Embedded numeracy teaching practices at an Institute of Technology:

Vocational tutors' characterisations of embedding numeracy and related professional development

A thesis

submitted in partial fulfilment

of the requirements for the degree

of

Master of Education

at

The University of Waikato

by

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THE UNIVERSITY OF
WAIKATO
Te Whare Wānanga o Waikato

2019

Abstract

As part of a multi-pronged strategy to improve levels of adult numeracy, embedded numeracy provision is a required element of foundation-level vocational education in New Zealand. Embedded numeracy, as it is interpreted in New Zealand, means learners develop numeracy skills within vocational courses and programmes rather than as a separate programme of study. This approach means that vocational educators, rather than numeracy specialist educators, are largely responsible for teaching numeracy. While it is acknowledged that the crux of embedding is good teaching practice, little is known about the embedded numeracy teaching practices of these vocational educators.

The purpose of this study was two-fold. Firstly, I set out to explore how vocational tutors characterise embedding numeracy as part of their teaching practice with respect to evidence-based knowledge of effective practice. Secondly, I wanted to consider how professional learning and development has impacted their practice, and what further professional development approaches might prove useful.

To that end, I undertook a case study research approach using qualitative analysis of data collected through semi-structured interviews. I conducted interviews with six foundation-level vocational tutors employed in the Institutes of Technology and Polytechnics (ITP) sector. Additionally, participants provided examples of teaching and learning resources, and I consulted official programme documentation for each sample case.

Approaches to embedding numeracy can be situated on a continuum from numeracy-saturated, practice-based embedding to opportunistic embedding. In all cases, tutors use authentic contexts that are meaningful to the learners, a key element of an embedded approach. Learners' anxieties towards mathematics are acknowledged and assuaged as tutors model numeracy practices in the vocational role that learners aspire to rather than as a teacher of a subject that

many learners loathe and fear. Collectively, tutors' characterisations of their practice align with evidence-based effective practices, but mathematical expertise and confidence varies between tutors and so, therefore, does the depth and quality of the mathematical tuition. Tutors gauge learners' numeracy development through their engagement in context-related numeracy practices; this development may not be captured by numeracy proficiency measures. Benefits of an embedded approach to numeracy development are compromised by a lack of resourced collaboration time between vocational tutors and numeracy specialists.

Given that good teaching is at the heart of embedded numeracy, further research that includes observations in teaching spaces and the voices of tutors and students is warranted. Tutors desire more collaborative opportunities to learn from each other and from numeracy experts. Professional learning going forward should include opportunities to create professional knowledge and skills through interaction with information and other people.

Acknowledgments

I feel incredibly privileged to have been supervised and mentored through this project by Professor Diana Coben. Thank you, Diana, for your wise and patient counsel, and for expertly guiding me to make my own discoveries in a topic where you already know so much.

Thank you to my teams at work for your encouragement and capable self-management while your manager was busy writing.

This project couldn't have happened without the honest and frank narratives provided by the participants. Thank you for your time and generous sharing.

Thank you, Martin, for your support through so many weekends of reading and writing.

Thank you, Coffee Girls, for the much-needed study breaks.

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List of Abbreviations

Abbreviation	*Explanation*
ALN	adult literacy and numeracy
ELN	embedded literacy and numeracy
FE	Further Education (UK tertiary setting)
ILM	Improving Learning in Mathematics (UK project)
ITP	Institutes of Technology and Polytechnics
LLN	literacy, language and numeracy
LN	literacy and numeracy
LNAAT	Literacy and Numeracy for Adults Assessment Tool
NCALNE	National Certificate in Adult Literacy and Numeracy Education (Vocational)
NZCALNE	New Zealand Certificate in Adult Literacy and Numeracy Education (Vocational)
NQF	National Qualifications Framework (NZ)
PD	professional development
PLD	professional learning and development
TEC	Tertiary Education Commission
TTM	Thinking Through Mathematics (UK project)

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Chapter 1: Introduction

New Zealand tertiary vocational education providers are required to embed literacy and numeracy education into their foundation-level programmes. An embedded approach means that literacy and numeracy is taught within the context of vocational training rather than in separate, generic literacy or numeracy courses. While the majority of foundation-level provision in the Institute of Technology and Polytechnics (ITP) sector is now considered to include embedded literacy and numeracy (ELN) practices (Tertiary Education Commission [TEC], 2015b, 2016), the measure and evaluation of those practices has not included examining vocational tutors' classroom practice. The measure of embeddedness for funding purposes has mainly relied on compliant use of the Literacy and Numeracy for Adults Assessment Tool (LNAAT). A recent TEC (2015a) investigation of the Assessment Tool's contribution to educational outcomes concludes that use of the Tool makes tutors and learners more aware of learners' literacy and numeracy skills, but concedes that good teaching practice is more important than the Tool.

The frameworks that have been developed for self-evaluating ELN practices and organisational benchmarks make only light mention of teaching and learning practice or activity, and no guidance on evaluating these (National Centre of Literacy and Numeracy for Adults, 2014b; Tertiary Education Commission, 2013). A recently released Adult Literacy and Numeracy Effective Practice Model (Ako Aotearoa, 2018a) appears to be a step towards a framework for evaluating teaching practices.

Professional development and attainment of literacy and numeracy (LN) qualifications for teaching staff are recognised indicators of ELN, and it is acknowledged that the crux of embedding is good teaching practice (Tertiary Education Commission, 2015a). However, little research has examined what embedded numeracy, particularly, looks like in everyday practice for vocational

tutors who have completed required LN training, and none has explored vocational tutors' perceptions of embedding numeracy in their teaching or their professional learning experience and needs. This study aims to begin to fill this gap.

Purpose of this research

The purpose of this study is to explore vocational tutors' perceptions of teaching numeracy within their vocational courses and programmes in an ITP, how they believe previous professional development and learning related to teaching numeracy impacts their practice and what further professional learning they believe would improve their practice.

Specifically, my research questions are:

- How do selected tutors' characterisations of their embedded numeracy teaching practice at an ITP align with evidence-based knowledge of effective practice?
- How do they consider any related professional development and learning has impacted on their practice with respect to their learners' numeracy development?
- What, if any, further professional development or support do they regard as useful to improve their practice in developing learners' numeracy in their teaching context?

Background

Embedding LN skills development in foundation-level vocational education has, since the early 2000's, been part of the New Zealand government's multi-pronged strategy to ensure that all New Zealanders have basic LN skills essential to participate fully in the modern world (Ministry of Education [MoE] & Ministry of Business, Innovation and Employment [MBIE], 2014). Adult literacy and numeracy development has been a priority in successive Tertiary Education Strategies in response to findings from the 2006 international Adult Literacy and Life Skills Survey (ALL) showing that a significant percentage of adult New

Zealanders had low literacy skills and a higher percentage had low numeracy skills (TEC, 2012, 2015b). The current Literacy and Numeracy Implementation Strategy (TEC, 2015b) was published before the release of results from the 2014 ALL survey follow-up, the international Survey of Adult Skills, part of the Programme for the International Assessment of Adult Competencies (PIAAC). These results showed no increase in numeracy skills between 2006 and 2014 for the New Zealand adult population, and while average numeracy scores for Māori and Pasifika increased, these groups continue to score on average below the total population (MoE & MBIE, 2016). These results are concerning, especially given that literacy skills gains were made for all groups over the same period.

ELN is based on the premise that adult learners' expertise will develop if tutors recognise learners' current knowledge and skills within their vocational context, develop and build on these, and transfer the learning to more and different contexts (Tertiary Education Commission, 2008a, p. 4). As will be shown in the Literature Review and Discussion chapters that follow, teaching numeracy in a conceptual, transferable manner is complex and demands high levels of numeracy from the teacher. While some New Zealand research has evaluated embedding at an organisational level and there has been limited research on the teaching practices of literacy specialists (Anderson, 2017; Benseman, Lander, & Sutton, 2005; Leach, Zepke, Haworth, & Isaacs, 2010; McDonald, Alkema, & Benseman, 2014), the teaching practices of vocational tutors tasked with embedding numeracy in New Zealand have remained largely unexamined. More still needs to be known about embedded LN practices at classroom and organisational level (Alkema & Rean, 2013, p. 41). My study is a start at exploring practices at the classroom level.

Personal interest

My interest in this research springs from my work over the last 15 years as a numeracy developer and teacher educator, roles in which I have worked with hundreds of vocational tutors coming to understand the demands, responsibilities, and opportunities related to embedded numeracy. In that work

my colleagues and I endeavour to model active teaching and learning that develops learners' conceptual mathematics understanding rather than rote memorisation of processes. While undertaking ELN professional development and training, tutors develop, implement and evaluate a small number of numeracy teaching resources relevant to their practice, including formative assessment and teaching and learning activities. The professional development outcome is introductory-level awareness of mathematics teaching practices that encourage conceptual understanding rather than rote memorisation and application of formulae and procedures. While many tutors appreciate learning about kinaesthetic and visual approaches to developing number sense, their own conceptual mathematics understanding is often tenuous. I have long held concerns that many vocational tutors do not have sufficient depth of understanding to effectively develop their learners' number sense.

As I will detail in my literature review, effective embedded numeracy involves collaboration between vocational educators and numeracy specialists. However, in New Zealand a collaborative model has proven difficult to resource and many vocational tutors in the ITP sector are solely responsible for embedding numeracy. Thus, I am interested in examining how tutors with minimal pedagogical training characterise their approaches to teaching numeracy in their vocational teaching contexts.

Overview of structure

This study firstly examines literature related to the elusive definition of adult numeracy, the notion of embedded numeracy teaching, effective mathematics and numeracy teaching practices, and professional learning and development (PLD) approaches. My research is a qualitative analysis of narratives gathered through semi-structured interviews from a small sample of vocational educators identified as embedded numeracy practitioners. Details of the methodology and design of this research is provided in Chapter 3 of this thesis, and the findings from my analysis are detailed in Chapter 4. In Chapter 5, I discuss my findings in

relation to current evidence-based knowledge of effective practice and PLD, and propose possible further research and PLD approaches.

Chapter 2: Literature Review

This chapter reviews international and New Zealand literature pertaining to adult numeracy teaching practices with particular reference to the meanings of and approaches to embedded numeracy education. I begin with reviewing current definitions of adult numeracy and numerate behaviour which evolved over time as numeracy became recognised as an autonomous subject of study, while still linked to both literacy and mathematics. I then look at the understandings, theories and principles associated with *embedded* numeracy, as New Zealand has adopted that approach almost exclusively in tertiary vocational education in its attempt to increase adult numeracy levels. This is followed by a review of the evidence-based knowledge of what constitutes effective adult numeracy and mathematics teaching practice. Lastly, I examine PLD related to embedding numeracy available in New Zealand, and local and international perspectives on desirable PLD.

Definitions of numeracy

Recognition of adult numeracy as a separate field of study and a skill-set in its own right gained momentum in the 1990s and early 2000s. The construct has always been closely associated with literacies and language development, and, historically, considered and measured as a subset of literacy, known variously as quantitative literacy, mathematical literacy, criticalmathematical [*sic*] literacy, statistical literacy, and financial literacy to name a few (Coben, 2003; Tout & Gal, 2015). Research and development of numeracy education is often combined, and indeed subsumed (Byrne & Maguire, 2013), by research around literacy and language education resulting in various acronyms related to numeracy education. The acronyms representing literacy and numeracy (LN), literacy, language and numeracy (LLN), literacy, numeracy and language (LNL), embedded literacy and numeracy (ELN), and adult literacy and numeracy (ALN) are used throughout this paper to correspond with acronyms used in the research consulted.

Developing effective teaching practices for numeracy and identifying appropriate provision types requires some shared understanding of the concept of adult numeracy (Byrne & Maguire, 2013). There is not one agreed definition of numeracy, but an understanding and consensus on the meaning of the term and its relationship to mathematics has been under consideration and evolving over the last two decades. It is generally agreed that it involves being able to choose and use appropriate mathematical skills in real-life contexts and that the required mathematical skills go beyond being able to add, subtract, multiply and divide with numbers. It is also understood that a numerate person will have the confidence and a disposition to use the appropriate mathematics when required (Berghella & Molenaar, 2013; Coben, 2003, 2006b; Coben et al., 2007; FitzSimons, 2009; Tout & Gal, 2015; Tout & Schmitt, 2002).

The New Zealand government uses the following definitions of numeracy and numerate behaviours in the Adult Numeracy Learning Progressions resources:

> To be numerate is to have the ability and inclination to use mathematics effectively in our lives – at home, at work, and in the community. (Ministry of Education, 2001, as cited in Tertiary Education Commission, 2008a, p. 34)

> To be numerate means to be competent, confident and comfortable with one's judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate and what the answer means in relation to the context. (Coben, 2003, as cited in Tertiary Education Commission, 2008a, p. 34)

> We believe that numeracy is about making meaning in mathematics and being critical about maths. This view of numeracy is very different from numeracy just being about numbers and it is a big step from numeracy of everyday maths that meant doing some functional maths. It is about using mathematics in all its guises – space and shape, measurement, data and statistics, algebra and of course, number – to make sense of the real

> world and using maths critically and being critical of maths itself. It acknowledges that numeracy is a social activity.
>
> (Tout, 1997, as cited in Tertiary Education Commission, 2008a, p. 34)

Adult numeracy is context-specific and context-dependent and relies on knowledge developed by individuals over their lifetimes. The problems to be solved are situated and set within constraints of the context, and for any particular individual or individuals a relatively original approach is needed. General mathematical principles can and should be drawn on, and the difficulty in teaching adult numeracy is in teaching generalities in ways that may be useful in specific contexts very different from the classroom situation (FitzSimons, 2009).

Common to all of these definitions is the everyday aspect of numeracy and the required confidence and understanding to apply mathematics to everyday situations. Rather than as a subject to be taught, numeracy development requires facilitating recognition of situations where mathematical and spatial reasoning is necessary. That is, helping learners identify a need for numeracy, and then supporting learners to develop the mathematical concepts to meet the needs. The integration of situational contexts and numeracy development is the premise for embedded numeracy.

Theories of embedded numeracy

In its *Theoretical Framework* for strengthening literacy and numeracy through embedding (henceforward "the Framework"), the Tertiary Education Commission (2009a) considered embedding literacy and numeracy in adult education provision such as vocational training to be the most effective and efficient way to provide adult numeracy instruction.

The Framework drew heavily on research conducted by and for England's National Research and Development Centre for adult literacy and numeracy (NRDC). The Framework identifies four features of effective embedded literacy and numeracy provision: clear and explicit links between the numeracy and vocational components of courses, team approaches between vocational tutors

and literacy and numeracy specialists, effective assessment and use of learning progressions, and whole-organisational approaches to embedded provision (Tertiary Education Commission, 2009a, p. 9).

Clear and explicit links between numeracy and vocation

The New Zealand fully embedded approach to teaching numeracy means that, generally, the numeracy and mathematics topics taught are directly applicable to the vocation learners are training for. The numeracy-related topics and skills to be embedded are identified by analysing the numeracy demands of the vocational context. The expectation is that the explicit link between vocation and the mathematics required for the vocation leads to greater learner engagement. This is in line with research in the UK which has shown that when mathematics learning is connected to students' vocational development and situated in a similar learning culture, it becomes more relevant, meaningful and coherent for the learners (Dalby & Noyes, 2015).

Fully contextualised mathematics learning may, however, lead to a limited utilitarian understanding of mathematics and although literature supports contextualised learning, piecemeal contextualisation can be limiting for learners (Dalby & Noyes, 2015; Education & Training Foundation, 2014). Focusing mathematics teaching on utilitarian aims rather than the underlying concepts and abstract structures produces students who may learn one or two "recipes" but who will not be able to transfer this knowledge or apply it in unfamiliar ways (Advisory Committee on Mathematics Education, 2011). The Education and Training Foundation (ETF) (2014, p. 20) suggests that "contextualising learning can reduce rigour and narrow learners' understanding of maths". A mathematics specialist interviewed gave an example of Motor Mechanics students who were adept at applying ratios when they related to cars, but were unable to apply the mathematics to recipes "because they didn't realise that what they were doing with the car had the same structure as what they were doing in the recipe".

Even so, ETF posits that contextualising mathematics is a proven way in which vocational students can be taught and describes embedding mathematics

teaching into vocational contexts as a vehicle to give students relevant contexts that enable them to understand abstract mathematics or concepts. Effective embedding described by ETF involves a mathematics specialist capitalising on real-life contexts to engage the learners and help them recognise situations in their lives which require mathematics. The specialists see high value in vocational tutors understanding mathematics and numeracy to "draw it out more clearly" in their contexts (Education & Training Foundation, 2014, p. 20).

A key desired outcome of an embedded approach is the link between the contextual mathematics and the underlying concept. The research suggests that embedded programmes should ensure that contexts are planned and aligned to a broader curriculum and that contextualised mathematical tools are applied across several areas.

Team approaches

Notably, a key understanding outlined in the Framework is that vocational programmes are more likely to effectively embed literacy and numeracy when a team approach between vocational educators and literacy and numeracy specialist educators is employed. The Framework references, for example, the Australian Course in Applied Vocational Study Skills (CAVSS) methodology for team-teaching between vocational lecturers and literacy teachers, and the various uses of the term "embedded" all share the concept of bringing together vocational or subject teaching with LLN teaching (Bates, 2005; Casey et al., 2006; Roberts et al., 2005).

An embedded approach to numeracy skills development is most effective when numeracy specialists and vocational tutors work collaboratively (Casey et al., 2007; Dalby & Noyes, 2015; Roberts et al., 2005; Tertiary Education Commission, 2009a; Thomas & Ward, 2009; Whatman, Potter, & Boyd, 2011). The ways they operate as a team can vary and may include shared planning, shared teaching time and space, team teaching, or observing each other. Regardless of the method of collaboration, time needs to be allocated sufficiently for this to happen. Casey et al. (2007, p. 5) found that learners on embedded courses had

better retention and success rates than those on non-embedded courses *except* where a single teacher was asked to take dual responsibility for teaching vocational skills and LN. They caution that the benefits of embedding may not be achieved by simply adding LN to vocational teachers' responsibilities.

Benseman et al., (2005, p. 61) note that even with the involvement of a LN specialist, "Many of the teachers of integrated provision find it challenging to cover their normal curriculum as well as respond to LNL needs of their learners." The authors were also unclear as to the extent of explicit LNL teaching occurring in the integrated programmes they observed. Effective embedding requires educators to design specific learning activities and implement deliberate acts of teaching aimed at improving LN skills (Whatman et al., 2011). Even so, a very recent survey found that liaising between mathematics and vocational teachers in the UK's Further Education (FE) colleges is more likely to be about student welfare and behaviour than about mathematics (Noyes, Dalby, & Lavis, 2018).

Despite the evidence indicating that best-practice for integrated or embedded LN happens when vocational tutors and LN specialists work in tandem, responsibility for embedding numeracy in New Zealand vocational education settings appears to fall largely to the vocational tutor. The reality in New Zealand mirrors what has been found in Australia where a team teaching approach is "more resource intensive, does not fit most apprenticeship training funding models and relies heavily on the availability of LLN specialists who are currently in short supply and aging" (Retford & Lawrence, 2013, p. 2). While some providers used a funding top-up (available to tertiary education providers during the embedded initiative implementation stage up to 2012) to support collaborative LN planning and teaching, that became untenable after that funding expired and embedded LN was expected to be "business-as-usual" (Tertiary Education Commission, 2012, p. 20).

Learning progressions without curriculum

Another notable difference between the embedded provision studied in the UK and Australia (Bak & O'Maley, 2015; Casey et al., 2006; Coben, 2003; Roberts et

al., 2005) and embedded provision in New Zealand is that provision here is not linked to a defined adult numeracy curriculum. England, at the time of the research regarding embedded approaches that informed New Zealand's Framework was undertaken, had a national curriculum for adult literacy and numeracy delivery, the Adult Numeracy Core Curriculum. It was informed by the National Standards for Adult Literacy and Numeracy, which aligned to a National Qualifications Framework (Basic Skills Agency, 2001; Qualifications and Curriculum Authority, 2005). Learners could obtain literacy and numeracy qualifications alongside vocational qualifications. Different assessment tools were used to measure learner proficiency, but all were referenced to a national curriculum for LN and the National Standards for Adult Literacy and Numeracy.

Current policy in England, requiring 16 – 18 year-olds who have not achieved a set minimum grade in the national General Certificate of Secondary Education (GCSE) in mathematics to continue mathematics education to support gaining the required minimum level, has led to an increase in students enrolled in GSCE mathematics courses with Further Education (FE) providers and impelled a need to find out more about the educators and effective practice in the sector (Education & Training Foundation, 2014; Higton et al., 2017; Minister of State for School Standards, 2017; Noyes et al., 2018). Not all educators agree that a GCSE qualification is appropriate for all 16 – 18 year olds because of a perceived lack of relevancy of some of the course content for students not seeking a traditional academic route, such as those pursuing a vocational career path (Higton et al., 2017; Smith, 2017). Functional Skills qualifications are offered to learners with very low GCSE results. Critics of these recently reformed qualifications do not view them as an effective stepping stone to GCSE and believe that the applied nature of the qualification has been diluted by the use of more abstract questions (Higton et al., 2017).

Embedded provision in vocational programmes in New Zealand does not include LN qualifications. The only national measure of Adult LN proficiency, and primary measure of the effectiveness of the embedded LN strategy, is the Literacy and Numeracy for Adults Assessment Tool (LNAAT) (Tertiary Education Commission,

2015b). The Adult Learning Progressions, the currently accepted benchmarking tool for embedded numeracy, "are neither a curriculum nor a teaching and learning programme" (Tertiary Education Commission, 2008b, p. 4). New Zealand vocational providers have been tasked with embedding numeracy without an adult numeracy curriculum to define outcomes or to measure success against (Ako Aotearoa, 2018a).

While LNAAT can provide educators with an indicative level of learners' numeracy proficiency, there is acknowledged tension between funding-linked compliance use of LNAAT, its use as a diagnostic tool to inform teaching and learning, and potential use as a measure of embedding success or otherwise. An unintended consequence of mandatory regimes such as the use of LNAAT may be to limit rather than expand learner outcomes through an over-emphasis on compliance (Coben & McCartney, 2016).

Whole-organisational approaches

Embedded provision and collaborative working between teachers tends to be most successful when supported by a whole organisation approach that incorporates a strong numeracy strategy (Alkema & Rean, 2013; Education & Training Foundation, 2014, p. 6). Organisational policies for embedding LLN should be documented to ensure the value of LLN, particularly numeracy, is understood and viewed as an integral part of vocational training with strong emphasis on educator training and professional development. Embedded provision requires a learner-centred culture in which everyone in the organisation has a knowledgeable and positive attitude to embedded provision. It involves devolving responsibility for identifying and marking elements of numeracy in students' work to teachers of all subjects. English and mathematics are promoted as relevant by vocational teachers, and appropriate management systems are in place to support effective LLN practice (Higton et al., 2017; Whatman et al., 2011).

Organisational challenges identified internationally include building a congenial environment for learning in business-focused settings, recruitment and induction

processes, and adequate funding for on-going development (Higton et al., 2017; Leach, Zepke, Haworth, Isaacs, & Nepia, 2009). While the use of non-scholastic, kinaesthetic activities linking numeracy to vocational interests and illuminating the importance of numeracy in a holistic way may be effective approaches, the compartmentalised delivery of numeracy topics imposed by competency-based training frameworks can make this difficult (Ewing, Baturo, Cooper, Duus, & Moore, 2007).

An evaluative framework was developed by the National Centre of Literacy and Numeracy for Adults in New Zealand to support organisations to critically evaluate organisational objectives, policies and structures related to embedded literacy and numeracy (ELN). The framework also provides direction in creating organisational benchmarks to judge learner outcomes (National Centre of Literacy and Numeracy for Adults, 2014a). Data on the uptake of the model in organisations is not in the public domain. The focus of embedded numeracy in New Zealand has remained largely at the level of the programme of study and the tutor (Coben & McCartney, 2016). This means that pockets of good ELN practices may not be generalised across the whole organisation.

Adult numeracy teaching practices

Distinctions between adult numeracy and school mathematics

An intended outcome of embedding numeracy is to engage youth and adult learners who have not succeeded or engaged with traditional school mathematics education. An embedded approach endeavours to minimise the tensions between abstract mathematics learning in formal teaching environments and the often contrasting learning cultures, environments and teacher-student relationships in practice-based vocational training (Dalby & Noyes, 2015). Minimum mathematics requirements in the workplace are often expressed against school qualifications, but employers often find that young people are unable to apply the mathematical concepts learned in school to workplace problems (Education & Training Foundation, 2014).

Numeracy has often been considered to deal only with numbers and the four basic arithmetic operations. Adult numeracy now, however, is understood to cover a range of additional skills including measurement, statistical reasoning, interpreting shape, space and design (Tout & Gal, 2015). FitzSimons (2009, p. 10) points out that while it is generally agreed that most workplace activities require low levels of school mathematics, the skills need to be used in often complex and unpredictable ways. "To be numerate […] is to actually be inside a situation – as distinct from studying a mathematical topic and finding so-called relevant applications."

A disconnect between problem-solving strategies used in real-life contexts and students' ability to apply the same strategies or methods in formal education settings has been observed in several studies (Coben, 2003). For example, school children in Brazil were able to calculate answers related to street vending that mathematically were often higher than what they were learning in school. Yet when given the calculations in mathematical representation and in similar written word problems the children fared poorly and often made mistakes by incorrectly applying algorithms taught in school (Carraher, Carraher, & Schliemann, 1985). Similarly, a study in Brazil comparing calculation strategies used by adult tradespeople to those used by school children doing similar problems found that the calculation methods used by the workers were totally different than those used by the schoolchildren. While the workers solved problems, mainly orally, fully within the context of the problem, the school methods require students to extract the numbers and operations from the context, perform algorithmic calculations on them and then apply the answers back to the context. In most cases the workers performed better than the students with similar or more schooling, with students often attempting algorithmic solutions but not remembering them well. Mistakes were far less likely in the work situation where the calculation values were meaningful to the problem solver (Nunes, Carraher & Schliemann, 1993, as cited in Coben, 2003).

Lave (1988) observed that adults in everyday life generate their own problems and use a wide range of strategies to solve them. Whereas, when adults were

given school mathematics questions contextualised as if they were everyday mathematics, they treated them as school mathematics questions and were much less likely to successfully solve the problems (as cited in Coben, 2003, p. 41). Similarly, few adult learners attending numeracy courses said that the mathematics they learnt in their classes helped them in their lives outside the classroom (Swain, Baker, Holder, Newmarch, & Coben, 2005).

Additionally, the term “mathematics” itself often has negative connotations among adult students who associate it with disliked school mathematics classes. These negative feelings toward mathematics are often referred to as “maths anxiety” (Education & Training Foundation, 2014; Tout & Schmitt, 2002; Whitten, 2012). School mathematics is often seen by students as a subject, a set of rules and procedures to be memorised, and separate from real-life applications and problem solving (Boaler, 2016; Whitten, 2012). The learners coming to foundation-level trades courses have all had years of school mathematics education, yet they often perceive that they cannot do mathematics or simply hate it. Failure at remembering how to apply algorithms or, as has been suggested by Whitacre and Wessenberg (2016) and Petty (2016) , being directly instructed in using “invented strategies” resulting in ineffectively learning them as rote procedures (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998), has resulted in maths-phobic and maths-averse adult learners. These are the very learners that are often directed to vocational training. Learners who have had negative learning experiences may need to be supported to develop an identity through which they can view themselves as being competent in real-life mathematics situations (Education & Training Foundation, 2014).

The implication is that teaching adult numeracy needs to differ from re-teaching school mathematics. Ideally, It involves teaching “generalities in ways that may be useful in specific contexts very different from classroom situations” (FitzSimons, 2009, p. 10). Effective embedded numeracy should provide a bridge for learners from their often unsuccessful experiences with school mathematics, to success in developing their numeracy skills and confidence in authentic and relevant contexts. The transition involves changing students’ perceptions of

mathematics from object to tool (Dalby & Noyes, 2015). Effective numeracy teachers are likely to “take into account the students' various informal ways of doing math, allowing the understandings and strategies amassed in and out of school to serve as valid resources” (Tout & Schmitt, 2002, p. 14).

Subsuming of embedded numeracy practice research

Research into embedded numeracy practice is muddied by the common practice of including literacy, language and numeracy, LLN, or literacy and numeracy, LN, as a single subject. For example, New Zealand case studies provide indicators of organisational practices that promote effective delivery of LLN within programmes (Leach et al., 2010), yet the findings refer either generically to LLN or to literacy specialists and literacy provision, never specifically to numeracy provision.

Research regarding numeracy teaching practice has largely concentrated on numeracy *specialists*, or often literacy specialists who are assumed to be numeracy specialists (Berghella & Molenaar, 2013; Byrne & Maguire, 2013), teaching numeracy and mathematics as a subject, either in mathematics courses, or to a lesser extent in specialist roles in integrated or embedded programmes (Benseman et al., 2005; Casey et al., 2007; Coben et al., 2007; Education & Training Foundation, 2014; Higton et al., 2017; Swain et al., 2005). Similarly, profiles of the numeracy workforce have largely focused on tutors and trainers who identify as specialists. The adult numeracy educator workforce internationally is largely female, aging, part-time, and often teaching other subjects, usually literacy-related, with a significant number holding no, or introductory only, qualifications (Byrne & Maguire, 2013; Cara, Litster, Swain, & Vorhaus, 2010; Medlin, 2016; Noyes et al., 2018). There is no summary information available about the New Zealand adult numeracy workforce (Coben, Kane, & Whitten, 2017) and while reports from the TEC contain information on literacy and numeracy, they do not specify numeracy as distinct from literacy.

Dearth of research regarding vocational tutors embedding numeracy

A small number of studies have included the voices of vocational tutors tasked with teaching numeracy. Berghella and Molenaar (2013) included both LLN specialists and vocational specialists in their examination of the Australian Vocational Education and Training (VET) workforce capacity to address numeracy skill needs. Benseman (2014, p. 108) remarked on the "dearth of research" about the professional practices of New Zealand tutors in the adult literacy and numeracy sector.

Benseman, Lander and Sutton's (2005) observational study of literacy, numeracy and language (LNL) pedagogy in practice concentrated on LNL specialists, some of whom were using an integrated (embedded) approach where the prime focus was on a vocational topic and literacy was a secondary focus. Although a "less than optimal" (p. 27) number of qualifications above certificate level specific to adult literacy, numeracy and language were held by the teachers in this study, they were considered LNL specialist teachers.

Mackay, Burgoyne, Warwick and Cipollone (2006b, pp. 33–34) note the need for a study to determine whether vocational trainers are in fact incorporating language literacy and numeracy (LLN) into their delivery. They assert that it is unlikely that most vocational trainers would be able to confidently incorporate LLN into their practice and that an integrated model of LLN delivery should not require vocational trainers to become experts in LLN. Rather they need to be aware of the LLN demands of their teaching and learning materials and assessment practice and collaborate with LLN specialists. Berghella and Molenaar (2013) found that vocational specialists delivered workplace numeracy skills training more often than LLN specialists, as the skills are specified in the unit requirements within vocational qualifications. The small study found that participants' confidence in their ability to deliver workplace numeracy skills contrasted with low levels of numeracy proficiency themselves. They suggest that the confidence stems from being required to deliver numeracy only rarely and only to a level, and within a context, at which they feel comfortable, rather than in the volume and at the level needed to improve learners' proficiency.

Given the paucity of research about embedded numeracy practice per se and the fact that vocational tutors have been largely left to teach mathematics and numeracy without the support of a numeracy or mathematics specialist, I have focused my review of effective practices on effective teaching practices in the adult numeracy and mathematics classroom. In doing so I assume that effective embedded numeracy practice for vocational specialists should parallel evidence-based strategies found to be effective for adult numeracy and mathematics specialists.

Evidence-based knowledge of effective practice

Given the complexities of defining numeracy and numerate behaviour, and the complications and difficulties in measuring numeracy or improvements in numeracy abilities, it is unsurprising that defining a measure of effective numeracy teaching practice is extremely elusive. The studies reviewed here refer to both mathematics and numeracy teaching practices.

In the early 2000's a concerted, government-funded effort was made in England to investigate adult mathematics and numeracy teaching practices, a previously largely-neglected area of research (Coben, 2003), resulting in evidence-based guidelines and resources. Swain, Baker, Holder, Newmarch and Coben (2005) investigated numeracy teaching from the learners' perspective, resulting in a set of understandings and recommendations for practitioners. The study concluded that the quality of teaching is at least as important as the content and that mathematics can be made meaningful without always making it directly applicable to a specific adult's everyday life. However, learning is linked to students' identities and biographical contexts, so it is vital that educators know their learners as well as possible. Although the study included only learners who attended discrete numeracy classes, a recommendation was made for embedded numeracy provision, with numeracy content made explicit from the outset. The guidelines for good practice produced from the study are summarised in Appendix F.

Around the same time, Coben et al. (2007) undertook a much larger study aiming to identify potential improvements to numeracy teaching, learning and assessing, and factors that contribute to successful learning. The teachers in this study were generally experienced and well-qualified, with 79 percent having a formal qualification in mathematics or a related subject and 88 percent a teaching qualification. Teachers used a wide range of teaching approaches, but whole class and individual work predominated. It was unusual to find learners collaborating and learning from each other; and more common to see teachers demonstrating procedures and learners working through worksheets. Observations of classrooms revealed little use of higher-order questions, collaborative work, or practical resources. Learners valued clear explanations and work broken into smaller steps and regular feedback. No particular set of teaching practices was confirmed as more or most effective in all forms of LLN provision, although the researchers believed that they did observe effective teaching practice. The features of effective lessons identified in the study are summarised in Appendix F.

The authors above reiterated that numeracy teaching is particularly complex. It is not a discrete set of skills and it is "intimately bound up with literacy and language" (Coben et al., 2007, p. 58). Transferability of numerate practices between contexts may be problematic, and levels of achievement may not be stable across time or under varying circumstances. There were few significant correlations between student progress and the extent of the different approaches used, the only significant, but not strong, positive correlation being with procedural teaching relying on whiteboard examples directed at the whole class. But given the number of counter examples, the correlation was not considered causative (Coben, 2006a).

Similarly, a study of a group of low-achieving, difficult to engage learners found that the students preferred a procedural, absolutist teaching approach to numeracy, even though that method had not been successful for them at school (Ewing et al., 2007). This may be because the expected procedural, algorithmic approach to numeracy allows the learners to keep and reinforce their

maladaptive beliefs about numeracy rather than being challenged with an exploratory, growth mindset (Boaler, 2016) approach to mathematics.

The “Improving Learning in Mathematics” (ILM) project developed and modelled research-based principles for teaching mainstream and more advanced mathematics classes (Swan, 2005). This was followed by the “Thinking Through Mathematics” (TTM) project to investigate how far the same pedagogical principles could be applied in the very different context of teaching adult numeracy to foundational level learners. The study used a design-research process that included professional development designed to challenge teachers’ existing practices and beliefs (Swain & Swan, 2009; Swan & Swain, 2007, 2010). The tasks and activities that were trialled and developed supported the following outcomes:

- Classifying mathematical objects
- Interpreting multiple representations
- Evaluating mathematical statements
- Creating and solving problems
- Analysing reasoning and solutions.

The complete set of modified activities were subsequently widely distributed to adult numeracy teachers as part of the *Thinking Through Mathematics* resources (Swan & Wall, 2007).

The research-based pedagogical principles that underpin the ILM and TTM projects and resources were widely consulted on and expanded slightly by the mathematics education community through the UK National Centre for Excellence in the Teaching of Mathematics (2007). The principles as presented in TTM are outlined in Appendix F.

A report by the UK’s Education and Training Foundation (2014) includes a review of international practices and concludes that no single approach is appropriate for teaching vocational mathematics and that culture and training ambitions influence both content and pedagogy. Learner engagement and a focus on identifying, through diagnostic and formative assessment, and addressing

specific gaps in learning is preferable to teaching an entire curriculum. The pedagogical models singled out by the study as being highly successful for post-16 vocational mathematics education in a mathematics-specific classroom centre on contextualised and real-world problem solving, linking current knowledge to social and cultural backgrounds and activity-based learning. The practices are summarised in Appendix F.

A cross-national (Scotland, Finland, The Netherlands and Denmark) collaborative project that set out to share best practice and expertise in mathematics education in the vocational classroom recommends four broad approaches (also summarised in Appendix F):

- Collaborative learning - to search for understanding, common meaning, solutions or creation of a product of learning
- Concretizing the mathematics – transferring real-world problems into mathematical problems and transferring abstract mathematical concepts into the solving of a real-world problem in an authentic, meaningful context
- Dialogic learning and teaching – conversation between students and teachers/trainers prompted and stimulated by effective questioning strategies
- Problem-based learning – learning through "real-world" problem-solving relevant to the vocational course of study

(Directorate-General for Education and Culture, 2013)

A recent study in England examined factors associated with effective teaching of English and mathematics to 16 – 18 year olds in Further Education (FE) settings (Higton et al., 2017). Key teaching practices that were identified as influencing learning are summarised in Appendix F. The authors conclude that there are no "best practice" solutions and there is a diverse range of solutions to contextual and delivery issues. The student- teacher relationship is, however, central to nearly all delivery. Systems need to be in place to help learners transition from school to a more mature learning environment where they are treated as adults.

One potentially contentious practice identified in the above study is grouping learners in levelled rather than mixed-ability classes. Others argue that ability grouping often does not have the desired effect of matching the work set to learners' strengths and weaknesses, and serves to reinforce the learners' poor perception of their mathematical ability (Boaler, 2005; Boaler, Wiliam, & Brown, 2000). This undermines learners self-belief which is considered one of the most impactful influences on learner success (Hattie, 2012).

A New Zealand research summary examining ways to optimise LLN learning reiterates the importance of knowing the learners' strengths, existing knowledge, and challenges in order to design tailored teaching that supports learners to construct meaning for themselves. The authors emphasise attending to relationships and the learning environment and modelling positive attitudes towards mathematics while having, and clearly communicating, high expectations (Whatman et al., 2011). The practices associated with numeracy teaching are included in Appendix F.

In New Zealand a *Knowings* framework is used to frame embedded literacy and numeracy teaching practice. The framework conceptualises a pedagogical model for planning and delivery of embedded teaching and learning through an inter-related triad of domains: Knowing the learner; Knowing the demands; and Knowing what to do (Tertiary Education Commission, 2008c). The framework guides educators to identify gaps in learners' knowledge relative to their vocational education context and plan embedded numeracy provision in the absence of a set numeracy curriculum. The framework is part of an infrastructure of supporting resources, and effective practice has been considered to be the competent integration of each aspect (Ako Aotearoa, 2018a). The three domains provide a useful structure for analysing embedded teaching practice and capability. As such, I will use the domains throughout this paper as an analytical framework.

Appendix F summarises the effective practices identified in the preceding paragraphs in relation to the three Knowings. The practices either require or lead to the associated Knowing.

An "ALN Effective Practice Model" providing guidelines for adult literacy and numeracy education practice in New Zealand has very recently been released (Ako Aotearoa, 2018a). The model endeavours to answer the question: "What skills, knowledge, and understanding do effective literacy and numeracy practitioners possess?" (p. 3). The model continues the use of the three Knowings pedagogical framework but breaks each of the Knowing domains into finer good practice attributes, making it a more holistic teaching model. Knowing the learner is expanded beyond identifying learners' LN strengths and weaknesses to acknowledging and responding to cultural and affective factors and individual learning challenges. Emphasis is placed on adopting culturally responsive practices to improve engagement of, and outcomes for, Māori and Pacific learners. Interestingly, the updated model is referred to as an effective practice model for *ALN* practitioners even though many of the educators who are tasked with embedding LN in New Zealand would not consider themselves literacy or numeracy practitioners. The implication is that vocational tutors are responsible for the dual roles of vocational educator and LN educator.

The recommended pedagogical practices for teaching numeracy (Ako Aotearoa, 2018a, p. 30) are outlined in Table 3 in the Findings chapter of this thesis. The practices provide an analytical framework for my findings.

Professional learning and development (PLD)

Most of the effective practices that I have associated with "Knowing the learner" in Appendix F and Table 3 relate to developing caring, respectful, adult-to-adult relationships with learners, and creating positive cooperative learning environments that aim to minimise the anxiety often associated with mathematics learning. These practices, along with being able to couch mathematics topics in everyday or familiar vocational contexts, do not require maths-specific pedagogical knowledge. The "Know the learner" practices, along

with most teaching strategies categorized in “Know what to do”, align with what is considered effective learner-centred pedagogy for any subject in an adult teaching context (Brookfield & Preskill, 2005; Fink, 2013; Hattie, 2012; Race, 2010).

The practices associated with “Know the demands”, however, are mathematics-specific pedagogies and require practitioners to have a deeper understanding of mathematical concepts and processes than just how to get an answer. Developing these practices, and more generic teaching strategies such as discussion, questioning and small group work, to good effect in numeracy teaching requires mathematics-specific subject knowledge and is likely to require specialised professional development. Numeracy teachers need subject, pedagogical and also subject-specific pedagogical knowledge to enable flexible teaching approaches that draw on students’ understanding, involve them in discussion, and engage all students in a variety of complex tasks (Advisory Committee on Mathematics Education, 2011; Coben et al., 2007). Tutors with little training in how to teach numeracy at the students’ level may focus more on achieving a friendly environment than on challenging students mathematically (Ewing et al., 2007).

A lack of skilled teachers and trainers in vocational mathematics settings continues to be a challenge internationally (Berghella & Molenaar, 2013; Education & Training Foundation, 2014; Higton et al., 2017; Smith, 2017). There has been little research carried out on what constitutes effective professional development for tutors in adult numeracy and countries vary in their approaches both to professional development and to the required professionalism of the workforce (Byrne & Maguire, 2013). The complexity of professional development for numeracy tutors is illustrated by the different dimensions they may inhabit. While undertaking professional learning, numeracy tutors can be considered to be, concurrently, students and tutors of numeracy, collaborators, modulators of change, and lifelong learners (National Adult Literacy Agency, 2015).

In New Zealand, the government tertiary funding provider, the TEC, has an expectation that all tutors teaching foundation-level learners know how to embed LN effectively in teaching activities. This is typically evidenced by completing a National Qualifications Framework (NQF) Level 5 Certificate in Adult Literacy and Numeracy Education (NZCALNE; NCALNE prior to 2016) qualification (Tertiary Education Commission, 2017b). While the effectiveness of the qualification training has not been independently evaluated, just over 80 percent of tutors who self-reported on the impacts on their practice responded that the Certificate has been highly or moderately influential on their practice (Benseman, 2014).

A stocktake of New Zealand numeracy provision, resources and professional development revealed a shortage of numeracy content and pedagogical knowledge in the adult and tertiary sector workforce, and an absence of coordinated professional development specialising in numeracy. There was also a lack of clarity for both learners and tutors around existing numeracy support in organisations (Coben & Kane, 2017). A recently released, nationally available, online short course targeted at tutors who cover numeracy in vocational courses represents a small step towards addressing the dearth of numeracy teaching professional learning available in New Zealand (Ako Aotearoa, 2018c). A small study evaluating the use of action research as a model for developing embedded LN pedagogical understanding in a NZ vocational education setting could provide a potential model for PLD going forward (Schwenger, 2017).

Changing beliefs and practices

Tutors' beliefs, attitudes, values and previous learning experiences will influence how they teach (Askew, Rhodes, Brown, Wiliam, & Johnson, 1997; Byrne & Maguire, 2013; Coben et al., 2007; Swan & Swain, 2007). They come to teaching with *theories of action,* "implicit beliefs, assumptions, values, knowledge and emotions that individual educators bring to their practice", and these will define and certainly influence their practice, regardless of whether the tutors acknowledge, or are even aware of them (Learning Media, 2008, p. 106). Timperley, Wilson, Barrar, and Fung (2007) contend that creating, acknowledging

and addressing *dissonance* between current values and beliefs and new information which challenges those beliefs is a core element of the learning and change process.

A study of teachers who taught numeracy in primary schools found that highly effective numeracy teachers operated from a particular set of coherent beliefs and understandings about mathematics, numeracy, and teaching and learning (Askew et al., 1997). Highly effective teachers demonstrated a connectionist view whereby mathematics is viewed as a network of related concepts that need to be acknowledged in teaching, and the concepts and ability to apply the concepts are learned alongside each other. A transmission view sees mathematics as a set of discrete methods and procedures, and problem-solving as the ability to identify and apply the required routine or technique to get an answer. Coben et al. (2007) classified adult numeracy classroom teaching approaches, based on observations, as "connectionist", "transmission" or "constructivist". The constructivist teacher works alongside learners, using questions and activities to co-construct conceptual understanding and raise learners' mathematical thinking. The researchers considered effective practice to be the use of a predominately connectionist and constructivist approach emphasising conceptual understanding and conceiving of mathematics learning as a network of constructed concepts.

Similarly, Swan (2006) contrasts a transmission orientation with a "collaborative" orientation towards teaching. A collaborative orientation is promoted using the constructivist set of teaching behaviours. Likewise, the Office for Standards in Education, Children's Services and Skills (Ofsted) (2006) reported that collaborative enquiry focused on developing students' understanding of mathematical concepts contributed to high achievement for 14 – 19 year olds. Research indicates that transmission approaches, where explanations, examples and exercises dominate, do not promote transferable learning or knowledge and skills that can be used in non-routine situations outside the classroom (Swan & Swain, 2010).

TTM (Swan & Swain, 2007) was a research and professional development programme attempting to transform educational practices in adult numeracy classrooms by helping teachers to develop more “connected” and “challenging” teaching methods. While teachers generally rated their practice before the project as learner-centred, learners tended to see the methods as more teacher-centred. Through the project, teachers’ practices changed making them more learner-centred in their approaches. Changes observed were more use of group work and a classroom ethos more accepting of mistakes, exploration and questions (Swan & Swain, 2010). There was wide variation in teachers’ own mathematics subject knowledge. Where there were gaps in teachers’ own knowledge and/or little understanding of how learners come to understand mathematics, discussion and teaching opportunities were missed and learning suffered. Successful use of some activities was directly related to knowledge of subject-specific pedagogy. For example, exposing and discussing misconceptions was generally not used effectively or consistently (Swain & Swan, 2009).

Changes in teachers’ beliefs were measured. Teachers reported a significant movement away from a transmission/discovery orientation towards a connectionist orientation. They recognised the value of learning mathematics through interpersonal activity and making connections with learners’ prior knowledge and between topics, although their own mathematical gaps may have limited effective use of connectionist approaches (Swan & Swain, 2010).

Desired PLD

A depth of subject knowledge is desirable for numeracy teachers to draw on to contextualise teaching, recognise and address misconceptions, connect related topics, question and respond to questions effectively and for variety of other reasons (Coben & Kane, 2017). A discrepancy between vocational tutors’ perceived and actual numeracy skills means they may not recognise a need for further development of their own numeracy or mathematics skills (Berghella & Molenaar, 2013). More often, it seems, tutors desire opportunities to share practice and more generic professional development related to teaching strategies and classroom management.

A recent survey of the Further Education mathematics teaching workforce in England revealed that while almost all teachers surveyed have or are training for a teaching qualification, less than half hold a mathematics- or numeracy-specific teaching qualification (Noyes et al., 2018). Even so, the most common PLD needs identified by teachers in the survey were for strategies to increase student engagement, motivation and behaviour, and secondly subject-specific approaches to teaching. They value opportunities for informal professional development, particularly discussion with colleagues, and report more benefit from observing their peers than from being observed. Likewise, tutors of adult numeracy surveyed in Ireland, while recognising the importance of developing a deeper understanding of elementary mathematics, expressed a favoured desire for PLD that expands their repertoire of teaching strategies and enables them to develop resources to suit their learners (Byrne & Maguire, 2013).

The desire to learn from each other was echoed by tutors in a small New Zealand study focused on developing embedded LN through sharing practice. The project authors concluded that tutors want to learn from each other and to see examples of embedded LN in different contexts. They also note that the formal language in the New Zealand Learning Progressions can be a barrier to tutors understanding of the concept of embedded LN (Reid, McLaughlin, & Cama, 2018).

It appears that although mathematics education specialists view a conceptualised and connected understanding of mathematics and numeracy as important for effective teaching and learning, practitioners may not share that view.

Chapter Summary

Defining numeracy is complex and, to a degree, context-dependent. What is certain is that numeracy is much more than arithmetic and that, for many people, school mathematics does not transfer to real-life numeracy capabilities. Effective numeracy educators will understand numeracy in its broader sense and are likely to take a connectionist and constructivist approach to teaching the

associated mathematics. No single set of teaching approaches or practices has been found to be most effective, but common themes arise related to creating learner-centred environments where experimentation and discussion are encouraged and curiosity and learning from mistakes is valued. Knowing the learner, knowing the demands and knowing what to do encapsulates well the domains that an effective numeracy educator needs to attend to.

New Zealand's interpretation of an embedded approach to adult numeracy development means that vocational tutors in the ITP sector are responsible for teaching numeracy without the benefit of a defined curriculum and with little collaboration or support from numeracy specialists. This runs counter to international research on effectively embedding numeracy in vocational education. Research on effective practice has concentrated on adult numeracy and mathematics classrooms and teachers. The practices of vocational tutors tasked with teaching numeracy are virtually unexplored. This study is a small step into the exploration of those practices.

Chapter 3: Methodology

Introduction

This qualitative study sets out to explore selected tutors' characterisations of their teaching practice with respect to the imperative to embed numeracy teaching and development into vocational education programmes. The study also explores how related professional learning and development has impacted tutors' practice from their point of view and seeks their views on what further development or support would be useful.

This chapter provides an outline of the methodological framing and overall approach, the research design, methods, considerations and processes used in this study.

Methodological framing and approach

My aim with this study was to develop a picture of what embedded numeracy looks like in practice from a vocational tutor's viewpoint. Case study research is suited to this aim as it is richly descriptive and encourages the creation of "mental images that bring to life the complexity of the many variables inherent in the phenomenon being studied" (Hancock & Algozzine, 2006, p. 16). Furthermore, a case study approach uses purposive sampling techniques and can capture multiple realities that are not easily quantifiable (Hancock & Algozzine, 2006). Merriam and Leahy (2005) suggest that research in Adult Education done from an inductive theory-building rather than theory-testing perspective using in-depth interviews is likely to uncover factors and interactions not thought of on pre-designed data gathering instruments.

Accordingly, a case study approach has been taken using qualitative analysis of data collected through semi-structured interviews.

Research design

This research uses an instrumental, multi-case study design conducive to providing insight into a complex phenomenon such as teaching practice (P. Baxter & Jack, 2008). The case, or "unit of analysis" as defined by Miles and Huberman (1994, p. 25), under study is the characterisation of embedded numeracy in practice for vocational tutors teaching foundation-level courses in an ITP who have received literacy and numeracy education PLD. An acknowledged proposition (P. Baxter & Jack, 2008) that guided this study, and stems from my professional experience, is that many vocational tutors do not have sufficient depth of mathematical understanding to effectively develop their learners' number sense. I chose to examine multiple cases in order to compare embedding practices and test my proposition in more than one context.

Sample case participants

Six participants were purposively selected based on the type of programme that they teach in and their completion of qualifications related to embedding numeracy. Half of the participants are from construction-related trades programmes which, I know from my experience as a numeracy developer, provide rich contextual opportunities for numeracy development, and have always explicitly included mathematics calculations in the curriculum. The other participants teach on programmes in Business, Hospitality and Hairdressing, where numeracy is less integral to the vocation. These were included in order to provide a broader view of approaches to embedding numeracy.

All participants have completed the minimum of a Level 5 qualification with outcomes equivalent to the National Certificate in Adult Literacy and Numeracy Education (NCALNE). The NCALNE, superseded in 2016 by the current New Zealand Certificate in Adult Literacy and Numeracy Education (NZCALNE), represents a minimum baseline literacy and numeracy qualification requirement for tutors teaching foundation-level courses introduced by the TEC (2012). The "NCALNE-equivalent" qualifications held by the majority of participants in this

study (see Table 1) were assessed by TEC as meeting the baseline literacy and numeracy education qualification requirements.

Two of the participants also completed additional numeracy education professional development (PD) that was funded by the Tertiary Education Commission to support the introduction of the Adult Numeracy Learning Progressions and national embedded numeracy initiative (labelled as Learning for Living and in-house PD in Table 1).

The participants were selected from lists of names given to me by managers contacted from several departments. I explained the research I intended to undertake and asked for their help in identifying tutors that were teaching Foundation (NQF Levels 1 – 3) groups and who had done some training in embedded LN. I asked them to nominate tutors who they believed had a good awareness of their responsibilities to embed numeracy. I stated that I was looking for those that were not necessarily "star" embedders but who were "giving it a good go". The six chosen participants represent a mix of genders, departments, programme levels, and a range of embedded numeracy training.

Participants' profiles, using pseudonyms, are detailed in Table 1 below.

Table 1 Participant profiles

Pseudonym	Trade / teaching context	Student tuition time per week	LN Qualification and Training
Monty	Construction Trade Skills NQF Level 3	3 days (full year)	• NCALNE-equivalent • Level 6 "Teaching Maths in Context" course (15 credits) • In-house embedded numeracy PD (20 hours tuition)
Therese	Construction Trade Skills NQF Level 3	3 days (half year)	• NCALNE-equivalent • Davis Methods (Dyslexia) training

Pseudonym	Trade / teaching context	Student tuition time per week	LN Qualification and Training
Noah	Foundation Skills - Trades context NQF Level 2	3 days (half year)	• NCALNE-equivalent • TEC-funded Learning for Living PD (18 hours tuition)
Abby	Hairdressing NQF Level 2	2 days (half year)	NCALNE
Lucy	Foundation Skills - Retail context NQF Level 2	3 days (half year)	NCALNE-equivalent
Sue	Supported Learning – Hospitality context NQF Level 1	Students attend 'practical' sessions 2 mornings/week. Sue teaches 'theory' 2.5 hrs per week	NCALNE-equivalent

Monty, Therese and Noah will be referred to collectively in this study as Building Trades tutors.

Methods

Data collection and iterative analysis took place over a period of several months, once project and ethical approvals were obtained. A brief summary of the timeline is provided in Table 2 below.

Table 2. Data collection timeline

Month (2018)	Data collection milestones
January	Ethics approval granted by University of Waikato
March	Ethics approval granted by participating ITP
March – May	Interviews conducted
July	All transcriptions complete and approved by participants Iterative analysis begins
August	Programme documentation obtained and analysed for relevant attributes and triangulation with participant programme descriptions
November	One participant revisited to obtain teaching resources Another participant revisited to clarify discrepancy between narrative programme description and programme document
December	Literature review revisited and expanded

Interviews

Semi-structured interviews were conducted with each participant individually; each lasted approximately one hour. The interviews were conducted either in the participant's or my office. A set of focus questions (see Appendix A), vetted through an ethics approval process, was used for each interview. Not all questions were used in every interview and additional questions were asked during some interviews to probe or further elucidate statements made by participants. Questions were related to five general topics of interest: teaching context and PLD undertaken, perceptions and beliefs about numeracy and the vocational tutor's role in developing numeracy, mathematics teaching pedagogies and practices, probes for issues and challenges to embedding numeracy, and PLD desires and perceived needs. I had short follow-up conversations with two of the participants several months after the initial interviews to seek clarification about assessment and resources.

While using semi-structured interviews to explore the participants' points of view, I accept, epistemologically, that the narratives are a co-construction between interviewer and interviewee, and will to some degree be what the respondent believes the researcher should hear (Mann, 2016). The interviews in many ways took the form of professional conversations and the narratives are thus considered as "accounts" attentive to the context of interviewing, rather than "reports" concentrating on experiences (Brinkman, 2013, as cited in Mann, 2016). The interviews provide a glimpse of pedagogy insofar as pedagogy is "fundamentally concerned with what people perceive to be meaningful, important and relevant as they engage in teaching-related activity" (Nind, Hall, & Curtin, 2016, p. 3). However, as Silverman (2007, p. 59) notes, "people's own perceptions are an inadequate guide to their behaviour." In framing my research question in terms of tutors' *characterisations* of their teaching practice I am adopting a constructionist research model, and not assuming that interviews can give direct access to experience, as might be presumed in a naturalist model (Silverman, 2013).

Additional information sources

Case study research is strengthened by accessing and examining information from multiple sources including existing documents and artefacts related to the phenomenon of interest (Eisenhardt, 2002; Hancock & Algozzine, 2006). In addition to interview data, for this study I examined the formal Programme of Study documentation for each of the programmes described by study participants. The documentation includes programme structure, graduate outcomes, course learning outcomes, assessment grids, and indicative curriculum. I used this information to triangulate programme details gleaned from interview data. I also examined teaching and learning resources supplied by participants (see Appendix E for examples). Both of these additional sources helped inform my findings and discussion.

Iterative thematic analysis

Each interview was audio recorded and fully transcribed by me. To analyse and make sense of the resulting text I used iterative thematic analysis, first analysing within-case, then searching across cases for patterns, similarities and anomalies (Eisenhardt, 2002; Hancock & Algozzine, 2006; Miles & Huberman, 1994). Using qualitative data analysis software NVivo, I coded each interview firstly to the high-level topical themes used to group my interview focus question. These codes were refined and changed as themes arose during multiple readings of the transcripts. I then created matrix frameworks to display the data in a form that allowed me to identify themes and begin to test hypotheses across cases.

At this stage I had decided that the previously described Knowings framework (Tertiary Education Commission, 2008c, p. 4) would provide a useful high-level framework for describing my findings. I also realised that I needed a more detailed framework for reporting data coded initially to “pedagogies and strategies” in my thematic analysis. The timely release of the Ako Aotearoa (2018a) recommended pedagogical practices for teaching numeracy, provided that needed framework, and ties the emerging theories closely to existing literature enhancing the internal validity (Eisenhardt, 2002). These analytical

frameworks provide the structure of discourse in the Findings and Discussion chapters that follow.

Considerations and processes

Ethics considerations and approvals

This research study received ethics approval from the University of Waikato and, because the intended study participants were colleagues at my place of work, ethics approval was also sought from our employer ITP. Approval was granted following minor changes to the proposal and participant information sheet clarifying safeguards to anonymity and stipulating the potential for identification of the organisation or individual participants. A statement was added guaranteeing that a decision to participate or not would have no bearing on participants' employment. See Appendix B for the full Participant Information Sheet and Appendix C for the Participant Consent Form. Each participant was provided with the full transcript of their interview and each signed an approval form agreeing that the transcript was accurate and approving it for analysis (see Appendix D).

Researcher bias

As declared in the information to participants, I come to this research from a history as a teacher educator, having worked in that role with some of these participants and many others across New Zealand. None of these participants were my students at the time of this research, so no direct and immediate conflict of interest exists. I am aware, however, that my role and position in the ITP and my previous work and professional experience as a teacher educator and numeracy developer represents a potential source of bias in this research.

I have endeavoured to take a neutral stance in analysing and reporting participants' narratives, but acknowledge that inevitable bias will exist (Denscombe, 2003; Miles & Huberman, 1994), and my current and previous professional experience influenced my sample selection and interview questions, and certainly informs my discussion of the findings. The use of semi-structured

interviews helps mitigate bias in that the collection of data is somewhat systematic (Hancock & Algozzine, 2006). Holstein and Gubrium (2004), however, contend that in accepting that all interviews involve active construction of reality and rejecting the positivist model of respondent as a vessel of answers, the idea of bias changes. All interview participants are involved in constructing meaning and the concern is not about contamination of data, but how the interview generates useful information for the specified purpose.

Summary remarks

This study is descriptive in nature rather than inferential (Hancock & Algozzine, 2006) meaning the results cannot be generalised in the sense of a positivist paradigm, and nothing is *proven* by this type of study. The fact that knowledge cannot be formally generalised does not diminish its value as part of the accumulation of collective knowledge about a phenomenon. While proof is hard to come by in Social Sciences, learning is certainly possible (Flyvbjerg, 2006).

The full and frank discussions I undertook with participants certainly added to my knowledge and understanding of tutors' interpretations of and strategies for embedding numeracy in practice. In the chapters that follow I endeavour to encapsulate my learning and thinking resulting from this study as it relates to my research questions around tutors' characterisations of their teaching practice and PLD.

Chapter 4: Findings

In this chapter I will outline the findings from my analysis of the interviews. The first part of the chapter aligns participants' narratives of their embedded numeracy practice with evidence-based knowledge of effective practice. To that end, my findings are framed using the Knowings framework adopted in New Zealand and the evidence-based pedagogical practices for teaching adult numeracy recently published by Ako Aotearoa (2018a, p. 30). I have categorised each Ako-recommended pedagogical practice according to the Knowing that supports or is supported by the practice in Table 3. Each practice requires a practitioner to attend to the related Knowing. Participants' narratives are analysed against each practice.

Table 3. Ako Aotearoa- recommended numeracy teaching practices

Know the learner	Determine, and build on, what learners already know about a topic Address and evaluate attitudes and beliefs regarding both learning and using mathematics
Know the demands	Expose and discuss common misconceptions Encourage reasoning, sense-making, and demonstrating interconnected nature of mathematics rather than emphasising rote learning and getting the answer Situate problem-solving tasks within familiar, meaningful, realistic contexts
Know what to do	Develop a community of discourse engaged in activity, reflection and conversation Use rich collaborative tasks and provide opportunities for group work Use effective questioning to generate deep thinking Develop understanding by providing opportunities to explore mathematical ideas with concrete manipulatives or visual representations and hands-on activities Select tools and representations to support learner thinking

The second part of this chapter reports participants' views on how professional learning and development has influenced their practice and what further support or professional development they see as useful.

Knowing the demands

Defining numeracy

Given that there is no prescribed curriculum to frame embedded numeracy in the ITP sector, the premise is that embedding numeracy will increase learners' overall numeracy proficiency. Therefore, an educator's understanding of what it means to be numerate will likely influence what they believe to be numeracy development. That is, recognising numeracy demands depends on recognising numerate behaviours and practices and mathematics that may in fact be invisible to those engaged in it because it is perceived as common sense or because it does not look and feel like school mathematics (Coben, 2000; Keogh, Maguire, & O'Donoghue, 2012, 2019). All study participants were asked what they believe it means to be numerate. The definition is relevant because it may influence what tutors teach as well as how generically or procedurally they teach the numeracy skills or concepts (Byrne & Maguire, 2013).

Monty and Lucy presented the most generic definitions, relating numeracy to everyday life skills. Monty described being numerate as being able to find information in everyday life and being able to look at news items "and things like that" in any context and understand that there is a numeracy component embedded into it somewhere, and understand what is going on. He did not mention any specific mathematical skills or strategies, rather, described numeracy as a way of understanding. Lucy described being numerate as "To have that number sense…some recognition on some core, some what we might call a core basic, you know like what is a third of a cup." She emphasises that it is not so much about knowing an answer but knowing that a strategy or formula can be found to get an answer. This belief was echoed by two other participants. Lucy attributes her philosophy to the numeracy development training she took, through which her attitude shifted from being highly maths-phobic to

recognising that she did not have to memorise a lot of procedures to solve problems. Lucy refers to numeracy as having a "feel for numbers" and understanding whether an answer is "about right or not".

Therese responded with: "For me, [it is] to be comfortable working with and thinking with foundational maths." She then states that she thinks it is important to expose students to that without making it a mathematics lesson, rather, using the mathematics as part of a process in a contextual project. She states, "I think someone [is numerate] that can work and understand, with help and instructions and some practice, that they can pick up the steps." Therese stated that for *her students* "numerate would probably be step 4 on LNAAT."

Noah states explicitly in his definition that numeracy is relative to context.

> Noah: *Understanding, sort of a basic level of numeracy that is applicable to the industry that you're in…I don't think you say being numerate; it needs to be numerate in 'what'. And so that would be bringing that back to what we're trying to achieve: their learning outcomes, their graduate profile, that sort of thing. So, being numerate at that level and in that industry.*

Sue, who works with disabled learners, some of whom are extremely illiterate, also ties her definition to individual demands: "To be able to use numeracy in the level that you need it to be...it can be really basic for my group; being able to recognise numbers."

The implication from the above three responses is a belief that numeracy is context-specific, and it follows that embedded numeracy provision will be relative to the contexts and demands of the trade or programme. The context determines both the level and the boundaries of embedded numeracy provision.

Abby had difficulty articulating what she believes it means to be numerate. Her initial response was, "I don't know how to answer that actually." After prompting her to relate it to what it means to be literate and then asking, "If you say you are embedding numeracy, what are we aiming for?" Abby responded:

So, they have an understanding of numbers, and what they mean and what they can and can't do and how to put them together and how to take them away from each other…They can add, and they can subtract, and they can divide.

This response is interesting because it appears as though Abby had never thought of this question before and she provides quite a vague, arithmetic-related definition, unrelated to any contextual demands. In later descriptions of embedded numeracy, she makes clear links between the mathematics and contexts, but her answer here indicates that she sees numeracy primarily as arithmetic. This is further indicated later in her narrative where she repeatedly refers to learners finding or knowing the "formula" for solving a problem.

Situating problem-solving tasks in meaningful contexts

Overseas programmes report that timetabling difficulties and curriculum constraints hinder the ability to deliver mathematics within a practical workshop setting in order to make it relevant to learners (Higton et al., 2017). Arguably, the key advantage of embedding numeracy education wholly within vocational education programmes in New Zealand is the set of ready-made, relevant and authentic contexts for situating learning. Students can readily see the relevance of mathematics to their career or study path of interest, and they have an incentive to learn the required numeracy.

The first step to embedding numeracy as prescribed in the TEC's embedding framework resources (Tertiary Education Commission, 2008c) is identifying the number, measurement, spatial and or statistical tasks or problems learners will encounter within the course and trade context. The identified course and trade demands determine the potential numeracy development outcomes that can be embedded in a course or programme.

Recognising demands and opportunities for numeracy learning

The tutors in this study all have a very clear idea of the numeracy demands of their trade and teaching context, which comes through early in their interviews in defining what it means to be numerate. All of the tutors accept that numeracy

is part of their teaching practice and are able to identify demands that are meaningful and related to the trade or to a context of interest to the learners.

The tutors recognise that numeracy skills are integral to competence in their trades:

> Monty: *Carpentry can't be done without numeracy and it's used every day and sometimes 1000 times a day.*
>
> Therese: *I'm from the trades and so everything here requires measuring and some really basic maths.*
>
> Abby: *Maths actually is embedded quite a bit into hairdressing, not that everyone knows that, because we're talking about angles of cutting and we're talking about lines and direction.*
>
> Noah: *When you do technical drafting, my view is you're using maths. Because it's all instruments, it's measuring instruments. To me it's so close to maths.*

All tutors clearly articulated contexts and topics that they include as part of their embedded numeracy teaching. Five of the six tutors described teaching using a money context. All of the Building Trades tutors specifically mentioned the topics measurement skills, metric conversion, and interpreting plans. Pythagoras' theorem was specifically mentioned by two of the Building Trades tutors as a difficult topic for students. The Hairdressing tutor identified numeracy demands as telling time, timetabling, angles, lines and direction, and ratios, in addition to measurement topics. Percentages and fractions were also mentioned by trades and retail tutors.

There appeared to be no doubt in any of the tutors' minds that embedding numeracy is an expected part of their teaching practice. The tutor working with special needs learners is teaching in a context less immersed in numeracy demands than in the trades but was still able to identify and describe teaching and learning experiences that develop numeracy.

Sue: *So I think it's my responsibility to at least know where they're at and try and build on that if I can or at least make them aware that numbers are out there and they need on some level to know*.

The Retail trades tutor uses a money and cash handling context, as well as finding opportunities for discussions and calculations in more general contexts. She describes, for example, using numbers derived from a "how do you feel" starter activity to start a conversation about percentages and fractions.

Lucy: *I do a rating system on the board. They have to score themselves out of 10 how they're feeling on any given day so that we can see...and even looking at, you know, you've got to be 50 percent OK to be able to get through your day and be a member of the team and, "What is 50 percent and what is half?" and, "If we had 18 students what would that look like?" And we turn our class numbers into a fraction as well because we've got that up on the board and then how can we simplify that fraction and where that might be useful and what might that look like if that was a piece of pie.*

Emphasis on contextualising and 'just in time' teaching

A commonality in all of the narratives is an emphasis on teaching mathematics within contexts that are meaningful and of interest to the learners. Participants also commented on the importance of timeliness, that is, teaching the mathematics as it is required for the tasks or projects.

Monty: *It has to be wrapped around what we're doing. If we're not doing that sort of thing that week there's no point covering it.*

Therese: *…when I can connect it to the trade it goes a lot better. They see a value to it, they want to learn it, they see that if they don't get those equations right or the area right or the quantities right, they could lose some money. So, then we've got the buy-in.*

Abby: *The assessment's ugly… it's not anything that they have understanding on. The assessment itself is way beyond what the students*

actually have in real life situations. So one of the things I do is I play a game with them… But it's real life reality and I think that is a really good, fun way of doing it.

Lucy: *So capturing that within something that they do every day anyway. And they're starting to notice what's happening out in industry…*

Sue: *One of the unit standards we do is eating out in a public outlet, and so they get a budget of $20 and we go, so we usually go to Denny's… and so they have to work out what they can buy on the day and what they can order for $20.*

Noah, too, stated that all of his numeracy teaching is done within a series of building projects set for the students.

Challenges to contextualising

A challenge to authentic, contextualised numeracy development discussed by both Therese and Abby was preparing students for mandatory assessments or assessment topics that are not perceived as relevant to the trade by the tutors or related to the students' interests. These assessments assess the only explicit numeracy outcomes documented in their respective programmes.

In Abby's case, the programme includes the learning outcome *Produce a balanced budget and adjust the budget to reflect changing financial circumstances*. The assessment she is required to use in the programme requires learners to produce a budget for a family of four, two parents and two children, budgeting for a trip and household expenses. Abby recognises that this context is outside of the scope of most of her young learners' life experience. To prepare students for the assessment and make the learning outcomes relevant for them, Abby sets up a project where students work in small groups and go through the budgeting scenario to set up a flat for the group. They do this over a period of five weeks in class, in groups, using the internet to investigate the various costs and potential take-home pay. The students display their budget planning and calculations using posters. Students were enthusiastic about the project because

it was a scenario they could relate to. The students were able to transfer their learning in a familiar context to the less familiar context used in the assessment. Abby says, “The students use the formulas and working from their posters to complete the assessment with some guidance.”

Therese strongly expressed her view of the necessity of contextualising all numeracy topics to make them *real* for the students and to get student “buy-in” throughout her interview. The numeracy assessment that she finds challenging to prepare learners for is for the learning outcome *Perform calculations to solve simple problems in a construction environment*. She expressed concerns about both the timeliness of the assessment and the contexts that are not directly relevant to her trade. The outcome is considered a core competency common to three strands (carpentry, allied trades and joinery) of the Level 3 Construction Trade Skills programme and they use a shared assessment. The outcome is meant to be taught and assessed early in the course (to allow learners to change strands if desired) and therefore the teaching may not be able to be linked to practical projects. Therese notes that there is a lot of assumed prior knowledge for some of the questions and big gaps between what students come with and what they need. So, requiring learners to complete the assessment early in the course is a big challenge. The questions are couched mainly in a carpentry context, and some of the problems presented are, in Therese’s view, outside of the scope of what students need to know for the allied trade. She reflects, “[I] try to connect it wherever I can but for me I think the most resistance is when they say, ‘what do we need this stuff for?’ and I can’t actually answer that honestly.”

Therese’s solution is to teach the types of problems in the assessment topic by topic and have students complete the assessment in parts rather than as a single assessment. She reflects that this gets the students through the assessment, but that for some learners and some topics, it is a matter of remembering the procedures long enough to pass the test. But when it comes to the problems that are integral to the trade, the actual on-the-job calculations that students will need, these are practised and assessed many times in projects and the tutor is much more confident that learners will retain the learning.

Reasoning, sense-making and connections over rote learning

I have associated the practice "Encouraging reasoning, sense-making, and demonstrating interconnected nature of mathematics rather than emphasising rote learning and getting the answer" with "Know the demands" (see Table 3) because it requires educators to have a solid understanding of the mathematical generalities related to the numeracy practices in their trade, in order to link a procedure or formula to associated generic mathematical concepts.

Most participants in this study used the terms "simple" or "basic" in their descriptions of the mathematics in their trades and courses. Literature supports this assessment of the level of mathematics required in many work contexts, but numeracy involves applying the mathematics in often "complex and unpredictable ways" (FitzSimons, 2009, p. 10). FitzSimons contends that traditional mathematics teaching often fails to produce a "transfer" of abilities required in a workplace context and that a difficulty in teaching adult numeracy is how to teach generalities in ways that may be useful in varying specific contexts. Conversely, teaching mathematics within a specific vocational context for a particular purpose, without exploring the connections to mathematical generalities is unlikely to lead to transfer of abilities either (Education & Training Foundation, 2014). Developing reasoning and sense-making for learning transfer requires problems to be presented and solved in authentic contexts and environments that bring together cognitive competence (*knowing that* and *knowing why*) and functional competence (*knowing how* and *skills*) (Coben & Weeks, 2014). Hence, Knowing the demands for effective, i.e., transferable, vocational embedded numeracy development must include not only identifying the contextual mathematics demands, but identifying and understanding the general mathematics concepts and principles that underpin the contextual problems.

This practice is arguably the one most likely to influence the effectiveness of a wholly embedded numeracy development programme. Whereas most of the practices in the analytical framework used in this study would be considered good practice in any teaching context, this practice and "expose and discuss

common mathematical misconceptions" are the ones most specifically related to teaching mathematics. There were no examples in the narratives of tutors using discussion of common mathematical misconceptions.

Encouraging reasoning and sense-making seems to be a practice that is likely to lead to desired learning transfer because it implies that learners are learning *why* an answer is what it is, i.e., understanding the basis of an algorithmic procedure or formula, rather than blindly applying it and hoping for a right answer. It is moving learning away from rote memorisation towards understanding concepts, methods and connections between them (Swain et al., 2005). Learners are able to use some number sense and numeracy strategies to evaluate the reasonableness of an answer and create connections between topics (Swan & Swain, 2007). Certainly, cultivating this practice was a major emphasis of the numeracy PLD contracted by the TEC early in the embedded delivery initiative, and of that provided in-house by the organisation in this study.

There is evidence that most of the tutors are making some attempt to teach some conceptual mathematics along with 'numeracy tools' required to complete the assessed trades-related projects, but it is unclear how strongly or explicitly a link is made between the generic mathematics principles and contextual problem-solving. Generally, all seem to be most focused on getting learners to use a formula or procedure, but there are some glimpses of at least an interest in teaching some general concepts, for example, fractions, percentages, and some proportional thinking.

Exploring interconnected mathematics concepts

Monty identified pricing and Goods and Services Tax (GST) calculations as transferable numeracy topics and I asked him to describe what an initial lesson on calculating the 15 percent GST would look like. He described using brochures (for realistic context) and "pricing things out and then teaching those that need to, how to use a calculator to get that [GST] in a very simple way so that they can understand how the 15 percent will work." He follows this statement with, "Teaching percentages on the whole using various things…" and describes using

linear objects or representations, including a carpenter's string line, and getting learners to mark the half-way point, quarter points and naming the points also as percentages and decimal values. Presumably other fractional points are identified as part of the activity although Monty did not say so specifically. He then went on to describe implementing ad hoc activities in the workshop to reinforce links between fractions and percentages and practice calculations. For example, students are asked to measure or count up something in the workshop then work out "what is a percentage of that or what is half of that".

The narrative indicates that the first strategy presented to learners is pressing the correct buttons on a calculator. Although asked to describe a lesson, the narrative did not describe a planned teaching and learning sequence that connected the calculator solution to the activity relating fractions and percentages. At the end of the description Monty states, "If they can make the connections between fractions and percentages, they can then use the calculator, and they do rely on the calculators a lot." Perhaps the connection between fractions, percentages and entering the problem in a calculator is discussed with the learners, but this was not articulated in Monty's narrative.

Monty shared with me visual resources that display parallel number lines relating fractions, percentages and decimals (Appendix E, Resource 1). He uses these to support the string line activity described above. Another of Monty's resources shows how to calculate a percentage of a value using proportional thinking strategies (Appendix E, Resource 2). This is a good example of presenting a topic in a visual, principle-based way. There are, however, a couple of weaknesses evident in the resource. The instructions for finding 10 percent and 1 percent of a value will only work for values in the hundreds and cannot be generalised. The decimal point is allocated an initial place column in the place value grid but assumed to be part of the tens or hundreds column after it is moved, which could potentially cause confusion for learners. The dialogue that Monty uses with the resource was not discussed in his interview.

Therese has a growing bank of visual, hands-on resources for helping learners explore foundational mathematics principles and for supporting learners to break a problem down into steps. Again, although learners have opportunities to explore and practice the underpinning concepts, an explicit link between the conceptual understanding and the procedure taught for getting an answer is not apparent. For example, the sequence of practice activities to prepare learners for waste percentage calculations starts with shading different percentages of circles, a fractional representation of percentages. The activity that follows provides a definition of percent unrelated to the fractional representations of percentages in the circles and provides a formula for calculating percentages (Appendix E, Resource 3). The teaching and discussion that goes with the resources may link the fractional representation of the percentage to perhaps evaluating the reasonableness of the answer. But it may also be a visual representation of percentages that students *know* but are unable to link to practice. This is where the support of a numeracy specialist could help improve these factually sound resources by making the links between underpinning knowledge and applications clearer.

Lucy's class starter activity described in the "Recognising demands and opportunities for numeracy learning" section of this paper requires learners to link percentages and fractions at a basic level. Lucy also describes teaching finding 10 percent "with a simple wee trick of moving the decimal" and then using the result and proportional thinking strategies to find 30 percent and 20 percent and then 15 percent.

The activities and resources are indicative of tutors' awareness of and considered attempts at embedding numeracy-supporting knowledge and concepts into their programmes beyond rote teaching *how* to get answers for the contextualised problems in their curriculum. There is a sense that this happens on a somewhat ad hoc basis, which is perhaps a natural outcome of the mandate to embed without a set mathematics or numeracy curriculum. It also means measuring the effect of the opportunities taken for developing number sense is elusive. Numeracy knowledge aspects, such as linking of fractions and percentages and

decimals, are not assessed as an outcome in these programmes, and it is not evident in either the narratives or resources that clear links are made between concepts and the formulae taught for the problem-solving that is assessed.

Creating connections between topics

Five of the six tutors related examples of teaching mathematics within the context of money. Solving problems related to money is an explicit learning outcome in some of the programmes, but it is also used as an accessible context for learning the mathematics required for other contexts. Therese expresses this succinctly:

> *Sometimes you do it without even intending to do that transferable thing because if they're not wanting to relate it to timber they will relate it to shopping. They will relate it to money and they'll relate it to taxes because all of those things affect them. And so often, I will use the examples in the practice, like if we're learning percentages, it doesn't matter if we're learning percentages and [using] how many people in this class and how many people are wearing red t-shirts.*

Connections are also made between vocational and everyday contexts that require money-related calculations. Monty relates building-related pricing to buying electronics and hire purchases. Abby, in the budgeting unit described earlier, teaches the budgeting elements and procedures in a familiar flatting context to prepare learners for preparing a budget in a less familiar context. She reflects that students are extremely engaged in the activity because it is immediately relevant to their real lives; some groups even calculated parameters into the budget beyond what was required and were able to successfully transfer the learning to the household budget context of the assessment. As well, students discovered and expressed surprise at the actual costs of flatting and learning that tax came out of gross wages, so the learning is immediately applicable to their lives.

This sequenced activity allowed students to successfully transfer procedures used in one context to a closely related one. It was apparent from the narrative,

however, that the mathematics calculations themselves were procedural and formula-based. When I asked Abby if she had discussions with learners about whether the answers make sense and how one answer relates to another, she stated, “We don’t talk about the calculation…I make sure I put the right formula.” So while the activity seems to engage learners with thinking about costs and the value of money, it would appear that strategies for calculating numerical answers are not discussed. Connections are made between contexts, but connections are not made to generalisable numeracy strategies or concepts.

Connections to school mathematics

Noah reflected on the fact that learners will have been taught the required numeracy related to measurement in school, but they may have learnt things in a theoretical, intangible way that they are not all able to apply. This observation echoes the findings of Baxter et al. (2006) where teachers lamented an emphasis on teaching unit conversions with little obvious purpose to learners. Noah describes exploring learners limited understanding of millimetres, centimetres and metres.

> Noah: *It’s a question and answer sort of thing and when they say “Yeah, we understand it,” but when it comes to applying it in the real physical world, they don’t. So it’s kind of talking in an abstract sense, but then we need to put it into a practical sense.*

He also referred to the divide between learners having arithmetic skills but not knowing how to apply them to interpreting or solving problems, in the context of interpreting building plans.

> Noah*: It might not be that they can’t subtract or multiply, it’s how they convert something visual into something sort of mathematical. You have to be pretty flexible about how you guide them – it gives you an opportunity to understand what sort of prior knowledge they come with, then you mix all that up, then work out some strategies for them.*

Noah talks about how young people come in from school having done calculations similar to what is required for a trade-related problem, “but there it’s all isolated.” He says, “When it’s in a real problem, they get overwhelmed, they get panicky.” So he sees embedding numeracy as breaking problems down and helping students isolate the parts of the solution to relate to something the learner knows.

Therese describes addressing the challenge when learners don’t understand the arithmetic methods they learned at school. Her intended aim is apparently to take the learners back to some basic principles to foster number sense.

> Therese: *It doesn't make sense to me the new ways that are taught at some of the high schools. I don't know where to start with all of those rows and rows of zeros and they* [*students*] *get lost and they can't talk me through it. I can't follow it so then I just try and teach them what I think is a simpler way, a way that they'll understand and a way that they can actually think with, but making sure they know what they’re actually looking at.*

Encourage reasoning and sense-making

The numeracy associated with measurement and spatial reasoning is a domain that logically should transfer to life outside the vocational workshop and classroom and it is a major component of the Building Trades as well as Hairdressing. It is a context in which the Building Trades tutors’ narratives exemplified encouraging sense making and some proportional reasoning.

Noah describes talking a boy through working out half the length of 490 mm. He encouraged him to break it down asking, “What’s half of 400?” When the student said he did not know half of 400, he asked him to find half of 4 in the hope that he could work up from there. When the student said he did not know half of 4, Noah is bewildered and chooses not to push the point so as not to risk hurting the student.

> Noah: *Now I don't know whether he was winding me up or he just didn't know or what the reason was. I found it difficult to believe that he didn't know what half of 4 was. I really felt that. So this is where you've got to, as a professional, you have to think, you know, you do not want to become or show signs that you are disparaging in some sort of way. These might be genuine learning disabilities; you don't know. So you have to* [pauses] *leave that situation where you hope you've given him something to build on. And so, leave it on a positive note.*

Noah reflected that, in hindsight, he could have gotten some paper out and worked through a visual activity folding halves. However, in reading the student's body language, decided to move on.

> Noah: *He had a kind of smile on his face and I thought he was just kind of being a little bit lazy, you know, that's where you have to kind of read the situation…with the time constraints you've got to navigate that one.*

Noah has created tutor resources to guide breaking down projects with question prompts for each arithmetic operation (Appendix E, Resource 4) and resources that identify the underpinning numeracy ideas in the project learning outcomes and teaching reminders about checking for sensibility in measurement and fit (Appendix E, Resource 5). Handouts and resources for learners combine pictures of the workshop process with the mathematics calculations in an effort to link the paper-based instructions and mathematical terminology to the workshop activity (Appendix E, Resource 6).

Monty and Therese both recounted asking questions to challenge learners to think beyond a calculated answer or measurement.

> Monty: "*Measure this. What's half of it? Work out the percentage." That can be in the workshop. Counting something up and figuring out what is a percentage or that or what is half of that… Main one is with linear measurement if they're having to measure. "How much if you only have to build half of that?"*

> Therese (after students have worked out calculation for two coats of paint)*: "What would you do if we were doing 3 or 4 coats? What if it's brand new board that's thirsty, how do we work out all the coats?"*

Therese and Noah described getting learners to think sensibly about metric units and measurement.

> Noah: *When I say cut something at 600 [mm] and they come back and it's 6 cm, you know. So I say, "We're talking in millimetres," and then I do that whole question and answer thing. I don't do as much of that as I would like.*
>
> Therese: "*What metric unit would we use and why?" The 'why' behind it quite a lot of the time because, and often it comes out and you have to laugh. Like I'll say, "Well why would you use metres? Why did you use metres for this metric unit?", "Because he did." So then you know that you need some more discussion around what would it look like if you used centimetres or millimetres for that big house out there? "Oh It'll have big equations and some big numbers to work out." That's all I want to know that they can actually see the advantage or see why in one situation you might use this metric measurement and in the other you're going to change to this.*

Noah described visual and kinaesthetic numeracy strategies that he sometimes shares with his students. One is for checking their calculations for halving a length.

> Noah: *You've got a stick of wood and you need to divide it in half, so I say, you know, whatever the length of timber is, "Do your maths in your head." And then I say, "Mark it, and then turn the ruler and use that same measurement from the other side. And if they don't meet then you know you've done* [something wrong]." *And I say, "Now you don't have to re-think that measurement if you don't want to; you can just halve that* [gap]

measurement." You know, there's varying strategies… so that's a physical and… that's how you use your tools to help you with your maths.

Another is a common-sense method for calculating area of a circle.

> Noah: *So we talk about a circle fits into a square; I always remember that 'cause it's a visual thing. A circle fits into a square. I know what a square is, I know that you, how you find the area. So if a circle fits inside the square then the area of a circle and the square are going to be close to each other. So I don't need to know about pi; all I need to know is that approximate. So when I find an equation and I do the math on that equation, if I get it slightly wrong, say back to front, I'm going to get numbers that are a long way away from that.*

When asked if he has these discussions with his learners, if he teaches the topic this way, Noah responded, "Not all of the time." He said that students are not always receptive or interested. "They're still finding their legs, especially the Level 2 [programme students]."

Therese articulated a belief in the importance of explicitly linking the mathematics lessons with practical activities.

> Therese: *I think that it needs that next step; it needs for them to not just put it away close the book and think, "Right, glad that's over, right, let's get into this work." I think it needs, you need to keep connecting that back to, "Remember yesterday when we were working out those areas? What did we have to look at the can for? What were we finding and is it gonna be the same all the time?"*

She notes that students truly understand, or perhaps trust, the mathematics procedure when they see the results of the calculation.

> Therese: …*sometimes they're like sort of, "I think I get it." But when we actually get the cans and do it on the real wall and mix the real paint up and then they paint it twice and have like two tablespoons left over, now*

*they think they're convinced (*laughs*). The formula worked and they worked it out well because they had enough paint.*

Knowing the learner

The Knowing the learner domain of the embedding triad refers to finding out learners' numeracy strengths and gaps relative to the identified numeracy in their programme of study. Effective educators will also know enough about their learners to respond to cultural and affective factors effecting learning. This may include managing learned helplessness and actively engaging learners who have developed a preference for passive learning (Ako Aotearoa, 2018a; Education & Training Foundation, 2014).

Determine, and build on, what learners already know

This section outlines diagnostic and formative assessment practices described by participants in this study.

Use of the Literacy and Numeracy for Adults Assessment Tool

A cornerstone of embedded numeracy practice in New Zealand is knowing the learners' abilities in relation to the demands of the course and teaching or training context (Tertiary Education Commission, 2008a). A key component of the New Zealand government's literacy and numeracy development infrastructure is the Literacy and Numeracy for Adults Assessment Tool (LNAAT). The expectation is that educators will use data from the Tool to inform effective teaching and monitor learner progress (Tertiary Education Commission, 2017a).

Learners in the Foundation programmes discussed in this study are required to undertake LNAAT reading and numeracy assessments at the beginning and end of their programmes of study (except the Hospitality programme whose learners are exempt due to learning disabilities). Four of the tutors referred to LNAAT results in their narratives, stating that they use the results in conjunction with their own diagnostic tests or processes. Their comments indicate that they view and use the LNAAT results as a sort of benchmarking tool to gauge learners'

numeracy, and that they are wary that results may not actually be indicative of learners' abilities in relation to their courses and demands.

> Monty: …*sometimes the numeracy result from LNAAT can be extremely low and you, with my own testing and with observations and things like that I can think, "Oh actually they* [students] *are a lot better than that*."
>
> Therese: *So that* [LNAAT result] *gives me a good level, however, personally I don't think that that is a true indication of maths abilities, from my observation.*
>
> Abby: *I get a group report. I actually get them* [*the students*] *to tell me before I have to wait for it… I do evaluate them all individually and then try to work with them, and then I can see some holes as well, and hopefully help them.*
>
> Lucy: *In the very beginning of the course we do LNAAT tests so that gives me a quick idea of where they fall* [with respect to the Learning Progressions].

There is evidence that the tutors use the results to identify students who may have particularly low numeracy skills, but no evidence to suggest that they use those results specifically to inform their teaching steps. In fact, when specifically asked if she would approach teaching a topic differently for someone who was assessed at step 2 of the Learning Progressions than someone assessed at step 6, Abby responded:

> *No 'cause I always teach from bottom up. They might require a little bit more tuition on that area in question…But I do look at who are the under* [Progression step] *threes and I have a look and a think about.*

There was no further articulation of planning or teaching actions in response to the "have a look and a think about" statement.

Therese reflected on the LNAAT and her use of the results significantly more than the other tutors interviewed, and she was the only one who related her

definition of numeracy to a LNAAT Progression step. She believes that assessment results are useful as part of knowing her learners, but she voiced concern that the LNAAT score does not necessarily give a true indication of mathematics abilities for students who are unable to read and comprehend the questions. This has been a commonly expressed criticism of LNAAT by vocational tutors and underscores the confusion between assessing and teaching numeracy versus arithmetic. Literacy skills are integral to undertaking a numeracy task. In cases where mathematical representations involve text, an individual's performance will depend on reading and comprehension skills as well as mathematical knowledge. Likewise, mathematical relationships described in words, for example, prices doubling or being twice as high, require interpretive literacy skills. Thus it is accepted that one's performance in a proficiency assessment such as LNAAT will include literacy-related factors (PIAAC Numeracy Expert Group, 2009).

Therese compares her students' reading scores with their numeracy scores for some context around numeracy scores.

> Therese: *I do a comparison so if they are high on reading and they are low on maths, it's a pretty good indication that their numeracy is an issue. But if they're high on reading and score a three or four on the maths it doesn't actually mean that they are going to be good at all of this stuff either.*

Abby commented on the challenge for students to answer questions in unfamiliar contexts that are presented in the LNAAT assessments.

> Abby: *The only thing with the LNAAT thing we've always said it would be really nice if they could relate it to something that the students know. 'Cause it's got building questions in there, you know, they've got questions that they've got no idea of their understanding of it* [the context].

Contextualised diagnostics

All tutors described contextualised diagnostic assessments or activities that they use with learners early in the programmes. Monty has a written test that he marks and then goes through with the learners. He emphasises that the numeracy in the test is fully within the context of his building trade. Therese and Lucy both designed and use attitudinal assessments, asking students to rate their confidence to solve types of problems that will arise in the course and trade, but not actually asking students to solve the problems. Noah described a process of task-setting, demonstration, one-on-one observation of students working out their solutions, and questioning, when asked how he establishes their numeracy level. He also uses a printed questionnaire with a project early in the course that assesses related numeracy understanding. Students complete and hand them in and Noah says he can "get a feel for their understanding". Abby did not describe any diagnostic assessment, apart from the LNAAT reports. She states that she does one-on-ones with students once a term to find "where the holes are".

The purpose of the diagnostic activities appears to be as much for warming students to upcoming topics as for determining next teaching steps. None of the tutors discussed planning learning based on diagnostic results. Their conversations suggest that one-to-one questioning and conversations with learners when mathematics topics are introduced is the main strategy for determining students' strengths and weaknesses.

Address and evaluate attitudes and beliefs about mathematics

Effective adult educators will create adult-to-adult relationship and environments that are different from school where learners may have failed (Higton et al., 2017). The narratives outlined below reflect how participants in this study address this aspect of engaging learners in numeracy learning.

Not school mathematics

In describing their practice of embedding numeracy, all of the tutors were keenly aware of anxiety, phobia, and even loathing towards mathematics that many of their learners come with. This sympathy, and for two of the tutors empathy, for

students' fears and feelings about mathematics leads to tutors making explicitly expressed efforts to camouflage or normalise the mathematics and numeracy teaching sessions. They make a concerted effort to make it not like school mathematics.

> Monty: *So we could be doing a project and we'll be using numeracy and they won't ever see, hopefully they won't be aware of how much numeracy we're using and it becomes a second nature thing to them.*
>
> Therese: *But in my practice I think it's important to take opportunities to expose students to that without making it a maths lesson.*
>
> Abby: *... and then I tell them we're doing maths and they say "oh no I hate maths!" and I go "But yeah now you do it!"* [laughs]*. But putting it into something they like and enjoy doing makes them understand it a lot easier.*
>
> Sue: *…that's kind of where that embedding thing comes in, it's just part of the, sort of, it flows naturally within the process.*

Lucy uses embedded numeracy activities as a way of normalising mathematics for her learners, and a way of removing some of the negative connotations that learners have regarding learning mathematics. She says of an embedded numeracy in the word processing activity which she has built up to through earlier numeracy activities, "If I started like that, it was just too much like maths and like school but now that they've got it, it's kind of exciting because they want to get it down because they know, 'I know how to do this!'"

Personal challenges

Both Lucy and Abby discussed their own anxieties and phobias related to mathematics and to teaching numeracy. When asked to tell about her own confidence about teaching numeracy and mathematics, Abby said, "I hate it. I get so embarrassed because my spelling is terrible, my maths is ugh, average…even as a tutor standing up there, you still feel a little bit thick sometimes when you get it wrong."

However, being honest about her own weaknesses, and laughing and allowing her learners to laugh about her mistakes helps create a class culture where mistakes are seen as learning, a desirable teaching practice (Swain et al., 2005).

Lucy recalls how her fear of her own numeracy deficit being "found out" when she undertook embedded numeracy training, and her need to manage her emotions in that situation, helps her empathise with her own learners, and that has influenced the way she teaches embedded numeracy. She recognises that it is normal for students to come with mathematics anxiety and talks with her learners about those feelings. She tells them that she has gone to mathematics support people herself to fill some gaps in her knowledge and walked out feeling "crikey it really wasn't hard at all." She encourages learners to "own" the bits they are not good at, and to push but also support each other. She believes in using humour and aims to create a safe environment where trying is safe. Attending to learners' fear of being found out, as Lucy has described, is identified as an aspect of effective practice for teaching young adults by Higton et al. (2017). They recommend using classroom assistants, peer-learning and self-rating systems to help alleviate this fear.

Permission to not know everything

Lucy and Abby both espouse the view that it is not so much about knowing the answer as knowing that one is out there and having skills and confidence to look for it.

Noah also sees his role as making learners aware that they do not have to know or remember all the mathematics and procedures or formulas that they are exposed to in his programme, if they can learn the principles of what the formula is trying to achieve.

> Noah: *You don't have to teach them exactly the maths, but they need to understand that there is a solution here. There is a way. If they understand the principle of an area of a circle, so they don't know... they've forgotten pi. But they think, "Now there's something out there that," you know, "It's got this name; I need to understand what that is."*

So then they can go online...and that's how I've done it in the past too. You go online and there's a few things that stick with you.

Likewise, Monty says, “We can’t teach them everything, but we try to expose them to a lot of stuff so that later they can go, ‘oh yes, I’ve seen this before’ and then they can learn that.”

Therese takes care to ensure that learners who are struggling can get extra help without drawing attention to themselves. She is sensitive to learners not wanting to be seen as dumb by their peers for asking questions or seeking extra help, so she is proactive in supporting them.

Therese: *I'm always in the classroom early and I'm always hanging around at the end and I'll just say, “Can I have a chat with you?” and say, “How are you finding the maths so far?” You know, “How was your experience of it at school? Did you ever like it?” You know, “What are your thoughts on it? Just be honest.” And normally it's, “I hate it because I can't do it,” and then I'll say, “Well would you like to take some of these* [mathematics worksheets] *home and just have a go at them? No one needs to see them and bring them back and I'll check them, or if you don't want to bring them back that's fine as well, but just attempt it.” And most of the response is pretty good and often they do bring them back because they've found it easy, and think it can't be (*laughs*); it might be wrong. So they do bring them back but it's just a good, and I think it just takes away that fear. I think if you can't manage to do that you haven't got them.*

Knowing what to do

Develop a community of discourse engaged in activity, reflection and conversation

Given the tutors’ concerns with contextualising and with helping learners overcome their fears of mathematics, it is not surprising that discussion and questioning feature strongly in their descriptions of numeracy teaching and

learning. Five of the six tutors spend two or three full teaching days per week with the learners, and a sense comes through strongly in the narratives that tutors strive to develop good relationships with the learners and respectful learning environments where learners feel comfortable contributing to conversations. The Building Trades and Hairdressing tutors all stated that the measure of learners' numeracy practice is the success or otherwise of their projects. Providing tutor feedback and requiring student self-evaluation and reflection on tasks or project results is, in my experience as a teacher educator working with vocational tutors, a core signature pedagogy (Shulman, 2005) for trades training. Development of communication and work-ready social skills is a component of all the foundation programmes that the participants teach on.

> Noah: *The emphasis is on groupwork, cooperation, and communication, reflection*, *within the context of the trade.*

Participants were asked as part of the interviews to tell me about unplanned mathematics discussions that occur with their learners. The Building Trades and Retail Skills tutors all expressed with certainty that these impromptu conversations happen regularly and are a major component of their embedded numeracy teaching. Both Monty and Noah stated that these types of discussions happen "all the time".

> Noah: *All the time. That is the majority of the learning, not in set lessons. All right let's sit down, go on the whiteboard and this is what, with this level of student, yeah all the time…you kind of want them to be not aware of what they're absorbing.*

Likewise, Lucy says that most of the numeracy teaching in her course arises from discussions about other topics that have a numeracy component.

> Lucy: *It's usually when we're discussing the topic and without realising it, all of a sudden, we are in a maths conversation. So we don't even call it that; we just carry on.*

Therese related several examples of numeracy-related conversations arising while students are working on practical projects. She also identified that unplanned discussions result from relating a trade-related numeracy skill to more general life contexts.

> Therese: *Little things come out of it like loans, student loans percentages. You know just in daily life and well, how would I work this out?*

Abby and Sue were less certain about the extent of unplanned numeracy discussions. For Sue's very low-level learners, impromptu discussions might be about the cost of things and money. Abby related how she reveals to students that they have been doing mathematics after they have learnt a numeracy related topic.

> Abby: *When we first start, with the measuring and monitoring of colours and things like that, that's a big discussion on it then, when I tell them at the end of the day that that this is maths. That's when they go, "No way!"*

Use rich collaborative tasks and provide opportunities for groupwork

Use of groupwork

All of the tutors make use of groupwork and will, at times, intentionally engineer the groups for specific purpose or dynamics. The tutors recognise that working as a team is a learned skill that many of their younger learners, especially, have not developed. Noah, who says he uses about a 50-50 split between individual and group work, reflected on the advantages and challenges of students working together. He finds that the learners he gets "tend to work in isolation" and find it difficult to work as a team. "So if they have a problem, they will come to me 90 percent of the time." Noah manages this by asking other learners in the group what they think and getting the individual that asked the question to use their ideas. He models valuing other's opinions when working with a group with responses like, "Yeah I never thought of that, I'm gonna use that. That's a good idea." Abby also mentions that in using group work she tries to get the learners

to take the lead more than her because, “otherwise I’m just like a school teacher, and with the feedback we’ve been given from them they don’t want that.”

The tutors talked of group dynamics and the need to monitor and support equitable participation in groups. Reasoned choices are made to group learners by both varying and similar abilities. Monty groups across abilities to encourage peer teaching, and to avoid having a group who would all be struggling with their numeracy. Abby uses LNAAT scores to group across abilities for the extended group project on budgeting. Although peer-led learning approaches are believed to be more effective in mixed-ability groups (Higton et al., 2017, p. 9), Noah reflects that while groupwork can potentially help elevate learners who are “a long way from the main level of knowledge of the group”, realistically the support they get from a group in that scenario is limited. Lucy groups learners with similar abilities “so they help to push each other but also to support each other.” She is sensitive to learners’ fears of being “found out” if they work with someone with higher ability than themselves.

A common issue identified is that of one learner taking the lead, and taking over the whole process, and others in the group coasting or disengaging completely from the activity. Therese notes that with her young learners, “they’re pretty quick to tell me if somebody’s not doing their part.” Conversely, Sue comments that groupwork for her supported learners provides opportunities for them to learn to take the lead, even for simple counting activities like getting enough chairs for a group.

Collaborative tasks

Analysis of the richness of tasks used with the groups and whether they lead to collaborative problem-solving is beyond the scope of my study. I can surmise, based on the narratives about group work and the resources shared by tutors, that on a continuum of cooperative to collaborative learning models, the groups are more likely to operate on the cooperative end. Cooperative group learning is more teacher-directed and learners work towards solving a specific problem or attaining a specific goal. At the collaborative end of the scale, learning is more

exploratory and involves learners in testing, proving, explaining, reflecting, and interpreting. The spoken and written interactions in collaborative learning should lead to greater understanding than would have occurred for a learner working independently, and collaborative group work requires of learners a sophisticated communication skill-set (Alipour, 2016; Panitz, 1999; Swan, 2005).

True cooperative learning has an element of positive interdependence where each individual's success is dependent on the success of others in the group (Johnson & Johnson, 2010). This is hindered by poor attendance issues, a common challenge noted by the tutors. Thus, some of the groupwork employed by tutors does not entail cooperative learning at all. As Swan (2005, p. 37) notes, "There is a clear difference between working *in* a group and working *as* a group."

As an example, Monty refers to an activity he uses early in the programme to support learning to read and interpret construction drawings and specifications. The related numeracy is around understanding scales used in the drawings, and the activity he describes is that learners identify the scale stated on a selection of sample plans and then use scale rules to check the measurements. The exercises in a supporting workbook are repetitive practice of drawing a length to a stated scale using a scale rule and calculating the scaled length using the given formula, followed by calculating an actual length given the scaled dimension and stated scale (Appendix E, Resource 7). Monty says the students generally sit in groups to complete the activity and exercises.

> Monty: *I generally set the tables up so they can be in groups of four to six and then they can work on this in the groups that they choose; so they just come in and sit down. And so, they can work through it individually because even in groups some of them will work individually through stuff. Lots of discussion. I try to manage it and it gets quite noisy.*

So, the students are working *in* a group, but probably not *as* a group and the tasks set require students to use a set procedure as opposed to collaboratively exploring strategies for working out solutions. While the Building Trades tutors all spoke of projects that students work on as a group, the indications are that

groupwork related to numeracy is best described as communally practising a procedure or formula for calculating an answer.

> Therese: *I've got some very basic maths sheets that I bring them out and I get them to do it in a group and I just watch and observe. I see who's sort of stand-off-ish and who's not really wanting any part of this. The ones that grab the sheet and this is easy well we don't need to worry about them.*

> Therese (referring to working out paint quantities): …*but making them work it out and making them do it themselves. And I often put them in groups and get them to do it and check in that, you know, they are participating and doing it.*

There do appear to be elements of collaborative or cooperative learning in the budgeting exercise implemented by Abby (described earlier in this paper) in that learners are encouraged to think through and explore many aspects of flatting and investigate the associated costs and are posed with *what if* and *what if not* type questions. The activity sparked genuine curiosity for the learners and opportunity for surprise. These are all hallmarks of rich collaborative tasks (Swan, 2005). It appears, though, that any exploration and unpacking of the supporting mathematical concepts such as percentages, rates and proportional thinking were given short shrift. The tutor understands numeracy as a set of formulae, and numerical problem solving as finding the appropriate formula to get an answer, and unsurprisingly, teaches it that way. This is apparent in her interview response to "describe an incident that has challenged you about embedding numeracy in your class".

> Abby: *When we were doing budgeting, my students, I gave them the wrong formula. So I found that embarrassing and funny. But I used it to my advantage that we all make mistakes. So I had given them the wrong formula to get to where they were and I had a bright spark, someone who's very good at numeracy, in my class. And she said, "No, no, no, Abby, you do it this way, this way, and this way." And I said, "Why is that?*

Tell me where to find that." So we all went together; so we used that as a learning thing, we all went together on this website so it showed us the right formula to use.

Use effective questioning to generate deep thinking

Tutors were not asked specifically about their use of questions in the interviews, and this study's methodology does not support analysing the effectiveness of questions used. Most of the questions that participants related in the interviews were fairly closed, variations on, "How would we work this out?" However, some of the narratives did provide examples of tutors using questions as a way of prompting learners to make sense of their calculations and encourage some proportional reasoning.

During a painting activity where learners work out an area then use a spread rate to calculate the required amount of paint, Therese uses questioning to get learner to think beyond the answer calculated by formula.

> Therese: "*Well what would you do if you were doing 3 coats or 4 coats?" And so then we ask, "Well what would we do, do you think?" Then that will lead to "Well what about if this was a brand new gib board wall because you were telling us, you said that if things were thirsty and they're going to take more paint, so how do we work out all of the coats?"*

As a warm-up to a project task that will involve numeracy, Monty describes using multi-choice questions that might have two correct answers expressed in different units or form. These types of questions could potentially lead to deeper thinking if a discussion were to follow, but that was not mentioned in the narrative.

Given that there are few or no mathematics or numeracy assessments in the programmes, the tutors say that they use question and answer as a gauge of learners' understanding of the numeracy they cover. That is, questioning is used as a formative assessment tool.

> Noah: *So I check that they've got the cutting list right and then I ask them some questions about the process, about the calculation and little things in there…And I look at their calculation, couple of questions about, just to check their knowledge before they go out there* [the workshop].

Half of the tutors mention their learners using mini-whiteboards to respond to questions posed. The questions that were posed are closed, single-answer questions, rather than "show me…" questions that lead to discussion and deeper thinking (Swan, 2005). The mini-whiteboards are used as a formative assessment tool, but there is no evidence in this study that they are used to encourage conceptual or generalised thinking.

Explore mathematical ideas with concrete manipulatives or visual representations and hands-on activities

All of the tutors interviewed recognise value in using hands-on activities. For the Building Trades and Hairdressing programmes, particularly, much of learning the trade is applying and practising hands-on skills. Their narratives indicate that they transfer that hands-on teaching practice to numeracy-related topics as well. The students use measuring tools, a ready-made hands-on activity, as part of learning the trades, but tutors also described how they use visual and hands-on materials and activities for teaching some numeracy concepts and calculations.

Therese articulated clearly her belief in the value of hands-on and visual supports for learners. She draws on training she undertook for working with dyslexic learners, and feels confident to create resources and works with her colleagues to create shared manipulatives. She has dedicated a quiet area away from the main teaching workshop where she can work with learners who are struggling, practising measuring skills as well as using visual resources to help learners visualise some basic number properties and the calculations required for materials and costing problems (Appendix E, Resource 8 and Resource 9). She describes using concrete and authentic aids while embedding mathematics discussions into practical teaching sessions. For example, after measuring and calculating the area of a room, Therese will direct the learners to tins of paint to

find spread rates and discuss how to use the information they have to calculate the amount of paint required, including consideration of how the spread rate is affected by surface characteristics. She also describes using physical "props" in her theory sessions:

> Therese: *In my theory sessions I will teach things that I struggle to connect to a trade like Pythagoras, and I will still teach it with things - hands on. I got* [name omitted] *to make me the 3 by 4 triangle, 3 by 4 by 5 and I've painted these ones this colour in that... They can actually visually see if we put all of them around there it equals out and they can see the colours and they... so we do a lot of that, a lot of practice worksheets. I do demonstrations with all kinds of weird and wonderful props up there and get them understanding it hopefully.*

Abby uses jigsaw puzzles to teach about working out appointment blocks and analogue time-telling for learners who have only ever used digital timepieces. The learners use protractors and pins with mannequin heads to learn about angles.

Noah includes labelled photos and diagrams in handouts to support mathematics-related vocabulary such as "square" and "parallel" (Appendix E, Resource 6). He also talks about relating metres to students' heights and the length of an arm, then relates that to the size of sheet of MDF board.

Monty uses the string line activity and accompanying visual paper resources described earlier in this paper for visualising fractions and linking to decimals and percentages. He uses 100 mm squares to get learners to visualise and experiment with building up area and exploring how perimeter changes when the shape of an area changes. He makes use of a foam model that can be built up to a cubic metre.

Lucy has a large stock of play money that she uses with role play activities to teach counting change. She also relates money to decimals and simple fractional thinking. Lucy stated that she tries to "turn everything into something visual or

activity-focused" although she does not yet have a toolkit of prepared numeracy resources. Rather, she seems to rely on using the whiteboard to create visuals during class discussions.

Professional learning and development (PLD)

In this section I will outline the views participants expressed regarding how professional learning they have already undertaken has impacted their practice, and what further professional learning and development they desire.

Impacts of PLD on practice

All of the tutors interviewed had a very clear awareness of the expectation that numeracy education be embedded in their courses, and an understanding of the ubiquitous nature of numeracy in education and everyday life. Several of the activities and resources described and shared in the interviews were introduced in embedded LN education courses that the tutors attended, evidence that they are applying what they learned to current practice. Tutors are also continuing to use and adapt resources, including contextualised diagnostic questions, that they developed themselves as part of the training.

Participants were asked to reflect on how they think they are applying the PLD they have undertaken to their practice or how they believe their practice has changed as a result of PLD. The list below is an amalgamation of their responses:

- Recognise the importance of visual representations and supports, especially for those with learning struggles
- Increase the level of activity for learners
- Relate numeracy topics to everyday contexts that students relate to
- Greater realisation and awareness of how much numeracy is in the trade, and therefore more likely to explicitly address it
- Using diagnostic assessments and not assuming what students know or do not know

- More confidence in their own numeracy skills and understanding - realising that filling in a few gaps in knowledge made mathematics much less daunting
- Remembering the anxiety that can accompany being a mathematics learner and using that empathy with students to improve the learning environment
- Using 'guesstimation' and physical models to increase learners' awareness of reasonability of answers
- Awareness of learners' numeracy levels in relation to course demand levels
- Not just teaching the way they were taught, i.e., "the old rote system where you just memorise it"
- Unpacking underpinning ideas, breaking down problems to understand the why and how
- Looking for potential difficulties – "how someone could get this wrong"

These changes in practice are situated within wider changes to programme design.

Programme changes

A whole-organisation approach was always the intent of the New Zealand embedded LN initiative (Tertiary Education Commission, 2009b). A framework for developing organisational benchmarks for LN learner outcomes was developed for tertiary education organisations. It was designed to support self-assessment and evaluation and encourage data-driven decision-making for embedding literacy and numeracy organisationally (Coben & McCartney, 2016; National Centre of Literacy and Numeracy for Adults, 2014a). It is beyond the scope of this study to examine this aspect of embedding numeracy, but there is evidence in participants' narratives that some organisation-wide shifts have affected numeracy teaching practice. Particularly, in the Building Trades programmes, there has been a shift from the compartmentalised delivery of numeracy topics imposed by competency-based training frameworks described

by Ewing et al. (2007) towards programmes designed using more holistic course and graduate outcomes, with greater emphasis on project-based learning and assessment. The result appears to be more integrated delivery of mathematics skills as they are required for projects and less specific assessment of mathematics topics. Weeks, Coben, Lum and Pontin (2017) refer to this intersection between cognitive and functional competences as theory-practice integration. The Level 3 Construction Trade Skills programme has only one numeracy-specific learning outcome, shared by the three programme strands, "perform calculations in a construction environment". The Level 2 programme has no numeracy-specific learning outcomes. The programme document lists "literacy and numeracy" in the indicative curriculum, and includes the learning outcome "use essential skills to solve a range of specified problems" as a vehicle for embedding numeracy in the programme.

Whereas earlier programme designs treated mathematics and calculations units as part of the *theory* to be taught and assessed and then applied in *practice*, the tutors describe a much more integrated approach now taken to teaching the numeracy skills required for projects.

> Monty: *Ten years ago we sat in the classroom for four hours and did theory, did a PowerPoint and then a handout or whatever, and then we'd go do four hours of practical in the practical room. And the practice and the practical might have had nothing to do with the theory, and the assessment was a paper-based assessment wrapped around the handout that they were given in the theory. So it is very, very different* [now]. *We've moved from a paper-based assessment to more practical-based. The theory around that has been that if they can do it in practice, they should be able to move into a different environment and carry on that practice, rather than in the paper-based one where they can just demonstrate on a piece of paper that they can do some calculations but have no idea how to read a ruler. So I guess that has probably been the biggest thing. Whether it's working yet or not, we're not sure.*

Perceived impact on learners

Measuring the impact of professional learning is complex and difficult, and learner outcomes are not a direct gauge of teaching practice (Coben et al., 2007). However, the aim of teacher PLD is to promote changes in teaching practice that ultimately have some positive impact on learners. Participants were asked how they know if what they are doing regarding embedded numeracy development has an impact on their learners' numeracy and how they know whether what they are doing is effective. I also asked their views on whether they think students transfer their numeracy learning to other contexts, given that the embedded numeracy approach is based on that premise.

Contextual application of numeracy

For the Building Trades tutors and the Hairdressing tutor, a stated key measure of learners' grasp of numeracy relates to the products that students produce.

> Noah: *The product. When they actually go out to the machine and they start cutting and when they start seeing that, and then you see mistakes, or you don't see mistakes… If they've done all of the maths right and they've used the instruments correctly, read them, then it should be right.*
>
> Abby: [regarding angles] *The haircut's correct.*
>
> Monty: *They have to produce the project…so if they've made a mistake in their plans we live with that. But they have to build the chair to their plans.*

Therese recognises that even where students have passed the paper-based calculations assessment the real test of their understanding is in applying the numeracy in practice.

> Therese: *I'm thinking that they've all got the maths and I've moved on and then we go out to project and I say, "Right we're going to work out our quantities, measure out our areas", and they're all sitting there looking at each other thinking, "What the hell is she talking about?" I*

would know that that's just tick the box for that assessment... we haven't got what we need. We can't put it into practice.

Therese also considers informal student feedback as evidence of impact. Students say things like, "I've learnt more in 2 hours than I have [ever] learnt," and "I actually knew how much I was going to save." She notices that students become able to "talk the language" and use appropriate mathematical terminologies. Therese recognises that not all students will reach the same level, but observes improvement in confidence, abilities and affective responses to mathematics.

The Level 2 Retail Services and Hairdressing programmes are both unit standard-based programmes. Each includes at least one unit that explicitly assesses some numeracy: a budgeting unit in Hairdressing and cash handling in Retail. Abby and Lucy's stated evidence of what effect embedded numeracy has on their students is related to those topics.

Lucy: *Students are happy to jump in and do the first role plays with the cash register instead of standing back hoping someone else will do that task… Students will often sit next to someone else as well and you know that they've got it because they know that they can teach it to someone else.*

Lucy also gauges success by learners' body postures that reflect confidence: eyes and shoulders are up, and energy is high. Students create a curriculum vitae (CV) as part of the programme and some choose to include numeracy-related skills that they are confident with on their CV, for example, "making up the banking", balancing a petty cash book and preparing a cash register.

Abby is confident that the five-week budgeting activity to prepare learners for the budgeting assessment develops students' numeracy, based on their feedback as well as assessment outcomes.

Abby: *Absolutely yes, because they say to me, "I didn't know that you had to take tax off your gross wage. I didn't know that that would cost me*

that much over that amount of time." So they're giving me that feedback straight away.

Sue stated that she cannot honestly say that her learners' numeracy understanding changes. She does observe increased inclination to participate in activities by most students, and some students demonstrate improved skills in spatial tasks like setting up a room. She recognises that, lacking any summative assessment, there is no way of knowing whether change happens across the cohort, or whether she is observing those who are already good getting better or choosing to participate.

Transfer of learning

Noah states that he has no evidence that learners transfer the mathematics that they learn to complete building projects in his programme to other aspects of their lives. He believes that learning mathematics is a cumulative process; some of what is learned in his course will "stick" and some will need to be re-visited.

Monty surmises that money and time management skills that are discussed and reinforced during the programme are likely to transfer into life skills. He refers to impromptu conversations about how to plan in order to arrive on time and simple budgeting conversations perhaps related to their student allowances. Monty says that these impromptu conversations happen "all the time" and in addition to planned teaching sessions on costings and timings related to projects. Monty also mentions positive feedback from employers and "losing students" to jobs in industry and feedback from previous students as measures of effectiveness. Monty notes that the students lost to industry are the most capable students. This may indicate, as Sue reflected above, that the already confident and capable learners make transferable gains but does not gauge what change happens across the cohort.

Therese and Abby referred to learners' LNAAT results as a measure of students' transfer of learning.

> Abby: *I don't know if I'm testing it* [numeracy] *as much as maybe I could. But I do know with the outcomes with my students, I know they're getting better as well. Looking at my* [LNAAT] *results from the beginning of last year to the end of last year, 70 percent of my students have jumped up at least one level. So I know what we have done had been working.*

Abby did not provide any supporting evidence for her stated claim of 70 percent of students increasing progression levels and based on wider examination of LNAAT reports, the proportion of learners making *statistically significant* gains is likely much lower.

Therese refers to students' comments after completing a second LNAAT assessment as evidence of their numeracy development.

> Therese: *Some of them will say, "I knew what they were doing when they were talking about percentages, I could work that out" … And it's just showing that now they understand it because the assessment hasn't changed, it's still full of words, but they've got an understanding of what they're talking about…*

Desired further PLD or support

All of the Building Trades tutors are confident in their mathematical abilities within the bounds of their trade, and Sue also expressed that she is confident within the bounds of the level she is teaching. They are confident that they understand and can teach the mathematics that students need to complete projects and assessments. Both Therese and Noah noted that while they are comfortable using and teaching the contextual mathematics at the level the of their programmes, they would require time and support to re-learn topics beyond that. They indicated a "use it or lose it" understanding of mathematics.

> Noah: *You tend to teach at a level that fits the level of student that you're at, so you don't tend to expand past that. So if you don't expand past that, you're not reinforcing your own professional knowledge.*

> Therese: *There is some maths that I haven't visited for a long time that I wouldn't, if I knew I was going to teach it, I would want to go and spend time with someone who is really confident and revisit that and get confident.*

Therese reflected, however, that not all tutors she has worked with have the confidence to teach the mathematics or "non-practical" components of the programme.

> Therese: *I've seen and talked to some of them* [other tutors] *and said, "What do you think of this new program?"* [They say] *"I'm just not teaching this rubbish... this is just nothing to do with the trade. It was fine the way it was before; they were learning how to build and they were learning." You know, "No employer wants them knowing about this." You know, and that isn't because they are not wanting, in my view, those comments aren't because they don't want to teach. Those comments aren't that they've suddenly lost their passion overnight. Those comments are "Oh my god I don't know what to do with this."*

She does not feel that there is a clear avenue of support when tutors are uncertain about teaching particular topics and believes there may be a stigma to asking for help. She recognises that the numeracy teaching professional development she has undertaken gives a "snippet" of teaching mathematics in a related and contextual way, but believes there should be more opportunities for professional development related to teaching numeracy in the Trades. She believes there would be value in identifying capabilities and strengths within teams and playing to each other's strengths.

> Therese: *Who in here is great with maths that could help the others? Who in here is great at making resources that could help share with that? Because if you are not actually great at any of them but you're just a good hands-on and relate, it doesn't mean that you should be struggling, standing there overwhelmed.*

Similarly, four of the other tutors expressed a desire for opportunities to observe and share ideas with other tutors about embedded numeracy resources and teaching strategies, both with those teaching in a similar context and between departments.

Also desired, are more opportunities for feedback and guidance from numeracy specialists on resources, sequencing and scaffolding learning, and for filling in gaps in their own mathematics knowledge. Numeracy specialists are available through a student learning support division to provide tutorial assistance for learners with very low LNAAT scores. All of the Building Trades and Hairdressing tutors make use of the support service for their learners. Two tutors mentioned getting a numeracy specialist to guest-teach particular numeracy topics or concepts in their courses. This service is, however, very ad hoc and limited.

Monty would like to better use technology to make digital resources that learners can access any time and that are related directly to student projects and tasks.

In summary, the tutors in this study desire opportunities to share practice and learn from others who have also been tasked with embedding numeracy, mirroring the PLD desires identified my literature review. They also value opportunities to consult with numeracy specialists for teaching ideas and support. The tutor who seems to have the most advanced skills and deliberate actions for developing learners' numeracy is also the one who recognises that PLD she has undertaken so far is only a beginning to understanding the complexities of teaching numeracy.

Chapter summary

In this chapter I presented my findings from interviews with six vocational tutors who are embedding numeracy as part of their practice. Their narratives indicate that they value and employ learner-centred teaching approaches and there is evidence that they are using, to a greater or lesser degree, teaching approaches and practices that align with evidence-based effective practice. A notable missing practice is "discussing and exposing mathematical misconceptions". This aligns

with the research of Swain and Swan (2009) who found that even teachers who perceived they were implementing the practice did not use it effectively or consistently. Misconceptions are not the same as mistakes and this advanced practice requires knowledge of "how learners come to understand mathematics and teaching strategies that might facilitate this" (Swain & Swan, 2009, p. 81). While a numeracy specialist may effectively employ this practice, it seems an unrealistic expectation for vocational tutors embedding numeracy.

This chapter also revealed the tutors' views of PLD that has impacted their practice, how they believe embedded numeracy impacts learners, and their desires for further PLD. The following chapter provides a full discussion of these findings with consideration of current relevant literature.

Chapter 5 Discussion

Introduction

This study set out to explore, firstly, how selected vocational tutors' characterisations of their numeracy teaching practice aligns with evidence-based knowledge of effective numeracy teaching practice. Secondly, the study examines how these tutors consider any professional development related to embedding or teaching numeracy has impacted on their practice, and what further professional development or support they regard as useful to improve their practice. My aim was to develop a picture of what embedded numeracy looks like in practice from the perspective of vocational tutors. The picture is filtered, admittedly, through my lens as a numeracy specialist who has worked extensively with vocational tutors developing skills to meet the expectations of New Zealand's embedded numeracy initiative.

The preceding Findings chapter provides an in-depth analysis of tutors' descriptions of their embedded numeracy practice framed by New Zealand's prescribed pedagogical model for planning and delivery of embedded teaching and learning: Know the learner, Know the demands and Know what to do. In this chapter I will summarise and discuss the findings using the same framework. I will then discuss the perceived impact of professional development on tutors' practices and their desired further professional development and implications of these findings, all in light of existing research.

Knowing the demands

The participants in this study are well-aware of the expectation to teach numeracy as part of their practice. They easily articulated examples of embedded numeracy teaching in their various programmes. They understand numeracy as a kind of number sense, expressed in relation to everyday or vocational contexts or demand. Although most of the tutors used the terms "basic" or "foundational" in their descriptions, only one of the tutors described

numeracy primarily in terms of arithmetic operations. Further, the tutor who defined numeracy arithmetically, later in the interview, identified non-arithmetic examples of numeracy demands in her programme. This indicates that the tutors recognise numeracy as more than basic arithmetic. Their descriptions are similar to definitions outlined in my literature review, although more bounded by their vocational contexts.

Situating the numeracy and problem-solving

Embedded numeracy is, by definition, situating numeracy problems and learning in a context that has meaning and familiarity for the learners. I consider there are three approaches to embedding demonstrated by the tutors in this study, defined in large part by programme context and design. They sit on a spectrum from *numeracy-saturated, practice-based* embedding to *opportunistic numeracy development* embedding (See Figure 1).

The numeracy-saturated end of the spectrum is demonstrated by the Building Trades tutors. Numeracy calculations and practices are recognised as regularly-used and essential aspects of the Building Trades. The tutors view numeracy as one of the necessary tools of the trade, as opposed to a subject or topic to be taught. Therefore, the problem-solving is nearly always placed within familiar, meaningful and realistic contexts. The teaching, learning and assessing in these recently redesigned programmes appear to be primarily problem- and project-based. The tutors visit and re-visit the mathematics and numeracy required to complete a project task as and when it is required. The learners' numeracy skills and knowledge are mostly assessed by their application in a project. Problems in the one paper-based assessment that are not directly related to the learners' trade of interest pose a teaching challenge because the learners balk at what they see as non-relevant problem settings. Unlike the problems that are directly related to the hands-on trades training, it is difficult for the tutor to make the problems *real* for the learners.

The second embedding approach is demonstrated in the Hairdressing programme. There are numeracy practices and understanding that are

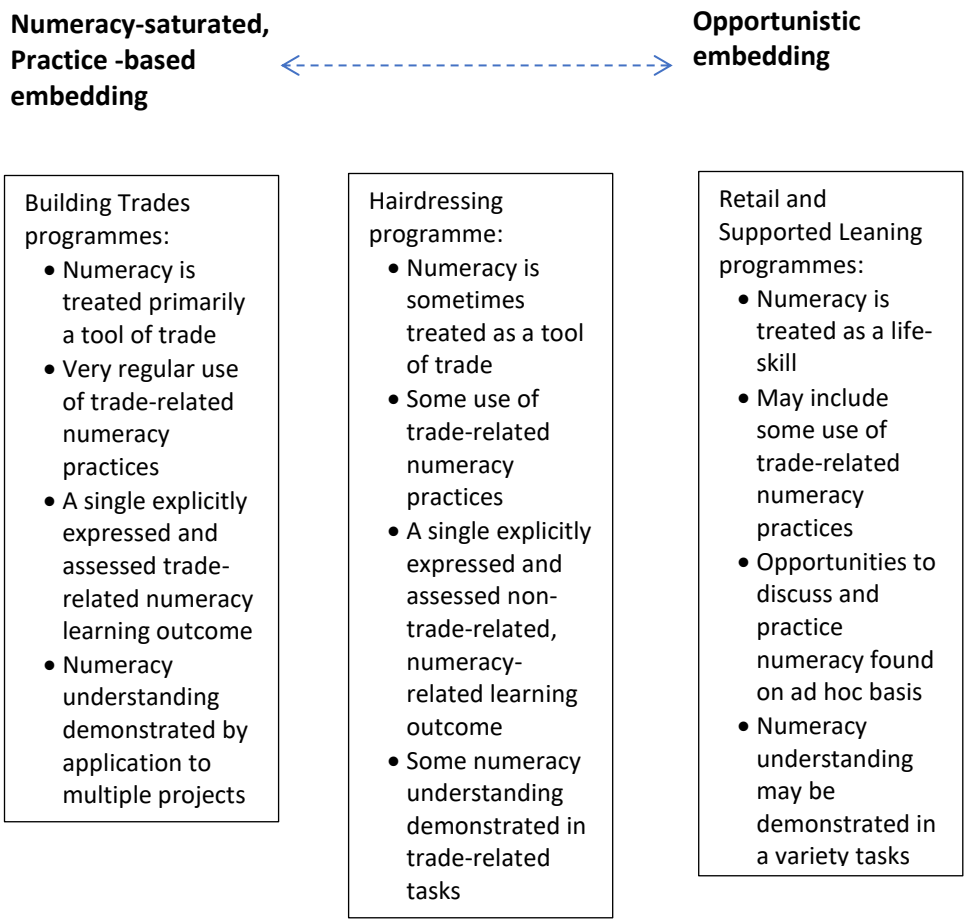
recognised as part of the trade. As in the Building Trades programmes these topics, for example, measuring angles, estimating lengths, timetabling, mixing to given ratios, are taught as tools for the trade, fully situated within the context. The Hairdressing trade is, however, less saturated with numeracy than Building Trades. Perhaps to more explicitly include a numeracy outcome, the programme design includes a numeracy-related, assessed learning unit on household budgeting that is quite unrelated to the trade. The tutor has devised a successful teaching and learning activity situated in a context that has meaning and relevance to the learners in order to prepare them for the assessment which is written in a less meaningful context.

The third approach, demonstrated in the Retail Services and Supported Learning programmes, is embedding by finding and creating opportunities for using and developing numeracy. The Retail Services programme design does include a numeracy related assessment of cash handling; however, the tutor never mentioned the assessment in her narrative, so I place her approach closer to the finding opportunities end of the continuum. The Supported Learning programme has no numeracy-specific assessed units or outcomes. The tutors in both of these programmes have identified situations and contexts that are of interest and relevance to the learners, and where numeracy practices and concepts can at least be discussed. Tutors in both programmes use money scenarios and cash handling as a numeracy-learning context. The Retail tutor related examples of finding opportunities for number and mathematics activities and discussions in very generic situations. For example, learners express class poll results about their emotional state as fractions or percentages.

All along the spectrum of approaches tutors make concerted effort to situate problems in contexts that are interesting and relevant to the learners. Contexts along the saturated end of the continuum are generally more authentic, while embedded numeracy at the opportunistic end is potentially more visible to learners.

Figure 1 below provides a summary of the spectrum of approaches.

Figure 1. Spectrum of embedding approaches



Reasoning and making connections

In preparing learners for the word-problem-based written assessments used in the Building Trades and Hairdressing programmes, tutors provide the students with practice calculations set in scenarios very similar to the problems in the assessment. The students then apply those steps in the written assessment independently, or semi-independently. There is, therefore, an element of rote learning in preparing for the assessment. There were examples provided from the Building Trades tutors, however, of attempts to encourage learners to use reasoning, particularly visual reasoning, strategies to make sense of problems

rather than applying formulae by rote. There were also examples of activities and resources attempting to link and help learners understand the mathematical concepts of fractions, percentages and place value using visuals that encourage proportional reasoning. These provide evidence of tutors' understanding and agreement that teaching numerate problem-solving should go beyond providing formulae for learners to memorise and/or apply. In casting my "numeracy specialist eye" over the tutors' resources, however, I could see potential points of confusion for learners in many of the resources, which could be easily remedied with minor editing. Collaboration between the vocational tutors and a numeracy specialist would improve the resources and could almost certainly improve the teaching sequences that are used with the resources (Casey et al., 2007; Dalby & Noyes, 2015; Roberts et al., 2005; Tertiary Education Commission, 2009a; Thomas & Ward, 2009; Whatman et al., 2011). The vocational teaching environment is context-rich, but time-poor. The benefits of an embedded approach to numeracy development are compromised by the lack of resourced collaboration time between vocational tutors themselves and between vocational tutors and numeracy specialists.

Where numeracy is embedded as a tool of the trade, as in the Building Trades programmes, tutors' narratives indicate that the strategies and formulae for completing a task or required calculation are introduced, taught and re-taught as they are required. Learners practise numeracy within the context of several projects, and they can see the result, right or wrong, of their calculations and measurements in the resulting project output. The Building Trades and Hairdressing tutors all described prompting the learners to think sensibly about measurement units and scale, noting a disconnect between how learners understand measurement concepts abstractly from school and the physical sense of measurement required in the trades. Tutors also use the numeracy embedded in workshop sessions to challenge learners to think beyond the stated problem, by asking how the answer or calculation would change if some aspect of the scenario changed. Non-conventional, builders' strategies for calculating or checking answers are sometimes shared with learners.

Research suggests that apprentices who learn the mathematics that they need gradually, through meaningful calculations are likely to be able to apply the knowledge in other similar contexts, as long as the contexts are meaningful to them. Pragmatic calculation knowledge was not shown, however, to transfer to mechanical context-free calculations of the same order (Coben, 2003). It seems likely, then, that learners will transfer the numeracy skills and understanding that they practice regularly in these programmes into other similar contexts. The skills associated with measurement practices, calculations and conversions, for example, would seem likely to transfer widely. But procedural calculations practised regularly in one context, for example calculating paint quantities from spread rates or mixing hair colour using ratios, are unlikely transfer to other contexts unless learners understand the proportional reasoning that underpins the calculation formulae and procedures they have practised. Transfer is most likely to occur when the learner understands both the facts and the underpinning principles that can be applied to problems in new contexts, and can detect and connect the similarities in problem-solving situations (Gillespie, 2002; Salomon & Perkins, 2015).

Proficiencies and practices

In evaluating the success of embedded numeracy, tutors most commonly referred to students producing a "good product" as evidence of their numeracy skills. In other words, learners' numeracy competency, and thus the impact of embedded numeracy, is assessed by their abilities to use numeracy skills in practice. Learners' increased willingness to participate in numeracy practices such as cash handling was also mentioned as a sign of successful embedded numeracy. Only one tutor referred to learners' successful completion of a numeracy-related assessment as evidence of successful numeracy development. Another mentioned that students may have shown competency on the assessment, but it becomes apparent when they cannot identify and use the same calculations in a practical session that they have not retained the learning. Tutors are gauging learners' numeracy abilities and development by their engagement in context-related numeracy practices. These implicit embedded

numeracy outcomes are what the National Centre of Literacy and Numeracy for Adults (2014b) describes as embedded numeracy practice objectives, tasks that cannot be completed if learners' numeracy practices are ineffective. Numeracy development is assessed implicitly through learners' ability to complete numeracy-dependent tasks.

The development of an explicit measure of adult numeracy practices is something that has been promoted both in New Zealand and internationally for some time (Ako Aotearoa, 2018b; Coben & Alkema, 2017; Reder, 2009). A proficiency assessment measures and scores a person's abilities against a defined set of tasks or behaviours that are ranked by difficulty (Earle, 2014). LNAAT, for example, measures numeracy proficiency against the Adult Learning Progressions. Longitudinal Survey results from North America found no relationship between participation in adult basic skills programmes and proficiency change, but a strong positive relationship between participation and changes in literacy and numeracy practices. The study also found that engagement in literacy practices, in turn, leads to growth in proficiency over a long period of time, although the same statistically significant effect was not observed between numeracy practices and proficiencies (Reder, 2009, 2012). There is often a gap between numeracy adults use in their daily lives and their performance on proficiency tests (Coben & Kane, 2017).

Proponents of the development of a literacy and numeracy practices measure to supplement current proficiency measures argue that programmes have their most direct and immediate effects on adults' LN practices, and that improving the ability of adults to engage in personally meaningful practices is a primary purpose of LN provision. So, there is a misalignment between the effects programmes have on students' LN development and the short-term proficiency gains for which programmes may be accountable (Ako Aotearoa, 2018b; Reder, 2009).

New Zealand's LNAAT was designed to provide robust and reliable information about the literacy and numeracy skills of adult learners, primarily to inform level-

appropriate teaching and learning for LN skills development. It was also designed to generate nationally consistent measures of learners' LN skill gain over time (Tertiary Education Commission, 2015a). Most students in foundation level courses are required to complete LNAAT reading and numeracy assessments at the beginning and end of programmes of study (Tertiary Education Commission, 2016). Similarly to international findings, analysis of LNAAT data shows little correlation between time on-programme and proficiency gain in the short term (Coben & Alkema, 2017). Nationally, while tutors value using the LNAAT results to inform teaching, they expressed concerns about using them to measure learner progress and are wary of potential expectations or targets for proficiency gains that might be set (Tertiary Education Commission, 2015a). To date, programme funding is dependent only on using the LNAAT, and is not linked to proficiency gains measured by the Tool.

The findings in this study support the potential usefulness of a numeracy practices measure, given that tutors from all three types of embedded approaches already rely primarily on practices and attitudes to practices in evaluating learners' numeracy skills and development. A measure of practices seems particularly relevant to New Zealand's fully embedded and contextualised approach to improving adult numeracy which is lacking an adult numeracy curriculum and currently, any indicators of the initiative's progress or success other than compliant use of the LNAAT.

Knowing the learner

This is a domain where vocational tutors embedding numeracy may have a real advantage over teachers teaching numeracy classes within a vocational or foundational programme. The vocational tutors in this study are with their learners for two to three days per week and come to know them well as individuals. They can evaluate and address learners' attitudes and beliefs and fears about mathematics regularly over an extended period of time. What is more, at least some of them strongly empathise with learners' struggles and fears towards mathematics.

Diagnostic and formative assessment

While the majority of tutors in this study use self-designed, contextual diagnostic assessments early in their courses, it is not apparent that they use the information gathered to design or adapt teaching plans. Rather, the information seems to be used primarily to identify learners who may need extra support, from either the tutor directly or through referral to learning support services.

Some of the diagnostics used are assessing self-efficacy, asking learners to identify how confident they are in completing particular numeracy tasks. These task-specific self-efficacy questions may serve to expose learners' attitudes and anxieties about mathematics, as well as serve as an indicator of their actual abilities. Strong correlations have been shown to exist between learners' self-efficacy in mathematics and performance where the self-efficacy is analysed at the task level and closely corresponds to the criterial task (Pajares & Schunk, 2001).

None of the tutors mentioned re-assessing learners using similar questions *after* the topics have been taught. Likewise, only one tutor referred to comparing beginning and end-of-course LNAAT results. It appears that neither the diagnostic nor the LNAAT assessments are used as a measure of learners' numeracy development progress. Those tutors that mentioned LNAAT results use them as a broad indicator only of learners' abilities. None mentioned analysing individual learner reports which can be generated by LNAAT to provide some detail on the types of questions learners got correct or wrong. Like tutors surveyed by the TEC (2015a), the tutors in this study view the LNAAT results with some scepticism, and use the results only within the context of other information they have about their learners.

Questioning learners, observing learners working, and listening to group discussions were mentioned as methods for finding out how learners are going and checking their understanding. These are research-recognised formative assessment strategies suitable for adult numeracy education (Hodgen, Coben, & Rhodes, 2009). Four programmes discussed in this study each include one

summative assessment of a numeracy-related learning outcome, and the other two have none. This means explicitly expressed and assessed numeracy outcomes make up only a small proportion of the programmes. The narratives indicate that tutors rely much more on the formative assessment activities above, used within the context of a vocational learning outcome, to gauge learners' numeracy understanding and progress. Application of numeracy within projects and attitudinal changes in learners towards mathematics are the main indicators of learners' progress. Thus, any quantifiable measure of the effectiveness or impact of embedded numeracy is absent.

Attitudes and anxieties

It is strongly evident that tutors place emphasis on attending to learners' anxieties and improving their attitudes to mathematics by increasing their confidence. They understand that teaching mathematics as a school-type mathematics class will not engage their foundation level learners. Tutors related methods for camouflaging the numeracy teaching and learning so students do not view it as school mathematics and disengage. Some spoke of creating learning environments where mistakes are valued as learning opportunities and where humour about their own mistakes and weaknesses is used in a positive way.

A potential strength of New Zealand's embedded approach of vocational tutors taking responsibility for teaching numeracy is that learners do not view them as mathematics teachers. Vocational tutors model using mathematics and numeracy in the role of a tradesperson that learners aspire to be rather than as a teacher of a subject that many learners loathe and fear. Vocational tutors can help learners see numeracy as an essential part of their lives and work. The tutors in this study are open with their learners about their own weaknesses and fears about mathematics, which can help learners accept and move beyond their own anxieties. Some tutors expressed their awareness of learners' fear of their numeracy weaknesses being "found out" by their peers and described strategies they use to protect these learners' self-esteem.

The tutors note that there are severe time-constraints limiting how much time they can spend on numeracy development. They expressed a pragmatic view that they are at least exposing learners to numeracy and they endeavour to make them aware that they do not need to remember everything, but to be aware and have the confidence to search out a solution.

Knowing what to do

There were examples in the narratives in this study of the use of each of the recommended practices related to the knowing what to do domain. Many of these practices are part of the tutors' general vocational teaching practice, and this study looks at those practices as they relate to embedded numeracy.

Conversational discourse

The narratives indicate that tutors often use a conversational approach to learning, where questions and questioning are normal. Given that there are few explicit mathematics or numeracy learning outcomes, it follows that much of the numeracy teaching is somewhat ad hoc. Tutors stated that numeracy teaching and learning often arises from discussions about projects or topics that require some numerate thinking or practices. Unplanned discussions about mathematics arise to a greater or lesser extent for all of the tutors. The common pedagogy in vocational teaching of teacher demonstration followed by student practice and reflective feedback and conversation support the desired community of discourse. Certainly, the environment described by the tutors is conducive to applying this practice to numeracy teaching. This study design does not include evaluation of the depth or breadth of the conversations and reflection related to numeracy or mathematics topics. It seems though, that at the very least, these conversations help normalise mathematics for learners, and participants commented on learners' increased confidence when discussing numeracy related topics evidenced through body language and use of mathematical terms.

Groupwork and questioning

Likewise, groupwork appears to be used regularly in these programmes, although tutors acknowledge that low levels of social skills mean that working

meaningfully in groups is difficult for some learners. For some of these foundation-level programmes group or team participation and interacting effectively with a variety of people are assessable outcomes. Groupwork for developing mathematical thinking is considered effective when centred on rich collaborative tasks that promote discussion, are accessible and extendable, and encourage creativity (Swan, 2006). Of the group activities focusing on numeracy described in this study, only the activity involving learners developing a budget for a flatting situation could be described as a rich collaborative task. While the context and discussions about unrecognised costs and considerations of flatting in this activity provided rich opportunities, there were apparently no rich discussions regarding numeracy strategies or mathematical principles. Rather, students identified and used appropriate arithmetic formulae to arrive at numerical answers. Opportunities for learners to experiment with and share different problem-solving or calculation strategies were missed. The tutor's acknowledged limited understanding of the related mathematics would preclude confidently facilitating these types of conversations.

It seems that often, while students may work in groups, these are better described as groupings because the learners are working independently, together. The numeracy tasks and questions set for groups described by the tutors are primarily for procedural practice, and the groupings promote primarily peer checking opportunities. There were examples in tutors' narratives of questions that encourage learners to think about the reasonableness of their calculated answers and the effects of changing some aspect of the problem. These were usually described as being part of a workshop activity.

Manipulatives and visual representations

The tutors in this study all described using visual representations and manipulatives for teaching and learning. They recognise that keeping foundation-level learners engaged is best achieved with active, hands-on activities supported by visual resources. Most of the numeracy demands in the Building Trades programmes are related to measurement, spatial reasoning and related calculations such as area and volume; a key skill is to read and interpret building

plans and drawings. Thus, calculation problems are often taught and practiced along-side visual representations of spaces, using the measurement tools that learners will use in the trade. This practical approach to teaching measurement and spatial reasoning represents a key advantage of an embedded approach to numeracy, especially if it includes genuine groupwork (M. Baxter et al., 2006). Building Trades tutors also shared examples of resources and described teaching sequences that visually or physically represent mathematical concepts including fractions, percentages, rates and ratios.

Tutors teaching in the less numeracy-saturated end of the embedding continuum also described using some visual and manipulative aids with numeracy topics. The Hairdressing tutor described, for example, manipulatives representing time blocks to support appointment timetabling and using mannequin heads to teach angles. However, the further the programme is towards the finding opportunities end of the continuum, the less developed the numeracy toolkit appears to be. In keeping with the apparent ad hoc nature of embedding numeracy in these programmes, much of the numeracy teaching appears to be discussion-based using the classroom whiteboard.

Professional development: impacts and needs

In this section I discuss how participants in this study consider they are applying PLD they have undertaken in their current practice, and what further professional development or support they regard as useful to improve their practice.

The evidence-based effective practices referenced in this study involve creating learner-centred, active teaching and learning environments using meaningful, authentic contexts for teaching numeracy. The practices related to “Knowing what to do” support teaching numeracy conceptually rather than as a large set of methods and rules to be remembered. While this approach to mathematics teaching is now advocated by some for teaching children (Boaler, Munson, & Williams, 2017), vocational tutors are likely to have learnt mathematics as rote methods and procedures. In order to teach numeracy conceptually tutors need

to understand and teach *how come* a particular formula or procedure works and not just *how to* use it to get an answer. The PLD provided for vocational tutors aimed to increase tutors' use of active, learner-centred teaching strategies and to encourage conceptual teaching of numeracy.

Perceived changes in practice

In reflecting on how they have changed their practice in response to PLD, responses from tutors in this study collectively covered all three "Knowing" domains. Changes related to "Knowing the learner" involve both explicitly finding out about learners' existing knowledge and gaps in knowledge using diagnostics and being more attuned to learners' anxieties related to mathematics. With respect to "Knowing the demands", tutors stated that they are more aware of how much numeracy is in their trade and are more likely to explicitly address it. Several of the changes related by the tutors indicate at least an increased awareness that good practice "Knowing what to do" involves teaching numeracy conceptually. Tutors specifically mentioned the following realisations and changes: unpacking and breaking down problems to understand the why and how, not teaching rote memorisation problem-solving, emphasising reasonability of answers, increasing the level of activity for learners, and recognising the importance of visual representations and supports.

These stated impacts on practice are encouraging because they represent the range of intended outcomes of the PLD provided. They are, however, collective responses from the six participants and the changes mentioned were not consistent between the participants. The tutors who themselves are not confident in mathematics were the ones who noted increased empathy with anxious learners, and those that are confident are the ones who spoke of unpacking underpinning ideas and teaching the why and how of calculations. Clearly the ability to teach numeracy conceptually, and not as a set of methods and procedures, is dependent on tutors having a solid understanding of conceptual connections themselves.

Recognised PLD needs

Although some tutors recognise unpacking and teaching underpinning numeracy concepts as a desirable practice, the narratives and resources shared indicate that clear links are often not made between the concepts and the numeracy practices taught. None of the tutors expressly stated a desire for PLD to increase their own mathematical knowledge, but most mentioned they would appreciate learning more strategies for teaching and engaging learners in numeracy, building on what was modelled in previous PLD. Properly designed PLD for increasing conceptual teaching of numeracy would also increase participants' conceptual mathematics understanding.

The most commonly expressed desire for PLD was opportunities to observe and share practice ideas with colleagues, both within their teaching context and across contexts. Tutors would also value more opportunities to consult and collaborate with numeracy specialists. Collaboration that was specifically mentioned was having resources and plans reviewed by a specialist, ideas for lessons and having a specialist teach numeracy topics so tutors can see teaching strategies modelled.

Concluding thoughts and recommendations

The evidence from this very small study indicates that vocational tutors who have undertaken PLD programmes aimed at developing the concept of embedded numeracy and some teaching and learning strategies that support it are aware and supportive of the expectation that embedding numeracy is part of their practice. Tutors provided examples of resources and teaching sequences that were a direct result of training they have received, indicating lasting impacts of the PLD. The level and amount of embedded numeracy varies widely, however, even across the six programmes included in this study. There was evidence that, collectively, the tutors' characterisations of their practice align with evidence-based knowledge of effective practice, particularly with respect to knowing their learners and contextualising and differentiating teaching approaches from "school maths" to encourage learners to engage and lessen

learner anxiety. Individually, however, much variation is evident in tutors' own mathematical confidence and knowledge and, accordingly, use of teaching practices that develop learners' conceptual understanding of mathematics.

Learners enrolled in different foundation programmes are likely to receive vastly different amounts of exposure to numeracy concepts and practices, and thus development opportunities. Learners in numeracy-saturated programmes like Building Trades will regularly participate in numeracy-related practices and problem-solving. Whereas in programmes where embedding numeracy requires seeking out opportunities to pose numeracy problems and explore numeracy concepts, learners are likely to receive less numeracy exposure and tuition. What is more, within programmes there will be variations in the volume, scope and quality of numeracy tuition provided, dependent on tutors' numeracy skills and confidence. The narratives indicate that numeracy discussion and teaching is often ad hoc rather than programmed into particular points in lessons or courses. Learners' numeracy outcomes, whether implicit or explicitly expressed in the programmes, must vary widely across foundation level delivery locally and nation-wide.

Measuring the outcomes or impact of embedded numeracy delivery is difficult. The LNAAT was designed primarily as a tool to indicate learners' numeracy levels relative to vocational numeracy demands in order to inform teaching plans. While it can also measure change in learners' proficiency, limitations of a proficiency-only measure of numeracy and progress are recognised, particularly over the relatively short duration of many foundation level programmes. Work is currently being undertaken on developing and trialling a practices measure of adult numeracy (Ako Aotearoa, 2018b) that may prove useful in measuring the impacts of embedded numeracy in vocational programmes. Certainly, indications from this small study are that development of numeracy *practices* in a vocational setting makes up the bulk of embedded numeracy delivery.

This study examines only self-characterisations of embedded numeracy practice for a small number of tutors in one organisation. Embedded numeracy practice

in New Zealand is woefully under-researched and under-supported with PLD. Research that includes classroom and workspace observations of tutors and students, examination of lesson plans and resources, and the voices of students is necessary to begin to understand the scope and quality of embedding within programmes. If we accept that effective numeracy development requires conceptual teaching beyond teaching formulae and processes for finding answers to particular types of problems, then we need to determine to what extent this is happening. This is relevant particularly because New Zealand currently lacks any measure of the effectiveness of embedded numeracy delivery in the ITP sector.

It is clear that collaborative opportunities for developing and teaching embedded numeracy are not readily available to tutors. The onus of embedded numeracy has been left solely with the vocational tutors, despite international research advising against this. Tutors have received professional development in the traditional sense of delivery of information and training intended to influence teacher practice. A professional *learning* approach would be useful now with a focus on learners, systematic inquiry into effectiveness of practice, and attention to requisite knowledge and skills (Timperley, 2011). Professional learning implies a tutor-navigated process where individuals create professional knowledge through interaction with information and other people that challenge previous assumptions and create new meanings. This aligns with tutors' expressed desire for opportunities to share practice and ideas with colleagues. Numeracy education focused communities of practice (Wenger-Traynor & Wenger-Traynor, 2015), if suitably resourced, could be a successful medium for this type of professional learning, providing a forum for collaboration with numeracy specialists and teaching colleagues.

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Whitten, D. (2012). Divergent learner pathways: Exploring the mathematical beliefs of young adult learners. In A. Hector-Mason & D. Coben (Eds.), *Synergy: working together to achieve more than the sum of the parts : proceedings of the 19th International Conference of Adult Learning Mathematics : a research forum (ALM-19)* (pp. 209–219). Auckland, New Zealand: National Centre of Literacy & Numeracy for Adults in association with ALM.

Appendix A: Interview focus questions

These focus questions will provide the framework for the semi-structured interviews

Teaching context and embedded numeracy PD or training

What courses and/or professional development have you undertaken related to embedding numeracy and/or teaching maths (prompts: Words add up or Learning for Living, Embedded Literacy and Numeracy course as part of CTLT or NCALNE, Teaching maths in context course)

What courses do you teach? In which of these would you say you 'embed numeracy' or teach embedded numeracy?

Perceptions and beliefs about numeracy tutor's role in developing numeracy

What do you believe it means to be numerate?

One definition of embedded numeracy is that learners experience their numeracy development as an integral part of their vocational studies. What does embedded numeracy look like in your courses? What do you see as your responsibilities for embedding numeracy?

What are you doing in your practice to help learners become more numerate? How do you know if what you're doing is effective?

Mathematics Pedagogy

Tell me about a couple of your numeracy or maths lessons/sessions. How do you plan them? If I walked into a class where you are embedding numeracy, what would I see? (prompts: room set up, resources, silent working, group working, completing workbooks, watching whiteboard examples)

Are there times when maths discussions take place that are unplanned; discussions that arise from problems or other discussions? Tell me about these. (prompts: student to student, student to teacher)

Probes for issues/challenges of embedding

Describe an incident that has raised issues for or challenged you about embedding numeracy in your class.

Can you give me an example of good practice of embedded numeracy in your department?

Professional learning and development needs

Tell me about your own confidence and feelings about teaching numeracy/maths. (probe: confidence in own about maths/numeracy)

Can you describe how you've developed your numeracy or maths teaching as a result of your PD? I.e., How are you applying what you learned to your current practice? (prompts: including any use of TEC-funded adult numeracy resources)

What further learning or support do you think would improve your practice of embedding numeracy? (probe: with reference to previous answers)

Not all questions will be used for every interview. Follow-up questions may be asked to elucidate participants' answers.

Appendix B: Participant Information Sheet

THE UNIVERSITY OF
WAIKATO
Te Whare Wānanga o Waikato

Project Title

Vocational tutor perspectives on embedding numeracy in practice

Purpose

This research is conducted as partial requirement for a Master of Education (professional learning specialisation). This project requires that I choose a topic and conduct research on the topic through using semi-structured interviews.

What is this research project about?

Following several years of TEC-promoted professional development initiatives, embedded literacy and numeracy teaching in foundation level courses is expected to now be 'business as usual' in the polytechnic sector. There is little research about how vocational tutors undertake embedding numeracy in their teaching practice. This research is to investigate how vocational tutors characterise their work in embedding numeracy in courses they teach, explore how related professional development has impacted their practice, and what, if any, further professional development they regard as necessary or desirable to improve their professional practice and learner outcomes.

What will you have to do and how long will it take?

I am seeking your consent to be interviewed by me to explore, from your perspective, what you consider to be embedded numeracy teaching in practice. The interview should take no longer than 1 hour. The precise timing and location of the interview will be negotiated with you.

I may also ask to see examples of resources you use and lessons you've developed to support learner numeracy development in vocational courses. The interview will be recorded, and the recording transcribed.

You will be asked to give consent prior to the interview.

What will happen to the information collected?

The information collected will be used by me to write a thesis for the credit of the University of Waikato paper DSOE593 – Education Thesis (90 credits). It is possible that articles and presentations may be the outcome of the research. Only myself and my thesis supervisor will be privy to the notes, documents, recordings and transcriptions used to develop the thesis paper.

I will transcribe the recordings myself, and use pseudonyms rather than real names in the transcription. You will be given the opportunity to review the transcript before it is analysed. All data will be stored on a secure server.

After the thesis is submitted and marked, notes and documents will be destroyed, and recordings erased. I will keep transcriptions of the recordings, but will treat them with the strictest confidentiality.

No participants or their employer will be named in the thesis or any resulting articles or presentations, and every effort will be made to disguise their identity. The organisation will be referred to generically as a large New Zealand ITP (Institute of Technology or Polytechnic), and only broad references will be made to teaching contexts, eg., building trades, hospitality, business. While every effort will be made to ensure confidentiality, this cannot be guaranteed due to the limited number of providers in New Zealand's ITP sector.

The completed thesis will become widely available, as the University of Waikato

requires that a digital copy of Masters theses be lodged permanently in the University's digital repository: Research Commons.

Declaration to participants

Although I am your colleague and may have been your teacher or facilitator on a Teacher Education or professional development programme, for the purposes of this research project I am a student at the University of Waikato and not in my role as Teacher Educator. The information gathered from you will be used only for the purposes of writing my thesis.

You are under no obligation to participate in this study. A decision to decline to participate or to share artefacts as part of this study will have no bearing on your employment at Ara.

If you take part in the study, you have the right to:

- Ask any further questions about the study that occur to you during your participation;
- Be given access to a summary of findings from the study when it is concluded; and
- Refuse to answer any particular question or request to view your resources, and to withdraw from the study or withdraw any information you have provided before analysis has commenced on the data;

Who's responsible?

If you have any questions or concerns about the project, either now or in the future, please feel free to contact either:

Myself as Researcher:

Lynette Winter

Phone: [redacted]

Email: [redacted]

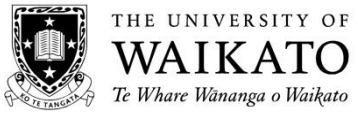
Thesis Supervisor:

Professor Diana Coben

University of Waikato Faculty of Education

Email: [redacted]

Appendix C: Participant Consent Form



Vocational tutor perspectives on embedding numeracy in practice

Consent Form for Participants

I have read the **Participant Information Sheet** for this study and have had the details of the study explained to me. My questions about the study have been answered to my satisfaction, and I understand that I may ask further questions at any time.

I also understand that I am free to withdraw from the study any time before analysis of data has commenced, or to decline to answer any particular questions in the study. I understand I can withdraw any information I have provided up until the researcher has commenced analysis on my data.

I agree to provide information to the researchers under the conditions of confidentiality set out on the **Participant Information Sheet**.

I agree to participate in this study under the conditions set out in the **Participant Information Sheet**.

Signed: _____

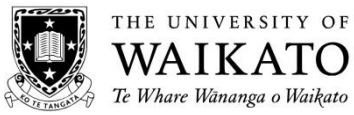
Name: _____

Date: _____

Researcher's Name and contact information:
Lynette Winter
Phone: [redacted]
Email:

Supervisor's Name and contact information:
Professor Diana Coben
University of Waikato Faculty of Education
Email:

Appendix D: Interview Transcription Approval Form



Project Title

Vocational tutor perspectives on embedding numeracy in practice

Approval of transcribed interview data

I have read the transcript of the interview conducted with me for this project on (*date)*.

- ☐ I agree that the transcript accurately reflects my statements and approve the transcript for analysis for this research project.

OR

- ☐ I approve the use of the transcript with the following amendments: (*please attach transcript with any required amendments)*

Signed: _____

Name: _____

Date: _____

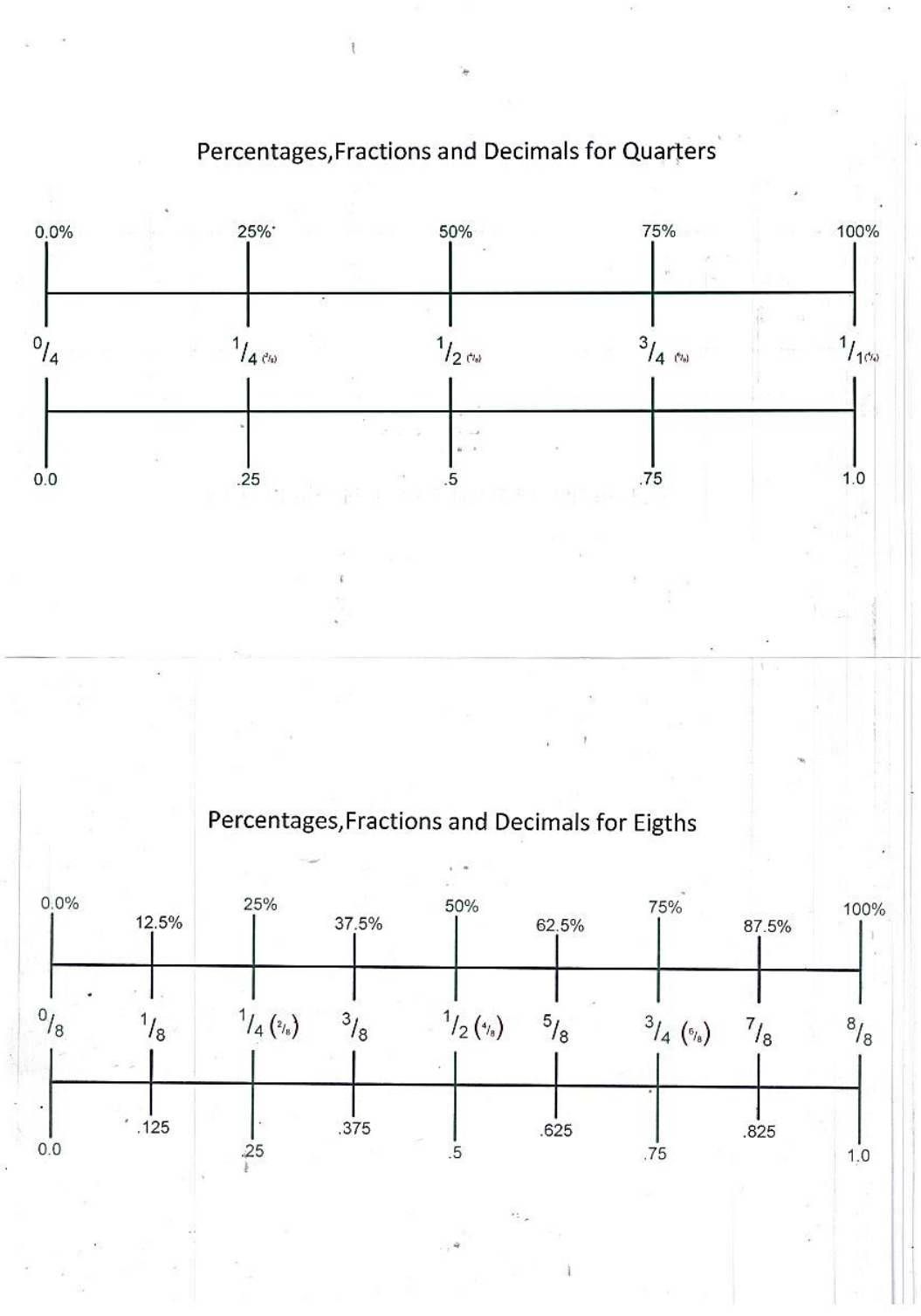
Please return this approval to the researcher within two (2) weeks of receiving the transcript.

Researcher's Name and contact information:
Lynette Winter
Phone: [redacted]
Email:

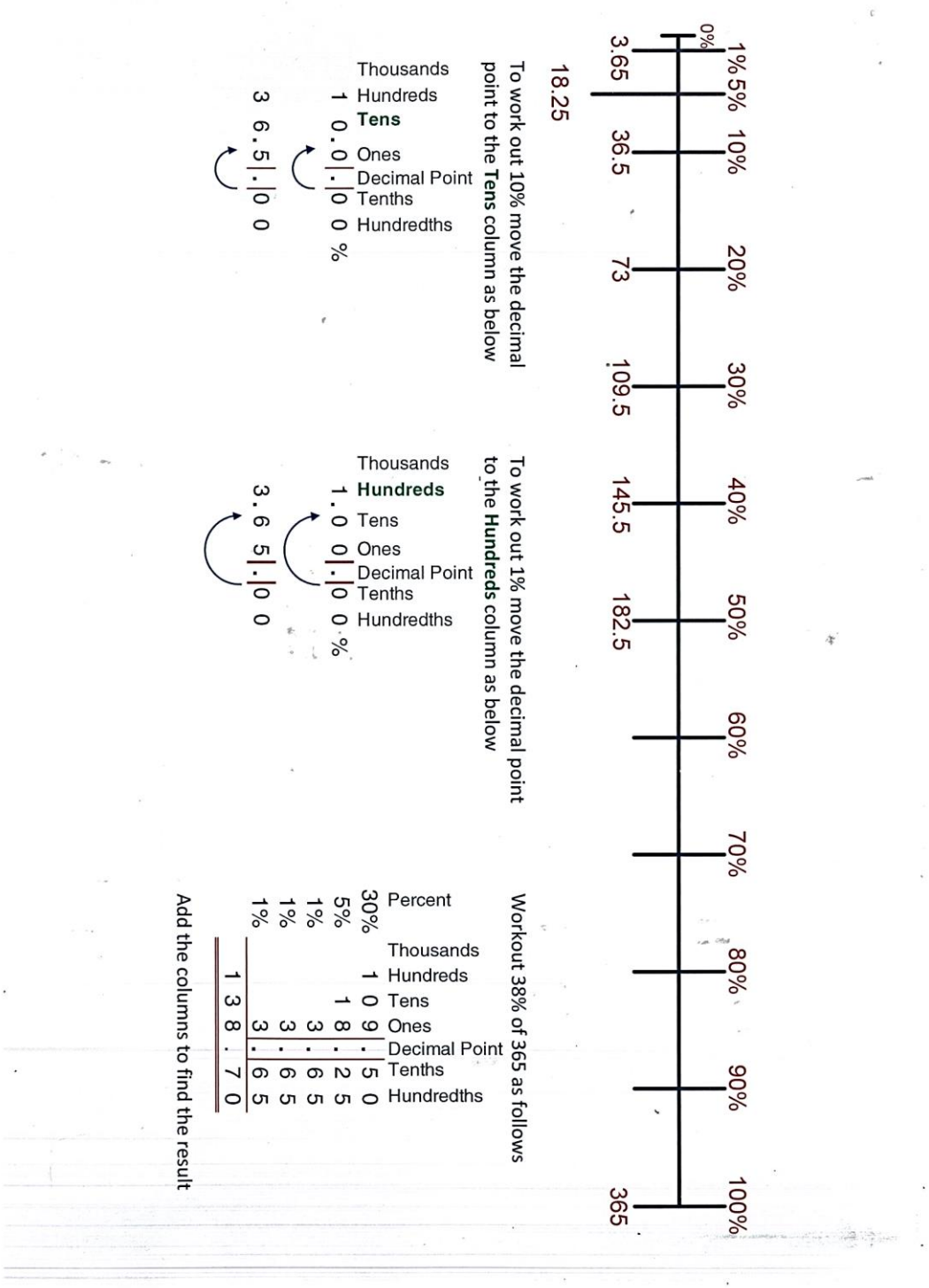
Supervisor's Name and contact information:
Professor Diana Coben
University of Waikato Faculty of Education
Email:

Appendix E: Teaching and learning resources

Resource 1. Visually linking percentage, fractions and decimals

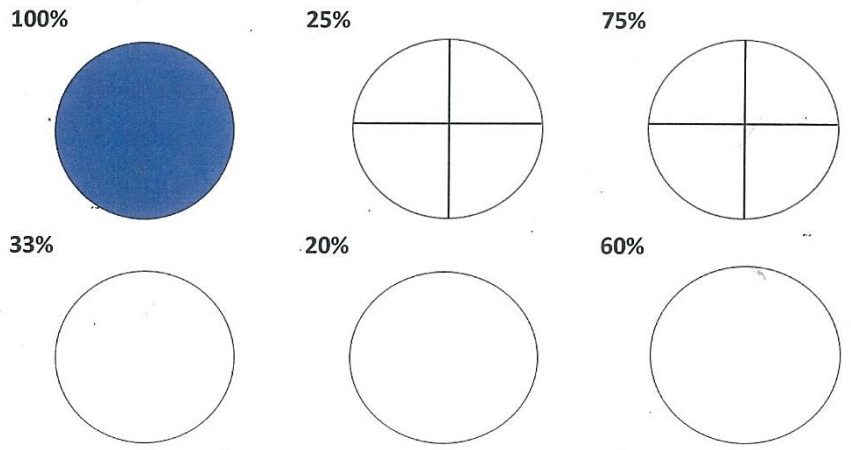


Resource 2. Proportional reasoning approach to percentage calculation



Resource 3. Sequenced worksheets missing link between visual and formula

Working with Percentages



ADDING % FOR WASTE

Percent means part of 100.

To find 10% of 4O metres of timber

$40 \times \frac{10}{100}$ or $40 \times 10 \div 100 = 4$

To add this on it is 40 + 4 = 44.

Calculator operations 40 + 10 %, 44.

Add 15% to 33 metres of timber = _____

Add 12% to 123metres of timber = _____

Resource 4. Guided activity tutor prompts

TUTORS INSTRUCTION SHEET MATHS IS NOT ALOUD TO BE GUIDED

Bread board instruction sheet

1. **Sketch bread board on project procedure sheet.**
2. **Calculate the number of strips needed of pine and beech and the sizes of each strip e.g. Length width and thickness.**
3. **Fill in the cutting list on the back of the procedure sheet**
4. **Plan how to machine the material out of the available timber your tutor will advise you what sizes are in stock.**
5. **Write down your Plan on your project procedure sheet in your operation and tools and equipment needed columns.**

Use numbers to solve problems 26623

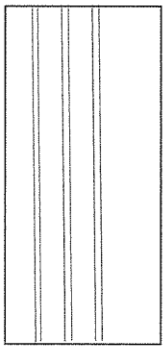
Division **how thick are the strips? 200 ÷ 9 = 22**

Multiplication **How wide will the pine timber need to be. Allowance needs to be made for 5mm loss in the width of the saw cut and enough for thicknessing the bread board down to size after it is glue? The timber sizes that are available are 150mm and 200mm wide.**

Addition

(40mm x 4 = 160mm) + (3 saw cuts x5mm = 15mm) =175

So we have enough timber to cut make our sizes bigger to allow for machining.



Subtraction, division, addition

200mm -15mm =185mm left over 185mm ÷ 4 pieces = 46mm

The strips can be cut to 46mm giving 6mm left over for machining down to the finished size 40mm.

Resource 5. Tutor guide to underpinning numeracy concepts

Problem	Underpinning idea	Teaching points
Interpret plans and produce drawings.	• Understanding 2D and 3D drawings. *To be able to produce a cutting list. View from different view pionts , end elevation plan elevation front elevation* • Understanding metric measurement. *May need to change mm into m. Understanding the prefix m= milli link this to prior knowledge millilitres milligrams.* • Understanding dimensioning principles. *Teaching that everything measures from the outside of the object to prevent confusion with the lines* • Understanding dimensioning symbols and terminology. *Dimensioning arrows point to the edge of the measurement.NZ standards related to the terms\ words used. Common language eg radius* • Understanding Scale. *Being able to measure the plan with a scale rule and then convert this measurement to actual size by multiplying by the scale of the drawing*	• Exercises producing working drawings relevant to the industry. • Dimensioning drawings. • Place a ruler on the line so you can see measurement it relates to • Place value • Moving the point • Divide by 100 • Power @ 10
Produce a cutting list of components from drawings.	• Be able to add and subtract and multiply measurements and quantities from plans. *Multiply the length of the component to see how many multiples will fit into the length \width of the sheet size 2440x1220 MDF .Component length x the number of components. Off cut length = MDF length – total length of components*	• 2 groups of student's .Each group has a cutting list and as a group they work out the sizes drawing them onto the sheet with chalk.
Produce a cutting schedule.	• Understand area. *Estimate sizes in relation to a sheet of MDF 2440x1220mm. Linked back to everyday objects. Like how tall are you.* • Be able to add and subtract and multiply measurements and quantities from plans. • Proportional thinking. *Factors of 12 24 .Visualise \ calculate quickly how many 300 fit into 1200. Understand that 1300 is bigger than 1200 so it won't fit. How they compare area used. Best way round to place component length wise or width wise.*	• Knock off the zeros • Reduce back to 12 x24 • Matrix • Ratio and comparisons • Stand the student up against a sheet of MDF

Resource 6. Student instruction sheet linking text to physical project

Check that all measurements match the plans and that the frame is square and parallel .

Measure from the 100mm point on the tape measure for accuracy.

Parallel

Square

Resource 7. Repetitive practice activity worked on in groups

ACTIVITY

ACTIVITY : SCALING

Complete the following scale exercises in the spaces provided.

Exercise 2

Use a ***scale rule*** to draw a lines from the starting points provided.

1.0 m to a scale of 1:10

0.1m or 100mm

Actual length 1.0 ÷10 = 0.1m or 100mm

a. 1.2 m to a scale of 1:10

Actual length _____

b. 2.40 m to a scale of 1:20

Actual length _____

c. 15 m to a scale of 1:100

Actual length _____

d. 22 m to a scale of 1:200

Actual length _____

Exercise 3

Now work out an actual true length by calculation using a ruler.

Example:

20mm on the ruler at a scale of 1:10

Formula: Dimension x scale

Calculation: **20mm x 10 = 200mm**

a. 35 mm to a scale of 1:10

Formula: Dimension x scale

Calculation: _____

b. 150 mm to scale of 1:20

Formula: Dimension x scale

Calculation: _____

c. 55 mm to a scale of 1:50

Formula: Dimension x scale

Calculation: _____

d. 124 mm to a scale of 1:100

Formula: Dimension x scale

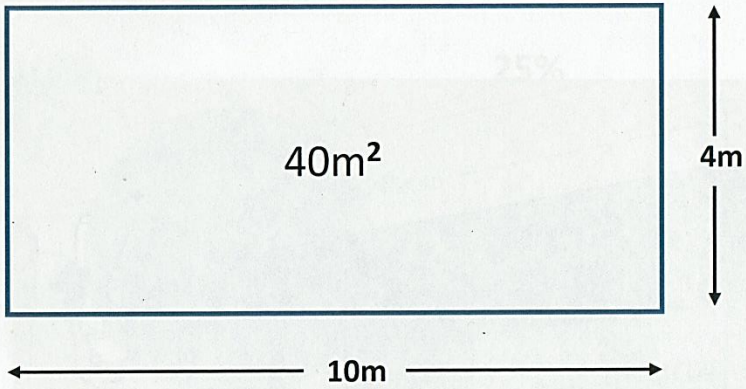
Calculation: _____

e. 215 mm to a scale of 1:200

Formula: Dimension x scale

Calculation: _____

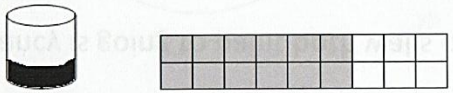
Paint Quantities



Spread rates: (S/R)

Every paint product has a **'spread rate'**. The spread rate can be found on the paint's container or by looking on the internet at the paint manufacturer's (maker's) website.

A **spread rate** of 12 m^2 per litre means 1 litre of paint will cover 12 square metres.



Working out paint quantities:

Step 1: Work out the area to be painted

10m x 4m = $40m^2$

Step 2: Divide the area by the spread rate.

Area ÷ S/R = litres of paint.

$40m^2$ ÷ $12m^2$ = 3.33 Litres of paint.

(per coat of paint)

Step 3: Round up to full litres.

3.33 litres = 4 litres.

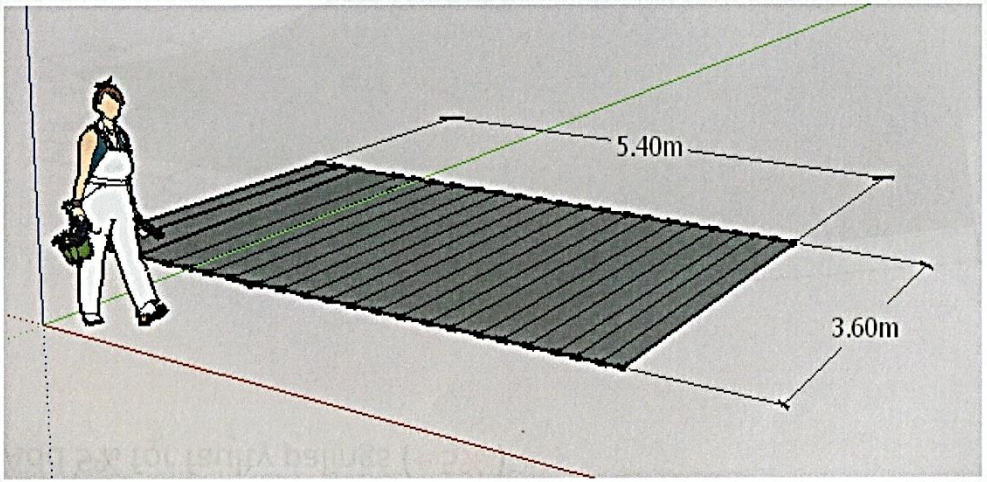
Resource 8. Visual support for calculation

LINEAL MEASUREMENT

Nancy is going to build a deck and needs to order enough decking to cover the 5.4m x 3.6m deck.

Allowing 90mm width for each board, how many boards will be required to cover 5.4 m?

If each board is 3.6m long , how many lineal metres will be required?



Show workings here:

Resource 9. Visual support for calculation

Appendix F: Effective practices

	Swain et al. (2005)	Cohen et al. (2007)	Swan (2005) Swan & Wall (2007)	Whatman (2011)	Education & Training Foundation (2014)	D-G Education and Culture (2013)	Higton et al. (2017)
Know the learner	Link learning to learners' identities and biographical contexts Find out as much as possible about the learners and their abilities in order to realistically challenge them with engaging activities without fearing failure Use contexts and resources that are interesting, relevant and appropriate to the students	Know where learners are with regard to the desired outcomes and extend them beyond their comfort zone	Build on learners' pre-existing knowledge	Use learners' strengths, existing knowledge, and challenges to design tailored teaching that supports learners to construct meaning for themselves Attend to relationships and learning environment while clearly communicating high expectations	Address and change learners' negative beliefs and self-images about mathematical abilities Link current knowledge to social and cultural backgrounds	Use authentic, meaningful contexts	Use diagnostic class-based formative assessment activities leading to personal goals and targets for learners Create group profiles for identifying general topics and concepts to target Develop adult-to-adult relationships
Know the demands	Move away from rote memorisation towards understanding concepts, methods and connections between them	Use subject, pedagogical and subject-specific pedagogical knowledge to enable flexible approaches Encourage learners to make their thinking explicit and to make connections between mathematical concepts	Expose and discuss common misconceptions Encourage reasoning rather than "getting answers" Create connections between topics	Relate numeracy to everyday work and life experiences and support learners to make connections between these experiences and mathematical concepts	Increase learners' perceptions of the relevance of mathematics to real-life and career progression	Concretize the mathematics using real-world problems	Demonstrate alternative methods, persisting until the concept or process is understood Contextualise learning, for example, maths for plumbers

	Swain et al. (2005)	Coben et al. (2007)	Swan (2005) Swan & Wall (2007)	Whatman (2011)	Education & Training Foundation (2014)	D-G Education and Culture (2013)	Higton et al. (2017)
Know what to do	Use social interactions and discussions to inspire curiosity Encourage cooperative and collaborative thinking, reasoning and problem-solving	Strive for clear explanations and break work into smaller steps Provide regular feedback. Plan well to enable flexibility and differentiated teaching Provide a variety of activities (including social learning activities) that engage and extend learners	Use effective questioning strategies Use cooperative small-group work Use rich collaborative tasks Use technology to present concepts in dynamic, visual ways	Provide timely feedback with a balance between challenge and support Use a mixture of teaching approaches including experiential and group learning, avoiding approaches that focus exclusively on correct answers and give little support to thinking	Use contextualised and real-world problem-solving Use activity-based learning	Incorporate collaborative learning for higher level understanding Develop dialogic learning and teaching using effective questioning strategies Use problem-based learning relevant to the vocational course of study	Organise levelled, rather than mixed-ability classes incorporating differentiated teaching approaches Use whole class discussions, student work and mark schemes in a formative approach Make learning different from school where students have failed Coach on exam-taking techniques to extract the most marks from a question Vary the activities and break learning into small chunks