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Original Article

Prediction of the mechanical properties of isotropic pure metal-based and two-phase alloy-based porous materials using modified analytical models

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ABSTRACT

This work investigates the applicability and accuracy of the five fundamental analytical models commonly used to estimate the thermophysical properties of porous materials for the prediction of the mechanical behaviour, both compressive and tensile, of isotropic pure metal-based and two-phase alloy-based porous materials. In literature, the prediction of the mechanical behaviour of these advanced engineering materials requires the development of semi-empirical models, which are material-specific and, thus, require empirical constants. The significance of the current investigation is the possibility to optimise the mechanical behaviour of porous metallic materials through the rapid and accurate prediction of their mechano-physical behaviour using non-empirical physically-based prediction models. The work is complemented with the derivation of new combined models with increased accuracy prediction of up to approx. 90% with respect to fundamental models. Although developed for porous materials, the derived combined models could be applied for the accurate and rapid prediction of the thermophysical and mechanical properties of multi-phase materials with unknown microstructure such as two-ductile-phase alloys, nano-composites, hetero- and harmonic-structured materials, and immiscible alloys.

1. Introduction

Porous metals and metallic foams (i.e. porous metallic materials) are important materials for a variety of engineering applications like prostheses for bone replacement [1], heat exchangers [2], energy absorption structural components [3], and stiff and light sandwich panels [4]. The combination of properties porous materials provide derives from the presence of a distribution of pores at micro and meso level and, therefore, from their microstructure composed of interconnected struts. The features of the interconnected struts are a direct consequence of the manufacturing process as both liquid metal [5,6] and solid-state methods can be used. Among the latter, the space holder method [7], where the metallic powder of the material of interest is mixed with another materials (either organic or inorganic) which will be removed at later stages, is a simple and effective way of obtaining porous metallic materials.

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Nomenclature

CC	Co-Continuous Model
d	Parameter of the general equation to derive the fundamental analytical models
E	Young modulus
EMT	Effective Medium Theory
ME1	Maxwell-Eucken 1 model
ME2	Maxwell-Eucken 2 model
P	Parallel Model
S	Series Model
v	Volume fraction of a phase
Δ	Difference between the overall mean squared errors of a fundamental and that of a modified model
ε	Strain
σ	Strength
χ	Generic property of a porous metallic material (e.g. thermal conductivity)
$\overline{\chi}$	Parameter of the general equation to derive the fundamental analytical models

Subscripts

1, 2	Phases composing the porous material
c	Compressive
FAM_1	Fundamental analytical model one
FAM_2	Fundamental analytical model two
i	ith phase
m	Material under investigation
t	Tensile

The physical properties such as thermal conductivity, electrical resistivity and magnetic permeability as well as the mechanical properties of porous metallic materials are strongly dependent on the composition and the structure. The optimisation of these properties for particular engineering applications can be a time- and resources-demanding exercise and, therefore, their effective prediction would be advantageous. Such prediction is generally performed using different approaches including modelling, (semi)empirical models, and analytical models. Prediction of the mechano-physical properties via modelling (e.g. finite element method) requires computing-intensive rigours simulations of the physical structure, which most of the time is unknown. Regarding empirical models, the applicability of purely empirical models is limited to a given material, the one for which experimental data were generated and used to develop the model itself, as we previously demonstrated [8]. In the majority of cases, semi-empirical models, like the Gibson and Ashby model [9], have a theoretical base but their accessibility is hindered by the presence of parameters that need to be empirically determined. Some of these semi-empirical models are based on the modification of fundamental analytical models to overcome two key limitations. These are the ability to deal with complicated physical structures, as fundamental models general work well only with simple physical structures, and the ability to make accurate predictions over the whole range of porosity (i.e. from 0 to 1).

Regarding the estimation of physical properties, analytical models are preferred over numerical models due to their physical basis, rapid and low cost of calculation as well as reasonable accuracy even when the microstructure is uncertain [10]. Concerning the prediction of the mechanical properties, semi-empirical models are favoured. However, due to mathematical analogy, methods used to estimate physical properties may also be applicable to predict mechanical properties if appropriate substitution of the variables concerned is performed [11]. Consequently, this work examines the applicability of fundamental analytical models via comparing the prediction of these models with experimental data available in literature considering two scenarios with progressing complexity. Firstly, the accuracy of the models in predicting the mechanical properties of isotropic porous metallic materials based on pure metals, where the material is composed of metallic struts and porosity, is analysed. Secondly, the prediction is extended to isotropic two-phase alloys where the metallic struts are composed of two metallic phases. The analysis is complemented with the development of modified analytical models with the ability to effectively predict the mechanical properties of isotropic pure metals and two-phase alloys' porous materials over the entire porosity range. It is worth specifying that, in the context of this work, two-phase alloys are materials composed of two metallic phases, like in $\alpha+\beta$ Ti alloys [12,13], and not materials where the second phase derives from a eutectic or eutectoid reaction, such as in Al–Si alloys [14,15], or through precipitation hardening treatments. It is also worth mentioning that proof of the prediction of the tensile behaviour of isotropic pure metals through some structural-analytical models, especially using the symmetric and interconnected skeleton structural (SISS) model [16], can be found somewhere else [8].

2. Analytical models and their applicability

Five fundamental analytical models are commonly used for the prediction of the physical properties of porous materials and they are: the Parallel Model (P, Eq. (1)), the Maxwell-Eucken 1 model (ME1, Eq. (2)), the Effective Medium Theory (EMT, Eq. (3)), the Maxwell-Eucken 2 model (ME2, Eq. (4)), and the Series model (S, Eq. (5)).

$$\chi_m = v_1\chi_1 + v_2\chi_2 \tag{1}$$

$$\chi_m = \frac{\chi_1 v_1 + \chi_2 v_2 \frac{3\chi_1}{2\chi_1+\chi_2}}{v_1 + v_2 \frac{3\chi_1}{2\chi_1+\chi_2}} \tag{2}$$

$$(1-v_2)\frac{\chi_1-\chi_s}{\chi_1+2\chi_s} + v_2\frac{\chi_2-\chi_s}{\chi_2+2\chi_s} = 0 \tag{3}$$

$$\chi_m = \frac{\chi_2 v_2 + \chi_1 v_1 \frac{3\chi_2}{\chi_2+\chi_1}}{v_2 + v_1 \frac{3\chi_2}{2\chi_2+\chi_1}} \tag{4}$$

$$\chi_m = \frac{1}{\frac{v_1}{\chi_1}+\frac{v_2}{\chi_2}} \tag{5}$$

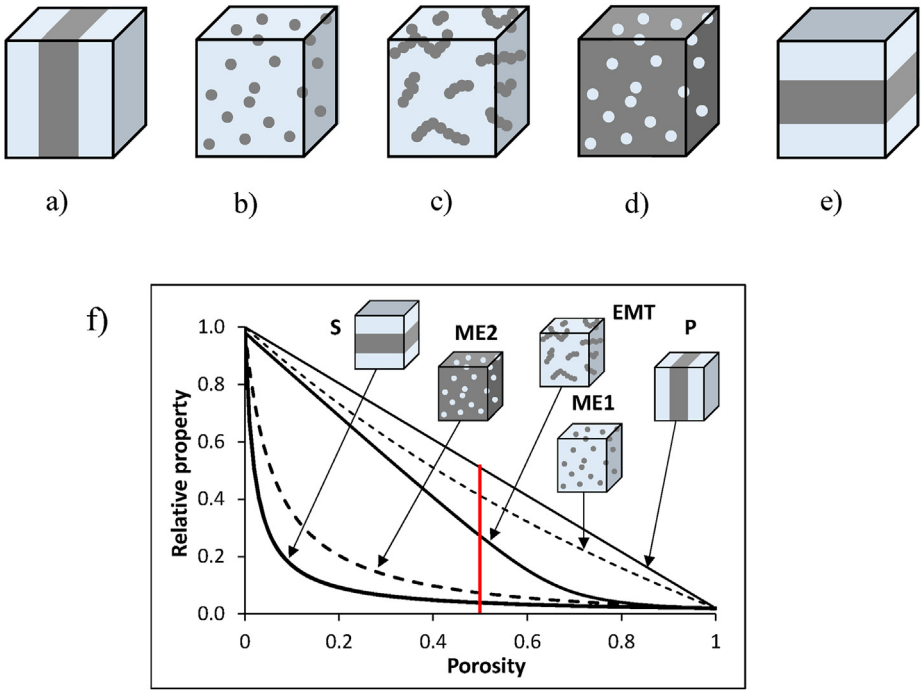

Fig. 1 – Assumed physical structure of the five fundamental analytical models: a) Parallel Model (P, Eq. (1)), Maxwell-Eucken 1 model (ME1, Eq. (2)), Effective Medium Theory (EMT, Eq. (3)), Maxwell-Eucken 2 model (ME2, Eq. (4)), and Series model (S, Eq. (5)), and f) prediction of the variation of a generic relative property. Note: $\chi_1/\chi_2 = 0.02$, which is the ratio between a generic property of the solid and porous phases composing an isotropic metallic porous materials.

These five fundamental analytical models were developed using an assumed physical structure, which is shown Fig. 1 a-e. The assumed physical structure of each model is, respectively, composed of two alternating phases with parallel orientation to the axis of interest (P, Fig. 1a), a dispersion of χ_2 small spheres within a continuous χ_1 matrix (ME1, Fig. 1b), a completely stochastic distribution of the two phases (EMT, Fig. 1c), a dispersion of χ_1 small spheres within a continuous χ_2 matrix (ME2, Fig. 1d), and two alternating phases with perpendicular orientation to the axis of interest (S, Fig. 1e).

A generic property of a porous material, such as its thermal conductivity, depends on the amount of porosity present in the porous material itself, where the higher the amount of porosity the lower the property. The prediction of the variation of a generic relative property, where relative means with respect to the bulk solid material, versus the entire porosity range for the five fundamental analytical models is shown in Fig. 1f. It can be seen that for the same amount of porosity, for example 0.5 in Fig. 1f, the predicted value of the property varies from 0.5 of the property of the solid phase composing the material for the P model to 0.04 for the SE model as each model has a different assumed physical structure that takes into account other features of the porosity, like its distribution within the porous materials, rather than just the porosity volumetric fraction. It is worth mentioning that the curve shown in Fig. 1 are for $\chi_1/\chi_2 = 0.02$, where χ is a generic property of a porous metallic material, as the analytical models were derived from the estimation of the thermophysical properties, for example the thermal conductivity, of porous materials. Moreover, it should be specified that, although each fundamental analytical model can mathematically predict the relative property value over the whole range of porosity, this does not necessarily correspond to a real physical situation. The ME models, which are based on the random dispersion of small spheres within a continuous matrix, lose their physical basis above the threshold for the packing factor of identically sized spheres (i.e. 0.74) [10].

2.1. *Isotropic pure metal-based porous metallic materials*

Quantification of the compressive behaviour of isotropic pure metal-based porous metallic materials is quite common, especially at the high porosity fraction end (i.e. metallic foams). Therefore, Fig. 2 shows the distribution of the relative compressive properties with the level of porosity plotted alongside the prediction of the five fundamental analytical models. The data are from different pure metals including Al, Fe and Ti available in literature [17–27] and, not surprisingly, show that the higher the porosity fraction the lower the relative property, i.e. modulus, strength and strain. It can be noticed that data about the Young modulus and the strength are much more numerous than those about the strain. It is worth mentioning that the relative properties were either directly reported in literature or calculated using the properties of the bulk pure metal as reference [28]. Regarding the compressive strain, this refers to the densification strain and its value increases with the deformability (and thus the porosity, Fig. 2d) of the material and, consequently, its

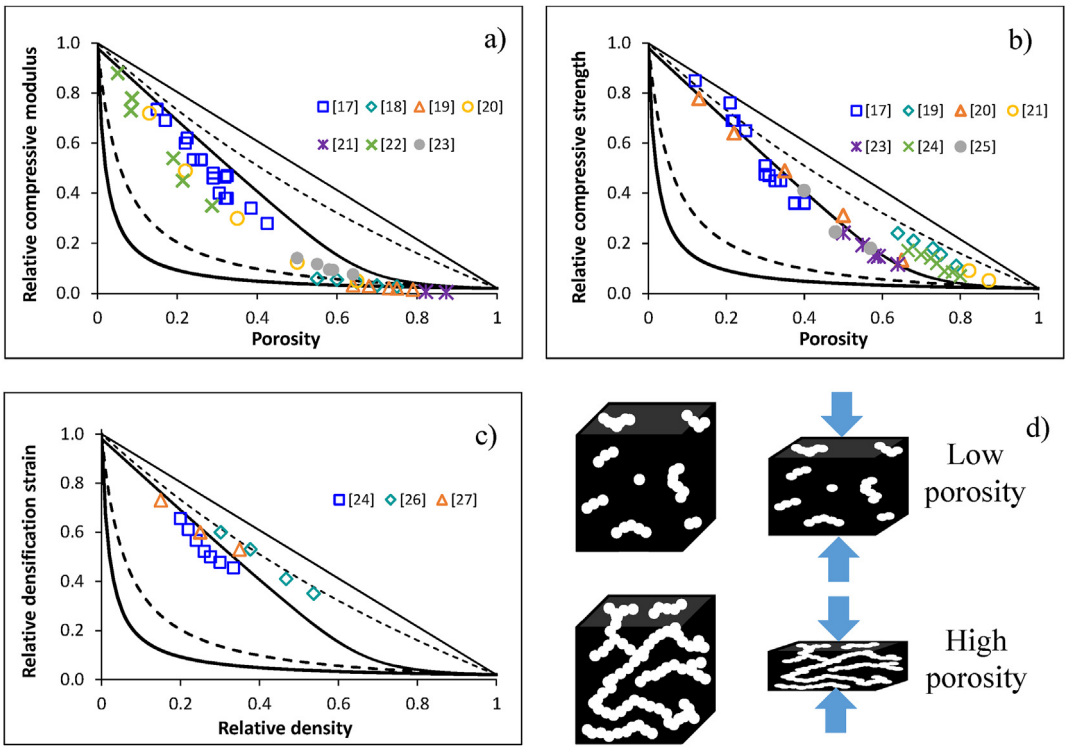

Fig. 2 – Analysis of the compressive behaviour of isotropic pure metal-based porous metallic materials via fundamental analytical models: a) relative modulus, b) relative strength, c) densification strain, and d) sketches of the maximum deformability of materials with different porosity levels. Note: $\chi_1/\chi_2 = 0.02$ (see Fig. 1).

prediction is done versus the relative density rather than the volumetric fraction of porosity (Fig. 2c).

From Fig. 2a, some of the fundamental analytical models are able to accurately estimate the variation of the compressive Young modulus at high porosity fractions (≥0.55) but cannot satisfactorily predict it over the full range of porosity. Even the closest model (i.e. EMT) will give a significant overestimated value, especially in the 0.2–0.4 fraction porosity range. In the case of the compressive strength (Fig. 2b), the scenario is the opposite where the prediction is much less accurate at high porosity fractions (≥0.65) rather than at low volume fractions. The EMT and ME1 models are the ones with the best fit but significantly underestimate and overestimate, respectively, the compressive strength at high volume fractions of porosity. Concerning the deformation strain, the data from the few studies reporting this property [24,26,27] seem to align with the trend of the compressive strength where the prediction of the EMT model becomes less accurate as the percentage of porosity increases, and the ME1 model becomes more accurate.

Conversely to compressive properties, quantification of the tensile behaviour of isotropic pure metal-based porous metallic materials is much more common for low volume fractions of porosity; but data are available for most of the porosity spectrum, as visible in Fig. 3. The number of studies reporting tensile strength and ductility is generally higher with respect to those discussing the Young modulus [8,29–31].

The analysis of the variation of the tensile properties of porous metallic materials (Fig. 3) primarily aligns with that of the compressive properties (Fig. 2). Indeed, the EMT and ME (either 1 or 2) fundamental analytical models can predict the properties at high fractions of porosity (≥0.55), but with a couple of differences. Firstly, the tensile properties, especially strength and strain, are much more negatively affected by the presence of the porosity. Therefore, the data are generally shifted to lower relative values for similar volume fractions of porosity with respect to relative compressive properties. This is due to the much higher stress concentration occurring at the surface of pores while the material is tested under a uniaxial tensile load with respect to when it is tested under a compressive load (Fig. 3d). Consequently, the values of the Young modulus obtained via quasi-static tensile testing are also lower in comparison to those determined using compressive testing. Secondly, the relative tensile strain is better predicted using the ME2 and S fundamental analytical models, rather than the EMT model. Consequently, the P and ME1 models significantly overestimate the mechanical behaviour of pure metal-based porous metallic materials.

2.2. *Isotropic two-phase alloy-based porous metallic materials*

The variation of the compressive behaviour of isotropic two-phase alloy-based porous metallic materials is shown in Fig. 4. In this figure it can be noticed that only Young modulus and compressive strength data are commonly reported in literature while no significant amount of data is available for the densification strain [32–38]. The variation of the

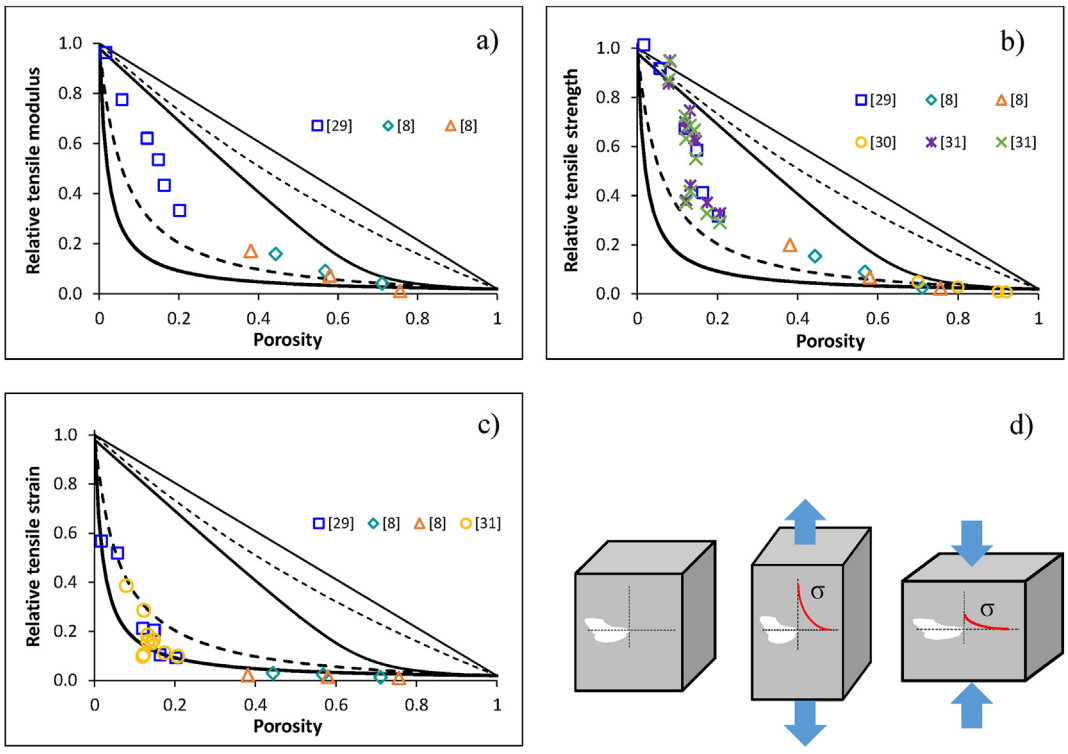

Fig. 3 – Analysis of the tensile behaviour of isotropic pure metal-based porous metallic materials via fundamental analytical models: a) relative modulus, b) relative strength, c) relative strain, and d) sketches of the stress concentration associated with porosity under a tensile or compressive load. Note: $\chi_1/\chi_2 = 0.02$ (see Fig. 1).

properties with the porosity fraction and the two fundamental analytical models that match with the experimental data are the same of the compressive properties of isotropic pure metal-based porous metallic materials (Fig. 2). The only exception is the compressive strength for which EMT and ME2 rather than ME1 and EMT have more accurate predictions. However, arguably, there is a general downwards shift of the relative properties towards lower values for similar porosity fractions when comparing the compressive properties of pure metal-based and two-phase alloy-based porous metallic materials. This indicates that the compressive properties of two-phase alloy-based porous metallic materials are more affected by porosity. Thus, harder two-phase alloy-based porous materials have intrinsically lower relative compressive properties with respect to softer pure metal-based porous materials for an equivalent porosity fraction.

Figure 5 shows the variation of the relative tensile properties of isotropic two-phase alloy-based porous metallic materials along with the prediction of the five fundamental analytical models. It can be noticed that data about the tensile strength are more readily available than for the Young modulus and the tensile strain [39–43]. Moreover, it was not possible to find tensile data for isotropic two-phase alloy-based porous metallic materials with porosity fraction $\geq$0.4. However, analysing the trends it is safe to assume that the distribution will be similar to that of isotropic pure metal-based porous metallic materials (Fig. 3) and the relative properties will be mainly <0.1 for porosity fractions $\geq$0.5.

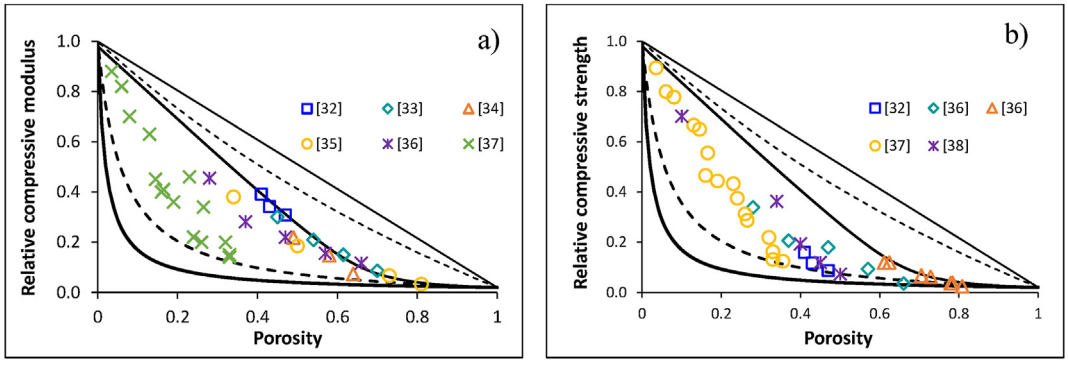

Fig. 4 – Analysis of the compressive behaviour of isotropic two-phase alloy-based porous metallic materials via fundamental analytical models: a) relative modulus, and b) relative strength. Note: $\chi_1/\chi_2 = 0.02$ (see Fig. 1).

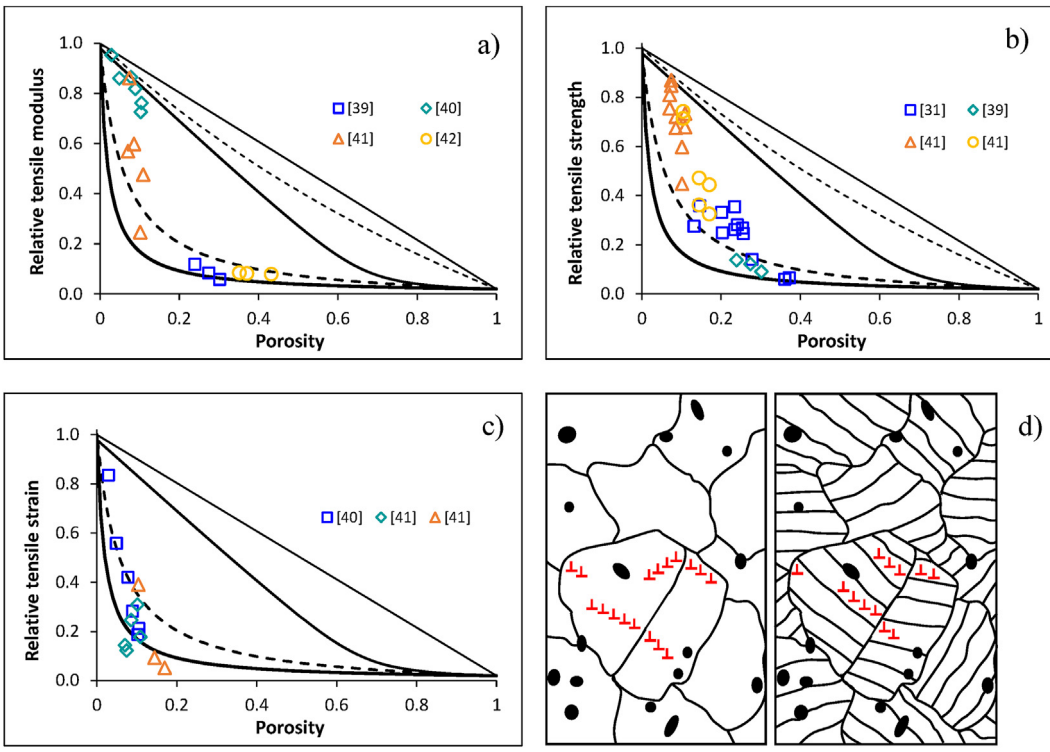

Fig. 5 – Analysis of the tensile behaviour of isotropic two-phase alloy-based porous metallic materials via fundamental analytical models: a) relative modulus*, b) relative strength*, c) relative strain*, and d) sketches of the influence of the microstructure of pure metals and two-phase alloys on the deformability of the material. Note: $\chi_1/\chi_2 = 0.02$ (see Fig. 1). *Some data are from austenitic stainless steel which is not a two-phase alloy per se but, in this instance, it is considered to be so due to the presence of a significant amount of twin grains.

Considering the general distribution of the representative data available in literature, the EMT and ME2 fundamental analytical models are the best fit for estimating the tensile Young modulus (Fig. 5a) and the tensile strength (Fig. 5b) of isotropic two-phase alloy-based porous metallic materials. The ME2 and S models provide a more accurate prediction of the tensile strain (Fig. 5c). As in the case of the compressive properties, the relative tensile properties of isotropic two-phase alloy-based porous metallic materials are slightly shifted downwards in comparison to isotropic pure metal-based porous metallic materials for a similar level of porosity fraction. The only exception is the tensile strength that is slightly shifted upwards. This is due to the intrinsic effect of the expected finer microstructural features of two-phase alloys with respect to pure metals processed under the same conditions and with a similar porosity fraction (Fig. 5d).

2.3. *Accuracy of the estimation of the mechanical behaviour of isotropic porous metallic materials*

The overall mean squared error of the prediction of the compressive Young modulus of isotropic pure metal-based porous metallic materials (Fig. 2a) is 6.39% and 1.07% for the ME2 and EMT fundamental analytical models, respectively. The fairly low value of the EMT model is affected by the distribution of the data with a higher number of them available for high porosity fractions. The ME1 and EMT fundamental analytical models are the best suited to predict the compressive strength (Fig. 2b) and they are, respectively, characterised by an overall mean squared error of 1.24% and 0.35%. The analysis of the few data available for the estimation of the deformation strain during quasi-static compression (Fig. 2c) indicates that the squared mean error is, respectively, 0.83% and 0.50% for the ME1 and EMT models; however, this is most likely biased by the insufficient amount of studies reporting densification strain values.

Regarding each individual tensile property of isotropic pure metal-based porous metallic materials, the overall mean squared error of the prediction of the relative tensile Young modulus (Fig. 3a) by means of the EMT and ME2 models is, respectively, 3.76% and 3.06%. The same models have overall mean squared errors of 4.98% and 8.13%, respectively, in the case of the estimation of the tensile strength (Fig. 3b). Regarding the tensile strain (Fig. 3c), the ME2 and S fundamental analytical models are the closest to the experimental data and their overall mean squared error is 1.06% and 1.12%, respectively.

On the one side, the analysis of the compressive properties of isotropic two-phase alloy-based porous metallic materials via the EMT and ME2 fundamental analytical models yields overall mean squared errors of 4.05% and 3.47%, respectively, for the compressive Young modulus (Fig. 4a), and 4.04% and 3.45%, respectively, for the compressive strength (Fig. 4b). On the other side, the overall mean squared error values of the

analysis of the tensile properties of isotropic two-phase alloy-based porous metallic materials via the EMT and ME2 fundamental analytical models are 10.43% and 7.05% for the tensile Young modulus (Fig. 5a), and 10.89% and 5.17% for the tensile strength (Fig. 5b), respectively. From Fig. 5c, the tensile strain is best predicted via the ME2 and S models and their overall mean squared error values are, respectively, 2.80% and 2.93%. With some few exceptions, mainly related to the prediction of the densification strain during compression testing, the five fundamental analytical models commonly employed to estimate mathematically equivalent thermophysical properties have relatively low accuracy, or high overall mean squared error, when used to estimate the mechanical behaviour of porous metallic materials.

3. Modified analytical models derivation and their accuracy

General equations are available in literature to derive the five fundamental models (Eqs. (1)–(5)) by using suitable choices of the generic d_i and $\overline{\chi}$ parameters (6) [44]:

$$\chi = \frac{\sum_{i=1}^{n} \chi_i \upsilon_i \frac{d_i \overline{\chi}}{(d_i-1)\overline{\chi}+\chi_i}}{\sum_{i=1}^{n} \upsilon_i \frac{d_i \overline{\chi}}{(d_i-1)\overline{\chi}+\chi_i}} \tag{6}$$

In literature, (6) has been used to derive new models where the common approach is to combine two fundamental analytical models. This is either done using empirical weighting factors or by mathematically combining the models. An example of the former is the harmonic weighting of the S and P models (7) as proposed by Krischer [45]:

$$\chi_m = \frac{1}{\frac{\upsilon}{\chi_S} + \frac{(1-\upsilon)}{\chi_P}} \tag{7}$$

Regarding the mathematical combination of different models, the work of Wang et al. [46] analysed different scenarios including the ME1+ME2, the ME1+EMT and the ME2+EMT binary-structure models. Although rigorous, this approach is not straightforward as it might require numerical iteration as part of the estimation routine because some of the mathematically derived models do not have an analytical solution.

The original work of Brailsford and Major (Eq. (6)) [44] also reported a mathematical procedure to derive models for materials composed of two diverse phases dispersed within a continuous phase of different nature. Assuming that the second dispersed phase is characterised by a value of the property of interest, for example thermal conductivity, equal to the average value of the property of the other two phases results in the derivation of a new model. This new model was proposed by Wang et al. [10] and it is named co-continuous model (CC) as the assumed physical structure comprises to co-continuous phases such as in some block copolymers. The model is mathematically expressed as (8):

$$\upsilon_1 \frac{(\chi_1 - \chi_m)(2\chi_1 + \chi_m)}{\chi_1} + \upsilon_2 \frac{(\chi_2 - \chi_m)(2\chi_2 + \chi_m)}{\chi_2} = 0 \tag{8}$$

Using the mathematical definition of the fundamental S (5) and P (1) models, (8) can be rearranged into (9), which is the CC model:

$$\chi_m = \frac{\chi_S}{2}\left(\sqrt{1 + \frac{8\chi_P}{\chi_S}} - 1\right) \tag{9}$$

Therefore, the CC model was used to analyse the variation of the mechanical behaviour of isotropic pure metal-based and two-phase alloy-based porous metallic materials. Nevertheless, at a higher level, the approach taken by Wang et al. [10] of creating a new assumed structure (i.e. two co-continuous phases) can be generalised into (10) to generate new non-standard assumed physical (complex) structures derived by the combination of two fundamental analytical models:

$$\chi_m = \frac{\chi_{FAM_1}}{2}\left(\sqrt{1 + \frac{8\chi_{FAM_2}}{\chi_{FAM_1}}} - 1\right) \tag{10}$$

The hypothesis tested in this work is whether the combination of the two fundamental analytical models with the closest prediction to the actual experimental values, using (10), is able to enhance the accuracy of the prediction of the mechanical behaviour of porous metallic materials over the entire porosity fraction range. The modified models derived and analysed in this investigation are summarised in Table 1.

3.1. *Isotropic pure metal-based porous metallic materials*

Figure 6 shows the results of the analysis of the variation of the relative compressive properties of isotropic pure metal-based porous metallic materials by means of the modified analytical models and the CC model. From Fig. 6a, both the CC and the $\left\{\begin{matrix} EMT \\ ME2 \end{matrix}\right\}$ models underestimate the compressive modulus at low porosity fractions much more than the $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$ model. However, the last two models are also more

Table 1 – Summary of the combined models derived and tested for the prediction of the mechanical behaviour of porous metallic materials.

Combined model	FAM_1 (Eq. (10))	FAM_2 (Eq. (10))	Property analysed
$\left\{\begin{matrix} ME1 \\ EMT \end{matrix}\right\}$	ME1	EMT	- Pure metals: σ_c, ε_c
$\left\{\begin{matrix} EMT \\ ME1 \end{matrix}\right\}$	EMT	ME1	- Pure metals: σ_c, ε_c
$\left\{\begin{matrix} EMT \\ ME2 \end{matrix}\right\}$	EMT	ME2	- Pure metals: E_c, E_t, σ_t - Two-phase alloys: E_c, σ_c, E_t, σ_t
$\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$	ME2	EMT	- Pure metals: E_c, E_t, σ_t - Two-phase alloys: E_c, σ_c, E_t, σ_t
$\left\{\begin{matrix} ME2 \\ S \end{matrix}\right\}$	ME2	S	- Pure metals: ε_t - Two-phase alloys: ε_t
$\left\{\begin{matrix} S \\ ME2 \end{matrix}\right\}$	S	ME2	- Pure metals: ε_t - Two-phase alloys: ε_t

Legend: E_c: compressive Young modulus, σ_c: compressive strength, ε_c: densification strain, E_t: tensile Young modulus, σ_t: tensile strength, and ε_t: tensile strain.

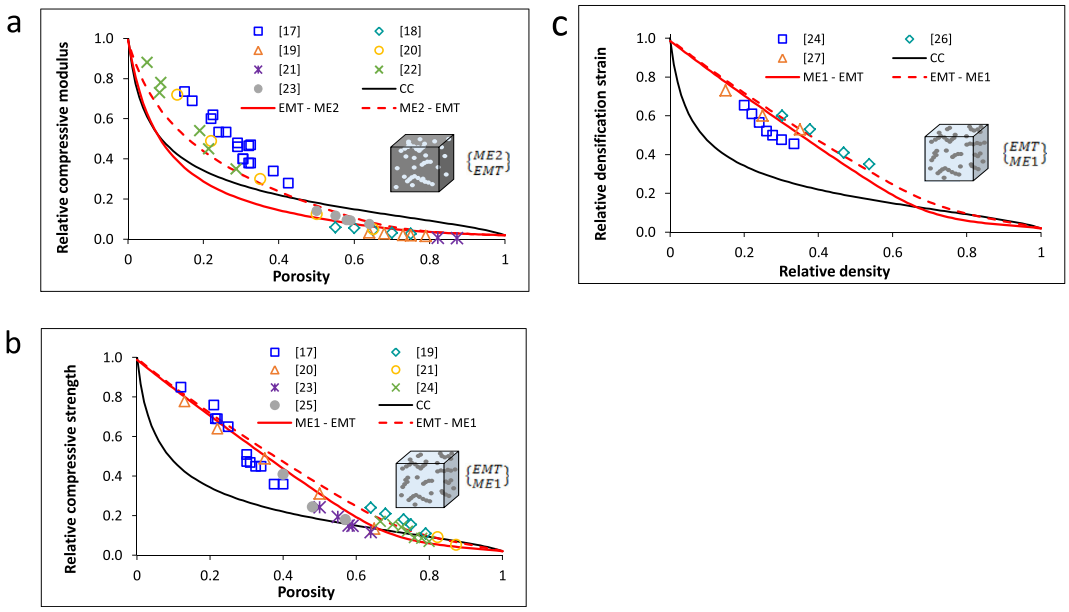

Fig. 6 – Analysis of the compressive behaviour of isotropic pure metal-based porous metallic materials via modified analytical models: a) relative modulus, b) relative strength, and c) densification strain. Note: The insets refer to the assumed structure of the combined analytical model with the best prediction.

accurate with respect to the CC model at high porosity fractions. The overall mean squared error values for the $\left\{\begin{matrix} EMT \\ ME2 \end{matrix}\right\}$, the $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$, and the CC models are, respectively, 3.88%, 1.07% and 2.80%.

In the case of the compressive strength (Fig. 6b) and the densification strain (Fig. 6c), the CC model always underestimates the property at low fractions of porosity and becomes more accurate at high porosity fractions. The $\left\{\begin{matrix} ME1 \\ EMT \end{matrix}\right\}$ and $\left\{\begin{matrix} EMT \\ ME1 \end{matrix}\right\}$ models have similar trends, are more accurate than the CC model, and seem to be fit to properly estimate the compressive property of concern, especially looking at the comprehensive trend of the compressive strength. The $\left\{\begin{matrix} ME1 \\ EMT \end{matrix}\right\}$, $\left\{\begin{matrix} EMT \\ ME1 \end{matrix}\right\}$ and CC models have, respectively, overall mean squared error values of 0.51%, 0.33% and 3.52% for the compressive strength, and 0.58%, 0.48% and 7.09% for the deformation strain.

The prediction of the variation of the relative tensile properties with the porosity fraction of the modified analytical models is shown in Fig. 7 along with the distribution of the experimental data. The $\left\{\begin{matrix} EMT \\ ME2 \end{matrix}\right\}$, $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$ and CC models were used to predict the tensile modulus (Fig. 7a) and the tensile strength (Fig. 7b) and they seem to be accurate, with minor differences, as the porosity fraction increases. Based on the analysis, the overall mean squared error values of these three models are, respectively, 1.25%, 0.35% and 1.34% for the tensile modulus, and 4.75%, 1.96% and 4.28% for the tensile strength. This indicates that the $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$ model provides the most accurate prediction. Concerning the tensile strain (Fig. 7c), both the $\left\{\begin{matrix} ME2 \\ S \end{matrix}\right\}$ and $\left\{\begin{matrix} S \\ ME2 \end{matrix}\right\}$ models seem to give a good estimation of the variation of this property. This estimation is more accurate than that obtained via the CC model, which always significantly overestimates the tensile strain. The overall mean squared error of the $\left\{\begin{matrix} ME2 \\ S \end{matrix}\right\}$, $\left\{\begin{matrix} S \\ ME2 \end{matrix}\right\}$ and CC models are, respectively, 0.44%, 0.46% and 4.69%.

3.2. *Isotropic two-phase alloy-based porous metallic materials*

The estimation of the relative compressive properties of isotropic two-phase alloy-based porous metallic materials shown in Fig. 8 yields similar outcomes to those of the analysis of the tensile properties of isotropic pure metal-based porous metallic materials as the same modified analytical models were used for their analysis. The three $\left\{\begin{matrix} EMT \\ ME2 \end{matrix}\right\}$, $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$ and CC models seem reasonably accurate as the experimental data are distributed both above and below the prediction curves. The overall mean squared error values of these three models are, respectively, 1.83%, 0.80% and 1.16% for the compressive modulus (Fig. 8a), and 1.68%, 0.59% and 1.51% for the compressive strength (Fig. 8b).

Regarding the estimation of the tensile behaviour of isotropic two-phase alloy-based porous metallic materials, the analysis reported in Fig. 9 shows that the $\left\{\begin{matrix} EMT \\ ME2 \end{matrix}\right\}$, $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$ and CC models fit with the data at low porosity fractions but

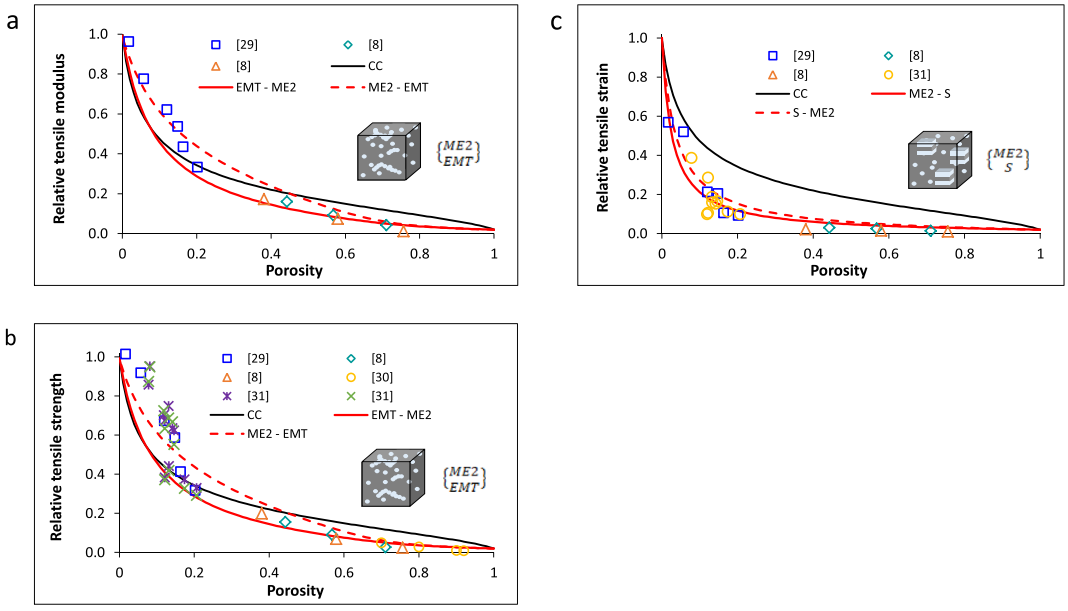

Fig. 7 – Analysis of the tensile behaviour of isotropic pure metal-based porous metallic materials via modified analytical models: a) relative modulus, b) relative strength, and c) relative strain. Note: The insets refer to the assumed structure of the combined analytical model with the best prediction.

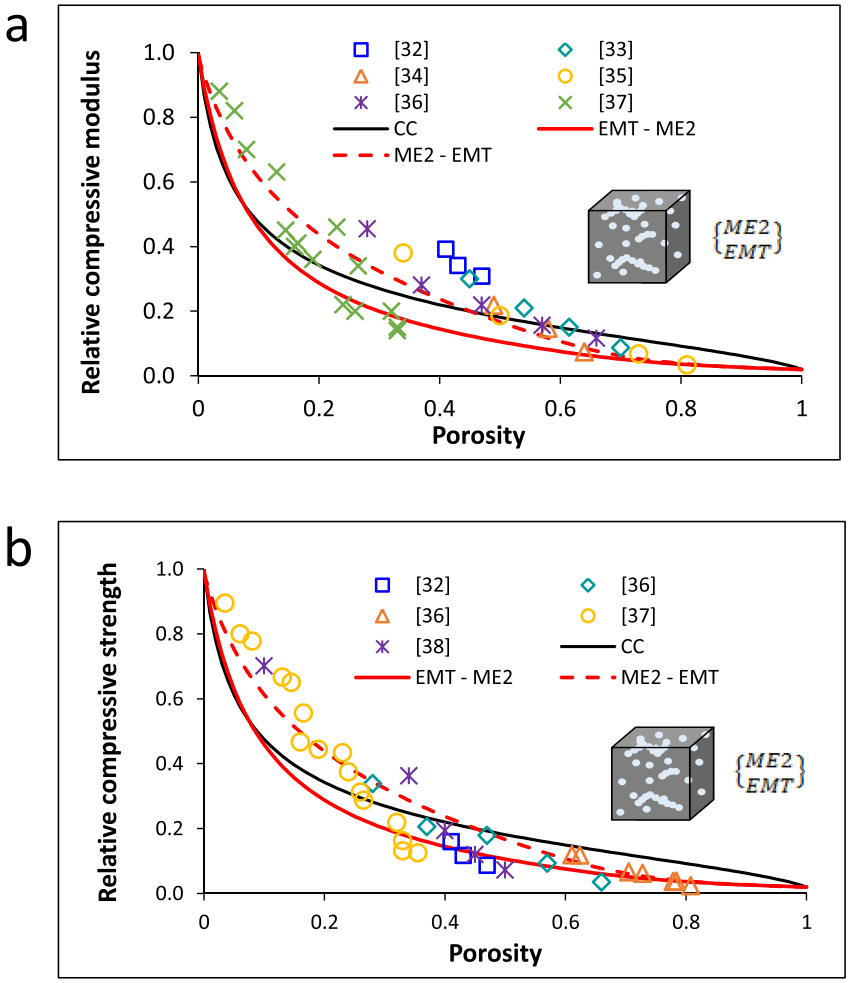

Fig. 8 – Analysis of the compressive behaviour of isotropic two-phase alloy-based porous metallic materials via modified analytical models: a) relative modulus, and b) relative strength. Note: The insets refer to the assumed structure of the combined analytical model with the best prediction.

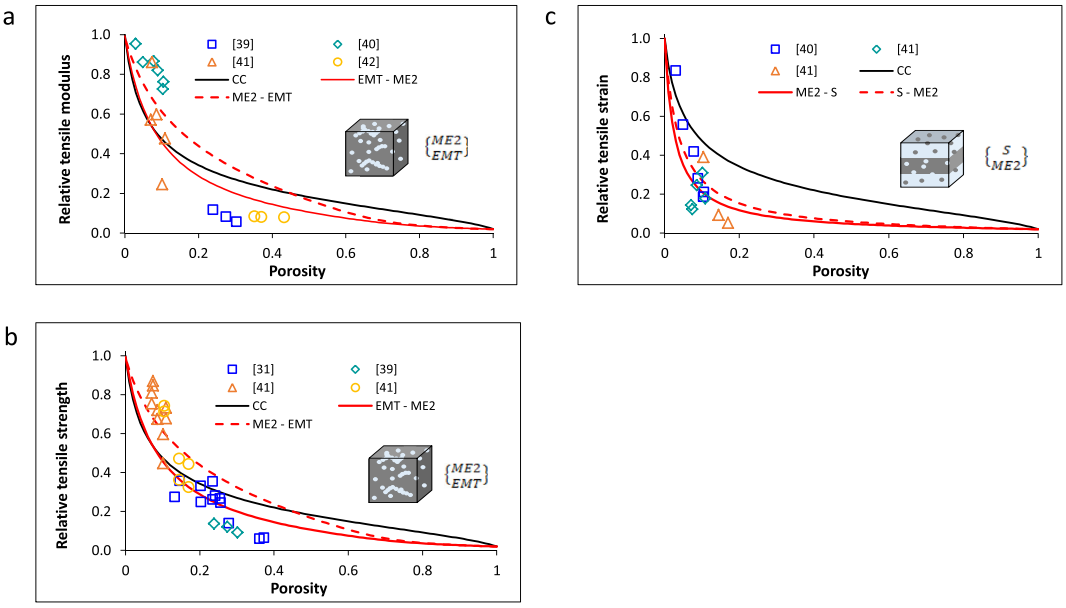

Fig. 9 – Analysis of the tensile behaviour of isotropic two-phase alloy-based porous metallic materials via modified analytical models: a) relative modulus, b) relative strength, c) relative strain. Note: The insets refer to the assumed structure of the combined analytical model with the best prediction.

their accuracy decreases as the porosity fraction increases, especially in the case of the tensile modulus. The overall mean squared error values of these three models are, respectively, 4.37%, 3.77% and 5.02% for the tensile modulus (Fig. 9a), and 2.65%, 2.34% and 2.86% for the tensile strength (Fig. 9b). As for the tensile strain of isotropic pure metal-based porous metallic materials (Fig. 7), the modified $\left\{\begin{matrix} ME2 \\ S \end{matrix}\right\}$ and $\left\{\begin{matrix} S \\ ME2 \end{matrix}\right\}$ models prediction is better than that of the CC model over the entire porosity fraction range (Fig. 9c). The overall mean squared error of the $\left\{\begin{matrix} ME2 \\ S \end{matrix}\right\}$, $\left\{\begin{matrix} S \\ ME2 \end{matrix}\right\}$ and CC models are, respectively, 2.16%, 1.86% and 6.68%.

3.3. *Accuracy of the modified analytical models*

Table 2 shows a summary of the best fundamental and modified analytical models analysed for the prediction of the mechanical behaviour of isotropic pure metal-based and two-phase alloy-based porous metallic materials. Either the EMT model, with an assumed physical structure composed of a random distribution of the metal phase and of the porosity, or the ME2 model, with an assumed structure composed of a dispersion of metallic phase (χ_1) within a continuous matrix (χ_2) constituted by porosity, is the best fundamental analytical model depending on the specific property considered. The majority of the compressive and tensile properties are better estimated by means of the modified $\left\{\begin{matrix} ME2 \\ EMT \end{matrix}\right\}$ model. Exception are the compressive strength/densification strain of isotropic pure metal-based porous metallic materials, for which the $\left\{\begin{matrix} EMT \\ ME1 \end{matrix}\right\}$ model is better, and the tensile strain. The latter is best estimated using the modified $\left\{\begin{matrix} ME2 \\ S \end{matrix}\right\}$ and $\left\{\begin{matrix} S \\ ME2 \end{matrix}\right\}$ models for isotropic pure metal-based and two-phase alloy-based porous metallic materials, respectively. The positive Δ values, which range from 0.004% for the prediction of compressive modulus of isotropic pure metal-based porous metallic materials to 3.28% for the estimation of the tensile modulus of isotropic two-phase alloy-based porous metallic materials, confirms that the modified analytical model generally give a more accurate prediction with respect to fundamental models.

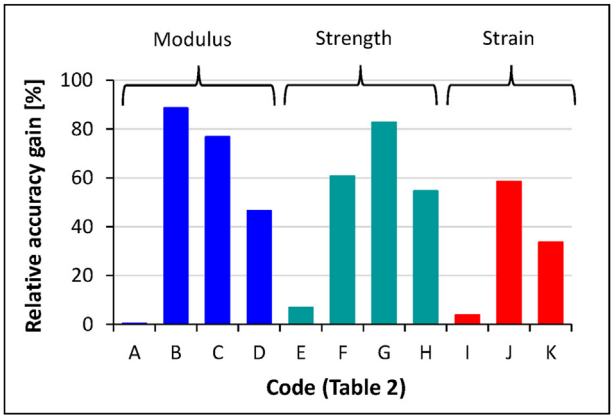

Fig. 10 – Relative accuracy gain of the prediction of the mechanical behaviour of isotropic porous metallic materials when using modified analytical models. Note: The letter code refers to Table 2.

Table 2 – Summary of the best fundamental and combined models for the prediction of the mechanical behaviour of porous metallic materials.

Property	Fundamental model	Modified model	Δ	Code (Fig. 10)
E_c (pure metals)	EMT	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	0.004	A
E_t (pure metals)	ME2	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	2.71	B
E_c (two-phase alloys)	ME2	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	2.66	C
E_t (two-phase alloys)	ME2	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	3.28	D
σ_c (pure metals)	EMT	$\left\{\begin{matrix}EMT\\ME1\end{matrix}\right\}$	0.02	E
σ_t (pure metals)	EMT	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	3.02	F
σ_c (two-phase alloys)	ME2	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	2.86	G
σ_t (two-phase alloys)	ME2	$\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$	2.82	H
ε_c (pure metals)	EMT	$\left\{\begin{matrix}EMT\\ME1\end{matrix}\right\}$	0.02	I
ε_t (pure metals)	ME2	$\left\{\begin{matrix}ME2\\S\end{matrix}\right\}$	0.62	J
ε_t (two-phase alloys)	ME2	$\left\{\begin{matrix}S\\ME2\end{matrix}\right\}$	0.94	K

Legend: E_c: compressive Young modulus, σ_c: compressive strength, ε_c: densification strain, E_t: tensile Young modulus, σ_t: tensile strength, ε_t: tensile strain, and Δ is the difference between the overall mean squared errors of the fundamental minus the modified model.

More in detail, Fig. 10 shows the relative gain in accuracy derived by using a modified model rather than a fundamental analytical model for the prediction of the mechanical behaviour of porous metallic materials. It is worth mentioning that the relative grain was calculated as the ratio between Δ and the accuracy of the fundamental analytical model of reference for each specific property (Table 2). From Fig. 10, the lowest gain in accuracy (0.34–6.82%) is obtained when using a modified model to estimate the compressive properties of isotropic pure metal-based porous metallic materials (A, E and I). The overall highest accuracy gain (58.52–88.55%) is for modified models to predict the tensile properties of isotropic pure metal-based porous metallic materials (B, F and J). With the exception of the compressive properties of isotropic pure metal-based porous metallic materials, a minimum gain in accuracy of 34% can be achieved by using modified analytical models to estimate the mechanical properties of porous metallic materials.

Due to their enhanced accuracy prediction, the modified analytical models presented in this work are valuable tools for the accurate and rapid estimation of the mechanical behaviour of isotropic pure metal-based and two-phase alloy-based porous materials. Applications where the prediction of the mechanical behaviour of porous metallic materials is beneficial for their optimisation, such as in impact energy absorbing devices, heat exchangers and structural porous filters, can be found in [47]. Let us suppose that a structural biomedical prosthesis with 50% volume fraction of pores has to be manufactured using a bioinert metal such as Ti-based alloys. If the prosthesis was to be made out of pure Ti (grade 3), the modified analytical model with the highest accuracy predicts that the prosthesis will have E_c: 16.7 MPa, σ_c: 64 MPa, ε_c: 32.1%, E_t: 18.3 GPa, σ_t: 75 MPa, and ε_t: 0.84%. If Ti–6Al–4V was to be used, the predictions will be: E_c: 17.5 MPa, σ_c: 142 MPa, E_t: 19.0 GPa, σ_t: 150 MPa, and ε_t: 0.47%.

Due to their physical bases, the modified analytical models derived in this study are also expected to be able to predict the variation of the thermophysical and mechanical properties of a wide range of materials composed of different chemical elements (e.g. immiscible alloys), different constituents (e.g. nanocomposites), different morphological distribution (e.g. 3D printed cellular structures), and different phases (e.g. architectured materials).

4. Conclusions

This work analysed the feasibility of using fundamental analytical models developed for the estimation of thermophysical properties, such as the thermal conductivity of porous materials, to predict the mechanical behaviour of isotropic porous metallic materials. From the analysis of the results, it can be concluded that fundamental analytical models like the Effective Medium Theory (EMT), the Maxwell-Eucken 2 model (ME2), and the Series model (S) give reasonably accurate prediction of the mechanical behaviour, like compressive and tensile properties, of porous metallic materials. The main limitation is that these models work well at high porosity fractions but their overall prediction is not sufficiently accurate for implementing the estimation for design and optimisation purposes. Some of the analytical models are well suited to capture intrinsic microstructural differences like, for example, the influence of the stress concentration factor associated with porosity on the ductility of the material. Consequently, they are a good base for the derivation of modified analytical models, which are generally more accurate than the five fundamental analytical models. From this study it is also concluded that the modified model derived from the combination of ME1 and EMT should be used to predict the compressive behaviour, with the exception of the Young modulus, of isotropic pure metal-based porous metallic materials whereas the combination of the ME2 and S models accurately estimates the variation of the tensile strain of isotropic pure metal- and two-phase alloy-based porous metallic materials. The modified $\left\{\begin{matrix}ME2\\EMT\end{matrix}\right\}$ model is the one that can effectively forecast most of the compressive and tensile properties of porous metallic materials. The derived combined models are expected to have broader applicability to a wide range of multi-materials with hetero and periodic unknown structures.

Author contributions

LB: Conceptualisation, Methodology, Investigation, Funding, Writing – Review & Editing. FY: Methodology, Investigation.

Data availability

All metadata pertaining to this work will be made available on request.

Declaration of Competing Interest

The authors declare no competing financial or non-financial interests.

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