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**Improving the Quality of Mathematics Education:
Two Teaching Modes and Taiwanese Student Learning**

A thesis

Submitted in fulfilment of the requirements for the degree

of

Master of Philosophy

in

School of Education

by

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at

The University of Waikato



THE UNIVERSITY OF
WAIKATO
Te Whare Wānanga o Waikato

Abstract

Students' and teachers' long-term (i.e. three years) experiences in three classes of the traditional direct instruction and constructivist class discussion approach to the mathematics teaching at a Taiwanese junior high school and at an experimental school in Taiwan were discussed in this study. This research utilized qualitative methods. The study adopted content analysis approaches from a qualitative perspective. This was combined with the perspectives of social constructivism and situated learning theories to interpret students' learning and growth.

The research findings of this study revealed differences in the group of students exposed to the constructivists teaching environment. These differences were evident in their mathematical competencies and richer students' autonomy. However, when compared to the traditional teaching environment there were several challenges such as time use, understanding all classmates' dialogue, mathematical writing ability in explaining and communicating their thinking and more teacher work.

Constructivist class discussion classrooms in this study appeared open, relaxed, lively, friendly, and supportive of each other in building new knowledge. This was apparent in School E where the environment provided more opportunities for students to develop their own mathematical ideas. This environment also produced a more social/collective/adaptive form of mathematical knowledge, with ongoing assessment of information provided by the teachers, to inform instructional practices. The data presented here show that students exposed to the constructivist discussion approach had richer learning experiences which may be viewed as a result of their active participation during instruction. Compared to the their peers in School T, the traditional direct instructional group, School E students had more learning roles - (knowledge explorers, knowledge producers, and knowledge adventurers). Student in School T acted mainly as knowledge receivers; they mostly received and followed the teacher's instruction and explanations of mathematical concepts, and then applied the received procedures to solve given mathematical problems.

The findings of the sequential relationship between teachers' perceptions of mathematics/learning, teaching practice, and students' knowledge/perceptions sheds new light on the social relationships between teaching and learning and the situated influences among classroom practices and students' knowledge/competencies/perceptions.

This investigation revealed that the constructivist approach seems to be an excellent medium to provide quality education. It is recommended that educators should re-introduce the use of a constructivist approach to teaching Mathematics because of its potential to enhance the quality of Mathematics education, which in turn augments students' competency as future Mathematicians.

This thesis is dedicated to my parents

Chao-Lien Chi 紀肇廉

and

Yin-Ying 銀英 & Li-Chin 麗琴

and

My husband: Rod and my daughter: Shana

and

All my family

Acknowledgements

*For the Lord is good and his love endures forever;
his faithfulness continues through all generations. (Psalm 100:5)*

All thanks and praise to God for guiding all my ways, and giving me health, strength and the opportunity to finish this thesis. God also answered my prayers by giving me a supportive family, supervisors, friends and colleagues. Their assistance and encouragement contributed to the successful completion of this study. Praise and thanks be given to the Lord, God Almighty!

I would like to acknowledge the contribution made by many people (directly or indirectly) towards this thesis. I am very grateful for the support I received from colleagues, friends, church members, and family. Thanks for so many precious people giving support in part of my discovery journey for my research. In particular, I would like to thank the following people for their assistance:

- My supervisors: Associate Professor Beverley Bell, Associate Professor Jenny Young-Loveridge, my ex-supervisor: Dr. Tony Brown, Dr. Garth Ritchie, Dr. Bronwen Cowie and Dr. Fred Biddulph, for their wisdom, kindness and patience in challenging and shaping my thinking, and in providing valuable comments and guidance through many drafts of my thesis.

- Special thanks to Professor Bill Jacob who supplied some needed resource material to start my study, Professor Jo Boaler for a short discussion of my work, and two articles from Professor Martyn Hammersler.
- The Taiwanese teachers and 49 students who participated in this research. Special thanks for your valuable opinions and time.
- Dr. Sandra Trotman and Miss LeAnn Eyerman for proofreading part of my thesis. In particular, I sincerely appreciate their views, valuable time, support and friendship forever.
- Staff and colleagues at the School of Education for their genuine friendship and support. Special thanks to Dr. Mira Peter, Dr. Elaine Khoo and Dr. Junjun Chen (Wilf Malcolm Institute of Educational Research) for discussing my research. Thanks to librarians: Mrs. Yvonne Milbank, Ms. Heather Morrell, and Mr. Alistair Lamb for advice on the format of my thesis. Thanks to Dr. Teresa Ritchie, Mr. Jason Yao, Mrs. Sylwia Rutkowska and all my colleagues in the graduate study office for their friendship and kind assistance in times of need.
- The Christian Fellowship and my Christian friends, Hooi yen, Ampy, Elaine & Jik, Kumalee, for the friendship and fellowship they provided. In particular, I want to express my sincere appreciation to Preetha, Lula, aunty Leela & uncle, Cathy, Jenny and Jenny's mother for taking care of my toddler daughter to give me free time for my study. What treasured friends you are to me, thank you.
- The former principal, Ms. Xiu-qin Gu 古秀琴, Mr. Jin-cheng Xie 謝金城, the current principal, Mr. Li-tai Li 李立泰, and the deans of Tai-Shan Junior High School in Taiwan who approved my leave for study.
- The Taiwanese Ministry of Education for granting me four years study leave to further my professional development.

- My family, my father, brothers (紀弘庠, 張育其), sister (張亞梅), my husband and my daughter for their unconditional love and support. Thank you.

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Chapter One: Introduction

1.1 Introduction

Development in mathematics education is informed by multiple learning theories (Cobb, 2007; Ford & Forman, 2006; Simon, 2009), illustrations of mathematical understanding (Skemp, 1976, 2006), reforms in curriculum or educational policies (Franke, Kazemi & Battey, 2007; Wey, 2007), and promising results from research on reforms to mathematics education (Boaler & Staples, 2008; Schoenfeld, 2002). This advanced or new knowledge of mathematical education has fuelled the chances of educational reform with regards to research on mathematics teaching and learning (Cobb, 2007).

In today's educational environment, many countries have and continue to undergo reforms; however the pendulum of educational reform movements keeps swinging in different directions (Chung, 2005; Lambdin & Walcott, 2007; Sfard, 2003). For example, during the period 2001 to 2004 some educators in Taiwan regarded the mathematics curriculum offered at the junior high level as a constructivist-based curriculum (Chung, 2005; Wey, 2007). Further examination of the educational practices revealed that instruction at the junior high school level did not truly reflect this paradigm shift. In fact, most Taiwanese junior high school teachers were still using the traditional direct teaching approach with the teaching method of 'chalk and talk' (Xu, 2004; Yu & Hang, 2009) combining a great amount of lecturing in lessons (Yu & Hang, 2009). This approach emphasised repetition, practice, and memorisation (Chou & Ho, 2007; Leung, 2014; Wei & Eisenhart, 2011), to deliver the content required to prepare students for examinations (Hsu & Silver, 2014; Jarvis, Holford & Griffin, 1998; Wei & Eisenhart, 2011). A similar trend also existed for Taiwanese primary school teachers (Xu, 2004; Yu & Hang, 2009). Thus, the educational practices showed evidence of shifts in different directions: constructivist vs. traditional direct teaching approaches.

Efforts have been made to adopt or fully embrace a constructivist approach to the teaching and learning of mathematics by Taiwanese educators. During the period

1996 to 2004, constructivist approaches to teaching and learning mathematics, have been experimented with and were mandated in Taiwan especially at the primary school level (Guo, 2004). However, this reformation path, especially at the primary school level, has not been viewed by the public of Taiwan as being successful (e.g., Chou, 2003a; Chung, 2005; Wey, 2007).

In general, it was perceived that the public was satisfied with the outcomes of using traditional direct teaching (Chou, 2003a). Methods of instruction place an emphasis on the transmission of facts and knowledge (Boaler & Greeno, 2000; Even & Tirosh, 2008). In this educational environment, Taiwanese students may still spend most of their mathematics classroom time on practising skills or developing procedural understanding (Xu, 2004; Yu & Hang, 2009). Similar learning patterns also existed in the USA (Kilpatrick, Swafford & Findell, 2001), and the United Kingdom (Boaler, 1996).

However, shifts from this delivery of the mathematics curriculum to a constructivist approach have not been embraced by the Taiwanese public. Dissatisfaction appears to stem mainly from the lack of research data on the effectiveness of constructivist approaches (Wey, 2007), and incomplete assessment practices (Chou, 2003a; Richardson, 2003). Researchers identified these concerns, among others, as the major cause for a backward movement of the educational reform pendulum in Taiwan (Chung, 2003b; Chung, 2005; Wey, 2007). The Taiwanese constructivist-based mathematics curricula from 1996 to 2004 as reported by Guo (2004) were replaced in 2005 (Chung, 2005). Hence, the focus of instruction shifted away from discussion or discourse teaching back to the traditional direct teaching (Xu, 2004).

Therefore, it may be argued that there currently exist two types of educational dilemmas in mathematics education in Taiwan. One dilemma is the traditional direct approach vs. the constructivist approaches to teaching and learning mathematics. This is the movement of the educational reform pendulum back and forth (or from one paradigm towards the other); from a procedural-centred focus on one end of the pendulum to a learner-centered focus at the other end of the pendulum (Chung, 2005). This shift in instructional approach is not unique to

Taiwan; it presents an area of concern for other nations such as the United Kingdom (Boaler, 1996; 2001), and to some extent the USA (Lambdin & Walcott, 2007; Weng, 2003).

The other dilemma stems from the first, that is, the teaching practices in Taiwan did not necessarily change to match the mathematics educational reform focus (e.g., Chen, 2003b; Liu, 2004; Wey, 2007). According to researchers, many teachers did not embrace this change, or saw the need to be change agents; they did not use the teaching approaches that were aligned with constructivism (e.g., Chen, 2003b; Liu, 2004; Wey, 2007). Hence, it was easier for these teachers to revert to the traditional direct teaching approach.

However, when compared to using a constructivist teaching approach, the use of the traditional direct teaching appears to be limited in scope. For example, the traditional direct teaching approach focuses mainly on the end product (Wei & Eisenhart, 2011) and little or no provision is made for developing communication skills. Since students' mathematical power comprises not only of end-products, but includes both conceptual and procedural knowledge, advanced abilities such as criticizing, generalizing, making connections, and positive mathematics values (Anthony & Walshaw, 2007; Kilpatrick et al., 2001), using only a traditional approach will, to some extent, limit students' ability to achieve mathematical competency. According to Lampert (2001, p.330), "mathematical competence is complex and multidimensional", therefore it stands to reason that using constructivist approaches to the teaching and learning of mathematics would better meet the needs of students than that of using traditional direct teaching approaches.

Some researchers agree that the use of reform-based or in the case of Taiwan, a constructivist approach has led to increased student learning outcomes on standards tests, (e.g., Boaler & Staples, 2008), and deeper mathematical understanding (e.g., Ball & Bass, 2000b; Boaler & Staples, 2008). While the literature has examples of constructivist studies conducted in primary schools (e.g., Lamon, 2007; Zeng, 1998), only few studies were based on long term constructivist research (i.e. three or more years) that were conducted at the high

school level (e.g., Boaler, 1997; Boaler & Staples, 2008). Given the differences in both teaching approaches, further research is needed to explore the differences in learning effectiveness (Confrey & Kazak, 2006; Richardson, 2003). In addition, a long-term research project would offer more chances to examine students' learning development (Boaler, 2000d). These concerns and the 1997-1999 longitudinal research conducted by Boaler served as the impetus for the present study.

This study focused on the mathematical instructional practices at an experimental school which was established towards the end of the 20th century. In 2004, this school's initiative was viewed by one scholar as a rare case of a school using an alternative teaching approach such as a constructivist approach at the junior high school level in Taiwan. Further, it is described as one of the best charter schools in Asia. This school emphasized autonomous learning. Thus, the teaching experimentation was valued and a constructivist teaching approach was used in mathematics classrooms by the Bureau of Education of Taiwan at the beginning of the 21st century. Two scholars reporting on the instructional approach of this school, during the early years of the 21st century, viewed enthusiastic class discussions as the dominant characteristic. Consequently, this school was eligible to serve as an example to reveal the principles of their constructivist approaches and research the long-term strength of teachers' and students' experiences and, in comparison to the traditional direct teaching approach. This study explored teachers' and students' long term experiences during the use of both the traditional direct teaching method and the constructivist approach of teaching mathematics at a Taiwanese junior high school for a period of three years.

This research draws upon Boaler's (1997-1999) longitudinal study conducted in the United Kingdom and the reform-oriented approaches or group work (Boaler and Staples, 2008). Boaler's longitudinal study monitored 300 students in two different schools with different teaching approaches. These students were exposed to the same instructional approaches at ages 11 and 12. However, at age 13, the teaching approaches differed. One group attended a school where the focus was on using traditional direct teaching, while the other group was exposed to instruction via problem solving and modelling. Based on the research findings of this study, Boaler (2001, p. 125) suggested the need to "examine the ways in

which students engage in different [teaching] practices ...if students only ever reproduce standard methods that they have been shown, then most of them will only learn that particular practice of procedure repetition...”

This study sought to explore ways in which using a constructivist approach can successfully build up students' mathematical thinking and understanding compared to using the traditional direct teaching approach, especially given that students in Taiwan face the highly competitive nature of tests (Chi, 2000) and a very full mathematics curriculum. This study also explored the nature of the developing constructivist pedagogy (Richardson, 2003), and developing a teaching model as a professional development tool through findings from teaching strategies and students' perceptions. In conducting this study, cultural issues in the Taiwanese context were also considered (Richardson, 2003).

The remainder of this chapter is devoted to:

- 1.2 Rationale and General Aims
- 1.3 Research Question
- 1.4 Definition and Standardization of Key Terms
- 1.5 Overview of the Thesis

1.2 Rationale and general aims

The need for this study arose from considering several factors of the Taiwanese educational system during the period 1996 through 2010. Mathematics in Taiwan was basically taught by the traditional teacher-centred method of instruction (Chi, 2000) or by direct instruction (Chi, 2000; Yu & Hang, 2009). It is firmly believed by many Taiwanese educators and the public that the level of student achievement on tests depends on the level of instruction; that is, the higher the level of teachers' instruction (i.e., coverage of content), and the higher will be the students' mathematics achievement (Wong, 1993). This belief is also supported by Gau's (1997) research.

In terms of student mathematics abilities, Taiwan when compared to the rest of the developed world is consistently ranked as one of the top four countries. For

example, top in 2007, third in 2011 (Third International Mathematics and Science Study, TIMSS) survey and fourth in 2012 (Programme for International Student Assessment, PISA) survey for the Grade 8 students (Mullis, Martin & Foy, 2008; Mullis, Martin, Foy & Arora, 2012; OECD, 2014), top in the PISA 2006 survey for the 15-year-old students (OECD, 2007), TIMSS 2000 report (Gonzales, et.al, 2000).

While these results may ‘paint a rosy picture’, Wu (2001) cautions that large-scale international studies do not always serve as the best tool for evaluating educational achievement. Further, one cannot completely assess students’ mathematical understanding by using standardized tests only (Richardson, 2003). Leung (2014) also reminds us of the need to address social or cultural background issues in order to interpret the strength of the achievements of students from various countries. This is geared towards avoiding a surface understanding of students’ achievements from large-scale international studies. However, it is sometimes quite difficult to find a clear relationship with those background or contextual issues and the students’ achievements, let alone a causal relationship (Leung, 2014). For example, in analysing the 2007 TIMSS survey, Leung (2014) found that there was no relationship between class size or parents’ educational background and students’ achievements. To cite another example to support Leung’s (2014) claims, among the top performing Asian countries, a high percentage of parents of students from Korea and Japan had more university degrees than the average of other countries, but not for parents from Hong Kong, Singapore or Taiwan.

Therefore, a country’s high performance on international surveys may not be the best indicator of good quality. Quality education includes providing opportunities for students to continually explore mathematics (Kilpatrick et al., 2001) and utilize knowledge (Franke et al., 2007; Lampert, 2001; Kilpatrick et al., 2001). This element of applying knowledge to new situations, argues Kickbusch (1996), is missing in traditional assessments of student performances. Since there is no guarantee that students with high scores on international comparison tests have a

good ability in applying their knowledge to new situations, it is important to further explore the mathematical competence of these students.

Besides the high achievement of Taiwanese students in international surveys, it was also found that Taiwanese teachers mainly focused on developing procedural understanding. This means that they taught rules/procedures but ignored conceptual understanding in problem solving (Taiwan Ministry of Education, 1992; “Examining Teaching of”, 1997). As a result, students lacked creativity (Stevenson & Stigler, 1992), and experienced problems such as a heavy study load and difficult content (see Table 1, p. 27).

Further examination of Taiwanese education has revealed that students are provided with a narrow kind of teaching. For example, in a typical junior high school setting where the direct instructional approach is used (Chi, 2000; Yu & Hang, 2009), the role of the student is like that of a follower. Students, rather than developing understanding through exploration, investigations or using problem solving strategies, mostly learn by copying the teacher’s problem solving methods. With regards to this, Boaler (2001, p.121) cautioned that:

... considerations of competency need to examine the ways in which students engage in different practices. Thus, it becomes important to engage students in opportunities to use and apply knowledge, not only because such opportunities may afford the development of deeper knowledge, but because students engage in practices that they will need to use elsewhere.

However, Boaler’s early PhD and later studies (2002b) failed to provide adequate comparisons of teaching approaches- traditional vs. constructivist. Hence, my work sought to add to the body of research in this area. It offered an opportunity to better understand the results from a long term teaching experiences (i.e. three years) on the influence on learning by using a constructivist approach. This study will therefore focus on the characteristics and influence of using two contrasting teaching approaches on student learning. Further, this study responds to calls for

research evidence from Taiwanese classrooms on the strength of using a teaching approach based on the implications of constructivism as it applies to learning and teaching mathematics (Wey, 2007) as well as students' competence (Chou, 2003a; Wei & Eisenhart, 2011), especially in a high school mathematics environment. It is therefore anticipated that the findings of this research may inspire all stakeholders about the value of using a constructivist approach to teaching.

1.3 Research Aim

The following research aim guided the present study:

Compared to traditional direct approaches to teaching mathematics, what benefits are there in using constructivist approaches in the mathematics classrooms of Taiwanese Junior high schools?

1.4 Definition and Standardization of Key Terms

Several terms used in this study are defined as follows:

Cram Schools: these are private organizations established to provide additional instruction to help students to pass national examinations (Chou & Ho, 2007).

Constructivist instruction: it does not specify a particular model of instruction (Greenes, 1995; Simon, 1995; Windschitl, 1999b), but a student-centre learning style. It aims at building up learners as skilled and thinking people (Hagg, 1991). Teachers minimize their direct instruction or lecture mode (Simon & Schifter, 1991), encourage and facilitate discussion (Brooks & Martin, 1999; Trotman, 1999; Windschitl, 1999b) and problem posing by students (Wheatley, 1991; Trotman, 1999) by creating a culture for inquiry (Windschitl, 1999b). Students learned through conducting their own approaches to problems (Lambdin & Walcott, 2007).

Resource books vs. text books

Resource books: all materials that can provide additional help in understanding the different mathematics concepts. These resource books are mainly used to help students become more successful at taking the school tests.

Text books: these are the required instructional material used by each grade level as specified by the Taiwanese Ministry of Education. These books, published by the Ministry of Education, are the only texts used in the classroom during the research.

Procedural vs. conceptual knowledge

Procedural knowledge: it refers to the formal or symbolic expression of mathematics (Haapasalo, 2003; Hursh, 2004), and includes rules and/or (problem solving) procedures (Haapasalo, 2003; Hiebert & Lindquist, 1990; Hursh, 2004; Skemp, 1976, 2006; Star, 2000) to carry out with routine mathematical tasks, and normally with automatic but not thoughtful reflection (Haapasalo, 2003). This knowledge could not be adapted into other situations (Alibali, 2005).

Conceptual knowledge: it is described as knowledge based on making meaningful connections and the usage of formula/algorithms among existing and new concepts or situations (Alibali, 2005; Haapasalo, 2003; Hursh, 2004; Skemp, 1976, 2006).

Problem solving: it is a cognitive exercise (Mayer, 2012) and refers to the seeking of solutions to problems through adapting mathematical formula or concepts (Bicknell, 2009). These solutions to problems may not be instantaneously apparent (Haylock & Thangata, 2007) nor straightforward. Problems could appear as entirely mathematical (for example: arithmetical or geometrical) or in some ways life-related in context (Haylock & Thangata, 2007).

Traditional teaching: teaching instruction carried out in Taiwan during the research period (Xu, 2004; Yu & Hang, 2009). It is based on elements of behaviourism (Wenger, 1998) and includes direct instruction (in term of didactic teaching (Boaler & Greeno, 2000)). The teaching strategies of direct instruction

emphasise on teachers' explanations of the content, primarily with a chalk-and-talk method (Zhang, 2002). This traditional teaching approach focuses mainly on the end product and prepares students to pass examinations (Wei & Eisenhart, 2011).

Class discussion: it is defined in this study as either the teacher encouraging students to discuss mathematical concepts, or one or two students coming to the front of the class to explain their mathematical concepts or problem solving, with opportunities available for the whole class to join discussions (Hunter, 2006b; Pontecorvo & Girardet, 1993; Rojas-Drummond & Zapata, 2004; O'Connor, 1998; Wood et al., 2006), in order to explore the students' own mathematical ideas along mathematical themes.

A sociocultural approach: it seeks to describe and explains relationships among the processes of learning and meaning-generating when participating in activities and environments of a sociocultural and historical context (Bell & Cowie, 2000; Bowers et al., 1999; Franke et al., 2007; Hanks, 1991; Lave & Wenger, 1991; O'Connor, 1998; Sfard, 1998; Voigt, 1994; Wertsch, del Rio & Alvarez, 1995).

School T (the traditional school or students in Tom's classroom), School E (the experimental school or students in Eve's and Ed's classrooms)

1.5 Overview of the thesis

This thesis is arranged into four main sections:

Section I: Overview and related literature

Chapter 1 has briefly set the context of this research and its rationale and aims. It also presents an outline of the structure of the thesis. This chapter is followed by the review of relevant literature, Chapter Two, where the focus is on mathematics education in Taiwan, theoretical models of pedagogy, learning theories, teaching styles, and a discussion on the need for quality mathematics.

Section II: Research design and data collection

Chapter 3 has provided the theoretical perspectives of the research framework. A description of the research design, research questions, nature of this research, methodology used, and discussion of analysis, reliability and validity of the data and data collection instruments are presented in Chapter 4.

Section III: Analysis and presentation of data

The research findings for the study are presented in the form of case studies of three teachers, their teaching practices, and the influences of the different instructional approaches on student learning. These cases are found in chapters 5, 6, and 7. Students' perspectives on mathematics, their interests and difficulties, the relationships between teachers and other colleagues, and three teachers' perceptions towards current mathematics education are discussed in Chapter 8.

Section IV: Conclusions and recommendations

A discussion of the research findings and comparisons of the findings with the literature are presented in chapter 9. Conclusions are drawn in chapter 10 from the present research in relation to the research questions. Additional discussions, recommendations and suggestions for further research are given in this final chapter.

Chapter Two: Literature Review

2.0 Introduction

Two long-term constructivist research projects with different teaching approaches at the high school level: one open project-based approach in the UK (Boaler, 1996), and the other a group work approach (Boaler & Staples, 2008), both indicated a sound impact on students' learning with better mathematics performance/competence, and that group work approaches benefited students' positive learning attitudes towards mathematics, when compared with the schools using traditional teaching. A concern of this study is whether the learning influences of using constructivist approaches to teaching over a long period, as experienced by Western/English speaking countries, could be reproduced in an Asian country: Taiwan. This concern is augmented by the challenges experienced by Taiwanese students and teachers of highly competitive testing (Chi, 2000; Jarvis et al., 1998), and a full mathematics curriculum (e.g., Hsieh, Huang, Shin & Li, 1996; Leung & Park, 2002). Moreover, these concerns served as the impetus for the present investigation that focused on the use of constructivist teaching approaches like class discussion, to enhance Taiwanese students' mathematical thinking and teachers' instructional practices. This chapter examines literature on mathematics education in Taiwan, theoretical models of pedagogy (e.g., teaching styles), and the relationship between teaching practices and student learning.

2.1 Taiwanese education

The educational system in Taiwan with a curriculum oriented focus is directed and supervised by the Taiwanese Ministry of Education (Taiwan Ministry of Education, 2014a). As far back in the early 80s a 6-3-3-4 system was applied in students' schooling (Chou & Ho, 2007; Kimbell, 1997; Lin, 1988). This system simply refers to i) six years at the primary level beginning at age six, ii) three years at the junior high level, iii) three years at the senior high level, and iv) normally four years at the university level (Kimbell, 1997; Lin, 1988). Students are required to attend both the primary and junior high schools, where there is free tuition (Chang, 1984). In Taiwan, Mandarin is the chief language of instruction in

schools. When the 2001 curriculum was implemented, English became a second language in primary schools (Chou & Ho, 2007).

Education is highly valued by parents and the society (Chou & Ho, 2007; Wei & Eisenhart, 2011); as such, schooling is therefore central to Taiwanese students' lives (Wei & Eisenhart, 2011). This is evident by the very long hours Taiwanese students spend in school (Wang, 2010). For example, the number of high school days in the classroom is about 200 days which is longer than the average 180 school days in the USA (Chou & Ho, 2007). Students at the Junior High public school attend an average of over 8 hours per day in class, with lessons lasting for 45 minutes. Most schools require students to attend one extra hour at school each day (Chou & Ho, 2007). Therefore some students may spend up to 9 hours per day in class. Students normally arrived to school at 7: 30 a.m. or earlier according to schools (Chou & Ho, 2007; Wei & Eisenhart, 2011). In this study the traditional participating school had the same schedule. The schedule for the experimental participating school was similar, but did not have the extra learning class; they began classes at 7:40 am and each class time took 50 minutes.

In addition to the length of the instructional day or a lesson, one must also consider the classroom learning environments. According to Wei and Eisenhart (2011, p.74), since no scientific evidence exists to indicate that American children are less intelligent than their Taiwanese counterparts, "the key difference must be the mathematical learning environment". Researchers have described Taiwanese classrooms as very conservative, where students mostly sat quietly and listened to teachers' instruction (Kimbell, 1997). It is within such environments that the mathematics curriculum is delivered.

2.1.1 Taiwanese mathematics curricula

The focus of Taiwanese mathematics education tends to change with time. Initially the focus was to train mathematicians, however, after the 1990s it changed to focus on serving most people. The mathematics curriculum went through a period of simplification and reformation by the Ministry of Education (Chen, 1998a). This change did not last for long because after 2003, the

curriculum focus changed back to what it was traditionally (Chung, 2003b). The details will be discussed in section 2.1.6.

In the early 90s the Taiwanese Ministry of Education appointed an expert committee to examine the national curriculum including how its content and goals would benefit students' success and future lives in the society (Chou & Ho, 2007). An examination of the national curriculum (i.e. for elementary and junior high schools) reveals that it does not provide detailed information on how to teach a lesson, but rather it presents suggestions to guide lesson planning (Kimbell, 1997). Teachers have the freedom to generate their own lessons as long as they are aligned with the focus of the curriculum. The Ministry of Education licensed supervisors to regularly visit schools to ensure that they confirm to the curriculum focus (Kimbell, 1997). Consequently, a centralized control arose that resulted in uniformity of the scope and sequence of the curricula, and the use of similar teaching approaches (Stevenson & Stigler, 1992).

To further compound the issue of control, the Ministry of Education exerted force to ensure that all schools adhered to the national standards and used the textbooks that are written and published under the supervision of the Ministry (Chou & Ho, 2007). All students attending primary and junior high schools used the same textbooks until 1996 (Guo, 2004). Only the senior high schools were allowed by the Ministry to choose their own textbooks. Since 1996, a new textbook system has been implemented where primary and junior high schools can now choose their textbooks (Chou & Ho, 2007). In 2001, the Ministry of Education discontinued the practice of producing the official textbooks for all subjects in junior high schools (Guo, 2004). Presently, schools are allowed to select textbooks approved by the government, from different local publishers.

This tight control that was exerted by the Ministry of Education has been criticized by many teachers. In a study by Hsieh et al. (1996), researching 6600 junior and senior teachers, most participants complained about the over loaded mathematics curriculum. This, they claimed, led to some students needing extra help. The same problem was earlier identified by Lo (1994) who stated that the

majority of teachers were against using a single uniformed content for teaching all students with different learning abilities.

The mathematics curricula of junior high schools in Taiwan still exhibit evidence of a strong teacher-centred approach, which is different from the 2000 to 2002 curriculum changes (Xu, 2004) that lasted until 2004. This 2000 –2002 curriculum was seen by some mathematicians as a constructivist-based mathematics curriculum (Wey, 2007). Although a constructivist focus informed the reform goals, the reality told a different story. Examination of mathematics teaching in Taiwanese classrooms revealed that these constructivist approaches to the curriculum had little influences on classroom practices (Xu, 2004; Yu & Hang, 2009) (see section 2.1.6(a)). To understand the influences of teaching approaches upon student learning, one must also examine teachers' attitudes towards the use of these instructional approaches.

2.1.2 Teacher attitudes

Teaching is a much respected occupation in Taiwan (Kimbell, 1997). Teachers are respected by students and parents. It was the norm for students to obey teachers in school (Kimbell, 1997). However, teachers' position of respect is gradually shifting in Taiwanese schools and society (Chi, 1999).

Teachers' attitudes towards the teaching and learning of mathematics play a vital role in the student learning environment and determine the level of student engagement and interactions (Wei, 2005). Most Taiwanese teachers perceive the traditional direct teaching as being an easier approach, especially when the focus is on the correct response (Wei & Eisenhart, 2011). Hence, their attitude would be more favorable to maintaining that format of teaching. Also, opponents of a constructivist approach to teaching have contended that this approach “should only be used for children with developmental delays” (Wei & Eisenhart, 2011, p.76).

Research on mathematical experiences and practices in Taiwan have described instruction as drills and repetition of skills (Huang, 2010; Wei, 2005). It also includes

memorization of rules and procedures (Leung, 2014; Wei & Eisenhart, 2011) without raising questions (Wei & Eisenhart, 2011). Teachers operating in this learning environment believed that by providing opportunities for students to develop mathematical skills, they were providing a basis for creativity. However, Wei and Eisenhart (2011) cautioned such teachers to reconsider their attitudes because a focus on developing mathematical skills void of conceptual understanding may lead to mathematics instruction that is “rigid and often boring”. In contrast, Western educators believe that students’ conceptual understandings should be developed before working on rules and procedures (Biggs, 1996). One may use the term “product versus process” to aptly sum up the comparison between the philosophy of East Asian and the Western mathematics education (Leung, 2001, p.35). Other polarizing terms are relational vs. instrumental (Skemp, 1976, 2006), or conceptual vs. procedure-oriented approaches (Boaler & Staples, 2008; Schoenfeld, 2002).

2.1.3 Current teaching approaches and learning in Taiwan schools

Traditionally, most Taiwanese mathematics teachers have adopted the teaching method of “chalk and talk” (Xu, 2004; Yu & Hang, 2009). Lecturing is still the major means of delivering instruction in Taiwanese mathematics classrooms (Yu & Hang, 2009). Most teachers focus on delivering the content, especially towards preparation for internal/external examinations (Wei & Eisenhart, 2011). Consequently, teachers emphasize the skills of memorization (Wong, 1993; Wei & Eisenhart, 2011) and repetitive practice (Fang & Chung, 2005). These above mentioned teaching emphases are consistent with the cultural values in educational fields from Asian countries of Confucian heritage (Leung, 2014). Moreover, these characteristics of the traditional approach to teaching are also highlighted in western students’ feedback on the so called “didactic” approaches in Boaler and Greeno’s research (2000, p.189).

Research conducted by Wong in 1993 revealed that so called “successful” teaching patterns by Taiwanese junior high teachers included: more lecturing time, less individualized work, and skills memorization. Wong’s (1993) work also noted the high reliance on ‘chalk and talk’ and the encouragement of memorization as

successful teaching patterns in Taiwan. Lo (1994) asserted that teachers used the “chalk and talk” method to explain problem solving for the entire class. Further, problems were solved on the blackboard or individually. Instruction was made up of teachers assigning and correcting homework and emphasising repeated practice (Yu & Hang, 2009).

In addition, many Taiwanese teachers viewed the prominent role of ‘chalk and talk’ and the rapid delivery of lectures as the most efficient way to receive good test results (Yoong, 1992). Many viewed the strong reliance on teaching algorithms to students as a direct result of insufficient time to meet the needs of each student in the large classrooms (Wei & Eisenhart, 2011). Consequently, teaching algorithms, with an emphasis on developing test-taking techniques, leaves no room for using alternative instructional methods (Chang, 1984). Similar “procedural” instruction also appears to be the delivery mode of instruction in Hong Kong and Korea, where teachers are required to closely follow the full curriculum and assigned textbooks (Leung & Park, 2002, p.128).

Examination pressures, as reported in other countries (Silver, 1992), affect the content of teaching in Taiwan (Wei & Eisenhart, 2011). School curricula are driven to help students to succeed in examinations (Chou & Ho, 2007). Thus, a fast teaching speed is assumed to be needed to cover the textbook and algorithms are used to help teachers cover the syllabus quickly (Chang, 1984). Formal teaching algorithms are more likely to be applied in tests and to elevate students’ scores (Lin, 1988). It is commonly believed that teachers’ failure to correctly judge their teaching time to cover the textbooks will adversely affect students’ achievement in mathematics. Gau (1997) conducted a study investigating 9702 Grade 8 students from 446 Taiwan junior high schools, and discovered that the coverage of textbook content was directly related to student achievement; that is, the more teachers covered textbook content, the higher students’ mathematics achievement was. Hence, the over loaded mathematics curriculum and competitive entrance examinations appear to affect and guide the teaching (Huang, 1994), and affect mathematics reforms (Wei & Eisenhart, 2011).

2.1.4 Examinations and assessment

The selection of candidates to senior high schools and universities, based on test performance, is supervised by the Taiwanese Ministry of Education (Kimbell, 1997). The availability of few spaces in these institutions has led to high levels of competition which have put pressure on teachers and students in the past (Jarvis et al., 1998). For instance, in 1997 of the three quarters of junior high school students who took the senior high school entrance examination, only about 40% were selected due to availability of places; the rest were placed at vocational schools. The entrance examination contains five test subjects: Chinese, mathematics, English, natural science, and social studies (Kimbell, 1997). In recent years, the acceptance rate for the universities has remained high (over 80% since 2002, about/over 94% since 2007) (Educational Department of Statistics in Taiwan, 2010; Zhang & Liu, 2010). Although the number of universities has increased in the education market (Chou, 2003a), the stress placed upon senior high school students to gain entrance into universities has not been reduced (Chou & Ho, 2007; Wang, 2010).

The examination system has undergone continuous reform. With regards to this, in 1995, there was an alternative option to select students to the tertiary study level besides from the entrance examination performance; for example, using school transcripts or teachers' recommendations to hand in applications (Kimbell, 1997). Since 2001 the National Entrance Examination for Senior High school (The committee of the Basic Competence Test for Junior high School Students, 2010) has increased its offering to twice a year in May and July. If students are satisfied with their results on the first attempt at taking the test, they can apply for a Senior High school placement. However, if they are not satisfied with the results or failed to enter their ideal Senior High school, students may have a second try. At present, the entrance examination for university has an additional feature; a Student Subject Ability Test administered around February.

In making these changes, policy makers expected a reduction in the study stress level; however, it worsened during the reform period -1996 -2004 (Chou, 2003a; Chou & Ho, 2007), and continued to increase even after the reform period ("The

Consequences Appear from”, 2008). This study’s stress factor may be due partly to the fact that junior high school students are still frequently tested. According to statistics, 42% of students reported daily testing while 68% reported having tests at least three times per week (The Humanistic Education Foundation in the 2010 National Survey, 2010). Further, many students and/or parents still felt the need for cram schools after a normal school day (Chou & Ho, 2007), or for students to stay at school for self-study classes. Self-study classes were approved by the Ministry of Education. Normally, Grade 9 students are permitted to remain until 9 p.m. in school for self-study. To meet the high quantity of requests for cram schools, four times the number of existing cram schools in 1999 were established in 2008 (Chou 2008). Taiwanese educators in an effort to reduce the study stress occurrence have continued to re-examine their examination policies. The 2011 examination policy now has provision for students, and that 70% of students will not need to sit the national examination for entrance to senior high schools (Ye, 2011).

Since 2014 the Ministry of Education has established a new policy of 12 years of national basic education. This new policy is expected to reduce study pressure, improve student learning and enhance the competence of students. This policy was also established in conjunction with a change to the national examination system for Senior high schools. The National Entrance Examination only occurs once every year in May. The achievements of students are divided mainly into three levels instead of by scores (Taiwan Ministry of Education, 2014b). The influences of this new system, along with other factors impinging on student learning, such as the role of homework need further evaluations.

2.1.5 Homework

The issue of the amount of time spent on homework is debatable. According to research, the optimal time for middle school students should not exceed one hour per night while high school ranged between 1.5 to 2.5 hours per night (Center for Public Education, 2007a). Clemmitt (2007) argued that this information must be considered in light of one’s cultural expectations and definition of homework.

According to Lapointe et al. (1992b), in the USA, students expended one or less hour per week for their mathematics homework. This is not the case in Taiwan where less than 25% of 13 year old students used at least 4 or more hours per week for their mathematics homework and more than 25% of students used 2 to 3 hours. Nearly 50% of students took one hour or less to finish homework (Lapointe et al., 1992b). Homework is given throughout the school year and extends to holidays. Assigning homework during holidays is meant to keep students working on academic pursuits. This becomes a challenge when one considers the heavy schedule of students, such as attending: a normal 8 hour school day, one extra 45 minutes school class, and attending a cram school; very little time if any, is left for Taiwanese students to fit in doing homework.

The relationship between the amount of homework and time spent on homework was examined by Cooper (2007), director of Duke University Education program. Of the 35 studies examined, 77 percent cited a positive relationship between the two factors (DeNisco, 2013). Evidence also exist that support movements towards reducing the time spent on homework, by addressing issues of quality rather than quantity of homework as a means of improving academic performance (Center for Public Education, 2007a; McPherson, 2005). Such a move will affect the belief within the Chinese culture that repetitive practice benefits understanding (Biggs, 1996; Watkins, 1996), and in Asian beliefs, that memorisation enhances understanding (Chalmers & Volet, 1997). Repetitive practice must not be viewed as rote learning that is absent of understanding (Biggs, 1994; Leung, 2014). Linver, Brooks-Gunn, and Roth (2005) argued that memorization should not be the objective for giving homework; rather the focus should be on providing creative and challenging tasks for reviewing the content. This suggestion will require changes in educational practices.

2.1.6 Educational Reform

(a) Taiwanese Experiences

Discussion on educational reforms in Taiwan involves two main points of views about how students learn. Primarily, this discussion will look at the experiences of adopting a traditional approach or constructivist approach to teaching and learning

mathematics (see sections 2.2.2). Beginning in 1993, a constructivist approach to teaching mathematics was introduced in *the Curriculum Standards for Elementary School Mathematics in Taiwan* (Taiwan Ministry of Education, 1993), and implemented from 1996 to 2004 (Guo, 2004). The new approach required teachers to pay more attention to how students learn, and placed more emphasis on the development of student conceptual understanding. This was in contrast to the traditional teaching which focused on getting the right answer (Wei & Eisenhart, 2011). Aspects of constructivism were evident in the 1996 mathematics curriculum for elementary schools wherein a student-centred approach to learning was introduced (Chen, 1998a). Some concepts of constructivism were interwoven into the mathematics textbooks (Guo, 2004), and all curricula appeared to reflect the constructivist teaching styles within different grades (Zheng & Wang, 2004). Teachers were encouraged to embrace and teach students using the constructivist teaching styles (Guo, 2004). However, this teaching change did not occur much at the junior high school level (Xu, 2004; Yu & Hang, 2009). The public of Taiwan viewed the 1996 – 2004 education reforms conducted in primary schools by the government of Taiwan as being not very successful (e.g., Chung, 2005; Guo, 2004; Wey, 2007).

With the exception of the temporary Grade 1-9 Curriculum Guidelines introduced from 2000 to 2004, an examination of the curricula for junior high schools still appeared to have a strong teacher-centred approach to teaching and learning (Xu, 2004). Compared to the 1995 mathematics curriculum, the 2001 Curriculum Guidelines for the junior high school level appeared easier than the previous curriculum (Chung, 2005). The goals of the 2001 curriculum emphasize the development of students' competence, including problem solving, analysing abilities, communication and appreciation of mathematics (Yang, 2003). This 2001 curriculum was perceived by some mathematicians and Wey (2007) as a constructivist-based mathematics curriculum. Although many teachers and parents welcomed the education reform, there appeared to be many difficulties and confusion for teachers and parents at the junior high (Xu, 2003), and primary level schooling (Chou & Ho, 2007). For example, due to minimal professional development opportunities provided by the government, many teachers were inadequately prepared to implement a constructivist-based mathematics

curriculum (Chou & Ho, 2007; “Critique from constructivist”, 2006; Wey, 2007). Thus, the constructivist approaches as applied to teaching and learning have been somewhat utilized at the primary school level, but not at the junior high school level. This may be due to either a lack of constructivist-based knowledge by Junior high school teachers to meet the expectations of the 2000 curriculum (Chen, 2003b), or confusion and challenges in applying the new focus in their classrooms (Xu, 2003). At the end of 2004, most teachers were found to be still using the traditional direct teaching approach (Xu, 2004; Yu & Hang, 2009) to apply the new focus of the textbooks. According to Xu and Chung (2004), in their research on primary school teachers who used the constructivist approach to teaching, the inclusion of discussion time during instruction was time consuming, and hence the coverage of content suffered. Thus many teachers reverted to traditional methods of teaching mathematics.

Kilpatrick et al. (2001) cautioned against laying the blame solely on teachers. Some scholars (Borko, 2004; Kilpatrick et al., 2001) felt that sufficient time was not provided for teachers to gain enough knowledge or practice about teaching from the new educational focus. Thus, they suggested that time should have been set aside from teachers’ daily work load to obtain such understanding about the policy changes and to implement it. Liu (2004) argued that it was very short sighted of stakeholders to expect teachers to change their practices, since i) they were not informed as change agents; and ii) changes towards a constructivist approach only occurred in the curriculum guide or textbooks. Therefore, like Kilpatrick et al. (2001), Liu (2004) identified the lack of fully preparing teachers to own this change as the critical factor which led to the demise of using a constructivist-based mathematics curriculum in Taiwan.

The influences of these educational reforms are still debated in the Taiwanese society (Chou & Ho, 2007). The change to a constructivist-based curriculum had its success and challenges. Some researchers claimed that the constructivist approaches benefit students’ thinking in mathematics (Chung, 2003a). Some teachers have commented that the new approaches to teaching and learning mathematics, especially the inclusion of class discussions, have led to students developing their own problem-solving strategies and thereby resulting in

meaningful learning, improved attitudes towards learning mathematics, interest in the subject, and self-confidence as problem solvers (Chung, 2005). Others, opponents to the reform change, have complained that students were forced to learn many alternative ways of problem solving in the reform period in contrast with the past when only one way of problem solving was learnt or provided (Fu, 2008).

Further empirical evidence from Taiwanese primary mathematics classrooms in support of the constructivist teaching, indicated that students were more able to (i) voice their opinions; (ii) ask teachers questions; (iii) cooperate with others and learn to appreciate other ways of thinking than in the traditional approach (Chung, 1997b, cited in Chen, 1998a, p.91), and (iv) the constructivist approach effectively reduced students' mathematics anxiety (Chen, 1998a).

Most primary teachers in Chen's (2007) study mentioned that students' mathematical reasoning and conceptual knowledge were enhanced by using a constructivist mathematics curriculum, but not students' overall mathematics achievement. One short-term (i.e., two months) investigation in Grade 7 mathematics classrooms found that students in the constructivist classrooms had better mathematical motivation, classroom atmosphere, and mathematical achievement in tests (however not on school examinations) than those in traditional classrooms (Yeh, 1998).

The role of the public in educational reforms should not be underestimated. The Taiwanese public was very critical of the new educational thrust, i.e., using a constructivist approach to teaching and learning. They felt that the change caused students to (i) perform low in calculation abilities (Chen, 2003a; Chou, 2003b; "Critique from constructivist", 2006; Wey, 2007); and (ii) be unable to use efficient methods to solve problems (Chung, 1997a, 2005). As a result, reliance on students' problem solving strategies may lead to them making mistakes (Chung, 2005).

Parents too were affected; they were uncertain about the education reforms after 1994. They worried over the simplicity of the content; primarily they felt that the easy mathematics content would decrease students' mathematical abilities (Liu, 2004). As such, many parents sent their children to cram schools for extra tutoring. More homework and textbooks were loaded onto students than in the pre-reform period (Chou & Ho, 2007)

Quite a few primary school teachers misinterpreted the elements of a constructivist-based curriculum, and prohibited students from memorizing multiplication tables. They followed only the complicated problem-solving methods presented in the new textbooks (Fu, 2008; Guo, 2004; Wey, 2007). The use of this approach was seen as making students inefficient when speed was required for problem solving (Chou, 2003a; Guo, 2004; Xu, 2003). Also, failure to use the problem-solving methods from their textbooks resulted in students not receiving any marks on the test (Cai, 2002; Lin, 2002a). Such complaints by parents led to the Ministry of Education in Taiwan redirecting teachers to allow the use of memorization, especially when learning the multiplication tables ("Critique from constructivist", 2006). Added to these concerns was the fact that some teachers did not regard students' problem-solving methods as being well developed. Rather than spending the time to provide opportunities for students to explore and develop higher order thinking skills, teachers complained that it was too time consuming to allow students' reasoning to develop. They also doubted whether students had the ability to discuss or present their thinking, and thus preferred the traditional or direct teaching to achieve efficiency (Chung, 2005).

Augmenting this challenge of implementing the new educational reform was the removal of several units from the primary textbooks (e.g., the calculations of fractions); and also from the 2000 junior high school curriculum (Guo, 2004). This removal led to a disconnection in the mathematics curriculum between the junior high level (Chou, 2003b; Guo, 2004), and later at the senior level (Chen, 2005). Consequently, in 2002 the Ministry of Education in Taiwan urgently requested primary schools and junior high schools to offer extra hours of mathematics lessons during the summer break for graduating students. The same situation happened in 2005 when Senior high schools were required to offer 18

extra lessons in each of the two semesters, to help bring Grade 10 students who had received the reform curriculum up to grade level expectations (Chen, 2005). Compared to students under the 2004 curriculum, students in the reform period had missed 8 mathematics units (Han & Jiang, 2005). Moreover, when compared to the 1997 California mathematics curriculum, the 2001 Taiwanese mathematics curriculum was found to be one or two years behind (Zheng & Wang, 2004).

It is within this setting that many parents became negative about the new approach to teaching (Chou & Ho, 2007; Qiu, 2002; Sun & Cai, 2002). Some primary school teachers were left in an indecisive position (Sun & Cai, 2002; Zhuang, 2002) where they felt that, using the new approaches might upset parents and confuse students; however if they did not use the advocated new approaches, it would be ignoring the policies of the Ministry of Education (Zhuang, 2002).

Debates about education reform in Taiwan have been going on from 1999 to the present with varying views (Chou & Ho, 2007); it is no surprise that the Ministry of Education replaced the constructivist-based mathematics curriculum in 2003 with a new approach. It is the view that the group of mathematicians who were given the task to establish a new national curriculum were opponents of the constructivism approach (Wey, 2007). This new curriculum shifted from learner centred to knowledge centred (Chung, 2005; Leung, 2011), and valued knowledge and students' calculation abilities (Chung, 2005; Wei & Eisenhart, 2011; Yang, 2003). Thus there appeared to be a backward movement as the new approach was similar to the 1978 curriculum focus (Chung, 2003b).

Despite the evidence from the large scale research conducted by the National Science Council in Taiwan in 2003 and 2004 (Li, 2003a; 2004), to investigate the influences on students' mathematics achievement of the constructivist curriculum, the new 2003 reform in mathematics were ushered in. The 2003 mathematics curriculum placed a high value on students' calculation abilities, as well as the connection to the senior high schools (Yang, 2003; Chung, 2003b, 2005). The curriculum differed from the 2000 mathematics curriculum by moving away from a focus on developing students' abilities, attitudes, and thinking and creative

power (Chung, 2003b), to one which added more content at the primary level (Chen, 2003c; Guo, 2004).

Further research findings of national studies pointed to the benefits of using a constructivist approach. Li (2003a, 2004) reported a decrease in the number of low achieving students in Grade 4 and Grade 8 who had been through the 2001 new curriculum. Moreover, there was an increase in the thinking and logical reasoning abilities of Grade 8 students, who had been through the 2000 curriculum and the constructivist-based curriculum at the primary level (Li, 2004).

Given the findings from different local studies, and a strong theoretical basis for implementing a constructivist approach to teaching and learning, the new shift in curriculum focus was heavily debated. The national government department of Taiwan, Control Yuan (a Government department), warned the Ministry of Education about rushing to enact policies in education reform. However, it may be argued that many Taiwanese stakeholders were caught up in trying to maintain the academic achievement status (DeNisco, 2013; Wei & Eisenhart, 2011). If this was their central goal, then it was no wonder that certain inherent weaknesses in the traditional approach were overlooked (Huang, 2004).

To further increase its centralized hold on education, in 2005 the Ministry of Education resumed producing textbooks for both primary and junior high schools (Guo, 2004). Thus, the reform of the mathematics curriculum in Taiwan in the last ten years has been like a pendulum (Chung, 2005; Wey, 2007), moving from difficult content to simple, then from simple content back to difficult (Wey, 2007). In more recent times, there has been a change in Taiwanese parents' views about the use of multiple problem-solving approaches. According to Huang (2010), 92.3% of 2051 parents supported "a multi-methods approach to instruction to spark children's interest in learning mathematics". Similar pendulum movements with a different mathematical focus have also happened in the past century in the USA (Lambdin & Walcott, 2007).

International performance

International research has shown that Taiwanese students continue to perform in the top five positions when compared to similar grades in developed countries. For example, Taiwan was ranked top of all countries in the PISA 2006 survey for the 15-year-old students (OECD, 2007), top of all countries in the Trends in Mathematics and Science Study (TIMSS 2007) report (Mullis et al., 2008), and fourth in the TIMSS's 2003 reports (Mullis et al., 2004) for Grade 8 students (Mullis et al., 2008). However, students' performance decreased and Taiwan was ranked fifth of all countries in the PISA 2009 survey (Lin, 2010; OECD, 2010).

It is noteworthy that although high mathematics achievement occurred in the international comparison studies, the disposition scores of Taiwan Grade 8 students were still below the international average or even near the bottom among all countries in the last ten years (TIMSS 2007, 2011). This was seen in the 2007 TIMSS report in the areas of students' positive affect (students' interest in learning mathematics), of students valuing mathematics and of students' self-confidence (Mullis et al., 2008), as well as in the 2003 and 1999 TIMSS reports which found low percentages of students in the high self-confidence category (Mullis et al., 2004).

Besides these challenges and issues there were other problems in the Taiwanese educational system, especially at the junior high level. Table 1 highlights some problems prior to the reform curriculum.

Table 1 Problems in Taiwanese mathematics education

Key issue	Events
Instructional approach	Teachers emphasized procedures and rules teaching, but neglected students' understanding (Taiwan Ministry of Education, 1992; "Examining Teaching of", 1997).
Mathematical Content	• Content was too difficult ("Examining Teaching of", 1997). • Parents had difficulties to understand all of their primary school children's mathematics homework in the reform periods ("Examining Teaching of", 1997). • There were only top (streamed) students in junior high schools, who understood all the mathematics content, but the others had difficulties in understanding even basic concepts. The situation was worse in senior high schools. The top students improved, but low ability students performed worse ("Examining Teaching of",

	1997).
Academic Achievement vs. Creativity	Many complaints to critique Chinese with high mathematics achievement but short of creativity (Stevenson & Stigler, 1992).
% dislike math	- About 33% of primary school students and 46% of junior high school students chose mathematics as the least liked subject (Taiwan Ministry of Education, 1992). - The higher the grade of students the higher was the percentage of students who disliked mathematics (Chen, 1998a; Mullis et al., 2008; Taiwan Ministry of Education, 1992).
Cram school (private organization for helping to pass examinations) (Chou & Ho, 2007)	Many junior high school students attended various cram schools. This laid a heavy study burden on them (Chi, 2000; Huang, 1996; Hsiao, 1994).
The Taiwan centralised curriculum	- Although the curriculum had the same content, it could not cater for students of different abilities; low ability students were marginalized (Lin, 1988). - Lack of individual attention for students. This was due to un-streamed classes with great numbers of students (Wong, 1996). - Low ability students remained passive; they just sat quietly and waited until the lesson finished (Wong, 1996).

The evidence as presented in Table 1 shed some light on the competitive and intensive nature of Taiwanese mathematics education. This is apparent in several areas, including: the mathematical content, long study hours, teachers' attitudes, instructional approach and assessment practices.

Some attempts try to link recent students' excellent mathematics performances in the international surveys with the 1996-2004 education reform of the constructivist-based curriculum (Lin, 2008; Lin & Huang, 2008). For example, the vice leader (Lin, 2008) and previous leader of Ministry of Education (Lin & Huang, 2008) both regarded students' excellent performance on the 2007 TIMSS's report as being a direct result of the most recent education reform. Based on this performance, they encouraged students and parents not to be anxious about the shift from the previous reform. However, teachers felt that there was not any hard evidence to link mathematics success in the PISA 2006 survey with the constructivist-based curriculum or to the recent education reform (Wey, 2007). Conversely, curriculum reform usually does not change classroom practices, for example, the American experiences in the last century (Ball, 2003; Franke et al., 2007; Hiebert et al., 2003; Labaree, 1999; Webb et al., 2006), show that the traditional pedagogies still governed mathematics classrooms and students still

spent most of their time practicing procedures (Boaler & Greeno, 2000; Kilpatrick et al., 2001). Although the recent curriculum (1993-2002) was regarded as a constructivist-based mathematics curriculum (Chung, 2005; Wey, 2007), this curriculum did not change classroom practice into constructivist teaching in Taiwan, due to a lack of teacher development (Wey, 2007). It still is a very complex task to explain the influence of the constructivism based curriculum on students' learning in Taiwan.

Confusion rose up in the public from different theoretical views expressed about constructivism from two groups of Taiwanese scholars. Mathematics educators supported the concepts of constructivism in the curriculum, but mathematicians abandoned it (Wey, 2007); a similar situation to the American mathematics wars (Boaler, 2002c; Liu, 2004).

However, without acceptable local education research evidence, it is hard to validate the arguments from each side (Wey, 2007). A call for research evidence from Taiwanese classroom experiences has been made (Wey, 2007). This shows the importance of this study in answering the recent needs in the education field of Taiwan about the long term influences of constructivist teaching on students' learning of mathematics.

(b) International research experiences

Several researchers have examined traditional and new mathematical teaching to improve mathematical abilities (Franke et al., 2007). Some sound research evidence in different countries have indicated that conceptually oriented mathematics curricula have provided higher and more equitable results than procedure-oriented approaches (e.g., Boaler & Staples, 2008; Briars & Resnick, 2000; Pesek & Kirshner, 2000; Schoenfeld, 2002). Specifically, some studies also indicated the benefits of reform-oriented approaches where they have helped students to apply their knowledge in new situations (Boaler, 1997, 2000b, 2002b; Carpenter et al., 1998; Lamon, 2007), and to increase mathematical reasoning (Corbett & Wilson, 1995) and flexibility (creativity) in problem solving (Lamon, 2007; Pesek & Kirshner, 2000). The higher levels of performance (Lamon, 2007;

Pesek & Kirshner, 2000) and standard examinations (Boaler, 1997; Briars & Resnick, 2000) of new approaches compared to traditional approaches have been researched. Further discussions of these above mentioned studies are stated in this section and small sections of (d) relevant studies and (e) long-term research in the Section 2.2.2.

At the international level, researchers of primary school level mathematics have argued that more practising of problem solving (i.e., procedures) does not benefit or enhance students' understanding (Franke et al., 2007; Carpenter et al., 1998; Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996). Rather, the focus should be on quality, such as spending time on fewer problems but with a deeper level of investigative thought and thinking (Franke et al., 2007).

Briars and Resnick (2000) researched three years of primary school students' achievements, and revealed that students performed better in strong reform approaches than weak reform approaches e.g. in problem solving and conceptual understanding (Schoenfeld, 2002). Another short term investigative study conducted by Pesek and Kirshner (2000), operated for several weeks in six fifth-grade mathematical classrooms where they compared two contrasting teaching approaches: pure conceptual instructions (3 days) vs. a mixed instructions (procedural development first 5 days, then conceptual instructions (3 days). The pure conceptual instruction classes outperformed the mixed instruction classes (Pesek & Kirshner, 2000). These findings are important to this study since they provide further insight into possible teaching approaches that may be used to enhance the mathematical competence of students.

(c) The backward movement

Over the last thirty years, research has been conducted that will continue to accumulate the knowledge to guide teaching for understanding (Ball & Bass, 2000b; Boaler, 1997; Boaler & Staples, 2008; Malara & Zan, 2008; Jacob & Akers, 2000). Despite the fact that many researched advantages of the constructivist classrooms have been explored, educational development seems to have contributed towards a polarized position (Leung, 2001). The teaching styles in

much of the world tend to be moving back (i.e. to some extent) towards some of the traditional teaching styles. Even when the teachers were using conceptually rich curricula in the U.S.A. (Ball, 2003; Franke et al., 2007), and attempted to teach based on the NCTM standards in the U.S.A. (Franke et al., 2007; Hiebert & Stigler, 2000), or a constructivist-based mathematics curriculum (Wey, 2007), and followed constructivist mathematics textbooks as in Taiwan (Guo, 2004), many existing classroom mathematics practices were still inconsistent with the constructivist reform (Ball, 2003; Franke et al., 2007; Wey, 2007).

In the United Kingdom, many university educationists espouse a reform and open form of thinking-based mathematics but the government has promoted a ‘back-to-basics’ policy, to pressure schools to move school mathematics back to closed approaches (Boaler, 1996). The primary result of California’s ‘back-to-basics’ push has brought back the instructional approaches that were common in the United States during the 1980s (Jacob, 2001). The “standards-based” policies in the USA drew attention back on students’ best achievements in the traditional measurement when aligned with new instructional approaches (Briars & Resnick, 2000). However, there are limitations to the traditional uniform assessment to evaluate students’ mathematical capability (Richardson, 2003). Some mathematical abilities cannot be assessed using the traditional assessment methods. This inability to assess the totality of a student’s academic ability might result in important mathematical competencies (for example, abilities in mathematical argument, creative mathematical ideas, or applying knowledge in new situations) being valued less in classrooms. Thus, the assessment accountability focus in the USA has been criticized for its role in driving classroom practices to facilitate the traditional standardized assessment examination purposes (Lambdin & Walcott, 2007). According to these authors, this focus drove down the quality of the curriculum and teaching.

Opportunities for studying the influences of constructivist-based reforms on teaching and learning appear to be limited. The open school studied by Boaler (1996), has closed in the UK and the Taiwanese alternative experimental school initiative conducted in junior high schools was discontinued in 2003. Such closures only increase the need for further opportunities to examine the

continuous learning influences of this reformed work. In summary, mathematics educational dilemmas with a knowledge centred focus appeared in (i) the maintenance of convention teaching practices in reform movement, (ii) the changes back to knowledge curriculum focus and (iii) the closure of reform schools. The backward movement of the educational reform pendulum towards a knowledge centre focus as in the past, has resulted in a shift away from learner centred to knowledge centred teaching in countries, such as Taiwan (Chung, 2005), and to some extent in the USA (Lambdin & Walcott, 2007; Weng, 2003). The backward mathematics educational dilemma could be improved when new light is shed (Sfard, 2003), especially from reformed-oriented research (Franke et al., 2007), that may introduce new definitions of students' competence/knowledge/understanding, or influence/generate new learning theories/pedagogy. New insight into students' competencies/capabilities or reformed research needs to be sought so that the public and mathematics educators are better informed about what quality of mathematics education is needed, with suggested ways to achieve it.

The following section examines the relevant literature for theoretical pedagogical models in mathematics education.

2.2 Teaching styles

Teaching is not only combinations of the teachers' own behaviors/arrangements in classrooms, but also a connection of many intertwining relationships among teachers, students, and the mathematics content in classroom instruction (Boaler, 2002c; Franke et al, 2007; Lampert, 2001; Kilpatrick et al., 2001). It is also a process of engaging together in generating mathematical meaning (Boaler, 2002c; Franke, et al., 2007; Kilpatrick et al., 2001). Effective teaching expects to continually elevate students' mathematical competencies and the level of a student's involvement in learning, also determines the quality of teaching (Kilpatrick et al., 2001).

Research shows that different concepts of teaching, learning, and classroom cultures influence the ways in which teachers teach and how students learn

(Dossey, 1992). According to Marx and Collopy (1995), teachers' teaching styles directly influence students' learning. Teachers who are sensitive to their students' learning are more likely to change their teaching practices and such changes are more likely to improve students' learning (Irwin & Britt, 1994).

There are many kinds of teaching approaches which can include the reciprocal, inquiry-oriented, traditional, progressive and constructivist. This study focuses on two kinds of teaching styles: the traditional or direct teaching which is closely link to elements of behaviorism, and constructivist teaching. These two teaching approaches were used in three mathematics classrooms in Taiwan (Chi, 1999), and in this study. Each style is discussed below.

2.2.1 Traditional teaching

In looking at the traditional approach to teaching, a discussion on direct instruction, didactic instruction, and features of behaviorism are included since these in essence describe “traditional teaching”.

The behaviourist approach still remains main stream in the educational field; from the concepts or adaptation of behaviorism (Wenger, 1998). Knowledge can be defined as a combination of facts and skills (Even & Tirosh, 2008). Behaviourist theories emphasize behaviour modification through stimulus – response connections and selective reinforcement (Fang & Chung, 2005). The intended behaviour is reinforced by repetitive practising and praising of the correct answers (Wei & Eisenhart, 2011). The behaviourist theory explains learning as passively receiving stimuli or information rather than mentally processing such information (Fang & Chung, 2005). These theories completely ignore issues of meaning; particularly social meaning. They address issues such as learning through rewards and practices and assess student learning based on observable behaviours (Fang & Chung, 2005; Wenger, 1998).

Here, the key assumption is that students learn what was taught or transmitted. As long as the knowledge was clearly communicated and received, then this knowledge could be generalised in other circumstances (Boaler, 2002a). Multiple

opportunities for drill and practice should be offered to reinforce certain behaviours (Boaler, 2002a).

Traditional teaching is based on behaviourism where the focus is on drill and practice (Fang & Chung, 2005), speed and accuracy of answers, with an outcome of automatic recall (Trotman 1999). The teaching is limited to the classroom context and the teacher has limited freedom from schools to arrange teaching activities (Chi, 1999). The teacher is assumed to know all the mathematics for students' learning (Begg, 1992).

Researchers agree that traditional teaching promotes teacher centered learning, where teachers control all the teaching discourse (McCarthy & Peterson, 1995). There is no room for student discussion (Threlfall, 1996). Thus, teacher-centred and quiet classrooms normally appear. Students are seen as learning as the teacher transmits the information (Even & Tirosh, 2008; Windschitl, 1999b), and often need to give up their individual decision making in obedience to the demands of the classroom teacher (Boaler & Greeno, 2000). Students do not participate in curriculum planning (Bennett, 1976).

The role of the teacher in the traditional approach is to adopt a clear and coherent presentation of instruction (Trotman, 1999), such as:

- lecturing through the “chalk and talk” method (Threlfall, 1996);
- before the beginning of an activity, giving very clear and detailed instructions for the procedures (Fang & Chung, 2005);
- correcting immediately students' incorrect statements (Threlfall, 1996); and
- ensuring that students know what to do in each stage (Sosniak, Ethington & Varelas, 1994).

Teachers follow the syllabus to transmit knowledge (Livingstone & Izard, 1994), monitor students' progress (Frederiksen, 1984), give regular tests (Werry, 1989) and to ensure that students retain this knowledge until the examinations are over

(Livingstone & Izard, 1994). Therefore, skill-based tasks would be given with an expectation of a uniformity of learning (Windschitl, 1999b), with an emphasis on rote memorization of mathematics rules (Wei & Eisenhart, 2011). Teachers use formulae and encourage students to use particular rules or formulae in most mathematics problems (Bennett, 1976; Silver et al., 1995).

Teaching emphasis is on "content" (Threlfall, 1996) with a speedy transmission of facts and knowledge (Even & Tirosh, 2008), such as:

- using basal texts in mathematics and many worksheets (McCarthy & Peterson, 1995);
- separating mathematical subject matter into small objectives within a sequence of tasks (Begg, 1996);
- asking convergent or factual questions for which they have prepared answers already and assessing students' work within the narrow domain of each unit (Carr & Ritchie, 1992; McCarthy & Peterson, 1995; Silver et al., 1995).
- focusing on the product of a student's work rather than including the processes (Trotman, 1999).

(a) Advantages of the Traditional Approach

Advantages of the traditional approach to teaching include:

1. Teachers can cover more mathematical content within a limited time.
2. Students may feel more secure in a structured teaching environment (Bennett, 1976).
3. Firm teacher discipline leads to good self-discipline by students (Bennett, 1976).
4. Students may perform better under traditional teaching rather than from a constructivist approach (Mousley, Clements & Ellerton's, 1992 study researching the mathematics learning of children in the United Kingdom, Australia, and the United States).

Some benefits of instrumental approaches are that students more easily learn and apply the rules/procedures knowledge in similar situations and receive senses of achievement. It is easier for teachers to adopt these approaches than relational methods, because less knowledge and technique in instrumental teaching (more rules/procedures) and students are easier to reach right answers on paper (Skemp, 1976, 2006).

Teaching approaches with great emphasis on procedures and memory are still commonly adopted in many classrooms (Pesek, Kirshner, 2000; Wei & Eisenhart, 2011). Leung and Park (2002, p. 127) argued that “procedural teaching does not necessarily imply rote learning or learning without understanding”. They researched nine mathematics teachers in each place - Hong Kong and Korea. They found that most of the teachers adopted very procedural teaching strategies but conveyed conceptual and procedural understanding to students and they also found support from Ma’s work (Ma, 2010). The structural teaching for core concepts and repetitive practices might benefit the high mathematics achievements of Asian students (Leung & Park, 2002). Leung & Park (2002) perceived that conceptual and procedural understandings are connected (Hiebert & Lefevre, 1986), especially, when students conduct repetitive practices (Dahlin & Watkins, 2000; Leung & Park, 2002) that provide various challenges (Leung & Park, 2002). These challenges help to strengthen students’ conceptual and procedural understanding (Dahlin & Watkins, 2000; Leung & Park, 2002).

Direct instruction is the instructional approach which is most prevalent in traditional classrooms. This approach entails reviewing, teaching and practising that which was taught. The “chalk-and-talk” method is mainly used in direct instruction classrooms. The teaching strategies of direct instruction place an emphasis on the teachers’ explanation of the content; also called explicit teaching (Zhang, 2002). The learning theories associated with direct instruction strategies do not come from a single theory but may be viewed as, a combination of behaviourism, the meaning learning theories and the information processing and transmission theories from the cognitive theory (Zhang, 2002). A direct instruction lesson has five steps:

- (1) learning new ideas from old experiences,
- (2) clearly explaining the content of the teaching material,
- (3) helping students to do practice in time, or guided practice,
- (4) adjusting mistakes from feedback, and
- (5) allowing students to complete their assignment individually (Zhang, 2002).

Didactic or instrumental approaches also commonly appear in the traditional classrooms. Students' participation in a didactic classroom normally is governed by textbooks, procedures and rules related to memorization and procedure duplication. They rarely negotiate or develop ideas, procedures or creativeness (Boaler & Greeno, 2000). Students' learning in the didactic and instrumental approaches is limited to passively absorbing and acquiring knowledge and procedures, then applying them (Boaler & Greeno, 2000; Silver et al., 1995; Skemp, 1976, 2006). Didactic approaches place an emphasis on memorization and procedural practice, but rarely develop mathematical ideas (Boaler & Greeno, 2000). To some extent, instrumental approaches are similar to didactic approaches, but they place more emphases on procedures and ignore the understanding behind the rules/procedures (Skemp, 1976, 2006).

Wenger (1998) also argued that if teachers regard knowledge as learning pieces of fact, then naturally they would present knowledge in a high structured manner. From that perspective, direct lecturing will be the teaching strategy (Wenger, 1998). Then, the most efficient way is probably to impart knowledge through demonstration and practice. This can be seen in the traditional mathematics classrooms in Boaler's (1997, 2002a) research.

To sum up, students all learned passively from teachers' explanations in the traditional approaches, direct instruction, didactic approaches and instrumental approaches (Boaler & Greeno, 2000; Skemp, 1976, 2006; Zhang, 2002). This traditional approach often combines teacher centred views of learning (McCarthy & Peterson, 1995), teaching strategies of a behaviourist approach and monitoring

of class events (including decisions of classroom learning task, tests given or students' learning progress). However, there is more meaning processing in the direct instruction (Zhang, 2002), but not much understanding processing in the didactic and instrumental approaches (Boaler & Greeno, 2000; Skemp, 1976, 2006).

(b) Disadvantages of the Traditional Approach

The traditional teaching approach may inhibit students' freedom to think. It fails to focus on mental processes (Romberg, 1993; Trotman 1999). Other disadvantages include over-emphasis on rote learning, insufficient emphasis on creative expression (Bennett, 1976), concern with academic standards and competition (Bennett, 1976) and use of external rewards such as grades. For example, external rewards were used when teachers reinforced right answers, corrected wrong ones and evaluated by right answers (Kamii, 1985). Hagg (1991) also argued that, the behaviorist teaching practice may result in students regarding learning with little enthusiasm or intellectual tension and it may fail to cater for the average students. It has been suggested that the emphasis on 'rule following' rather than 'rule learning' is anti-mathematical (Hagg, 1991; Neyland, 1994).

These learning behaviours may lead to many students developing negative feelings toward passively receiving abstract knowledge (Boaler & Greeno, 2000). Additionally, they can result in students developing over-dependency on the authority of the teachers (Boaler & Greeno, 2000). Some scholars have viewed Asian learners as being passive learners with a heavily reliance on teachers' instructions (Beaver & Tuck, 1998; Samuelowicz, 1987). The limitation of the behaviourist approach becomes more apparent, particularly in the teaching of higher-order skills (Hagg, 1991; Neyland, 1994).

Research conducted by Baker, Czarnocha and Prabhu (2004) showed that when using the traditional curriculum, with its focus on the computational modelling of procedural knowledge, the knowledge students acquired was not long term. In summary, in traditional teaching, students work on graded exercises, memorise content and formulas, and are continuously tested throughout a unit of work and at

the end of the unit. No emphasis is placed on processes, so hence the need for an alternative approach to teaching.

2.2.2 Features of constructivism

Another view of learning is from a cognitive perspective, i.e., constructivism. An individual's reasoning and cognitive growth is emphasized from perspectives of cognitive psychology (Fang & Chung, 2005; Voigt, 1994). Here, learning is interpreted as a growth in the internal cognitive areas (Wenger, 1998). Learning is typically described inside the mind of the individual from acquiring knowledge (Ford & Forman, 2006; Greeno, 2003; Peressini et al., 2004), or growth in conceptual understanding (Ford & Forman, 2006; Peressini et al., 2004). It is understood that knowledge is thought to be able to be transposed/generalised to other situations (Peressini et al., 2004) but the characteristics of tasks and contexts might affect the transformation of knowledge in other situations (Peressini et al., 2004).

Cognitive theorists argue that what is learned can also be independent of the context, even while learning takes place in a social context (Peressini et al., 2004). In contrast, some regard cognition as situated in the context, as a process of conceptual construction from reasoning information (Wenger, 1998). The focus here is on the “processing and transmission of information through communication, explanation, recombination, contrast, inference, and problem solving” (Wenger, 1998, p. 279). Prior experiences are significant and benefit students when making sense of new information (Wenger, 1998).

Cognitive psychology, on the other hand, concerns how children connect mathematics with their world in order to make sense out of both. It assumes that children bring knowledge and experiences to the classroom and when presented with a problem, through grappling with it and finally realizing that there are many possible paths that can be taken to arrive at a “satisfactory” solution, they develop their understanding.

The term constructivism has been interpreted from pedagogical, psychological, philosophical (Bettencourt, 1993) or sociological tendencies (Wood, Cobb & Yackel, 1991). For example, some scholars considered that constructivism is a theory of knowing (von Glasersfeld, 1993), a theory of knowledge (Bettencourt, 1993) related with personal construction (Wood et al., 1991), an epistemological theory, a theory about learning, teaching and administration of education (Matthews, 2000), and a theory of cognitive development (Confrey & Kazak, 2006; Greenes, 1995; Noddings, 1990). Therefore, cognitive enhancement is central for constructivist teaching (Kickbusch, 1996). An examination of the theory of constructivism reveals that learning is actively constructed by students (Cobb, 2007; Lesh, Doerr, Carmona & Hjalmanson, 2003) rather than passively received by teachers' transference (von Glasersfeld, 2005). So, the ownership of learning belongs to the learners and not to the teachers (Hong, Li & Lin, 2005; von Glasersfeld, 1993).

Students need to make sense of different ideas and activities and organize them into their own cognitive schemas, selecting and adapting (Boaler, 2002a; Confrey & Kazak, 2006) and reorganizing knowledge as part of their own constructions (Even & Tirosh, 2008). Their prior ideas affect the ways in which they make sense about new experiences (von Glasersfeld, 1995; Windschitl, 1999a) and these experiences are also influenced by the students' social and cultural contexts (Windschitl, 1999b).

Constructivism is one possible way of thinking and knowing, and is a model that can never be claimed as "true" but more so a personal interpretation of reality (Confrey & Kazak, 2006; Hammersley, 2009; Lesh et al., 2003; Liu, 2004; Malara & Zan, 2008). Constructivism is one of the theories (for example, symbolic interactionism, the distributed view of intelligence) which emphasize student thinking development (Cobb, 2007). The recent development of constructivism was closely incorporated with a school of psychology and sought to explore the characteristics of learning (Lerman, 2001). Constructivism can serve to interpret the teaching or learning model and lead to the explanation of educational practices

such as individual development or analyses among groups within a specific unit (Confrey & Kazak, 2006); for example Gravemeijer's work (1999).

According to Wenger (1998, p.279), "constructivist theories focus on the processes by which learners build their own mental structures when interacting with an environment". Self-directed activities are favoured by teachers or researchers in classroom practices and lead to the development of students' conceptual thinking abilities, especially in individual design and discovery (Papert, 1980; Wenger, 1998).

The different types of constructivism are individual/radical constructivism and social constructivism. The work of Piaget has great influences on constructivism and cognitive theorists (Confrey & Kazak, 2006); especially for individual constructivism (Scott, Cole & Engel, 1992; Smith, 1999). The followers of Piaget perceived constructivism as an individual learning independent from cultural and people influences (Scott et al., 1992; Smith, 1999). Although it is impossible to understand inside of a person's mind, individual constructivists claimed that the creating models or metaphors of an individual's thoughts enhance the ways to interpret learning (Smith, 1999). The majority of constructivists can be termed as close to individual and radical constructivism (von Glasersfeld, 1995; Smith, 1999).

In summary, constructivism may be viewed as a way of thinking and knowing, where knowledge is a personal construction (Cobb, 2007), and interpretation of reality rather than an objective truth (Hammersley, 2009; Malara & Zan, 2008; von Glasersfeld, 1993). This theory places a focus on cognitive, epistemological and knowledge development (Matthews, 2000; von Glasersfeld, 1993). Further, constructivism as it applies to teaching and learning, has a student-oriented focus. The ownership of learning belongs to students rather than the teacher (Hong, Li & Lin, 2005; von Glasersfeld, 1993).

Social constructivism is highly influenced by Vygotskian's and Bruner's concepts (Hartas, 2010). Lerman (2001) comments that there are differences between these

two scholars. He stated that Vygotsky emphasizes sociocultural views of learning that generates a meaning closely associated with culture while Bruner highlights the importance of actions in learning and emphasizes the behaviour of exploring meanings in culture. Social constructivists apply cognitive perspectives to interpret individuals' development in social interactions (Lesh & Doerr, 2003). Individuals, based on their experiences and previous knowledge, actively construct knowledge, especially concepts and hypotheses (Ernest, 1991), through interacting with people or cultural and social worlds (Hartas, 2010). Opportunities for learning occur during social interaction/dialogues such as teacher-student and student-student dialogues, students' explanations and justifications (Hong, Li & Lin, 2005; Ernest, 1991; Wood et al., 1991), argument, negotiation and mediation that will produce a consensus or a social form of knowledge (Confrey & Kazak, 2006; Jaworski, 1994).

It should be noted that, mathematical discussion has been emphasized in constructivist teaching (Richardson, 2003; Threlfall, 1996), a social perspective of learning (Peressini et al., 2004; Van der Lindendagger & Renshaw, 2004), and in recent education reforms such as in Taiwan or the USA (NCTM, 2000; Taiwan Ministry of Education, 2001). A social perspective on learning recognizes the importance of students presenting a collective form of knowledge through discourse in classrooms (Driver, Asoko, Leach, Mortimer & Scott, 1994; Wood, 1999). Discourse is not a tool to shape ideas into some 'material' actions (expected content knowledge), but rather a collective form of inference (Pontecorvo & Girardet, 1993, p. 366; O'Connor, 1998). Discourse also presents ways of thinking and serves as a social knowledge construction (McNair, 1998; Pontecorvo & Girardet, 1993; O'Connor, 1998); especially a synthesis on connecting core mathematical concepts (Romberg, Carpenter & Dremock, 2005). Mathematics learning has been considered as "a trajectory of participation in the practices of mathematical discourse and thinking" (Boaler & Greeno, 2000, p. 172). To some extent, each classroom is a unique social environment, and teachers use discourses to deliver their goals/lessons (O'Connor, 1998).

One remarkable character of constructivist teaching is that an individual or group generates “meaning-making” through the process of classroom conversations (Richardson, 2003, p. 1623). Classroom discussions have been recognized as important elements to improve students' mathematical conceptions (Wood, 1999) through spoken and written communication (Taiwan Ministry of Education, 2001; NCTM, 2000).

In conclusion, social constructivism places a focus on the fact that students learn via social interactions (Hartas, 2010; Hong, Li & Lin, 2005; Lesh & Doerr, 2003), through constructing their knowledge and interacting with social dialogues among students and the teacher (Hong, Li & Lin, 2005). Students are engaged in activities which allow them to select, adapt and make sense of ideas and activities into their own cognitive schemas (Boaler, 2002a; Confrey & Kazak, 2006). Thus, this environment provides the impetus for students to actively construct their own learning through social dialogues rather than passively receive teachers' transference (Cobb, 2007; von Glasersfeld, 2005). Their arguments and negotiations produce a consensus or a social form of knowledge (Confrey & Kazak, 2006). Students' previous learning experiences and the influence of their social and cultural contexts also affect their learning (Windschitl, 1999a).

Furthermore, mathematical classroom discussions afford opportunities to students to present their mathematical ideas through expressions, agreements, and disagreements (Peressini et al., 2004), while engaging in “sense-making” and problem solving practices (Boaler & Greeno, 2000, p.172). Class discussion is a continuous negotiation between members (Pontecorvo & Girardet, 1993; O'Connor, 1998). Students can practice evaluating their own work and that of others to make sense or arguments during class discourse (in small group time or in whole-class discussions) (Lamberg, 2013; Lampert, 2001). The conceptual structure of subjective mathematical knowledge is achieved through the functions of language (Ernest, 1991). Through this process, students are likely to identify conflict and restructure their own thinking (Hiebert & Wearne, 1993). As students understand and learn about the discourse, they will improve their own mathematical dialogue (Rittenhouse, 1998). Moreover, students' higher-order

thinking skills, including skills of discovering, reasoning, organizing and arguing (Torff, 2003), can be achieved in mathematical class discussions.

Opportunities in classroom discourses offered chances for students to assess their understanding in solving problems (Webb, 1991), and chances for receiving support from others (a teacher/students) for misunderstood or incomplete answers (O'Connor, 1998; Webb, 1991). Students have opportunities to control the pace and content of the teaching activities (Webb, 1991).

Thus, opportunities for class discussions are offered to allow students to contribute to “the judgement of validity, and to generate questions and ideas” (Boaler & Greeno, 2000, p. 189). As Resnick (1988) described it, whole class discussion is likely to employ a large group as a medium to empower individual students to formulate their ideas for conflict and development of ideas. The strategy of students sharing or explaining provide opportunities for other to get further clarifications and understanding (Franke et al., 2007). This strategy will therefore bridge the growth of “connected knowing” among individuals (Boaler & Greeno, 2000). The classroom base knowledge will be enriched (Brown & Campione, 1994) and will lead to the development of collective public knowledge (Ball & Bass, 2000b; Franke et al., 2007; Kazemi & Stipek, 2001). Student discourses also can be regarded as verbal forms of thought about relations of mathematical ideas, reasoning, asking questions, making of plans (Franke, et al., 2007) and correlated with students’ ability to use conceptual knowledge while explaining a phenomenon (Van Boxtel et al., 1997).

Classroom discourse is therefore regarded as the key principle for the educational design and instructional tools (Cazden, 2001). Researchers believe that “Students in these learning communities are capable of deep, sustained, complex thinking, both in whole-class discussions and in their small groups” (Brown & Campione, 1994, p. 261). Lively open-class discussions represent normal class patterns (Pirie, 1988) that benefit the development of a student’s mathematical understanding (Boaler & Greeno, 2000; Cazden, 2001; Franke et al., 2007).

Embedded in a discourse is exploratory talk. It is used to develop collective

mathematical reasoning (Hunter, 2008). Further, teachers can know students' thinking from class conversations and this is essential for teaching for understanding (Franke et al., 2007). Thus, class discourse can also be regarded as an important part of ongoing classroom evaluations (Kahan, Cooper & Bethea, 2003). In addition, the teacher can generally teach students not only mathematics but also how to study mathematics, by asking students to reason, to explain, to interpret the assumptions of their peers, and to explore mathematics together (Lampert, 2001). Another benefit of exploratory talk is that class discussions are also able to foster students' participation in thinking (reasoning) in the whole class discussion (Nathan & Kim, 2009), such as shown in Nathan & Kim's work (2009), and Hunter's (2008) work.

The characteristics of instruction that promote classroom discourse are not well documented in the literature (Franke et al., 2007). However, some key elements that foster class discourse have been pointed out by several scholars. Generally, teachers are mindful to allow conversations to serve as a source of students' ideas (Walther, 1982; Lampert, 1990a). To discuss this in detail, in order to guide class discourse, a teacher needs to (1) select and offer discussion questions, (2) coach, explain, respond and challenge students' conversations, (3) address mathematical meaning or norms in time, and (4) maintain the engagement of all students (Franke et al., 2007; Peressini et al., 2004). Another detail that could be added to support class discourse is that of problem posing by teachers to provide a range of answers; not just right or wrong (Franke et al., 2007; Lamberg, 2013; Lampert, 2000). In addition the teacher can allow some time for students to explore their own ideas as well as those of others (Hunter, 2005; Nathan & Knuth, 2003), question students' thinking (Ford & Forman, 2006; Lampert, 2001), explore students' mistakes to offer chances for them to reflect on their learning by explaining and challenging their own arguments (Ford & Forman, 2006), and managing the coverage of the content (Lampert, 2001). Students gaining ownership of their learning will better manage the coverage of the content to be learnt (Lampert, 2001). As a result, through discourse (class discussion), a teacher can grasp the mathematical needs of the class and understand students' mathematical thought. Specifically, they can find out what students know, their misconceptions, and how these misconceptions might have developed (Franke et

al., 2007; Romberg et al., 2005) and apply students' responses to instruction (Romberg et al., 2005). This is demonstrated in Lampert's work (2001). This will also benefit teachers' question asking, to connect to students' ideas and extract multiple strategies to assist the development of students' mathematical proficiency (Franke et al., 2007).

Teachers may be called upon to perform different roles such as facilitators (BRAP, 2003), where they are engaged in fostering students' participation and mathematical discourse amongst each other. This helps students to develop their comprehension and it helps them to use the discourse to deepen their mathematical understanding (Franke et al., 2007; Rittenhouse, 1998). Teachers may also function as mediators to reconcile differences in students' inner knowledge and understanding of mathematics (Walther, 1982; Lampert, 1990a). Teacher talk will support and develop students' mathematical command as they move from legitimate peripheral participation of class discussions to enhance engagement (Lave & Wenger, 1991; Rittenhouse, 1998).

On the other hand, the process of discourse lays the foundation to transform the classroom practices into a supportive learning community (BRAP, 2003; Hartas, 2010), to establish a collective understanding through the class discourse and students' justification (Hunter, 2008) from the multiple input from the teacher and students. Besides this, seating arrangement can help to balance supportive social interaction and support to clarify students' spoken ideas (Lampert, 2001). However, some challenges can arise from class discussions. For example, new students often find it difficult to make sense of what is being said, even at a normal rate of speed for conversations (Rittenhouse, 1998). Many scholars have discussed the two core elements: justifications and arguments inside classroom discourse that lift up high level of mathematical thinking and understanding. The next sections will further explore these two factors.

(i)Justifications and Arguments

A constructivist approach to teaching offers teachers several opportunities for students to engage in activities that require them to justify and establish reasonable arguments. The rich information (justifications) is contained in class

discourse while developing and explaining ideas in classes about their problem solving strategies (Webb, 1991; Wood et al., 1991). Justification can be defined as the value of something to be true or certain (Ball, 2003).

Mathematical arguments offer individuals opportunities for reasoning (Wood, Williams & McNeal, 2006), to criticise and justify ideas from a collective point of view and to generate new perspectives (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006) and conceptual understanding (Wood, 1999). Moreover, students can create a public knowledge from different forms of mathematical explanations in the class discourses that are aligned with the content and students' inspections/inquiries. This will also develop the mathematical identities of students (Franke et al., 2007). In addition, mathematical content discussions and debates can also lead to the development of student autonomy (Hunter, 2006b) and competence (Hunter, 2006b; Lambdin & Walcott, 2007).

In conclusion, class discussions can foster mathematical arguments that benefit students' mathematical understanding (Ball & Bass, 2000b; Boaler & Greeno, 2000; Franke et al., 2007; Lampert, 2001), knowledge (Franke et al., 2007; Wood et al., 2006) or reasoning (Hunter, 2006b). Moreover, informal discourse can enhance a higher-level of thinking (Franke et al., 2007; Hunter, 2008; Nathan & Kim, 2009; Wood et al., 2006). For example, Hunter (2008) reported that four teachers challenged students through questioning, in-depth explanations, and justification. This form of discourse led to the development of collective reasoning and views. Other studies also have indicated the positive relationship between classroom discourse and students' learning outcomes (Hiebert & Wearne, 1993; O'Connor, 1998; Webb, 1991). For example, high achievement correlates with the behaviour of giving explanations to classmates (Webb, 1991).

(ii) Two Patterns of Classroom Discourse

Classroom discourse has been classified according to two models (Cobb, Yackel & Wood, 1993; Peressini et al, 2004). One type is that arriving at a solution is the driving force for class discussions; typically found in the traditional school mathematics classrooms (Peressini et al, 2004). Classroom interactions can be

illustrated as three steps: the teacher starts first to pose a known-information question (Cobb et al., 1993; Peressini et al., 2004), students respond, and then the teacher evaluates the feedback (Peressini et al., 2004). These steps match an “IRE (initiate–respond–evaluate)” pattern (Cross, 2009, p.340).

In contrast, in the other type of classroom discourse, the students’ dialogue drives the mathematics teaching and learning flow in an inquiring classroom. Information-seeking questions are raised first from the teacher and it is expected that students give an explanation of their interpretation and problem solving (Peressini et al, 2004).

Other strategies when used appropriately are possible to increase the level of class discourse. This can be seen for example in cooperative groups or revoicing strategies for students’ mathematical conversations (involving explanation, rephrasing or reporting) (Franke et al., 2007). Teachers facilitate discourse around mathematical ideas through support and monitoring or extracting students’ ideas from discussions (Franke et al., 2007).

The above section has illustrated how different teaching styles lead to different class practices. The following section is going to introduce constructivist teaching, the role of a teacher and student in constructivism, advantages of constructivism, relevant studies long-term research and disadvantages of constructivism

(a) Constructivist Teaching

The constructivist learning approach, when applied to teaching, is aimed at producing life-long learners. It is intended to build up learners as skilled and thinking people (Hagg, 1991). However, constructivism as is applied to teaching, is relatively less developed than the views of constructivist learning (Prawat, 1992). This is also true for the factors that contribute to effective constructivist teaching which are still under investigation (Richardson, 2003).

Most research on developing constructivist pedagogy, concerns the relationship between teachers’ actions (including teachers’ beliefs, values, behaviour and activities) and students’ learning (Richardson, 2003). The other important area of

developing constructivist pedagogy is linked to theory building. Research experiences will release information of effective teaching practices/pedagogy to benefit teacher education and professional development (Richardson, 2003). Investigations of effective teaching practices might suggest to go back to the focus of classroom practices relating to teaching and learning (Boaler, 2002c). Researchers can start from a subject or a general level (Richardson, 2003). For example, some researchers have discussed the effective teaching practices with respect to students' learning outcomes from the constructivist pedagogical perspectives, such as standardised tests (Boaler, 1997; Boaler & Staples, 2008; Hiebert & Wearne, 1993), students' deep mathematics understanding (Ball & Bass, 2000b; Boaler, 1997; Boaler & Staples, 2008) and some disciplines of establishing constructivist classrooms (Boaler, 1997; Boaler & Staples, 2008; Malara & Zan, 2008). Moreover, teachers' knowledge of the subject matter and their awareness of cultural issues are also addressed in the theory building of developing constructivist pedagogy (Richardson, 2003).

When constructivism is applied to teaching, it does not specify a particular model of instruction (Windschitl, 1999b). Constructivism states that students learned best through conducting their own approaches to problems in reaching mathematically competence (Lambdin & Walcott, 2007), and students will learn from different forms of instruction (Richardson, 2003). It is rather a set of beliefs, norms and practices that contribute to the culture in classrooms and in the school, but new relationships exist between teachers, students and mathematical ideas (Windschitl, 1999b). The constructivist view of learning and its application to teaching has the following characteristics:

- Teachers minimise their direct instruction or lecture mode (Simon & Schifter, 1991), and promote discussion and problem posing by students (Wheatley, 1991; Trotman, 1999).
- Teachers develop their own curricula according to their students' current conceptions or needs (Begg, 1996; Windschitl, 1999b). It is possible that curricula developed from theses are not driven by external curriculum such as school schemes or national syllabi (Steffe, 1990). Teachers need to be experienced in applying diverse strategies to help students' understanding, such as explaining, demonstrating, and advising etc

(Windschitl, 1999b).

- Teachers encourage and facilitate discussion (Brooks & Martin, 1999; Trotman, 1999; Windschitl, 1999b) by creating a culture for inquiry (Windschitl, 1999b); guiding and framing an issue which is realistic and open-ended for students' discussion (Brooks & Martin, 1999; Threlfall, 1996; Windschitl, 1999b). Teachers select activities to facilitate discussions (Gravemeijer, 1994). Teachers allow a certain waiting time after giving questions (Brooks & Martin, 1999). It places an emphasis on students' explaining their thoughts (Kazemi & Stipek, 2001). Some other reform studies also valued the waiting time, beside the advantages mentioned above, and added that students could explore their own ideas and those of others (Hunter, 2005; Nathan & Knuth, 2003). So, a constantly quiet classroom cannot be expected as in the traditional teaching approach, if these skills are practiced (Barton, Begg, Butel & Ellis, 1995).
- The classroom social norms are established and negotiated so that the teacher and students can remain focused on following a constructivist perspective as it applies in teaching and learning (Confrey & Kazak, 2006). An example of this can be seen in the work of diSessa & Cobb (2004). Moreover, the norm of respecting each other's ideas is expected (Windschitl, 1999b).
- The emphasis from the constructivist views of learning is placed on discovery (Threlfall, 1996), reproduction (Windschitl, 1999b), understanding (Greenes, 1995), student autonomy and initiative (Brooks & Martin, 1999), and problem solving. Similar arguments are stated as below:
 - teachers encourage students to conceptualize situations in different ways (Confrey & Kazak, 2006; Windschitl, 1999b). They are encouraged to think and develop their own ideas (Carr, 1993; Threlfall, 1996; Lampert, 2001) and to explore misconceptions and conflicting ideas in order to develop broader and more resilient concepts (Simon & Schifter, 1991). It is an ongoing process of students' concept constructions and corrections (Windschitl, 1999b);

- teachers expect student learning with less memorization and imitation (Simon & Schifter, 1991);
- “problem-based learning” is suggested (Windschitl, 1999b, p.752). Students find their own questions through the procedure and try to work problems out (Carr, 1993). Teachers act as research leaders to help students plan and carry out their own investigations of their questions (Begg, 1991). Real-world examples and problems are used with an emphasis on process problem-solving processes (Threlfall, 1996; Wheatley, 1991). Students are encouraged to use their own or a variety of methods for solving problems (Carr, 1993). Teachers allow student responses to direct the lessons and alter teaching strategies/content (Brooks & Martin, 1999).
- Literature supports, from the constructivist view of learning and its application to assessment, that teachers assess both the processes and products of student thinking and assist students' own efforts to assess what they have learnt (Carr & Ritchie, 1991, 1992; Trotman, 1999). These might include:
 - assessment approaches such as observing, listening, investigations and self-assessment. In this way, teachers can gain ideas about students' mathematics knowledge, conceptual misunderstanding (Trotman, 1999), prior ideas (Begg, 1996), and strategies from their description of problem solving to teachers or peers (Carr & Ritchie, 1991);
 - “cooperative learning” in classrooms (Windschitl, 1999b, p.753), e.g., teachers let students solve problems collaboratively in pairs or small groups with little monitoring (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti & Perlwitz, 1991; Hagg, 1991; Windschitl, 1999a). The power of cooperative peer learning has been broadly exposed (Yackel, Cobb & Wood, 1991; Pea, 1993; Van Boxtel, Van der Linden & Kanselaar, 1997; Van der Linden & Renshaw, 2004; Dekker & Elshout-Mohr, 2004). Thus, students will benefit as they learn to explain, argue and defend their mathematical thinking during peer interactions. It also has advantages in fostering students' full

participation in class dialogues (Kazemi & Stipek, 2001; Wood, Cobb & Yackel, 1991).

- students present and discuss their work to the whole class (Mayers & Britt, 1998). Abundant resources could appear in the classroom (Windschitl, 1999b). Teachers encourage discussions either among students or with the teacher (Brooks & Martin, 1999; Windschitl, 1999b). Teachers help students to draw sensible and useful conclusions from their findings (Begg, 1991). Alternatively, teachers initiate discussions and reformulate students' mathematics contributions (Gravemeijer, 1994);
- students can explore the limits to their constructed knowledge, compare their solutions with others with regards to similarities or differences, and actively test and integrate their ideas. Teachers might encourage students to investigate why their ways of understanding differ from others (Windschitl, 1999b), and the reasonableness of their solutions or responses (Trotman, 1999).

Some challenges may appear for teachers with regards to how to support students developing key concepts of the subject, so students can also succeed in standardized tests, because in project learning, students may choose diverse focus topics that develop different concepts. Those developed concepts and students' understanding do not always match/benefit the focus of standardized tests (Windschitl, 1999b).

(b) Role of Teacher and Student in Constructivism

This view of learning necessitates a shift in teaching. Teachers are not seen as authorities but rather as facilitators of learning (Barton et al., 1995; Mayers & Britt, 1995; Trotman, 1999; von Glaserfeld, 1987), as challengers, encouragers (Confrey & Kazak, 2006), consultants (Barton et al., 1995), “guides ... and critics” (Confrey & Kazak, 2006, p.335). Vygotskian's cognitive learning perspectives highly influence the development of recent constructivist learning theories (Zhang, 2002). Particularly, from a Vygotskian perspective, teachers act

as a guide in the zone of proximal development and are responsible for choosing the tasks and strategies to develop classroom discourse and interactions (Malara & Zan, 2008). Teachers' rich knowledge and experiences would benefit them to know when/how to offer good guidance according to students' responses (Chen, 2001; Richardson, 2003), and implement teaching strategies (Windschitl, 1999a).

In a classroom based upon constructivist beliefs, teachers prepare an environment to allow students to construct their own learning (Malara & Zan, 2008). Individual differences in constructs, knowledge and abilities are recognised and supported by teachers (Carr & Ritchie, 1992). The focus is placed on enhancement of conceptual understanding and also individual learning (Even & Tirosh, 2008).

Although there is no particular model of teaching instruction from constructivism, the following summarizes some principles when constructivism is applied to teaching. It is very much a student-centred approach to teaching and learning where students construct and develop their own knowledge, rather than absorb from teachers' transmission. Curricula are not determined from outside, but are developed from the students' current conceptions and arguments or from a specific focus or activities from students or the teacher. It is a set of norms and practices that contribute to an inquiry and open discussion culture in classrooms. Multiple teaching approaches (class or peer discussion, cooperate learning, investigation, students' self assessment, waiting time given in classes...and so on) develop students' observation, autonomy, discovery and responsibility for their own learning. Teachers minimise their dialogue but encourage and facilitate students' conversation and open discussion through open-ended problems or problem posing by students. These ideas are highlighted in social constructivist views, emphasising that social dialogues and arguments lead students to argue and explore new ideas. Students reorganize and integrate information from social dialogue or activities into their own cognitive schemas to construct their own knowledge or as a social collective form of knowledge (Confrey & Kazak, 2006). This knowledge is kind of personal (Cobb, 2007) or social and collective interpretation of reality (Confrey & Kazak, 2006) rather than an objective truth (Hammersley, 2009; Malara & Zan, 2008).

(c) Advantages of Constructivism

When students are involved in "sense-making" discussions and are able to employ their knowledge to solve problems, they develop a deeper understanding of core mathematical ideas rather than learning from recalling a list of facts (Windschitl, 1999b, p. 752). Moreover, when students are actively involved in solving meaningful tasks on their own (Hagg, 1991) and have gained a measure of success, they become more motivated towards further learning (Carr & Ritchie, 1992; Hagg, 1991) and as a result, learning can be more effective. Thus, this teaching method lets students have more opportunities to think for themselves, encourages responsibility and self-discipline, and allows students to develop their full potential (Bennett, 1976).

In addition, when students work independently, they think mathematically (Higgins, 1994). Through activities and group work, students can continually focus their concentration on an activity (Norman, 1993), and have more motivation to learn (Barton et al., 1995). The investigative approach is an effective way to learn mathematics (Boaler & Staples, 2008; Briars & Resnick, 2000; Carpenter et al., 1998; Hagg, 1991; Schoenfeld, 2002). Evidence from several reform studies indicate the benefits of students ability to apply knowledge in new situations (Boaler, 1997, 2000b, 2002b; Carpenter, et al., 1998; Lamon, 2007), or the flexibility (creativity) in problem solving (Lamon, 2007) (see section 2.1.6(b)). Lamon (2007) in examining the effect of reform approaches on student learning described students' progress in learning as appearing slow at the beginning of the reform process. However, after they developed and internalized their own mathematical understanding, they produced more powerful ways of thinking and creative methods than their peers in the traditional approaches. Moreover, the assessment in the constructivist ongoing classrooms can help the teacher to continually have feedback of students' knowledge and reasoning, and this will benefit the teacher's diagnostic instruction or curricular changes (Confrey & Kazak, 2006). The emphasis here is on "a way of knowing", or "a way of seeing the world", rather than "a way of doing" (Neyland, 1994, p. 451). Consequently, the constructivist perspective, when applied to teaching,

encourages students to develop their mathematical thinking and understanding of mathematics during the learning process (Greenes, 1995).

(d) Relevant Studies

A number of studies have shown that when a constructivist approach to teaching is applied, (i) students attain better achievement (Boaler, 1997; Briars & Resnick, 2000; Schoenfeld, 2002; Silver et al., 1995; Thomas, 1993; Zeng, 1998); (ii) mathematical understanding is enhanced (Chen, 2007; Cobb et al., 1992; Boaler, 1997; Briars & Resnick, 2000; Schoenfeld, 2002); (iii) attitudes toward mathematics improve (Cobb et al., 1992; Yeh, 1998; Zeng, 1998); and (iv) both their motivation and/or confidence in doing mathematics are enhanced (Thomas, 1993; Yeh, 1998).

Zeng (1998) found that Sixth Grade students' mathematics achievements and attitudes toward mathematics learning in constructivist classrooms were better for students in direct instruction classrooms. Most primary school teachers in Chen's study (2007) noticed that students' mathematical reasoning and conceptual knowledge were enhanced in the constructivist mathematics curriculum. However, students' overall mathematics achievement was not as expected. The gender favouring differences in Zeng's research (1998) did not make any obvious difference on students' learning in both the sixth grades constructivist classrooms and the classrooms applying direct instruction. There was also no gender difference in Boaler and Staples' (2008) work.

Although overall teaching practices in mathematics at the junior high level in Taiwan could not be regarded as constructivist teaching styles (Wey, 2007), Yu and Hang (2009) analyzed the Taiwanese data from TIMSS 2003 and found some positive relationships between teaching styles and students' learning. The authors found that teacher-centred instructions benefited students' achievement. The relevant constructivist instructions enhanced students' mathematical value and interest. Here, I refer to "relevant" constructivist instruction because of a lack of strong evidence to indicate student-centred classroom instruction as a normal

classroom practice. For example, Yu and Hang (2009) categorized constructivist instruction only in terms of three factors of a students' survey regarding the frequency of lessons involving classroom activities: how frequently students were asked to explain their answers to the class, deciding on their own procedures for solving complex problems and relating mathematics learning to daily life (Martin, 2005).

Of the few long-term constructivist research projects, are those conducted at primary mathematics level (ex. Carpenter et al., 1998; Lamon, 2007) and at the high school level (Boaler, 1996; Boaler & Staples, 2008). For instance, Boaler (1996) researched mathematics learning for three years at an alternative school and a traditional school in England. The outcomes of this open project-based approach, wherein the constructivist tenets were applied to teaching in the alternative school, indicated a better performance than traditional approaches in the national examinations (Boaler, 2002b) and in applying knowledge into new situations (Boaler, 1997, 2000a). The students in the traditional approaches believed that mathematical success came from memory rather than thought (Boaler, 1996).

Cobb et al. (1992) investigated five project second-grade classes with constructivist teaching and six non-project classes with normal teaching for a year in New Zealand. They found that the project students' procedural and conceptual challenging tasks were superior to the non-project students. Students' attitudes were also seen as a reason for success in mathematics. For example, the project students believed in the importance of working hard, being interested and trying to understand in mathematics. They also understood the need for collaboration. These students found it less important to conform to the methods of solution of others (Cobb et al., 1992).

Yet, another scientific study from New Zealand compared both constructivism and Empiricism (Hashweh, 1996). It was found that compared to non-constructivist teachers, (i) the constructivist teachers had more ways of teaching; and (ii) teaching strategies were better at improving students' conceptual knowledge

growth (Hashweh, 1996). The latter belief was consistent with the mathematics research of Britt, Irwin, Ellis and Ritchie (1993). The findings of these studies point to the need for reforms in the education system.

Some researchers did not clearly state that their work was under the disciplines of constructivism but pedagogically (Bettencourt, 1993) their teaching strategies appeared to support student-centred learning, indicating that they belonged to the body of constructivist work (Carr, 1993; Simon & Schifter, 1991; von Glasersfeld, 1990, 1993). For an example, the studies of Boaler and Staples (2008), Lamon (2007), Wood et al. (1991), Hiebert and Wearne (1993), and Lampert (2001) are discussed in the section below.

(e) Long-term Research

One five-year long-term research conducted in a high school within (constructivist) reform-oriented approaches used less lecturing (i.e., 4% of class time), mostly group work (72% of class time), high levels of interactions with students, and less coverage of content than in the traditional approaches (Boaler & Staples, 2008). When compared with two other traditional teaching schools, the findings showed that students had better mathematics performance/competence and positive attitudes toward learning mathematics. Students also had a more open perspective to achieve success in mathematics learning, than students in the traditional classes.

Lamon (2007) reported of a 4-year long-term study that investigated five reform teaching classes and one traditional teaching class from Grade 3 to Grade 6. Students of the reform classes without any mathematical rule teaching were encouraged to share their thinking at any time. These students performed better than their peers from the traditional approaches in ways including: computation abilities, achievements, creative methods in problem solving and applying their knowledge in new situations (Lamon, 2007).

Another long term study was conducted over a period of 3 years. This study examined the development of 82 Grades 1 to 3 children's mathematics concepts in multi-digit numbers (Carpenter et al., 1998). They found that students indicated better knowledge of base-ten number perceptions, generalized their understanding

without supplying formal algorithms instruction and encouraged their own invented strategies.

Undeniably, teachers too were affected by their instructional setting. One teacher in a year-long reformed experiment of second-grade mathematics suggested that her beliefs about the teacher role, the students' role and the nature of mathematics changed and she recognized the strength of social mathematical discussions/interaction that benefited students' learning (Wood et al., 1991).

One short-term research (one year) investigated six second-grade classrooms, about conceptual understanding instead of algorithmic skills (Hiebert & Wearne, 1993). Students were required to explain alternative strategies and were given more time for each problem in the alternative classrooms, with more frequent question asking and reviewing fewer problems than the traditional classrooms. Students performed higher, when compared with students within the more traditional instruction (Hiebert & Wearne, 1993).

Lampert (2001) focused on problem-based instruction and adopted some teaching strategies which were consistent with the constructivist view of learning. Those approaches included whole-class discussion, group work (Mayers & Britt, 1998), explorations of students' own ways of thinking by promoting discussion (Lampert, 2001; Wheatley, 1991, Mayers & Britt, 1998), public reasoning to make sense of the public mathematics discussion together, and a longer waiting time for students to explore their own mathematical thought (Carr, 1993; Threlfall, 1996; Lampert, 2001).

Lampert (2001) revealed that, besides demonstrating knowledge and skills, mathematical competence is complex and multidimensional. The "within-student variations" existed in a class (Lampert, 2001, p. 362). Some students performed competently on tasks but were not always good at explaining their reasoning or representing relationships among ideas. Some students were able to contribute productively in small-group problem solving but did not perform competently on the quiz. Moreover, students were found to reach a diversity of levels of

understanding. Some students offered more proof than the researcher's expectations, but some did not show understanding (Lampert, 2001).

(f) Disadvantages of Constructivism

Some disadvantages of applying a constructivist view of learning to teaching are:

- Time - teachers need more time (Chou, 2003b; Knight & Meyer, 1996; Trotman, 1999), knowledge (Chou, 2003b; Irwin & Britt, 1994), and confidence to process this type of teaching. The use of open-ended questions means that teachers cannot prepare answers in advance.
- Assistance - teachers might not know when to give assistance or the nature of the assistance to be given;
- Ownership - constructivists feel that if teachers explain mathematical methods to students, it would deny students' ownership of the methods. However, when no instructions are given most people cannot re-invent and acquire a sufficient portion of the whole of mathematics knowledge (Hagg, 1991). Further, it could create discipline problems or let students feel unsure of what to do (Bennett, 1976).
- Effectiveness - discovery methods tend to be less effective than directed teaching over the short term (Barton et al, 1995; Chou, 2003b; Hagg, 1991). For example, Taiwanese primary students' overall mathematics achievements were not as high as expected in the reform period from Teachers' perspectives (Chen, 2007); this included inefficiency in speed to solve problems (Chou, 2003b; Guo, 2004; Xu, 2003).
- Uncertainty – while the constructivist method may have a greater potential to cater for average students than the traditional teaching method, Hagg (1991) doubts that the full potential can be realized; Hagg was concerned that because the method is too complex and requires too much expertise to operate, it might be unlikely to be widely accepted (Hagg, 1991);
- Assessment - when teachers want to assess students' self selected work, it may be complex and lack objectivity (Hagg, 1991); it is more complex for teachers to help or assess students' learning. For example, students can

have different choices in selecting content and teachers will need to support them in these different directions (Hagg, 1991; Hu, 1996).

- Level of thinking - in this approach it is felt that individuals' prior ideas might only question at a very basic level (Begg, 1996), and also children's methods may lead to mistakes (Chung, 2005).

Moreover, similar teaching time and assessment challenges also appeared in the relational approaches (Skemp, 1976, 2006). If most teachers were still to adopt the instrumental approaches, it would be hard for a teacher to insist on the relational/reformed approaches in a school and face criticism of different pedagogical views (Skemp, 1976, 2006).

Some challenges also arise from the switch of the teacher's role to meet the expectations of constructivist classrooms. For example, one case appeared in an experimental class of a Taiwanese primary school under the constructivist mathematics teaching. After four years' effort, many teachers were not used to being facilitators instead of authorities in that class, this resulted in discontinuing that experimental class in 1992 (Fu, 2008). The findings from this case highlight the importance of having good support to assist teachers with coping with the changes of educational focus and practices (Fu, 2008) to benefit (or guarantee) the long term educational reform development.

Some disadvantages of applying a constructivist view of learning to teaching include time consuming, not enough knowledge to promote students' further discovery of knowledge, students' methods inadequate to cope with the needs of school tests or acceptances for other students, difficulties to assess students' wide range of mathematics knowledge, difficulties to assist students in a timely manner and difficulties to conquer the traditional school culture with regards to teaching expectations.

Based on the foregone discussion, both constructivism and behaviorism have their place in the learning of mathematics. Teachers need to be aware of these theories

and the implications they have on instruction, assessment and student learning.

The relationship between teaching practices and knowledge will be presented in the next section.

2.3 Knowledge and teaching practice

This following discussion focuses on mathematics classroom practices, cultures and norms, and their relationship to teachers' beliefs and students' knowledge.

2.3.1 Mathematics classroom practices, cultures and norms

(a) Classroom Practices and Cultures

Mathematics classrooms can be defined as particular kinds of social contexts (Boaler & Greeno, 2000) where learning activities are taking place which involve mathematical content, students and how learning occur (Franke, et al., 2007). Mathematics classroom practices can be interpreted as all activities that occur in the mathematics classroom under the classroom norms (i.e., expected classroom behaviour patterns) (Cobb & Yackel, 1996). Some specific classroom discussion practices include the use of symbolizations, arguments, and verifications of problem solving between teacher and student (Bowers et al., 1999). According to Cobb and Yackel (1996), the development of classroom practices occurred especially when students restructured their personal mathematical activities.

Mathematics classroom culture is the product of invisible beliefs, values and knowledge from classroom teaching and learning activities that influence the social interactions between the teacher and students (Nickson, 1992). Every classroom culture is unique because of the different participants (Nickson, 1992), the content, and the teaching designs and strategies that result in many different variations on classroom culture (Lampert, 2001). Although classroom cultures vary, it is still possible to categorize the classroom practices or the research focuses, for example, from the teaching content and methods, and the teaching objects (Nickson, 1992). Two additional dimensions of classroom cultures are

students' participation and mathematical thinking (Wood et al., 2006; Wood & Turner-Vorbeck, 2001).

Here the relationship among classroom cultures, teaching practices and students' learning will be discussed further. According to Lampert (2001), "The establishment of a classroom culture that can support studying is a fundamental element of teaching practice" (Lampert, 2001, p.53) and they will occur whether or not the teacher promotes it (Windschitl, 1999b). Establishing a classroom culture involves creating and sustaining norms within the teachers' teaching and students' learning (Donovan & Bransford, 2005; Lampert, 2001) and the norms will characterize the classroom culture (Franke et al., 2007). However, norms are often not mentioned (Windschitl, 1999b). Classroom cultures have been reported as supporting the progress of students' mathematics understanding through continuing mathematical discourse (Boaler, 2002a; Peressini et al., 2004; Franke et al., 2007). By providing opportunities for mathematical discussions to flourish, the classroom practices help to sustain the development of students' understanding of mathematics (Boaler, 2000b; Franke et al., 2007). Research evidence is illustrated in Section 2.3.3 (page 68).

Wood et al. (2006) argued that the traditional classroom culture was informed by the teacher's given information and instructions related to the textbook. In comparison, the reform classroom culture ("inquiry/argument" and "strategy reporting") consists of class discussions, students' pair work and students' instruction explaining (Wood et al., 2006). Students' activities in the strategy reporting classroom culture are mainly representing their problem solving methods and responses to the teacher's questions. The inquiry/argument classroom culture supports students justifying their reasoning when sharing their problem solving strategies and from the challenges of others, occurring during inquiry and discourse. The teachers provide stimulating environments by challenging and questioning students' understanding (Wood et al., 2006).

(b) Classroom Norms

A norm may be viewed as a general accepted pattern or behavior in a group (Cambridge Advanced Learner's Dictionary, 2008). Several factors affecting the

growth of classroom cultures include classroom norms, teachers' teaching styles, the history of school and communities and students' and family histories and identities. To further support the development of student participation, teachers may structure classroom norms or create learning contexts (Cobb & Yackel, 1996; Franke et al., 2007; Kazemi & Stipek, 2001; Simon, 1995). Teachers need to consider:

- participation or limitations of individual actions in classroom activities (Franke et al., 2007) (e.g., in what circumstances students are allowed to talk, or raise their hands),
- cooperation among students, supporting participation with language, e.g., in discussions (Franke et al., 2007; Kazemi & Stipek, 2001),
- showing respect for each other's ideas (Franke et al., 2007; Silver & Smith, 1996; Windschitl, 1999b) and acknowledging their mistakes (Kazemi & Stipek, 2001),
- exhibiting non-judgemental attitudes for students' right or wrong answers, or conflict of thought (Wood et al., 1991),
- persisting to find out the depth of students' understanding (Kazemi & Stipek, 2001), and
- using tools or manipulation to promote discourse (Franke et al., 2007).

The classroom norms can be structured first by the teacher (Cobb & Yackel, 1996; Lampert, 2001), and are jointly established through the teacher and students' ongoing and constant renegotiations to maintain regularities (Cobb & Yackel, 1996; Franke et al., 2007; Yackel & Cobb, 1996). Classroom norms can reflect the influences of classroom social interactions (Franke et al., 2007). Thus, classroom norms are established with common beliefs in the classroom about what teachers and students should do with respect to behaviour (including boundary) and accepted standards (Franke et al., 2007; Simon, 1995).

Moreover, the social norms of discussion include making sense of others' explanations, dealing with agreement or disagreement, justifying or questioning solutions and sharing different strategies (Cobb, Yackel & Wood, 1995; Cobb & Yackel, 1996; Kazemi & Stipek, 2001).

Regarding the characteristics of mathematical learning, the socio-mathematical norms allow us to explicitly address the mathematical aspects of teachers' and students' activities in classrooms (Franke et al., 2007; Kazemi & Stipek, 2001; Yackel & Cobb, 1996). Examination of such activities looks at ways in which they help to build students' mathematical thinking (Kazemi & Stipek, 2001) such as maintaining a classroom atmosphere to support problem solving and inquiry (Yackel & Cobb, 1996). Discussing different mathematical solutions or seeking a compromise among mathematical arguments is a socio-mathematical norm (Kazemi & Stipek, 2001). Socio-mathematical norms can also be illustrated according to the qualities or characteristics of mathematical solutions. For example, they may be viewed as homogeneous/related, reasonable, or efficient explanations (Cobb & Yackel, 1996; Franke et al., 2007; Yackel & Cobb, 1996). These aspects of the norms are more parallel with Lampert's ideas of the mathematical meanings when dealing with disagreement or to re-justify a mathematical explanation (Franke et al., 2007). The socio-mathematical norms can be regarded as extensions of general classroom social norms (Yackel & Cobb, 1996). These norms can reveal inner perspectives of the mathematical micro-culture in classrooms (Yackel & Cobb, 1996). Hence, the socio-mathematical norms might be substantially different from one classroom to the other (Yackel & Cobb, 1996).

Classroom practices and classroom norms are intertwining factors that develop and result in each other. Different classroom practices will result in different norms (Boaler, 2002c). Different classroom norms including social and socio-mathematical norms are useful in understanding how classroom practices progress (Franke et al., 2007; Kazemi & Stipek, 2001). Hence, it is expected that the norms will differ in the traditional and constructivist classrooms (Boaler, 2002c).

Beside the mathematical practices, classroom norms influence students' intellectual learning (Franke et al., 2007). Research evidence supports the idea that socio-mathematical norms benefit students' mathematical conceptual thinking in fourth and fifth grade. The classroom practices include (i) mathematical discussions that are more than mere description of procedures, (ii) encouraging multiple solutions to develop understanding, (iii) seeing mathematical mistakes as

an opportunity to sharpen students' thought, and (iv) promoting cooperation, individual student's accountability, and dialogue (Kazemi & Stipek, 2001).

The social norms within inquiry-based mathematics classrooms can foster the development of social autonomy and also intellect (e.g., Franke et al., 2007; Yackel & Cobb, 1996). According to Cobb and Yackel (1996), the analysis of socio-mathematical norms can help to better understand how teachers develop students' intellectual autonomy or participation in the classroom practices (Cobb & Yackel, 1996). Intellectual autonomy can be interpreted as students' willingness to apply their intellectual abilities to make mathematical decisions, judgements or arguments (Cobb & Yackel, 1996). While it is agreed that unproductive discussions can happen in classrooms, students need to employ their personal ways of judgment (Cobb & Yackel, 1996; Yackel & Cobb, 1996) about "what counts as a different solution, an insightful solution, an efficient solution, and an acceptable explanation" (Yackel & Cobb, 1996, p.473). These kinds of judgements are built up, when socio-mathematical norms are being established (Cobb & Yackel, 1996; Yackel & Cobb, 1996). Other norms that may develop include what contributes to mathematical reasoning: making assumptions, conjectures or reasoning arguments and revising conjectures (evidence-base proof) (Lampert, 2001). Promoting the formation of such norms might lead students to achieve ways of developing their intellectual qualities (Franke et al., 2007).

Another aspect of classroom norms to be examined is the structure of the interactions among the teacher, students and content in school (Lampert, 2001). The teacher acts as the source of institutionalized authority to establish social norms for students through initiating, guiding, and organizing students' renegotiation processes in classrooms (Cobb & Yackel, 1996; Lampert, 2001). Since social norms are established prior to social-mathematical norms, the teacher's provision of a classroom environment that is safe and comfortable would greatly increase the likelihood of student participation in class discussions, including proposing their ideas (Hunter, 2006a; Hunter, 2006b; Silver & Smith, 1996).

Lampert (2001) valued the idea that classroom mathematics instruction should be suitable for everyone. Further, mathematics is more than simply getting the right or wrong answers; it requires a high level of student engagement. The author suggested several ways to avoid discouraging students' academic self-confidence, such as creating a 'fair' grading system. The norm also will set up expectations for students' thinking and social roles in classrooms such as listeners or explainers (Wood & Turner-Vorbeck, 2001) while teachers may be called upon to wear several hats including that of supporters (Franke et al., 2007; Kazemi & Stipek, 2001), directors, guiders, and organizers (Cobb & Yackel, 1996).

A norm can be viewed as a general pattern of a group (Cambridge Advanced Learner's Dictionary, 2008) or common beliefs in classroom behaviour patterns (Franke et al., 2007). Mathematics classroom practices can be interpreted as normalised classroom activities (Cobb & Yackel, 1996). Mathematics classroom culture is the social interactive patterns of teaching and learning among teachers and students from invisible beliefs (Nickson, 1992). Therefore, norms, classroom practices and classroom culture point out certain forms of patterns with different focuses in classrooms and influence on each other. Interactions among norms, classroom practices, classroom culture, teachers' values and students' learning can be concluded.

To sum up, teachers' values and teacher-student interactions and renegotiations influence the structures of classroom norms (Cobb & Yackel, 1996; Franke et al., 2007; Yackel & Cobb, 1996). The sustaining norms of classroom actions and interactions will build up a classroom culture (Franke et al., 2007). The classroom culture is an essential element of instructional practices (Lampert, 2001). While classroom cultures (Boaler, 2002a; Peressini et al., 2004; Franke et al., 2007), norms or teaching practices support the development of students' mathematics understanding (Franke et al., 2007), one needs to understand the role teachers' beliefs play in students learning mathematics.

2.3.2 Teachers' Beliefs Influence Teaching Practice

According to Franke et al., "teaching is a principled decision-making that emerges from complex interactions between teachers' knowledge, beliefs and goals" (2007,

p.227). A teacher is important in cultivating the mathematics environment in the classroom (Franke et al., 2007) which is also supported by the socio-mathematical normative perspectives, especially in establishing socio-mathematical norms for students' activity (Yackel, Cobb, Wood, Wheatley & Merckel, 1990; Yackel & Cobb, 1996). Teachers' beliefs about knowledge will inform their teaching practices (Trotman, 1999; Anthony & Walshaw, 2007), as does teachers' pedagogical knowledge and teachers' mathematical competence/knowledge along with their classroom teaching experiences (Ma, 2010; Kilpatrick et al., 2001). Teachers' beliefs about the nature of mathematics are a key influence on their teaching practices (Cross, 2009; Sullivan, 2003; Szydlik, Szydlik, & Benson, 2003; Thompson, 2004). Some research has suggested different levels of consistency for this finding (Cross, 2009).

For instance, Cross' (2009) research indicated that teachers' perceptions of the nature of mathematics influenced aspects of their students' learning and classroom instruction. When mathematics was viewed by teachers in the study as mathematical formula operations, they perceived students' learning as successful in terms of their use of algorithms and the importance of memory and practice. Their classroom practices involved a focus on procedures and operations. One teacher viewed mathematics as "a way of thinking" and viewed learning as students developing their own concepts/knowledge individually or in a group setting (Cross, 2009, p. 338). This teacher's classroom practices were consistent with his beliefs wherein he acted as a facilitator and created chances for students to explain their discovery or problem solving processes to him (Cross, 2009). Another teacher acknowledged mathematics as a mixed type of perspectives with a focus on both conceptual and procedural knowledge, problem solving and building critical thinkers (Cross, 2009). This teacher illustrated learning as developing students' own concepts/knowledge while participating in process and valued both the importance of students' doing and participating for learning. Thus, her classroom practice used mixed methods: direct instruction and group work; both requiring students' explanations. She acted as a facilitator to help develop the students' own ideas (Cross, 2009).

2.3.3 Teaching practice influences on students' knowledge

Students' learning opportunities are substantially shaped by a teacher enacted curriculum and instruction in classrooms (e.g., Boaler, 2002a; Boaler, 2002c; Boaler & Staples, 2008; Lamon, 2007; Wood et al., 2006). Several studies have focused on the influences of one's teaching practices on student knowledge (e.g., Boaler & Greeno, 2000; Lamon, 2007; Pesek & Kirshner, 2000; Wood et al., 2006).

One such study was conducted by Boaler (2002a), who employed this situated lens in her 1997 research and opened two important avenues of exploration and understanding. She focused on classroom practices in a traditional and constructivist/alternative school to consider the relationship between students' knowledge production and the characteristics of their teaching and learning environments (Boaler, 2002a). When discussing her 1997 research, Boaler concluded that students' knowledge development consisted of the pedagogical practices in which they engaged (Boaler, 2002a; Boaler, 2000b), and further suggested that different classroom practices foster different students' understanding (Boaler & Greeno, 2000) consistent with other studies (e.g., Cobb, 2007; Lamon, 2007; Peressini et al., 2004). Cobb and Bowers (1999) through investigating a third grade mathematics classroom found that teaching practices impacted on students' thinking abilities, which in turn influenced classroom practices.

Boaler (2002a) in comparing the traditional and constructivist teaching environments found that, compared to the constructivist environment, the constructivist mathematics classroom practices increased students' thinking abilities allowing them to better apply their mathematical knowledge in diverse situations. The different students' mathematical abilities were linked to the differences in their classroom practices. According to Boaler (2002a, p. 43), "studies of learning need to go beyond knowledge to consider the practices in which students engage, and in which they need to be engaged in the future". Therefore one needs to look at how schooling empowers students in the integration and implementation of their knowledge to fit into society (Ford & Forman, 2006).

2.3.3.1 Transferable Abilities

Students learn to follow standard procedures of mathematical proof. This way of learning can be referred to as the ‘agency of the discipline’ (Boaler, 2002a, p. 45). Boaler (2002a) observed that ‘traditional’ classrooms are commonly associated with agency of the discipline, whereas reform classrooms are associated with student agency. Student agency implies that students use their own ideas and methods to solve problems (Boaler, 2002a). The term ‘Dance of agency’ refers to students’ flexibility to switch agencies based on the students developing and adjusting standard methods to match new situations when solving problems (Boaler, 2002a, p. 46). Boaler (2002a) concluded that the Phoenix, alternative/constructivist school, encouraged students to use mathematics in different situations or to ‘transfer’ mathematics, partly because of their knowledge, partly because of the practice in which they engaged and partly because an active and productive relationship with mathematics was developed.

In contrast, the lack of abilities of students in the traditional teaching school to apply learning to new situations, stemmed primarily from the procedural forms of knowledge they had developed in the school (Boaler, 2000a). From a situated perspective, the traditional teaching practices do not provide the opportunity for building up students’ mathematical concepts. According to Cobb, Yackel and Wood (1992a), if classroom practices do not allow students to converse, debate, alter and adjust their ideas or problem solving methods or provide chances for interaction with classmates and the environment, then students’ ability to transfer new information may be greatly hindered. This belief is also supported by a number of scholars (Boaler, 2000a; Greeno, 1991; Grouws & Cebulla, 2000).

A situated perspective proposes that teaching strategies are relevant with practice (situations), “not only enhance individual understanding, they provide students with opportunities to engage in practices that are represented and required in everyday life” (Boaler, 2000a, p.6). Herrenkohl and Wertsch (1999) in their research concluded that many analyses of student learning have focused only upon students’ mastery of knowledge but the ‘appropriateness’ of their knowledge was overlooked. They felt that students needed not only to develop the skills for

critical thinking, but they also needed to develop the skills necessary to make a connection between the content that they learned and the ways they relate to that knowledge (Boaler, 2002a). It therefore must be emphasized that one's mathematical ability is not only a function of knowledge, but also their capability that results from the complex relationship between knowledge and practice (Boaler, 2002a). The next section discusses further key characteristics to enhance high quality of education besides knowledge and practice.

2.4 Some perspectives on quality in mathematics education

The concepts of high-quality instruction, students' mathematical proficiency (or competency) (Silver et al., 1995), and teaching for understanding are connected and related to each other. This implies that, high-quality instruction always places an emphasis on teaching for students' understanding, and leads to students developing mathematical proficiency.

In looking at the reform movement in some countries (e.g., Taiwan) one can see that many teachers while going through the change process or the reformation, did not significantly change their teaching practices. Even when the teachers were using conceptually rich curricula in the U.S. (Ball, 2003; Franke et al., 2007) and attempted to teach NCTM standards in the U.S. (Franke et al., 2007; Hiebert & Stigler, 2000), or when a constructivist-based mathematics curriculum was implemented (Wey, 2007), or when the use of constructivist mathematics textbooks were implemented in Taiwan (Guo, 2004), many existing mathematics practices were still inconsistent with the reform (Ball, 2003; Franke et al., 2007; Wey, 2007). Thus, good resource or curricula guidelines cannot promise changes in classroom practices leading to good learning. However, good teaching practices might be the key to achieving it. Research on reform-based classroom practices can offer further information about the knowledge development that supports mathematical proficiency, including classroom practice or teachers' work (Franke et al., 2007), which leads to quality instruction.

High-quality instruction focuses on important mathematical content. It is expected that students have the ability to represent, integrate and develop the core mathematical content. Similarly, teachers also have abilities to detect students'

mathematical thinking, knowledge and developing ideas, and to encourage students to participate in classroom activities and utilize the knowledge gained (Franke et al., 2007; Lampert, 2001; Kilpatrick et al., 2001). Thus, high-quality instruction is able to inspire students to continually explore mathematics and advance students' mathematical knowledge and proficiency (Kilpatrick et al., 2001).

The close relationship between high-quality instruction and teaching for understanding can also be observed from reform-oriented empirical evidence of their consistency with each other. For example, Franke et al. (2007) examined Lampert's (2001) work and felt that her work was characteristic of teaching for understanding. Further, Lampert's (2001) work meets the criteria of high-quality instruction mentioned above (Kilpatrick et al., 2001). According to the National Research Council, the criteria include having in place structures for students to explore their own thinking, and cooperatively evaluate their thinking/assumptions during class discussions (Lampert, 2001). Moreover, some criteria include teachers coordinating mathematical conversation in class (Franke et al., 2007; Lampert, 2001), students' explanations of their ideas, cooperatively correcting wrong mathematical concepts, students making inferences and testing them, forming collective mathematical consensus, or students' generalizations all help to contribute to teaching for understanding (Franke, et al., 2007). The importance of students' representation for teaching for understanding was also mentioned (Franke, et al., 2007). Hiebert and Wearne (1992) concluded that "representation is one viable form of teaching for understanding" (p. 121). Promoting questioning offers opportunities for students to verbally reformulate and explain their ideas in detail; this helps to enhance the development of students' understanding (Franke, et al., 2007).

Some scholars (Franke et al., 2007; Webb et al., 2006) found that only giving high level mathematical questions or just arranging students into cooperative groups would not lead to changes in classroom practices that would improve students' mathematical understanding. The researchers found that having a great amount of problem solving in classrooms did not lead to enhancing students' mathematical

understanding (Franke et al., 2007; Carpenter et al., 1998; Fennema et al., 1996). Engaging students in classroom practice is therefore the key factor (Franke et al., 2007; Webb et al., 2006) that will enhance students' mathematical proficiency (Franke et al., 2007).

To conclude, high-quality instruction (i.e., teaching for student understanding) cultivates productive classroom practices that support the development of students' proficiency. Hence it echoes the importance of classroom social practices and students' learning from the situated learning perspectives (Boaler, 2002c; Peressini et al., 2004). Nevertheless, teachers have a key role in nurturing classroom practices that lead to students' productive learning, especially from their mathematical competence/knowledge along with their classroom teaching experiences (Kilpatrick et al., 2001; Ma, 2010). From this view, it also points out the need for ongoing teacher professional development to support teachers in promoting productive classroom practices (Borko, 2004). If the goal of the teacher's guidance is to generate students' understanding rather than train specific performance, then the teacher's task should be one of facilitating a mathematics environment wherein focus is placed on cultivating mathematical competence (von Glasersfeld, 1987).

2.4.1 Competence and Proficiency

Many countries seek to develop the quality of education provided to their citizens. Lampert (2001, p. 330) viewed mathematical competence as being "complex and multidimensional". Competence according to Wenger (1998) is the ability to perform some task well. Competence may also be viewed as products of the individual's conceptual organization of experiences (von Glasersfeld, 1987). Constructivism states that students establish mathematical competence through discovering their own approaches in problem solving (Lambdin & Walcott, 2007). Moreover competence is more than an individual possession; it develops interactively in practice (Lampert, 2001). For example, students may perceive themselves or their peers as "good or not good at mathematics" during discussion with classmates, either in groups or the whole class (Lampert, 2001, p. 358).

An examination of the American curriculum identified five strands that are related to students' mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al., 2001, p. 116). These "strands are interwoven and interdependent in the development of proficiency in mathematics" (Kilpatrick et al., 2001, p. 116). The strands point towards portraying students' ability to use what they know productively in solving problems (Kilpatrick et al., 2001). Once more, the evidence to identify an individual's mathematical understanding to support their mathematical competence, through confirmation to apply students' knowledge in new situations, also echoed other scholars' theoretical views (e.g. Gardner, 1994; Kickbusch, 1996; Perkins & Blythe, 1994; Sfard, 1998; Steinberger, 1994).

Further exploration of core competencies revealed similarities between the New Zealand (NZ) curriculum (New Zealand Ministry of Education, 2007) and the Taiwanese curriculum (Taiwan Ministry of Education, 2003, 2008). A comparison of both curricula is presented in Table 2.

Table 2 Similarities in Taiwanese and New Zealand Curriculum Competencies

New Zealand Curriculum Guidelines 2007	Taiwanese Grade 1-9 Curriculum Guidelines 2003, 2008
• Thinking • Using language, symbols, and texts	• Appreciation of beauty, performance, and creative abilities • Use of technology and information • Automatically explore problems and to research them • Independent thinking to solve problems.
• self-management	• Self-understanding and developing personal potentials • Organize personal plans for life and lifelong learning • Organize, make plans and apply the plans • Automatically explore problems and to research them
• Relating to others • Participating and contributing	• Share, communicate and express their views • Aware of cultural and international aspects • Cooperate with others and respect different opinions in team work

As shown in Table 2, the New Zealand curriculum identifies five key “competencies for learning and life” (New Zealand Ministry of Education, 2007, p.7). The first two competencies of the New Zealand curriculum and the top column of Taiwanese curriculum guidelines indicate students’ critical, integrated and applicable mathematical abilities to apply their knowledge/understanding in (new) situations and develop knowledge. Self -management in the NZ curriculum and the middle column of Taiwanese curriculum guidelines relates to students’ ability to independently or autonomously design their own learning. These competencies provide chances to develop student leadership. The last two competencies of the NZ curriculum and the bottom column of Taiwanese curriculum guidelines are linked to students’ proficiency in mathematical social learning (Taiwan Ministry of Education, 2003, 2008; New Zealand Ministry of Education, 2007).

Thus, the goals of education should be aimed at fully developing students’ abilities, including their content knowledge and processes knowledge. This view is shared by the National Council of Teachers of Mathematics (NCTM) standards which focuses on not only content knowledge but also on the inclusion of processes such as “problem solving, reasoning and proof, connections, communication and representation” (NCTM, 2000, p. 7), along with student products. To sum up, the mathematics key competencies have been pointed out from curricula guidelines in three countries (USA, New Zealand and Taiwan), including students’ procedural and conceptual understanding/knowledge, applying/integrating knowledge, (for example, Kilpatrick et al., 2001; New Zealand Ministry of Education, 2007; Taiwan Ministry of Education, 2003), interacting/relating with people (New Zealand Ministry of Education, 2007; Taiwan Ministry of Education, 2003) and positive mathematical values (Kilpatrick et al., 2001). However, the new goals from reform curricula still do not have enough power to establish reform mathematics teaching practices overall, such as seen in the experiences in USA or in Taiwan and teaching even went backwards to the conventional classroom teaching practices (Ball, 2003; Franke et al., 2007; Wey, 2007).

Based on the preceding discussion, it is necessary to examine ways to establish and promote good mathematics teaching practices to enhance Taiwanese students' mathematical abilities. In this regard, I support Fancy's (2006) comment, that if students are equipped with the knowledge, competencies, and confidence from schooling, they will be more likely to succeed in a constantly changing world (New Zealand Ministry of Education, 2006). While academically, Taiwanese students may appear to be performing well, the concern should be how best can we as educators prepare them to sustain a lifetime of learning.

In conclusion, this study of a Taiwanese perspective has attempted to explore student learning and the learning outcomes within the traditional and experimental/constructivist approaches. Educational background, theoretical learning and pedagogical perspectives of this study were presented in this chapter. Students' competencies in mathematics were explored to better understand their learning. Although the use of present curricula guidelines or national standards could not promise or bring about changes in classroom practices, they have shed light and provided important directions for future educational research. Thus, this research examined curricula to better understand students' proficiency in mathematics. The next chapter describes the research framework and design used in this project to explore the influence of different teaching practices on students' learning.

Chapter Three: Research Theoretical Perspectives

3.0 Introduction

This research focused on the influence of using contrasting teaching approaches. It started with a review of related studies, and developed by drawing information from relevant areas (Strauss & Corbin, 1990; Boaler, 1996). As a qualitative study, the use of content analysis provided a way in which the researcher could categorize data and search for emerging patterns and themes, in order to generate a comprehensive report (Alaszewski, 2006; Basit, 2010) and highlight new information in the field of study (Cohen, Manion & Morrison., 2007; Krippendorff, 2004). Thus, the categories and analyses of the teachers and students' perceptions, students' performance, classroom practices and the researcher's interpretations of the data will be interwoven throughout the discussion to a holistic account of the findings.

The theoretical perspectives as they relate to the study undertaken by the researcher are discussed in this chapter. Further, it provides an explanation of the rationale behind using a multi-faceted approach or triangulation of theoretical perspectives. Consequently, this chapter has addressed aspects of learning theories: situated learning and social constructivism, and the research framework that support the researcher to examine the quality of a mathematics education programme.

3.1 Research framework

This study adopted a content analysis approach from qualitative perspectives which were combined with the theoretical perspectives of social constructivism and situated cognition to develop theoretical insights for this research project. The use of a qualitative approach gives the researcher more freedom to be able to acknowledge and work with the different classroom dynamics and activities (Boaler, 1996; Demerath, 2006; Johnson & Onwuegbuzie, 2004). Discussion of a

theoretical model as it applies to acquisition and participation metaphors, along with sociocultural views of learning are discussed in the sections: 3.2.2.2 and 3.2.2.3.

3.2 Theoretical models of pedagogy and learning theories

This literature review provides a base for understanding the nature of mathematics learning. Attention directed to knowledge, understanding and meaning and different learning theories all have profound implications for the teaching of mathematics to all students, and at all levels.

3.2.1.1a Knowledge

Different perceptions about knowledge have been proposed. These include beliefs of knowledge as: (i) “a matter of competence with respect to valued enterprises”, for example, fixing machines and writing poetry (Wenger, 1998, p. 4); (ii) individual stable characteristics (Lave, 1988; Boaler, 2002a), and (iii) the individual’s conceptual product from learning (Voigt, 1994; von Glasersfeld, 1987). Hedegaard (1988) established a distinction between external knowledge (i.e., acquired knowledge from outside environments) and internal knowledge (i.e. individual inner knowledge). As individuals construct new knowledge, they integrate both internal and external knowledge to make sense of any given situation (e.g., Boaler, 2002a; Confrey & Kazak, 2006; Sfard, 1998).

Researchers have discussed the importance of language and social influences on students’ learning by addressing the issues of objectivity and subjectivity of knowledge (e.g., Brown, Collins & Duguid, 1996; Ernest, 1991). Ernest (1991), from a social constructivist perspective, described ‘objective mathematics’ as knowledge consisting of socially accepted or shared forms of linguistics expressions. These linguistic expressions evolved over time through processes. He perceived an individual’s knowledge as subjective and being shaped by one’s social environments. Ernest (1991) argued that subjective knowledge and objective knowledge influence and support each other’s development.

Another view of mathematical knowledge presented by Ball and Bass (2000b, p. 201) is ‘public knowledge’. This type of knowledge is described as knowledge

that is cooperatively constructed from collective and publicly shared views through explanations and justifications. Thus, developing communication skills is an essential element in classroom discussions, since it can lead to students constructing new mathematics knowledge (Confrey & Kazak, 2006). Public knowledge includes mathematical terms, procedures, concepts, expressions and problem solving (Ball & Bass, 2000b). Empirical research evidence supporting this type of (collective/public) knowledge has emerged from class discussions (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006).

An additional view of knowledge is that knowledge does not serve as a personal learning product but knowledge is socially constructed among people, activities, environment (e.g., Cobb, 2000; Boaler, 2002a), context and culture (Brown et al., 1989, 1996; McLellan, 1996; Wenger, 1998). This view aligns with tenets of situated learning. Here, rather than seeing knowledge as an individual acquisition, it is regarded as collective learning produced among individuals when reacting to situations (Greeno, 1997).

Supporters of traditional teaching methods may claim that students' knowledge could be increased by receiving and absorbing from their teacher and the textbook (Boaler & Greeno, 2000). According to Belenky, Clinchy, Goldberger and Tarule (1986), this way of learning or 'received knowing' does not give ownership of the learning to the student. Rather the source of power remains outside for the students; the teaching is teacher centred and rests with teachers and textbooks. Thus, this instructional approach is authoritative in nature, and students' knowledge comes from outside inputs (Boaler, 2002a).

Compared to proponents of the use of traditional approaches, reform-oriented teachers may argue that students' knowledge would be better developed through mathematical discussions. These teachers assist constructivist pedagogy and view mathematical learning as being socially constructed through meanings and explorations (Confrey & Kazak, 2006). To sum up, underlying these two different forms of teaching styles (reformed and traditional), students acquire two distinct types of knowledge - received knowledge vs. constructed knowledge (Boaler & Greeno, 2000).

3.2.1.1b Theoretical concerns about conceptual and procedural knowledge

Knowledge of mathematics consists of both concepts and procedures. In order to understand students' mathematical performance patterns in this study, it is necessary to explore the isolated areas (Haapasalo, 2003) and overlapped areas (Alibali, 2005; Haapasalo, 2003; Mason, Stephens & Watson, 2009) of conceptual and procedural knowledge. Over the last two decades, mathematics educators have seen fit to distinguish between these two forms of knowledge: procedural and conceptual (NCTM 2000).

- Procedural knowledge refers to the formal or symbolic expression of mathematics (Haapasalo, 2003; Hursh, 2004), and includes rules and/or (problem solving) procedures (Haapasalo, 2003; Hiebert & Lindquist, 1990; Hursh, 2004; Skemp, 1976, 2006; Star, 2000). It is used when carrying out routine mathematical tasks and normally requires automatic and not thoughtful reflection (Haapasalo, 2003). However, procedural knowledge could not be adapted into other situations because this knowledge is attached to particular problem modes (Alibali, 2005).
- Conceptual knowledge of mathematics may be described as 'knowing mathematics'. It is described as knowledge based on making meaningful connections and the usage of formula/algorithms among existing and new concepts or situations (Alibali, 2005; Haapasalo, 2003; Hursh, 2004; Skemp, 1976, 2006). This kind of knowledge can be generalised in new circumstances (Alibali, 2005; Hursh, 2004) or be presented in diverse structures (Haapasalo, 2003). It cannot be learned by rote but by thoughtful, reflective learning (Hursh, 2004). Thus, it is important to develop student conceptual understanding of mathematics (Hiebert & Carpenter, 1992; Hursh, 2004).

The distinction between procedural and conceptual knowledge continues in the classroom. Some researchers (e.g., Kadijevich, 2000; Sfard, 1994; Vygotsky, 1978) assumed that procedural knowledge occurs before conceptual knowledge. Thus, when applied to the classroom, teachers tend to begin with developing

procedural understanding and then reflect on the outcome (Baker, Czarnocha & Prabhu, 2004; Davis et al., 2000; Kadijevich, 2000). This can be referred to as the procedures first theories where students begin with the procedures, then after repeating practices of procedures, it is expected that repetition will lead to students developing conceptual understanding (Alibali, 2005). One example indicated students' preference for the procedures first approaches. Although, Pesek & Kirshner (2000) found that pure conceptual instruction was very effective for students' learning than the mixed method. However, there were half the number of students in the mixed instruction method group than the pure conceptual instruction group, who felt they learnt more from the procedural first teaching (Pesek & Kirshner, 2000).

On the other hand, it is generally accepted that developing conceptual understanding should be at the forefront of teaching and learning of mathematics (Kadijevich, 2000; Pesek & Kirshner, 2000; Resnick & Omanson, 1987). Thus, when applied to instruction, teachers would begin by building meaning for procedural knowledge before mastering it (Kadijevich, 2000; Resnick & Omanson, 1987). This can refer to the concepts first theories that students start learning first with conceptual understanding/knowledge and later, this understanding will lead to the development of students' procedural understanding/strategies in problem solving (Alibali, 2005).

Recent research shows that conceptual and procedural knowledge appears to be linked to each other and enhance each other (Alibali, 2005; Mason et al., 2009; Siegler, 2003). Examples can be found in the studies by Alibali (2005), Donovan and Bransford (2005) and Ma (2010). Ma's investigation (2010) of primary school teachers revealed that most Chinese teachers' instruction appeared to convey conceptual instructions through explaining the reasoning around procedural steps. By explaining each procedural step it is expected that students would develop their overall mathematical conceptual thought (Ma, 2010). When applied to developing high order thinking skills, some Chinese teachers would adopt conceptual strategies; some will only use procedural teaching, while others would combine both forms of knowledge (Ma, 2010).

To discuss further, procedural knowledge and conceptual knowledge have isolated areas (Haapasalo, 2003) and also overlapped areas (Alibali, 2005; Haapasalo, 2003; Mason et al., 2009; Siegler, 2003). Regarding the overlapped areas of knowledge, for example, students need to have conceptual understanding of the procedural rules to be able to successfully apply the combining knowledge into situations (Haapasalo, 2003). The conceptual knowledge can help students to understand mathematical procedures (Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993; Walston, 2000).

Considering the isolated and overlapped characteristics of procedural and conceptual knowledge, when analyzing students' knowledge, besides interpreting them as procedural and conceptual types, I also need to include the overlapped areas of knowledge and define it as conceptual-procedural knowledge.

To conclude, knowledge can be interpreted as (i) individual possessions (Ernest, 1991; Hedegaard, 1988; Lave & Wenger, 1991) including: competence (Wenger, 1998)/relative abilities (Lave, 1988; Boaler, 2002a), procedural (Haapasalo, 2003; Hursh, 2004) and conceptual operations (Alibali, 2005; Haapasalo, 2003; Hursh, 2004; Voigt, 1994; von Glasersfeld, 1987), or (ii) as not an individual's possessions but a socially collected form of production (knowledge) (Ball & Bass, 2000b) which is linked to classroom social function from situated perspectives, including from environmental factors (people, activities and systems (Lave, 1988; Greeno & MMAP, 1998; Cobb, 2000; Boaler, 2000a, 2000b), culture (Brown et al., 1989, 1996; McLellan, 1996), classroom instruction/curriculum (Boaler & Greeno, 2000), and class discussion (Ball & Bass, 2000b; Ernest, 1991).

3.2.1.2 Understanding

Any attempt at discussing student mathematical knowledge must take into account the role of understanding. Researchers have identified and classified different forms of understanding (e.g., Franke et al., 2007; Herscovics & Bergeron, 1988; Skemp, 1976, 2006). These forms of understanding include “relational and instrumental, concrete and symbolic, and intuitive and formal” (Pirie & Kieren, 1994, p. 165).

Relational understanding is similar to that of conceptual knowledge. It emphasizes making connections, understanding and managing relations (including rules and problem solving) (Skemp, 1976, 2006). This kind of understanding when developed empowers students to continuously and independently discover new thoughts and this understanding endures for long period of time (Skemp, 1976, 2006).

Compared to relational understanding, instrumental understanding is similar to procedural knowledge. It highlights understanding as formula-base without explanations, without generalization and textbook-methods (Skemp, 1976, 2006). Skemp (2006) perceived students' instrumental understanding as occurring due to their ability to apply mathematical formula and with an emphasis on memorization.

Lampert (2001, p. 5) defined understanding mathematics “as a matter of reasoning”. This involves a student’s ability to make and test conjectures and hypothesis. The author strongly argued that it is only when students have developed a strong foundation of arithmetic skills, will they make the reasoning progress smoothly. She suggested that “teaching mathematics would have to engage students in doing mathematics as they were learning it” (Lampert, 2001, p.5). This perception is also supported by Franke et al. (2007).

Perkins and Blythe (1994, pp. 5-6) described the difference between knowing and understanding:

When a student knows something, the student can bring it forth upon demand – tell us the knowledge or demonstrate the skill. Understanding is a subtler matter, which goes beyond knowing.... Understanding is a matter of being able to do a variety of thought-demanding things... like explaining, finding evidence and examples, generalizing, applying, analogizing, and representing a topic in a new way.... In summary, understanding is being able to carry out a variety of “performances” that shows one’s [knowledge] of a topic, and at the same time, advances it.

Gardner (1994) interprets concepts of understanding (1994) that are consistent with the above. His definition of an individual’s understanding includes the ability to apply what was learned to new situations, rather than just merely recalling what

was taught in the classroom (Steinberger, 1994, p. 1). Further, Gardner (1994) argued that one can only measure a student's level of understanding based on the individual's response to a given task. He said:

We can only really determine whether a student understands when we give the student something new and they can draw upon what they have learned to answer a question, illuminate a problem, or explain a phenomenon to someone else (Steinberger, 1994, p. 1).

Other views of understanding from a constructivist perspective describe it as the ongoing and dynamic growth of an individual's knowledge structure (von Glasersfeld, 1987; Pirie & Kieren, 1994).

3.2.1.3 Meaning

Mathematical learning can and must have meaning. This statement is viewed as the 'cornerstone' of all instructional planning and teaching. Wenger (1998) perceived "meaning" as the capability to meaningfully interpret the world. This is evident as we talk about our experiences and engagement with our social setting, whether individually and/or collectively. Meaning therefore, may be viewed as the ultimate product from what learning is to be produced.

Mathematical meaning is interpreted through various theoretical perspectives. Individual's reasoning and cognitive growth is emphasized from perspectives of cognitive psychology (Fang & Chung, 2005). Mathematical meaning is interpreted as independent in an individual's inner world from several philosophical perspectives (Voigt, 1994). However, mathematical meaning is defined as a synthesis of social interaction/ processes within a sociological aspect. Mathematical meanings are assumed to develop among individuals, rather than existing in an individual's inner world (Voigt, 1994).

Based on the different views of meaning, the concept can be interpreted from different theoretical perspectives with different focuses, as being constructed in individual minds, interactions and negotiations with people and social environments regarding subject matters.

3.2.1.4. Generality

Generality can be interpreted from both cognitive and situated perspectives.

Generality is often associated with abstract representations, with decontextualization.... The generality of any form of knowledge always lies in the power to renegotiate the meaning of the past and future in constructing the meaning of present circumstances. (Lave & Wenger, 1991, pp. 33-34)

Generality can be defined as an individual's ability to apply conceptual understanding of past experiences into future tasks (Brown et al., 1989, 1996). Generality, as seen from a cognitive perspective, looks at the individual receiving abstract forms of knowledge and procedures, then representing them to other situations (Greeno, 1997). From a situated perspective, "generality depends on learning to participate in interactions in ways that succeed over a broad range of situations" (Greeno, 1997, p. 7). Lave and Wenger (1991) stated that generality differs from knowing. They perceived knowing as an ordinary perspective that does not promise to lead to generality (Lave & Wenger, 1991).

3.2.2 Theoretical views of learning

Learning and knowing are not solely rational or logical activities. These concepts involve more than social renegotiation and reconstruction of meaning (Bell & Gilbert, 1996; Ford & Forman, 2006; Wood et al., 2006). Therefore, the theoretical concerns of learning in this study do not only address cognitive theory but also include social and situated perspectives. Sfard (1998) also supported the combining use of several learning theories, as is utilized in this study.

Currently, there are several views about learning which influence upon the learning of mathematics. These views include behaviourism, cognitive theory, constructivism, social learning and situated learning. The discussion that follows situates this research in a body of knowledge, incorporating different views that may be applied or used to inform teaching, curriculum and student learning. This section focuses on the following theoretical views of learning:

3.2.2.1 Social constructivism

3.2.2.2 Acquisition and Participation Metaphors

3.2.2.3 Sociocultural views of Learning

3.2.2.1 Social constructivism

Social constructivists interpret learning within social and cultural settings from a situated perspective (Smith, 1999). Here the focus is on interpreting learning within language and the social/cultural background, and might include the progress of individual learning (Smith, 1999). Smith used a metaphor to differentiate social and individual constructivism. That is, with the social constructivists “individual constructivists cannot see the forest for the trees”, while for the individual constructivists “social constructivists cannot see the trees for the forest” (Smith, 1999, p. 413). Thus, according to Smith (1999) both forms lack the ability to see the big pictures of what students learned.

The focus on constructivism has been debated by researchers. Confrey and Kazak (2006) critiqued several points of constructivism. For example, some researchers view constructivism as a theory of knowledge and as such, one has to apply its implications for instruction. They argued that researchers or teachers lack maturity in applying the tenets of constructivism into instructions. There are shortages of systematical summaries of constructivist research findings. Social cultural factors are over emphasised among constructivist research (Confrey & Kazak, 2006). While others also raise the concern that not all concepts need to be constructed (“Critique from constructivist”, 2006; Lesh & Doerr, 2003), such as some procedural or imitating work (Lesh & Doerr, 2003). Moreover, there are too many concepts in mathematics curricula and it is hard for students to construct all of them in classrooms (“Critique from constructivist”, 2006). Confrey and Kazak (2006) suggested that these areas of concern can be used as future research objectives that may lead to bridging the perceived gaps.

3.2.2.2 Acquisition and participation metaphors

The use of metaphors in this study supports the need of especially adopting situated learning theories to fully explain participants/students’ learning during the research process, along with the use of other learning theories. Combining metaphors provides a more robust way of explaining learning and or teaching (Richardson, 2003; Sfard, 1998).

Sfard (1998) described two methods of learning: acquisition and participation metaphors. As defined, the acquisition metaphor places emphasis on concept development and gaining possession of knowledge. Moderate or radical constructivism, interactionism and sociocultural theories tend to fall in this category (Sfard, 1998). From an analytic perspective, behaviourism and cognitive theories also belong to the category of the acquisition metaphor. Evidence of behaviourism which may be linked to the acquisition metaphor includes 'grasping knowledge' (Even & Tirosh, 2008; Greeno, 2003; Neyland, 1991; Peressini et al., 2004; Young-Loveridge, 1995), and passive concept development (Romberg, 1993; Young-Loveridge, 1995). Evidence supporting these emphases of cognitive theories of grasping knowledge can be found in the works of Greeno (2003) and Peressini et al (2004). Evidence claiming these emphases on constructivist theories of grasping knowledge are revealed in the arguments of Cobb (2007) and von Glasersfeld (2005) who state that knowledge is actively constructed by students. Evidence supporting these emphases on sociocultural theories of grasping knowledge are found in the arguments of Lave & Wenger (1991) and O'Connor (1998) stating that learning occurs not only in individuals but also when interacting within a social context.

The second metaphor, participation can be viewed as "part-whole relation" (Sfard, 1998, p. 6). Learning can be interpreted as a process of participating or taking part in the whole (Sfard, 1998). Hence, one examines the interaction between the part and the whole. The participation metaphor can offer alternative ways to interpret learning and help to avoid labelling people from their achievement, such as in the acquisition metaphor, because people's actions differ each day (Sfard, 1998). For instance, a smart student is not necessarily to be labelled as excellent every day; it is dependant each time on how well that student interacts while learning. However, the single use of this framework does not support interpreting learning, because it refuses the objectivity knowledge (Sfard, 1998). For example, Sfard (1998) argued that it cannot explain carrying knowledge in different contexts. Whereas, applying knowledge in new situations is essential in learning or explaining one's competence. Moreover, this participatory framework does not support the weak points in constructivism (including the moderate, radical or social constructivism), which is a lack of understanding of student agreement or consensus with others or

the connections of individual concepts with the public ideas, simply because it rejects the objectivity knowledge (Sfard, 1998), such as the social collective form of knowledge which is constructed from students.

It is therefore hard to separate these acquisition and participation metaphors, because the actions of acquisition are often combined with the actions of participation (Sfard, 1998). It is also not advisable to only choose one of these conceptual frameworks, since they each serve a different role in learning and a single focus may result in the loss of important meanings (Sfard, 1998). A disadvantage of only valuing the acquisition metaphor occurs when one labels an individual's product as a "quality mark" based solely on achievement. A participation metaphor does not explain knowledge applied in different contexts (Sfard, 1998). Hence, a focus on just one metaphor is insufficient in explaining learning such as constructivism. The strength lies in combining the advantages of both forms of metaphors (Richardson, 2003; Sfard, 1998).

3.2.2.3 Sociocultural views of learning

The acquisition metaphor was highly adopted in educational mathematics research in the last century (Forman, 2014). However, since the late 1980s, there have been new shifts of theoretical frameworks focusing on the social prospects of learning in the mathematics education field (Lerman, 2001). The new growth of theoretical focuses especially, has embraced language and social practices as fundamental and constitutive elements of "consciousness, behaviour and learning" (Lerman, 2001, p.97). Several frameworks have attempted to explain sociocultural views of learning and practice, including ethnographic frameworks (Greeno, 2003), participation metaphor versus acquisition metaphor (Sfard, 1998), discursive psychology (Lerman, 2001), social constructivist perceptions of learning (Lesh & Doerr, 2003; Smith, 1999), communities of practice (Wenger, 1998), situated learning (Lave & Wenger, 1991) and situated cognition (Lave & Wenger, 1991; Graven & Lerman, 2003) and practices (Boaler, 2002c; Lave & Wenger, 1991). Some of the above categories are common in many ways and are lacking in clarity, because they are established according to different ideologies which include education (Bell & Cowie, 2000), anthropology, sociology and psychology (Bell & Cowie, 2000; Greeno, 2003; Peressini et al., 2004). However,

Lerman (2001) has integrated some of the above theoretical frameworks, especially those which take account of language and social practices as essential elements of learning, as “social practice theory (also called situativity, communities of practice and situated cognition); sociology; and Vygotskian theories (p.97)”.

This sociocultural theory of learning can be considered in addition to that of cognitive learning theories, because mathematical meaning-generating and learning occurs not only in individual minds but also, it includes participating in social complex interactions among people and environments (Lave, 1988 ; Lave & Wenger, 1991; O’Connor, 1998), and culture and history (Wenger, 1998). Vygotsky (1978) claimed that learning stems from sociocultural interaction. He (1978) asserted "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). ... All the higher functions originate as actual relationships between individuals" (p.57). Meaning is generated when participating in sociocultural interaction, then the knowledge and understanding is intergrated into personal consciousness. Students’ mathematical abilities (i.e., including interpretation, explanations, solutions and justifications) should therefore not be seen as being merely individual competence but rather, their abilities should be viewed as simultaneous acts of participating in collective or communal social classroom processes (Bowers et al., 1999; Lampert, 1990b; Simon, 1995). According to Lave and Wenger (1991), “learning is never simply a process of transfer or assimilation”. Rather, it is complex because “learning, transformation, and change are always implicated in one another” (p. 57).

Learning taken in a social context occurs during classroom interactions (Boaler & Greeno, 2000; Franke et al., 2007), through participation in communities and organizations (Lave & Wenger, 1991) and through social/discourse practices (Boaler & Greeno, 2000; Wenger, 1998). Therefore, learning occurs from multiple dimensions of an individual’s integrated activities that include an individual’s everyday life experiences (Wenger, 1998), combining both

experiences outside and inside of school; collaborative interactions and collective constructive knowledge (Brown et al., 1989, 1996; Mclellan, 1996).

The instructional process in the social learning paradigm is measured by the social interactions (Voigt, 1994) that lead to logical progress (Doise & Mugny, 1984) and the growth of mathematical thinking (Hiebert & Wearne, 1993). Learning is viewed as reproducing and transforming the social structure (Wenger, 1998). Thus, within a culture, people communicate and modify ideas. Social conversation and interaction are significant in developing an individual's belief and learning (Brown et al., 1989, 1996). A social learning theory can inform academic investigations and is also relevant to design activities, organizations and educational policies (Wenger, 1998).

Several scholars viewed learning from a participatory metaphor rather than from individualism (Franke et al., 2007; Hanks, 1991; Sfard, 1998; Wenger, 1991). For example, Hank (1991) viewed learning as “a process that takes place in a participatory framework, not in an individual's mind” (Lave & Wenger, 1991, p. 15), and people engage in sense-making while participating together (Franke et al., 2007). Further, “participation is always based on situated negotiation and renegotiation of meaning in the world. This implies that understanding and experience are in constant interaction-indeed, are mutually constitutive” (Lave & Wenger, 1991, p. 51). However, participation is not easy to be identified, because of the often unspoken underlying purposes of the teacher, school or society (Franke et al., 2007).

Group activities during mathematical instruction provide an opportunity for students to engage in discussions. According to Brown et al., group activities promote “social interaction and conversation” to occur (Brown et al., 1989, 1996, p. 39). The authors summarized some features of group learning including: “collective problem solving”, “displaying multiple roles”, “confronting ineffective strategies and misconceptions”, and “providing collaborative work skills” (Brown et al., 1989, p.40). More details about mathematical discussion were documented in section 2.2.3.

The role of the environment in learning cannot be ignored. Voigt (1994) refers to Vygotsky's view of the environment. He claimed that one's environment and cultural practices seem to directly and tremendously influence their learning of mathematics (Voigt, 1994), and benefit their development (Kersaint, 2007). Boaler's (2000b) study also supports this statement. The individual internalizes given mathematical knowledge, which is influenced by cultural practices (Voigt, 1994). Thus, teachers must consider the importance of a learning environment, social practices, and the influence of these social practices on an individual's learning.

The aim of a sociocultural approach is consistent with the nature of this sociocultural view of learning to "explicate the relationships between human action, on the one hand, and the cultural, institutional, and historical situations in which this action occurs, on the other (Wertsch, del Rio & Alvarez, 1995, p. 11)." In order to sum up several of the scholars' theoretical frameworks (for example, Bell & Cowie, 2000; Bowers et al., 1999; Franke et al., 2007; Hanks, 1991; Lave & Wenger, 1991; O'Connor, 1998; Sfard, 1998; Voigt, 1994; Wertsch, del Rio & Alvarez, 1995), a sociocultural approach seeks to describe and explain relationships among the processes of learning and meaning-generating when participating in activities, environments, sociocultural and historical contexts. Research about social interaction and mathematics learning has been conducted in different countries. The social interaction patterns in classrooms were found to influence students' knowledge within the cultural context (Wood et al., 2006). Learning occurs during mathematical discussions (Driver et al., 1994; Voigt, 1994; Wood, 1999) as the learner negotiates meanings and develops mathematical ideas (Voigt, 1994).

From a behaviourist perspective, the teacher might assume that a students' weak performance when learning is due to insufficient opportunities to practice solving problems, whereas a constructivist might refer to the same problem as being due to insufficient opportunities for the student to develop their own understanding. Both views maybe accurate and there might be no single explanation to adequately understand students' weak performances when learning (Boaler,

2000a). One therefore may use another learning theory to explain more carefully the nature of the problem or rather students' failure in transitioning their knowledge to different situations, for example, a cognitive or situated learning perspective (see discussions in a later section of this study). Moreover, the concepts of social constructivism are different from sociocultural learning perspectives, for example, the theories of Vygotsky (Lerman, 2001). Vygotsky (1978) claimed that learning stems from sociocultural interaction and the generating of meaning is closely associated with culture first, then the new understanding is intergrated into an individual level. In contrast, social constructivists emphasize the learning behaviour within the learning processes. They announce that individuals, based on their experiences and previous knowledge, actively construct knowledge (Ernest, 1991) through interacting with people or cultural and social worlds (Hartas, 2010). However, sociocultural perspectives have changed the attention of constructivist research with claims of students' agency, beliefs and abilities in successful learning instead of social cultural issues (Confrey & Kazak, 2006).

Situative Learning

The term 'situative' has been coined from several fields including anthropology, sociology and psychology (Greeno, 2003; Peressini et al., 2004). It emphasizes contextually, that mathematical knowledge is situated in activities (Boaler, 2000b; Confrey & Kazak, 2006), especially from a constructivists viewpoint (Confrey & Kazak, 2006); for example, the work of Brown et al. (1989, 1996). Research focusing on situative learning is categorized by their analytic focuses: (i) psychological perspectives such as individual behaviour and cognition performance in successful learning (Greeno, 2003; Wenger, 1998); and (ii) social perspectives which include processes of interaction (discourse), especially under an ethnographic framework (Greeno, 2003). Or, sociocultural perspectives which comprise a crowd as an analytical unit, for instance, a classroom group, to interpret the social context and the patterns while students are involved in learning (Borko, 2004).

Researchers have expressed their views about the situative theories from the (ii) category, the social/interactional perspectives. For example, Hanks (1991) stated that the situative theory emphasizes learning linked with social practices and so does the relationship between human understanding and communication. Social/interactionists “mostly address the interactive relations of people with their environment” (Wenger, 1998, p. 13). The perspective of situative learning also supports the view that learning occurs with respect to the cultural environment learners are engaged in (Lave & Wenger, 1991) and practice (Boaler, 2002c; Lave & Wenger, 1991). Moreover, several scholars (Brown et al., 1989, 1996; Greeno, Collins & Resnick, 1996; Lave & Wenger, 1991; McLellan, 1996) summarized the situated characteristics of learning and suggested that learning occurs as an operation of the classroom tasks, context and culture (Brown et al., 1989, 1996; Greeno, Collins & Resnick, 1996; McLellan, 1996). Within these perspectives learning is viewed as a process that occurs due to changes in participation in socially structured activities (Even & Tirosh, 2008; Lave & Wenger, 1991; Peressini et al., 2004).

This situative perspective of learning has shifted away from the research focus of cognitive and behaviourist perspectives, towards the individual acquisition and use of knowledge when participating in social practices (Greeno, 2003; Peressini et al., 2004). However, it places less emphasis on students’ acquisition of mathematical knowledge but recognizes students’ informational representations in interactions as their contributions (Greeno, 2003). Even, the most extreme of situated perspectives ignore written documents (Wenger, 1998) or written tests (Greeno, 2003).

Furthermore, situated learning theories within a participatory framework will need to indicate socio-cultural perspectives (Lave & Wenger, 1991). Situative research normally investigates the consistency of patterns that include (i) interaction between individuals, groups, or materials (Greeno, 2003), or (ii) participation in the process of dialogue development and transformation resources across different situations (Peressini et al, 2004). This stems from the belief that learning is tied to the context or situation or environment that guides students’ learning (Boaler, 2000b; Kersaint, 2007; Voigt, 1994), and also to the belief that students’ culture

shapes their cognitive development, and that learning is extremely social.

Legitimate Peripheral Participation

Further, Lave and Wenger (1991) regarded all theories of learning as involving relations within the person and the world, especially in social practices. Legitimate peripheral participation places a situative focus on how learning occurs (Lave & Wenger, 1991). Brown et al. (1989, 1996) explained the ideas of Lave and Wenger and suggested that legitimate peripheral participation infers that individuals do not directly learn from a specific activity. Rather, they learn through internalizing and integrating from the surrounding social and cultural environment, even including political and historical background (Lave & Wenger, 1991). The key feature of legitimate peripheral participation is its focus on the kinds of social practices to prepare for learning to occur (Hanks, 1991; Boaler, 2000a). Thus, one may conclude that “there is no activity that is not situated”, even learning the curriculum (Lave & Wenger, 1991, p. 33). Boaler (2000c) also supports this view and states that “all learning is situated, and greater or lesser degrees are unavailable” (p 4).

Here one can also sense that situated learning has its roots in the work of social learning, e.g., Vygotsky’s theory. However, while one’s social practices influence on the individual’s learning, it does not necessarily result in the same level of individual learning. As a result, viewed from a situative perspective, participation and social relationship are key factors in learning (Cobb & Yackel, 1996). Further examination of the social context is needed.

Situated cognitive perceptions connect the cognitive and social perspectives (Lave & Wenger, 1991; Graven & Lerman, 2003). Cognition is generally influenced from the social and physical environment (Brown et al., 1989, 1996). The situated cognitive perspectives highlight the significance of the situation, context and culture in which learning occurred (Boaler, 1996; Lave & Wenger, 1991). Knowledge within situated cognitive perceptions is perceived as a product, which is principally shaped through the activity, context, and culture (Brown et al., 1989, 1996).

Mclellan (1996, p. 14) in reference to Brown's (1989) study stated that "situated cognition involves reasoning with causal stories, acting on situations, resolving emergent dilemmas, producing negotiated meaning and socially constructed understanding, and making sense out of complex, unclear data to solve problems". As Mclellan (1996, p. 9) pointed out, "situated learning provides a model for achieving a greater integration and balance between experiential and reflective cognition". Thus, critical characteristics of situated cognition appear in multiple practices and reflection on the learning process (Mclellan, 1996). In addition, situated cognition theories support the concepts of transferring an individual's knowledge and notions across different social situations (Boaler, 1996, see 3.2.3 section).

Other critics of situated learning looked at learning that occurs outside of cultural settings. Sociocultural features are not causal items for learning (Confrey, 1992; Smith, 1999), but they can be referred to as intrinsic (Lave & Wenger, 1991; O'Connor, 1998). Mathematical learning occurs when individuals participate in social complex interactions among people, environments (Lave, 1988; Lave & Wenger, 1991; O'Connor, 1998), and culture (Wenger, 1998). For example, Smith's (1999, p. 423) study raised a rebuttal question: "if all learning is situational, how could they explain for the inventiveness of people to resolve problems using methods unseen in their cultural traditions?" Therefore, Smith highlighted the critical role that students' inner and creative abilities plays on their academic performances. Thus, any evaluation of students' performance would be incomplete if the evaluation is seen only through examination of the environmental/cultural factors. This point is also argued by Confrey (1992), who argued about the risk of developing research solely on one theory. She doubted whether all learning was linked with the surrounding practices (social and cultural). In examining the social constructivist perspective, she argued that it may be insufficient to interpret educational events. The reasons may relate to the strong emphasis on social and cultural factors from social constructivists, however, student individual's learning may be ignored (Confrey, 1992; Smith, 1999).

Examination of the literature revealed that learning occurs within a social context (Cobb & Yackel, 1996; Lave & Wenger, 1991; Wenger, 1998), and that social dialogue has a general influence on students' learning and understanding (Boaler & Greeno, 2000; Wood et al., 2006). Social practices, especially classroom practices, influence on students' learning and bring out individual cognitive changes (e.g., Boaler & Greeno, 2000; Boaler, 2002b; Cobb, 2007; Lamon, 2007; Peressini et al., 2004). For example, when teachers provide opportunities for students to communicate, develop and negotiate ideas through dialogue, they are enhancing student learning through the social function of their classroom (Boaler, 2002b). Consequently, like Confrey, the author believes that one needs to explore all the alternatives to get the best mix. Hence, this research is grounded in several learning theories to explain the findings and also to find support from the combinations of metaphors.

Participation metaphor, situated perspectives and transformation

The participation metaphor (Sfard, 1998) and situated perspectives (Even & Tirosh, 2008; Lave, 1988; Lave & Wenger, 1991; Peressini et al., 2004) both view learning as participation in the whole. However, situated theories also value learning in the process of changes during participation (Boaler, 2000c; Lave, 1988; Greeno, 2003; Lave & Wenger, 1991; Peressini et al., 2004) towards individual's acquisition and use of knowledge (Greeno, 2003; Peressini et al., 2004). As a result, situated theories appear combining acquisition and participation metaphors, thus the use of situated theories in this study adds support to explain students' growth and application in learning.

In addition, it is argued that situated learning does not fully account for the role of transformation of knowledge in the learning process (Anderson, Reder & Simet, 1997; Peressini et al., 2004). Instead, the transformation of knowledge to different situations is better addressed from the cognitive perspective (Cobb, 2007; Peressini et al., 2004). Knowledge from the cognitive perspective is viewed "as an entity that is acquired in one setting and then transported to other settings" (Peressini et al., 2004, p69). Further, the cognitive perspectives offer the ways to explain the interactions among individual minds within the social context and also recognized the importance of the information-processing approach in acquiring,

analysing and transferring knowledge (Anderson et al., 1997).

Some have commented that these situated perspectives do not offer the analytic power to explain the transformation of knowledge in different situations, because it is hard to take into account the social contexts in these theoretical perspectives (Anderson et al., 1997). The others viewed that the situated perspectives do not value knowledge (Even & Tirosh, 2008).

However, some scholars (Boaler, 1996; Greeno, 1997; Peressini et al., 2004) are in opposition to this previous statement that these situated perspectives could not offer the possibility to explain the transference of knowledge in different situations. Boaler (1996) and Peressini et al. (2004) mentioned situated theories as offering perspectives on the relative influence/process of transferring knowledge to different situations of individuals. However, 'transfer' may not be appropriate to explain the generality of learning from a situative perspective (Adler, 2000; Greeno, 1997; Peressini et al., 2004), but "generality of knowing" is a better way of explaining about transformation of knowledge among situations in this perspective (Greeno, 1997, p.11). Nevertheless, the detailed discussion on differences in knowledge transformation (Peressini et al., 2004) or generalization (Greeno, 1997) from the cognitive and situative perspectives to interpret participants' applications of their knowledge across situations, are beyond the scope of the present study. Some (Greeno, 1997; Peressini et al., 2004) noticed that the situative perspective seeks successful participation to assist with different types of situations. The standpoint of this study was closer to the latter scholars' views through situated theories to understand the relative influence from different learning environments affecting the transformation or application of knowledge in new situations – seeking disciplines to reuse knowledge in different circumstances (Markus, 2001).

In summary, this literature review provides a base for understanding the nature of mathematics learning in a changing environment and the influence of such changes upon the teaching and learning of mathematics. Attention directed to knowledge, understanding and meaning and different learning theories all have

profound implications for the teaching of mathematics to all students at all levels. A summary of the different views of learning is presented in Appendix A (see my comments on p. 352). The combination of several learning theories will provide thorough theoretical perspectives to interpret students' learning from classroom practice/teaching styles. The details will be further illustrated in the next section.

3.2.3 Theoretical and analytical perspectives

This research adopted theoretical input from social constructivism. It was further supported by input from situated cognition. The learning theories of constructivism and situated theories will be used to interpret students' learning and classroom instruction processes. Moreover, the scope of this study did not draw from the views of cognitive psychology that focus on an individual's internal inferred interpretations (Cobb, 2007) but from a macro view to investigate the group performance such as the reasoning of different class group in the class activities. This study draws on research from experimental psychology to assess the relative influences of teaching approaches through collective measurement of students' particular knowledge (concepts) (Cobb, 2007; Lambdin & Walcott, 2007). Some might critique the study of experimental psychology that neglects in depth theoretical interpretations (Cobb, 2007). To complement this shortcoming, a multiple theoretical perspective was adopted to collect and analyse the data emerging from this study.

As indicated before both constructivism and situated theories give credence to this study. However, these two theories differ in the way in which learning is viewed. Constructivist and situated cognition perspectives appear differently in theoretical and practical conclusions (Boaler, 2000c). The supporters of constructivism regard learning as being shaped by the social world, while the followers of situated cognition theory consider learning as related to the world (Boaler, 2000c). Situated theories discuss learning relations among people, activities (Boaler, 2000c; Even & Tirosh, 2008; Lave & Wenger, 1991; Peressini et al., 2004), environments (Boaler, 2000c; Wenger, 1998; Voigt, 1994), practice (Boaler, 2002c; Lave & Wenger, 1991), and culture (Brown et al., 1989). Learning is also viewed as occurring due to active participation in the learning process (Even &

Tirosh, 2008; Lave & Wenger, 1991; Peressini et al., 2004). However, students' acquisition and use of knowledge when participating in social dialogue are under the analytical scope of situative perspectives (Greeno, 2003; Peressini et al., 2004). Situated learning looks at the relative influence and/or process of transferring knowledge to different situations (Boaler, 1996; Peressini et al., 2004) or generalising knowledge to other circumstances (Greeno, 1997).

Situated learning plays an important role in this study. If students did not engage in classroom activities that promoted discussion of mathematical thinking or investigation of new concepts, how would be the best way for learning occur? Students' mathematical knowledge would be developed similar to types of procedural usage (Boaler, 2000c). The use of situated learning is therefore used to address students' learning development and knowledge/competency as it relates to their participation practices (Boaler, 2000b; Confrey & Kazak, 2006). Moreover, since these aspects of learning are not discussed in constructivist theories (Boaler, 2000c) and are of importance to this study, the use of situated learning will play a vital role in this study. Moreover, participation practice as it is interpreted in learning may target students' involvement in class discourse (Boaler & Greeno, 2000), students' involvement in classroom activities (Sfard, 1998), relationship from the surrounding learning environment as explained in the previous section of legitimate peripheral participation (Lave & Wenger, 1991), or the process of students' concept changes (Boaler, 2000c; Greeno, 2003; Peressini et al., 2004). Data evidence that supported students' participation in learning were documented in the results chapters, for example, see Sections 6.2.4 (page 172) and Sections 7.2.4 (page 200) for the average time of class discussions, see Sections 5.2.6, 6.2.6 & 7.2.6, Section 8.3.1-the time interval count analyses for typical lessons of each classroom.

Further, using a situated perspective will offer a wider scheme to interpret educational practices in which individuals participate along with other people, material and learning relationship, for example, representational and conceptual material, and the awareness as contributors and learners (Greeno, 1997). Some scholars insist on including the social factors or activity as the situated analysis

(Borko, 2004; Peressini et al., 2004). Others have disagreed with the above statements but agree with the wider aspects claiming the scope of the situated perspective analysis as it relates to the purposes used (for example, Bowers et al., 1999; Cobb & Bowers, 1999; Peressini et al., 2004). Thus, the very nature of the situated perspective allows one to better examine classroom mathematical practices including interpersonal learning (Bowers et al., 1999; Peressini et al., 2004). An example of this is given by Bowers et al. (1999). They conducted an analysis of mathematics classroom practices and found that students' mathematical progress related to the social factors. Further, they were able to analyze students' activities as individuals and documented the various ways in which students engaged in practices. It is necessary as we investigate student learning from contrasting classroom practices, as in the case of this research, to adapt the focus to zoom in on classroom practices "as an integral part of generative social practice" (Lave & Wenger, 1991, p.35) to interpret their social context in classrooms and to analyze students' learning patterns (Borko, 2004). Therefore, the focus would be to better address and interpret students' learning and growth (Bowers et al., 1999; Lave & Wenger, 1991).

Based on the foregoing discussion, both learning theories are therefore important to this research. In an attempt to broaden our perspective of what we know about how learning occurs, this study will utilise situated learning theory to supplement constructivism. This combination will be significant in assisting the interpretation of students' learning. Additionally, the author believed that incorporating elements from both theories provided an in-depth theoretical framework for understanding student learning in the three classrooms investigated in this research.

3.3 Summary

This chapter describes the perspectives employed by the researcher in an attempt to understand student learning of mathematics and the need for quality education. Learning theories such as situated learning, and social constructivism are used to explain the findings with the expectation to gather new information to add to the body of knowledge on student learning and instructional approaches that promote quality education.

Situated learning when combined with a social constructivist perspective to interpret individuals' (i.e. teachers and students) construction of knowledge or interpersonal events such as activities and conversation (Ernest, 1991; Wenger, 1998), assist the researcher to better examine and explain the emerging patterns in teaching strategies, and students' learning attitudes and achievements in the classroom. It is the belief that using focuses from both perspectives would be advantageous to the researcher by helping to more adequately interpret students' cognition development through socially constructed understanding (Clancey, 1997; Mclellan, 1996; Wenger, 1998). In addition, the researcher was able to make sense out of data (Mclellan, 1996; Peressini et al., 2004) within a social context in order to make suggestions and/or recommendations for providing quality mathematics education (Mclellan, 1996). It is expected that the research findings of this study will serve to reduce the shortage of research evidence from Taiwanese classroom experiences about the strength of constructivist teaching (Wey, 2007).

This study's research framework is presented in Figure 1. It outlines the procedures for obtaining the data and the sources where information would be collected from. This study adopted the content analysis approaches from qualitative perspectives which were combined with the sociocultural and theoretical perspectives (for example, social constructivism and situated learning theories) to instruct/sharpen research methodology (triangulation methods including classroom observations, videotaping, interviews, student questionnaire, and tests). The use of the content analysis approaches from qualitative perspectives provided numerical analyses from/cross categories to focus on emerging patterns and themes, that led to the output of a synthesized report (Alaszewski, 2006; Basit, 2010) of new knowledge (Cohen et al., 2007; Krippendorff, 2004). The findings from this study wherein two different teaching approaches were contrasted (i.e., constructivism and the traditional direct teaching) at junior high level in Taiwan, would add new knowledge into the ongoing development of constructivist pedagogy to indicate the strength of long term

constructivist classroom teaching practices and norms of student mathematics competences and views.

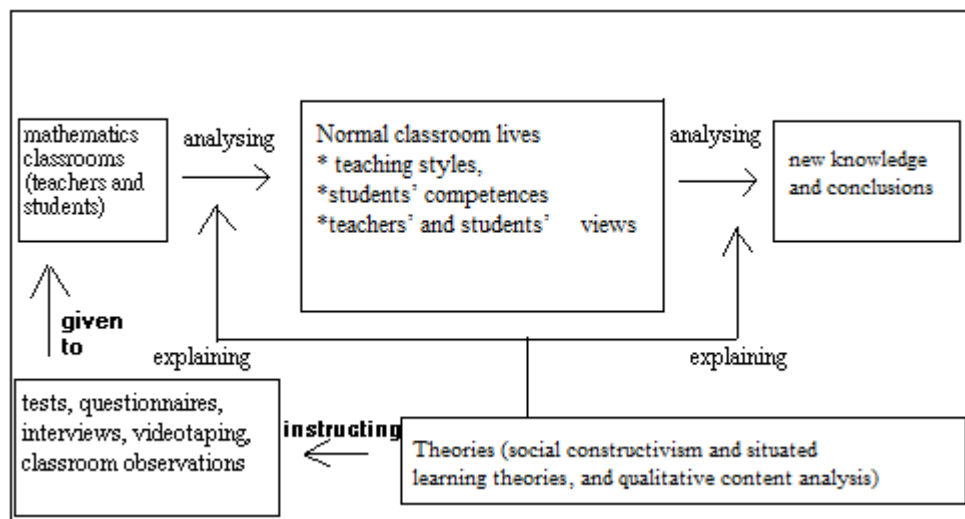


Figure 1 Research framework

Chapter Four: Methods

4.0 Introduction

This chapter introduces the research paradigm, the overall approaches of collecting and analysing data, and the ways of maintaining the quality of the research which are adopted in this study.

4.1 Nature of this study and research questions

The interpretivist paradigm is used to inform this qualitative research (Brooke & Parker, 2009; Lather, 2006). A number of scholars have used the term interpretivist paradigm as interpretive paradigm (Cohen, Manion & Morrison, 2011; Moll, Major & Hoque, 2006; Rubin & Babbie, 2008), or as interpretivism (Collis & Hussey, 2009; Rubin & Babbie, 2008). The word paradigm, here, refers to “a set of beliefs, values, and assumptions” about doing research, including epistemology, ontology (Greene & Caracelli, 2003; Johnson & Onwuegbuzie, 2004, p. 24; Rubin & Babbie, 2008; Willis, Jost & Nilakanta, 2007) or methodologies (Greene & Caracelli, 2003; Johnson & Onwuegbuzie, 2004; Niglas, 2000). The interpretivist paradigm seeks to comprehend the meaning of people’s behaviour and their inner subjective perspectives regarding the outside world (Cohen et al., 2011; Moll, et al., 2006; Rubin & Babbie, 2008). This form of meaning/knowledge building is from an epistemological perspective of interpretivism (Goldkuhl, 2012). The word interpretivism refers to recognizing or hypothesising the meanings of individuals’ subjective experiences of their social reality (Goldkuhl, 2012; Willis, et al., 2007). Epistemology explains how knowledge is constructed (Denzin & Lincoln, 2013; Gall, Gall & Borg, 2010).

Moreover, interpretive research intends to expose the participants' inner and personal views of reality (Cohen et al., 2011; Collis & Hussey, 2009; Rubin & Babbie, 2008). In the interpretivist paradigm, reality is subjective and hence there can be multiple realities (Check & Schutt, 2012; Cohen et al., 2011; Collis & Hussey, 2009; Lather, 2006; Moll, et al., 2006; Willis, et al., 2007). This form of

reality in the ontological perspective of interpretivism (Collis & Hussey, 2009) is consistent with social constructivism (Hartas, 2010). Individuals based on their experiences and previous knowledge actively construct knowledge (Ernest, 1991). Learning occurs during social interaction/dialogues such as teacher-student and student-student dialogues, students' explanations and justifications (Cobb, Wood, Yackel & Perlwitz, 1992; Ernest, 1991; Wood et al., 1991), arguments and negotiations that will produce a consensus/a social form of knowledge (Confrey & Kazak, 2006). Therefore, knowledge/reality of constructivist research is subjective and cooperatively constructed from human's multiple and social interactions (Hartas, 2010).

There is no wonder therefore that interpretative researchers are also called “constructivists” or qualitative researchers (Johnson & Onwuegbuzie, 2004, p. 14). Therefore, this form of interpretivist paradigm adopted in this research is also categorized as constructivist-interpretivist paradigm (Denzin & Lincoln, 2013) which is in contrast with positivist paradigm (Hennink, Hutter & Bailey, 2011). Positivist paradigm seeks proofs and generalizations (Johnson & Onwuegbuzie, 2004) or a causal relationship (Collis & Hussey, 2009). The other diverse forms of interpretivist paradigms can be seen in Denzin & Lincoln's work (2013) for a review. Moreover, ontology refers to “the nature of reality” (Creswell, 2007, p. 16).

Furthermore, interpretive analyses are often embedded in qualitative study (Ary, Jacobs, Sorensen & Walker, 2010; Collis & Hussey, 2009; Hennink, et al., 2011; Rubin & Babbie, 2008) in order to continuously explore in-depth understanding of the data, emerging themes and theories, existing theories and relationships among them (Ary, Jacobs, Sorensen & Walker, 2010; Collis & Hussey, 2009; Rubin & Babbie, 2008). The philosophical nature of qualitative study links to epistemology, but mainly performs in the phenomenological region (Taylor & Bogdan, 1998). This is related to the fact that a qualitative study involves interpreting specific social on-going phenomena or behaviours to develop deep understanding of human/personal experiences (Johnson & Onwuegbuzie, 2004). The knowledge of

qualitative work is constructed from people's subjective views of contexts/phenomena or their experiences (Holosko, 2001). This form of qualitative knowledge is consistent with the phenomenologist claims of human behaviour/knowledge "as a product of how people interpret their world" (Taylor & Bogdan, 1975, p.13). Therefore, this epistemology of qualitative knowledge appears to be subjective rather than objective such as from positivism (Holosko, 2001). The ways of subjective knowledge conducted in a qualitative study are essentially based on a phenomenological rather than positivist realm.

Principally, a qualitative research often includes profuse and deep illustrations and explanations/interpretations (Johnson & Onwuegbuzie, 2004). It also allows flexibility for the researcher to shift the research focus while conducting the work (Demerath, 2006; Johnson & Onwuegbuzie, 2004). The findings might be presented as synthesis of new knowledge, a theory or theory-related discussions (Gay, Mills & Airasian, 2012; Holosko, 2001; Gall, et al., 2010; Rubin & Babbie, 2008). Thus, the characteristics of a qualitative study can be described as subjective, flexible, meaningful and contextual, and enabling rich descriptions (Gall, et al., 2010; Rubin & Babbie, 2008). Additionally, a qualitative researcher performs as a researching "instrument" to continuously question the data and process within the whole research journey and inventively collect data and interpret them (Ary, et al., 2010; Gall, et al., 2010; Johnson & Onwuegbuzie, 2004, p.18).

The purpose of this study is to attain and interpret the research participants' views of their personal long term experiences of experimental-constructivist junior high school mathematics lessons in contrast to the traditional teaching in Taiwan. To attain the data from the individual teaching and learning experiences of a number of Taiwanese teachers and students, the interpretivist paradigm is used in this qualitative study to generate appropriate data to answer the research questions. The research questions for this study are:

1. What are the differences between the traditional and experimental approaches to teaching mathematics in Taiwanese classrooms and their influences on teaching practices and student learning?
2. How do classroom practices in the alternative school benefit students' mathematical learning attitudes, thinking ability, knowledge and achievement compared to the classroom practices in the traditional school?
3. What are the relationships between teachers' beliefs/perspectives relating to mathematics and teaching strategies, and the education provided for students?

Although qualitative and interpretive work is quite often challenging with excessively narrow focus to interpret the world from a social perspective (Cohen, et al., 2011), this interpretivist paradigm enables the researcher to draw deep understanding (Gay, Mills & Airasian, 2012; Holosko, 2001; Gall, et al., 2010; Rubin & Babbie, 2008) of the participants' thoughts, attitudes or performances regarding their classroom practices and their classroom phenomena (Rubin & Babbie, 2008) from rich data and also to conduct an in-depth study (Gall, et al., 2010).

To address the research questions, descriptive research and case studies are utilized as the “general approaches” or methodology of this study (Ethridge, 2004, p. 4). The approaches are discussed below.

Descriptive research

Descriptive research seeks to describe or explain relationships in words or numbers of the educational/social phenomena to identify features, patterns, practices or problems (Collis & Hussey, 2009; Ethridge, 2004; Reiss, 2011; Rubin & Babbie, 2008; Thomlison, 2001) from interpreting a rich collection of data and practices or documentation (Collis & Hussey, 2009; Ethridge, 2004; Reiss, 2011; Rubin & Babbie, 2008; Thomlison, 2001). Moreover, a good quality descriptive study or case study can be evaluated from pursuing in-depth, meanings of great amount of information (Cohen, et al., 2011; Reiss, 2011).

Descriptive research outcomes seek to add new knowledge from a research question and the results may offer a general image of phenomena (Thomlison, 2001), or conduct and present detailed in depth analyses from interpreting quality data (Rubin & Babbie, 2008; Thomlison, 2001). In consequence, these quality and detailed findings of descriptive study can add new knowledge to future research (Thomlison, 2001). An example of this is the adoption of ethnographic approaches to examine large qualitative data (Rubin & Babbie, 2008; Thomlison, 2001). That is, ethnographic researchers can analyse and describe from any form of quality data source to seek themes. This leads to the conducting of abundant interpretations of the context (Thomlison, 2001) and to provide “a phenomenological understanding” (p.20), for example, classroom practice (Delamont & Hamilton, 1984). A description technique in qualitative research has frequently been used. It provides quality and profound understanding of a practice/context, through a researcher intensively examining research context and data, and interweaving with participants’ subjective perspectives and experiences with regards to their surroundings (Rubin & Babbie, 2008).

For instance, I adopted words and frequency counts to describe the characteristics of participants’ opinions and performances, classroom behaviour and students’ tests, researching the meaning and understanding for their teaching and learning. The findings of the descriptive study from describing and interpreting participants’ opinions and performances can benefit the understanding of teaching and learning in the classrooms of mathematics (Thomlison, 2001).

Knowledge is generated from descriptive research located to enhance “practice, policy, and program services” (Thomlison, 2001, p. 131) and theories or theoretical directions (Reiss, 2011). For example, exploratory work aims at searching new understandings or themes of a new field, or a field where rare research studies were done (Collis & Hussey, 2009). It expects to elicit profound comprehensions of objects/participants with open research methods, focusing on a flexible nature to collect great amounts of qualitative data, such as, through a case study or open interviews to explore a deeper understanding of participants’ ideas (Collis & Hussey, 2009; Rubin & Babbie, 2008). The outcomes of exploratory work are possible to offer new knowledge for different research directions, for

example, to attain a primary results for guiding future studies, to examine an existing theory, to assist the development of a theory/hypotheses, rather than to provide a solution/conclusion in answering questions or to test hypotheses (Collis & Hussey, 2009).

In contrast, explanatory research is a more advanced analysis than descriptive research which constantly describes the characteristics of a study (Collis & Hussey, 2009). It goes further to unceasingly interpret/explain and identify inferential or causal procedure/reason of a study over a period of time and often involves the use of statistical/quantitative methods (Check & Schutt, 2012; Collis & Hussey, 2009; Rubin & Babbie, 2008). The findings of an explanatory research often examine and explain causal relationships among contexts, for example, discussing the influence of a variable (Collis & Hussey, 2009). However, descriptive research does not allow for interpretation of causal relationships between variables (Lauer, 2006; Suter, 2006; Thomlison, 2001), such as, the effect of an intermediation (Thomlison, 2001), or identifying a causal factor cross groups (Rubin & Babbie, 2008). Rather, it describes or explains relationships between variables (Collis & Hussey, 2009; Ethridge, 2004; Reiss, 2011; Rubin & Babbie, 2008; Thomlison, 2001.). Thus, the generalizability of descriptive studies is limited and it can possibly be achieved by the researchers if they specify the context and justify the findings. Generalizability may even be left to the readers to verify (Reiss, 2011).

Case study

The case study method is a powerful research tool to illustrate deep reasoning, disciplines or phenomena in real situations while focusing on a single or plural amount of people, groups, institutions or procedures (Ary, et al., 2010; Cohen, et al., 2007, 2011; Collis & Hussey, 2009; Gall, et al., 2010; Rubin & Babbie, 2008). A case study offers a descriptive, explanative and theoretical scope for analysing data (Cohen, et al., 2007, 2011; Gall, et al., 2010; Rubin & Babbie, 2008) of all aspects including qualitative sources (Rubin & Babbie, 2008). However, the criteria of qualitative data analyses places an emphasis on clear descriptive or interpretive direction towards research inquiry rather than finding causal relationships of the data (Cohen, et al., 2011; Gall, et al., 2010).

A case study could be conducted for various purposes, for example, to answer specific research questions, to explore a theory and to search a holistic view of a study (Cohen, et al., 2007, 2011). Thus interpretive paradigm is often adopted to meet the inquiry (Cohen, et al., 2011; Collis & Hussey, 2009; Denzin & Lincoln, 2013). Participants' opinions and the researcher's interpretations are interwoven to give rise to new knowledge (Gall, et al., 2010) and often present an in-depth and complex synthesis (Gall, et al., 2010; Rubin & Babbie, 2008). In this way, it is easy for readers to recognize the same features of a case to generalise to other cases that benefit from generalization (Gall, et al., 2010; Cohen, et al., 2007, 2011; Rubin & Babbie, 2008). Moreover, examination of the consistency from case study outcomes with a theory is frequently used (Rubin & Babbie, 2008). Sometimes, according to the analytical focus, noteworthy rare findings are more meaningful to acknowledge the inner perspectives of participants' subjective experienced worlds than the high quantity of data within a case study (Cohen, et al., 2007, 2011).

To sum up, the power and flexibility of the interpretivist paradigm within this qualitative study (Demerath, 2006; Johnson & Onwuegbuzie, 2004; Rubin & Babbie, 2008) enables the researcher to acknowledge the changeable characteristics of school/classroom activities (Boaler, 1996) and to continuously explore in-depth understanding of the rich data and their relationships (Ary, et al., 2010; Collis & Hussey, 2009; Rubin & Babbie, 2008). A case study provides a good means to present and comprehend the participants' world/reality and perspectives through their eyes and voices when interwoven with the researcher's in-depth interpretations (Cohen, et al., 2011; Gall, et al., 2010). The descriptive, explanatory and theoretical scopes of a case study (Cohen, et al., 2007, 2011; Gall, et al., 2010; Rubin & Babbie, 2008) allowed me to describe and interpret participants' opinions, performances, and classroom context by continuously questioning and comparing the data, process and theories for this study (Ary, et al., 2014; Gall, et al., 2010). The findings are discussed and summarized in this thesis (Chapters 4 to 10). Thus, the knowledge gained from this study (for example, the characteristics of contrast teaching and learning in mathematics classrooms, and the potential relationships within teachers, teaching practices and students'

learning) is expected to shed light (Holosko, 2001; Rubin & Babbie, 2008; Thomlison, 2001) or provide suggestions to future research or policy makers (Cohen, et al., 2011).

4.2 The context of the study

This research was conducted in two junior high schools in Taiwan, a traditional school and an experimental school during 2002 and 2003. When I started this study, students were just turned to Grade 9 year. In order to understand teachers' and students' long term teaching and learning experiences including their early Grade 7 and 8 years to shape their typical teaching practices. I need to adopt multiple research methods to evaluate and interpret their opinions (see Section 4.4).

The traditional school is referred to as School T in this study, and the experimental school, School E. The teacher in the traditional school is called Tom, while the male teacher at the experimental school is given the name Ed, and the female teacher is called Eve. The teacher (Tom) in the traditional school (School T) taught using the traditional direct teaching approach. The teachers (Ed and Eve) in the experimental school (School E) taught based on a constructivist view of learning. All three teachers taught the same curriculum and covered the same mathematics content.

The Traditional School (School T)

This junior high school is defined as a traditional school in this research. Based on observations of this school by the researcher for more than ten years, it was felt that the main aims of the school were similar to those of most schools in Taiwan. Teaching was focused on helping students successfully pass the entrance examination to the senior high school; it also focused on developing students' moral values and other talents besides academic achievement. This is a rural school. The size of the school is relatively small when compared to other schools in Taiwan. There were about 23 class cohorts from Grade 7 to Grade 9 in this school during the academic year of 2002-3. In order to improve students' achievement, each year this school - like other schools in Taiwan - offers opportunities for Grade 9 students to study late until 9:00 p.m. at the school. Every year there is one special talented class in this school.

Tom, the male teacher involved in this study, is considered a very good mathematics teacher by the school. Tom has been working for 24 years as a mathematical teacher in School T; when interviewed in 2002 (T1Ihp1Q1), he was a homeroom teacher and also taught four mathematics classes (T1Ihp9Q22). He normally would teach at least one special talented class cohort each year. Students in School T who participated in this study were in a special talented class in Grade 9 in 2002. Of the 27 students in this class, 26 participated in this research.

Evidence from my personal surveys held with other grade 9 students in 2001 had revealed that Tom helped students to better understand mathematics. Thus, it was expected that it would be interesting to examine how this experienced and excellent teacher conducts his mathematics teaching.

The Experimental School (School E)

The experimental school, established in the end of 20 century, was a rare case of a school using alternative teaching approaches at the junior high school level in Taiwan. However, the junior high level of this school was discontinued in early years of 21 century, while the senior high level finished after three years. The goal of this school included giving back the ownership of learning to students while promoting students to collaborate and cooperate with others. Further examination of the school revealed that a constructivist teaching style was used in mathematics classrooms by a Bureau of Education. Two scholars in early years of the 21 century regarded that teachers of School E did not automatically give direct answers, so students could develop independent thinking abilities from multiple dimensions, and abilities to solve problems and to make judgements (references removed for confidentiality). Students could choose their homeroom teachers, arrange their own learning schedule, and learn at their own speed. They used a school court and discipline cooperatively structured by teachers and students, instead of school rules, to maintain justice on campus (references removed for confidentiality). Students normally took nearly two years to learn self-management, self-acceptance and responsibility in such an open environment (references removed for confidentiality). The Ministry of Education and the Ministry of Justice in Taiwan have commented that this school performed well in

democracy and law education. The Union of Education Reform for Secondary School Students recommended this school as the best campus with respect to respecting students' rights (references removed for confidentiality). Students' learning styles were consistent with the spirit of self-learning and cooperative learning. However, mathematics teachers in this school had been asked by the Bureau of Education in an early year of 21 century to provide more mathematical subject matter knowledge in classrooms (references removed for confidentiality).

Eve, the female teacher who engaged in this research, earned a Master's degree in mathematics, had taught in primary school for seven years, and was a mathematics teacher in this junior high school for three year at the time of this study. (Of1Ihp1Q1t). Eve was also a homeroom teacher of one of the Grade 9 class cohorts (Of1Ihp14Q12m) and taught four mathematics classes for Grades 8, 9, 10 and 11, when participating in this study (Of1Ihp21Q21). Ed, the male teacher involved in this study, had been a mathematics teacher in an army school teaching mathematics at the junior high level (Om1Ihp9Q7e) for more than 20 years. Then he retired and came to this experimental school (Om1Ihp4Q1b). When Ed participated in this research project, he taught mathematics, science and sociology (Om2Ihp1Q1t). Further information on the schools, teachers and the coding systems used in this study are listed in Appendices N & O.

4.3 Participants

The participants in this study comprised a total of three mathematics teachers, one from a traditional junior high school (School T) and two from an experimental junior high school (school E), and one homeroom teacher of School T in Taiwan, and their Grade 9 mathematics classes. There were 27 students in Tom's class (26 students participating in this study), 12 in Ed's and 11 in Eve's. Tom had taught his class since Grade 7. Eve had taught theses 23 School E students in Grade 7, and Ed had taught these students in his Grade 8 mathematics class. In order to easily identify both groups of students in this study, School T will also refer to students in Tom's classroom, while School E refers to students in Ed's and Eve's classrooms.

All student participants were between 15 and 16 years or more than 16 years old, and had attended their junior high schools for at least two years. On this basis, they were able to share the learning experiences and the mathematics teaching practices during those two years and give feedback on the specific teaching style in their classrooms.

In order to present clear and short descriptions about the frequency of students providing given information in this study, a bracket is used, where n is the mean number of students who contributed the information. When presenting the comparison information, the first number in the bracket always represents School T, the second represents School E. For example, 14 students from School T and 13 students from School E contributing information would be represented in a short description as (n=T14, E13).

The students participating in this study were being instructed under the previous mathematics curriculum (the 1994 curriculum (Chung, 2005) prior to the education reform period, 1996-2004). Most of the students of both schools in this research had taken intelligence quotient tests just before starting Grade 7, and the IQ test results were comparable with 99% of coefficient accuracy, with $p < 0.05$ between the results in the Intelligence Quotient Test in Junior High School Level (the Third Edition), and the Intelligence Quotient Test in Academic Aptitude in Junior High School Level (Kuo, 1989), and with correlations between 0.7 ~0.9 (Kubiszyn & Borich, 2003). Thus, the students' results in the intelligence quotient tests served as an index to show the students' initial learning ability. Students in Tom's classroom of School T had, on average, a higher IQ in percentile rank than Eve's and Ed's classrooms of School E (58.40, n=T25 vs. 53.67, n=E18) (Lu, Cheng & Lu, 1991; Xu & Chu, 1986).

In addition to the Grade 9 students, there were six Grade 10 students attending the Grade 9 mathematics classes in School E, three in Eve's class, and the other three in Ed's class. Four of these six students were attending the Grade 9 mathematics classes for the first time, because of the freedom given by School E to students to choose subjects to study (SyQ1p.1). They had chosen to delay their attendance in

the Grade 9 mathematics class by one year. The other two students were attending the Grade 9 mathematics classes for the second time (one in Ed's class vs. the other in Eve's class (SyQ1p.1), because they failed their previous Grade 9 mathematics classes. Five students in School E chose not to sit the national examination. Student E3 self-studied for his Grade 7 mathematics lessons, so he did not attend Grade 7 classes in that year. Student E4 was absent from Grade 8 classes, because he was abroad in that year.

Students of both schools in Grade 9 gave information about one of their parents' educational background and careers. The data shows that the students' parents of School E had received a higher education (see Appendix Q) and more of them worked in middle class occupations than students' parents of School T (see Appendix P).

4.4 Research methods

As mentioned previously, this qualitative research has used an interpretivist paradigm design and case study. Therefore, multiple data generation techniques to encompass qualitative research data have been employed. Research within qualitative perspectives requires detailed and deep descriptions of personal perspectives and experiences (Johnson & Onwuegbuzie, 2004) to interpret complicated relationships of the data (Boaler, 1996). It allows for the voices of those being researched to be heard. Rather, the numeric data is used to inform and/or to provide a widespread understanding of the complex relationships in the data (Boaler, 1996). For this study, both types of data are important to supplement each other, in order to describe and explore the reality and the practice of mathematics teaching and learning within the three different mathematics classrooms.

The terminology of method here is identified as utilizing "specific techniques, tools" to answer the research inquiry (Ethridge, 2004, p.25). Hence, in researching these three junior secondary mathematics classrooms in Taiwan, multiple data generation techniques were used: mathematical tests, student questionnaires, interviews, videotaping and classroom observations and document collection. The collected data represents the range and variety of the practices in the sample

classes with their different teaching approaches, together with the importance of these in mathematics education. “Teaching is relational. Teachers, students and subject matter can only be understood in relation to one another” (Franke et al., 2007, p. 227). Thus, this study sought to view classroom practice as a whole to evaluate different perspectives from teachers and students, students’ performances and students’ relationships to mathematics. It was hoped this would bring out some possible influences of different classroom practices on students’ mathematical abilities including their mathematical understanding in relation to mathematics as defined from Franke et al. (2007), and forms of mathematical knowledge.

Multiple research methods were used in this study. Data collection involved classroom observations, video-taping, audio-recording, interviews with teachers and students, questionnaires, quizzes and tests given to students, and students’ results on the Intelligence Quotient test and on the National Entrance examinations from the Student Affairs Office in each school. The focus of each classroom observation, interview, questionnaire and quiz was influenced/adjusted by the continuous feedback of data (Boaler, 1996). Each of the data collection and generation techniques is discussed below in turn.

As a result, these multiple sources of data were analysed, and provided the opportunity to triangulate the findings (Boaler & Staples, 2008) to achieve good validity (Boaler, 1996; Cohen et al., 2011; Franklin & Ballan, 2001) and reliability in the understanding of teaching and learning within the two schools.

4.4.1 Questionnaires

Questionnaires are used to collect data about students’ perceptions with a quick contact to save time (Cohen et al., 2007). The purposes of the three questionnaires used in this study are described below:

Questionnaire 1:

The first questionnaire was adapted from Yeh’s study (1993) and was designed to explore classroom atmosphere, including students’ views about teacher support, peer support, satisfaction with the mathematics class and the strength of a class

group (Yeh, 1993). Students' feedback was sought to portray the strength of different classroom practices in the two schools and to answer the research question about the general picture of classroom practices and student learning. According to Yeh' category (1993), the first questionnaire could be summarized into four sub-areas as Table 3 illustrates:

Table 3 Four sub-areas of the first questionnaire (classroom atmosphere)

Sub-areas	Question	The number of questions
teacher support	1 to 7, 9 and 12	9
peer support	13 to 22, and 24 , (28)	11, 1
satisfaction with the mathematics class	8, 10, 11, 23, and 25 to 27	7

Yeh (1993) categorized the twenty eighth question of the first questionnaire as to see the strength of class unity. However, the strength of class unity is beyond the scope of this study, so here this twenty-eighth question is categorized as peer support. The first questionnaire is documented in Appendix E.

Questionnaire 2:

Most of the questions in the second questionnaire were open-ended questions, and sought to answer the first and second research questions about the students' perceptions (Cohen et al., 2007; Rubin & Babbie, 2008) of classroom practice, relationships with their teachers, the nature of mathematics, their learning attitudes, and self assessment of their learning. Some questions were revised from Boaler's study (1997) and some from this researcher's design. One question came from the work of Gonzales et al. (2000). The second questionnaire is documented in Appendix F.

Questionnaire 3:

The third questionnaire contained both multiple choice questions and open-ended questions, and was designed to answer all three research questions about students' perceptions of teaching strategies and attitudes toward mathematics learning, for example, factors to improve mathematics learning, their favourite factor to enhance mathematics learning, mathematics value and motivation, and homework. These questions were designed by this researcher and some questions were adapted from those other researchers such as Boaler (1996 & 1997), Chang (1995),

Flockton & Crooks (1998), Gonzales et al. (2000) and Wong (2000). Teachers' teaching skills (Flockton & Crooks, 1998), students' valuing of mathematics and their motivational beliefs towards mathematics (Chang, 1995) were also investigated. The third questionnaire is documented in Appendix G.

The answering of some questions on the first and third questionnaire used the Likert-type five equal measure tools (Ary et al., 2002). The possible answers were 'totally agree', 'agree in some ways', 'no comment', 'disagree in some ways', 'totally disagree,' rated respectively as 5, 4, 3, 2 and 1 points. Hence, higher average points showed that a student had a higher agreement with the statement in question on the questionnaire (Yeh, 1998), and the higher scores meant that, from the student's view, he or she perceived himself or herself to have a better classroom atmosphere and motivation. (Only Question 1 and Question 2 in the third questionnaire used 7 point scales.) The three separate questionnaires were given to students to complete after their school examinations, at the teachers' convenience at the end of 2002 or early 2003. Either mathematics teachers or other teachers administrated the questionnaires, or students took the questionnaires home to complete them.

4.4.2 Interviewing

Interviewing is a powerful way of eliciting the ideas of people (Gay, Mills & Airasian, 2012; Rubin & Babbie, 2008), their knowledge, values and attitudes (Gay, Mills & Airasian, 2012; Tuckman, 1988). "An interview is often a verbally administered questionnaire" (Bainbridge, 1992, p. 74). Interviewers can use oral questions to get personal information: for example teachers' interviews, the opinions or beliefs from informants concerning some specific topics. Teachers' interviews served to answer the first and second research questions about teachers' beliefs and perspectives on mathematics, learning, education and teaching strategies. This study adopted individual face-to-face interviews and telephone interviews (Rubin & Babbie, 2008). One advantage of telephone interviewing appears that participants have freedom to raise their opinions, because they could not see the researcher face to face, they do not worry to conflict the researcher's opinions or not to limit their talk (Rubin & Babbie, 2008).

There were four different face-to-face interviews with each of the three teachers, except one last interview through the telephone to teacher Ed on May 11, 2005. Firstly, in 2002, the mathematics teachers were interviewed for approximately forty-five (45) minutes about their views of mathematics and mathematics teaching. The interview schedule is documented in Appendix B. Secondly, when necessary, participant teachers' brief comments were sought after every class lesson. At this time, the focus was on the teaching plans, their thoughts about the delivery of instruction and any suggestions about the research. Thirdly, at the end of a sequence of lessons for the video-taping class observation, a post-interview of twenty minutes was conducted to find out if there were any changes to their initial perceptions (see Appendix C). Finally, in May of 2005, I re-approached and interviewed the three teachers to elicit any further changes or suggestions of their perceptions towards mathematics education and mathematics teaching. This last interview lasted from 10 minutes to an hour, depending on how much the teacher wanted to share (see Appendix D).

Prior to every interview, participants were asked for available places and choice of time for the interview sessions. Pre-structured open-ended questions were used to solicit responses (Best & Kahn, 2006). The questions were asked in the same order during each teacher's interview. All answers were accepted without any comment to avoid researcher prediction or bias, and to ensure that the researcher would not lead the interviewee's thinking or responses. Any misunderstandings encountered by interviewees' were readily clarified. When the interviewees' responses moved too far away from the interview questions, they were guided back to the focus question.

All interviews were conducted in Mandarin Chinese, that is, the mother tongue of the teachers and students interviewed, and the researcher. The translations are the researcher's interpretation of what the interviewees said. In some instances, follow-up short interviews were used to clarify some of their ideas and to explore some points, which they did not mention in the oral interviews. When teachers sought the researcher's views on particular practices, no comment was offered about what was involved in being a good mathematics teacher, and the researcher

tried not to influence or change their thinking (Cohen et al., 2007; Cohen & Manion, 1994).

Student Interviews

Short interviews of less than 10 minutes were conducted with the students. The interview was used to answer the first and second research questions about students' perceptions of classroom practices. They were also used to clarify students' thoughts, and any mismatches between their responses to the questionnaire and their behaviour in classes. The audio-taped interviews were conducted in school between classes or by a short telephone interview. The interview schedule is documented in Appendix H. If a student refused to participate, his/her wish was respected. Again, the researcher conducted a follow-up interview, if needed. After each interview, a brief verbal summary was given to the participant in order to check the accuracy of the researcher's perception. For example, telephone interviews were conducted to explore 12 students' opinions about how they use things from everyday life to solve mathematics problems.

4.4.3. Observations: video and audio

Observations

Observation is a useful tool to collect information on classroom events, even insight information in circumstances, but it may be time consuming, especially in data analysis (Cohen et al., 2007, 2011). In this study, the researcher's role in the mathematics classrooms was that of observer. Besides observing the classes, the researcher made field notes (Gall, Gall & Borg, 2010) also recording the time of key activities of teachers and students) and made sure the video was working properly. If students sought the researcher's assistance, I would show respect but remind them of the tasks that their teachers had set them. This helped me to develop positive relationships without altering classroom dynamics. Whatever my level of involvement the researcher had in a classroom, my presence could be an influence on the classroom, so it was incumbent on to interpret what was experienced relative to this involvement (Jaworski, 1994). The classroom observations served as supplemental tools to answer the three research questions of classroom practice, a check on the data from teachers and students about

classroom practices, and an exploration of new perspectives that teachers and students did not reveal in the interviews (c.f. Cohen, et al., 2007).

The classroom observations were also video- recorded and audio-recorded and the information is summarized in Table 4 below:

Table 4: Classroom observations

Teacher's name	Number of lessons video-taped	Mathematical geometry unit
Tom	26	2-2, 3-1, 3-2, 3-3
Ed	14	3-3
Eve	20	2-2, and part of 3-1

(Sy.all.vt.p1t)

The video and audio data provided an extensive record of classroom transactions and provided rich analysis opportunities for the researcher (Stigler et al., 1999). The data source of video and audio was in addition to the field notes and helped to answer the three research questions on classroom practice including teachers' teaching strategies, classroom management, the time spent on each of the teaching and learning tasks, the frequency of questioning and the coverage of classroom curriculum.

Three geometry units from Grade 9 mathematics syllabus were the focus of instruction during the period September 2002 to January 2003. The number of classroom lesson observations is summarised in Table 4. One video camera and audio tape-recorder were placed in the mathematics classroom. The video camera was placed at the back of the classroom to capture the teacher's instruction and students' reactions and interactions. The researcher consulted with the mathematics teacher to ensure that the placement of video cameras and my presence would have minimal influences on classroom teaching and learning.

At least two weeks prior to any classroom observation, the video camera and an observer were present in the classroom. This was to ensure that the teacher and students became familiarized with the presence of the video camera and an observer and, as such, treat them as part of their class. It was intended that their behaviour would have returned back to normal classroom practices by the time the data collection began.

One student in School T did not consent to participate in this research, and so the video camera was not used to shoot that student's learning situation, but only the teacher's teaching in the classroom and the rest of students. Initially, the researcher recorded the students' learning situation by Boaler's 'time on task' technique (1996, p. 37), as well as gathering full field notes of classroom transactions. The number of students, who appeared to be working, appeared to be not working at three points of class time (the first 10, 25 and 35 minutes in a lesson) were counted. The field notes recorded the time of each event. For example, the length of time was used by each person that who gave a talk (hours, minutes and seconds) and key content. It also recorded the time (hours, minutes and seconds) of some important/special events (for example, one student came to help another student or several students helped each other). I often need to quickly draft students' seat map in my field notes to indicate students' interactions, besides writing up classroom practices at three points of class time. As well as this numeric data, records of student interactions were also included. I was always busy in observing and writing up issues when conducting classroom observations. Normally, every minute or two or three minutes within a class discussion, I needed to trace and write down who gave a talk and key content, and when possible, others' reactions. Very soon, I found my field notes convey much more information than the records of the three times on task method. Therefore, I analysed my full field notes rather than the selected results of three times on task. I referred this type of field notes conducting as "event time recording". This method is beneficial because it enables the researcher to trace the duration of certain tasks and look at issues such as "time consuming" or diagnose key events in field notes. For instance, one advantage of using these detailed notes helped the researcher later to be able to calculate the time of teacher/student talk in a class from my field notes instead of examining all video tapes.

4.4.4 Tests and quizzes

Two forms of written assessments, tests and quizzes were given to students. Quizzes and test can collect information on students' knowing and what they are able to do (Lampert, 2001; Cohen et al., 2007). Tests convey a number of mathematical questions related with the mathematics textbook in order to assess

students' mathematical knowledge regarding the textbook content. Three 45-minute tests were administered by the three teachers in their mathematics classes to evaluate students' short term mathematical content abilities within three mathematics units of the textbook (2-2, 3-1, 3-2) in 2002. Each teacher designed one unit's test, so as to reduce test design bias from favourites or certain design of test. The three tests assessed a combination of both procedural and conceptual problems (see Appendix F1). The results of the tests could indicate students' content knowledge abilities in mathematics.

The quiz items assessed conceptual (including conceptual-procedural) understanding and included problems applied in everyday life. There were 15 problems in total and 11 of these 15 problems were problems applied in daily life (see Appendix Z & B1) to investigate students' mathematical ability. All these mathematics problems covered the mathematical skills which students had learned. In contrast to Boaler's work (1996), Boaler adopted two mathematics activities and work sheets related to life context to assess students' mathematics abilities in applying into life context. I intended to use more questions to offer me more chances to interpret students' capabilities. Thus, I revised one of Boaler's (1996) life applied mathematics activities and her two other mathematics assessment questions related to life context as three quiz items. Six other quiz items revised from other mathematics books one of which is adopted from my previous supervisor and another one is revised one from an early version of mathematics textbook (see Appendix Y). However, the above 11 problems applied in daily life could not really specify students' mathematics abilities in dealing life issues. This is mainly because these were only conceptualized written problems related with life context and students were situated in classrooms for these quizzes. Nevertheless, the results could offer beneficial information about how well can they apply their knowledge in life related context (c.f. Boaler, 1996).

There were four quizzes in total, which were administered by three teachers and the researcher, before the teachers taught the mathematics textbook. In this way, the researcher expected that there would be more new questions for students on these quizzes. Hence, students' ability in applying their mathematics learning in

new situations could be evaluated. These quizzes took about 8 minutes and were carried out in mathematics class.

In addition, two other quizzes related to geometry units in Grade 7 or Grade 8 were also given to the students in 2002 and early 2003. Students took twenty minutes for each of these quizzes. These assessed students' understanding and long-term memory of previous learning. All three tests and quizzes were evaluated by the researcher. Therefore, students' performances can be understood by the same criteria.

Student results on the National Entrance Examination, which tested all mathematics content at the junior high level, were collected in May, 2003. This was to understand students' achievement patterns of three classes among two schools. With respect to the qualitative nature of the work, this study did not research the causal or correlation relationships between students' Intelligence Quotient (IQ) and their performance in learning. Rather, the data was collected as a way of understanding of background issues to indicate some starting points of students' ability of two schools.

The information from three tests, six quizzes and the national examination was obtained to answer the research question on students' ability in applying mathematical knowledge in situations, knowledge and achievement. The quizzes used in this research are presented in Appendix Y. Appendix Z shows students' working time for quizzes, and Appendix A1 contains the assessment criteria. The tests (textbook focus), quizzes (related life applied ability focus) and the national examination data were obtained to describe and comprehend the characteristics or potential influences of the students' performances rather than to pursue achievement excellence of cross school comparison. In order to understand the characteristics of students' mathematics performances, the overall average performances of the students from tests, quizzes and certain category of quizzes in the different classroom environments of two schools would be presented. For instance, a discussion related with students' average performances in each item of conceptual-type quizzes indicated any differences to illustrate different strength of two schools.

Analyzing students' performance and understanding

This research also expected to look at whether students could apply their mathematics thinking to new situations, if they could carry ideas into new circumstances that identify their mathematical understanding to support their competence (Gardner, 1994; Kickbusch, 1996; Perkins & Blythe, 1994; Sfard, 1998; Steinberger, 1994). Therefore, quite a few of these 15 quiz items were selected from other resources to avoid repetition with the textbook. It was supposed that students had not seen these quizzes before. If they could answer well, that could mean that they have the ability to adopt mathematics thinking to new situations (cf. Boaler, 1996).

Comparing Data from Three Classes

“Classroom research is part of social science” (Delamont & Hamilton, 1984, p.22), and class groups and settings never have equal circumstances with each other (Delamont & Hamilton, 1984). Especially, students of School T had higher IQ than students of School E, therefore the chances to interpret students' performances within non-similar IQ class groups are limited. For example, it is a challenge to interpret the better student performances of a high IQ class, when compared with a low IQ class. The reasons are fuzzy, because the excellent student performances of a high IQ class might link with students' own abilities, not with other factors, for example, a teaching style. However, if the average lower IQ classes (School E) could perform better than the higher IQ class (School T), that indicated that teaching styles or other factors of School E might have some potential influences to elevate students' mathematical ability. This kind of instance would be the chance to acknowledge some potential ways of the teaching power within dissimilar average IQ classes for this study.

The research findings from the main study of the three teachers' teaching styles are presented in the form of three case studies in Chapters 5, 6 and 7. This will lead to an understanding of the cultural environment (classroom teaching practices) that the students encountered and in which the learning is taking place (Lave & Wenger, 1991).

In these three chapters, curriculum enacted from three teachers' own views (teachers' interviewing data) and from the researcher's classroom observations (field notes and video-taping data), teachers' views about mathematics and teaching styles/practices (teachers' interviewing data), and students' perceptions about their teachers' teaching styles (students' data from the second and third questionnaire) were discussed, followed by comparisons with and discussions of the literature in Sections 9.1.4 and 9.2. The influences of three teachers' teaching practices on students' knowledge (students' data from the second questionnaire)/understanding (students' data from the second and third questionnaire), achievement (students' results of the three tests, 15 conceptual quiz items and the National Entrance Examination), and students' views (students' data from three questionnaire and following up interviews) were presented and discussed in Chapter 8.

4.4.5 Data Collection Procedure and Timeline

Procedures for recruiting participants and obtaining informed consent

Three mathematics teachers were approached to be involved in the research project in 2001. Letters of information and consent forms were prepared. Letters explaining the nature and purpose of the research and soliciting interest were presented to the two school principals in September 2002, with verbal explanations and the guarantee of confidentiality and anonymity, asking them for permission for the teachers to be involved in the research, and permission for access to student information (see Appendix J). Once permission had been given by the principals, the teachers were approached for their informed consent. Personal approaches were also made to the homeroom teacher of School T to assist by administering student questionnaires or allowing the researcher to administer tests or short quizzes during her class times. Letters of information and consent forms were given to the students in these three teachers' classes (see Appendix K) and consent was sought.

To recruit the class students, the researcher explained the research project to the class and gave a guarantee to maximise confidentiality and anonymity (Rubin & Babbie, 2008). Letters outlining the nature of the research and seeking informed consent (see Appendix L) were given to the students to take home and discuss

with their parents, with expectations of receiving their consent.

Procedures and timeline for collecting data

After all had consented, in September 2002, the data collecting began. Firstly, students' background information (the results of Intelligent Quality tests, and their family background) was obtained. Teachers' and students' interviews; three questionnaires, three mathematics tests and six quizzes to students, videotaping, sound recording and field note taking of classroom instruction were conducted during the period of late September 2002 to early January 2003. The results of students' national tests were obtained after May 2003. Follow-up telephone interviews were done with three teachers and 12 students to clarify their ideas during the first half year of 2005. One class observation and document collection were conducted during the first half year of 2006. The details about time frame of research methods are summarized in Table 5.

Table 5 Time frame of research methods

Time Period	Methods	Purposes
September 2002	Giving consent letters to principals, other assistant teachers, teachers, students and students' parents.	Collecting students' backgrounds (the results of Intelligent Quality tests, the results of students' national tests, students' family background) Collecting the background of schools (the location, the size of school, the size of classes)
September 2002 to January 2003	Videotaping, sound recording and taking field notes of class teaching Interviewing teachers Giving questionnaire to students; (some short interviews with students) Tests and quizzes to students	Teachers' teaching patterns (classroom management, the time spent on tasks (teaching, students' discussion), the frequency of questioning) The coverage of classroom curriculum Teachers' teaching strategies Teachers' teaching philosophy and attitudes about education and teaching Students' views about mathematics (the nature of mathematics, the enjoyment of this subject and the mathematics classrooms, teaching preferences, classroom atmosphere and learning difficulties and advantages) Students' procedural knowledge Student's conceptual knowledge
August to September 2003	Collecting documents	Collecting students' results of national examination
January to March 2005	Interviewing students through the telephone	Clarifying students' ideas

April to June 2005	Interviewing teachers	Teachers' perceptions of their teaching and teaching approaches
January 4, 2006	Observing Student E2's presentation about his learning experiences of his own self designing curricula in senior level of School E.	Examining the consistency of Student E2's presentation with Teacher Eve's self-reflections about the growth of student learning
June 7, 2006.	Collecting Grade 9 documents of School E students	Clarifying information of School E students in Grade 9 mathematics learning

4.5 Analysis of data

The data analysis is discussed in this section by including the introduction and analyses of the content analysis approach, data analysis and coding, and Likert-type five equal measure items.

4.5.1. The content analysis approach

Content analysis serves as qualitative analyses for this study (Basit, 2010; Mayring, 2004). The study of content analysis benefits in interpreting/understanding psychological (Mayring, 2004), educational or social practices (Best & Kahn, 2006; Mayring, 2004) and commonly supports historical and ethnographic study (Wallen & Fraenkel, 2001).

Depending on research purposes (Alaszewski, 2006), the content analysis approaches within an inductive analysis nature of broad ranges (Merriam, 1998) can vary extremely according to the researcher's perspectives (Basit, 2010) or holistic/comprehensive perspectives (Krippendorff, 2004) and enhance the text analysis without previous structured code systems (Wallen & Fraenkel, 2001), towards strict formularized procedures or rigorous goal focus (such as testing a theory/hypothesis), examining and interpreting numeric patterns/relationships (Wallen & Fraenkel, 2001) from oral or written communicative data texts to conduct a study (Basit, 2010; Cohen, Manion & Morrison, 2007; Mayring, 2004). The analysing processes of the content analysis approaches are also similar to the "interpretational analysis"- continuously seeking, describing and explaining patterns from data (Gall, Gall & Borg, 2010p. 350). The data source includes recorded materials (tapes) (Ary, Jacobs & Razavieh, 2002), written (painting) material, papers, themes (Ary et al., 2002; Krippendorff, 2004) and the context-related information (Krippendorff, 2004). The findings of content analysis emerge

from data and data analyses, even the researcher cannot predict the results in the beginning (Merriam, 1998).

Content analysis historically started to serve as a quantitative approach, and in the 1970s developed as a qualitative approach, cooperating with statistical analysis (Mayring, 2004). Quantitative content analysis approaches targets by analysing frequency, for example, the frequency of events or codes (Mayring, 2004) through statistical methods (Krippendorff, 2004; Scott & Sutton, 2009), to produce a summary report (Cohen et al., 2007). However, these quantitative approaches were criticised for lacking holistic awareness/interpretation of the data context (Mayring, 2004). The content analysis approaches from qualitative perspectives investigate the frequency of texts (coding), (for example, sentences, phrases) to produce categories and to surface themes to generate a synthesis report (Alaszewski, 2006; Basit, 2010).

Content analysis does not seek to answer research questions directly, but rather analysing data. The data not only includes the revealed recorded or written materials, but also the researcher's interpreted data with a holistic view of the whole context. What the researcher meets, observes, and interprets of the context serves as data within content analysis (Krippendorff, 2004). When in the early stage of conducting content analysis, the researcher needs to define these context units (Cohen et al., 2007). However, there are no fixed rules to determine the range for the context units; this depends on the research purposes (Krippendorff, 2004). Generally, the context unit can be "a word, phrase, sentence, paragraph, whole text, people and themes" (Cohen et al., 2007, p. 477), different group of participants or others (Krippendorff, 2004).

The content analysis approaches from qualitative perspectives provided the researcher in this study with opportunities to examine different sources of data (interviews, questionnaire) to identify categories (Merriam, 1998), then descriptive numerical frequency analyses from/across categories generate themes (Alaszewski, 2006; Basit, 2010; Rubin & Babbie, 2008). This research adopted a combination or triangulation of research methods to collect rich and in-depth information, incorporating classroom observations, videotaping, interviews,

student questionnaire, and assessments of students' performance. Examining the themes/ (triangulation) data and analysing the relationships from holistic views/interpretations (Krippendorff, 2004) led to emergence of new knowledge in this research (Cohen et al., 2007; Krippendorff, 2004). These new findings/reports of content analysis are still not yet to be claimed as a theory that needed extra efforts to examine, for example, the extra analyses from a grounded theory or if these content analyses work catered for testing and generating from a hypotheses (Cohen et al., 2007). The results of this study are rather a synthesis report (Alaszewski, 2006; Basit, 2010) and discussions/arguments lead to new light of knowledge (Cohen et al., 2007; Krippendorff, 2004).

In order to understand students' perceptions of classroom teaching and their learning by using a questionnaire, the researcher analysed a thought (the meaning of words)/ (thematic item) (Rourke, Anderson, Garrison, & Archer, 2001; Strijbos, Martens, Prins & Jochems, 2006; Wallen & Fraenkel, 2001) as one analysis context unit, by searching out meaning from one or several sentences of students' responses to generate a content analysis unit. Then the frequency data of each thematic unit from students' opinions were categorised. I compared each data set and triangulated all the data sets to seek themes and meaning of each mathematics class teaching for this study (Krippendorff, 2004; Scott & Sutton, 2011).

According to Wallen and Fraenkel's (2001) arguments, there are both benefits and weak points of the content analyses approaches. Content analyses offer beneficial ways for studies such as this one to interpret relationships between categories or to see themes emerge. This method also benefited the researcher to assess the data at any time and to conduct repeated analyses to discover relationships within the work from the written/recorded data. On the other hand, content analyses are criticised as not being able to address complicated relational relationships of a phenomenon due to the simple logical analysis nature. Some critique that it is not easy to understand certain perspectives of information only through written/recorded data, for example, students' competencies (Wallen & Fraenkel, 2001). Moreover, if the nature of a study is open and exploratory, that will restrict

the quality of the content analysis approaches from qualitative perspectives, because of the challenges in categorizing the data (Mayring, 2004).

However, the multiple research approaches of this study helped the researcher to avoid disadvantages of content analyses. The qualitative and interpretive nature of work allowed me to continuously question the data and themes began to emerge. For example, classroom observations gave the big pictures of students' daily class teaching and learning practices and wove together with information of teacher interviews and student questionnaires to portray detailed phenomenon of teaching practices (see section 5.2.4). Data evidences supported from teachers' perspectives, students' views and classroom observations which are summarised in Table 10. The table illustrates Teacher Tom's normal teaching procedures and followed up an in-depth discussion of Tom's intended curriculum, teaching practices and other aspects. Although the research questions of this study were open and exploratory (see section 1.3), they focus on inspecting three areas: students' learning, learning attitudes, and teaching practices. Thus, the multiple research methods of this study (see section 4.4) could produce rich and detailed data to fit into the inquiry in this study.

4.5.2 Data analysis and coding

Firstly, the data was coded using codes to signify the data sources, place and time as listed in Appendix N. The data of mathematics tests was coded and is listed in Appendix O. For example, the code of Sy.Tv.p1er1213 means that the information came from video-taping Tom on December 13, 2002 and written on the first page of the summary sheet on the bottom right hand side. The code of NQ 2 to 5 means new questions from the second quiz to the fifth quiz.

Secondly, the majority of data sources, including teacher interviews, field notes, questionnaires, quizzes and tests, results of Intelligence Quotient test and a national examination were interpreted and analysed. Data from audio tapes and all videotapes in classroom observations were treated as existing evidence to triangulate the findings with other sources of data. For example, if classroom teaching patterns of a teacher or students' problem solving styles (group work or individual work) were consistent from triangulation data (teacher interviews,

student questionnaires, classroom observation), then the data of audio tapes and all videotapes would be kept as extra sources. I only transcribed the data collected in Mandarin on the first day. Then I translated it into English, in order to double confirm the consistency of findings from triangulation data. Since teachers would perform better on the first day, so I decided to collect the data to analyze the teaching styles of these three teachers.

Further, all information was categorised from multiple methods (class observations, tests results and the data from each teacher and his or her students) and then responses were organised and categorised/triangulated to discover themes. Theoretical categories that emerged were continuously cross compared with other data. The documentation of the data analysis is found in chapter five to chapter eight.

To analyse the classroom observation data as recorded on the audio and video tapes and field notes, in order to find out teaching patterns, the following foci were used:

- teaching sources, teaching speed and coverage of content,
- teachers' written notes on the blackboard,
- suitable and timely responses given to encourage students' thinking and caring,
- types and amount of feedback,
- small or large group work,
- skills of disciplining students' behaviour,
- multiple-methods or single method of assessing students' achievement,
- types of problem solving methods,
- strategies of motivating students in learning,
- observing and exploring what mathematics ideas teachers considered were most important.

These aspects were focused on in order to understand (i) the patterns/norms of teaching practices; (ii) what kind of mathematics was revealed in class teaching (Stigler et al., 1999).

To analyse the classroom observation data in the audio and video tapes and field notes, in order to find out students' classroom learning practices, the following were focused on:

- students' classroom activities (groups or individual; problem solving or teaching),
- students' classroom practice (understanding, interested or concentrated (involved) in learning, note writing, chatting, off topic, dazed),
- students practicing routine procedures or investigating new concepts or solutions, students' apparent concentration, students' discussions (communications), and students' learning attitudes.

These aspects were focused on in order to understand what kind of mathematical thinking and learning practices students were engaged in during the lesson (Stigler et al., 1999).

Moreover, videotaping could only collect students' visible learning behaviour. Students' feelings and mathematics thinking were difficult to ascertain from the observation data. The information on students' attitudes was elicited from the questionnaire or short interviews. The indication on students' mathematical competencies were provoked from quizzes, tests, and students' results on the Intelligence Quotient test and on the National Entrance examination.

In theorising or explaining data analyses, the teaching strategies and teachers' attitudes were discussed and explained with reference to the teachers' philosophies of education and teaching, and their perceptions of social-cultural expectations.

4.5.3. Analysing Likert-type five equal measure items

Likert's Five Equal Measure Items (Rubin & Babbie, 2008) were adopted in questionnaires; for example, the ninth and tenth questions on the third questionnaire (see Appendix G). The response choices included totally agree, agree in some ways, no comment, disagree in some ways, totally disagree, with points ranging from 5 to 1. Also the eighth question on the third questionnaire investigated students' ideas about the frequencies of different teaching methods in mathematics classes. Responses and associated points for this item were: 5 points

for ‘in every lesson’, 4 points for ‘almost always during their class time’, 3 points for ‘most of their class time’, 2 points for ‘sometimes’, and 1 point for ‘hardly ever or never’. If the average scores were high, that indicated high frequencies of that specific teaching behaviour in their mathematics classes.

However, if the data only included the leading school and the average score categories from the Likert-type scales but ignored the differences of the Likert-type average scores between both schools, students’ opinions could be misjudged. For example, the average students score of Tom’s classroom in School T was 4.13 and Eve’s and Ed’s classrooms of School E was 3.83 for the fifth question of the first questionnaire (see Appendix I). It appears that students in Tom’s classroom of School T performed better than students in Eve’s and Ed’s classrooms of School E, because their answer average scores were located in a higher level category (Level 4 vs. Level 3), (Tom’s classroom averaging 0.3 points higher than Eve’s and Ed’s classrooms of School E). In order to easily identify both groups of students in this study, School T will also refer to students in Tom’s classroom, while School E refers to students in Ed’s and Eve’s classrooms. In another case, students in School T could be assumed to perform similarly with students in School E (the third question on the first questionnaire) with both average scores in the same level 4 category (School T: 4.12 vs. School E: 4.83) (see Appendix I), although School E was 0.71 points higher than School T. Thus, in these two examples, the categories of Likert-type average scores were not sufficient to indicate the differences of students’ opinions in the two schools. The difference of 0.3 points pushed School T to a position of performing better according to Likert’s average scores in the first case, but 0.71 points failed to give power to School E as performing better in the Likert-type categories of the second case. Thus, the categories of Likert-type average scores were not sufficient to indicate the results and might cause bias. Therefore, this study presented the differences of the Likert-type average scores (see Appendix I) to interpret and discuss students’ opinions in both schools rather than presenting the categories of Likert’s average scores.

Only a few questions on the third questionnaire did not use a positive statement. For example, the seventh question of the ninth question on the third questionnaire

stated that “I wish that I do not have mathematics lessons”. One point was given for the answer ‘totally agree’, 2 points for ‘agree in some ways’, 3 points for ‘no comment’, 4 points for ‘disagree in some ways’, and 5 points for ‘totally disagree’. In this way, higher scores can directly show students’ positive attitudes towards learning mathematics, that students are more willing to take mathematics lessons. So, the scores are positive relative with their positive learning attitudes. This allowed the results to be easy to analyze. This kind of questions also included the sixth question of the ninth question and the (h) part of the eighth question on the third questionnaire (see Appendices E, F & G for all three questionnaires).

Moreover, the analysed numeric data in this study were “quantified” data from the qualitative data (Collis & Hussey, 2009, p. 7), such as the numbers of themes/frequency (Check & Schutt, 2012) of my research. The frequency count (numeric) data was from the content analysis of students’ opinions in the questionnaire and interviews including five Likert-type items.

4.6 Maintaining quality

Some scholars suggested that quality of qualitative research data can be evaluated by using the concepts of trustworthiness, accuracy and free of prejudices/biases, (for example, triangulation) (Gall, Gall & Borg, 2010; Rubin & Babbie, 2008) and “transferability” (findings apply into similar situations) (Denzin & Lincoln, 2013, p. 28; Rubin & Babbie, 2008). These terms are better than discussing reliability and validity which are often pointed as criteria in a quantitative study (Rubin & Babbie, 2008). Moreover, a criterion of social constructivist views towards qualitative research also value “trustworthiness” (Rubin & Babbie, 2008, p. 432). However, repeated and consistent data evidences are employed from multiple methods (triangulation) to support the reliability of qualitative interpretive claims (Rubin & Babbie, 2008). Moreover, triangulation method excellently benefits validity, especially for qualitative study (Campbell & Fiske, 1959; Cohen et al., 2007; Franklin & Ballan, 2001), for example, content analysis (Wallen & Fraenkel, 2001) and case study (Cohen, Manion & Morrison, 2011). Triangulation is an approach to understand the data from at least two research methods (Cohen et al., 2007; Rubin & Babbie, 2008), and is also powerful to treat complex/holistic phenomena (Jick, 1979), for example, of a case study (Adelman, Kemmis &

Jenkins, 1980; Cohen et al, 2007), and avoid the insufficient validity of a sole research approach (Cohen et al., 2007). Triangulation benefits the validity in content analysis, as it inductively examines the processes of analysing data for the generation of category, themes and arguments (Wallen & Fraenkel, 2001), and so does in case study (Cohen et al., 2011). That is the consistency of various data sources advantages internal validity (Cohen et al., 2011). A perspective of social constructivists also supports that triangulation reveals multiple aspects of realities (Rubin & Babbie, 2008). Thus, triangulation would be an important support for reliability (Rubin & Babbie, 2008) and validation of this case study research (Boaler, 1996; Cohen et al., 2000, 2007, 2011; Franklin & Ballan, 2001).

Another suggestion to enhance quality of qualitative work can be achieved by reducing the bias (subjectiveness) to the greatest extent (Gall, Gall & Borg, 2010; Rubin & Babbie, 2008). To do this, the researcher compared the data from multiple methods to find common points in the participants' responses (Rubin & Babbie, 2008), and tried to keep a neutral attitude during the process of data collection and data analysis. Further arguments on enhancing reliability and increase validity are discussed below.

The additional information of research approaches was also examined in other occasions, for example, the reliability or validity of the first and part of the third questionnaires and the IQ test, to address trustworthiness of the research tools. The reliability of the first questionnaire (classroom atmospheres) had been examined in two researchers' studies: Yeh (1993) and Yeh (1998). The Cronbach α on the reliability of their research was calculated as shown in Table 6. Classroom atmospheres in this research related with the psychological side of the classroom learning environment. Classroom atmosphere is used here in the sense of students' feelings from their classroom activities that might inspire students' intrinsic motivation in learning (Chang, 1995). Information was collected from three questionnaires (see Appendices E, F & G). The first questionnaire was designed by Yeh (1993). If students' scores were higher on the questionnaire, that would represent better classroom atmospheres (Yeh, 1998).

Table 6 Cronbach α of the first questionnaire (classroom atmospheres) from Yeh's (1993) and Yeh's (1998) studies

Sub-areas	Cronbach α (Yeh, 1993) N=98	Cronbach α (Yeh, 1998) N=83	Relationship with the first questionnaire (classroom atmospheres) (Yeh, 1998) N=83
teacher support	.8993	.8996	0.776
peer support	.8574	.8447	0.555
satisfaction with the mathematics class	.8015	.7943	.837
(the strength of class unity)	.8687	.7917	.778
the whole scores of the classroom atmospheres questionnaire	.9242	.9130	1.000

(Yeh, 1993, $p < 0.01$) (Yeh, 1998)

The good inter-consistency of the sub-areas in the classroom atmospheres questionnaire was supported from the Cronbach α (Yeh, 1993; Yeh, 1998). The consistency of the sub-areas with the whole classroom atmospheres questionnaire was considered as well (Yeh, 1998). The whole scores of the classroom atmospheres questionnaire reached to .9130 in the Cronbach α , which proved good inter-consistency of this classroom atmospheres questionnaire (Yeh, 1998).

Moreover, the validity was built up by Yeh's (1993) careful design, through literature review and five professors' examinations (Yeh, 1998). As a result, the reliability and validity of the classroom atmospheres questionnaire has been proved, so the questionnaires were capable to offer information to assess the classroom atmospheres (Yeh, 1998). This study's first questionnaire only adopted one question related with the strength of class unity from Yeh's (1998) classroom atmospheres questionnaire, so this research does not discuss the strength of class unity, but explores the sub-areas of teacher support, peer support and satisfaction with the mathematics class. These three sub-areas are able to offer good quality information about the classrooms in two schools.

The reliability of the parts of the third questionnaire that included the two sub-

areas of motivational belief (students' inner value and students' motivation related with achievement towards mathematics) were examined in two researchers' studies: Yeh (1998) and Chang (1995). Chang (1995) defined motivational belief as these following two attitudes. Students considered mathematics was a meaningful or valuable subject. Students were willing to do their best to learn mathematics well. If students have higher scores in the motivational belief questionnaire that means that they have higher learning motivations or considered learning mathematics would benefit their future (Chang, 1995).

Yeh (1998) and Chang (1995) also investigated students' self assessment of their learning efficiency in their motivational belief questionnaires; however, because that was not the focus of this research, and their learning performances could be understood from tests and quizzes of this research, this research did not investigate that sub-area (students' self assessment of their learning efficiency) as they did.

Yeh (1998) used the Cronbach α and the score relationship with the motivational belief questionnaire to support the reliability of the motivational belief questionnaire as in Table 7.

Table 7 Cronbach α of the motivational belief questionnaire in Yeh's (1998) study

Sub-areas	Cronbach α (N=83)	Score relationship with the whole scores of the motivational belief questionnaire
students' inner values	.8150	.847
students' motivation related with achievement	.7197	.880
(students' self assessment of their learning efficiency)	.7495	.858
the whole scores of the motivational belief questionnaire	.8929	1.000

(Yeh, 1998)

Chang (1995) used the Cronbach α and the retesting reliability (after two weeks) of the motivational belief questionnaire to support the reliability of the motivational belief questionnaire as shown in Table 8.

Table 8 Cronbach α of the motivational belief questionnaire in Chang's (1995) study

Sub-areas	Cronbach α (N=131)	the retesting reliability (N=126)
students' inner values	.8283	.8156
students' motivation related with achievement	.8355	.8056
(students' self assessment of their learning efficiency)	.8538	.8173

The good inter-consistency of the sub-areas in the motivational belief questionnaire was proved from the Cronbach α (Chang, 1995; Yeh, 1998). The consistency of the sub-areas with the whole motivational belief questionnaire was considered as well. That the whole scores of the classroom atmospheres questionnaire reached to .8929 for Cronbach α proved good inter-consistency of this motivational belief questionnaire (Yeh, 1998).

The validity of this motivational belief questionnaire was built up by Chang's (1994) careful design. For example, in her questionnaires, the questions researching students' inner values were adapted from Pintrich & DeGroot's (1990) theories (Chang, 1995).

As a result, the reliability and validity of the motivational belief questionnaire has been provided, so these questionnaires were capable of offering useful

information to assess students' motivational beliefs (Yeh, 1998). The third questionnaire in this study, which only adopted two sub-areas of the motivational belief questionnaire (students' inner value and students' motivation related with achievement towards mathematics), would retain reliability and validity from the support of the above scholars' cronbach α results. So, data from the two sub-areas of the motivational belief questionnaire would be able to offer good quality information on students' inner values and students' motivation related with achievement at both schools.

Moreover, one question was not adopted from their motivation questionnaire, because that question could appear to indicate high value of students' motivation from their answers. That question asked students if they felt they did not understand mathematics, would they ask another person immediately. Students' answers of 'totally agree' would get 5 points. However, I considered that this did not necessarily mean that students would have good motivation in learning, because some students might like to think by themselves first. I would consider this latter action also means that they have good motivation in learning and deserves 5 points as well, but this action might lead students to choose the answer 'disagree in some ways' or 'totally disagree', only worth 1 or 2 points. So, I considered this question was not suitable to interpret students' learning motivation and did use this question to assess students' motivation.

The reliable measurement of students' IQ scores

Furthermore, the reliable measurement of both school students' IQ scores was achieved in this study. The definitions of intelligence have been hypothesized and discussed for decades and these debates carry on continuously. For example, intelligence was considered as inherited and unchangeable through education in the past, but recently, some characterized intelligence was considered as improvable through education (Kubiszyn & Borich, 2003). The nature of IQ tests can be defined as i) instinctive nature: testing students' generic intellectual skills (i.e. general thinking skills); and ii) learnt thinking skills: testing students' scholastic ability (in the home or school). The two IQ tests referred to in this research belong to the latter type.

Several scholars have commented on the reliable measurement of IQ scores. For example, generally, IQ scores maintain reasonable stability, after children reach about six years old (Kubiszyn & Borich, 2003), or within different ages (Dancey & Reidy, 2004). High correlations have been identified between IQ tests and standardized achievement tests or school grades, e.g., .70 to .90 (IQ vs. standardized tests), and .50 to .60 (IQ vs. school grades) (Kubiszyn & Borich, 2003). However, this high correlation of IQ scores does not denote accuracy. For example, students' IQ scores are not exactly the same every week (Dancey & Reidy, 2004).

The IQ scores used in this study were measured prior to this research, right before Grade 7. Although students of the two schools took two different Intelligence Quotient tests, the Intelligence Quotient Test at Junior High School Level (the Third Edition) and the Intelligence Quotient Test of Academic Aptitude at Junior High School Level, the accuracy and validity of these two tests are highly correlated with each other (Kuo, 1989). The validity in the same time period is about .47~.85 between these two tests (Xu & Chu, 1986). High reliability is seen in these two tests, .77~.94 for the third edition IQ test (Lu, Ching & Lo, 1991) and .67~.94 for the Academic Learning IQ test (Xu & Chu, 1986). These two tests are very commonly used in Taiwan. As a consequence, students' IQ scores in this study were reliable.

4.6.1 Trustworthiness of qualitative data

Enhancing the reliability of qualitative data

Cohen and Manion (1994) suggested that an interviewer achieves more reliable data by utilising the important questions in the beginning, accommodating the participants with optimistic relationship and reducing any factors of un-reliability to the greatest extent. These suggestions are similar to those applied to qualitative data.

Thus, several strategies were applied in the present study to maintain trustworthiness and minimize bias of qualitative data. Information on the study was provided to principals, teachers, students (see Appendices J, K & L) and combined with verbal explanation. Letters outlining the nature of the research and

seeking informed consent (see Appendix L) were given to the students to take home and discuss with their parents. The purposes of the research were clearly communicated to participants through oral explanations and written documents (Stigler et al., 1999). The researcher tried to build up a positive relationship with the participants; carefully sensed the interviewees' attitudes and asked further questions to clarify their misunderstandings and made sure that they did not misinterpret the questions; and used carefully pre-structured interview questions and questionnaires, and a planned data collection procedure. These latter two strategies also enhance the validity for this type of research, according to Best and Kahn (1993).

Repeated experiments for the same group or different group participants are good ways to achieve reliability of content analysis (Wallen & Fraenkel, 2001) and interviews, for example, using a slightly different form for the interview at a later time (Best & Kahn, 1993). For example, there are slightly different ways of getting students' answers about their opinions towards the teaching style in the fourth question of the second questionnaire and in the second question of the third questionnaire (see Appendices F & G). This is expected to establish better reliability on students' views. This strategy combined "within method" and "between method" triangulation (Jick, 1979, p.603; Boaler, 1996) and also "across time/respondent triangulation" (Smith and Robbins, 1982; Boaler, 1996, p.30). The definitions of 'within method', 'between method' and across time/respondent triangulation are discussed in the end of this section.

Because of time constraints in these interviews, the researcher could not restate all the questions in a slightly different ways to the interviewees; instead, the researcher compared the interviewees' responses with the other data (questionnaire, behaviour in classrooms) or to similar items in the literature. By doing this, the researcher expected to check the reliability to some extent. However, participants were approached again to clarify some contrasting points or unclear opinions.

Enhancing the validity of qualitative data

Recently, validity has appeared in many forms (Cohen et al., 2000, 2007). Validity in qualitative research does not mean certainty in the results but pursuit of maximum validity (Cohen et al., 2007). Representation of reality is structured from a researcher's interpretation in ethnography (Hammersley, 1992; Cohen et al., 2000) and qualitative research (paradigm) (Johnson & Onwuegbuzie, 2004). Validity can be increased with clear sequential analysis from data coding, theme producing, then into a report/theory (Demerath, 2006), such as content analysis (Mayring, 2004); detailed analyses from different sources of data (Demerath, 2006; Wood et al., 2006), such as triangulations (Boaler & Staples, 2008) to achieve validity (Boaler, 1996; Campbell & Fiske, 1959; Cohen et al., 2000, 2007, 2011; Franklin & Ballan, 2001) and to explore the depth of learning situations in understanding of teaching and learning within two schools.

Cohen et al. (2011, p.179) also addressed qualitative data validity as requiring consideration of "honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation" (Cohen et al., 2000, 2007, 2011; Winter, 2000).

Qualitative data is subjective to participants' expressions with their perceptions and attitudes. Such data produces bias to some extent (Cohen et al., 2007). It is not possible to achieve perfect validity from qualitative (Cohen & Manion, 1994) and quantitative data (Cohen et al., 2000, 2007).

A carefully designed structure of qualitative research enhances the validity (Best & Kahn, 1993; Cohen et al., 2000, 2007; Winter, 2000), for example, "carefully sampling, appropriate instrumentation and appropriate statistical treatments of the data" (Cohen et al., 2000, 2007, p. 133). In order to enhance validity, this project has carefully structured research plans in choosing samples and coding, using multiple research methods and triangulation. Moreover, the triangulation methods and examining the coding increase trustworthiness of data (Franklin & Ballan, 2001; Gall et al., 2010).

Students might try to be on their best behaviour with video cameras and an

observer present, so normal classroom teaching practices may be hard to capture (Stigler et al., 1999). However, the researcher supposed that once students got used to video cameras and a observer present for a period of time, and treated them as part of a class, their behaviour would likely turn back to normal routines. So, before officially starting a classroom observation, the video cameras and an observer were present for at least two weeks in advance, in order to increase reliability and validity. A sufficient amount of time of inside classroom observation is needed to achieve a valid/reliable picture of classroom teaching (Stigler et al., 1999) and normal classroom practices.

Triangulation enhances validity

Examination of the rich triangulation data could shape/raise the researcher's arguments/theories for this study from firm data evidence (Boaler, 1996). However, some critiques of triangulation were mentioned in the work of Cohen et al. (2007), including inconsistent or invalidated data. If such happened, this could be a good opportunity to lead to further discussions to clarify situations.

The findings of this research were triangulated as documented in Table 9.

Table 9 Triangulation Data

	Teachers' self-assessment (interviews)	Students' self-assessment (questionnaires or (interviews))	Researcher's analysis from information collected from classroom
Teaching strategies (also the criteria of choosing material)	√	√	√
Classroom management (discipline of students' behaviour)	√		√
Coverage of mathematics content			√
Teachers' philosophy about education and teaching	√		√ (also teacher's attitudes)
Nature of mathematics, enjoyment (the subject of mathematics)	√	√	√ (also students' enjoyment in the classes)
Students' concentration			√

and thinking actions in classes (discussion or quiet thought)			
Students' understanding and thinking, different forms of knowledge (results from students' quizzes, tests and examinations)		√	√
advantages, disadvantages, difficulties	√ (Inferred from all the above data)		

Validity can be acquired in this study from supreme combination of detailed analyses/conclusions (Jaworski, 1994) including several types of triangulation: within method, between method (Jick, 1979; Boaler, 1996) and “across time/respondent” (Smith and Robbins, 1982; Boaler, 1996, p. 30). Within method triangulation examines the consistency of data from one method, while between method triangulation examines the consistency from multiple methods (Jick, 1979). For example, if a student's response on a questionnaire is that students always keep quiet and listen to a teacher's teaching, my sequential classroom observations can also confirm that opinion.

“Across time/respondent triangulation” means that if the consistency of data came from different participants or from the same respondent over a period of time (Smith & Robbins, 1982; Boaler, 1996, p. 30). For example, if many students' responses on a questionnaire are that students always keep quiet and listen to a teacher's teaching, then this opinion would be examined over a period of time. Nevertheless, across time triangulation was rarely used in this study, because of time constraints, only when needed to clarify some contrasting or unclear points.

Moreover, although reliability and validity are rarely examined in content analysis (Wallen & Fraenkel, 2001), the arguments are still raised. For example, the analysis of thematic content units has been criticized as achieving poor reliability, because of the interpretable nature of thematic units (Krippendorff, 2004). However, the reliability of the content analysis approaches from qualitative

perspectives can be enhanced through triangulation methods (Wallen & Fraenkel, 2001) or systematic analysis procedures (Mayring, 2004).

4.6.2 Generalisation

Generalisation is the goal for most researchers to apply findings/results of educational research to theory building (Bell, 1993). However, generalisation is a challenging issue for small sample in-depth qualitative studies (Bell, 1993). Interpretive research outcomes appear to have high validity but low reliability, and can possibly be applied to other similar circumstances (Collis & Hussey, 2009). For example, generalisation may occur in qualitative research, if a study can present thick and in-depth description, so other researchers could determine how much can be related or translatable from it (Cohen et al., 2000, 2007; Rubin & Babbie, 2008; Schofield, 1990). A criterion of social constructivist views towards qualitative research also supports the above arguments (Rubin & Babbie, 2008). Therefore, the current research did not have a specific focus and used content analysis approaches to interpret/analyse the rich data from multiple methods to generate thick and in-depth summary. It is expected that the findings (the characteristics of contrast teaching and learning in mathematics classrooms, and the potential relationships within teachers, teaching practices and students' learning) from the rich description and report of this case study can offer possibilities for generalisation to further research (Cohen et al., 2007, 2011; Rubin & Babbie, 2008). The findings might yield more valuable information about strengths in mathematics teaching at these two contrasting schools and also provide in-depth information on the student learning and attitudes. Some of the problems and issues identified in this study might nevertheless be recognized by mathematics educators and might suggest some changes in curricula or teaching practice.

Regarding ethical considerations, the researcher followed the guidelines of the University of Waikato Human Ethics Committee (University of Waikato, 1997; Human Research Ethic Regulations, 2000) to protect the rights and confidentiality of participants, to inform about conflicts of interest, and the use of information, copyright and ownership of data. The details are documented in Appendix M.

4.7 Summary

The interpretivist paradigm is adopted to inform this qualitative research (Brooke & Parker, 2009; Lather, 2006). In order to investigate the differences of junior high school students' three-year long-term learning experienced via both the traditional and the constructivist teaching approaches, this study adopted multiple research methodologies. Data collection involved classroom observations, videotaping, audio tape recording, (teachers and students) interviews, questionnaires, quizzes (mathematics real-life problems), tests given to students, and students' results on both the Intelligence Quotient test and the National Entrance examination. The analysis of classroom practices, students' and teachers' perspectives and students' learning were drawn from classroom observations and video-taping, student questionnaire, and several kinds of assessment to target students' different forms of knowledge. It was expected that teachers' beliefs and their intended and implemented teaching strategies influenced students' learning, moreover, that these perceptions would be understood and emerge from the data. Thus, the multiple research methodologies of this study can serve as an alternative research mode to examining teaching and learning in a holistic context.

Moreover, frequent examinations and tests are used by Taiwanese teachers to evaluate students' abilities (Chi, 1999). However, considering the limitations and narrow assessment of school tests, this study adapted fifteen mathematics conceptual problems (the second to sixth quizzes) to interpret students' mathematical abilities and their abilities to apply prior knowledge in new situations. This broadening of the assessment practices is to compensate for limitations in the use of written tests in schools. These alternative assessments are used to indicate students' abilities to solve mathematics problems in a wider context rather than focusing only on a narrow range of skills and procedures (Carr & Ritchie, 1991; Mayers & Britt, 1998).

The next chapters will present findings, arguments and recommendations.

Chapter Five: Case One: Tom's Teaching in School T

5.1 Introduction

This chapter looks at the data regarding Tom's teaching style in his mathematics classes in a Taiwanese junior high school. This chapter explores the teaching practices/ patterns/pedagogy of one successful and respected mathematics teacher in his school. This case study also uses a behaviourist and cognitive perspective to interpret Tom's teaching practices and style.

In this chapter, curriculum enacted from Tom's own views and from the researcher's classroom observations, Tom's views about mathematics and teaching styles/practices, and students' perceptions about his teaching style will be discussed. It followed by comparisons, which is presented in Chapter 9. The chapter concludes with a summary. Students' knowledge/understanding, achievement and students' views will be discussed in Chapter 8.

5.2 Tom's teaching practices

5.2.1 Tom's perceptions about students' learning, mathematics and his intended curriculum

Tom indicated that his views about mathematics and his teaching style were built up from his teaching experiences of 24 years (T1Ip7Q14'). He felt that students were naive and needed help from the teacher to find the important points which would appear in tests (T1Ihp5Q9t). He viewed that students could learn well by giving them fast solutions for problem-solving and by using direct instruction (T1Ihp3Q6b, T2Ihp3Q5e), thus his preferred teaching style (intended curriculum) were consistent with his views of learning (T1Ihp1Q3). He explained:

If you do not deliver direct instruction, how could students understand mathematics? So, your first priority would be direct instruction. Direct instruction is very important. When you encounter a mathematics problem, you need to show students your problem-solving method, explaining clearly

why and how that problem could be solved in that way. So, I emphasize both problem-solving and direct instruction. (T1Ihp1Q3).

Therefore, he felt that direct instruction and fast problem-solving methods were best teaching strategies for students' learning (T1Ihp3Q6b). Tom also viewed that students needed the stress from tests to push them to study hard, unless they were very disciplined themselves or their families cared about their learning (T2Ihp2Q4m). He added that when students got the problem solving methods and applied them on other questions, they would be practicing their mathematical thinking (T2Ihp3Q5e). Students also needed to do lots of practice in problem solving to improve their mathematics abilities and speed in problem solving, and thus succeed on tests (T1Ihp7e,8tQ15).

Tom viewed mathematics as tools used in daily life, especially at the junior high level (T1Ihp1Q2).

Regarding the intended curriculum, he ranked small-group work, team teaching and investigations as his second choice of preferred teaching style (T1Ihp2Q3). Testing is the third of his preferred teaching styles (T1Ip2Q3). However, he viewed that his actual teaching style placed an emphasis firstly on direct instruction, next problem-solving, and then testing (T1Ip2Q4). As a result, his views revealed his intention to help students to understand mathematical knowledge and use that as a tool in problem solving.

5.2.2 Tom's perceptions of his teaching practice

Tom felt that his teaching methods were especially suitable and beneficial for students whose mathematics abilities were in the top one third of a class (T2Ihp1Q1e). He gave as a reason that: "I could teach more and do wider problem solving, and then students could learn more (T2Ihp2Q3t)". In contrast, when he faced students with poor mathematics abilities, he slowed down the progress of lessons to address students' needs. Although the mathematics content in these few years was less than before, he still felt that the class time was insufficient to cover all the content for those lower-achieving students (T2Ihp2Q3t).

Tom tried to find chances to connect content with other units, to benefit students in reviewing previous concepts. For example, when he mentioned parabola, he connected with many other concepts such as quadratic equations (T1Ihp6Q11).

Tom mentioned that if his class size is small, he retains the same teaching strategies, and the small size class would offer better opportunities for him to build up relationship with students. He could have better chance to notice students' learning reactions, understanding and memory of mathematics concepts (T2Ihp1Q3e). He considered this kinds of caring could not happen in a large-sized class, with over 30 students (T2Ihp1Q3e). Tom perceived that the class in School T participating in this study had good learning attitudes and abilities, but a few students had low mathematics abilities (T1Ihp6Q12m).

He said that his views towards mathematics and teaching strategies did not change (T2Ihp2Q4m) during his working career (T1Ihp1Q1); only the teaching material that he used in classes was different. In recent years, he mainly used textbooks to teach students, whereas before, he mainly used resource books (T2Ihp2Q4m).

5.2.3 Tom's emphasis

These three case studies show evidence that what teachers emphasise was integrated into their teaching and shaped the different characteristics of each of the classroom practices. This section introduces Tom's emphases in his teaching.

Tom (i) emphasized process and understanding over the result, (ii) focused on students' reactions, and (iii) encouraged students' alternative solutions. Each of these is discussed in turn.

Emphasizing process/understanding over the result

(i) Tom emphasized that "mathematics teaching should emphasize process over result, and also understanding over result (T1Ihp2Q5e)". He explained that "...through the process of teaching he could understand the level of students' understanding" and students' confusions (T1Ihp2Q5). So, the best tests should include the process and he gave marks for process as well as for the answer (T1Ihp2Q5b).

He applied these emphases into his teaching through testing to find out students' levels of understanding, and also suggested an alternative way through inviting them to conduct problem solving on the blackboard (T1Ihp2Q3t). For example, after he had clearly explained problem-solving and felt that students understood the methods, he would give a test to see how well they did their problem solving (T1Ihp2e,p3tQ5).

(ii) Students' reactions in the mathematics lessons were also one of his focuses. For instance, when he did problem-solving, he would inquire about students' understanding individually or as a class (T1Ihp5Q9b).

(iii) Tom appreciated very much when students used alternative solutions in problem-solving. He viewed that there is more alternative problem solving in geometry than algebra. He preferred to teach students the fastest solutions, and also felt impressed when he learned some other fast problem solving from students. He would use those students' methods to improve his teaching in the next term (T1Ihp3Q6be). However, due to the time limit (five classes in a week), he felt there were not many chances to encourage students' alternative justifications in classes (T1Ihp3Q6e).

5.2.4 Teaching styles and practices

From interviews, students' responses from the first and second questionnaires, and sequential classroom observations of Tom's 26 lessons from November 18, 2002 to January 14, 2003 (Sy.vt.Tom,p.1-2) for four mathematical units revealed Tom's teaching style as including the steps presented in Table 10.

Table 10 Tom's teaching steps from triangulated data

Teaching steps	Data from		
	Tom	students	class observation examples
when starting a new unit, giving direct instruction of the textbook/teaching notes first.	T1Ihp4Q8e	n=12 TQ2hp1tl	Sy.Tvt.p1tl1118 Tvt.p1mr1206
directly pinpointing the important points and	T1Ihp4Q8e T2Ihp1Q1t	n=5 TQ2hp1tl	Sy.Tvt.p1tl1118, Tvt.p1mr1206

summarizing into key points or key content on the blackboard				
important points include:	mathematical formulas and definitions		n=6 TQ2hp1tl	Sy.Tvt.p1tl1118, Tvt.p1mr1206.
	skills in problem solving or textbook content to cope with tests	T1lhp5Q9t		Sy.Tvt.p1tl1126
through constant questions, demonstrating problem solving by using given rules and explaining reasons (Tom answering)		T1lhp3Q6be		Sy.Tvt.p1tl1118, 1126, 1206, 1210 Appendices H1 & I1
through constant questions demonstrating problem solving by using given rules and explaining reasons (students answering)				Sy.Tvt.p1tl1118 Appendices H1 & I1
teaching fast solution strategies		T1lhp3Q6be	n=3 TQ2hp1tl	Tvh1118p5e
emphasizing students' calculation speed				Sy.vt.Tp1el.1210
requiring students to memorize rules		T2lhp1Q1t	n=8, TQ3hp2	Tvh1118p3e
requiring students to take notes			n=2 TQ2hp1tm	Sy.vt.p1el.1206, 1213
requiring students to read the mathematics questions from the textbooks aloud together and also underlining the important points				Tvh1118p7b
students practicing problem-solving		T1lhp4Q8e T2lhp1Q1t	n=2 TQ2hp1tl	all lessons
reviewing mathematics content of the Grade 7 and 8		T1lhp6Q11	n=2 TQ2hp1t	Dec 23, 2002 to Jan 1, 2003
teaching content (Tom's teaching notes, textbook, practice book, and resource book)		T2lhp2Q4m T2lhp1Q1t	n=4 TQ2hp1tl	Sy.Tvt.prel210 (all lessons)
tests/quizzes for a unit		T1lhp4Q8e	n=17 TQ2hp5t	Sy.vt.p3ml.1129, 1228 Sy.vt.p3ml.1209, 0107

n: the number of students

The three sources of data referenced above illustrated Tom's teaching steps/strategies and also confirmed the consistency between Tom's teaching

practices and his intended curriculum: problem-solving and direct instruction (see section 5.2.1), e.g. pinpointing the important points and problems, giving direct instructions, introducing fast problem solving strategies and covering the subject content. Some teaching steps were only recorded in the class observation data, without data support from the teacher or students (as in Table 10, Table 11, Table 12), because students' and the teacher's perceptions were elicited from open questions of this research. Every individual's concerns were different, so every detail of Tom's teaching steps would not necessarily be covered.

Tests were given at a high frequency by Tom. When finishing a unit, one or two tests of that unit would be given (perceived by three students, TQ2hp1tl), e.g. two tests for the unit 2-2, one test for the whole chapter 2 (Nov29(5)), and one test for the whole chapter 3 test (Jan7(8)) (e.g. Sy.vt.p3ml.1129, 1228; perceived by student T12, TQ2Q(1)a). Seventeen students reported a high frequency of mathematics tests in school T, for example, every three days. (TQ2hp5t). Even if he had only finished teaching the key points, sometimes he would give a test for few minutes (Sy.vt.p3ml.1209,0107).

Tom said that he would use different strategies if there was an overload of content in one unit. For example, through pinpointing, writing and explaining important points of unit 3-3 on the blackboard, he states, "I required students to understand those mathematics points and then memorize them. Then we went to do problem solving in the textbook. In this way, it is simpler. Students could absorb the content better (T2Ihp1Q1t)." Class observation showed that he frequently delivered his lessons in this way, three of four mathematics units of my videotaping periods provided this information, e.g., units 2-2, 3-1, 3-3 (Sy.Tvt.p1tl1118, Tvt.p1mr1206). Some students asked Tom questions (Tvt.p1mr1126).

He rarely used other teaching strategies, because of the limited class time (T1Ihp4Q8e). Some of Tom's other teaching strategies were seldom used as illustrated below:

- As could be seen in class observation from time to time, students automatically had small group discussions, but their discussion time normally lasted no more than two or three minutes (e.g. Sy.vt.Tp1el.1213, 1217, 1210).
- Few chances were given to let students do seat work during the class observation period, (three times, Sy.vt.Tp1el.1217,1121,1220) or to write their problem solving on the blackboard (two times). Rather, Tom explained problem solving to the whole class (Sy.vt.Tp1el.1205, 1217).
- Personal interaction between Tom and individual students was observed. For example, Tom reminded students individually when checking students' seat work (e.g. Sy.vt.Tp1el.0109) or asking students to come to the front to mark and talk individually about students' homework (Sy.Tvt.p1el1220, p2tl.0109). He called on individual students to answer questions during (e.g. vt.T.1119).
- It was not easy to discover students' thinking or alternative thinking, because there were not many chances for students to explain their own mathematics concepts. There were parts of the two lessons when some students conducted problem solving on the blackboard (Sy.all.vt.p1m).

Other characteristics of Tom's classrooms were the fast teaching pace, style of question asking and classroom atmosphere which are illustrated as below.

Fast teaching pace

Eight students felt that Tom's teaching speed was too fast (TQ2hp2mr). Five complained that they could not understand some of the mathematics content because of the fast teaching (TQ2hp2mr, TQ2Q(1)a). In contrast, student T7 marvelled that Tom could finish one unit in a lesson period (TQ2hp1t). However, the fast teaching pace led to a great amount of content coverage (perceived by two students) (TQ2hp1tm).

Style of question asking

The constant questioning of students to apply given rules or explain reasons in demonstrating problem solving was Tom's major teaching activity. The questions that Tom asked were very frequently related to his teaching content

(textbook/practice book) (e.g. Tvh1119.p5t, Tv1119). Students gave responses together, such as short answers related to his questions such as “yes”, “no”, or short answers related with the content such as “90⁰” (e.g. Tvh1118p7t), or long answers as applying mathematics formula. He asked students to answer his questions, e.g. answers from (1) individuals: by pointing to individual students for answers (e.g. Tvh1118p5t) and (2) as well as the whole class. He repeated his questions or knocked on the blackboard to pressure students to answer (e.g. Tvh1118p3t). Quite often, Tom answered the questions himself or a group of students answered together (e.g. Tvh1119.p5t, Appendix E1).

Authoritative, humorous and demanding attitudes/atmosphere

Tom’s authoritative and humorous attitudes both appeared in his classes. For example, one case showed his authoritative attitudes. He said “why I asked you all to come to the front [checking students’ work] - when you are scored - is so that you would remember [mathematics] (Tvh1206epr)”. Tom’s humorous attitudes also appeared in classes. For example, Student T7’s scores on one test increased more than 20 points and he looked happy. Tom said to the student that “you are finally saved” (Tvh0114p.1’rtpr).

Tom appeared highly dominating and managing of students’ work. For example, Tom asked all students to come to the front and he checked student’s work in their textbooks, practice books, test books (Sy.vt.Tp1el.0109), and students’ tests (Sy.Tvt.p1tr1128) one by one (Sy.Tvt.p1er1213), and also marked them (Sy.Tvt.p1tr1121, Tvt.p1tr1217), e.g., students’ test books (Sy.vt.Tp1el.0109), and students’ tests (Sy.Tvt.p1tr1128). Homework (e.g. the resource book) was frequently given to students and checked by Tom.

Students’ responses also reflected Tom’s attitudes. For example, eight students felt that Tom’s teaching style was very authoritative (TQ2hp1tl). Students felt that Tom looked serious (n=6, TQ2hp1tl), horrible (n=2, TQ2hp1tm), strict (T15, TQ2hp1ml) and talked loud (n=2, TQ2hp1tl&hp2t). Individual students felt that he rarely talked about things besides mathematics in classes, but sometimes chatted with some students (T19, TQ2hp2t) or made jokes (perceived by two students, TQ2hp1t,2t).

Due to his teaching style and personality, four students reported Tom's class atmosphere as serious (TQ2hp1tl), while another three thought of it as quiet (TQ2hp1tl). Eleven students felt that students concentrated in Tom's classrooms (TQ2hp1tl); however, student T22 complained that her classmates in Grade 8, when answering Tom's questions, did so in a very weak voice (TQ2hp1t). For instance, student T3 explained "All the classes are very quiet and we are concentrating on listening in mathematics classes. When giving his lessons, he talks very loud, makes his jokes and wears sunglasses. No students dared not to concentrate in his classes" (TQ2Q1(a)stT3). Class observations also confirmed that Tom's classroom was constantly quiet and students listened to Tom's teaching, and sometimes students together answered Tom's questions (e.g. Sy.vt.Tp1el.1213, 1217, 1210).

On the other side, four students complimented Tom as having a sense of humour (TQ2hp2tl) and student T5 expressed that Tom was kind in Grade 8 and Grade 9 (TQ2hp1t). Student T8 viewed the classroom atmosphere as relaxed and vital (StT8, TQ2hp1t). This might be explained from Tom sometimes joking around (TQ2hp1t,2t).

To summarize Tom's typical teaching styles and practices, when he started a unit, he would do direct instruction of the textbook first, but did not closely follow the order of the textbook within a unit (perceived by three students, TQ2hp1tm), then pinpointed the important points and problems and asked students to underline those important points in the textbooks (T2lhp1Q1t). He expected students to understand those mathematics points and memorize them, and then they went on to do problem solving in textbook, practice and the resource book.

During instruction, he authoritatively organized all the class activities to support students' learning. He gave direct instruction with a fast teaching speed, frequent tests, and checked students' individual work in his classes. Tom did not only teach the Grade 9 mathematics content but also reviewed the previous years. Thus, Tom delivered lots of mathematics content. It was quiet in his classrooms, with few chances for student discussions. He emphasized students' calculation speed by

teaching fast solution strategies. Besides the direct teaching, from time to time, Tom also gave chances to challenge students' thinking, by giving time to allow them to form their answers through several rounds of questioning (e.g. Tvh11186e).

5.2.5 Students' perceptions

Twenty two of Tom's 26 students viewed that Tom's teaching style from Grade 7 to Grade 9 was very similar. Three of these students explained that they only had Tom as their mathematics teacher for these three years (TQ2Q(1)). Student T8 interpreted the main differences existing in Grade 9 as: "More students asked question. The progress of a lesson was very fast and there were many difficult questions" (TQ2Q(1)c).

Eleven students complimented Tom for doing a good job in teaching mathematics (TQ2hp2t). For example, three students felt that Tom spoke to the point (TQ2hp2b), and two of them expressed that he saved them time (TQ2hp2mm). One student viewed that Tom taught mathematics very clearly (T25, TQ2hp2mm), step-by-step (T6, TQ2hp1tl) and in detail (T20, TQ2hp2mt), that benefited understanding (n=3, TQ2hp1t, TQ2hp2mm). Another student felt that the formulas given by Tom were very useful (T7, TQ2Q(3)a), that lead to a great amount of learning (T17, TQ2Q(9)a), even if students did not understand, they could ask again and Tom explained in detail (St T20, TQ2Q(3)b). Student T18 concluded that Tom's teaching was good for excellent students (TQ2Q(3)a). Hence, the student feedback was the same as Tom's comments on his own teaching (T2lhp1Q1e).

In contrast, ten students complained about Tom's fast teaching speed (TQ2hp2b) and seven students felt that they did not understand the mathematics content, but three of these seven addressed cases which occurred occasionally (TQ2hp2b). Student T6 felt that Tom made high and strict demands and student T11 also felt pressure from Tom. Both students still fell positive toward Tom despite feeling uncomfortable (TQ2hp2t). Moreover, Student T19 added that Tom also cared for individual learning, for instance, arranging peer support for a poor performing student in Grade 8 (TQ2Q)(3)a).

5.2.6 Classroom observations

In this section, the classroom observation of Tom teaching is reported. Tom's class conversation for 6 minutes offers a good sense of his teaching style.

A Geometry Lesson

Grade 9

Duration: 3:09 – 3:15 pm

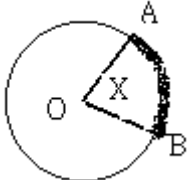
Date: November 18,

2002

Mathematical Content: Geometry – the beginning of unit 2-2.

Method: Direct teaching, questioning and problem Solving

Questions:

 Diagram 1	The teacher's writing on the blackboard was: 2-2 A central angle, an angle in circular segment, an angle of a circular segment $\widehat{AB}$: 1 The length 2 The degrees 3 The location The length of an arc = $2\pi r \times \frac{\text{deg rees}}{360}$ The area of a sector of circle = $2\pi r^2 \times \frac{\text{deg rees}}{360}$ (1) a central angle: the top of an angle located at the centre of a circle The degrees $\Rightarrow$ The degrees of an arc subtended by a central angle (Tvh1118p1,2t)
---	---

This example shows how he lectured on the concept of a central angle and two formulas. He spent six minutes on this.

Tom: You all look at the blackboard, I will tell you all the important points first. There are five kinds of angles: a central angle, an angle in circular segment, an angle in a chord and tangent, an angle located inside of a circle, an angle located outside of a circle. Regarding the latter two kinds of angles, the textbook does not discuss these two

kinds of angles, but there are some mathematical problems dealing with these two kinds of angles. So, you still need to pay attention to these two. After I have lectured and given you these important points, you can organize them and I might test you on these important points.

Before I mention these five angles, please look at this arc of AB which represents three meanings. Firstly, what is this? (He pointed at this arc of AB on the blackboard.) This is the length of an arc. The first one means the length. What does the second mean? How many degrees are there in a circle?

At least six students responded at low volume: Three hundred and sixty degrees.

Tom: Three hundred and sixty degrees. Can this arc represent a small part of the degrees? Secondly, it represents degrees. Today what we will use most are these concepts about degrees. Thirdly, what does it mean in

this place? It means location. So, the sign of $\widehat{AB}$ represents three meanings: the length, degrees and location. How about the length? The formula of the length of an arc! The formula of the length of an arc has been mentioned in our textbook. There were two formulas, one is about the length of an arc and the other is about the area of a sector of a circle. The formulas of the length of an arc and the area of a sector in a circle! Here we can review the formulas of the length of an arc and the area of a sector of circle. What is the formula of the length of an arc?

Students: ...

Tom: Louder! I cannot hear you.

Students: $2\pi r$ times $\frac{\text{degrees}}{360}$

Tom: OK! $2\pi r$ times $\frac{\text{degrees}}{360}$. The area of a sector of circle (pointing to

the blackboard) will equal to that $2\pi r^2$ times $\frac{\text{degrees}}{360}$.

One student: degrees.

Tom: We should generally know this concept. Now we will talk about five kinds of angles. The first angle is a central angle. Why do we call it a central angle? The reason is that the top of the angle is located in the centre of a circle, O. [He pointed at O on the blackboard.] This point O means the centre of a circle. The top of an angle is located at the centre of a circle; so, we call it "a central angle". So, we define a central angle as one where the top of an angle is located at the centre of a circle (Tvh1118p1). [Tom always pointed at the blackboard since here.] Then, how many degrees are there in a central angle? This is an important point. The first important point of today is that the degrees of a central angle equal the degrees of an arc subtended by a central angle. You need to pay special attention to this. If the degrees of an

arc are 40 degrees, pay attention for the degrees of a central angle are 40° . [He wrote $X=40^{\circ}$ on the blackboard.] When you consider the degrees of five kinds of angles, you need to pay special attention to the definitions of five kinds of angles. The degrees of a central angle equal the degrees of an arc subtended by a central angle. The degrees of a central angle equal the degrees of an arc subtended by a central angle. [Tom watched and waited for students to finish their writing.] Have you finished writing? The first angle is a central angle. The second angle is an angle in circular segment. ... (Tvh1118p2t)

The example above shows one of his typical ways of teaching through directly delivering and explaining his lessons. This example also indicates his fast teaching speed through direct instruction, then moving to the next mathematics concept. For example, he clearly explained the key point of the definition of an arc through several short questions but mostly he answered his own questions when questioning the class. Next, he asked two mathematics formulas without explaining, and then he shifted to directly explain the definition a central angle. In the later part of this lesson, he used lots of mathematics formulas to explain problem solving in the textbook. Moreover, Tom's authoritative attitude also could be sensed from his demands on students by directly giving orders without asking students' opinions and from the pressure of a test.

Another example showed how Tom used questioning skills to teach a concept and solved the problem with students' responses. For instance:

Tom: Keep writing and keep listen to me. ... If the length of an arc is longer, does it means that a central angle is bigger?

One student: Yes [in a quiet voice].

Tom: Are my statements right or wrong?

One other student: It is not necessarily like this.

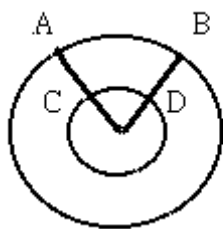
Tom: If it is not necessarily so, that means it is wrong. If the length of an arc is longer, then a central angle is bigger. Is this right or wrong? If the length of an arc is longer, then a central angle is bigger. Is this right or wrong?

Some students: Wrong! Wrong! [Students answered at different times].

The teacher: If the length of an arc is longer, then a central angle is bigger. It is wrong. Tell me where it is wrong. Is it right or wrong? Tell me where is wrong? How do you judge this?

One student: Check the radius.

[Tom drew a picture on the blackboard and asked students.]



Tom: Please tell me. Is $\widehat{AB}$ bigger than $\widehat{CD}$?
Some students: Yes!

Tom: IS $\widehat{AB}$ bigger than $\widehat{CD}$ or not?
Some students: Yes!

Tom: $\widehat{AB}$ is bigger than $\widehat{CD}$, right?
Please tell me, is $\angle AOB$ bigger than $\angle COD$?
Some students: No!

Tom: Is $\angle AOB$ bigger than $\angle COD$, or not?
Students: No!

Tom: It is not. The two angles are equal to each other, right? The longer length of an arc does not mean a bigger central angle. What is the key to judge this? Radius, right? The longer or shorter radius decides the length of an arc. So, you need to pay special attention to this. The longer the length of an arc does not mean the bigger a central angle. Please don't be cheated by this! This is a key point if it is located in a same circle. Before when I mentioned this question, I did not mention the same circle. So, I did not give you this condition that the two circles are the same circle. I only said that the longer the length of an arc means the bigger a central angle. This statement is wrong. Please pay attention about this! (Tvh11186e).

This example indicates another type of teaching instead of directly asking formulas to solve problems. This showed how Tom challenged students' thinking in a big class. Through several times of questioning and waiting, students gradually formatted the correct answers and gave short responses. Then Tom concluded the main mathematical ideas and explained the reasons himself. These teaching strategies might echo Tom's emphasis on teaching students' understanding. Tom gave chances for students to think and adjust their ideas and later used teacher's explanations to develop their understanding.

Tom emphasized the importance of using formulas. (1) He felt that some parts of the textbook used too many steps to solve a problem (Tvh1118p4e,5t) and reminded students to avoid those methods in the textbook (Tvh1118p5e). He

recommended students to use a formula e.g. $2\pi r \times \frac{\text{degrees}}{360}$ to speed up the time

(Tvh1118p4e). (2) He gave some short words to help students to remember the relation between some mathematics concepts (Tvh1118p4t, 7t).

Tom emphasized the importance of students' concentration in his classes. He encouraged students that if they concentrated in classes, they would learn very quickly. Even if they did not do the practice in the textbook; they could easily understand it (Tvh1118p5t).

Student being engaged

Students mostly appeared to concentrate in Tom's classes during my class observations. That indicated students were either listening or writing notes. For example, on November 18, 2002:

- the first 10 minutes

All students were either listening or copying from the blackboard (Tvh1118p2e).

- the next 23 minutes

All students were listening and at least seven students were both listening and writing (Tvh1118p3b).

- the next 35 minutes

All students were looking at Tom and listening to his sharing (Tvh1118p5b).

5.3 Discussion and Summary

The findings of this case study have presented Tom's teaching strategies and emphases. He practised direct instruction with a fast teaching speed and emphasized problem solving, students' understanding, memorization and calculation speed. Tom also challenged students' thinking in his classes by frequent questioning. Frequent tests were given and he covered lots of mathematics content in his classes. Eleven students complimented Tom's teaching, but on the other side, the fast teaching speed and difficulties in understanding were noted by ten students (see section 5.2.5).

This chapter has outlined Tom's teaching practices. Students' perceptions of mathematics, learning, and the class teaching styles; students' performances and

teachers' perceptions have discussed in Chapter 8. The next chapter will discuss Eve's teaching practices in the alternative school in Taiwan.

Chapter Six: Case Two: Eve's Teaching in School E

6.1 Introduction

This chapter looks at the data regarding Eve's teaching style in her mathematics classes. In this chapter, Eve's implemented curriculum will be discussed from her own views and from classroom observations, especially her views about mathematics and emphases, teaching styles/practices, and her students' perceptions about her teaching style. School E was a rare case in Taiwan and Eve's teaching style was also unusual with respect to a general traditional teaching perspective. The details will be explored and discussed in each section. After these sections, the chapter concludes with a summary. Students' views about mathematics learning will be presented in Chapter 8.

6.2 Eve's teaching practices

This section will present and discuss several topics regarding Eve's teaching practices.

6.2.1 Eve's perceptions about students' learning, mathematics and her intended curriculum

Eve believed that it was the students' own responsibility to build up their mathematics abilities and not rely solely on the teacher (Of2Ihp3Q1m). She emphasized students' talk in students' learning (Of1Ihp12Q9e). She really liked the class discussion method when students, through discussions, produced many ways of thinking (Of1Ihp9Q6b). She believed that when class discussion methods were applied successfully in a class, the teacher's role is not necessarily needed in students' learning, because students themselves could accept, judge, and discuss each other's ideas and make conclusions. That would lead to establish students' own learning (Of2Ihp3Q1e). She said even she did not need to do any instruction, as through the continuous discussion students could find some conclusions (Of1Ihp2eQ3).

Students' talk would help her to understand students' mathematical concepts (Of1Ihp12Q9e) or discover chances to help them to clarify/correct their wrong ideas (Of1Ihp9Q6b). She said that if students could express mathematical concepts clearly, this mostly meant that students understood those concepts (Of1Ihp12Q9e). She perceived that if students understood what they had learned, they would be able to apply it in other situations (Of1Ihp13Q10t).

Eve believed that "when students have interest in learning, they will gradually improve their abilities" (Of2Ihp3Q1m). When she cooperated with their learning pace, she felt happy to see the students' joy in learning (Of1Ihp3eQ3). She noticed that most of her students in this research, when they were in Grade 7, feared and rejected mathematics. So, she focused her efforts on "helping them not reject mathematics and become interested in mathematics, then turn to students themselves building up their own abilities, not her building up their abilities" (Of2Ihp3Q1m). Eve had faith in these Grade 9 students, even supposing that if they did not perform well on the national examination, they would not feel very upset. She thought the students would keep on trying to learn (Of2Ihp3Q1m).

She felt that her students had abilities to think and analyse situations to produce their own arguments, and then to test their own hypotheses in real life (Of1Ihp12Q9t). She also felt touched by their alternative (Of1Ihp9Q6b) and creative thoughts (Of1Ihp12Q9e), but their abilities in doing mathematics were very weak (Of1Ihp12Q9t). Therefore, she tried to give them more tests in Grade 9, to encourage students to focus on mathematical writing (Of1Ihp5Q4t). She considered that tests would increase their opportunities to practice mathematics and make up for the shortage of homework.

Moreover, from the class discussion method, Eve found students' progressed at the senior high level more than at the junior high level in autonomous learning attitudes (Of3Ihp2eQ3pr, hp4eQ5pr) and independent/critical thinking abilities (Of3Ihp4mQ5pr).

She gave more responsibilities to students to run mathematics classes at the senior high level and found students progressed and her teaching role could remain at the third line at the senior high level (Of3Ihp2eQ3pr), whereas before she stayed at the second line in junior high (Of3Ihp4eQ5pr). (Eve assumed teacher's role on the first line that means a transmissive role to deliver knowledge to students.) She found that students relied less on the teacher and the teacher role was one of posing questions at the senior high level. She found that students started to learn independent thinking by reading books themselves, setting up their own goals, working cooperatively, engaging in critical thinking and arguing, and proving simple facts (Of3Ihp4mQ5pr). Although the pace of building students' abilities was slow, students' thinking, arguing and expressing abilities were built up (Of3Ihp2eQ3) and their expressions were improved more than before (Of3Ihp3mQ3pr).

Regarding Eve's mathematics perceptions, she viewed that the whole picture of mathematics contains many characteristics (Of1Ihp2Q2t). These are:

- logical inference (Of1Ihp1Q2t),
- absolute truth (Of1Ihp1Q2b),
- a tool (Of1Ihp1Q2e),
- a training of thinking ability (Of1Ihp1Q2e),
- and a human face on it (Of1Ihp1Q2e).

She could accept the statement of mathematics being content knowledge and a field of knowledge composed of theorems and formulae (Of1Ihp1Q2e).

Because she viewed mathematics as problems, logical inference (Of1Ihp1Q2t), a training of thinking ability (Of1Ihp1Q2e), and a tool (Of1Ihp1Q2e), this might influence her practice to build up the content of mathematics while helping students engage in logical reasoning, debate, and find contrasts in class discussion intended to develop students' own mathematics concepts and problem-solving (see section 6.2.4). The body of students' mathematics was built up through these kinds of class discussions.

6.2.2 Eve's perceptions of her teaching practices

Eve felt that all these methods of small-group work/teaching, self-paced learning, testing, direct instruction, problem-solving and investigations were included in her actual teaching, but in different quantities (Of1Ihp4Q4b). Her actual teaching style emphasized, in order of preference: small-group work and team teaching, self-paced learning (Of1Ihp4Q4e), testing, and direct instruction, and lastly problem-solving and investigations (Of1Ip2Q3).

She mainly preferred small-group work/teaching, because she found that students learned through the process of discussion. Normally, she drew from students' questions, then expanded these questions mixed with students' past experiences, and encouraged students to think and discuss these new questions. She felt that through the process of discussion, students' personal ideas would be extended from the challenge of other's ideas. This discussion helped the students to arrive at some conclusions. Sometimes, she was not sure what conclusions and directions the students would expand to; so if need be, she would start to challenge students' ideas (Of1Ihp2beQ3). She shared:

At that time, I would play the role of a 'bad' person questioning them every day about why this happened in this way! Why this happened in this way! I played this role every day to try to help students to expand their thinking wider and wider of. Of course, this is my personal assumption about students' learning; I could not do it very well when I started to use these methods. Actually, I am still trying out these methods. I quite like it (Of1Ihp2e,3tQ3).

She believed that she did not even need to do any direct or explicit instruction, as through the continuous discussion, the students could find some conclusions (Of1Ihp2eQ3). She also quite enjoyed listening to students' conversation during the discussion, because their conversation indicated their background and life experiences (Of1Ihp3bQ3).

She chose investigations as her second choice of preferred teaching style, because of the limited class time. Although she did not worry about the school timetable, time pressure came from her own teaching plan to complete several big mathematics units in a semester (Of1Ihp3Q3t). Sometimes, when students discussed too broadly, she would stop and refer them to investigations in groups

after mathematics class. Students could do some simple investigations and submit them as small research reports from their groups. She liked this method, because she thought that through this investigation method students could have extra and expanded learning opportunities (Of1Ihp3Q3b), satisfaction of achievement (Of1Ihp7Q4e), and deeper understanding to nourish their learning journey (Of1Ihp3Q3b).

She would let students do reports in groups in some units, especially when the content of the units was not very difficult (Of1Ihp7Q4e) and for those “which were close to the characteristics of operations, observations, and vital experiences that I supposed students would be capable to do those units by themselves” (Of1Ihp7Q4t). For example, she adopted this teaching approach in a unit in the textbook that was dealing with the relationships between points, lines and circles (Of1Ihp7Q4t).

However, sometimes she felt very disappointed with the students’ non-preparation for class discussions; even after reminding them, the same situation happened again (Of1Ihp7Q4e). However, the students performed very well sometimes, even without preparation in advance; perhaps due in part to Eve’s teaching strategy.

Eve shared that she quite liked to give tests to students, and also give investigations. Direct instruction was used some. Problem-solving was the last teaching approach for her to use (Of1Ihp2Q3t). However, Eve tried to avoid the use of direct instruction. She said:

Direct instruction actually is quite commonly used. I feel very bad for this. So, I definitely would improve this (Of1Ihp4Q4b). ... I could not stop myself from giving direct instruction. For example, like today, I felt that I have a little pressure, and then I gave direct answers. Actually, I could try a suspecting way or questioning students or comparing the problem with a similar situation to let students to clarify the situation (Of1Ihp4Q4e).

She found that she needed to go with the learning pace of a class and some classes were not easy. For example, she felt that it was difficult to speed up their learning pace, especially for her Grade 8 class in 2002. She felt okay about the Grade 9 class which participated in this research (Of1Ihp3eQ3). She enjoyed the feedback when she cooperated with their learning pace and students felt happy in learning

mathematics (Of1Ihp3eQ3). She said that “students’ learning feedback is a great reward for my achievement. So, I try my best to slow down their speed. If they could go fast, we [I] would go fast. If they go slow, we [I] would progress slowly” (Of1Ihp3e, 4tQ3).

She added that she had given more tests to students in this semester because she found that students could think but could not write well. “It was a big difference between what they think and what they write” (Of1Ihp5Q4t). However, these more frequent tests brought more time pressure for Eve, because she felt that the usage of class discussions consumed lots of her class time and she needed to find extra time for students to take tests (Of1Ihp19e,20t Q18).

She spent less time on problem-solving in her classes, but spent most of the time on clarifying mathematical concepts with several easy solutions to problems and practice exercises. If students had problems, normally they would come to ask her or other teachers or other classmates after classes (Of1Ihp5Q4b). She could accept that they asked around, because her aim was for the students to do more practice (Of1Ihp5Q4b).

When she was teaching, she would do her best to connect the mathematical ideas with the other units. This also helped students to review previous concepts, but this needed some inspiration. Sometimes, she did not have inspiration, and forgot to connect with the other units. For example, when she taught about proportion equation, she would review and use problem solving of quadratic equations with one variable (Of1Ihp13Q11). Sometimes, she used examples to connect to the next unit and tried not to separate mathematics ideas between different units. For example, one question in unit 3-1 linked to the ideas of unit 3-2 (Of2Ihp1Q1m).

Eve felt that she had the freedom, with no stress, to plan her teaching while at the senior high level in School E. But she felt the stress of not having enough time for her to do mathematics lessons after classes (Of 3Ihp1tQ1pr). Eve’s opinions point out the burden of time for planning constructivist or alternative classrooms as teachers need lots of time for creative thinking to plan lessons, because no textbooks or curriculum guidelines are available to inform constructivist

classrooms. Eve felt that every day she needed to think and figure out the students' situations (Of2Ihp5Q5t), then plan her teaching according to the students' situations or her ideas (Of1Ihp10Q6e, Of2Ihp5Q5t).

A final teaching strategy used by Eve was that she tried to train a student tutor. She asked one of her Grade 11 students to come into her Grade 9 class to observe and learn some skills of teaching mathematics, e.g. how to help and challenge students to inspire their thinking and encourage their discussions (Of1Ihp6Q4e).

Eve shared that "class discussion method absolutely could not be used in a big class (Of2Ihp7Q3e)" and pointed to the problem that class discussion slowed down the teaching speed and resulted in parent's criticism. She shared her teaching experiences as below:

I taught in a private school [in 2001]. That is, I was responsible for speaking and students were responsible for listening. There were 50 students in a class. You could not do class discussion, because that (would) absolutely delay the learning speed of a whole class and you could not catch up the learning speed of a whole school. When the test time was coming, which was a whole school test, you would surely be dead and not alive if the students' scores were poor. Parents would blame you, blame you! Blame you! Blame you! (Of3Ihp3Q4e).

When she was teaching a big class, she said she needed to focus on the majority of students' needs and it was not like in a small class where she could care about differences between individuals (Of2Ihp7Q3e). She shared her strategies for a big size class:

I generally focus on those students who abilities rank in the middle of the class. ... I do not need to worry the top students, because they have good abilities. I cared for middle ability students, but I could not care too much for lower ability students. If I cared for them, over half of the students will feel impatient. So, I need to focus on middle ability students and cannot do class discussion. If I wish students could have some interactions with each other, there are actually some difficulties to apply this intention. So, in the arrangement of my classes I mostly ask students to listen to my instructions. When I give instructions, if I ask them questions, mostly I answer my questions myself. Mostly, I answer my questions myself. It is different from here. I can ask students questions and they will answer or I just wait for them to answer. In big classes, I asked questions, and then I answered myself. With too many students in a class, it is more difficult to let interactions occur (Of2Ihp8Q3m).

So, Eve felt it was difficult to use class discussions or have interactions in big classes. Later in those big classes, she adjusted her teaching by giving students chances to ask questions when she finished a period of teaching. In this way, she could understand where the students were having difficulties. She gave more tests to students in big classes than students in School E, to understand their learning situations (Of2Ihp8Q3m).

Eve also critiqued big size classes for placing more focus on solving problems, and that it was more difficult to build up some additional mathematical ability that included “expression ability, independent thinking ability, problem-solving ability..., the ability of appreciating the beauty of mathematics” (Of3Ihp4m).

If the number in a class reached 50, Eve recommended not using the class discussion method. If the number of a class reached 30, she felt it was all right to use the class discussion method although she felt it was still a little too many, but she needed to divide the class into five or six groups,. Eve added that in a small class of about 10 persons e.g. in School E, everyone could share in the discussion at anytime, but not in a class of 30 students, as it would be unworkable for 30 students to talk at any time in class (Of2Ihp8Q3m). Eve shared one successful experience when she taught Grade 1 in a primary school. Different groups of a class presented to the whole class on different topics or sub-topics of a main topic. Each time when a group presented their ideas, only one person spoke. If everyone talked at a same time, that would confuse their audience (Of2Ihp9Q3m). She shared that “They need to learn order (in class), how to talk to make people understand” (Of2Ihp9Q3m).

Eve felt the class discussion method was more productive in 2002 as she was more aware of students’ learning difficulties than in the year 2000. For example, Eve felt that her students of Grade 9 in this research had gotten used to her class discussion styles, and she was more encouraging of students to promote their mathematical thinking in classes than two years ago (Of2Ihp7Q5e). She also better understood some students with special learning difficulties, and could respond to them more patiently. For example, student E1 often could not understand the meanings from questions or mathematical signs, but “his logic

performed very well to infer some findings (Of2Ihp6Q5m)”. She tried to find ways to help those students, such as student E1, to accept and better understand mathematical signs (Of2Ihp6Q5m).

Eve’s perceptions about her students

Eve felt students chose her classes because she emphasized clear understanding of mathematics concepts with no overloading of homework (Of2Ihp9e, 10tQ3). Eve felt her students in this research “were outgoing and willing to talk, express themselves and willing to think” (Of1Ihp13Q10t). “They really like talking; if I give them a question, they will carry on discussing it (Of1Ihp15Q12e)”. Because of the characteristics of her students, she could apply the class discussion method in the classroom. But she mentioned one weak point: It appeared to her that they were “lazy to use their hands” (Of1Ihp13Q10t) – doing too little practice after classes (Of1Ihp19Q18). So, she gave them more homework and tests in Grade 9 (Of1Ihp13Q10t).

Eve’s perceptions about teacher’s duties in School E and the traditional school

Eve felt that when teachers finished lessons in the traditional schools, they did not have burdens in their hearts. In contrast, Eve, as did Ed, felt that there were lots of challenges in School E (Om3Ihp2tQ4pr, Of3Ihp4tQ4pr). Eve often needed to think how to plan the next lesson to build up students’ expression abilities, independent learning abilities, appreciation of the beauty of mathematics and cooperative learning (Of3Ihp4tQ4 pr).

6.2.3 Eve’s emphasis

In her teaching, Eve (i) emphasized that both process and results are important in students’ learning, and that understanding is more important than the result, and (ii) encouraged students’ alternative solutions or justifications.

(i) Both process/understanding and result are important

Eve valued both process and result as important in students’ learning and specified that understanding is more important than getting the result. She also emphasized the process in learning; through this she could find students’

misconceptions or common mistakes. So, she could have opportunities to clarify some concepts again (Of1Ihp8Q5t).

Eve viewed that results are relevant to right and wrong, so they are important, but she could not accept valuing the result or the process, but valued both (Of1Ihp8Q5t). If students could infer a result through the process, then she would assume that students really have mathematical ability (Of1Ihp8Q5b).

She applied this emphasis to her teaching strategy, by inviting students to come to the blackboard to share and infer their thinking of problem solving or concepts in a class discussion. She could perceive students' understanding through mathematical writing on the blackboard, how students developed their processes, and how students talked about their concepts (Of1Ihp8Q5e). If she discovered students' blind points, she would try to help to clarify them (Of1Ihp9Q5t).

(ii) Encouragement of students' own methods but not alternative solutions

Eve encouraged students to transfer mathematical language into their own signs and language, but did not emphasize students' alternative solutions (Of1Ihp9Q5b).

In order to encourage students' thinking, she used an alternative way of encouragement. She tried to build up an image that she was naughty and very lazy about mathematical writing. So, she would press students for a simpler way when they did problem-solving, because she felt too lazy to write more. For example, when students solved a problem, she asked, "Could you simplify your method more? I feel very lazy about this, really feel lazy about this" (Of1Ihp10Q6t). She reflected that, "I feel that laziness matches very well with the spirit of mathematics, because mathematics is very simplifying" (Of1Ihp10Q6b).

Then, she would lay the responsibility on the students to let them think. She reflected that she used this way of asking to encourage students' alternative solutions and, through a long-term process, to build up a tacit understanding between herself and students (Of1Ihp10Q6b).

In conclusion, her pedagogical emphases on students' thinking/understanding appeared in her teaching strategy. Further discussions are documented in the following section.

6.2.4 Teaching styles and practices

The classroom observations of Eve's 20 lessons from October 30, to December 11, 2002, for the mathematical units 2-2 and some parts of the unit 3-1, indicated that class discussion was Eve's main teaching method (Sy.Of.vt.p4). In examining the structure of Eve's 20 lessons, the students appeared highly involved in the class discussions in sixteen of her 20 classes. The average time of one or two students standing in the front of the class to explain their mathematics ideas or leading the class discussions was at least 24.3 minutes of a 50 minutes class time during those 16 lessons (Sy.Of.vt.p2', see Appendix S). The other four lessons showed different teaching patterns, two lessons for testing, one lesson for Eve's direct instruction to explain problem solving for a test, and one lesson of students' seat work for practicing Eve's organized material (Sy.Of.vt.p2'e).

If Eve leaded the class discussion, she would come to the front of the class giving lots of opportunities to let students explore their ideas through questioning or challenging them or inviting them to explain their thinking in the front of the class (16 lessons, Sy.Of.vt.p3t). Therefore, class discussions continuously flew between the lead students **or Eve** and the rest of the class to explore mathematics ideas.

Students' discussions were very vital in Eve's classes. The reason partly came from Eve's teaching style as she was very encouraging of students' talking or inviting them to the front to share, e.g., student E7 (2002/Oct30(1), Ofvthp6m), and student E3 (2002/Oct30(2), Ofvthp11e). The other reason might partly come from the students being used to talking in Eve's classes. They automatically asked questions, added comments or explained ideas to the lead student or to the teacher at any time of the class conversation, even automatically came to the front to explain to the whole class (2002/Dec 4(1), Sy.Of.vt.p2). For example, student E1, E8 and E11 automatically joined in Eve's conversation in the class (2002/Oct 30(1), Ofvthp9e, 10t, Sy.vt.p2r).

Examining the sixteen classes in which Eve frequently used the class discussion method, the person standing in front of the class leading discussions often swapped between students and the teacher (see Appendix T). The pattern often appeared as:

a student →Eve →a student →Eve →a student ...etc (12 of 16 lessons, Sy.Of.vt.p3).

Or, if a lesson started with Eve, the pattern often appeared as:

Eve→ a student →Eve ...etc (4 of 16 lessons, Sy.Of.vt.p3).

Students were used to going to the front to do logical deduction to persuade the other students in Eve's classes. The data indicated that in six of these 16 lessons (see Appendix T), (1) at least four students continuously came to the front to share in three lessons and (2) at least two students continuously came to the front, then Eve came to share in three other lessons.

The three sharing patterns described above indicated that (i) students automatically went or were willing to go the front to share, (ii) students were highly involved in the class discussions and (iii) Eve frequently encouraged students to share their mathematical thinking with the class.

Generally, Eve's teaching style could be broken down into the steps shown in Table 11 using triangulated data of the interviews with Eve, responses from students' questionnaires and from the class observations.

Table 11 Eve's teaching steps from triangulated data

Teaching steps	Data from		
	Eve	students	class observation examples
class discussions	Of1Ihp2beQ3	n=9 OQ2hp1re& tl	16 lessons, Sy.Of.vt.p3t
Eve emphasizing students' understanding.	Of1Ihp8Q5t	n=3, OQ2hp1er	Sy.Of.vt.p3t
teaching of key concepts	Of1Ihp11Q6t	n=1, OQ2hp1tl&re&me	Sy.Of.vt.p3t

students (one or two) presenting their ideas and solutions on the blackboard and explaining			16 lessons, Sy.Of.vt.p3t
Eve standing to the side of the classroom, listening to and observing the class	Of1Ihp11Q6b		16 lessons, Sy.Of.vt.p3t
Eve giving short challenges through questioning or giving hints, as needed	Of1Ihp2e,3tQ3	n=1, OQ2hp1tl&re	9 lessons, Sy.Of.vt.p3m
Students or Eve challenging the rest of students through asking for understanding	Of1Ihp11Q6b		Appendices H1 & I1
Students asking questions to the lead students		n=3, OQ2hp1tl	Appendices H1 & I1
Lead students or other class members answering those questions			Appendix H1
New questions sometimes resulting in discussions among the class (students and Eve helping those students who did not understand)			16 lessons, Sy.Of.vt.p3t
Eve summarizing students' talk, or posing problems that are thought provoking, or story telling mathematical concepts, or giving brief and direct explanations/teaching	Of1Ihp4Q4e	n=2, for Grade 7, OQ2hp1tl; n=4 for Grade 9, OQ2hp1re	at least 12 lessons, Sy.Of.vt.p5t
A student automatically going to the front to share his/her solutions			at least 6 lessons, Sy.Of.vt.p3,
Eve inviting students to the front to explain mathematical ideas to the whole class	Of1Ihp8Q5e		at least 2 lessons, Sy.Of.vt.p5m
investigations/research reports	Of1Ihp3Q3b	n=9 OQ2Q(1)	
group discussions	Of1Ihp2beQ3	n=5	Sy.Of.vt1111p1e,

		OQ2Q(1)	1118p1e
tests	Of1Ihp13Q10t	n=5, OQ2hp5t	

n: the number of students from the sample at School E

The consistency of the triangulated data is revealed as above. Underlying Eve's key mathematical themes for each lesson, Eve allowed her classes to progress on a journey for students to share and discover their mathematical concepts. In a lesson, there was always one focus in the front on the blackboard, no matter whether a student or the teacher was leading the class discussion. However, small group discussion could be seen in sequential class observations from time to time during the class discussion (e.g. Sy.Of.vt1111p1e, 1118p1e).

Normally, her classes started with a student's presentation of his or her problem solving and the student also asking for feedback from the class about their understanding (12 of 20 lessons, Sy.Of.vt.p3), although some lessons first started with the teacher's discussion (4 of 20 lessons, Sy.Of.vt.p3).

Eve generally posed an exaggerated question to attract students into discussion and felt very touched by the students' creative thoughts (Of1Ihp11Q6b, Of1Ihp12Q9e). She supported students' discussions and challenged them, for example, by asking "What would happen next?" (Of1Ihp11Q6b)? In each class, she maintained a main mathematical theme from her own curriculum (Of1Ihp11Q6t, Of2Ihp1Q1t). Students expanded their ideas from the main theme and sometimes they generated some mathematical findings earlier than she expected. Eve used students' findings to continue their discussion, but still within the main theme (Of2Ihp1,2Q1e). If the students' discussions went too far from the main theme, at some stage of their discussion, she would help students come back to the focus (Of1Ihp11Q6t). Eve felt that most of her teaching style was very similar. Sometimes, she felt that she had no time to let students continue discussions in class. So, she would let students hand in reports in groups, then share their reports with the whole class (Of2Ihp2Q1m).

She criticized herself in two ways about her questioning. (1) Sometimes, she posed a question which was too broad or not clear. That would bring out a lot of student questions. Then she could help to clarify their thinking through the

discussion process. Because when the question was too broad or not clear, that inspired students' creative thinking (Of1Ihp12Q9e), e.g., see section 6.2.6.2 (e). (2) Some questions she posed were connected to students' experiences or conversations, but not real life issues (Of2Ihp1Q1m), for one example of posing questions from students' conversations, see section 6.2.6.1 (b) teaching episode.

Very frequently, the other students added their opinions (alternative thinking) or asked questions of that student. Eve was also involved in the discussions at anytime, either asking many questions to challenge some unclear concepts or to clarify ideas through re-explanations. (5 of 12 lessons, Sy.Of.vt.p3).

Mostly, after a student lead discussion, Eve would summarize the ideas or move on to discuss and explain another mathematics concept (7 of 12 lessons, Sy.Of.vt.p3) through posing questions or story telling (e.g. a spider story relating to the concept of an arc, Ofvthp4,Oct30). She encouraged students to think and speak through posing questions, giving hints, inviting students to answer or to explain to the whole class. Or, she even pretended that she did not know the answers. That drew students to think and they shared their ideas in their seats or at the front. Through the continuous class discussions, students' mathematics ideas were continuously revealed and explored.

When Eve or a student was leading a class discussion, they both checked out students' understanding, so questioned them frequently. For example, Eve questioned students five times in the first twelve minutes and thirteen times in the remaining 33 minutes of a lesson (Ofvth1030p1~10). Student E5 questioned thirteen times in his twelve minutes of sharing with his classmates to check that they understood (or agreed with) what he said (Ofvth1030p1~3). If a student sharing in the front forgot to ask for the other students' feedback, Eve would help him or her to ask them. If students showed no understanding, Eve or students would try to help. For example, student E4 did not understand student E5's sharing; Student E5 explained again, and even the other classmates and Eve also tried to help him as well (Ofvth1030p2~3).

Note-taking was required and would be examined at the end of each semester. Eve encouraged but also accepted that one student was not willing to write, e.g. student E2 (Ofvh1030p3e).

Eve gave time and allowed students to explore their own thinking in her classes, for quite a period of time. For example, to her surprise, student E3 performed excellently in his presentation in front of the class (October 16, 2002), although his group had not prepared. He patiently stood in front of the blackboard, and although made mistakes several times while working out the mathematical signs, he achieved presenting his own findings (Of1lhp7Q4b) for about twenty minutes or more.

Two other lessons were observed. Those classes were quiet, because the student in the front took quite a period of time writing on the board. The rest of students were doing their own things and the students appeared to think and to find solutions, e.g. 10 minutes and 22 minutes (Sy.Of.vt.p2', Nov 4 (4), Nov 18 (3)).

Class observations are shown to be consistent with Eve's descriptions of her teaching, except for one statement. She felt that direct instruction was commonly used in her teaching (Of1lhp2Q3t), but the researcher felt that direct instruction was just one of her teaching methods, as she used the class discussion method more frequently to discover students' own methods. Students' views also supported the researcher's class observations (n=2, OQ2hp1t).

The mathematical content of Eve's teaching could be observed from the researched classroom observations. The data came from her personal understanding of mathematics content and also possibly from different resources (e.g. resource books, the practice book, textbooks, and some content from web search) for students to study and to guide the class discussion (Sy, vt, Eve, p1t, Nov11,2002). Her teaching content might change to respond to students' learning conditions (Of3lhp4m). Also, the mathematics content was included in the conversations that were produced from students and Eve about mathematical concepts and problem solving through the class discussion. She also designed her own tests for students.

6.2.5 Students' perceptions

Students responded that Eve's teaching style in Grade 7 and Grade 9 was very similar (n=7, OQ2 1st Q), but with more questions (n=1, StE8, OQ2Q1st) and Eve's explanations in Grade 9 (n=1, StE11, OQ2Q1st), and more reports and group discussions in Grade 7 (n=1, StE12, OQ2 Q1st).

In both Grade 7 and Grade 9, Eve assigned mathematics problems to students and students did problem solving (n=4, for Grade 7, OQ2hp1tl; n=2, for Grade 9, OQ2hp1re). Several teaching approaches were adopted, including class discussions (n=9 for Grade 7, OQ2hp1tl; n=5 for Grade 9, OQ2hp1re) about theories (n=1, St E11 for Grade 7&9, OQ2hp1tl) or some difficult problems (n=1 St E19 for Grade 7, OQ2hp1tl).

In Grade 7, the students said that Eve separated students into different groups (n=5, OQ2hp1tl) and assigned them to report to the whole class (n=9, OQ2hp1tl) on different units related to the textbook (n=3, OQ2hp1tl). When students shared and explained the mathematics content to the class, they performed the teacher's role in front of their class. Students said that they also shared their own problem solving with the class (n=1, St 9, OQ2hp1t). Eve encouraged students to challenge and question those students who gave reports to the class (n=3, OQ2hp1tl). The students also said some other approaches were in use, including group discussion (n=2, OQ2hp1tl), some tests or students asking questions (n=1 StE15 for Grade 7, OQ2hp1tl). Students said they had good interactions in Grade 7 classes (n=1, St E17, OQ2hp1tl) and chances to chat in classes (n=1, St E18, OQ2hp1tl). However, student E7 criticized some students when reporting to the class, of ignoring that other students might feel lost (n=1, OQ2hp1t). Five of her 12 students felt that the frequency of giving a test was about once every ten days, four other students felt it was once in ten to twenty days in Grade 9 (OQ2hp5t).

In Grade 9, the students said Eve let students have more chances to explore and generate their own methods (n=2, St E3&E9, OQ2hp1re). Students came to the front to do problem solving and reported their methods to the whole class (n=4, OQ2hp1re). Eve supplemented some ideas if she felt that students' reports were

insufficient (n=1, St E16, OQ2hp1t). The class mathematics content still followed the textbook order (n=1, St 6, OQ2hp1el). Moreover, Eve did not mark students' work by herself. She also asked other students for their involvement. If a student finished his or her textbook or exercise books, he or she could find another classmate to check and sign it (n=1, St 6, OQ2hp1el). Student E11 observed that students' reactions in Grade 9 class discussions were slower than in Grade 7 (n=1, OQ2hp1er).

Two students said that Eve's classes benefited students to have a thorough understanding of mathematics (n=2, St 7,11, OQ2, 3(a)Q).

Student E18 confessed that she quite often fell asleep in Grade 7 classes (Eve's), but not in Grade 8 and 9 classes (Ed's) (OQ2Q1st&3(a)). In contrast, student E11 preferred the discussion time in the mathematics classes, or she would fall asleep (OQ2, 3(c)Q).

6.2.6 Classroom observations

Presented here are three cases of Eve's class conversations to offer a good sense of her classes and her teaching strategies.

6.2.6.1(a) A student leading classroom discussion

A Geometry Lesson

Grade 9 Duration: 8:40 –8:46am Date: October 30, 2002

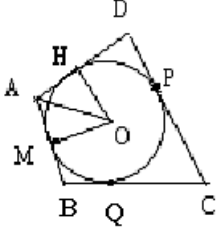
Mathematical Content: Geometry – the end of unit 2-1.

Method: class discussion method lead by student E5: explaining, questioning and problem solving

This case below showed how a student promoted mathematics learning in the classroom and the close involvement of other students and Eve in classroom discussion.

Question: Please describe the relationship between $\overline{AB}$, $\overline{BC}$, $\overline{DC}$ and $\overline{AD}$?

Student E5 wrote his solution methods on the blackboard first as:

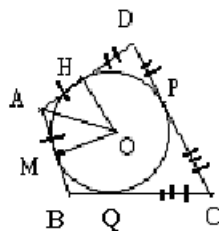
 Diagram 1	In $\triangle AOM$ and $\triangle AOH$ $\therefore \angle OHA = \angle OMA = 90^\circ$ $\overline{AO} = \overline{AO}$ $\overline{MO} = \overline{HO}$ $\therefore \triangle AOM \cong \triangle AOH$ So $\overline{AM} = \overline{AH}$ In the same ways, we could find: $\overline{HD} = \overline{DP}$, $\overline{BQ} = \overline{BM}$, $\overline{CP} = \overline{CQ}$ $\therefore \overline{HD} + \overline{AH} + \overline{BQ} + \overline{CQ}$ $= \overline{DP} + \overline{CP} + \overline{AM} + \overline{BM}$ $\therefore \overline{AD} + \overline{BC} = \overline{DC} + \overline{AB}$ (Ofvh1030p1r)
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Then the conversation was:

Student E5: Please use mathematical statements to describe the relationship between $\overline{AB}$, $\overline{BC}$, $\overline{DC}$ and $\overline{AD}$? That means in these two triangles $\triangle AOM$ and $\triangle AOH$, if I want to find what relationship existed, I need to find the relationship which exists among the sides. So, I found that these two sides were possibly the same length (pointed at $\overline{AM}$, $\overline{AH}$ and drew one short line on each side). I wanted to prove that the lengths were the same. In these two triangles $\triangle AOM$ and $\triangle AOH$, $\angle OHA = \angle OMA = 90^\circ$, because these lines were tangent with the circle at the points M and H. So, they were vertical to the circle. They were both vertical. Could you understand? $\overline{AO} = \overline{AO}$, because they shared the same sides. $\overline{MO} = \overline{HO}$, because they both were radius. So, a radius is equal to a radius. So, $\triangle AOM \cong \triangle AOH$. So, I could know that $\overline{AM} = \overline{AH}$. My assumption was right.

Student E9: Those two triangles are completely the same as each other.

Student E5: OK, I could write it down as RHS. (He added RHS to his previous writing as $\triangle AOM \cong \triangle AOH$ (RHS) So, in the same ways, I could prove that this side would be equal to this side. (He marked the picture as below.)



Do you have problems about this? So, I tried to find out that what would happen? $\overline{HD}$ plus $\overline{AH}$ plus $\overline{BQ}$ plus $\overline{CQ}$ equals $\overline{DP}$ plus $\overline{CP}$ plus $\overline{AM}$ plus $\overline{BM}$. So, $\overline{AD}$ plus $\overline{BC}$ equals $\overline{DC}$ plus $\overline{AB}$.

Student E11: 1 plus 2 plus 3 plus 4 equals 1 plus 2 plus 3 plus 4.

Student E5: I will use colour.

Eve: Let me suggest this to you. $\overline{HD}$ is this side, then $\overline{AH}$. (She painted different colours on them.)

Student E5: The equal lines. [He painted the same colour on $\overline{HD}$ and $\overline{DP}$, another colour for $\overline{BQ}$ and $\overline{BM}$, and yet another colour for $\overline{CP}$ and $\overline{CQ}$.]

Eve: This is a very good method!

Student E5: It is fine.

Student E2: A mathematics lesson

Student E5: I did not write wrong statements. Could you understand? [He pointed at the picture.] Could you understand the mathematical writing?

Eve: Student E11, Student E5 asked if you understand.

Student E11: Of course, I understand these.

Student E5: That is fine. [He walked to the other side of platform and pointed at the blackboard.] People on this side, could you understand?

Student E4: I do not understand.

Student E5: People on this side do not understand. I do not know the reasons. OK, because on that side $\overline{HD} = \overline{DP}$. That means $\overline{AM} = \overline{AH}$. Because the two triangles were the same, so, two sides would be the same, same length. Do you understand, on that side? [He pointed at the blackboard. Student E4 and student E3 concentrated to look at the blackboard.]

Student E11: 1 plus 2 plus 3 plus 4 equals 1 plus 2 plus 3 plus 4. [The other students laughed.] It is very useful!

Student E5: Could you understand?

Eve: I know. (She pointed at $\overline{HD}$ and $\overline{DP}$ on the picture.) $\overline{HD}$ is this yellow line, then so is $\overline{DP}$. (Ofvh1030p1tb)

In summary, student E5 read the problem first, then step by step directly explained his problem-solving on the board. The other classmates followed his explanation and raised questions or alternative thinking anytime during of the progress of his talk. Eve also assisted to paint colour on the picture on the blackboard and challenged the other students' to pay attention. Whenever a student raised a question, student E5 answered immediately.

6.2.6.1(b) Eve assisted students' discussion while a student lead classroom discussion

The second class conversation is given to illustrate Eve's teaching practices. Eve assisted the students' conversation closely, whenever she saw a need to help the other students understand better, but used the students' ideas not hers. Examining

the above example, Eve's teaching strategies in managing a student leading classroom discussion were: (i) assisting the student's thinking without adding her opinions, (ii) challenging students to think, (iii) re-explaining to a student again about the problem-solving, and (iv) following students' opinions and giving further suggestions. More details are described in the following.

(i) In the first part of the above geometry class in October 30, 2002, student E5 was leading a class discussion on his problem solving method. Eve did not add her opinions in the first ten minutes, but followed student E5's talk and assisted him by colouring the mathematics picture (eight lines) on three occasions, so the other students could more easily distinguish the differences in the mathematics picture on the blackboard (Ofvh1030p1e). For example,

Student E11: (Line) $1 + 2 + 3 + 4 =$ (Line) $1 + 2 + 3 + 4$

Student E5: I used colours.

Eve: Let me suggested to paint $\overline{HD}$ (She painted a yellow colour on the line.), then add $\overline{AH}$ (She painted a red colour on the line.).

(ii) She followed up student E5' talk to challenge the other students three times in the first ten minutes. For instance,

Student E11, student E5 asked if you understand (Ofvh1030p2t)? ...

Student E4, you looked at the problem-solving again yourself ...

Student E5: What was the relationship in this problem?

Student E4: AHN ...

Eve: Student 4, what was your question (Ofvh1030p3t)?

(iii) After ten minutes, she noticed that one student still felt confused after several students' tried to help him. She directly re-explained the problem-solving again. Student E4 finally understood. In that case, she still asked student E4 twice if he understood (Ofvh1030p3be).

(iv) Student E5 later said that he would use colour. Eve suggested he colour two lines of the mathematics pictures. Student E5 followed and coloured the rest of pictures (Ofvh1030p1e). Eve's further suggestions made sure that students could distinguish the differences more easily.

6.2.6.1(c) Eve leading classroom discussion

The third conversation illustrates that while Eve was leading the classroom discussion, students' vital and lively participation in classroom discussion was

very common. In the example below, she worked with students and let students raise their voices to together produce students' own mathematical solutions.

A Geometry Lesson

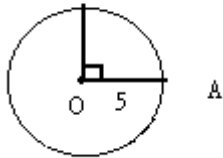
Grade 9

Duration: 8:50–9:48am Date: October 30, 2002

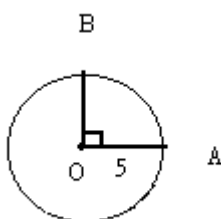
Mathematical Content: Geometry – the beginning of unit 2-2

Method: class discussion method: explaining, questioning and problem solving

Questions: The radius is 5 [units of length]. The angle is 90 degrees. What are the degrees and the length of this arc?

 Diagram 2	The teacher's writing on the blackboard while the class progressed with discussion: The arc = 90 deg The length of the arc = $5 \times 2 \times \pi \times \frac{90}{360}$ (Ofvh1030p11m)
---	--

Eve: What I am asking you now will connect the concepts that we have learned in the past. The radius is 5. The angle is 90 degrees. What degree is this arc? Let me name these points on the arc A, B. I am asking the degree on this arc now.



Student E1: 90 degrees

Student E2: It is an arc again!

Student E1: It is 90 degrees!

Student E7: It is 90 degrees!

Eve: The arc is 90 degrees. I want to ask the second question. What length is the arc?

Student E1: Radius.

Eve: The length of the arc!

Students E1, student E2, student E9: A quarter.

One student: Radius divided by 10.

Student E2: Let it times 3.14, then divide 4.

Student E10: 2.5π .

Eve: Talk slower! Is it 5 times 2 or 2 times 5?

Two students: 5 times 2.

Eve: A diameter times π , then?

Three students: It's divided by 4.

Eve: What do you mean: one fourth? I could not see that.

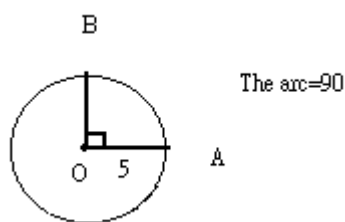
Student E2: That means times $\frac{1}{4}$.

Student E11: Student E2, you are so smart!

Student E1: 360 divide...

Student E2: 360 equal $\frac{1}{4}$.

(Eve was pointing her writing on the blackboard at $\frac{90}{360}$)



$$\text{The length of an arc} = 5 \times 2 \times \pi \times \frac{90}{360}$$

Student E9: Ninety degrees exists inside of the three hundred sixty degrees.

Eve: Do you agree with me? Three hundred sixty degrees divided into ninety degrees or three hundred sixty pieces. There are ninety pieces in it. Do you

agree with me? So, do you know that what are an arc and the length of an arc?

Are you clear about this?

Students [indefinite number of students responded]: Mm... Yes!

(Ofvh1030p11b)

These three examples of Eve's classroom conversations have given a picture of her teaching.

6.2.6.2 Eve's teaching norms

In analysing Eve's teaching, Eve is seen as developing thinking and exploring mathematics concepts through these observed teaching norms as described below.

(a) Not using direct teaching but posing questions

One way Eve lead classroom discussion was by posing questions. Eve's common teaching skill, as in the above example, was that instead of giving direct answers,

Eve asked lots of questions to elicit students' thinking (at least 4 of 16 lessons, Sy.Of.vt.p3).

(b) Using students' feedback to pose new questions or to expand them

Eve used students' feedback to understand students' learning, and then posed new questions continuously to carry on the conversation, and students were inspired to think more (Of1Ihp2beQ3). Then students or students, along with the teacher, found the mathematics conclusions.

In some instances, Eve used and expanded students' alternative thinking and invited students to explore it. For example, in the lesson of October 30, 2002, student E11 found out that if we added one more point C on the arc AB, that would help people to distinguish which arc AB on a circle that we want to talk about (Ofvh1030p10t). Eve followed and explained what she said and asked student E2 a question. She made sure that he understood, and answered student E10's question. Then she moved on to talk about a new mathematics problem (Ofvh1030p10e).

(c) Encouraging students' sharing

Although students vitally participated in class discussion, the teacher still tried hard to make the students talk more. For instance, she said this to a student: "If you shared in the class, you would pass the course. Please share more! Please share more!" (Ofvh1030p7b).

She often invited students to come to the blackboard to share their ideas about mathematical concepts or problem solving. For example, she invited student E7 and student E2 to share in a lesson (Ofvh1030p6b, Ofvh1030p7t) and invited student E2, student E11 and student E3 in the next lesson but she accepted that student E11 did not want to go the front to share (Ofvh1030p11e).

(d) Using hints/questions to develop students' own mathematical ideas

When students came to the front to share their ideas but did not do so clearly, Eve would try to give some hints to help the students to more easily point out the mathematical meaning and clarify their concepts. For example, when student E2

came to share his thoughts about the arc, at the beginning he just talked and pointed out a part of the circumference of a circle as arc. Then Eve did not directly share her thought, but just said draw clearly and re-draw two radiuses on the circle (as the below diagram). Student E2 was able to continue, saying “...This is the arc between these two lines”. Eve marked the angle in the centre of a circle on the blackboard and asked student E2 if that was the angle that he meant? He agreed. (Ofvh1030p7b). So, in this way Eve used hints to help the student to develop his own mathematical ideas.



Diagram 3

(e) Pretending not to know in order to draw out students' ideas

In order to let students have more chances to think, sometimes Eve pretended that she did not know some mathematics concepts “to make students continuously clarify their points and explain themselves clearly” (Of1lhp16Q13m). For example, one student concluded one point by herself:

Student E7: Is the degree of an arc the same as the degree of the angle of the centre of a circle?

Eve: What is the angle of the centre of a circle? Wow! I did not know about this. Someone has read the textbook, but not me. (Ofvh1030p11e)

(f) Speeding up the class discussion by giving support or hints

When some situations arose, Eve gave support or hints to speed up the class progress. When she sensed that students felt confused, she would support those students immediately who were leading or were sharing with the class. For example in one lesson, student E5 just mentioned that he would use colour. Eve came to colour parts of the picture for student E5 to show to the class (Ofvh1030p1b) as in the previous example.

In another example, when student E2 came to the front to share his ideas but did not do so clearly, Eve would give one hint and wait for his next response to give another hint, continuously in this way, instead of letting the student and the whole class wander around (Ofvh1030p7b) and waste time.

However, Eve still felt anxious about the delay of class time in discussions. For example, she shared in an after-class interview that she had already explained to student E2, but student E2 still needed student E5 to teach him again (vt.af.1104).

(g) Using her mathematics themes to direct the class

While Eve was leading the classroom discussion, she seemed to have some mathematics content in mind that she wanted students to know in that class. She made lots of opportunities to invite students to be involved in a class discussion or to share their thinking in public, and used the class discussion to come close to the main mathematics ideas that she had prepared to lead (4 lessons, Sy.Of.vt.p3).

(h) Summarizing mathematical ideas

If, after students' sharing in the class, the other students still felt confused, Eve would summarise the mathematical ideas (Ofvh1030p8b) or problem solving (Ofvh1030p3b) and directly lecture to the students (Ofvh1030p3, 8b). But she only spent a short time giving the direct summary, for example, about one minute for summarizing the arc ideas (Ofvh1030p8b) and about two minutes for re-explaining a problem solution (Ofvh1030p3b).

(i) Having gentle attitudes

Although Eve felt disappointed that students had not prepared their group presentations, she did not criticize the students, but reminded them in a gentle way. She gave them a clear hint when she noticed that they had not prepared. For example, she said: "Student E3, you really shared very excellently and presented it with good organization, but you know, in your lesson is so easy for us. Sometimes, we could have a break and have a rest" (Of1lhp7Q4b). That is, because of student E3's little preparation, most people felt bored. In another case, Eve gently reminded a student who was talkative with her classmates. She said: "Student E11, you could not always be talking". Student E11: "OK! I am sorry." (Ofvh1030p11e). She also accepted that a student did not want to write notes or students did not want to go to the front to share.

(j) Maintaining supportive classroom atmosphere

A supportive atmosphere was evident in Eve's classrooms. Either Eve or students would help those students who faced difficulties or gave praises to those students did a good job. Four examples are given below.

(i) For example, student E5, student E8, student E11 and Eve came to help student E4 when he had difficulty in understanding (Ofvh1030p1~3). At the end of student E5's second time of explanation to help out student E4, who still had difficulties to understand the problem solving, student E8 added in and came to help by adding, "Student E4, you could think about this. There are two 1s, two 2s, two 3s, and two 4s. Then $1 + 2 + 3 + 4 = 1 + 2 + 3 + 4$ " (Ofvh1030p2e).

(ii) For instance, Eve felt touched when all students in her class wanted to help student E3 to understand in one class discussion (OfItelephone2003/1/13p.2e).

(iii) In one section of classroom conversation, one student complimented student E2:

Student E11: Student E2, you are so smart! (Ofvh1030p11b)

(iv) Eve complimented student E5's method. "This is a very good method, Student E5. He responded: It is OK, la (Ofvh1030p1e)".

(k) Students challenging the teacher's authority

Students felt able to challenged Eve's authority. Four students (StE2, E5, E7, E11) questioned Eve's sharing, because Eve used an exaggerated example that her dog used a thread ball to hit and then kill a spider, to connect to a mathematical concept of an arc. One student shouted loudly that the teacher told lies (Ofvh1030p5t&e). (This behaviour is normally considered as rude in Taiwanese culture.) This shows that Eve allowed students to express their ideas freely. So, students expressed their feelings honestly.

(l) Giving students' freedom in the classroom

Eve gave some freedom to students in class. Evidence is given in the following examples. Students had freedom to eat in the classroom during class time (e.g. student E9, student E3, Ofvh1030p1, 2e). Student E2 stood up and stretched while

student E5 was leading class discussions (Ofvh1030p2e). One student did not ask Eve and just ran out to get water and came back to the classroom after half a minute (Ofvh1030p6e).

Eve respected students' right to refuse some of her requests. For example, student E2 was not willing to copy notes from the blackboard (Ofvh1030p3e). Student E11 refused Eve's invitation to go to the front to share her thought (Ofvh1030p11e).

On one hand, the freedom in Eve's classes might be explained by the teacher's personal teaching styles and her classroom management. On the other hand, this freedom in class might be influenced by the characteristics of School E, which places an emphasis on students' self-learning and respecting their choices.

(m) Sharing freely

Students in Eve's class had good imagination and freely shared their thoughts (Ofvh1030p4). The classroom had a friendly and relaxed atmosphere. When the teacher used the exaggerated example to connect to the mathematics concept of arc, about her dog using a thread ball to hit and then kill a spider, four students (stE1, stE2, stE8, stE10) performed the dog running, barking, and tracking the spider, and how the spider died (Ofvh1030p4).

(n) Enabling vital class discussions

Given Eve's teaching style and norms, vital and lively student discussions were very common in Eve's classroom (e.g. Ofvh1030p5), no matter whether she or a student lead the classroom discussion. Some supporting data was revealed in ten minutes of a classroom discussion analyses, for instance, 33 times of students' volunteer sharing their thought (see Appendix U), 16 times of students' volunteer sharing their thought (see Appendix V). Two students thought that her classes benefit students' complete understanding in mathematics (n=2, St 7, 11, OQ2, 3(a)Q).

To sum up, in order to meet Eve's teaching goals for building up students' thinking abilities (Of1Ihp5Q4t, Of1Ihp16Q15e), her teaching norms (e.g. (a), (c),

(d), (e), (f), (m)) were consistent with her goals to help students to construct their own mathematical thinking.

6.3 Discussion and summary

This chapter has described Eve's teaching practices. The class discussion approach was Eve's main teaching method (Sy.Of.vt.p4). Data supported came from the triangulated data (Of1Ihp2beQ3; n=9, OQ2hp1re&tl; 16 lessons of class observation, Sy.Of.vt.p3t) and averagely long time for class discussions within a lesson (at least 24.3 minutes of a 50 minutes class time, Sy.Of.vt.p2'). Eve's teaching styles strongly emphasize on student-centred learning that was reflected in her perceptions of students' learning (see section 6.2.1) and teaching practices (see section 6.2.4), for example, emphasizing students' talk, (see section 6.2.6.2(c)) and using class discussion to develop and expand students' own mathematics thinking (see section 6.2.6.2(b)). Eve's case showed evidence that her teaching content and teaching strategies were related to her views about mathematics, and her teaching strategies are related to her pedagogy (see section 9.3).

The next chapter will discuss Ed's teaching practices in School E.

Chapter Seven: Case Three: Ed's Teaching in School E

7.1 Introduction

This chapter looks at the data regarding Ed's teaching style in his mathematics classes in School E. It discusses Ed's implemented curriculum from his own views and from classroom observations, his views about mathematics and emphases, and teaching styles/practices; also discussed are students' perceptions about his teaching styles. Ed's teaching style is unusual from the general traditional teaching perspective in Taiwan. The details will be explored and discussed in each section. A conclusion will end this chapter.

7.2 Teaching Practices

Ed felt that in School E teachers could develop their own teaching material/content and did not need to follow the syllabus as compared with other schools (Om2Ihp6Q7m).

7.2.1 Ed's perceptions about students' learning, mathematics and his intended curriculum

Ed's view was that students are the centre of learning (Om1Ihp7Q3b) and he tried to see mathematics from students' perspectives (Om1Ihp6Q2). He perceived that the function of mathematics classes was to help students learn how to think, and how to play with mathematics (Om1Ihp6Q2). He also emphasized that students could actively be involved in classes and had good interactions with the teacher (Om1Ihp11Q8e). He aimed at improving students' abilities, including mathematical thinking abilities and problem solving abilities (Om1Ihp14Q15e).

He felt that teachers should only do their teaching to a certain level. The rest should be the students' own responsibility, including doing problem solving and preparing for tests (Om1Ihp6e,7tQ3).

He wanted his students to use their own words to describe mathematical concepts, as that could help students to remember them for longer (Om1Ihp11Q8tb). He

viewed that if students were able to explain their ideas or generalise their understanding to other situations (Om1Ihp6Q2), that would identify their real understanding in mathematics (Om3Ihp2Q5m).

He encouraged students to do more practice, if they wanted to beat the other students (Om1Ihp5Q2). Moreover, he viewed that students could not understand 100% of the mathematics content through teachers' lecturing. They needed to also do problem-solving to clarify their understanding (Om1Ihp10t Q7), using not only the textbooks but also the resource books (Om1Ihp10t Q7).

He valued students' learning attitudes rather than just their scores, and also encouraged students in this way. For example, he said: "You see, I spent the same amount of ink to mark your 80 scores, 90 scores or 60 scores. That did not bother me at all. You did not need to care for teachers' marking of you, but care for your own effort" (Om1Ihp5Q1b).

Regarding Ed's mathematical perceptions, he viewed mathematics as

- a collection of problems (Om1Ihp1Q0),
- a kind of puzzle (Om1Ihp5Q2), and
- as a game (Om1Ihp6Q2).

Because of his perceptions of mathematics, he would introduce students to "the rules, the content inside the game, what terms are inside the game, and then how to play the game" (Om1Ihp6Q2). So, the content of mathematics (mathematics classrooms) from his perspective was to help students "to think and learn how to play mathematics" (Om1Ihp6Q2), to know the mathematical content, and to solve problems (Om1Ihp10t Q7). Further, he emphasized that mathematics needs to be linked to life experiences. He illustrated that "Each unit is like a problem to students. If you treat mathematics as mathematical problems, it would be very painful for students. If you treat mathematics as a kind of life problem, how will you solve this problem" (Om1Ihp1Q0)?

For his pedagogy, he shared that students are the centre of learning. Through small-group work, he could hear and see what students were thinking, and provide

them with new and varied stimulating activities to develop their thinking ability (Om1Ihp7Q3b). This focus, he believed, influenced and directly impacted upon his teaching strategies. Further, rather than seeing passing the examination as the main goal when he educated students, he used this as a guide and tried to help students to develop better understanding of the problems. If passing the examination was the focus, then he would choose to use direct instruction all the way in his classes (Om1Ihp8tQ4).

It seems that he also viewed direct instruction as an effective way to help students to pass the examination (Om1Ihp8tQ4). However, he valued students' thinking in their learning (Om1Ihp7Q3b), so he adopted other strategies as a priority.

Intended Curriculum

He ranked small-group work, team/peer teaching and self-paced learning as his first choice of teaching strategy, investigations as his second choice of preference (Om1Ip2Q3) and his actual teaching style (Om1Ip2Q4). Problem solving was third, while testing was last (Om1Ip2Q3).

He explained, "Through small-group work/teaching, I could hear and see what students think about their learning content. So, I could focus on their thinking to give them new stimulation. This was intended to help them to expand their ideas" (Om1Ihp7Q3b).

He perceived that direct instruction was good for a small amount of people who were not willing to think and hoped for direct answers (Om1Ihp7Q3t). He explained:

I have met this kind of person. Whatever you wanted to discuss with him/her, he/she just did not want to discuss. He/she only wanted you to tell him/her how to solve the problem. After your sharing, he/she could do problem-solving quite well later on" (Om1Ihp7Q3t). "You could give him/her guidance by several examples... No, he/she just wanted one example. He/she only wanted the correct one (Om1Ihp7Q3t).

7.2.2 Ed's perceptions of his teaching practices

Ed viewed the functions of junior high schools as being to inspire students' learning interests and enhance their thinking abilities, so they could think and

make wise judgments (Om2Ihp9Q7m). To meet his focus, Ed felt that when he taught his classes, he aimed at delivering knowledge, used a heuristic method and did not use games (Om2Ihp1Q1t).

As outlined by Ed, when preparing a typical lesson, he first clarified his students' needs and their learning difficulties that might occur in a unit. He considered "why his previous students felt scared and what in the unit may scare them, e.g., confusion over the definitions. Then he would focus on those learning difficulties and try to avoid them happening" (Om1Ihp10Q8e). He suggested that by slowing his pace and observing students' reactions, such problems might be reduced (Om1Ihp11Q8t).

Additionally, when he started a new unit, he would give students about five minutes to read the textbook themselves, because then he knew that his students did not do pre-study at home. Then, he would use the textbook to instruct students. He encouraged students to use their own words to describe mathematics concepts, so they could remember them for a longer period (Om1Ihp11Q8tb). Sometimes, Ed also informed students of some possible developments in some units in the senior high schools.

Ed considered that teachers should have several methods to help students' learning. He said, "When a teacher teaches, he/she need to understand what kinds of students he/she is teaching and what goals he/she wants to achieve. So, a teacher should have ideas. You could not use only one script to teach all students" (Om2Ihp1Q1e).

Ed pointed out that every teaching method had advantages and disadvantages and teachers' strategies should respond to students' characteristics. He said, "You need to understand the characteristics of your students, and then you will find your teaching ways" (Om2Ihp5Q7m). Although Ed was very familiar in using the class discussion method, he still warned of the risk for introvert students. For example, he said, "If a student is an introvert and a teacher forces him/her to stand in front of the class and he /she had not prepared, from that time on, he/she would feel scared to death about mathematics" (Om2Ihp5Q7t).

Emotional Care

Ed cared about students' feelings in classes. He tried to chat with students at any time (in classes or after classes) to understand their learning situations and their feelings, so he could know better the ways to help students and improve his teaching (Om2Ihp9Q7t). "Teachers' experiences are built up and accumulated from many small issues. You need to often sense students' feelings, and then a teacher can make progress" (Om2Ihp9Q7t). Ed recommended the best ways to communicate with students were when students were solving problems in their seats and teachers came and shared his/her caring for students. For example, he touched students' heads or shoulders or orally encouraged students with comments such as "Well done! You have made progress". He felt that in these ways students would feel warm and close to teachers and they could also sense teachers' respect and trust (Om2Ihp4Q5e).

Building up students' confidence was Ed's first priority in teaching mathematics. If students studied mathematics for a long time and did not achieve learning success, they might quit their study (Om2Ihp7Q7t) or lose their confidence in mathematics (Om2Ihp6Q7e). In order to build up the students' confidence, Ed found chances to praise their work and encourage them personally in or after classes (Om2Ihp4Q5e). If students did not meet the passing standard and asked help from him, he could lend them some points and asked them to work hard to return those points to him on the next examination (Om1Ihp5Q1t). For example, Ed called one excellent student a mathematical prince. He said, "When he grows up and remembers that he was called a mathematical prince, would he not keep going in mathematics? I use this ways to bless their learning" (Om2Ihp10Q7t). Ed tried to give students high scores if they had good learning attitudes. Through this encouragement, he found that some students, who never brought textbooks into classrooms, changed and started to bring textbooks. In another example, student E20 started to hand in assignments and did not need reminding (Om2Ihp4pr).

Ed's strategies for big size classes

Ed would give a simple test to the whole class first to know students' abilities, and then focus his teaching on those students whose abilities ranked in the middle of

the class. He would give challenging and difficult questions to those top students in classes, so they would not feel bored. For slower learners, Ed used peer teaching (Om2Ihp2Q3t). He said,

I would find one of his good friends to sit beside the student, and ask the excellent student in advance. ‘You need to help me to teach this classmate. If he/she does not understand, you need to help me to teach him/her and teach him/her slowly.’ Let the slow student know that he/she was not been given up by the teacher. He/she would not be looked down on by his/her classmates (Om2Ihp2Q3m).

In the interview of 2005, he was not working in School E because the size of school became small, so the school could not afford two mathematics teachers. He taught some small classes of mathematics and science at the senior high school level in his home and other places. He felt that his teaching methods were the same as before (Om3Ihp2Q5m). For example, one of his strategies was peer-teaching, to let students sit in pairs and let the excellent student to teach the slower student. This can help students to develop their expressive abilities to show students’ real understanding of mathematics. If students really understand the mathematics concepts, they would be able to express themselves. He encouraged students with prizes, e.g. free drink. He found that slower students’ learning attitudes also improved (Om3Ihp2Q5).

7.2.3 Ed’s emphasis

This section introduces Ed’s emphases in his teaching. These emphases affected his choices of teaching strategy, which in turn shaped the characteristics of his classroom practices.

Ed emphasized (i) both process and results are important in students’ learning, and understanding is more important than results, (ii) focus on definitions, (iii) students’ alternative solutions or justifications are to be encouraged, and (iv) students should actively be involved in classes and have good interactions with teachers. He tried to apply his emphases to his teaching.

(i) Process/understanding and result

He valued both process and result as being important in students’ learning. Although he emphasized the process in learning, he realized that if a student’s

achievement was not good, those low scores would result in a lowering of students' motivation in learning (Om1Ihp8bQ5). So, the learning process and students' achievement were both important to him.

He perceived that understanding was more important than test results in a student's learning, because mathematics can very easily be copied from the blackboard, but that does not mean that students understand (Om1Ihp8bQ5). He applied this emphasis to his teaching strategy in order to elevate students' understanding, by making connections between concrete and abstract. He thought that if teachers ignored what abstract concepts the students did not know and the background that the students had, but kept carrying on teaching, the students would not understand. That would result in "a teacher teaching at the front of a class very happily, but students suffering a lot, because they could not understand" (Om1Ihp8eQ5).

He gave examples of teaching for student understanding, by making connections between concrete and abstract. For example,

Ed: Today, I want to introduce the concept of a circle. I would ask students to actually make a circle and measure the length.

The researcher: So, you would bring a rope. (I had seen him bring ropes into his classroom.)

Ed: Yes! Yes! You knew that π equal 3.1416, right? I could measure it to about 3.1. So, students could know that they could calculate π . Early in my teaching career, I did not know how to show this concept, but now I use the simplest method. After this kind of sharing, students have got a very deep impression (Om1Ihp9tQ5).

(ii) Focusing on definitions

Because he viewed the teacher's role as helping students to understand "the rules, what is the content inside the game, what terms are inside the game, and then how to play the game" (Om1Ihp6Q2e), it is understandable that he considered definitions as the focus in his mathematical lessons. He viewed that definitions are the basic rules. Students needed to know the rules and should not violate them, so that they could play the mathematical games (Om1Ihp11Q8e). This applied to his teaching when he gave students about five minutes to read the textbook themselves, then some instruction, and then aimed for his students to use their own words to describe their mathematics concepts (Om1Ihp11Q8tb).

(iii) Encouraging alternative solutions or justifications

In order to improve students' thinking ability, he encouraged alternative solutions or justifications and applied this thinking to his teaching by offering opportunities to elicit students' thoughts and discussions. When he started a class, he would not do direct teaching because that gave only one way of answering. He would propose a question first (Om1Ihp6Q2e), then offered chances to let students to share their mathematical concepts with the whole class. He would let the whole class think and check for any problem in those concepts shared (Om1Ihp9Q6e).

(iv) Students being actively involving in classes and having good interactions with the teacher

When he thought of students' learning, he emphasized that students should be actively involved in classes and should have good interactions with the teacher. He also viewed that teachers needed to know how to pose a good question, one that would bring out good interactions in classes (Om1Ihp11Q8e).

7.2.4 Teaching styles and practices

Classroom observations by the researcher investigated Ed's 14 lessons from December 11, 2002 to January 8, 2003 that included 9 lessons for the mathematical unit 3-3, two lessons for the school examination, and three lessons for students to compare questionnaire, quizzes, and a test related to this research (Sy.T.vt.p2e). Generally, Ed's teaching style could be analysed as in Table 12.

Table 12 Ed's teaching steps from triangulation data

Teaching steps	Data from		
	Ed	students	examples of class observation
group discussion (giving chances for students' seat work to work on problems (peer teaching also involved))	Om1Ihp7Q3b Om2Ihp2Q3t	n=2, OQ2hp1ml	6 lessons, Sy.Om.vt.p4m Omvh1211p7t, Omvh:1216,1218,1225
adopting class discussions	Om1Ihp6Q2e		4 lessons, Sy.Om.vt.p4m
short direct instruction (giving brief and direct explanations or summarizing the points)	Om2Ihp4Q5e	n=6, OQ2hp1eml	4 lessons, Sy.Om.vt.p4m
direct instruction (for one			1 lesson,

whole class time)			Sy.Om.vt.p4m
emphasizing students' understanding and also making sure that students did understand	Om1Ihp8bQ5 Om1Ihp8bQ5	n=3, OQ2hp1me	Omvh1211p5e&7t
reviewing previous lessons	Om1Ihp6Q2e	n=1, OQ2hp1el	
posing problems that are thought provoking	Om1Ihp6Q2e	n=13, OQ2hp1eml	4 lessons, Sy.Om.vt.p4m
assigning some students to present their ideas and solutions on the blackboard	Om1Ihp6Q2e; Omvh1211p3b;	n=3, OQ2hp1ml	3 lessons, Sy.Om.vt.p4m
asking students to explain/report mathematical ideas/strategies to the whole class (Om1Ihp6Q2e). In the mean time, the whole class checking those ideas (Om1Ihp9Q6e)	Om1Ihp6Q2e Om1Ihp9Q6e	n=3, OQ2hp1ml	2 lessons, Sy.Om.vt.p4m
moving around the class checking students' work.			3 lessons, Sy.Om.vt.p4m
questioning students or teaches students (one by one)			4 lessons, Sy.Om.vt.p4m
challenging students or giving hints, (such as while they were writing on the blackboard, or working at their seats, presenting their problem solving)			4 lessons, Sy.Om.vt.p4m
encouraging students	Om2Ihp4Q5e		3 lessons, Sy.Om.vt.p4m
asking students to take notes		n=4, OQ2hp1mm	6 lessons, Sy.Om.vt.p4m
asking students to hand in homework (from the textbook, exercise book and 3 questions a day)			3 lessons, Sy.Om.vt.p4m
giving tests		n=6, OQ2hp5t	4 lessons, Sy.Om.vt.p4e
giving homework		n=2, OQ2hp1ml	

n: the number of students.

It is noted that the number of students in Ed's classes were small (n=1 to 13).

There was consistency between Ed's perceptions of his teaching, students' perceptions and the researcher's class observations of his teaching. Summarizing, Ed's teaching steps closely followed the focus of the small group discussion and class discussion methods that aimed to develop and explore/discover students' mathematics ideas, apart from the steps of the teacher's direct explanations and giving tests. If we consider the frequencies of teaching strategies in Ed's 9 lessons of the 3-3 unit, there was one lesson for Ed's test, one lesson mostly for Ed's direct instruction of his own three-page summarized notes to students (the second

class, Dec 30, 2002, Sy.Om.vt.p4t), and 7 lessons for students' seat work, the class discussion method and the teacher's brief and direct explanations (see Appendix W). For example, there was an average of 38.1 minutes per lesson for the 7 lessons for students' seat work with small group discussion, some class discussions and Ed's individual challenges from students' seat work (checking students' understanding or giving guidance) at all times (Sy.Om.vt.p4m) (see Appendix W). Ed's brief and direct explanations were average 8 minutes for one lesson of these 4 of these above 7 lessons (Sy.Omvtp4e &4e').

During instruction, Ed took the main role of organizing the lesson structure to aid students' learning (Om1Ihp6Q2e; Omvh1211p7t, Omvh:1216,1218,1225). He posed questions first, then often invited students to share their ideas orally or by writing on the blackboard (Om1Ihp6Q2e; Omvh1211p3b), with the other students working in their seats (peers or individual work) (Om1Ihp6Q2e; Omvh1211p7t, Omvh:1216,1218,1225). Ed and the whole class checked the problem solving of those students' sharing (Om1Ihp9Q6e; Omvh1211p7t, Omvh:1211,1216,1218,1225,1230). When a student had difficulties in problem solving at the blackboard or in his or her seat, Ed mostly would give students hints but not direct answers, or ask for peer support to help that student (Om2Ihp2Q3t; Om3Ihp2Q5; Omvh1211p2t,p2m,4e,5e,6t). For example, student E16 helped student E15 at the blackboard (Omvh1211p2t). Ed gave frequent chances for students to do seat work and Ed also tried to understand and assist each individual student's progress and understanding (Om2Ihp4Q5e; Omvh:1211,1216,1218,1225,1230), for example, by checking each student's writing twice in the lesson of December 11, 2002 (Omvh1211p4t, 6e). He also used manipulation to move them through the stages of thinking and problem-solving strategies, e.g. using an easier example or concrete material (Om1Ihp9tQ5, a rope for measuring the length of a circle). At all times, they were required to infer, justify and adapt their own conclusions. Also, through the students' work and explanation on the blackboard, all students learned from each other (Omvh:1211,1216,1218,1225,1230). Through the learning processes, the students' arguments and justifications would produce a synthesis of mathematical consensus (see section 6.2.6 and teaching examples in section 7.2.6.1) within class discussions. When needed, Ed also gave brief and direct explanations or

summarized the points (Omvh:1211p3be,1216,1218,1225,1230), for example, if a student had difficulties in explaining clearly his/her mathematical thinking (Omvh1211p3be). In one case, student E15 could not explain clearly her ideas, so Ed summarized the two main points of two mathematical lines (Omvh1211p3b). Through this, Ed helped students to understand the main ideas of those students' writing on the blackboard (Omvh1211p3e) and meet his mathematical theme (Omvh1211p7t). As his own statements, "I always use problems to very slowly bring [focus] into a theme (Omvh1211p7t)".

During this lesson, the focus of the class came through the teacher's organization of the lesson structure. For example, sometimes attention was directed to the blackboard, or there was small group discussion. That is, when a student explained what he/she thought (or when Ed challenged the student to come to the blackboard), the class' focus would be towards the front and the blackboard. Ed also walked around the class to understand all students' work and questions.

Ed's teaching of mathematics content could be observed from this sequence of classroom observations. His personal notes guided the class discussion and were from his summarization of some important conceptions and questions from the textbook, the practice book, and the resource book. He also gave other organized notes to students to study (Sy.Om.vt.p4t) and designed tests for students (Sy.Om.vt.p4t). The mathematics content also came from the conversations in classes or peer teaching between students and from Ed through the class discussion.

7.2.5 Students' perceptions

Students' responses on the questionnaire were consistent with Ed's intended curriculum and implemented curriculum. Students responded that Ed's teaching style in Grade 8 and Grade 9 was very similar (n=6, OQ2Q1st), with students doing more tests in Grade 9 (n=1, StE18, OQ2hp1el). The frequency of giving a test was about once every seven days in Grade 9 (n=6, OQ2hp5t). Ed directly lectured (n=6, OQ2hp1ml) to give main and basic concepts first (n=1, St E11, OQ2hp1ml). He asked students to practice mathematics questions in classes (n=13, OQ2hp1ml) and some questions being quite difficult (n=2, OQ2hp1ml). Some

students added that Ed gave a lot of mathematics questions in Grade 8 (n=5, OQ2hp1ml) and Grade 9 classes (n=2, OQ2hp1el), and so does great amount of homework in Grade 8 (n=2, St E8,20, OQ2hp1ml). Sometimes, when he started a class, he gave different quizzes to students according to their progress in learning mathematics, and let students practice in classes (n=1 StE6, OQ2hp1mm). The students said that the testing was sometimes different from the other teachers. He would ask students to give ten questions to him, and he would select some of them for students to test themselves (n=1, StE23, OQ2hp1el).

He chose some questions that students thought were difficult and discussed them with students together in class (n=6, OQ2hp1ml). If students made mistakes with their sharing to the class, the other students could correct them at any time (n=1, St E14, OQ2hp1ml). If students had problems, they could freely ask the teacher or students (n=2, OQ2hp1mm). He divided students into different groups and asked them to discuss with and help each other (n=2, OQ2hp1ml).

The students said that Ed emphasized students' abilities in problem solving and speed (n=3, OQ2hp1mm). His teaching was very lively (n=1, St E16 OQ2hp1ml). He was very patient in helping students (n=1, St E17, OQ2hp1mm). Students also supported each other (n=1, St E21, OQ2hp1e). Sometimes, he chatted with students in classes (n=1, StE18, OQ2hp1e).

To sum up, some advantages of School E pointed out by students were that teachers in other schools guided students how to think, but teachers in School E were like helpers to let students explore their thinking (n=2) (OQ2hp1tr). Student E9 said, "Our teachers let students explore and discover the solutions by themselves. Teachers just stand beside students and give hints" (OQ2hp1tr). Teachers emphasized students' understanding and would give more time (n=1, OQ2hp2b) or a lot of opportunities to try and made sure that students understood (n=3). Teachers did not like students who could only do problem solving but without understanding (n=1). The atmosphere in School E was open (n=1) (OQ2hp1tr, E4). Teachers' teaching styles brought relaxed atmosphere in classes (n=2) and students did not have pressure from classes (n=2) or from examinations (n=1, OQ2hp1).

7.2.6 Classroom observations

This section presents Ed's class conversations to give a good sense of his classes, his teaching skills, and his teaching emphases. This example shows that Ed kept questioning students to know their understanding.

7.2.6.1 A geometry lesson

Grade 9 Duration: 9:01 – 9:11 am Date: December 11, 2002

Mathematical Content: Geometry – the beginning of unit 3-3.

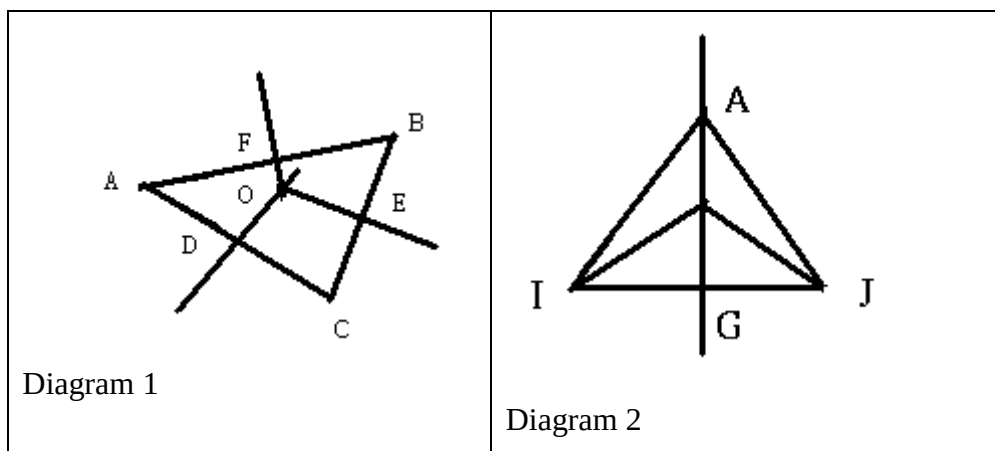
Method: mathematics discussion (Ed questioned a student's mathematics ideas in front of the whole class as a way to help the learning of the whole class) and problem solving

Questions: What is the characteristic of the central and vertical line?

Which lines are equal to each other? (See Diagrams 1 & 2)

What are the characteristics of the line that equally divides one angle?

$$\overline{OA} = \overline{OB} = \overline{OC}$$



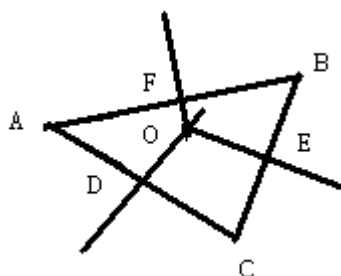
The first example occurred as below (Omvh1211p5m) (Duration: 9:01 – 9:02 am).

Ed: What is the characteristic of the central and vertical line, student E20?

Student E20: There are two lines. Their length is equal to each other.

Ed: The length of which lines is equal to each other? Is it to this point? Is it to that point? (Ed pointed to two positions. Very soon in the next step, he marked them as point D and point E.) I start to name each point. (Ed marked point D and point E and also named other points.)

The length of which lines you see is equal to each other? Which lines are equal to each other?



Student E20: To...

Ed: To where? OK! You come here and measure it. (All students were quiet and watching the blackboard. Student E20 came to the front). You can use the triangle board to measure it. We can suppose this point is point O.

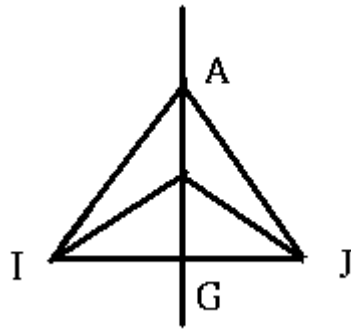
Student E20: [She pointed at $\overline{OD}$, $\overline{OE}$, $\overline{OF}$ and thought their length was the same.] This one, this one and this one are the same. (Omvh1211p5b) (Duration: 9:03 – 9:04am)

Here we can see that Ed used students' feedback to pose new questions, and that helped students to explore their thinking.

Ed did not do direct teaching, but developed the students' own curricula according to their current conceptions. For example, he encouraged student E20 to investigate why her ways of understanding differed, and how she could actively test and integrate her ideas.

The next three examples also illustrate the characteristics of Ed's teaching. For example, following up on their conversation, student E20 made mistakes in the beginning (Omvh1211p5b). Ed came to help her and used the triangle board as a ruler to measure $\overline{OD}$, $\overline{OE}$, $\overline{OF}$ and marked the length of these three lines on the same side of the triangle board. Then, student E20 realized that the lengths of these three lines were different (Omvh1211p5e).

The second example appeared when Ed then asked students to answer his questions about a diagram that student E19 had drawn on the blackboard (Omvh1211p5e). (Duration: 9:03 – 9:04am)



Ed: What is the characteristic of the central and vertical line? Is this a central and vertical line? [He pointed at $\overline{AG}$.] Then?

Student E20: [standing in the front of the blackboard]: These two sides [pointing at $\overline{AI} = \overline{AJ}$, in a small voice]

Ed: Point I and Point J of these two sides are similar with which points of the previous triangle? [Student E20 just continuously looked at these two pictures.] Let us move these two pictures (in our minds) together.

Student E20: [pointed at point A, point B and point C]

Ed: What is the characteristic of the central and vertical line?

Student E20: [pointed at $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$] (Omvh1211p5e)

Ed often used peer support to help students when he sensed students had difficulties. He noticed that student E20 was still not confident about her ideas. He invited student E19 to come to the front to help student E20 understand the characteristics of the central and vertical lines and also explain to the whole class (Omvh1211p6t). However, the explanations given by student E19 were not very clear (Omvh1211p6m).

The third example (Duration: 9:06 – 9:10am) appeared when Ed tried to explain the concept again, by using comparisons rather than direct teaching. For example, he mentioned $\overline{OA} = \overline{OC}$ and $\overline{IG} = \overline{GJ}$ and asked student E20 to the blackboard to write the relationship. She discovered and wrote $\overline{OB} = \overline{OC}$ and $\overline{CE} = \overline{BE}$; $\overline{OA} = \overline{OB}$ and $\overline{AF} = \overline{BF}$. Then Ed asked the class together to find relations between those equations. Student E20 successfully found out the relationships and referred to the new finding by herself that $\overline{OA} = \overline{OB} = \overline{OC}$ (Omvh1211p6e) (The same conclusions as stated in the textbook). Moreover, while Ed and student E20 were conducting their mathematical conversations, students' small group discussions also occurred quietly; for example, student E19 and student E21 discussed with each other (Omvh1211p5e).

In summary, these episodes indicate how a learning journey helped a class to learn together and how student E20 successfully found out the relationships herself with Ed's inquiring and teaching. Ed's teaching norm in this case was not direct teaching but posing questions or hints (comparisons) to develop students' own mathematical ideas, using students' feedback to pose new questions or to expand them and using peer support.

In Ed's classrooms, it was very frequent that Ed used students' ideas and group discussion with students' helping others who had mathematical difficulties, to guide, develop and build up the body of students' mathematics concepts. His other teaching norms and classroom interaction patterns were also documented during the class observation period and are discussed below.

7.2.6.2 Ed's teaching norms

The teaching activities that Ed used over the research period were:

(a) Using students' own methods and not necessary the teacher's method

Ed encouraged students to use their own methods and not necessarily the teacher's method (Omvh1211p1e,2b). For instance,

Student E20: I do not know how to draw.

Ed: It is just a [mathematics] sign. You write down your thought. It is not necessary to use my method.

(b) Using students' feedback to pose new questions or to expand them

Ed used students' own conclusions to understand their learning, and then he linked this to a new concept. For example, Ed used the finding of student E20 to quickly explain to all of the students and expanded a new concept (a circle passed through point A, point B and point C) to the class (Omvh1211p6e).

(c) Teacher's role as a facilitator

Ed acted as a facilitator by reminding students about better ways by which they could help students to solve problems. For example, when he asked students to draw three middle and vertical lines on the three sides of a triangle, he checked each student's diagram and advised them to: "enlarge your triangle", "make your triangle smaller" and "only one interception" (Omvh1211p4t). He also modelled

the desired behaviour and challenged students to check their understanding. For example, when student E18 came to the blackboard to draw three middle and vertical lines on the three sides of a triangle, before she finished the first middle and vertical line on one side of a triangle, he wiped the extra two arcs but kept the two interception points. He reminded student E18 that fewer lines would be better. Student E18 closely followed his suggestion. (Omvh1211p4e).

(d) Encouraging students' sharing

During the lesson, it was clear that Ed cared about students' emotions and feelings. To help students, he allowed them to share; intervening only when he felt they needed him (Omvh1211p3t; Omvh1211p7t). He also gave specific praise by complimenting students' work as "nice" "excellent", "you are doing a good job". Ed offered prizes to students to encourage them to solve some difficult questions from him, e.g. student E19 won two bottles of beverage (Omvh1211p1b).

(e) Requiring homework and note taking

Ed believed that there should be more practice activities to improve students' understanding of mathematics. For example, he said, "Students need to do problem solving to clarify their understanding about the mathematical definitions and theories" (Om1Ihp10t Q7), "not only from the textbooks but also from resource books" (Om1Ihp10t Q7). Further, "if students want to beat others, they needed to invest their time" (Om1Ihp5Q2). This might explain the reason why Ed required students to do 3 questions every day and to hand in the textbook and exercise book as homework to him (Sy.T.vt.p4e). Note taking was a requirement in Ed's classes (Omvh1211p2t). Ed had a high expectation of students' homework; if students did not do homework, they were not allowed to enter the classroom (Sy.Of.vt.p2m).

(f) Classroom interaction patterns

In consequence, Ed's teaching styles and students' reactions contributed to the classroom interaction patterns. According to classroom observations, interactions between Ed and the students occurred often, but the classroom interaction patterns were started mainly by Ed who challenged his students to develop higher order thinking skills. Ed said that the interactions between himself and the students were

different from his other previous classes. Students in his previous classes were more willing to actively share during class time. Students in this class were quieter (Om1Ihp12). Thus, this might explain why Ed needed to invite students to the front to solve problems or explain their ideas. Otherwise, they might keep quiet and that might disadvantage Ed's intentions to bring students into class discussions to explore mathematics together.

(g) Small group discussions

Generally, Ed's students were very quiet, gentle and obedient in Ed's classes. Students' small group work could often be found in his classes. For example, when student E18 was invited to write on the blackboard, student E20, student E21 and student E22 were discussing with each other at their seats (Omvh1211p4e, Omvh1211p5e).

(h) Mathematics themes

One of Ed's concerns was the selection of instructional material. He said, "I always use selected problems and gradually guide students into the mathematical theme" (Omvh1211p7t).

To sum up, in order to meet Ed's teaching goals for building up students' thinking and problem solving abilities (Om1Ihp14Q15e), his teaching norms (e.g. (a), (b), (c), (d), (g)) were consistent with his goals to help students to structure their own mathematical thinking, and the (e) teaching norms were aligned with helping build students' problem solving abilities.

7.3 Discussion and summary

This chapter has outlined Ed's teaching practices. His teaching styles strongly emphasize on student-centred learning that are reflected in his perceptions of students' learning (see section 7.2.1) and teaching practices (see section 7.2.4). Ed's case has given evidence that the teaching content and teaching practices match his views about mathematics, and his teaching strategy.

Findings to support the use of the small group discussion and class discussion approaches and along with some challenges will be discussed in Chapter 9. The

next chapter will present comparison data from students' views, teachers' views, and mathematical knowledge/understanding of the three teaching cases.

Chapter Eight: Perspectives of Teachers and Students

8.0 Introduction

This chapter will present comparisons of students' views (8.1) in three classes at two schools, mathematical knowledge/understanding (8.2), a comparison of the teaching skills of three teachers (8.3), and teachers' perceptions of difficulties/challenges faced (8.4). A conclusion will end this chapter.

8.1. Students' Views

In this section, the viewpoints of students in all three case studies are compared. These viewpoints were gathered from three questionnaires given to the students (see Appendices E, F & G).

8.1.1 Students' views about family support, classroom atmosphere and mathematics learning

8.1.1.1 Family support to Students

Questions on the first questionnaire asked the students about their parents' educational background (see Appendix E) and family support with homework (see the twelfth question of the third questionnaire, Appendix G). Although the parents of students' in School E had higher educational backgrounds than students in School T, students of both schools indicated that they received similar support from their parents. There were a similar number of parents of students from both schools, helping with students' homework: 14 students from School T and 13 from School E (n=T14, E13), and specifically for problem solving (n=T11, E9). However, some students did not receive any help (n=T12, E9) or were just sent to a cram school (n=T3, E1), or received help from a private teacher (n=T1, E0) (TQ3hp8e, OQ3hp8e).

8.1.1.2 Classroom atmosphere

In answering each question in the questionnaires used in this study, the higher the average scores especially from the first questionnaire (classroom atmospheres) and the parts of the third questionnaire including the two sub-areas of motivational belief, the more the students felt that they had a better classroom atmosphere and intrinsic motivation.

Data related to the student-perceived class atmosphere are presented below. See Appendix I for students' feedback from the first questionnaire.

- (i) Students in School E viewed their mathematics classes as more caring and friendly and felt that they had a better relationship with their teachers than students in School T. Students in School E had a higher level of agreement than students in School T about the following statements regarding their mathematics teacher: (i) treated students as friends (1.18, more than School T), (ii) cared about students' learning situation in mathematics classes (0.7) (iii) liked every student in the mathematics classes (0.65), (iv) cared about students' feelings while in mathematics classes (0.35), and (v) most students liked their mathematics teachers (0.28) (TQ1, OQ1, Appendix I). More students in School E than in School T were willing to be friends with their mathematics teacher (n=T14, E20) (TQ2hp4el, OQ2hp4el).
- (ii) Students in School E had a higher level of agreement with statements about teachers' support than students in School T. For example, they felt that their mathematics teacher encouraged students to discuss mathematics problems with each other in mathematics classes (0.71), and often offered opportunities to let students ask questions during mathematics classes (0.38) (TQ1, OQ1, Appendix I). Students' feedback from the first questionnaire in School E confirmed that question posing and discussions were norms of their mathematics classes.
- (iii) Students in School T showed slightly higher levels of agreement than students in School E that their mathematics teacher helped them to learn more mathematical content than other mathematics classes (0.30, more than School E students, TQ1Q(5), OQ1Q(5), Appendix I). Students in both schools appeared to have similar levels of satisfaction for their mathematics classes (School E had a higher agreement of 0.07 more) (TQ1Q(5), OQ1Q(5),

Appendix I).

- (iv) Students in School T showed a slightly higher agreement on peer support: sharing experiences (0.15) and resources with classmates (0.25), caring for each other's learning and improvement (0.11) and caring for classmates' attendance (0.21). These slightly higher differences might be linked to the different school systems that may have influenced students in School T; they having a longer time to meet with their peers than students in School E. Students in School E agreed slightly more on praise or encouragement of each other in learning (0.22) (TQ1, OQ1, Appendix I).

Students from School T valued mathematics learning slightly more than students in School E (T3.74, E3.47), and had a slightly better motivation to make an effort to study mathematics than students in School E (T3.10, E3.01). For example, students in School T indicated that they had higher value for these statements than students in School E: (i) what I have learned in mathematics classes will benefit my future (0.66, more than students in School E); (ii) mathematics learning is useful when applied in life (0.41); (iii) I am willing to do mathematics assignments and do not care about the time (0.35); (iv) I like to have mathematics lessons (0.17); (v) mathematics is a subject that enhances one's thinking ability (0.12); (vi) I study hard in order to improve my mathematics ability instead of (just) pleasing my parents or other persons (0.1); and (vii) Learning mathematics is a joyful thing (0.07) (TQ3, OQ3, Appendix R).

Students in School T indicated that they were slightly better motivated to make an effort to study mathematics when compared to students in School E, e.g. spending time to study mathematics (0.32, more than students in School E), solving difficult mathematics problems (0.24), learning mathematics even if they did not feel interested (0.18), staying up late at night to improve mathematics homework (0.14), and setting a high standard for my mathematics achievement (0.08). However, students in School E indicated that they were more active in finding resource books to study than students in School T (0.15) (TQ3, OQ3, Appendix X).

However, in contrast, School T students had a slightly higher inner value of and motivation for mathematics learning than students in School E. When regarding students' actions in current mathematics learning, students in School E indicated that they were more willing to take mathematics lessons than students in School T (n=T16(62%), E23(100%), TQ3Q(9)8a, OQ3 Q(9)8a), specifically for their current mathematics classes than students in School T (n=T20(77%), E20(87%) TQ3Q(9)8b, OQ3 Q(9)8b).

One point to remember here is that the high percentage (100%) of students in School E willing to take mathematics lessons might be explained by the fact that students in School E were able to choose classes that they wanted to study and had already made the decision to take mathematics classes.

Although students in School T indicated that they valued mathematics learning, liked mathematics better and had a slightly higher motivation to study mathematics than students in School E, when they had the choice to choose either to learn mathematics or not to in their school schedules, this higher interest, motivation and valuing in mathematics did not necessary translate into students in School T taking mathematics lessons. Students in School T did not show higher intentions to take mathematics lessons than students in School E (T62%, E100%).

Students' given reasons for studying mathematics may explain their attitudes towards mathematics learning. Students' given reasons in School T were fun (n=5) or improved thinking abilities (n=3), while the given reasons in School E considered the useful functions of mathematics learning such as helping them to go to better schools (n=7) and being useful in the future (n=4).

Students in School E appeared to adopt more practical attitudes in choosing to take mathematics lessons in their school or not. This may be linked to the educational environment in School E that helped students to think more or further, as well as the frequent opportunities to arrange their own school syllabus in each semester. They were forced to think about what each subject meant to them. Nevertheless, even students in School E did not have a higher value of mathematics learning or interests in mathematics or motivation to study

mathematics than students in School T, but when considering their futures, they chose to take mathematics lessons.

Regarding students' interest in learning mathematics in the future, from the fourth question of the third questionnaire (see Appendix G), confusing results appeared. There were slightly more students in School E who were willing to keep learning mathematics after they had finished their school education ($n=T7(27\%), E7(30\%)$) than students in School T. However, a slightly higher percentage of students in School E than School T felt that they would not keep learning mathematics after finishing their school education ($n=T10(38\%), E10(43\%)$). Therefore this researcher prefers not to comment, under the contrasting teaching styles, on the interest of students to continue learning mathematics in the future.

Ed and Eve had small classes and showed that they cared personally about their students. They encouraged students to share, discuss and explore mathematics in classes, with friendly and respectful attitudes. Their students felt that their teachers were friendly, supportive and had good relationships with their students. In consequence, the mathematics class atmospheres in School E appeared to students to be friendlier, supportive and had a better relationship between teachers and students than the class atmospheres in School T.

8.1.1.3. Students' views about their interests and difficulties in mathematics

Question 1 and Question 2 of the third questionnaire indicated that students from School T appeared to like mathematics better than students in School E ever since primary school ($T4.32, E3.70$) and in junior high school ($T4.96, E4.48$) (the full marks on these two questions were 7 points). While students in junior high at School T had slightly improved their preferences (0.64) in mathematics, students in junior high at School E had higher preferences for mathematics (0.78) than School T. This might suggest that mathematics classes in School E helped students to increase their interest towards mathematics.

The most interesting part of mathematics for students of both schools was geometry ($n=T9, E9$), with reasons as pictures ($n=T5, E4$) and proof questions ($n=T3, E1$). Three students in School E liked the multiplication formula of

quadratic equations. More students in School T liked solving problems (n=T5, E2) and working their brains (n=T3, E0) than students in School E.

Regarding mathematics learning difficulties in schools, more students in School E felt they had faced some difficulties in learning than students in School T (n=T16 (62%), E19(83%)). More students in School T had felt anxious than students in School E (T2.04, E 1.79) (TQ3hp4el, OQ3hp4el). (The full marks are 5 points.) In School T, ten students complained that their teacher taught too fast and seven students had difficulties in understanding the teacher's lecturing (TQ2hp2b). More students in School T had specifically faced difficulties in mathematics content than students in School E (n=T12, E6, TQ2hp3bl,4er). A small number of students complained about bad scores (n=3) and felt scared of their teacher in School T (n=2). In School E, more students complained that they did not do enough practice in mathematics classes (n=5, OQ2hp2br, 3bl) and had difficulties in problem solving (n=5, OQ2hp3e, 3eb). Slightly more students in School E than students in School T echoed that they did not work hard enough (n=T2, E4) and some felt they did not do a good job in learning mathematics (n=T2, E3).

Students viewed that the mathematics class atmospheres in School T appeared to help them to learn more mathematical content than other mathematics classrooms (see section 8.1.1.2(iii)) but were more anxious than the class atmospheres in School E. These characteristics of class atmospheres in both schools were consistent with the researcher's class observations of both schools, as documented in field notes. These students' perceptions echo the differences of mathematics classrooms at both schools. Tom gave direct-teaching and tried hard to impart knowledge to students, with the students indicating that they felt anxious but learned more content.

The difficulties experienced by students suggest the weak points of mathematics teaching in both schools. Students in School T reported having difficulty with the fast pace of teaching and had difficulties in understanding the content. Students in School E reported difficulties in not having enough practice in mathematics classes and difficulties in problem solving. These difficulties might lead to suggestions to improve mathematics classes in both schools. A suitable teaching

pace and decreased difficulty level of content could help to improve mathematics classes in School T. More practice in mathematics classes for problem solving might improve mathematics classes in School E.

To sum up, mathematics classes in both schools could cater more for students' mathematics learning. According to the students, mathematics classes in School E led to a higher motivation for mathematics learning than School T. Students' feedback of difficulties revealed the weak points of mathematics teaching in both schools, and where changes could be made.

8.1.2. Students' views on enhancing mathematics learning

8.1.2.1a Students' ideal design of mathematics lessons

When students responded to the open question (the Question 3(b) of the second questionnaire) about their ideal mathematics lessons, some students gave multiple answers. In order to understand and outline the students' opinions in two schools, their ideas were first examined by summarizing the key ideas given by the students. Their key ideas can illustrate the character of their class group and school. More students in School T gave mathematics content as their key ideas (n=12 vs. 1), more students in School E proposed teaching styles (n=9 vs. 12); some key ideas related with students' efforts (n=T1, E2), and no responses (n=T4, E8) were also given.

Generally, schools in Taiwan emphasize students' success in academic achievement. So, coverage of content and test giving are emphasized in a traditional mathematics classroom. More students in School T viewed the teaching content as helping them learn mathematics well when compared to students' opinions in School E (n=T12, E2). For example, some students in School T felt that interesting questions or lessons (n=6), the content design (n=4), giving simple and quicker problem solving (n=1) and practical examples (n=T2, E1) were their ideal designs. Some students of both schools in this study expected contrasting teaching practices from their current mathematic classes (n=T4, E2). Moreover, Student T22 in School T mentioned test giving. Whereas, student E17 in School E suggested that summarize formulas can serve as one of teachers' teaching strategies.

More students in School E emphasized the teachers' teaching styles as their ideal design of mathematics lessons than students in School T (n=T10, E12) (TQ2hp2e, OQ2hp2e). For example, students in School E mentioned the design of teaching as their teacher's teaching style (n=5), more discussions in classes (n=2), allowing students to learn in their own ways (n=1), allowing students to find their own ways to prove formulas (n=1), and checking for thorough understanding (n=2). Student E11 concluded that there should be a mixing 75% of Eve's teaching style with 25% of Ed's teaching style, because students in School E had experienced Eve and Ed's teaching in Grade 7 and Grade 8. In comparison, students in School T suggested that the ideal design was teaching as Tom did (n=5), explaining formulas thoroughly to benefit students' memories (n=1), more discussions (n=3) and more interactions (n=1). Hence, the students' ideal teaching in the above first five items in School E and the first two items of School T matched the character of their own school's teaching. This can be explained as the situated influence on students' ideal mathematics teaching.

Regarding the ideal teaching including self-study and students' efforts, student T18 in School T mentioned concentrating in classes, and four students in School E suggested that students themselves study hard doing lots of problem solving in and after classes. This self-study concept met the expectation of School E for students to become autonomous learners.

It would appear that influences of situated school and classroom practices have been elicited in students' responses. School E emphasized that students have ownership of their learning and become independent self-motivated. The unique school educational philosophy has motivated teachers to develop their own teaching styles in order to empower students to become autonomous learners. These unique characters of School E may have influenced students in School E to appreciate the teaching styles in their classrooms (n= 8 (35% of students)) and student efforts (n= 4 (17%)) as their ideal design of mathematics lessons. Students in School E appreciated the content (n= 12 (46%)), their teaching classroom styles (n= 6 (23%)) and tests given (n= 1(4%)) as their ideal design.

However, some students of both schools expected different teaching practices from their current mathematic classes. For instance, three students in School T wished to have class discussions and student T24 hoped to have more interactions in classes. Student E15 in School E suggested that the teacher review previous content first, then practice easier questions before difficult questions (TQ2hp2e). Student E17 advised that the teacher summarize formulas (TQ2hp2e).

8.1.2.1b Ways to improve students' learning interest

Question 3(c) on the second questionnaire (see Appendix F) asked students what they thought would improve their interest in mathematics. More students in School T suggested improving the classroom atmosphere, to be a lively atmosphere (n=T10, E4); more students in School E proposed some factors related to School E teaching styles (n= T3, E8); some students in School T mentioned exactly what happens in the teaching styles in School T (n=T6, E0); more students in School E suggested improving content (n=T2, E5) and there were no comments from the rest of students (n= T5, E6).

In reference to classroom atmosphere to increase their learning interest (n=T10, E4), fun was the main concern mentioned by the students. For example, quite a few students suggested increasing the use of games (n=T5, E2), jokes (n=T3, E0), interactions (n=T2, E1) or a relaxed classroom atmosphere (n= T0, E1).

Here we can see the situated influences of students' classroom practices towards their mathematics learning interest. For example, some students in both schools regarded some characteristics of their teaching styles as improving their learning interest (n= T6 (23%), E8 (35%). However, there were a few students from both schools who suggested different class practices to increase their interest in learning mathematics. For instance, three students in School T suggested discussions in classes rather than direct lectures (n= 2). Two students in School E felt lots of practice would benefit their learning interest.

In summary, a more exciting classroom atmosphere (n=10) and School T teaching styles (n=6) were main suggestions by students in School T to increase their interest in learning mathematics. Some characteristics of the School E teaching

styles (n=8), content (n=5), and classroom atmosphere (n=4) were the main suggestions that students in School E considered.

8.1.2.2 Student ratings of the first to the fifth important mathematics learning factors

Question 7 on the third questionnaire (see Appendix G) asked students to suggest and rate the important learning factors. Students of both schools had varied opinions of the first five important factors for mathematics learning. Table 13(TQ3hp2, OQ3hp2) summarizes students' perceived important learning factors which were raised at least by two students in each school.

Table 13 Rating given for each important mathematics learning factor in each school

Reasons	As the ____ important learning factor in School T (the number of students)	As the ____ important learning factor in School E (the number of students)
doing more problem solving	first, second, third (n=7,5,5)	first, second, fifth (n=5,6,3)
being able to understand mathematics lessons	first, second, third, fourth (n=4,3,2,3)	first, second (n=4,2)
paying attention (concentrating) in classes	first, second, third, fourth (n=5,2,3,3)	first, fifth (n=2,2)
students liking mathematics	third, fifth (n=3,2)	first (n=2)
doing more thinking		first, second (n=3,2)
discussing in class		second, third, fourth (n=2,2,3)
letting students learn freely without any requirement		second, third (n=2,2)
memorizing mathematics formulas	second, third (n=3,5)	
practicing many questions from resource books	fourth (n=2)	
students learning from a cram school	fourth, fifth (n=2,3)	
doing more mathematics activities	fifth (n=2)	
students willing to ask questions	fourth, fifth (n=2,2)	fourth (n=3)

Students of both schools suggested some common important mathematics learning

factors including doing more problem solving, understanding, paying attention/concentrating in classes, students' liking mathematics, and being willing to ask questions. However, there were more students in School T than School E who suggested that hard work helped their learning in geometry units ($n= T12, E6$) (retrieved from Question 13 of the second questionnaire).

Students from School T highly rated these reasons as influencing their mathematics learning. These were consistent with some characteristics of traditional classrooms such as: memorizing mathematics formulas, practicing many questions from resource books, learning from a cram school and doing more mathematics activities.

If combined, students' responses from their ideal design of mathematics lessons reconfirmed that more students in School T than School E valued memorization of mathematics formulas or methods ($n=T10, E2$). However, more students in School T than School E stated negative feelings toward memorizing mathematics formulas or methods in mathematics learning as one of their top three most disliked factors in learning mathematics ($n=T6, E3$).

Students from School E valued factors that were consistent with some characteristics of their mathematics classrooms such as: doing more thinking, discussing in class and letting students learn freely without any requirements. Combined students' responses from the first or second factors to succeed in mathematics learning and their perceptions of mathematics (retrieved from Question 7 of the second questionnaire in Appendix F) reconfirmed that more students in School E than School T valued thinking/understanding in mathematics learning ($n=T10$ (38%), $E17$ (74%)).

Moreover, students' responses on frequencies of teaching behaviors in mathematics classes also confirmed School E teachers' emphases on students' mathematical thinking/understanding more than School T, ex. explaining the reasoning behind an mathematical idea ($T3.54, E4$), even challenging thinking tasks such as more chances to tackle mathematics projects ($T1.27, E1.65$), and working on open problems without fixed/certain solutions ($T2.39, E2.48$) (see

section 8.3.3). (The full marks are 5 points.)

Mathematics classes have a distinctive character of thinking compared to other school subjects, and more students in School E than School T perceived this character. Students' responses echoed Ed's and Eve's emphases on mathematical thinking in classroom practices and indicated the situated influences from teaching practices upon students' values.

In conclusion, students' perception and rating of important mathematics learning factors once again echoed the characteristics of their classrooms.

8.2 Mathematical Knowledge and Understanding

The questionnaire (in Appendix F) also asked students to comment on their perceptions of mathematics itself.

8.2.1 Students' views about mathematics

When students responded to Question 7 on the second questionnaire, about what is mathematics in their opinion, some students (n=T4, E4) gave more than one description, and only one was selected. For example, student T18 indicated that mathematics is a subject and is a troublesome thing. There were also eight other students in School T who viewed mathematics as a subject. I only selected one of student T18's ideas: mathematics as a subject. Thus, there were nine students in School T that viewed mathematics as a subject. The high frequency of students' opinions from a class can illustrate the character of that class. A noticeable difference is that more students in School T interpreted mathematics as a subject (n=T9(35%), E1(4%)) and more students in School E interpreted mathematics as a way of thinking (n=T5(19%), E8(35%)).

For example, referring to mathematics being a way of thinking, student E8 said, "The knowledge can be applied in life and is able to improve people's logical thinking" (OQ2Q7). When regarding mathematics as a subject, Student T7 explained, "It is a subject in which students need to work hard on problem solving and rely on their own abilities" (TQ2Q7). Student T3 said, "It is a subject that students love it and hate it" (TQ2Q7).

8.2.2 Time in class and self-assessment of understanding

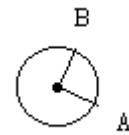
Question 14 on the second questionnaire (see Appendix F) asked students about their studying time in a cram school. The data indicated that students in School T received more teachers' instructions than those in School E. For example, they had more time with a teacher in the school and a cram school. For instance, there were five mathematics classes (including one extension class) in a week in School T, compared with four mathematics classes in a week in School E. There were more students in Grade 9 in School T than School E who went to a cram school after classes ($n=T21(81\%), E3(13\%)$) (TQ2hp5) (OQ2hp5).

Question 15 on the second questionnaire (see Appendix F) inquired about students' understanding within and after a mathematics class. From students' self-assessment, more students in School T thought that they could understand more than 60% of mathematics content after a lesson than students in School E ($n=T19(73\%), E15(65\%)$). Thus, this might suggest that direct teaching in School T brought better understanding in classes, but students in School E indicated that they faced more challenges in thinking/understanding through class discussion methods. However, after studying on their own after classes, slightly more students in School E thought they increased their understanding of their class mathematics content than School T (students ($n=T19(73\%), E20(87\%)$)). It is to be remembered that School E expected students to be independent learners. Students' effort after class overcame the challenge of difficult understanding in the constructivist classrooms of School E. Eve was also concerned about the challenge of understanding within the constructivist teaching of the class discussion method (see section 8.4.1).

8.2.3 Students' creative operations

The experimental classrooms encouraged students to explore and discuss mathematics that might benefit their creative abilities. For example, students' creative thought was observed in Eve's and Ed's classrooms. In one of Eve's classes, student E8 came to a conclusion herself and said "the central angle of a circle is the angle of an arc" (Ofvh1030p8b). Student E11 found out that if you added one more point, C, on the arc AB, it would help people to distinguish which

arc AB on a circle that we want to talk about (Ofvh1030p10t).



In one of Ed's classes, student E20 referred to a new finding by herself that $\overline{OA} = \overline{OB} = \overline{OC}$ with Ed's continuous questioning and some hints (Omvh1211p6e).

Thus, class discussion and class time in School E allow students quality chances to explore/argue and to create mathematics concepts. In contrast, students of School T learned mathematics concepts as told by the teacher. For example, as in the instance above, student E8 herself found the mathematics relationship that the central angle of a circle equals the angle of an arc, but students of School T learned this concept by being told by Tom (see the first example of section 5.2.6).

Chances still existed in the School T class to allow students to create/expose their own ideas, e.g. through Tom's questioning of students for quick answers on problem solving (see the second episode of section 5.2.6) and a few chances appeared in students' seat work or discussion (see section 5.2.4). However, from class observations it is clear that less class time was allowed to wait for students' responses/thinking in School T than School E; thus, this reduced School T students' chances to develop their own creative thinking abilities compared to those in School E.

8.3 Comparison of the Three Teachers' Teaching Styles

8.3.1 Time interval count analyses

In this section, comparisons between the three teaching styles as presented in Chapters 5, 6, 7 are discussed. The video tapes of lessons from the three teachers were analysed using time interval count analyses. The time interval count analyses of these three lesson videotapes is provided only as supportive evidence for the different emphases of the two schools' teaching that were consistent with the data from teachers' interviews about their teaching styles, students' feedback and classroom observations. The frequencies of every event and explanation were

recorded every 30 seconds in a lesson of 43.5 minutes. A detailed account of the categories is documented in Appendix H1, with results in Appendix E1.

The time interval count analyses confirmed the differences of class norms and especially Tom's teaching style compared with those two teachers in School E. For example, the following teaching practices were more frequent in Tom's teaching than in that of the other two teachers. The frequencies of every 30 seconds were presented in a bracket:

- Tom more frequently talked to the whole class (Eve: 68, Ed: 45, Tom: 84) and used the textbooks (Eve: 0, Ed: 0, Tom: 29).
- He was more often positioned at the front of the class (Eve: 39, Ed: 52, Tom: 83).
- Students were more frequently making notes in his class through listening to him talking (Eve: n= 10, Ed: 5, Tom: 31).

These frequency counts on the teaching characteristics support that students are seen as learning by passively absorbing the teachers' delivered knowledge and Tom focused on the task (textbook). Tom mainly use these chalkboard/lecture methods.

These teaching practices were more frequent in Ed's teaching:

1. inviting students to share and talk about their problem solving in the front of the classroom (Eve: 1 , Ed :40, Tom: 0), though students did not talk a long time
2. teaching individual students (Eve: 15 , Ed :48, Tom: 0)
3. asking students to do seat work to solve problems by themselves (Eve: 0 , Ed :46, Tom: 2)
4. walking around the class to check students' seat work (Eve: 23 , Ed :29, Tom: 1)

These teaching practices were more frequent in Eve's teaching:

1. giving opportunities for a student to share his/her problem solving with the whole class (Eve: 35 , Ed:5, Tom: 0) and with an individual student, to clarify questions (Eve: 5 , Ed:1, Tom: 0)

2. individual students initiating talking with Eve about their mathematics thinking in the whole class discussion (Eve: 14 , Ed:5, Tom: 0)
3. individual students teaching their classmates in front of the whole class by speaking their ideas out loud (Eve: 5 , Ed:1, Tom: 0)
4. discussing in small groups (slightly more than Ed's classes) (Eve: 8 , Ed:7, Tom: 0)
5. individual students asking (Eve: 14 , Ed:9, Tom: 5) or answering questions (more than in the other two teachers' classrooms) (Eve: 29 , Ed:15, Tom: 3)

To conclude, Eve applied the class discussion method (as the above first and second points of Eve's teaching practices) and Ed let students work on seat work (as the third points of Ed's teaching practices) to let students explore their own problem solving and mathematics concepts, but there was more in Ed's classes and Ed discussed more with individual students or a student group (as the second points of Ed's teaching practices). Eve's classes appeared livelier and more students participated in class discussion (as the above the fourth point of Eve's teaching practices). Student peer discussion appeared in both teachers' classrooms (as the above the fourth point of Eve's teaching practices).

8.3.2 Students' perceptions of their teacher's teaching behaviours and attitudes

Teacher's teaching behaviours

Students of both schools shared their opinions about the frequencies of several teaching behaviours in mathematics classes through the eighth question on the third questionnaire. That the teaching styles of both schools are different is supported by the higher frequencies of varied teaching behaviours that appeared in their mathematics classrooms; and these higher frequencies pointed out the characteristics of different mathematics teaching styles in both schools. The average point of students from School T is shown first in the bracket and next is the average of students from School E (OQ3&TQ3summary p.1re). The data from Tom's interviews and classroom observations are also addressed here in the bracket. Students in School T gave higher average points to the teaching behaviour in mathematics classes of showing how to do mathematics problems than students in School E (T4.54, E3.79) (T1lhp1Q3; T1lhp3Q6be; Tvh1118p5e,

Sy.Tvt.prel210), asking students to work independently from worksheets or textbooks on their own (T3.23, E2.26) (T2Ihp2Q4m), writing equations to represent mathematical relations (T3.62, E3.00) (T1Ihp1Q3; T1Ihp3Q6be; Tvh1118p5e, Sy.Tvt.prel210) and practicing computational skills (T3.81, E3.18) (T2Ihp2Q4m; T2Ihp2Q4m, Sy.vt.all.Tom,p.3t, Sy.vt.p3ml.1129, 1228; Sy.vt.p3ml.1209,0107).

Among these comparisons, higher number of students in School T than in School E indicated high frequency of individual work in mathematics lessons and of working on worksheets, exercise books or textbooks in classrooms. These findings suggested that School E appeared not to use much individual work as School T, but appeared to employ a style more similar to a corporate study style in mathematics classrooms than School T (including group discussions and class discussions), because of less frequency of working on their own in classes, on worksheets, exercise books or textbooks (T3.23, E2.26) (Of1Ip2Q3, Of1Ihp4Q4e). (The full marks are 5 points.) That finding was also confirmed through class observations, e.g. Eve 16 lessons, (Sy.Of.vt.p3t; Om1Ip2Q3) and Ed 9 lessons (Sy.Om.vt.p4m).

Students in School E indicated higher frequencies than students in School T of these teaching behaviours in mathematics classes, such as explaining the reasoning behind a mathematical idea (T3.54, E4) (Of1Ip2Q3; at least 12 Eve's lessons, Sy.Of.vt.p5t) (Class observations found this in 5 of Ed's 9 lessons, Sy.Om.vt.p4m), more mathematics projects (T1.27, E1.65) (Of1Ip2Q3), (TQ3hp3rt, OQ3hp3rt), and working on open problems without fixed/certain solutions (T2.39, E2.48) (e.g. Ofvh1030p5e, Of1Ip2Q3, Om1Ihp9Q6e).

The three teachers in School T and School E all allowed students to use multiple ways to do problem solving, not just follows the teachers' ways. However, more students in School T than in School E felt they used several methods to solve a particular problem (n=T17, E7) (TQ3hp2tl, OQ3hp2tl). Here address the benefit of School T teaching. .

Student E11 commented the benefit of Ed's and Eve's mathematics teaching as below:

The advantages of Eve's classes are that students can completely understand the content and not forget it easily. The disadvantage of Eve's classes is that if students want to improve the speed in problem solving, they need to do extra practice at home. The advantages of Ed's classes are that students' speed in problem solving is fast. The disadvantage of Ed's classes is that if students want to remember clearly (the mathematics concepts); students need to do extra reading at home (OQ2Q3st11).

Teacher's Attitudes

Students in School E agreed more that their mathematics teacher often praised students in mathematics classrooms than students in School T (T3.16 vs. E3.48) (Sy.OQ3&TQ3p.1'e). (The full marks are 5 points.)

These three teachers were all very supportive and helped students. When they sensed students' confusion, they would come to help, either giving hints to guide students to think as Eve and Ed did (reported in sections 6.2.6.2 (a) & 7.2.6.1), or give direct statements as Tom did (Sy.Tvt.p1el1220, p2tl.0109). However, Eve and Ed still gave direct instructions when needed.

8.3.3 Teachers' speed, the coverage of content and practices

Based on what was observed in the classroom none of the three teachers closely followed the mathematics textbooks to conduct their teaching. They all summarized the core points from the textbooks but used different teaching methods to help students to learn mathematics content.

Tom delivered his mathematics lessons at a quick speed and covered lots of content. Some evidence of this included: (i) He sometimes asked students questions, but he did not wait for students to answer. For example, he answered himself eight times in a lesson (Tvh1118p1tb~6). (ii) Tom urged students to hurry and write down the answers. For example, he urged students three times, as in the below examples (Tvh1118p7e).

Tom: Hurry up and write down the answer. How many degrees is it? How many degrees in a quarter?

Some students: 90 degrees.

Tom: 90 degrees. Hurry up and write it down (Tvh1118p7e).

Another example shows three teachers' different teaching speed. In Tom's classes, he asked a great number of questions from the textbook that showed his intention to cover more content in a short period of time. As a result, Tom's speed in content coverage was quicker than Eve's. For example, Tom took 6 lessons to finish unit 2-2, but Eve took 16 lessons to finish it. Tom spent 6 lessons to finish unit 3-3; Ed spent 9 lessons (Sy.vt.all.p.4). Generally, Tom taught faster than the other two teachers, and he covered more content in a lesson.

A great amount of problem-solving practices occurred in Tom's classrooms than Eve's and Ed's classrooms. After finishing coverage of the textbook, and he arranged time to give tests (ex. one extra unit 2-2 test, one whole chapter 2 test (Nov29(5)), and one whole chapter 3 test (Jan7(8))), to correct/explain tests for at least four lessons (Sy.vt.all.p.3ml), to do seat work on some extra material (two lessons) (Nov26(2), Dec17(2), Sy.vt.all.p.3ml) and to correct/explain those extra materials (Sy.vt.all.p.3ml).

8.3.4 Mathematics activities and seating arrangements

The differences in the usage of mathematics activities and in the seating arrangements in the three classes added to the teachers' different teaching styles in both schools.

Students' responses at both schools on the first question of the second questionnaire (see Appendix F) indicated the following information. For example, in School T, Tom rarely used mathematics activities in his classes, and 15 students mentioned no activities in classes. Student T13 mentioned one mathematics activity as paper cutting to make a cube in Grade 8 classes (TQ2hp4t). That activity was included in the Grade 8 textbooks.

On the other hand, Ed and Eve carried out mathematics activities in their classrooms (n=12). For example, they divided students into groups to report on the content in textbooks especially in Grade 7 and 8 (n=3), reported mathematician stories (n=9), tossed dice (n=1), played cards (n=1), solved

problem in groups (n=1) or had their mathematics class in a cafe with the teacher treating every student to a drink (n=1) (OQ2hp4t).

The students in School E perceived some benefits from the mathematics activities. They felt they had better understanding about the mathematical content (n=3), had connections between mathematics and real life (n=1), had more fun in learning (n=1), and felt they learned more extra knowledge such as the history of mathematics (n=3). Two students learned co-operation, such as organizing notes for classmates (OQ2hp4t).

The three teachers had different seating arrangements. Tom had students sit individually, but Ed and Eve allowed students to sit together (Omvh1211p1e, Ofvh1030p4t). This indicated the intention of teachers in School E to encourage students' cooperation in learning and of Tom, to encourage students' individual efforts.

8.3.5 Teacher roles

Tom and Ed both felt that they had the characteristics of an authority and a helper in their classes, and teachers' roles should be flexible between these two (Om1lhp14Q13m, T1lhp6Q13e). Tom thought that if a teacher was too authoritative, the students would be too scared to talk, so a teacher needed to be encouraging and helpful as well (T1lhp6Q13e). Ed said, "When I talk about mathematics theories or start to talk about the core concepts, I feel that I need to be authoritative" (Om1lhp14Q13m). "When students are explaining their problem solving, I need to be a helper. My roles are always changing" (Om1lhp14Q13m).

Eve felt that she was like a supporter, helper and questioner in her classes, but not an authority figure. In her classes, Eve always questioned students to make them think deeply or encouraged students to make clear statements about their understanding. She even said, "Students treated me as having no authority, but teased me, made fun of me" (Of1lhp16Q13m).

The three teachers all used ways to change students' unsociable behaviours. Tom used his authoritative position to discipline students. If students did not

concentrate in Tom's classes, he would scold and correct them (T1Ihp8Q18m). Ed had basic respect rules in his classes. For example, if students disturbed Ed's class, he would say to them that "you should not disturb my classes and disturb the other classmates' learning". Hence, the students' unsociable behaviours would be required to stop (Om1Ihp15Q18t).

Eve used a gentle way to discipline her students but still made high demands on their behaviour. For example, she asked students, "What is your situation now?" Later, she understood her students better and knew their learning situations so she took a sterner attitude towards them, e.g. scolding them or warning them that they might fail; 17 students in Eve's Grade 7 class were failed by Eve (Of1Ihp19Q18).

School E teachers had more school meetings than teachers in School T. All teachers needed to attend teacher meetings once per week in School E, to discuss students, or parents or school issues, usually for two or more hours (Of1Ihp21Q23). In School T, mathematics teacher meetings occurred three times in one semester, each lasting less than one hour. Homeroom teacher meetings occurred three times in one semester, normally lasting one or two hours (T1Ihp9Q22).

The three teachers all worked hard for their classes. According to class observation, Tom tried to give students more knowledge in his classrooms as best he could. Eve felt that she needed to spend lots of time planning and preparing lessons (Of2Ihp5Q5t). Ed perceived that teachers should have several methods to help students' learning (Om2Ihp1Q1e).

8.4 Teachers' Perceptions of Difficulties/Challenges

Eve felt that when teachers finished lessons in the traditional schools, they did not have burdens in their hearts. In contrast, Ed and Eve felt that there were lots of challenges in School E (Om3Ihp2tQ4pr, Of3Ihp4tQ4pr). Eve often needed to think how to plan the next lesson to build up students' expression abilities, independent learning abilities, appreciation of the beauty of mathematics and cooperative learning (Of3Ihp4tQ4 pr).

Eve shared that when a teacher gave direct instruction, students only needed to focus on the teacher and the teacher's problem-solving strategies (Of1Ihp9Q6b). After reasoning about the teacher's talk and methods, students just followed the teacher's methods. However, more challenges were raised in class discussions. That is, after several different class discussions, Eve felt that a few students were unable to catch up with the shifts in the discussions and failed to understand the mathematical content of other students' conversation (Of1Ihp11Q6b). Eve suggested that might be a result of the students' logical reasoning ability still not being mature (Of1Ihp2Q2e) and hence weak (Of1Ihp9Q6e; hp11Q6), so they would be unable to catch up with the shifts of focus in students' discussions (Of1Ihp2Q2e; hp11Q6) or understood the mathematical content of other students' talk (Of1Ihp11Q6b).

Eve indicated that she faced challenges/difficulties while doing class discussion such as posing questions, students unable to catch up with the shift focus of class discussions (Of1Ihp2Q2e; hp11Q6), that she would spend extra work to follow up on those students' learning (Of1Ihp16Q12t), students' passive learning attitudes (e.g. students' late assignment and non-preparation), heavy work load (lesson planning (Of3Ihp5Q4t), too many school meetings (Of1Ihp14Q12m), time arrangement of classes, great range of students' ability due to the school policy to allow students attend their age group mathematics classes even without taking mathematics lessons in previous years (Of1Ihp6Q4b) and parents' conflicts (Of2Ihp4Q1m).

For example, it takes time to see the growth of students' abilities, but not every parent has patience to wait for them. Parents did not necessarily value Eve's teaching goal to help students "conduct self-learning in the future and they could discover knowledge themselves" (Of2Ihp4Q1m). Some parents forced their kids to transfer to other senior high schools and students suffered (Of2Ihp4Q1m).

Tom criticized students' problem solving methods from the influence of constructivism in the primary schools in Taiwan, and worried for students' mathematics abilities in 2002. He found that students used complicated methods to do division; hard for even him to understand. He also found some primary

students had difficulties to understand the calculations of a longer mathematical sentence and made mistakes, e.g. $\Pi \times 20^2 \times \frac{60}{360} = \Pi \times 200 \times \frac{1}{3}$. Tom concluded that there was a need to teach the traditional calculation methods to those primary school students to replace the methods that they had learned (Tvh1118p5e).

Another common difficulty occurred, when practicing constructivist teaching in a normal school, pressure came from parents or the school, forcing teachers to change back to the traditional direct teaching. Several cases are described here.

Case 1: teacher Eve, in this research taught at a private school in 2001 where she used the constructivist teaching method. However, after three days the head of academic affairs asked her to change her teaching style. After that, she did some adjusting. She described the experience as follows talked:

I was responsible for talking, and students were responsible for listening. There were fifty students in a classroom. Group discussion could not be used because that would have seriously delayed the learning speed of the whole class and they would not have been able to keep pace with the learning speed of the whole school. With the approach of examinations (examinations in that school were always whole school competitions) the use of the class discussion method would really put you down. If the students' performance were not satisfactory, the parents would complain, complain incessantly (OfPIhp2Q4e).

8.5 Summary

The chapter compared data on students' and teachers' views about their teaching and learning, and mathematical knowledge/understanding. The next chapter will argue on the different learning influences of the two contrasting teaching modes.

Chapter Nine: General Discussion

9.1 Introduction

The research findings from the main study of the three teachers' teaching styles are presented in the form of three case studies in Chapters 5, 6 and 7. Teachers' and students' perceptions of teaching and learning are documented in Chapter 8. The analyses of instructional practice and pedagogy can help to explain students' learning. Moreover, the perspective of situated learning views that learning occurs with respect to the cultural environment (Lave & Wenger, 1991) and practices that learners were involved in (Boaler, 2002c). Based on these factors, this research focused on the classroom practices "as an integral part of generative social practice" (Lave & Wenger, 1991, p.35) and as an analytical unit (Borko, 2004) in these three case studies. A combination of behaviourist, cognitive and constructivist perspectives were used to interpret these teaching practices and styles. Drawing on the different views or perceptions of learning will lead to a better understanding of the classroom teaching practices or culture that students encountered and in which their learning is taking place (Lave & Wenger, 1991).

The findings of this study indicate that there were several benefits to be derived from using teaching based on constructivist views of learning in these two cases of Taiwanese Junior high school mathematics classrooms. While the results emanating from both the traditional and experimental instructional approaches point to students problem solving patterns, the use of the experimental or constructivist approach suggested the following:

1. Classroom Dialogue: More chances were offered in the classroom for student dialogue about mathematics. Students made new connections during class or mathematics discussions. The benefits of this might be: a) students receiving supportive relationships in the classroom - this was helpful to many confused classmates, b) more opportunities to practice and develop their communication skills, including more oral explanations of

their thinking or problem solving processes, c) instruction being more student centred with more ownership by student than teacher (e.g., teacher and students sharing talk time; and teacher valuing student input), d) opportunities to allow students to share their thinking and openings for teachers to find out what the students are thinking.

2. Developing Critical Thinking Skills: Students were more involved in problem solving activities: different kinds of student thinking emerged – students engaged in exploring, producing and creating flexible solution strategies. The benefit of this was: a) greater student-directed thinking, b) possible chances of developing higher-order thinking skills, and acquisition of more conceptual and procedural knowledge, and c) improving the quality of students' mathematics knowledge, competence and situated influence.
3. Increases student motivation: The results also indicated that students exposed to two schools with similar teaching styles showed high motivations in mathematics learning in different ways.

Further findings will be presented and discussed in each of the following sections.

These sections will be followed by the chapter conclusion.

9.1.1 Quality opportunities for student dialogue about mathematics and new relationships in the classroom

9.1.1.1 Supportive relationships in the classroom

Based on students' responses from the first and second questionnaire, Eve's and Ed's classrooms appeared to provide an environment that was open, relaxed, vital, friendlier, supportive and had better teacher-student relationships than Tom's traditional classroom environment. For example, the data supported that the teachers, Eve and Ed, appeared friendlier and supportive and had better teacher-student relationships than teacher Tom. Eve's and Ed's students in School E had a higher level of agreement than students in Tom's classroom in School T. The data also show that teachers in school E treated students as friends more than School T; cared about students' learning situation, and students' feelings in mathematics

classrooms (Appendix I: TQ1, OQ1). More students in School E than School T were willing to be friends with their mathematics teacher (n=T14, E20).

Ed and Eve encouraged students to share, discuss and relate mathematics (see Tables 11 & 12) engage in other activities (see section 8.3.4) and provide report (see section 6.2.4) in class with a friendly and respectful attitude (see sections 6.2.6.2 & 7.2.2). Ed felt that students of School E were more open or willing to ask teachers question than in other schools (Om1Ihp1Q0e). Thus, at School E, the participating students felt that teachers were friendly, supportive and had good relationships with their students. Further, they felt that theses teachers allowed them to freely support and explain to each other. They were also provided with many chances for questioning and discussion. This supports Engle and Conant (2002) the criteria that leads to dynamic participation in classroom while learning.

The reform classroom provided opportunities to foster better peer support in School E. If a student appeared to not understand in-class discussions, he/she would receive support from classmates. A supportive atmosphere was evident in Eve's classrooms. For example, either Eve or the students would help those students who faced difficulties (Ofvh1030p1~3, OfItelephone2003/1/13p.2e) or praised those students who did a good job (Ofvh1030p11b). Opportunities for small group work were evident in Eve's classes. According to Eve, she felt touched in one lesson when all the students in her class wanted to help student E3 to understand a particular concept (OfItelephone2003/1/13p.2e). Similarly, students' small group work collaboration was common in Ed's classes (i.e., in 6 of 9 lessons for one mathematics unit, Sy.Om.vt.p4m). Ed and his students' opinions also supported that there were greater agreements among Eve's and Ed's students than in School T (n=2, OQ2hp1ml). There is evidence that this classroom environment supported students becoming confident as their mathematics teacher often praised students during mathematics activities (T3.16 vs. E3.48, Sy.OQ3&TQ3p.1'e). In contrast to this classroom, Tom rarely used other teaching strategies (T1Ihp4Q8e) but on many occasions, students automatically had small group discussions. However, their discussion time normally lasted no more than two or three minutes (see Sy.vt.Tp1el.1213, 1217, 1210).

Therefore, students taught in the experimental group, as in Eve's and Ed's classrooms, appeared more autonomous in sharing their mathematical thoughts and offering support to classmates in classes than Tom's classrooms from the students' perspectives. The three teachers at both schools were all very supportive in helping to resolve students' confusion (see section 8.3.2). Students modelled the observable behaviours; they got used to talking and helping each other as they collaborated to solve problems in the class discussion teaching approaches of School E in Eve's and Ed's classrooms. The differences of frequency in classmate support might be a direct result of the different teaching styles and classroom culture (Wood et al., 2006; Wood & Turner-Vorbeck, 2001). Moreover, the social norm of students' supportive cooperation in Eve's and Ed's classrooms further supports the development of student participation in mathematic learning (Franke et al., 2007; Kazemi & Stipek, 2001).

Evidence of Eve's and Ed's classrooms having a more open and relaxed environment than Tom's classrooms is also seen from the students' perspectives. For example, student E4 felt that the atmosphere in School E was open (OQ2hp1tr). This viewpoint might relate to the regulations or approach to learning taken by School E and Eve's and Ed's classroom atmosphere. School E emphasized autonomous learning and allowed students to have freedom to choose the subjects to study for every academic year (SyQ1p.1). Both Eve and Ed believed that students were responsible for their own learning (Of2lhp3Q1m, Om1lhp6e, 7tQ3). Ed and Eve encouraged students to openly share their ideas (See the Table 11 & Table 12), and their ideas were respected and valued by Ed and Eve (see sections 6.2.6.2 & 7.2.2). For example, Ed encouraged his students to use their own language to describe mathematical concepts (Om1lhp11Q8tb). Students' responses and willingness to participate were also respected. This is seen in the instance when student E2 and Student E11 refused Eve's invitation to go to the front to share their thoughts (Ofvh1030p3e, Ofvh1030p11e). Eve responded that students' rights were always foremost in her thoughts even when "Students treated me as having no authority, but teased me, made fun of me" (Of1lhp16Q13m). This type of rapport led students in School E to comment that teachers' teaching styles provided a relaxed atmosphere in classes, and students did not have pressure from classes or from examinations (OQ2hp1).

In contrast, although students reported that the mathematics class atmosphere or established culture in Tom's classrooms provided some flexibility towards learning the mathematic content than other mathematics classrooms in School T (Appendix I), they felt slightly more anxious (TQ3hp4el, OQ3hp4el, see section 8.1.1.3) than students in Eve's and Ed's classrooms of School E. This may be viewed as a result of the way that Tom gave direct teaching and how hard he tried to impart knowledge to students (see section 5.2.4).

Based on the preceding discussion, Eve's and Ed's classrooms appear to have provided an environment that was open, relaxed, vital, friendlier, supportive and had better teacher-student relationships. This environment differed from that of Tom's traditional classroom environment where the instructional style predicated student learning style of being more passive during mathematics lessons. The social normative behaviours, such as participation in class discussion (Franke et al., 2007) and providing support to encourage student cooperation as was evident in Eve's and Ed's classrooms further support the need to promote student participation in learning mathematics (Franke et al., 2007; Kazemi & Stipek, 2001).

9.1.1.2 More chances to practice communication abilities

The results of this study are also aligned with the perceptions of other researchers who support the use of class discussion as a means of transforming classroom practices in schools (for example, School E), or having a more supportive learning community (McLain & Cobb, 1998). The data in this study revealed that students' mathematical communication in Eve's classrooms were encouraged and flourished. Further, the number of students actively and automatically raising their opinions in Eve's class was more than those in the other two similar classes. Eve also expressed that she emphasized student talk and student learning (Of1lhp12Q9e). She utilized class discussion to help her better understand student thinking (Of1lhp9Q6b). Eve also found that students' dialogue or expressing their mathematical ideas at the senior high level increased more than at the junior high level (Of3lhp4mQ5pr). This finding is consistent with some scholars' beliefs (Lave & Wenger, 1991; Pimm, 1987; Peressini et al, 2004; Cobb et al, 1991;

Webb, 1991). These researchers agree that the use of classroom discussion promotes student ability to orally explain their thinking.

In comparing both classrooms and the teaching styles, Eve's and Ed's classrooms differed from Tom's classrooms in terms of: students' behaviour while participating in class discussions, frequency of student dialogue and time for class discussions. For example, Eve's students automatically asked questions, added comments or explained ideas to lead student or teacher's dialogue during class discussion. Some students even automatically came to the front to explain to the whole class (Sy.Of.vt.p2). For instance, student E1, E8 and E11 automatically joined in Eve's class discussion (Sy.vt.p2r). Ed used this method to encourage students to demonstrate understanding of mathematical problems by using their own words to describe mathematical concepts (Om1Ihp11Q8tb), and tried to see mathematics from students' perspectives (Om1Ihp6Q2).

Regarding the frequency of student talk, the data revealed that there were at least two instances during the first ten minutes of Eve's class discussions, students voluntarily shared their thoughts 33 times (see Appendix U); in another class, students voluntarily shared their thoughts 16 times (see Appendix V). The time allocated for class discussion varied; student talk or explanation of their mathematics ideas was on the average 24.3 minutes of a 50 minutes lesson (Sy.Of.vt.p2, see Appendix S).

Generally, Ed's students were very quiet, gentle and obedient in his classes (Omvh1211p4eEd). Ed felt that students in this class were quieter than his previous classes (Om1Ihp12). Therefore, Ed encouraged students to go to board to solve problems or explain their ideas. Students' small group work was often evident in his classes (Omvh1211p4e, Omvh1211p5e). For example, in seven of Ed's lesson an average of 38.1 minutes per lesson was used for students' seat work with small group discussion, and class discussions. Ed used this opportunity to challenge individual students (see Appendix W).

In comparison, Tom adopted direct instruction (T1Ihp3Q6b, T2Ihp3Q5e) which required students to follow the direct transmitting of mathematical information

from Tom instructions. This mode of instruction did not provide many opportunities for his students to communicate their mathematics ideas. According to three students, they were constantly quiet in the classroom (TQ2hp1tl). Further, Tom believed that students would learn better when provided with fast solutions for problem-solving and using direct instruction (T1Ihp3Q6b, T2Ihp3Q5e). In Tom's classes, where the traditional approach was used, it was not easy to discover student thinking since not many chances were given to these students to explain their mathematics thinking (see section 5.2.4). Students mostly responded when they were questioned or asked to explain the mathematics concepts or complete problem solving on the blackboard. Further, discussion occasions were limited. These students, in comparison to students in Eve's class, were not otherwise encouraged to readily share their alternative thinking, (see sections 6.2.6.1 & 6.2.6.2).

Further analysis of the findings of differences in student communication abilities in the two classes of School E could be explained as the differences of the personalities of these two groups. The two teachers involved in this research both taught Grade 9 mathematics; before that they had taught Grade 8 and Grade 7 mathematics in this school (see section 4.3). So, students are familiar with both teachers. For example, Eve liked to encourage and inspire students to share their mathematics thoughts with the class. She required students to hand in reports and to present their reports in groups to share with the class (see section 6.2.4). Ed also encouraged students to share their mathematics thoughts with the class, and strongly required students do their homework (see section 7.2.4). As a result, students who like discussion would choose Eve's class. Students who did not like discussion but felt all right about homework would go to Ed's class. If students did not like to talk by nature, naturally their communication abilities in mathematics could be limited or encouraged depending on the classroom teacher's expectations or the culture of the classroom. Thus the student learning experiences generated from this research from using class discussions could provide teachers with a tool to foster ongoing classroom evaluation (Confrey & Kazak, 2006; Kahan et al., 2003).

According to research, when this strategy is effectively utilized it will:

- help students clarify their thinking, and develop their ideas, questions and justifications (Boaler & Greeno, 2000; Confrey & Kazak, 2006; Franke et al., 2007). This suggestion is supported by student E9: “Our teachers let students explore and discover the solutions by themselves” (OQ2hp1tr);
- provide good feedback to teachers (Kahan et al., 2003) and students (Lampert, 2001), and understanding of student mathematical concepts (BRAP, 2003; Franke et al., 2007; Romberg et al., 2005). Teacher Ed reflected on the role of class discussion:

I let students go to the blackboard to share their ideas about what is proportion, what is parallel, and what is the string inside of a circle. I will let students share first, then I will read what they write on the blackboard to let students check if there are any problems in those students’ ideas, then we discuss this as a class (Om1Ihp9Q6e).

Teacher Eve found that class discussion could lead to students establishing ownership of their own learning based on the process of accepting, judging, valuing, and discussing each other’s ideas to make conclusions (Of2Ihp3Q1e). Moreover, students could have a chance to check the reasonableness of their and others’ answers or raise questions to leading student responses (Ofvh1030p1tb). These findings support the work of researchers (e.g., Fu, 2008; Hunter, 2005; Nathan & Knuth, 2003) who promote the notion of appreciating students’ use of alternative strategies, and their creative or reproductive thoughts (Windschitl, 1999b).

9.1.1.3 More student centred learning

The findings of this study point to students in the experimental school developing ownership of their learning. This finding could be attributed to differences in teaching style. As seen in Eve’s class, student led discussion was the main teaching style (Sy.Of.vt.p2’). While students’ seat work and class discussion were part of Ed’s teaching styles (Sy.Om.vt.p4m). Even when direct instruction was occasionally adopted by both teachers, there was still evidence of student- centred learning (Om2Ihp4Q5e, Sy.Of.vt.p2’e).

Further evidence of a student centred learning environment comes from multiple inputs from School E. Apart from the teacher's adopting a teaching style supportive of this desired outcome, the students themselves demonstrated the ability to take ownership of their learning. This was evident by i) them listening to others and checking the reasonableness of their explanations and solutions as they explored mathematics, ii) the class building up a body of students who were actively engaged in their own learning – researching, discussing, challenging, and problem solving, and iii) peer teaching. It can be said that students' knowledge of mathematics was built up by the joint efforts of the teacher and students themselves. The teachers used student answers to pose questions in order to better understand their thoughts and to challenge them to develop their own thought (Franke et al., 2007; Rittenhouse, 1998).

This finding strongly contrasts with Tom's use of the traditional approach to instruction. He adopted a direct instructional approach to facilitate his teaching style preference; that is to give mathematical definitions/rules, and constantly asked questions for students to apply given rules or to solve problems. The majority of instruction time was taken up in explaining or answering his own posed questions (Tvh1119.p5t). Student talk occurred mostly when responding to the teacher's questions (Appendix E1). Zhang (2002) stated that students' mathematics knowledge in such an environment (e.g., School T) is mostly built up through a lecture style of instruction.

9.1.2. Different kinds of student thinking

9.1.2.1 Students' exploring, producing and creating

Student thinking and the role it plays in developing conceptual and procedural thinking should be at the forefront of all instruction. Emanating from this study is the role of student thinking in the instructional process. The changes that occurred in student thinking may be discussed under two categories: i) richer learning roles and ii) exploring mathematics classrooms.

Richer learning roles

Different teaching strategies shape different teaching practices that were consistent with different student learning patterns/roles. As discussed before, the teaching styles in both classes differ. Students receiving direct instructions were mainly viewed as followers or receivers. That is, they followed the teacher's methods and reasoning, and rarely had opportunities to discuss their own mathematics thinking in classes. This classroom environment served to maintain the status quo. In comparison, the experimental teaching approach used by two teachers in School E facilitated the growth of students moving from traditionally passive learners to active engagement. The new roles taken on by students include class and group discussions, researching and validating their responses and the responses of their peers. Students had rich opportunities to explore mathematics through teachers' posing questions and classmates' discussions (even though occasionally student talk was off task). Eventually, Eve would help students to refocus. There were many instances during these classes when students were encouraged to think hard for answers. Students explored mathematics ideas by themselves and were welcome to share their ideas with the class (Of1lhp2beQ3). During these sessions, students made conjectures, tested their ideas and then produced their own mathematical knowledge.

These activities provided opportunities for cultivating creative abilities (Ofvh1030p8b, Ofvh1030p10t). Within these classrooms every student had the chance to become i) a knowledge explorer, to discover or reason class ideas, ii) a knowledge producer, to contribute their thinking to the public, and iii) a knowledge adventurer to promote/test his/her ideas in the class. Students learning roles were never static; they were always swapping among being explorers, adventurers, producers and followers. Thus, the different learning roles help students to be more able to interpret diverse situations and develop mathematics ideas (Boaler, 1997). Therefore, student engagement in the learning activities is viewed as a direct result of the teaching style. Tom's teaching practices can be referred to as a way of knowing and doing mathematics based on observation of teacher demonstrations and practices. Ed and Eve's teaching/learning practices appeared as a way of exploring, discovering, negotiating, knowing and understanding from constant classroom discussions, arguments and inferences.

Exploring mathematics classrooms

Class discussion can transform a mathematics class into an exploring and enquiry-based mathematics learning experience (Hunter, 2008). The classroom culture of School E thrived because it was supported by the students' abundant and vital contribution of their explanations, reasoning, arguments, justifications, representations of their problem solving strategies (see sections 6.2.4 & 7.2.4; Wood et al., 2006) and creative thinking which emerged during the learning process (see section 8.2.4). Thus, student E9 concluded that within the exploring and discovering environment of School E mathematics classrooms students could learn through their own learning styles" (OQ2hp2b). In contrast, the classroom culture of School T, traditional in its approach to learning was built by the teacher's given instructions and sole focus on the textbook (see section 5.2.4). Based on the preceding discussion, the classroom learning environment provided opportunities for student thinking to be stimulated and enriched, or for students to remain in the same mode as passive learners.

9.1.2.2 student-directed thinking

The importance of thinking and opportunities for students to construct and communicate their own knowledge in learning cannot be overemphasized. Students have to confront the problem, explore and construct meanings, communicate and negotiate these new ideas as they seek to understand the concept or procedure. These actions allow students to become active participants in their own learning ((Boaler, 1997; Nathan & Kim, 2009; O'Connor & Michaels, 1996). These characteristics of students' thinking as they constructed and communicated their own knowledge were evident in the constructivist classrooms of School E, but were missing in the traditional classrooms of School T.

Class discussions benefited students' thinking in ways that surpass direct instruction such as:

- inspiring students to think hard and share their thinking with the class (Nathan & Kim, 2009; O'Connor & Michaels, 1996);
- making sense from other students' mathematical explanations (Boaler

& Greeno, 2000; Lampert, 2001);

- testing other students' mathematical concepts, hypotheses and strategies (Boaler & Greeno, 2000; Lampert, 2001) through listening or thinking each time students shared or engaged in dialogue;
- structuring their own mathematical thinking (Hiebert & Wearne, 1993) and constructing their own knowledge (Cobb, 2007; Lesh et al., 2003);
- building their communication abilities (Brooks & Martin, 1999; Trotman, 1999; Windschitl, 1999b), negotiation (Confrey & Kazak, 2006), discovery (Threlfall, 1996), and creativity and critical thinking (Franke et al., 2007).

Evidence of students' ability to re-direct their thinking occurred whenever the progress of the lesson slumped because either the student leading discussion did not know how to complete the problem solving process, or when the teacher and students explained something to a student who felt confused. On such occasions other students got more time and chances to think or to discover contradictions in their classmates' methods, and to share their new understandings.

9.1.2.3 Higher thinking skills

The inclusion of class discussion promoted higher-order thinking ability in students from School E as compared to what is required with the direct instruction approach used by School T. Primarily, developing higher order thinking skills require School E students to concentrate, reason, and find contrasts to the content and shifts in different classmates' explanations and mathematical methods. As the student progresses, the focus moves to creating and drawing out their personal arguments. According to Eve, when direct instruction is used students only need to focus or understand the teacher's problem-solving strategies (Of1Ihp9Q6b), and follow or apply what was modelled. This finding is supported by Zhang (1994).

Of interest here is the fact that not every student can expertly and clearly share their thoughts or problem-solving methods; or be able to give correct explanations

or methods. So, in the discussion classroom environment it is reasonable to provide more opportunities for students to make more effort to understand, reason, and find contrast from each shift of their discussions (Boaler & Greeno, 2000; Lampert, 2001). If students' logical reasoning abilities are not strong and quick, they might be stumped while trying to make sense of their classmates' explanations. This hinders their progress and they fall behind in the class conversation. Eventually, they might succeed or fail to find more clues in the class conversation to be able to catch up with the others (see section 8.2.2). Rittenhouse (1998) work partly supports these findings and suggested other problems which may arise for new students when the focus is on developing higher order skills during class discussion.

Although requiring higher-order thinking skills (Torff, 2003), and possibly more knowledge appearing in class discussion approaches than in direct instruction, it does not mean that the class discussion approaches are only suitable for high IQ students. Evidence in this research supports the idea that despite the fact that students in School E had lower average IQ than School T, the use of a constructivist approach to teaching might have influenced students' mathematical abilities in applying their knowledge to new situations and developing conceptual knowledge (see section 8.2.3).

Using a constructivist approach to learning not only promoted higher learning but it also inspired higher-level thinking. The teaching approach of School E might indicate more opportunities for inspiring student higher-order thinking skills. According to Torff (2003), being exposed to discovering, reasoning, organizing and arguing through multiple class dialogues and during social interactions led to these students developing higher order skills (see section 9.1.2.2). For example, as class discussions increased, so did students ability to reason and discover mathematical meanings which led them to organize their own thinking and share, question, or contribute to class discussions/arguments. When a student raised an idea that triggered the class thinking (see section 6.2.4), the class thinking/talking cycle shifted to the next topic or level of understanding. Compared to the direct instruction approach, mathematics activities, reports and students' presentations used by School E increased students' chances to discover new ideas and organize their thoughts. While direct instructions might include the higher-order thinking

skills of reasoning, and organizing based on the teacher's instructions and textbook information, small amounts of discovering tasks along with few chances for mathematical arguments within peer discussions exist (see section 5.2.4). The benefits of inferring discourse for stimulating higher-level thinking are also supported by many scholars (Franke et al., 2007; Hunter, 2008; Nathan & Kim, 2009; Wood et al., 2006).

9.1.2.4 Students' mathematics knowledge, competence and potential situated influences

The incomplete practice accountability of students' competencies from traditional assessment (Richardson, 2003) or large-scale international studies have raised concerns (Boaler, 1988; Wu, 2001). The need for interpreting students' mathematical competencies (Chou, 2003a; Kickbusch, 1996; Richardson, 2003), e.g., applying knowledge into new situations, (Kickbusch, 1996) should be included in assessment that can indicate rich aspects of students' performances. This research adopted 15 conceptual question items (i.e., quizzes 2 to 7) to interpret student mathematics performances patterns besides the three traditional school types of tests and one national examination. However, based on two unequal cases of students from two schools, it is not easy to interpret the influences on students' learning from many different issues. Within Boaler's (2002b) study, students of her both contrast teaching style schools had similar social background. Thus Boaler, based on statistical analysis could directly claim students' high achievement as a result of the teaching styles. This situation differs; due to small number of students and several unequal issues (e.g., IQ, family background) in both schools, and the interpretive nature of this qualitative study, I would describe and interpret potential influential issues which might influence on students' learning.

Regarding students' competencies in curriculum and students' learning during class discussion and group teaching as is the case of School E, more chances were offered to practice all key competencies of the New Zealand curriculum (New Zealand Ministry of Education, 2007). In contrast, the learning behaviour characteristics of students' in the direct instruction supported only two of the five key competencies as (1) critical or logical thinking, and (2) using language,

symbols, and text. These findings are supported from that students of both schools had good performances in certain types of tests and quizzes that shows characters of critical or logical thinking, and uses language, symbols, and text in meaningful ways (New Zealand Ministry of Education, 2006). Students in School E had good chances to practice the competency of managing self (New Zealand Ministry of Education, 2007), because they were given chances to choose subjects to study each year and expectations from School E to exercise their autonomous learning that brings about great potential for students to take responsibility for their learning. They also had good chances to practice the competencies of “relating to others”, and “participating and contributing” (New Zealand Ministry of Education, 2007, p. 7) due to the class discussion teaching style.

These students worked in co-operative ways to achieve common goals: solving problems or answering questions. They supported each other’s needs or helped classmates who still felt confused and thus built up students’ relationship/friendship with each other. Students were explaining, participating, and contributing their ideas within class discussions (see sections 6.2.4 and 7.2.4). Further, student leadership was also developed, as evidenced in Eve’s classes, where some students led the class discussions. (Sy.Of.vt.p2’; see Appendix S). In contrast, students in the direct instruction setting mostly participated in their classes by just responding to Tom’s questions.

Three of the ten key competencies which are identified by the Grade 1-9 Curriculum Guidelines in Taiwan (Taiwan Ministry of Education, 2003; 2008) commonly appear in students of both schools as (1) competencies to share, communicate and express their views (see section 9.1.1.2); (2) competencies to take initiative to explore problems and to research them (data evidenced by students asking each other questions after classes), and (3) competencies for independent thinking and to solve problems (data support from through tests or challenging questions in classes). However, rich opportunities in the School E classrooms (through class discussions, debate, explanations and projects) allow students to develop the former two competencies more than in the traditional classrooms. More opportunities allow students to communicate and express their views through the class discussions in the constructivist classrooms.

Constructivist teaching, through not giving direct answers or teaching, brings great opportunities and space for students to explore their problems and own answers (Boaler, 2002a; Confrey & Kazak, 2006; Even & Tirosh, 2008). For example, Eve gave short challenges through questioning or giving hints (Of1Ihp2e,3tQ3). My 9 lessons of class observations (Sy.Of.vt.p3m) and one student's feedback from the second questionnaire (OQ2hp1tl&re) supported the previous statements.

Two other of the ten key competencies which were identified by the Grade 1-9 Curriculum Guidelines in Taiwan (Taiwan Ministry of Education, 2003, 2008) also appeared in constructivist teaching as (4) competencies to cooperate with others and respect different opinions in team work; (5) competencies to organize, make plans and apply the plans. The former of these was shown in students' seat work discussions, group presentations to the class and also a student leading class discussion. A student leading class discussion was like a big team work; through cooperation students offered ideas to find solutions (n=2, OQ2hp4t). For example, Student E 5 led the class discussions explaining, questioning and solving problem and other students also automatically shared their ideas at 8:40 –8:46am, October 30, 2002 (see Section 6.2.6.1(a)). That could indicate how a student organized and apply his plans to show his knowledge and respected other students' opinions.

The latter of these were shown through completion of teachers' assigned reports, as students needed to organize, make plans and apply the plans. For example, Eve required reports (Of1Ihp3Q3b) and 9 students also commended request of reports from the second questionnaire (OQ2Q1st). The vital class discussions within the constructivist teaching approach of School E also met the emphases of the 2001 Taiwanese mathematic curriculum. Accordingly, students were encouraged to discuss and share their thoughts, and learning would occur through social interaction (Taiwan Ministry of Education, 2001).

9.1.3 Better attitudes towards mathematics learning in different ways

Students of both schools showed high motivations in mathematics learning in different ways. For instance, students with higher IQ from the direct instruction of School T appeared to like mathematics better than students from the experimental

group or School E (see section 8.1.1.3). Students of School T with higher IQ had higher inner value, and were intrinsically motivated to make the effort to study mathematics than students of School E (see section 8.1.1.2, Appendix R). It is therefore hard to conclude which students had higher value or motivation for learning mathematics. For example, if students were given a choice to take more mathematics lessons, School E students would willingly do so than students in School T (n=T16(62%), E23(100%), TQ3Q(9)8a, OQ3 Q(9)8a). Further, School T students' high motivations in mathematics learning might come from the direct instruction or that they already liked mathematics than School E students in the beginning of Grade 7. School E students' high motivations in mathematics learning might come from the the reformed teaching approaches (class and group discussion) or parents' high education background influences. Moreover, students of School E developed a liking or preference for learning mathematics in junior high school compared to when they were in primary school than students of School T(0.64) (OQ3Q(1), OQ3Q(2)).

To sum up, students of both schools showed high motivations in mathematics learning in different ways. Students with higher IQ from the direct instruction of School T appeared to have higher inner value, preference and higher motivation to make effort to study mathematics than students of School E. Students from the experimental group (class and group discussion) or School E had high motivation in learning mathematics and more willing to attend the mathematics classes thereby making more progress towards developing a liking or preference for mathematics in junior high school than students of School T.

9.1.4. Teaching styles

Previously alluded to, one's teaching style might influence upon the mathematics classroom learning environment. The findings on the triangulation of data are linked to the different teaching styles. The details are discussed in the sections below.

9.1.4a Traditional direct instruction teaching styles

Students in the direct instruction approach of School T mostly acquired knowledge from the teacher's given information and students' receiving the

information and acting on it as directed by the teacher. Tom's teaching practices were found to be consistent with his intended curriculum: problem-solving and direct instruction, as supported by the triangulation of data (e.g., teacher interviews, students' opinions, class observations) as reported in Table 10. A summary of Tom's teaching steps from triangulated data and supporting literature is presented in Table 14.

Table 14 Tom's teaching steps from triangulated data and literature support

Literature support	Literature support the cognitive learning focus	Teaching steps	Data from		
			Tom	students	class observation examples
• behaviourist approach (McCarthy & Peterson, 1995) • traditional direct teaching style (Boaler & Greeno, 2000; Even & Tirosh, 2008) • direct instruction (Silver et al., 1995) • traditional approach (a clear and coherent instruction) (Trotman, 1999)		• direct instruction of the textbook/teaching notes first.	T1lhp4Q8e	n=12 TQ2hp1tl	Sy.Tvt.p1tl1118 Tvt.p1mr1206
• traditional approach, (Trotman, 1999) • behaviourist approach, (Fang & Chung, 2005) adopting a clear and coherent presentation (Fang & Chung, 2005) • the traditional approach separating the mathematics content into		• directly pinpointing the important points and summarizing into key content	T1lhp4Q8e T2lhp1Q1t	n=5 TQ2hp1tl	Sy.Tvt.p1tl1118, Tvt.p1mr1206

small objectives (Begg,1996)					
- the traditional approach formulae used in most mathematics problems (Silver et al., 1995) - direct instruction, clearly explaining the content (Zhang, 1994)	the cognitive learning perspectives stress on understanding (Peressini et al., 2004)	demonstrating problem solving by using given rules and explaining reasons	T1lhp3Q6be		Sy.Tvt.p1tl1118 ,1126,1206,1210 Appendices H1 & I1
the traditional approach the focus is speed in problem-solving (Trotman 1999)		teaching fast solution strategies	T1lhp3Q6be	n=3 TQ2hp1tl	Tvh1118p5e
the same as above		emphasizing students' calculation speed			Sy.vt.Tp1el.1210
- behaviourist approach (Fang & Chung, 2005) - traditional approach - the focus is on drill and practice (Fang & Chung, 2005) - direct instruction - practising which was taught (Silver et al., 1995)		students practicing problem-solving	T1lhp4Q8e T2lhp1Q1t	n=2 TQ2hp1tl	all lessons
the traditional approach) stress memorisation of mathematics rules (Silver		requiring students to memorize rules	T2lhp1Q1t	n=8,TQ3hp2	Tvh1118p3e

et al., 1995).					
the traditional approach- using fundamental texts in mathematics (McCarthy & Peterson, 1995)		teaching content (Tom's teaching notes, textbook, practice book, and resource book)	T2lhp2Q4m T2lhp1Q1t	n=4 TQ2hp1tl	Sy.Tvt.prel210 (all lessons)
the direct instruction reviewing the connection (Silver et al., 1995)	reviewing the connection (Wenger, 1998)	reviewing mathematics content (of the Grade 7 and 8)	T1lhp6Q11	n=2 TQ2hp1t	Dec 23, 2002 to Jan 1,2003
the traditional approach assessing students' work in each unit (McCarthy & Peterson, 1995; Silver et al., 1995).		tests/quizzes for a unit	T1lhp4Q8e	n=17 TQ2hp5t	Sy.vt.p3ml.1129, 1228 Sy.vt.p3ml.1209, 0107

Examining Tom's teaching practice in School T from the perspectives in the literature, his teaching styles appears to be closer to a traditional approach and a direct instruction approach and also appeared to be mixed with an influence of the cognitive learning perspective. The cognitive learning focus is for understanding (Peressini et al., 2004), and its application to his teaching as follows.

A traditional approach is based on behaviourism where the focus is on drill and practice, and speed (Fang & Chung, 2005). This approach often combines teacher centred views of learning (McCarthy & Peterson, 1995) and a behaviourist approach. Students spend most of their time passively listening and receiving knowledge from Tom's lectures (McCarthy & Peterson, 1995). Teacher Tom had authority to manage all teaching discourse and activities (McCarthy & Peterson, 1995). The clear and highly structured, coherent presentation of instruction appears in classes, mainly using "chalk and talk" (T1Ihp4Q8e; Fang & Chung, 2005) with the focus on knowledge and content transmission (Even & Tirosh, 2008), for example,

- using fundamental texts in mathematics (T2Ihp2Q4m; McCarthy & Peterson, 1995); (Here Tom used a textbook, a practice book, and a resource book.)
- separating the mathematics subject matter into small objectives within a sequence of tasks (Fang & Chung, 2005) such as key points;
- asking convergent or factual questions for which the teacher has prepared answers already, and assessing students' work in each unit (T1Ihp4Q8e; Silver et al., 1995).

The traditional approach included the teaching strategies for pinpointing the key content and summarizing into important points (T2Ihp1Q1t; Begg, 1996). He taught fast solution strategies (T1Ihp3Q6be; Trotman, 1999) and emphasized students' calculation speed in problem-solving (Sy.vt.Tp1el.1210; Trotman, 1999) and students' practice in problem-solving (T1Ihp4Q8e; Silver et al., 1995).

The traditional approach stresses memorisation of mathematics rules and formulae used in most mathematics problems (n=8, TQ3hp2; Silver et al., 1995). Tom followed the syllabus and transmitted knowledge and tried to ensure that students

retained this knowledge, for example, assessing students' work in each unit (n=17,TQ2hp5t; Silver et al., 1995).

Tom' teaching strategies were consistent with a direct instruction approach according to the characteristics of clear explanations of the content by using given rules and explaining reasons (Silver et al., 1995) and processing of meaning (Zhang, 2002). He reviewed the previous content to build up mathematical connection and stimulate students' understanding (Silver et al., 1995). Students in this learning environment are seen as learning by passively absorbing the teachers' delivered knowledge (Boaler & Greeno, 2000; Even & Tirosh, 2008) and focusing on the task or textbook (Boaler & Greeno, 2000).

Direct instruction strategies also come from behaviourist and cognitive learning perspectives (Zhang, 2002). The cognitive theoretical learning perspectives from Tom's teaching could be concluded from his emphasis on students' understanding, mathematical processes and alternative solutions in his class. Further discussion will be presented in section 9.1.4c.

The second teaching example in section 5.2.6 showed how Tom challenged students' thinking in a big class for teaching students' understanding, instead of directly asking formulas to solve problems. Through several times of questioning and waiting, students gradually formatted the correct answers and gave short responses (Tvh11186e). Tom concluded the ideas and explained the reasons to students. Tom gave chances for students to think and adjust their ideas and later used teacher's explanations to develop their understanding. Tom's questioning skills of giving several chances to allow students to format their answers from students themselves are consistent with the cognitive learning perspectives that "they are useful for designing sequences of conceptual material that build upon existing information structures" (Wenger, 1998, p. 279).

As a result, Tom's teaching style is close to traditional and direct instruction approaches with cognitive learning focus for understanding (Peressini et al., 2004). His teaching style was consistent with Wong's (1993) report that the behaviour of successful Taiwanese teachers of junior high schools appeared as: spending more

time in lecturing, and spending less time to let students work individually. This finding can help to clarify some misconceptions that Asian teachers adopt a teaching approach close to the behaviourist teaching style, and emphasize practices but ignore understanding.

9.1.4a.1 Chances for students to practice mathematics thinking

Although the students spent most of their time passively listening and receiving knowledge presented in the form of lectures from Tom, there were still chances for students to practice their own mathematics thinking/ideas in his classes. For example, this was done through the opportunities to answer Tom's questions; even though Tom did not wait for students' answers before answering the questions himself.

A significant amount of mathematics content delivered in his classes offered rich challenges to students' thinking. Many frequent tests also offered students chances to examine their own learning, thinking and practices. Through the corrections of tests, students were offered chances to review their learning and learn new problem solving. However, most of the learning narrowly focused on answering or doing problem solving from the textbook/resource books. Another benefit of the frequent tests was that they offered chances for Tom to know students' levels of learning and understanding.

Students mostly passively listened to understand and learn Tom's mathematics ideas and problem solving methods. Mathematical meanings were generated from the teacher (Cross, 2009). This type of teaching practice influenced students learning styles. Since they simply follow the teacher's steps to interpret the mathematical ideas or procedures, their ability to develop full conceptual understanding is of concern. This type of knowing is different from accepted knowing (Boaler, 2002a) as it does not include much of students' own thinking (Belencky et al., 1986).

Students' conceptual knowledge was also being practiced, when they faced challenging questions in their learning of the content such as the textbook, tests and so on, or while Tom was lecturing. The more challenging the content given by Tom, the more increased were the opportunities to develop students' conceptual

knowledge. Thus, Tom's teaching practices offered lots of opportunities to let students practice both procedural and conceptual knowledge and opportunities for them to be familiar with that knowledge. This set of teaching practices might influence students' learning.

9.1.4b Constructivist Teaching Styles

The constructivist approach, when applied to teaching, is intended to build up learners as skilled and thinking people (Hagg, 1991). Eve's and Ed's teaching styles are considered to be based on a constructivist view of learning, because of their strong emphasis on student-centred learning (Confrey & Kazak, 2006) that was reflected in their perceptions of students' learning (see Sections 6.2.1 & 7.2.1), and teaching practices (see Sections 6.2.4 & 7.2.4). The multiple data sources came from Ed's and Eve's interviews, classroom observations, students' perceptions (see Chapters 6 & 7), and also have been discussed regarding students' views of the frequency in teaching behaviours (see section 8.3.2) and the time interval count analyses (see section 8.3.1). The data all confirm that Ed's and Eve's teaching styles are consistent with the constructivist view of learning and its application to teaching as follows. Ed's and Eve's:

- (a) Classrooms practices placed an emphasis on students' explanations of their thoughts (Hunter, 2005; Nathan & Knuth, 2003). Ed and Eve encouraged students' sharing (Cobb et al., 1992; Ernest, 1991; Wheatley et al., 1990) and discussion (Mayers & Britt, 1998; Windschitl, 1999b). They minimized their direct instruction or explanations (Simon & Schifter, 1991) and promoted discussion and problem posing by students (Trotman, 1999). Students presented to and discussed their work with the whole class, for example from averagely about near half of Eve's class time (i.e., 24.3 minutes) and five of Ed's nine lessons for the mathematical unit 3-3 (Mayers & Britt, 1998). Or, teachers initiated discussions (4 of Eve's 20 lessons, Sy.Of.vt.p3; Om1Ihp6Q2e) and reformulated students' mathematics contributions (Gravemeijer, 1994). Question posed attracted students' explanations, ideas (Cobb et al., 1993) and creativity (Of1Ihp12Q9e, Ofvh1030p8b, Ofvh1030p10t). Eve and Ed used students' feedback to pose new questions continuously (16 Eve's lessons, Sy.Of.vt.p3t; Omvh1211p6e); those made students explore their thoughts,

then - through the teacher and students' cooperation - produce mathematics' ideas (Cobb, et al., 1991; Hagg, 1991; Mayers & Britt, 1998; Windschitl, 1999a, 1999b). For example, in one of Ed's classes, student E20 referred to a new finding by herself that with Ed's continuous questioning and some hints (Omvh1211p6e).

Moreover, Ed's and Eve's usages of students' feedback to pose new questions continuously (Sy.Of.vt.p3t; Omvh1211p6e) to build up their class curriculum were consistent with one characteristic of constructivism; that is, teachers develop their own curricula according to their students' current conceptions (Windschitl, 1999b). Both teachers still followed the national syllabi, but they designed their own curricula (Of1Ihp10Q6e, Of2Ihp5Q5t, Om1Ihp7Q3b). Teachers assessed both the processes and products of student thinking and assisted students' own efforts to assess what they have learnt (Carr & Ritchie, 1991, 1992; Trotman, 1999).

Furthermore Ed and Eve encouraged students to transfer mathematical language into their own signs and language (Of1Ihp9Q5b, Om1Ihp11Q8tb). This fulfils one principle of constructivism: adapting and reorganizing knowledge as part of students' own constructions (Boaler, 2002a). Both teachers allowed certain waiting time after giving questions to students (Brooks & Martin, 1999).

- (b) The emphasis is on discovery of students' mathematical ideas (Greenes, 1995; Threlfall, 1996) and problem solving (Om1Ihp14Q15e). That is, teachers encourage students to conceptualise situations in different ways (Windschitl, 1999b). Students are encouraged to think and develop their own ideas/ knowledge (Boaler, 2002a; Confrey & Kazak, 2006; Even & Tirosh, 2008) within interactions in class discussions (Wenger, 1998), to explore misconceptions and conflicting ideas in order to develop broader concepts (Simon & Schifter, 1991). Such practices were seen in these two teachers' classrooms, e.g. encouraging students' talk to discover their own or alternative solutions (n=2, OQ2hp1t).

Mathematics problems which were related with world examples were seen in the two teachers' classroom, e.g. Ed used a rope to introduce the concept of a circle (Om1Ihp9tQ5). Moreover, more mathematics

projects or reports were completed in these constructivist mathematics classes than those in School T (Of1Ip2Q3).

- (c) Students' feedback also supported the more corporate study style in these mathematics classrooms than in School T (see section 8.3.2). For example, students solved problems collaboratively in small groups (Cobb et al., 1991; Hagg, 1991; Windschitl, 1999a). Students automatically participated and cooperated in class discussions, for instances, asking questions, adding comments or explaining ideas (2002/Dec 4(1), Sy.Of.vt.p2, 2002/Oct 30(1), Ofvthp9e, 10t, Sy.vt.p2r).
- (d) Students found their own questions through the procedure and tried to work problems out (Of1Ihp2beQ3, at least 4 of 16 lessons, Sy.Of.vt.p3) (Carr, 1993).
- (e) Ed's assessment approaches included observing, listening, and self-assessment such as tests, students' learning attitudes (Om1Ihp11Q8t, Om2Ihp9Q7t), students' homework, students' self-chosen questions (n=4, OQ2hp1mm, Sy.Om.vt.p4m.) and Ed's teaching notes (Sy.Om.vt.p4). Eve's used assessment approaches such as observing, listening (Of1Ihp2beQ3), and investigations such as class individual or group presentation, homework, tests, learning notes and (investigation) reports (Table 11). In this way, teachers gain ideas about students' mathematics knowledge, conceptual misunderstanding (Carr & Ritchie, 1992), prior ideas (Begg, 1996) and strategies from students' description of problem solving to teachers or peers (Carr & Ritchie, 1991).

Eve described aspects of her teaching style as: using students' questions in her teaching, expanding those questions to stimulate discussion by encouraging students to think and discuss these new questions (Of1Ihp2beQ3). Both student E3 and student E9 reported that Eve allowed students several chances to explore and generate their own solution methods (OQ2hp1re). One example of this occurred when they were working on the area of a circle. Eve asked students to explain that how they arrived at that answer.

Student E2: Let it times 3.14, then divide 4.

Student E10: 2.5 Π .

Eve: Talk slower! Is it 5 times 2 or 2 times 5?

Two students: 5 times 2.

Eve: The diameter times Π , then?

Three students: It's divided by 4.

Eve: What do you mean: one fourth? I could not see that.

Student E2: That means times $\frac{1}{4}$ (Ofvh1030p11b).

(The discussion continued with Eve getting students to refine their thinking.)

In comparison, Ed mostly gave students hints, but not direct answers (Om2lhp2Q3t; Om3lhp2Q5; Omvh1211p2t,p2m,4e,5e,6t). For instance, Ed posed the question: "What is the characteristic of the central and vertical line?" Student E20 answered that the length of two lines is equal to each other. Based on this student's response, Ed realised her confusion and challenged her thinking by posing several questions regarding the location and the length of the lines (see details in Section 7.6.2.1). Those questions encouraged student E20 to investigate why her ways of understanding differed; test and integrate her ideas, and look at alternatives ways. The use of in-depth probes and getting students to refine their thinking through questioning are supported by the works of Carr and Ritchie, 1992 and Windschitl, 1999b.

Students in the experimental classroom of School E mostly acquired knowledge from a social or collective adaptive form of ongoing developing mathematical knowledge. Knowledge was produced from their own creative production/thought within class discussions and occasionally from the information provided by the teacher. These students actively participated in the learning process through social dialogue and interactions with the class community.

To sum up, Ed and Eve functioned as helpers to let students explore their thinking (n=2, OQ2hp1tr). They spent a big part of their lessons challenging students (Begg, 1995; Confrey & Kazak, 2006) and gave many chances to develop/explore students' own knowledge (Boaler, 2002a; Confrey & Kazak, 2006; Even & Tirosh,

2008; Lampert, 2001; Sfard, 1998; Wenger, 1998) and creative thinking (Greenes, 1995), for example, using students' own methods but not necessarily traditional teachers' methods (Boaler, 2002a; Even & Tirosh, 2008; Lampert, 2001). They got students to think, talk (Brooks & Martin, 1999; Windschitl, 1999b) and draw (the geometry pictures) what they understood. That resulted that students thinking all the time in this class (cf. Boaler, 1997); they must reason, draw, explore the drawings, look for clues and follow these clues...they had to explain. Those strategies and foci successfully establish a collective understanding through the class discourse and build up their mathematics classrooms as thinking and exploring environments (Hunter, 2008). Mathematics flows in these two teachers' classrooms were from the interactions of teacher and students and thoughts of the teacher who make up the class (Lampert, 2001).

As a result, the mathematics classrooms (of Grade 9) in School E closely fit the model of constructivist classrooms. These findings are similar to those in the 2001 report from the Bureau of Education about the constructivist teaching styles that appear in the grade 7 and 8 mathematics classrooms of School E (The Taipei City Bureau of Education, 2001).

9.1.4c Student mathematical process and understanding consistent with a cognitive learning perspective

The view of learning from a cognitive perspective is seen as focusing on the growth of internal cognitive structures including the acquisition of knowledge (Ford & Forman, 2006; Greeno, 2003; Peressini et al., 2004), or growth in conceptual understanding (Peressini et al., 2004). The "Pedagogical focus is on the processing and transmission of information through communication, explanation, recombination, contrast, inference, and problem solving" (Wenger, 1998, p. 279). Therefore, the importance of process as pointed out is on "the processing and transmission of information" (Wenger, 1998, p. 279) which looks at the transformations of knowledge or conceptual understanding in the personal cognitive structures. So, one can conclude that the learning process and understanding can be explained as characteristic of a cognitive learning perspective.

The three teachers in this study all emphasized students' mathematical understanding and process which are close to the foci of a cognitive learning perspective.

Moreover, prior experiences is significant as one makes meaning of the new information from a cognitive learning perspective (Wenger, 1998), therefore mathematical connection is essential to help students reach understanding in their growth of cognitive structures. Three teachers all tried to make connections in students' mathematical learning. For instance, Tom and Eve tried to find chances to connect content with previous other units (T1Ihp6Q11, Of1Ihp13Q11). Ed tried to make connections between concrete and abstract. For example, when he wanted to introduce the concept of a circle, he would ask students to actually make a circle and measure the length of a circle with a rope (Om1Ihp9tQ5).

To sum up, three teachers in this research all emphasized students' mathematical understanding, process and mathematical connections, which are close to the foci of a cognitive learning perspective.

9.1.4d High quality of teaching approaches in School E

The high quality of class and group discussion teaching approaches in School E is revealed in three areas, as below.

(1) The teaching approaches of School E also met the high-quality instruction foci with awareness and development of students' current knowledge/thinking from Eve's and Ed's observations of students' feedback in classrooms (Of1Ihp2beQ3, Omvh1211p6e), representations (Ofvh1030p8b, Omvh1211p3t, Omvh1211p7t, Omvh1211p6e) and students' engagement in and use of the integrated and core mathematical tasks (Franke et al., 2007; Kilpatrick et al., 2001; Lampert, 2001). Such instruction would benefit students' mathematical knowledge and proficiency (Kilpatrick et al., 2001). The teaching approaches in School E were consistent with some criteria of teaching for understanding that include Eve and Ed's coordinating class mathematical conversation (Franke et al., 2007; Lampert, 2001) with students' representations,

explanations, making inferences and testing them and forming collective mathematical consensus (Franke, et al., 2007). For example, Eve and Ed selected the mathematical theme (Omvh1211p7t, Sy.Of.vt.p3), students involved and shared in a class discussion (Omvh1211p6e, Sy.Of.vt.p3), and teachers used the class discussion to come close to the main mathematics ideas (Sy.Of.vt.p3, Omvh1211p6e) or expanded a new concept (Omvh1211p6e).

(2) Students' arguments and negotiations produced a consensus/social form of knowledge (see section 6.2.6.1(c), Ball & Bass, 2000b; Confrey & Kazak, 2006) within class discussions (Driver et al., 1994; Wood, 1999). This social/collective form of knowledge was built up by the joint efforts of the teacher and students themselves through a collective form of inference and justifications (see section 6.2.4; Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006) to generate new mathematical knowledge together (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006) and increase the chances for students' creative thought (Ofvh1030p10t, Omvh1211p). For instance, in the example of section 6.2.6.1(a) and 6.2.6.1(c), several students automatically raise their opinions to solve a problem. Students tried persuaded others of their mathematical thought. Their ideas were weaved together by Eve (section 6.2.6.1(c)) and Student E5 (section 6.2.6.1(a)) into a collective form of mathematical solutions.

The preceding discussion provides evidence which supports both the direct teaching style and the constructivist teaching style lends itself towards increasing the quality of mathematics education that students are offered in Taiwanese junior high schools. The inclusion of class discussion not only requires higher-order thinking ability from students but it can help share students' thinking ability, and establish an exploring and thinking classroom environment. This may be due to students within a supportive learning community of class discussions (BRAP, 2003) having many more chances/challenges to develop and concretize their own thinking and strategies rather than just following or copying their teachers' methods, as is the case of traditional classes. This type of social supporting,

discovering/exploring/arguing learning within the class discussion approach is missing in the direct instruction approach.

In conclusion, the characteristics of class and group discussion teaching approaches in School E met the high-quality instruction foci for developing students' current knowledge/thinking in core mathematical tasks (Franke et al., 2007; Lampert, 2001; Kilpatrick et al., 2001), revealed more chances for practicing students' autonomy (see section 9.1.1.1) and competence (see section 9.1.2.4) (Hunter, 2006b; Lambdin & Walcott, 2007), higher-order thinking skills, and met some criteria of teaching for understanding. These evidences/arguments reveal the class and group discussion approaches to have potential to offer high quality of teaching/education.

9.2 Disadvantages of School E (constructivist teaching)

Like any classroom approach, the use of discussions has its flaws. Seven disadvantages of using class discussions were identified in this research: (i) students lagging behind by several minutes into other students' discussion, (ii) time consuming, (iii) focus on oral explanations might lead to poor mathematical writing ability in students' explanations of their thinking (Of1Ihp5Q4t), (iv) students' sharing skills not mature enough to bring thorough understanding to their classmates, (v) possible creation of some emotional pressure when a student leads a class discussion (though this case did not frequently occur), (x) expectation gaps and (xi) more teacher work. Points (i), (ii) and (iii) were mentioned before by Eve, so that discussion will not be repeated here, but additional discussion about the time consumed by class discussion method follows.

(ii) Time consuming

As Eve used the class discussion method in her classes more often than the other two teachers, the disadvantages of consuming time appeared more in her classes than those of the other two teachers. The evidence appeared in the teaching rate, e.g., Eve spent 16 class periods, Ed 9, Tom 6 to cover a unit (see section 8.3.3). The time consumed issue within the constructivist approaches has been addressed in Eve's comments (see section 8.4) and several other studies (Chou, 2003b; Hiebert & Wearne, 1992; Lampert, 2001; Xu & Chung, 2004). Eve felt that it took time to see the growth of students' abilities. These challenges were similar with

Gardner's comments in Steinberger's interviews (1994, p. 5) that "understanding takes time, and the greatest enemy of understanding is coverage". It is always a challenging task to balance the constructivist approaches and content coverage, because it always take more time to build one concept in constructivist approaches (through students' presenting ideas, arguments, negotiations to develop collective public knowledge) than through direct instruction. Possible solutions will be given in the second part of section 10.5.1 and part (iii) of section 10.5.2 (b).

The great use of time in Eve's classes not only resulted from the use of the class discussion method, but also from the characteristics of her classes. Eve emphasizes students' understanding, and that teacher and students are supportive to help each other for understanding. For example, some students and the teacher tried hard to help student E4 in one problem-solving. During twelve minutes, the problem solving of the same question was explained three times. Student E5 explained to the whole class the first time; then because student E4 did not understand. Student E5 explained it again more thoroughly. However, student E4 still felt confused. Student E8 helped by reminding "Student E4, you could think about this. There are two 1s, two 2s, two 3s, and two 4s. Then $1 + 2 + 3 + 4 = 1 + 2 + 3 + 4$ (Ofvh1030p2e)." However, student E4 still did not understand. Then the teacher explained it again to him the third time. Finally, he understood (Ofvh1030p3e).

(iii) Eve suggested that class discussions benefited students' thinking ability in real life, but not necessarily aided written explanations of their problem-solving strategies. For example, Eve found that while her students had abilities to distinguish and argue to arrive at solutions using real-life experiences, their written explanations in mathematics were very weak (Of1Ihp12Q9t). According to Eve, "students could think but could not write well in mathematics" (Of1Ihp5Q4t). As a result, she tried to give them more tests in Grade 9 to promote their problem solving writing skills. This, she believes, will help encourage students to focus on mathematical writing more (see section 6.2.1) (Of1Ihp5Q4t).

These findings are consistent with Lampert's (2001, p.362) findings that "within-student variations" exist in a class. Students are not uniformly competent or

incompetent across the class and their strength or weaknesses do not follow any simple patterns. Some students performed competently on tasks, but were not always good at explaining their reasoning or representing relationships among ideas. Some students could contribute productively in small-group problem solving, but did not perform competently on quizzes (Lampert, 2001).

(iv) Students' sharing skills not mature

The evidence can be seen in the two teachers' classrooms in School E. In student E5's sharing of his problem-solving in one of Eve's classrooms, he explained clearly about these reasons for each step that

$$\angle OHA = \angle OMA = 90^\circ, \overline{AO} = \overline{AO}, \overline{MO} = \overline{HO}$$

and led to the conclusion that $\triangle AOM \cong \triangle AOH$. He did not explain clearly that

$$\overline{HD} = \overline{DP}, \overline{BQ} = \overline{BM}, \overline{CP} = \overline{CQ}.$$

He also ignored these in his second explanation of problem solving (Ofvh1030p2tr, Ofvh1030p3e).

Ed asked student E15 to explain the main points of her writing on the blackboard. However, she just explained the pictures on the blackboard but not the reasons or relationships in the picture (Omvh1211p3b).

(v) Possible creation of some emotional pressure

Sometimes, when a student led a classroom discussion he would not be as careful as an adult to avoid bringing negative pressure to the other students. In this case, student E5 mentioned that student E4 might quickly forget what he learned; that might bring some pressure to student E4, but student E5 still tried to offer helpful support and warm smile to that student. For example,

Student E5: He [Student E4] was not clear about the aim of this question.
Teacher, later when you ask him again, he will forget it. (Student E5 looked at student E4 with a gentle and smiling face.) (Ofvh1030p3b).

Although disadvantages may exist, these could be overcome or their occurrence reduced. For example, when addressing the issue of students lagging behind and being lost or students' sharing skills being immature, teachers could add brief instruction, set aside free discussion time in classes, reduce the discussion issues,

or use mixed teaching strategies. For example, Eve and Ed gave brief instructions in problem-solving (Ofvh1030p3b, Omvh1211p6e) or concepts (e.g., an arc, in Ofvh1030p7e, 8b) to help students' understanding (see section 10.5.2).

Teachers encouraging class discussions need to be aware of these disadvantages. If some emotional pressure occurs from the class discussions, the teacher can try to encourage the class towards positive and respective attitudes. This alternative approach to teaching mathematics in junior high schools will work if we, as teachers, provide positive direction and modelling to develop a classroom culture that facilitates enquiry, discussion and the acceptance of varying point of views.

(x) Expectation gaps

There was a gap between the expectations of the teachers in School E and students' performances (Black & Wiliam, 1998; Boaler, 2002b). For example, students in School E were expected to be independent and responsible for their own learning, but both teachers complained that some students did not meet the expectations, e.g. non-preparation of class work or late assignments (see section 8.4).

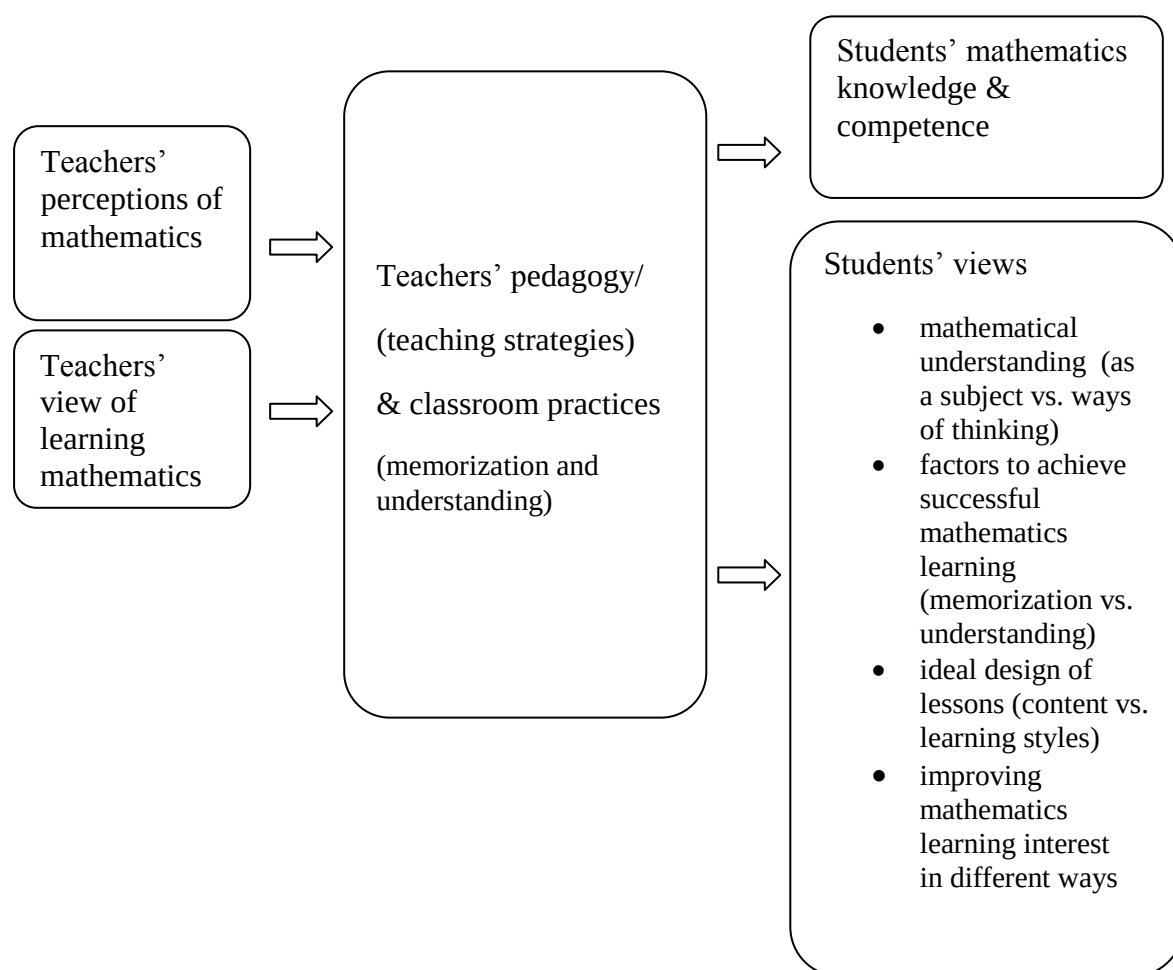
(xi) More teacher work

Constructivist teaching brings challenges and more work to a teacher. For example, Eve felt pressure to target each individual's learning pace in the class, to check students' understanding after classes, because in class discussions some students lagged behind, and to plan lessons to bring students into class discussions (see section 8.4).

9.3 Teachers' perceptions of mathematics and learning, teaching styles, and students' mathematics knowledge

A sequential relationship among the teacher's perceptions, classroom practices and student learning has been exposed in this research. The relationship mode and discussions of supportive data are presented in Figure 2.

Figure 2: A diagram of the relationship between teachers' perceptions of mathematics, pedagogy, and classroom practices, and students' mathematics knowledge and views



(1) The relationships between a teacher's perceptions and classroom practices

Teacher Tom views mathematics as a tool that assists students in quickly learning problem solving methods as modelled by him (see section 5.2.1). His pedagogy and applied teaching strategies are consistent with his claims that he favours direct instruction (n=12, TQ2hp1tl) and emphasizes problem-solving methods (T1Ihp3Q6be) to transmit the teacher's knowledge. Instruction given this way will help students understand the mathematical tools and be able to memorize and use them (see the triangulation data in Table 14). However, Tom also emphasizes understanding (T1Ihp5Q9b) and was keen to see students' alternative solutions to

given problems (T1Ihp3Q6be).

In comparison, Ed's and Eve's perspectives of learning are associated with classroom social interactions. They support and encourage the idea of having students at the centre of learning. For example, Ed emphasizes students' involvement and interaction in classes (Om1Ihp9Q6e) while Eve encourages students' discourses (Of1Ihp12Q9e). They both believe that it is the student's responsibility to build up their mathematics abilities, and not to rely solely on teachers (Of2Ihp3Q1m, Om1Ihp6e,7tQ3). Cobb (2007) supports this perspective as being consistent with elements of constructivism (see sections 9.1.4b).

Ed and Eve viewed mathematics as mainly having to do with logical inferences and as a collection of problems (Of1Ihp1Q2t, Om1Ihp1Q0). They aim to help students themselves to build up their abilities (Om1Ihp6Q2, Of2Ihp3Q1m) and to increase students' learning interest (Om1Ihp5Q1b, Of1Ihp3eQ3). Therefore, mathematics thinking (Of1Ihp9Q6b, Om2Ihp9Q7m), problem solving and social interactive learning (Om1Ihp11Q8e, Ofvh1030p7b) were highly valued in these teachers' classrooms. In particular, they mainly used class and group discussions as teaching strategies to better understand their students' thought processes, and to further challenge students' mathematical thinking (Of1Ihp12Q9e, Of1Ihp2beQ3, Om2Ihp9Q7t).

For instance, because of Ed's perceptions of mathematics as problem solving, he would use games and puzzles to introduce different concepts (Om1Ihp1Q0). Students would be introduced to "the rules, the content inside the game, what terms are inside the game, and then how to play the game" (Om1Ihp6Q2). So, the content and context of mathematics from his perspective was to help students "to think and learn how to play mathematics" (Om1Ihp6Q2), to know how to apply their knowledge into problem-solving to clarify their understanding (Om1Ihp10tQ7).

In Eve's case, viewing mathematics as problem solving and logical inference (Of1Ihp1Q2t), a training of thinking ability (Of1Ihp1Q2e), and a tool

(Of1Ihp1Q2e), influenced her arrangements of classroom teaching practices to help students engage in logical reasoning, debates, and find developing analytical skills that went beyond compare and contrast during class discussion. The focus of this was to develop students' mathematical thinking abilities. The data evidences were supported from the class discussion approach was Eve's main teaching method (see details in section 6.3) and her views of the class discussion approach that benefited students' logical inference and mathematical thinking ability. She believed that when class discussion methods were applied successfully in a class, students themselves could accept, judge, and discuss each other's ideas and make conclusions (Of2Ihp3Q1e). She noted that students would begin the session without her having to do any instruction. Students took ownership of the lesson and through continuous discussion they arrived at reasonable conclusions (Of1Ihp2eQ3). Eve felt very touched by the students' creative thoughts (Of1Ihp11Q6b, Of1Ihp12Q9e). She felt that her students had abilities to think and analyse situations to produce their own arguments. They would make and test their hypotheses as they made connections to real life situations (Of1Ihp12Q9t). Moreover, from the class discussion method, Eve found students' progressed at the senior high level more than at the junior high level in autonomous learning attitudes (Of3Ihp2eQ3pr, hp4eQ5pr) and independent/critical thinking abilities (Of3Ihp4mQ5pr). She found that students started to learn independent thinking by reading books themselves, setting up their own goals, working cooperatively, engaging in critical thinking and arguing, and proving simple facts (Of3Ihp4mQ5pr). She observed that students' thinking, arguing and expressing abilities were built up (Of3Ihp2eQ3) and their expressions were improved more than before (Of3Ihp3mQ3pr). She concluded that her teaching role could remain at the third line at the senior high level (Of3Ihp2eQ3pr) as mainly posing questions to students (Of3Ihp4mQ5pr), whereas before she stayed at the second line in junior high (Of3Ihp4eQ5pr).

The three teachers' perceptions of learning and teaching pedagogies/strategies are all linked to their perceptions of the nature of mathematics (Cross, 2009; Sullivan, 2003). Moreover, through theoretical perspectives, Tom's perceptions of learning and pedagogy/teaching strategies and emphases are close to behaviourists' and cognitive points of views (Even & Tirosh, 2008; Greeno, 2003; Peressini et al.,

2004). Tom's emphasis on memorization of rules is consistent with the characteristics of the behaviourist approach (Boaler & Greeno, 2000; Wei & Eisenhart, 2011).

(ii) The influences between classroom practices and students' knowledge/understanding/views

Franke et al. (2007, p.227) argued that "...mathematical understanding involves students' relation to the mathematics - how they see themselves as doers of mathematics". The findings below reflect the influences from the different nature of mathematics classrooms on students' understanding and views to achieve successful mathematics learning, and echo the emphases of teaching practices in teachers' classrooms (Boaler, 1996).

More students in School T regard their mathematics understanding as a subject (n=T9, E1) and more students in School E regard mathematics as a way of thinking (n=T5, E8, TQ2Q(7), OQ2Q(7)). Students in the two schools all learned similar core content from the textbooks, but their ideas about mathematics were different. These differences appeared to be consistent with their long-term daily mathematics classroom practices involving different mathematical norms and culture. It may suggest that different class teaching styles and practices influence students' concepts about mathematics. Teachers' emphases or actions in classrooms, even unintentional or unspoken, might influence students' concepts of mathematics. Students in School T worked hard in classes to learn from the teacher and to solve problems for most of their class time. It was not unexpected that students in School T, more than in School E, considered mathematics as a subject (n=T9, E1). The two teachers in School E gave lots of chances in class for students to present their thinking and to discuss and debate mathematics ideas, with the teachers giving little structure or guidance. Rich thinking and exploring in mathematics classes was critical in School E (cf. Boaler, 1997). It is not unexpected that more students in School E than in School T considered mathematics as a way of thinking (n=T5, E8). This finding matches the research literature that the situated influences in students' mathematics classes are consistent with students' interpretations to mathematics (Boaler, 1997).

Students in both schools gave suggestions about their ideal design of mathematics lessons that were situated in the classroom context and the emphases of the schools (Boaler, 2002c; Lave & Wenger, 1991). High numbers of students' perceptions of their ideal mathematics lessons in both schools (n=T10, E12, TQ2hp2e, OQ2hp2e) were consistent with their school/classroom practice. Students' responses on their ideal design for mathematics lessons echoes the classroom practices of School T where most of the content is covered, than School E (Sy.vt.p.4). Being able to coverage more course content might affect students' views. More students in School T viewed the teaching content (n=T12 (46%), while more students in School E suggested the teaching styles (n=T10 (38%), E12 (52%)) as their ideal design.

Students' perceptions of improving their mathematics learning interest also illustrates the situated influences from students' classroom practices. For example, several students in both schools regarded some characters of their teaching styles as improving their learning interest (n=T6 (23%), E8 (35%), TQ2Q(3(c)), OQ2Q(3(c))) but a few expressed opposite views from their class practices to increase their mathematics learning interest (n= T3, E2, TQ2Q(3(c)), OQ2Q(3(c))).

Some students in both schools (n=T2 to 5, E2 to 3, TQ3Q(7)hp2, OQ3Q(7)hp2) showed strong support of their mathematics class characteristics to achieve successful mathematics learning such as: memorizing mathematics formulas, learning from a cram school, and handling many questions from resource books at School T and doing more thinking, class discussion, and letting students learn freely without any requirement at School E (see section 8.1.2.2). These provide evidences that mathematics class characteristics influence students' views of important mathematics learning factors.

(iii) The sequential links among teachers' perceptions, classroom practices and students' knowledge/understanding/views

Three data evidences indicate significant connections from teachers' perceptions towards class practices that are linked to students' views.

First example:

The three teachers' beliefs of practices to improve students' mathematical abilities (T1Ihp7eQ15, Of1Ihp13Q10t, Om1Ihp5Q2) consisted of the emphasis on repetition practiced in Taiwanese teaching approaches (Fang & Chung, 2005) and in Chinese culture (Leung, 2014); these content might overlap in noted by Leung (2006). They all applied this emphasis in their classrooms, but from students' responses it appeared more frequently in Tom's classrooms than in mathematics classrooms at School E. For example, students in School T reported higher frequencies of teaching behaviours such as practicing computational skills (see section 8.3.2) in mathematics classes than students in School E. These classroom practices also affected students' views of successful mathematics learning. For instance, more students in School T than School E value working hard (see section 8.1.2.2) in successful geometry learning, and slightly more students in School T than School E value more problem solving as their first factor in successful mathematics learning (see section 8.1.2.2).

Second example:

Students' emphasis on memorization is also consistent with teacher's pedagogy and emphases in classroom practices. Tom values memorization (T2Ihp1Q1t) and requires students to memorize rules ($n=8$, TQ3hp2, see Table 10). More students in School T than School E ($n=T10$, E2, TQ3Q(7), OQ3Q(7)) value memorizing mathematics formulas (or the methods of solving mathematics) in their ideal design of mathematics lessons and the top five factors in successful mathematics learning.

A higher number of students in School T than School E appreciate memorization of rules and procedures, which are consistent with Tom's emphases in the classroom and also the characteristics of a behaviorist approach that values memorizing the rules (Boaler & Greeno, 2000; Wei & Eisenhart, 2011) in contrast with less memorization in constructivist approaches (Simon & Schifter, 1991).

Third example:

The three teachers all valued understanding, but students' responses on frequencies of teaching behaviours (T1Ihp2Q5e, Of1Ihp8Q5t, Om1Ihp8bQ5) in

mathematics classes also confirmed School E teachers' greater emphasis on students' mathematical thinking/understanding than at School T, (e.g. explaining the reasoning behind an mathematical idea), even challenging thinking tasks such as mathematics projects (Landau & Everitt, 2004), and working on open problems without fixed/certain solutions (see section 8.3.2).

More emphasis on students' mathematical thinking/understanding in classroom practices of School E than School T affected students' views and mathematics understanding. For instance, more students in School E than School T valued thinking/understanding in mathematics learning from students' feedback in open questions of their concepts about mathematics and the first or second factors to succeed in mathematics learning (see section 8.1.2.2).

To sum up, this study has confirmed the situated sequential relationship wherein the teachers' perceptions of mathematics influence teacher' classroom practices (Cross, 2009; Sullivan, 2003; Thompson, 2004). Teachers' views of learning significantly influence their teaching practices. Situated influences from different teaching practices influence different forms of mathematics knowledge (Boaler, 2002a; Franke et al., 2007), students' mathematical understanding (Boaler, 2002b; Franke et al., 2007), mathematics competence (Boaler, 2002b), perceptions of how to succeed in mathematics learning (Boaler, 1996), ideal design of mathematics lessons, and views of improving student interest in learning mathematics.

The situated sequential relationships from this study highlight the importance of teacher education and professional development, because those shape teachers' perspective of mathematics and might influence classroom practices (Cross, 2009) teaching approaches and students' competence and mathematical views. How we prepare teacher professional development (Borko, 2004; Steele, 2001), and how we prepare the proper social context for learning to take place (Boaler, 2000a) could be two major focuses of the next educational development, because they both might influence students' mathematics knowledge/understanding, competencies, and views.

9.4 Summary

It is necessary as we investigate student learning and growth from contrasting classroom teaching practices, using a situated perspective that will offer a wider scheme to interpret educational practices, for example, learning relationship with respect to (conceptual) material, the awareness as contributors and learners (Greeno, 1997), classroom context, activities and culture (Brown et al., 1996). Therefore, multiple aspects of information from classroom teaching and learning practices were collected and discussed to interpret the characteristics and patterns of student learning from long term of two contrast teaching styles.

Compared with the literature, Tom's teaching is consistent with the traditional direct instruction teaching styles that have been discussed in several sections of this study. Multiple data sources including Tom's interviews, classroom observations, students' perceptions of the frequency of teaching behaviours, the time interval count analyses of classroom observations, and discussions of the literature have been used to aptly placed Tom's view of mathematics and hence his teaching style within the traditional view of learning.

Tom's teaching appears structured, emphasized procedural/understanding with fast speed, and often includes calling for students' answers in applying a given formula or their thinking on problem solving. Frequent tests and teacher's explanations were given to provide a great amount of problem-solving practice in his classroom. It was noted that Tom's traditional and direct instructional teaching styles are similar to the traditional teaching methods used by most Taiwanese junior high school teachers (Xu, 2004; Yu & Hang, 2009).

The findings of this case study about Tom's teaching practices suggested that the traditional and direct instruction approach was mainly in use with mixed influences from the behaviourist and cognitive learning perspectives. These can be seen from his teaching strategies and emphases. He practised direct instruction with a fast teaching speed and emphasized problem solving, students' understanding, memorization and calculation speed. Tom also challenged students' thinking in his classes by frequent questioning. Frequent tests were given and he covered lots of mathematics content in his classes. Eleven students

complimented Tom's teaching, but on the other side, the fast teaching speed and difficulties in understanding were noted by ten students.

These findings on Tom's teaching practices clarify a misconception that Asian teachers adopt a teaching approach close to a behaviourist teaching style and ignore understanding. From Tom's emphases, it could be seen that he gave direct teaching but still worked on questioning students' understanding.

Ed's and Eve's teaching was the class discussion approach mainly in use and sometimes applied the group discussion approach that were consistent with the constructivist and cognitive learning perspectives and also very infrequent use of direct instruction. The free and open spirit of mathematics classes in School E and expectations for students' independent learning from one scholar's comments in 2001 are similar to the open school in the UK in Boaler's study (2002) but with different teaching approaches, (e.g. project-based approaches in UK, the class discussion approaches in Taiwan). Ed's classes also frequently used the group discussion approach.

The variance between Ed's and Eve's pedagogy indicates the flexibility of School E teachers to respond to different students' characteristics to alter their teaching strategies. Eve's students easily talk/share, thus class discussion was the main approach. Ed's students were quiet in class, so class discussion approaches were and mixed with the group discussion approaches.

The experiences of long-term mathematics teaching and learning in the class discussion approaches of a Taiwanese experimental school produced some findings and insights that were consistent Boaler's (1997) findings. The class discussion method used in School E, which appeared supportive, provided an encouraging environment from students' perspectives that offered rich opportunities for students to explain, debate and explore/create their own ways to interpret mathematics concepts/strategies. This brought up vital class conversations and students' creative/deep thoughts (Brown & Campione, 1994). The free-flowing explanation/support and question asking led to dynamic participation (Engle & Conant, 2002).

Direct instruction approach teaching practices in School T might influence students' conceptual-procedural knowledge and good performance on a small range of school tests, because students in School T mostly received and followed Tom's mathematical concepts and logical explanations and then applied formula/procedures in class problem solving. However, students in School E explored mathematical concepts together with their teacher through class discussions (Lampert, 2001).

Thus, beside students at both schools targeting to learn the same mathematics subject knowledge, students in School E had more learning and thinking chances to develop/create their own mathematics ideas in the class discussion approaches than students in school T under direct teaching approaches (c.f. Lamon, 2007). While participating in the stages of developing a collective form of knowledge within the class discussions, every student has chances to become a knowledge explorer to discover/reason class ideas, a knowledge producer to contribute their thinking to the public, a knowledge adventurer to promote/test his/her ideas in the class, and as a knowledge receiver to summarize all information. Those students' (or the teacher's) ideas and contrast arguments interweave the ongoing developing form of the collective classroom knowledge. Those strategies and foci offered chances to build up thinking and exploring classrooms in school E (c.f. Hunter, 2008). In contrast, students' roles in the traditional classrooms are as followers to follow and reason the teacher's given information/knowledge and methods. Learning mostly occurred following classroom problem-solving, as students applied knowledge and methods to tasks.

The characteristics of class discussion approaches in School E met more the emphases of high-quality instruction (Franke et al, 2007; Lampert, 2001; Kilpatrick et al., 2001), some criteria of teaching for understanding (Franke et al, 2007; Lampert, 2001) and higher-order thinking skills (Torff, 2003).

Class discussion provides opportunities for dialogic argumentation (Boaler & Greeno, 2000; Franke et al, 2007; Lampert, 2001) and supports establishment of a collective understanding among students and the teacher (Hunter, 2008). The class

discussions/dialogic argumentation might challenge School E students' intelligence more in classes, but in the long term that might benefit students' mathematical understanding. For example, when applied in new situations School E students did better in some assessment items than School T students. School E students felt that they had better understanding about the mathematical content, learned more extra knowledge, such as the history of mathematics, co-operation with classmates, good connections between mathematics and real life, and more fun in learning from the school mathematics activities.

Moreover, a number of studies also have illustrated that a constructivist approach benefits students' mathematical understanding/thinking (Briars & Resnick, 2000; Chen, 2007; Schoenfeld, 2002) and competence (Lambdin & Walcott, 2007).

More evidence supports that the class discussion approach may offer students an opportunity to get quality education, because that brings about more supportive and student centred learning, competencies/thinking skills/abilities, and not just learning mathematics content knowledge. Rather, it meets the big educational picture to develop more abilities in life, such that students have rich opportunities to develop a broad range of key competencies in constructivist classrooms to face their future lives, and that meet the educational curriculum goals, no matter in New Zealand or Taiwan.

Besides targeting the same mathematics subject knowledge, the use of a direct instruction approach did not have the power to allow students to create their own mathematics. It also had low chances for ongoing development of social/collective/adaptive form of mathematical knowledge. This form of instruction stands in stark contrast to the rich diets available to students when elements of a constructivist approach (i.e., class and group discussions; questioning, reflection, making and testing hypotheses) as identified in this study are used.

On the other side, the class discussion teaching approaches have the disadvantage of consuming more class time, so the direct teaching methods in School T covered more content quicker than the approaches of School E. However, teachers' concerns at School E were placed on quality of rather than quantity of students'

learning. These concerns are similar with teachers in the open school of the UK (Boaler, 2002).

The importance of meaning or understanding in learning cannot be overemphasized. Becker and Jacob (2000, p. 536) argue that “Content knowledge is no substitute for knowledge of how students’ understanding develops”. A student has to confront the problem, explore and construct meanings (Voigt, 1994) to develop his/her thinking/understanding. The student also has to be able to communicate these new ideas, through whatever source (e.g., drawings, discussions, text). These processes allow the student to actively participate (Nathan & Kim, 2009) or construct their own learning.

This investigation revealed that the teaching approach used by a teacher can affect student learning. The approaches used both provided students with an opportunity to get a quality education. The fact is that more supportive relationships and communication occurs, more students own their thinking/creative opportunities (Lamon, 2007), more ongoing assessment information is available for the teacher to respond to students’ needs in this approach (Kahan et al., 2003; Confrey & Kazak, 2006), the social/collective/adaptive form of mathematical knowledge is developed (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006), students’ autonomy and competence are cultivated in this approach than through the direct instruction approach.

Moreover, the findings of situated sequential relationships among teachers’ perceptions, classroom practices and students’ mathematics knowledge or competencies, and views supporting situated theories highlight the importance of teacher education and professional development. This is needed because those perceptions shaped the three teachers’ views of mathematics, and influenced their classroom practices (Cross, 2009) thereby defining the quality of students’ learning.

These research findings propel me to act. There is a need for the results to be carefully analysed. In light of the results, the majority of our students could lose

out if we fail to look at the different evidences and provide them with quality education as seen in the alternative school. Hence, this research is important.

Chapter 10 extends the discussion comparing the research findings with the literature. Conclusions are drawn from the present research in relation to the research questions, and recommendations and suggestions for further research are given.

Chapter Ten: Conclusions

10.1 Introduction

The value of my research is based on a rare case of a long term teaching and learning experiences of reform styles (mainly class discussion approaches) at the high school level. It must be mentioned that this case in today's world is still quite rare. Thus, teachers and students' opinions are valuable because no one have their experiences as so long term as long as three years. The value of this study is enshrined in the identification of teaching patterns (styles), teachers' experiences (opinions), and students' opinion patterns. Students' opinions about mathematics show their views on knowledge. Boaler's work explored project-oriented approaches (1996) and group discussion approaches (2008) but teachers' and students' experiences were based on Western countries and not from a highly developed Asian country. My study presented and discussed the long term experiences of teachers and students with regards to reform styles (mainly class discussion approaches). The cases were located in a Taiwan, a country of high study pressure and top performances in TIMSS and PISA studies.

This chapter concludes the investigation and discussion on the improvement of the quality of Mathematics Education. The focus of the investigation was to examine how two long-term teaching modes in Taiwan, influenced the perceptions and practices of teachers and students. The research examines the role that constructivist class discussions and traditional instruction approaches play in mathematical learning and the quality of education of students' at junior high school levels. The key findings will be summarized, paying special attentions on extending the scope of the discussion on the cycle of educational development. This will be followed by the limitations, suggestions and summary of the research.

10.2 Summary of the research and key findings

In order to investigate the strength of the teaching approaches, especially at the junior high school level, this study incorporated the long-term use of both the

direct/traditional and constructivist modes of teaching (e.g., lecture whole class vs. class discussion). Participants in this study were drawn from three classes, grades 7 to 9, from two Taiwanese schools. This research utilised qualitative approaches to describe and analyse questions. Both the traditional/direct and constructivist teaching approaches were adopted in one junior high and one experimental school over a period of three years (2000-2003). Data collected were analysed to address the following research questions:

1. What are the differences between the traditional and experimental approaches to teaching mathematics in Taiwanese classrooms and their influences on teaching practices and student learning?
2. How do classroom practices in the alternative school benefit students' mathematical learning attitudes, thinking ability, knowledge and achievement compared to the classroom practices in the traditional school?
3. What are the relationships between teachers' beliefs/perspectives relating to mathematics and teaching strategies, and the education provided for students?

The long-term experiences of two contrasting teaching approaches (class discussion approaches and traditional/ direct instruction) - on students' learning, relative to the perceptions of teachers and students in relation to mathematics/learning, teaching practices have been examined in this research. The findings of this research are summarized below. Number1 addresses the first research question, numbers one to four addressed the second question and numbers five to seven speak to the last research question.

1. The constructivist (class discussion) approaches of Ed's and Eve's classes in School E and direct instruction approaches of Tom's classes in School T were identified in this research from literature and multiple data sources (see sections 9.1.4a & 9.1.4b). According to the responses of students from both schools in this study, their families supported them with their learning of mathematics (see section 8.1.1.1). Students under the traditional

teaching approach followed the teacher's direct teaching. These students had practical experience in problem solving through the teacher's frequent appeals for answers in applying given knowledge on problem solving (see section 9.4).

2. The class discussion teaching approach of Ed's and Eve's classes in School E promoted a great amount of class discussions. Such opportunities encourage and promote students' thinking, dialogue, mathematical communication, debates, and negotiation of their mathematical ideas as they formed social collective knowledge (see section 9.1.1.3) (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006).
3. Long term class discussion approaches allowed students to experience multiple learning roles. Instead of being seen as receivers/followers, they were viewed as knowledge explorers, producers, and adventurers (see section 9.1.2.1). Such active engagement in the learning process offered students lots of chances to demonstrate higher-order thinking skills: discovering, reasoning, organizing and arguing (Torff, 2003).
4. Students in the experimental classroom were operating in an open environment where thinking and exploring mathematics influenced them through many areas in mathematics (see sections 9.1.1, 9.1.2 & 9.1.3). For example, there were key competencies developed (see section 9.1.2.4), increased creative thinking (see sections 8.2.4 & 9.1.2) (Lamon, 2007) as well as the use of a social/collective/adaptive form of mathematical knowledge (see section 9.1.1.3) (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006). Students' autonomy was pronounced (see section 9.1.1.1), and there was an open, supportive and friendly class atmosphere with close teacher-students relationships (see section 9.1.1.1).
5. Students had different views of mathematics. Students in School T interpreted mathematics as a subject (n=T9(35%), E1(4%)) and more students in School E interpreted mathematics as a way of thinking (n=T5(19%), E8(35%)). This finding supported the opinion that the situated influences of students' mathematics classrooms are consistent with students' interpretations to mathematics (Boaler, 1997).
6. Moreover, the situated sequential relationship in this study has suggested that the potential relationships, during the teachers' perceptions of

mathematics and learning, were consistent with teachers teaching pedagogies/strategies (e.g., classroom practices) and students' mathematics knowledge, competencies, or understanding and views (see section 9.3).

7. The situated sequential relationships from this study also endorsed the importance of teacher education and professional development (Borko, 2004; Steele, 2001). The aforementioned might influence the teachers' perspectives of mathematics, classroom practices (Cross, 2009) and mathematical views.

The following conclusions are made based on the above findings:

This study has revealed the prospecting future of using a class discussion approach as in Eve's and Ed's classrooms, to provide high quality classroom instruction and students' mathematical competencies. "...content knowledge is no substitute for knowledge of how students' understanding develops" (Becker & Jacob, 2000, p. 536). Students need cultivate some key competencies (Lambdin & Walcott, 2007). This study addressed the call for research evidence from Taiwanese classroom experiences, to examine the benefits of using a constructivist teaching approach (Wey, 2007; Chou, 2003a; see sections 9.1.1 & 9.1.2). It also responded to the need to understand students' views (see sections 8.1, 8.2.1, 8.2.2, 7.2.5, 8.3.2, 9.1.3). These findings answered research questions 1 and 2.

The findings revealed that teachers' perspectives of mathematics/learning were consistent with their teaching practices and different types of classroom teaching practices, revealed different student mathematics competencies and mathematical understanding (see section 9.3). These findings addressed the third research question. To sum up, this piece of work can contribute new understanding with regards to the ongoing development of constructivist pedagogy (Richardson, 2003) based on the Taiwanese experiences from the class discussion approaches.

Emanating from the results of this study are the advantages of using the constructivist approach; that is, using this approach to build students' mathematical thinking abilities and understanding, which allows them to gain

mathematical power. However, it must be noted that, acquiring these abilities should be viewed as a long-term goal rather than a short-term one, since it takes time to develop such power, students' mathematical thinking abilities, understanding, and competencies.

This research used the social constructivist perspective and sociocultural learning perspective (Bell & Cowie, 2000; Wertsch, del Rio & Alvarez, 1995), for example, situated cognition, to interpret students' learning, classroom instruction processes and the relationship between students' learning and classroom teaching practices. The findings support using an interpretivist perspective to provide a framework for understanding teachers' instruction and student learning patterns. A situated perspective may be used to provide a framework to address learning with respect to the cultural environment (Lave & Wenger, 1991) and practices (Boaler, 2002c; Lave & Wenger, 1991) while using a social constructivist perspective to interpret the teaching and learning of mathematics during social interaction/dialogues (Cobb et al., 1992).

One may therefore conclude that dilemma in mathematics education appears from the inconsistencies informed by the strengths of the traditional/direct and constructivist teaching approaches. According to Boaler (2002c), the main problem within the traditional approach is that of ignoring the complexity of teaching/learning. However, we see that many Asian countries have adopted this approach and are still performing excellently (Leung & Park, 2002).

When educational policies began to be informed by a constructivist pedagogy, the general teaching practice, (in Taiwan and the USA), was still unable to fully realize the reform focus (Ball, 2003; Ford & Forman, 2006; Franke et al., 2007; Hiebert et al., 2003; Webb et al., 2006; Wey, 2007). Later, incomplete practice in accountability of general students' competencies (Chou, 2003a), caused a backward movement of the educational reform pendulum towards the previous knowledge centre, in countries such as Taiwan (Chung, 2005) or the USA to some extent (Lambdin & Walcott, 2007). Thus, Taiwan was among those countries that reverted to the traditional approach to teaching (Chou, 2003a; Chung, 2005). In looking at the results, one needs to be careful in criticizing the traditional teaching

approach since the research findings point out that both approaches have their merits.

While the outstanding results of adopting and implementing a constructivist approach have been revealed in many studies (see section 2.2.2), missing from the body of literature is a wealth of long-term constructivist studies (Carpenter et al., 1998) especially at the secondary school level. For example, the open project-based approach in England (Boaler, 1996), and the (cooperative) group work approach in USA (Boaler & Staples, 2008) are good examples of this approach. To this end, the researcher is calling for the implementation of long-term constructivist studies with different approaches that will better guide educators.

It is therefore suggested that the main findings in this research may provide a profound understanding of mathematical learning from the direct instruction and constructivist teaching modes that may explore the quality education in the field of mathematics. This research offers exemplary constructivist teaching models that the public can see and understand and support. This can lead to a supportive culture that empowers teachers to engage all students in quality and challenging mathematics learning.

While educators continue to search for ways to better meet the needs of all students, teachers too must be ready for action. The shared vision should be one where "... students [can] achieve a high standard while at school and leave equipped with the knowledge, competencies, and confidence that they will need for success in a constantly changing world" (Fancy, 2006, p.3). The call goes out for schools to lay the foundation by equipping students for success in their future lives. If this call is to be heeded, then mathematics classrooms need to meet educational needs to build up students' knowledge and competencies. What better way to do so than to expose students to other instructional modes such as using a constructivist approach. The educational functions need to serve the big picture to aim at attaining future success in mathematics and lifelong learning.

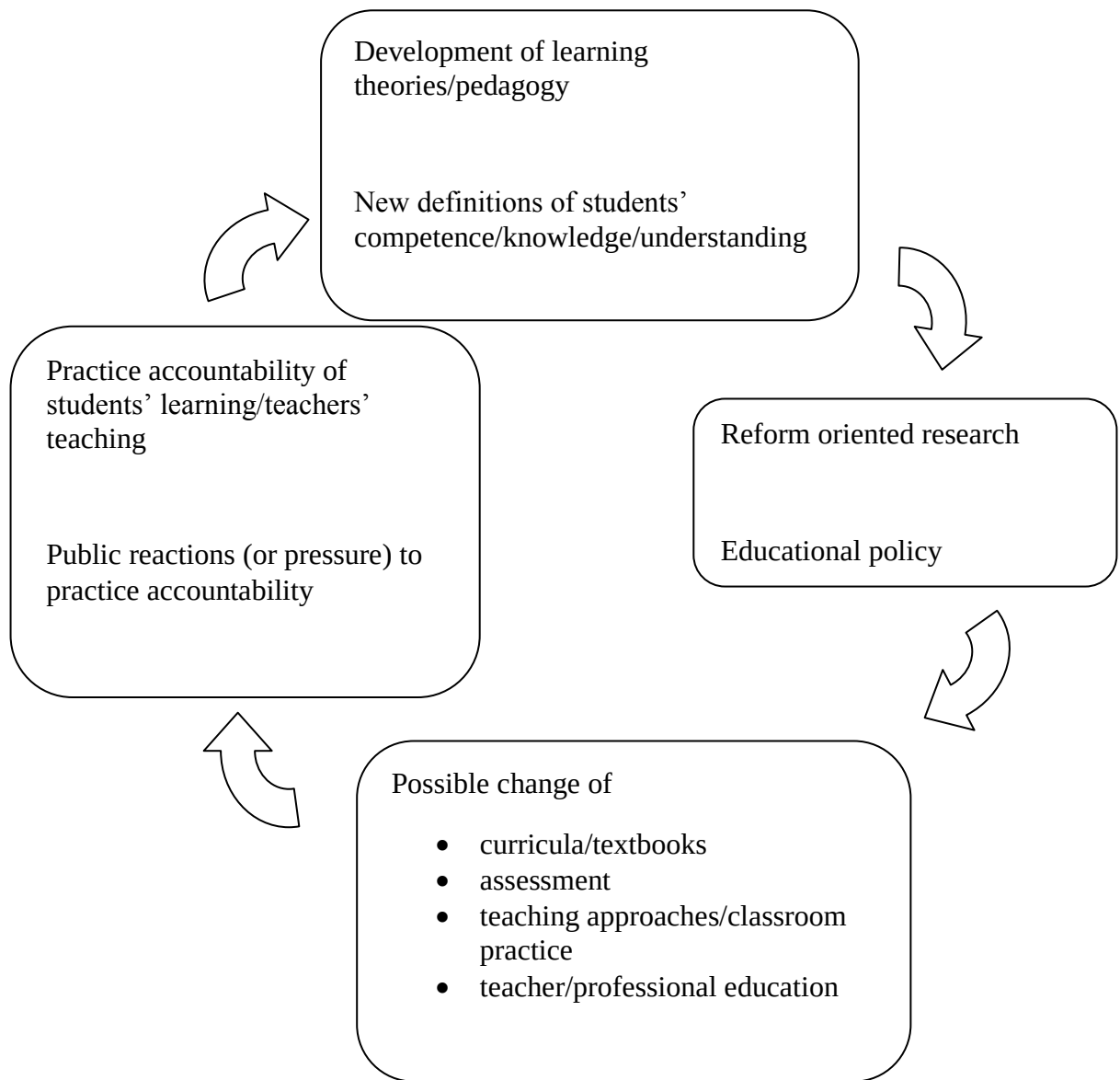
As I examine the findings from this research, and attempt to put the pieces together to better understand students' learning and the influences of being in the

constructivist classrooms, I feel impelled to act. That is, one should not sit idly by and let the constructivist classrooms disappear because they do offer better potential to develop students' abilities, in mathematics and abilities to face the future than the traditional direct teaching classrooms. Evidence presented in this study (see sections 9.1.1 to 9.1.3 and 9.1.4d) are supported from other research, and constructivist long-term studies at the high school level (Boaler, 1997 & 2002b; Boaler & Staples, 2008). Ensuring the continuance of constructivist classrooms would offer students an opportunity to get quality education. How to react to practical challenges in today's school environments will be discussed in the latter sections.

10.3 The cycle of ongoing educational development

The circular ongoing relationship between instructional theories and classroom-based research has been explored (Cobb & Yackel, 1996). Franke et al. (2007) focused especially on reformed research that provided information on classroom practices/teachers' efforts to support development of students' mathematical competencies. To some extent, the relationship between theories and classroom practice can be summarized as in Figure 3, based on reviews of mathematics educational movements in the USA through the past century (Lambdin & Walcott, 2007) and a reflection on the education reform in Taiwan.

Figure 3 The circular ongoing educational development concept maps/model



In some ways, the reform experiences in Taiwan can explain the circular educational relationship. Constructivism influenced the focus of Taiwanese mathematics discipline (Chung, 2005). This brought change to the educational policy/curriculum from 1996 to 2004 (Chung, 2005; Guo, 2004) including changes in textbooks (Chou & Ho, 2007; Guo, 2004) and teaching approaches/classroom practice (Chou, 2003b; Fu, 2008; Guo, 2004; Weng, 2003; Xu, 2003). Scholars criticized the government for the lack of proper teacher professional development (Borko, 2004; Chou, 2003b; Chou & Ho, 2007; Teng, 2001; Wey, 2007). However, the traditional standard tests were still the main method to report students' achievements during the reform period. Later, the

unsatisfied public (Chen, 2003a; Chung, 2005; Guo, 2004; Xu, 2003; Wey, 2007; Zhuang, 2002) and incomplete practice accountability of students' competencies from the traditional uniform assessment (Richardson, 2003) caused a redirection of educational policy. This paradigm shift of an immature reform movement, replaced the curriculum in 2005 (Chung, 2005) (which had a backward focus similar to the 1978 curriculum (Chung, 2003b)). Although research evidence did reveal that the 1996 curriculum resulted in some benefits with regards to constructivist approaches (Chen, 1998a; Chen, 2007; Yeh, 1998; Zeng, 1998) or achievements from constructivism (Li, 2003a; Li, 2004), those findings have not been able to change the regressive movement in education since 2005.

This study reveals this circular educational development model by contrasting teaching approaches and the influences on students' knowledge/competence. The practice of accountability of students' learning and teachers' teaching contribute to the development of constructivist pedagogy from understanding factors such as the patterns of students' learning knowledge/competencies, teaching approaches/class norms, methods/assessments or materials. The finding of the sequential relationships (see Section 9.3) among teachers' perceptions, teaching practice of mathematics/learning, and students' knowledge/perceptions shed new light on the sequential social relationships between teaching and learning and the situated influences between classroom practices and students' knowledge/competencies/perceptions. Therefore, the importance of teacher professional education is highlighted in this study because it might bring influences on students' competencies/perceptions. It is expected that the findings of this study will raise (i) the awareness of the norms, benefits and possibility of constructivist approaches in junior high mathematics level to be reintroduced elsewhere, even within the competitive educational environment in Taiwan, and (ii) the influences in the ongoing development of global education, teacher or professional education.

The educational development cycle is ongoing. Within the different historical periods of the educational development cycle, there are different educational foci. The educational pendulum movement is not just back and forth, as in the case of Taiwan in recent decades, between the learner-centred and knowledge-centred

focus (Chung, 2003b, 2005). The Taiwanese educational reform movement has now determined that the centre factor that caused the backward focus of the curriculum (Chung, 2003b) and direct instruction (Xu, 2004), was because of immature constructivist approaches practiced in primary education in general (Chung, 2003b). During the past century in the USA, it was a knowledge-centred focus with drill and practice (1920-30, after the 1970s to present), and the focus was learner vs. accountability (1990s to present) (Lambdin & Walcott, 2007). These knowledge- and learner-centred foci in both countries are different from the definitions of Donovan & Bransford's work (2005). Upon examining the curricula in Taiwan (Chung, 2005; Wey, 2007) and the USA (NCTM, 1989), one can see that this learner-centred focus is similar to constructivist perceptions. The knowledge focus that appeared in Taiwan valued knowledge and students' calculation abilities (Chung, 2005; Yang, 2003) but the knowledge focus in the USA included (i) drill and practice (1920-30, post 1970s to present in general classrooms), and (ii) the learner vs. accountability focus (1990s to present) (Lambdin & Walcott, 2007). The accountability focus has been criticized as being responsible for lowering the quality of curriculum and teaching in school/classroom practice for examination purposes (Lambdin & Walcott, 2007).

As more information becomes available (ex. new definitions of students' competence/knowledge/understanding from reform-oriented research, new learning theories) the practice of accountability of students' learning/teachers' teaching, will trigger the next movement of the educational pendulum (Sfard, 2003) and continue the dynamic journey of circular, ongoing educational development.

10.4. Limitations of this Study

During this study a number of limiting factors were evidenced: one important bias that was evident was the researcher's personal prejudices or a lack of appreciation of the alternative school practices. This might have affected the objectivity when interpreting the data. To reduce such limitations the researcher examined the literature (about the alternative school practices), found more ways to understand the school (conversing with other teachers and students), and peer reviewed some findings with the two mathematics teachers in the study or other educators. It is

important to note that the researcher also maintained a neutral attitude during the process of data collection.

The sample size was another limiting factor (the small size of sample giving a lack of precision (Bell, 1993) which has been noted in section 4.6.2). A limitation exists from a small sample size of this study; however, the respondents produced a lot of in-depth information from detailed analyses of multiple sources of data that provide important insight between teaching and learning relationships (Wood et al., 2006).

This study may serve as an example of an unequal comparative study. For example, there are quite a few unequal conditions between participants at both schools. A critique might rise that a small class size of Eve's and Ed's classrooms might benefit relationship in their classrooms. However, students in Tom's classrooms might be advantaged from long period of classroom time for developing peer or teacher-student supportive relationship. The reason was drawn from different school systems. Students in School T always stayed with the same classmates while studying every subject, but it was not the same case for students in School E. Students were given the freedom by School E, to choose subjects to study (SyQ1p.1), so students may come cross different classmates in different subjects. For example, there were 34 grade 9 students in School E, but only 17 students chose to attend Grade 9 mathematics, the other 17 students either attended Grade 8 mathematics class, Grade 7 mathematics class or did not attend any mathematics class. School T offered 5 mathematics lessons per week, but School E only offered 4 mathematics lessons per week.

Moreover, Anderman & Mueller (2010) illustrated that a small class size is not necessary to increase relationships in classroom. They noted that there are other considerable and important issues which might influence relationships, for example, teachers' pedagogy knowledge, classroom practices, students' participation or cognitive enhancement within classroom learning. Therefore, this study has drawn on the sociocultural perspective (Bell & Cowie, 2000; Wertsch, del Rio & Alvarez, 1995) to interpret and discuss the relationship patterns in three classrooms of two schools as above arguments from data evidences (including

classroom practices, students' and teachers' perceptions) and literature. The triangulation methods and data from classroom practices, students' and teachers' perceptions increase trustworthiness of the finding (Franklin & Ballan, 2001; Gall et al., 2010).

Addressed here are the different background issues in two schools such as small class size in School E, that might benefit the student-teacher relationship. However, student's long gathering time in classes in School T might also benefit the student-teacher relationship. Therefore, readers can understand that these two schools have two different background issues and each might benefit each student-teacher relationship. Hence, the comparison and interpretation of student-teacher relationship within two teaching modes of this study is discussed in a relative balanced way also with the triangulation methods and data and is trustworthy.

Another limitation evidenced is that the data came from a macro view of classroom practices (discussion), so this study did not give much focus to the individual development of students' mathematics understanding (Wood et al., 2006).

Some limitations of class observations appeared. For example, in my class observations, no student was observed checking the other students' homework. Some events happened before my class observation periods. Therefore, I am not aware if a student fell asleep in Grade 7 mathematics classes, or there was no way of checking Eve's teaching strategies for a big class (more than 50 students in a class). Students felt that Eve emphasized more class discussions and problem solving in Grade 9 than in previous years (SyOfvtp.4).

Translation from Mandarin to English proved to be another limiting factor, mainly evident during the data analysis. The researcher used some strategies to address such limitations. For example:

(i) Some Chinese vocabulary cannot be translated directly into English word by word. In such cases, the researcher used two or more English words to convey interviewees' opinions;

(ii) Sometimes the people did not specify number difference in Mandarin when they used nouns, but in the custom of Chinese, later they would use pronouns to represent the nouns and the pronouns showed the number differences clearly. So, in this study, the researcher asked interviewees to clarify the numbers whenever necessary.

It must be noted that generalization is another issue to be addressed, because most reform work is still linked with curriculum or a cultural background (Richardson, 2003) but it is still possible to advise some disciplines through research beyond the limitations of curriculum or culture. Educators need to pay attention to this before applying the results of this study to their local or national context.

10.5 Recommendations

Polarized teaching approaches developed polarized student competence (see section 10.2). Students' competence in mathematical abilities requires children to develop and link their knowledge of concepts and procedures (Alibali, 2005). If a single teaching approach is adopted, it limits the development of students' abilities, as no single approach is adequate enough to develop students' mathematical abilities.

Consequently, the recommendations given would advise on some general principles related to these two contrasting teaching styles.

First of all, the long-term support of teacher communities is needed, since this will generate opportunities for teachers to share experiences/strategies for inquiry instruction (Romberg et al., 2005). This will lead to teachers' growth and professional development and fuel teachers to support each other when facing challenges.

10.5.1 Recommendations for the Constructivist Classrooms

Recommendations offered here cover certain mathematics content/curriculum and requirement of the traditional/standard assessments. Time management and content coverage (see section 9.2) are the challenges for the constructivist (class discussion) approaches. The first three suggestions below are given to address

these concerns.

1. Adding more class time

Students in School E had difficulties in problem solving (n=5) and not having enough practice in mathematics classes (n=5) (see section 8.1.1.3). Student E22 commented “although teaching styles bring relaxing class atmosphere and students feel less pressure, when we are facing a test, we realize that we actually did not learn much, but only have basic understanding. So (we are) unable to solve problems in depth” (OQ2Q2). However, student E19 felt that in Grade 9, with more chances to practice problem solving this improved the situation (OQ2Q2).

Mathematics conceptual and procedural knowledge are interwoven and interdependent in developing students’ mathematics competence (Alibali, 2005; Kilpatrick, et al., 2001). Teaching mathematics has to engage students in doing mathematics as they are learning it (Franke et al., 2007; Henningsen & Stein, 1997; Lampert, 2001, p. 5). Besides developing students’ conceptual knowledge, improving mathematical rules and procedures should also be another important focus in the constructivist classrooms. In order to develop this type of knowledge, students need to have more chances for practice in classes.

Students exposed to direct instruction performed better in conceptual-procedural quiz items than students exposed to class (group) discussion approaches, and that might connect with greater amount of classroom practice in problem solving. I recommend that students in School E may need more opportunities to do problem solving in classes, e.g. adding at least one more class period for group problem solving (to cover the content of textbooks and resource books). Thus, students would have more chances to practice conceptual-procedural knowledge in class and to become more proficient.

2. Timely guidance

One reminder here is that, in the constructivist teaching classrooms, it is still necessary for teachers to give students hints or guidance (maybe direct teaching) during the progress of students’ presentation, argument or discussion. The

aforementioned is true because children's mathematical thought process is immature (as Eve's comments, see section 9.2) and giving hints will save on class time. Teachers need to be careful about their choices of when to give guidance, because it is still hoped that students could have abundant opportunities to develop their own mathematical ideas. The times for the teacher to give guidance could be under the following conditions: (i) the whole class is bemused for a long time; (ii) the whole class made wrong conclusions; and (iii) students discussed for a long time and still could not reach the mathematics conclusion at which we hope they will arrive.

3. Treatment of individual understanding

Each individual's opinions are valuable to contribute to the development of mathematical flow (including mathematical concepts or problem solving strategies) in classes through the constructivist/class discussion approach. However, if a student got stuck in mathematical concepts/problem solving within class discussion, the teacher should be aware of the time and focus of competencies for the majority of students to decide either to move on to the next step or pause. This is a conflict point. If a long amount of time is given, there is the potential to include more students' ideas to form collective perceptions to develop mathematics concepts together (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006), but this might come at a high cost of class time.

Some other recommendations are offered here for consideration. It is recommended if a student has difficulty understanding and the class discussion cannot clarify the problem, it might be time for that class to move to a new mathematics target, and encourage peer support (or teacher support) to help that confused student after the class. Eve and Ed also adopted similar procedures.

For example, after the explanations of student E8 and student E11 were expressed to student E4, and he had difficulty in understanding student E5's discussion, and solution strategies (see section 6.2.6.2 (j)), Eve came to support and gave direct instructions that helped to solve student E5's confusion (see section 6.2.6.1 (b)).

Ed also adopted the same method (Om1Ihp12, 13Q10). For example, Ed asked

student E16 to help student E15, and student E18 to help student E17 (Omvh1211p2t). Ed gave hints to help Student E23 (Omvh1211p2t).

4. The need for long-term constructivist classrooms

Whatever class practices that students are involved in, those practices will affect the development of student abilities. As revealed in this study, if we want to build mathematics thinkers, we need to afford students opportunities in classes to think, communicate and to construct their own knowledge as in the constructivist approaches. If we want to students to be skilful in their problem solving, then we can offer lots of direct teaching and problem solving as in the traditional approach.

Though the alternative experimental school in this study was closed in 2006; we can see the success of developing students' thinking abilities, and competencies. The next step is to investigate how to improve the constructivist approaches, to benefit students' procedural knowledge to cope with the competitive school tests and the full mathematics curricula at junior high level in Taiwan.

In reality, it is very rare and very difficult that a mathematical constructivist classroom existing in a junior high school for a long term in Taiwan. The difficulties can be seen in section 10.5.2(a), from sharing by Eve and other teachers. The reasons for this are explained in section 10.5.2(a).

Further research should be made of the constructivist teaching styles. Given the difficulties of a mathematical constructivist classroom existing in a normal school in Taiwan, it is highly recommended to start with another experimental school; or a long-term project in a school with approval from both parents and the school. However, these two suggestions cannot occur just by the passions of mathematics teachers but need support from other parties. For example, establishing an experimental school needs support from the Ministry of Education, and a long-term project needs support from parents and the school.

5. Establishing Classroom Norms

Establishing classroom norms in advance are important. This practice can prepare students for participation in classroom activities (Cobb & Yackel, 1996; Kazemi

& Stipek, 2001). The norms will also shape the classroom culture (Franke et al., 2007) and classroom practices (Cobb & Yackel, 1996) in a healthy and friendly way to meet and retain the reform focus.

For example, establishment of the (social) class norms of respect (Franke et al., 2007; Silver & Smith, 1996; Windschitl, 1999b) with non-judgemental attitudes toward their peers' right or wrong answers (Wood et al., 1991). Acceptance of different thoughts, might avoid potential emotional harm or conflicts from too much debating, or overly strong attitudes toward their ideas to persuade others.

Professor Huang suggested alternative solutions in problem solving are needed in order to pursue students' creative abilities. This is not to demand that students learn every possible method, which was the genesis of the confusion raised in primary school mathematics education during the reform period of time (Fu, 2008).

The norm of the classroom authority needs to be established in advance, attributing roles to the teacher or students or joint role of teacher and students, to avoid conflicts in schools. From my point of view, there is still some distance between 'student-centred learning' and 'student-directed learning'. Establishing the norm early and accepted consensus between the teacher and students might avoid conflict such as the case in one Taiwanese primary experimental class in 1992 (Fu, 2008). One disappointing reform experience in the past, occurred when the authority of a Taiwanese experimental classroom was built on students' decisions. This conflicted with many other teachers' ideas about teachers' authority and led to an end of that experimental class after four years of effort (Fu, 2008).

The norm of the classroom authority still can be set up as the teacher, even in student-centred learning classrooms, for example, as Lampert's (2001) work. She invited students and facilitated the open discussions and also reformulated students' ideas. The classroom authority of evaluation of students' achievement still depended on the teacher (Lampert, 2001).

10.5.2 Recommendations for mathematics educators in normal schools

In Taiwan there are many difficulties that surmount in teaching. In fact, it is very rare that a mathematical constructivist classroom can exist in junior high schools for a long time. This is because of the full syllabus carried by the school and the parent' expectations of teachers, teachers wanted to help children succeed in school and the national examination. Normally, the teachers' time only allows for covering the mathematical content from the textbooks, practice books and correcting students' tests through direct teaching in classes, therefore not much time is left over. Also, most teachers may not be aware of the constructivist method and the benefits of this style of teaching.

There are different expectations with regards to the responsibility of learning from the constructivist (discussion) classrooms and the typical mathematics classrooms in Taiwan. In constructivist learning, the onus of learning is on the learners and not the teachers. Students actively construct their learning rather than passively learning through the teachers' transference (Simon & Schifter, 1991; von Glasersfeld, 1990, 1993).

In contrast, the majority of parents in Taiwan expect teachers to transfer mathematics knowledge and skills to students. If teachers did not see it as their jobs to transfer mathematical knowledge, then the responsibilities of students' learning might shift to students themselves. However, if the whole class did not perform well on average, some parents may complain about the teacher. As a result, the onus of learning still partly belongs to the teachers, even after the teachers have transferred the mathematical knowledge. This contrasts constructivism where the learners are responsible for their learning. Consequently, in the past, the differences in expectations about the responsibility of learning between the constructivist classrooms and the normal mathematics classrooms in Taiwan have brought conflicts between parents and teachers.

It must be mentioned that understanding takes time, and this is disadvantageous to the coverage of content in classes (Gardner, 1994). These time and content coverage challenges exist in Taiwan (Chou, 2003b) and have also affected other reform practices in many countries (Cross, 2009). It is difficult to cover all the

content in detail from the textbook and the practice books, in the constructivist (discussion) classroom. Therefore, when practicing constructivist teaching in Taiwan at junior high level, teachers face difficult challenges from parents, then parents pressure schools (see examples in section 8.4).

This investigation demonstrated that the constructivist approach, when compared to the traditional approach provided a higher quality education to successfully build up broad areas of their abilities such as thinking capacity, understanding, key competencies and positive learning attitudes. Therefore, the constructivist approach has more potential to develop students' abilities to attain future success in life and lifelong learning.

It is also important to address the shortcomings of the traditional direct approach, especially as Bennett (1976) comments on insufficient emphases on students' creative production. Although aligned with the teacher's given mathematics definitions/formula (Hagg, 1991; Neyland, 1994), students' learning roles were mostly like followers or receivers in classes where they focused on problem solving with the teacher. Although, it was still possible to have a few chances to produce their own mathematical productions/problem solving in classes including seat work (see section 5.2.4). However, these chances for students' own mathematical productions are relatively less when compared to the high freedom given in the learner-centred approaches of constructivist instruction.

How to cultivate an educational environment in order to build up mathematical thinkers in a classroom? The problem-centred and learner-centred classroom practice has powerful potential to achieve this (Cross, 2009). The constructivist instruction often includes one of these two categories of classroom practice (Confrey & Kazak, 2006; Windschitl, 1999b). The constructivist approaches which allow students to discover, explain, discuss and argue mathematical ideas, thus successfully building up students' mathematical thinking abilities, understanding and competencies, may offer a good solution. If this method is to be adopted the following needs to be considered:

(i) Support from parents and schools

If the class discussion approach is to be applied smoothly in schools, one needs to communicate well with parents and gain their support. Parents can accept an exploring classroom, and that students need to be responsible for their own learning and not just rely on teachers for knowledge. However, if one parent of a class is against the teaching method, it would be difficult to apply this method in that class. Then, mixed teaching strategies could be a good choice as mentioned in suggestions (iii) and (x).

(ii) Strengthen teachers' beliefs and knowledge

One argument of this research shows that teachers' views about mathematics and their pedagogy influence their teaching content and teaching strategies. A sequential relationship among the teacher's perceptions, classroom practices and students' learning was exposed (see section 9.3). As a result, teachers' beliefs and knowledge could influence the quality of mathematics education learnt by students. Therefore, in order to carry on educational reform and to have a greater influence on students' learning, teachers will be the key factor. There is a need to strengthen teachers' knowledge and beliefs to prepare for the changes (Bell, 2007c).

Teachers need to be aware of (1) what constructivism is, (2) how to prepare the educational environment to let students have opportunities to explore and find their own ways to learn mathematics through constructivist teaching, and (3) the benefits and weak points of constructivist teaching. With good understanding of constructivist teaching approaches, teachers can be motivated to apply these approaches in their classrooms. Bell and Gilbert (1996) states that teachers will view the changes as challenges rather than problems. There is no fixed way to do problem solving in a constructivist classroom. For example, if children's methods are reasonable then they can be accepted, rather than only valuing textbook or teacher methods. These factors also decrease some of the misinterpretation of constructivist teaching by Taiwanese teachers' (e.g. Lin, 2002a).

(iii) Mixed teaching strategies

Not all teaching approaches are suitable for all students (Boaler, 1997). Although

many research evidence has indicated that concept-oriented mathematics curricula have provided higher and more equitable results than procedure-oriented approaches (Boaler, 1997, 2002b; Boaler & Staples, 2008; Schoenfeld, 2002), some students opined that they learn more from the traditional procedural teaching (Pesek & Kirshner, 2000). A few students of this study also expressed opposite views from their class practices with regards to increasing their mathematics learning interest (n= T3, E2, TQ2Q(3(c)), OQ2Q(3(c))).

Direct instruction and constructivist teaching strategies could possibly be applied in combination in classrooms (Xu, 2004) to help to overcome the weak point of being time consuming when only class discussion method is in use, and to cover more mathematics content. Evidence from data of the study supported the above arguments, for instance, in Eve's intended curriculum (see section 6.2), students' reports of direct lecturing in use of Eve's teaching (see section 6.2.5), class observations of Eve's classes (see section 6.2.4), Ed's perceptions of the need for teachers' multiple teaching approaches (see section 7.2.2), students' reports of Ed's direct lecturing (see section 7.2.5), class observations of Ed's classes (see section 7.2.4). Moreover, although within the constructivist teaching styles in School E (see section 9.1.4b), during my class observations of Ed's 14 lessons and Eve's 20 lessons, Ed and Eve still adopted direct instruction for one lesson to cover the key concepts of the unit and to speed up the class teaching. For example, Ed taught his own three-page summarized notes to students (see section 7.2.4) and Eve explained problem solving of a test (see section 6.2.4).

Another suggestion, when addressing the issue of students lagging behind and being lost or students' sharing skills not being mature, teachers could add brief instruction, set aside free discussion time in classes, reduce the discussion issues, or give some direct instructions. Moreover, a reasonable amount of practices could be included to overcome the weak point of poor mathematical writing ability from Eve's experiences, e.g. giving homework or tests.

(iv) Upgrading children's mathematics into mathematician mathematics

Tom has specified the poor students' mathematics methods and mistakes in solving complex problems in constructivist teaching (see section 8.4). This points out that some of the children's own methods developed in the primary constructivist classrooms were inadequate when learning the mathematics content at the junior high level and these students needed to improve their skills. The class discussion method of School E may offer good examples to indicate upgrading students' methods into mathematician/textbook-like mathematics thereby assisting them to cope with national tests.

For example, students presented/explained their methods/concepts from their own previous independent study and received challenges and questions from the whole class and the teacher that led to dynamic class discussions together. Also, Eve often challenged students to simplify their methods (see section 6.2.3) to maintain the quality of students' methods. Later, Eve found that this developed students' debating ability and critical thinking (Of3Ihp4mQ5pr, see section 8.4), and the discussions improved the students' own methods.

(v) Add extra mathematics class time

Even if parents accept constructivist teaching, the other challenge to be addressed is assessment. One solution to this challenge might be to increase the number of mathematics classes, to allow students to have more time to practice their conceptual knowledge when dealing with mathematical rules and procedures. Because of the discussion nature of the constructivist teaching style, students learned less problem solving through classes compared with that learnt in classes using the direct teaching style. When faced with procedural type questions in school examinations, if the students themselves did not practice problem solving after classes, they tended not to perform as well as students in the traditional classrooms. This results in teachers being faced with pressure from schools or parents to incorporate more direct teaching in classes. Therefore, more class time might help to diminish this potential problem. However, this requires approval from the school in question or the Ministry of Education.

(x) Changes start from a small step

If parents or the school do not agree with the constructivist teaching method, teachers may use it as a mathematics activity as part of a mathematical unit, e.g. once a semester. Then, teachers can examine the feedback from students and parents to decide how frequently this approach may be used. Teachers can ask students for feedback on the areas needed to be improved or explained more, and the teaching method can be adjusted accordingly. These approaches can help to assuage the fears of students and parents. This might help teachers to apply this approach more smoothly in classrooms.

Teachers need to use experiences from their own classrooms to determine how frequently they can use the constructivist teaching approach in their classes. The more use of the constructivist teaching approaches, will result in more possibilities to build students' thinking abilities, understanding and key competencies.

10.5.3 Suggestions for the national examination

Most assessments in schools evaluate students' abilities to use mathematical formulas, facts, and procedures to do problem solving. However, students' mathematical thinking abilities in new situations are commonly ignored in mathematics assessment in Taiwan and many other countries. Students' ability to use their learned mathematics knowledge and concepts in new situations to solve problems will demonstrate how possible it will be for them to use that knowledge in real life situations. This clearly shows how vital it is to develop students' mathematical thinking abilities/competencies. After students graduate from school, they will face many challenges or issues in life that were not taught in schools. Therefore it is important to build students ability to be independent thinkers. This will allow them when faced with new challenges, to think, transform and use their knowledge to solve problems, rather than panic because of a lack of thinking ability.

Fast problem solving is necessary for Taiwanese students to be able to perform well in the national mathematics examination, to allow them to have more time to solve difficult questions. Otherwise, they may not finish the examination and lose

marks. For example, there were thirty one questions on the national examination of 2003 and students needed to solve them in 60 minutes. If students wanted to get high scores, they needed to solve all problems speedily by having a clear grasp of all mathematical concepts.

As a result, if students habitually practice problem solving, their speed in problem solving would increase and they should perform better on tests like the national examination. The design of the national examination is likely to encourage teachers to let students increase their practice in problem solving and tests in classes, so students may develop fast problem solving speed and have more experience with more questions. This could explain why most schools in Taiwan adopt the traditional teaching method with direct teaching, more tests, and more classes (including extension classes) to help students to practice. In order to cover more content or problem solving, the time for students to discover or communicate their own methods is normally ignored or limited in classes. Thus, in this way, the discovery development and communication of students' mathematics abilities are restricted.

The following are suggestions that can be used to ameliorate this problem.

(i) Extend the time of the national mathematics examination, so those students who have good mathematics understanding but not necessary speed could reach their potential scores. Therefore, teachers would not need to emphasize speed but rather may place emphasis on the other issues related to students' learning, for example, understanding or discovering the students' own methods. (ii) "New" questions can be added in the examination, these questions can be related to the students' learned mathematics concepts not found in the textbooks, practice books or resource books. Students would have more chances to practice their mathematics understanding and use this knowledge in new circumstances. Yu and Hang (2009) suggest that improving assessment methods and quality (Chen, 2003b) to evaluate students' high level thinking will benefit reform in classroom instruction.

10.5.4 Suggestions for the development of education reform

The experiences of curricula reform or curriculum guideline in the Unit States and

Taiwan did not bring many changes in general classroom practices to be consistent with the reform (Ball, 2003; Franke et al., 2007; Wey, 2007). The curriculum development needs to change teachers, parents and students' ideas, not just change on paper. Regarding teachers' growth, continuing reform-oriented professional development, and updating new findings of reformed approaches are important (Borko, 2004; Steele, 2001; Tao, 2003; Visnovska & Cobb, 2013) for teachers to receive support (knowledge and strategies) to attain the new focus of the curriculum (Tao, 2003; Romberg et al., 2005).

Appropriate assessment tools are needed to analyse students' mathematical knowledge/competencies (see section 10.5.3). It is also important to inform all parties (including policy makers, educational administrators, teachers, parents and students) about the benefit or challenges of implementing the new curriculum. This heightened awareness and better understanding of the curriculum can help the implementation of teaching techniques and therefore educational reform.

A sequential relationship among the teacher's perceptions, classroom practices and students' learning has been exposed in this research (see section 9.3). Thus, high quality teaching (practices) might be the key to facilitate greater learning (Kilpatrick et al., 2001). A sequential relationship among the teacher's perceptions, classroom practices and students' learning has been exposed in this research.

The experiences in Taiwan and the U.S.A. indicate that: 1. curricula guidelines or textbooks alone cannot guarantee changes in classroom practices or influence the way students learn. 2. Teachers' mathematical competencies/knowledge along with their classroom teaching experiences can influence students' learning (Ma, 2010; Kilpatrick et al., 2001). 3. Teachers' pedagogical content knowledge develops the class discussion that cultivates students' thinking within social interaction (Lin, 2002b). 4. Reform must have a sound information base. 5. Some reform-focused research based on classroom practices, can offer information on knowledge development that supports mathematical proficiency, including classroom practice or teachers' work, and so on (Franke et al., 2007). The next stage of educational development should not only focus on improvement of curricula, but should seek experiences learnt from reform studies. It should also

acknowledge new classroom practices that would empower and develop professional teaching development or advise future curriculum developers. This might influence the change in classroom practices that are necessary and the enrichment of the students' mathematical competencies.

10.5.5 Suggestions for further research

Researchers might play an important role by providing guidance in ongoing educational curricular development. The conducting of new research, especially on reformed classroom practices, has the potential to fuel teachers' continued professional development, to sharpen their teaching approaches and change classroom practices to meet curriculum goals, and to offer new knowledge to the public (see section 10.3). The reason for specializing in reform research is that it reveals how classroom practice or teachers' work supports mathematical proficiency (Franke et al., 2007) and benefits teachers' professional development. The (research) data has more meaning than scholars' theoretical debates which indicate the advantage of diverse teaching approaches. For example, the over focus on the curriculum debates as in the "math wars" of the U.S.A. (Boaler, 2002c), or over theoretical debates that lack research evidence, such as in Taiwan mathematics education field (Wey, 2007).

The long-term learning influences from the constructivist class discussion teaching approaches in Taiwan, have been examined carefully in this study and the findings coincide with those of the few long-term constructivist research studies at secondary school levels, including Boaler's study (1997) on the open, project-based methods and Boaler & Staples's study (2008) on group work. These long-term constructivist research projects, on different teaching methods at high school level, showed a higher quality of constructivist approaches when compared to the traditional approaches.

However in my study in Taiwan, students within the constructivist approaches did not perform better on the school tests than students within the direct instruction approaches, as was shown in the abovementioned studies. These differences might have been influenced by the fact that there were unequal conditions of participants in my research (students who participated in the directed instructions

had higher IQ, greater amount of practices in classes, and one extra mathematics lesson weekly), different (or more procedural) mathematics content in Taiwan compared with the western mathematics curriculum, or different constructivist approaches. For example, are constructivist class discussion methods more or less beneficial than open project or group work approaches in a normal junior high school within a long-term period? How can we develop a long-term constructivist classroom to build up students' mathematics thinking abilities, as well as their procedural type of knowledge?

One critical thought is whether or not a good teaching approach can cater to all students' learning. Students' opinions were considered in some constructivist studies. For example, a small number of students complained in the open school of Boaler's research (1997). Some students of both schools in this study expected contrasting teaching practices from their current mathematic classes (n=T4, E2) (see section 8.1.2.1a). Further research could also be carried out on this issue. These questions leave the gates open wide for future research.

10.6 Summary

This research focuses on a single long term (i.e. three years) Taiwanese high school case study of a mathematical teaching and learning experiences based on reformed teaching styles such as class discussions. Although this study draws on Boaler's work, the class discussion approaches used in this study differ from Boaler's (1996) project-oriented approaches and Boaler and Staples' (2008) work and group discussion approaches. Further, while Boaler's work focused on teachers' and students' experiences from Western countries, this study presents findings from a highly developed Asian country, Taiwan.

The study presented and discussed teachers' and students' long term experiences of using reform styles in the Mathematics classroom. Even, this long term case of using mathematical class discussions in high school is still rare in Asian countries. Stemming from this study is the knowledge that teachers and students' opinions are valuable. Therefore, the value of this study is based on identifying mathematical teaching patterns or styles, teachers' experiences, and students' performance and opinion patterns from two classes - a direct teaching approach versus constructivist reform teaching approaches. Based on the data collected, the

constructivist teaching approach of class discussions when compared to the traditional direct instruction, provided an environment that was more conducive towards facilitating quality student learning and teaching. For example, students exposed to the constructivist teaching approach had more learning roles than those in the traditional teaching group. These roles include students as knowledge explorers, producers, and adventurers. This is in direct contrast with students being only knowledge receivers. Class discussions provided more opportunities for students to clearly present and evaluate the thinking of their peers and themselves. This environment which focused on facilitating student thinking and explorations, allowed students to develop the social/collective/adaptive form of mathematical knowledge (Hunter, 2006b; Rojas-Drummond & Zapata, 2004; Wood et al., 2006).

Further, students in the constructivist environment appeared to be more opened, relaxed, lively, friendlier, supportive of each other and willing to share ideas than their counterparts in the traditional group. This type of social interactive learning and collective/adaptive form of mathematical knowledge was missing from the traditional direct instruction environment.

The findings presented in this study are in accordance to other similar research. That is, the constructivist teaching approach led to high-quality instruction, developing understanding (Franke et al., 2007) and higher-order thinking skills (Torff, 2003). Students became empowered with mathematical thinking as through class discussions, they were able to practice and adapt their own thinking or problem solving methods. Class discussions provided a forum for students to engage in activities such as debating, interacting and negotiating with others in the social practice/environment (Greeno, 1991). Such activities may have influenced their understanding/knowledge and ability to apply their learning to other situations (Boaler, 2002b; Lamon, 2007).

The discovery of situated sequential relationships in teachers' perceptions of their teaching practices and students' learning in this study, highlighted the importance of teacher education and professional development. These factors, to some extent, influence teachers' perspectives of mathematics teaching and learning, and their

influences on students learning mathematics (Cross, 2009). As such, researchers need to be cautious when comparing student performances in different pedagogical settings.

Some weaknesses emerged when applying the constructivist teaching approaches in this study. For example, the management of class time consumption, content coverage, understanding all the class discussion, and teachers' heavy work load were perceived as areas of concern. Thus, educators will need to consider ways in which to minimize or remove the occurrence of such challenges. Due to the scope of this study, it would be feasible to conduct similar research focusing on whether a long-term constructivist classroom can build up students' mathematical thinking abilities, as well as their procedural type of knowledge.

When discussing future research, it is also important to consider the mathematics learning environment that should be offered to future students. Should it be one that promotes a student centred approach or one that is traditional with heavy reliance on the teacher for transmitting the knowledge? What learning roles do we wish for future students? Is it to become only knowledge followers or to be explorers, producers and adventurers that results in building up more thinking and creative ability? Students can be equipped through the constructivist approaches as flexible thinkers (Boaler, 2002b) and with competencies to attain future success in life and lifelong learning.

It is my view that Taiwanese educators should pay more attention to introducing this constructivist discussion model as they rethink their educational goals towards providing quality education. There is a need for all stakeholders to better understand the valuable promises of constructivist approaches for enhancing quality education. Constructivist classrooms should not be allowed to disappear; instead they should be encouraged as the Taiwanese government continues to seek alternative solution paths towards developing students' abilities and competencies in mathematics.

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Appendices

Appendix A Views of Learning

Summary views of learning are presented as below. The main features are classified based on knowledge, teaching and learning. Though these theories may differ in nature, effective teachers are usually informed by using a combination of learning theories to apply to all students at all levels.

Views of Learning

	Behaviourism	Constructivism	Situated Learning
Knowledge	A collection of facts and skills (Even & Tirosh, 2008; Neyland, 1991; Young-Loveridge, 1995) and being transmitted (Boaler, 2002a).	Students construct their own knowledge (Boaler, 2002a; Confrey & Kazak, 2006; Even & Tirosh, 2008; Lampert, 2001; Mayers & Britt, 1995; Sfard, 1998; Threlfall, 1996; Wenger, 1998; Windschitl, 1999b) with influences from their prior ideas (von Glasersfeld, 1995; Windschitl, 1999a) and the social and cultural contexts (Windschitl, 1999b).	Socially constructed knowledge (Brown et al., 1996) Built on what participants contribute, construct together (Lave & Wenger, 1991) and is contextually situated and is influenced by the activity, context, and culture (Brown et al., 1989; Mclellan, 1996)
Learning	Passively receiving stimuli or information rather than mentally processing (Fang & Chung, 2005). Occurs through drill, guided practice (Fang & Chung, 2005)	Actively constructed by students (Cobb, 2007; Lesh et al., 2003; Simon & Schifter, 1991; von Glasersfeld, 1990, 1993; Windschitl, 1999b).	Discussion of learning relations among people, activities (Boaler, 2000c, Even & Tirosh, 2008; Lave, 1988; Lave & Wenger, 1991; Peressini et al., 2004) and environments (Boaler, 2000c; Wenger, 1998; Voigt, 1994), practice (Boaler, 2002c; Lave & Wenger, 1991) and culture (Brown et al., 1989). Especially, learning occurs in the participating process (Even & Tirosh, 2008; Lave, 1988; Lave & Wenger, 1991; Peressini et al., 2004). Acquisition and use of knowledge are under the analytical scopes (Greeno, 2003; Peressini et al., 2004), including transferring/generalising knowledge (Boaler, 1996; Greeno, 1997; Peressini et al., 2004).
Teaching	Transmission, lecturing (Threlfall, 1996) Focus on drill and	not specify a particular model of instruction (Greenes, 1995; Windschitl, 1999b). Challenging or/and	Prepare the kinds of social practices for learning to occur (Boaler, 2000a)

	practice and speed and accuracy of answers (Fang & Chung, 2005).	guiding thinking through facilitate discussion (Brooks & Martin, 1999; Windschitl, 1999b) and inquiry (Windschitl, 1999b).	
Criticism	Completely ignoring issues of meaning or social meaning (Skinner, 1974; Wenger, 1998). Limitation in developing higher-order skills (Hagg, 1991; Neyland, 1994).	May leave out the individual's learning of mathematics, with over focus on language and social interaction (Confrey, 1992; Smith, 1999). lacks of understanding students' agreements/consensus with others or the connections of individual concepts with the public ideas (Sfard, 1998.).	If all learning is situational, how could they explain for the inventiveness of people to resolve problems using methods unseen in their cultural traditions? (Smith, 1999)

Appendix B The First interview Questions to Teachers (October, 2002)

1. Could you please generally share with me about your mathematics teaching history in your carrier?
2. What are your views about mathematics? (What will you describe about mathematics? e.g. content knowledge or logical thinking ability;" Mathematics as a field of knowledge is composed of theorems and formulae." (Bishop, 2000) Do you like mathematics?
3. What would your preferred teaching style emphasize: (Please ranks these items, "1" indicates your first choice, "2" indicates the second choice. You can have a same ranking among different items.) (Bishop, 2000)

problem-solving ()

direct instruction ()

testing ()

self-paced learning ()

small-group work ()

team teaching ()

investigations ()

other ()

Why do you rankas the first choice, second choice?

4. What does your actual teaching style emphasize: (Please ranks these items, "1" indicates your first choice, "2" indicates the second choice. You can have a same ranking among different items.) (Bishop, 2000)

problem-solving ()

direct instruction ()

testing ()

self-paced learning ()

small-group work ()

team teaching ()

investigations ()

other ()

Why do you rankas the first choice, second choice?

5. Do you agree that mathematics teaching should emphasize process/understanding over product/result? (Bishop, 2000) Could you please briefly your reasons? How do you apply this thinking into your teaching?
6. Will you encourage alternative solutions and/or justifications, where possible?

(revised from Bishop, 2000) How do you apply this thinking into your teaching?

7. What are your opinions about the mathematical content in secondary schools? What are your opinions about the geometry content in secondary schools?
8. Normally, how will you arrange your teaching plans in your mathematics classes? (What is a typical lesson to you)? (What are your teaching strategies (one main method or multiple ones)? Why do you choose those strategies?) (revised from Stigler, Gonzales, Kawanaka, Knoll & Serrano, 1999)
9. What are your focuses in the lessons or classes in generally?
10. Could you please tell me about are there any special characteristics of the students of your Grade 9 class in your opinions? Because of these, do you intend to make any changes in your normal teaching?
11. Will you link to different areas of mathematics, when you teach one unit? (Do you link different areas of mathematics to give students an overall picture?) Could you please give me some examples?
12. Do you feel satisfaction about your mathematics classes in what points? What advantages or difficulties do they feel in teaching those lessons?
13. How do you think that your major role is it in the classroom? (e.g. authorial or helpful attitudes to help students learn) (e.g. teaching the students mathematical rules, procedures (problem solving methods))
14. Do you think that your mathematics classes are common or different with the classes of the other teachers in what ways?
15. How does the government assessment affect your teaching?
16. How can we improve teachers' teaching in mathematics in your opinions?
17. Have you ever observed other mathematics teachers' classes? Did that influence you?
18. What do you do to change students' un-sociable behaviour?
19. What do you think what way is a good way to improve students' learning (understanding) in mathematics? (e.g. more discussions, at their own pace, work in open ways)
20. What kind of help or freedom that you expect schools or the Ministry of education can give you?
21. Do you do your lesson preparation and marking at your home or school? How do you do?
22. What are your responsibilities of your job (e.g. school duties)? What are your other school duties which besides your teaching related responsibilities (pastoral care for kids)? How often is your school meeting time during a month?
23. How often do the meetings of the mathematics teachers occur in the school within a semester?
24. Is there anything else that you would like to tell me?
25. Where do your views of mathematics come from? (This question was given to teacher Eve in question 2, teacher Ed in question 25 and teacher Tom in

question 14')

Appendix C The Post-interview Questions to Teachers (December, 2002 & January, 2003)

1. Were those three units which have been video-taped typical lessons to you? Have anything that you have been doing in mathematics classes, but have not been showed in those three units video-taping? (Stigler et al., 1999)
2. Could you please tell me that in any aspect of those lessons were not typical lessons to you? (Stigler et al., 1999)
3. When you teach a small class or a big class, do you have different teaching styles?
4. Could you please tell me that is there any change in your attitude from initial perceptions about mathematics or mathematics education, while you join this research project?
5. It is near the end of the research project. Do you have any suggestions or reflections on your mathematics teaching? Do you have any suggestions that you want to give to me about the research or any comments about mathematics education in Taiwan?
6. Is there anything else that you would like to tell me?
7. (This was an extended question only given to Teacher Ed, because he said that when he visited Teacher Eve's classes. He has already used those teaching skills in his classes.) Could you please explain more what did you mean?

Appendix D The Third Interview Questions to Teachers (May, 2005)

1. Could you please share your views or feelings of the current Mathematics educational situation?
2. What better could have been done?
3. Can (alternative) schooling be revived? , in what ways?
4. How do you feel back in the traditional mathematics classrooms?
5. Do you still teach the same way, compared with 2002? What have changed? Why do you make those changes?

Note: The Questionnaires (Appendix E, F & G) are to be administered in Chinese. Consequently, the translation between Chinese and English in these examples is only approximate.

Appendix E Questionnaire in Mathematics (Junior High Level) (I)

The Name of School: _____

Full Name: _____ Student Number in a class: _____

Gender: () Male () Female

Parent/Guardian's occupation: _____

Parent/Guardian's education backgrounds (under junior high level, high school level, a Bachelor degree, a Master or Doctor degree): _____

Class Atmosphere (selected and revised from Yeh, 1993)

Please circle one answer from the below questions from your opinions.

1. My mathematics teacher cares students' learning situation in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
2. My mathematics teacher treats students as friends. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
3. My mathematics teacher encourages students to discuss mathematics problems with each other in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
4. My mathematics teacher often offers opportunities to let students inquire in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
5. My mathematics teacher helps students to do effective learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
6. My mathematics teacher often praises students (e.g. students' improvement) in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
7. My mathematics teacher like every student in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
8. Most of students like my mathematics teacher in my mathematics classroom. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
9. My mathematics teacher cares about students' feeling in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
10. My mathematics teacher offers clear learning goals in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
11. My mathematics classrooms are structured and organized. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
12. We like to share our personal feeling with my mathematics teacher. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)

disagree)

13. Students help each other when learning mathematics or face difficulties in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
14. Students share with each other about their mathematics learning experiences in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
15. I am willing to share mathematics resource with my classmates. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
16. Students care each other about their improvement in mathematics learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
17. Students encourage with each other in mathematics learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
18. My classmates care about my improvement in mathematics learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
19. My classmates care about my personal feelings in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
20. My classmates wish that I can perform well in mathematics learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
21. My classmates will praise me if I perform well in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
22. I will not feel pressure if my classmates study hard. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
23. I can learn a lot of things in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
24. I can learn some important experiences from my classmates in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
25. Students feel satisfied in mathematics learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
26. Students feel interested when taking mathematics classes. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
27. Students feel happy about their own performances in mathematics learning in mathematics classrooms. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)
28. If one student is absent, most of students will care about him/her. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree)

Appendix F Questionnaire in Mathematics (Junior High Level) (II)

The Name of School: _____

Full Name: _____ Class Number : _____

Gender: () Male () Female Parent/Guardian's occupation: _____

Dear students

I will be very grateful for your honest and detailed opinions about mathematics education in Taiwan. Your contribution would be very helpful in providing me valuable information for this research. Thank you for your co-operation.

1.(a) Could you please write down some sentences to describe a typical junior high school mathematics lesson in Grade 7 in your school to someone from another school? (How did your teacher teach? What did students do in classrooms? Did your teacher have other alternative teaching method? (revised from Boaler, 1997)

1.(b) Could you please write down some sentences to describe a typical junior high school mathematics lesson in Grade 7 in your school to someone from another school? (How did your teacher teach? What did students do in classrooms? Did your teacher have other alternative teaching method?) (revised from Boaler, 1997)

1.(c) Could you please write down some sentences to describe a typical junior high school mathematics lesson in Grade 7 in your school to someone from another school? (How did your teacher teach? What did students do in classrooms? Did your teacher have other alternative teaching method?) (revised from Boaler, 1997)

1.(d) Do you notice that are the teaching in your mathematics classroom different or similar from another mathematics classrooms in your school or other schools in what ways?

2. Please tell me how you feel about your mathematics teachers of Grade7, Grade 8, and Grade 9?

3.(a)What advantages and disadvantages in learning mathematics did you face in mathematics classrooms of Grade7, Grade 8, and Grade 9? (Teaching methods or other parts)

(b) In your opinions, how do mathematics lessons need to be designed or be changed, so that they can help you learn well in mathematics? (revised from Boaler, 1997)

(c) In your opinions, how do mathematics lessons need to be designed or be changed, so that mathematics will be more interesting for you? (revised from Boaler, 1996)

4. () Generally, Do you like your mathematics lessons?

(1) I like mathematics lessons very much. (2) I like mathematics lessons most of the time. (3) I generally like mathematics lessons. (4) I sometimes like

mathematics lessons. (5) I have no feelings about mathematics lessons. (6) I don't like mathematics lessons. (7) I don't like mathematics lessons very much. (8) Other opinions (Please describe that) :_____ (revised from Boaler, 1997)

Could you please briefly describe the reasons that how you choose your answer from the above question?

5. Have you faced any difficulties in learning mathematics in junior high school level?

6. Please use the space below to draw a picture about your feeling towards mathematics.

7. What is mathematics in your opinions? Please describe it.

8.(a) How often did you employ your life experiences to solve problems during your mathematics lessons? (almost always, most of the time(pretty often), sometimes, hardly ever or never) (Gonzales, Calsyn, Jocelyn, Mak, Kastberg, Arafeh ...Tsen, 2000)

(b)Please tell me that did you have any opportunity to use mathematics concepts outside of mathematics classrooms? (revised from Boaler, 1997)

9. Did you have any opportunity to do mathematics activities in mathematics classrooms? If you have, please tell me that when (in what grade) did you do that activities? What is that activity?

If you have, do you think you learn different things – doing activities and working from a book? (Boaler, 1997)

10. Are you willing to contact or keep friendship with your mathematics teachers of Grade 7, Grade 8, Grade 9 outside of mathematics classrooms? Could you please tell me the reasons?

11. How would you evaluate your mathematics learning in Junior high level?

12. How do you feel about the content of geometry units?

13. What factors helped your learning in these geometry lessons?

What difficulties do you feel in these lessons?

14. (a) Have you ever attended a cram school for mathematics subject? (yes/no)___

(b) How long did you attend a cram school for mathematics subject? _____

Do you still attend a cram school in Grade 9? _____

15. () Generally, when you finish a mathematics lesson, how much percentage of mathematics content that you can understand? (1) 80% to 100% (2) 60% to 80% (3) 40% to 60% (4) 20% to 40% (5) below 20%.

After a mathematics lesson, did you do any effort to increase your understanding

to mathematics content? () Yes. () No.

What do you do?

Afterwards, how much percentage of mathematics content that you can understand? _____%

16. Generally, how many days will you have a mathematics test? _____ days.

17. Generally, how do your mathematics achievement rank in your class? (1) the first one third. (2) middle (3) that last one third.

Do you think that are there any room to improve your mathematics achievement? (Yes ____, No ____)

If you have any comments about mathematics education or suggestions about this research, you are very welcome to talk to me after classes. Thanks very much for your co-operation.

Appendix G Questionnaire in Mathematics (Junior High Level) (III)

The Name of School: _____

Full Name : _____

Class Number : _____

Gender: ()Male ()Female

1. () Generally, Do you like your mathematics when you study in a primary school?

(1) I like mathematics lessons very much. (2) I like mathematics lessons most of the time. (3) I generally like mathematics lessons. (4) I sometimes like mathematics lessons. (5) I have no feelings about mathematics lessons. (6) I don't like mathematics lessons. (7) I don't like mathematics lessons very much. (8) Other opinions (Please describe that) : _____

Could you please briefly describe the reasons that why you choose your answer from the above question?

2. () Generally, Do you like your mathematics when you study in a junior high school?

(1) I like mathematics lessons very much. (2) I like mathematics lessons most of the time. (3) I generally like mathematics lessons. (4) I sometimes like mathematics lessons. (5) I have no feelings about mathematics lessons. (6) I don't like mathematics lessons. (7) I don't like mathematics lessons very much. (8) Other opinions (Please describe that) : _____

Could you please briefly describe the reasons that why you choose your answer from the above question?

_____ (revised from Boaler, 1997)

3. Could you please share to me the most interested piece of mathematics that you had ever had in classes? (Boaler, 1997)

4. Do you want to keep learning mathematics, when you finish your schooling? (Flockton & Crooks, 1998)

5. In your mathematics classrooms can solve a particular problem using more than one method, or must they use only one method?

6. Is it important in mathematics lessons to use your imaginations? (Boaler, 1997)

7. Please write down five important reasons which can help you to learn mathematics well? Please place the most important factors in the first place then next.

(1)_____ (2)_____ (3)_____ (4)_____
(5)_____

Please write down three things that you do not like in a mathematics classroom? Please place the most disliked factors in the first place then next.

(1)_____ (2)_____

(3)_____

If you are not sure about your answers, you can use the factors below, please place the item number. For example, you may give answer 1(a), or 1(b), but you will not only give an answer 1.

1. I like that when my teacher deliver a lesson, he/she (a) require us strictly in many parts. (b) let us learn freely without any requirement.
2. I like that when my teacher deliver a lesson, he/she (a) deliver a lot of mathematics content. (b) does not need to deliver a lot of mathematics content, but help us understand mathematics concepts clearly. (c) allows classroom discussion. (Teachers lead the whole class to discuss. Classmates sharing inspires the whole class to discuss. Or students discuss in a small group.)
3. I like that when my teacher deliver a lesson in the mathematics content part, he/she (a) repeat the content several times. (b) focuses and explains more in a textbook and a student practice book. (c) gives students a lot of mathematics questions from resource books.
4. In a class, there are more opportunities to let students (1) to do mathematics activities. (2) to let students exercise mathematics problems.
5. I like that (a) the more frequency of tests is better (b) the less frequency of tests is better. (c) there are more mathematics concepts of mathematics problems in a test . (d) there are more mathematics problems from a textbook and a student practice book in a test. (e) there are more mathematics problems from resource books in a test. (f) there are more creative mathematics problems in a test.
6. The classroom atmosphere is quiet.
7. Students can do investigation or research project in mathematics.
8. Teachers give more homework.
9. Students learn from a cram school. (item revised from Wong, 2000)
10. Students' own efforts: (a) Students study by themselves after school (at home or other place). (Students do more mathematics problems by themselves). (item revised from Wong, 2000) (b) Students pay attention in classes. (c) Students revised mistakes.
11. Personal attitudes: (a) I like mathematics. (b) I like my mathematics teacher. (item revised from Wong, 2000)
12. I memories mathematics formulas or the methods of solving mathematics problems.
13. I am able to understand mathematics lessons.

8. Did you mathematics teachers perform this following behaviour in your classes? Please circle the frequency behind every item. (revised from Gonzales et al., 2000)

Teachers were

- (a) showing how to do mathematics problems (in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never). (revised from Gonzales et al., 2000)
- (b) explaining the reasoning behind an idea (in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never).
- (c) asking students to independently study mathematics materials by themselves

(in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never). (revised from Gonzales et al., 2000)

- (d) asking students to work on mathematics projects (in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never). (revised from Gonzales et al., 2000)
- (e) working on open problems with certain solutions (in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never)
- (f) writing equations to represent relations (in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never)
- (g) practicing computational skills (in every lesson, almost always, most of the time(pretty often), sometimes, hardly ever or never)
- (h) do you ever feel anxious about work in mathematics lessons (in every lesson, almost always, most of the time (pretty often), sometimes, hardly ever or never)? (Boaler, 1996)

9. Mathematics Motivation (student internal value)

Please circle one answer from the below questions from your opinions.

- 1. I think that mathematics is a subject that benefits the training of thinking ability. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 2. I think that what I have learned in mathematics classes will benefit my future. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 3. I study hard in order to improve my mathematics ability, instead of pleasing my parents or other persons. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 4. No matter how much time that I will spend, as long as they are mathematics assignments, I am willing to do them. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 5. I consider that mathematics learning is a joyful thing. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 6. I consider that mathematics learning is useless when applied in life. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 7. I wish that I do not have mathematics lessons. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
- 8. (a) If you had a choice, would you choose to take mathematics lessons? Could you please tell me your reasons?
- 8. (b) If you would choose to take mathematics lessons, could you please tell me your reasons?

10. Mathematics Motivation (student motivation of achievement)

Please circle one answer from the below questions from your opinions.

1. When I study mathematics, I will set up a high standard of my mathematics achievement. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
2. I will study hard, even to those mathematics problems which I do not feel interested in. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
3. When I face very difficult mathematics problems, I will do my best or try to find some ways around to solve them. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
4. I always do my mathematics homework first, then do the homework in other subjects. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
5. I am one of those students who like to spend time to study mathematics, compared with my classmates in my class. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
6. If I do not feel satisfy about my mathematics homework, I will stay up late in a night to improve it. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
7. I often find some mathematics resource books to do more mathematics practice. (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree) (Selected from Chang, 1995)
8. My mathematics teacher often praise a student's/students' good behaviour or performance (totally agree, agree in some ways, no comment, disagree in some ways, totally disagree).

11. Have you discussed or done (begun) your homework outside of classes (Gonzales et al., 2000)?_____ Have teachers demanded you to do homework? What will teachers do? _____

12. Have your families help your homework? (yes/no)_____

Or, what kind of support will they offer to you? _____

Appendix H The short interview questions to students

There are no fixed questions to interview students. The interviews are short follow-up probes. The aim is to clarify students' deep thoughts, or to understand unclear or contrasting points from the responses of students to the questionnaire or student behaviour in classes. The short interviews will be conducted in school between class times. The researcher would through the focuses of the research (e.g. the nature of mathematics, the teaching style, students' achievement and thinking ability) decide whether there is a need to conduct follow-up interviews or not.

For example,

1. When a student has different views about some points (e.g. the nature of mathematics, the preference of the teaching styles, the factors benefit their learning) from most of students in his/her class. I will approach to him/her to ask more about his/her thinking. Can he/her talk more about his/her ideas about mathematics? What made him/her think about mathematics in that ways?
2. If students do not show clearly in questionnaires their thoughts (e.g. the nature of mathematics), then I will interview them further about their ideas in order to understand the student's perceptions about mathematics.
3. If a student ticked having a low percentage of understanding in his/her mathematics lessons, but considered his/her mathematics achievement as in the first one third among his/her classmates. I will interview them further about their ideas in order to understand this contrast.

Appendix I Students' results of the first questionnaire (classroom atmospheres)

The number of questions	School T Average students included, (25 one absent)	School E Average (23 students included)	Heading School (differences)
Q1	4.08	4.78	E(0.7)
Q2	3.56	4.74	E(1.18)
Q3	4.12	4.83	E(0.71)
Q4	4.36	4.74	E(0.38)
Q5	4.13(24 students answered)	3.83	T(0.30)
Q6	3.88	3.96	E(0.07)
Q7	3.44	4.09	E(0.65)
Q8	3.76	4.04	E(0.28)
Q9	3.52	3.87	E(0.35)
Q10	4.04	4.22	E(0.18)
Q11	4.04	3.96	T(0.08)
Q12	3.12	3.61	E(0.49)
Q13	4.52	4.52	E(0.002)
Q14	4.24	4.09	T(0.15)
Q15	4.6	4.35	T(0.25)
Q16	3.76	3.65	T(0.11)
Q17	4.04	4.17	E(0.13)
Q18	3.72	3.57	T(0.15)
Q19	3.52	3.48	T(0.04)
Q20	3.68	3.57	T(0.11)
Q21	3.56	3.78	E(0.22)
Q22	3.52	3.65	E(0.13)
Q23	4.24	4.17	T(0.07)
Q24	4.32	4.17	T(0.15)
Q25	3.64	3.43	T(0.21)
Q26	3.56	3.65	E(0.09)
Q27	3.5(24 students answered)	3.39	T(0.11)
Q28	3.65(17students answered)	3.43	T(0.21)

(Each question had five items as totally agree, agree in some ways, no comment, disagree in some ways, totally disagree and giving 5, 4 , 3 , 2 , and 1 points according from students' answers.)

Appendix J Consent Letter for Principals

Dear principal (Mister/Misses/Miss):

I am a school teacher in a Junior High School, and am also a graduate student in the School of Education, University of Waikato in New Zealand. I will be carrying out research for my PhD study. In particular, it is hoped that this research will inform those efforts that are being made to implement reforms in mathematics education. The research I have embarked on will look in-depth into the characteristics of mathematics classrooms in Taiwan. Taiwanese students have performed excellently in international comparative studies, but the factors that exist in the teaching methods in Taiwan that result in high mathematics performances are puzzling. The intention of the research is to produce productive explanations and evaluations in order to identify possible factors, which may contribute to a better learning environment in mathematics education.

I would like to invite you to give permission for your school to participate in my research project. The project will involve a mathematics teacher, his/her Grade 9 mathematics class and some classrooms of other teachers. The mathematics teacher has showed high interest in participating the research project. Two or three classes of other teachers will be used for carrying out students' mathematics testing in this semester. Two possible quizzes less than 20 minutes could be taken in the part of other teachers' classrooms in this semester. Teachers, students and their parents will be invited to participate in this research. All participation is voluntary.

Details of what the research project entails:

I will choose three geometry units during the period of September 2002 to January 2003. Data collection will involve classroom observations, videotaping, sound recording, interviews given to teachers and students, questionnaires, quizzes and tests given to students, and students' results of the Intelligence Quotient test and on the National Entrance examinations. (The students' results on the national examination in mathematics will be collected in May or June, 2003.)

One video camera and one audio tape-recorder will be placed in the mathematics classroom. I will consult with the mathematics teacher to ensure that the placement of video cameras and my presence will have minimal impact on classroom teaching and learning. These cameras will be used to record the teacher's teaching and students' interactions. These video recordings will later be analyzed to find out more about the mathematics classroom behaviours.

Initially, the mathematics teacher will be interviewed for approximately 45 minutes about his/her views of mathematics and mathematics teaching. If possible, his/her brief comments after the classes will be sought. At this time, the focus will be on the teaching plans and his/her thoughts about the delivery of instruction or any suggestions about the research. At the end of a sequence of lessons, another twenty minutes interview will be conducted to find out any changes from his/her initial perceptions.

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Short interviews may be done with students. These interviews will be to clarify students' deep thoughts, or to understand unclear or contrasting points from the responses of students to the questionnaire or student behaviour in classes. The interviews will be conducted in school between class times to avoid intrusion upon students' valuable time. Three separate attitude questionnaires will be given to students to complete after the school examinations at the teachers' convenience.

Two or three 45 minutes tests from the other participating mathematics teacher will be given to your students in another teacher's classroom. The chances of giving these tests are dependent on the other teachers' convenience and there being no disturbance of students' learning in the other subjects. If the mathematics teacher allows, five small quizzes related to life applied mathematics problems from the mathematics textbook or practice book will be given to students before the teacher solves those mathematics problems in his/her class. These are about six minutes or less and will be carried out in the mathematics class. If possible, in addition two short quizzes related to geometry units of Grade 7 or Grade 8 of the mathematics content will also be given to the students out of mathematics class time. The chances of giving the latter quizzes are dependent on the other teachers' convenience. Students should take less than twenty minutes for each of these quizzes.

For the study, I will also access student information about the participating students' results for the Intelligence Quotient test and the National Examination results in mathematics in 2003. I will need to obtain these from your school's student affair office.

The teacher and students have the right to access or withdraw their data at any time. You have right to complain to me if anything is disturbing you or you are uncomfortable because of this research. You may also contact my chief supervisor at University of Waikato, Dr. Garth Ritchie, gritchie@waikato.ac.nz. Participants' concerns will be respected and individual wishes will be respected.

Students who decline to participate will not be disadvantaged. If students do not wish to be videotaped, it will be arranged for them to take seats where they will not be shot on video. It is important that they do not receive less teacher attention, and they should not be disadvantaged because they have not agreed to participate.

The findings of this research will be published in a PhD thesis and possibly in research journals. If you wish to receive more information after the thesis is completed, an executive summary of this study will be posted to you.

Can I ask for your schools' participation in this research? Your contribution would be very important in providing me with the valuable information I need for this research. The depth of analysis made possible by this study may help to challenge policy makers, teaching practice, and implement reforms in mathematics education.

You can be assured that all the information provided from the schools, teachers and students will be kept private and confidential. You and your students' anonymity will be maintained by use of pseudonyms when reporting results. Schools will also be referred to by pseudonyms. The researcher and her

supervisors will have access to the data. The participants own their data. The data will not be shared with other people. Data will be securely stored in locked cupboards.

Thank you for your co-operation.

Yours sincerely,

Hsiao-Li Chi (Ms)

REPLY SLIP

I have read the letter describing the research project by Hsiao-Li Chi, and have understood the research procedures. I understood that:

All the information provided from the schools, teachers and students will be kept private and confidential and anonymity. Any recorded of information of participants or reporting finding of the study will utilize pseudonym or code numbers.

My school, teachers and students have the right to withdraw from this research at any time. If participants, withdraw any material collected from them will not be analysed or reported on.

Regarding any question of my involvement for this research, I may contact Hsiao-Li Chi or her chief supervisor at University of Waikato, Dr. Garth Ritchie, gritchie@waikato.ac.nz.

Hsiao-Li Chi

I consent to participate the research project.

Principal's signature: _____

Date: 2002 _____

Appendix K Consent Letter for Teachers

Dear teacher (Mister/Miss):

I am a school teacher in a Junior High School, and am also a graduate student in the School of Education, University of Waikato in New Zealand. I will be carrying out research for my PhD study. In particular, it is hoped that this research will inform those efforts that are being made to implement reforms in mathematics education. The research I have embarked on will look in-depth into the characteristics of mathematics classrooms in Taiwan. Taiwanese students have performed excellently in international comparative studies, but the factors that exist in the teaching methods in Taiwan that result in high mathematics performances are puzzling. The intention of the research is to produce productive explanations and evaluations in order to identify possible factors, which may contribute to a better learning environment in mathematics education.

I am very grateful for your high interest in my research project. You are invited to participate in this study along with your Grade 9 mathematics class. Some classrooms of other teachers will be involved for carrying out students' mathematics testing. Teachers, students and their parents will be invited to participate in this research. All participation is voluntary.

Details of what the research project entails:

If you agree to participate, the research will involve you in the following procedure. I will choose three geometry units during the period of September 2002 to January 2003. Data collection will involve classroom observations, videotaping, sound recording, interviews given to you and students, questionnaires, quizzes and tests given to students, and students' results of the Intelligence Quotient test and on the National Entrance examinations. (The students' results on the national examination in mathematics will be collected in May or June, 2003.)

One video camera and one audio tape-recorder will be placed in the mathematics classroom. I will consult with you to ensure that the placement of video cameras and my presence will have minimal impact on classroom teaching and learning. These cameras will be used to record your teaching and students' interactions. These video recordings will later be analyzed to find out more about the mathematics classroom behaviours.

Initially, you will be interviewed for approximately 45 minutes about your views of mathematics and mathematics teaching. If possible, your brief comments after the classes will be sought. At this time, the focus will be on the teaching plans and your thoughts about the delivery of instruction or any suggestions about the research. At the end of a sequence of lessons, another twenty minutes interview will be conducted to find out any changes from your initial perceptions.

Short interviews may be done with students. These interviews will be to clarify students' deep thoughts, or to understand unclear or contrasting points from the responses of students to the questionnaire or student behaviour in classes. The interviews will be conducted in school between class times to avoid intrusion

upon students' valuable time. Three separate attitude questionnaires will be given to students to complete after the school examinations at the teachers' convenience.

Two or three 45 minutes tests from the other participating mathematics teacher will be given to your students in another teacher's classroom. The chances of giving these tests are dependent on the other teachers' convenience and there being no disturbance of students' learning in the other subjects. If you allow, five small quizzes related to life applied mathematics problems from the mathematics textbook or practice book will be given to students before you solve those mathematics problems in your class. These are about six minutes or less and will be carried out in the mathematics class. If possible, in addition two short quizzes related to geometry units of Grade 7 or Grade 8 of the mathematics content will also be given to the students out of mathematics class time. The chances of giving the latter quizzes are dependent on the other teachers' convenience. Students should take less than twenty minutes for each of these quizzes.

You and students have the right to access or withdraw your data at any time. You have right to complain to me if anything is disturbing you or you are uncomfortable because of this research. You may also contact my chief supervisor at University of Waikato, Dr. Garth Ritchie, gritchie@waikato.ac.nz. Participants' concerns will be respected and individual wishes will be respected.

Students who decline to participate will not be disadvantaged. If students do not wish to be videotaped, could it please be arranged for them to take seats where they will not be shot on video. It is important that they do not receive less teacher attention, and they should not be disadvantaged because they have not agreed to participate.

The findings of this research will be published in a PhD thesis and possibly in research journals. If you wish to receive more information after the thesis is completed, an executive summary of this study will be posted to you.

Can I ask for your participation in this research? Your contribution would be very important in providing me with the valuable information I need for this research. The depth of analysis made possible by this study may help to challenge policy makers, teaching practice, and implement reforms in mathematics education.

You can be assured that all the information provided from the schools, teachers and students will be kept private and confidential. You and your students' anonymity will be maintained by use of pseudonyms when reporting results. Schools will also be referred to by pseudonyms. The researcher and her supervisors will have access to the data. The participants own their data. The data will not be shared with other people. Data will be securely stored in locked cupboards.

Thank you for your co-operation.

Yours sincerely,

Hsiao-Li Chi

REPLY SLIP

I have read the letter describing the research project by Hsiao-Li Chi, and have understood the research procedures. I understood that:

All the information provided from the schools, teachers and students will be kept private and confidential and anonymity. Any recorded of information of participants or reporting finding of the study will utilize pseudonym or code numbers.

My school, teachers and students have the right to withdraw from this research at any time. If participants, withdraw any material collected from them will not be analysed or reported on.

Regarding any question of my involvement for this research, I may contact Hsiao-Li Chi or her chief supervisor at University of Waikato, Dr. Garth Ritchie, gritchie@waikato.ac.nz.

Hsiao-Li Chi

I consent to participate the research project.

Teacher's signature: _____

Data: 2002 _____

Appendix L Consent Letter for Parents and Students

Dear parents (Mister/Misses) and students:

I am a school teacher in a Junior High School, and am also a graduate student in the School of Education, University of Waikato in New Zealand. I will be carrying out research for my PhD study. In particular, it is hoped that this research will inform those efforts that are being made to implement reforms in mathematics education. The research I have embarked on will look in-depth into the characteristics of mathematics classrooms in Taiwan. Taiwanese students have performed excellently in international comparative studies, but the factors that exist in the teaching methods in Taiwan that result in high mathematics performances are puzzling. The intention of the research is to produce productive explanations and evaluations in order to identify possible factors, which may contribute to a better learning environment in mathematics education.

You are invited to participate in this study along with the rest of your class. You and your parents will be invited to give consent for your involvement in this research project. Your participation is voluntary.

Details of what the research project entails:

I will choose three geometry units during the period of September 2002 to January 2003. Data collection will involve classroom observations, videotaping, sound recording, interviews given to teachers and students, questionnaires, quizzes and tests given to students, and students' results of the Intelligence Quotient test and on the National Entrance examinations. (Your results on the national examination in mathematics will be collected in May or June, 2003.)

One video camera and one audio tape-recorder will be placed in the mathematics classroom. I will consult with your mathematics teacher to ensure that the placement of video cameras and my presence will have minimal impact on classroom teaching and learning. These cameras will be used to record your teaching and students' interactions. These video recordings will later be analysed to find out more about the mathematics classroom behaviours.

I will also invite you to take part in the short interviews. These interviews will be to clarify your deep thoughts, or to understand unclear, or your mathematics learning in classes, or contrasting points from your responses to the questionnaire. The Interviews will be conducted in school between class times to avoid intrusion upon your valuable time. If you are not available, your wishes will be respected. Three separate attitude questionnaires will be given to you to complete after the school examinations at the teachers' convenience.

Small tests related to mathematics content freely given to you. The scores of tests will not be concerned by the school. Two or three 45 minutes tests from the other participating mathematics teacher will be given to you along with the rest of your class in the another teacher's classroom. The chances of giving these tests are dependent on the other teachers' convenience and there being no disturbance of students' learning in the other subjects. If your mathematics teacher allows, five

small quizzes from the mathematics textbook or practice book will give to you to practice before your teacher solves those mathematics problems in your class. These are about six minutes or less and will be carried out in the mathematics class. If possible, in addition two short quizzes related to geometry units of the mathematics content will also be given to you out of mathematics class time. The chances of giving the latter quizzes are dependent on the other teachers' convenience. You should take less than twenty minutes for each of these quizzes.

You have the right to access or withdraw your data at any time. You have right to complain to me if anything is disturbing you or you are uncomfortable because of this research. You may also contact my chief supervisor at University of Waikato, Dr. Garth Ritchie, gritchie@waikato.ac.nz. Your concerns will be respected and individual wishes will be respected.

Students who decline to participate will not be disadvantaged. If you do not wish to be videotaped, your teacher will arrange you to take seats where you will not be shot on video. It is important that you do not receive less teacher attention, and you should not be disadvantaged because you have not agreed to participate.

The findings of this research will be published in a PhD thesis and possibly in research journals. If you wish to receive more information after the thesis is completed, an executive summary of this study will be posted to you.

Can I ask for your participation in this research? Your contribution would be very important in providing me with the valuable information I need for this research. The depth of analysis made possible by this study may help to challenge policy makers, teaching practice, and implement reforms in mathematics education.

You can be assured that all the information provided from the schools, teachers and students will be kept private and confidential. Your anonymity will be maintained by use of pseudonyms when reporting results. Schools will also be referred to by pseudonyms. The researcher and her supervisors will have access to the data. The participants own their data. The data will not be shared with other people. Data will be securely stored in locked cupboards.

Thank you for your co-operation.

Yours sincerely,

Hsiao-Li Chi (Ms)

REPLY SLIP

I have read the letter describing the research project by Hsiao-Li Chi, and have understood the research procedures. I understood that:

All the information provided from the schools, teachers and students will be kept private and confidential and anonymity. Any recorded of information of participants or reporting finding of the study will utilize pseudonym or code numbers.

My school, teachers and students have the right to withdraw from this research at any time. If participants, withdraw any material collected from them will not be analysed or reported on.

Regarding any question of my involvement for this research, I may contact Hsiao-Li Chi or her chief supervisor at University of Waikato, Dr. Garth Ritchie,gritchie@waikato.ac.nz.

Hsiao-Li Chi

I consent to participate the research project.

Student's signature: _____

I consent to _____ (the name of the student) participating in this study.

Parent /Guardian signature: _____

Data: 2002 _____

Appendix M Ethical Considerations

For ethical considerations I followed the guidelines of the University of Waikato Human Ethics Committee (Student Guidelines, 1997; Human Research Ethic Regulations, 2000).

All participation was voluntary. There was no pressure to be involved this research.

The interviews with students happened in schools or through telephoning. The interviews with teachers happened in schools, or other public places of their choosing.

1. Confidentiality

The researcher is aware of the need to keep all data provided by participants confidential. Participant anonymity is maintained by using of pseudonyms. Schools were also be referred to pseudonyms (because of the nature of the alternative school, it is likely that some readers will be able to identify which school has been involved in the research.)

Any data/data analysis provided to a participant for comments, related only to that participant. Data information was not be shared with people, except the participants who own the data and supervisors until in draft/final form. Participants had opportunities to respond to transcripts and draft documents or alter their transcripts. Data is securely stored in locked cupboards in my home from completion of the project in ten years period of time, then data will be destroyed.

If the researcher needs to enlist an assistant to transcribe, the assistant must be a person of integrity and maintain confidentiality.

2. Potential harm to participants

I avoided any stressful situations and responses in the interview situation and acted accordingly, including termination of the interview if necessary.

Schools and parents might worry that students' mathematics learning or learning effect could be disturbed by a video camera and a researcher into classrooms.

Teachers may change their normal performance because of videotaping or an observer involved in a classroom. These changes may affect students' learning. For example, teachers may speed up or slow down their normal speed to deliver a lesson, because they noticed the video camera or the researcher. These changes may influence students' learning quality or result students less understanding in some parts of lessons.

Students may have less effective learning, because they may feel disturbed or lose their attention by videotaping or an observer appeared in a classroom. Students may feel kind of pressure to try to perform well in front of video camera. Students may notice that teachers try to perform well in front of video camera and a researcher, so students feel stress to be expected to perform well as well. That might bring feel kind of learning pressure to them.

Every effort was made to help teachers and students feel comfortable during the videoing. I checked after each lesson to see how teachers and/or students found

the experience and their suggestions for improvements where possible, these suggestions would be adopted. If they feel stressful, I would take steps to deal with those issues.

Students might need to spend extra time to answer questionnaire and tests, or be interviewed. This time consuming might affect their feeling to this research, especially when they were under stress to face the coming national examination in May, 2003. However, the students in junior high school in Taiwan were used to be tested, especially in Grade 9 tests could be seen as part of school daily routine. So, these extra tests or questionnaire should not surprise them. The researcher reminded students the benefits of tests and questionnaire. Extra tests could offer extra opportunities for students to practice that might benefit students to improve their problem solving ability. Questionnaire could let their perspectives about mathematics be understood by other.

Or some research questions might offend their feeling to this research. Some mathematics tests might cause them some negative feeling towards the mathematics subjects. So, the researcher carefully designed questionnaire and tests to avoid bringing any negative feeling from those tests to students.

A participant who declined was not disadvantaged. She still remained in the same seat in the class and to received equal teacher attention same as other students. Class activity carried on as usual, and the video camera videotaped class events except her.

3. Participants' right to decline

Participants were voluntary. There was no pressure placed on teachers or students to participate. They had right to refuse to answer any particular questions in interviewing, tests or questionnaire. They had right to decline this research at any time and any material collected from them would not be analysed or reported on.

4. Arrangements for participants to receive information

Principals, teachers, students or parents could approach to the researcher to receive information in informal meetings (e.g. meeting in schools or calling the researcher). Teachers or students could also meet the researcher in interview and asked more details when they needed. If the participants wished to receive more information after the thesis is completed, an executive summary of this study would be posted/emailed to them.

5. Use of the information

This research would not be used for school monitoring (e.g. by the principal, government inspection agencies). This study will be published as a PhD thesis. Some parts of this research could be presented in conferences or in journals or other publication.

6. Conflicts of interest

When conducting this research project, I was still working in a school. My teaching program was arranged. So, my students are not disadvantaged by my involvement in the research.

At the beginning of classroom observation period, my role was explained to avoid that the students maybe pay more attentions for me and tried to act more actually, and treated me as a teacher in a classroom more than an observer.

Be aware of that the other mathematics teachers in the school might have negative feelings toward the participant teacher's role, when needed I would explain the

reasons to them in each school.

Any other ethical concerns relevant to the research

The finding of this research will not be used in such a way to advertise the advantages of one school over another.

7. Legal Issues

7.1. Copyright

The participants have copy right from the data provided by them, such as data from interview, questionnaire, and tests. The researcher has copyright of the data analysis, the thesis, and any papers which eventuate from it.

7.2. Ownership of data or materials produced

Participants own the original data which provided by them. The researcher has the ownership of the analysis of this research, the thesis, and any papers which eventuate from it.

Appendix N My coding system for my raw data

Code	Meaning
T	the male teacher in the traditional school was given a name as Tom
Om	the male teacher in the experimental school was given a name as Ed.
Of	the female teacher in the experimental school was given a name as Eve.
St	Student
T1 to T 26	There were 26 students in Tom's class, participating in this study and coding as T1 to T 26.
E1 to E 23	There were 12 students in Eve's class, E1 to E12. There were 11 students in Eve's class, coding as E13 to E23.
1I	the first interview with teachers before videotaping classes in 2002
2I	the second interview with teachers after one unit videotaping classes in 2002 or early 2003.
3I	the third interview with teachers in May, 2005
pr	hand writing when interview teachers
Q1	the first questionnaire given to students (e.g.: OQ2 : the second Questionnaire; OQ3 : the third Questionnaire.)
h	handwriting transcribe
vh	handwriting transcribe or the field notes from the video taping
p	page
Q	Question (e.g. OQ2Q(1) : the first question in the second questionnaire) If the information also indicated from the handwriting transcribe, then the number of question may not be shown in brackets (e.g. Of1Ihp2beQ3 : the feedback from the third question which was located in the bottom part of the second page handwriting transcribe from Eve's first interview)
v or vt.	video-taping
vt.af.	An interview given after video-taping
all	comparison of three teachers
Sy	summary
the location coding	t : the top part. b or m : the middle part. e : the bottom part. r : the right hand side. l : the left hand side
the coding of time	e.g. 1118: 11: month, 18:date Nov29(5): Nov: month, 29: date, (5):the fifth lesson

Example : (Sy Tvt.p)

Sy: Summary. T: Tom. vt: video tape. P: page number.

Appendix O The coding system for IQ and mathematics tests

Code	the content
IQ PR.	the percentages of the results of Intelligent Quality tests compared with students in the same age and same gender
Na all100	the percentile of students' scores from all subjects, compared with all students who set the National examination
Na Math	students' scores from the mathematics subject in the National examination, the full marks were 60 points.
Test 1	this first test (2-2 unit) (given by Tom: the male teacher in the traditional school)
Test 2	the second test (3-1 unit) (given by Ed: the male teacher in the experimental school)
Test 3	the third test (3-2 unit) (given by Eve: the female teacher in the experimental school) (p.s. Eve stated that in that time students did not have good mood to set this test.)
Ave 3tests	students' average scores in the first, second and third tests
Q 2 to 5	the second quiz to the fifth quiz
Q 6 to 7	the sixth quiz to the seventh quiz
Q 2 to 7	the second quiz to the seventh quiz
NQ 2 to 5	new questions from the second quiz to the fifth quiz
NQ 6 to 7	new questions from the sixth quiz to the seventh quiz
NQ 2 to 7	new questions from the second quiz to the seventh quiz

P.S. The full mark is 100 points in all tests, except Na Math (the National examination) .

Appendix P The Ratios of Student Parents' Socio-economic Statues

Students' parents' careers can be divided into two categories as middle and working class (Boaler, 1996). The middle class includes professional or skilled but not manual jobs. The working class includes skilled, unskilled and manual jobs. The data are summarized as below.

Table 4.6 The Ratios of Student Parents' Socio-economic Statues

	School T the number of students(%)		School E the number of students(%)	
Middle class	10	38%	18	78%
Working class	6	23%	4	18%
House work	3	12%	1	4%
Not known	7	27%	0	0%

(SyQ1hp1,2)

Appendix Q Student Parents' Educational Background

	School T the number of students(%)		School E the number of students(%)	
Graduate School or above	3	12%	11	48%
University	4	15%	7	30%
Senior High or College	12	46%	2	9%
Junior High School	6	23%	2	9%
Not known	1	4%	1	4%

(SyQ1hp1,2)

Appendix R Students' results of the third questionnaire in question 9 (mathematics motivation: student internal value)

The number of questions	School T Average (26 students)	School E Average (23 students included)	Differences (School E Average-School T Average)
Q9(1)	4.60 (one absent)	4.48	-0.12
Q9(2)	3.62	2.96	-0.66
Q9(3)	3.89	3.78	-0.10
Q9(4)	3.31	2.96	-0.35
Q9(5)	3.46	3.39	-0.07
Q9(6)	3.76 (one absent)	3.35	-0.41
Q9(7)	3.58	3.41(one absent)	-0.17
Average	3.74	3.47	

(Each question had five items as totally agree, agree in some ways, no comment, disagree in some ways, totally disagree and giving 5, 4 , 3 , 2 , and 1 points according from students' answers, except Q9(6) and Q9(7). These two exceptions did not use positive ways to state those question, thus points were given in a reverse way, for example, totally agree, agree in some ways, no comment, disagree in some ways, totally disagree and giving 1,2,3,4 , and 5 points)

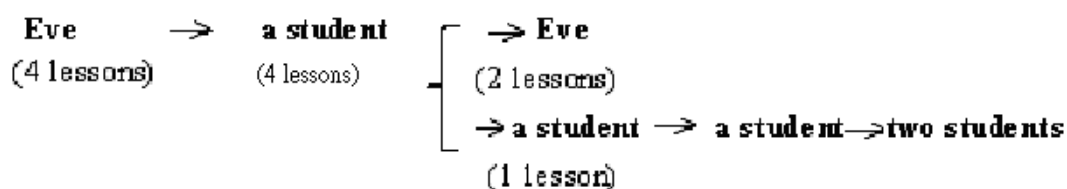
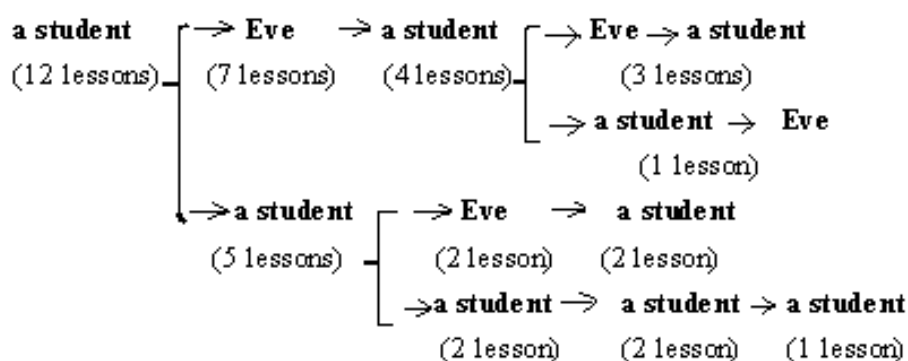
Appendix S The time of students sharing or leading discussions in front of the class in a lesson

Date (2002)	Minima explaining Time(minutes)
Oct 30 (1)	12
Oct 30 (2)	26
Nov 4 (3)	15
Nov 4 (4)	Writing 10, explaining 27
Nov 6 (1)	18
Nov 6 (2)	11
Nov 11(4)	27
Nov 13 (2)	29
Nov 18 (3)	Writing 22, explaining 16
Nov 18 (4)	19
Nov 20 (1)	26
Nov 20 (2)	34
Dec 2 (3)	40
Dec 2 (4)	13
Dec 4 (1)	7
Dec 4 (2)	37
sum	389
Average	$389 \div 16 = 24.3$

(Sy.Of.vt.p2')

P.S. (1): the first lesson of a day, (2): the second lesson of a day, (3):the third lesson and (4):the fourth lesson. (Sy.Of.vt.p2')

Appendix T The pattern of a person standing in front of the class leading discussions in Eve's classes



(Sy.Of.vt.p3)

(1) at least four students continuously came to the front to share in three lessons (Dec4(2), Nov11(4), Nov20(2), Sy.Of.vt.p3) and

(2) at least two students continuously came to the front, then Eve came to share in three other lessons (Dec2(4), Dec4 (1), Nov18(3), Sy.Of.vt.p3).

Appendix U Ten Minutes of a Classroom Discussion Analyses

Grade 9

Duration: 8:40–8:50am

Date: Oct 30, 2002

Mathematical Content: Geometry – the beginning of unit 2-2.

Background: Student E5 leading classroom discussion

I used abbreviated codes with s5 representing Student E5, and tr representing Eve.

The order of persons who contributed to discussions were s5, s9, s5, s11, s5, tr, s5, tr, s5, s2, s5, tr, s11, s5, s4, s5, s11, s5, tr, s5, tr, s4, s5, tr, s5, s4, s5, s4, s5, s8, s4, s5, s4, s5, s4, s5, s4, s11, tr, s5, and tr.

Appendix V Ten Minutes of a Classroom Discussion Analyses

Grade 9

Duration: 9:31–9:41am

Date: Oct 30, 2002

Mathematical Content: Geometry –Unit 2-2.

The order of persons who contributed to discussions were tr, s5, tr, s8, tr, s11, s9, tr; s11, s5; tr, s11, tr, s11, tr; s8, s5; tr, some students, tr, s8, tr, one student, tr, s1, tr, s8, tr, s8 and tr.

Appendix W The frequencies of Ed's teaching strategies appears in 9 lessons

Teaching strategies		The number of lessons
students' seat work (including group discussion)& students' class discussions (38.1 minutes for average one lesson of these 7 lessons)		7
Ed's brief and direct explanations (average 8 minutes for one lesson of these 4 of these above 7 lessons), one other lesson for mostly direct instruction in the whole class		5
class discussion methods	● Ed poses questions ● students' class discussions (assigns students to present their ideas on the blackboard) ● Ed moves around and questioned /taught students individually	4
	● Ed assigns students to present their ideas on the blackboard ● Ed moves around the class checking students' work.	3
Ed asks students for homework		3
Ed asks students to explain mathematical ideas to the whole class (class discussion methods) Ed explains and gives hints (at least 2 lessons) Ed encourages students Ed gives tests		2

(Sy.Omvtp4e &4e')

P.S. Here cannot calculate the average time of the class discussion method, because class discussion and seat work were overlapped for some period of time and hard to count the average time.

Students' seat work and the class discussion methods are more often used than the teacher's direct explanations.

Appendix X Students' results of the third questionnaire in question 10 (mathematics motivation: student internal value)

The number of questions	School T Average (26 students)	School E Average (23 students included)	Differences (School E Average-School T Average)
Q10(1)	3.39	3.30	-0.08
Q10(2)	3.31	3.13	-0.18
Q10(3)	3.50	3.26	-0.24
Q10(4)	2.81	2.57	-0.24
Q10(5)	2.58	2.26	-0.32
Q10(6)	2.92	2.78	-0.14
Q10(7)	3.15	3.30	0.15
Q10(8)	3.15	3.48	0.32
Average	3.10	3.01	

(Each question had five items as totally agree, agree in some ways, no comment, disagree in some ways, totally disagree and giving 5, 4 , 3 , 2 , and 1 points according from students' answers.)

Appendix Y The samples of Quizzes

The mathematics problems of each quiz show as below. English translation has put under each quiz.

The first quizzes did not give to students in this research, so did not present it here.

(The Second Quizzes)

校名：____國民中學____

____年__班__號

姓名：_____

日期：____年__月__日

School name: _____

Class: _____

Your name: _____

Date: ____month____day____year

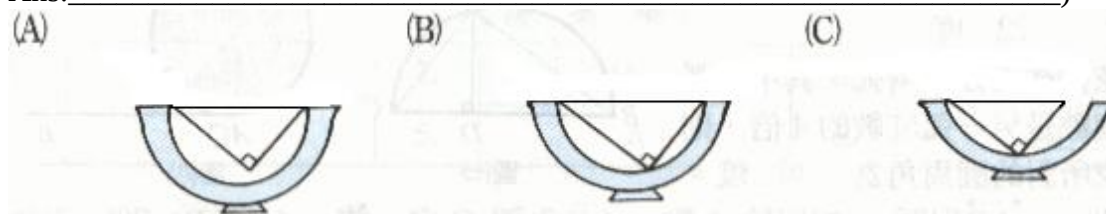
答題時間: 5 分鐘 (It is allowed 5 minutes to answer this question.)

1. 有人將三角板放於下列的三個碗中，三個碗的內壁曲線都是圓弧，三角板碰觸碗的內壁的情況如下。可否請你說明一下，關於 B 碗，內壁曲線的數學性質。(Tien, 2002, 圖片取自 p. 37)

答: _____

- (1. Someone put a set-square into three bowls and touch inside of bowls as below. The curves inside three bowls are a part of a circle. Could you please explain the mathematical meaning of the curve inside the bowl in chart (B) (Tien, 2002, and the three figures from Tien, 2002, p. 37)?

Ans. _____)



你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過

自己答題時間約_____分鐘

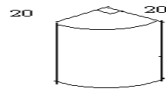
(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no

How many minutes do you answer this question? _____)

2. 阿強家中客廳角落有個扇形置物架，半徑為 20 公分，圓心角為 90 度。

他用防滑塑膠板，剪出一個扇形當止滑墊，此扇形剛好覆蓋此置物架的表面，請算出此止滑墊的面積與周長？(請寫出計算過程) (Tien, 2002)

- (2. John has a shelf in a sector shape which is placed in a corner of a living room. The radius is 20 cm, and the central angle is 90 degrees. He cut a piece of plastic as the same shape of the sector and put on top of the shelf to avoid things drop from the shelf. Could you please tell me the area of this piece of plastic and the circumference of the sector? Please write down your mathematical procedures (Tien, 2002)



你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過

自己答題時間約_____分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no

How many minutes do you answer this question? _____)

(The Third Quizzes)

校名：____國民中學____

____年____班____號

姓名：_____

性別（男/女）：_____

日期：____年____月____日

School name: _____

Class: _____

Your name: _____

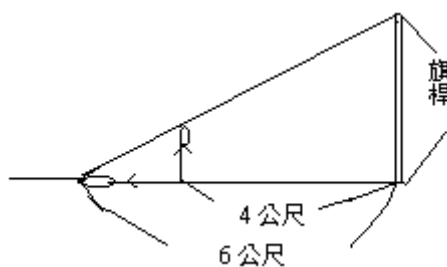
Date: ____month____day____year

Gender(male/female): _____

答題時間: 4 分鐘(It is allowed 4 minutes to answer this question.)

1. 小華要測量學校升旗桿的高度，發現陽光下旗桿的影長 6 公尺，當時人離旗桿底 4 公尺，本人影長的前端，剛好和桿影的端點疊合(如下圖)，已知小華的身高 160 公分，則旗桿高多少公尺？(請寫出計算過程)

(1. George wanted to measure the high of a flag rod of his school. He found that if under the sun, the length of the shape of a flag rod is 6 meters. When he walked 4 meters away from the rod, the top of his shadow were just overlap with the top of the rod(shown as the diagram as below). George are 160 cm tall, could you please tell me that how many meters are the flag rod? Please write down your mathematical procedures.)



你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過

自己答題時間約_____分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no

How many minutes do you answer this question? _____)

(The Forth Quizzes)

校名：_____國民中學_____

_____年_____班_____號

姓名：_____

日期：_____年_____月_____日

School name: _____

Class: _____

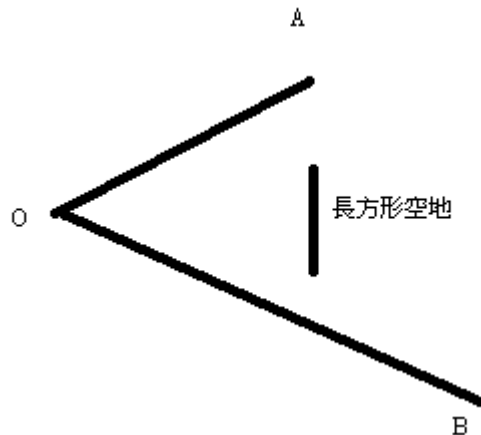
Your name: _____

Date: _____month_____day_____year

答題時間: 6 分鐘 (It is allowed 6 minutes to answer this question.)

1. 一年一度的長距離慢跑比賽即將開始，有 $\overline{OA}$ 方向和 $\overline{OB}$ 方向的兩條路線，大會工作人員想在圖中的長方形空地上，設立裁判休息處。此裁判休息處需與 $\overline{OA}$ 和 $\overline{OB}$ 道路的距離相等。(1) 請用直尺和圓規，畫出此裁判休息處的位置？(2) 並說明為什麼？(revised from Tien, 2002)

(1. Annual long distance jogging race is going to start. There were two jogging routes: $\overline{OA}$ and $\overline{OB}$. The workers of the committee want to build up a resting place for judges and the place is located in the rectangle area of the diagram as below, but the rest place need to remain a same distance with the two jogging routes. (1) Please use a compasses and ruler to draw a suitable point on this rectangle to build up a resting place for the judges in the diagram below. (2) Please give reasons and explain why your drawing is right (revised from Tien, 2002).



你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過
自己答題時間約_____分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no
How many minutes do you answer this question? _____)

2. 在平直的海岸線上，想要建立一座燈塔，並使此燈塔的位置距離到海中 A，B 兩小島的距離要相等。(1) 請用直尺和圓規，畫出此燈塔的位置？(2) 並說明為什麼？(revised from Tien)

(2. If want to build up a lighthouse on the straight beach line, and make the lighthouse keep a same distance with two islands: island A and B in the ocean as the diagram as below. (1) Please use a compasses and ruler to draw a suitable point on the straight beach line to build up a lighthouse in the diagram as below. (2) Please give reasons and explain why your drawing is right (revised from Tien, 2002).



你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過。答題時間約__分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no

How many minutes do you answer this question? _____)

(The Fifth Quizzes)

____年____班 ____ 號 姓名：____ 日期：____年____月____日

Class:____ Class number:____ Your name: _____

Date: ____month____day____year

答題時間: 4 分鐘 (It is allowed 4 minutes to answer this question.)

1. 有 A, B, C 三個村莊，位置如下圖。想要建立一所車站，使此車站到三個村莊距離相等。(1) 請用直尺和圓規，畫出此車站的位置？(2) 並說明為什麼？

(There were three towns, Town A, Town B and Town C. Their locations were showed as the diagram as below. (1) If you want to locate a train station and the station would reach three towns with equal distance, please use a ruler and compasses to draw the location of a train station (Tien, c 2002). (2) Could you please explain your reasons, why make your draw reasonable to meet the requirement of this question?)

A

、

B

、

、 C

你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過

自己答題時間約____分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no

How many minutes do you answer this question? _____)

____年____班 ____ 號 姓名：____ 日期：____年____月____日

Class:____ Class number:____ Your name: _____

Date: ____month____day____year

答題時間: 4 分鐘 (It is allowed 4 minutes to answer this question.)

2. 若 A, B, C 三個村莊各相距 10 公里，想要建立一所車站，使此車站到三個村莊距離相等，則此距離為 _____ 公里。(請寫出計算過程) (revised from Tien, 2002)

(There were three towns: town A, town B, and town C, and all keep 10 miles distance with each other. If want to build up a station and make the location of the station keep a same distance with the three towns. Please find out what is the distance and write down your mathematical procedures (revised from Tien, 2002)

你曾看過此題，或此題的類似題嗎？ ☐ 看過 ☐ 沒看過

自己答題時間約_____分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes ☐ no

How many minutes do you answer this question? _____)

幾何動動腦(一) (The Sixth Quizzes)

____年____班 ____ 號

姓名：_____

日期：____年____月____日

答題時間: 20 分鐘。(可使用圓規、直尺)

Class:_____ Class number:_____ Your name: _____

Date: ____month____day____year

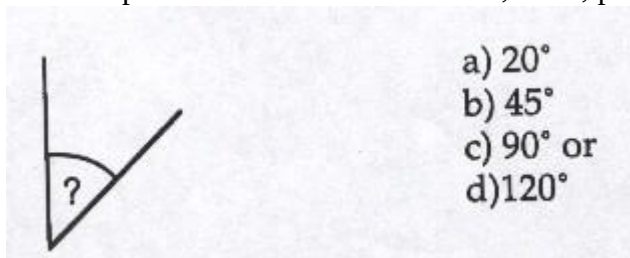
(Time: 20 minutes. You are able to use a ruler or compass)

1. () 請寫出下圖角的度數為? (不可使用量角器) (並請簡略敘述理由。)

(Boaler, 1996, 圖片及選項取自 p. 394)

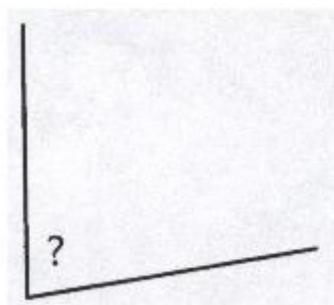
(1. () If a protractor was not allowed, could you tell the angle shown below is?)

(Please simplify explain your reason.) (revised from Boaler, 1996, but the figure and four possible choices from Boaler, 1996, p. 394)



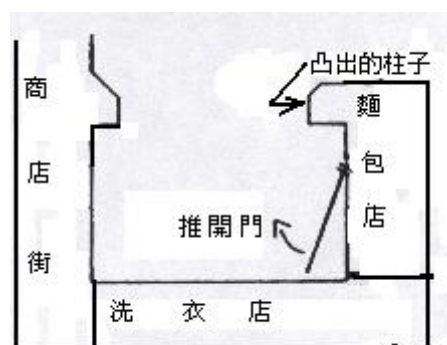
2. () 請問下圖角的度數，大於或小於 60 度? (不可使用量角器)，並請簡略敘述理由。(Boaler, 1996, 圖片取自 p. 400)

(2. () If a protractor was not allowed, is this angle below more or less than 60° ? Please simplify explain your reason.) (revised from Boaler, 1996, but the figure from Boaler, 1996, p. 400)



3. 街道上的門，依規定向外開啟時，需至少能打開 105 度，才算合乎規定。請問，如下圖的麵包店 (門前的右側有凸出柱子)，此麵包店的門，是否合乎規定？並請寫出理由？(GAIM, 1988, cited in Boaler, 1996, 圖片取自 p. 398)

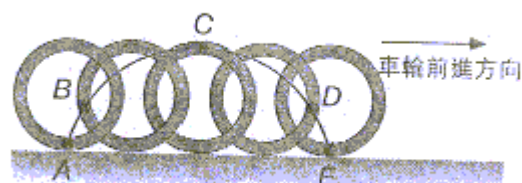
(3. Streets doors need to open to an angle of at least 105° . Will the door of the bakery pass this regulation? Please write down your explanations.) (revised from GAIM, 1988, cited in Boaler, 1996, and the figure from GAIM, 1988, cited in Boaler, 1996, p. 398)



4 小華腳踏車的車輪沾上牛糞，他觀察到，在地面上第一個有牛糞痕跡的位置，到下一個有牛糞痕跡的位置，距離是 150 公分。請問車輪的直徑是多少公分？(請用分數表示) (Lo, 1997, 圖片取自 p. 96)

(4. The tire of Well's bicycle touched a piece of shit. He found that there were 150 cm between the first sign of shit on the ground to the next one. Please find out the length of diameter of the tire? Please show the answer in a fraction form. (revised from Lo, 1997 and the figure from Lo, 1997, p. 96)

牛糞的軌跡圖：



你曾看過上面這些題目，或這些題目的類似題嗎？ ☐ 看過，看過第_____題

☐ 沒看過 自己答題時間約_____分鐘

(Have you ever seen this problem or a similar problem before? ☐ yes, Which question have you ever seen? _____)

☐ no

How many minutes do you answer this question? _____)

幾何動動腦(二) (The Seventh Quizzes)

____年____班 ____ 號

姓名：_____

日期：____年____月____日

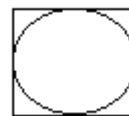
Class:_____ Class number:_____ Your name: _____

Date:____year ____month____day

答題時間: 20 分鐘 (It is allowed 20 minutes to answer these questions below.)

1. 一個正方形內，有一個內切圓，則正方形面積：內切圓面積 = _____：

(1. There is an inscribed circle inside a square. Please find out the ratio of the two areas. The areas of a square: the areas of an inscribed circle = _____：_____)



2. 一個內接正方形位於一個圓形中，則 面積比為? 圓形：內接正方形 = _____：

(2. There is a square inside a circle. Please find out the ratio of the two areas. The areas of a circle: the areas of the inside square = _____：_____)



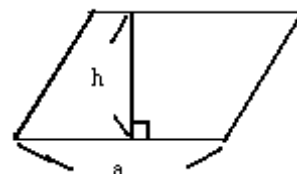
3. 一潛水夫要勘查海底世界的情況，已知此處海域，海底平坦，海底皆距離海平面 30 公尺。為避免潛水夫走失，潛水夫身上綁著 50 公尺的繩子，連接於船上。此時風平浪靜，船不移動。

- (1) 請問當潛水夫自船上跳入海中，垂直碰到海底以後，向東方直走，最遠可在海底走多少公尺？(Ritchie, 2002)
(2) 請問此潛水夫在海底，最大可走動的面積範圍，為多少平方公尺？

(3. A diver wants to investigate the situation under the sea. It is known that the bottom under the sea is flat. There are 30 meters between the sea level and the bottom under the sea. The diver is tied up a 50 meters ropes connecting with the ship, avoiding to lose the diver. At this moment, this ship is still above the sea and the wind is not blowing. (1) When the diver jumps into the sea and vertically touches the bottom under the sea, then he go ahead to the east. How far can he walk (Garth, 2002)? (2) What is the biggest area that he can walk on the bottom of the sea?)

4. 小學學過長方形面積 公式為邊長乘上邊長，可否請你解釋為何平行四邊形面積 = 底邊長 $\times$ 高? 如下圖示例，平行四邊形面積 = ah 請寫出你的證明方法。(Thomas, 1993)

(4. Most people know that the areas of a rectangle = the length of one side $\times$ the length of the other side. Could you please explain the areas of a parallelogram = the length of the bottom side $\times$ the height = ah . It is shown as the diagram as below. Please write down your method to prove this mathematical rule (Thomas, 1993)



你曾看過上面這些題目，或這些題目的類似題嗎? ☐ 看過，看過第 _____ 題

☐ 沒看過

自己答題時間約 _____ 分鐘

(Have you ever seen this problem or a similar problem before?

☐ yes, Which question have you ever seen? _____

☐ no

How many minutes do you answer this question? _____)

Appendix A1 Assessment criteria for quizzes

Student performances were assessed by the researcher and divided with five ranges as below. Therefore, the average scores of schools in each quiz were presented to show the excellence of students' achievement.

Scores (points)	criteria
100	students made complete explanations and gave a correct answer
75	students made partly correct explanations or just visual reasons, and gave a correct answer
50	students made correct explanations, or a correct mathematical equation, or gave a correct answer
25	students made partly correct explanations
0	students failed to make correct explanations and failed to give a correct answer

Appendix B1 Students' average scores in each quiz of both schools

[Note] C: conceptual question, C-P: conceptual-procedural question, *: problems related with everyday life

Q2-1: the first question in the second quizzes

NQ5-2: the second question in the fifth quizzes is a new question for students

I	Quiz	Question	School	number of students answered	Average	Leading
	Q2-1	C	T	24	54	T
			E	23	52	
	Q2-2	*C-P	T	24	71	T
			E	22	67	
	Q3-1	*C-P	T	25	56	E
			E	21	67	
	Q4-1	*C	T	26	43	E
			E	23	55	
	Q4-2	*C	T	26	54	E
			E	23	55	
	Q5-1	*C	T	26	86	T
			E	21	38	
	Q5-2	*C-P	T	26	64	T
			E	23	42	
II	Quiz	Question	School	number of students answered	Average	Leading
	Q6-1	*C	T	26	90	T
			E	22	75	
	Q6-2	*C	T	26	87	E
			E	22	90	
	Q6-3	*C	T	26	67	E
			E	22	74	
	Q6-4	*C-P	T	26	60	T
			E	22	57	
	Q7-1	C-P	T	26	62	T
			E	20	45	
	Q7-2	C-P	T	26	46	T
			E	20	18	
	Q7-3	*C-P	T	26	69	T
			E	20	65	
	Q7-4	C	T	26	58	E
			E	20	74	
III	Quiz	Question	School	number of students answered	Average	Leading
	NQ2-1	C	T	13	46	E
			E	19	50	
	NQ2-2	C-P	T	8	53	T
			E	7	36	
	NQ3-1	C-P	T	6	50	E
			E	6	67	

	NQ4-1	C	T	18	21	E
			E	16	44	
	NQ4-2	C	T	17	51	E
			E	16	53	
	NQ5-1	C	T	10	75	T
			E	8	50	
	NQ5-2	C-P	T	12	58	T
			E	8	28	
IV	Quiz	Question	School	number of students answered	Average	Leading
	NQ6-1	C	T	25	92	T
			E	20	76	
	NQ6-2	C	T	25	86	E
			E	20	89	
	NQ6-3	C	T	25	67	E
			E	21	77	
	NQ6-4	C-P	T	22	52	E
			E	21	55	
	NQ7-1	C-P	T	23	57	T
			E	18	50	
	NQ7-2	C-P	T	21	38	T
			E	19	18	
	NQ7-3	C-P	T	25	68	E
			E	19	68.4	
	NQ7-4	C	T	26	58	E
			E	16	67	

Students in School E performed better than students in School T in the areas that they were able to use their mathematical knowledge in new situations. The data supported from (1) their long term memory of mathematic knowledge (School E leading 5 questions vs. 3 questions of School T) (NQ 6 to 7), and also (2) their current learning content (School E leading 4 questions vs. 3 questions of School T) (NQ 2 to 5).

Appendix C1 Students' performances in new situations of their long term learning content (NQ 6 to 7)

	Leading Questions	The number of leading questions
School T	NQ6-1, NQ7-1, NQ7-2,	3
School E	NQ6-2, NQ6-3, NQ6-4, NQ7-3, NQ7-4	5

Students' performances in new situations of their current learning content (NQ 2 to 5)

	Leading Questions	The number of leading questions
School T	NQ2-2, NQ5-1, NQ5-2,	3
School E	NQ2-1, NQ3-1, NQ4-1, NQ4-2	4

Appendix D1 Students' performances in conceptual questions and conceptual-procedural questions of quizzes

Leading Questions	conceptual (n= 8)		conceptual-procedural (n= 7)	
	School T	School E	School T	School E
Q 2 to 5	2	2	2	1
Q 6 to 7	1	3	4	0

Leading Questions: the number of questions that students performed better than the other School.

School E students performed better than School T students in 5 of 8 conceptual questions.

School T students performed better than School E students in 6 of 7 conceptual-procedural questions,

Appendix E1 Time interval counts in teaching activities (sequence) (every 30 seconds)

Teaching activities		Frequencies		
		Eve(43.5m) (2002/10/30)	Ed(43.5m) (2002/12/11)	Tom(43.5m) (2002/11/18,19)
tr talk to	whole	68	45	84
	groups	0	0	0
	individual	15	48	0
st talk to	whole	35	5	0
	groups	0	0	0
	individual	12	0	0
st talk to	teacher	14	5	0
st teach	a st publicly	5	1	0
Small group	discussion	8	7	0
	chat	3	0	2
questions asked	teacher (short)	21	9	12
	teacher (long)	18	36	30
	student	14	9	5
	students	2	1	3
questions answered	teacher	5	10	8
	student	29	15	3
	students	12	4	19
ask understanding	teacher to a st	2	2	0
	tr to students	3	2	4
	st to a student	8	0	0
	st to students	4	0	0
tr's	encouragement	2	1	0
tr ask sts	to share front	1	40	0
St automatics	to share front	2	0	0
Tr walk	In front	39	52	83
	around	23	29	1
seat	work	0	46	2
	sts took notes	10	5	31
a test		0	0	0
material	textbook	0	0	29
	tr's worksheet	75	87	58
	publications	0	0	0

Tr: teacher, st: student. sts: students. tr's encouragement (include praises, gifts).

The shadow: the leading class of the three teachers.

P.S. The information of this above table was from three teachers' first lesson of the 43.5 minutes, when I entered into their classrooms to do videotaping. Tom's and Eve's first class started late, so I need to also take some time from their second class to make up enough for 43.5 minutes.

Appendix F1 Structures of three tests

The number of questions		1 st test (2-2 unit)		2 nd test (3-1unit)		3 rd test (3-2unit)	
Multiple-choice questions		9		8		23	
Blank questions		0		6		0	
Calculate questions		2		2		0	
Given teacher		Tom		Eve		Ed	
Question analyses of a test							
Procedural questions		1(Q4)		1(Q2)		2(Q1 and Q8)	
Conceptual questions	Pure	10	0	15	10	21	1
	-procedural		10		5		20

NB: Q: the multiple-choice question, e.g. Q4: the forth multiple-choice question.

Three tests have totally had 35conceptual-procedural questions, 4 procedural questions, 11 conceptual questions.

Appendix G1 Students' average scores in tests, the national examination and quizzes

Average	IQ PR.	Test 1	Test 2	Test 3	Average of 3 tests	National math exam	All subject PR (National Exam)	Q 2 to 7	NQ 2 to 7
School T	58.40 (n=25)	86.3	69.2	81.1	78.6	40.0 (n=26)	70.9 (n=26)	64.6	60.1
School E	53.67 (n=18)	65.1	60.4	67.5	64.2	37.4 (n=17)	63.1 (n=17)	58.5	57.9
Differences (T-E)	4.73	21.12	8.84	13.63	14.45	2.6	7.76	6.1	2.2

P.S. (1) (T-E): Student average score in the traditional school minus student average score in the experimental school.

(2)

See the Appendix O for the coding system.

(3) The full marks of the national mathematics examination are 60 points.

Appendix H1 Explaining the detail account of categories in the time interval count table

Teaching	activities	explanations
tr talk to	whole	A teacher talked a whole class, group or individual in front of the class.
	groups	
	individual	
st talk to	whole	A student talked a whole class, group or individual in front of the class.
	groups	
	individual	
st talk to	tr	A student explained his/her thinking or gave suggestions but not questions to the teacher publicly in the class when the teacher lecturing.
st teach	a st publicly	A student explained his/her mathematical thinking to the other student publicly in the class during the teacher or a student's lecturing or class discussion.
questions asked	tr (short)	A teacher asked an easy question and students could answer easily, e.g. what is the degree of a right angle? Or direct answers e.g. yes, no, numbers can be easy to be figured out.
	tr (long)	A teacher asked a complicated question that demanded more of thinking, but could not answer by direct response e.g. how to solve this problem?
	st	One student asked a question.
	sts	More than one student asked a question.
questions answered	tr	The teacher answered a question.
	st	A student answered a question.
	sts	More than one student answered a question.
ask understanding	tr to a st	The teacher asked a question to make sure students understood, e.g. "do you understand?", or "are you clear about this point" etc.
	tr to sts	The same questions as above, but toward a few of students.
	st to a st	A student leading a class discussion asked a question a student to make sure students understood during a class discussion, e.g. "do you understand?"
	st to sts	A student leading a class discussion asked a question to a few of students to make sure them understood during a class discussion, e.g. "do you understand?"
Small group	discussion	A few of students discussed mathematics together.
	chat	A few of students chatted together.
tr's	encouragement	The teacher encouraged students.

tr ask sts	to share front	The teacher asked one or several students to share their ideas in the front.
St automatics	to share front	A student automatically went to the front to share his/her thought to the class.
Tr walk	In front	The teacher's position stayed in the front.
	around	The teacher walked around during the class time.
seat	work	Students were doing seat work.
	sts took notes	Students took notes from the teacher's teaching.
a test		The teacher gave a test to the class.
material	textbook	The mathematics content of the class was from the textbook.
	tr's worksheet	The content was from teachers' worksheet.
	publications	The content was from some resource books.