of technology and other digital resources.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

There are several digital resources on the Internet, it is difficult to decide which one to use to teach a particular mathematics topic.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I know how to write math equations and expressions using equation editor.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I can teach using dynamic graphing software (e.g. GeoGebra, Maple etc.).

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I can solve geometry or trigonometry problems using dynamic graphing software.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I can use dynamic mathematics software for calculus and algebra.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I teach mathematics better when my students use pen and paper for solving problems.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I can create images and videos for teaching purposes.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I want to improve my ability to teach mathematic with dynamic graphing software (e.g. GeoGebra).

- Strongly agree

- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I know how to use video conferencing (calling) applications (e.g. Zoom, Skype etc.).

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I know how to screencast (screen recording), share screen and resources during video conferencing.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

If required, I can teach mathematics in a fully online environment (e.g. School closure during COVID-19).

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

Please tell us little bit about your COVID-19 teaching strategy. What tools you used for teaching mathematics during COVID-19?

Knowledge/skills of using DR in curriculum planning and assessments

Curriculum and Assessment Planning using Digital Technologies

The next section of this questionnaire is consist of questions regarding your knowledge and skills of using digital resources in curriculum planning and assessments.

Please Note: Digital Resources (DR) means videos, images, graphics, digital graphs, info-graphics, simulations, animations and dynamic mathematics software that can be used for teaching and learning of mathematics.

I know how to design and organize lessons that include digital resources.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I can locate and search databases/websites containing digital resources.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I can download and install digital applications for teaching mathematics.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I know how to create and evaluate assessments using online tools (e.g. Google Forms).

- Strongly agree
- Somewhat agree
- Neither agree nor disagree
- Somewhat disagree
- Strongly disagree

I recommend the use of digital resources for teaching and learning.

- Strongly agree
- Somewhat agree
- Neither agree nor disagree

◯ Somewhat disagree
◯ Strongly disagree

If you want to mention anything in relation to teaching mathematics with Digital Resources, please use the space provided below.

Thank you for participating in the survey. You can submit your response by clicking the SUBMIT button. Clicking on SUBMIT will provide your consent to use this information for academic purposes. If you would like to volunteer for the second phase (an individual interview lasting up to an hour) of this research project, you can contact me by email or the phone number given below.
Thank you
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