

http://researchcommons.waikato.ac.nz/

Research Commons at the University of Waikato

Copyright Statement:

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

The thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of the thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from the thesis.

ESTIMATION IN TONGAN SCHOOLS

A thesis submitted in fulfilment of

the requirement for the Degree of

Doctor of Philosophy

by

'Ana Hau'alofa'ia Koloto

University of Waikato

Te Whare Whānanga O Waikato

1995

ABSTRACT

In recent years, estimation has become increasingly prominent in the mathematics curriculum statements and recommendations in western countries including the United States, United Kingdom, Australia, and New Zealand. Tonga, too, has produced a draft syllabus on mathematics in which estimation has a prominent place (Tonga Ministry of Education, 1992a). Much of the research on estimation has been conducted in the United States, and a small number of studies in the United Kingdom, Japan, Australia and New Zealand. In Tonga, research on mathematics education is almost non-existent and no research on estimation has been conducted in Tongan classrooms. A belief that estimation is one of the most useful mathematical skill both in the classroom and everyday life, together with a concern about the lack of research data on this important topic determined the focus of the present study.

The study reported in this thesis involved a review of estimation in both formal and informal learning in Tonga, a review of the research literature on estimation and research in three secondary and two primary schools in Tonga.

This study was designed to investigate estimation in Tongan schools by focussing on students' measurement and computational estimation skills and strategies, and to explore ways of teaching these topics. A small sample of 18 form three (i.e. 13–15 year olds) and 12 class six (i.e. 10–11 year olds) students from three secondary and two primary schools in Tonga took part in this study. These students were interviewed before and after taking part in an instructional unit on measurement estimation activities. In addition, the computational estimation ideas of the 12 primary students were explored through individual interviews before and after an instructional unit on computational estimation strategies. Students' ideas and strategies identified from the first individual interview data were used to design an instructional unit on measurement and computational estimation. Thus a study of measurement estimation of both primary and secondary students preceded that of computational estimation of primary school students. Furthermore, a questionnaire was used to collect data on teachers' views regarding estimation. In particular, the present study investigated the following:

i) What estimation strategies are used by Tongan students in measurement and computation? How have the students constructed these strategies? How effective are the students' estimation strategies?

ii) What other mathematical skills and/or concepts do students need in order to become proficient estimators?

iii) What are the mathematics teachers' views regarding estimation? What estimation strategies are currently taught and encouraged by mathematics teachers in Tongan classrooms?

iv) How can students' existing ideas about estimation be utilised to develop teaching strategies which will further develop this mathematical process?

The interview data showed that students' existing ideas of standard measures formed the basis for their strategies for estimating measures of length. On the other hand, ideas and benchmarks developed in everyday life experiences were the basis of the strategies for estimating the measures of weight and capacity. The data also showed that class six students relied on 'conventional algorithms' to solve computational estimation tasks suggesting a lack of appropriate and effective computational estimation strategies.

The data showed that the instructional unit on computational estimation strategies helped students shift from the reliance on 'conventional algorithm' to the use of appropriate estimation strategies, particularly the 'rounding procedure'. Data also showed that many experiences with a variety of benchmarks and standard measures, making comparisons and judging relative sizes of objects, and reflections on their estimates helped students develop appropriate measurement estimation skills and strategies.

Even though teachers viewed estimation as useful, it was not a major part of their pre-service and in-service training and as well as current classroom practices. The needs for specific pre-service and in-service programme on estimation activities were identified by the teachers.

The thesis argues that measurement estimation tasks provide the means by which students' variety of ideas and experiences (both in and out of school) could be used in the mathematics classroom. By their very nature estimation tasks require the use of a variety of procedures, answers and assessment procedures. Reflection on their answers and strategies allow students to ascertain the reasonableness of their ideas. Both teachers and students need to be involved in making decisions regarding acceptable ranges for the answers and the criterion to be used for the assessment of estimates. These should encourage students to become more actively involved in their mathematics learning. Estimation has the potential to provide much needed variety in the Tonga mathematics classroom.

This thesis ends with a discussion of the implications of the main results for learning, teaching, resource and curriculum development, assessment and future research.

ACKNOWLEDGMENTS

I would like to acknowledge the contribution made by so many people without which this thesis would not have been completed.

- My supervisors at the Centre for Science, Mathematics, and Technology Education Research, Dr. Andy Begg, Dr Fred Biddulph, Dr Ken Carr, and Associate Professor Beverley Bell, for their guidance, encouragement, advice, comments, patience and understandings. My advisors in Tonga, Hon. Dr Langi Kavaliku and Dr 'Ana Taufe'ulungaki of Tonga Ministry of Education, for their support of this study.

- My brothers, Tui'onetoa Koloto and Sione Molitika Koloto, who were my main source of finance during the course of this study. The rest of my sisters and brothers, for their financial support, which had allowed me to pursue this study on a full-time basis.

- The students, teachers and principals of the five schools in Tonga in which this study was conducted, for their support in facilitating the data collection phases of this investigation. Many thanks go to the five teachers who participated in this study. Special thanks to the 30 students who took part in the interviews and teaching activities, for their ideas not only form the basis of this thesis but enhance my understanding of the estimation process. Your ideas and cooperation will always be valued.

- To a dear friend and colleague, Halahingano Tu'akolo Rohorua, formerly of the Tonga National Form 7, for her moral support and giving much of her time to discuss my ideas of mathematics learning in the Tongan classrooms.

- Staff of the Distant Education Centre, Sitiveni Finau, 'Ofa Malafu, Silia Tupou and Naua Paongo for their support during the data collecting phase in Tonga; 'Ofa Pongi and the mathematics curriculum writers of the Curriculum Development Unit, for their positive responses to my many requests during the course of this study

- My friends and colleagues at Centre for Science, Mathematics, and Technology Education Research, for helping to provide a supportive environment for research. Special appreciation goes to Raewyn Oulton, for her willingness to assist in times of needs.

- My sister, Mele'ana Taufa, for her assistance in typing various drafts of this thesis and for being there whenever help was necessary.

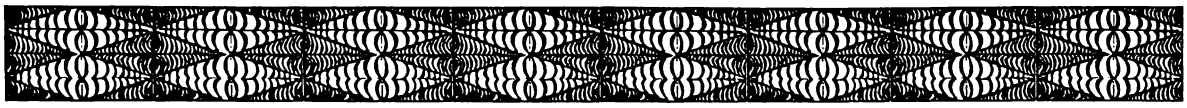
- My parents, for their love, encouragement and prayers. I am forever grateful that both have lived to see this work completed.

DEDICATION

This thesis is dedicated to my parents

Rev. Semisi Koloto
and Vika Siulolovao Koloto

TABLE OF CONTENTS



			Page
Abstract			ii
Acknowledgements			iv
Table of Contents			vii
List of Tables			xi
List of Figures			xii
Chapter 1	INTRODUCTION AND OVERVIEW		1
	1.1	Introduction	1
	1.2	Definitions of estimation	3
		Mental computation	3
		Approximation	3
		Estimation	4
	1.3	Rationale for the present study	5
		Type of estimation	5
		Assessment of estimation	6
		Lack of research data in Tonga	7
	1.4	Research questions	7
	1.5	Theoretical framework	8
		Constructivist view of learning	8
		Social constructivism	9
		Constructivist views on teaching	9
		Implications of constructivism for research	10
		Views of learning and teaching in Tonga	11
	1.6	Overview of chapters	12
Chapter 2	ESTIMATION IN TONGA		13
	2.1	Introduction	13
	2.2	Tongan society, way of life and education prior to contact with Europeans	13
	2.3	Estimation in Tongan society	14
		Counting	14
		Measurement of lengths	15
		Weaving	17
		Tongan view of estimation	19
	2.4	Estimation in the formal school curriculum	20
		Early mission schools (1820–1860)	20
		Formal schooling between 1860–1900	21
		Schools between 1900–1944	21
		Curriculum development from 1945 to the present time	21
	2.5	Textbooks and resources	24
		Mathematical instruments	24
		Calculators and computers	24
	2.6	Teacher pre-service education/training	25

	2.7	Assessment and research	26
		External examination	26
		School examinations	28
		Research on estimation	28
	2.8	Summary and comment	29
Chapter 3		REVIEW OF THE LITERATURE	32
	3.1	Introduction	32
	3.2	Estimation skills and their relationship with other abilities	33
		Measurement estimation skills	33
		Computational estimation skills	35
	3.3	Strategies used by estimators	38
		Measurement estimation	38
		Computational estimation strategies studies	41
	3.4	Teaching estimation	52
		Teaching measurement estimation	52
		Teaching computational estimation	53
	3.5	Summary and comment	57
Chapter 4		RESEARCH DESIGN	62
	4.1	Introduction	62
	4.2	Research design	62
	4.3	Familiarisation phase	63
		Setting-up period	63
		Making contact with schools	64
		Rationale for selecting class 6 and form 3 students	65
	4.4	Phase 1: pilot study	65
		Pilot survey	65
		Pilot interview	66
		Comments and discussion	66
	4.5	Phase 2: main study	67
		Classroom observation	67
		The sample	68
		First individual interviews (measurement)	69
		The interview process	71
		Teaching activities (measurement)	72
		Second individual interviews (measurement)	72
		Individual interviews (computational)	72
		Teaching activities (computational)	74
		Teacher questionnaire	74
	4.6	Data analysis	75
		Interview data	75
		Assessment Procedure	75
		Effectiveness of teaching activities	76
	4.7	Limitations, problems and difficulties	76
	4.8	Summary and comment	76
Chapter 5		RESULTS AND DISCUSSION : TEACHERS' VIEWS ABOUT ESTIMATION	78
	5.1	Introduction	78
	5.2	Teachers' definition(s) of the term 'estimation'	79
	5.3	Usefulness of estimation in mathematics	80

	5.4	Teaching estimation	82
		Measurement estimation strategies	82
		Teaching computational estimation strategies	85
		Teaching numerosity or quantitative estimation strategies	86
	5.5	Estimation in Teachers' preservice training	87
	5.6	Inservice training needs on estimation	89
	5.7	Teachers' views on assessment of estimation	90
	5.8	Teachers' other comments	92
	5.9	Summary and comment	93
Chapter 6		RESULTS AND DISCUSSION : MEASUREMENT	95
	6.1	Introduction	95
	6.2	Students' definitions of the term estimation	95
	6.3	Strategies for estimating lengths	98
		Multiple of a measuring instrument	99
		Multiple of a benchmark	100
		Part of a measuring instrument	103
		Part of a benchmark	104
		Just a guess	105
	6.4	Strategies for estimation weights	106
		Comparison to a benchmark	107
		Feel for the size of the weight	110
		Visualise this scale's needle	111
		Light or no weight	112
		Just a guess	112
	6.5	Strategies for estimating capacities	115
		Comparison to a benchmark	115
		Just an estimate or guesses	116
		Measuring instrument used in class	118
		Amount of money spent to fill up the petrol tank	118
	6.6	Students' ideas	120
		How have students constructed their ideas?	120
		Assessing reasonableness of an estimate	121
	6.7	Class differences	121
		Class differences in estimating lengths	121
		Class differences in estimating weights	122
		Class differences in estimating capacity	123
	6.8	Summary and comment	124
Chapter 7		MEASUREMENT ESTIMATION TEACHING ACTIVITIES: RESULTS AND DISCUSSION	127
	7.1	Introduction	127
	7.2	Participating classes	127
	7.3	The measurement activities	128
	7.4	Effectiveness of measurement estimation activities	138
		Case 1: A student who adopted appropriate strategy types	139
		Case 2: A student who showed some inappropriate strategy	143
		Case 3: A student who showed some changes in strategy types used	146
		Limitations of the activities	150
		Components involved in measurement estimation	151
	7.5	Summary and comment	154

Chapter 8	RESULTS AND DISCUSSION : COMPUTATIONAL ESTIMATION STRATEGIES		157
	8.1	Introduction	157
	8.2	Strategies involving addition with whole numbers	157
		Conventional algorithm	158
		Adding compatible numbers	159
		Rounding procedures	160
	8.3	Strategies involving subtraction with whole numbers	161
		Conventional algorithm for addition	161
		Front-end strategy	162
	8.4	Strategies involving multiplication with whole numbers	164
		Conventional algorithm	164
		Rounding procedure	165
		Subtracting numbers	166
	8.5	Strategies involving division with whole numbers	167
		Conventional algorithm	167
	8.6	Summary and comment	168
Chapter 9	TEACHING COMPUTATIONAL ESTIMATION		171
	9.1	Introduction	171
	9.2	Computational estimation activities	171
	9.3	Main results of the second individual interviews	180
	9.4	Effectiveness of the teaching activities	181
		Case 1: A student who adopted different strategy types	183
		Case 2: A student who adopted a variety of strategies	186
		Case 3: A student who showed little change in strategy types	188
	9.5	Summary and comment	189
Chapter 10	SUMMARY, CONCLUSIONS AND IMPLICATIONS		193
	10.1	Introduction	193
	10.2	Summary of main findings	194
	10.3	Conclusions	201
	10.4	Implications for learning	202
	10.5	Implications for teaching	204
	10.6	Implications for resource and curriculum development	206
	10.7	Implications for assessment and future research	208
	10.8	Concluding Thoughts	211
APPENDICES			212
Appendix A	Trial questionnaire for form 3		213
Appendix B	Trial questionnaire for class 6		217
Appendix C	Instructional unit on measurement estimation		220
Appendix D	Instructional unit on computational estimation		239
Appendix E	Teacher questionnaire (English Version)		251
Appendix F	Teacher questionnaire (Tongan Version)		255
BIBLIOGRAPHY			259
GLOSSARY			272

LIST OF TABLES

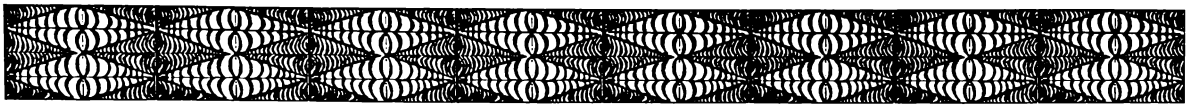
		Pages
3A	An analysis of the components involved in computational estimation	49
3B	Summary of assessment procedure	59
4A	Sequencing of the phases of the investigation	63
4B	Summary of the five schools selected for this study	64
4C	Details of the sample of class 6 students	68
4D	Details of the sample of form 3 students	69
4E	Measurement estimation tasks for first and second interview	70
4F	Computational estimation tasks for first and second interview	73
5A	Summary of teachers' background	79
6A	Terms used by students to describe "fakafuofua"	96
6B	Strategies types for estimating lengths	99
6C	Frequencies with which strategy types for lengths were used	99
6D	The actual distances between school and hospital/town	101
6E	Types of strategies used for estimating weights	107
6F	Frequencies with which the strategy types for weights were used	107
6G	Strategy types for estimating measures of capacity	115
6H	Strategy types for capacity and the number of times it was used	115
6I	Class comparison of the strategy types for estimating lengths	122
6J	Numbers and percentages of strategy types for estimating weights	123
6K	Numbers and percentages of strategy types for estimating capacities	123
7A	Summary of the mini-classes who participated in the activities	128
7B	Comparison of Lahaina's responses to the length items in the two interviews	139
7C	Comparison of Lahaina's responses to the weight items in the two interviews	141
7D	Comparison of Lahaina's responses to the capacity items in the two interviews	142
7E	Siosiua's responses to item on length in the two interviews	144
7F	Siosiua's responses to weight item in the two interviews	145
7G	Siosiua's responses to capacity items in the two interviews	146
7H	Itama's responses to length in the two interviews	147
7I	Comparison of Itama's responses to the weight items	148
7J	Comparison of Itama's responses to the capacity items	149
7K	Analysis of the components involved in measurement estimation	151
8A	Strategy types for tasks involving addition	157
8B	Strategy types for tasks involving subtraction	161
8C	Strategy involving multiplication	164
8D	Strategy type for tasks involving division	167
8E	Summary of strategy types for computational estimation	168
9A	Outline of the teaching unit on computational estimation	171
9B	Summary of students' strategies in the second interview	180
9C	Comparison of strategy types used in the first and second interview	181
9D	A summary of the change and non-change in strategy types used by students	182
9E	Summary of Lofa's pre-teaching and post-teaching strategies	183
9F	Summary of Pauliasi's pre-teaching and post-teaching strategies	186
9G	Summary of Jamie's pre-teaching and post-teaching strategies	188

LISTS OF FIGURES

		Pages
Figure 2A	Positions of the hand in spanning a hanga-fesi	16
Figure 10A:	Distinctions between, 'intended', 'implemented' and 'attained' curriculum	207

CHAPTER ONE

INTRODUCTION AND OVERVIEW



1.1 Introduction

It is generally accepted that one of the goals of mathematics education is to help students use mathematics in everyday life (Cockcroft, 1982; Davis and Maher, 1993; Ministry of Education, 1992; Onslow, 1991; Tonga Ministry of Education, 1992a; Usiskin, 1993). One practical topic that has become increasingly prominent in the mathematics curriculum is **estimation**. It has been suggested that in real life problems and situations, estimation is used far more often than exact computation (Bestegen and Reys, 1982; Harte and Glover, 1993; Hope, 1986; Mcintosh, Reys and Reys, 1992; Threadgill-Sowder, 1984) and is at least as important as the ability to carry out formal calculations (Knight, Arnold, Carter, Kelly and Thornley, 1992). For instance, estimation is used in such diverse activities as deciding how much time to allow for travel to an appointment with the doctor; assessing whether there is enough space between two parallel parked vehicles to park your car; ascertaining whether you have enough cash to cover a purchase, or determining how much money a student can expect to live on during a year of university study. In a large number of situations, such as in trade, construction, business and politics, there is no choice but to give an estimate because exact values are often unobtainable.

Although most situations involving estimation share a common feature of finding an approximate or close answer to a problem, they differ in many ways. Some involve measuring, others calculating, surveying, or predicting. Estimations may also differ in complexity, context, the amount of information required, the numbers involved, format of presentation and in the type of answer(s) expected (Rubenstein, 1986).

Despite estimation's importance, relevance and usefulness in everyday life and in the workplace, some claim that estimation is one of the more neglected and less effectively taught topics in the mathematics curriculum (Hope,1986; Hope, 1987; Mitchell, 1991; O'Daffer, 1979; Reys, Bestgen, Rybolt and Wyatt, 1982; Reys,1986). Until recently curriculum developers have given little attention to estimation. Prior to 1980, few mathematics educators spoke in support of the role of estimation in the mathematics curriculum, and by and large the topic was ignored. It appears that computational estimation, at least, did not fit the objectives of the "new math" movement (Sowder, 1992). This situation is now changing, due in part to the increasing use and acceptance of calculators, and estimation has become increasingly

prominent in recent mathematics curriculum statements and recommendations in countries including the United States (National Council of Teachers of Mathematics, 1989), United Kingdom (Department of Education and Science and the Welsh Office, 1991; Department for Education and the Welsh Office, 1995), Australia (Australian Education Council, 1990; 1991), New Zealand (New Zealand Ministry of Education, 1992) and Tonga (Tonga Ministry of Education, 1992a). The National Council of Teachers of Mathematics 1986 yearbook "Estimation and Mental Computation" (Schoen and Zweng, 1986) was a response to the growing recognition among mathematics educators in the United States of the importance of teaching and learning estimation.

A number of issues, problems and difficulties associated with estimation in part account for its neglect over the years. A central issue involves the commonly held view that mathematics is a subject concerned with **exactness**. On the other hand, estimation is a process which produces inexact answers. Estimates are often inexact answers arrived at because exactness is not required or conditions such as a lack of time or information do not allow exact calculation. Estimation draws attention to an aspect of mathematics which is **not so exact.** As such, estimation seems antithetical to the very nature and purposes of mathematics (Usiskin, 1986).

One problem associated with teaching and assessing this topic is that an estimation exercise often leads to a variety of strategies for formulating an answer, as well as many possible answers. Thus, one needs to adopt an assessment procedure which may be in direct contrast with traditional mathematics classrooms procedure which implies one "right" answer, and one right method. The assessment procedure needs to identify student estimation strategies as well as their estimations. It is doubtful whether the ability to estimate can be validly assessed if only the answer obtained (i.e. estimate) is considered. For instance, students could arrive at an estimate by using inappropriate procedures, such as mentally computing an answer and then rounding it off to give an estimate. Thus, various formats have been used by researchers for the assessment of estimation. These have included an open-ended format, reasonable versus unreasonable, asking students to decide whether the estimation item is over or under a given reference number, interval estimation, and multiple choice items which require students to choose from given answers. Research has shown that students' performance varies significantly depending on which format was used (Reys and Bestgen, 1981; Rubenstein, 1985; Schoen, Blume and Hoover, 1990).

Another problem associated with estimation is that it has been defined in many ways. The terms 'estimation', 'mental computation' and 'approximation' have often been used interchangeably. Even though these terms are related in some way, very often no clear distinction is made between them. This problem is discussed later in this chapter.

Concerns about the above problems, together with an added interest in the usefulness of estimation in everyday life's activities, as well as in the development or construction of other mathematical ideas in the classroom, convinced the researcher that this topic needed further investigation. Thus the present study is an attempt to investigate students' estimation skills, and explore ways of teaching and assessing estimation, in Tonga.

The remainder of this chapter is in four parts. First it examines various definitions of estimation and outlines the ones adopted for this study (1.2). It then outlines the rationale for the present study (1.3), and specifies the research questions (1.4). The next section (1.5) describes the theoretical framework underlying this study. Finally, it presents an overview of the succeeding chapters of this thesis (1.6).

1.2 Definitions of Estimation

Definitions of estimation are many and varied (Sowder, 1992). It is important to distinguish between the terms 'estimation', 'approximation' and 'mental computation'. Although these are related in some ways, too often no clear separation is made between them (Payne & Seber, 1959; Trafton, 1978). Distinguishing between these terms will help clarify the meaning of the term 'estimation' as it is used in this study.

Mental computation

The term 'mental computation' and the alternatives 'mental arithmetic' and 'mental calculation' have been used interchangeably in the mathematics classroom and mathematics education literature. Mental computation has been defined by Reys (1984)

> as a procedure that produces an exact answer, and the procedure is performed mentally, without using external devices such as pencil and paper. (p.548)

The above definition leads to the conclusion that mental computation is an inappropriate term for estimation because it deals with exactness whereas estimation has to do with inexactness.

Approximation

Approximation like estimation results in inexact answers, but some authors claim that estimation and approximation are not synonymous and that there are important differences between the two (Hall, 1984; Thompson, 1979). For instance, in Hall's (1984) view, *whereas estimation is usually a mental exercise, approximating usually requires a tool of some kind* (p.517). And later:

> Estimation is the mental skill of making an educated guess. Approximating is finding a result precise enough for a specific purpose. (p.517)

Like Hall (1984), Thompson (1979) defines estimation as *an educated guess* whereas approximating is *attempting to close in on a target value*. Thompson (1979) claims that it is often difficult to assess the magnitude of error in an estimate, while on the other hand, it is often possible to get as close as one desires without ever reaching the exact value. Approximation has its own "arithmetic" and rules (Hilton and Pedersen, 1986).

Estimation

Most definitions of estimation refer to it as a process involving various components. For instance, Siegel, Goldsmith and Madson (1982, p.211) described it as *a process that gives a rough solution to a problem in counting or measurement* , Hall (1984, p.516) considered it to be *the mental process of making an educated guess*, while, Harte and Glover (1993) suggested that:

> Estimation is a process: it involves comprehending the problem, relating it to information that is already known, judging and verifying reasonableness, and revising as necessary.(p.76)

Whereas Hall (1984) and Siegel, et al (1982) did not specify how the "educated guess" or "rough solution" is formulated, Harte and Glover (1993) focused on what the estimation process entails. However, they did not specify the nature of the solution or end result.

Four characteristics of estimation can be identified from the above definitions: firstly, it is a mental process; secondly, the answer obtained is not exact; thirdly, it involves various component skills and concepts; and finally, there is more than one type of estimation, two are counting and measurement estimation.

For this study, two types of estimation are selected for further investigation—namely measurement and computational estimation. The reasons for this selection are outlined in section 1.3.

The nature of computational estimation is outlined succinctly by Reys (1984) who defined four characteristics of computational estimation:

> (1) it is performed mentally, generally without paper and pencil; (2) it is done quickly; (3) it produces answers that are not exact but are adequate for making necessary decisions; and (4) it often reflects individual approaches and produces various estimates as answers. (p.551)

Measurement estimation, on the other had, is defined by Reys and Reys (1987) as:

> the mental process of (a) arriving at a measurement without direct use of a measuring instrument (b) selecting a representation of a given measurement or (c) comparing two quantities without measuring. (p.16)

Like mental computation, computational estimation is usually performed mentally, but unlike mental computation, it produces answers that are not exact. Computational estimation may seem similar to approximation but approximation involves working to a required degree of accuracy, whereas computational estimation often leads to many inexact answers. By and large estimation utilises unconventional and often idiosyncratic procedures leading to inexact answers. The degree of accuracy for an estimate is often based on the context and the purpose of the estimation exercise; consequently the criteria for assessment of these exercises may vary. This contrasts with the end result of a mental computation exercise which can be marked either 'right' or 'wrong', and likewise, with the result of an approximation by determining whether the result falls within the required degree of accuracy. This can be a problem for researchers and teachers. The 'right' or 'exact' answer and the required degree of accuracy for the mental computation and approximation respectively provide a basis for assessment, but often no such guideline is available for an estimation exercise. Thus, a challenge for teachers and researchers is to determine what criteria to use when assessing an estimate.

1.3 Rationale for the Present Study

With the increasing emphasis on estimation in mathematics curricula statements in countries such as United Kingdom, United States of America, Australia, New Zealand and Tonga, the need for research in this area cannot be over emphasised. This section discusses the rationale for the present study.

Types of estimation

The various types of estimation are useful in many domains of mathematics as well as in everyday life. Estimation can be taught as part of measurement, computation, numerosity and other domains of mathematics such as probability and statistics. As Whitin (1994) suggested, estimation is *a crucial mathematical strategy that can be woven throughout the entire mathematics curriculum* (p.436).

Research efforts, however, have concentrated on only a few types of estimation with the majority focussing on computational estimation. Although the research base is growing, much additional research addressing computational estimation is needed (Reys, 1993) as well as other types of estimation (Benton, 1986; Sowder, 1992). Very little research has explored teaching strategies that would promote the development of estimation skills. The present study is an attempt to address these issues, by focussing on measurement estimation and some aspects of computational estimation.

Measurement was selected because it is a universal and significant activity for the development of mathematical ideas. Measuring is concerned with comparing, ordering, and quantifying qualities (Bishop, 1988). Tongans of course have traditional measurement systems, particularly for lengths. Because measuring is a familiar activity (both within and beyond school) to Tongan students, the present study was designed to explore the type of measurement estimation skills developed and used by Tongan students in their mathematics classroom.

Computation is entrenched as the basis of the mathematics curriculum, with the 'four rules' gradually being developed to handle more and more complicated numbers. It has been the main focus of mathematics education for many years. With the availability of calculators and computers, which were developed in order to carry out these computational techniques faster and more accurately, some mathematics educators have called for a change from the emphasis on computation in the curriculum to computational estimation skills and concepts. This is based on the belief, supported by research data, that estimation skills will equip students to assess whether results given by calculators and computers are reasonable (Edwards, 1984; Hall, 1984; Reys, 1993).

As mentioned in section 1.1, it has been suggested that in everyday life estimation is used far more often than exact computation and is at least as important as the ability to carry out formal calculation (Bestegen and Reys, 1982; Harte and Glover, 1993; Hope, 1986; Knight, et al., 1992; Mcintosh et al., 1992; Threadgill-Sowder, 1984). Given that computational estimation is a useful mathematical skill for everyday life problems and situations, and that this topic has a prominent place in the Tongan Draft Mathematics Syllabus (Tonga Ministry of Education, 1992a), computational estimation was also selected for this investigation. A related purpose is that the research would provide relevant and useful data on primary students' computational estimation skills as well as possible teaching strategies for developing this skill in Tonga.

Assessment of estimation

As discussed above, one problem associated with estimation is that of determining whether answers are reasonable or acceptable. This involves making decisions regarding which criteria to use in assessing what is a reasonable estimate.

Researchers have used several approaches which suggest that the answers to an estimation exercise cannot be taken on its own as a measure of students' ability to estimate; the strategies used to formulate an estimate also need to be taken into account as does the context of the question. This adds to the complexity of the task involved in evaluating or assessing an estimate. Perhaps this is why, in Reys' (1986) observation:

> testing programs have often omitted estimation completely or at best made very shallow efforts to assess estimation skills. (p. 225)

Lack of research data in Tonga

The past five years has seen new curriculum statements in many western countries such as the United Kingdom, United States, New Zealand and Australia. Tonga, too, has produced a draft syllabus on mathematics in which estimation has a prominent place (Tonga Ministry of Education, 1992a). Much of the research on estimation has been conducted in the United States, with a small number of studies in the United Kingdom, Japan and Australia. It is problematical whether findings in these developed countries could be applied to students in the developing country of Tonga.

In Tonga, research on mathematics education is almost non-existent and no research on estimation has been conducted in Tongan classrooms. A concern about the dearth of research data on this important topic in the mathematics curriculum, as well as an interest in the development in students of estimation skills and processes, determined the focus of the present study. Also of interest to the researcher was (i) the link between students' development of estimation skills and strategies and teachers' teaching strategies and (ii) utilisation of research findings in classroom practices. Hence, another objective of this study was to explore and identify teaching strategies that could help enhance students' estimation skills and strategies. The research questions addressed are outlined in the next section.

1.4 Research Questions

Learning

i) What estimation strategies are used by Tongan students in measurement and computation? How have the students constructed these strategies? How effective are the students' estimation strategies?

ii) What other mathematical skills and/or concepts do students need in order to become proficient estimators?

Teaching

iii) What are the mathematics teachers' views regarding estimation? What estimation strategies are currently taught and encouraged by mathematics teachers in Tongan classrooms?

iv) How can students' existing ideas about estimation be utilised to develop teaching strategies which will further develop this mathematical process?

It was anticipated that the results of the present study would provide information on how Tongan students learn estimation during mathematics lessons in which effective teaching and assessment techniques are used to develop estimation strategies. Such information could be of considerable value to teachers, curriculum writers, and text-book authors.

1.5 Theoretical Framework

The theoretical framework underlying this study is the constructivist model of learning, a theory which is becoming a dominant paradigm in mathematics education. This section outlines the main assumptions of this theory and the implications for mathematics learning, teaching and research.

Constructivist view of learning

The constructivist theory contends that learners actively construct their own knowledge for themselves. The notion that knowledge is the result of a learner's activity, rather than of passive reception of information or instruction, goes back to Socrates and is today embraced by all who call themselves "constructivists" (von Glasersfeld, 1991). The theory of constructivism is based on two main principles. These are:

> a. *Knowledge is not passively received but actively built up by the cognising subject;*
>
> b. *the function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality.*
>
> von Glasersfeld, 1989a, p.162

The first principle draws attention to the learner's responsibility for what he or she learns. It implies that learners actively construct their own meanings in unique ways as they interact with their environment. Mathematical ideas and thoughts are 'made' by the learner and are not simply accepted from others (Clements and Battista, 1990), that is, the learner actively interprets and imposes meaning through the lenses of his or her existing knowledge structures, working to make sense of the world. Knowledge has to be constructed or built up by each individual learner; it cannot be packaged and transferred from one person to another (von Glasersfeld, 1991). Thus, for the constructivist, "knowledge of the world is 'phenomenal', only representing how things are constructed by the knower" (Dubinsky and Lewin, 1986, p.58).

The second principle implies that the mind organises knowledge so that it fits personal experiences rather than matches an ontological reality which can never be really known (von Glasersfeld, 1989a; 1989b). This principle is important because it distinguishes the 'trivial' from the 'radical' constructivists. Those who accept only the first principle are considered trivial constructivists whereas those who accept both principles are called radical constructivists (Jaworski, 1988; von Glasersfeld, 1989a). The second principle can be

viewed as saying that people do not find truth but construct viable explanations of their experiences (Wheatley, 1991). That is, if one accepts the second principle, then one can no longer maintain the traditional idea of knowledge as knowing the 'external' reality that exists independent of the knower. Thus, radical constructivists (of whom von Glasersfeld is one) do not deal with the traditional conception of "truth", which they maintain would require that people know, or at least believe that an idea, a theory, or any conceptual construct is necessarily an accurate representation of something beyond their experiential world. The notion of truth is replaced with that of viability (von Glasersfeld, 1992). Radical constructivism is intended as a model (not as the description of the real world) it is a working hypothesis which may or may not turn out to be viable (von Glasersfeld, 1990).

Social Constructivism

Although individuals construct their own knowledge and understandings, they do not do so in isolation because this process is subject to social influences (Cobb, Wood and Yackel, 1990; Wheatley, 1991). The importance of social interaction and negotiation of social norms leads to what is referred to by Noddings (1990) as social constructivism. The process of constructing mathematical knowledge involves cognitive conflict, reflection and cognitive reorganisation. Social interaction is therefore crucial to the process of learning mathematics (Cobb et al., 1990).

Constructivist views on teaching

Constructivism is a theory of learning or knowing, not a model of teaching. However, since it is generally accepted that the aim of classroom teaching is to promote learning one needs to consider what this theory of learning implies for teaching.

Constructivism leads to methods of teaching that attribute the student the power to become an active learner. From an educator's point of view, an important feature of constructivism is the distinction it draws between teaching and training (Noddings, 1990; von Glasersfeld, 1990). Whereas teaching aims at generating understanding, training aims at the repetition of behaviours for competent performance. So from a constructivist perspective the teacher's prime concern and interest will be focussed on what can be inferred to be going on inside the student's head, rather than overt behavioural responses.

It follows from the first assumption of constructivism that mathematical knowledge cannot be given ready-made to students. That is, the teachers cannot put mathematical knowledge and ideas into students' heads; rather, students actively construct their own knowledge and meanings in unique ways as they interact with their environment. One of the implications of this is that the students' ways of thinking and knowing rather than a predetermined body of 'content' knowledge should be the starting point of the instructional process (Wheatley,

1992). In discussing the implications of constructivism for teaching Confrey (1990) outlined the following teaching role.

> An instructor should promote and encourage the development for each individual within his/her class of repertoire for powerful mathematical constructions for posing, constructing, exploring, solving and justifying mathematical problems and concepts and should seek to develop in students the capacity to reflect on and evaluate the quality of their constructions. (Confrey, 1990, p.112)

Accepting that learning mathematics requires construction, not passive reception, on the part of students it follows that mathematics teachers need to establish a truly mathematical environment in their classrooms (Davis, Maher and Nodding, 1990; Gadanidis, 1994). The constructivist view implies a need to take seriously what students say, for this can lead to new understandings of how students think. The knowledge that students have constructed forms the basis on which they build further knowledge.

Thus in order to teach well, teachers need to understand what students are thinking, how they produce the "chain of little marks" seen on their papers, and what they can do with material presented to them (Nodding, 1990). To understand and appreciate student's thinking, the teacher needs to be flexible because very often students start from premises that may seem inconceivable to teachers. As Confrey (1990) suggests teachers ought to be prepared for the likelihood that the ideas and knowledge constructed by students will not correspond with their own. Understanding of the students' ideas and ways of thinking can help teachers create a tentative path upon which students may move to construct mathematical ideas more consistent with accepted mathematical knowledge.

Furthermore, Noddings (1990) suggests that mathematical learning occurs most effectively through guided discovery, meaningful application, and problem solving, as opposed to imitation and reliance on the rote use of algorithms for manipulating formal symbols. Thus, the teacher is viewed as a facilitator of learning, a person who provides a classroom learning environment which (i) encourages the development of diverse and creative problem solving processes in students, (ii) reduces the exclusive emphasis on mathematically correct responses (Wheatley, 1992); (iii) allows students to acquire basic concepts, algorithmic skills, heuristic processes, and practices of cooperation and reflection (Davis, et al., 1990), and (iv) encourages reflection as part of a community of inquirers .

Implications of constructivism for research

It has been suggested that acceptance of the premise that knowledge and reality itself are constructed by the individual leads to methodological constructivism (Noddings, 1990). In research this implies that researchers investigate the subjects' perceptions, purposes, premises, and ways of working things out in order to understand their behaviour.

One implication of constructivism for research in mathematics education is a need to use research techniques such as interviews and small group studies in order to understand what goes on in students' heads; that is, the constructivist model supports research techniques that probe students' thought processes. Mulhern (1989) discussed two general approaches to studies of mental processes. These are the (i) thinking aloud procedure and (ii) clinical interviewing. Individual interviews have been widely used by researchers in mathematical education research, both overseas and in New Zealand to tap into students' mathematical knowledge (Carr, 1986; Erlwanger, 1973, 1975; Koloto, 1989, Sharma, 1993). Thus, to gain an insight into students' estimation strategies, the interview approach was used in the present study.

The idea that the learner constructs knowledge on the basis of prior ideas provided the theoretical basis for the present study. It was thought that students would develop understanding and increase knowledge of estimation strategies if they could relate new knowledge to relevant existing ideas and concepts. The first and second series of individual interviews aimed to gain insight into students' estimation strategies and related concepts before and after instruction. The data collected from the first series of interviews formed the basis of the instructional programme.

It is acknowledged that although students construct their own mathematical knowledge, they do not do so in isolation. As Cobb (1990) and Zevenbergen (1991) have suggested interactions with both the teacher and other students provide important learning opportunities. Thus, the present investigation also explores teaching activities on measurement and computational estimation based on the assumptions and implications of the constructivist theory. It also aims to assess how such teaching activities influence the development of estimation ideas and strategies in Tongan students.

Views of learning and teaching in Tonga

From the researcher's observations, the teacher in Tonga is still regarded as the 'expert', the person with knowledge who passes it on to the learner in the classroom through didactic instruction. At the same time, however, the learner is viewed as the person responsible for the learning. This view of the learner is consistent with the constructivist premises adopted in this study. Accepting that the learner is responsible for his or her learning may seem contradictory to the strongly held view that teachers are the experts with the knowledge to be transmitted to students, especially for examination purposes. However, as Ellerton, Cowan and Clements (1992) have noted, quality mathematics learning has occurred in many whole class teaching and learning environment, in classrooms where teachers have never heard of the constructivist view of learning. As a Tongan researcher, aware of the possible discrepancies between her view(s) of learning and those held by Tongan parents and educators, the author nevertheless used constructivist ideas as a framework for investigating learning and teaching in the Tongan classrooms.

1.6 Overview of Chapters

This investigation involves (i) a series of individual interviews with a small sample of primary and secondary students before and after participation in instructional activities on measurement and computational estimation and (ii) an analysis of the effectiveness of the instructional activities. The remainder of this thesis reports further background to this investigation, the methodology used, results obtained and conclusions drawn.

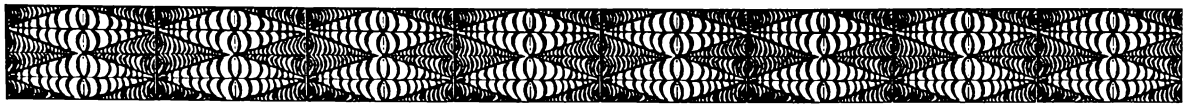
Chapter two provides background information on estimation in Tongan society and further presents the case for the present study. This is followed in chapter three by a critical review of the literature on estimation. Chapter four details and explains the research design used for the investigation. There were several phases to the investigation, including interviews with students, a survey of teachers' views and a teaching intervention.

Chapter five discusses the results collected from a questionnaire on teachers' views about estimation. The results of the first individual student interviews on measurement estimation, are presented and discussed in chapter six. The teaching activities on measurement estimation, including their success and limitations, are analysed and discussed in chapter seven. Chapter eight presents and discusses the first individual interview on computational estimation. Chapter nine reports on teaching activities on computational estimation, and their effectiveness and limitations.

Chapter ten summarises the main findings of the research, draws conclusions about these, and outlines possible implications for learners, teaching and teacher education, curriculum and resource development, assessment, and future research.

CHAPTER TWO

ESTIMATION IN TONGA



2.1 Introduction

Estimation is not a foreign concept to Tongans. In some form it has always been part of Tongan life. For instance, estimation was and is used in assessing the length of time to allow for food to be cooked in an **umu** (an earth oven or **hangi** as used by the Māori people of New Zealand); determining the number of days it would take to weave a mat; ascertaining the time to travel to the sea for low tide fishing; and deciding the width of a ngatu (tapa cloth). Despite being a major part of activities in everyday Tongan life, estimation has not found its way into the school curriculum until very recently, and probably not yet into most Tongan classrooms. This chapter starts with some background information on the Tongan way of life and education (2.2) and then focuses on estimation in various aspects of Tongan society (2.3). It examines the place of estimation in the Tongan school mathematics curriculum (2.4), textbooks and resources (2.5), teacher education (2.6), and assessment and research (2.7).

2.2 Tongan society, way of life and education prior to contact with Europeans

Prior to contact with Europeans in the late 1700s it is estimated that about 30 000 people lived in Tonga. Like other Polynesian societies of the Pacific, a highly organised socio-political system existed, based on the rules of the chiefs. Tonga was unique in Polynesia and in the Pacific as a whole in that it had a traditional monarchy under a king, the Tu'i Tonga (Leyard, 1982). Thus, the society was characterised by a three-tiered social system that comprised the king, nobles or chiefs and commoners. Membership of each social class was determined by birth. Although there have been some changes over the years this hierarchical nature of the society still exists today.

The Tongan people lived by the seasons, tides, planting and harvesting of crops, fulfilment of their obligations to the chiefs and Tu'i Tonga (King of Tonga) as well as individual events of birth, marriage and death. Their way of life was stable. The people did not live in villages as they do nowadays but had their houses on the land that they worked (Campbell, 1992). The main daily activities included fishing, house and canoe building, yam planting, weaving and tapa making. These were all occasions for communal activity. Children's lives followed the same pattern as that of their parents and grandparents. What people did depended on their place and function in society (Cummins, 1977). Consequently traditional education was

informal, flexible, and based in the family and the environment (Koloto, 1993; Tupouniua, 1977).

Before contact with the European missionaries the training and education of the children in the skills and knowledge necessary for their roles in the society were the responsibility of the extended family. Occupations and access to knowledge required for each occupation were determined by class. What was considered to be important knowledge, who it was taught to, and how it was taught were controlled by the chiefly groups (Mara, Foliaki and Coxon, 1994). For example, canoe and house building, navigation, fishing and weapon making were occupations for two intermediate auxiliary classes known as the **mu'a** and **matapule**. Knowledge about these remained within this occupational class and was passed on from generation to generation within that family.

People were educated in informal ways for these occupations. Learning occurred through observing, listening, memorising and practical application. Children learned by observing their parents and relatives (Cummins, 1977; Mara et al, 1994; Tupouniua, 1977). The roles of the mu'a and matapule class included gathering young people, especially the young chiefs, and lecturing them on customs and behaviour. They also administered, organised and mobilised the labour of the non-chiefly people when it was required by the chiefs. In addition, they organised the presentations of produce by the people to their chiefs and were the spokespersons for the chiefs at particular ceremonies. Thus, specialist knowledge was controlled by the mu'a and matapule.

Traditionally, the teacher was viewed as the person in possession of knowledge and skills superior to that of the learner. A relationship of respect and deference by the learner towards the teacher was demanded (Mara et al., 1994).

2.3 Estimation in Tongan Society

Various aspects of traditional life involved estimation. These included calculation of time, navigation, canoe and house building, planting and harvesting of crops and fishing. It is beyond the scope of this chapter to deal with all of these activities. Three activities considered central to the life of the Tongan people and relevant to the present study are discussed in this section. These are counting, measurement of lengths, and weaving.

Counting

Counting was an important part of daily life including counting items for presentations at specific ceremonies, such as the **'inasi** ceremony (an annual ceremony whereby people presented products of their labour to the Tu'i Tonga). Before contact with Europeans Tonga had a developed number system as far as 100 000; items could be counted from **taha** (one) to **kilu** (one hundred thousand). However, the demand of everyday life did not usually require the use of extremely high numbers (Cummins, 1977).

Special ways of counting coconuts, yam, fish and certain other items were developed and used (Churchward, 1953; Koloto, 1993). Counting was done mostly in twos or in pairs. In discussing numerals and their accessories, Churchward (1953) referred to two terms: **tupu** and **malie**, which are especially relevant to estimation. When **tupu** (to grow) is used after **hongofulu** (ten) or a multiple of ten such as **hongofulu tupu,** it means 'odd' or 'and more', while **malie** means 'exactly' or 'no more'. Thus **uofulu tupu** means twenty-odd while **uofulu malie** means exactly twenty. Specifically uofulu tupu refers to the range of numbers from twenty-one to twenty-nine. One might ask a teacher the question: *'Oku toko fiha ho'o kalasi?* (How many students have you got in your class?). *'Oku toko tolungofulu tupu.* (There are thirty-odd). The use of these terms, particularly **tupu**, might be helpful in expressing the results of estimations.

Various language expressions indicate the use of division and multiplication concepts in the everyday life of the Tongan people. Simple division was called for when chiefs distributed presentations from the 'inasi. Terms such as **fakatolu** (divide into three portions) and **fakavalu** (divide into eight portions) were used. Multiplication concepts are implied by the words **lo, tu'o** and **liunga**. For instance, lotaha, loua and lotolu, meant one-layered, two-layered and three-layered or three-fold respectively. Tu'o taha means once, tu'o ua means twice, and tu'o tolu means three times. When liunga appears before a numeral, such as liunga ua (twice as big as it was before), it indicates that the object has grown in size by that numeral. These expressions are important because their use shows some of the mathematics that was and is used in the everyday life of the Tongan people.

Measurement of lengths

For measurements of lengths, **ofa** (arm span, later known as a fathom); **hikova'e** (100 fathoms); and **hanga** (hand span, about 20 centimetres) and **hanga-fesi** (augmented handspand, about 30 centimetres) were used. When measuring rope in **ofa** the ordinary numerals were used. A rope made from coconut fibre (plaited in many ways) known as a **kafa**, was normally measured to 100 **ofa**. In gardening, ofa was used in measuring garden plots. However, in this case ofa is not necessarily six feet. It is simply the distance between two consecutive rows of yams or tapioca, as judged to be appropriate and agreed upon for this particular plot (Churchward, 1953). Nowadays, other units such as one spade-length are used to measure the distance between rows of yams or tapioca and between consecutive crops.

Hanga-fesi or simply hanga was and is used for measuring the lengths and widths of **fala** (mats) and **ngatu** (tapa cloth). This is measured by spanning and then rolling the hand over to the knuckles thus adding to the span the length of either the forefinger or the middle finger.

Position 1 **Position 2**

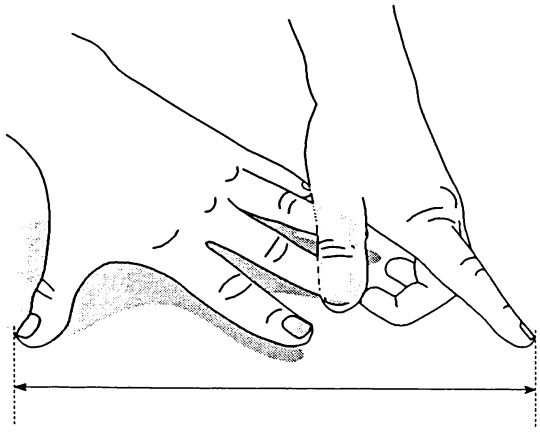


Figure 2A : Positions of the hand in spanning a hanga-fesi

Hanga-fesi may be regarded as the smallest and basic unit for measuring both length and breadth. It has been reported that a woman is chosen to set the measure because of her **nima mafola**, that is, one whose hand extends flat on the surface of the cloth (Collocott, 1925a). When measuring mats, the following numerals are used.

Tekumi	*Ten*
Tekumi ma ua	*Twelve*
Tekumi ma nima	*Fifteen*
Uongokumi	*Twenty*
Tolungokumi	*Thirty*
Tefuhi	*One hundred*

For ngatu (tapa cloth) making, the breadth of the printed part of the tapa cloth is 16 or sometimes 14 hanga, divided by lines down the middle into two equal portions of eight or seven hanga-fesi. Down each side of the cloth runs a border which is unpainted, known as the tapa. The width of each border measures from one to two hanga (hand spans). Thus, in their everyday life interactions the above names are used by Tongans on the assumption that other Tongans have an understanding of the unit lengths involved. Thus, when present gifts of tapa, a lautefuhi (100 langanga) is normally presented to members of the Tongan royal family, a launima (50 langanga) to nobles and people of higher social status, and a fola'osi (4 langanga) to social occasions involving the common people. The sizes of the lengths of the tapa are not only significant for the development of mathematical ideas, that is, quantifying objects, but they also have social significance in illustrating the hierarchical view of the society held by Tongans.

The unit of measurement for the length of the ngatu (tapa cloth) is the langanga, which is two hangafesi (augmented spans) long (60 centimetres). The following are the terms used for measuring the size of the ngatu. It should be noted that the size of the ngatu is described by its length only. The following are measurements of the lengths of ngatu.

Name of the piece of ngatu	**Length**
fola'osi	*4 langanga (2.4m)*
fatuua	*8 langanga (4.8m)*
hongofulu'i ngatu	*10 langanga (6m)*
uofulu'i ngatu	*20 langanga (12m)*
lautolu	*30 langanga (18m)*
laufa	*40 langanga (24m)*
launima	*50 langanga (30m)*
lau tefuhi	*100 langanga (60m)*

As mentioned earlier, the size of the ngatu is normally a launima (50 langanga). The smallest piece of ngatu that can be cut from a launima is a fola'osi (4 langanga). It is not socially acceptable to have one, two or three langanga cut from the tapa; therefore, any piece of tapa has to be at least 4 langanga. The names given to both mats and tapa cloth are derived from their lengths.

Weaving

Among the tasks for Tongan women was weaving. Mats of various kinds were made and these are still main activities of many women, particularly those in the Ha'apai group and the outer islands of Tonga.

The researcher collected data about weaving via a group interview with five weavers from the island of Pukotala in the Ha'apai group. It reveals the kind of "mathematics" used by weavers and the role of estimation in weaving.

The first task for the weavers is to prepare the fe'unu (strips of pandanus leaves ready for weaving) from the dried tofua, kie, paongo, or tutu'ila (four different kinds of pandanus). In a society with a hierarchical social structure and a hierarchical view of life, pandanus too were ranked according to this hierarchy. The most chiefly of these kinds of pandanus is the paongo, followed by kie, tofua and then tutu'ila. Thus, if a mat is required for presentation to the King or chiefly people paongo is used, and if it is for the common people, tofua is used. Even though the term 'fala' is used as the Tongan translation for mat, fala is a name given to a special type of Tongan mat. In Tonga the different kinds of mats can be identified from the size of the fe'unu. For instance, the fe'unu for the fala ranges from 5 to 8 cm, for the lotaha 9 to 12 cm, and for the papa 18 to 25 centimetres. Moreover, a fala is loua (two layers of

fe'unu facing opposite sides), while lotaha (one-layer) is as its name suggests one layered fe'unu, and papa can be either lotaha or loua (one or two layered).

To prepare the fe'unu, the piece of dried pandanus (about 4-6 cm wide) is ha'alo (scraped with a shell to make it clean, soft and smooth) and then tohi (split with a shell) into strips of four, five, six for a fala, three or four for the lotaha and one or two for a papa. It is important that these strips are of equal width. Through experience in splitting the pandanus leaves the women have learnt to prepare the fe'unu without measuring; estimation skills are used. For the purposes of the present study, the discussion is now limited to the division of labour and work involved in weaving a fala.

Once enough fe'unu has been prepared for the fatu (to begin the making of the mat), a decision is made about how they are going to divide up the task of weaving. An explanation of this division of labour by one of the weavers, is provided below:

*Tau pehe, tau taki **fitunga'ahoa** pe. Pea tau palapala pe. Taki pala 'e nima ki he 'aho.*	*Say, let's use **seven pairs**. Let's use palapala (portions of rows). Five portions each per day.*
Kapau ko e tekumi ma ua ko e 'aho ia 'e tolu. 'Okapau ko e pala 'e nima he 'aho. 'Ai e fo'i hala ko e 'o tau, 'aia ko e fo'i pala ia 'e taha. Pea toki ha'u e fo'i hala ko e 'o tau pea toki lau hake leva ia. Taha, ua, tolu, fa, nima. 'Aia ko e tahaua ia 'oku fe'unga ia mo e fo'i 'aho ia 'e tolu. 'Ai ko e tahanima ko e 'aho ia 'e fa pea ko e tekumi 'e 'aho pe ia 'e ua.	*For a twelve span mat, that would take us three days, if it is five rows a day. One would work on each portion of the row. When one reaches the end of her allocated portion, then a new row is started. The women would continue until five hala (rows) are completed. So a twelve span mat would take three days. A fifteen will take four days whereas a ten span mat will only take two days.*

In this context, **fitunga'ahoa** refers to the use of seven pairs of strips for each **pala** or **hala** (row). For the actual weaving of mats the terms pala or hala are used to refer to the row and palapala means more than one row. The size of the pala, that is, the number of fe'unu is then determined by the size of the mat to be made, and the ease with which women could work with it. Often these do not necessarily correspond to one hangafesi (argumented span, 30 cm), and this explains the need to have a fourth day to complete a tekumi ma nima (fifteen). Although the fifteen pala might be completed in three days, these do not necessarily correspond to 15 widths required for this mat.

At first one pala or length of a mat is made, the women taking their place equally spaced out along the length of the mat. Each will concentrate on her portion of the mat. The hala or row (consisting of seven pairs of strips) is then woven, one hala at a time. Once this is completed a new hala (again consisting of seven pairs of fe'unu) is begun. The other women will carry on from where the first ones left off. The process of making is repeated until slightly over half of the width of the mat is completed. The width of the mat is then measured using a

hangafesi. For a tekumi ma ua (12 spans) the width would be about 6 hanga and therefore the unfinished mat is folded after three hanga has been completed. The women will then continue to weave the second half of the width of the mat.

The above explanations reveal several mathematical concepts used by the weavers in the process of weaving. These are learned through observation of older people's work. For instance, counting skills, particularly counting by two or in pairs, is evident in the use of **fitu nga'ahoa**. In the everyday life of the Tongan women, counting in twos appears to be the norm. Division, fractions and ideas about rate are all part of the language of the weavers. Estimation is used for determining the size of the fe'unu, the number of fe'unu in a hala, and the size of the width of the mat. As well, estimation is used to determine the number of rolls of pandanus and the number of days required to complete a mat.

Tongan view of estimation

The above discussion suggests that fakafuofua (estimation), in traditional activities is important and useful. The skills involved are acquired through many experiences in working in a particular activity. A person, such as a weaver, would look at the length of a mat and decide the appropriate measure for its width. Fakafuofua is determining a solution to a problem depending on one's experiences in that area. The purpose of the estimation has to be taken into account. For traditional use of fakafuofua, there is a purpose for the estimation task and it is this purpose that determines the solution to be obtained.

The skills involved in estimation were acquired from considerable experience in a variety of activities. Knowledge of the environment through observation of nature, moon and stars, patterns, behaviours of animals and birds, and apparent changes in the development of crops were all significant in people's estimation and calculation of time. It appears that fakafuofua (estimation) was a major part of Tongan people's everyday life.

Other issues to have come out of the above discussion are related to the use of various methods of counting and measuring lengths. The use of measures such as **ofa** (arm length) and **hanga** (span) will lead to inexact answers compared with using measuring devices such as the ruler. However, these were measuring instruments used by Tongans and measures obtained cannot be considered as estimation. It was only after the introduction of standard measuring instruments such as rulers that use of the above measures were considered estimation. The use of traditional measuring devices are limited to out-of-school activities where they may be the only measuring instrument available for use, but they could and perhaps should form the basis for estimation in the classroom.

2.4 Estimation in the Formal School Curriculum

The mathematics curriculum used in Tonga since the establishment of schools gave little attention to estimation. Discussion of estimation in the school curriculum requires reference to developments in mathematics education and education in general.

Early mission schools (1820–1860)

Because their objective was to convert people to Christianity, the first task of the missionaries was to develop a written form of the Tongan language and then translate the Bible into it. Selected verses from the bible were then learnt until they could be recited from memory. The most elementary forms of reading, writing and arithmetic were taught.

In the field of numeracy the missionaries found that they had a sound foundation upon which to build. They simply listed the vernacular names for the numbers from 1 to 100,000. Once the symbols had been learnt, arithmetical tables were taught. For many years little more than repeating numbers and chanting tables were taught in schools. A common method of teaching was to have the whole class recite orally or chant the times tables (Cummins, 1977; Tupouniua, 1977).

It is evident from the writings on Tongan education that the mathematics curriculum in these early schools centred on counting, times tables and the basic arithmetical rules.

Because the aims of the missionaries were to 'civilise' as well as 'convert' the people, 'civilised' concepts such as numeracy and the accurate calculation and recording of time found a place in the school curriculum. Cummins (1977) reported that the books used in the schools introduced the word taimi (time) as there was not an appropriate word in the vernacular to describe this concept. The traditional expressions that described time in terms of changes in the environment, were replaced by new words such as **momeniti** (second), **miniti** (minute), **houa** (hour), **uike** (week) and **moniti** (month). The vernacular terms **'aho** (day) and **ta'u** (12 month seasonal cycle) were retained. It ought to be noted the terms **moniti** for month and **momeniti** for second were later replaced by **mahina** (moon) and **sekoni** respectively.

The curriculum of the missionary schools represented the knowledge that missionaries considered to be important and of value. This period saw the beginning of the undervaluing of traditional knowledge and skills. Traditional knowledge possessed by the Tongan people before contact with the European was not included in that taught in schools, so that traditional knowledge was replaced over time with the technological knowledge of the West (Mara, et al., 1994). The Tongans were eager to shun their own customs and material goods for things European, unaware until much later of their effects on their way of life (Kavaliku, 1966).

Formal schooling between 1860–1900

With the establishment of a government and the introduction of a Constitution in 1875, the aim of education changed from teaching children to read and write to that of preparing people for government employment and for teaching in the newly established system of education. The curriculum was broadened to include mathematics and other subjects. As far as mathematics is concerned, it was not until the arrival of Dr Moulton in the 1860s that any significant advances were made in introducing mathematical knowledge. Tonga's first secondary school was established by the Wesleyan mission in 1866 and was named after the King, Tupou College. Dr Moulton and subsequently Dr H.H. Roberts introduced courses such as Euclidean geometry and algebra in their school's curricula. Much of the syllabus was the same as in many English Grammar schools (Tonga Ministry of Education, 1992b).

Schools between 1900–1944

Tupou College set the general pattern for post-primary education until 1947, with a strong emphasis on academic subjects. The curriculum Moulton used reflected the English public and grammar school tradition. Moulton was not encouraged to introduce 'practical subjects' into the curriculum (Kavaliku, 1966).

In 1912 the first Director of Education, W.H. Gould, a New Zealander, was appointed and apparently arrived in Tonga to find the whole educational system in a *disorderly and disorganised condition* (Kavaliku, 1966). An Education Ordinance was passed in 1913 which represented the first attempt by the government to introduce a set syllabus into Tongan schools. Prior to 1914 there is no record of any set syllabus in schools. The schools had their own curriculum depending upon what the teachers were capable of teaching. The theme of the 1914 syllabus was the usefulness of school subjects to the everyday life of the child (Kavaliku, 1966).

The 1934 mathematics syllabus for the Leaving Certificate Examination included (i) mental arithmetic, (ii) arithmetic problems involving the four simple rules, money, weights and measures, decimals fractions, proportion, simple and compound interest, trade discount, averages and percentages, (iii) mensuration of plane figures, rectangular solids and cylinders and (iv) algebra including simultaneous equations (Tonga Government, 1934). Estimation did not have an explicit place in the mathematics curriculum.

Curriculum development from 1945 to the present time

The Tongan Government Gazette (1946) listed the arithmetic syllabus for the Public Service Examination as *consolidation of arithmetic as in the Leaving Certificate Examination Syllabus with special attention to speed and accuracy* (p.47). The syllabus for Leaving Certificate Examination included *mental arithmetic based on rules and methods indicated and prescribed*

for written arithmetic. Once again, it appears that there was no attention given to estimation in this syllabus either.

In 1947, a new government secondary school, named initially Matriculation School and later Tonga High School, was established. Its main purpose was to prepare students for the New Zealand University Entrance Examination with a view to sending the best of them for further education overseas. This school adopted the New Zealand Syllabus and recruited teachers from New Zealand. This school continued the tradition of putting more emphasis on academic subjects.

The churches continued to play an important role in education. However, one of the major problems associated with the provision of education by various churches was the adoption of different syllabi for their schools. The Wesleyan education system followed the Victorian (Australia) syllabus, the Roman Catholic secondary schools used the New Zealand syllabus, the Latter Day Saints followed an American pattern while the Church of England, Seventh Day Adventists and other churches used the local Tongan syllabus (Kavaliku, 1966).

In 1969, a new primary schools syllabus was implemented, the first major change in curricula since 1942. The mathematics Scope and Sequence Chart (Tonga Ministry of Education, 1974) divided the syllabus for Classes one to six under six headings. These were *set, numbers and numeration, operations, measurement, shapes* and *representation*. For the first time estimation appears in the mathematics syllabus, under the heading measurement. *"Estimations of size of sets - larger, smaller; of elements - size weight, length; and description of elements"* was listed in this chart. This chart also listed *"estimation of fractional parts of objects."* However, estimation appeared only in the syllabus for class one. Even though estimation appeared in the official curriculum, the extent to which it was emphasised in classroom teaching is unknown.

In 1972, a new curriculum was introduced to the secondary schools, which was a slight modification of the New Zealand syllabus. Its objective was to introduce a subject range which more specifically addressed the circumstances of Tongan students, rather than educating them as if they were all destined for overseas university study. Mathematics syllabi used by New Zealand secondary schools were also used in Tongan classrooms. A significant development was the establishment of the Curriculum Development Unit (CDU) of the Ministry of Education in 1977. The Unit's brief was to develop curriculum materials meaningful to Tongan students.

Recent developments and changes in examinations, for instance the abolition of the New Zealand University Entrance Exam in 1986 and the adoption of the Tonga School Certificate and Pacific Senior Secondary Certificate courses and examinations in 1989 for forms five and

six respectively, added a sense of urgency to the demand for appropriate curriculum materials. The establishment of the Tonga National Form 7 in 1991, which prepares pupils for the New Zealand Bursary Examination, also suggested a need to review the whole curriculum from primary through Secondary to Form 7 to ensure good linkages between each level of education (Tonga Ministry of Education, 1992b).

At present the mathematics curriculum writers in the Tonga Curriculum Development Unit are headed by a senior education officer. There is one full-time writer each for secondary and primary school mathematics. Several part-time writers who work full-time as mathematics teachers are called upon to assist in the development and writing of curriculum materials. All writers are very experienced mathematics teachers. Another education officer works primarily on the trialling of the use of the soroban, a project funded by the Japanese government.

In 1992, a draft mathematics syllabus for Class 1 up to Form 5 was prepared by the Curriculum Development Unit in association with the Australian International Development Assistance Bureau, Queensland Education Consortium and the Australian Government. Not only did the Australian International Development Assistance Bureau finance the development of the new draft syllabus, they also provided personnel and other resources. Australian consultants visited the Tongan curriculum writers at the Curriculum Development Unit from time to time to advise on and evaluate materials being written and trialled. In comparison to the mathematics syllabus of the 1970s, there is more emphasis on estimation in the new syllabus. One of the aims of mathematics education specified in the draft syllabus is to help students *estimate appropriately, and recognise the reasonableness of results* (Tonga Ministry of Education, 1992a, p. 5).

Ideas on estimation feature in the trial material for both primary and secondary schools. Informal discussions with the curriculum writers indicated that although they are very experienced mathematics teachers, an overall theoretical framework on which they could base the development of the mathematics curriculum seems to be lacking. The theoretical framework for the Draft Syllabus was provided by the consultants. Ideas on estimation as well as materials and guides developed for teachers, were also based on the advice and expertise of the overseas' consultants.

Reliance on the Australian consultants is reflected in the use of English in writing the maths syllabus for classes one to six, with a planned translation into the Tongan language. Guides for teachers and students in class one were written in Tongan. Materials for other primary school classes are also being developed in the Tongan language. Trialling of this syllabus is being carried out in selected schools with a plan for full implementation in 1996.

2.5 Textbooks and Resources

By the 1850s an arithmetic textbook was prepared and used in the schools (Cummins, 1977). One of the problems encountered by the researcher in collecting material for this chapter was the lack of old textbooks in the Ministry of Education and the Wesleyan church Department of Education office. As one of the workers put it, "*Ko e 'osi pe hono 'aonga e 'u tohi kuo laku kinautolu ia he veve* ." (Once books were outdated they went to the rubbish).

Except for a few booklets and ideas written in the Ministry of Education publication, Tokoni Faiako (Teacher's Help or Help for Teachers) and the curriculum materials, there has been very little mathematics material written by Tongans for use in the Tongan secondary schools. Over the years, Tonga has relied on textbooks from overseas countries such as the United Kingdom, Australia, New Zealand, United States (for Mormon schools) and Fiji.

Dr Moulton wrote books for the curriculum he prepared and implemented at Tupou College and the Wesleyan Mission schools. One of these was entitled "Eukaliti" or "Euclid" (Moulton, 1900). An analysis of the text shows Euclidean geometry was a major component. The Tongan Government Gazette (1934) listed Pendlebury's Arithmetic, Longman's Junior School Mensuration, and Hall and Knight Algebra as the textbooks to be used for arithmetic, mensuration and algebra respectively.

Most of the imported textbooks used in mathematics classrooms in Tonga contain examples which are irrelevant to the experiences of Tongan students. Too often these are the only examples used by teachers to illustrate mathematical concepts. At present, most secondary schools in Tonga use a New Zealand series of six textbooks "Moving with Mathematics" written by Begg, Mackintosh and Thomson (1974) and this series does not include estimation.

Mathematical instruments

Nowadays, rulers and other mathematical instruments used in western countries are used by Tongan students. Rulers and tape measures have replaced the traditional use of the **ofa** and the **hanga**. In some classrooms there are weights and measuring cylinders for capacity. However, in many primary school classrooms, the only other resources for the teacher are the blackboard and the one-metre ruler.

Calculators and computers

For students up to form three, the use of calculators is not permitted in the classrooms. It is believed that the use of a calculator will lead to a decline in students' knowledge and understanding, particularly their mental computational skills, with a reliance on the machine rather than the use of their heads to work out mathematical problems. In forms 5 to 7, calculators are considered necessary tools to be used in the classroom. Estimation skills

become even more important at this level for determining the reasonableness of results given by a calculator (Carpenter et al, 1976; Reys, 1984).

To date no computers are used in the mathematics classrooms in Tonga. The use of computers in the classroom is not a priority. For many classrooms, particularly in the outer islands, a major priority for the teacher is to ensure that there is chalk available.

2.6 Teacher Pre-service Education/Training

Teacher education in Tonga began in the 1840s with the establishment of the "Friendly Island Wesleyan Academy for the Training of Native Assistant Missionaries." Again, this institution was established by the missionaries to educate the most promising teacher-preachers. Initial enrolment was only five and by the third year the enrolment increased to fifteen. The curriculum included: Scripture reading; spelling; writing; catechism; geography; a short course on the leading doctrines of Christianity; a few elementary rules in arithmetic; and some of the more able students were also taught English.

Reports indicate that there was a familiar diet of rote learning and catechising. The students chanted almost everything, a familiar and popular mode of receiving instruction (Cummins, 1977; Tupouniua, 1977). This mode of receiving instruction is very difficult to displace (Tupouniua, 1977) and tends to persist in primary schools. It can still be heard in the mathematics classroom, where students chant the times table.

Ua 'e taha 'oku ua	*Two ones are two*
Ua 'a e ua 'oku fa	*Two twos are four*
Ua 'a e tolu 'oku ono	*Two threes are six*
Ua 'a e fa 'oku valu	*Two fours are eight*
Ua 'a e nima 'oku hongofulu	*Two fives are ten*

and so on.

In 1944 the Tonga Teachers Training College was established, although this was first provided for in the 1927 Education Act. The aim of the College was to train primary school teachers. The college provided Tongan teachers for the first time with formal, professional training. This was seen as an important step in improving the standards of schools. The course was of two years' duration, with admission based on examination. The mathematics education programme at the College was an academic one. In recent years the programme at the college was seen as an extension of the secondary school curriculum with some emphasis on teaching strategies for mathematics. Strategies for teaching estimation are not found in these courses.

In 1986 a Diploma in Teaching was introduced. This is a three-year, full-time preservice training programme that provides training for both primary and secondary teachers. All students follow a common programme in their first year and then in the second year, students wishing to undertake secondary teaching are required to take two teaching majors, while those who prefer primary school teaching are required to do a general programme. Again, the programme consists of an academic course in mathematics. Moreover, in general, estimation does not feature at all in the preparation of teachers for either primary and secondary mathematics.

Although attempts are being made by the Ministry of Education to upgrade teachers' professional skills and knowledge through a series of in-service courses, this is hindered by lack of funds. The Ministry of Education Report for 1989 stated that the inservice programme for "*primary teachers was determined by the [total] budget of $1500 per annum for the 102 government primary schools in the kingdom, from Tongatapu in the south to Niuafo'ou in the far north. The financial constraints meant that the three main centres, where the field officers were stationed, benefited whilst Niuafo'ou and Niuatoputapu could only be inserviced through the radio* "(p.5). Inservice training for all subjects including mathematics was covered by the above budget. The same budget for inservice training was allocated for 1992. *"As usual the shortage of the annual budget of $1,500 for the whole kingdom was the major problem"* (Tonga Ministry of Education 1992b, p.4). A one-hour weekly radio programme for teachers of primary schools is conducted by the Ministry of Education. These programmes provide the means of disseminating information on teaching methods to all primary teachers, and for those in the outer islands these could be their only form of receiving inservice training.

An analysis of the pre-service and inservice courses for Tongan mathematics shows a lack of attention to estimation.

2.7 Assessment and Research

External Examination

By about the 1840's public examinations were carried out in schools. In the everyday classroom lessons, number work was usually confined to counting and chanting the multiplication tables. These were the most popular items on examination days. Cummins (1977) reported a typical problem in an examination in 1850's was of the type:

> If a piece of tapa measures so much, how many will it require to reach from Tongatapu to London, the distance being, say fifteen thousand miles? (p.259)

An account by Lawry (1851) of an examination day at Nuku'alofa in 1847 reported that students worked the sum in the first five rules of arithmetic and their examination was concluded by the simultaneous repetition of the arithmetical tables. Lawry did not specify what these five rules of arithmetic involved.

Since estimation was not part of the curriculum until the early 1970s, its assessment was not an issue until recently. Almost all of the assessment of students' mathematical skills focused on placing a tick (√) beside 'correct' or 'right' answers. Assessment also concentrated on one right procedure or method to obtain the right answers. More recently assessment has become slightly more flexible. For example, students are now given points for part of a procedure followed correctly, even if the answer shown is wrong.

The Secondary Entrance Examination at the end of class six is the first external examination that all Tongan students sit. Mathematics is one of the four subjects in which students are examined. An analysis of previous years papers indicates that estimation has not been a major focus of these exams. However in recent years, for instance, in 1987 and 1990 one question on estimation appeared in the secondary school examination papers. These items were;

SECTION A

10. Which of the following things has a height closest to one metre?

A. Table

B. School Bag

C. Church

D. Door

(Tonga Ministry of Education, 1987)

SECTION ONE

19. Which of the following can be filled with 50 litres of water ?

A. Spoon

B. Bucket

C. Bath Tub

D. Concrete water tank

(Tonga Ministry of Education, 1990)

It is interesting to note that both items on estimation were presented using a multiple choice format. Research studies have shown that it is questionable whether estimation skills are adequately assessed by multiple choice items (Reys and Bestgen, 1981; Reys, 1993; Rubenstein, 1985). However, the inclusion of these items in the paper suggests some recognition of the importance of estimation in the mathematics curriculum. It is also interesting to note that both items were presented in the English language, possibly

disadvantaging students without a good command of the English language. Furthermore, only estimation items on measurement were used; computational estimation items have not been used in the mathematics examination.

The secondary schools adopted the New Zealand School Certificate and University Entrance Examination in forms 5 and 6 respectively until 1987. Since 1989 the form 6 students sat the Pacific Senior Secondary Certificate examination and since 1991 students from the National Form 7 have sat the two New Zealand Bursary Mathematics examinations. Inclusion of estimation in the mathematics curriculum was confined mainly to the primary schools; the secondary school mathematics examinations both internal and external did not include questions on estimation. It appears that the inclusion of estimation in the new draft syllabus (if implemented) will require teachers and examiners to address the issue of assessing students' estimation skills in mathematics.

School examinations

Many of the assessment practices in the New Zealand and the Australian systems of education were adopted in the Tongan classrooms. Mid-year and end-of-year exams are used in most secondary schools. In the primary schools, end of term exams and prize-giving are held at the conclusion of each of the three terms in the academic year. Thus, ranking of students in mathematics begins in Class 1 and continues until they leave high school. By six years of age, Tongan students come to know their respective positions in the class in comparison to their peers, in terms of their performance in school internal tests. As with external examinations, estimation has not been a focus of internal school examination, a direct result of the lack of emphasis on this topic in the mathematics curriculum.

Research on estimation

In Tonga, as in some developing countries of the world, virtually no research in mathematics education has been conducted and as far as estimation is concerned the present study is the first in Tonga. Apart from the few items which appeared in the Secondary Entrance Exam papers, from which some general trends in students' estimation skills could be inferred, there is no available data on Tongan students' performance on this topic. Item analyses of the 1987 and 1990 mathematics examination papers were not available from the Examination Unit of the Tonga Ministry of Education at the time of data collection. Therefore, to date there is no available data on Tongan students' performance on estimation tasks.

This lack of research data on estimation in Tonga is not surprising given that research efforts on this topic in overseas countries have only began to emerge and develop during the past fifteen years. As in other developing countries the Tonga Ministry of Education does not have the funds to finance research on mathematics education let alone studies on a particular topic such as estimation. A research unit within the Ministry of Education is yet to be established so

that research on mathematics education would appear to rely on the interest and concern of Tongan postgraduate students selecting thesis topics for research in their personal study.

2.8 Summary and Comment

The purpose of this chapter has been to (i) discuss some of the historical background information about the place of estimation in the Tongan society particularly in informal and formal education and (ii) draw attention to those facets of Tongan society which would support the teaching of the new mathematics curriculum.

Education in pre European Contact

As discussed in section 2.2, traditionally education took place within the extended families. Learning occurred in informal ways through observing, listening, memorising and practical application. The content of what was taught and how it was taught were determined by an individual's membership in one of the social classes of the stratified Tongan society. Membership in each social class was through birth and traditional education ensured conformity to it. Thus, the inculcation of leadership at the top and obedience at the bottom were key principles; loyalty to one's group and respect for elders were enforced ideals.

Estimation in the Tongan society

The everyday activities of fishing, house and canoe building, planting and harvesting of crops, weaving and tapa-making provided the means by which Tongans learnt appropriate skills and behaviour for their roles in society. Thus, relevant estimation skills were acquired through everyday experiences. Such experiences provided people with opportunities to use and practice their skills of estimation, and as such estimation was a major part of Tongan everyday life activities. However, some of the skills and knowledge involved in these activities seem to have been lost due to their lack of use, particularly after the introduction of various technological devices through contact with the western world.

Estimation in schools

Sections 2.4 to 2.7 reported on the role of estimation in formal schooling, highlighting the lack of emphasis on estimation in the mathematics curriculum, textbooks and resources used in the mathematics classroom, teacher education, and assessment and research. The lack of attention to estimation mirrors what occurred in other developed countries such as the United States, United Kingdom, Australia and New Zealand prior to 1980. This lack of attention to estimation in school mathematics is a direct result of lack of attention to this topic in the mathematics curriculum. It should be noted that the mathematics curricula used in Tonga have been imported from other countries since the formalisation of schooling in Tonga. Initially British missionaries set up schools and therefore the English Grammar Schools mathematics curriculum were used. Between the 1940s and 1980s the New Zealand mathematics curricula

were adopted, and more recently from the 1980s Australian mathematics educators have supported curriculum development in Tonga. All these had major influences on mathematics education in Tonga.

Case for the present study

This chapter has highlighted the lack of emphasis on estimation in the formal school curriculum since the formalisation of schooling in about 1828. The mathematics that was taught involved chanting numbers and the times tables, with major emphasis on speed and accuracy. As discussed in chapter one, estimation deals with inexact answers, the approximate solutions to problems. This is more in line with the types of mathematics Tongans used in traditional ways of measuring. It is interesting to note that although these methods of measurement are not found in the mathematics curriculum, they are included in the Tongan studies curriculum. However, these are not referred to as methods of measuring lengths; rather they are called methods of **lau e fala** (counting mats) and **lau e ngatu** (counting tapa cloth). Their relevance to mathematics learning in the classroom has not been identified or emphasised.

As mentioned earlier in this chapter, the Tongans were quick to adopt things European (Kavaliku, 1966). However, there is a growing awareness among Tongans of the need to preserve and maintain traditional skills. As a result, a committee was set up in the late 1950s to conduct research into and preserve Tongan customs and traditions. This change in attitude and the aim of this committee was outlined in the late Queen Salote's opening speech to Parliament in 1959, in which she said:

> I wish that all Tongans should realize the importance of full knowledge of our customs and traditions... The customs of a people are its heritage and the possession of a complete knowledge of their customs will enable them to modify those customs to meet the demands of modern progress along lines most beneficial to their well being. It is the aim of this committee to stimulate among the Chiefs and People of our land a desire to hold on to what is best in our customs (Tonga Government, 1959).

The ideas expressed above, if applied to mathematics education, will conceivably lead to proper identification of the types of mathematical skills and concepts inherent in the activities of the Tongan people. It should be noted that in comparison to Tongan studies, Mathematics has a higher status as a academic subject. Achievement in Mathematics is highly valued whereas a pass in Tonga studies is desirable but not highly valued. It seems that what is required is to shift topics such as Tongan ways of counting and measuring from the Tongan studies curriculum to relevant units in the mathematics curriculum to give them the status they deserve and to enable students to value them as mathematical topics worthy of study.

Identification of these mathematical skills is part of ethnomathematics. The development of studies in ethnomathematics in Tonga could lead to incorporation into the mathematics curriculum of mathematical ideas used in the activities of the Tongan people, among these are estimation skills.

Such efforts will be valuable for the teaching and learning of important topics such as **estimation**. As discussed earlier in this chapter, it is possible that **fakafuofua** (estimation) as used by people in their everyday life could be different from that used in the classroom. Therefore, there is a need to conduct further research in estimation as it is used in the classroom as well as estimation used in the everyday life of the Tongan people. The present study investigates estimation in the formal schooling of both primary and secondary school students. It is anticipated that the present study will identify useful information for teaching and learning estimation in the schools.

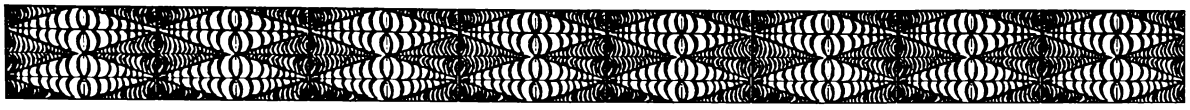
The 1992 Mathematics Draft Syllabus (Tonga Ministry of Education, 1992a), which is to be implemented in 1996, includes estimation as an important mathematical skill for both primary and secondary schools. Possibly estimation has been included in the curriculum because it appears in Australia's mathematics curriculum, and Australian mathematics educators assisted in the development of the draft Tongan document. Whatever the reason, estimation is an important Tongan traditional skill that was and is useful in the everyday life of the Tongan people. Unlike other mathematical concepts, such as differentiation and integration, estimation is not a 'foreign' topic. There is considerable potential for this topic to provide the vehicle by which Tongan traditions and ideas are incorporated into the classroom mathematical experiences of Tongan students.

As mentioned in section 2.7 the lack of research on estimation is due mainly to lack of funds for research in mathematics education in general in Tonga. Moreover, the Tongan Ministry of Education does not have a research unit. The present study is designed to contribute to the development of research in mathematics education in Tonga by focusing on estimation, a traditional Tongan skill which is useful in everyday life as well as a worthwhile mathematical process.

This chapter has provided a background of the role of estimation in both informal and formal learning in Tonga relevant to the present study. The next chapter focuses on what the research literature has to offer about estimation.

CHAPTER THREE

REVIEW OF THE LITERATURE



3.1 Introduction

As noted in chapter one, historically, estimation has received very little attention in mathematics curricula and classroom learning and teaching. This lack of attention to estimation was also reflected in a lack of research on this important topic over the years. During the last fifteen years, however, there has been growing recognition of the importance of estimation resulting in growing attention to research on this mathematical process. Most studies have focused on computational estimation, with a few studies on measurement, and numerosity or quantitative estimation. Although there exist some overlaps in the main focuses of previous studies, they concentrated on three main areas:

(i) estimation skill or ability and its relationships with other mathematical abilities (Bestgen, et. al., 1980; Corle, 1960, 1963; Gliner, 1991; Herman, Norton and Roth, 1983; Levine, 1982; Rubenstein, 1985);

(ii) identification of strategies used by estimators (Dowker, 1992; Hildreth, 1983; Koloto, 1989; Levine, 1982; Reys, Reys, and Penafiel, 1991; Reys., Bestgen, Rybolt, Wyatt, 1982; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991; Siegel, et al., 1982; Sowder and Wheeler, 1989; Threadgill-Sowder, 1984); and

(iii) investigating methods of teaching estimation (Bestgen, Reys, Rybolt and Wyatt, 1980; Bobis, 1991; Case and Sowder, 1990; Hildreth, 1983; Schoen, Freisen, Jarrett, and Urbatsch, 1981).

It is in terms of the above broad classification that this chapter reviews previous studies on measurement and computational estimation. The next section (3.2) reviews studies that focused on estimation skills and other mathematical abilities and/or concepts. Section 3.3 considers studies aimed at identifying strategies used for estimation. Section 3.4 centres on studies that investigated the effects of instruction on the development of estimation skills and strategies. The final section (3.5) summarises major findings in the research data and their implications for the present study, limitations in previous studies, and further outlines the case for the present study.

3.2 Estimation Skills and Their Relationship With Other Abilities

Although there are several types of estimation, three of which were described by Sowder (1992) (i.e. computational, measurement, and numerosity or quantitative estimation) and that researchers have attempted to investigate these different aspects of estimation. This review of the literature will focus on studies of measurement and computational estimation, the two types of estimation relevant to the present study.

Measurement estimation skills

Among the earliest efforts to study the estimation skills of students in the United States were those of Corle (1960, 1963), who sought to investigate the accuracy of subjects' estimates of quantities such as weight, linear distance, temperature, time and capacity. Ten objects (e.g. weight of a blackboard eraser, weight of a block of wood, length of a piece of rope) were used by Corle (1960) to study the accuracy of the estimates and measures provided by 108 sixth grade (55 males and 53 females) and 39 fifth grade (17 males and 22 females) students.

Indices of estimate error and measurement were calculated for each estimate and measure given by each student. These indices of error were obtained by dividing the amount of error by the actual measurement of the quantity. For example, if a block of lead (3 1/2 pounds) was estimated to be 10 1/2 pounds, the error would be 7 pounds. This error was then divided by 3 1/2 giving a quotient of 2, the *index of estimate error.*

Errors in measurement by sixth graders averaged 0.78 of the actual values whereas for fifth graders the error was 2.4 times the actual values. The greatest discrepancy in estimating and measuring occurred in weights, whereas, the least was found in temperature. Subjects provided more accurate estimates for length measures and temperature. The results of the study showed that sixth graders were more effective in estimating quantities than were fifth graders, suggesting that estimating and measuring were learned skills which would increase with an additional year of school experience.

A second study by Corle (1963) investigated the estimation of quantities by 368 elementary school teachers and 96 college students. As with the previous study (Corle, 1960) ten problems involving estimation of weight, linear measure, time, capacity, volume, temperature, and barometric pressure were used. Items were presented to groups of teachers or groups of college students. Again all of the estimates were considered in terms of the ratio of the error of estimate to the actual quantity;estimates within 10 percent of the actual value were considered to be 'correct'.

The data showed that estimates given by teachers and college students were more accurate than those provided by sixth and fifth grade students in the earlier study by Corle(1960). The

greatest difficulty occurred in estimating the cubic content of a basketball. Only 4 percent of those who participated in the study gave reasonable estimates. Lack of success with estimating air pressure indicated lack of familiarity with terms describing air pressure. The results also showed that teachers were not superior to college students in their ability to estimate.

Herman, et al. (1983) conducted two studies which examined time and distance estimates as a function of the number of objects between locations. Subjects were twenty-four males and twenty-four females at each of 7, 9, 11 and 19 years of age. Results showed that subjects who took the first half of a walk in a shorter amount of time compared to the second half tended to slightly underestimate the first half distance relative to the second half distance. In contrast, subjects who took the first half of the walk in a longer amount of time than the second half overestimated the first half distance relative to the second. Moreover, subjects who made time estimates then distance estimates were more accurate on the distance estimation. The results showed that at all age levels the actual time travelled and the subjects' perception of this time were significantly related to distance estimates.

In a New Zealand study of 13 and 14-year-olds, Koloto (1989) conducted a survey of a sample of Form 3 students (n=208) as well as individual interviews with a small representative sample (n=24). This procedure was used to (i) obtain quantitative data about how well students performed computational and measurement estimation, and (ii) investigate in greater depth the **strategies** that the students used when formulating their estimates. Students' scores in the survey was taken as measures of estimation skills. The results of the survey showed that students were competent at estimating measures of length. Tasks involving estimation of weights and capacities of large objects proved difficult for students. Therefore, they tended to make guesses. Students had difficulty estimating capacity of 50 litres and possessed inaccurate ideas about the capacities of larger objects such as a milk tanker or the school swimming pool.

Taking these measurement estimation studies together, it is clear that the researchers have focused on different measurement estimation tasks.

An important issue that needs consideration is the type of assessment procedures used by these researchers. Whereas Corle (1960, 1963) used indices and percentages of estimate error as measures of the subjects' estimation skills, and values within 10% of the actual value as the 'correct' response, Koloto used various ranges for each item, depending on what seemed reasonable for each task, with the total score in the survey as a measure of students' estimation skills. The Herman, et al. (1983) study did not specify the criteria used for assessing the estimates.

It seems that the use of percentage error gives an unrealistic picture of the errors involved. For instance, providing 14 pounds as an estimate for a 5 pounds brick involved an error of 180%. This could have been better presented and analysed by considering the actual error of 9 pounds. Using 180% has magnified the error, thus percentages seems inappropriate for assessing results of estimation. The emphasis by Corle (1960, 1963) on the errors for the estimates is another issue of concern because it does not give recognition to one of the characteristics of estimation, that is, it almost always involves some error.

Whereas Corle (1960, 1963) used estimation problems involving weight, linear measure, time, capacity, volume, temperature and barometric pressure, Herman et al. (1983) focused on just distance and time estimation. Koloto (1989) used a subset of the Corle list involving length, weight and capacity tasks. Since the tasks differed greatly between these studies, it would be unwise to draw any general conclusion from the above findings regarding a particular quantity. However, it appears that in general students tend to lack measurement estimation skills.

Computational estimation skills

One of the main sources of data on students' computational estimation skills are large scale surveys in countries such as the United States, United Kingdom and New Zealand. A number of these surveys of students' mathematics performance revealed important findings regarding their estimation skills. For instance, the results of the first, second and third National Assessment of Educational Progress (NAEP) studies in mathematics in the United States provided ample evidence that students lacked basic and necessary computational estimation skills (Carpenter et al., 1976; 1980a; 1980b; Reys, 1985; Reys & Bestgen, 1981). To illustrate, only 54% of 17 year olds and 64% of young adults who took part in the first survey could estimate to the nearest million dollars the sum of four addends: $ 11 954 164 + $ 1 126 005 + $ 4 170 522 + $ 750 572. For this exercise several different strategies could have been used by respondents. As Carpenter et al. (1976) pointed out it was impossible to determine whether individual respondents actually estimated the answer using some kind of shortcut procedure or simply added the four numbers and rounded the sum. This is one of the problems associated with assessing students' estimation skills through large scale surveys.

Similar low achievement patterns were obtained from the results of the Assessment of Performance Unit (APU) in the United Kingdom and from the second international mathematics study conducted in New Zealand under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) (Department of Education, 1982, 1987; Robitaille and Garden, 1989).

Data from the APU survey showed that only 54 percent and 27 percent of 15-year-olds, could round a number involving tens of millions (eg. 56 283 893) to the nearest million and nearest thousand respectively. The poor performance of students in tasks involving decimal numbers

was shown in the data from the New Zealand sample in the second IEA Survey (Department of Education, 1982, 1987; Robitaille and Garden, 1989). For example, only 18 percent of 13-year-olds gave the appropriate option (0.006) as the product for 0.2131 x 0.02958.

It should be noted that even though these survey results have been taken as measures of students' estimation skills, they may be more appropriately referred to as studies of approximation skills because the tasks required a specific degree of accuracy such as rounding to the nearest million. In addition, large scale surveys have relied on a multiple choice format for presentation of estimation tasks and it is doubtful whether estimation is the only skill being assessed by this method.

Rubenstein (1985) conducted a study aimed at (i) developing instruments to measure four types of computational estimation and several related mathematical skills, (ii) assessing computational estimation performance within several dimensions, and (iii) exploring the relationships among the computational estimation tasks and mathematical skills. Subjects were 309 eighth grade students (144 females and 165 males).

A 64-item test was designed to measure performance on four types of tasks: open-ended, reasonable versus unreasonable, reference number estimation, and order of magnitude scales. The test consisted of four 16-item scales (one for each type of task) and was designed to provide information on four dimensions of estimation: type, form (i.e. numerical or verbal), numbers (whole or decimal) and operation (addition, subtraction, multiplication and division). A series of analysis of variance (ANOVA) tests were carried out to investigate possible interaction effects between gender and the four dimensions of estimation being studied.

The Analysis of the data revealed significant effects for type, number and operation but not for form. With respect to the *type* dimension, these ranged from relatively hard for open-ended items, moderately hard for reference number items, to relatively easy for order of magnitude items. On the *number* dimension, decimals were more difficult than whole numbers, a finding supported by the results of the studies by Threadgill-Sowder (1984), Vance (1986) and Koloto, (1989). Addition and subtraction did not differ in difficulty, but division was more difficult than multiplication. The estimation tasks in numerical form were not more difficult than those presented in verbal form. That is, no significant difference was found in eighth graders' performance on estimation tasks in the applied format compared with the computational format.

The regression analysis revealed that operating with tens (i.e. multiplying and dividing by powers of 10) had an especially strong relationship with estimation performance. Students who lacked this skill had little understanding of the size of a number, a skill considered by Carpenter et al. (1976) to be essential for estimation. *Making comparisons* and *judging relative size* were mathematical skills shown to have contributed most to the prediction of

estimation performance. *Knowing place value, knowing number facts, rounding* and *operating with multiples of ten* - explain no more than 1% of the variance in any of the stepwise regression analyses. These skills might be necessary for estimation but the findings suggested that other skills (e.g. making comparisons and judging relative size) contributed more to the difference in performance. This finding implied that instruction on estimation should not be limited to instruction in rounding, place value, and operating with rounded numbers. It must be pointed out that the multiple regressions generated in this study accounted for about 47% of the variance in computational estimation performance. The factors which accounted for the remaining 53% are unknown.

In another study which concentrated on computational estimation, Gliner (1991) investigated the performance of 119 female and 22 male college students on estimation problems involving various operations and types of numbers (whole, fractions and decimals) when presented in a word problem format versus a computational format. In addition, this study involved investigating what personal attributes (e.g. mathematical background) might be related to success in estimation.

The results of the study revealed that students' self-perception of success in mathematics and college point average were positively correlated to estimation score (0.34). Students' average mathematics grade was negatively correlated with estimation score, which suggested that better mathematics students were not necessarily better at estimation. The number of participants answering problems correctly in the application format was significantly greater than the number answering problems correctly in the computation format, a finding inconsistent with that of Rubenstein (1985). Six of the eight problems in which there were significantly more correct answers in the application format involved money, indicating perhaps that participants were used to estimating the cost of items, and that these might be a very relevant type of problem. Estimation appeared to be a natural process when problems were presented in an applied format. Even though participants were asked to estimate rather than mentally compute answers, they appeared to have often tried to mentally compute answers when problems were presented in the computation format. This finding does not support findings of Rubenstein (1985) who found no difference in eighth graders' performance on the two formats.

The results from the computational estimation tasks in the study by Koloto (1989) revealed that students in the 'high competency' group were competent in estimation exercises involving whole numbers, fractions, decimals, determining the reasonableness of results given in a calculator, and estimating involving costs. Data from the 'average competency' group revealed that students were competent in estimating answers to items involving addition of whole numbers, determining the reasonableness of results given on a calculator, and exercises involving costs. Students in this group showed difficulty with multiplication of three-digit factors, addition of fractions and multiplication with decimals. The results also revealed that

students in the 'low competency' group were competent with estimation tasks involving subtraction of whole numbers and estimation costs but had difficulty estimating in the following topics: addition and multiplication with whole numbers, addition of fractions, operations with decimals. There was also evidence to suggest that students were hampered by weaknesses in the following areas: lack of understanding of basic number facts, incorrect ideas about rounding, difficulties with carrying, place value errors and multiplication by zero. The inability of students in the 'low competency' group to provide reasonable computational estimates might be accounted for by their lack of understandings of fractional and decimal numbers, lack of knowledge of their times tables, and part-remembered rules or algorithms. Results from this group indicated that links and ideas that students generated were not always what mathematics teachers had intended.

Comment

One of the largest sources of data on students' estimation and approximation skills has come from large scale surveys such as the NAEP studies in the United States (Carpenter et al., 1976; 1980a; 1980b; Reys, 1985; Reys & Bestgen, 1981), APU studies in the United Kingdom (Assessment of Performance Unit, 1985) and IEA studies (Department of Education, 1982, 1987; Robitaille and Garden, 1987). Other studies have provided important findings regarding students' performance on estimation tasks as well as other mathematical skills and abilities related to estimation. In a review of these data, Benton (1986) concluded that *these ability studies show, on the whole, across all age levels and for both computational and measurement estimation a disappointing inability to estimate.*

Many of the studies discussed in this section assumed that scores generated from students' performance on estimation tasks could be taken as measures of their estimation ability. However, assessment procedure which considers only the final result of calculation cannot be taken on its own as a valid measure of estimation ability (Edwards, 1984). In order to assess whether estimation skills have been used to derive answers, the processes and strategies used by respondents need to be explored and taken into account in assessing students' estimation skills. Some researchers have focused studies wholly or in part on the strategies or processes used by estimators. Such studies are reviewed in the next section.

3.3 Strategies Used by Estimators

As with the majority of the studies reported in the previous section, most of the studies on estimation strategies concentrated on computational estimation. Only a few studies focused on measurement estimation strategies.

Measurement estimation

One study which focused specifically on strategies, processes and skills involved in measurement (i.e. length and area) estimation was that conducted by Hildreth (1983). Twenty-four students from each of grades 5, 7 and college freshmen, were individually

interviewed to identify the strategies used when estimating length and area tasks. Nine appropriate and five inappropriate strategies were identified from the results. Appropriate strategies for estimating measures of length included:

(i) *Unit Iteration* - successively applying the unit mentally to the length of the object and determining a count of the unit;

(ii) *Use of subdivision clues* - subdividing the length using information that is available within the problem. For instance, to estimate the length of a hallway in metres, one could use the distances between doorways and the doors as subdivisions. These lengths are then used, making the iteration process easier to use;

(iii) *Prior knowledge* - using information that the estimator has about the stimulus object or unit;

(iv) *Comparison* - comparing the stimulus object to another object either in sight or out of sight;

(v) *Chunking* - To estimate the length of a line in inches, the length could be subdivided mentally into segments (chunks) and then the lengths of the segments could be estimated; and

(vi) *Squeezing* - making an initial estimate and then adjusting it to make a better.

Other strategies for estimating area were:

(i) *Repeated Addition* - a term used to distinguish unit iteration for area;

(ii) *Lengths times width, that is,* using length estimation strategies to estimate the dimensions of a polygon and then using an algorithm to obtain the area of the region; and

(iii) *Rearrangement* - rearranging part of the target figure to obtain an area which is easier to estimate.

Some inappropriate strategies used were: (i) finding the area of a rectangle by adding estimates of the length and width (*length plus width);* (ii) reporting the area of an object as being the estimated length of one of its dimensions (*centring)*; and (iii) wild guessing.

In summarising some of the skills and concepts involved in measurement estimation Hildreth (1983) suggested that measurement estimation is a complex skill which involves several concepts and abilities. These include :

- an understanding of the attribute (length or area) to be measured, and concept of unit;
- a mental image of the unit that is being used in the estimation task; and
- ability to compare objects on the attribute to be measured, perform unit iteration, select and use appropriate strategies for making estimations, and to check the appropriateness of the estimation.

In order to examine more closely the processes involved in measurement (i.e. length and height) and numerosity estimation Siegel, Goldsmith and Madson (1982) proposed a model of

the estimation process based on Gagne (1968) and Resnick, Wang & Kaplan (1973) models of learning involving rational task analysis. The model distinguished between two hierarchically related procedures or processes: *benchmark estimation* and *decomposition/ recomposition.* The term *benchmark estimation* refers to the application of a known standard to the To-Be-Estimated (TBE) item. The model proposes that for many estimation problems no benchmark is available so an additional process must be applied. In these cases, the estimation task involved first *decomposing* the TBE item to obtain samples small enough so that a benchmark could be applied and then *recomposing* these samples to arrive at a final estimate. For instance, if one needs to solve a problem involving the width of a parking lot, then the length of the parking lot decomposes into parking spaces. Since all of the spaces are the same width, this is a *regular* decomposition problem.

Sometimes an item does not decompose easily into smaller samples, as in estimating the number of people in a large crowd. The model predicts that the order of increasing difficulty of the problem types would be benchmark, regular decomposition, and irregular decomposition, and that this sequence would be reflected in age-related differences in accurate estimates.

Twenty children (10 males and 10 females) each from grades 2 through 8 and ten adults (6 females and 4 males), ranging in age from twenty to forty years took part in this study. Students were interviewed individually and were requested to 'think aloud' as they worked on the estimation problems. The procedure was modified for the adults who were presented the problems in small groups. Estimates were assessed for both reasonableness and accuracy, where accuracy was defined as an estimate within 50% of the actual value. Analysis of responses from students revealed six strategies being used.

The strategies were:

(1). *Guess* : The respondent indicated that it was just a guess or that she or he just thought of the answer;

(2). *Eyeball* : The student gave a perceptual description as an explanation for the estimate. For example, "*It looks like it*" ;

(3). *Range* : Boundaries for range for locating the estimate were given by the student, for instance, "*150. It looks more than 100 and less than 200*";

(4). *Benchmark* : The estimate was obtained by comparing the stimulus to another object or distance;

(5). *Pseudo-Decomposition*: The subject revealed an awareness that the problem could be broken down into parts but clearly did not use this information to formulate his or her estimate. For example, in explaining the answer of 300 feet for the length of a bowling alley, the student responded "*I guess each alley was about 5 to 10 feet across, who knows, the number of alleys would be about that much*";

(7). *Decomposition/Recomposition* : The student suggested a strategy of breaking the problem into parts and then recombining them to arrive at an estimate. For example, in responding to the task, "how many names are there in this page of the phone book ?", one student explained *"There's a lot on a page and they're small...so there's probably 100 in a row and there are 5 rows"*.

As a result of this study, a revision of the model was carried out. One of the significant changes made for the revised model was the entry point into an estimation task, which was originally proposed to involve asking the question: *"Is a benchmark available?"*. The researchers suggested that the initial 'move' in an estimation problem was sometimes affective rather than cognitive, in the sense that there seemed to be some conditions when students' emotional reactions specified where they entered the estimation process rather than automatically entering the process by searching for a benchmark. This was often the case where the numerosity or extent of TBE is huge; rather than trying to apply some benchmark, children usually gave a response *"Oh my !"* or *"You've got to be kidding!"*

The findings also implied that poor understanding of units of measurement, fractions or multiplication would make it difficult for children to make accurate estimates for the fractional benchmark problems or to recompose decomposition problems. Thus, careful attention should be paid to children's knowledge and ability in computation, number sense and measurement.

In the study by Koloto (1989) the following strategies for estimating measures of lengths, weights and capacities were observed: (i) *Use of objects of known quantities to compare with the stimulus object.* This procedure is described by Siegel et al. (1982) as "benchmark comparison"; (ii) *use of mental images of a unit of measurement or measuring instrument* , referred to as 'benchmark' (Siegel et al., 1982) or 'unit iteration' (Hildreth, 1983); (iii) *recalling the number or measurement seen on the stimulus object.*

Computational estimation strategies studies

In another study in the United States, Reys et al. (1982) identified and described computational estimation processes used by "good" estimators, and characterised their thinking strategies and techniques when estimating. A computational estimation test consisting of 55 open-end numerical and application items was developed and administered to over 1 200 students in Grades 7 - 12 and selected adults. The test scores for each grade level were rank ordered and students in the top 10 % of each group were defined as "good" estimators. These participants formed the sample who were interviewed individually. The interview provided the vehicle by which the researchers could study the strategies and processes the participants used in formulating their estimates. The estimation exercise used for the interview contained fifteen items (5 straight computation and 10 applied computation).

The researchers reported that the strategies used by "good" estimators seemed to have resulted from an interaction of several complex variables. These included the experiential background of the individual, the mathematical operation involved, the size of the numbers, the relationship of the numbers within a given problem, the exact number of digits used when formulating an estimate and the accuracy desired. Three of the key processes that appeared to be closely associated with good estimation skills were identified and described: these were *reformulation*, *translation* and *compensation.*

Reformulation refers to the process of changing the numerical data within a problem to a mentally manageable form. Several different strategies were revealed, the three key ones being (i) *rounding* addends to the nearest multiple of ten, (ii) a *front-end strategy*, which involved computation using the leading or left-most digits and then determining appropriate place value, and (iii) *substitution* which refers to the use of an equivalent form (fractional, decimal and percent) of the numbers involved to formulate their estimate.

Translation is described as changing the mathematical structure of the problem to a more mentally manageable form. An important strategy used was averaging, that is, estimating an average and using it to translate from an addition problem to a multiplication problem and the use of *benchmarks* (i.e. the use of reference points as bounds in forming as estimate). A response of a 9th grader illustrates this strategy.

$$\begin{array}{r} 87\ 419 \\ 92\ 765 \\ 90\ 045 \\ 81\ 974 \\ +\ 98\ 102 \end{array}$$

"All numbers are close to 90 000; so it would be 90 000 times 5 or about 450 000."

Compensation refers to the process of adjusting numbers to compensate for changes due to reformulation or translation. *Intermediate compensation* involved adjusting numerical values to systematically correct for error prior to their being operated on. *Final compensation* refers to adjusting an initial estimate to more closely convey the person's knowledge of the error introduced by the strategy used.

Other characteristics of good computational estimators have to do with number skills, cognitive process and affective attributes. These included:

(i) possession of quick and accurate recall of *basic facts* for all operations;

(ii) understanding of *place value* in determining accurate results of different arithmetic operations;

(iii) quick and efficient use of *mental computation* to produce accurate numerical information with which to formulate estimates;

(iv) knowledge and understanding of the meaning, purpose and concept of estimation enabled good estimators to be *tolerant for error* ;

(v) knowledge and use of *arithmetic properties* including distributive, associative and commutative properties;

(vi) possession of a *variety of strategies and techniques*;

(vii) strong *self-confidence* in the strategies they were using and the estimates given. Their confidence in their own ability influenced their consistency, quickness and choice of strategy.

The above research design was used by Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu (1991) to study the computational estimation skills of Japanese students in Grades 5 and 8. Respondents were 279 fifth graders and 187 eighth graders in 12 Japanese schools. Students were administered an open-ended 39 item screening test, 25 of these items being taken from the Assessing Computational Estimation Test (Reys et al., 1982). Response time was 10-15 seconds for each item. Students were asked whether they considered themselves "good estimators" and whether estimation was important by selecting one of the options YES, NO, or NOT SURE.

Unlike the study in the United States which used the top 10%, students scoring in the top 5% of each grade level, 10 fifth graders and 11 eighth graders were defined to be "good" estimators. Again these students were interviewed individually to ascertain the strategies and processes used while estimating.

The scores in the screening test ranged from 0 to 24 and 0 to 26 for the fifth and eighth graders, respectively. These scores suggested that the screening test was difficult for both grade levels and even for the best estimators. Discussion with some of the students following the test confirmed that students perceived the test to be very difficult. Some students seemed confused, and others did not seem to understand what they were expected to do when asked to estimate. None of the students reported having previously taken an estimation test.

Overall, the results showed that students' performance was poor, and rarely did a majority of students give an acceptable estimate on any item. Straight computation items were found to be more difficult than items in an applied context. This finding is consistent with the results of other studies in the United States (Reys et al., 1982; Gliner, 1991).

About 75% of the fifth and the eighth graders perceived estimation as important. However, only 12% of the fifth graders and 4% of the eighth graders rated themselves as "good" estimators. For the fifth graders, over three times as many males as females rated themselves as good estimators. In the eighth grade, six times as many male as female students identified themselves as good estimators. About 60% of students at each grade level responded that they were not good estimators, a self-assessment that parallels closely the level of low performance on the estimation test.

Data from the interviews revealed that respondents used a variety of estimation strategies. In general, the three general estimation processes, namely, *reformulation*, *translation* and *compensation* were observed in varying degrees. As in the United States study (Reys, et al., 1982) reformulation was the most frequently used strategy, followed by compensation and translation. Among the reformulation strategies used were the front-end strategy, rounding and the use of a benchmark. Averaging was the translation technique most frequently observed. Another common strategy was to use written algorithms mentally to "estimate" by producing an exact answer. Data suggested a reliance on straight rounding to a certain place value. For example, with the item involving 474 257 ÷ 8127, it was observed that none of the students who used rounding (500 000 ÷ 8 000) made any adjustments to their estimates, whereas the majority of the students who rounded to compatible numbers (400 000 ÷ 8 000, 480 ÷ 8, or 500 ÷ 10) made adjustments to their estimates.

These findings suggested to the researchers that the use of straight rounding is more methodical and therefore less likely to be accompanied by the process of compensation. Compatible rounding seemed more likely to leave the estimator with a sense of the direction of the exact answer and therefore more likely to be accompanied by compensation. In addition, the results appeared to reflect the emphasis on straight rounding in the mathematics curriculum.

The application of paper-and-pencil algorithms became most pronounced with fractions. For instance, for the problem 12/13 + 7/8, only two of the fifth graders and four of the eighth graders gave an acceptable response in the screening test but nearly all of those interviewed produced an acceptable estimate. Even though the majority of students at each grade level recognised 12/13 and 7/8 as being close to one, this observation was seldom immediate. For example, one fifth-grader first found a common denominator of 104 and began mentally computing the answer using the traditional fraction addition algorithm. After one minute, the student observed that each fraction "was near 1".

The findings implied that estimation does not necessarily evolve from the development of traditional written computation algorithms. It was possible that over learning of written algorithms might inhibit a number of important factors that contribute to success in estimation, namely, flexible use of numbers, tolerance for error, use of multiple strategies and adjustment techniques.

The majority of Japanese students interviewed approached the problem as a mental computation task using the numbers and operations as given and were therefore reluctant to change the basic structure of the problem to an equivalent form. Although no direct assessment of mental computation skills was made in this study, the interviews revealed that this sample of Japanese students possessed excellent mental computation skills. Of all the characteristics of good estimators (knowledge of basic facts, understanding of arithmetic

properties, use of a variety of strategies, skills at mental computation, tolerance for error and self-confidence as an estimator), tolerance for error was the least visible among Japanese students.

One of the strengths of the Japanese students was their knowledge and use of place value techniques, as shown by few order-of-magnitude errors when large numbers (values greater than 10 000) were involved. The researchers suggested that this might be a reflection of the frequent opportunities for working with large whole numbers provided by the Japanese monetary system. Moreover, it might be due in part to the Japanese use of "mang" (Japanese term for 10 000) as a standard referent point as opposed to the American referent of 1 000. For instance, the number 50 000 is read as 5 mang in Japan as opposed to 50 thousand.

The study also showed that many students did not understand what they were supposed to do when they were asked to estimate. Although all of the "good" estimators referred to rounding as a useful skill, few of them recalled being taught to estimate in school. Most of them attributed their success in estimation to skills and techniques they had developed on their own. The research data suggested that these strategies were seldom taught in the classroom.

The above research design was again replicated by Reys, Reys, and Penafiel. (1991) with a sample of Grades 5 and 8 students from 12 Mexican schools (6 elementary and 6 secondary). A computational estimation test consisting of 38 items was used to screen 432 students. Based on students' performance on the screening test, the top 5% of the grade 8 students were defined as "good" estimators and chosen for the interview sample. The same criterion was used for the fifth graders but these children found the estimation tasks to be very difficult, so no further interviews were conducted with this group.

These results showed the test scores of the Grade 5 students ranging from 0 to 7 and for the Grade 8 students ranging from 0 to 18, indicating that the test was difficult for students. As expected, a significant difference between grades was observed. However, there was no significant difference in the performance between males and females in either grade. This finding contrasts with gender differences on estimation performance across all age groups shown in research conducted in the United States (Reys et al., 1982; Rubenstein, 1985). A high percentage of students said that estimation was important, but only a few students rated themselves as good estimators.

The interview data revealed a limited number of estimation strategies. Among the reformulation strategies described by students were the *front-end strategy*, *rounding* and the use of *compatible numbers*, the front-end strategy being the most common technique used. Another common strategy used to produce an estimate was *mentally applying a pencil and paper algorithm.* This process appeared to be the first choice of some students. The findings of this study further documented the difficulty involved in assessing estimation skills. Several factors were suggested as contributing to have influenced the low scores of the Mexican

students. Among these factors were: (i) a lack of understanding by some students of what "estimate" meant, (ii) they were not used to quizzes or tests where the time allowed per item was very short and there was no opportunity to review a problem, (iii) frustration at the fast pace of the test and their inability to produce a response in the time allowed, and (iv) lack of experience of a test administered with a slide projector.

Another study in the United States which focused on the processes used by estimators was that conducted by Threadgill-Sowder (1984). Twenty-six students from Grades 6-9, who represented a variety of school backgrounds, socioeconomic status and ability levels were interviewed individually. Twelve computational estimation problems, representative of the types of items used on the NAEP tests were chosen for the interview. Students were to select either the closest answer and give reasons, or they were to give an approximate answer with a rationale. The problems contained whole, fractional and decimal number operations.

The results demonstrated that the requirement to have problems done without a pencil in hand was a difficult one for students to accept. Some students went so far as to use a combination of finger writing and visual imagery to perform the computation. They would then round their answers and give this result as their estimate. The researcher suggested that this appeared to be a result of the need to get the "right" answer and their desire to find "rules" to guide them. To illustrate, for Item 4 (9/31 + 4/9 is approximately...) some students wanted to find a common denominator as large as or larger than 31, leading to answers such as 21/31 and 46/62. Other students seemed to feel they were justified in guessing at an answer, as though this was somehow synonymous with estimation.

The study showed that "rounding" was the most commonly used strategy. In addition, students who consistently gave acceptable responses demonstrated good "number sense" whereas those who gave unacceptable responses seemed to have little feel for numbers, as suggested by their indiscriminant use of rounding procedures. As an example, for Item 6 (789 x 0.52 is approximately...), some students rounded 0.52 to 1 and 0.

One of the focuses of the study by Koloto (1989) discussed in the earlier was to identify students' computational estimation strategies. The findings revealed that students in the 'high competency' group used a variety of strategies for computational estimation including: (i) *Front-end strategy* - The essential feature of this strategy was that students focused on the front-end or left-most digits; (ii) *Rounding;* (iii) *Adjustment of Initial Estimate* - This is the process described by Reys et al (1982) as 'compensation'; (iv) *Compatible Numbers;* (v) *Changing Fractions to Decimals* ; and (vi) *Use of the Distributive Property* .

In addition to the strategies used by those in the 'high competency' group, students in the 'average competency' group used *groupings of numbers into thousands.* They also exhibited similar inappropriate strategies. These comprised (i) *rounding numbers down when it was more appropriate to round them up*; (ii) *rounding part of a number which the student had*

already rounded; (iii) *place value error in reading numbers*; (iv) *always subtracting the smaller from the larger digit*; (v) *adding numerators and denominators as if they were separate identities*; and (vi)*proceeding algorithmically.*

The results revealed that students in the 'low competency' group were hampered by weaknesses in the following areas: lack of understanding of basic number facts, incorrect ideas about rounding, difficulties with carrying, place value error and multiplication by zero. In instances where estimation strategies were used by students, the most common strategies were *rounding,* the *front-end strategy* and *compatible numbers.* The majority of students in this particular group used inappropriate strategies. In addition to the inappropriate strategies identified for the 'average competency' group the following strategies were identified from students' responses: (i) *multiplying using vertical algorithms;* (ii) *multiplying the decimal parts of the numbers and then adding this answer to the whole number part of the decimal*; (iii) *reading the exercise as division instead of multiplication;* and (iv) *dividing the decimal part of the divisor with that of the dividend and the whole number part of the divisor with that of the dividend.*

The inability of students in the 'low competency' group to provide reasonable computational estimates might be accounted for by their lack of understanding of fractional and decimal numbers, lack of knowledge of their times tables, and half-remembered rules or algorithms. Results from this group indicated that the links and ideas that students constructed were not always those that the mathematics teachers had intended.

Whereas most studies used primary and secondary school students, Levine (1982) studied the strategy use and estimation ability of college students. Subjects were 89 (34 males and 55 females) undergraduates college students who were not majoring in mathematics. A test of estimation ability containing 10 multiplication and 10 division problems was used in this study. The School and College Ability Test quantitative subtests were used to measure students' quantitative ability.

Research data showed that overall *rounding both numbers* and *proceeding algorithmically* (i.e. mentally applying a pencil and paper algorithm) were the types of strategy most frequently used. They were also the types that occurred most frequently as a subject's primary strategy type. The popularity of proceeding algorithmically might be a reflection of students' dependence on the well-defined algorithms and their relative unfamiliarity with estimation. Subjects reported having been taught little if anything about estimating. In cases in which students reported having received instruction, it almost always concerned 'rounding' numbers.

Lower quantitative ability was found to be associated with the use of "proceeding algorithmically". Students of relatively high quantitative ability used a greater number of different estimation strategies and were better estimators than students of relatively low

quantitative ability. These findings suggested that quantitative ability or "number sense" was closely related to estimation ability.

Levine's (1982) Test of Estimation Ability and the procedure employed in her study were used by Dowker (1992) to investigate the strategies of people with exceptional knowledge of, experience with, interest in the properties of and relationships between numbers. Unlike the subjects who participated in Levine's (1982) study, who were not majoring in mathematics, participants in this particular study were 44 professional pure mathematicians (39 males and 5 females) in a university in the United Kingdom. Participants were asked to *'think aloud'* while formulating their estimates. Eighteen of the 44 participants were given the same task again after an interval of 6 to 9 months. The strategies used on the two occasions were then compared.

The percentage error of the estimates were calculated and points were scored for each estimate. For an estimate that was no more than 10% away from the exact product or quotient, three points were scored, two points were scored if the percentage error was greater than 10% but less than 20% and one point if the percentage error was greater than 20% but not more than 30%. A zero was scored if the percentage of error was greater than 30%.

The types of strategies identified from the interview data were classified according to Levine's (1982) categories but modified to take into account the strategies used by respondents. Eight strategy types were identified: (i) *use of fractions*, for example, restating the problem 482 x 51.2 as 482 x 100 x 1/2; (ii) *using known or "nicer" numbers*, which involved converting the numbers in the problem to more compatible numbers in ways not subsumed under other strategy types such as rounding strategies e.g. 9208÷32 to 9600÷32 to make division much easier; (iii) *rounding two numbers*; (v) *factorisation*; (vi) *proceeding algorithmically;* and (vii) *distributivity*, that is, the distributive principle of multiplication was used; and (viii) *other*. This last category included rare and idiosyncratic responses and those that were only used for one or two problems.

Analysis of the data showed that out of the 1240 responses given, 117 were scored as 0, twenty as 1, seventy-three as 2, and the remaining 1030 were scored as 3 points, suggesting that these mathematicians were accurate estimators. Not only were the subjects very accurate estimators but they also exhibited a very large number of strategies. A high level of accuracy with occasional errors, a tendency to use strategies involving knowledge of arithmetic properties and relationships rather than strategies involving the use of school-taught techniques, and great flexibility and diversity with regards to the strategies used were characteristics of the mathematicians. The interview data revealed that the minimum number of strategies used for a single problem was seven. Ten or more strategies were used for nineteen problems and in six cases twenty or more strategies were used. Moreover, when retested, the subjects often used different strategy types for the same problems.

In contrast to the performance of college students in Levine's (1982) study, mathematicians in this study were more likely to make inaccurate estimates (i.e. more than 30% away from the correct answer) than mildly inaccurate estimates (i.e. more than 10% but less than 30% away from the correct answer). Moreover, subjects in this study rarely used the strategy of "*proceeding algorithmically*". The strategy types most frequently used were "*using fractional relationships* ", "*using known or nicer numbers* ", "*rounding two numbers* " and "*factorisation*".

Making use of previous research findings, Sowder and Wheeler (1989) made a useful analysis of the components of computational estimation prior to the commencement of their study. Table 3A below is the outcome of this analysis.

Table 3A: An analysis of the components involved in computational estimation.

I. Conceptual Components
   1. Role of Approximate Numbers
      1.1 Recognition that approximate numbers are used to compute
      1.2 Recognition that an estimate is an approximation
   2. Multiple Process/Multiple Outcome
      2.1 Acceptance of more than one process for obtaining an estimate
      2.2 Acceptance of more than one value as an estimate
   3. Appropriateness
      3.1 Recognition that appropriateness of process depends on context
      3.2 Recognition that appropriateness of estimate depends on desired accuracy

II. Skill Components
   1. Processes
      1.1 Reformulation: Changing the numbers used to compute
         1.1a Rounding
         1.1b Truncation
         1.1c Changing the form of a number
      1.2 Compensation: Making adjustments during or after computing
      1.3 Translation: Changing the structure of the problem
   2. Outcomes
      2.1 Determination of correct order of magnitude of the estimate
      2.2 Determination of the range of acceptable estimates.

III. Related Concepts and Skills
   1. Ability to work with powers of ten
   2. Knowledge of place value of numbers
   3. Ability to compare numbers by size
   4. Ability to compute mentally
   5. Knowledge of basic facts
   6. Knowledge of properties of operations and their appropriate use
   7. Recognition that modifying numbers can change outcome of computation.

IV. Affective Components
   1. Confidence in ability to do mathematics
   2. Confidence in ability to estimate
   3. Tolerance for error
   4. Recognition of estimation as useful

(Sowder and Wheeler, 1989, p.132)

Twelve students (six males and six females) of average ability from each of Grades 3, 5, 7 and 9 were randomly chosen from a pool of average ability students identified by teachers. Students were individually presented the estimation problems with solutions from hypothetical students, accompanied by questions asking them to compare and contrast the solutions. In the

first part of the interview, nine tasks which focused specifically on the Conceptual and Skills components, as listed in Figure 3A were used. Students were asked whether they found the hypothetical estimation processes and answers acceptable and appropriate. In the second part of the interview, three open-ended tasks were presented to Grades 3 and 5 students while those in Grades 7 and 9 were given six and five tasks, respectively.

On the task which involved understanding the "*role of approximate numbers*", the majority of students at all grade levels found acceptable both the procedure of rounding-then-computing, and the resulting values as estimates. Students' responses to the "why" question revealed that the third graders were very hesitant about rounding numbers whereas the seventh and ninth graders were more confident to give answers. When students were shown a hypothetical estimate that was obtained by the computing-then-rounding procedure, the numbers of students who preferred this procedure to rounding-then-compute increased by grade level. By Grade 9, only four of the twelve students interviewed preferred the round-then-compute strategy over the compute then-round-strategy. The ninth graders appeared more concerned with accuracy and wanted to view concurrently both the exact answer and estimate so they could determine how "far off" their estimate was.

For the "multiple process and multiple outcomes" tasks, a clear developmental trend emerged in students' willingness to accept more than one estimation process and more than one value as equally valid. The researchers suggested that these findings appeared to be a result of years of instruction where only the exact answer was acceptable and were related to the usual emphasis on 'one right answer' and also the 'one right strategy' syndrome in mathematics classes.

For the "skills variables", the students' ability to choose the process that gave a closer estimate was related to their age. Although the importance of the use of *compensation* to obtain better estimates was recognised by more students at successively higher grade levels the complexity of the compensation process used in the tasks affected its ultimate acceptability. Students were more reluctant to use compensation in the open-ended problems than to agree with the final result when they observed it in some hypothetical work. The *reformulation* of problems using *averaging* was shown to be an acceptable method of estimation. However, no students at Grades 3 and 5 and only half of the students at grade 7 indicated that they would solve the problem that way.

The results also showed that the *order of magnitude* was the only type of skill-level problem that the majority of third graders could successfully answer. However, the procedure predominantly used by students was to compute mentally using the exact numbers and then to choose the closer answer. Clearly this is not computational estimation but perhaps a combination of mental computation with comparison to the numbers involved in the options given.

Results of the study by Sowder and Wheeler (1989) showed that the most frequently used technique to *determine an acceptable range* was to compute mentally the sum (not always correctly) and subsequently compare that sum with the target rather than to find a range for the estimates.

For the open-ended items, the third graders did not demonstrate any estimation skills. It was evident that students at this grade consistently preferred exact computation. Even though six of the fifth graders attempted to round numbers when estimating sums and several used the averaging procedure, overall the fifth graders preferred exact computation. The performance of the seventh graders showed a limited repertoire of estimation skills and in general they were unsuccessful at tasks such as estimating the sum of three four-digit addends. The data showed that students preferred to compute with exact numbers given in the tasks and then round off the sum to obtain an estimate. Sowder and Wheeler (1989) suggested that the attractiveness of this procedure might be associated with students' tolerance for error. In some cases, students often refused to accept approximations obtained by the rounding procedures that differed from what they had learned in mathematics classes. For example, some fifth and seventh graders objected to rounding 267 to 250 rather than 300, arguing that "*You are always taught to go up if it's past five*" or "*Seven is above five, so you have to go up, not down.*"

Comment

Although estimation ability exists in a continuum and that much is yet to be learned about estimation and related mathematical skills and concepts, the picture that emerges from the above studies appears to emphasise the extremes (Sowder, 1992). The studies showed that good computational and measurement estimators use a variety of strategies and are flexible and versatile in their thinking. Good estimators demonstrate a deep understanding of numbers, arithmetical properties and operations, and continually draw upon that understanding. In addition, their knowledge and understanding of the meaning, purpose and concept of estimation allow them to be tolerant of errors (Dowker, 1992; Koloto, 1989; Levine, 1982; Reys, Reys and Penafiel, 1991; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991). On the other hand, poor estimators vaguely understand the nature and purpose of estimation by equating it to guessing, and exhibit a limited number of strategies. Very often this is limited to one strategy type, namely that of mentally applying algorithms more suitable for paper-and-pen exact calculation. Furthermore, their lack of understanding of basic number facts, units of measurement, arithmetical operations and properties tends to inhibit their ability to make reasonable estimates (Koloto, 1989; Reys, Reys and Penafiel, 1991; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991; Threadgill-Sowder, 1984).

Sowder (1992) suggests that several instructional questions are at issue. These questions are:

(i) can the particular strategies used by good estimators be taught to less skilled estimators?

(ii) should the flexibility of thinking and inventing of strategies be the goal of instruction? and

(iii) if good estimators could develop a variety of estimation strategies on their own without specific instruction on estimation does this imply that students can learn estimation strategies on their own?

These questions need to be taken into account in planning instructional programs on estimation. Several researchers have addressed these issues by conducting research on the effects of instruction on the development of estimation skills. It appears that these researchers attempted to address the issue addressed in the first question, that is, teaching appropriate estimation strategies used by "good" estimators. The findings of these studies are reviewed in the following section.

3.4 Teaching Estimation

As noted earlier the studies reviewed so far suggest that a possible reason for students' low performance on estimation tasks is lack of instruction on estimation in mathematics programs. Only the rounding procedure is reported to have been taught. Other strategies appeared to have been developed by students on their own. Whether estimation can be taught is an important question. Several studies have investigated the effects of instruction on the development of estimation skills and strategies.

Teaching measurement estimation

In addition to identifying strategies used in estimating length and area, another focus of the Hildreth (1983) study involving 24 fifth and seventh grade students was to investigate the effects of two methods of teaching estimation. One half of the students at each grade level were taught to use a "guess-then-check" method for estimating lengths and areas in the metric system, while the other half were taught to use an "appropriate estimation strategy" (listed earlier in section 3.3), which included several strategies identified in the first phase of the study. The instructional programs consisted of five 40-minute lessons. Pretest and posttest scores were obtained to determine the effects of the two treatments.

Estimation ability was found to be positively correlated with strategy use. Data suggested that participants who were taught to use several strategies exhibited more strategy use on the posttest than subjects who were taught the guess-then-check method.

The data had several implications for teaching measurement estimation. It was suggested that initial instruction on measurement estimation should include a discussion of the *unit iteration*, *comparisons*, *chunking* and *squeezing* strategies. In order to use these strategies, students

need to acquire a set of reference objects for which they know the measure, for example, children's body measurements, the height of the children's desk. Moreover, students should not be taught a length-times-width rule for finding the area of a rectangle until they have an understanding of the concept of area. Students should have practice estimating the measurement of objects using the standard and non standard units of measure.

Teaching computational estimation

One of the three main purposes of a study by Bestgen, Reys, Rybolt and Wyatt (1980) was to investigate the effects of instructional lessons demonstrating techniques that can be used to estimate solutions to computational problems. The subjects were 187 preservice elementary teachers. In all, four Algebra, three Methods and two Geometry sections were involved in this twelve-week study. While the two Geometry sections acted as the control group (T 1), the four Algebra and three Methods of Teaching Elementary School Mathematics sections were randomly assigned to one of two treatments, T 2 and T 3.

The researchers developed ten weekly instructional lessons focused on certain estimation skills and strategies relevant to one of the four fundamental operations. Ten weekly quizzes consisting of 16 computation items were administered to students in the two experimental groups (T 2 and T 3). The first five quizzes involved the four operations (4 items each) with whole numbers. The 6th and 7th quizzes involved each of the four operations with decimals. The last 3 quizzes contained both whole and decimal number items.

Students in the second treatment group, T 3 (n=58) were taught specific estimation strategies each Monday during the 10-week instructional period. The following Wednesday both T 2 and T 3 students were given estimation quizzes. Strategies taught included rounding numbers and the front-end process. Acceptable intervals for estimates for each item were presented to students at the conclusion of the quizzes to provide immediate feedback. The instructor also discussed with the T 3 students four items related specifically to the operations and further exemplified the operation presented in the instructional lesson the previous Monday. The control section, T1 (n=50) took all the pretests and posttests but did not receive the instructional lessons or the weekly quizzes.

Pretest and posttest scores were collected to determine the effectiveness of the lessons. Two semantic differential attitude instruments, followed by parts A and B of the computation section of the Standford Achievement Test, and a 60-item computational speed test were administered to the participants. To identify the relationship between attitude toward mathematics and estimation, items were correlated with scores on the Estimation Pretest.

The group who received weekly quizzes showed greater gains in estimation performance than the group receiving no practice. Instruction on strategies accompanied with practice resulted in improved attitudes. Participants who received weekly instruction on specific estimation

strategies generally showed more favourable attitudes than those who either had weekly practice quizzes on estimation for 10 weeks or those who had no systematic practice at all. However, there was no significant difference between the performance of the group which received regular instruction on estimation strategies and the group which had only weekly practice. The results of this study provided evidence of the value of providing preservice teachers with computational estimation activities. Brief systematic practice over a 10-week period increased performance. Moreover, when such practice was accompanied with specific estimation strategies greater understanding and respect for estimation process occurred.

Schoen, Friesen, Jarret and Urbatsch (1981) used two fourth-grade classes with 21 students in each to study the effectiveness of systematic instruction in estimation. Five 45 minute lessons on estimating one-digit and two-digit, one-digit and three-digit, and two-digit and two-digit factors were developed for this study.

The front-end process (as described by Reys et al., 1982) was taught in the first two lessons. For example, a first estimate of 573 x 4 is 500 x 4 or 2 000. A closer estimate can be found by also considering 70 x 4 = 280, and adding 2 000 + 280 = 2 280. The rounding process was taught in the third and fourth lessons. For example, a first estimate of 573 x 4 would be 600 x 4 = 2 400. Throughout the first four lessons the analogy of shooting at a target was used for finding closer estimates, and teacher-led instruction and student worksheets were followed by drill and practice. The fifth lesson involved practice sheets in which either process could be used.

In a follow-up study, referred to as Study II, the same researchers used 46 fifth and 54 sixth grade students to investigate the effectiveness of instruction on estimation. Students in each grade were assigned to one of three treatments. Treatment 1 (T1) completed two lessons (10-15 minutes each) of individual computer-assisted drill and practice on exact computation of sums of two-digit, three-digit and three two-digit whole numbers; two lessons on exact computation of products; and one review practice lesson combining sums and products. Treatment 2 (T2) completed the five computer-assisted drill and practice lessons as those in T1 except that the practice items required estimation rather than exact computation. Items were presented in multiple-choice format. Participants assigned Treatment 3 (T3) completed hand calculator-aided worksheets individually, one per day for 5 consecutive days. In contrast to the worksheets used in T2, the aim of the worksheets in T3 was to present a sequence of lessons leading to an understanding of the use of the rounding process in estimating sums and products. The instructional method in the worksheet was essentially an individualised version of the rounding lessons in Study I. Again, each student was scheduled to complete his/her individual lesson each day for five consecutive days. On the 6th day, all eight posttests and transfer tests were administered by four classroom teachers. Three weeks later, the classroom teachers administered the four retention tests.

Taken together these two studies indicated that estimation with whole numbers could be taught in a short period of time. Not only did fourth, fifth and sixth graders become better estimators, they did so by adopting valid estimation strategies. These skills and strategies were retained for at least a 3-week period which suggested that retention and maintenance would involve minimal time.

The data showed no evidence to suggest that estimation instruction either improved or interfered with computation skills. In addition, after instruction estimation groups could estimate better and compute as well as a group that spent the same amount of time on computation drill. Estimation instruction did not affect the number of word problems students could solve correctly. As might be expected, there was evidence that students who were taught to estimate sums and products were better able to estimate the solutions to word problems. Interviews with "good" estimators in Study I provided data on the students' estimation processes. The good estimators were usually able to judge the magnitude of their estimates relative to the exact answer.

Estimation has also been taught as a heuristic strategy for verbal problem solving. De Corte and Somers (1982), in a teaching experiment with 20 twelve-year-old students in an experimental class and a control group of 21 students, directed students through a solution procedure that consisted of reading and understanding the task, estimating the solution, representing the solution, working out the problem, then verifying the solution by comparing it to the estimate.

Although some design problems prevented the researchers from claiming conclusive proof of the effectiveness of this strategy, the results showed evidence to warrant consideration of teaching estimation as a heuristic strategy in problem solving.

With the growing body of research data on children's estimation competencies, Case and Sowder (1990) were concerned about how it should be organised for effective curriculum development. They conducted a study to address this issue. As a first step, a developmental model of the concepts and processes involved in one form of computational estimation, namely, the estimation of multidigit sums was developed, using a top-down approach. When using a top-down approach, a pre-existing general theory of cognitive or mathematical development is adopted. One then articulates a set of assumptions that would allow the theory to be operationalised for the set of tasks to be studied, using existing data as a guide. The model is then tested and refined by creating new tasks for which it can make predictions.

The model developed was expressed as a series of six hypotheses, one for each of the substages that span the period of formal schooling (ie. from 6 to 15 years of age). The researchers reported the results which confirmed the following hypotheses:

1. During the first substage of the dimensional period (at about 6 years of age), children should become capable of computing single-digit sums or making single-column nearness judgements. The reason for this hypothesis is that both of these mental computations can be undertaken with the aid of a single mental number line, which is hypothesized to underlie mathematical competence available during this period.

2. During the second substage of the dimensional period (at about 8 years of age), children should become capable of mentally computing double-digit sums or making two-column nearness judgements because both of these operations involve using a single number line for one calculation, then decentring and moving to a second number line to make a second calculation.

3. During the third substage of the dimensional period (at about 10 years of age), children should become capable of computing double-digit sums in which some form of carrying or regrouping is necessary, because this entails moving back and forth between two number lines and appreciating the relationship between them. At the same time, they should become capable of making more complex double-digit nearness judgements, that is, ones in which the decision about which way to round requires taking some additional factor into account.

4. At the beginning of the vectorial stages (i.e., around age 11 or 12), the first sign of genuine multidigit sum estimation should appear. This hypothesis may seem somewhat counter intuitive, because, for adults, estimating a sum is easier than computing it exactly. According to the foregoing analysis, however, (a) estimating a sum requires coordinating the act of double-digit rounding with the act of generating a sum large enough to run to two columns, and (b) any mental operation that requires the coordination of two bi-dimensional competencies must, by definition, be vectorial competence. Thus, even though the only thing children must do is to keep track of a single, running, rounded total, it was predicted that this ability would not emerge until 11 or 12 years of age.

5. At the middle substage of the vectorial period (approximately 13 or 15 years of age), children would be able to keep track of two running, rounded totals rather than just one. Among other things, this means that they should begin to display the first sense of "how much off" their first running, rounded total is (a result of systematic alterations of the second total). They should also be able to introduce some compensation as a result.

The result of this study would seem to suggest age related factors associated with computational estimation processes. In considering these hypotheses it would be unwise to base instruction on the results of one study. Replication of this study is necessary before any conclusion can be drawn about age related factors in estimation skills. It could be argued that adopting a developmental approach may limit teachers to focusing on limited and specific processes. The model does not take into account the many processes generated by subjects in the age groups used in the above study.

In an Australian study by Bobis (1991), a sample of 101 fifth grade boys were used to investigate the effectiveness of teaching computational estimation strategies. The results showed that after instruction the students tended to adopt valid estimation strategies. The results revealed that one of the most difficult obstacles for students to overcome was the reliance on paper and pencil techniques. This obstacle was partly overcome when new material was introduced via estimation strategies. The results of this study implications about the order in which instruction should occur in the classroom and the placement of estimation strategies in the mathematic curriculum.

3.5 Summary and Comment

This chapter has focused on a review of the literature on measurement and computational estimation. These studies were reviewed under three main categories:

(i) estimation skill or ability and its relationships with other mathematical abilities ;

(ii) identification of strategies used by estimators; and

(iii) investigating methods of teaching estimation.

This section summarises the main findings in the research literature, outlines the limitations of previous studies and further presents the case for the present study.

Measurement estimation

Reasonable measurement estimation seems to be a complex skill that involves the use of several strategies and concepts. In summarising some of the skills and concepts involved in measurement estimation Hildreth (1983) suggested they include :

- an understanding of the attribute (length or area) to be measured, and concept of unit;
- a mental image of the unit that is being used in the estimation task; and
- ability to compare objects on the attribute to be measured, perform unit iteration, select and use appropriate strategies for making estimations, and to check the appropriateness of the estimation.

Furthermore, researchers (Siegel et al, 1982 and Koloto, 1989) have found that in some cases the initial move in the estimation processes is affective rather than cognitive. That is, subjects' emotional reactions to the estimation tasks specified where and when they entered the estimation process.

Data also showed that poor understanding of units of measurement, fractions or operations involved would make it difficult for subjects to make 'reasonable' estimates (Hildreth, 1983; Koloto, 1989; Reys et al., 1982; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991; Siegel, et al., 1982).

Low performance on estimation tasks

Students' low performance on estimation, as revealed by large scale surveys and investigations by Levine (1982), Reys et al. (1982), Reys, Reys, and Penafiel (1991), Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu (1991), Threadgill-Sowder (1984), possibly resulted from them being taught little about estimation in the classroom. Since the Tongan mathematics curriculum has not included estimation as a major focus until 1992 it is likely that Tongan children are also lacking in estimation skills.

Computational estimation and its relationships with other skills

Various attributes and abilities were shown to be highly correlated with performance on computational estimation. The following skills were revealed to be related to computational

estimation: *operating with tens*, that is, multiplying and dividing by powers of 10; *making comparisons* and *judging relative sizes* (Rubenstein, 1985); *an individual self-perception of success in mathematics* (Bestgen et al., 1980; Gliner, 1991) and *self-confidence* (Reys et al., 1982); *college grade point average* (Gliner, 1991); *experiential background of the individual; possession of quick and efficient use of mental computation; tolerance for error; knowledge and use of arithmetic properties including distributive, associative and commutative properties; possession of a variety of strategies and techniques* (Reys et al., 1982).

The research data indicated that students' performance on computational estimation tasks varied depending on the types of numbers and operations involved in the problems. On the number dimensions, exercises involving decimals were more difficult than those involving whole numbers (Bestgen et al., 1980; Carpenter et al., 1976; Koloto, 1989; Rubenstein, 1985; Reys et al., 1982; Schoen et al., 1980). Addition and subtraction did not differ in difficulty, and division was more difficult than multiplication (Rubenstein, 1985).

Estimation strategies and processes

Studies of "good" estimators showed on the whole that estimators used a variety of strategies (Dowker, 1992; Koloto, 1989; Levine, 1982; Reys et al., 1982). The key processes used by good estimators were *reformulation, translation* and *compensation.* One of the most useful reformulation strategies was the *front-end strategy* in which estimators focused on the left-most digit. Another commonly used strategy was *compatible numbers.* In general, good estimators often ignored school-taught rules of rounding and rounded to numbers more suitable for solving the problem at hand.

The strategies used involved understanding of arithmetical properties and relationships rather than school-taught techniques. The implications of these findings for the present study in the Tongan classroom would appear to be (i) focus on developing children's number sense and (ii) include the above strategies in an estimation instructional program.

Research techniques

The studies employed a variety of research techniques to gather data and assess students' estimation skills. These ranged from large scale surveys, at least some of which identified student' approximation skills rather than their estimation skills, to individual interviews. Individual Interviews was the most frequently used research technique for identifying strategies and cognitive processes used by estimators (Dowker, 1992; Hildreth, 1983; Koloto, 1989; Levine, 1982; Reys, et al., 1982; Reys, Reys and Penafiel, 1991; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991). In order to identify the estimation strategies of Tongan students, it would be appropriate to use the interview technique in this investigation.

The number of items and the formats in which the estimation tasks were presented varied from study to study. It is possible that differences in students' performances on estimation tasks

might be accounted for by the different formats used. For instance, it has been suggested that different strategies and skills were required for solving open-ended tasks as opposed to those presented as multiple choice items. It also seemed to be the case that the number of students answering problems appropriately in the application format (word problem type) was significantly greater than the number who answered the problems appropriately in the computation format (Gliner, 1991; Reys et al., 1982; Reys, Reys and Penafiel, 1991; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991). There was evidence to suggest that performance on the two formats differed because the two formats required different skills. The application format, for instance, gave greater purpose to the estimation and provided some indication of the degrees of accuracy required. For the present study open-ended items only will be used.

Criteria and measures used for assessing estimation skills.

An important area that needs to be carefully considered by researchers is the criterion and/or measures used for assessing an estimate. One difficulty involved in drawing conclusions from previous studies is that different criteria were used for measuring and assessing estimation skills. Table 3B summarises the assessment procedures used by previous studies.

Table 3B: Summary of assessment procedure

Researchers	Type(s) of Estimation studied	Definition of good/proficient estimators	Criteria for acceptable reasonable est.
Reys et al. (1982)	Computational	top 10% in a rank ordered test score	Not specified
Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu (1991)	Computational	top 5% in rank ordered test score	No specified
Reys, Reys and Penafiel (1991)	Computational	top 5% in rank ordered test score	No specified
Threadgill-Sowder (1984)	Computational	Not Applicable	Not specified
Levine (1982)	Computational	Not Applicable	Estimate score 3 if within 10% of exact value score 2 if 10-20% - 30% and 0 if the percentage error was greater than 30%.
Dowker (1992)	Computational	Not applicable	Used Levine's (1982) criterion.
Koloto (1989)	Computational and measurement	Top 10%	Various Acceptable range based on the numbers involved in the problems.
Corle (1960; 1962)	Measurement	Index of estimate error.	Estimates within 10% of actual value.

Most researchers defined 'reasonable' and 'accurate' estimates as answers which fell within a specified percentage of the actual or exact answer. For instance, Siegel et al (1982) defined estimates that were within 100% and 50% as 'reasonable' and 'accurate' estimates,

respectively, while Baroody and Gatzke (1991) defined 'accurate estimates' as answers which were within 25% of the actual answers. In Corle's (1960, 1963) studies, estimates within 10% of the actual value was considered to be 'correct'. In addition, Corle used an 'index of estimate error' to determine the 'accuracy' of estimates given by respondents, where an index of estimate error was obtained by calculating the difference between the 'estimated value' and the 'actual value' and then dividing that answer by the 'actual value'. Levine (1982) and Dowker (1992) used a scoring system. However, it is important to point out that such measures cannot be used in situations where the actual value cannot be obtained and an estimate is the only option.

With respect to percentage error, its use is perceived as an inappropriate measure of estimation skills. As Dowker (1992) suggested :

> *Percentage of gross error sometimes distorts the results of the estimation study. Sometimes a rather small discrepancy resulted in a high percentage of error and in other cases a relatively low percent failed to describe the error* (p.325)

Language used

Related to the above issue is the language used to describe the estimate. Researchers have used different terms to label what they have judged to be "good" or "poor" estimates. One can observe a change in the language used by researchers as the years progressed and more information was collected. Whereas Corle (1960, 1963) used "correct" estimate, Siegel et al. (1982) used "reasonable" and "accurate" estimate, while Threadgill-Sowder (1984) used "acceptable" responses.

Gender differences

Research on gender differences in estimation provides contradictory findings. For example, the study Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu.(1991) showed no significant difference in the performance between males and females. On the other hand there were consistent gender differences on estimation performance across all age groups in earlier research conducted in the United States (Reys et al., 1982; Rubenstein, 1985). On computational estimation performance the difference favoured males. Gender as an influencing factor will be left aside in this study.

Affective factors and purposes of estimation

Among the characteristics associated with good computational estimation was students' self confidence. Students' confidence in their own ability influenced their consistency, quickness and choice of strategy (Reys et al., 1982). Siegel et al (1982) suggested that efforts to include estimation in the mathematics curriculum must take into account affective factors. Students needs to be aware of the relevance of learning estimation and must believe that they are capable of doing it. An instructional program should help students in both respects. Thus, the

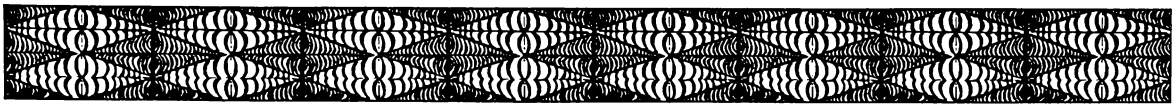
implication for the present study is that students and teachers ought to be involved in developing a learning environment in which the above goal can be met.

One important implication has been mentioned by Sowder (1992) and Dowker (1992) who emphasise the need for students to overcome the belief that there is always one right answer and one procedure for obtaining it. Success in terms of an exact answer is just **one** answer whereas success rate in terms of making an estimate can be many answers that fall within an acceptable range. The need to get the right answer is in conflict with the very purpose of estimation.

The research literature shows that most studies have been conducted in the United States, with a few in the United Kingdom, Japan, Mexico Australia, and New Zealand. To provide wider cross cultural data on students' estimation, and teaching of estimation, a decision was made to investigate estimation skills and strategies of Tongan students in Tonga. The design of the Tongan investigation is the focus of the next chapter.

CHAPTER FOUR

RESEARCH DESIGN



4.1 Introduction

The review of the literature together with the problems and rationale presented in chapters one, two and three determined the research questions which guided this investigation. The main purposes of this study were to investigate estimation skills and processes of a sample of Tongan students and to explore how instruction on measurement and computational estimation strategies affects students' understanding of and strategy use in estimation. Justification of the research methodology, and the design of the investigations are the focal points of this chapter.

First, the research design is discussed (4.2). Second, the steps taken in setting up the research in Tonga are outlined (4.3). Third, the pilot survey and interviews are detailed (4.4). The chapter then describes the main study (4.5) which involved individual interviews, an intervention phase involving trialling of teaching activities on measurement and computational estimation, a second round of individual interviews, and a questionnaire for teachers. The next section (4.6) outlines the method of data analysis. The chapter concludes with a discussion of the limitations, problems and difficulties associated with the research design (4.7).

4.2 Research Design

In order to address the research questions (outlined in chapter one), a decision was made to adopt a qualitative approach involving interviewing, case study and action research methodologies. This approach was considered appropriate to investigate students' thought processes about estimation. The findings would serve as a basis for exploring the effectiveness of the teaching activities on measurement and computational estimation.

The research design comprised four main phases:

Phase 1 : Familiarisation and Pilot Study (4.4)

Phase 2 : First Individual Interviews (4.5)

Phase 3 : Trial of Teaching Activities (4.6)

Phase 4 : Second Individual Interviews (4.7)

Phases 2 to 4 involved measurement estimation. They were repeated for computational estimation but in two primary schools only (see 4.5).

The sequencing of the different phases of the investigation are outlined in Table 4A below.

Table 4A: Sequencing of the Phases of the Investigation

Year	Term	Data collection
1992	TERM III (Sept - Nov)	Pilot survey 10 - 12 year olds (n= 63) 13 - 16 year olds (n= 95) Pilot of interview tasks (n= 20)
1993	TERM I (Feb - April)	Classroom observation Selection of sample First interview primary (n= 12) (Measurement estimation)
	TERM II (June - August)	First interview secondary (n=18) (Measurement Estimation) Teaching activities (Measurement estimation)
	TERM III (Sept - Dec)	Second interviews (Measurement estimation) First interview primary (Computational estimation) Teaching activities Second interviews primary (Computational estimation) Administration of Teacher Questionnaire

Details of these phases of the study are provided in the following sections.

4.3 Familiarisation Phase

The familiarisation phase in Tonga formed an important part of the research design. Processes involved in the setting up period, making contact with schools, various government departments and people in the community are documented in this section. A description of these processes might be of value to future researchers conducting research in Tonga.

Setting-up period

This phase of the project consisted of making contact with the Minister of Education, who agreed to be an adviser to the researcher while she collected data in Tonga. A formal request was also required by the Tongan Cabinet for permission to conduct research in Tonga. This request was made through the Prime Minister's office and approval was granted by Cabinet. On the invitation of the Minister of Education, one of four Deputy Directors of Education agreed to be another adviser to the researcher during the data collection phase. A circular letter by this Deputy Director of Education was sent to the principals of the schools selected for the study, heads of various divisions within the Tongan Ministry of Education (i.e. Curriculum

Development Unit, Examination Unit, Primary Schools Division, Distance Education Centre and Community Development and Training Centre, and Tonga Teachers Training College) and directors of relevant church education departments. In addition to the above letter, the researcher also visited and met with the above people to formally ask for their support and assistance during the course of the study.

Making contact with schools

Three secondary schools (one co-educational, one boys only, and one girls only college) and two primary schools (co-educational) were selected for this study. The schools represent a wide cross section of the Tongan school population. The schools were also selected to represent both government and church schools. Table 4B below summarises the five schools.

Table 4B: Summary of the five schools selected for this study

SCHOOL CODE	SCHOOL LEVEL	NATURE OF SCHOOL POPULATION
I	Secondary	boys
II	Secondary	co-educational
III	Secondary	girls
IV	Primary	co-educational
V	Primary	co-educational

The principals and/or deputy principals of each of the schools were personally approached by the researcher for permission to conduct research in their schools. All school principals were willing to help except one primary school principal who wanted a letter from the Ministry. However, once the formal letter from the Deputy Director of Education was provided this principal was fully supportive of the research.

The researcher's meetings with the principals and/or deputy principals focused on the research proposal, data to be collected and the data collection method. The purposes of the research, details of what would be required of both students, teachers and researcher, and the duration of the study were clarified. The principals decided which teacher and class the researcher could work with. In the case of one primary school, the principal was the class 6 teacher, and therefore there was no need to meet with other teachers. One might argue that this was a biased sample of teachers; however, the criteria used by the principals to select the teachers were sensible and acceptable to the researcher. The main criteria used by principals were (i) teachers whom they thought would not mind being observed while teaching, and (ii) those whom the principal thought would be very supportive to the researcher in terms of arranging time for observation and getting students organised for the main study.

The principals' choice of teachers for this study were respected and accepted by the researcher. The principals and/or deputy principals introduced the researcher to the chosen teachers and further discussions of the research proposal took place. A copy of the research proposal was given to each of the teachers involved. Because the teachers had not been involved in a research project, it was necessary for the researcher to make time to explain and discuss with the teachers the rationale, objectives and the research methodology for the investigation.

Rationale for selecting class 6 and form 3 students

A decision was made to focus the investigation on learners from class 6 (10 - 11 year olds) and form 3 (14 - 15 year olds). These students were selected for a number of reasons. First, class 6 students are in the final year of primary schooling so it seemed important to find out what estimation skills they would take with them to secondary school. It then seemed useful to investigate the effect of an extra three years of mathematical instruction on the students' estimation skills. Hence the decision to include a form 3 sample.

4.4 Phase 1: Pilot Study

The pilot study involved two dimensions: a survey in all of the five classrooms selected for the observation, followed by individual interviews with a randomly selected sample. The main purposes of the survey were to determine and verify the appropriateness of the tasks for Tongan students, and to establish the appropriateness of the two methods of data collection (survey and interviewing) for assessing students' estimation skills and strategies.

Pilot survey

The questionnaire used by Koloto (1989) in a New Zealand study was used for the pilot survey with the form 3 students, with some questions modified to suit Tongan students. Analysis of the curriculum statements indicated that the questions would be at an appropriate level of difficulty for Tongan students. Discussions with the teachers involved confirmed that the questions used were appropriate for Tongan students. For the secondary students, an English version of the questionnaire was used. A copy of this questionnaire is provided in Appendix A. The results of the survey with the secondary students showed that computational estimation tasks did not elicit the use of estimation. The students worked out the exact answers using their pen. Thus, a decision was made to omit computational estimation tasks from the questionnaire for primary students. For this questionnaire the questions were given in the Tongan language (see Appendix B).

The decision to use the English language was based on the researcher's form 3 classroom observation that all notes and most of the writing on the blackboard used English as the medium of instruction. It was observed that in two of the secondary classrooms, teachers used the Tongan language for explaining ideas and in discussion with students, but since the

survey was in written form, English seemed appropriate. A copy of the questionnaire was given to teachers for comments and there was no indication from teachers that using English would be inappropriate for form 3 students.

Pilot interview

Four students were selected from each of the five classrooms for the trial of interview questions. Twenty items from the survey questionnaire were selected for the interviews. The Pilot Interviews showed that too many items were used, so a decision was made to reduce the number of items from twenty to twelve for the main study. The pilot study also revealed that the use of the questionnaire-type survey, with spaces for students to write down explanations of the strategies they used, could be useful for collecting data on students' measurement estimation ideas and strategies. There was clear evidence to suggest that these would be inappropriate for assessing students' computational estimation skills and strategies. The availability of pen and paper led to almost all students working out the exact answer using a written algorithm. This highlights one of the disadvantages or limitations of using questionnaires as a data collection tool.

Comments and discussion

The pilot study revealed that presenting the questions to form 3 students in the English language was most inappropriate for Tongan students. The data from the pilot study showed that the students had not mastered the English language well enough to allow them to answer the questions on their own using the English language. The researcher was continually asked to translate some of the questions during the trialling of this questionnaire. This finding suggested that even though the teachers used the English language as a medium of instruction it would be unwise to assume that all Tongan form 3 students had sufficiently mastered this language to warrant its use in the research. Thus, a decision was made to use the Tongan language in all interactions with students in the main study.

Another important result of the pilot study related to the number of items used in individual interviews. Although the interview has a number of advantages over other data collection techniques such as the questionnaires, one of its limitations is that it is time consuming. Thus, the number of interview items needed to be reduced to allow the researcher time to explore with students each item, particularly the computational estimation problems. As a consequence, decisions were made to (i) have two separate series of interviews, one each for computational and measurement estimation, and (ii) trial the measurement and computational estimation instructional activities separately to allow the researcher to monitor the development of students' ideas and skills during the course of the intervention phase. The next section details the processes and procedures followed in the main phase of this study.

4.5 Phase 2: Main Study

The main study took place between February and December 1993. It consisted of classroom observations, individual interviews focussing on measurement estimation, trialling of measurement estimation activities, and follow-up individual interviews. The same procedure was used for computational estimation activities, but due to time limitations this was restricted to the primary school students.

The review of the literature also suggested that to assess students' estimation skills, one needed to consider both the strategy used to derive an estimate together with the estimate itself. To ascertain whether respondents use an estimation strategy, individual interviews have been the most widely used research technique (Dowker, 1992; Hildreth; 1983; Koloto, 1989; Levine, 1982; Reys et al., 1982; Siegel et al., 1982; Threadgill-Sowder, 1984). Thus, the present study employed interviewing as its main data collection technique. Data were also collected from the researcher's observations and discussions with classroom teachers, students' worksheets, and a questionnaire for teachers. The rationale for using a questionnaire rather than the interview technique to collect data from teachers is discussed later in this section.

Classroom observation

The main study began with classroom observation in each of the five classrooms in the five schools chosen for the main study. This allowed both the researcher and students in these classrooms to get used to one another. It also gave the researcher and two new teachers (who did not participate in the pilot study) the opportunity to get to know one another and to discuss the aims of the investigation, the data collection procedures that would be used for the investigation, and any queries that they might have.

During the classroom observation the researcher became a participant observer by helping out in the classroom to explain to students some of the questions involved in their worksheets, problems and the notes given on the blackboard. It was hoped that this would help make students feel at ease with the researcher's presence in their classroom, and to convey the message that the researcher was an individual interested in students' mathematics learning. This was also viewed as an appropriate step to take in putting back something to the school involved in the study.

During the observation phase, each of the five teachers provided a list of students in his or her mathematics class. Six names were randomly selected from each class list to be included in the sample (n=30) for the main study. The teacher and researcher then arranged times for individual interviews with these students.

The Sample

As indicated previously 30 students (12 primary and 18 secondary) were selected for the main phase of the investigation. Table 4C and 4D summarise the selected sample.

Table 4C: Details of the sample of class 6 students

NAME	STUDENT CODE	AGE	GENDER	SCHOOL CODE	TEACHER'S CODE
Aina	P1	10 years	Female	V	T5
Keleni	P3	10 years	"	"	"
Siale	P5	10 years	"	"	"
Seini	P7	11 years	"	IV	T4
Sosefina	P9	11 years	"	"	"
Tolofi	P11	11 years	"	"	"
Elisaia	P2	10 years	Male	V	T5
Heivakaola	P4	10 years	"	"	"
'Ofa*	P6	12 years	"	"	"
Manoa*	P8	11 years	"	1V	T4
Peni	P10	10 years	"	"	"
Saia	P12	11 years	"	"	"

Note - * second year class 6 student

It should be noted that the primary schools students were preparing for the Secondary School Entrance Examination at the end of the year, the first external examination faced by Tongan students in their schooling. Therefore, in both schools, extra classes after school hours, normally from 4 to 6 pm, 7 to 9 pm and 6 to 8 am were carried out by both teachers without extra pay and in School V, the principal of the school was also involved in running these extra classes. There were huge demands on teachers' time at class 6 level, for the preparation of students for the examinations. The number of students who would pass to Tonga High School from one' school was perceived as a measure of the teacher's performance. Thus, the secondary school entrance examination papers dominated classroom activities in both schools.

As a government funded school, School V was better equipped in terms of the materials and equipment available in the classroom, and the state of the school buildings, compared to those available in School IV, a church funded primary school. Whereas School V had three streams in class 6, School IV had only one class. The A stream from School V took part in the study because the teacher was selected by the principal for this study. Both classrooms involved in the study were overcrowded with 38 pupils in School V and 37 in School IV.

Table 4D: Details of the sample of form 3 students

NAME	STUDENT CODE	AGE	GENDER	SCHOOL CODE	TEACHER'S CODE
Belinda	S1	14 years	Female	III	T3
Heleni	S3	14 years	"	"	"
Litia	S5	15 years	"	"	"
Molavia	S7	15 years	"	"	"
Mosiana	S9	16 years	"	"	"
Rowen	S11	14 years	"	"	"
Katalina	S13	14 yeas	"	II	T2
Losalia	S15	15 years	"	"	"
Maletina	S17	15 years	"	"	"
Timote	S2	14 years	Male	I	T1
Latavao	S4	15 years	"	"	"
Makoni	S6	14 years	"	"	"
Sailosi	S8	14 years	"	"	"
Semisi	S10	13 years	"	"	"
'Utu'atu	S12	14 years	"	"	"
Fisipuna	S14	14 years	"	II	T2
Sione	S16	14 years	"	"	"
Pita	S18	15 years	"	"	"

First individual interviews (measurement)

The items used for the first individual interviews about measurement estimation are shown in Table 4E below. Twelve items were used, four items each on length, weight and capacity. A decision was made to use the same items with both class 6 and form 3 students to allow comparison of the ideas of the students at the two class levels. One question (Item 4) regarding long distance was modified to make it more suitable to the students' experiences. Whereas students in schools I, II, III and V were asked to estimate the distance between school and Vaiola hospital, students in School IV were required to estimate the distance between their school and Nuku'alofa town. All items were presented in the Tongan language. To facilitate discussion an English translation of these items is also provided in Table 4E below.

Table 4E: Measurement Estimation Tasks for first and second interview

AREA	ITEM NUMBER	TASK IN TONGAN	ENGLISH TRANSLATION
Length	1	'Ai ange ha'o fakafuofua ki he loloa e ki'i peni ko 'eni? (senitimita 'e 13.8)	Estimate the length of this pen (13.8 cm)
	2	'Ai ange ha'o fakafuofua ki he falahi 'o e matapa sio'ata 'i lokiako	Estimate the width of the classroom window (various)
	3	Ko e ha nai e ma'olunga 'o e tepile 'a e faiako	Estimate the height of the teacher's desk (various)
	4	'Ai ange ha'o fakafuofua ki he mama'o mei heni 'apiako ki falemahaki Vaiola	Approximately how far is it from here (school) to Vaiola hospital?
Weight	5	Ko e ha nai e mamafa 'o e ki'i peni (kalami 'e 4)	Weight of this pen (approx. 4 grams)
	6	Fefe nai e mamafa 'o e kapaika ko 'eni (kalami 'e 450)	Weight of a tin of fish (approx. 450 grams)
	7	Ko e ha ha'o fakafuofua ki he mamafa 'o e tohi (kalami 'e 1500)	Weight of a book (approx. 1500 grams)
	8	'Ai ha'o fakafuofua ki he mamafa 'o ha tamasi'i pe ta'ahine ako kalasi 6/foomu 3.	Weight of a class 6/form 3 student (various ranges)
Capacity	9	Ko e ha nai e lahi e vai e fonu ai e ki'i sepuniti	How much liquid could a teaspoon hold? (5 millilitres)
	10	Ko e ha nai e lahi 'o e huhua'i niu te ke lava 'o ma'u mei ha fo'iniu mata.	How much juice can you get from a green coconut? (600-1000 mls)
	11	Ko e ha nai e lahi e vai 'e fonu ai 'a e sioki ko 'eni	How much water would you need to fill this jug? (2 litres)
	12	Ko e ha nai e lahi 'o e penisini/lolo 'e fonu ai e tangike 'o ha veeni.	How much petrol can fill up the petrol tank of a van?

The tasks were selected by the researcher to obtain information about.

- students' strategies for estimating familiar measures of length, weight and capacity. (The objects selected were based on the researcher's observation of the students' environment and everyday life activities);
- students' use of benchmarks and any evidence of the use of traditional methods of measuring length;
- students' ideas of the basic metric units of measurement, such as a centimetre, metre, gram, kilogram, millilitre and litre; and
- contexts from which students have developed their ideas and strategies.

After consultation with the teachers, a time was arranged for individual interviews with each of the subjects in the five schools selected for the study. Each interview lasted about 40 minutes.

The Interview Process

Students were interviewed individually in a room or under a tree away from the rest of the class. Each student sat alongside the researcher. The interviews were tape recorded and pen and paper were also available for the researcher for recording student responses.

The following procedures were followed during each interview.

1. An introduction whereby about 5 minutes was spent in making the subject feel at ease. Non-threatening personal questions which often involved asking the student whether she or he was related to a person with the same surname or people from his or her village that the researcher knew or had heard of. These led to students asking similar questions to the researcher. Establishing a person or place known to both the researcher and student helped the student feel at ease.

2. The aim of the present study and the interview were explained to the student. It was stressed that the session was not a test but an opportunity for the student to explain as fully as possible the strategies and thought processes he or she used to arrive at his or her estimates. It was also emphasised that only an estimate for the answers were required and therefore pen and paper would not be needed.

3. Permission for the interview to be tape-recorded was then requested from the student, because the researcher would otherwise find it difficult to record everything that she or he might say. The confidentiality of the students' responses was stressed.

4. For measurement estimation tasks, the objects were laid out in front of the student and he or she was asked to decide the order in which estimation tasks (i.e. length, weight or capacity) would be answered. The students also decided the order in which the four items within each type of tasks were answered. All students' responses were accepted. The flexibility of the interview method allowed the researcher to rephrase a question in cases the student seemed to misunderstand it. Throughout the interview the following probing questions were used when necessary: Why? Why do you think that? How did you get that? What strategy did you use to obtain that answer? Can you explain that again?

   For the computational estimation tasks, the items were presented on stimulus cards. These were read to the student and proceeded from addition to, subtraction, multiplication and finally division tasks.

5 Finally the researcher thanked the student for participating in the study.

Students' responses were analysed, the results of which are presented and discussed in chapters six and eight. Based on students' responses in the interview tasks, strategies for measurement estimation were incorporated into the design of the teaching activities on measurement estimation.

Teaching activities (measurement)

A twelve-lesson programme was designed by the researcher to trial in the classrooms. An hour-long lesson was taught once each week over 12 weeks. In terms of students' responses to the estimation problems, students were encouraged to use 'reasonable range', both students and researcher deciding on the upper and lower bounds for these ranges, depending on the problem involved and the strategies used by students to arrive at their estimates. Appendix C details the teaching programme that was used.

A percentage error was not used as a criterion with which to assess students' responses. The limitations of such an approach were discussed in the review of the literature. As discussed in chapter three, this method of assessment was considered not suitable for the present study.

Second individual interviews (measurement)

All students in the selected sample were reinterviewed to identify the strategies that they used. For the follow-up interview, a decision was made to reduce the number of estimation tasks from twelve to nine. The main determining factor was the time involved in using the same twelve. Eight of the original twelve items were retained, while a new Item was introduced. Items 4, 7 and 12 were omitted, whereas Item 11 was replaced with another task considered more appropriate. The reason for not using item 4, relating to estimating long distance, was that estimating long distance, and/or developing students' concept of a kilometre was not included in the 12-lesson teaching unit.

Item 7 was eliminated from the second interview because it was decided that one item (i.e. Item 6) could be used to investigate students' estimate of objects between 100 and 2000 grams. Because students' everyday life shopping activities involve purchasing a tin of fish from the shop a decision was made to use this item in the second interview.

Again, there was little attention given in the unit to activities which might help students estimate larger capacities so Item 12 was no longer relevant. The 2-litre jug of Item 11 was replaced by a more familiar everyday object, a bottle of lemonade, in the hope that students would be encouraged to use a variety of benchmarks as the basis of estimation.

Individual interviews (computational)

Because of limited time and unavailability of secondary students, a decision was made to limit the study of students' computational skills and strategies to primary students only. Details of the items used in first and following individual interviews are specified in Table 4F.

Table 4F: Computational Estimation Tasks for the First and Second Interview

Area	Item number	Tongan version	English translation
Estimating involving addition	1	Na'e 'alu 'a Mele, Lupe, Lola mo Lose 'o fato'o. Na'e ma'u 'e Mele 'a e fo'i to'o 'e 50, Lupe 'a e to'o 'e 200, Lola 'a e 150. Na'e lahi 'a e va'inga 'a Lose pea ne ma'u pe 'ene fo'i to'o 'e 20. Ko e fo'i to'o nai 'e fiha na'e ma'u 'e he tamaiki ?	Mele, Lupe, Lola and Lose collected some shellfish from the reef - Mele 50, Lupe 200, Lola 150. Lose spent most of the time playing and therefore collected only 20. Approximately how many shellfish did the girls collect?
	2	Ko e maaka 'o Mele 'i he'ene ngaahi lesoni 'i he sivi ko e: English 52, Fika 56, Saienisi 54, Tonga 67. Ko e ha nai e katoa e maaka 'o Mele ?	Mele's marks in the exams were: English 52, Maths 56, Science 54, Tonga 67. Approximately how many marks did Mele get in the exam?
Estimating involving subtraction	3	Na'a ku 'alu ki fale koloa mo 'eku $10.00 ke fakatau 'a e ngaahi me'a ko 'eni: Kapaika 'e 1 $1.60, Mapakupaku kilo 'e 1 $2.30, Pata $1.00, Suka kilo 'e 1 $1.80 mo e puha hu'akau 'e 1 $1.00. 'E lahi fe'unga nai 'a e $10.00 ki he 'eku fakatau.	I went to the shop with a $10.00 note to buy the following items: tin of fish $1.60, 1 kilogram of crackers $2.30, $1.00 worth of butter, 1 kg of sugar and a carton of milk which costs $1.00. Would $10.00 be enough to cover my purchase ?
	4	Na'e 'alu 'a Pita mo 'ene $500 'o fai 'a 'ene hu koloa. Ko e katoa e totongi 'o e koloa ko e $312. Ko e fiha nai 'a e pa'anga 'a Pita na'e toe 'o foki mo ia ki 'api.	Pita went to the wholesalers with $500. He spent $312. Approximately how much money did he have left?
Estimation involving multiplication	5	Ko e ngoue 'a Sione 'oku 'iai 'a e 'otu talo futuna 'e 28. Ko e fu'u talo 'e 60 'i he 'otu. 'Oku fakafuofua 'oku ofiofi ki he fu'u talo 'e ____ 'i he ngoue 'a Sione.	Sione has 28 rows of taro in his plantation. There are about 60 taro plants in each row. There are about ____ taro plants in Sione's plantation.
	6	Ko e mama'o mei he 'api 'o 'Ana ki 'apiako ko e kilomita 'e 24. 'Oku 'alu pasi 'a 'Ana he kilomita 'e 48 'i he 'aho. Ko e kilomita nai 'e fiha fakakatoa 'oku 'alu ai 'a 'Ana ki he ako he uike 'e taha?	The distance between Ana's home and her school is 24 kilometres. 'Ana travel 48 kilometres by bus daily. Approximately, how many kilometres does Ana travel in one week?
Estimation involving division	7	Na'e fili 'a e toko 13 ke kau ki he timi netipolo 'a Tonga. 'Oku fiema'u ke kumi 'e he kau va'inga 'a e $3 890 ke tokoni ki hono fakapa'anga 'o 'enau folau ki Fisi. 'E taki pa'anga nai 'e fiha 'a e tokotaha va'inga kae ma'u 'a e pa'anga 'oku fiema'u?	Thirteen people were selected for the Tonga national netball team. These players need $3 890 to help finance a trip to Fiji. How much does each player need to raise to get the total amount required?
	8	Na'e 'alu 'a e tamaiki 'e toko 9 'o fai 'enau tufi niu. 'Oku fakafuofua ki he fo'i niu 'e 550 na'e ma'u. Kapau 'e vahevahe 'ehe toko 9 'a 'enau niu na'e ma'u, te nau taki fiha nai e fo'i niu?	Nine children collected coconuts from the plantation. It is estimated that they collected 550 coconuts. If these coconuts are divided among the children, approximately how many coconuts would each person have?

Twelve students who participated in this phase of the study were interviewed before and after taking part in the teaching activities.

Teaching activities (computational)

A ten-lesson unit on computational estimation activities was developed and trialled with the same sample of primary students. Appendix D provides details of these activities. Unlike the previous trial of measurement estimation activities, which took place on a weekly basis, this trial took place on a daily basis over a period of two weeks.

Results of the first and second individual interviews and teaching activities on computational estimation are reported and discussed in chapter eight and nine respectively.

Teacher Questionnaire

Even though the main focus of the present study was to investigate the estimation strategies and skills of Tongan students, the view(s) held by their teachers on estimation was considered an important factor in the teaching and learning of this topic. Therefore, a questionnaire was designed to be used for collecting data from the teachers. Appendix E details the English version of the questionnaire that was administered to the five teachers in the five classrooms selected for the main study.

There were several reasons for using a questionnaire rather than individual interviews. First, it was important that the researcher was not seen to be teaching or giving advice to teachers by discussing with them their practices in the classroom regarding estimation. Three of the five teachers were much older than the researcher. It is considered culturally inappropriate for a younger Tongan person to appear to give advice to these teachers. Such a situation may be best explained by Foliaki (1994) in her description of similar experiences of working with other Polynesian adults. Foliaki (1994) said:

> When an older person talks to the Polynesian parents and attempts to advise them, there is no loss of face on the part of the parents. When a young person comes along and attempts to play this role, it is just negative and counter-productive for everyone. (p.105)

Secondly, given that most teachers regarded the researcher as an outsider (i.e. a person who collects information for university study), it was considered inappropriate for the researcher to discuss with teachers matters relating to the teaching practices because the researcher had no "authority" and/or "status" in the Ministry of Education. Moreover, as a university student the researcher was also perceived to be knowledgable about her work and as a consequence ought to be allowed to carry out her research work as she knew best. Thus, a relationship of respect and deference existed between teachers and the researcher which resulted in both focusing on their own role in the classroom. The teacher was seen as a person with considerable experience in teaching and the researcher has experience in conducting research. In other

words it was culturally appropriate for the teachers and researcher to focus on their respective roles in the classroom.

4.6 Data Analysis

The data collected in the investigation were mainly qualitative. Information obtained through individual interviews, the teachers' questionnaire, and teaching activities were all qualitative in nature, while some data from the students' worksheets and some data drawn from the interviews were quantitative. This section provides more details on the method of data analysis.

Interview data

All interviews were recorded on audio tape and transcribed for analysis. Students' estimation strategies evident in the data were categorised according to the nature of the strategies and processes used by the students. Categories used by previous researchers on measurement (Hildreth, 1983, Siegel et al, 1982) and computational (Reys et al 1982, Reys, Reys and Penafiel, 1991; Reys, Reys, Nohda, Ishida, Yoshikawa, Shimizu, 1991) estimation strategies served as a guide, but in this study names which reflected the processes used in each strategy type were constructed for the categories. The number of times a particular strategy type was used by students was also recorded. A decision was made to eliminate data from 3 out of 30 students involved in the study due to incomplete data collected from these students. Thus responses to the 4 measurement items by 27 (15 secondary and 12 primary) students led to 108 possible answers each for length, weight and capacity. A comparison of the responses by class 6 and form 3 students was conducted by analysing the percent of strategy types used by the students.

Assessment Procedure

The analysis of the data focused on identifying the strategies used by students to derive their estimates. The assessment of the estimates was based on the processes and strategies that respondents had used. Although a reasonable range were established for the measurement tasks, the analysis of the data showed that students' estimates often fall just outside the range established and therefore a decision was made to use students' strategies in determining the reasonableness of the estimates. For example, the reasonable range established for Item 1 (i.e. length of the pen) was 11-18 centimetres. An estimate of 8 centimetres based on the width of the middle finger (i.e. student's concept of one centimetre) would be considered unreasonable if the above range is taken as the criteria for assessing students' estimates. Because the student had used the middle finger to work out the given estimate a decision was made to take it as a reasonable estimate. Thus, the strategies used were also taken in account in assessing the reasonableness of the estimates. The same procedure was used for the computational items. In cases where the student made errors (e.g. place value, reading the tasks) the estimates were considered unreasonable.

Effectiveness of teaching activities

The effectiveness of the teaching activities in helping students was determined by (i) reviewing the trial of the teaching activities, and (ii) data drawn from students' interview responses before and after the teaching activities. Observation and field notes based on the responses of four case study students were collected, analysed and reported to illustrate the possible effects of the teaching activities. Chapters seven and nine report and discuss the outcome of the teaching activities for measurement and computational estimation, respectively.

4.7 Limitations, Problems and Difficulties

Certain difficulties existed in implementing a research design to address the research questions which guided this study. Although a Tongan, the researcher had been absent from Tonga for more than twelve years. She therefore endeavoured to conduct research in Tongan classrooms as a relatively unknown person to most people in the Tonga Ministry of Education, teachers and students. Being unknown has both its advantages and disadvantages. The study was not initiated and/or funded by the Tongan Ministry of Education, so the Ministry needed to be consulted about this study.

The problem associated with the researcher's perceived status led to the use of the questionnaire rather than interviewing to collect data from teachers. It also meant that the researcher had to teach the measurement and computational estimation activities that she had developed. Although the original plan had been for the teachers to trial the teaching activities, teaching the units allowed the researcher to become more aware of their strengths and weaknesses as vehicles for developing students' estimation skills.

4.8 Summary and Comments

This chapter has focused on a discussion of the processes and procedures involved in the research design used in this study. The data collection techniques were mainly qualitative using individual interviews to probe students' ideas and strategies about estimation. The main study consisted of individual interviews, trial of teaching activities and follow-up individual interviews of a small sample of primary and secondary students.

The use of the interview technique has been largely influenced by previous studies on estimation and the constructivist view of learning. As indicated in chapter one the constructivist theory of learning assumes that an individual's existing ideas are important in responding to and making sense of incoming information and that the individual makes sense of his or her experiences by actively constructing meaning. Thus, research that is based on this theoretical position inclines towards a methodology that will assist the researcher to probe beneath the surface and to move beyond observable behaviour in order to understand the phenomenological perspectives of learners.

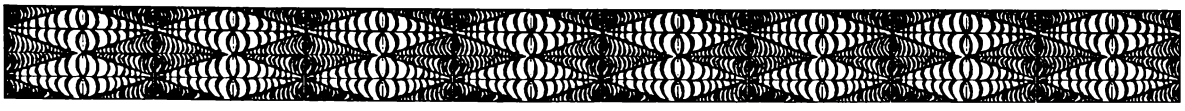
The research design also involved the trialling of estimation activities with the small groups of students in the five schools selected for the investigation. Given that estimation had not been a major part of the mathematics curriculum and teaching in the Tongan classrooms, it was anticipated that the results of the teaching activities would provide useful information for the teachers.

The problems and difficulties associated with the research were also discussed. Some of the these difficulties, for instance, the unavailability of secondary students for the computational estimation study, were a direct result of the nature of the school programme in an academic year. Others, for example, the use of the teacher questionnaire to obtain data from teachers and the use of the researcher to trial teaching activities were consequences of the researcher's attempts to observe some of the expected behaviours in the culture where the study took place. It is important to note that had a European conducted this study in the Tongan classroom, he or she might not have encountered the same problem.

Data obtained from the teachers' questionnaires are reported and discussed in the next chapter. The results of the first individual interviews on measurement estimation, are presented and discussed in chapter six. The teaching activities on measurement estimation, including its successes and limitations are analysed and discussed in chapter seven. Chapter eight presents and discusses the first individual interviews data on computational estimation. Chapter nine reports on teaching activities on computational estimation, and its effectiveness and limitations.

CHAPTER FIVE

RESULTS AND DISCUSSION: TEACHERS' VIEWS ABOUT ESTIMATION



5.1 Introduction

Of interest to the researcher were the views about 'estimation' held by teachers in the five classrooms from which the students were selected for the present study. It was expected that these would influence their work with students in this area of mathematics. As reported in chapter four, three of these classrooms were at the secondary level (form 3) and two at the primary level (class 6).

The research questions addressed in this phase of the study were:

- What are the mathematics teachers' views about estimation?
- What estimation strategies are considered to be currently taught and encouraged by mathematics teachers in Tongan classrooms?

Data about these were collected through a Teacher Questionnaire (see Appendix E). The questionnaire sought information about:

(i) the teachers' meaning(s) for the term 'fakafuofua' (estimation);

(ii) their views about the usefulness of estimation in mathematics learning;

(iii) the estimation strategies for measurement, computational and numerical estimation that they may have tried to teach;

(iv) the teachers' perceived preparation to teach estimation, while at teachers training college;

(v) their views of what would be helpful preparation to enable them to implement the new draft syllabus in which estimation has a prominent place; and

(vi) their ideas regarding assessment of estimation in school internal and external examinations.

The responses of the three secondary and two primary teachers to the questionnaire are reported and discussed in the succeeding sections of this chapter. Given a choice of using

either an English (Appendix E) or Tongan version (Appendix F) of the questionnaire, all teachers opted for the Tongan version. As outlined in chapter four, T1, T2 and T3 are the secondary school teachers, and T4 and T5 are the primary teachers whose classrooms were chosen for the main phase of this study. Table 5A below summarises background information about the sample of teachers who participated in this study.

Table 5A: Summary of teachers' background

TEACHER	SCHOOL	GENDER	NUMBERS OF YEARS IN THE TEACHING PROFESSION	GROUP FOR MICRO TEACHING
T1	Secondary (boys only)	Female	22	I
T2	Secondary (co-educational)	Male	1	II
T3	Secondary (girls only)	Female	5	III
T4	Primary (co-educational)	Male	17	IV
T5	Primary (co-educational)	Male	20	V

5.2 Teachers' definition(s) of the term 'estimation'

The teachers' definitions of the term 'estimation' are given below.

T1: Ko 'ete sio ki ha me'a pea te fakakaukau ki hono lahi, hono mamamfa, hono loloa 'o fakaofiofi ki hono ***mo'oni.***

T1: To look at something and to think about its size, mass, length, and to work out a value closer to the exact answer.

T2: Ko e fakafuofua kiate au ko e fakamahamahalo mo e fakaofiofi

T2: Estimation means a conjecture or to work out a close answer.

T3: Ko 'ete fakaofiofi 'ete fua ki he ofi taha 'o fakatefito 'i he me'a 'oku fai ki ai e fakafuofua 'o ngaue'aki hoto ngaahi ongo'anga (sio, fanongo, ala, 'ahi'ahi'i etc) 'o 'ikai ngaue'aki ha me'a ngaue fua.

T3: To work out a measure close to the correct value based on the purpose of the estimation exercise, using one's senses (sight, hearing, touch taste, etc), without using a measuring instrument.

T4: Ko 'eku faka'uhinga ki he fakafuofua ko e fakatatau ofiofi pe tatau 'o ha me'a ki ha toe me'a kehe.

T4: To compare the size of two objects to ascertain whether their sizes are close.

T5: (Abstract thinking)
Ko hoto mata pe mo e 'atamai 'oku na ngaue ke kumi'aki ha'ate mo'oni fakaofiofi pe tonu 'aupito, he taimi 'e taha ta'e ala hoto nima pe me'a ngaue ke fua'aki mo fakapapau'i 'ete mo'oni.

T5: (Abstract thinking)
To use one's eyes and brain to find an answer close to the right answer, without using one's hands or a measuring instrument to check one's approximate answer.

The definitions provided by the five teachers suggest that they considered estimation as a process largely involving the use of one's senses to mentally work out an approximate answer which is close to the actual value, without using a measuring instrument to obtain the answer or measure. In other words, estimation is seen by teachers as a mental process of working out a close answer. The teachers' definitions are similar to that of Siegel et al (1982) reported in chapter one, which considered estimation as "*a process that gives a rough solution to a problem.*" Except for T2 whose response does not refer to a particular aspect of estimation, responses from the other four teachers indicate that they were thinking of measurement estimation.

In defining fakafuofua (estimation), teachers used the following terms: *fakaofiofi* (closer to), *fakamahamahalo* (conjecture), *fakatatau ofiofi* (close comparison), and *fakakaukau* (to think). Like fakafuofua (estimate), fakaofiofi refers to the process itself as well as the outcome or end result. The definitions proposed by the teachers illustrate some of the processes involved in estimation. It is interesting to note that for some teachers fakafuofua (estimation) is fakakaukau (to think). It appears that in some cases the Tongan translation of the term 'thinking' is fakafuofua. This is in line with Whitin (1994) suggestion that *estimation is mathematical thinking.*

The term **mo'oni** (answer) appears in two of the teachers' definitions of estimation. Mo'oni is one of the terms used for an answer to an exercise; for instance, ko e ha e mo'oni ki he fo'i fika ko 'eni 2 + 2? (what is the answer to this exercise 2 + 2?). In the everyday use of the term it also refers to true, truth or fact. In mathematics there is a commonly held view that the answer to a mathematical exercise or problem is the mo'oni or the truth. In this respect, it seems that the use of the term mo'oni might not be helpful in the development of an appreciation of estimation in mathematics. Very often in mathematics education, the term **mo'oni totonu** (exact answer) is used. Mo'oni totonu (exact answer) could also be interpreted as exact truth. As estimation is a process that often leads to an inexact answer, a language problem could arise for both students and teachers, in explaining the end result of an estimation exercise. It seems that the use of other terms such as tali fakaofiofi (approximate answer) needs to be encouraged in the classroom when working on estimation.

5.3 Usefulness of Estimation in Mathematics

With respect to the usefulness of estimation in mathematics learning, all five teachers agreed that estimation is indeed useful in the mathematics classroom. The main reasons for its perceived usefulness are given below:

T1: 'Oku vave 'aupito 'a e fakafuofua. 'Oku malava ke fikaloto'i 'o ma'u ha ngaahi tali 'uhinga lelei 'i he fakakaukau fakafuofua.

T1: Estimation is quick. It can be done mentally to obtain reasonable or sensible answers through the estimation process.

T2: *He 'oku lava heni ke ngaue 'a e 'atamai 'o e tangata fe fifili mo toe viviliange 'o 'ikai ke fakafalala ki ha me'a ngaue pe fakamo'oni pau, ka ko 'ene to'o mai pe ha ngaahi 'ilo kuo ne 'osi a'usia 'o fakakaukau'i mo fakama'opo'opo 'o fakahoko'aki leva 'ene fokotu'utu'u faka'osi 'i he founga fakafuofua.*

T2: *Estimation allows one's mind to seek and search for answers and not to rely on any measurement or calculating devices to obtain an exact answer, but uses ones knowledge and experiences to think of and use estimation strategies.*

T3: *'Oku 'aonga lahi 'aupito he taimi lahi ia 'o e fengaue'aki 'a e ki'i ta'ahine ako mo hono 'atakai 'o a'u pe ki he'ene ngaue 'i lokiako tautefito ki he'ene fakakaukau'i ha tali 'o ha fehu'i 'oku 'ikai tene fakapapau'i.*

T3: *It is very useful when the student interacts with his or her environment and in the classroom, particularly when required to provide an answer to a question that she is not quite sure about.*

T4: *Ko e 'uhi ko e taha 'eni ia ha topic 'oku fu'u mahu'inga 'aupito 'i he fanau koe'uhi ko honau kaha'u.*

T4: *Because this is one important topic which is most valuable for the students' future.*

T5: i. *Lava ai ke ilo'i 'e kita faiako a e mahino'i 'ehe fanau mo e tukunga pe ngata'anga 'enau 'ilo ki he fa'ahinga me'a ko ia.*

T5: i. *Allows the teacher to ascertain the limitations of students' knowledge regarding estimation.*

ii. *Lava ai ke 'ilo 'ehe faiako 'a e kamata'anga 'o e me'a ke ako'i.*

ii. *Allows the teacher to work out where teaching should begin.*

iii. *Lava ai ke hanga pe 'ehe tamasi'i 'o ako 'iateia pe 'ene fehalaaki 'o 'ikai ko e faiako' ke ne 'oatu 'aki 'ene fakafuofua pea fua 'o 'ilo hono faikehekehe pe 'oku fu'u lahi pe si'isi'i.*

iii. *Allows the students to learn from their mistakes but not from the teacher. The student will provide an estimate then measure then he would know the difference, whether his estimate is larger or smaller.*

The above responses indicate some of the perceived use of estimation in the mathematics classroom. Estimation is seen as a (i) quick mental process for obtaining reasonable answer; (ii) mental process whereby students could use their knowledge and experiences to formulate answers rather than relying on measuring devices; and (iii) vehicle by which students could assess the reasonableness of their answers and hence learn from their mistakes.

Four teachers provided answers which point to various ways estimation is used in classroom activities but there was no specific reference to its usefulness in everyday life. Although the fifth teacher, T4, responded that estimation would be useful for the students' future, he did not specify in what ways estimation would be of value to students in the future.

The link between an individual's everyday life activities and estimation was not shown in the data. It is possible that the teachers saw a link but did not express it. The questionnaire, of course, did not provide opportunity to probe further on this issue.

5.4 Teaching Estimation

The responses of the five teachers indicated that they thought estimation strategies could be taught in the classroom but that they teach estimation only sometimes (taimi pe 'e ni'ihi). The lack of attention given to estimation in their mathematics teaching was also reflected in the limited number of strategies for measurement, computational and quantitative estimation reported by the teachers. This may in part be accounted for by the lack of emphasis on estimation in standard text books used in the Tongan classrooms in assessment in mathematics. As discussed in chapter two both internal and external examinations gave little attention to estimation problems and in the two years (i.e. 1987 and 1990) in which an estimation problem was included in the Secondary School Entrance Examination mathematics paper, these were limited to items on measurement estimation. The remainder of this section discusses the estimation strategies reportedly taught by teachers in their classrooms.

Measurement estimation strategies

The main strategy type reported by teachers for teaching measurement estimation involves getting students to estimate, then measure and finally compare the estimates to the actual measurement. Explanations given by two of the teachers (T1 and T4) illustrate this strategy type.

T1: Teuteu ha ngaahi naunau: maka, fua'i'akau, kapa'one'one, puha, hina, etc.

1. *Fekau ke nau to'o tahataha ki 'olunga pea tala honau mamafa 'i he fakafuofua.*
2. *Fua tahataha kinautolu 'i ha me'afua.*
3. *Lekooti 'a 'enau fakafuofua mo e mo'oni pe fua totonu. Fakafehoanaki [e fakafuofua mo e fua totonu].*

T1: Prepare the following materials: stone, fruits, can of sand, boxes, bottles, etc.

1. *Ask students to pick up each of these and estimate its weight.*
2. *Measure each of these on a scale.*
3. *Record their estimate and the exact or correct measurement. Compare [the estimate and the exact answer].*

T1 provides an explanation regarding the way measurement estimation is taught in the Tongan classrooms. The researcher's classroom observation confirms this as a commonly used strategy by teachers, particularly at primary school level where some estimation activities were part of the mathematics program. What is missing from this explanation is how the comparison of the estimate with the exact measure is carried out. T4 explained how the estimate was assessed in the extract below.

T4: 'Oku 'uluaki 'oange 'a e faingamalie ki he fanau ke nau 'uluaki fakafuofua'i e fua vai pe me'a huhu'a mo e fua loloa mo e fua mamafa pea mau toki ngaue fakataha ki hono fua'aki ha me'afua ke lava kenau ako mei he'enau fakafuofua kapau na'e ***fehalaaki.***

T4: Students are first given the opportunity to estimate the capacity, length and mass. Then they would work in groups to measure these so that they would be able to learn from their estimates if these were ***wrong.***

The comment by T4 reflects the notion of wrongness (fehalaaki) attached to estimation, following comparison with the actual measure. During the researcher's classroom observation the students were told to work out how **wrong** their estimates were from the actual measure obtained. No attempts were made to inform students that all measures were approximations. The difference between the estimate and the actual measure was calculated (i.e. by finding the difference between the two values). However, no attempts were made to repeat the process, whereby students would use their first estimates to make further estimates and hence improve on their initial estimate. Researchers (Edwards, 1984; Reys, 1986; Trafton, 1986) have suggested that in order to develop an appreciation of and foster skills in making estimates, plenty of experiences are needed in making estimates, making comparisons with the actual values, followed by further estimates to improve on the initial estimates. The practice of judging an answer as **wrong** seems to be more appropriate to mental computation in mathematics. For estimation problems the term **reasonable** appears to be a more appropriate term. Emphasis on **wrong estimates** will only lead to the perception that an estimate is inferior to the right answer.

The comments of two teachers (T2 and T5) suggested that they saw prior experiences with actual units of measurement as important to the development of estimation.

T2: 'Oku ou fa'a ako'i ke nau maheni mo e lahi 'o e ngaahi fua taautaha eg 1cm, 10cm, 1m, 1l, 2l, 1kg etc. Nau toutou sio mo fua e ngaahi fua anga maheni ko 'eni pea 'osi ko ia 'oku ou toki ha'u leva 'o 'eke fefe 'a e loloa 'o e lokiako mita nai 'e fiha? To'o mai ha fo'i maka 'o 'eke ko e kilo 'e fiha?

T2: I often provide them with opportunities to have experience and be familiar with a unit of measurement e.g. 1cm, 10cm, 1m, 1l, 2l, 1kg, etc. Provide them with plenty of opportunities to measure these basic units of measurement. I would then ask them how about the length of this classroom, how many metres would that be? I would take a stone and ask, how many kg would this be?

T5: ***Loloa***
Ko e ha nai hono loloa homou tesi? 'osi, pea fekau ke nau fua.

T5: ***Length***
What is the length of your desk? and, then ask them to measure.

Mamafa
'Oange ha ngaahi object ke nau ala ki ai 'o milihi ngaohi mo to'o. Tohi 'enau fakafuofua 'osi tohi 'enau mo'oni. Fua'i 'i ha me'afua ke nau 'ilo pe 'oku lahi pe si'isi'i 'enau estimate.

Weight
Provide them with various objects to hold and feel their weights. Estimate, then record their estimates. Measure the object so that they may ascertain whether their estimates are greater or less than the measures.

Kakano
To'o ha fa'ahinga 'ai 'anga me'a pe ko hono fakafuofua pea tali ki mu'a pea toko fua.

Capacity
Empty container of liquid or else estimate then write answer before measure.

The strategy described by T2 and T5 showed that the teachers intended students to have plenty of experiences with the basic units of measurement before they were asked to estimate the length and weight of familiar objects such as the length of the classroom and the weight of a particular stone. It should be pointed out that T5 provided his answer for kakano (capacity) in English and therefore a Tongan translation is provided.

The responses above represent the reported intentions of teachers. It does not always follow that they put these intentions into practice. Considering that these teachers had earlier reported having taught estimation taimi pe ni'ihi (only sometimes) care needs to be exercised in drawing conclusions from the data regarding teachers having provided 'plenty of experience' for estimation. One of the questions required teachers to rate the estimation skills of each of the six students selected for this study. It is interesting to report that T2 did not provide any ratings other than to explain that he had not taught estimation to these students and therefore was not in any position to answer the question. It appears, therefore that there is a gap between the teachers' intentions to teach estimation and actual practice of teaching this topic.

As discussed in chapter three, research has shown that one of the skills involved in measurement estimation is an understanding of the attribute (e.g. length or area) to be measured and concept of unit of measurement (Hildreth, 1983). It is a move in the right direction if students are encouraged to make themselves familiar with basic units of measurement. It is only through familiarity with these units that one would then be expected to make reasonable estimates using those units. Moreover, the assessment procedure for estimating weight explained by T5, *"ascertain whether their estimates are greater or less than the measures"* is an appropriate approach to adopt in the context of estimation. However, as mentioned earlier in this section, there is a need to take this a step further and use initial estimates together with the knowledge that it is greater or less than actual measure to improve their estimates.

Informal measures and/or benchmarks should also be used to help students develop and value estimation. It appears that this would involve two stages, that is, moving from the unfamiliar

to benchmark and then to measure. Data later reported in chapter six, revealed that students used a variety of benchmarks from everyday life experiences particularly for estimating weights (e.g. a kilo of sugar, flour or mutton). Data suggest that for teaching estimation, teachers need to use students' benchmarks for tasks where students are unfamiliar with units of measurements but have reference points or benchmarks of their own acquired through their own experience. Students' existing ideas of various benchmarks derived from their experiences could be used by teachers to further develop the students' estimation and measurement skills.

It is important to examine what was reported by teachers; equally important is what was not reported. It is interesting to point out that none of the methods of length measurement (e.g. hanga or ofa) discussed in chapter two were reported by teachers. The standard metric units of measurement were emphasised in their responses without any reference to the traditional Tongan units of measuring. There are two possible interpretations of this. First, as discussed in chapter four one of the limitations of the questionnaire as a data collection tool is that it does not allow the researcher to probe further. Therefore, the lack of reference to the traditional Tongan methods of measurement as estimation tools was to be expected given that there were no specific questions on these. A second interpretation of this is that traditional Tongan methods of measurement are not part of most teachers' thinking in mathematics. Given that traditional methods of measurement are part of the Tongan studies curriculum but not the mathematics curriculum it possible that these are perceived as non mathematical and therefore do not have place in the mathematics classroom.

Teaching computational estimation strategies

Two of the teachers (T2 and T4) did not provide any strategy for computational estimation, indicating that they perhaps were not sure about this question. The other three teachers reported on computational estimation strategies. The strategy types described by the teachers were: (i) the front-end strategy and (ii) working out the place value of the numbers involved.

T1: Fakafuofua fai ha ngaahi nomipa mu'omu'a
$2.15
+ $3.39
$4.26
2+3+4=9
Ko e fakafuofua 'e laka $9.

T1: Use the front-end digits
$2.15
$3.39
+ $4.26
2+3+4=9
The estimate would be bigger than $9.

T3: Feinga pe ke fakatokanga'i pe 'oku 'i he tu'unga fe ia 'o e mahu'inga fakafo'i mata'ifika (matelau, teau, afe, etc) pea toki faingofua ai pe 'ene li ki ai 'ene fakafuofua.

T3: Work out the place value of each of the numerals (ones, hundred, thousand, etc) and then it would be easy to make an estimate.

T5: Fakafuofua, ngaue'i, fakatatau pe fakafehoanaki.

T5: Estimate, work out, compare and contrast.

Of these five teachers, only T1 was able to suggest a useful computational strategy. The responses or lack of responses of the others imply a lack of familiarity with this teaching estimation. T1 reported one of the most powerful computational estimation strategies (i.e. fronted strategy) but, as explained in chapter four, T1 is a secondary school teacher who has also been a part-time curriculum writer in the curriculum development unit. It is possible that she acquired this knowledge through her involvement with the development of the new draft mathematics syllabus, but it is doubtful whether she has taught this strategy in the classroom.

It should be noted that the limited number of computational strategies reported by teachers could be a result of the data collection technique used. The use of the questionnaire rather than interviewing did not allow for probing questions to be asked. T2 and T4 did not provide any strategy which suggests that they were not sure about "fakafuofua fakafika" (computational estimation). Perhaps it was an unfamiliar term to them. Re-phrasing and further explanation of the question were not possible. This finding suggests that the translation of the concept of "computational estimation" to the Tongan language needs to be carefully considered. The Tongan term used for computational is fakafika, which refers to the four mathematical operations. The researcher's definition, however, was not clearly communicated to these two teachers as the term 'fakafika' in general means mathematical without specific reference to the four operations. It is possible that T1 and T4 could have explained computational estimation strategies had the question been rephrased to indicate that "fakafuofua fakafika" (computational estimation) involves the four mathematical operations.

Teaching numerosity or quantitative estimation strategies

As with computational estimation, the data suggests a lack of familiarity by teachers with quantitative estimation. The strategies reported included (i) rounding numbers; (ii) think of a similar task in one's experiences; and (iii) using a portion of the object to be estimated to work out an estimate.

The responses of the five teachers are shown below.

T1: Fakaofiofi
Fakaofiofi 'a e 5400 ki he 1000 ofi taha.
Faka'ilonga 'a e vaeuamalie.
Faka''ilonga 'a e 5400
Ofitaha ki he 5000 pe 6000.
Fakaofiofi 5000

5000 5500 6000

T2: Fokotu'unga mapu ke fakafuofua'i pe ko e tu'unga mapu nai 'e fiha?

T3: 'Uluaki feinga pe ke nau fakaofiofi ki he lahi 'o e me'a 'e taha pea toki li ai pe fakafuofua ki ha to e me'a 'oku lahiange 'o ngaue'aki e fua 'o e me'a 'e taha.

T1: Rounding.
Round 5400 to the closest thousand. Mark the halfway.
Mark 5400.
Is it closer to 5000 or 6000.
Closer to 5000

5000 5500 6000

T2: A pile of marbles to estimate how many marbles in the pile?

T3: First estimate a small section and then use that to estimate anything that is larger.

T4: 'Oku 'uluaki fai'aki 'a e sio ke fakapapau'i pea toki fai hono fua.

T4: Just look at it to make sure and then measure.

T5: Nau sio ki ai Takatakai'i 'o ala ki ai pea fakakaukau'i ha toe me'a 'oku fofolinga tatau pea kuo ke feangaianga mo ia.

T5: Look at it. Touch it and then think of something in your experiences that is similar to it.

The above explanations show that only one teacher, T2, provided an example of quantitative estimation as it is perceived by the researcher. While the responses given by T3 and T5 could refer to quantitative estimation strategies, these could also be used as strategies for measurement estimation. T4's explanation specifically refers to measurement rather than quantitative estimation. This result indicates the inter-connectedness of measurement and quantitative estimation–perhaps, this is a consequence of using the same number system for both measurement and counting. While both use the number system for quantifying, measurement differs in that it has a developed standard unit of measurement.

The lack of other examples of numerical estimation implies that for these teachers it was difficult to think of relevant instances. The example given for the rounding procedure (i.e. rounding 5 400 to the closest thousand) shows the use of a number line to determine which thousand, 5 000 or 6 000, it is closer to. Because of the required degree of accuracy specified (i.e. the nearest thousand) this example may be more appropriately labelled as an approximation exercise, as the term 'approximation' is defined in chapter one. The teachers' responses suggest that they may have had few experiences with differentiation of estimation into various types. The terms employed in this question appeared to be new to most of the teachers involved. As well, the result once again highlights the limitations of using a questionnaire as a data collection technique.

5.5 Estimation in Teachers' Preservice Training

When asked whether they were well prepared to teach estimation as a result of their preservice teacher training three teachers responded "No" while the remaining two said "Yes".

Well prepared to teach estimation

One secondary teacher, T3 and one primary teacher, T5 said that they were well prepared at teachers' college to teach estimation. Their responses are reported below.

T3: Taimi 'e ni'ihi 'ete faiako 'oku pau kete fe'ao mo e palani e lesoni faka'aho kae fakaofiofi 'ete fua ki he tu'unga lelei 'e a'u ki ai e kalasi

T3: In teaching one would be required to prepare daily lesson plans. The teacher has to figure out an acceptable standard to be achieved by the class by the end of a lesson.

T5: 'Oku 'i he founga faiako pe 'a e me'a ko e 'oange [ki he fanauako] ha faingamalie lahi ke nau lea he discussion pea 'oku mau lava ai ke nau 'omai 'enau ngaahi ideas pe fakakaukau ki ha me'a 'oku ke fehu'i atu ko 'enau tali 'oku 'omai kiate koe ko 'enau fakafuofua ia .	*T5: Methods of teaching should include provision of opportunities [for students] to take part in discussions, through which they could share their ideas pertaining to teachers' questions and that the responses given would be their estimates.*

T3 describes general estimation problem for teachers, which involves planning and implementing a lesson plan based on an estimation of an expected and acceptable standard or to be attained by students. T5 describes a particular teaching/learning approach (i.e. taking part in discussions) which would allow students to make their estimates known.

Despite their feeling of adequate preservice preparation, the explanations given by both teachers do not mention student estimation strategies. Indeed, the secondary school teacher, T3 mentioned only teacher estimation. These responses suggest that specific estimation procedures were either not a major focus of their preservice teacher training programs or not incorporated into the teachers' thinking about mathematics education.

The other three teachers indicated that estimation was not a major part of their preservice teacher education programs.

T1: Ko e ngaahi ngaue mo e ngaahi fehu'i na'e fiema'u pe ke fekumi ki he tali tonu.	*T1: All work and questions required an exact answer.*
T2: Na'e si'isi'i 'aupito ke 'iai ha me'a pehe ia ai. Ko e fua pe mo a'u tonu ki ai. Ko e konga faka'osi na'a mau to'o ai e "statistic". Ka koe konga si'i pe ia 'Ikai fai ha savea, etc.	*T2: There was very little opportunity for estimation. Measure and work out an exact answer. Statistics was covered towards the end of the course but even that was only a small part of the course. There was no survey, etc.*
T4: Koe'uhi na'e te'eki ke fu'u ma'olunga 'a e silapa 'a e kolisi fakafaiako 'i he ta'u 1970's.	*T4: Because the standard of the teachers' training college's syllabus in the 1970s was not high.*

The above responses suggest that estimation was not a significant focus of the Tongan preservice teacher education program, even as recently as 1992, when T2 was a student teacher. It is evident from the teachers' responses that mathematics courses in the preservice education programs did not adequately prepare teachers to teach estimation in mathematics. As T1 said "Ko e ngaahi ngaue mo e ngaahi fehu'i na'e fiema'u pe ke fekumi ki he tali totonu" (*"All work and questions required an exact answer".*)

5.6 Inservice Training Needs on Estimation

One notable change of emphasis in the content of the new mathematics draft syllabus (Tonga Ministry of Education, 1992a) is the inclusion of estimation in various topics from form 1 up to form 5. Considering that the data showed a lack of emphasis on estimation in their preservice teacher training programs, it was considered helpful to find out the perceived professional needs of the teachers. The teachers responded to a question about what preparation they thought they needed to implement the draft syllabus and to teach 'estimation' well. Relevant responses are given below.

T1: 1. Teuteu'i kinautolu 'i ha ngaahi ako ke nau 'ilo mo maheni mo e silapa fo'ou.

2. Fakatokanga'i kinautolu ke nau teuteu 'a e ngaahi naunau 'oku fiema'u ke ngaue'aki.

3. Fai mo'oni 'a e ako'i 'o e fakafuofua 'o toutou ngaue'aki pea toki fai mei mui 'a e ngaue'aki pe ko e fua ke ma'u 'ae mo'oni tonu.

T2: Ko e fakalotolahi'i e fanau ke nau ngaue'aki e taukei ko ia pea 'oua 'e ilifia he ko e natula 'o e fanauako he taimi ni ko e kata'i e hala mei he fo'i mo'oni totonu. Kae ta ko e na'e fai e ki'i fakafuofua ia e leka ako ko ia. 'I he'ene pehe 'oku lotosi'i leva e leka ako ke to e fai ha fakaofiofi pe fakafuofua. Ako'i e fua ke nau ma'u pea toutou 'ahi'ahi'i 'aki leva 'a 'enau fakafuofua.

T3: 'Uluaki ke lava 'e he faiako 'o fai e fakafuofua pea toki fakahoko ki he fanauako. Kae ako'i mo fakafehu'i ma'u pe he ngaahi me'a honau 'atakai eg. anga 'o e 'ea 'o e 'aho kotoa pe, lahi 'ene kai pongipongi pe 'e a'u ki he tuku 'a e ako, taimi mohe, 'e lava he sivi etc. Ngaahi fehu'i 'aonga ki he ta'ahine ako pea fekau'aki mo 'ene mo'ui faka'aho.

T1: 1. Prepare them through inservice courses so that they are familiar with the new syllabus.

2. Prepare resources, and relevant materials required for teaching estimation.

3. Make an honest attempt to teach estimation and to use it frequently before attempting to measure or obtain the correct answer.

T2: To encourage students to use this skill and not to be afraid because there is a natural tendency for students to laugh at another person who provides an answer that deviates from the correct answer. But this student could have provided an estimate. In that case, students would be discouraged from making or attempting to make estimates. Teach units of measurement and then use them to make estimates.

T3: First, teachers should be able to make reasonable estimates before they teach it to students. Teach and ask questions based on students' environment e.g. the weather, amount of food for breakfast to last a school day, time required for sleep, whether they would pass the exam, etc. Use questions that are useful for and relevant to the everyday experience of the student.

T4:	*'Oku totonu 'aupito ke toe fai 'ehe Potungaue Ako 'a e Pule'anga mo e siasi ha ngaahi workshop ki he kau faiako ke lava 'ehe kau faiako 'o ako'i lelei 'a e fanau.*	*T4: More workshops run by the Ministry of Education as well as the churches to prepare teacher effective in teaching students.*
T5:	*1. Faiako 'oange ha taimi ke fanongo pea tali lelei e mo'oni 'oku 'oatu 'ehe fanau he taimi discussion.*	*T5: 1. Teachers should make time to listen and accept the responses given by students during discussions.*
	2. Kau faiako fiema'u ke 'iai ha'o taukei mavahe he six basic skills of questions and analysis.	*2. Teachers need to be knowledgable on the six basic skills of questions and analysis.*
	3. Teuteu'i ke lahi pea anga hono ngaue'aki me'a e real objects he taimi fika.	*3. Prepare teachers on using real objects in mathematics lessons.*
	4. 'Uluaki fai hano Inservice kau faiako kimu'a pea fakahoko 'a e Draft Mathematics Syllabus.	*4. Inservice training of teachers before the Draft Mathematics Syllabus is implemented in the classrooms.*

Teachers' responses reveal a need for more workshops and inservice training programs to help and support them with the implementation of the new curriculum, and in particular to teach estimation.

One of the important suggestions from T1 was for teachers to *"make an honest attempt to teach estimation"*. Teachers need to make a commitment to teaching estimation. It is likely that this can happen only if teachers have been helped to develop estimation strategies themselves, and are well prepared with relevant resources for teaching estimation.

Another important suggestion made by T2 involves encouraging students to *"use this skill and not to be afraid"*. This teacher went on to explain one reason that would discourage students from attempting to make estimates and that is the tendency for Tongan students to laugh at others if their answers given in class differ from the correct or exact answer. This is an important issue which is supported by the findings of the trial of teaching activities. This is further discussed in chapter seven.

5.7 Teachers' Views on Assessment of Estimation

Teachers' responses to the question regarding assessment of estimation in school internal and external examinations reveal that they considered little attention was given to estimation in assessment. This is hardly surprising given that estimation has only been included as a major part of the draft mathematics syllabus in 1992 and is yet to be fully implemented in the Tongan classrooms.

T1:	*'Oku 'ikai fu'u fakakau ia he sivi, ka 'oku ou pehe 'oku totonu ke fakakau he 'oku fu'u mahu'inga ke ako'i mo sivi'i 'a e fakafuofua.*	*T1: It is not included in examination but I think it should because it important to teach and examine estimation.*
T2:	*'Oku si'i hono ngaue'aki he pepa sivi. 'Oku totonu ke fakalahilahi pea 'oku fakatupu fakakaukau.*	*T2: There is little on estimation in examination papers. There should be increased emphasis.*
T3:	*'Oku ou tui 'oku kau lelei 'aupito ki he meimei 3/4 'o e ako ka ko e toenga 'e malava pe kapau 'e ngaue'aki ma'u pe 'i lokiako faka'aho pea 'e toe leleiange.*	*T3: I believe that this would be good or useful to about 3/4 of student but for the remaining students it would be useful if they encounter estimation in the daily mathematics program.*
T4:	*'Oku mahino pe 'a e 'ikai lava ke ako'i lelei 'a e fanau 'i he "fakafuofua" pea si'i si'i 'a e taimi koe 'uhi he 'oku 'ikai ke fu'u 'atamai'ia 'a e tokolahi 'o e fanau.*	*T4: It is clear that it is not possible to teach estimation and there is very little time because many students are not capable.*
T5:	*'I he sivi 'oku si'isi'i. Kae fai ha fehu'i 'o ngaue'aki application & analysis etc. Ngaue'aki compare ha fa'ahingame'a 'e ua pe lahiange.* *Multiple choice eg.* *Ko e sioki ko 'ena 'oku 'iai nai ha vai lita 'e fiha?* *(a) lita 'e 5 (e) ml 200 (f) lita 'e 1.* *Ko e ha e fua 'engikolo ko 'ena* *(a) 90 degrees (e) 50 (f) 180* *(h) 30*	*T5: There is little in examination. There is a need for more questions requiring application and analysis. Use comparison of two or more objects.* *Multiple choice :* *For this jug, how many litres of water?* *(a) 5 litres (b) 200 mililitres* *(c) 1 litre* *What is the size of this angle?* *(a) 90 degrees (b) 50 (c) 180* *(d) 30.*

It appears from these responses, that there are few estimation problems in examination papers. This is not surprising given the recency of its emphasis in the syllabus, and the apparent lack of attention to estimation in preservice teacher education programs and classroom practices. The lack of attention to estimation in examinations both internal and external confirms what was discussed in chapter two. Given the current examination oriented nature of schooling in Tonga, it is possible that if estimation is included in examinations this might help people treat it more seriously.

Furthermore, teachers' attitudes towards students perceived ability might be a hindrance to the successful implementation of the new mathematics syllabus. If teachers perceived estimation to be useful to only some of the students then it would be necessary to focus inservice workshops on teachers' expectation of students before any attempts are made to teach estimation.

5.8 Teachers' Other Comments

The teachers were invited to offer any other thoughts about estimation which came to mind. Their responses are provided below.

T1:	*'E ako'i lelei 'a e fanau 'o kapau 'e ngaue'aki 'e he kau faiako 'a e silapa, teuteu 'a e ngaahi naunau kotoa 'oku fiema'u, toutou fai 'a e fakafuofua pea 'e tokoni ia ki he'enau ako fika.*	*T1:*	*Students will be taught well if teachers use the syllabus, prepare necessary materials and teaching resources, frequent use of estimation as these will help the learning of mathematics.*
T2:	*Fakalotolahi'i e fanau ke nau tali e ngaahi fehu'i he 'oku mahino ko e tali 'e taha ko e fakafuofua. Pea fehu'i ke ne fakamatala e founga na'a ne ma'u 'aki 'ene mo'oni pea te nau lotolahi ke fai pehe.*	*T2:*	*Encourage students to answer questions as it clear that an estimate is one possible answer. Students should also be asked to explain the procedures and processes used to derive their answers as these would help encourage students to speak up in class.*
T3:	*Faka'amu ke toe hakule mai e ngaahi 'ilo fakafuofua he kuonga mu'a na'e ngaue'aki 'ehe kau folau vaka, matu'a vaivai he taimi koe 'o toe ako'i pea toki fakafelave'i mo e ngaahi 'ilo fakafuofua pau 'o e 'aho ni pea 'oku ou tui 'e 'aonga lahi 'aupito pea 'ikai ngata ai ka e kei pukepuke e ngaahi 'ilo fakaofiofi ko ia na'a mole eg. hoko e 12 ho'ata 'oku matala lelei e te'ehoosi (plant)*	*T3:*	*I hope that the estimation knowledge and skills used in olden days in navigation , for example, will be identified and taught as these are very useful, and not only that but to preserve these knowledge before they are lost. e.g. 12 noon is when the petals of the te'ehoosi flowers are wide open.*
T4:	*'Oku ou tui ko e topiki matu'aki mahu'inga 'eni pea 'oku totonu ke lahi 'a e ngaahi me'angaue 'oku fiema'u ki ai (resources).*	*T4:*	*I believe that this is a very important topic and there is a need for resources.*
T5:	*Ke fai ha fengaue'aki 'a e va'a fo'ou ki he silapa fika mo e ngaahi P.T.A. ke nau tokoni hono supply e equipment ki he kau faiako ki hono ako'i 'o e estimation 'i he ako. Ke loto lelei kiai e faiako kae lava ako'i lelei e estimation.*	*T5:*	*The P.T.A. and curriculum development unit should co-operate to supply the equipment required by the teachers to teach estimation. That teachers are willing to teach estimation.*

The teachers' further comments reaffirm the need for teaching materials, textbooks and relevant resources that would help teachers teach estimation. There is also some recognition of the need to include many estimation experiences in the mathematics program, have students communicate orally their estimates and estimation strategies, and relate estimation to everyday life instances and applications.

The successful implementation of the draft syllabus, and in particular estimation will largely depend on teachers' attitudes and willingness to teach this topic. As T5 puts it "Ke loto lelei ki ai e faiako kae lava ke ako'i lelei e estimation" (Only when teachers are willing can estimation be taught properly). This is an important point, because although estimation now has a prominent place in the curriculum, its place in the mathematics classroom will depend on teachers' willingness to accept that this is an important topic that ought to be fostered. Furthermore, teachers' perceptions of their students ability might also affect their decisions to teach estimation.

5.9 Summary and Comment

This chapter has reported and analysed teachers' responses to the teacher questionnaire on estimation. As specified in the introduction two research questions were addressed in this questionnaire. These were:

- What are the mathematics teachers' views about estimation?
- What estimation strategies are considered to be currently taught and encouraged by mathematics teachers in Tongan classrooms?

Despite the limited nature of the sample (n=5) and the limitation associated with the questionnaire as a data collection tool, the data obtained provided useful information to answer the above research questions. These questions were further asked under six categories, as outlined in section 5.1. The results of the analysis of the data collected are summarised below.

Teachers' meanings for the term fakafuofua (estimation)

The following terms were used by teachers to define estimation:

i. fakaofiofi (closer to);
ii. fakamahamahalo (conjecture);
iii. fakatatau ofiofi (comparison); and
iv. fakakaukau (to think)

It is possible that fakafuofua (estimation) as it is used in the Tongan language could mean much more than the term estimation in English. It some cases, estimation could also refer to thinking.

Usefulness of estimation, perceived preparation to teach estimation, and its assessment in school internal and external examinations.

Despite the generally held belief that estimation is useful, the data indicated that the lack of preparation and resources on estimation seemed to prevent the teachers from doing justice to this important mathematics process in their classrooms. Data also showed that estimation has not been a major part of their preservice training, teaching, or examinations. The lack of emphasis in teaching was also revealed through the limited number of strategies for measurement, computational and numerosity estimation specified by teachers. This result is

not surprising given that estimation strategies and techniques were not part of their preservice and inservice teacher education and training programmes.

Preparation to enable them to implement the new draft syllabus

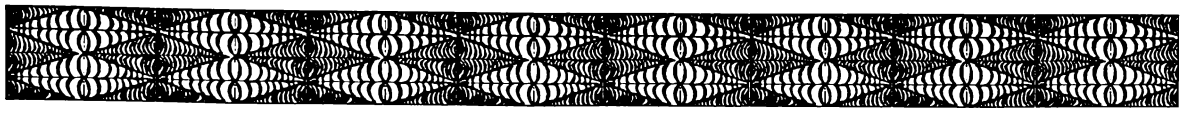
To teach estimation the need for teaching materials, textbooks and relevant resources was identified by the five teachers. This is a very general statement regarding the need for teachers to be fully supported in order to implement the new draft syllabus, an in particular to teach estimation. Thus, it was timely that this study was undertaken.

The above findings have important implications for the implementation of the new draft syllabus in Tongan classrooms, in which estimation is emphasised throughout. If estimation is to be fostered and developed in the classroom then teachers will need adequate preparation through appropriate preservice and inservice programs. The data also implies that in an examination orientated education system, the inclusion of estimation in the examinations could result in the topic being taught in the classroom.

The above findings also point to the importance of investigating the effectiveness of teaching strategies on this topic. In this study, prior to exploring teaching strategies for estimation, the ideas that students have were investigated through individual interviews with the sample of class 6 (n=12) and form 3 (n=18). The results of students' responses to measurement estimation tasks involving lengths, weight and capacity form the basis of the next chapter.

CHAPTER SIX

RESULTS AND DISCUSSION: MEASUREMENT ESTIMATION



6.1 Introduction

The previous chapter has reported and discussed data regarding teachers' views about estimation. As reported in chapter five, according to the five teachers who responded to the questionnaire, estimation has not been a major component of classroom teaching, teacher education, and examination in the Tongan schools selected for the present study. Unlike the data obtained from teachers, which were obtained through a questionnaire, the data gathered from the students were collected through individual interviews. Moreover, the teachers' questions focused on their views regarding estimation, whereas questions used with students centred on identifying skills and strategies used for estimation. Given that estimation has not been a major focus of classroom teaching, it was important to find out what estimation skills and processes relating to length, mass and capacity students possessed. The following research questions were addressed:

i) What estimation strategies are used by Tongan students?

ii) How have the students constructed these strategies?

This chapter presents and discusses estimation data obtained from the individual interviews with a small sample of Tongan students. Section 6.2 discusses the students' definitions of the term 'fakafuofua' (estimation). This is followed by an examination of the strategies used by students to estimate measures of length (6.3), weight (6.4), and capacity (6.5). The next section reports data on students' construction of ideas on estimation (6.6). Section 6.7 then examines possible class differences revealed in the data from the class 6 and form 3 students. The final section (6.8) summarises and comments on the main focuses of this chapter.

6.2 Students' Definitions of the Term 'Estimation'

Because the researcher was interested in the meaning(s) held by students for the term 'estimation', she first asked the students to tell her the meaning they held for the term **fakafuofua** (estimation). Terms used by students to describe 'fakafuofua' and the number of students who gave them are summarised in Table 6A below. It ought to be noted that some students gave more than one responses. Incomplete data led to the omission of data from 3 students. Thus the data analysis was based on the responses of 27 students (12 primary and 15 secondary).

Table 6A: Terms used by students to describe "fakafuofua"

Terms/phrases in Tongan	English translation	Number of students who used it (n=27)
1. Fakamatala ki he ngaahi feitu'u 'oku fai ai 'a e fakafuofua.	1. Explanations of situations or contexts where estimation takes place	7
2. Fakafuofua	2. Estimate/Estimation	4
3. Fakakaukau'i pe 'i loto	3. Mentally figure out something	4
4. Fakaofiofi	4. Closer or nearer to	3
5. Fakamahamahalo	5. Conjecture	3
6. Mate'i	6. Guess	2
7. Fua pe 'i loto	7. Mentally work out the measure	2
8. Sio ke ma'u e anga 'o e me'a ko ia	8. Look at an object and work out its characteristics	2
9. Kumi ha mo'oni pe 'oku ofiofi ki he mo'oni totonu	9. To find an answer which is close to the correct answer	1
10. Fakamahu'inga'i ha me'a	10. Working out how much something is worth	1
11. Meimei	11. Nearly	1

Explanations of situations requiring estimation

Table 6A above shows that although seven students were not able to provide any explanation for the term fakafuofua (estimation) they did provide several everyday life situations in which fakafuofua (estimation) is used. An extract from the interview with S9 illustrates this:

S9: Ko 'ete hanga 'o fakafuofua'i ha me'a 'oku te hanga 'o ngaahi. Hange ko e 'utu ha vai 'oku te hanga 'o fakafuofua'i 'a e vai pe 'e 'uluaki fonu e fo'i hina pe 'ikai.	*S9: To estimate something that one is working on. For example, working out whether there is enough water to fill a bottle.*

Two of the seven students who were not able to provide any explanation for the term fakafuofua (estimation) were able to provide estimates when they were asked to estimate the height, weight or capacity, although they did not say how they formulated these estimates. For example S4 :

I: Ko e ha koa e 'uhinga e fo'ilea 'fakafuofua' kiate koe.	*I: What does the term 'estimation' mean to you?*
S4: 'Ikai keu 'ilo ia 'e au.	*S4: I do not know.*
I: 'Oku 'ikai te ke 'ilo ia 'e koe. Fefe 'eni kapau teu pehe atu 'ai ange ha'o fakafuofua ki he ma'olunga e matapa. Ko e ha leva ho'o me'a 'e fai.	*I: You do not know. What about if I say to you to make an estimate of the height of the door. How would you make that estimate?*
S4: Ko 'eku fakakaukau... Fakakaukau'i pe anga e loloa e matapa. Pehe pe mita 'e tolu	*S4: I would think ... Think about the length of the door; say three metres*

I:	*Na'e anga fefe ho'o ma'u e mita 'e tolu.*	*I:*	*How did you get three metres?*
S4:	*'Ai noa'ia pe ia 'a'aku.*	*S4:*	*Just made it up.*
I:	*Na'e pau pe na'e 'iai ha fa'ahinga me'a na'a ke ngaue'aki ke ma'u e mita 'e tolu.*	*I:*	*But you must have used something to come up three metres.*
S4:	*'Ikai*	*S4:*	*No*

It is important to note that *I* refers to the Interviewer, and *I* will be used throughout the remainder of the thesis to refer to the Interviewer.

Estimation is when you estimate

Four students responded that "Ko e fakafuofua ko ha'ate fakafuofua" (Estimation is when you estimate). Again, they could not provide alternative terms for estimation. For example, one student, S5 responded:

S5: Kapau 'oku 'ikai ke te 'ilo'i fakapapau'i ha me'a, pea 'oku te hanga pe 'e kita 'o fakafuofua atu pe.	*S5: When you are not so sure about something, I would just make an estimate*

Another response by a secondary student, S7, suggests that for a some situations, the term fakafuofua would be the most appropriate term to use, that is, an alternative Tongan term appears inappropriate for the context. For example,

S7: Ko 'ete hanga 'o fakafuofua'i ha me'a 'oku te hanga 'o ngaahi. Hange ko 'ete hanga 'o fai ha haka. 'Oku te fakafuofua'i pe ki he'ene lahi lelei.	*S7: When you estimate something that you are making or working on. For example, when you cook. I would try to make sure that there is enough food for everyone.*

Other expressions for estimation

The data reveals that students used a range of acceptable alternatives for estimation:

- fakaofiofi (closer or nearer to);
- fakamahamahalo (conjecture);
- mate'i (guess);
- fakakaukau'i pe 'i loto (mentally figure out something);
- fakakaukau (think);
- fua pe 'i loto (mentally work out the measure); and
- sio ke ma'u e anga 'o e me'a ko ia (look at an object and work out its characteristics).

The above definitions were used by both primary and secondary students. The following two extracts from interviews illustrate the variety of ideas and terms used by students for the term 'fakafuofua' (estimation).

S12:	*Ko e 'uhinga e fakafuofua ko 'ete hanga 'o fakafuofua ha lahi 'o ha me'a pea 'oku te hanga 'o fakaofiofi atu pe kiai.*	*S12:*	*The meaning of estimation is when I estimate the size or amount of something and I would work out an answer close to that amount.*
S2:	*Ko e fua 'o ha me'a ka 'oku 'ikai ko hono fua totonu ia. Ko 'ete fakamahamahalo pe, pe ko 'ete faka'uhinga pe 'o fakaofiofi.*	*S2:*	*To measure something but not its exact measurement. It is a conjecture or trying to understand and then work out a closer answer.*

Both students used the term fakaofiofi (closer to) as another term for fakafuofua (estimation).

The data seems to suggest that in some situations fakafuofua (estimate) is the most appropriate term to use. As mentioned earlier, the example given by S7 illustrates one situation in which fakafuofua (estimate) is the most appropriate term to use. Any attempts to replace it with any other terms will certainly change the meaning of the situation being explained. 'Fakafuofua'i' (to estimate) means 'to make sure there is enough' of the thing that is being prepared. In other cases, fakafuofua means 'to think' or 'the act of making the estimate'. This has important implications for research focusing on estimation strategies. For example, if students are asked to explain the strategies that they had used to formulate their estimate then one possible explanation would be that they 'thought of it'. Whereas in the first case fakafuofua (estimate) is a combination of mental and physical skills, the second case would seem to involve mental activity only.

The above definitions of estimation needed to be taken into account when working with students on this topic. When rephrasing questions and probing for clarifications the researcher used various terms proposed by students to describe 'fakafuofua' (estimation). Thus, using the students' own definitions of fakafuofua (estimation) helped the researcher to clarify her ideas to students.

6.3 Strategies for Estimating Lengths

As indicated in chapter four, four tasks involving estimating measures of lengths were used to elicit students' ideas and processes. Analysis of the interview data revealed that the students used six types of strategies to estimate the lengths involved in the tasks presented to them. While some of these categories are similar, they are separated out to highlight the variety of processes and ideas used by students, and to illustrate the contexts in which these ideas developed. The strategies, numbers of students who used them, and the number of times these strategy types were used are summarised in Table 6B below.

Table 6B: Strategy types for estimating lengths

Strategy type	Number of students who used the strategy (n= 27)	Number of times it was used
1. Multiple of a measuring instrument	21	50
2. Multiple of a benchmark	23	32
3. Part of a measuring instrument	14	19
4. Part of a benchmark	4	4
5. Just a guess	2	2
6. No response	1	1

A summary of the number of students who used particular strategy types for the length measures is provided in Table 6C below.

Table 6C: Frequencies with which strategy types for lengths were used

Strategy type	Length of a pen	Width of classroom window	Height of a desk	Distance from school to hospital
1. Multiple of a measuring instrument	12	19	18	1
2. Multiple of a benchmark	3	3	5	21
3. Part of a measuring instrument	12	4	3	-
4. Part of a benchmark	-	1	1	2
5. Just a guess	-	-	-	2
6. No response	-	-	-	1

Each of the above strategy types are discussed below and illustrated with extracts from the interview data.

Multiple of a measuring instrument

Analysis of the interview data showed that the most commonly used strategy type was the "multiple of a measuring instrument". This strategy involved taking a familiar length of a standard measure or unit of measurement, then counting or working out the number of times this length seemed to fit the total length of the object. For example, one student explained that the height of the teacher's desk was *"ninety centimetres, because three 30-centimetre rulers will fit that"*. Another student, S4, consistently used this strategy more than once as the following extract shows.

S4:	*Koe matapa sio'ata 'oku senitimita 'e fanima. 'Oku meimei loloa tatau pe ia mo e lula senitimita 'e tolunoa 'e taha mo e konga pea ko e tesi 'oku mita 'e taha.*	*S4:*	*The width of the window is about forty-five centimetres. It is about the same length as one and a half 30 - centimetre ruler, and the desk is one metre.*
I:	*Ko e ha 'oku ke pehe ai 'oku mita 'e taha e tesi?*	*I:*	*Why do you think that the desk is one metre?*
S4:	*He 'oku vaofi pe mo e mita 'e taha he tepi fua mita.*	*S4:*	*Because its length is close to one metre in a measuring tape.*

The data shows that two thirds of the students used the "multiple of a measuring instrument" strategy to estimate the width of the classroom window and the height of the teacher's desk. It appears that the 30-centimetre ruler is a familiar measuring instrument which most of these students could use to estimate these lengths. As well, 12 students used ideas of a centimetre.

It is also evident from the data that only one student used this "multiple of a measuring instrument" strategy to estimate the distance between school and Vaiola Hospital. Analysis of this student's response showed that her idea of the length of a centimetre was used to make an estimate of 1000 centimetres for this distance. When asked to draw a one-centimetre line, she drew a line which measured approximately 12 centimetres. This response suggests that her mental image of a centimetre is about 12 times the size of a centimetre. If this mental image of a centimetre is then taken as basis of her estimation, then 1000 centimetres would provide about 120 metres. This is still an under-estimation of the task but the comparison between the mental image of a centimetre and the distance to be estimated is a skill considered by Hildreth (1983) to be involved in measurement estimation.

The interview tasks consisted of mainly familiar and short lengths which were manageable by students, but the standard measures (one centimetre and 30-centimetre ruler) used by students for shorter lengths would have been inappropriate for longer lengths or distances. For longer distances a "multiple of a benchmark" strategy was required. This strategy type is discussed in the next section.

Multiple of a benchmark

Unlike the previous strategy type where a mental image of a measuring device such as a ruler or a unit of measurement was used, students with this strategy used a **benchmark**. A benchmark is an object (other than unit of measurement) with a known measurement which the student uses as a reference point or basis for making an estimate. It goes beyond a mental image of a unit of measurement to a mental image of an object of known length.

This strategy was most commonly used for estimating long distances. Item 4 required students to estimate the distance between their school and a well-known and/or frequently visited place, namely Vaiola hospital. Because the distance between school IV and the hospital is greater than the distance between the other schools and Vaiola hospital, this

question was altered when presented to the students in School IV. Instead, these students were asked to estimate the distance between their school and Nuku'alofa, the capital of Tonga. Table 6D provides details of the distances involved in Item 4.

Table 6D: The actual distances between schools and hospital/town

School Code	Distance between school and hospital/town
I	5 kilometres
II	6 kilometres
III	3 kilometres
IV	5 kilometres
V	10 kilometres

Various benchmarks were used by students to derive an estimate. For instance, S6 used a familiar distance.

I:	*Fefe nai e mama'o mei heni (apiako) ki falemahaki Vaiola?*	*I:*	*What about the distance from here (school) to Vaiola hospital?*
S6:	*Kilomita 'e hiva.*	*S6:*	*Nine kilometres.*
I:	*Na'e anga fefe ho'o ma'u e kilomita 'e hiva?*	*I:*	*How did you get nine kilometres?*
S6:	*Ngaue'aki pe 'alu mei Malapo ki Toloa.*	*S6:*	*I just used the distance between Malapo and Toloa (ie. Tupou College)*
I:	*Ko e ha e loloa mei Malapo ki Toloa?*	*I:*	*What is the distance between Malapo and Toloa ?*
S6:	*Kilomita 'e tolu.*	*S6:*	*Three kilometres.*
I:	*'Aia 'oku kilomita 'e tolu mei Malapo ki Toloa pea ke 'omi pe ia 'o fakafehoanaki mo e mama'o mei heni ki Vaiola.*	*I:*	*So you think that it is three kilometres from Malapo to Toloa and you used that distance to work out the distance from here to Vaiola,*
S6:	*'Io*	*S6:*	*Yes*

Another benchmark used by students was Mailetaha (Onemile), as evidence in S8's responses.

S8:	*'Atele ki Vaiola 'oku meimei maile 'e ua mo e konga. Ko 'eku fakafuofua ki he 'alu ko e mei he hala Vuna ki Mailetaha ko e maile 'e taha pea toe tanaki ki ai mo e konga.*	*S8:*	*It is nearly two and a half mile from 'Atele to Vaiola. My estimate is that the distance from Vuna Road to Mailetaha is one mile and you add on about half a mile.*

In explaining the reasons for thinking that the distance between his school and Vaiola hospital was two miles, S2, responded :

S2:	*Mei kolo ki Mailetaha ko e maile 'e taha, pea toe maile taha pe mei Mailetaha ki falemahaki, 'aia 'oku tatau pe ia mo e mama'o mei heni ki falemahaki*	*S2:*	*From town to Mailetaha is one mile , and it would be one from Mailetaha to the hospital, so that would be the same as the distance from here to the hospital*

Mailetaha (one mile) derived its name from the fact that it is one mile from the flagpole in town to this place. Therefore, this knowledge was used by S8 and S2 as the 'benchmark' for estimating the distance between their school and Vaiola hospital.

As shown in Table 6C; 21 out of 27 students (i.e. 77%) used a **benchmark** for estimating the task contained in Item 4. The benchmarks used appear to have developed from their everyday life experiences. It is interesting to note that for long distance the use of the imperial unit, such a mile, is still prevalent today.

Three students used 'multiple of benchmarks' to estimate the width of a window and the length of a pen. For instance, S8 used a one foot piece of leather to work out the width of a window.

S8:	*Fute 'e taha. Ko eku fa'a 'alu koe' 'o fa'a kumi leta koe' 'i kolo.*	*S8:*	*One foot; I used to go to town to buy some leather.*
I:	*Leta ha?*	*I:*	*Which kind of leather?*
S8:	*Leta koe' ki he fanaika, 'oku fute 'e taha.*	*S8:*	*The leather that is used for making a fishing rod. It is one foot.*

Likewise, a primary pupil (P2) used the length of a block used in class to estimate the length of a pen. In explaining the reason for giving 10 centimetres as an answer, P2 said:

P2:	*Na'e 'iai 'a e ki'i fo'i poloka na'e 'omai 'e Saia (faiako) ke mau ngaue'aki, 'oku senitimita 'e 5. Peau hanga pe 'e au 'o sio ki he loloa 'o e peni 'o fakatatau ki he fo'i poloka ... 'Oku nounou ange e fo'i poloka ka ko e peni 'oku senitimita 'e 10.*	*P2:*	*There was a little block that Saia (teacher) gave us to work with, which was 5 centimetres. I looked at the pen and then compared it to the length of the block ... The block is shorter and therefore the pen is 10 centimetres.*

Another primary pupil (P4) used the length of the lid to estimate the length of a pen.

P4:	*Ko e ki'i peni 'oku senitimita 'e fa pe tolu.*	*P4:*	*That little pen is four or three centimetres.*
I:	*Na'e anga fefe ho'o ma'u 'a e senitimita 'e fa pe tolu ?*	*I:*	*How did you get four or three centimetres ?*
P4 :	*Ko 'ete hanga pe 'o 'ai e peni o tu'u pehe pea te hanga pe 'o fakafuofua pe ha me'a 'e taha hange ko e tapuni e peni.Mei he ki he 'oku tatau ia mo e senitimita 'e taha, ua, tolu ki he 'ene 'osi. 'Aia ko e senitimita 'e fa.*	*P4:*	*I would put the pen this way and then estimate one part say the lid of the pen. From there to there one end of the lid to the other is equivalent to one centimetre. Then I would go one, two, three until it finishes.*

The above response by P4 illustrated that measurement estimation often involves using objects in hand, the lid of a pen in this case. It does not always involve thinking of a mental image of an unit of measurement and using that as the basis of estimation.

Part of a measuring instrument

This strategy involved a student thinking about a measuring device such as a ruler or a known unit of measurement and working out a fraction or part of that as his or her estimate. The main criteria for responses to be included in this category was that the unit of measurement or measuring instrument selected was longer than the item to be estimated. This required the respondent to work out a fractional relationship between the two. For example, when explaining an estimate of fifteen centimetres for the length of a pen, two students S11 and P10 responded *"Na'a ku fakakaukau ki he vaeua 'o ha lula 'aia ko e senitimita 'e tahanima" (I thought about half of the ruler and therefore fifteen centimetres).*

Sometimes responses were given not as a proper fraction but as something less than the unit of measurement used as the basis of estimation. For instance, P8, a primary pupil's explanation for giving ninety centimetres for the height of the desk was: *"Na'a ku fakakaukau ki he ma'olunga 'o e mita 'e taha 'oku ma'olalo ange 'eni [tesi] ia" (I thought about the height of one metre and I thought that this [desk] would be shorter*).

Students' responses suggest that the 30-centimetre ruler and the one-metre ruler were the measuring instruments most frequently used as the basis of their estimates. In a sense they were 'benchmark' measures.

Even though the same standard measuring instrument was used by different students to derive their estimates, the results were not necessarily the same. For example, whereas S11 and P10 used the 30 centimetre ruler to arrive at 15 centimetres another secondary student, S8, said,

S8: Ko e peni 'oku senitimita 'e uanoa pe tahavalu. Ko e fa'a fua koe' he initasi, 'oku ou sioloto pe ki he loloa e lula senitimita 'e tolunoa, pea 'oku meimei hange pe ha senitimita 'e uanoa pe ki'i si'i hifo pe ai 'aia ko e senitimita 'e tahavalu.

S8: The pen is about twenty or eighteen centimetres. From my experience in measurement in industrial arts, I can visualise the length of a 30-centimetre ruler, and it is about or close to twenty centimetres or slightly less than that and that is eighteen centimetres.

Thus the students used the 30-centimetre ruler to work out estimates of 15 and 18 centimetres, respectively. Both were reasonable estimates.

Not all students used the 30-centimetre ruler as the basis of their estimate. In response to the same question, another secondary student, S2, based his estimate on a mental image of a 15-centimetre ruler.

S2:	*(To'o e S2 e peni). Senitimita 'e valu mo e konga. Ko e fakafuofua'i pe ki he loloa e lula .*	*S2:*	*(S2 picked up the pen). Eight and a half centimetres. That is an estimate based on a comparison with the ruler.*
I:	*Fakafuofua'i ki he loloa e lula. Lula senitimita 'e fiha?*	*I:*	*Compared to which ruler? How many centimetres is this ruler?*
S2:	*Lula senitimita 'e taha nima?*	*S2:*	*The 15 - centimetre ruler.*
I:	*Ko e lula fe ia?*	*I:*	*Which ruler is that ?*
S2:	*Lula ko e he kapasa*	*S2:*	*The ruler in the compass (or mathematical instrument set).*

Although this student, S2, used an acceptable strategy, an estimate of eight and a half centimetres is an underestimate of the length of the pen. Further probing suggested that his idea of the length of a centimetre might have influenced the size of his estimate.

I:	*Hanga ange 'o tohi'i he e loloa 'o e senitimita 'e taha. Laine'i ange ho'o fakafuofua ki he senitimita 'e taha.*	*I:*	*Can you draw a line which is about one centimetre?*
S2:	*Tohi'i*	*S2:*	*Drew a line (approximately 2 cm long)*
I:	*Ko e fe senitimita 'e taha?*	*I:*	*Which is one centimetre?*
S2	*Mei he´ ki he´*	*S2:*	*From there to there.*
I:	*Ko e ha e ki'i laine ia 'i loto?*	*I:*	*What is that line in the middle?*
S2:	*Ko e vaeua ia.*	*S2:*	*That is half of a centimetre.*

It is evident that his idea of a centimetre was twice what it should be, and this therefore resulted in an estimate that was approximately half the length he intended.

Part of a benchmark

For this strategy, a fraction of a benchmark was used to work out an estimate. In some cases, the estimates of lengths given earlier during the interview were used as benchmarks. Responses by S6 indicate that the student worked back from the length of a known object. Moreover, answers to items presented earlier were used as benchmarks for later responses.

S6:	*Ko e tesi 'oku mita 'e taha mo e konga pea mita 'e taha e matapa sio'ata.*	*S6:*	*The desk is one and a half metres and the window is one metre.*
I:	*Ko e ha e founga na'a ke ngaue'aki ke fai'aki ho'o fakafuofua?*	*I:*	*What kind of strategy did you use to make your estimate?*
S6:	*Ngaue'aki pe 'eku tali koe´ 'anenai ki he ma'olunga e matapa.*	*S6:*	*I used my earlier answer about the height of the door.*
I:	*Na'e fiha koa e ma'olunga e matapa?*	*I:*	*What was the height of the door?*
S6:	*Mita 'e tolu.*	*S6:*	*Three metres.*

Just a guess

For two students the strategies and processes involved in making an estimate for the distance from school to Vaiola hospital could not discerned. When estimating the distance between his school and Nuku'alofa town, an estimate of six miles was given by one student. In probing for an explanation for the strategies used to obtain six miles, the student, S16, responded:

S16: *Ko e fo'i fakafuofua pe ia.*

I: *Na'e tala atu pe ako'i koe 'e ha taha ko e mama'o ko ia 'oku maile 'e ono?*

S16: *'Ikai.*

I: *Ka ko e ha e me'a 'oku ke pehe ai 'oku maile 'e ono ?*

S16: *Ko 'eku fakafuofua pe ia 'a'aku.*

S16: *Just a guess.*

I: *Were you told or taught that this distance is six miles?*

S16: *No.*

I: *But what makes you think that it is six miles ?*

S16: *It is just a guess.*

It is possible that when presented with an estimation task, these student either did not have, or could not think of an estimation strategy, such as comparing the item to be estimated to some mental images of measurement units and/or benchmarks, but rather made a guess. Another student put it "*'Ilo pe ia 'e au*"(*I just know it*). A possible explanation for this is that some students thought that to estimate means to guess.

No response

A few students could not provide estimates for some items. They did not even attempt a guess. When the researcher was unable to elicit any response and was faced with a complete silence or a long pause despite prompting, a decision was made to move on rather than prolong the silence and risk alienating these students.

It is interesting to note that the "just a guess" and "no response" strategies were used only when estimating the distance from school to the hospital. This suggests that this task presented problems to some students, perhaps because they lacked a suitable benchmark.

Discussion

This section has reported on the variety of strategies that the students used to formulate answers to tasks involving estimates of lengths. Six strategy types were identified and discussed above. The data show that it is possible to arrive at the same estimate using different procedures. Similarly, different students can use the same procedure to arrive at different estimates, indicating individual differences in their constructions and use of ideas. Given that the teachers who worked in these classrooms reported having taught estimation "only sometimes", it seems that students by and large have developed these estimation strategies on their own. It is possible that estimation is a mathematical skill that could be developed without being taught in the classroom.

Data revealed that students' experiences with measuring instruments such as a 30-centimetre or one-metre rulers aid the formulation of reasonable estimates. Multiples of these measuring

instruments were the most common strategy used in 50 instances. Parts or fractions of these measuring instruments were employed in another 19 instances. Overall, 69 of 108 responses (i.e. 64% of the estimation tasks), involved using a standard measure as the basis of the estimates. The remaining 36% used a multiple or a fraction of a benchmark. These strategy types were more prevalent when estimating long distance. Even the benchmarks used a unit of imperial or metric measurement. The dominance of the standard units of measurement is a direct result of the students' everyday life experiences in the classroom.

One interesting finding was that students did not use or mention traditional Tongan measures such as hanga (handspan) or ofa (fathom). It is possible that students perceived these as "non-school" or "non-mathematics" measures and therefore did not use these even though they were likely to be familiar with them. This was not unexpected, given that their teachers too did not mention these. As discussed in chapter five, the traditional methods of measurement were not specified by teachers as a measurement estimation strategy taught in the classroom. It would appear that in the mathematics classroom context, the standard units of measurement have become the students' dominant or 'natural' mode of thinking. The traditional Tongan methods of measurement do not appear to be in the students' thinking repertoire in the mathematics classroom.

The above findings suggest that for Tongan students, there appear to be two separate types of measures; the traditional methods of measuring referred to above as the "non-mathematics" measure and the standard units of measurement which may be referred to as the "school" measure. It is possible that these two types of measures are viewed as belonging to two different contexts and subject areas. As indicated in chapter two the traditional methods of counting and measuring are taught as part of the Tongan studies curriculum. Their relevance to mathematics education remains to be established.

6.4 Strategies for Estimating Weights

Strategies for estimating weights were categorised to highlight the variety of processes and procedures involved in formulating an estimate. In contrast with the task involving measurement of lengths, where a measuring unit and/or instrument such as a ruler was used daily by students, an equivalent instrument for measurements of weights was not readily available. In other words, whereas students could think of a 30-centimetre or a one-metre ruler as a basis for formulating an estimate, students were not able to use similar devices for estimating weights. The result was that strategies involving the use of a multiple or fraction of a standard measure were not frequently used when estimating weights. The strategies used by students fell into six main categories. These, together with the number of times they were used and the number of students who used them are summarised in Table 6E.

Table 6E: Types of strategies used for estimating weights.

Strategy type	Number of students who used strategy (n=27)	Number of times it was used (n=108)
1. Comparison to a benchmark	26	67
2. Feel for the size of the weight	12	14
3. Light or no weight	8	8
4. Just a guess	8	12
5. Visualise the scale's needle	3	3
6. No response	4	4

The types of strategies used for each of the four tasks and the number of times these were employed to formulate estimates for these tasks are summarised in Table 6F below.

Table 6F: Frequencies with which the strategy types for weights were used.

Strategy type	Weight of a pen	Weight of a tin of fish	Weight of a book	Weight of a form 3/class 6 student
1. Comparison to a benchmark	6	17	25	19
2. Feel for the size of the weight	4	9	-	1
3. Light or no weight	8	-	-	-
4. Just a guess	4	1	2	5
5. Visualise the scale's needle	3	-	-	-
6. No response	2	-	-	2

Comparison to a benchmark

As shown in Tables 6E and 6F, "comparison to a benchmark" was the most frequently used strategy type. Sixty-seven out of the total 108 possible responses revealed that a benchmark was used. Unlike the strategy types used for estimating lengths, where two processes were used (i.e. multiple *of a bench mark* and *part of a benchmark*) the benchmarks used for estimating measures of weight were considered to have the same weight as the object involved.

The following extracts from the interviews provide examples of the various benchmarks used by students.

S8: Ko e kapaika 'oku tatau ia mo e alanga'imoa 'e ua 'aia ko e kuata kilo.

S8: The tin of fish is about the same weight as two chicken legs and that is about a quarter of a kilo.

Another student, S4, used the weight of a tin of milk. As he explained:

S4	*Ko e kapaika kalami 'e nimangeau.*	*S4:*	*The tin of fish is five hundred grams.*
I:	*Na'e anga fefe ho'o ma'u 'a e kalami 'e nima ngeau?*	*I:*	*How did you obtain five hundred grams?*
S4	*Fakatatau pe ki he kapa hu'akau 'oku mililita 'e nimangeau 'oku na mamafa tatau pe.*	*S4*	*Compared to the five hundred millilitres can of milk, they have the same weight.*

Similarly, S12 used the mass that he thought he had seen on a can of drink as a benchmark to estimate the weight of the tin of fish.

S12:	*Kalamai 'e teau*	*S12:*	*One hundred grams*
I:	*Kalami 'e teau.*	*I:*	*One hundred grams.*
S12:	*Fa'a 'ai koe' e kapainu 'o inu 'oku ou fa'a sio ai he kalami 'e teau 'oku tohi he kapa peau pehe 'e au 'oku mamafa tatau mo e kapaika.*	*S12:*	*I often see one hundred grams written on the can of drink and I thought that would be the same weight as the tin of fish.*
I:	*'Aia 'oku mamafa tatau ia mo e kapa inu.*	*I:*	*So it is about the same weight as the can of drink?*
S12:	*'Io.*	*S12:*	*Yes.*

These responses revealed many benchmarks were recalled by students and used as a basis of the estimates for the weight of a tin of fish. Not all measures of the benchmarks recalled were correct, for example, 100 grams for a can of drink. Thus estimates based on incorrect measures were unreasonable. Other benchmarks used included the quarter kilograms of salt, sugar, mutton and flour. These were items that students were familiar with because shopping in the local village shops involves the customer going in, asking for the items needed and then waiting while the shop assistant weighs the items. As such, the students have many experiences observing things being weighed in the shop, how the needle of the scale moves as more weight are added and then carrying these item as they walk home. Thus, students have many experiences of buying one or two items such as a kilo or half a kilo of sugar, flour or mutton. An example of this may be illustrated using this response by a secondary student, S5:

S5:	*Falekoloa ko e fua mamafa. Kuo ou fa'a 'alu 'o fua e sipi kilo 'e taha pea ko 'ete fa'a to'o koe' 'a e kilo 'e taha 'oku ou ongo'i 'oku [mamafa] tatau pe ia mo e kapaika*	*S5:*	*In the shop there is weight measurement. I often went to purchase one kilo of mutton and when I hold that one kilo I feel that has the same [weight] as the tin of fish.*

Even though appropriate benchmarks were used by students to formulate their estimates, these did not always lead to a reasonable answer. The weight of the tin of fish was 450 grams, but the responses of the four students reported above shows that their estimates ranged from 100 grams to 1 kilogram, while the estimate of the full sample of 27 students range from 1 gram to 150 kilograms.

Once the weight of the tin of fish was obtained, this was then used as a benchmark to estimate the weight of the book. For example;

S12:	*Ko e tohi 'oku kalami 'e teau nimanoa.*	*S12:*	*The book is one hundred and fifty grams.*
I:	*Ko e ha e founga na'a ke ngaue'aki ?*	*I:*	*How did you get that ?*
S12:	*Ko 'eku fakafuofua 'oku mamafa ange e tohi he kapaika.*	*S12 :*	*My estimate is the book is heavier than the tin of fish.*

A "comparison to a benchmark" strategy was used by 25 out of the 27 students to estimate the mass (1 kilogram) of a book. The benchmark used by most students was the weight of the tin of fish that they had previously estimated. It seems that the close proximity of the weight of these two objects affected the variety of benchmarks used by students as basis for the estimation of this object.

As shown in Table 6F, nineteen students used a benchmark to derive an estimate for the weight of a student. It was evident from the data that male students, particularly those in form 3, were at an advantage in comparison to their female counterparts when estimating the weight of a form 3 student. The boys were able to provide reasonable estimates because of their weighing-in experiences and participation in the inter-college rugby competition. For example;

S12:	*Kilo 'e ononoa e ki'i tamasi'i ako.*	*S12:*	*Sixty kilo for the student.*
I:	*Ko e ha leva e founga na'a ke ngaue'aki ke ma'u ai e kilo 'e ononoa ?*	*I:*	*How did you obtain sixty kilo?*
S12:	*Fa'a fua koe' he 'akapulu oku ou kilo 'e ononoa.*	*S12:*	*When I was weighed for rugby. I was sixty kilo.*

In responding to the same question, S8 provided the following explanation:

S8:	*Kilo 'e fa nima.*	*S8:*	*Forty-five kilo.*
I:	*Na'a ke ma'u mei fe 'a e kilo 'e fa nima ?*	*I:*	*How did you get forty-five kilo?*
S8:	*Ma'u ia mei he taimi fua koe' he 'akapulu. 'Oku 'iai e taimi 'e ni'ihi 'oku nau 'osi fu'u lahi kinautolu ia ka 'oku nau kei grade one pe kinautolu ia.*	*S8:*	*Got that from having to be measured for rugby. Very often some students are older but are still in grade one.*
I:	*Ko e grade one ko e ha ia ?*	*I:*	*What is grade one?*
S8:	*Ko e pauni ma'ama'a ia, ko e fa'ahinga ia 'oku ma'u lalo hifo he kilo 'e fanima.*	*S8:*	*That is the lowest grade, and that is for the students whose weight is under forty-five kilo.*

Not all students gave their estimate in kilograms. One student, S2, used pounds as the unit of measurement for his answer.

I:	*Ko e ha nai e mamafa 'o ha tamasi'i ako foomu three?*	*I:*	*Approximately how much does a form three student weigh?*
S2:	*'E sai pe keu ngaue'aki pe 'a e pauni?*	*S2:*	*Is it all right for me to use pounds?*
I:	*Fa'iteliha pe koe.*	*I:*	*It is up to you.*
S2:	*Pauni 'e nimaua.*	*S2:*	*Fifty-two pounds.*
I:	*Te ke lava 'o 'omai ho'o tali he kilokalami?*	*I:*	*Can you give me your answer in kilograms?*
S2:	*Longo*	*S2:*	*Silence*
I:	*Ko e ha e founga na'a ke ngaue'aki 'o ma'u ai 'a e pauni 'e nimaua?*	*I:*	*What procedure did you use to get fifty-two pounds?*
S2:	*Fakafuofua atu pe ki he pauni koe 'oku fa'a 'oatu koe' he fua he 'akapulu. Tamaiki koe' he grade he "akapulu. Ki he grade tolu ko e pauni 'e nimanoa ki he nimenima.*	*S2:*	*Using the measurement for rugby. Students' weights in grade three are between fifty and fifty-five pounds.*

It should be noted that 50 to 55 pounds is an unrealistic range for the weight of a form 3 student. It is possible that this student has mixed up the imperial unit of measurement with the metric unit, thus using pounds instead of kilograms. Fifty to fifty kilograms is a more reasonable estimate for the weight of a form three student.

Although this student gave an unreasonable range (i.e. 50 to 55 pounds) as his estimate, giving answers as a range is a desirable and appropriate skill to develop and use in the context of estimation. One of the problems associated with working on this topic in mathematics, raised in chapter one, is the view that mathematic deals only with exact answers whereas estimation is a process which produces inexact answers. Giving answers as a range suggests that S2 recognised this as the appropriate type of answer for the estimation tasks, an important component of the skills involved in what Trafton (1986) described as "establishing an estimation mind-set". Recognising that estimation in mathematics does not require an exact answer would in some way help students to accept the teaching of estimation as an important and useful skill in their mathematics classroom.

Feel for the size of the weight

The student's feel for the size of a kilogram was used by a few students to determine the weight of a tin of fish. This 'feel' for the size of a kilogram may have been developed in experiences in everyday shopping. This is suggested by the type of benchmarks used. For example, one secondary student, S7, considered the weight of a tin of fish by how it felt, although there was no reference to a standard weight.

I:	*'Aia na'a ke ngaue'aki 'a e mamafa 'o e kapaika ke ma'u e mamafa 'o e tohi. Ka ko e ha e me'a na'a ke ngaue'aki ke ma'u e mamafa 'o e kapaika ?*	*I:*	*So you used the weight of the tin of fish to work out the weight of the book. What did you use to estimate the weight of the tin of fish ?*
S7:	*Anga pe 'eku ongo'i 'i he'eku to'o 'a e kapaika.*	*S7:*	*Just how it feels when you hold it.*

A similar instance occurred with a primary pupil estimating the weight of a tin of fish.

P2:	*Kalami 'e teau nimanoa*	*P2:*	*One hundred and fifty grams.*
I:	*Na'a ke ma'u mei fe kalami 'e teau nimanoa?*	*I:*	*How did you get one hundred and fifty grams ?*
P2:	*Na'a ku fakafuofua pe he anga 'eku to'o pea mo 'eku fa'ahinga ongo'i he anga 'eku to'o.*	*P2:*	*I estimated it using how heavy it felt when I hold it.*

Another secondary student considered the tin of fish was about half of his idea of a kilogram.

S6:	*Ko e kapaika 'oku haafe kilo.*	*S6:*	*The tin of fish is half a kilo.*
I:	*Ko e ha e me'a 'oku ke pehe ai 'oku haafe kilo ?*	*I:*	*What makes you say that it is half a kilo ?*
S6:	*Me'a' he 'oku mamafa ange e kilo ia ai.*	*S6:*	*Because a kilo is heavier than this.*

A primary pupil, on the other hand, considered that the same tin of fish was nearer to his idea of a kilogram, and that the book was much more.

I:	*Na'a ke pehe 'anenai 'oku kalami 'e taha e kapaika.*	*I:*	*You said earlier that the tin of fish is one gram.*
P4:	*Ko 'eku 'uhinga 'oku kilokalami 'e taha.*	*P4:*	*I mean one kilogram.*
I:	*Ko e ha leva e me'a na'a ke ngaue'aki ke vakai'i e mamafa 'o e ongo me'a ko 'eni, 'a e kapaika mo e tohi ?*	*I:*	*So what did you use to work out the weight of these two objects, the tin of fish and the book?*
P4:	*'Aki pe 'eku to'oto'o mo 'eku ongo'i. 'Oku ki'i mamafa ange e tohi he kapaika.*	*P4:*	*My own feeling when I hold it. The book feels heavier than the tin of fish .*

In contrast to the length estimates, in which a mental image of the unit of measurement was used, the weight estimates relied on the students' 'feel for' the size of the weight. Both the mental image and the feel for the size of the objects are forms of benchmark that appears to depend on many experiences with these objects. Whereas the 'mental image' skill depends on visual imagery together with comparison about relative sizes of the objects, the 'feel for the size' seems to involve what might be called tactile imagery.

Visualise the scale's needle

A visual image strategy used by three secondary students was to imagine how far the needle on a scale would move if the item involved was weighed on shop scales.

S11: Ko e ki'i peni 'oku kalami 'e ua. Kapau te u fua e ki'i peni he me'afua he'ikai ke fu'u loko ngaue e hui ia.	*S11: The little pen would be two grams. If I weigh this little pen the needle of the scale would hardly move.*

This estimate was probably also based on the student's feel for the pen.

Light or no weight

An interesting finding to have come out of this study is students' perception of the weight of a pen. Eight students thought that a pen did not have any weight because it was light. S6 and S7 gave very similar responses regarding the weight of a pen.

S6: Ko e peni 'oku 'ikai ke 'iai hano mamafa ia 'o 'ona.	*S6: The pen does not have any weight*
I: Ko e ha e me'a 'oku ke pehe ai 'oku 'ikai ke 'iai hano mamafa?	*I: What makes you say that it does not have any weight ?*
S6: 'Oku 'ikai ke 'iai hano mamafa ia he 'oku ma'ama'a pe ia.	*S6: It does not have any weight because it is light.*

P4 gave the same explanation for giving half a gram as an estimate for the weight of a pen. *"The little pen is light".*

A possible confusion could have arisen with this item because the term 'mamafa' is the Tongan term for 'heavy' as well as 'weight'. Therefore, for this primary student the question "Ko e ha nai e mamafa 'o e peni ?" (*What is the weight of this pen?)* could be interpreted as "Is this pen heavy?" Because this pen is not mamafa (heavy), it is therefore ma'ama'a (light). Ma'ama'a (light) can be thought of as the opposite of mamafa (heavy) thus a few students could have thought that 'ikai ke mamafa (not heavy) is the same thing as ma'ama'a (light). Thus the pen does not have any mamafa (weight) because it is ma'ama'a (light). Another term 'pauni' (pound) was used by one student instead of mamafa (weight). Again, because the pen did not have any pauni (pound), it was ma'ama'a (light) and therefore had no weight.

Just a guess

Four primary students and one secondary provided the responses which are classified in this category. Once again an explanation of the procedure used was not given; rather the students indicated that the answer provided was a 'fakafuofua' or a guess.

I: 'E lava ke 'omai ha'o fakafuofua ki he mamafa 'o ha tokotaha ako foomu tolu?	*I: Can you give an estimate for the weight of a form three student ?*
S7: Kilokalami 'e ua nima.	*S7: Twenty-five kilograms.*
I: Ko e ha e founga na'a ke ngaue'aki 'o ma'u ai e ua nima.	*I: What strategy did you use to come up with twenty-five kilograms ?*
S7 : Ko 'eku fo'i fakafuofua pe ia 'a'aku.	*S7: Just a guess.*

A very similar explanation was given by a primary student.

I:	*Fefe nai ha'o fakafuofua ki he mamafa 'o e ki'i tamasi'i pe ta'ahine ako kalasi 6.*	*I:*	*How about an estimate for the weight of a class 6 student ?*
P4:	*Mahalo pe 'oku kilokalami 'e tahavalu ki he uanoa.*	*P4:*	*Maybe it is eighteen to twenty kilograms.*
I:	*Sai ko e ha leva e founga na'a ke ngaue'aki ki ho'o fakafuofua ?*	*I:*	*Good. How did you come up with that estimate ?*
P4:	*Ko 'eku fakafuofua'i pe ia 'e au.*	*P4:*	*I just estimated that*

The interview data showed that these students have not been weighed before. Unlike most New Zealand children who have bathroom scales in their homes, these Tongan students do not have such scales in their homes. Scales in shops provide the context in everyday life where students experience objects being weighed. Unlike the sample of secondary school boys who had been weighed at school for rugby, the primary school students had not had the same sporting experiences.

The data suggest that for a few secondary students a kilogram is the smallest unit of measurement used for estimating, even when the tasks involved a small object such as a pen. Students' ideas of a kilogram were used to formulate their estimate. For instance,

S16:	*Ko e peni ko e haafe ia 'o e kuata kilo (1/8 kilokalami).*	*S16:*	*The pen is half of a quarter of a kilo (1/8 kilogram).*

Perhaps for these students kilogram rather than a gram is the smallest unit of weight measurement in their experiences.

Discussion

The results reported in this section showed that the 'multiple or fraction of a benchmark' was not frequently utilised for estimating measures of weights. This contrasts with those strategies used for estimating lengths. The data suggested that students tended to look for or think of benchmarks that were similar in size to the weight of the stimulus object. Thus, rather than working out a multiple or a fraction of the benchmark the students made a direct comparison of the objects. This finding suggests that the mental processes involved in estimating weights differ from those used for estimating lengths. The lack of experiences with equivalent standard measuring devices for weights might also explain the apparent differences in strategy types revealed by students.

'Comparison to a benchmark' involves looking for benchmarks of equivalent or similar sizes. This appears to be an easier process than trying to figure out a multiple or a fraction of a benchmark. The reasonableness of the estimates would depend on the students' perceptions

and/or judgement regarding the weights of the objects involved. As mentioned earlier the close proximity of the stimulus objects in part also accounted for the frequency with which 'comparison to a benchmark' was used. It is also possible that working out a multiple or a fraction of linear measures could be easier than working out the multiple or fractional relationships of other quantities.

'Comparison to a benchmark' is also a natural process for estimating answers to problems. As part of everyday life problems, one appropriate strategy is to think of objects which are similar in sizes to those involved in the problems at hand. Making judgements about relative weights of objects may involve one of three decisions: that it is bigger, smaller or about the same size. Thus, it appears that students have tended to use an everyday life problem solving strategy for making their estimates. Students' everyday life experiences have provided the main source of ideas for estimating weight measures. These experiences may differ from student to student and from males to females. Unlike the form 3 girls, the boys were more successful in providing reasonable estimates for the weight of a form 3 student. The data also demonstrated that ideas developed from out-of-school mathematics have assisted students in formulating their estimates.

Students' estimates for the weight of small objects such as a pen highlighted one of the difficulties associated with the language used as well as the lack of familiarity with small units of measurement such as a gram. The term 'mamafa' refers to 'weight' as well as 'heavy'. It is therefore very important that the question requiring estimates of weights are stated very clearly to avoid misunderstandings. Lack of everyday life experiences with scales for smaller and larger weights also partly explains the unreasonableness of the estimates for the weights of a pen and a class 6 or form 3 student. This may also account for the non responses by four students.

It is important to note that students who could not think of an answer did not make any attempt to make guess. However, it was not clear whether this was due to students' lack of confidence in making reasonable guesses or a belief that making guesses was an inappropriate response. As discussed in the review of the literature, 'tolerance for errors' is one of the affective component skills involved in computational estimation (Reys et al. 1982; Reys, Reys and Penafiel, 1991; Reys, Reys, Nohda, Yoshikawa, Shimizu, 1991; Sowder and Wheeler, 1989) and for estimation in general. Therefore it appears that the students who failed to provide an estimate may benefit from a teaching programme which stresses the need for tolerance for error. As Trafton (1986) suggested, establishing an estimation mind-set would involves accepting the idea that estimation almost always involve some errors. Thus, tolerance for error should be part of any teaching programme on estimation.

6.5 Strategies for Estimating Capacities

Four main strategies for estimating capacity were identified in the students' responses. These, together with non-responses, the frequency with which each strategy type occurred, and the number of students who used them are presented in Table 6G.

Table 6G: Strategy types for estimating measures of capacity

Strategy type	Number of students who used it	Number of times it was used
1. Comparison to a benchmark	19	38
2. Just a guess	19	35
3. Measuring instrument used in class	10	22
4. Amount of money spent to fill it up	8	8
5. No response	5	5

The types of strategies employed by students to estimate the capacities of the four objects included in the interview tasks are presented in Table 6H below.

Table 6H: Strategy type for capacity and the number of times it was used.

Strategy type	Teaspoon	Green coconut	Jug	Petrol tank
1. Comparison to a benchmark	3	9	16	10
2. Just a guess	15	9	4	7
3. Measuring instrument used in class	8	8	6	-
4. Amount of money spent to fill it up	-	-	-	8
5. No response	1	1	1	2

Comparison to a benchmark

The most commonly used procedure for estimating capacities, used on 38 occasions, was comparison of the item involved to a benchmark. All benchmarks included in this category were objects that students had used outside school.

This strategy type was most often used when estimating the capacities of the jug, petrol tank of a van and the green coconut. Various benchmarks were used, including a kerosene bottle, a can of milk, a cup of water, and a jug used for milk delivery. Responses by a secondary student, S4, illustrate the use of this strategy type.

S4 : Ko e sioki 'oku lita 'e ua. Ko e fa'a 'alu ange koe´ veeni hu'akau 'a Liahona 'oku fa'a 'ai e lita 'e taha ki he sioki 'e taha, ka 'oku fa'a 'ai 'emau sioki 'e ua. 'E tatau pe ia [sioki lita taha 'e ua] mo e sioki ko 'eni.	*S 4: The jug is two litres. When Liahona's milk van delivers milk they use a one litre jug, but we normally buy two jugs of milk. Those [two one - litre jugs] would be the same as this jug.*
S4: Ko e sepuniti ko e vaeua ia 'o ha mililita. Fakatatau ia ki ha vaeua 'oha ipu, 'aia ko e mililita ia 'e ua.	*S4: The teaspoon is about half of a mililitre. Compare that to half of a cup which is two mililitres.*
I : 'Aia 'oku fonu e ipu he mililita 'e fiha?	*I : So how many mililitres would fill the cup?*
S4: Mililita 'e fa.	*S4: Four mililitres.*
I : Fefe nai e sioki vai ?	*I : How about the jug of water?*
S4: 'Oku lita ia 'e taha. 'Oku lahi tatau pe mo e ngaahi puha hu'akau 'aia 'oku lita 'e taha....Fo'iniu mata 'oku mililita ia 'e tahaua he'e fonu tu'o tolu ai ha ipu mililita 'e fa.	*S4: That would be one litre. It is about the same size as the one-litre carton of milk...The green coconut is twelve mililitre because a four-mililitre cup can be filled up three times.*

Even though a benchmark, in this case the capacity of half a cup was used, it is clear that this (i.e. 2 millilitres) is an unrealistic measure. In turn this misconception of the capacity of a cup was also used to provide an unreasonable estimate for the capacity of a green coconut. Although the data shows that an appropriate strategy (i.e. multiple of a benchmark) was used this did not guarantee a reasonable estimate. This example highlights an important factor regarding the use of a benchmark when formulating an estimate—reasonably accurate information about this known measure needs to be stored in memory. Unreasonable and inaccurate information about a benchmark will inevitably be unhelpful to the estimation process even when an appropriate approach is being used.

To illustrate;

S2: 'Oku meimei kalami 'e nimanoa e fo'i niumata, 'aia ko e mililita 'e nimanoa.	*S2: The coconut is about fifty grams and therefore it is fifty millilitres.*
I : Ko e ha 'oku ke pehe ai 'oku mililita 'e nimanoa?	*I : What makes you think that it is about fifty millilitres*
S2: Ko e taimi koe´ ''oku lingi ai e fo'i niumata ki he hina kalasini lita 'e taha 'e 'ikai ke fonu ia 'e mililita pe ia 'e nimanoa.	*S2: If you pour this coconut into a one-litre kerosene bottle that bottle will not be full , it will only fill about fifty millilitres.*

In this case the student seems to think that 100 millilitres is 1 litre.

Just an estimate or guesses

The second most commonly used strategy was for students to make a guess. This strategy was used on 35 occasions by the 19 students. As shown in Table 6G, 15 of the 27 students employed this strategy when formulating an estimate for the capacity of the teaspoon,

suggesting perhaps that a benchmark for small capacities could not be readily recalled, or that students were not familiar with capacities of small objects.

"Just a guess" or "just an estimate" included all responses where students did not specify any benchmark or made refer to any measuring instrument. Whereas most students just looked at the object and then provided an estimate with the explanation *"na'a ku fakafuofua'i pe" (I estimated that)*, two students consistently made guesses saying that it was the *"ko e 'uluaki me'a ia na'e ha'u ki hoku 'atamai" (that was the first thing that came to mind).*

For some students the units of measurement used for capacities were either not well understood or forgotten. One student's (S6) response indicates this.

S6:	*Ko e voliume*	*S6:*	*The volume*
I:	*Feinga mu'a ke fakamatala'i mai pe ko e ha e lahi e vai 'e fonu ai e ngaahi me'a ko 'eni. Ko e sepuniti, sioki vai, tangike 'o ha veeni mo e fo'iniu mata.*	*I:*	*Can you explain how much water can fill the following objects: teaspoon, jug of water, tank of a van and a green coconut.*
S6:	*'Oku 'ikai keu 'ilo 'e au ia e 'iuniti 'oku 'ai ke ngaue'aki?*	*S6:*	*I do not know the unit to use?*
I:	*'Ikai teke 'ilo 'e koe e 'iuniti ke ngaue'aki. 'Oku ke 'ilo 'e lita, pe ko e mililita.*	*I:*	*Do not know the unit. Do you know about litre or millilitre?*
S6:	*'Io. Ko e fo'i niumata 'oku lita 'e taha.*	*S6:*	*Yes. The green coconut is one litre.*
I:	*Lita 'e taha. Ko e ha e founga na'a ke ngaue'aki ke ma'u ai e lita 'e taha?*	*I:*	*One litre. What procedure did you use to obtain one litre?*
S6:	*'Uluaki me'a pe ia na'e ha'u ki hoku 'atamai.*	*S6:*	*That was the first thing that came to my mind.*
I:	*'Io*	*I:*	*Yes*
S6:	*Koe tangike 'oku 'ai ha veeni ko e lita ia 'e teau Ko e sepuniti 'oku 'ikai keu 'ilo ia 'e au pea ko e sioki vai 'oku lita 'e tolu.*	*S6:*	*The petrol tank of a van is 100 litres. I do not know the capacity of the teaspoon and the jug of water is three litres.*

It is evident from the above responses that S6 was not sure about the capacity of a small object such as a teaspoon. He also overestimated the capacity of a large container such as the petrol tank of a van which suggested that it was just a guess. Given that S6 initially responded that he did *"not know the unit to use"*, indicating perhaps a lack of familiarity and experiences working with capacity, it is not surprising that he resorted to giving the first number that came to his mind and guessing at answers. The lack of understanding by this form 3 student of the basic unit for capacity suggests that one cannot assume that by form 3, students would have developed appropriate understanding and ideas pertaining to basic unit of measurement. The

data suggests that understanding of the units of measurement ought to have been developed by students before these units could be used as benchmarks for estimation.

An important question to have come out of the above response is: at what point does a student who did not know decide to make a guess and/or provide no response? It is possible that the student felt obliged to provide an answer even though he did not know. On the other hand, a guess was made since it is only an estimation exercise. Further probing could have identified some answers for the above question.

Measuring instrument used in class

Ten students (8 primary and 2 secondary) thought of a measuring instrument they had used in their classrooms to estimate the capacities involved. As shown in Table 6G this strategy was used in 22 instances, 15 of which belong to six students in one primary school. Only two students in the other primary school involved in this study used this strategy on four occasions. As well, only two out of 15 secondary school students used this strategy. An explanation given by one primary student, P3 is one example of the responses grouped under this category.

P3:	*Ko e fo'i niu 'oku 'iai 'a e ki'i sioki lita 'e taha pea ka 'utu ia ki he fo'i niumata, 'e fonu ai pe fo'i niumata ia.*	*P3;*	*For the green coconut, there is this one litre jug and if that is poured into the coconut it will be full.*
I:	*Ko e ki'i sioki lita 'e taha ko 'eni 'oku ke fakamatala ki ai na'a ke ngaue'aki ia 'i fe?*	*I:*	*This one litre jug that you are talking about, where have you used it?*
P3:	*'I lokiako.*	*P3:*	*In the classroom.*

Students in the two primary schools had access to various measuring jugs (250 and 500 millilitres, and one and two-litres) and have had experiences making estimates of capacities and measuring capacities of various containers using these in their classrooms. Analysis of the data, confirmed by classroom observations, revealed differences in the classroom experiences of students in the two primary schools (i.e. Groups IV and V) involved in the investigation, a reflection of the different emphasis given to teaching measurement of capacities in the two classrooms. Whereas the researcher observed classroom activities in Group V involving the above measuring instruments, she did not observe these being used in Group IV. However, students in this latter group reported having used these with their teacher in the classroom. There was a tendency for students in Group V to used these measuring instruments as benchmarks for formulating their estimates.

Amount of money spent to fill up the petrol tank.

Eight students gave an estimate based on the amount of money that they have seen used to fill up the tank of a van.

S4:	*Ko e tangike 'oku lita 'e uanoa.*	*S4:*	*The petrol tank is twenty litres.*
I:	*Lita 'e ua?*	*I:*	*Twenty litres.*
S4:	*Ko e fa'a 'utu koe' e me'alele 'oku fa'a fonu he pa'anga 'e uanoa. 'Aia ko e lita ia 'e uanoa.*	*S4:*	*Often when a vehicle is filled up it is often full for about twenty dollars. That is twenty litres.*

A further instance is the reply of another secondary pupil.

S12:	*Ko e tangike 'o e veeni 'oku ua nima.*	*S12:*	*The petrol tank of a van is twenty -five.*
I:	*Uanima. Ko e uanima ha ia?*	*I:*	*Twenty-five. What is this twenty-five ?*
S12:	*Uanima lita (Longo) koe 'uhi ko e taimi koe' 'oku fa'a 'utu fonu ai 'emau veeni 'oku fa'a pa'anga 'e uanima.*	*S12:*	*Twenty-five litres. (Pause). Because very often twenty-five dollars would fill up our van.*

Although it was not possible to identify from the data whether students have had experiences measuring or estimating measures of larger capacities, it seems that recalling the amount of money spent to fill up a petrol tank of the van was the only experience which eight students could draw upon when making their estimates.

No response

Again the "no response" category consisted of students who failed to provide an estimate and whose reponse was a complete silence. Five students provided non-responses, two for the petrol tank of a van and one each for the teaspoon, green coconut, and jug of water. Students' lack of ideas of capacities in part accounted for their non-responses. Again, students did not attempt to make any guesses.

Discussion

The data reported in this section showed that students were just as likely to use 'comparison to a benchmark' as to make 'guesses'. The 19 students who used a 'comparison to a benchmark' strategy recalled different objects from their everyday life experiences as the basis of their answers. This demonstrates one of the strengths of estimation tasks as a mathematical topic. That is, estimation allows students to generate ideas based on their different life experiences as tools to solve problems in mathematics. That as many students made guesses could be viewed as a both a strength and a weakness. It could be considered a strength if students made guesses because they were confident enough to give it a try. On the other hand, this could be considered a weakness as it can also illustrate that students had not had many experiences measuring and/or estimating measures of capacity.

Although students used acceptable strategies for estimation, the results were not always reasonable. The results suggested that students in different classes have had different experiences estimating capacities in the classroom. In comparison to secondary students, primary students revealed more school taught ideas in the strategies that they had used. This could be a reflection of the difference in content of the mathematics curriculum in the two classes used for the present study. Whereas measurement is a main component of the class 6 mathematics syllabus, it does not have a prominent place in that used in form 3. It should be noted that the extent to which the form 3 students have been exposed to estimation in their primary schooling is unknown. The data seems to suggest they had not had much experience working with capacities in their classroom. Had these form 3 students been taught estimation involving capacities at class 6, the data would suggest that by form 3 the students had forgotten what they learnt in class 6.

6.6 Students' Ideas

This section considers how students constructed their strategies as well as issues relevant to assessing the reasonableness of an estimate.

How have students constructed their ideas?

Both school and out of school ideas were used by students when estimating. It is apparent from the interview data that for short lengths such as the length of a pen, school ideas dominated the types of strategies used. The 30-centimetre and one-metre rulers are school apparatus most frequently used by students when estimating lengths. On the other hand, for a long distance, such as the distance from school to Vaiola hospital, ideas from out of school experiences were used. The distance from the village of Malapo to Toloa, Nuku'alofa town to Mailetaha (One Mile), and distances on sign posts were the three benchmarks most frequently used.

Out-of-school experiences were the main source of ideas for estimating the weight of objects. A kilogram of sipi or mutton, sugar and salt, and can of drink or can of milk, were items that students often purchase from shops and from which their "feel for" the size of a kilogram developed. The only school experience that appears to provide some students, form 3 male students in particular, with an opportunity to formulate relevant ideas of weight, was weight measurement for rugby.

For estimating capacity, both school an out of school experiences provided students with benchmarks with which to base their estimates. For larger capacities such as the petrol tank of a van, prior experiences in out of school situations appeared to be the only context in which students developed ideas about the capacity of this object. All of these seems to depend on where the students had their prior experiences.

Assessing reasonableness of an estimate

Assessing students' estimation skills requires one to take into account both the strategies used to make the estimate as well as the estimate itself. Analysis of the interview data showed that it was possible to use appropriate estimation strategies but yet come up with an unreasonable estimate.

S12:	*Ko e mama'o koe´ mei heni ki falemahaki mahalo 'oku kilomita pe 'e tolunoa.*	*S12:*	*The distance from here to the hospital would probably be thirty kilometres.*
I:	*Na'e anga fefe ho'o ma'u e kilomita 'e tolunoa?*	*I:*	*How did you get thirty kilometres?*
S12:	*Sio pe ki he papa koe´ 'oku 'asi he hala. Ko e papa koe´ 'oku fa'a tu'u 'i Te'ekiu ko e ha'u koe´ mei, 'oku meimei kilomita 'e fanoa mei Te'ekiu ki Ha'atafu peau ngaue'aki pe ia ke fakafuofua e mama'o mei heni ki falemahaki.*	*S12:*	*Just look at the sign on the road. There is a sign at Te'ekiu which says that it is forty kilometres from Te'ekiu to Ha'atafu and I just used that to estimate the distance to the hospital.*
I:	*'Ai ange ha'o fakafuofua ki he loloa 'o e kilomita 'e taha, mei heni nai ki fe?*	*I:*	*Can you give an estimate of one kilometre, say from here to where?*
S12:	*Falahi e lokiako ko e kilomita ia 'e taha.*	*S12:*	*The width of this classroom is one kilometre.*

In assessing the reasonableness of an estimate, one needs to set the criteria for a reasonable or an acceptable response. It seems that in assessing the reasonableness of an estimate one cannot avoid having to assess other concepts used by students. These include students' concepts of basic units of measurement; association of these concepts with familiar object; flexibility in their use of a variety of estimation strategies; accuracy of information about benchmarks; and making decisions on whether to accept the estimates as being reasonable.

6.7 Class Differences

As indicated in chapter four, the same tasks were used with both primary and secondary students. The tasks were considered appropriate for both classes of students. Strategy types used by class 6 (10-12 year olds) and form 3 (13-16 year olds) students are presented in Table 6I. The responses from 12 primary and 15 secondary school students who participated in the interviews are compared and discussed in this section.

Class differences in estimating lengths

Table 6I below summarises the number of times and percentages of each of the strategy types used by primary and secondary students for estimating length measures.

Table 6I: Class comparison of the strategy types for estimating lengths

Strategy type	Number of times strategy was used by primary students $N^*=48$	Number times strategy was used by secondary students $N^*=60$
1. Multiple of a measuring instrument	19 (40%)	31 (52%)
2. Multiple of a benchmark	14 (29%)	18 (30%)
3. Fraction of measuring instrument	10 (21%)	9 (15%)
4. Fraction of a benchmark	2 (4%)	2 (3%)
5. Just a guess	2 (4%)	-
6. No response	1 (2%)	-

N^* refers to the total number of possible responses.

Due to the difference sample size for the two groups (i.e. 12 for primary and 15 for secondary) the percentages rather than frequencies are used in the analysis and comparison of the data. Table 6I shows that for 52% and 40% of the total number of responses by secondary and primary students respectively, a "multiple of a measuring instrument" strategy was employed. Combining the percentages for the "multiple of a measuring instrument" with that for the "fraction of a measuring instrument" strategy types shows that 61% and 67% of the primary and secondary students respectively used a measuring instrument. Thus, standard measuring instruments such as a 30-centimetre and one-metre rulers formed the basis for more than 60% of the length estimates. Moreover, students tended to work out multiples of, rather than fractions of, these standard measures.

The data showed no differences between the frequencies with which "multiple of benchmark" and "fraction of a benchmark" were used by the two groups. This is indicated by a combined total of 33% for these two strategy types, for each group. This means that the primary students were just as likely as the secondary students to use a benchmark to formulate their estimates. Only a tiny minority 6% of primary students' responses were either a guess or non-responses while none of the secondary students' responses were classified under these categories.

Class Differences in estimating weights

As with the lengths estimates, the percentages of the respective strategy types for estimating weights were calculated and analysed to allow for comparison of the types of strategy used by the two groups of students. Table 6J summarises both the number of times and percentages of particular strategy type for estimating weights. Again the data were obtained from the same sample of primary ($n=12$) and secondary ($n=15$) students. Given that 4 items were used, there are 48 and 60 possible numbers of responses from the primary and the secondary samples respectively.

Table 6J: Number and percentage of strategy types for estimating weights.

Strategy type	Number of times used by primary students $N^*=48$	Number of times used by secondary students $N^*=60$
1. Comparison to a benchmark	29 (60%)	39 (65%)
2. Feel for the size of the weight	4 (8%)	10 (17%)
3. Little or no weight	3 (6%)	5 (8%)
4. Just a guess	8 (17%)	3 (5%)
5. Visualise the needle of the scale	-	3 (5%)
6. No response	4 (8%)	-

N^* refers to the total number of possible responses.

Both group of students tended to use a benchmark. This is demonstrated by 60% and 65% of responses by primary and secondary students respectively. Slightly more responses given by secondary students used the "little or no weight" strategy type while primary students made more guesses. "Feel for the size of the weight" of the object involved appeared in 17% of the secondary students' responses compared to 8% of the primary students'. This indicates that more secondary students used their "feel for the size" as a basis of their estimates. In addition, 5% of the secondary responses involved visualising the needle on the scale to make their estimates. Furthermore, primary students made more guesses, 17% compared to 5% of secondary students as well as had more non-responses. It could be that because secondary students were older and perhaps have had more life experiences with a variety of objects, these provided them with opportunities to develop "feel for the size" of these weights.

Class differences in estimating capacities

As reported earlier in this chapter five strategies for estimating capacity were revealed by students. The respective number of primary and secondary students' responses categorised under each of the strategy type for estimating capacity are summarised in Table 6K below.

Table 6K: Number and percentage of strategy types for estimating capacities.

Strategy type	Number of times used by primary students ($N^*=48$)	Number of times strategy was used by secondary students ($N^*=60$)
1. Comparison to a benchmark	10 (21%)	28 (47%)
2. Just a guess	17 (33%)	19 (32%)
3. Measuring instrument used in class	19 (40%)	3 (5%)
4. Amount of money spent to fill up the petrol tank	2 (4%)	6 (10%)
5. No response	1 (2%)	4 (7%)

N^* - refers to the total number of responses.

About the same number of guesses were made by both groups of students. There was a tendency for secondary students to use benchmarks from everyday life experiences, 47% compared to 21% by primary students. On the other hand, more responses by primary students made use of measuring instruments as a basis of estimation, 40% compared to only 5% of secondary responses. This result suggests that there was more emphasis in the use of units of measurement in class 6 mathematics compared to form 3 mathematics. In addition, secondary students seems to have had more experiences from which they could recall benchmarks for their estimates.

6.8 Summary and Comment

This chapter reported and discussed the results of individual interviews with a sample of 27 (15 form 3 and 12 class 6) students. Tasks requiring students to estimate measures of length, weight and capacity provided the vehicle by which students exhibited their strategies and ideas for measurement estimation. This section summaries the main points of discussion for this chapter by first summarising the students' definitions of **fakafuofua** (estimation). It then outlines the strategy types used; and the areas of strengths and weaknesses identified from the data.

Various definitions of estimation

The interview data revealed a variety of terms and phrases for fakafuofua (estimation). These included: fakaofiofi *(closer or nearer to)*; fakamahamahalo (*conjecture*); mate'i (*guess*); fakakaukau'i pe 'i loto *(mentally figure out something)*; fua pe 'i loto (*mentally work out the measure*); fakakaukau (*to think*); and sio ke ma'u e anga 'o e me'a ko ia (*look at an object and work out its characteristics*).

All of these definitions, except for *"fua pe 'i loto"* (mentally work out the measure) which may be limited to measurement estimation, could be applied to other domains of mathematics. In some situations, the term fakafuofua (estimate) would be the most appropriate term to use, although it can mean much more than estimation or estimate.

Estimation strategies used by students

After discussing the definitions for 'fakafuofua', the chapter then answered the first question outlined in section 6.1. That is,

1. What estimation strategies are used by Tongan students?

Five strategy types used for estimating lengths were identified. These were: (i) multiple of a measuring instrument; (ii) multiple of a benchmark; (iii) fraction of a measuring instrument; (iv) fraction of a benchmark; and (v) guess. The use of a standard measure or a measuring instrument dominated the type of strategies employed by students for estimating lengths. Data showed that 69 out of the total 108 responses, that is about 64%, involved using a standard measure. Moreover, for the benchmarks used for the remaining 36%, standard units for

length measurement were utilised. There were no references to the traditional Tongan methods of measuring suggesting perhaps that these might be considered "non-school" or "non-mathematical" measures and therefore inappropriate to be used to solve tasks in mathematics.

Five strategies were used for estimating measures of weight. These included: (i) comparison to a benchmark; (ii) feel for the size of the weight; (iii) light or no weight; (iv) guess, and (v) visualise the behaviour of the scale's needle. The use of a benchmark dominated the type of strategies employed by students. Analysis of the data revealed that 67 out of 108 responses (ie. 62%) involved using a benchmark as a basis of estimation.

Four strategy types were used for estimating capacities. These were: (i) comparison to a benchmark; (ii) measuring instrument used in class; (iii) guess; and (iv) amount of money spent to fill up a container. Data showed that comparison to a benchmark and making guesses were the most frequently used strategy types. Thirty-eight and 35 out of 108 responses (35% and 32%) involved comparison to a benchmark and making guesses, respectively.

The results showed that out of school experiences provided students with ideas and concepts used for estimating measures of weights and capacity. That students could develop strategies for estimation on their own with very little teaching is one of the main strengths of the results. That students bring to the classroom many ideas from out-of-school experiences, these could be used to help students to further develop the skills involved in estimation.

Areas of strengths

In this context areas of strengths included items in which a large number of students used an estimation strategy to provide a reasonable estimate.

Data reveals that estimating lengths was one area of strength. This was suggested by the higher number of strategy types as opposed to estimating measures of weight and capacity. Students' ideas of the length of a centimetre, and 30-centimetre and one-metre rulers, were used when estimating length measures. Clearly, these were ideas and concepts developed from classroom experiences. In contrast, the weight measure estimates used the weight of objects in their everyday life experiences. Here, ideas developed outside their school experiences dominated the types of strategies and benchmarks used to formulate estimates.

Students' alternative ideas and areas of difficulties

Analysis of the interview data suggests areas of difficulties caused by students' alternative ideas or misconception. Considering all three measures of lengths, mass and capacity, data suggest that students' understanding of the units of measurement, particularly for mass and capacity hindered their ability to make reasonable estimates using standard measures. The terms 'kilo' (kilo) and 'kalami' (gram) were sometimes viewed as the same thing. Confusion

of these terms together with lack of familiarity with them and other units of measurement led to unreasonable estimates.

For longer distance, both 'kilometre' and 'mile' were used. The use of 'mile' suggests that the everyday experiences of some students and more specifically the unit of measurement used by the parents has influenced the unit used by the students. Moreover, the incorrect ideas about the measure of a benchmark affected the size of their estimate. Thus, knowledge of a 'benchmark' does not always lead to a reasonable estimate. Long distances provided some difficulties for students. Most distances were given in miles and not kilometres, indicating the influence of parent's ideas of the unit of measurement. Despite the use of the metric system since about 1969, use of the imperial units for lengths is still prevalent today in Tonga.

From the researcher's perspective, another area of difficulty was that some students could not explain the strategies used to arrive at their estimate. Perhaps they arrived at their estimates intuitively, or used the procedures so rapidly that they were not conscious of them. The challenge for researchers is to find ways of unravelling these unexplained procedures. On the other hand, considering that one of the definitions for "fakafuofua" (estimation) given by students was "fakakaukau'i pe 'i loto" (mentally work out an answer) or "fakakaukau" (to think), it could be that students thought about the tasks but saw no need to itemise the processes used other than to say *"Na'a ku fakakaukau'i pe ia 'e au"* (I thought of it).

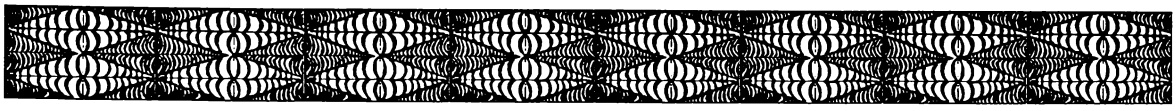
The data suggest that in designing instructional activities on estimation the following points need to be taken into account.

- students' and teacher's definition(s) for the term **fakafuofua**;
- students' prior experiences;
- links between classroom and everyday ideas; and
- the need for traditional as well as metric units of measurement.

This chapter has focused on students' strategies and existing ideas for estimating measures of length, weight and capacity, prior to participating in an instructional unit on measurement estimation. The findings from these interviews point to the need to provide students with a variety of experiences in order to develop the use and understanding of the estimation process. Even though students appeared to have developed estimation strategies without them being a major focus of their classroom experiences, the review of the literature suggested the need for estimation to be taught rather than leave it for students to pick up on their own. The next chapter reports and discusses the data on the instructional units on measurement estimation and their effects on students' estimation skills, ideas and strategies.

CHAPTER SEVEN

MEASUREMENT ESTIMATION TEACHING ACTIVITIES : RESULTS AND DISCUSSION



7.1 Introduction

As outlined in chapter four, one of the aims of this study was to explore teaching activities on measurement estimation. The thirty students who were individually interviewed were again used for this phase of the study. As discussed in chapter one, one of the purposes of the present study was to explore the question:

- How can students' existing ideas on estimation be utilised to develop teaching strategies which will further develop these skills?

The strategies and ideas revealed by students in the interview phase were used as a basis for the design of an instructional programme. The review of the literature and the implications of the constructivist theory of learning also guided the teaching strategies employed.

This chapter reports on the trial of these estimation activities; the results of these are presented and discussed. The next section (7.2) describes the nature of the five groups or classes who participated in the trial of the teaching activities. The measurement estimation activities are then outlined and discussed in section 7.3. Three case studies are then used to demonstrate the effectiveness of the teaching activities (7.4). A summary of and comments on the main points are provided in section 7.5.

7.2 Participating Classes

The groups of six students from each of the five schools used for this phase of the investigation were described in chapter four. The researcher had anticipated that teachers in these classrooms would trial the activities designed. However, due to practical difficulties, such as lack of time needed for the researcher to explain and discuss with teachers the objectives of the activities, preferred mode of presentation, and its rationale, a decision was made for the researcher to teach these units. The five mini-classes, each consisting of six students, worked with the researcher who acted as the teacher in small group activities at regular times during the twelve week programme. A summary of the classes involved in this phase of the investigation is provided in Table 7A below.

Table 7A: Summary of the mini-classes who participated in the activities

MICRO TEACHING GROUP	SCHOOL	GENDER COMPOSITION OF MICROTEACHIG GROUP
I	Secondary (single sex)	6 Male
II	Secondary (co-educational)	3 Male 3 Female
III	Secondary (single sex)	6 Female
IV	Primary (co-educational)	3 Male 3 Female
V	Primary (co-educational)	3 Male 3 Female

7.3 The Measurement Activities

The activities used (see Appendix C) were organised under three headings, (i) estimating lengths, (ii) estimating weights, and (iii) estimating capacities. The following concepts were included under each heading where relevant.

- traditional methods of measurement;
- understanding basic units of measurements, such as a centimetre, gram or millilitre;
- comparison and making judgements based on sizes as important estimation skills;
- use of benchmark as a basis of estimation;
- solving estimation problems;
- concept of reasonable range;
- deciding and agreeing on the criteria for reasonable range of estimates.

Small group and whole group discussion were the predominant method of interaction, accompanied by demonstrations. Ideas and estimation strategies were written on the blackboard (where appropriate) and worksheets for students. Students were encouraged to discuss and reflect upon the answers and strategies they had used.

Activity One: What is estimation?

The purpose of the first activity was to explore the meaning of 'fakafuofua' (estimation) in the domain of measurement held by students. This was an important part of the lesson in which a shared meaning of the characteristics of measurement estimation was established by the students and the teacher/researcher.

Students' responses to the question *"what is the meaning of the term estimation?"* were recorded on the blackboard and a discussion about these followed. The students' responses included.

Terms/Phrases in Tongan	English Translation
Fakafuofua	*Estimate/Estimation*
Fakakaukau'i pe 'i loto	*Mentally figure out something*
Fakaofiofi	*Closer or nearer to*
Fakamahamahalo	*Conjecture*
Mate'i	*Guess*
Fua pe 'i loto	*Mentally work out the measure*
Fakamahu'inga ha me'a 'i loto	*Working out how much something is worth.*

The definitions given by students during the interviews were also used to initiate and facilitate discussions, particularly in groups where students were initially reluctant to take part. Whole class discussions allowed the students and the researcher to reflect on the various meanings of the above terms or phrases and to identify the contexts or situations in which these would be used. The discussions led the teacher to identify the following:

(i) **Mate'i** (Guess) would be utilised when students had very little or no information about the problem and therefore the only option available is to **mate'i (guess** at an answer). As one secondary student explained *"Oku te mate'i 'okapau 'oku 'ikai te ke 'ilo'i" (You'd make a guess when you don't know);*

(ii) **fua pe 'i loto (mentally work out the measure)** occurs when a measurement or estimate of a measure is required but no measuring instrument is available;

(iii) **fakakaukau'i pe 'i loto (mentally figure out an answer)** although similar to mentally work out the measure, this is used for computational estimation problems; and

(iv) **fakamahu'inga ha me'a 'i loto** (involves ascertaining the cost of a purchase).

The discussions revealed that some of the definitions given by students for the term estimation could be situation specific, for instance, *fua pe 'i loto* (mentally work out the measure) is specific to measurement estimation while *fakamahu'inga'i ha me'a 'i loto* (working out how much something is worth) refers to estimating costs. Moreover, there is evidence to suggest that terms such as *mate'i (guess at an answer)* and/or *fakamahamahalo* (conjecture) are more appropriate to situations in which lack of information, or lack of experiences or familiarity with the context of the problem allow only for a guess to be made. On the other hand, *fakafuofua* (estimate/estimation) is used when situations are familiar to students and therefore *"na'a ku fakafuofua'i pe 'e au" (" just estimated that").* Thus, familiarity with the context of the problem or the situation involved allowed students to make estimates, although the strategies could not be verbalised.

Using the students' responses, the teacher highlighted the main features or characteristics of estimation. These were:

- measurement estimation is a mental skill, which involves working out a measure without the use of a ruler or measuring instrument;
- there are a variety of acceptable strategies for working out an answer; and
- there are a variety of possible answers.

In classes III and IV, the students were very responsive to the questions without much encouragement from the teacher. However, in classes II and V the reverse was true. When questions were asked, students had difficulty talking and in Class V in particular it became evident that these students had very little experience in discussion. There was also a tendency for students to make judgements about other students' responses. For instance, when one primary student gave an answer, the remainder of the group said *"He loi" (That is not true)* or *"Hala ia" (That is not correct).* In all five classes there was a tendency for students to laugh at others' answers thinking they had made mistakes. The comments made by students in Class V were particularly offensive to students to whom these comments were directed. The boys tended to band together and direct their comments at the girls' responses, and the girls in return made negative comments about the boy's responses. With this particular class, the teacher had to continually explain to students the need to allow other students to talk and share their ideas, and also the importance of respecting other's responses as these were what the teacher was interested in. The inexact nature of the answers to estimation problems led to negative comments from some students, suggesting the need to emphasise that estimation involves inexact answers.

Activity Two: Reasonable range

The concept of **'reasonable range'** was introduced using a non-standard measure, namely a handkerchief. The researcher began the activity by holding up a handkerchief and posing the question. *"How many handkerchiefs will make up the length or height of a horse?"* and "length of the blackboard." This question was modified in different groups, depending on objects which were visible to students at the time of the lesson. For instance, with Class IV, the question was changed to *"what is the length of a spade?"* and *"width of the school house".* The number of handkerchiefs given by the students was recorded and then the researcher demonstrated these lengths using the handkerchief. As she demonstrated various lengths given by students, these estimates were used to judge the reasonableness of answers.

This activity was enjoyed by students. The height of a horse given by some students, for instance 10 handkerchiefs, led to comments such as *"you would need to use a ladder to get on top of Seini's horse"* and *"you would need to climb up a tree to get on to that horse".* When P3 gave 25 handkerchiefs as the length of the spade, there were comments such as *"you'd need two or three people to carry that spade to the* plantation". Many students began to change their answers as the teacher demonstrated each of the lengths that they had provided.

The number of handkerchiefs were determined just by looking at the handkerchief and the object, comparing the two and then making a judgement on how many handkerchiefs would fit the given object. The number of handkerchiefs given by students for the height of a horse ranged from 8 to 16. Demonstrations of these lengths using the handkerchief were a major source of amusement and/or amazement. The unfamiliar nature of the exercise led to students making guesses. It provided the opportunity for students for the first time to think about the reasonableness of their answers.

The concept of reasonableness was introduced by considering the responses given. Students were encouraged to observe the demonstrations of the heights or lengths and then to decide what would be a range for acceptable or reasonable estimates. The rationale for using a range rather than one value was discussed by demonstrating the various lengths given by students. For example, the lengths of a spade given by students in Class IV were 6,7,8,10,16,25. As the teacher demonstrated these lengths, students observed that 6 handkerchiefs would be a reasonable number for this length. The teacher then asked the question, "What about 7,8,10 handkerchiefs; does 7 or 8 seem reasonable?"

Students who thought that seven seemed reasonable were asked to justify their answer. Those who thought that 10 and 16 handkerchiefs were too much, were also asked to justify their decisions. Seven and 8 were considered reasonable because *"some spades are longer than six and others shorter than six"* Ten, 16 and 25 did not seem reasonable because these were *"too long for one person to handle"*. After discussion, the students and teacher decided that 5 to 8 handkerchiefs would be the acceptable or reasonable range for this exercise.

An important issue that arose from this activity concerned the language to be used as the medium of instruction. A review of the mathematics draft syllabus prior to teaching these activities revealed that under the heading *"students learn mathematics through"* an outline of the language(s) for medium of instruction for the new maths syllabus, was presented as follows:

It is essential that:

Teachers are familiar with the language patterns appropriate to the different mathematical processes:

- *In Class 1 to 4, all teaching in Tongan*
- *In Class 5 and 6, teachers should teach in Tongan but should give the English words and pronunciation for important concepts, and students should write and say them,*
- *In Forms 1 and 2, the teaching should be in English, but concepts and ideas should be explained both in Tongan and in English.*
- *In Forms 3 to 5, all teaching will be in English.*
- *From Form 1 on, an explanation in English which is not understood by students should be repeated in Tongan, and then repeated again in English.*

(Tonga Ministry of Education, 1992a, p.3).

The intention of the syllabus is clearly set out in the above statements, that is, the introduction of mathematical concepts in English in Class 5 and 6 to full use of the English language in the secondary schools, that is, Forms 3 to 5. Thus, the teacher decided to introduce the following terms in English; *estimation, reasonable range, under-estimation* and *over-estimation.* These were explained in Tongan and students were encouraged to use these terms in their discussion as well as when giving answers and explanations for their answers. All discussions were conducted in Tongan with both primary and secondary school classes. It was found that third formers had not mastered the English language well enough for any fruitful discussion to take place in that language alone.

Another important issue to arise from the activities concerned the use of the terms over-estimation, under-estimation and determining a reasonable range. An 'overestimation' was defined as 'estimates that fell above the range of acceptable values' whereas 'underestimation' referred to 'estimates that were under or below this range'. It was evident that presenting items as problems or in a context rather than as an exercise required changes to the upper and lower limits for the reasonable range. As an example, with Class I one of the exercises was to estimate the height of the door. The reasonable range established for this exercise was 160 to 240 centimetres. Putting this exercise in the context of a problem involving material for a curtain for the door changed the reasonable range to 225 to 250 centimetres. Explanations given by students included "Ko e mita 'e 2 'e fu'u si'isi'i koe'uhi 'e pelu e tupenu" *("2 metres would be too small because the material will be folded or hems will be needed").* The actual height of the door 1.8 metres in this case was taken into consideration in both cases. However, this measure would lie within the reasonable range for the exercise, whereas in the problem it would be outside the range and therefore would be regarded as an underestimation. In other words, for the estimation exercise: *what is the height of the door?* The actual height of 1.8 metres would lie within the reasonable range established, that is 1.6 to 2.4 metres. On the other hand, for the estimation problem: *how much material do we need for a curtain for the door?* The actual measure of 1.8 metres would lie outside the reasonable range (ie. 2.25 to 2.5 metres) and may be considered an underestimation.

This activity promoted the following:

- For the first time, some students came to realise that their responses to estimation tasks involve much more than relating the first number that came to mind.
- It allowed students to reflect on and assess the reasonableness or unreasonableness of their answers.
- It generated a lot of discussion about the concept of reasonable answer and hence the need for a reasonable range rather than one 'right' estimate.
- For each estimation task, the students and researcher would decide what would be a reasonable range. Thus, a new concept was introduced into students' mathematical

experience and thinking, that is, taking part in making decisions about assessment and evaluation of their answers.

Activity Three: Methods of measurement used by Tongans in the olden days.
The use of estimation in everyday life activities was the focus of this activity. It was anticipated that students would have ideas about various methods of measurement used in traditional Tongan life as well as the present everyday life of parents and grandparents. In order to initiate discussion the following questions were posed:
How did our grandparents and great grandparents measure the lengths of the following:

(i) a fala (mat)
(ii) a ngatu launima (tapa cloth)
(iii) a kafa (rope made from coconut husk)
(iv) distance between two rows of ufi (yam) or talo (taro).

The students worked in pairs to discuss the above questions. This was followed by a whole class discussion.

This activity generated a lot of interest, and discussion of ideas revealed that only a small number of students (i) could demonstrate a **hanga** (hand-span) even though they knew that a **hanga** was used for measuring lengths of a mat, and (ii) knew how a langanga (distance or space between two consecutive transverse stripes on a piece of tapa cloth) for the tapa cloth was measured. The home-work task provided a vehicle whereby the students and researcher learnt more about these methods of measurement used by Tongans in the past. Once all ideas were discussed, a task was set as a homework. Details of this task is given in Worksheet 1 in Appendix C.

Activity Four: Estimating using different parts of the body
The ideas and information about traditional methods of measuring lengths were discussed. These were used to (i) increase students' understanding and awareness of the use of different parts of the body for measuring and estimating lengths; and (ii) develop an awareness and appropriate use of traditional Tongan methods of measuring lengths in the mathematics lessons.

Students demonstrated the use of a hanga (span), hangafesi (argumented handspan), ofa (full arm length), va'e (foot) and manga (paces) for measuring lengths of mats, rope, distance between two rows of taro or yam; and related to the group the ideas and information that they collected from parents and/or grandparents.

These are useful measuring instruments that people have at their disposal all the time. The use of the following parts of the body for measuring various lengths formed the basis of this activity. These included the width of the middle or little finger, hand, hand span, foot, pace

and the arm length. Students were encouraged to use these body parts to measure various lengths and to use these to formulate estimates when no measuring device is available to them.

Students worked in pairs to estimate the following:

- width of an exercise book
- height of the desk
- width of their window
- height of a student
- width of the blackboard
- width of the classroom

Activity Five: Purposes of estimation and further discussion on reasonable range

The aims of these activities were to (i) encourage students to value estimation as an important mathematical and practical skill, (ii) increase students' awareness of the contexts and/or time where estimation is required or the only viable method to use, and (iii) further promote the concept of reasonable range.

The following questions were asked:

- How many metres of materials do we need to purchase to make a tupenu (skirt worn by boys) for a class 6 student?
- How many metres of material do we need to make a puipui (curtain) for the classroom door?
- How long is an adult's kafa (sennit, i.e. a rope made of coconut fibre)?

The discussion that followed revealed the need to further establish the concept of reasonable ranges. Students had been used to finding one answer to mathematical problems. Thus, the concept of reasonable range of answers had to be re-emphasised and further discussed.

The teacher and students continued to decide the reasonable range for the answers to the problems. The reasons for choosing a particular range were required from each student. For instance, the length of material required for the tupenu (skirt worn by boys) was suggested by a class 6 boy as one metre because *"the boy is not very big, therefore one metre would be enough"*. Another student reasoned that because *"two pockets are required then we would need more; therefore it would be more than one metre, say one and a half"*. Two metres was considered to be too much, as it would be a waste of money to buy the extra half a metre. However, others saw the need to have two metres as there were some big boys in class 6. It was decided therefore that a reasonable range for the tupenu would be one to two metres.

Despite the emphasis given to establishing a reasonable range for these estimation problems, some students persisted in their use of one answer. The use of the reasonable range allowed many answers to be accepted as good estimates, rather than being rejected on the basis of one

correct answer. Discussions of the purposes of the estimation problems provided the vehicle whereby students could observe the rationale for establishing a range for assessing and evaluating students' estimates, and the inappropriateness of having one right answer for problems involving estimation.

Activities Six - Seven: Benchmarks for estimating lengths

The aims of these activities were to (i) assess students' ideas of the sizes of a millimetre, centimetre and a metre, and (ii) encourage the use of a variety of benchmarks for estimating lengths. Based on the results of the strategies used by students in the interviews reported in chapter six, a decision was made to use various objects which were familiar to students as benchmarks for making estimates. These benchmarks included the width of the little finger, length of hand span, fathom, feet. These activities involved asking students to name objects which were 1 millimetre, 1 centimetre, 50 centimetre and 1 metre long. Students' responses showed they had difficulty naming objects which were 1 millimetre long. Because of this apparent difficulty the students were asked to show on a ruler the size of a millilitre, centimetre, 30 centimetres and one metre.

The activities showed that while most students recognise that a student's ruler is 30 centimetres and the teacher's ruler is one metre, only one of the six students in Class III was able to show the length of a millimetre on a ruler. The primary students on the other hand had little difficulty demonstrating this length.

This result was somewhat surprising to the researcher, who had assumed that form 3 students would have relevant ideas regarding these lengths. Apparently they lacked experiences working with the very small millimetre unit. The discussions that followed allowed for sharing of ideas, and students were learning from one another about objects that could be one millimetre in length, for example, a *needle, sharp end of a pen, a dot marked with a pen, a thin piece of wire, width of a thread.* These were used by the researcher in working with other classes to initiate and/or promote further discussion.

The activity showed that the opportunity to think about and reflect on their responses as well as those of other students, can help students validate their ideas as well as develop or construct new ideas. Small group and whole class discussions once accepted as a medium of teaching and learning mathematics provided a vehicle for students to think of other possible answers.

The results of the activities showed that an environment in which reflecting on responses, and actions was encouraged was quite effective in getting students to think about the reasonableness of their estimates. However, as Wheatley (1992) noted:

> Establishing an environment which encourages reflection is a complex enterprise. In addition, to selecting tasks, the teacher must negotiate social norms in which students can become a community of inquirers. Both students and teacher must learn to talk mathematics and learn to listen. (p.539)

The experiences of working with a Class of six students supports the above ideas suggested by Wheatley. The need for both students and teacher to "learn to talk mathematics and learn to listen" is of utmost importance if an environment which encourages reflection and negotiation of social norms is to be established. One might argue that the experience of working in a class of six cannot be used to generalise about what could happen in the normal classroom situations, in which a class of 35 to 45 is common in most church secondary schools. However, the purpose of these teaching activities was to explore the effectiveness of encouraging a learning environment which emphasises discussion about and reflection on responses to estimation tasks.

Activity Eight: Developing a feel for the sizes of units for weight

These activities were designed to (i) increase the variety of benchmarks used by students for estimating weights, (ii) further develop students' feel for the sizes of standard units of measurement, and (iii) encourage the use of, or help students generate links to, ideas developed in everyday life.

The interview data reported and discussed in chapter five showed that out of school experiences in buying items such as 1 kilogram of sugar, flour, mutton and salt provided the means by which students had developed a good sense of the feel for the weight of a kilogram. In addition, the types of responses given for smaller objects such as weight of a pen, revealed the use of the size of a kilogram as a basis of their estimates. Responses such as "half or quarter kilogram" imply a lack of prior experiences with mass of smaller objects and/or working with a "gram".

Two kinds of tasks were presented in a worksheet for students. Task 1 involved students estimating the weight of objects, while the second required students to name three objects whose weights were close to given measures. Worksheet 4 (Appendix C) details these tasks. Again students' responses were discussed in class.

Students' responses showed a lack of familiarity with very small weights, such as 5 grams, 100 and 200 grams. Small weights of 1, 2, 5, 10, 20, and 50 grams were then given to students to hold and they were then helped to name objects that they thought were similar in weight.

Activity Nine: Developing benchmarks for estimating weight

In addition to the weight (1, 2, 5, 10, 20, 50 grams) used in Activity Eight, kitchen scales and a bathroom scales were provided to students in this activity. Certain objects were also

provided and students were asked to weigh these and select various benchmarks which they could use as a basis of comparison with other objects and then to formulate their estimates.

This activity showed that 23 out of 30 students, with the exception of one primary child and the six students in Group I had remembered weighing themselves prior to this activity.

CAPACITY ESTIMATION ACTIVITIES

Following the procedures used to provide activities in estimating length and weight, the last three lessons aimed at developing a feel for the amount of the liquid in various units of measurement for capacity, such as one, 10, 200, 500 millilitres and 1 litre.

Activity Ten - Eleven: Developing a 'feel' for the size of various capacities.
A variety of objects used in everyday life was provided. These were a teaspoon, tablespoon, glass, cup, bowl for soup, jug, and a five-litre bucket. A ten-litre bowl of water was also provided. Each of the students was asked to select any one of the objects and use it to draw one millimetre of water from the bowl of water. A range of measuring cylinders was then provided and students asked to measure the water that each had drawn earlier. The measuring cylinders were borrowed from a school science laboratory, and the Mathematics Education resource room at Waikato University.

These activities provided students with the opportunity to observe the amount of water in a millilitre. Both class 6 and form 3 students found this exercise interesting and challenging. Some students thought that it would be a small amount so used a teaspoon or table spoon. Others used a cup. It was evident that most students had not had experience measuring one or ten millilitres. The researcher helped these students to read the measures. Each of the amounts of water for one millilitre was displayed in a set of test tubes, and beakers were used for amounts that did not fit in any of the test tubes provided. The students' initial ideas of a millilitre were discussed.

The amount of water drawn by students ranged from 5 to 250 millilitres. From the display of these amounts of water students were encouraged to reflect on their initial estimates (i.e. amount of water drawn). They were then asked to take any of the objects and again draw one millilitre from the bucket of water. Students who had used the cup now tended to wait for either the teaspoon or the table spoon to be free so that they could use that to draw the water out from the bucket. The second measurement provided more reasonable estimates which ranged from 2 to 18 millilitres. Students repeated the exercise until they had drawn one millilitre. Those who reached the one millilitre tended to repeat the activity.

The exercise was repeated with 10, 200 or 250, and with 500 millilitres and one litre, and any other measures that students chose such as 100, 400 or 800.

For measurement of small measures such as one or ten millilitre(s), both primary and secondary students reported having no experience in such activity. It was expected that form 3 students would have had some experience in using the measuring devices for small measures in their Science lessons. However, it became clear that students in one particular secondary school had not had any hands-on experiments or lessons in the science laboratory. As one student explained: "*'Oku hanga pe 'e he faiako ia 'o fakamatala'i mai e me'a 'e hoko 'okapau 'e fakahoko e experiments*" (*"The teacher would explain what would happen if the experiments were carried out"*). Students were more successful with measures such as one litre. For primary students measuring 250 and 500 litres were activities that they had prior experience with. However, the form 3 students did not have access to these measuring devices in their classrooms.

The students' responses to the activity described above were very similar to the experiences described by Broekman and Hoffmann (1994) in an activity which required students to 'estimate the volume of the "small" box. They found that it was not easy to start estimating. "To get the first estimation, we had to wait for a daredevil".

The students' responses to these estimation activities confirm the significance of the 'first estimate'. Once one of the students made a first estimate, it was easier for the remainder of the group to make their estimates. The 'first estimate' provided the respondent and the rest of the group with a reference point from which to base further estimates.

Activity Twelve: Review

A review of the activities involving the benchmarks used in previous lessons was the focus of this activity. This final lesson gave students opportunities for more practice in making estimates, and using the benchmarks for lengths, weights and capacity. Ability to make comparisons and to judge the relative size of the benchmarks in comparisons to the objects to be estimated has been shown to be an important skill involved in measurement estimation (Crites, 1993; Hildreth, 1983; Siegel et al., 1982). Thus, the decision to provide students with opportunities for making comparisons and judgements about relative sizes of objects as well as determining reasonable or acceptable ranges for estimates seemed justified.

7.4 Effectiveness of measurement estimation activities

The teaching activities, in general, were effective in providing opportunities for students to develop a variety of benchmarks for estimating measures of length, weight and capacity. In addition, students' understanding of basic units of measurement and appropriate units for the three measures used were enhanced through participation in the activities. The effectiveness of the programme is illustrated using specific cases studies.

The effectiveness of the activities can be inferred from an analysis of the data of students' responses in the first and second interviews. The strategy types employed to formulate the estimates as well as the reasonableness of results are examined. Responses from three students are used to illustrate the changes or lack of changes that occurred in students' ideas regarding estimation. The names used in the case studies are pseudonyms to protect the identities of the actual students.

CASE 1 : A student who adopted appropriate strategy types

Lahaina was a first year class 6 female student. She was one of the 6 students in Group V and a very attentive and co-operative member of the group. Table 7B below presents the responses by Lahaina for the items involving estimates of length.

Table 7B: Comparison of Lahaina's responses to the length items in the two interviews.

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Length of a pen	6 centimetres Fakakaukau ki he senitimita 'e taha peau pehe 'oku senitimita 'e 6 *Thought about one centimetre and then reckon it is 6 centimetres.*	10 centimetres Ngaue'aki 'a e lula senitimita 'e 30 peau pehe mahalo pe 'oku senitimita 'e 10. *I used the 30-centimetre ruler and reckon maybe it is 10 centimetres.*
Height of the teacher's desk	1 metre Tatau pe [hono ma'olunga] mo e lula mita 'e taha. *Same [height] as a one-metre ruler*	70 centimetres Ngaue'aki 'a e lula senitimita 'e 30 peau pehe ko e lula 'e 2 mo e senitimita 'e 10 e lahi [ma'olunga] e tepile. *I used the 30-centimetre ruler and I thought that the size [height] of the table would be 2 rulers and 10 centimetres.*
Width of the classroom window	15 centimetres Ngaue'aki pe 'a e senitimita 'e taha [2.4 senitimita] peau pehe senitimita 'e 15 *Used one centimetre [2.4 cm] and reckoned that it is 15 centimetres.*	1 metre Fakatatau 'a e la'iluva mo e lula palakipoe 'a e faiako. *Compare the window to the teacher's ruler.*

In the first interview Lahaina provided 6 centimetres as an estimate for the length of a pen. As she explained, the length of a centimetre was used to work out the length of the pen. The student was asked to draw a line which she thought was one centimetre long. I later measured this line and found it to be 2.4 centimetres long. It was evident that the student's idea of the size of a centimetre (2.4 cm in actual length) led to an underestimation of the length of the pen.

Her idea of a centimetre was also used to determine the width of the classroom window giving 15 centimetres as an estimate. For the width of the window, the student was asked to think of the length of her ruler and use that to make her estimate. Sixty centimetres was given as an answer *"because two rulers would fit the width of the window"*. The student was then asked to compare this with the answer that she had given earlier in the interview, that is, 15 centimetres. Through this activity the student came to realise that her first answer of 15 centimetres was not reasonable given that her ruler was 30 centimetres.

During the instructional activities, students were encouraged to use a variety of benchmarks in addition to the standard measuring devices that they use daily in their classroom. In spite of having the knowledge that a ruler was 30 centimetres long, this was not used by Lahaina when required to make estimates of lengths. The inconsistencies between ideas used to formulate an estimate (e.g. length of a centimetre) and other measures already known by the student need to be confronted and addressed by both student and teacher.

The second interview revealed that her idea of a centimetre had been modified during the course of the lessons. The line given for the length of a centimetre was later measured and found to be approximately one centimetre. Data from the interview also showed a change in the strategy used. Unlike the first interview, in which her idea of the size of a centimetre was used to formulate her answer, the length of a 30-centimetre ruler was used as the basis of her estimates in the second interview.

In the first interview, Lahaina had used the length of one metre to estimate the height of the desk, thinking this desk height the same as one metre. This latter strategy was used in the second interview to estimate the width of the window giving one metre as an answer.

Lahaina's responses during the first and second interviews to the tasks involving estimation of weight are summarised in Table 7C.

Table 7C: Comparison of Lahaina's responses to the weight items in the two interviews.

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Pen	50 grams Me'a na'a mau fua 'i lokiako. *We measured that in class*	5 grams Na'a ku fakafuofua pe ki he fanga ki'i naunau na'a tau ngaue'aki he uike kuo maliu atu. *I estimated using the weights that we used in class in previous weeks.*
Tin of fish	2 kilograms Ongo'i pe hono mamafa he'eku to'oto'o. *How it felt when I hold it.*	1 kilogram Na'a ku fakaofiofi 'eni ki ha mei sipi 'oku fua mai mei he falekoloa peau ma'u ai pe kalami 'e 1000 pe ko e kilokalami 'e taha. *I used the weight of mutton that you purchase from the shop, and I obtained 1000 grams or one kilograms*
Class 6 student	10 kilograms later changed to 50 kilograms. Ko e fakafuofua pe ia 'a'aku *Just an estimate or a guess.*	45 kilograms Fakafuofua ia ki ha topu kaukau 'oku fonu vai peau ma'u ai ha mo'oni ko 'eni ko e kilo kalami 'e 45. *I compared that to a bath tub that is full of water and obtained this answer 45 kilograms.*

It is evident in the above data that the student has had experiences in working with weights in the classroom. Ideas developed in class were used in the initial estimate of the weight of a pen giving 50 grams. The use of various weights 1, 2, 5, 10, 20, 50, and 100 grams in the instructional activities has also helped this student develop her ideas of small weights, thus giving five grams as an answer in the second interview. A shift from trying to remember the weight of a pen that they had obtained when measured in class to that of using the weights of objects that were used in this study led to the formulation of a more reasonable estimate in the second interview.

The responses presented above show that emphasis on using weights of everyday objects as benchmarks has been adopted by the student. In the first interview the student provided 2 kilograms based on how the tin of fish felt when she held it. It the second interview, the weight of a *'kilogram of mutton'* was used to provide an estimate of one kilogram. This result highlights the importance of helping students to generate links to appropriate ideas developed in their everyday life. This can be further illustrated by her responses to the task involving estimation of the weight of a class 6 student. Initially she gave 10 kilograms as an answer saying that was *"just an estimate"* or *"just a guess"*. When asked whether she knew

something that is about one kilogram in weight, she replied *"sipi kilo 'e taha" (one kilo of mutton).* She was then asked to think about her response of 10 kilograms and the kilogram of mutton, and to decide whether 10 kilograms of mutton would be about the same weight as the class 6 student. After giving it some thought she changed her initial response of 10 kilograms to 50 kilograms, saying that *"one can carry a ten-kilogram mutton but not a student"* and therefore 10 kilograms for the student was not reasonable. Furthermore, in the second interview there was an apparent change in strategy type used from "just an estimate" to comparison to a benchmark, namely the weight of a tub of water. It is also interesting to note that this was not a benchmark used in the activities but one the student developed herself.

Table 7D presents a comparison of the answers estimates and strategies provided by Lahaina in the first and second interviews.

Table 7D: Comparison of Lahaina's responses to the capacity items in the two interviews.

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Teaspoon	1 millilitre Na'a mau fa'a fua 'i lokiako e ngaahi me'a peau pehe 'oku mililita 'e taha. *We used to measure objects like that in the classroom.*	10 millilitres Na'a ku fakafuofua'i pe 'o fakatatau ki he vai na'a mau 'osi ngaue'aki ki ai 'i lokiako. *I estimated that using the amount of water that we had used in the classroom.*
Green Coconut	500 millilitres Tatau pe mo e founga na'a ku ngaue'aki ki he sepuniti. *The same method as for the teaspoon.*	1 litre Na'a ku fakafehoanaki e fo'iniumata mo e lita 'e taha he 'oku na mei mamafa tatau pe. *I compared the coconut to one litre and they are about the same weight.*
Bottle of lemonade	N/A	950 millilitres Na'a ku fakaofiofi ki he lita 'e taha he ko e lita 'e taha he hina lemani ko e fo'i hina 'e taha mo e 'oho'oho peau pehe ko e hina 'e taha 'oku tatau ia mo e mililita 'e 950. *I compared that to one litre, one litre would fill one bottle and another which only partially filled and I thought that would be the same as 950 millilitres.*

The responses given by Lahaina to the items requiring estimates of capacities showed that she had experiences in measuring various capacities in the classroom, and these ideas developed in the classroom were used to make her estimates.

Ideas about the amount of water in 10 millilitres and the mass and capacity of one litre were used in the second interview. Responses in the second interview were more detailed indicating perhaps that she has become used to explaining her strategies. Whereas in the first interview she indicated that she employed the same strategy that she had used for the teaspoon to estimate the capacity of a green coconut, in the second interview she gave a more detailed

explanation. This suggests that talking about the strategies used in formulating estimates in the activities has helped the student to verbalise her thoughts and ideas.

The responses given in the first interview and data collected from classroom observations showed the following measuring jugs of various capacities available to students in their classroom; 250 mls, 500 mls, 1000 mls or 1 litre and 2 litres. Therefore, a decision was made not to use a jug as in the first interview but rather to use a container from the everyday life of the student, namely a bottle of lemonade. Lahaina's response showed that for this item, one litre was used to compare with the capacity of the bottle of lemonade. As she rightly explained, if you put one litre in that bottle *"you will fill up one bottle and another one which is partly filled"*. She therefore reasoned that one bottle of lemonade will be about 950 millilitres, that is, less than one litre.

For Lahaina, there is evidence to suggest that she had developed more strategies for estimation and was willing to explain her strategies and processes. Data showed a shift in the types of strategies used from making a guess and trying to remember measures obtained in class to adopting a variety of benchmarks. Generating appropriate links to ideas developed in the classroom as well as outside of the classroom, making appropriate comparisons to benchmarks and judgements about the relative sizes of the item to be estimated and the benchmarks, were shown to have been adopted by the student.

But not all students who took part in the activities showed the appropriate strategies in all three measures used in the activities as intended by the researcher. This is illustrated in Case 2.

CASE 2: A student who showed some inappropriate strategies

Siosiua is an 10 year old boy and, like Lahaina, this is his first year in class 6. When he was selected for the first interview he was in the same class as the other five students in Group V. However, in term 1 all the class 6 students were streamed into three different classes for their preparation for the Secondary School Entrance Examination. As a result, Siosiua was put in the second stream with a different teacher, while the other five students remained in the first stream. Therefore, the emphasis given to estimation in Siosiua's second classroom was relatively unknown. Despite having been moved to the different class Siosiua continued to be a member of Group V for the teaching activities.

Like Lahaina, Siosiua used his existing idea of a centimetre to work out the length of the pen. In the first interview, he used the length of the lid of the pen, which he thought was about one centimetre, and as a result came up with an estimate of 4 centimetres. When asked to draw a line which was one centimetre long, two lines were drawn, and then he pointed out that the second line was one centimetre. During the analysis of the interview data, the researcher measured these two lines and obtained 3.5 and 2.2 centimetres, respectively. In the second

interview, however, Siosiua drew a line which was approximately one centimetre long. Moreover, his estimate of 14 centimetres for the length of the pen was based on his knowledge of a 15-centimetre ruler. The data revealed that Siosiua's idea or mental image of a centimetre had changed during the course of the school year.

Siosiua's responses to the tasks involving estimates of lengths in the two interviews are presented in Table 7E.

Table 7E: Siosiua's responses to items on length in the two interviews

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Length of a pen	4 centimetres Mei he ki he [loloa 'o e tapuni] 'oku tatau ia mo e senitimita 'e taha. Pea te toki lau pe, taha, ua, tolu ki he'ene 'osi. 'Aia ko e senitimita 'e fa. *From there to there [one end of the lid to the other] is equivalent to one centimetre. Then I would count one, two, three until it finishes. That is, four centimetres.*	14 centimetres Na'a ku fakafehoanaki ia ki ha lula senitimita 'e tahanima pea na'e fe'unga mo e senitimita 'e 14. *I compared it to a 15-centimetre ruler and it was 14 centimetres.*
Height of a teacher's desk	1 metre Mita 'e taha pe ofiofi ki ai. 'Oku te sio pea te fakafuofua pe ki he lula 'a e faiako. He 'oku meimei tatau pe 'a e tepile mo e lula 'a e faiako ['aia 'oku mita 'e taha]. *Compare to the teacher's ruler [1 metre].*	1 metre 20 centimetres Fakafehoanaki pe ki he lula mita 'e taha peau ma'u ai e mita 'e taha mo e senitimita 'e 20. *Compared to the one-metre ruler and I obtained 1 metre and 20 centimetres.*
Width of the window	1 1/2 metres Fakafuofua ki he lula 'a e faiako. *Compared to the teacher's ruler [1 metre].*	70 centimetres Fakafehoanaki ia ki he lula senitimita 'e 30 peau pehe 'e li tu'o 2 mo e senitimita 'e 10 pea ma'u ai 'a e senitimita 'e 70. *Compared that to the 30-centimetre ruler and I reckon that it will be twice the length of a ruler plus another 10 centimetres and obtained 70 centimetres.*

Siosiua's estimation of the width of a window also improved with a change of benchmark from the teacher's one-metre ruler to the 30-centimetre ruler. Instead of using a fraction of a benchmark (half of a one metre ruler) he used a multiple of a benchmark, that is two 30-centimetre rulers and another ten centimetres. His estimates for the height of the desk showed that this student needs to be encouraged to reflect upon the reasonableness of his estimates.

His response of 1 metre 20 centimetres in the second interview led to a request to demonstrate the height of a metre, and hence the estimate was changed to about a metre. This was the case when asked in the first interview to estimate the width of the window. The one and a half

metres given was obtained through comparison to the length of the teacher's ruler. However, when asked to think about his estimate (one metre) of the height of the teacher's desk, Siosiua changed his response to half a metre.

The estimates and strategies used by Siosiua for the tasks involving estimates of weight measures are summarised in Table 7F below.

Table 7F: Siosiua's responses to weight items in the two interviews

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Pen	half (1/2) gram Ko 'ete sio pe 'oku ma'ama'a e ki'i peni. *When you look at it the pen is very light.*	quarter (1/4) kilo Kapau 'e li 'eku peni ha me'afua pea 'e tu'u e hui he vaeua malie 'o e haafe kilo mo e 0, 'aia ko e kuata kilo *If my pen is put on a scale the needle will stop halfway between half a kilo and 0, that is quarter (1/4) kilo.*
Tin of fish	1 kilo 'Aki pe 'eku to'oto'o mo 'eku ongo'i. 'Oku ki'i mamafa ange e tohi he kapaika. *Just by the way it feels when I hold it. The book is heavier than the tin of fish.*	half (1/2) kilo Kapau 'e tuku ha kapaika ha me'afua pea 'e 'alu e hui 'o tu'u he vaeua malie 'o e kilo 'e taha pea ko e haafe kilo ia. *If you put a tin of fish on a scale, the needle will stop halfway of one kilo and that is half a kilo.*
Class 6 student	18 - 20 kilograms Ko 'eku fo'i fakafuofua pe ia 'a'aku. *That was just an estimate*	7 kilo Ka tu'u e ki'i tamasi'i kalasi ono he me'afua pea 'e 'alu e hui 'o tu'u he 7 pea ko e kilo ia 'e fitu. *If the class 6 boy stands on a scale, the needle will move and stop at 7 and that is seven kilo.*

The second interview responses show a shift in strategy type, from using his 'feel for' the weight of the object to that of imagining what value on the scale the needle would stop if the objects were to be measured on the scale.

This strategy type was described in chapter six as "visualise the needle of the scale". As mentioned in section 6.7, this strategy type was used by 5% of secondary students. However, the responses given by Siosiua demonstrated that this strategy type could be adopted by primary students. Data from the second interview reveal that benchmarks and standard measures used in the activities for estimating weights were not used by this student, indicating that participation in the instructional activity did not guarantee that the expected results would occur.

Siosiua's responses to the items involving estimating measures of capacities are presented in Table 7G, below.

Table 7G: Siosiua's responses to capacity items in the two interviews

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Teaspoon	1 centilitre Ko e ki'i ipu sakalameniti 'oku tatau pe e vai 'e lava 'o hu 'i loto mo e sepuniti 'e tolu, 'aia 'oku senitilita 'e 3 'a e ipu sakalameniti pea senitilita 'e taha leva 'a e sepuniti. *The little sacrament or communion glass can be filled with three teaspoonful of water. The sacrament glass is 3 centilitres and therefore the teaspoon is 1 centilitre.*	4 ml Ka lingi e vai mei he sepuniti ki he me'afua vai pea 'e fe'unga mo e mililita 'e 4. *If you pour the water from the teaspoon into a measuring instrument, that will be enough for 4 millilitres.*
Green Coconut	1 litre Ko e sioki lita 'e taha ka 'utu ia ki he fo'iniu mata 'e fonu ai pe fo'i niu mata ia. *A one litre jug, if you pour that into the coconut that would enough to fill it up.*	1 litre Kapau na'e lingi ha huhua'i niumata ki he me'afua vai 'e pea 'e 'alu hake 'o tu'u he mililita 'e 1000 ko e lita 'e 1. *If the coconut juice is poured into a measuring instrument, it would reach 1000 millilitres and that is 1 litre.*
Bottle of lemonade	N/A	750 millilitres Ka fua e vai he hina lemani pea 'e fe'unga ia mo e mililita 'e 750. *If you measure the liquid inside a bottle of lemonade that would be 750 millilitres.*

Table 7G above shows that Siosiua had developed and used ideas from the teaching activities. Whereas in the first interview he used a 'comparison to a benchmark' strategy to formulate an estimate of 1 centilitre for the capacity of a teaspoon, he used a 'visual image of a measuring device' strategy in the second interview. This later strategy was consistently used to estimate all three capacity measures. Taking together Siosiua's responses to the weight and capacity tasks, it is apparent that he tended to rely on visual imagery to formulate his answers.

CASE 3: A student who showed some changes in strategy types used

Itama is form 3 male students who was reluctant to provide answers in the first individual interview. However, as the study progressed he became more confident and was more involved in the activities. He showed no hesitation providing answers to the tasks in the second interview. Itama revealed some changes in the strategies used in the two interviews.

Table 7H provides a summary of Itama's responses to the tasks requiring estimates of length measures.

Table 7H: Itama's responses to length item in the two interviews

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Length of a pen	13 centimetres Na'a ku fakatatau pe 'a e peni ki he lula 'aia ko e senitimita 'e tahatolu *I compared the pen to a student's ruler and that is about 13 centimetres.*	9 1/2 centimetres Na'a ku fua 'aki hoku fo'i tuhu lekeleka 'aia ko e senitimita 'e taha. *I measured that using the width of my little finger which is one centimetre.*
Height of a teacher's desk	1 1/2 metres Na'a ku ngaue'aki 'eku tali 'anenai ki he matapa [mita 'e 3]. *I used my earlier answer about the height of the door [3 metres].*	80 centimetres Fakafuofua'i 'a hono ma'olunga fakatatau ki he lula 'e 2 pea 'oku 'iai mo e ki'i 'ova ia mahalo pe ko e senitimita 'e 20 ia. *I compared that height to two rulers and there is a left over; may be that would be about 20 centimetres.*
Width of the window	1 metre Ngaue'aki 'eku tali 'anenai ki he matapa [mita 'e 3]. *I used my earlier answer about the height of the door [3 metres].*	60 centimetres Na'a ku ngaue'aki pe 'a e fakafuofua 'a hono loloa fakatatau ki he lula 'e 2, 'aia ko e senitimita 'e 60. *I compared that width to two rulers, that is 60 centimetres.*

The above responses showed a change in the benchmarks used for estimating lengths. For the pen, instead of using the 30 centimetre ruler as he had used in the first interview, in the second interview Itama used the width of his little finger. In considering the reasonableness of the two estimates given in the two interviews (i.e. 13 centimetres and 9 1/2 centimetres) one needs to judge these estimates on the basis of the actual length of the pen (i.e. 15 centimetres) or on the basis of the benchmark used for formulating an estimate. An estimate of 9 1/2 centimetres might be considered an underestimation of the task, but if one takes into account the benchmark used (i.e. width of little finger) then 9 1/2 of this benchmark is a reasonable estimate for the length of the pen. Thus, in assessing students' estimates, the benchmark used needs to be taken into account.

For the height of the desk and width of the classroom window, reasonable estimates were given based on the length of the 30-centimetre ruler.

Table 7I: Comparison of Itama's responses to the weight item

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Pen	No Weight 'Oku 'ikai ke 'iai hano mamafa ia 'o'ona he 'oku ma'ama'a pe ia. *It does not have any weight because it is light*	10 gram Fakafuofua'i pe 'of fakatatau ki he'eku ongo'i hono mamafa. *I estimated that using my own feeling about its weight*
Tin of fish	1/2 kilo Mea'a he 'oku mamafa ange 'a e kilo ia ai. *Because a kilo is heavier than that.*	1 kilogram Fakafuofua'i pe ia 'e au fakatatau ki he mamafa 'o e kapa mo e me'a 'oku 'i loto ai. *I estimated that using the weight of the tin and the things it contains.*
Form 3 student	45 kilo Mea'a ko 'eku fa'a fua he akapulu, ko hoku mamafa ia. *Because that is my weight when I was weighed for rugby.*	53 kilo Ko hono 'uhinga ko hoku mamafa na'e kilokalami 'e 53. *Because my weight was 53 kilograms*

Itama's responses showed that the same strategy type(i.e. his own weight) was used to estimate the weight of a form 3 student. In the first interview he suggested that the pen did not have any weight because it was light. However, in the second interview he used his 'feel for the weight' of the pen to provide an estimate of 10 grams. The use of small weights 1, 2, 5, 10, 20, 50 grams) in the teaching activities provided students with some experience in holding these weights and to differentiate the feel between 1, 5, and 10 grams as well as the other grams involved. It was evident from data in the second interview that students such as Itama whose initial response to the weight of the pen had been "no weight" all changed their answers giving some estimates ranging from 1 to 10 grams. This finding points to the value of providing students with weights of small objects, and a variety of experiences in comparing weights of small objects, such as a pen, piece of chalk, marble, box of matches, lid of a bottle, in order to develop a feel for the weight of a gram and consequently the weight of very light objects.

The responses given by Itama in the first interview revealed that he was either not sure about or lacked experiences in measuring or estimating capacities. Details of the responses given by Itama for the tasks involving estimates of capacities are presented in Table 7J below.

Table 7J: Comparison of Itama's responses to the capacity items

TASK	FIRST INTERVIEW	SECOND INTERVIEW
Teaspoon	No Response Ko e sepuniti 'oku 'ikai teu 'ilo ia 'e au. *For the teaspoon, I don't know*	5 millilitres Ko 'eku hanga pe 'o fakafuofua ki he me'i vai 'oku 'i he sepuniti peau pehe 'oku mililita 'e 5 *I just estimated using the amount of water in a teaspoon and I reckon it would be 5 millilitres*
Green Coconut	1 litre Ko e 'uluaki me' pe ia na'e 'uluaki ha'u ki hoku 'atamai. *That was the first thing that came to my mind.*	800 millilitres Na'a ku hanga pe 'e au 'o fakafuofua'i 'a e lahi 'o e vai he fo'i niu. Fakafehoanaki mo ha sioki lita 'e 1. 'Oku lahi ange pe 'a e sioki ia he fo'i niumata. *I estimated the amount of water in the coconut by comparing it to a one litre jug. The jug is bigger than the green coconut.*
Bottle of lemonade	N/A	1 litre Na'e 'ikai ha fua ki ha fa'ahinga me'a na'a ku fakafuofua'i pe ia 'e au. *I did not compare that to any other objects I just estimated that.*

In the first interview, Itama suggested that he did not know which unit of measurement to use for capacity. When he was asked whether he knew about litres or millilitre he responded 'yes'. However, when one analyses the estimates provided by Itama, the data suggest that guessing, using the first numbers that came to his mind, was his main strategy. Thus, for the teaspoon he gave no answer, one litre for the green coconut because *"that was the first thing that came to my mind"* and 100 litres for the petrol tank of a van.

Participation in the activities seems to have influenced his responses in the second interview. Table 7J shows that in the second interviews, Itama has moved from relating the first number that came to his mind to assessing the amount of water in each of the interview items, and therefore providing reasonable estimates for the tasks. Whereas, he used a 'comparison to a benchmark' strategy to estimate the capacity of a green coconut, he just estimated the remaining tasks.

Assessment of answers of estimation activities

The experiences of working with students in the activities showed that even though students had been exposed to mathematics that deals with one right answer, the concept of 'reasonable range' used as criteria for assessing students' estimates was easily accepted by students and therefore ought to be a major part of working in the area of estimation.

Determination of the limits or end points for this range provided the vehicle by which students and teacher decided and agreed upon a reasonable range for the estimates. This discussion allowed students to reflect on their answers, it determined whether these were reasonable and within the range, and it allowed them to justify their answers and/or explain why they felt the range should be increased to include their estimates. The results of these activities suggest that estimation activities based on a reasonable range provide a means by which students can be in control of their mathematics learning.

The concepts of over-estimation and under-estimation were also introduced. Estimates that fell above the range were considered overestimation while those which fell below were considered underestimation. Taking into account the purposes of the estimation, particularly when a decision had to be made based on the estimates, the students realised that the exact value did not always have to be included in the reasonable range. Values that are reasonable in one context (e.g. actual measure in an estimation exercise) would be considered underestimation in others (e.g. actual measure in an estimation problem).

Limitations of the activities

The main limitations of the activities used in the present study are outlined below.

1. Limited Scope

As discussed above the activities used focused on three measures; estimating lengths, weights and capacities. Measurements in other areas specified in the Draft Syllabus, (Tonga Ministry of Education, 1992a) such as area, perimeter, circumferences and angles were not included in this study. The activities were also limited to students in the two levels used in this study, that is, class 6 and form 3.

2. Limited Number of Lessons

Availability of time was a major determining factor in the number of lessons that could be conducted with students. Just 12 activities could be managed in the time available.

3. Limited Number of Strategies

The main estimation strategy type emphasised in the activities was the use of the benchmark. Despite these limitations, this phase of the investigation has provided the researcher with important ideas regarding teaching activities on estimation. These are summarised and further discussed in the next section.

Discussion

The results of the first individual interviews, the teaching activities together, and findings of previous research studies (Hildreth, 1983; Siegel et al. 1982) were used to make an analysis of the skills involved in measurement estimation. The headings used by Sowder and Wheeler (1989) for their analysis of the components involved in computational estimation (see chapter three section 3.2) were used for this analysis. The result of this analysis is presented in Table 7K.

Components involved in measurement estimation

The main strength of the results of these activities involves the identification of important components involved in measurement estimation.

Table 7K: Analysis of the Components Involved in Measurement Estimation

I. Conceptual Components

1. Role of Estimation
   - 1.1 Recognition that estimation is a useful mathematical skill.
   - 1.2 Recognition that estimation is a useful practical skill.

2. Role of Approximate Numbers
   - 1. Recognition that approximate numbers are used for estimation.

3. Multiple Process/Multiple outcome
   - 1.1 Acceptance of more than one process/strategy for obtaining an estimate.
   - 1.2 Acceptance of more than one value as an estimate.

II. Skills Components

1. Processes
   - 1.1 Comparison to a standard unit of measurement.
     - 1.1 a. Equivalent to a unit of measurement.
     - 1.1 b. Equivalent to a fraction or portion of a unit of measurement.
     - 1.1 c. Equivalent to a multiple of a unit of measurement.
   - 1.2 Comparison to a benchmark.
     - 1.1 a. Equivalent to a benchmark.
     - 1.1 b. Equivalent to a fraction or portion of a benchmark.
     - 1.1 c. Equivalent to a multiple of a benchmark.
   - 1.3 Checking reasonableness of estimates.

2. Outcomes
   - 2.1 Determination of a reasonable range of estimates.
   - 2.2 Determination of appropriate units of measurement for the estimates.
   - 2.3 Purposes of estimation as determining factor for reasonable range.
   - 2.4 Reasonableness depends on the benchmarks used.

III. Related Concepts and Skills

1. Appropriate mental image of units of measurement.
2. Appropriate knowledge of measures of benchmarks.
3. Ability to compare objects by size.
4. Ability to make judgements about relative sizes of objects.
5. Reflection on procedures used to determine their answers.
6. Experiences in a variety of estimation problems.

IV. Affective Components

1. Recognition that estimation requires thinking.
2. Recognition of estimation as a useful mathematical and practical skill.
3. Acceptance that estimation leads to inexact answers.
4. Confidence in ability to estimate.

I. CONCEPTUAL COMPONENT

As discussed in chapter one, estimation has been one of the most neglected topics in the mathematics curriculum. A central issue involves the commonly held view that mathematics is a subject concerned with exactness. On the other hand, estimation is a process which produces inexact answers. As such, estimation seems antithetical to the very nature and purposes of mathematics (Usiskin, 1986).

One of the most important aims of any instructional program on estimation would be to help students recognise that estimation (i) is a useful mathematical skill; and (ii) involves multiple processes and multiple outcomes. The measurement estimation tasks used in the first individual interviews and teaching activities suggest that measurement estimation activities 'naturally' require students to formulate answers based on existing ideas and prior experiences. As illustrated in chapter six the tasks resulted in the formulation of many answers using a variety of viable strategies. The new Tongan mathematics syllabus (Tonga Ministry of Education, 1992a) gives a prominent place to estimation. This new syllabus is due for implementation in 1996—in mathematics classrooms which view mathematics as a subject which deals with exactness. The results of the teaching activities on measurement estimation suggest that these type of activities would help both primary and secondary students accept that estimation 'is a useful mathematical skill'. Acceptance of this would assist in obliterating the myth that 'mathematics is a subject concerned with precision and exactness'. Given that most of the terms and phrases used by both students and teachers (i.e. fakaofiofi, fua pe 'i loto, fakakaukau) refer specifically to measurement estimation, establishing an estimation mind-set via measurement estimation activities seems to be an appropriate step to take in the Tongan classroom.

II. SKILLS COMPONENTS

Data reported in chapter six revealed many processes and answers that could be generated from one estimation task. Again, an increased emphasis in the use of measurement estimation tasks in the classroom would encourage students and teachers to accept the inexact aspect of mathematics. The requirement to explain and relate to the researcher the strategies used to make their answers led students to reflect upon these and check the reasonableness of their estimates. Thus, the following questions are essential in any mathematics classroom:

i. How did you get that?
ii. Can you explain the procedures used to derive your estimate?
iii. What makes you think that?
iv. Does this seems reasonable?

The act of reflecting upon answers would assist students to develop the skills and habits of checking the reasonableness of their estimates. It has also been suggested that reflecting on

answers help students become better problem solvers and reasoners (Garofalo and Bryant, 1992; Krulik and Rudnick, 1994).

Two important questions which arose from the activities are related to the development of the benchmarks for estimation.

i. Is it always necessary to use standard units and measures for estimating?

ii. How can traditional Tongan methods of measurement be incorporated into mathematics education?

The first question draws our attention to students' use of standard and non standard measures for estimating. These have important implications for the criteria used for assessment. Data showed students often used benchmarks with incorrect ideas about their actual measures. For example, a student who give 8 centimetres as an estimate for the length of a 15-centimetre pencil, based this on the width of his/her middle finger. The width of the middle finger was considered one centimetre. Should the students' estimate be assessed as 8 centimetres or 8 fingers. It is possible that 8 centimetres could be judged unreasonable if an acceptable range is set at 10 to 20 centimetres. If the student's answer is taken as 8 fingers this could be considered a reasonable estimate. This example raises some important issues for teachers. To what extent are the standard units of measurement going to be used as a basis of assessment. In the everyday life experiences of students, it is possible that traditional methods of measurement are used. For example, if asked to make a kafa (sinnet), the student might ask how many ofa (arm-spans) do you need? In addition, when asked how tall is Sione? The answer could be "just as tall as Mele". Although non standard measures were used in the teaching activities, these were not used by students in the second interview. The use of non standard measures (particularly those used in everyday life) in the mathematics is an issue that ought to be carefully considered. It appears that two separate measures exist in Tongan students' thinking: "school" measures and "non-school" measures.

The results of the teaching activities indicate the need not only to develop an estimation mind-set but to develop an awareness and acceptance of traditional Tongan methods of measurement as an important mathematical skill. The attempt by the researcher to develop non-standard measures using body parts was not successful. The data in the second interview show students relied heavily on the standard units for estimating lengths. Given that the students have had six years of mathematical learning using the standard metric units, perhaps it is not surprising that they failed to use the non-standard benchmarks used in the teaching activities as basis of their estimates.

III. RELATED CONCEPTS AND SKILLS

The data points to the need for students to develop appropriate mental images of units of measurement and benchmarks. One of the weaknesses which hindered students' ability to provide reasonable estimates was incorrect ideas about units of measurements and benchmarks. The data suggest that experiences in a variety of estimation exercises and problems are all important in the development of appropriate ideas regarding standard units of measurement. These could also provide students with opportunities to develop skills in comparison and making judgements about the relative sizes of objects. Reflecting upon the procedures used to formulate their estimates encouraged students to determine their appropriateness.

IV. AFFECTIVE COMPONENTS

The affective components are just as important as the other three components discussed above. Students need to be confident in their ability to estimate. Familiarity and experiences with estimation exercises and problems serve as a means by which students can develop confidence in their ability to estimate. Acceptance that estimation leads to inexact answers may lead to the recognition that estimation is a useful mathematical skill. An understanding and awareness of the processes involved in measurement estimation would help students recognise that estimation requires much more than making a wild guess or relating the first number that comes to mind. Estimation requires thinking and using one's existing ideas to generate answers to estimation tasks.

7.5 Summary and Comment

Teaching activities on measurement estimation have been the focus of this chapter. The effectiveness of the programme in terms of helping students construct appropriate estimation strategies was assessed using specific case studies, and observation of students' responses to the activities involved.

Areas of strengths

The main strength of the activities was the opportunity for students to explain their thought processes and thus reflect on their initial and subsequent responses. The requirement for students to relate the procedure(s) used to formulate their estimates allowed students to think about the strategies they had used. This often led students to modify their estimates in order to provide more reasonable answers.

Another important feature of these activities involved using a reasonable range as criteria for assessing students' estimates. This encouraged students to: (i) attempt to make an estimate, (ii) value one's answer and not to modify it or copy others as soon as other students gave their

answers, (iii) accept the idea that estimation is a process that can lead to a variety of answers, (iv) recognise the need for 'reasonable range' to be negotiated by students and teachers based on the purposes of the estimation problems, and (v) take a more active role in their own learning.

Effects of the activities on students' strategy use

Increasing the number of benchmarks (i.e. objects with known measure) resulted in increased use of the different benchmarks. However, knowing the measure of a benchmark did not guarantee that this would be used to produce a reasonable estimate. These should be accompanied with plenty of experiences in making comparisons and judging whether the benchmark is equivalent to the item to be estimated.

In general, learning through humour is an important feature of the Tongan classroom that the teacher can take advantage of. As discussed earlier students tended to laugh at other students' responses. The experienced teacher can determine which kind of laughter and comments ought not to be tolerated in the classroom as these would be detrimental to other students' self-esteem. In other cases, the humour and comments could encourage students to think about their answers and to be more cautions about making unreasonable estimates. This is an aspect of the Tongan classroom that needs further investigation.

Areas of weaknesses

The activities showed that students' misconceptions of the basic units of measurement partly accounted for the many unreasonable estimates given. In particular students' ideas or mental 'image' or 'feel for' the size of a millimetre, gram and millilitre determined the reasonableness of the estimates given. The results of students' participation in the activities showed that using these units of measurement did not guarantee that students' feel for the size of units of measurement, particular small units such as a gram or millilitre, could be developed over a short period of time. Data suggested a need to provide students with plenty of opportunities to work with these units of measurement.

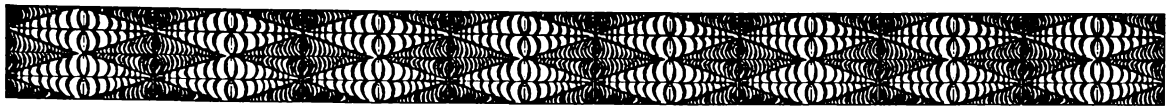
Emphasis on discussion, particularly whole group discussion, revealed that some of the characteristics of Tongan students if not properly managed will inevitably lead to the failure of this method in the Tongan classroom. As mentioned earlier students had a tendency to laugh at other students' responses. This an important issue raised by one of the secondary teachers, T2, as reported in chapter five. Moreover, in one primary school class there was a tendency for the boys to gang up on the girls, which provided a most uncomfortable learning environment for the girls, who in return made negative comments about the boys' responses. A combination of individual work and group work was adopted. The nature of the interaction in this particular classroom necessitated the frequent emphasis on the need for students to listen to, as well as to respond appropriately to others' answers. By its very nature, estimation provides answers that could deviate from the actual measure. Students need to

learn to accept that such answers are not 'wrong' but the most appropriate type of answers for estimation.

This chapter has reported and discussed (i) the instruction on estimation involving measures of length, weight and capacity, and (ii) case studies that illustrate how students' ideas of measurement estimation changed as a result of the teaching activities. Students' ideas of computational estimation are the focal point of the next chapter.

CHAPTER EIGHT

RESULTS AND DISCUSSION: COMPUTATIONAL ESTIMATION STRATEGIES



8.1 Introduction

In addition to measurement, computational estimation was another form of estimation explored in this study. Due to the limited time available in term III a decision was made to focus on just the sample of primary students (n=12). Again, data on computational estimation strategies were obtained through (i) individual interviews, (ii) in the course of a ten-lesson instructional unit on computational estimation strategies (see Appendix D), and (iii) follow-up individual interviews soon after the completion of the instructional unit.

This chapter reports on and examines the findings of the initial individual interviews with the sample of primary students. As discussed in chapter four the main sample of primary students was interviewed to determine both the strategies employed to formulate computational estimates with whole numbers, and the consistency and/or flexibility with which they used a particular estimation strategy. Results are reported under four categories: strategies used in addition (8.2), subtraction (8.3), multiplication (8.4), and division (8.5).

8.2 Strategies involving addition with whole numbers

The strategy types used for the two tasks involving addition (Items 1 and 2) are summarised in Table 8A and examined below. The data showed that students used three main strategy types to formulate answers for the tasks. These strategies, together with the number of students using them, and the frequency with which they were used are presented in the Table 8A below.

Table 8A: Strategy types for tasks involving addition

Strategy type	Number of students who used it (n=12)	Number of times it was used
1. Conventional algorithm	9	13
2. Adding compatible numbers	8	10
3. Rounding procedure	1	1

An explanation of these strategies for addition, together with an analysis of their use by students, is now provided.

1. Conventional algorithm

This strategy involves students mentally computing an exact answer using a 'conventional algorithm' taught in the classroom. This strategy type was described in the review of the literature as "proceeding algorithmically" (Reys et al., 1982).

Analysis of the data showed that more than half of the responses (i.e. 13 out of 24) used this strategy type, and it was employed by nine of the twelve students in the sample. All of the answers provided were the exact answers to the exercises rather than estimations. No attempts were made to give rough answers to the exercises. The dominance of the classroom taught rules was evident in the students' responses. For instance, in responding to Item I (which involved adding 50, 150, 200, 20), P11 said:

Na'a ku hanga 'o tanaki 'a e noa, noa, noa, noa 'o tu'u. Peau tanaki leva 'a e ngaahi mata'ifika tu'unga 'o e hongofulu 'o tu'u peau tanaki leva 'a e tu'unga 'o e teau 'o ma'u ai 'a e 420.	*I added zero, zero, zero, zero and wrote that down. And I added the numbers in the tens and then I added those in the hundred and got 420.*

The above response shows that although P11 formulated her answer mentally she carried it out as if she was doing it using pen and paper in written algorithm form, proceeding from the units to the tens and then to the hundreds. This result suggests that the 'conventional algorithm' dominated the classroom and therefore has become the most 'natural' strategy type for students to use. The result also implies that estimation strategies for computation might not have been used in this primary classroom This is consistent with the data regarding computational estimation reported earlier in chapter five, in which the child's teacher, (T4) said, "*'oku 'ikai ke fu'u 'uhinga kiate au 'a e fehu'i ko 'eni (I do not fully understand this question)* ". This response suggested that perhaps the use of computational estimation strategies is not a familiar concept to students and teacher in this class.

Other students first arranged the numbers in columns before proceeding to mentally add numbers using a 'conventional algorithm' producing an exact answer. Despite the researcher's attempts to point out that all that was required was an estimate, students continued to obtain exact answers for these tasks. The dominance of using a 'conventional algorithm' is a reflection of the emphasis given this strategy in the primary school mathematics classroom. This also highlights students' relative unfamiliarity with estimation in the context of computation. Thus, one possible reason for students' reliance on a 'conventional algorithm' is that estimation has not been part of their learning experiences in the classroom. This finding is consistent with the results of a study of Mexican students' estimation performance by Reys, Reys and Penafiel (1991). As reported in chapter three, a common strategy used by Mexican

students was to *mentally apply a pencil and paper algorithm.* This procedure appeared to be the first choice of some students. One of the reasons for students' low performance on estimation tasks is that they did not understand what "estimate" meant. It seems that for the Tongan students in this sample, "estimate" in the context of computation was also not understood.

2. Adding compatible numbers

Adding or combining compatible numbers was the second most frequently used strategy type for addition tasks. It accounted for under half the responses (i.e. 10 out 24) and was used by two-thirds of the students. This strategy involved adding pairs of numbers which are easier to handle or compatible, and then working out the sum of these. Eight out of the twelve students exhibited this procedure particularly for Item 1. For example, P5 gave the following explanation for providing 420 as an answer.

Na'a ku tanaki pe 'a e 150 mo e 50 'o tu'u 'a e mo'oni ko ia [200]. Peau tanaki pe 'a e 200 mo e 20 pea tu'u 'a e mo'oni ko ia [220] peau tanaki leva 'a e ongo mo'oni 'o ma'u ai 'eku mo'oni ko 'eni [420].	*I added 150 and 50 and put down that answer [200]. And then I added 200 and 20 and put down that answer [220] and then added both answers to get my answer [420].*

Another student, P2, combined 20 and 200 first and then proceeded to formulate an exact answer.

'Ai pe ua noa (20) 'o tanaki mo e uangeau (200). Pea tanaki leva 'a e nima noa (50) mo e teau nimanoa (150) pea tanaki e ongo mo'oni [220 mo e 200] 'o ma' ai 'a e fangeau uanoa (420)	*You take twenty (20) and add that to two hundred (200). Then add fifty (50) and one hundred and fifty (150) and then add the two answers [220 and 200] to obtain four hundred and twenty (420)*

The above response suggests that students were flexible enough to use a strategy which they constructed themselves depending on the numbers involved. It is also possible that the numbers involved in the tasks were easy enough for students to handle and therefore there was no need to look for an estimate because exact answers could be easily obtained. In comparison to the use of a 'conventional algorithm' which suggested a written algorithm was used, the 'adding compatible numbers' strategy type does not appear to rely on a written algorithm. The combination of compatible numbers seems to assist in formulation of answers mentally. The student is also taking numbers as whole entities and proceeds in the order in which we speak of numbers, that is from hundreds to ten to units. In contrast, the use of 'conventional algorithm' follows the opposite order, with computation going from units to tens then to hundreds.

The above responses by P5 and P2 demonstrate a different order of combining numbers. This is something that is not possible when using a 'conventional algorithm' where one is required

to follow the order in which the numbers are presented. 'Adding compatible numbers' allows for flexibility and a variety of ways of dealing with numbers.

The use of this strategy type has important implications for the development of students' estimation skills and strategies. Flexibility in using numbers seems to be related to good 'number sense' as students who used this strategy type responded that they had "made it up" themselves. It is possible that the requirement to carry out the task mentally prompted students to use this strategy type rather than following a 'conventional algorithm'. It is also possible that the requirement for students to relate their answers verbally encouraged the use of strategy types such as 'adding compatible numbers'. Research by Nunes (1992) has shown that *oral arithmetic allows for different types of manipulation than written arithmetic* (p.443). Whereas in written arithmetic students would tend to follow the same procedure, the need to relate answers orally allows for procedures other than 'proceeding algorithmically' to be used.

3. Rounding procedures

Only one student in the sample used a rounding procedure to obtain an answer, and this to only one of the items. Item 2, required the student to estimate the sum of 52, 56, 54 and 67. This student, P5 explained this process:

P5: Na'a ku fakalahi e ngaahi mata'ifika 52 ki he 50, 56 ki he 50, 54 ki he 50 pea 67 ki he 70 pea toki tanaki ma'u ai e 220.	*P5: I rounded the numbers, 52 to 50, 56 to 50, 54 to 50, 67 to 70 and then add them all together to get 220.*
I: Na'a ke ako e founga ko 'i fe?	*I: How did you learn that procedure?*
P5: Fakakaukau pe ia 'a'aku	*P5: I just thought about it?*

Perhaps this student attempted to copy the procedure used by the researcher in the introductory phase to demonstrate what is meant by computational estimation. The procedure used by P5 shows that all numbers in the 50s (i.e. 52, 56 and 54) were rounded to 50, indicating perhaps that the standard rounding procedure which would involve rounding 56 to 60 was either ignored or that it has not been part of the student's repertoire of skills and strategies. Given that an estimate was required, the rounding procedure used by P5 is appropriate for the task.

Comment

It is interesting to note that important computational estimation strategies identified by previous research and discussed in the research literature, for example, the 'front-end' procedure, did not feature in the data obtained. The data showed that the two main strategy types used were 'conventional algorithm' and 'adding compatible numbers'. The third strategy type used by one student was the 'rounding procedure'. Because there was no time limit for students to make their estimates, this could have encouraged them to use 'conventional algorithms' to produce exact answers. In addition, it is possible that the numbers involved were easy

enough for students to handle mentally and therefore there was no need to use estimation techniques.

An important question that arose from the analysis of the data involved making decisions on the reasonableness of the estimates. As outlined in chapter four, rather than establishing a reasonable range for the estimates, the assessment procedure adopted for the present study required that the strategies used were to be considered in determining the reasonableness of the estimates. Twenty-three of the responses given by students were the exact answers, 13 and 10 of which were generated using 'conventional algorithm' and 'adding compatible numbers' respectively. Only one response was an approximate answer or an estimate. As Sowder and Wheeler (1989) suggested an important conceptual component of computational estimation is the *recognition of the role of approximate numbers.* It appears that the students who produced exact answers were not aware of the role of approximate answers in estimation. Given the criteria used for assessment one would consider the 23 exact answers as unacceptable although they are reasonable. On the other hand, one of the important skills involved in computational estimation is the *ability to compute mentally.* In that case, the 23 exact answers would be considered reasonable and acceptable.

Because this phase of the investigation is an exploration of students' computational estimation strategies, and the results showed an emphasis on exact mental computation, the analysis of the data focused on the strategies used rather than the reasonableness of the estimates. Thus, no overall data on the reasonableness of the estimates provided were collated.

8.3 Strategies involving subtraction with whole numbers

Two strategy types for items involving subtraction were identified from the students' responses to Items 3 and 4. These, together with the number of students using them, and the frequency with which these were used are summarised in Table 8B below.

Table 8B: Strategy types for tasks involving subtraction

Strategy type	Number of students who used it	Number of times it was used
1. Conventional algorithm	12	22
2. Front-end strategy	2	2

1. Conventional algorithm for subtraction

The data showed that in 22 out of the total 24 responses students mentally used a 'conventional algorithm' for subtracting whole numbers in the way many had done for addition. All twelve students used this strategy to formulate an answer to Item 4 (the difference between $312 and $500). Explanations by P4 and P7 illustrate this strategy.

P4:	*Pa'anga 'e teau valuvalu ($188)*	*P4:*	*One hundred and eighty-eight dollars.*
I:	*Ko e ha e founga na'a ke ngaue'aki ke ma'u ai e $188?*	*I:*	*How did you get $188?*
P4:	*Na'a ku kole ua ki he noa pea kole taha ki he noa pea kole leva 'a e tolu ki he nima 'o toe ai 'a e pa'anga 'e $188.*	*P4:*	*I subtracted two from nought, and took away one from nought and three from five and left with one hundred and eighty-eight dollars.*

Similarly P7 explained:

P7:	*Na'a ku kole leva 'a e ua ki he noa 'o 'ikai lava pea kole mai leva 'a e fo'i taha [teau] mei he nima kae toe pe 'a e 4 'o 'ohake ia ki he 'uluaki noa. Pea to e to'o mei ai mo e fo'i hongofulu ['e 9] 'o 'ave ki he fo'i noa hono ua, 'o kole mei ai mei he hongofulu e ua kole leva 'a e taha mei he hiva pea kole leva 'a e tolu mei he fa.*	*P7:*	*I subtracted two from zero and couldn't do that and borrowed one [hundred] from five and left with 4. And take ten put in the first zero and then another tens [9] to the second zero, and minus two from the ten and one from nine and three from four.*

P7 used the renaming subtraction method to change

500	to	4 9 10	giving 188
-312		-3 1 2	

Both students said that these rules or algorithms were taught by teachers. Although all twelve students used a school taught algorithm to obtain an answer to Item 4, four of these students could not come up with the exact answers. It appears that for these students the requirement that the task to be done mentally led to difficulty in handling the numbers and they therefore came up with answers such as $168 and $228. The remaining two students put a point between one and eight (i.e. $1.88) with the explanation that *"ko e 'uhi ko e pa'anga kuopau ke 'iai 'a e seniti"* (*because you are dealing with money you need some cents*). When asked to think about the reasonableness of their answers, both students changed their answers to $188.

2. Front-end strategy

In using this strategy the subject first focuses on the left-most digits rather than the right-most digits (as when using a conventional subtraction algorithm). The front-end strategy has been identified by researchers as one of the most powerful computational estimation strategies (Dowker, 1992; Edwards, 1984; Lee, 1991; Levin, 1981; Reys et al., 1982). This strategy was used by two of the students but on two occasions only .

For example, P5 used the strategy for Item 4 (500 - 312).

To'o e fo'i teau mei he nimangeau (500) 'o 'ai ia ki he teau (100) pea kole leva 'a e tolungeau tahaua (312) mei he fangeau (400) 'o ma'u ai 'a e valuvalu (88) pea toki tanaki leva ia [100 mo e 88] 'o ma'u ai 'a e pa'anga 'e teau valuvalu ($188).

Take a hundred (100) from that five hundred (500) and put it there hundred (100) and then subtract three hundred and twelve (312) from four hundred (400) to obtain eighty eight (88) and then add those [100 and 88] and obtained one hundred and eighty-eight dollars ($188).

From the above explanation it seems that P5 is treating the numbers as whole entities rather than focussing on the ones, tens and then hundreds. In using the 'front-end strategy', P5, focussed on the 500 and 300 and reasoned that 100 could be taken from 500, then the 312 was then subtracted from the 400 (that was left over from the 500) giving 88. This 88 was then added to the 100 therefore obtaining $188 as the answer. It seems that taking numbers as whole entities is a skill associated with front-end strategists. Understanding of how numbers change when mathematical operations are applied is also another characteristic skill associated with the flexibility by which subjects use the 'front-end strategy'.

Further analysis of the strategy used by P5 suggests that even with the use of the 'front-end strategy' different approaches are possible. For instance, in centring attention on the front-end or left-most digits, P5 focussed on 500 and 300. At first she reasoned that 100 could be taken from the 500 leaving 400 from which 312 was later subtracted. The 'front-end strategy' as described in the research literature would involve the subject focussing on 500 and 300, subtracting these numbers giving 200 as an initial estimate and then adjusting this answer by subtracting 12 from it giving 188 as an answer. The procedure used by P5 demonstrated a flexibility in use of both numbers and operations, involving successive subtraction and addition to formulate her answer. The 'front-end strategy' as it is described in the research literature seems to suggest the use of one operation throughout the whole process. That is, there is an assumption that subtraction would be used to find an initial estimate as well as in the adjustment or compensation process. The procedures used by P5 illustrate a viable alternative procedure for the use of the front-end strategy. Furthermore, it helps illustrate one of the strengths of the 'front-end' procedure, that is, the subject would have some idea of the magnitude of the answer after the first operation (unlike those who rely on a 'conventional algorithm', and have to wait until the end of an operation to find out what the magnitude of the answer might be).

Comment

Overall, the data showed the students consistently used 'conventional algorithms' to formulate estimates. The answers given in most cases were the exact answers. These results showed that students did not possess effective estimation strategies but rather relied on conventional algorithms to mentally compute answers. Only two students exhibited the use of the 'front-end' procedure and as with those who used a 'conventional algorithm' they too provided exact

answers to the tasks. It appears that 'exact mental computation' was viewed as synonymous with 'estimation'. The strive to obtain exact answers suggests a lack of experiences with the type of answers required for computational estimation, that is, only rough solutions are necessary. It seems that for these students computational estimation had not been part of their mathematical experiences. It is therefore not surprising that they exhibited a very limited number of strategies and relied heavily on the use of a 'conventional algorithm', the main strategy for computation emphasised in the classroom.

8.4 Strategies involving multiplication with whole numbers

Three main strategy types were employed by students to formulate answers to Items 5 and 6 involving multiplication. These strategies, the number of students using them and the frequency of their use are shown in Table 8C below.

Table 8C: Strategy types for tasks involving multiplication

Strategy type	Number of student who used it	Number of times it was used
1. Conventional algorithm	11	20
2. Rounding procedure	1	2
3. Subtracting numbers	1	1
4. No response	1	1

Again the data showed that using a 'conventional algorithm' was the most frequently used strategy type. The data also revealed that one student, P7, did not provide any response to Item 5, even though he was able to provide an exact answer for Item 6 after mentally computing the answer. It is possible that the numbers involved (28 and 60) discouraged this student from attempting to formulate an answer to this item. The most he could managed was, *"'oku 'ikai teu 'ilo'i ia 'e au" (I did not know).* An explanation of the two strategy types for multiplication are given below.

1. Conventional algorithm

The data showed that eleven students mentally computed an answer using a 'conventional algorithm'. Responses by students revealed that for Item 5 which involved multiplying 28 by 60 (i.e. 28 x 60), students used one of two procedures: (i) recognising that it involved multiplying by zero and therefore first put down one zero and then proceeded to multiply 6 and 28; (ii) using a combination of finger writing and mental computation to formulate an

exact answer. The following responses by P4 and P6 illustrate these two procedures respectively.

P4: Na'a ku tu'u pe 'a e noa kae 'uluaki liunga 'a e ono (6) mo e valu (8) pea toki liunga 'a e ono (6) mo e ua (2) 'o ma'u ai 'a e taha afe onongeau valunoa (1680).

P4: I put down the nought and first multiplied six (6) by eight (8) and then multiplied six (6) by two (2) and got one thousand six hundred and eighty (1680).

Like P4, P6 gave an exact answer (i.e. 1680).

P6: Fokotu'u pe uavalu (28) mo e ononoa pehe'i

$$\begin{array}{r} 28 \\ \underline{x\ 60} \end{array}$$

[tohi'i 'aki hono fo'i tuhu]

Pea tu'u leva e noa 'e ua pea tu'u mo e noa 'e taha

$$\begin{array}{r} 28 \\ \underline{x\ 60} \\ 00 \\ 0 \end{array}$$

[Tohi 'aki pe hono tuhu]. Pea liunga leva 'a e ono ki he valu [ma'u ai e] favalu ono ki he ua ma'u ai e taha ua, tanaki 'o ma'u ai 'a e fu'u talo 'e tahaafe onongeau valu noa

$$\begin{array}{r} 28 \\ \underline{x\ 60} \\ 00 \\ \underline{1680} \\ 1680 \end{array}$$

P6: I arranged twenty-eight and sixty like this

$$\begin{array}{r} 28 \\ \underline{x\ 60} \end{array}$$

[writes using her finger].

Then I put down two noughts and another nought

$$\begin{array}{r} 28 \\ \underline{x\ 60} \\ 00 \\ 0 \end{array}$$

[writes using her finger]. Then I multiplied six into eight [and got] forty eight, six to two and obtained twelve, added those together and got one thousand six hundred and eighty taro plants.

$$\begin{array}{r} 28 \\ \underline{x\ 60} \\ 00 \\ \underline{1680} \\ 1680 \end{array}$$

Although 11 of the 12 students used a conventional algorithm it did not always lead to the exact answer. Place value error led to students giving unreasonable answers. This is illustrated by P5's response to Item 6 (48 x 5).

P5: Na'aku ngaue'aki pe liunga pe 'a e nima mo e valu 'o ma'u 'a e fanoa (40) pea liunga 'a e nima ki he fa 'o ma'u ai 'a e uanoa peau tanaki kinautolu (40 mo e 20) 'o ma'u ai 'a e ononoa (60).

P5: I just multiplied five and eight and got forty (40). And multiply five and four and got twenty and then added those (40 and 20) together to get sixty (60).

2. Rounding procedure

Only one student, P8, attempted to use a rounding procedure which she used for both tasks. Although the answers given were unreasonable, her estimation for Item 5 illustrates this strategy.

P8: Na'a ku 'ai e uavalu (28) ke tolunoa (30) pea liunga'aki e ononoa (60) peau ma'u ai e teau valunoa (180).	*P8: I rounded twenty-eight (28) to thirty (30) and multiplied by 60 and got one hundred and eighty (180)*
I: Ko e ha na'a ke ngaue'aki ai e 30 kae 'ikai ko e uavalu (28)?	*I: Why did you use 30 but not 28?*
P8: Koe uhi ke faingofua ange ai . Peau ma'u ai e teau valunoa (180).	*P8: Because it is easier to work with. And I got one hundred and eighty (180).*

It is evident from P8's response that an attempt was made to round one of the numbers (28) involved to make it *"easier to work with."* However, a place value error led to the formulation of an unreasonable estimate.

3. Subtracting numbers

In addition to place value errors, another student made errors in the type of operations used to formulate an answer. This strategy type clearly showed that they student read the problem as a subtraction rather than as a multiplication task. As mentioned above, P5 had difficulty with the two items involving multiplication. In providing 48 as an estimate to Item 5 (ie. 28 x 60), she used a subtraction strategy.

P5: Na'a ku kole pe 'a e uavalu (28) ki he ononoa (60) 'o ma'u ai 'a e favalu (48).	*P5: I subtracted twenty eight (28) from 60 and got forty-eight (48).*
I: Na'e anga fefe ho'o fai e fo'i kole ko ia?	*I: How did you do that subtraction?*
P5: Kole noa (0) ki he valu (8) pea kole 'a e ua (2) ki he 6 'o ma'u ai 'a e favalu.	*P5: Eight (8) minus Zero (0) and six (6) minus two (2) and you got forty eight.*

When the researcher pointed out that this task involved multiplication rather than subtraction P5 said *"Ko e founga pe ia"* (*That is the only method*).

Comment

Overall, the subjects' responses to tasks requiring multiplication showed that 11 out of the 12 students consistently used a 'conventional algorithm'. However, this did not always lead to reasonable answers as place value errors often led students to formulate unreasonable estimates. Understanding of what the magnitude of the answers might be would have helped students such as P5 recognise the unreasonableness of their answers. It is evident that students such as P5 needed help to understand what the multiplication operation does to numbers. The use of the 'conventional algorithm' was also a time consuming process. This added to the difficulty experienced by students in formulating their answers mentally.

8.5 Strategies involving division with whole numbers.

An analysis of students' responses to the tasks involving division of whole numbers (Items 7 and 8) showed that only one estimation strategy was used to solve these problems. Table 8D below summarises the number of students who used the strategy as well as number of times the strategy type was used. The table also indicates that three-quarters of the students were unable to respond to two-thirds of the items.

Table 8D: Strategy type for tasks involving division

Strategy type	Number of student who used it	Number of times it was used
1. Conventional algorithm	*6*	*8*
2. No response	*9*	*16*

1. Conventional algorithm

Half the students mentally employed a 'conventional algorithm' on eight occasions. For instance, P6 used this strategy to work out an exact answer to Item 8 (550 ÷ 9).

P6: Na'a ku vahevahe 'a e hiva (9) ki he nimenima (55) pea toki vahevahe ki he noa (0) 'o ma'u ai 'a e onotaha (61) toenga taha (1).

P6: I divided the nine (9) into fifty-five (55) and then to the zero (0) and got sixty-one (61) remainder one (1).

I: To e fakamatala ange ki he founga na'a ke ngaue'aki.

I: Can you explain that again?

P6: Uluaki vahevahe 'a e hiva ki he nimenima (55). Hiva 'a e ono (6) 'oku nimafa (54), kole ia ki he (55) to e ai 'a e taha (1). 'Omi 'a e noa ki he taha ma' ai 'a e hongofulu (10). Vahevahe 'a e hiva ki he hongofulu. Hiva 'a e taha 'oku hiva (9), kole ki he hongofulu (10) ma'u ai e taha. Ko e mo'oni ko e onotaha (61) toenga taha.

P6: First divide nine into fifty-five (55). Nine times six is fifty-four (54), take away that from fifty-five (55), you are left with one (1). Bring the zero down to the one making it ten. Then divide nine into ten. Nine times one is nine, take that away from ten [10 - 9] and got one. The answer is sixty-one (61) remainder one.

The application of a 'conventional algorithm' appears to dominate this student's thinking although not necessarily with an understanding of the reasonableness of results. The estimate given to Item 7 (3890 ÷ 13) was 241331 remainder one which was read out by P6 as *ua fa taha tolu tolu taha toenga taha (two four three three one remainder one).*

2. No response

Analysis of the data showed that of the nine students who provided non-responses seven provided no-response to both Items 7 and 8. The high number of non responses (i.e. 16 out of 24) suggested that students had difficulty handling mental computation involving division with large numbers. However, when asked to compute answers with pen and paper, students could compute the exact answers for the same problems using pen and paper. It appears that the problems involved were not beyond the capabilities of these primary students.

It was the need to carry out the tasks mentally that hampered the performance of these students. The data also suggested that computational estimation was not a familiar concept to these students. The researcher found she needed to continually emphasise that the tasks required only an estimate. The above results indicate that students had little or no experiences with any estimation strategies for division. The use of the 'front-end strategy' or the 'rounding procedure' were not shown in the results suggesting that estimation techniques for division were not part of the students' repertoire of calculation skills. Again, students relied on 'exact mental computation'.

The data reported above show that 'conventional algorithm' was the most frequently used strategy by students. This finding is consistent with the results of a study of Mexican students' estimation skills by Reys, Reys and Penafiel (1991). As reported in chapter three one of the factors identified by Reys, Reys and Penafiel (1991) to have contributed to Mexican students' low performance on estimation tasks was lack of understanding of what "estimate" meant. It is possible that these primary Tongan students too did not understand what "estimate" meant in the context of computation, although they understood what it meant in the context of measurement.

The high number of non-responses (i.e. 16 out 24) also points to the difficulty experienced by students in division. Researchers have shown that estimation tasks involving division are generally more difficult than tasks involving multiplication, subtraction and addition (Levine, 1982, Threadgill-Sowder, 1984). This could perhaps explain the difficulty experienced by students in tasks requiring division of whole numbers.

8.6 Summary and comment

This chapter has focused on a discussion of the strategies used by the sample (n=12) of primary students to tasks involving computational estimation. The main strategy types exhibited by students reported and discussed in this chapter, the number of times a particular strategy type was used, and the number of students who used it, are summarised in Table 8E.

Table 8E: Summary of strategy types for computational estimation

Strategy Type	Addition	Subtraction	Multiplication	Division	Total
1. Conventional algorithm	13	22	20	8	63
2. Adding compatible numbers	10	-	-	-	10
3. Rounding procedure	1	-	2	-	3
4. Front-end strategy	-	2	-	-	2
5. subtracting numbers	-	-	1	-	1
6. No response	-	-	1	16	17

Conventional Algorithm

The data reported and discussed in this chapter show that in general students lacked effective strategies for computational estimation. In general, students initially tended to mentally compute exact answers using a 'conventional algorithm' and often first arranged numbers in columns before proceeding from the right-most column to the left-most column producing exact answers. Strategies used were dominated by the use of a 'conventional algorithm' taught by teachers, particularly for tasks involving subtraction, multiplication and division of whole numbers. As shown in Table 8E above, 63 out of 94 (i.e. 67%) responses employed a 'conventional algorithm'.

This tendency to seek exact answers suggests lack of familiarity or experiences with computational estimation strategies. This is also consistent with their definitions of computational estimation. As reported in chapter six of the definitions given to estimation was "fika'i pe 'i loto" (mentally figure out an answer). It seems that students saw computational estimation and mental computation as synonymous. Estimation in the context of computation is perceived as the mental process of finding an exact answer, unlike measurement estimation where students could provide a range of answers.

This result has important implication for teaching strategies other than the 'conventional algorithm' for subtraction, multiplication and division. It appears that students ought to be provided with a variety of other viable strategies to allow them to be more flexible in their approaches to problems involving multiplication and division of numbers of more than one digit.

Students' responses to the interview tasks revealed that those involving division of a four-digit number by a two-digit number, and a three-digit number by a single-digit number were difficult for students. Students who had difficulty with these tasks tended to either provide no response or attempted to make mental calculation but ended up giving no estimate. The results showed the ineffectiveness of the 'conventional algorithm' for multiplication and division particularly when the numbers involved could not be handled mentally by students. The time involved in trying to mentally compute an answer for tasks involving multiplication and division also discouraged some students from attempting to provide answers. Instead they opted to provide no response. These results point to the need to encourage the use of other techniques which should help students manage these numbers mentally and assess the reasonableness of their estimates. It is also possible that *tolerance for errors* was not yet part of students' thinking in mathematics. Rather than providing an answer that they were not sure about its accuracy they opted not to provide an answer.

Combining compatible numbers

For addition, the results showed that adding or combining compatible whole numbers was the second most frequently used strategy type suggesting that the use of this strategy type ought to be developed and used by students. It is possible that the numbers involved in the task (e.g. Item 1: 50, 150, 200, 20) encouraged the use of this particular strategy type. This strategy type was consistently used by students for this task indicating that students were flexible enough in their thinking to depart from using a 'conventional algorithm'.

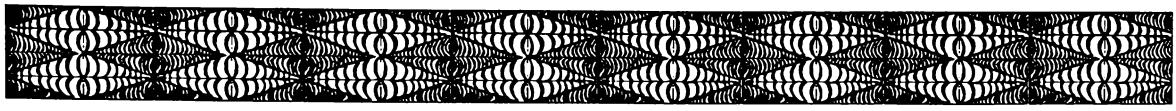
Non Responses

The data reported in this chapter also revealed that 'non responses' was the most common response to tasks involving division. That is, two-thirds of the responses fell into this category, highlighting both the difficulty involved in this operation as well as students' lack of experiences with effective strategies for estimating involving division. This result also illustrates that once numbers involved are too big for students to handle mentally the 'conventional algorithm' for division would be time consuming and therefore very ineffective to be used. This finding implies that teaching strategies other than the 'conventional algorithm' that is currently the main focus of mathematics education at the primary classroom should be developed and fostered at this level.

Taking the main findings from the interview data into consideration an instructional unit on computational estimation strategies was designed and trialled with the two primary mini-classes used for the measurement estimation activities (see chapter seven). The results of these activities are reported and discussed in the next chapter.

CHAPTER NINE

TEACHING COMPUTATIONAL ESTIMATION



9.1 Introduction

The data reported in the previous chapter indicated that in the two primary school classrooms used in this study, students showed lack of familiarity with the concept of computational estimation and exhibited a very limited number of strategies for computation. The main strategy type used by students for the four mathematical operations was the 'conventional algorithm', more appropriate for pen and paper calculations. Following the first phase of interviews a unit of instruction on computational estimation was developed (see Appendix D), to help students gain an understanding of various computational estimation procedures. The researcher had anticipated that the unit would be taught by the teachers but for reasons explained in chapter four, the researcher undertook this task.

This chapter reports and discusses the (i) instructional activities on computational estimation (9.2), (ii) main results of the second individual interviews (9.3) and (iii) effects of the instructional activities on students' estimation using three case studies (9.4). The chapter ends with a summary of the points of discussion in this chapter (9.5).

9.2 Computational Estimation Activities

Effective strategies for computational estimation identified from the research literature were selected and incorporated into a whole number instructional unit on computational estimation. Appendix D provides details of the instructional unit developed. The main purpose of the teaching activities was to explore ways of helping students adopt a variety of computational estimation strategies. Time constraints resulted in only a limited number of strategies being taught in this phase of the investigation. Table 9A below provides a summary of the units.

Table 9A : Outline of the teaching unit on computational estimation

Activity	Content
1	Introduction on computational estimation
2	Front-end strategy: initial estimate
3	Front-end strategy: with compensation
4-5	Rounding procedure
6-7	Use of distributive property of numbers
8-9	Use of compatible numbers and other techniques
10	Review of computational estimation strategies

Other ideas and concepts used in the activities were:

- understanding place value;
- mental computation;
- reasonable range;
- determining reasonableness of an answer; and
- estimation exercises and problems.

Groups IV and V, each with six students, were the mini-classes used for this phase of the investigation. Unlike the measurement estimation activities (see chapter seven) where working in pairs, use of whole group discussion and activities combined with discussion and reflection on answers were the main teaching approach, a decision was made to use mainly an expository teaching method, whole class discussion and students as teachers or demonstrators. The rationale for using this teaching approach were:

i) Expository teaching particularly in computation was most familiar to students.

Expository teaching was the most commonly used strategy for teaching mathematics in the Tongan classrooms. Indeed the researcher's mathematical learning in Tonga was through an expository teaching approach. Twelve years later the classroom observations showed the same teaching approach being used in the mathematics classroom. For the students in this study expository teaching was the most familiar mode of classroom instruction. Therefore, rather than exploring unfamiliar ways of teaching computational strategies, the researcher decided to use the method with which students were most familiar.

(ii) Using students' existing ideas

One of the implications of the constructivist model (see chapter one), the theoretical framework underlying this study, is that students' ways of thinking and knowing should be the starting point of the instructional process. Students actively construct their own meanings in unique ways as they interact with their environment. Given that the interview data revealed students' reliance on conventional algorithms, a lack of familiarity with the type of answers required for computational estimation (i.e. inexact answers) and limited number of computational estimation strategies, an important issue was how to provide a learning environment in which students could actively construct for themselves appropriate ideas about computational estimation and adopt a variety of estimation strategies. As Cobb (1990) and Zevenbergen (1991) have suggested, although students construct their own mathematical knowledge, they do not do so in isolation. Interactions with both the teacher and other students give rise to essential learning opportunities. Recognising that the dominance of the 'conventional algorithm' had resulted in students focusing on the right-most digits for computation, it seemed to the researcher that the expository approach should help students to quickly shift their focus and attention to the left-most digits in numbers. This shift in focus

might never happen if students were left to develop these skills on their own, in the length of time (i.e. two lessons) allocated for the front-end strategy. Thus, a decision was made to teach estimation techniques which could add to the 'conventional algorithm' strategy type already developed and used by students.

ACTIVITY ONE: COMPUTATIONAL ESTIMATION

The interview data reported in chapter eight showed estimation in the context of computation as an unfamiliar concept to students in both classes. Thus, one of the main objectives of this first activity was to clarify students' definitions of fakafuofua (estimation) in the context of estimation. The phrase 'fakafuofua fakafika' and 'fakafuofua 'i he fika' were used in the teaching activity to refer to computational estimation. However, throughout the discussion the term 'fakafuofua' (estimation) was used. Students' definitions of estimation were written on the blackboard. The definitions given by students included:

fika'i pe 'i loto	*mental computation*
kumi pe ha tali 'oku ofiofi ki he tali totonu	*finding an answer close to the exact answer*
kesi	*guess*
mate'i	*guess or figure out*
fakamahamahalo	*conjecture*

A discussion that followed revealed that students perceived fakafuofua (estimation) as fika'i pe 'i loto (mental computation). Five examples were put up on the blackboard and students were asked to mentally formulate estimates to these exercises. The answers provided by students were the exact answers. These were consistent with the types of answers given in the individual interviews. Thus, a major task for this activity was to encourage students to think of and accept the type of answers involved in estimation exercises (i.e. often inexact answers).

A second objective for Activity One was to ensure that students had 'good' understanding of the place value of numbers. For example, students were asked identify the 'mahu'inga fakatu'unga' (place value) of the underlined digits in the following set of numbers .

1.	4<u>3</u>	5.	<u>3</u>2 856
2.	2<u>3</u>4	6.	7<u>3</u> 543
3.	<u>3</u>24	7.	32 8<u>5</u>6
4.	<u>3</u> 567	8.	73 <u>5</u>43

For the above examples, students had no difficulty identifying and verbalising the appropriate place value of the underlined digits. The responses given by students were:

1. *matelau (ones)*
2. *hongofulu (ten)*
3. *teau (hundred)*
4. *afe (thousand)*
5. *mano (ten thousand)*
6. *afe (thousand)*
7. *hongofulu (ten)*
8. *teau (hundred)*

Students responses to this activity showed that they came to the instructional activity with proficiencies in naming the place values of numbers. As one student suggested *'Oku mau fa'a fai 'emautolu 'i lokiako* (We often do those type of activities in the classroom). Thus, students were familiar with such activities. However, a few of the students later showed that their ability to name place values did not guarantee an understanding of the order of magnitude of answers in later activities.

Using the data given by ten students in the first interview, the teacher pointed out to students that 10 of the 12 students who participated formulated strategies that they had developed on their own, mainly the 'adding compatible numbers' strategy type. The use of this strategy type illustrated one of the possible alternative procedures for working out answers to a computational exercise. The remaining activities emphasised the use of the *front-end*, *rounding*, *distributive properties of numbers*, and *compatible numbers.*

To begin all activities, a five-minutes mental computation exercise was used whereby students mentally computed answers to 10 tasks. This aimed at encouraging students to use mental computation rather than using pen and paper calculation type techniques. The limited time available for students to carry out the exercises led to some students managing to complete only a few tasks, missing items in the middle order. Initially many students complained about the fast pace of the exercise and the limited amount of time available. Among the common complaints were: *'Oku fu'u vave ho'o lau mai 'a e ngaahi mata'ifika* (You're reading out the task too fast); *'Oku si'isi'i e taimi ia'*(There is not enough time); *Toe lau mai ange fo'i fehu'i nima* (Can you read out exercise five again). These mental exercises highlighted the need to use other strategies and the limitation of relying on the 'conventional algorithm', a strategy type considered more appropriate for pen and paper calculation.

ACTIVITY TWO: FRONT-END STRATEGY - INITIAL ESTIMATE

The main purposes of Activity Two were to introduce the:

(i) front-end strategy as a method of obtaining an initial estimate; and

(ii) concept of reasonable answer by establishing lower bounds for addition and upper bounds for subtraction, that is, the idea that the answer to a computational exercise/problem would be greater or less than a specified value.

The activity was introduced using this example:

Na'e 'alu 'a Sione 'o tanaki niu mei he'ene ngoue'anga. Na'a ne 'uluaki tanaki 'ene fo'i niu 'e nimangofulu ma ua (52) 'o Fokotu'u. Pea ne to e 'alu 'o tanaki 'ene fokotu'unga 'e taha ko e fo'iniu 'e tahanima (15). Ko e fo'iniu nai 'e fiha na'e tanaki 'e Sione.	*Sione collected some coconuts from his plantation. He collected fifty-two (52, coconuts and put them into one pile. Then he collected another fifteen (15) coconuts and put them under the mango tree. About how many coconuts did Sione collected altogether?*

Students responded by giving sixty-seven (67) as an answer, obtained by mentally computing the answer.

In order to discourage the use of a conventional algorithm, the numbers selected should be big enough to encourage students to attempt to use an estimation technique. Thus, the numbers involved in the first example were changed to:

(2).	78	(3).	207
	+ 64		+ 109
	130		300

In order to have some ideas of the number of coconuts collected by Sione, students were required to think about the leftmost digits of the numbers involved and to determine the lower bound or a reference number for the answer. For instance, 60, 130 and 300 would be the lower bounds for Example 1, 2, and 3 respectively.

An initial estimate for Example 2 would be 130, that is, a lower bound for the answer. Thus the students were encouraged to use answers such as:

Ko e tali 'e lahi ange ia he 130	*The answer would be bigger than 130.*

The responses of the students in both groups were very positive. The technique were adopted and used by students. In line with the objectives of the draft mathematics syllabus (Tonga Ministry of Education, 1992a), the term "front-end strategy" was used in addition to the Tongan explanation of the term "*fakafuofua ngaue'aki e mata'ifika ki to'ohema*". (Estimation using the left-most digits). In a discussion of the use of this technique students responses included:

vave ange ia	*it is quicker*
faingofua ange ke fikaloto'i	*easier to manage mentally*
faingofua ange ke ma'u ha mo'oni	*easier to obtain an answer*
faingofua ange hono ngaue'aki e mata'ifika lalahi (e.g. 43694 + 24789)	*easier to use big number (e.g. 43694 + 24789)*

The students were given opportunities to demonstrate their newly acquired skill by sharing their ideas during whole group discussions and/or demonstrating the use of a particular

technique on the blackboard to the whole group. One of the value of the exercises was that the focus on the front-end or leftmost digits encouraged students to think of multi-digit numbers as whole entities, rather than focusing on single digits.

ACTIVITY THREE: FRONT-END STRATEGY - WITH COMPENSATION

Again a five-minute mental exercise preceded the activity, whereby students were asked to use the front-end technique in formulating their estimates. Results of this exercise showed that students could manage more of the exercises involved. The use of this technique in the mental exercise demonstrated its value as a computational estimation tool.

The aims of Activity Three were to (i) develop the use of a compensation process with the front-end process, (ii) determine upper/lower bounds for an estimate, and (iii) determine the order of magnitude for the estimates.

The compensation process involved the adjustment of the initial estimate to make more reasonable estimates. For example,

Example 3

$$\begin{array}{r} 207 \\ +109 \end{array}$$

Using the front-end strategy would give *300*

Considering 7 and 9 one can add another 10 to give a better estimate $\underline{+10}$ *310*

Example 4

$$\begin{array}{r} 579 \\ +\ 344 \end{array}$$

Ngaue'aki 'a e mata'ifika taupotu ki to'ohema, ko e mata'ifika taupotu ia ki lalo he tali (800)	*Using the front-end strategy gives 800 i.e. a lower bound for the answer*
Kapau te tau vakai ki he kolomu hono hoko (7 + 4) 'e ma'u ai 'a e teau hongofulu (110).	*If we consider the numbers in the next column (7 + 4) that would another one hundred and ten (110).*
Toe tanaki atu ia [110] ki he 800 'o ma'u ai 'a e 910.	*If we add that [110] to 800 you would get 910.*

A improved estimate would be about 910. The whole group discussion focused on establishing an initial estimate as was the focus of Activity Two. This was followed by shifting the focus to the next column of numbers to determine the adjustment that needed to be made to the initial estimate in order to give a better estimate. Students worked in pairs or groups of three to discuss these processes involved. This was followed by individual work on a worksheet to formulate an estimate using the front-end strategy with necessary

compensation or adjustment. Students could seek out the help of their peers and/or the teacher for confirmation of their techniques.

The demonstrations given on the blackboard by students revealed that all students had little difficulty using this process. It is important to note that in a few cases students continued to make errors in their estimation, particularly place value errors.

One of the advantages of the front-end strategy compared to the use of a conventional algorithm is that focusing on the leftmost-digit allowed the respondents to have some idea of what the magnitude of the answer might be, whereas, one would need to wait until the end of calculation using a conventional algorithm to have some idea of the magnitude of the answer (Reys et al., 1982; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991; Reys, Reys and Penafiel, 1991; Reys, 1993).

ACTIVITY FOUR-FIVE: ROUNDING PROCEDURE

The five-minute exercise was again used to stress the importance on mental work. Students were encouraged to use the 'front-end' procedure to formulate answers to the ten metal exercises, thus reinforcing the use of the strategy type used in the previous activity. More experiences in carrying out mental exercises led to less complaint as students changed their focus to the tasks at hand.

The 'rounding procedure' was the second technique used in the activity. One of the skills involved in an estimation process is the use of approximate number. The rounding procedure is an approximation technique used to find approximate numbers which could be used in the estimation.

The teacher demonstrated and explained the use of the rounding procedure. The examples used earlier for the front-end strategy were again used to demonstrate this procedure.

Example 1 52 + 15 Round to the nearest ten

'Oku ofiofi ange 'a e 52 ki he 60 pe ko e 50? *Is 52 closer to 60 or 50?*

All students recognised that 52 was closer to 50, so 52 was rounded down to 50.

'Oku ofiofi ange 'a e 15 ki he 10 pe ko e 20? *Is 15 closer to 10 or 20?*

'Oku tu'u ia he vaeua, mama'o tatau pe. *It is half way, same distance.*

Given that 52 was rounded down to 50, 15 would be rounded up to 20 to compensate for the 2 lost in rounding 52 to 50. Thus, the exercise became: 50 + 20 which would give an estimate of 70.

One of the advantages of using the rounding procedure was that it produced numbers that would be easier to manage mentally. Rather than encouraging students to always round up numbers that end with five, students were encouraged to always consider the numbers involved in the tasks before making a decision on whether to 'round up' or 'round down'. Thus, the concept of 'compatible rounding' or 'flexible rounding' was also introduced.

For example,

$$\begin{array}{r} 57 \\ +\ 15 \end{array} \quad \text{became} \quad \begin{array}{r} 60 \\ +\ 10 \end{array}$$

Subtraction exercises were also used to further demonstrate the use of the rounding procedure.

Example 1:

$$\begin{array}{r} 73 \\ -\ 49 \end{array} \quad \text{became} \quad \begin{array}{r} 70 \\ -\ 50 \end{array}$$

giving 20 as an answer.

$$\begin{array}{r} 82 \\ -\ 15 \end{array} \quad \text{became} \quad \begin{array}{r} 80 \\ -\ 10 \end{array}$$

giving 70 as an estimate

A worksheet on addition and subtraction exercises was used by students to practice the technique of rounding. Students worked individually and then discussed their answers with the whole group. The teacher moved around the group to help students.

Rounding numbers involved in multiplication and division exercises were used to:

(i) determine the order of magnitude of the answer; and

(ii) establish an estimate.

Exercises such as

$$21 \div 3 = 7$$

$$210 \div 3 = 70$$

$$2100 \div 3 = 700$$

$$21000 \div 3 = 7000$$

were used to explore ideas about magnitude of the answers.

One student showed weakness in determining the magnitude of the answers, particularly for tasks involving multiplication and division. Although he had no difficulty with naming place value the student consistently made place value errors in his responses. For example, for tasks involving 848 ÷ 7 and 9256 ÷ 8 the student provided 10 and 100 respectively. The teacher encouraged the student to reflect upon his answer and check for the reasonableness of his estimates by multiplying the estimate by the divisor thereby allowing the student to self-correct his estimate.

ACTIVITY SIX-SEVEN: USE OF DISTRIBUTIVE PROPERTY OF NUMBERS

To help students use the compensation process, the distributive property of numbers was introduced. It became evident that this was not a new concept to students it had been covered in their mathematics classroom. For example,

$$4 \times 15 = 4 \times (10 + 5)$$
$$= 40 + 20$$
$$2 \times 39 = 2 \times (30 + 9)$$

The use of this process was to encourage students to focus on the leftmost digits rather than the right most digits.

ACTIVITY EIGHT-NINE: USE OF COMPATIBLE NUMBERS AND OTHER TECHNIQUES

The interview data revealed that students used or combined numbers which are easier to combine to formulate their answers particularly for addition problems. Activity eight concentrated on activities encouraging students to use compatible numbers, mainly those that add to ten or multiples of ten.

The 'adding compatible numbers' strategy was another procedure used by students which would increase the flexibility with which students think about numbers and their computational estimation strategies. Another technique used for division was:

Vahevahe nima 'oku tatau ia mo e vahevahe hongofulu pea liunga'aki 'a e tali 'e ua.	*Dividing by five is the same as dividing by ten and then multiplying the answer by two*

ACTIVITY TEN: REVIEW OF COMPUTATIONAL STRATEGIES

The main purpose of Activity Ten was to review students' understanding of the estimation techniques used in the activities, namely:

- front-end strategy;
- rounding procedure;
- compatible numbers; and
- distributive property of the operation involved.

Discussion

The students' responses to the activities showed the importance of providing a variety of viable computational estimation strategies to class 6 students. Despite the limited nature of the activities, the data suggest that students whose computational strategies were limited to the use of 'conventional algorithm' could be encouraged to develop and use a variety of other

strategies for computational estimation. The first interview data showed that using a 'conventional algorithm' might not be the most effective strategy for computational estimation particularly for tasks involving division and multiplication of large whole numbers. The estimation strategies (i.e. front-end, rounding, distributive) taught in the instructional unit appeared to help students develop understanding of the order of magnitude of the answers and hence its reasonableness. The effectiveness of the instructional activities is illustrated using case studies of three students. These are presented and examined in the following section.

9.3 Main results of the second individual interviews

The main strategy types used by students in the second interview were again categorised under the same categories identified in the analysis of the data from the first interview. Data from five students who completed all the activities and who took part in the second interview are examined in this section to illustrate the types of strategies that they used in the second interview. Table 9B below, summarises the number of times each strategy type was used in the second interview.

Table 9B: Summary of students' strategies in the second interview

Strategy Type	Addition	Subtraction	Multiplication	Division	Total
1. Conventional algorithm	2	7	6	3	18
2. Rounding procedure	2	2	4	7	15
3. Adding compatible numbers	5	-	-	-	5
4. Front-end strategy	-	1	-	-	1
5. Flexible rounding	1	-	-	-	1

* Flexible rounding involved rounding to numbers other than multiples of ten.

The data presented in the above table shows that in 18 out of the 40 total number of responses, a conventional algorithm was used. This suggests a shift to a different strategy. Another 15 used the 'rounding procedures' and a further 5 used the 'adding compatible numbers' strategy type. The 'front-end' and the 'flexible rounding' procedure were used only on one occasion each. Data suggest that students continued to use the 'conventional algorithm' as their main strategy types for subtraction. It could be that the nature of the task involved in Item 3 (i.e. determining whether one has enough money) lead students to use a 'conventional algorithm' in order to ensure that they have enough money to make the purchase. The data suggest that the 'rounding procedure' was the main strategy types for division. Moreover, it was used in four occasions to solve the multiplication tasks.

9.4 Effectiveness of the teaching activities

Data from the second interview showed a major increase in the use of the 'rounding procedure' particularly for tasks involving subtraction, multiplication and division. The first individual interview data showed that the students responded to the computational estimation tasks mainly by using a 'conventional algorithm'. The second set of interviews revealed that some students had changed the types of strategy used for estimation. It is important to point out that no formal teaching took place in the classroom after the Secondary School Entrance Examination. Given that the study of computational estimation was conducted after the Secondary School Entrance Examination, it is reasonable to claim that the changes in the strategy types identified in the data were a direct result of their participation in the teaching activities discussed above.

A comparison of the frequencies with which the strategy types were used by students (n=5) in the first and second interviews are summarised in Table 9C below.

Table 9C: Comparison of the strategy types used by students in the first and second interview

Strategy Type	Addition		Subtraction		Multiplication		Division	
	1st	2nd	1st	2nd	1st	2nd	1st	2nd
1. Conventional algorithm	6	2	10	7	10	6	8	3
2. Rounding procedure	-	2	-	2	-	4	-	7
3. Adding compatible numbers	4	5	-	-	-	-	-	-
4. Front-end strategy	-	-	-	1	-	-	-	-
5. Flexible rounding	-	1	-	-	-	-	-	-
6. Non Response	-	-	-	-	-	-	2	-

As previously reported and discussed in chapter eight, students tended to rely on the 'conventional algorithm' to formulate their answers. However, in the second interview, the data showed a decrease in the frequencies with which the 'conventional algorithm' was employed by students and increased use of the 'rounding procedure'. Whereas in the first interview, 34 out of the 40 responses employed a 'conventional algorithm', the second interview data revealed this frequency was almost halved (i.e. 18 of the 40). The long term effect of this instructional programmes is relatively unknown. However, the results provide evidence of the value of specific teaching activities on computational estimation strategies.

The results also showed the 'front-end' strategy was used by only one of the five students in the sample. This suggests that the two one-hour sessions on the 'front-end' process did not make any major impact on the skills and strategies developed by students.

Given that the study of computational estimation skills was carried out after the Secondary School Entrance Examination, it is reasonable to assume that the changes that occurred in the types of procedures used by students were a direct result of the instructional activities. The nature of the change and non-change in strategy types are summarised in Table 9D below.

Table 9D: A summary of the change and non-change in strategy types used by students

Change or non-change in strategy from pre-teaching to post-teaching	Number of students	Number of times the change or non-change occurred
1. 'Conventional algorithm' to 'Conventional algorithm'	5	18
2. 'Conventional algorithm' to 'Rounding procedure'	4	13
3. 'Conventional algorithm' to 'Flexible rounding'	1	1
4. 'Conventional algorithm' to 'Front-end strategy'	1	1
5. 'Conventional algorithm' to 'Compatible numbers'	1	1
6. 'Compatible numbers' to 'Compatible numbers'	4	4
7. 'Non response' to 'Rounding procedures'	2	2

Table 9D shows that in 18 instances five students who used a 'conventional algorithm' in the first interview continued to use the same strategy in the second interview. In addition, four students who used the 'adding compatible numbers' in the first used the same strategy in the second. The above table also shows that the major shift in strategy types was from 'conventional algorithm' to 'rounding procedure'. This indicates that the 'rounding process' became a main strategy type for students in the second interview.

Detailed analysis of the responses of each student suggested individual differences in the nature of the change in strategy types that occurred. Three categories were identified from the students responses. These were:

1. Two students who consistently showed a shift from 'conventional algorithm' to the 'rounding procedure'.
2. One student who showed a variety of strategy types.
3. Two students who continued to used the 'conventional algorithm' and showed little change in the strategy types used.

Three cases studies of students representing each of the above categories in turn are now reported to illustrate the impact of the teaching program. The names used in the case studies are pseudonyms to protect the identities of the actual students.

CASE 1: A student who adopted different strategy types

Lofa was a first year class six female student. She was a very attentive student, and had no difficulty discussing her views and ideas. Table 9E below summarises the type of strategies used by Lofa before and after participating in the teaching/learning activities.

Table 9E: Summary of Lofa's pre-teaching and post-teaching strategies

ITEMS	PRE-TEACHING STRATEGY	POST-TEACHING STRATEGY
Addition		
1	*Compatible number*	*Compatible number*
2	*Conventional algorithm*	*Rounding procedure*
Subtraction		
3	*Conventional algorithm*	*Conventional algorithm*
4	*Conventional algorithm*	*Rounding procedure*
Multiplication		
5	*Conventional algorithm*	*Rounding procedure*
6	*Conventional algorithm*	*Rounding procedure*
Division		
7	*No response*	*Rounding procedure*
8	*No response*	*Rounding procedure*

The shows that Lofa was already using a sound strategy for one item (Item 1) and retained this. For three-quarters of the items, however, she changed from using an inappropriate strategy to an effective one.

Estimating involving addition and subtraction

Data showed that Lofa's responses to Item 1 remained the same in both interviews, that is, she combined the larger addends first and then the smaller addends. Since all the numbers were multiple of 10 (ie. 150, 200, 50, and 20) it seems that the numbers could be easily combined and therefore 'combining compatible numbers' strategy was used in the first and second interviews. In hindsight, rounding to the nearest ten and/or the use of the 'flexible rounding' strategy type were not an option, since the numbers contained in the exercise were multiples of ten. It seems then that the numbers involved in the tasks did not allow for a variety of strategies to be used. For the second interview, the numbers involved in Item 2 were changed to 75, 80, 65 and 72. Whereas in the first interview Lofa used a conventional algorithm to add the numbers (52, 56, 54, 67) she used the rounding procedure in the second interview. In providing 290, Lofa explained:

Na'a ku fakalahi e 75 ki he 80 peau tu'u pe e 80 hoko kau fakalahi e 65 ki he 70 mo fakasi'isi'i e 72 ki he 70 'o tanaki leva ia 'o ma'u ai e 290 ...To'o pe ha hongofulu [mei he 300] koe'uhi na'e fakalahi e 75 mo e 65 [ke 80] 'o ma'u ai e 290.	*I rounded 75 to 80 left the next 80 as it is and rounded 65 to 70, rounded 72 to 70 and added those to obtain 290...I took off 10 [from 300] because I rounded 75 and 65 [up to 80] and got 290.*

Thus, a 'rounding procedure' was employed to produce an initial estimate of 300 which she then compensated for by subtracting 10 from this giving 290 as a final estimate. The above response suggests that the teaching unit has lead to a change in strategy type used. The use of the 'rounding procedure' accompanied by the adjustment of the initial estimate (i.e. 300) shows not only an understanding of the rounding procedure but also the effect of this procedure on the initial estimate in relation to the exact answer. Moreover, the student was satisfied with giving an estimate rather than an exact answer, indicating an awareness and understanding of the type of answers required for estimation tasks. It appears that acceptance of the type of answers required for estimation is necessary for adopting an estimation procedure or technique for computation.

Item 3 actually involved both addition and subtraction so this item is not considered in the analysis. Thus, Item 4 is the only subtraction tasks that is considered in this discussion. As with the addition tasks, Lofa's response to Item 4 in the second interview revealed a change in strategy type from 'conventional algorithm' to a 'rounding' strategy. In the first interview she provided $188 as an answer, but in the second interview she obtained $200 as an estimate as shown in the following explanation:

'Ai e $312 ki he 300 'o kole ia ki he 500 'o toe 200.	*Take $312 as 300 and subtract that from 500 and left with 200.*

Estimating involving multiplication and division

The responses given by Lofa to the multiplication and division items also showed a change to use of the 'rounding procedure'. Although a 'rounding procedure' was used by Lofa in the second interview she did not always round numbers to the nearest 10. For Item 6, she used rounded 48 to 40 and then multiplied 40 by 5 resulting in 200. As she explained:

'Ai pe 48 ke tatau mo e 40 pea liunga ia (40) mo e 5 'o ma'u ai e kilomita 'e 200.	*Take 48 and make it equal to 40 and multiply that (40) by 5 and got 200 kilometres*

For Item 5, however, she rounded 28 to 30 then multiplied this by 60 giving 1800 as her estimate. As she explained:

'Ai pe 28 ke tatau mo e 30 ,pea liunga 'aki ia e 60 'o ma'u ai e fu'u talo 'e 1800.	*Take 28 and make that equal to 30, then multiply that by 60 and got 1800 taro plants.*

The above responses suggest that the student did not consistently use the standard 'rounding procedure' (i.e. rounding to the nearest ten).

In contrast to the multiplication tasks, Lofa's responses in the first interview showed the tasks involving division of whole numbers were difficult to solve without pencil and paper. No response was made to both tasks. The length of time required to mentally compute an answer was another key factor in Lofa's decision not to give a response. In the second interview, however, Lofa showed no difficulty with these tasks and solved the problem using a rounding procedure. Lofa's response to Item 7 illustrates the use of the rounding procedure for the division tasks.

Na'a ku fakalahi e 3890 ki he 3900 'o vahevahe'aki ia e 13 'o ma'u ai e 300.	*I rounded 3890 to 3900 and divided that by 13 to get 300.*

An important issue that arose from the analysis of this student's responses is the various Tongan terms or phrases used for the concept of 'rounding' numbers. As reported earlier, in rounding \$312 to 300, Lofa used the phrase " *'ai pe \$312 ki he 300* " (*take \$312 as 300*). Later on she used the phrase " *'ai pe 48 ke tatau mo e 40* " (*take 48 and make it equal to 40*). For addition and division items, Lofa used the terms "fakalahi" (make it bigger) and "fakasi'isi'i" (make it smaller) to refer to 'rounding up' and 'rounding down', respectively. These terms and phrases demonstrate various ways in which this student has conceptualised the 'rounding' procedure.

Two interpretations of the 'rounding procedure' could be taken from the above responses. First, the phrase "'ai pe" or "'ai pe ke tatau" suggest that the student is saying "imagine that 312 is 300" or "imagine that 48 is equivalent or the same as 40". In this case the numbers given in the problem are replaced by numbers that would make the operation easier to carry out in the student's head. Secondly, the terms "fakalahi" and "fakasi'isi'i" suggest a different process is involved, that is, the numbers given in the tasks are made "bigger" or "smaller" before they are used in the operations. It appears that these two different processes may account for the lack or, on the other hand, use of a compensation procedure in the estimation process. Whereas, in the first case (i.e. imagining that the numbers involved were replaced by or are equivalent to other numbers) did not require the student to adjust the answer to compensate for the replacement process, in the second case making the number bigger or smaller necessitated a compensation process.

Na'a ku fakalahi e 75 ki he 80 peau tu'u pe e 80 hoko kau fakalahi e 65 ki he 70 mo fakasi'isi'i e 72 ki he 70 'o tanaki leva ia 'o ma'u ai e 290...To'o pe ha hongofulu koe'uhi na'e fakalahi e 75 mo e 65 'o ma'u ai e 290.

I rounded up 75 to 80 left the next 80 as it is and rounded up 65 to 70, rounded down 72 to 70 and added to obtain 290...I took off 10 [from 300] because I rounded 75 and 60 [up to 80]. And got 290.

It ought to be pointed out that the use of a compensation process was not consistent throughout. Although for the division tasks the student also used the terms "fakalahi" and "fakasi'isi'i", a compensation process was not used.

CASE 2: A student who adopted a variety of strategies.

An example of a student who used rounding procedures and the front-end strategy in the second interview was Pauliasi, a first-year class 6 boy who consistently used a conventional algorithm in the first interview. The strategy types used by Pauliasi before and after the teaching activities are summarised in Table 9F below.

Table 9F: Summary of Pauliasi pre-teaching and post-teaching strategies.

ITEMS	PRE-TEACHING STRATEGY	POST-TEACHING STRATEGY
Addition		
1	*Compatible number*	*Compatible number*
2	*Conventional algorithm*	*Flexible rounding*
Subtraction		
3	*Conventional algorithm*	*Conventional algorithm*
4	*Conventional algorithm*	*Front-end strategy*
Multiplication		
5	*Conventional algorithm*	*Rounding procedure*
6	*Conventional algorithm*	*Rounding procedure*
Division		
7	*Conventional algorithm*	*Rounding procedure*
8	*Conventional algorithm*	*Rounding procedure*

As with Lofa, Pauliasi retained the useful compatible number strategy for Item 1, but changed from the inappropriate conventional algorithm to acceptable strategies for three-quarters of the tasks.

Estimating involving addition and subtraction

For Item 2, Pauliasi changed his strategy type from the use of a conventional algorithm to the use of a 'flexible rounding' procedure. The 'flexible rounding' procedure is differentiated from a 'rounding procedure' as they appear to involve different processes. The 'flexible rounding' involved the student rounding the numbers involved to numbers other than the usual nearest ten. As Pauliasi explained:

Na'a ku fakafuofua'i pe 'a e 76 ke fitunima (75) peau tanaki leva ia mo e 80 [155] peau toe tanaki pe 'a e 65 [ma'u ai 220] peau fakafuofua'i 'a e 72 ki he 80 'o tanaki fakataha kinaua [220 mo e 80] 'o ma'u ai 'eku mo'oni [300].

I estimated 76 to be 75 and I added that to 80 [getting 155] and then added 65 [giving 220] and I estimated 72 to be 80 and I added those two [220 and 80] to obtain my answer [300].

Rather than rounding to the nearest ten, Pauliasi rounded 76 and 72 to 75 and 80, respectively, suggesting that he had adopted compatible rounding procedures, to numbers that he would find easier to work with. Flexibility in the choice of numbers and in the use of the rounding procedure appeared to be a feature of Pauliasi's repertoire of skills. His flexibility in thinking and in the type of strategy used was also evident in the adoption and use of strategies other than the 'rounding procedure'. As evident in his response to Item 4, Pauliasi used the 'front-end strategy', a procedure which was a main focus of Activity Two and Three. Unlike most students, who adopted the 'rounding procedure', Pauliasi used the 'front-end strategy' to formulate his estimate of $200.

Na'a ku kole pe tolu (3) ki he nima (5) 'o ma'u ai 'a e uangeau (200) pea na'e 'ikai keu to e fai ia 'e au ha fakalelei.	*I subtracted three (3) from five (5) and got two hundred (200) and I did not make any compensation.*

Although he was aware that a compensation process could be applied he chose not to use this.

Estimating involving multiplication and division

Table 9F shows a clear change in strategy types used in the two interviews for tasks involving multiplication and division. Whereas in the first interview, exact mental computation or 'conventional algorithm' was used, in the second interview both multiplication estimation tasks were solved using the 'rounding procedure'. Similarly, in the first interview he used a 'conventional algorithm' for division. This was changed to the 'rounding procedure' in the second interview, providing reasonable estimates for the two tasks. A comparison of the responses given by Pauliasi for the multiplication showed that although he produced exact answers to Items 5 and 6 (i.e. 240 and 1680) in the first interview, he used the 'rounding procedure' to produce estimates (i.e. 250 and 1800) in the second interview. The data suggest an acceptance of the types of answers required for estimation tasks.

As shown in Table 9F, Pauliasi used 'conventional algorithm' during Interview 1 to formulate answers to the two tasks involving division (i.e. Item 7 and 8). For instance, Pauliasi gave "2912 remainder 24" as an answer to Item 7 after mentally applying a conventional algorithm. Providing both the whole number part as well as the remainder part of the answer, indicates lack of familiarity with the type of answers for estimation tasks. A place value error in the answer (2912) as well as an unreasonable remainder (24) were provided, indicating a lack of awareness about the reasonableness of his answer. Given that the dividend was 13, it would be unreasonable to obtain a remainder of 24. Using and continuing the process of looking for a number which 13 could be divided into, led the student to focus more on carrying out the process without consideration of the reasonableness of the answer obtained. In the second interview, he quickly formulated an estimate of 300 using a 'rounding procedure'. As he explained:

Na'a ku fakaofiofi pe 'a e toluafe valungeau hivanoa (3890) ki he toluafe hivangeau (3900) pea vahevahe ki ai [3900]'a e tahatolu (13) 'o ma'u ai 'eku mo'oni [300]. Taki pa'anga 'e tolungeau.	*I rounded three thousand eight hundred and ninety (3890)to three thousand nine hundred (3900)and divided thirteen (13) into that [3900] to obtain my answer [300]. Three hundred dollars each.*

For Item 8, he mentally computed an exact answer (i.e. 61 remainder 1) in the first interview. However, in the second interview a shift in strategy type occurred. The 'rounding procedure' was utilised to produce 55 as an estimate. Again, the term 'fakaofiofi' was used to describe the rounding process. As he said:

Na'aku fakaofiofi pe 'a e hiva (9) ki he hongofulu (10) peau vahevahe 'aki ia [10] 'a e nimangeau nimanoa (550) peau ma'u ai 'a e mo'oni ko 'eni [55].	*I rounded nine (9) to ten (10) and I divided five hundred and fifty by that [10] and got this answer [55].*

The responses given by Pauliasi to the division tasks indicate that the term used (i.e. fakaofiofi) in the instructional unit had been adopted by the student. This contrasts with the terms (i.e. 'ai pe ke tatau (make it equal to), fakalahi (make it bigger), fakasi'isi'i (make it smaller)) used by Lofa in the second interview. This suggests that whereas Pauliasi had adopted the terms which the researcher introduced, Lofa chose to use her own terms which explains the rounding process as she perceived it.

Cases 1 and 2 discussed above were used to illustrate cases in which the student had shifted in the strategy types used. Although both had shown a major shift from the use of the 'conventional algorithm' there were also differences in the processes and language used by the students to describe the strategies that they had used. The next case, Case 3, shows a student who exhibited very little change in the types of strategies used for computational estimation.

CASE 3: A student who showed little change in strategy types

Jamie was another first year class 6 boy whose responses illustrate those who showed little change in the strategy types used in the two interviews.

Table 9G: Summary of Jamie's pre-teaching and post-teaching strategies

ITEMS	PRE-TEACHING STRATEGY	POST-TEACHING STRATEGY
Addition		
1	*Adding compatible number*	*Adding compatible number*
2	*Conventional algorithm*	*Conventional algorithm*
Subtraction		
3	*Conventional algorithm*	*Conventional algorithm*
4	*Conventional algorithm*	*Conventional algorithm*
Multiplication		
5	*Adding Numbers*	*Conventional algorithm*
6	*Conventional algorithm*	*Rounding*
Division		
7	*No Response*	*Rounding procedure*
8	*Conventional algorithm*	*Conventional algorithm*

Jamie's response to Item 8 (which involved 550 ÷ 9) showed no change in the strategy used. In both interviews Jamie used a 'conventional algorithm' to derive an exact answer (i.e. 61 remainder 1). However, unlike the first interview in which he included the remainder (i.e. 1) in his answer, in the second interview he gave only 61 suggesting that he saw no need to include the remainder in his response. Perhaps this indicates that Jamie had developed some understanding of the types of answers required for estimation tasks.

Considering Jamie's responses to the two division tasks, one cannot assume that the instructional unit did not have any major effects on Jamie's estimation skills. His response to Item 7 suggests an understanding of the 'rounding procedure'. In giving 300 as an estimate he explained:

Na'aku fakaofiofi pe 'a e toluafe valungeau hivangofulu (3890)ki he toluafe hivangeau (3900). Pea tolu (3) 'a e tahatolu (13) 'oku toluhiva (39) pea 'e fakafofonga noa leva 'a e tu'unga ki mui.	*I rounded three thousand eight hundred and ninety (3890) to three thousand nine hundred (3900). And three (3) times thirteen (13) is thirty-nine (39) and noughts will represent the place values of the last two digits.*

It is possible that the numbers contained in Item 7 (i.e. 13 and 3 890) discouraged the student from attempting to apply the 'conventional algorithm' as he had done for Item 8. It appears that the student found the numbers (i.e. 9 and 550) in Item 7 simple enough for him to apply the 'conventional algorithm'. This result appears to support the findings which showed that the numbers involved in the estimation tasks ought to be complex enough to encourage respondents to apply estimation strategies (Edwards, 1984; Reys et al., 1982; .Reys, 1986).

Like Pauliasi, Jamie used the term 'fakaofiofi' (closer to) in his explanation of the strategy used for multiplication and division. This suggests that 'fakaofiofi' (rounding) could be an appropriate term to use in the context of computational estimation. Thus, Jamie exhibited limited change in the variety of strategies he used but the data suggest that he developed a better understanding of the nature of the answers required in estimation.

9.5 Summary and comment

This chapter has examined data on computational estimation obtained from the instructional unit and second individual interviews. Despite the limited nature and scope of the activities and the tasks involved in this phase of the investigation, the data showed some important findings regarding class 6 students computational estimation skills and strategies. These are summarised and discussed in this section.

1. Initially, for the small sample (n=12) of primary students used in this phase of the study 'computational estimation' (fakafuofua fakafika) meant 'exact mental computation' (fika loto).

2. After instruction the 'rounding procedure' was the most readily adopted estimation strategy type.

3. The term 'fakaofiofi' seemed to be the most appropriate term for fakafuofua (estimation) in the context of computation.

Given that 'computational estimation' was perceived initially as synonymous with 'mental computation', the data suggested that this perception together with students' unfamiliarity with estimation in the context of computation resulted in reliance on a 'conventional algorithm'. It was reasonable for students to use the only computational strategy that they were familiar with, that is, 'conventional algorithm'. Thus, in order to increase the variety of skills that students' could use for computation estimation, experiences in a variety of computational estimation strategies were required.

The instructional units on computational estimation strategies showed that these need to be a major focus of the students' mathematical learning experiences. During the trial of the instructional units on computational estimation, the students adopted and used the strategy types taught. All students demonstrated use of the strategy types used during the instructional activities. These included the front-end strategy, rounding procedure, distributive properties of numbers, and the compatible numbers. However, after its completion the second individual interview data showed that the 'rounding procedure' was the main strategy type adopted by the students. In contrast to the first interviews (see chapter eight) which reported 'conventional algorithm' as the most familiar and frequently used strategy, the second interview revealed a major shift from the use of this process to that of the rounding process.

The data also showed that students became more aware of the types of answers required for computational estimation (i.e. estimates). The use of the 'rounding procedure' allowed students to quickly produce answers in shorter lengths of time, particularly for the multiplication and division tasks. This also helped students understand the value of this procedure as an estimation technique. In addition, this also reduced the number of unreasonable answers provided by students. The 'conventional algorithm' for division and multiplication often led to incorrect answers, sometimes caused by the students focusing more on executing the processes involved while neglecting to check for the reasonableness of their answers. Moreover, they tended to focus more on each digit. It appears that in the rounding process students treated numbers as whole entities rather than focusing on single digits.

For instance, as discussed in the previous section, in the first interview Pauliasi proceeded to divide 3890 by 13 by focusing first on 38 and 13. He then looked for a number close to 38 which 13 could be divided into. The process was continued which resulted in 2912 remainder

24. In the second interview, however, 3890 was rounded up to 3900, then divided by 13 providing a reasonable estimate of 300. Thus, focusing on digits as in the 'conventional algorithm' may have led the student to not consider the place values of these digits and consequently overlook the magnitude of the answer generated.

Analysis of the interview data which concentrated on identifying the first strategy that students used to formulate estimates revealed three main types of changes in strategy types occurred. These were: (i) two who showed a shift from the 'conventional algorithm' to the 'rounding procedure'; (ii) one who showed a variety of estimation strategies; and (iii) two who showed little change in the use of estimation strategies taught in the instructional phase. Thus, the outcome of this small investigation has provided data on the nature of the changes that could occur if students are provided with learning activities on computational estimation. It is possible of course, that the students may have developed understanding of other strategy types.

As one of the objectives of the instructional unit was to explore effective teaching strategies for computation, one important outcome of these activities was the exploration of the language that could be used for teaching computational estimation strategies.

The responses given by students revealed the term 'fakaofiofi' used in the instructional unit was readily adopted by students. 'Fakaofiofi' (closer to) was used to refer to the rounding process. This was a direct result of the exercise on 'fakaofiofi ki he hongofulu ofi taha' (round to the closest ten) or 'afe ofi taha' (closest thousand). The term 'fakaofiofi' was adopted and used in the instructional phase to refer to the rounding process. It is important to note that not all students used 'fakaofiofi'. As discussed in the previous section of this chapter, terms such as 'fakalahi' (make it bigger), fakasi'isi'i (make it smaller), 'ai pe (take it as), and 'ai pe ke tatau (make it equal or is equivalent to) were also used by students to describe 'rounding up' and 'rounding down' processes. Thus, the students actively generated their own terminology based on their understanding of the processes involved. It is important to emphasise that these terms adequately reflect the processes involved in the rounding procedure. Hence, it is possible that they too could be used in the teaching of estimation strategies.

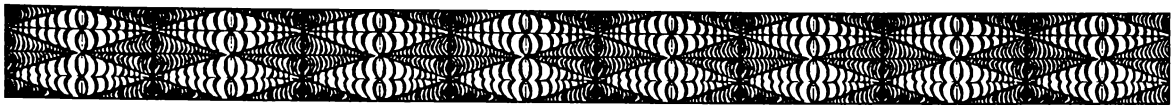
Although other strategies for computational estimation (i.e. front-end strategy and distributive property of numbers) were also emphasised in the teaching activities, only a few students used these strategies in the second interviews. It is possible that students saw the front-end strategy as procedures that lead to answers that may be too far off from the exact answer. Because the instructional activities focused on the use of the front-end strategy as a technique for producing initial estimates, it could be that students saw this as an inappropriate procedure for the estimation tasks in the second interview.

The responses of students to activities on the 'front-end' strategy suggest that 'fakafuofua ngaue'aki e mata'ifika taupotu ki to'ohema' (estimating using the left-most digits) could be a more appropriate phrase to use in the classroom. A direct translation of the term 'front-end' (e.g. mata'ifika taupotu ki mu'a) does not clearly convey the processes involved in the front-end strategy. Careful consideration of the terminology used in the mathematics classroom is needed to identify the most appropriate terms or phrases for this important and powerful mathematical process.

Given that the instructional unit only lasted ten lessons, it is not surprising that students failed to use strategies other than the 'rounding procedure'. However, the analysis of the data seems to suggest that the numbers involved in the tasks might have limited the choice of strategies for particular tasks. Moreover, the focus of the present study on the first strategy type used by students might also have affected the exhibition of other strategy types.

The main findings of the present study together with their implications for mathematics education in Tonga are discussed in the next chapter.

CHAPTER TEN
SUMMARY, CONCLUSIONS AND IMPLICATIONS



10.1 Introduction

The present study was designed to investigate estimation in Tongan schools by focussing on students' measurement and computational estimation skills and strategies, and to explore ways of teaching these topics. A small sample of form 3 (n=18) and class 6 (n=12) students from three secondary and two primary schools in Tonga were interviewed before and after taking part in an instructional unit on measurement estimation activities. In addition, the computational estimation ideas of the twelve primary students were explored through individual interviews before and after an instructional unit on computational estimation strategies. Students' ideas and strategies identified from the first individual interview data were used to design an instructional unit on measurement and computational estimation. Thus a study of measurement estimation of both primary and secondary students preceded that of computational estimation of primary school students. In particular, the present study investigated the following:

Learning

i) What estimation strategies are used by Tongan students in measurement and computation? How have the students constructed these strategies? How effective are the students' estimation strategies.

ii) What other mathematical skills and/or concepts do students need in order to become proficient estimators?

Teaching

iii) What are the mathematics teachers' views regarding estimation? What estimation strategies are currently taught and encouraged by mathematics teachers in Tongan classrooms?

iv) How can students' existing ideas about estimation be utilised to develop teaching strategies which will further develop this mathematical process?

This final chapter summarises the main findings of the study (10.2) and draws conclusions (10.3). It then examines the implications of the results for learning (10.4); teaching (10.5); resource and curriculum development (10.6); assessment and future research (10.7). This chapter ends with the researcher's concluding thoughts about the present study.

10.2 Summary of Main Findings

This section summarises the main results of this investigation. It also discusses how the research findings inform the research questions.

QUESTION 1: STUDENTS' ESTIMATION STRATEGIES

Estimation strategies used by Tongan students in measurement and computation

The results from the first individual interviews involving estimating measures of length, weight and capacity showed students utilising a variety of strategies. The main strategy types and the percentage of responses which used a particular strategy are summarised below.

STRATEGIES FOR ESTIMATING LENGTH.

i. Multiple of a measuring instrument (46%);
ii. Multiple of a benchmark (30%);
iii. Part of a measuring instrument (17%);
iv. Part of a benchmark (4%); and
v. Guess (2%).

STRATEGIES FOR ESTIMATING WEIGHTS

i. Comparison to a benchmark (62%);
ii. Student's feel for the size of the weight (13%);
iii. Just a guess (11%);
iv. Light or no weight (8%); and
v. Non responses (6%).

STRATEGIES FOR ESTIMATING CAPACITY

i. Comparison to a benchmark (35%);
ii. Just a guess (32%);
iii. Comparison to a measuring instrument used in class (20%);
iv. Amount of money used to fill up a petrol tank (7%); and
v. Non responses (6%).

STRATEGIES FOR COMPUTATIONAL ESTIMATION

The sample of primary students (n=12) provided data on the computational estimation strategies. The results showed students relied on the use of a 'conventional algorithm'. This strategy was used in 65% of the responses. Non responses made up 18% of the responses, while the remaining 17% of the tasks used appropriate estimation procedures. These were:

i. Adding compatible numbers;
ii. Rounding procedure; and
iii. Front-end strategy.

How the students constructed these strategies

Both school and out-of-school ideas were used to formulate answers to measurement estimation tasks. The results showed that the ideas and concepts of standard measuring instruments developed from classroom experiences were the main sources of ideas for estimating measures of lengths. In contrast, ideas developed in everyday life experiences provided the basis for the strategies and benchmarks used for estimating measures of weight and capacity. The 'multiple of a measuring instrument', 'part of a measuring instrument' and 'comparison to measuring instrument used in class' were strategies based on ideas developed in the classroom. On the other hand, strategies involving the use of benchmarks, 'feel for the size of the weight', 'amount of money used', 'light or no weight' were developed mainly from out of school experiences. That students bring to the classroom various ideas and concepts such as the weight of *kilo of sipi* (kilo of mutton), and the distance from the flagpole in town (Nuku'alofa) to Mailetaha being one mile, suggests that estimation benchmarks and skills acquired by students in out of school contexts were used to generate answers to measurement estimation tasks. Using benchmarks was also a key measurement estimation strategy identified by Hildreth (1983) and Siegel et al. (1982).

The primary students' computational estimation strategies, on the other hand, were dominated by school taught algorithms. Students were unfamiliar with estimation in the context of computation, and therefore relied on the only computational strategy with which they were familiar (i.e. the conventional algorithm) to produce exact answers. This finding is similar to the results of a study by Reys, Reys and Penafiel (1991) of the computational estimation strategies of Mexican students. Data revealed that some students had developed the 'adding compatible numbers' strategy on their own as a response to the numbers contained in the tasks (e.g. 50, 150, 200, 20). Multiples of ten seem to lead students to be more flexible in the use of numbers.

Effectiveness of the students' estimation strategies

The effectiveness of the above measurement estimation strategies in providing reasonable estimates seems to depend on the accuracy and appropriateness of the information regarding standard measures and benchmarks. Incorrect information regarding benchmarks and standard measures, inappropriate mental images of standard measures, and errors in comparing relative sizes of the objects were not useful to the estimation process.

The use of a 'conventional algorithm' for estimation was effective only if the numbers involved were simple enough for students to handle mentally; it became ineffective for tasks involving multiplication of two-digits factors, and division of four-digits by a two-digit number. 'Adding compatible numbers' was a strategy developed by students to handle addition of multiples of ten.

This appropriate strategy was effective in the first interview and was retained by students in the second interview.

QUESTION 2: SKILLS AND CONCEPTS ASSOCIATED WITH PROFICIENT ESTIMATORS

Other mathematical skills and/or concepts that students need in order to become proficient estimators

The research literature has provided data on the mathematical skills and concepts required for computational estimation. These were summarised by Sowder and Wheeler (1989) and presented in Table 3A. Data from both the first and second interviews and the teaching activities revealed that in order for students to become proficient in estimation the following skills and concepts are necessary.

1. Understanding of the nature and the type of answers required for estimation.

The students' reliance on 'conventional algorithm' for computational estimation items even when these were ineffective and time consuming suggests that understanding the nature of the estimation process and the type of answers required is necessary. As outlined by Sowder and Wheeler (1989) some of the important conceptual components of computational estimation involve the recognition that approximate numbers are used for computation, and the acceptance of more than one process for obtaining an estimate and more than one value as an estimate. One of the affective components identified by researchers was *tolerance for error* (Reys et al., 1982; Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991; Sowder and Wheeler, 1989). Acceptance that approximate numbers are used for estimation and that estimation may lead to may answers could help students become more flexible in the use of numbers given in the estimation tasks.

The language used in mathematics could also hinder the development of understanding and appreciation of estimation in mathematics. The Tongan term for 'answer' in mathematics was an important issue in the context of estimation. As discussed in chapter five the term **'mo'oni'** (answer) is one of the terms used for an answer to an exercise; for instance, ko e ha e mo'oni ki he fo'i fika ko 'eni 2 + 2? (what is the answer to this exercise 2 + 2?). The term **mo'oni totonu** (exact answer) is also used in mathematics. In everyday life the term **mo'oni** also refers to true, truth or fact. In mathematics there is a commonly held view that the answer to a mathematical exercise or problem is the mo'oni or the truth. Mo'oni totonu (exact answer) could also be interpreted as exact truth. Thus, it seems that the use of the term mo'oni might not be helpful in the development of an appreciation of estimation in mathematics. As estimation is a process that often leads to inexact answers, the results showed that terms such as tali (answer) or tali fakaofiofi (approximate answer) were more appropriate for estimation.

2. Many experiences in estimation activities.

The results revealed that students had very limited strategy types for estimating measures of capacities. Lack of experiences working with units of measurement for capacities in part accounted for the limited numbers of strategies and a tendency to make guesses. Hildreth (1983) has identified some of the important skills and concepts in measurement estimation. These include an understanding of the attribute (capacity or weight) to be estimated, concept of unit of measurement, and a mental image of the unit that is being used in the estimation task. It is also important for students to have the ability to compare objects, select and use appropriate strategies for making estimations, and to check the appropriateness of the estimation.

3. Possession of a variety of estimation strategies.

The results of the teaching activities highlighted the need to help students develop a variety of strategies for computational estimation. Specific teaching activities on computational estimation strategies (e.g. rounding and front-end strategies) have the potential to increase the variety of students' computational estimation strategies, flexibility in the use of numbers, and assessing the reasonableness of their estimates.

4. Correct information regarding benchmarks and units of measurement.

Students' incorrect ideas regarding the measures or unit of measurement particularly for weight and capacities affected the reasonableness of their estimates. Confusion about the terms kilo (kilo) and kalami (gram) and misunderstanding about which of the units to use for weight and capacities also added to the difficulty experienced by students in making estimates. Thus, in order for students to become proficient estimators it is necessary that they posses correct ideas regarding the units and benchmarks.

QUESTION 3: TEACHERS' VIEWS REGARDING ESTIMATION

The mathematics teachers' views regarding estimation

Despite the small number of teachers (n=5) involved in this phase of the investigation the researcher considers this sample as representative of the teacher population. Data from the teachers' questionnaire showed that in general estimation has not been a major part of their preservice teacher education programmes, teaching practices, and examination content. Although estimation was seen as a useful skill by teachers, data suggested a need for them to have adequate preservice and inservice programs. This is particularly important with the implementation of the new mathematics curriculum, which emphasises estimation. Teachers also saw the need for appropriate teaching resources and textbooks. More importantly there was a need for teachers to accept that estimation is an important topic that ought to be fostered in the mathematics.

Estimation strategies currently taught and encouraged by mathematics teachers in Tongan classrooms

All five teachers thought estimation strategies could be taught in the classroom but that they taught estimation only sometimes (taimi pe 'e ni'ihi). The main strategy types reported for teaching measurement estimation involved three steps:

i. estimate the quantity involved (e.g. length, mass or capacity of the object);
ii. measure these quantities, and
iii. compare the estimate and the measures to ascertain if the estimates were fehalaaki (wrong).

A limited number of computational and quantitative estimation strategies were reported by the five teachers. Only one teacher provided a useful strategy for computational estimation, namely the front-end strategy. Thus, appropriate estimation strategies and assessment procedures are not yet part of mathematics teaching.

QUESTION 4: USING STUDENTS' IDEAS TO TEACH ESTIMATION

How students' existing ideas about estimation can be utilised to develop teaching strategies which will further develop this mathematical process

Data from the first interview and the teaching activities provide some strong pointers to an approach that would be likely to enhance learning of estimation.

1. Students' various definitions of the term fakafuofua (estimation).

Various terms used by students to define fakafuofua (estimation) were used to clarify the researcher's questions and initiate discussions in the teaching activities. These definitions included:

fakaofiofi	*closer or nearer to*
fakamahamahalo	*conjecture*
mate'i	*guess*
fakakaukau'i pe 'i loto	*mentally figure out something*
fua pe 'i loto	*mentally work out the measure*
sio ke ma'u e anga 'o e me'a ko ia	*look at an object and work out its characteristics*
meimei	*nearly*

The use of the above terms and others identified from the interview data helped facilitate the discussion of ideas and concepts in the teaching activities.

2. Variety of benchmarks for estimation

Students' different life experiences were the main sources of ideas for measurement estimation skills. Their ideas of benchmarks and standard units of length, weight, and capacity provided the means by which students learn from one another. Many experiences with these benchmarks and

measures are necessary for developing skills in making comparisons, judging the relative sizes of objects, justifying their estimates, and making decisions on an appropriate range for their answers. These not only helped promote learning of estimation but students' feel for the sizes of standard measures. These also promoted the use of ideas developed in everyday life experiences to enhance learning in the mathematics classroom.

3. Understanding place value, operations with numbers and mental computation.

Even though students showed a lack of familiarity with computational estimation, their understandings of place value, algorithms for the addition, subtraction, multiplication and division, and mental computation skills assisted the shift from reliance on conventional algorithms to the use of appropriate estimation strategies. These prior knowledge and understandings allowed the researcher to focus teaching on the appropriate estimation techniques rather than developing students' understanding of these mathematical concepts and skills. Although the instructional unit was limited in nature, the results point to the value of specific teaching activities on computational estimation techniques. Data from the second individual interviews showed that participation in the instructional units on computational estimation strategies resulted in students adopting appropriate estimation strategies, particularly the use of the 'rounding procedure'. The results of the teaching activities showed students' ideas of place value and operations helped in the development of estimation skills and strategies. At the same time estimation activities also assisted in the development of ideas about the magnitude of answers; flexibility in the use of numbers, rounding procedures and the operations involved; and determining the reasonableness of their answers.

4. Ideas for teaching estimation

The exploration of the teaching activities revealed some important results for teaching estimation. These include:

i. Language used for teaching estimation

One of the main strengths of the instructional activities on measurement estimation is that it allowed the researcher as teacher to explore appropriate language and terminologies for teaching estimation in the Tongan language. The initial interviews revealed various definitions used by students to define fakafuofua (estimation). Consequently these terms were used in the instructional units for the discussion of ideas and concepts relevant to estimation. Unlike other mathematical topics such as geometry, trigonometry, integration and differentiation, estimation is one that has a rich source of associated terms and phrases in the Tongan language. These would help the teacher teach this topic well.

Teaching activities explored ways of teaching computational estimation techniques for addition, subtraction, multiplication and division. 'Fakaofiofi (rounding), and fakafuofua ngaue'aki 'a e mata'ifika ki to'ohema (front-end strategy) were introduced and used in the classroom. In order to convey the appropriate meaning of the front-end procedure, it was important also to use this term in the instructional activities. Terms such as 'fakalahi' (make it bigger) and fakasi'isi'i (make it smaller) were used by students to describe 'rounding up' and 'rounding down' respectively. Thus, students' invented terminology could be used in the teaching of estimation.

That student constructed their own meanings and strategies for estimation supports the contention that learning is a constructive process. Although the researcher had used the term fakaofiofi to refer to the rounding procedure, students had adopted their words (e.g. fakalahi, fakasi'isi'i, mate'i), which indicated the way in which they had conceptualised the rounding process.

ii. Practical assessment procedures for measurement estimation.

Another important issue explored in the teaching activities was a practical assessment procedure for assessing estimates. The shared decision making process involving both the teacher and student in determining the reasonableness of estimates based on the purposes of the estimation tasks was an appropriate and effective approach to take. Shared decisions regarding the criterion for assessment provided students with opportunities to justify their answers and hence take a more active role in their own mathematics learning. The concept of 'reasonable range' was shown by previous research (Edwards, 1984; Reys, 1986; Trafton, 1986) to be an appropriate method of assessing answers to estimations. This approach was used in this study and was found to assist the development of proper appreciation of the types of answers required for estimation, that is, estimation is a process that can lead to a variety of answers.

iii. Reflection upon strategies and answers as an effective teaching/learning tool.

The teaching activity which involved students explaining their thought processes and strategies for making estimates encouraged other students to think about their answers and to realise that estimations involve much more than relating the first number that comes to mind. Reflecting upon the strategies used also promoted thinking and caring about the reasonableness of their estimates. The act of reflection upon answers would assist students to develop the skills and habit of checking the reasonableness of their estimates; this might lead to students self-correcting their ideas.

10.3 Conclusions

The following conclusions are drawn from the results of the present study.

Students' Measurement and Computational Estimation Skills and Strategies

Both school and out-of-school experiences of Tongan students provided opportunities to develop estimation skills. Whereas ideas about standard measuring instruments were used for estimating lengths, different everyday life experiences provided the benchmarks used for estimating weight and capacity. Understandings of unit measures and benchmarks do not guarantee reasonable estimation; a variety of experiences in making comparisons with these are required for the development of estimation skills. Moreover, the traditional Tongan methods of measurement were not part of the skills used in the mathematics classroom.

The computational estimation concept, skills and strategies of primary students were limited to exact mental computation. Specific teaching activities are necessary to help students develop the use of a variety of estimation techniques, and a better understanding of the nature and type of answers required for computational estimation.

Teaching Activities on Estimation

The instructional unit on measurement and computational estimation provided variety in the mathematics classroom. By their very nature the measurement estimation tasks provided the means by which a variety of procedures, skills, concepts, and answers were promoted in the mathematics classroom. In addition, a variety of assessment procedures involving students and teacher in the decision making process helped students to take a more active role in justifying their ideas and consequently in their mathematics learning. Furthermore, the measurement estimation tasks required the teacher to take into account the variety of ideas and different life experiences that students brought with them to the classroom.

The computational estimation activities helped shift students' reliance on one strategy type (i.e. conventional algorithm) to other strategy types, particularly the rounding process. In developing computational estimation strategies, other important mathematical concepts were also developed. These included: variety of computational strategies, understanding of the order of magnitude of the answers involved and reasonableness of answers, place value, properties of numbers, and making decisions regarding the appropriate time and place for estimation.

Students' ways of conceptualising the estimation and rounding processes provided a rich source of ideas for teachers. For estimation the terms and phrases used include: fakaofiofi (closer or nearer to), fakamahamahalo (conjecture), mate'i (guess), fakakaukau pe 'i loto (mentally figure out something), fua pe 'i loto (mentally work out the measure) and meimei (nearly). The

following terms were used for the rounding procedure: fakaofiofi (closer or nearer to), fakalahi (make it bigger), fakasi'isi'i (make it smaller), 'ai pe ke (take it as), 'ai pe ke tatau mo e (take it as equal to). These ideas need to be taken into account in teaching estimation in Tongan classrooms.

The outcomes of the present study have very important implications for learning, teaching, resource and curriculum development, and future research into estimation. These are discussed in the remaining sections of this chapter.

10.4 Implications for Learning

The implications of the results for learners and the learning process are considered in this section. These include links between school and out-of-school ideas, measurement estimation skill and strategies, computational estimation, and guessing as an estimation strategy.

Generating links between school and out-of-school ideas

The results suggest that although estimation has not been a major part of school learning, students' experiences in everyday life provided a useful base on which to develop their estimation skills and strategies. The need to generate links between school and out-of-school mathematics has been recommended and argued for by researchers in mathematics education over the years (Gay and Cole, 1967; Carr, 1986; Cockcroft, 1982; Onslow, 1991). Estimation is a mathematical skill that is embedded in the everyday life experiences of Tongan students. These experiences tend to differ among individuals and this accounts for the variety and differences in the benchmarks used for measurement estimation. This further suggests that estimation is a skill that can be fostered and developed both in and out-of-school. Therefore, both learners and teachers need to be consciously aware of various activities in everyday life which could assist the development of estimation skills.

In developing estimation skills and strategies in the classroom, care ought to be exercised not to standardise the procedures and strategies involved as this could undermine their very nature and the ways they are used in everyday life activities.

Measurement Estimation Skills and Concepts

Results of students' measurement estimation discussed in chapter six revealed that the inability of some students to make reasonable estimates could be a result of misconceptions regarding the units of measurements. It appeared that students' conceptions and/or mental images of basic unit of measurement such as a centimetre affected the estimates of lengths. The accessibility of a ruler as well as its everyday use did not guarantee that students would develop appropriate conceptions

of the standard metric units of measurement. It seems that for some students the main use of a ruler is for ruling or drawing lines.

The absence of the traditional Tongan units of measurement from the students' and teachers' responses implies that these were either not seen as appropriate for use in the classroom tasks, or not even considered. This absence is consistent with the analysis of the role of estimation in formal learning discussed in chapter two which concluded that the relevance of these traditional units of measurement to mathematics learning has not been identified or emphasised. As suggested in chapter two a shift of these topics from the Tongan studies curriculum to relevant units in the mathematics curriculum could give them the status they deserve and provide students with the opportunity to value them as mathematical topics worthy of study.

Computational Estimation Skills

Students' reliance on conventional algorithms and exhibition of very limited computational estimation strategies in the first interview highlights the need for students to be aware of the many viable estimation strategies for computation. The reluctance of students to give approximate answers to questions again highlights the dominance of obtaining one "right" answer in their mathematical learning experiences. Students need to be exposed to a variety of viable estimation strategies. As well, students need to be aware that estimation is a process that leads to a variety of answers and consequently there is not a mo'oni or truth but rather a range of answers (ngaahi tali fakaofiofi).

Guessing as a measurement or computational estimation strategy

The findings of this study points to the qualitatively different network of skills and concepts involved in measurement and computational estimation. The results suggest that whereas students could make thoughtful guesses for measurement estimation tasks, they did not attempt to use this strategy for computational estimation tasks. It seems that the use of concrete objects in measurement estimation tasks as opposed to abstract concepts associated with computational tasks (although these were put in a context of a real life situation) makes measurement estimation a more accessible task to students. It is also possible that whereas students could visualise and recall objects in their everyday life experiences as benchmarks for estimation and judging whether a benchmark is similar in size to the to-be-estimated items, computational estimation requires skills other than recalling facts and making appropriate judgements about relative sizes of numbers before and after they are used in an operation. This implies that students' cannot be expected to develop computational estimation strategies unless these are taught and fostered in the classroom. The current mathematical learning experiences of these class 6 students point to the limitations of exposure to one method of computation (i.e. conventional pen and paper algorithm).

10.5 Implications For Teaching

Even though only a small sample (n = 5) of teachers was involved in the study, they represent typical Tongan teachers. Results from the teachers' questionnaires and the teaching activities have implications for teaching estimation. One of the most important implications of the results is that estimation exercises have the potential to provide and/or help promote a constructive learning environment for learners. However, it seems that this potential may not be fully realised if current teaching practices continue. Three main implications of the findings for teachers relate to

i. provision of ideas for teaching activities on estimation,
ii. using students' existing ideas and out-of-school experiences for developing estimation, and
iii. using appropriate language.

Provision of ideas for teaching activities on estimation

Even though all five teachers view estimation as being useful, there is evidence to suggest that they only teach the topic sometimes or 'taimi pe ni'ihi'. The results indicated a lack of familiarity with a variety of estimation strategies (particularly for computational estimation) and appropriate assessment procedures for answers to estimation tasks. These suggest that activities and teaching strategies on estimation may need to be explicitly discussed in both inservice and preservice teacher education programmes. In addition, the results suggest that practical assessment procedures for assessing estimations (e.g. determining reasonable ranges) may be needed in teachers' preservice and inservice education programmes.

Use of students' ideas and out of school experiences to develop estimation

The answers to Question 4 discussed previously in section 10.2 have important implications for teaching estimation. Among the students' ideas that ought to be taken into account in developing teaching activities are benchmarks, experiences in everyday life, place value, many definitions of estimation and the rounding process.

Moreover, flexibility is required when using the rounding procedure in estimation. For instance, when estimating 13 x 2542, students could round 13 down to 10 and 2542 to 2000 as an initial estimate. It appears that students ought to learn to use flexible rounding for estimation purposes before they learn rounding as an approximation technique, (which requires rounding to a required degree of accuracy). To teach students rounding as an approximation technique and then later try to get students to forget about these rules when estimating will only hinder the development of

appropriate estimation skills. In other words, it appears that it would be more appropriate to introduce rounding as an estimation technique before it is taught as an approximation technique.

There is evidence to suggest that the development of students' learning in the classroom can be facilitated by teachers using what their students already know. As discussed in chapter one, the belief that teachers are the experts with knowledge that is to be passed on to students is strongly held and accepted by parents, students and teachers. Listening to and accepting students' ideas goes against this accepted belief in Tonga. At the same time, students are expected to be responsible for their own learning. If taking responsibility for their own learning means using out-of-school ideas in the classroom, then students have not been allowed to take full responsibility for their learning. Both teachers and students as a consequence have much to lose if students ideas' are not taken seriously in the formal setting of the mathematics classroom. By their very nature estimation exercises provide the means and rationales whereby students' ideas have to be taken into account in teaching, learning and assessment in mathematics.

Appropriate Language Used for Estimation

As discussed in chapter one various terms such as mental computation, approximation and estimation are often used interchangeably to mean the same thing. In the Tongan classroom, it is vital that the terms used for estimation are clearly defined and separated to avoid or lessen the chances of possible confusion by both teachers and students. The term 'fakafuofua' is used for estimation (the process as well as the product), approximation, guess and figure out. It appears that various other terms could also be used and should be encouraged to be used in the classroom—terms such as fakaofiofi (nearer to or closer to), fakamahamahalo (conjecture), fakafehoanaki (comparison), fakatatau (comparison). Kumi ha mo'oni fakaofiofi (to find an answer that is close enough) is used by students and teachers to describe fakafuofua or estimation. The terms used in the teaching activities could provide useful ideas for teachers who have to make decisions about what terms to use. As previously discussed, the terms tali (answer) and tali fakaofiofi (approximate answer) also seem more appropriate to estimation.

Two important reasons for the adoption and use of the above terms are proposed here. First, these would begin to highlight the many aspects of mathematics in which estimation is involved. Secondly, adoption of such terms would help students better understand the processes involved in each of the above estimations. Translation of the English terms such as numerosity estimation ought to reflect the process inherent in the estimation. The phrase "fakafuofua e tokolahi" seems to be an appropriate definition of the process. Care needs to be exercised to avoid direct translation of the English terms such as 'niumelositi' for 'numerosity'. This type of translation

which often appears on curriculum statements and in the classroom may not help students to understand the ideas and processes inherent in these terms.

10.6 Implications for Resource and Curriculum Development.

An important implication of the results from the teaching activities is the need to seriously question whether teaching of estimation should precede teaching of measurement and computation. Results showed that basic ideas in measurement and computation could be developed via estimation. Conventional algorithms could inhibit students' flexibility in the use of numbers and the variety of strategies used for computation. The findings of the present study suggest that estimation should be taught before students learn the conventional algorithms.

It is vital that resources are provided to help support teachers in the implementation of estimation activities. The role of resources in the dissemination of ideas for teaching estimation cannot be undervalued. The implications for resource development are examined in this section.

Textbooks

The treatment given to estimation in textbooks currently used in the classrooms in Tonga is inadequate for the implementation of the new mathematics curriculum. Minimal attention is given to estimation in current textbooks. However, with the increased attention given to estimation in the latest mathematics curriculum, similar attention ought to given to the content of textbooks. One of the problems with mathematics education in Tonga is the use of textbooks written in overseas countries such as Australia and New Zealand. Lack of local context in these texts is one of their major shortcomings. With estimation being a very practical everyday life skill, the use of local examples in textbooks is of utmost importance. Contexts for estimation problems ought to be relevant to Tongan students' experience. Thus, carefully selected local materials should be included in any text written on estimation. The nature of estimation exercises, the various situations in which estimation is used, and the estimation strategies used by people in everyday life need to be incorporated into the writing of Study Guides and resource materials for learning and teaching estimation.

Curriculum Content and Implementation

In discussing the implication of this study for the implementation of the mathematics curriculum it is important to refer to the various distinctions about curriculum made by the Department of Education Report on the results of the IEA Mathematics Study in New Zealand (Department of Education, 1987). A distinction is made between 'intended curriculum', 'implemented curriculum' and 'attained curriculum'. This distinction is outlined in Figure 10A below.

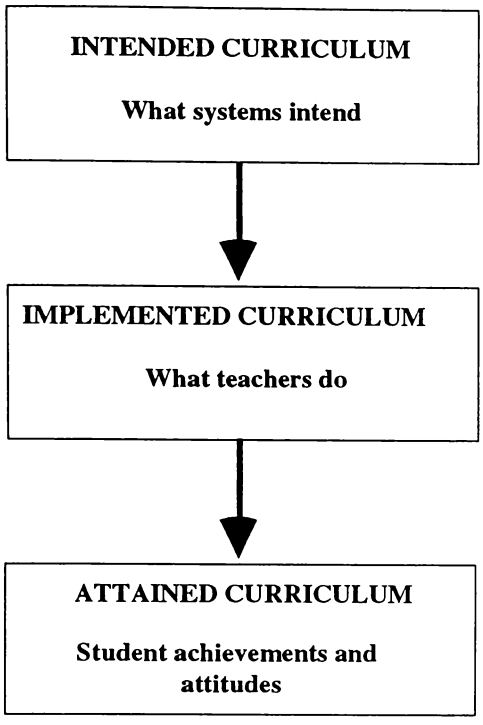


Figure10 A: Distinctions between, 'intended', 'implemented' and 'attained' curriculum.

National guides usually outline requirements for the curriculum, syllabuses, and prescriptions. These provide the basis for what authorities (e.g. curriculum writers and overseas consultants) intend the curriculum in mathematics to be—the 'intended curriculum'. In implementing the 'intended' curriculum teachers interpret these national syllabuses in terms of the abilities of students they are teaching, the time available, their own knowledge and experiences, topics covered in examinations and other factors. The 'implemented' curriculum (i.e. what teachers actually teach in mathematics classes) is thus not necessarily what is recommended in the intended curriculum. Furthermore the mathematics curriculum received by students is not necessarily the same as that which their teachers believe they are implementing. Students must also make interpretations and it is they who individually decide, either consciously or unconsciously, what they will make an effort to understand. The 'attained curriculum' (i.e. what mathematics students know and understand and are able to use) depends on these factors, which is turn depend on the many variables which are influential in the teaching/learning process.

In a small country such as Tonga there is great potential to develop an innovative curriculum relevant to the everyday life of students. For many years now, Tonga has relied on overseas ideas, resources and funds for curriculum statements to be written. In an endeavour to obtain an

internationally recognised curriculum content there is a possibility that the relevance of the curriculum to the very students whom it is intended for may be overlooked.

If the content of the curriculum is to be selected on the basis of its usefulness in everyday life then estimation should be among the first topic to be on the list. Very often the 'official' or 'intended' curriculum is a document that is not accessible to teachers and even if it is accessible it is often not referred to. Teachers are often given a scheme of work where topics are listed with the order in which they are to be covered. With the practice of having common examinations between schools, these topics are covered in the order that it is given. All too often the content of the text books as well as previous examination papers determine what is taught in the classroom (i.e. the implemented curriculum).

It is time that traditional ways of doing things are valued, and incorporated into the mathematical learning experiences of Tongan students. Careful examination and analysis of topics such as 'counting system' may lead to its inclusion in the mathematics syllabus rather than the Tongan studies curriculum. For so long, mathematics in the Tongan classroom has centred on foreign mathematics curriculum. It seems that the attitude of 'what is foreign is good or superior' has influenced decisions affecting the content of the curriculum that has been the major focus of mathematics education in Tonga.

Given that estimation has been emphasised in the new mathematics curriculum due for full implementation in 1996, and that teachers in this study showed limited strategies for teaching estimation, it is possible that although estimation is stressed in the 'intended' curriculum teachers may not be sufficiently equipped to teach these topics in their classrooms. The 'intended' curriculum may therefore not be implemented, and consequently not become the 'attained' curriculum. Thus it is vital that teachers are well supported to implement this new mathematics syllabus. It is important that estimation activities trialled in the Tongan classrooms, such as those explored in this study, are disseminated to teachers.

10.7 Implications for Assessment and Future Research

It is apparent from the results and the research literature that one of the most difficult aspects of teaching and research on estimation is assessment. The implications of the findings for assessment and research are discussed in this section.

Assessment in the Classroom

One of the implications involves teachers having to make decisions about appropriate assessment procedures to be used in the classroom. Previous studies have not been successful in identifying

a manageable assessment method for classroom teaching and learning. The results of this study support previous researchers (Clements, 1989; Edwards, 1984; Trafton, 1986) in their contention that one of the most important criteria for assessment is the purpose of the estimation exercise. Moreover, that the use of reasonable range rather than one value seems a more appropriate approach for estimation. For instance, in estimating how much material is required for a curtain for a door, a reasonable range in relation to the actual height of the door would not be an appropriate assessment procedure. This is because estimates that are less than the actual height of the door would not be enough. In this situation, an overestimation would be considered more suitable for the purpose of the exercise, that is, it would be more desirable to provide an estimate higher than the actual height of the door in order to guarantee there is sufficient material to cover it.

It is also important for teachers to be clear about the skill that they intend to assess because they would influence how the estimates are evaluated. This study suggests that one effective assessment procedure would require both students and teachers to make joint decisions on 'acceptable range' for answers to estimation tasks. There are several advantages in adopting this approach. First, students would be actively involved in determining the reasonableness and appropriateness of their answers as well as the strategies used to derive their answers. Secondly, students would become more aware of the problem-solving skills involved in formulating an estimate from the time the problem is posed to assessment and evaluation of the answers formulated. Finally, students would develop appropriate attitudes and appreciation of the estimation process and the types of answers required.

The traditional emphasis on comparison of the estimate with the exact measure may not help students appreciate the value of an estimate. The teacher needs to strike a balance between use of the exact measure to help students develop 'a feel for' the size of a measure as well as the reasonableness of their estimates on the one hand, and the development of an appreciation of the usefulness or value of an estimate on the other. This involves using a different language, for instance, tali fakaofiofi (approximate answer) rather than a mo'oni or tali totonu (true or exact answer).

National Assessment

As discussed in chapter three various assessment procedures and testing techniques have been trialled by researchers with no clear evidence of an agreed criteria for assessment. One important finding from the review of the literature was that the use of a multiple choice format does not necessarily assess estimation skills. While it could be useful for assessing approximation skills, it has been found to be ineffective in evaluating estimation. Thus, the current use of the multiple

choice questions in Secondary School Entrance Mathematics Examinations seems inadequate for assessing students' estimation skills.

Areas for Future Research

The results of this study provide some possible answers for the research questions but at the same time raise more questions. The following are possible future research questions:

1. What is a reasonable estimate?

More research is required to address this question. It appears that different criteria for assessing the reasonableness of a result are called for, particularly when dealing with different aspects of estimation. Effective and useful assessment techniques for everyday classroom activities need to be investigated. Practical assessment procedures that could be useful for teachers in the everyday world of the mathematics classroom are needed.

2. How can students' skills of estimation in one domain of estimation be used to facilitate and increase performance in estimation in another domain of mathematics?

The present study focused on two different aspects of estimation only; measurement and computational estimation. It would have been useful to investigate how students' measurement estimation skills and strategies are related to computational estimation skills, for instance, whether students who were proficient in measurement estimation tasks were also proficient in computational estimation.

3. What are the estimation skills and strategies used by students in everyday life activities?

The present study briefly considered some adult Tongan estimation skills, but the study has mainly focused on estimation skills and strategies in the schools. The question of what estimation skills and processes are used by students in everyday life activities would be worth investigating.

Conducting Research in Tonga

As described in chapter four, the present study was not initiated or sponsored by the Tonga Ministry of Education but was an attempt by a Tongan student researcher to conduct research in the Tongan classrooms in the hope that the data would provide useful information for teachers, teacher educators, curriculum writers, and resource developers. The researcher's experiences in the field work suggest that the results of such a personally initiated research project are unlikely to be useful and worthwhile unless the teachers, curriculum writers and teacher educators are aware of the contribution that research can make to the overall development of mathematics education in Tonga.

The decision to trial teaching activities with only a small sample of Tongan students allowed the researcher to collect data without having to teach teachers the rationale for and the activities. Such teaching would have seriously undermined the ability of teachers and their work as teachers. The researcher did not have the status (e.g. of a curriculum writer trialling some activities) within the Ministry of Education to allow her to ask teachers to trial the activities in the classroom. Given that the Tonga Ministry of Education does not have a research unit, there is potential for research students to make a significant contribution to the overall development of mathematics education in Tonga.

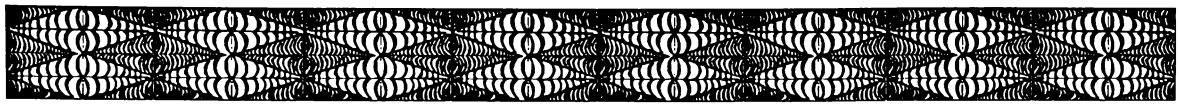
10.8 Concluding Thoughts

The present study was a response to the researcher's belief that estimation is one of the more useful mathematical skills in everyday life. A concern about the dearth of research data in Tonga and the emphasis given to estimation in the new mathematics syllabus determined the focus of the present study. The results of the present study have shown the value of exploring appropriate language for teaching estimation in the Tongan language; the effectiveness of adopting reflection and group discussion as a teaching approach; the need to increase the variety of strategies for computational estimation; and the inclusion and participation of students in the decision-making process in determining the reasonableness of their answers in mathematics.

Estimation has always been part of traditional Tongan life. It is therefore timely that the new mathematics syllabus gives this topic the emphasis it deserves in the classroom. The study has shown estimation as a vehicle for developing mathematical thinking, skills and concepts and vice versa. At the same time estimation is a means of checking the reasonableness of results in mathematics. A focus on the uses of estimation not only in the classroom but in everyday life experiences would provide the learner with "a more balanced perspective about the nature of mathematics" (Whitin, 1994, p. 436). Estimation has the potential to provide much needed variety in the Tongan mathematics classroom.

Ko e fakafuofua (estimation) ko e poto'i ngāue fakafika ia 'oku lahi ange hono 'aonga 'i he mo'ui faka-aho 'i Tonga, talu mei ono'aho. Ko e fekumi 'oku ha 'i he tohi ni na'e fakataumu'a ia ki hano vakai'i 'o e ngaahi 'ilo mo e poto'i fika fakafuofua 'a e fanau form 3 mo e kalasi 6, pea pehe ki hano 'ahi'ahi'i ha ngaahi founga faiako'i 'o e fakafuofua. Ko e silapa fika fo'ou 'oku fakamamafa'i ai 'a e fakafuofua ka ko e fehu'i: 'oku mateuteu nai 'a e kau faiako ke ako'i ' a e fakafuofua? 'Oku ou fakatauange pe ke 'aonga 'a e ngaahi fakakaukau 'i he tohi ni ki he fanauako mo e kau faiako fika 'i Tonga.

APPENDICES



Appendix A	Trial questionnaire for form 3	213
Appendix B	Trial questionnaire for class 6	217
Appendix C	Instructional unit on measurement estimation	220
Appendix D	Instructional unit on computational estimation	239
Appendix E	Teacher questionnaire (English Version)	251
Appendix F	Teacher questionnaire (Tongan Version)	255

Appendix A: TRIAL QUESTIONNAIRE FOR FORM 3

ESTIMATION EXERCISE

Name: _____

School: _____

IN THIS EXERCISE I WOULD LIKE YOU TO ESTIMATE ANSWERS TO QUESTIONS. YOU WILL **NOT** HAVE ENOUGH TIME TO WORK OUT THE ANSWERS EXACTLY.

WRITE YOUR ANSWERS IN THE SPACES PROVIDED.

JUST ESTIMATE THE ANSWERS DO **NOT** WORK THEM OUT EXACTLY, WORK QUICKLY AND CAREFULLY.

---

1. The length of this line _____ is about

.....

2. The height of your desk is most likely to be

.....

3. Estimate the distance between Nuku'alofa and Fu'amotu airport

.....

4. Approximately how far a car travels in 2 hours

.....

5. The mass of a ripe mangoes is about

.....

6. A tennis ball weighs

.....

7. A new pencil weighs about

.....

8. The weight of an adult is about

.....

9. A bottle of lemonade holds about ..... of lemonade

10. Name an object or a container which would hold about 50 litres

.....

11. Estimate the volume of a teaspoonful of medicine

.....

12. Name of container which would hold close to 300 millilitres

.....

13. A bucket holds about

.....

14. Estimate the answer to

```
  452
  547
  703
+  98
 -----

 -----
```

15. Estimate the answer to

87 419
92 765
90 045
81 975
+ 98 102

16. Estimate the answer to

4 236
- 2 517

---

17. Estimate the answer to

12 x 28 .....

18. Estimate the answer to

67 x 41 .....

19. Estimate the answer to

$\overline{60)\ 4\ 325}$ .....

20. Estimate the answer to

$\overline{34)\ 3\ 701}$ .....

21. Estimate the answer to the nearest whole number

$\frac{12}{13} + \frac{7}{8}$ .....

22. Estimate the answer to the nearest whole number

$\frac{1}{2} + \frac{1}{3}$ .....

23. 4.12 x 12 is about .....

24. Estimate the answer to

0.21 x 029 .....

25. Estimate the sun

4.92 + 5.3 + 5.14 + 4.7 .....

26. Estimate the sum

192.3 + 65.5 + 527.9 .....

27. My calculator shows the sum of 1 856 and 2 743 to be 5 091 008. Could that be correct?

Yes ( )
No ( )
Why do you think that?

.....

.....

28. My calculator says 786 x 9 equals 6184. Does this seem reasonable?
Yes ( )
No ( )

Why?

.....

.....

29. Sione bought four items from the shop
bread costs $0.59
butter costs $1.50
sugar costs $1.40
milk costs $1.55
Approximately how much money did he spend on the four items?
Give your answer to the nearest dollar?

.....

30. If you pay for a $1.29 purchase with a five dollar note, about how much change should you expect? (to the nearest dollar)

.....

31. About how much money would you take to the shop to buy two books costing $4.95 and $3.69.

.....

32. You only have $10 to spend. You need 5 notebooks. Each note book costs $1.85. You would also like to buy a 35c rubber. Do you have enough money?
Yes ( )
No ( )

Appendix B: TRIAL QUESTIONNAIRE FOR CLASS 6

FAKAFUOFUA ‘O E NGAAHI FUA (MEASUREMENT ESTIMATION)

HINGOA: _____

TA‘U MOTU‘A: _____

HINGOA HO ‘APIAKO: _____

‘OKU FIEMA‘U KE KE FAI HA‘O FAKAFUOFUA KI HE TALI ‘O E NGAAHI FEHU‘I KO ‘ENI.

‘E ‘IKAI KE ‘IAI HA TAIMI KE KE KUMI AI ‘A E TALI TOTONU, KO IA AI NGAUE KE VAVE PEA KE TOKANGA FOKI.

‘OKU FIEMA‘U KE KE TOHI HO‘O TALI ‘I HE NGAAHI FEITU‘U ‘OKU FAKA‘ATA ATU.

KO E ‘OSI PE HO‘O TALI ‘A E NGAAHI FEHU‘I KOTOA PEA KE HIKI HO NIMA KE TANAKI LEVA HO‘O PEPA.

MALO ‘AUPITO

Tongan Version **FAKAFUOFUA E TALI KI HE NGAAHI FEHU'I KO 'ENI:**	English Version **ESTIMATE THE ANSWER TO THE FOLLOWING:**
1. Loloa 'o e laine ko 'eni	Length of this line
2. Falahi ho fo'ituhu si'i si'i taha	Width of the little finger
3. Ma'olunga 'o e palakipoe 'i loki ako	Height of the blackboard
4. Ma'olunga 'o e tepile 'a e faiako	Height of the teacher's desk
5. Falahi 'o e lula	Width of a ruler
6. Loloa 'o ha mala'e 'akapulu	Length of a rugby field
7. Ma'olunga 'o e matapa ho lokiako	Height of the classroom door
8. Loloa 'o e la'ipepa ko 'eni	Length of this page
9. Mama'o mei Nuku'alofa ki mala'evakapuna	Distance from Nuku'alofa to the airport
10. Loloa 'o ha peni	Length of a pen
11. Mamafa 'o ha penivahevahe fo'ou	Weight of a new pencil
12. Mamafa 'o ha fo'i mapu	Weight of a marble
13. Mamafa 'o ha fo'i mango momoho	Weight of a mango
14. Mamafa 'o ha tangata lahi	Weight of an adult
15. Mamafa 'o ha fo'i meleni $5.00	Weight of a $5.00 watermelon
16. Mamafa 'o ha kato manioke	Weight of a basket of tapioca
17. Mamafa 'o ha fo'i pulu tenisi	Weight of a tennis ball
18. Mamafa 'o ha kapapulu	Weight of a tin of corned beef
19. Mamafa 'o ha puha 'OMO	Weight of a packet of OMO washing powder
20. Mamafa 'o ha puha hina	Weight of a carton of squash pumpkin
21. Lahi 'o e hu'akau ke fakafonu'aki ha hinahuhu 'a e pepe	Amount of milk in a baby's bottle
22. Fakahingoa mai ha fa'ahinga koniteina pe me'a 'oku lava ke fonu he lita 'e 50	Name an object or a container which holds 50 litres
23. Lahi 'o e ti pe kofi 'i ha ipu	Capacity of a cup of tea or coffee
24. Lahi 'o e penisini 'e fonu ai ha motoka	Capacity of a petrol tank of a car
25. Lahi 'o e lemani ha fo'i hina lemani	Capacity of a bottle of lemonade

26. Lahi ‘o e vai ‘i ha kane ‘oku fonu vai	Capacity of a bucket
27. Fakahingoa mai ha fa‘ahinga koniteina pe me‘a ‘oku fonu ‘i mililita ‘e 300	Name an object or container that would fill 300 millilitres
28. Lahi ‘o e vaipanatolo ‘i ha sepuniti ‘e taha	Capacity of a teaspoon
29. Lahi e huhua’i niu ‘i ha fo‘iniu mata	Capacity of a green coconut
30. Lahi ‘o e hu‘akau ‘i ha kapa-hu‘akau melie	Capacity of a can of condensed milk

Appendix C: INSTRUCTIONAL UNIT ON MEASUREMENT ESTIMATION

LESONI 1:
KO E HA 'A E FAKAFUOFUA?

NGAAHI TAUMU'A
1. Ke vakai'i 'a e ngaahi 'uhinga 'oku ma'u 'e he fanau fekau'aki mo e 'fakafuofua'.

2. Ke ngaue'aki 'a e ngaahi founga fakafuofua 'a e fanau ke fakamahino'i pea mo fakamamafa'i 'a e 'ulungaanga 'e 2 'o e fakafuofua, 'aia ko e -
   i) lava ke ngaue'aki 'a e ngaahi founga kehekehe ke tali ha fehu'i pe 'e taha;
   ii) lava ke ma'u ai mo e ngaahi tali kehekehe.

FAKAIKIIKI:
I. 'Oku mahu'inga ke kamata'aki 'a e lesoni 'a hono vakai'i 'a e 'uhinga 'o e fo'i lea **fakafuofua** ki he fanauako.

*Ko e ha 'a e 'uhinga kiate koe 'a e fo'i lea ko e **fakafuofua?***

Ko 'enau ngaahi tali ke tohi'i ki he palakipoe pea ke talanoa'i 'a e ngaahi 'uhinga ko 'eni. Pea ke fakapapau'i 'oku mahino ki he fanau 'a e 'uhinga 'o e fanau 'i hono ngaue'aki 'i he leson ko 'eni.

II. Ke ngaue'aki 'a e ngaahi tali 'a e fanau 'i he 'uluaki fepotalanoa'aki (first interview) ke faka'ai'ai'aki 'a e fanau ke nau lea'aki pea ke vahevahe mo e kalasi 'a e 'uhinga 'o e fo'ilea fakafuofua. Ko e ngaahi 'uhinga 'eni ne 'omai 'ehe fanau:

Fakafuofua

Fakakaukau'i pe 'i loto

Fakaofiofi

Fakamahamahalo

Mate'i

Fua pe 'i loto

Fakamahu'inga ha me'a 'i loto

ACTIVITY 1:
WHAT IS ESTIMATION?

OBJECTIVES
1. To explore the meaning(s) of 'estimation' held by students.

2. To use students' estimation strategies to highlight two important features of estimation,
   these are:
   i) a variety of strategies can be used and
   ii) a range of answers is possible?

PROCEDURES:
I. It is important that the lesson begins with a discussion of meaning(s) held by students about **estimation.**

*What is meant by the term **estimation**?*

Students' responses to the above question will be recorded on the blackboard and discussion of these meanings should lead to students' understanding of the term as it is used in this context.

II. The definitions given by students during the interviews will also be used to initiate and facilitate discussions. These definitions include:

Estimate/Estimation

Mentally figure out something

Closer or nearer to

Conjecture

Guess

Mentally work out the measure

Working out how much something is worth.

III. To'o ha ngaahi fo'i fehu'i 'e 2 pe 3 mei he ngaahi fehu'i na'a ngaue'aki 'i hono faka'eke'eke taautaha 'o e fanau 'o to e 'oange ki he fanau ke fai ha'anau tali fakafuofua ki ai. 'O hange nai ko e ngaahi fehu'i ko 'eni:

1. *Ko e ha nai 'a e loloa 'o ha peni?*
2. *Ko e ha ma'olunga 'o e tesi 'oku ngaue'aki ?*
3. *'Ai ha'o fakafuofua ki he falahi 'o e matapa sio'ata?*

III. Take 2 or 3 of the tasks used in the first individual interview to initiate discussions of the strategies used for estimation. Students will be asked to provide estimates and the strategies used. For example:

1. *Approximately how long is this pen?*
2. *Approximately how high is a student's desk?*
3. *Can you make an estimate of the width of the window?*

Ngaue'aki 'a e ngaahi fehu'i 'i olunga ke vakai'i 'a e ngaahi founga 'oku ngaue'aki 'e he fanauako ke fakafuofua e tali 'a e ngaahi fehu'i, pea ke talanoa'i mo e kalasi 'a e ngaahi founga kehekehe na'a nau ngaue'aki ke. 'I he taimi tatau ke feinga 'a e faiako ke ma'u mei he fanau 'a e founga mo e feitu'u na'a nau ako ai 'a e ngaahi founga fakafuofua.

The above questions will be used to initiate discussions of the various strategies used by students to formulate estimates. At the same time the teacher will try to obtain from students how they have learned these strategies. It is possible that students would have learnt these strategies on their own and in out-of-school experiences.

Ke hanga 'ehe faiako 'o ngaue'aki 'a e ngaahi fakakaukau 'a e fanau ke fakamahino mo fakamamafa'i'aki 'a e ngaahi tefito'i 'ulungaanga 'o e fakafuofua. 'Aia ko e:

- *fakafuofua 'i he fua ko ha ngaue pe 'oku fai 'i loto 'i he 'atamai 'o 'ikai ke ngaue'aki ha lula pe ha to e me'afua;*
- *'oku lahi 'a e ngaahi founga 'oku lava ke ngaue'aki ke ma'u ai ha tali fakafuofua;*
- *'oku lahi foki 'a e ngaahi tali 'e ala ma'u; pea*
- *ko e ngaahi tali 'oku ma'u 'oku 'ikai ko e mo'oni totonu .*

Using the students' responses, the teacher will highlight the main features or characteristics of estimation.
These are:

- *measurement estimation is a mental skill, which involves working out a measure without the use of a ruler or measuring instrument;*
- *there are a variety of acceptable strategies for working out an answer;*
- *there are a variety of possible answers; and*
- *that the answers obtained are not the exact answers.*

LESONI 2:
FAKAFUOFUA LELEI

NGAAHI TAUMU'A

1. Ke potalanoa'i 'a e fo'i fakakaukau ko e fakafuofua lelei.

2. Ke fakamahino'i ki he fanauako 'a e fo'i fakakaukau ko e **reasonable range** pe ko ha ongo mata'ifika 'e fokotu'u pea ko e ngaahi tali 'e to 'i hona vaha'a 'e ui ia ko e fakafuofua lelei.

3. Ke ngaue'aki 'a e ha me'a (holoholo) 'oku 'ikai ko ha me'afua loloa ke fakamahino'i'aki 'a e ongo fakakaukau 'i 'olunga.

FAKAIKIIKI

I. Kamata 'a e lesoni 'aki 'a e fehu'i ko 'eni:

Ko e holoholo nai 'e fiha 'e fe'unga mo e:
- *ma'olunga 'o ha hoosi,*
- *loloa 'o ha huo, pe*
- *falahi 'o ha lokiako.*

Ke hanga 'ehe faiako 'o to'o hake ha holoholo ke sio ki ai 'a e fanauako pea ke nau fakafuofua'i pe ko e holoholo nai 'e fiha 'oku tatau mo e loloa 'o ha hoosi.

II. 'E toki hanga leva 'ehe faiako 'o fakatataa'i 'a e loloa 'o e ngaahi loloa takitaha 'o ngaue'aki 'a e ngaahi mata'ifika na'e 'oange 'ehe fanau. 'Oku 'iai 'a e faka'amu 'e lava 'ehe ki'i fo'i ngaue ko 'eni 'o 'oange ki he fanauako ha faingamalie ke nau sio ai mo fakakaukau'i 'enau ngaahi fakafuofua takitaha pe ko ha fakafuofua lelei pe 'ikai.

'E hoko atu leva 'a e potalanoa mo e fanau ki he fakakaukau 'o ha **fakafuofua lelei.** Pea ke fokotu'u 'e he fanauako mo e faiako ha ongo mata'ifika (range) 'aia kapau 'e to 'i hona vaha'a e tali 'a e fanau pea 'e lau ia ko e ko ha **fakafuofua lelei.**

Ko e ha e fakafuofua lelei?
Ko e ha ngaahi fakafuofufa lelei?
Ko e ha ha fakafuofua 'oku fu'u si'isi'i?
Ko e ha fakafuofufa 'oku fu'u lahi?

ACTIVITY TWO:
REASONABLE RANGE

OBJECTIVES

1. To introduce the concept of **reasonable estimate.**

2. To develop an understanding of the concept of **reasonable range.**

3. To introduce the concepts of reasonable range and reasonable estimate using a non-standard measure (e.g. handkerchief).

PROCEDURES

I. Begin the activity with the following question:

How many handkerchiefs fill make up the:
- *height of a horse,*
- *length of a spade, or*
- *width of the school house.*

The teacher will hold up a handkerchief and students will then be asked to give their responses to the above exercise.

II. The teacher will then use the handkerchief to demonstrate the students' the length given by the students. This exercise should allow students to observe the reasonableness of their answers. Again, the students and teacher will then decide what would be a reasonable range for the number of handkerchiefs for the lengths of a horse.

This will then lead to a discussion of the concept of **reasonable estimate** or **acceptable** range for students' answers. Students and teachers will then determine an reasonable range for each questions, which would be referred to as **reasonable estimates.**

What is a reasonable estimate?
What is a reasonable range?
What is an underestimation?
What is an overestimation?

'E fokotu'u leva ha ongo mata'ifika ke hoko ia ko e reasonable range ki he ngaahi fakafuofua 'e 'omai 'ehe fanau. 'O hange nai ko e ma'olunga 'o ha hoosi, ko e holoholo ia 'e 4 ki he 8. Ko e ngaahi fakafuofua 'e ma'ulalo hifo he 4 'e ui ia ko ha **fakafuofua 'oku fu'u si'isi'i**, ka ko e fakafuofua 'oku fu'u lahi he 8 'e ui ia ko e **fakafuofua 'oku fu'u lahi**.

The students and teacher will then establish a reasonable range for each of the task. For example, 4 to 8 handkerchiefs will be the reasonable range for the height of a horse . Estimates less than 4 will be considered **underestimation**, while those greater than 8 will be considered **overestimation**.

NGAAHI FAKATOKANGA

1. 'Oku mahu'inga ke to 'a e fakamamafa ki hano tokoni'i 'a e fanauako ke mahu'inga malie kiate kinautolu 'a e 'uhinga 'oku ngaue'aki ai 'a e fakafuofua pea pehe foki ki he ngaahi **fakafuofua lelei**. 'Oku makehe 'a e **fakafuofua** mei he tali 'oku anga maheni ki ai he fika koe'uhi he 'oku **'ikai ke taha pe 'a e tali** ka 'oku lahi 'a e ngaahi tali 'e ala ma'u. 'I he'ene anga pehe kuopau ai ke fokotu'u ha ongo mata'ifika (range) 'aia kapau 'e to 'a e tali 'a e fanau 'i he vaha'a 'o e ongo mata'ifika ko 'eni pea 'e lau ia ko e **fakafuofua lelei**. Ko e mahu'inga 'o e **fakafuofua lelei** 'oku totonu ke fakamamafa'i 'i he taimi kotoa pe koe'uhi ko e tokolahi 'o e fanauako kuo nau anga ki he fika 'aia 'oku ngaue'aki ha founga ke ma'u ai 'a e **tali totonu** pe **tali mo'oni**. Ko e ngaahi fakafuofua 'e fai 'ehe fanau 'e tokoni ia ke fakalahi atu 'a 'enau ngaahi founga 'oku ngaue'aki 'i he fika pea 'ikai ke ngata ai ka ke nau a'usia ai 'a konga 'o e fika (fakafuofua) 'oku 'ikai ke fiema'u ai ha tali pe mo'oni pe 'e taha.

NOTES

1. Emphasis will be given to a range of answers rather than one right estimate, therefore, the use of **reasonable** and **acceptable** estimates rather than **one right, real** or **correct answers.** The concept of **reasonableness** is central to all work involving estimation and therefore this should be consistently emphasised throughout the teaching/learning activities. Most students are used to trying to get **exact** or **right** answer or the **'real answer'** as some students put it. Students' current experiences in mathematics should be widened to include opportunities which emphasise another aspect of mathematics, (estimation) which does not involve obtaining exact answers but rather a range of answers.

2. 'E 'ikai foki ke ngaue'aki 'a e lea ko e **fakafuofua hala**, ka 'e fakamamafa'i 'a e **fakafuofua lelei**, fakafuofua 'oku fu'u si'isi'i pea mo e fakafuofua 'oku fu'u lahi.

2. The phrase **wrong estimate** will not be used in the discussion, rather the emphasis will be on **reasonable estimate**, underestimation and overestimation.

3. 'E **'ikai** totonu ke ngaue'aki ha lula pe ha me'a fua 'i he lesoni 1 mo e 2. 'Oku mahu'inga ke 'oange ki he fanau ha faingamalie ke nau fakamahu'inga'i 'enau ngaahi fakafuofua pe mo fakalahi 'a 'enau 'ilo ki he ngaahi 'uhinga 'oku ngaue'aki ai 'a e fakafuofua. 'E lava nai heni ke faka'ai'ai kinautolu ke nau fakamahu'inga'i 'a e ngaahi founga fua na'e ngäue'aki 'ehe kakai Tonga 'i he mo'ui faka-'aho.

3. A ruler or tape measure will **not** be used during activities 1 and 2. It is important to give students time to value their estimates and to increase their awareness of the purposes of estimation. This should also help encourage students to use 'traditional' methods of measuring lengths. Moreover, such an exercise would increase students' awareness of the purposes of and processes involved in estimation.

LESONI 3:
NGAAHI FOUNGA FAKAFUOFUA NA'E NGAUE'AKI 'EHE KAKAI TONGA

TAUMU'A
1. Ke vakai'i pea ke fakalahi 'a e 'ilo 'a e fanauako ki he ngaahi founga fua pe fakafuofua na'e ngaue'aki 'e he kakai Tonga 'i mu'a pea 'oku kei ngaue'aki pe 'i he'e ngaahi 'aho ni.

FAKAIKIIKI
I. 'E hoko atu leva 'a e lesoni ki he ngaahi founga na'e ngäue'aki 'ehe kakai Tonga ki he fua e loloa 'oha fa'ahinga me'a. Pea 'e kamata'aki leva 'a e ngaahi fehu'i ko 'eni.

Ko e ha e founga na'e ngäue'aki 'he'etau ngaahi kui ke fua'aki 'a e ngaahi me'a ko 'eni:

i. loloa 'o ha fala,
ii. loloa 'o ha ngatu launima,
iii. loloa 'o ha kafa, mo e
iv. vaha'a 'o ha ongo 'otu 'ufi pe talo

II. 'E vahevahe leva 'a e kalasi 'e toko 6 ki ha kulupu 'e 3 'o tautau toko 2. Pea ke nau potalanoa'i 'a e ngaahi fehu'i ko 'eni 'oku ha 'i 'olunga ki mu'a pe nau toki lipooti mai ki he kalasi. Ke ngaue'aki 'e he faiako 'a e 'ene 'ilo fakataha mo e ngaahi 'ilo mo e fakakaukau 'a e fanauako ke fakamahino'i'aki 'a e ngaahi founga na'e ngaue'aki 'e he kakai Tonga 'i ono'aho pea 'oku kei ngaue'aki foki 'i he mo'ui faka'aho.

III. 'I ha lava pe ke 'osi a' e ngaahi fakakaukau hono talanoa'i pea ke 'oange leva ki he fanau 'a e ki'i ngaue ko 'eni ke nau fai mei 'api (PEPA NGAUE 1). Pea ke ngaue'aki 'a e toenga 'o e lesoni ke fakamatala'i 'a e ngaahi me'a 'oku fiema'u ke fai 'e he fanau ako. Koe 'uhi 'oku fai fakauike 'a e kalasi ko 'eni pea pehe 'e fe'unga pe 'a e 'uike 'e taha ke fai ai 'a e ngaue ko 'eni.

ACTIVITY 3:
METHODS OF MEASUREMENT USED BY TONGANS IN THE OLDEN DAYS

OBJECTIVES
1. To explore and increase students' understanding of the measurement strategies used by the Tongan people in the olden days some of which are still being used today.

PROCEDURES
I. The lesson will then focus on the ways used by the Tongan people to estimate lengths of various objects. The following questions will be posed to students.
What did our grandparents or great grandparents use to measure the following:
i. length of a mat;
ii. length of a tapa-cloth (launima);
iii. length of rope; and
iv. distances between rows of yams or taro

II. The class (of 6 students) will be divided into 3 groups of 2. Students will be asked to discussed the above question in groups before reporting back to the class. The teacher will then use her own knowledge and understanding together with those of the students to clarify ideas and to identify some of the methods of measurement used by Tongans in the olden days and in everyday life.

III. Once all ideas are exhausted a task will be set for students to do as home-work (WORKSHEET 1). The remaining time in this lesson will be spend explaining and clarifying what this homework task is involved. Since these estimation activities will be held on a weekly basis, the students will have a week to work on this task.

PEPA NGAUE 1

FILI HA TAHA 'O E NGAAHI ME'A 'I LALO.

i. loloa 'o ha fala,
ii. loloa 'o ha ngatu launima,
iii. loloa 'o ha kafa, mo e
iv. vaha'a 'o ha ongo 'otu 'ufi pe talo

FEKUMI KI HE NGAAHI FOUNGA NA'E NGAUE'AKI 'EHE KAKAI TONGA KE FUA E LOLOA 'O E ME'A KO 'ENI. FAKA'EKE'EKE HO'O ONGOMATU'A, NGAAHI KUI PE HA TAHA PE 'I HOMOU KOLO FEKAU'AKI MO E ME'A NI.

Fa'u ha'o lipooti peesi 'e taha 'o fakaha ai 'a e ngaahi fakamatala na'a ke ma'u, pea ke ngaue'aki 'a e ngaahi 'ulu'i tohi ko 'eni 'oka 'oku ke fiema'u.

- Taumu'a 'o e fekumi
- Na'e anga fefe ho'o tanaki ho'o ngaahi fakamatala
- Ngaahi fakamatala fekau'aki mo e founga fua na'a ke tanaki mai
- Ha to e me'a pe 'oku ke pehe 'oku mahu'inga.

WORKSHEET 1

SELECT ONE OF THE FOLLOWING:

i. length of a mat;
ii. length of a tapa-cloth (launima);
iii. length of rope; and
iv. distances between rows of yams or taro

FIND OUT FROM PARENTS, GRANDPARENTS OR SOMEONE IN YOUR VILLAGE, HOW THE TONGAN PEOPLE MEASURE THE LENGTH THIS OBJECT.

Write up the result of your investigation using the following headings if you wish:

- Title
- How did you collect your information ie. who did you interview?
- Describe the method used by people to measure the length of the chosen object.
- Other important or interesting information.

LESONI 4:
FAKAFUOFUA NGAUE'AKI 'A E NGAAHI KONGA 'O E SINO

NGAAHI TAUMU'A

1. Ke fakalahi 'a e 'ilo 'a e fanauako fekau'aki mo hono ngaue'aki 'o e ngaahi kongokonga 'o e sino ki he fakafuofua e loloa 'o ha fa'ahinga me'a.
2. Ke fakamahu'inga pea mo ngaue'aki 'ehe fanau 'a e ngaahi founga fakafuofua na'e ngaue'aki 'ehe kakai Tonga 'i lokiako. Ke fua 'e he fanau 'a e loloa 'o ha ngaahi me'a 'aki 'a e **hanga, ofa, va'e'** pea mo 'enau **manga.**
3. Ke fakalahi 'a 'enau 'ilo ki ha ngaahi benchmarks ke tokoni ki he fakafuofua loloa.

FAKAIKIIKI

I Ko e potalanoa mo e kalasi fekau'aki mo e ngaahi kongokonga kehekehe 'o e sino na'e ngaue'aki 'e he kakai Tonga pea pehe ki he ngaahi fonua kehekehe 'o mamani ki hono fua 'o e loloa 'o ha ngaahi me'a. Hange ko hono ngaue'aki 'o e **hanga** ki hono fua 'o e loloa 'o ha fala; **ofa** ki he loloa 'o ha maea; pea pehe ki he **manga** pe **va'e** ki hono fua 'o e vaha'a 'o ha ongo 'otu 'ufi.

'Oku mahu'inga ke 'oange ki he fanau ha faingamalie ke nau vahevahe mo e toenga 'o ke kalasi 'a 'enau ngaahi fakakaukau fekau'aki mo e ngaahi fua ko 'eni, 'aia ko e fa'ahinga 'iate kinautolu 'oku kei ngaue'aki pe 'i he ngaahi 'aho ni. 'Oku mahu'inga foki ke 'oange kiate kinautolu ke nau 'omai 'enau ngaahi fakakaukau fekau'aki mo e ngaahi 'uhinga nai na'e ngaue'aki ai 'a e konga kehekehe 'o e sino ki hono fua. 'O hange nai ko hono ngaue'aki 'o e **hanga** ko hono fua 'o e loloa 'o ha fala kae ngaue'aki 'a e "ofa" ki he fua 'o e loloa 'o ha maea. Ke ngaue'aki 'ehe fanau 'a 'enau ngaahi fakamatala na'e ma'u mei he'enau ngaahi fekumi (PEPA NGAUE 1).

II. Ngaue'aki 'a e fehu'i ni ke faka'ai'ai 'a e fanau ke 'omai 'enau ngaahi fakakaukau.
'E anga fefe ha'o fua 'a e fa lahi 'o loki ni pe loloa 'o e palakipoe 'okapau 'oku 'ikai ke 'iai ha'o me'afua hange ko e lula pe tepi?

III. Ngaue tautau tokoua ke fakafonu 'a e tepile ko 'eni (PEPA NGAUE 2)

ACTIVITY 4:
ESTIMATE USING DIFFERENT PARTS OF THE BODY

OBJECTIVES

1. To promote students' understanding and awareness of the use of different part of the body for estimating length.
2. To develop an awareness of the appropriate use of the traditional Tongan methods of measuring length in the mathematics classroom. By measuring length using the **hand-span, arm-span, foot,** and **step.**
3. Top develop benchmarks for estimating length.

PROCEDURES

I. Discuss with the class different parts of the body used by Tongans as well as people in other countries, to measure lengths of different objects. For instance, the Tongans used **argumented-handspan** to measure the length of a mat; and **arm-span** for the length of a rope; and **pace** or **foot** for the distance between rows of yams or taro.

It is important that students are given time to discuss and share with the rest of the class their ideas about these ways of measuring mats, tapa, rope and distances between rows of yams, some of which are still being used today. They should also be given time to share their ideas about possible reasons for using different parts of the body for measuring different objects. For example, why did they used a **hand-span** to measure the length of a mat but use an "arm-span" to measure the length of a rope? Students will be encouraged to use the information and result of their small investigation (WORKSHEET 1) during this discussion.

II. Use the following question to initiate discussions of students' ideas.
How would you measure the width of this room or length of the blackboard if you did not have a measuring device such as a ruler or a tape measure?

III. Work in pairs to complete this table (WORKSHEET 2)

PEPA NGAUE 2

ME'A KE FUA	ME'A KE FAI'AKI 'A E FUA
Falahi 'a ho'o pepa ako	tuhu fosi'i 'e --- nima 'e
Ma'olunga 'o ho'o tesi	nima 'e --- hanga 'e
Falahi 'o e matapa sio'ata	nima 'e --- hanga 'e
Ho ma'olunga	hanga 'e --- ofa 'e
Ma'olunga 'o e matapa	hanga 'e --- ofa 'e
Falahi 'o e palakipoe/kopate 'a e faiako	hange 'e --- ofa 'e
Falahi 'o e lokiako	va'e 'e --- manga 'e

WORKSHEET 2

OBJECTS	UNIT OF MEASUREMENT
Width of my exercise book	little finger --- hand
Height of my desk	hand --- hand-span
Width of the window	hand --- hand-span
My height	hand-span --- arm-span
Height of the door	hand-span --- arm-span
Width of the blackboard	hand-span --- arm-span
Width of the classroom	foot --- pace(s)

IV. Fakafehoanaki 'a e ngaahi tali ko 'eni 'a e fanau. Vakai'i pea fakamatala'i e 'uhinga 'oku kehekehe ai 'a e ngaahi tali 'e ni'ihi. Pea ke hanga 'e he fanauako 'o fokotu'u mai ha founga 'e lava ai ke tatau 'a e ngaahi tali. 'E fakamatala'i leva heni 'e he faiako 'a e anga hono fokotu'utu'u 'o e ngaahi 'iuniti 'oku ngaue'aki 'ehe fua metuliki (metric units for measurement of lengths).

IV. Compare students' responses. Explain why some may be different from yours and the need to standardise units of measurements such a the metric units for measurement of lengths.

LESONI 5: TAUMU'A 'O E FAKAFUOFUA MO E FAKAFUOFUA LELEI

NGAAHI TAUMU'A

1. Ke fakamahu'inga 'a e fakafuofua ko e poto'i ngaue faka-fika 'oku mahu'inga he lesoni fika pea pehe ki he mo'ui faka'aho.
2. Ke fakalahi 'a e 'ilo 'a e fanauako fekau'aki mo e feitu'u pea mo e taimi 'oku ngaue'aki ai 'a e fakafuofua pea ko e founga pe ia 'oku malava ke ngaue'aki.
3. Ke toe fakamalohi'i ange 'a e ilo mo hono ngaue'aki 'a e reasonable range.

FAKAIKIIKI

I. 'E ngaue}aki 'a e ngaahi fehu'i ko 'eni:

i. Ko e mita nai 'e fiha 'e fiema'u ke tuitui'aki ha tupenu 'o ha ki'i tamasi'i kalasi 6.?
ii. Ko e mita 'e fiha 'e fiema'u ke tuitui'aki ha puipui 'o e matapa 'o e lokiako?
iii. Ko e ha nai e loloa 'o e kafa 'o ha tokotaha lahi?

II. Ke fokotu'u 'ehe faiako mo e fanauako ha reasonable range ki he ngaahi fehu'i ko'eni.

III. Ke fakamamafa'i 'i fepotalanoa'aki mo e fanau 'a e 'ikai taau ke ngaue'aki pe ha tali 'e taha 'oku totonu.

ACTIVITY 5: PURPOSES OF ESTIMATION AND MORE ON REASONABLE RANGE

OBJECTIVES

1. To encourage students to value estimation as an important mathematical and practical skill.
2. To increase students' awareness of the contexts and/or time where estimation is required or the only viable method to use.
3. To further promote the concept of reasonable range.

PROCEDURES

I. The following questions were asked:

i. How many metres of materials do we need to purchase to make a tupenu (skirt worn by boys) for a class 6 student?
ii. How many metres of material do we need to make a puipui (curtain) for the classroom door?
iii. How long is an adult's kafa (sennit, i.e. a rope made of coconut fibre)?

II. The teacher and the students will then establish reasonable range for each of the above tasks.

III. Discussions will focus on the inappropriateness of one answer for the above tasks.

LESONI 6:
NGAAHI BENCHMARK KI HE FAKAFUOFUA E LOLOA

NGAAHI TAUMU'A

1. Ke faka'ai'ai 'a hono ngaue'aki 'a e 'u kongokonga 'o e sino ke fai'aki 'a e fakafuofua.
2. Ke faka'ai'ai 'a hono ngaue'aki 'a e ngaahi founga fakafuofua.

FAKAIKIIKI

I. 'E ngäue faka-kulupu leva 'a e fanau ke fakafonu 'a e tepile ko 'eni (PEPA NGAUE 3)

ACTIVITY 6:
BENCHMARKS FOR ESTIMATING LENGTH

OBJECTIVES

1. To encourage use of different part of the body for estimating length.

2. To encourage the use of a variety of strategies for estimating length.

PROCEDURES

I. Students will then work in pairs to complete this table (WORKSHEET 3)

PEPA NGAUE 3

ME'A 'OKU KE 'ILO HONO FUA (BENCHMARK)	FAKAFUOFUA E LOLOA
Falahi 'o e ki'i fo'i tuhu si'isi'i	
falahi ho nima	
loloa ho'o hanga-fesi	
falahi ho va'e	
loloa ho'o manga	
loloa ho'o ofa	

WORKSHEET 3

BENCHMARK	ESTIMATE LENGTH OF BENCHMARK
Little finger	
hand	
hand-span	
foot	
pace	
arm-span	

II. Talanoa'i mo e fanauako 'a e 'enau ngaahi fakafuofua.

III. Ke fua 'ehe fanau 'a e ngaahi kongokonga 'o e sino 'o ha 'i he PEPA NGAUE 3. Pea ke hoko 'a e ngaahi me' ko 'eni ko ha ngaahi benchmarks ke fai'aki ia 'a 'enau fakafuofua'i 'o ha ngaahi me'a hange ko ia 'oku ha he PEPA NGAUE 2.

IV. Ke faka'ai'ai 'a e fanau ke nau fakafehoanaki 'a e ngaahi me'a 'oku fiema'u ke fakafuofua mo 'enau ngaahi benchmarks. Ke toutou ngaue'aki 'a e fakafehoanaki pea vakai'i mo e tali 'oku ma'u pe ko ha fakafuofufa lelei. Vakai'i foki pe ko fe 'a e benchmarks 'oku lelei ange ke ngaue'aki.

II. Discuss with the class their estimates and the reasonableness of their estimates.

III. The students will then be required to measure the lengths of the various body parts specified in WORKSHEET 3. These will then be used for estimating various lengths such as those used in WORKSHEET 2.

IV. Students will make comparisons of the items to be estimated with the benchmarks, reflect on their estimates and modify their estimates if necessary. This process of making comparisons and reflecting on the estimates will be used throughout this activity. Students will also be required to determine which benchmark is appropriate for a particular task.

LESONI 7:
LOLOA 'O HA NGAAHI ME'A KE NGAUE'AKI 'E HE FANAU KE FAKAFUOFUA'I'AKI 'A E LOLOA.

NGAAHI TAUMU'A

1. Ke lava 'e he fanau 'o ma'u ha fakakaukau pea mo e ongo ki he loloa 'o milimita, senitimita 'e taha, mita 'e taha mo e kilomita 'e taha.

2. Ke fua 'a e loloa 'o ha ngaahi me'a 'oku lahi hono ngaue'aki 'ehe fanauako ke tokoni kiate kinautolu ke fakamanatu'i'aki 'a e loloa 'o e senitimita, mita pe kilomita.

3. Ke kei hokohoko ai pe hono tokoni'i 'a e fanauako ke mahu'inga malie 'a e ngaahi fakafuofua lelei 'aki hono ngaue'aki 'a e tali 'oku 'i he vaha'a 'o ha ongo mata'ifika, ka 'oka 'ikai ko e mata'ifika pe 'e taha.

FAKAIKIIKI

I. Ke 'oange ki he fanau ha faingamalie ke nau fakahaa'i 'i ha lula 'a e loloa 'o e milimita, senitimita, senitimita 'e 50, mo e mita 'e 1.

II. Ke potalanoa'i mo e fanau 'a e ngaahi fehu'i ko 'eni.
Fakahingoa mai ha me'a ko hono loloa pe falahi 'oku ofiofi ki he:
- *milimita 'e 1*
- *senitimita 'e 1*
- *mita 'e 1*

III. Ke ngaue 'a e fanau ki he ngaahi me'a ko 'eni , pea ke toki talanoa'i 'e he kalasi mo e faiako 'a e ngaahi tali mo e founga na'e ngaue'aki 'ehe fanau.
1. Fai ha'o fakafuofua ki he ngaahi me'a ko'eni:
- *falahi 'oha lokiako*
- *ma'olunga 'o ha tamasi'i kalasi 6 pe foomu 3*
- *loloa 'oha peni vahevahe*
- *loloa 'o ha huo sipeiti*

ACTIVITY 7:
BENCHMARKS FOR ESTIMATING LENGTH

OBJECTIVES

1. To assess and help develop students' ideas and mental image or a feel for the units used for measurement of lengths of such as a millimetre, centimetre, metre and kilometre.
2. To promote the use of a variety of benchmarks. To provide students with opportunities to measure the lengths of different objects, which they are very familiar with. Students will then be encouraged to use the lengths of these objects as benchmarks for estimating lengths.
3. To further help develop students' understanding of reasonable estimates using reasonable ranges.

PROCEDURES

I. Students will be asked to show on a ruler the length of a millimetre, a centimetre, 50 centimetres, and 1 metre.

II. A discussion will then take place focusing on the following question.
Name an object which length or width is about:
- *1 millimetre*
- *1 centimetre*
- *1 metre*

III. The students will then work on estimation activities before the whole class discuss the estimates and strategies used to formulate their estimates
1. Estimate the following:

- *width of a classroom*
- *height of a class 6 or form 3 student*
- *length of a pencil*
- *length of a spade*

2. *Fakahingoa mai ha ngaahi me'a 'oku ofiofi honau loloa pe maukupu ki he:*
- *milimita 'e 1*
- *senitimita 'e 1*
- *senitimita 'e 10*
- *senitimita 'e 20*
- *mita 'e 1*
- *mita 'e 2*
- *kilomita 'e 1*

NGAAHI FAKATOKANGA

1. Ke 'oua na'a ngaue'aki ha lula pe tepi ke fua 'a e ngaahi me'a 'oku ha 'i 'olunga. Ke faka'ai'ai 'a e fanau ke nau fakafuofua 'a e ngaahi tali.

2. 'Oku mahu'inga foki ke 'oange ki he fanau ha faingamalie ke nau toutou fai ai 'a e fakafuofua pea ke talanoa mo e faiako mo e toenga 'o e kalasi 'a e founga fakafuofua 'oku nau ngaue'aki.

3. Ke faka'ai'ai 'a e fanau ke nau ngaue'aki 'a 'enau ngaahi fakakaukau mo e fakakaukau 'a e kau memipa 'o e kalasi ke fakalelei'i'aki 'enau ngaahi fakafuofua pea mo vakai'i pe ko ha ngaahi fakafuofufa lelei ia.

2. *Name objects which lengths or widths are close to:*
- *1 millimetre*
- *1 centimetre*
- *10 centimetres*
- *20 centimetres*
- *1 metre*
- *2 metres*
- *1 kilometre*

NOTES

1. Rulers will not be used to measure the lengths given above. Students will be encouraged to estimate.

2. It is very important that students are given opportunities to make estimates and to observe the teacher and the members of the class make estimates.

3. Students should be encouraged to use their own ideas and those used by other members of the class to modify their estimates and determine the reasonableness of their answers.

LESONI 8:
NGAAHI 'IUNITI KI HE FUA MAMAFA

NGAAHI TAUMU'A

1. Ke fakalahi 'a e ngaahi benchmark 'oku ngaue'aki 'ehe fanau ke fakafuofua e mamafa.
2. Ke tokoni'i 'a e fanau ke nau ma'u ha fakafuofua lelei ki he mamafa 'o e ngaahi 'iuniti 'oku ngaue'aki ki he fua mamafa.
3. Ke faka'ai'ai 'a e fanau ke nau ngaue'aki 'a 'enau ngaahi 'ilo 'oku nau ma'u mei he'enau mo'ui faka'aho.

FAKAIKIIKI

I. Ngaue'aki 'a e ngaahi me'a ngaue (standard weights) ('oku nau kalami 'e 1, 2, 5, 10, 20, 50) ke tokoni'i'aki 'a e fanau ke nau ma'u 'a e fa'ahinga ongo 'o e mamafa takitaha. Ke to'o 'ehe fanau 'a e ngaahi me'a ngaue ko 'eni ke nau ongo'i ai 'a honau ngaahi mamafa takitaha.

II. Ke nau ngaue ki hano tali 'o e ngaahi fehu'i 'oku ha he PEPA NGAUE 4.

III. Ke 'omai ki he kalasi 'a e ngaahi tali ke talanoa'i ai mo e faiako.

ACTIVITY 8
DEVELOPING A 'FEEL' FOR THE SIZES OF THE METRIC UNITS FOR WEIGHT

OBJECTIVES

1. To increase the variety of benchmarks used for estimating weight
2. To help further develop students' feel for the sizes of standard units of measurement.
3. To encourage the use of, or help students generate links to, ideas developed in everyday life.

PROCEDURES

I. Use standard weights (1, 2, 5, 10, 20, 50 grams) to help students develop a feel for the size of these weights. These weights will pass around so that students can hold them and get a feel for their sizes.

II. Students will then be required to complete WORKSHEET 4

III. Students' responses will then be used in a class discussion.

PEPA NGAUE 4

I. FAI HA'O FAKAFUOFUA KI HE MAMAFA 'O E NGAAHI ME'A KO 'ENI:	II. FAKAHINGOA MAI HA ME'A 'E TOLU 'OKU OFIOFI HONAU MAMAFA KI HE:
1. peni vahevahe	1. kalami 'e 5
2. puha masi	2. kalami 'e 200
3. kapa pulu	3. kalami 'e 500
4. fo'iniu mata	4. kilokalami 'e 1
5. kato manioke	5. kilokalami 'e 5
6. tokotaha kalasi 1	6. kilokalami 'e 20
7. motoka	7. kilokalami 'e 50
8. veeni	8. kilokalami 'e 100
	9. kilokalami 'e 500
	10. kilokalami 'e 1000

WORKSHEET 4

I. ESTIMATE THE WEIGHT OF THE FOLLOWING OBJECTS:	II. NAME THREE OBJECTS WHICH WEIGH CLOSE TO:
1. pencil	1. 5 grams
2. box of matches	2. 200 grams
3. can of corned beef	3. 500 grams
4. green coconut	4. 1 kilogram
5. basket of tapioca	5. 5 kilograms
6. class 1 student	6. 20 kilograms
7. motorcar	7. 50 kilograms
8. van	8. 100 kilograms
	9. 500 kilograms
	10. 1000 kilograms

LESONI 9:
MAMAFA 'O HA NGAAHI ME'A KE NGAUE'AKI 'E HE FANAU KE FAKAFUOFUA'I'AKI 'A E MAMAFA.

NGAAHI TAUMU'A

1. Ke fakalahi 'a e ngaahi benchmark 'oku ngaue'aki 'ehe fanau ke fakafuofua e mamafa.
2. Ke tokoni'i 'a e fanau ke nau ma'u ha fakafuofua lelei ki he mamafa 'o e ngaahi 'iuniti 'oku ngaue'aki ki he fua mamafa.
3. Ke faka'ai'ai 'a e fanau ke nau ngaue'aki 'a 'enau ngaahi 'ilo 'oku nau ma'u mei he'enau mo'ui faka'aho.

ACTIVITY 9:
BENCHMARKS FOR ESTIMATING WEIGHT

OBJECTIVES

1. To increase the variety of benchmarks used for estimating weight
2. To help further develop students' feel for the sizes of standard units of measurement.
3. To encourage the use of, or help students generate links to, ideas developed in everyday life.

FAKAIKIIKI

I. Ngaue'aki ha ngaahi me'afua (mei he loki saienisi, falekai mo e falekaukau) ke fua'aki 'ehe fanauako ha ngaahi me'a ke hoko ia ko ha'anau ngaahi benchmark ke fakafuofua 'o e mamafa ha fa'ahinga me'a.

II. Ngaue'aki 'a e ngaahi me'a ngaue na'e ngaue'aki 'i he LESONI 8 ke fai'aki 'a e fakafuofua 'a e ngaahi me'a ma'ama'a ange ke kalami 'e 100.

III. Ngaue'aki 'ehe fanauako 'a e me'afua mei he falekaukau (provided by the researcher) ke nau takitaha fua hono mamafa ke hoko ia ko ha benchmark ke fakafuofua 'a e 'a e ngaahi me'a 'oku mamafa 'a e he kilokalami 'e 50.

IV. Ke fakalelei'i 'ehe fanau 'a e ngaahi fakafuofua mo e tali ki he PEPA NGAUE 4.

PROCEDURES

I. Students will use scales (from the science lab, kitchen, and bathroom) to measure weight of various objects of their choice which they will then use as benchmarks for estimating weights.

II. Use the standard weights used in ACTIVITY 8 to estimate the weights of objects below 100 grams.

III. Students will weigh themselves using the bathroom scale and the students' weights will then be used for estimating weights greater than 50 grams.

IV. Students will again reassess their estimates and answers given to WORKSHEET 4.

LESONI 10:
NGAAHI 'IUNITI KI HE FUA 'O E HUHU'A PE LAHI 'O HA VAI

NGAAHI TAUMU'A

1. Ke tokoni'i 'a e fanau ke nau ma'u ha fakafuofua lelei ki he ngaahi 'iuniti (mililita mo e lita) 'oku ngaue'aki ki he fua ha huhu'a pe ha vai 'i ha fa'ahinga 'ai'anga me'a.

2. Ke 'oange ha faingamalie ki he fanauako ke nau ngaue'aki ai 'a e ngaahi me'afua kehekehe ki he fua 'o e vai

FAKAIKIIKI

I. Ngaue'aki ha tini vai, pea mo e ngaahi me'a 'ai'anga vai (sepuniti, poulu supo, sioki, kane lita 'e 5) ke kamata'aki hano faka'ai'ai 'a e fanau ke nau fai ha fakafuofua ki he fua 'o ha huhu'a. Ke kamata'aki 'a e lesoni 'a ki'i ngaue ko 'eni.

Ngaue'aki ha taha 'o e ngaahi 'ai'angame'a 'oku 'oatu ke ohu'aki ha vai mililita 'e taha meihe tini vai.

Pea ke talanoa'i leva 'ehe fanauako mo e faiako 'a e lahi 'o e ngaahi me'i vai na'e ohu 'ehe tokotaha kotoa pe. Ke vakai'i pea fakafehoanaki 'a e ngaahi tali mo e fakakaukau 'a e fanau.

II. 'E ngaue'aki leva 'ehe fanau ha ngaahi me'afua vai ke fua'aki 'a e lahi 'o e me'i vai na'a nau ohu mei he tini. 'E lau leva 'a e lahi 'o e me'i vai ko 'eni ko 'enau **'uluaki fakafuofua.**

III. Ke toe fai 'e fanau 'a e ki'i ngaue ko 'eni pea fakalelei 'enau **'uluaki fakafuofua.** Ke hokohoko atu pe hono fai kae 'oua leva kuo ofi 'aupito 'a 'enau fakafuofua ki he mililita 'e taha.

IV. Ke hokohoko atu 'a e ngaue ko 'eni 'aki hano ohu ha me'i vai mililita 'e 10, 200, 250 mo e 500. Pea ke tau'ataina foki 'a e fanau ke fili ha ngaahi lahi te nau fiema'u ke nau ngaue'aki.

'Oku faka'amu ke hoko 'a e ki'i ngaue ko 'eni ke faka'ai'ai 'a e 'ilo 'a e fanau ki he ngaahi 'iuniti 'oku ngaue'aki ki hono fua 'a e lahi 'o e vai pe huhu'a 'i ha fa'ahinga 'ai'angame'a pehe foki ki honau ngaahi lahi takitaha.

ACTIVITY 10:
DEVELOPING A 'FEEL' FOR THE SIZE OF VARIOUS CAPACITIES

OBJECTIVES

1. To help develop a 'feel for' the sizes of standard units (millilitre and litre) for capacity.

2. To provide students with opportunities to work with a variety of measuring devices for capacity.

PROCEDURES

II. Use a large bowl or container of water and familiar containers or objects used in everyday life (teaspoon, bowl, jug, 5-litre bucket) to encourage students to make estimates of capacity. The lesson will begin with this activity.

Select any of the objects provided and draw 1 millilitre from the container of water.

A discussion of the amount of water that each student have drawn will then follow. The students and teacher will assess compare the amount of water and the object used by students.

Students will then measure the amount of water that they would have drawn from the bowl of water. These measures will be considered their **initial estimates**.

III. This activity will be repeated to allow students to reassess and modify their **initial estimates**. Students will continue the process until they can draw out an amount of water which is very close to 1 millilitre.

IV. The activity will be repeated with other measures such as : 10, 200, 250 and 500 millilitres. Students will be use other measures of their choice in this activity.

It is expected that these activities will help students develop understandings of the units and a 'feel for' the sizes of the units for measuring capacity.

LESONI 11:
UHO PE LAHI 'O HA NGAAHI ME'A KE NGAUE'AKI 'E HE FANAU KE FAKAFUOFUA'I'AKI 'A E MAMAFA.

NGAAHI TAUMU'A

1. Ke tokoni'i 'a e fanau ke nau ma'u ha fakafuofua lelei ki he ngaahi 'iuniti (mililita mo e lita) 'oku ngaue'aki ki he fua ha huhu'a pe ha vai 'i ha fa'ahinga 'ai'anga me'a.

2. Ke tokoni'i 'a e fanau ke nau 'ilo mo ngaue'aki ha ngaahi 'ai'angame'a kehekehe pe ke fai'aki 'enau fakafuofua 'a e lahi 'o e vai 'i ha fa'ahinga 'ai 'angame'a.

FAKAIKIIKI

I. Ke faka'ai'ai 'a hono ngaue'aki 'o e ngaahi benchmarks ki he fua vai 'e ngaue faka-kulupu tautau toko tolu 'a e fanau ki he ngaahi fehu'i ko 'eni (PEPA NGAUE 5).

II. Ke potalanoa'i leva 'ehe kalasi 'a e ngaahi tali mo e fakakaukau 'a e fanau.

ACTIVITY 11:
BENCHMARKS FOR ESTIMATING CAPACITY

OBJECTIVES

1. To help develop a 'feel for' the sizes of standard units (millilitre and litre) for capacity.

2. To help develop the use of a variety of benchmarks for estimating capacity.

PROCEDURES

I. To further develop the use of benchmarks for estimating capacity to the following tasks (WORKSHEET 5) will be carried out by students in groups of three.

II. The whole group will then discuss the ideas and responses to the given tasks.

PEPA NGAUE 5

I. FAKAHINGOA HA NGAAHI 'AI'ANGA ME'A 'OKU:

1. mililita 'e 10
2. mililita 'e 50
3. mililita 'e 100
4. mililita 'e 200
5. mililita 'e 500
6. mililita 'e 1000 pe lita 'e 1
7. lita 'e 2
8. lita 'e 5
9. lita 'e 50
10. lita 'e 100

II. FAI HA'O FAKAFUOFUA KI HE VOLIUME 'O E NGAAHI ME'A KO 'ENI:

1. sepuniti
2. ipuvai
3. hina huhu 'a e pepe
4. poulu supo
5. fo'iniu mata
6. sioki vai
7. kane
8. tangike 'o ha ka
9. topu
10. sima vai.

WORKSHEET 5

I. NAME AN OBJECT WHICH CAPACITY IS CLOSE TO:

1. 10 millilitres
2. 50 millilitres
3. 100 millilitres
4. 200 millilitres
5. 500 millilitres
6. 1000 millilitres or 1 litre
7. 2 litres
8. 5 litres
9. 50 litres
10. 100 litres

II. ESTIMATE THE CAPACITY OF THE FOLLOWING OBJECTS:

1. tea spoon
2. cup of tea
3. baby's bottle
4. bowl of soup
5. green coconut
6. jug
7. bucket
8. petrol tank of a car
9. tub
10. tank of water.

LESONI 12: LOLOA, MAMAFA, UHO PE LAHI 'O E VAI

TAUMU'A

1. Ke faka'ai'ai mo 'oange ha faingamalie ki he fanauako ke nau ngaue'aki 'a e ngaahi benchmarks ki he fakafuofua 'o e loloa, mamafa, mo e voliume.

FAKAIKIIKI

I. Ngaue'aki 'a e ngaahi benchmarks na'e 'osi ngaue'aki ki mu'a ke nau to e fai'aki ha fakafuofua 'a e loloa, mamafa, mo e voliume 'o ha ngaahi me'a.

II. Ke fakapapau'i 'oku mahino ki he fanauako fakafaikehekehe'i 'a e ngaahi 'iuniti 'oku ngaue'aki ki he ngaahi fua takitaha.

ACTIVITY 12: BENCHMARKS FOR ESTIMATING LENGTH, WEIGHT AND CAPACITY

OBJECTIVES

1. To further develop and practice the use of a variety of benchmarks for estimating length, weight and capacity.

PROCEDURES

I. This lesson will focus on a review of the use of the variety of benchmarks used in previous activities for estimating length, weight and capacity.

II. The teacher will help students to understand the differences in the units used for measurement of length, weight and capacity.

Appendix D: INSTRUCTIONAL UNIT ON COMPUTATIONAL ESTIMATION

TALATEU

Ko e ngaahi ngaue na'e fai na'e fakatefito ia 'i he vakai 'a e faiako 'oku lahi ange pe 'a hono ngaue'aki 'ehe fanau 'a e ngaahi founga maheni (conventional algorithms) ke fikaloto'i 'a e tali totonu ka 'oku 'ikai ko ha tali fakafuofua. Na'e ngaue'aki leva 'a e ngaahi founga fakafuofua fakafika ke fakalahi pe tanaki atu ki he ngaahi founga ngaue 'a e fanauako. Na'e 'ikai ke ngaue'aki ha ngaahi palopalema fakafika na'e ngaue'aki pe ha ngaahi fo'i fika ke 'ahi'ahi'i pe vakai'i ai 'a e 'aonga 'o hono ako'i 'o e founga fakafuofua fakafika 'aki 'a hono ngaue'aki 'a e lea faka-Tonga. Koe'uhi kuo mahino meihe ngaahi fekumi ko e front-end strategy ko e taha ia he ngaahi founga fakafuofua 'aonga taha, na'e kamata'aki 'a e 'u lesoni 'a hono ako'i mo fakafe'iloaki ki he fanau 'a e founga ko 'eni.

INTRODUCTION

The following activities was based on the results of the first interview which showed students relying on the use of the 'conventional algorithms'. Thus, in order to increase the variety of strategies that students could use, computational estimation strategies were incorporated into these teaching activities. Only a few estimation problems (which would require decisions to be made on the basis of the estimates) were used. The focus had been on estimation exercises in order to explore the effect of teaching specific estimation techniques in the Tongan language. Because researchers have found the front-end strategy to be an effective estimation procedure a decision was made to focus on this.

LESONI 1: FAKAFUOFUA 'I HANO FIKA'I HA PALOLEMA FAKAFIKA

Ko e tefito'i taumu'a 'o e lesoni ke vakai'i 'a e 'uhinga 'o e 'fakafuofua' (estimation) 'i hano fika'i ha fo'i palopalema fakafika. 'oku mahu'inga 'aupito 'a e lesoni ko 'eni koe'uhi ke fakapapau'i ai 'oku tatau 'a e 'uhinga 'a e faiako fekau'aki mo e fakafuofua fakafika mo e 'uhinga 'oku ma'u 'ehe fanauako.

Na'e kamata'aki 'a e lesoni ha fepotalanoa'aki 'a e faiako mo e fanauako ki he ngaahi fakakaukau 'a e fanau fekau'aki mo e fakafuofua. Na'e ngaue'aki 'a e ngaahi 'uhinga 'a e fanau ki he fakafuofua 'i he fua (vakai ki he chapter seven).

Na'e ngaue'aki 'a e fehu'i ko 'eni:

Ko e ha nai e 'uhinga 'o e fo'i lea fakafuofua kiate koe 'i ha tali 'o ha fo'i fika 'oku fiema'u ai ke ke fai ?
Ko e ngaahi tali 'eni 'a e fanau:

fika'i pe 'i loto

kumi pe ha tali 'oku ofiofi ki he tali totonu

kesi

ACTIVITY 1: INTRODUCTION TO COMPUTATIONAL ESTIMATION

The main purpose of the first activity was be to explore the meaning for 'fakafuofua' (estimation) in the domain of computation. This was an important part of the activity in which an understanding and a shared meaning of the characteristics of computational estimation was established.

This activity began with a discussion of students' ideas about computational estimation. A review of the definitions given for measurement estimation (see chapter seven) was used by the teacher to introduce the activity.

This question was posed:

What is the meaning of computational estimation?

The responses by students were as follows:

mental computation

finding an answer close to the exact answer

guess

mate'i

fakamahamahalo

FAKATOKANGA

1. Ko e taha 'o e ngaahi palopalema na'e fehangahangai mo e faiako e 'ikai ke 'iai fo'i leva Tonga pe 'e taha ke ne fakamatala'i 'a e computational estimation. Ke fakafaingofua'i ange 'a e fakamatala 'a e faiako na'a ne ngaue'aki pe 'a e fo'i lea 'fakafuofua' (estimation) fakataha mo ha fakamatala ko 'eni, ko e fa'ahinga fakafuofua ia 'oku fai 'i he ngaahi palopalema fakafika 'oku ngaue'aki ai 'a e tanaki (addition), kole (subtraction), liunga (multiplication) mo e vahevahe (division).

2. 'Oku mahu'inga 'aupito ke fakamamafa'i ko e fa'ahinga tali 'oku fiema'u ko ha **tali fakaofiofi** ka 'oku 'ikai ko hano fikaloto'i ha tali pe **mo'oni totonu**.

'I he fepotalanoa'aki mo e fanau na'e fakapapau'i ai 'a e ngaahi tefito'i 'ulungaanga ko 'eni 'o e fakafuofua fakafika. 'Aia 'oku kau ai 'a e:

- fakafuofua (computational estimation) ko fikaloto'i ha tali fakaofiofi (approximate answer) ki ha fo'i fika;
- ngaue'aki 'a e ngaahi founga kehekehe ka 'o 'ikai ngata pe 'i ngaahi founga maheni (conventional algorithm) kuo 'ilo ki ai 'a e fanauako;
- 'oku lahi 'a e ngaahi tali 'e ala ma'u;
- 'oku 'ikai ha mo'oni totonu ko ia ai 'e tali 'a e ngaahi tali fakaofiofi; mo e
- the concept of reasonable range used in the measurement estimation activities would also be used for assessing their estimates.

Ko e ngaahi fakamatala 'a e fanau na'e fai 'i he 'uluaki faka'eke'eke (interview) na'e ngaue'aki ai 'ehe fanau ha ngaahi mata'ifika 'oku faingofua ange hono fakataha'i (compatible numbers) ke fai'aki 'a e tanaki. Na'e ngaue'aki 'eni ke fakamamafa'i'aki 'a e malava ke ngaue'aki 'a e founga kehekehe 'i he fakafuofua fakafika. Koe'uhi na'e fakafalala 'a e tali 'a e fanau 'i he 'uluaki faka'eke'eke ki hano ngaue'aki founga maheni kuo ako'i 'i lokiako (conventional algorithm), na'e mahu'inga ke fakahaa'i ki he fanau ko e fa'ahinga 'iate kinautolu na'a nau ngaue'aki 'e kinautolu ha founga kehe, 'o hange nai.

guess or figure out

conjecture

NOTES

1. One problem encountered by the teacher was that the phrase 'computational estimation' did not have an equivalent Tongan phrase. Therefore, to facilitate discussion the term 'fakafuofua' (estimation) was used with an explanation of the relevant context involved, that is, estimation involved in problems or exercises requiring tanaki (addition), kole (subtraction), liunga (multiplication) and vahevahe (division).

2. It is very important to discuss and stress the need to obtain **approximate answers** rather than mentally computing **exact answers**.

The discussions that followed led to the teacher/researcher identifying the main characteristics of computational estimation. These included:

- fakafuofua (computational estimation) involves mentally working out an approximate answer to an exercise or a problem;
- use of a variety of strategies other than the conventional algorithm already known by some students;
- variety of possible answers;
- no one right answer thus acceptance of inexact answers; and
- the concept of reasonable range used in the measurement estimation activities would also be used for assessing their estimates.

Interview data showed that students used compatible numbers to formulate answers to addition tasks. This was used to introduce the concept of variety of strategies for computation. As most students' thinking in mathematics (as suggested by the interview data) were dominated by school taught rules it was important to point out to students that some of them had used their own strategies, for example.

Na'a ku tanaki pe 'a e 150 mo e 50 'o tu'u 'a e mo'oni ko ia [200]. Peau tanaki pe 'a e 200 mo e 20 pea tu'u 'a e mo'oni ko ia [220] peau tanaki leva 'a e ongo mo'oni 'o ma'u ai 'eku mo'oni ko 'eni [420].

I added 150 and 50 and put down that answer [200]. And then I added 200 and 20 and put down that answer [220] and then added both answers to get my answer [420].

Ko e taumu'a hono ua 'o e LESONI 1 ke fakapapau'i 'oku mahino ki he fanau 'a e mahu'inga fakatu'unga 'o e ngaahi mata'ifika.

A second purpose for the ACTIVITY 1 was to ensure that students had a good understanding of the place value of numbers.

Koe ha 'a e mahu'inga 'o e ngaahi mata'ifika 'oku laine'i atu?

What is the place value of the underlined digit?

1.	4<u>3</u>	5.	<u>3</u>2 856
2.	2<u>3</u>4	6.	7<u>3</u> 543
3.	<u>3</u>24	7.	32 8<u>5</u>6
4.	<u>3</u> 567	8.	73 <u>5</u>43

Ko e mahu'inga 'o e 3 'i he 4<u>3</u> ko e 3

Ko e mahu'inga 'o e 3 'i he 2<u>3</u>4 ko e 30

Ko e mahu'inga 'o e 3 'i he <u>3</u>24 ko e 30

Ko e mahu'inga 'o e 3 'i he <u>3</u> 567 ko e 3 000

Ko e mahu'inga 'o e 3 'i he 7<u>3</u>2 434 ko e 30 000

Ko e mahu'inga 'o e 5 'i he 32 8<u>5</u>6 ko e 50

Ko e mahu'inga 'o e 5 'i he 73 <u>5</u>43 ko e 500

LESONI 2: FAKAFUOFUA NGAUE'AKI 'A E MATA'IFIKA TAUPOTU KI TO'OMATA'U.

NGAAHI TAUMU'A

1. Ke ngaue'aki 'a e front-end strategy ko ha founga ke ma'u ai ha 'uluaki fakafuofua.
2. Ke tokoni'i 'a e fanau ke mahino kiate kinautolu 'a e fo'i fakakaukau ko e reasonable range. 'Aia ko hano fokotu'u ha ongo mata'i fika pea ko hona vaha'a 'e ui ia ko e reasonable range pe ko e ngaahi fakafuofua lelei.

Na'e kamata'aki 'a e lesoni 'a e fo'i fakatata ko 'eni.

ACTIVITY 2: FRONT-END STRATEGY - INITIAL ESTIMATE

OBJECTIVES

1. To introduce the use of the front-end strategy as a method of obtaining an initial estimate.
2. To help develop understanding of the concept of reasonable range by establishing lower bounds for addition and upper bounds for subtraction. That is, the idea that the answer to a computational exercise/problem would be greater or less than a specified value.

The activity was introduce using this example.

Na'e 'alu 'a Sione 'o tanaki niu mei he'ene ngoue'anga. Na'a ne 'uluaki tanaki 'ene fo'i niu 'e nimangofulu ma ua (52) 'o Fokotu'u. Pea ne to e 'alu 'o tanaki 'ene fokotu'unga 'e taha ko e fo'iniu 'e tahanima (15). Ko e fo'iniu nai 'e fiha na'e tanaki 'e Sione.

Sione collected some coconuts from his plantation. He collected fifty-two (52, coconuts and put them into one pile. Then collected another fifteen (15) coconuts and put it under the mango tree. About how many coconuts did Sione collected altogether?

Ko e tali 'a e fanau ko e 67, 'aia na'a nau fikaloto'i pe.

Students responded by giving 67 as an answer, obtained by mentally computing the answer.

Koe'uhi ke faka'ai'ai 'a e fanau ke nau fai ha fakafuofua na'e pau ke fakalahi leva 'a e ngaahi mata'i fika he fo'i palopalema. Ko ia ai na'e liliu leva 'a e ngaahi mata'ifika 'i he fo'i fakatata ki he:

In order to discourage the use of a conventional algorithm, the numbers selected should be big enough to encourage students to attempt to use an estimation technique. Thus, the numbers involved in the first example were changed to:

(2). 78 - 64 = 130

(3). 207 + 109 = 300

Na'e pau ke faka'ai'ai 'a e fanau ke 'oua te nau kumi ha tali pe mo'oni totonu, ka kenau fakakaukau ki he ngaahi mata'ifika pea kumi ha 'uluaki fakafuofua ngaue'aki 'a e front-end strategy.

Students needed to be encourage not to compute an exact answer, but rather to think of the numbers involved and make an initial estimate using the front-end strategy.

Hange nai ko 'eni, 78 + 64 = 130

For example, 78 + 64 = 130

'Aia ko e 'uluaki fakafuofua ko e 130, 'aia ko e mata'ifika taupotu taha ia ki lalo ki he ngaahi tali. Na'e faka'ai'ai 'a e fanauako ke nau ngaue'aki 'a e ngaahi tali hange nai ko e:

An initial estimate would be 130, a lower bound for the answer. Thus the students were asked encourage to use answers such as:

Ko e tali 'e lahi ange ia he 130

The answer would be bigger than 130.

Vakai ki he kolomu hoko (8 mo e 4), ko e tali 'e laka hake ia he 10. Ko ia ai ka tanaki 10 ki he 130 pea koha fakafuofua lelei ko e 140.

By considering the next column of numbers (8 and 4) it would seems that the answer would be more than 10. Therefore, if 10 is added to 130 than a better estimate would be 140.

Another example used was:

Example 3 207 + 109

Using the front-end strategy would give 300

Considering 7 and 9 one can add another 10 to give a better estimate +10 = 310

Example 4

579
+ 344

Ngaue'aki 'a e mata'ifika taupotu ki to'ohema, ko e mata'ifika taupotu ia ki lalo he tali (800)

Using the front-end strategy gives 800 i.e. an lower bound for the answer

Pea ka tau to e vakai 'a e ..... ki he kolomu hono hoko (7 + 4) 'e ma'u ai 'a e 110.

If we consider the numbers in the next column (7 + 4) that would another one hundred and ten (110).

Toe tanaki atu ia [110] ki he 800 'o ma'u ai 'a e 910.

If we add that [110] to 800 you would get 910.

Ko ha to e fakafuofua lelei ko e 910. Ko e fepotalanoa'aki mo e fanau na'e fakatefito ia 'i hano kumi ha **'uluaki fakafuofua** ka 'oku 'ikai pea toki ngaue'aki leva 'a e kolomu hoko ke vakai'i pe 'oku totonu ke fai ha fakalelei. Na'e ngaue tautau tokoua pe tolu 'a e fanau pea fepotalanoa'aki mo fevahevahe'aki 'a e' enau ngaahi fakakaukau pea nau toki ngaue fakafo'ituitui leva ki hano fai 'a e ngaahi fika 'o ngaue'aki 'a e front-end pea fakataha mo hano fakalelei'i 'o e ngaahi tali.

A improved estimate would be about 910. The whole group discussion focused on establishing an **initial estimate** followed by focusing on the next column to determine an adjustment that needed to be made to the initial estimate to give a better estimate. Students worked in pairs or groups of three to discuss these processes involved and then work individually on their exercises to formulate an estimate using the front-end strategy with necessary compensation or adjustment.

Na'e lelei 'aupito 'a hono ngaue'aki 'ehe fanau 'a e ngaahi founga. Na'e vave ''aupito 'a 'enau tali ke ngaue'aki 'a e front-end strategy. Na'e ngaue'aki 'a e fo'ilea "front-end strategy" pea fakamatala'i mo hono 'uhinga 'i he lea faka-Tongan *"fakafuofua ngaue'aki e mata'ifika ki to'ohema"*. Na'e pehe 'ehe fanau ko e founga ko 'eni 'oku:

The responses of the students in both groups were very positive. The technique were adopted quickly. The term "front-end strategy" was used in addition to the Tongan explanation of the term *"fakafuofua ngaue'aki e mata'ifika ki to'ohema"*. (Estimation using the left-most digits). In a discussion of the use of this technique students responses included:

vave ange ia

it is quicker

faingofua ange ke fikaloto'i

easier to manage mentally

faingofua ange ke ma'u ha mo'oni

easier to obtain an answer

faingofua ange hono ngaue'aki e mata'ifika lalahi (e.g. 43694 + 24789)

easier to use big number (e.g. 43694 + 24789)

Na'e ''oange ki he fanau ha faingamalie ke nau faka'aonga'i 'a 'enau 'ilo fo'ou. Na'a nau fakahaa'i foki 'a e ngaahi 'ilo ko 'eni 'aki ha'anau fepotalanoa'aki mo e toenga 'o e kalasi pea pehe foki ki hano fakahinohino 'o e founga ko 'eni 'i he palakipoe.

The students were given plenty of exercises to practice the use of the front-end strategy. They were given opportunities to demonstrate their newly constructed knowledge by sharing their ideas during whole group discussions and/or demonstrating the use of this technique on the blackboard to the whole group.

LESONI 3: FAKALELEI'I 'O HA FAKAFUOFUA NA'E MA'U 'AKI 'A E FRONT-END STRATEGY.

ACTIVITY 3: FRONT-END STRATEGY - WITH COMPENSATION

TAUMU'A

Ko e tefito'i taumu'a 'o e lesoni ko 'eni:

1. ngaue'aki 'a e front-end strategy ke kumi ha fakafuofua ki he liunga mo e vahevahe;
2. kumi ha fakafuofua taupotu taha ki lalo pe ki 'olunga; mo e
3. vakai'i 'a e lahi pe ko e mahu'inga faka-tu'unga 'o e ngaahi mata'ifika.

OBJECTIVES

The purpose of this activity were to:

1. use the front-end strategy to estimate multiplication and division problems/exercises;
2. determine upper/lower bounds for an estimate; and
3. determine an order of magnitude for the answer.

Na'e toe ngaue'aki pe 'a e front-end strategy ki he lesoni. 'Aia fakamamafa'i ai 'a e kumi ha mata'ifika ke hoko ke ne faka'ilonga'i (reference number) ko e tali 'oku lahi ia pe si'isi'i ia 'i he mata'i fika ko 'eni.

The front-end strategy was again the focus of the instruction in this lesson. Again, the emphasis was placed on determining a reference number to which the answer is bigger or smaller.

Kumi ha mata'ifika 'e lahi ai pe si'isi'i ai 'a e tali ki he fo'i fika ko 'eni

Find a number which the answer to this exercise is less than or greater than.

Kumi ha mata'ifika 'e ofiofi ki ai 'a e tali 'o fika ko 'eni.

Find a number to which the answer to this exercise is close.

Ko e taha 'o e ngaahi lelei 'o e front-end strategy 'i hono fakafehoanaki atu ki he ngaahi founga fika maheni (conventional algorithm) ko e malava ke ma'u ha fa'ahinga fakakaukau fekau'aki mo e lahi 'o e tali 'i ha'ane ngaue'aki 'a e ngaahi mata'ifika taupotu ki to'ohema. 'I he ngaue'aki 'o ngaahi founga maheni kuopau ke te tatali kae'oua ke 'osi lelei hono fika'i pea te toki ma'u ha fakakaukau fekau'aki mo e lahi 'o e tali.

One of the advantages of the front-end strategy compared to the use of a conventional algorithm is that focusing on the leftmost-digit allows the respondents to have some idea of what the magnitude of the answer might be. On the other hand, one would need to wait until the end of calculation using a conventional algorithm to have some idea of the magnitude of the answer.

Ko e taha ‘o e ngaahi vaivai‘anga ‘o e fanau ‘i he‘enau ngaahi tali na‘e ‘omai ‘i he ‘uluaki faka‘eke‘eke ko e mahu‘inga fakatu‘unga ‘o e ngaahi tali. Ko ia ai na‘e fakatefito ‘a e lesoni ‘i hano ngaue‘aki ‘a e front-end strategy ke vakai‘i pe kumi‘aki ‘a e mahu‘inga fakatu‘unga ‘o e ngaahi tali.

The order of magnitude of the answer was shown as an area of weakness in the data collected from the first interview. Thus, one of the focuses of the activity, was to help students determine the order of magnitude of the answers to multiplication and division exercises using the front-end strategy..

Ko e ‘uluaki fakafuofua ‘oku ma‘u mei hano ngaue‘aki ‘o e front-end strategy ‘e ui ia ko upper pe lower bounds ‘o e tali. Ngaue‘aki leva ‘a e ngaahi upper pe lower bounds ke vakai‘i‘aki pe ko e ha koa ‘a e mahu‘inga fakatu‘unga ‘o e tali pe ko e lahi ‘o e tali.

The initial estimates obtained by the front-end strategy could be taken as an upper or lower bounds for an exercise depending on which operation is involved. These upper/lower bounds was then used as guide for determining the place value or magnitude of the answer.

Na‘e fiema‘u ‘a e fanau ke nau “lea‘aki” ‘a ‘enau ngaahi fakakaukau, ‘o fakamatala'i ‘a e ngaahi founga na‘a nau ngaue‘aki ke ma‘u ‘enau ngaahi tali.

Students were required to “think aloud”, that is, to explain the procedures used to formulate their answers.

Ko e ngaahi ngaue ko ‘eni na‘e fakatefito ia he liunga‘aki ‘o e ngaahi mata‘ifika tu‘u lo-ua ha matai‘ifika tu‘u lo-taha, tu‘u lo-ua ‘aki ha matai‘ifika tu‘u lo-ua, tu‘u lo-tolu ha matai‘ifika tu‘u lo-tolu.

The activities included multiplication of two-digit by one-digit, two-digit by two-digit, three-digit by two-digit, three-digit by three-digit.

For example, (1) 31 x 5 would become 3 x 5 giving 150 as the lower bound

(2) 312 x 9 would become 3 x 9 giving 2700 as the lower bound.

Ko e ngaahi ngaue ko ‘eni na‘a ne fakamahino ki he fanau ‘a e natula ‘o e ngaahi tali ki he fakafuofua. ‘Oku mahu‘inga ‘aupito ke mahu‘inga malie ki he fanau ‘a e ngaahi tali ‘‘oku fiema‘u ki he fakafuofua faka-fika.

Such activities helped students to develop an awareness of the nature of the answers for computational estimation. Developing an understanding of the types of answers involved in computational estimation is an important component of this activity.

Na‘e ngaue‘aki ‘a e fo‘i lea **tupu** ‘o hange ko ia ko e **fitungeau tupu** ke fakamahino ki he fanau ko e ngaahi tali fakaofiofi pe ‘oku fiema‘u. Ko ia ko e tali leva ki he 312 x9 ko e **uaafe fitungeau tupu.**

The use of the term **tupu** (odd) as in **fitungeau tupu** (seven hundred odd) also showed students that all that was required was an .estimate. Thus, the answer to 312 x 9 would be about uaafe fitungeau tupu (two thousands seven hundred odd).

LESONI 4-5: FOUNGA FAKOFIOFI

Ko e founga fakaofiofi (rounding procedure) na'e ngaue'aki ia 'i he lesoni hono ua. 'Oku fiema'u ke ngaue'aki ha ngaahi mata'ifika fakaofiofi (approximate numbers) ki he fakafuofua. Pea na'e ngaue'aki leva 'a e rounding procedure ke kumi'aki ha ngaahi mata'ifika fakaofiofi ke fai'aki 'a e fakafuofua.

Na'e toe ngaue'aki pe 'ehe faiako 'a e fakatata na'e ngaue'aki ki he 'i he ngaahi lesoni ki mu'a ke fakamatala'i'aki 'a e founga fakaofiofi.

ACTIVITY 4-5: ROUNDING PROCEDURE

Rounding Procedure was the second technique used in the activity. One of the skills involved in an estimation process is the use of approximate number. The rounding procedure is an approximation technique used to find approximate numbers which could be used in the estimation.

The teacher demonstrated and explained the use of the rounding procedure. The examples used earlier for the front-end strategy were again used to demonstrate this procedure.

Example 1 52 + 15 Round to the nearest ten

The teacher posed the question.

'Oku ofiofi ange 'a e 52 ki he 60 pe ko e 50? *Is 52 closer to 60 or 50?*

All students recognised that 52 was closer to 50, therefore 52 was rounded down to 50.

'Oku ofiofi ange 'a e 15 ki he 10 pe ko e 20? *Is 15 closer to 10 or 20?*

'Oku tu'u ia he vaeua, mama'o tatau pe. *It is half way, same distance.*

Koe'uhi na'e fakaofiofi 'a e 52 ki he 50 'e fakaofiofi leva 'a e 15 ki he 20 koe'uhi na'e mole 'a e 2 he fakaofiofi 'o e52 ki he 50. Ko ia ai ko e 50 + 20 'e ma'u ai 'a e fakafuofua ko e70.

Given that 52 was rounded down to 50, 15 would be rounded up to 20 to compensate for the 2 lost in rounding 52 to 50. Thus, the exercise became: 50 + 20 which would give an estimate of 70.

Example 2 78 + 64 Round to the nearest ten would lead to an estimate of 80 + 60 140

FAKATOKANGA

1. Ko e taha 'o e ngaahi lelei 'o e founga fakaofiofi (rounding procedure) ko ha lava ke ma'u ai ha ngaahi mata'ifika fakaofiofi 'e faingofua ange ai 'a 'ete ngaue. Na'e faka'ai'ai 'a e fanau ke nau tomu'a vakai'i 'a e ngaahi mata'ifika pea toki fai 'a 'enau fakafuofua kae 'oua 'e ngaue'aki 'a e founga maheni 'aia ko hano fakaofiofi pe liliu ma'u pe 'a e mata'ifika 'okapau ko e nima pe 'oku lahi hake he nima 'a e mata'ifika ki to'omata'u.

2. Na'e ngaue'aki 'ehe faiako 'a e fo'ilea **founga fakafuofua** ke fakamatala'i'aki 'a e **rounding procedure**, kae 'ikai ko e fo'ilea **launi** 'aia koe 'oku fa'a ngaue'aki 'i he lokiako Tonga.

NOTES

1. One of the advantages of using the rounding procedure discussed was that it would produce numbers that would be easier to manage mentally. Rather than encouraging students to always round up numbers that ends with five, students were encouraged to always consider the numbers involved.

2. The teacher used the term **founga fakafuofua** to refer to the **rounding procedure**, rather than the term **launi** which is often used in the mathematics classroom.

Example, 57 + 15 — Round to the nearest ten — 60 + 10

Example 3: 207 + 109 — Round to the nearest hundred — 200 + 100

giving 300 as an estimate

Na'e lava ke fakahaa'i heni ki he fanau ko e front-end strategy mo e rounding procedure 'oku fakatou ma'u ai pe 'a tali pe fakafuofua tatau ('aia ko e 300)

It was pointed out that for this particular example the front-end strategy and the rounding procedure led to the same estimate (i.e. 300).

Na'e ngaue'aki foki ha ngaahi fika kole ke fakamahino'i'aki 'a e rounding procedure.

Subtraction exercises were also used to further demonstrate the use of the rounding procedure.

Example 1: 73 - 49 — Round to the nearest ten — 70 - 50

giving 20 as an answer.

82 - 15 — becomes — 80 - 10

giving 70 as an estimate

Na'e ngaue 'a e fanau 'i he ngaahi fika tanaki mo e kole ke faka'aonga'i ai 'a e founga. Na'e ngaue tautau tokotaha pe 'a e fanau pea nau toki talanoa'i mo e toenga 'o e kalasi mo e faiako 'a e ngaahi tali mo e founga na'a nau ngaue'aki. Pea na'e tokoni foki 'a e faiako ke fakamahino ki he fanau 'a e ngaahi me'a na'e 'ikai ke mahino.

A worksheet on addition and subtraction exercises were used by students to practice the technique of rounding. Students worked individually and then discussed their answers with the whole group. The teacher moved around the group to help students.

Na'e ngaue'aki foki 'a e founga FAKOFIOFI ki hano fai ha liunga mo ha vahevahe:

(i) vakai'i 'a e mahu'inga fakatu'unga 'o e ngaahi tali.
(ii) kumi'aki ha fakafuofua.

Rounding numbers involved in multiplication and division exercises were used to:

(i) determine the order of magnitude of the answer; and
(ii) establish an estimate.

Exercises such as $21 \div 3 = 7$

$$210 \div 3 = 70$$

$$2100 \div 3 = 700$$

$$21000 \div 3 = 7000$$

Na'e ngaue'aki foki mo e ngaahi fika kehe ke faka'ai'ai 'aki 'a e fanau ke nau fai ha fakaofiofi ki he lahi 'o e tali 'o ha ngaahi mata'ifika.

Other exercises were used to help students develop ideas about order of magnitude the answers.

Ko e ha e mahu'inga fakatu'unga 'o e tali ki he ngaahi fehu'i' ko 'eni?

What is the place value or order of magnitude of the answers to the following exercises?

For example $6\overline{)684}$ the answer would be in the hundred.

LESONI 6-7: NGAUE'AKI 'A NATULA DISTRIBUTIVE O E NGAAHI MATA'I-FIKA

NGAAHI TAUMU'A

1. Ke tokoni'i 'a e fanauako ke nau ngaue'aki 'a e distributive property ke fakalelei'i'aki 'o 'enau fakafuofua (compensation process). Fakatata'aki 'eni,

ACTIVITY 6-7: USE OF DISTRIBUTIVE PROPERTY OF NUMBERS

OBJECTIVES

1. To help students use the compensation process via the distributive property of numbers. For example,

$$4 \times 15 = 4 \times (10 + 5)$$
$$40 + 20$$
$$2 \times 39 = 2 \times (30 + 9)$$

Na'e mahino mei he ngaue mo e fanau kuo nau maheni ki hono ngaue'aki 'a e distributive property. Ko ia ai na'e ngaue'aki 'a e 'ilo 'a e fanau ke nau ngaue'aki 'eni ki hono fai 'a e fakafuofua pea ke fakalelei'i foki 'a 'enau fakafuofua.

It became evident that this was not a new concept to students as these have been covered in their mathematics classroom. Thus, students' prior knowledge of the distributive property of numbers were used to facilitate discussion and work on this activity.

Na'e ngaue'aki 'a e founga ko 'eni ke faka'ai'ai'aki 'a hono ngaue'aki 'a e ngaahi mata'ifika taupotu ki to'ohema ka 'oku 'ikai ko e ngaahi mata'ifika taupotu ki to'omata'u pea pehe foki ki he ngaahi founga fakalelei (compensation process).

The use of this process was to encourage students the focus on the leftmost digits rather than the right most digits and the use of compensation procedures.

LESONI 8 - 9: COMPATIBLE NUMBERS MO E NGAAHI FOUNGA KEHE

NGAAHI TAUMU'A

1. Ke faka'ai'ai 'a e fanau ke nau ngaue'aki 'a e ngaahi mata'ifika 'oku faingofua ange 'a hono tanaki (compatible numbers) ki he fakafuofua.
2. Ke tokoni'i 'a e fanau ke nau ngaue'aki 'a e founga kehekehe ki he fakafuofua. Hange nai ko 'eni,

Vahevahe nima 'oku tatau ia mo e vahevahe hongofulu pea liunga'aki 'a e tali 'e ua.

Na'e mahino mei he 'uluaki faka'eke'eke 'o e fanau, na'e 'iai 'a e fa'ahinga 'iate kinautolu na'a nau ngaue'aki 'a e ngaahi 'a hono fakataha'i 'a e ngaahi mata'ifika ke faingofua ange ai 'a hono tanaki. Ko e lesoni hono valu na'a ne fakamamafa'i 'e ia 'a e ngaue'aki 'o e compatible numbers, tautau tefito ki hono ngaue'aki 'o e hongofulu pea mo e ngaahi mata'ifika 'oku lava 'o vahevahe ki he hongofulu.

Ke fakataha'i ha ongo mata'ifika 'e 2, ko e taha ia 'o e ngaahi founga fakafuofua na'e ngaue'aki ke faka'ai'ai 'a e fanau ke lahi 'a e ngaahi founga 'oku nau ngaue'aki 'a e mata'ifika.

Ko e taha 'o e ngaahi founga na'e ngaue'aki ko e vahevahe 65 'oku tatau ia mo e vahevahe 10 liunga 2.

Ko e founga leva faka'osi na'e ngaue'aki 'a e fakatata ko 'eni.

$$
\begin{array}{r}
105 \\
95 \\
100 \\
108 \\
110 \\
+\ 93 \\
\hline
\hline
\end{array}
$$

'Oku ofi kotoa 'a e mata'ifika takitaha ki he 100, ko ia ai 6 x 100 = 600

Ko e fakafuofua ki he tali ko e 600

ACTIVITY 8-9: USE COMPATIBLE NUMBERS AND OTHER TECHNIQUES

OBJECTIVES

1. To encourage the use of compatible numbers for estimation.

2. To help students use other techniques for estimation. For example,

Dividing by five is the same as dividing by ten and then multiplying the answer by two.

The interview data revealed that students used or combined numbers which are easier to combine to formulate their answers particularly for addition problems. Activity eight concentrated on activities encouraging students to use compatible numbers, mainly those that add to ten or multiples of ten.

Combining pairs of numbers when adding two digit-factors was another strategy used by students which would increase the flexibility with which students think about numbers and their computational estimation strategies.

Another technique used for division was that dividing by 5 is the same as dividing by 10 and then multiplying the answer by 2.

A final strategy was introduced using this esmaple

All numbers are close to 100, so 6 x 100 = 600

An estimate to this exercise is 600.

LESONI 10: FAKAMANATU 'A E NGAAHI FOUNGA FAKAFUOFUA FAKA-FIKA

Ko e tefito'i taumu'a 'o e Lesoni 10 ko e to e fakamanatu 'a e:

(i) ko e toe fakamanatu 'o e ngaahi founga fakafuofua faka-fika na'e ngaue'aki 'i he ngaahi lesoni ki mu'a.
- fakafuofua ngaue'aki 'a e mata'ifika taupotu ki to'ohema;
- founga fakaofiofi;
- ngaahi mata'ifika 'oku faingofua ange ai 'a e ngaue;
- distributive property 'o e ngaahi mata'ifika.

FAKATOKANGA

1. Ko e taha 'o e ngaahi me'a mahu'inga na'e ma'u mei he ngaahi lesoni na'e fai ko hano 'ahi'ahi ha ngaahi fo'i lea faka-Tonga ke ngaue'aki 'i he lokiako 'i Tonga ki hano ako'i 'o e topiki fo'ou ko ia ko e fakafuofua fakafika. 'I he taimi tatau pe 'oku 'iai 'a e tokanga ki he ngaahi lea 'i he lesoni fika 'i he faka-Pilitania 'oku 'ikai ke 'iai hano faka-Tonga lelei. 'O hange nai ko e,

- distributive property of numbers;
- compatible rounding;
- rounding up;
- rounding down;
- number sense;
- upper/lower bounds

Na'e ngaue'aki pe 'a e ngaahi fo'ilea faka-Pilitania ko 'eni ki hono faiako'i 'o e ngaahi 'o e ngaahi founga fakafuofua fakataha mo ha fakatata mo e fakamatala'i 'o e anga 'o hono ngaue'aki. 'Oku matu'aki mahu'inga ke fai ha tokanga ki he ngaahi lea Tonga 'e ngaue'aki koe'uhi ko e 'amanaki ko'eni ke fakalele 'a e silapa fika fo'ou.

2. Ko e fakafuofua faka-fika ko e taha 'eni ha founga 'oku malava ai ke ngaue'aki ha ngaahi founga kehekehe ke ngaue'aki 'i lokiako. 'Oku mahu'inga ke ngaue'aki 'a e poto'i faka-fika ko 'eni he 'oku lava ai ke tokoni ke fakamahino 'a e ngaahi 'ilo faka-fika ke hange nai ko e (e.g. numbers sense, mahu'inga fakatu'unga, magnitude of the answers, lelei 'o e tali) pehe foki ki hano tokoni'i 'o e fanau ke nau fakakaukau faka-fika.

ACTIVITY TEN: REVIEW OF COMPUTATIONAL STRATEGIES

The main purposes of Activity 10 were to:

(i) review students' understanding of the estimation techniques used in the activities, mainly.
- front-end strategy;
- rounding procedure;
- compatible numbers; and
- the distributive property of the numbers involved.

NOTES

1. One of the values of the teaching activities on computational estimation was the identification of various Tongan terms and phrases that could be used in teaching a relatively new topic in the Tongan classrooms, that is, computational estimation. At the same time the activities raised some concern about mathematical terms that do not have an equivalent meaning in the Tongan language. For instance,

The approach taken in this study was to use the English terms together with examples and an explanation in the Tongan language. Development of appropriate Tongan terms for the above mathematical terms is an issue that needs careful consideration in the implementation of the new draft syllabus.

2. Computational estimation activities has the potential to provide variety in the mathematics classroom. It is important to utilise this mathematical skill in the classroom. Not only does it assist in the development of understanding of other mathematical concepts (e.g. number sense, place value, magnitude of answers, reasonableness of answers) but it could also help students to think mathematically.

Appendix E: TEACHER QUESTIONNAIRE (ENGLISH VERSION)

Name of School: _____

Total Number of years
teaching at primary schools: _____

Total Number of years
teaching at secondary schools: _____

Age

20 - 29	
30 - 39	
40 - 49	
51 and over	

(Tick √ only one box)

THESE QUESTIONS SEEK INFORMATION ABOUT YOUR VIEWS AND EXPERIENCES REGARDING ESTIMATION.

THIS INFORMATION WILL **ONLY** BE USED FOR MY RESEARCH REPORT AND **WILL BE TREATED STRICTLY CONFIDENTIAL.** THE OVERALL FINDINGS, HOWEVER, MAY HELP WITH MATHEMATICS CURRICULUM DEVELOPMENT IN TONGA.

THANK YOU FOR YOUR CO-OPERATION.

1. **Teachers may have different meanings for "estimation". What is the particular meaning for "estimation" that you hold at the moment?**

.....

.....

.....

.....

.....

.....

2. **Do you think that estimation is useful in mathematics?**
(Tick √ only one box)

YES	
NO	
NOT SURE	

My reasons for thinking this?

.....

.....

.....

.....

.....

.....

3. **Do you think that it is possible to teach estimation strategies?**
(Tick √ only one box)

YES	
NO	
NOT SURE	

4. **Do you try to teach estimation strategies to your students?**
Tick √ only one box)

YES	
NO	
NOT SURE	

5. If you try to teach estimation strategies to your students then can you think of any you have tried to help your students develop? Use the following headings if you wish?

a. Measurement estimation strategies:

.....

.....

.....

.....

.....

.....

b. Computational estimation strategies:

.....

.....

.....

.....

.....

.....

c. Numerical estimation strategies:

.....

.....

.....

.....

.....

.....

6. What do you think of the estimation skills of the following students?

Student Name	Not sure	Unsatisfactory	Satisfactory	Good	Excellent

7. **Do you think you were well prepared at Teacher's Training College to teach estimation in your mathematics class?** (Tick √ only one box)

YES	
NO	
NOT SURE	

Why ?

.....
.....
.....
.....
.....
.....

8. **In the Draft Mathematics Syllabus 1992 which is being trailed in some of the schools on Tongatapu, emphasis is given to the teaching of estimation skills in Class 1 to Form 5. What do you think teachers need in order to teach this well?**

.....
.....
.....
.....
.....
.....

9. **What do you think of the way students' estimation skills are being examined and/or assessed in your school examinations and in the Secondary School Entrance Exam?**

.....
.....
.....
.....
.....
.....

10. **Any other comments you would like to make about the teaching and assessment of students' estimation skills.**

.....
.....
.....
.....

Appendix F: TEACHER QUESTIONNAIRE (TONGAN VERSION)

PEPA FEHU'I 'A E FAIAKO

Hingoa 'o e 'apiako 'oku ke faiako ai: _____

Lahi 'o e ta'u kuo ke faiako ai he lautohi si'i: _____

Lahi 'o e ta'u kuo ke faiako ai he kolisi: _____

Ta'u motu'a:

20 - 29	
30 - 39	
40 - 49	
50 pe laka hake ai	

(Tiki'i √ pe puha 'e taha)

KO E TAUMU'A 'O E NGAAHI FEHU'I KO 'ENI KE MA'U HA NGAAHI FAKAMATALA FEKAU'AKI MO HO'O NGAAHI FAKAKAUKAU MO E MAHENI FAIAKO'I 'O E **"FAKAFUOFUA"** (ESTIMATION)

KO E NGAAHI FAKAMATALA TE KE 'OMAI 'E NGAUE'AKI PE IA KE FAKAMA'OPO'OPO 'A E LIPOOTI KI HE FEKUMI 'OKU OU LOLOTONGA FAI PEA 'E 'IKAI 'AUPITO KE LAU PE VAKAI KI AI HA TAHA KEHE.

MALO 'AUPITO HO'O TOKONI'I 'A E FEKUMI NI

1. **'Oku lava pe ke kehekehe 'a e 'uhinga 'oku ma'u 'e he kau faiako ki he fo'i lea "fakafuofua" (estimation). Ko e ha 'a e 'uhinga 'o e "fakafuofua" kiate koe?**

---

2. **'Oku ke pehe 'oku 'aonga 'a e "fakafuofua" 'i he leson Fika?**
(Tiki'i √ pe 'a e puha 'e taha)

'IO	
'IKAI	
'IKAI FAKAPAPAU'I	

b) Ko hono 'uhinga 'o 'eku tali 'i 'olunga ko e ....

---

3. **'Oku ke pehe 'oku malava ke ako'i 'a e ngaahi founga fakafuofua?**
(Tiki'i √ pe 'a e puha 'e taha)

'IO	
'IKAI	
'IKAI FAKAPAPAU'I	

4. **'Oku ke fa'a feinga ke ako'i ha ngaahi founga fakafuofua ki ho'o fanauako?**
(Tiki'i √ pe 'a e puha 'e taha)

TU'OLAHI	
TAIMI PE 'E NI'IHI	
'IKAI	

5. **Kapau 'oku ke fa'a feinga ke ako'i ha ngaahi founga fakafuofua ki ho'o fanauako, ko e ha nai 'a e ngaahi founga fakafuofua ko 'eni? Ngaue'aki 'a e ngaahi 'ulu'i tohi ko 'eni 'okapau te ke fiema'u.**

   a. **Ngaahi founga fakafuofua ki he loloa, mamafa, kakano pe lahi 'o ha ngaahi me'a:**
   (Measurement estimation strategies)

---

e) **Ngaahi founga fakafuofua 'o ha mo'oni 'o ha ngaahi fika pe fakafufofua fakafika (Computations estimation strategies):**

---

f) **Ngaahi founga fakafuofua ha lahi 'o ha fa'ahinga me'a (Numerical estimation strategies):**

---

6. **Ko e ha ho'o fakakaukau ki he tu'unga 'oku 'iai 'a e poto'i fakafuofua (estimation skills) 'a e fanau ako ko 'eni?** (Tiki √ pe 'a e puha 'e taha 'o fakatatau ki he tamasi'i pe ta'ahine ako takitaha)

Hingoa	'Ikai Fk-papau'i	'Ikai Fk-fiemalie	Fakafiemalie	Lelei	Lelei 'Aupito

7. **'Oku ke pehe na'e teuteu'i lelei koe 'i he 'apiako fakafaiako ki hono faiako'i 'o e fakafuofua 'i ho'o kalasi Fika?** (Tiki √ pe 'a e puha 'e taha)

'IO	
'IKAI	
'IKAI FAKAPAPAU'I	

Ko hono 'uhinga 'o 'eku tali 'i 'olunga ko e:

---

8. **'I he silapa fika fakaangaanga 'o e ta'u 1992 (Draft Mathematics Syllabus 1992) 'aia 'oku lolotonga 'ahi'ahi'i 'i he ngaahi ako 'e ni'ihi 'i Tongatapu ni, 'oku fakamamafa'i ai 'a hono ako'i 'o e "fakafuofua" mei he kalasi 1 ki he form 5. Ko e ha ha ngaahi me'a 'oku ke pehe 'oku totonu ke tokoni'i'aki 'a e kau faiako ke nau ako'i lelei 'a e "fakafuofua"**

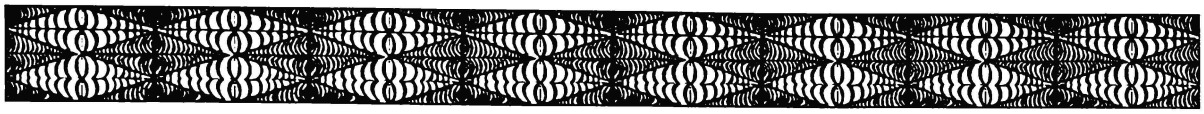
---

9. **Ko e ha ho'o fakakaukau ki he anga hono sivi'i 'o e "fakafuofua" 'i ho'omou sivi fakata'u pea pehe ki he sivi-hu ki he ngaahi kolisi?**

---

10. **'Oku 'iai ha toe me'a 'oku ke fie lave ki ai fekau'aki mo hono ako'i pe sivi'i 'o e "fakafuofua".**

BIBLIOGRAPHY



Assessment of Performance Unit (1985). *A Review of Monitoring of Mathematics 1978 to 1982 Part 1* Great Britain: Department of Education and Science.

Atweh, B. (1982). Developing mental arithmetic. In L. Silver and J.R. Smart (eds.). *Mathematic for the Middle Grades.* 1982 Yearbook for the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 50–58.

Australian Education Council. (1990). *A National Statement on Mathematics for Australian Schools.* Melbourne: Curriculum Corporation for the Australian Education Council.

Australian Education Council. (1991). *Mathematics in Our School. A Guide for Parents and Community.* Melbourne: Curriculum Corporation for the Australian Education Council.

Baggett, P. and Ehrenfeucht, A. (1992). What should be the role of calculators and computers in mathematics education. *Journal of Mathematical Behaviour,* 11(1), 61–72.

Baroody, A.J. and Gatzke (1991). The estimation of set size by potentially gifted kindergarten-age children. *Journal for Research in Mathematics EducatioN,* 22(1), 59–68.

Baroody, A.J. and Ginsburg, H.P. (1990). Children's learning : a cognitive view. In R.B. Davis, C.A. Maher and N. Noddings (eds.). *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education, Monograph Number 4.* Reston, VA: National Council of Teachers of Mathematics, 51–64.

Bauersfeld, H. (1992). Classroom cultures from a social constructivist's perspective. *Educational Studies in Mathematics,* 23(5), 467–481.

Begg, A.J.C., Mackintosh, N.P. and Thomson, D.B. (1974-76). *Moving with Mathematics Volumes 1-6.* Auckland: Longman Paul.

Begg, A.J.C. (ed.) (1991). *Mathematika Pasefika.* Hamilton: Centre for Science and Mathematics Education Research, University of Waikato.

Benton, S.E. (1986). A summary of research on teaching and learning estimation. In H.L. Schoen and M.J. Zweng (eds.). *Estimation and Mental Computation..* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 239–248.

Bestgen, B. and Reys, R. (1982). Teaching kids to estimate. *Instructor,* 91, 58–60.

Bestgen, B.J., Reys, R.E., Rybolt, J.F. and Wyatt, J.W. (1980). Effectiveness of systematic instruction on attitudes and computational estimation skills of preservice elementary teachers. *Journal for Research in Mathematics Education,* 11(2), 124–136.

Bishop, A.J. (1988). *Mathematical Enculturation A Cultural Perspective on Mathematics Education.* Dordrecht: Kluwer Academic Publishers.

Bobis, J. (1991). The effect of instruction on the development of computational estimation strategies. *Mathematics Education Research Journal,* 3(1), 17–29.

Bockarie, A. (1993). Mathematics in the Mende culture: Its general implication for mathematics teaching. *School Science and Mathematics,* 93(4),208–211.

Bodner, G.M. (1986). Constructivism: a theory of knowledge. *Journal of Chemical Education,* 63(10), 873–879.

Broekman, H. and Hoffmann, A. (1994). Learning to communicate about mathematical activities. *Cherods 8,* 42–46.

Brown, T. (1993). Do you get my meaning? *Cherods 6,* 16–21.

Bryant, L. (1986). *Pageant of the Pacific Tonga.* Melbourne: Longman Cheshire.

Campbell, I.C. (1992). *Island Kingdom: Tonga, Ancient and Modern.* Christchurch: Canterbury University Press.

Carlow, C.D. (1986). Estimation and mental computation early focus on number awareness. *Arithmetic Teacher,* 34(4), 16–17.

Carpenter, T.P., Coburn, T.G., Reys, R.E., and Wilson, J.W. (1976). Notes from national assessment: estimation. *Arithmetic Teacher,* 23(4), 296–302.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S., Lindquist, M.M. and Reys, R.E. (1980a). Results and implications of the second NAEP mathematics assessment: elementary school. *Arithmetic Teacher,* 27(8), 10–12, 44–47.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S., Lindquist, M.M. and Reys, R.E.(1980b). Results of the second NAEP mathematics assessment: secondary school, *Mathematics Teacher,* 73, 329–338.

Carpenter, T.P., Mathews, W., Lindquist, M.M. and Silver, E.A. (1984). Achievement in mathematics: results from the national assessment. *The Elementary School Journal,* 84(5), 485–495.

Carr, K.C. (1986). *Understanding Decimal Numbers: An Investigation of Students' Meaning Construction in Intermediate and High Schools* Unpublished D.Phil. Thesis. Hamilton: University of Waikato.

Case, R. and Sowder, J.T. (1990). The development of computational estimation: A neo–Piagetian analysis. *Cognition and Instruction,* 7(2), 79–104.

Churchward, C.M. (1953). *Tongan Grammar.* Nuku'alofa: Government Printing Press.

Churchward, C.M. (1959). *Dictionary Tongan–English, English–Tongan.* Nuku'alofa: Government Printing Press.

Clayton, J. (1988). Estimation. *Mathematics Teaching,* December 1988, 18–19.

Clements D.H. and Battista, M.T. (1990). Constructivist learning and teaching. *Arithmetic Teacher*, 38(1), 34–35.

Clements, M.A. (1992). Strategies for making estimating a central focus of the mathematics curriculum. In Irons (ed). *Challenging Children to Think When They Compute.* Conferences Papers, Centre for Mathematics and Science Education. Brisbane, Queensland University of Technology.

Cobb, P. (1990). Multiple perspectives. In L.P. Steffe and T. Woods (eds.). *Transforming Children's Mathematic Education: International Perspectives.* Hillsdale, New Jersey: Lawrence Erlbaum, 125–132.

Cobb, P and Steefe, L.P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education*, 14(2), 83–94.

Cobb, P., Wood, T., and Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R.B. Davis, C.A. Maher and N. Noddings (eds.). *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education Monograph Number 4.* Reston, VA : National Council of Teachers of Mathematics, 125–146.

Cobb, P. Wood, T and Yackel, E.(1992). Follow–up assessment of a second grade problem–central mathematics project. *Educational Studies in Mathematics,* 23(5), 467–481.

Coburn, T.G. and Shulte, A.P. (1986). Estimation in Measurement. In H.L. Schoen and M.J. Zweng (eds.). *Estimation and Mental Computation.* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 195–203.

Coburn, T.G. (1987). Estimation and mental computation. Measurement estimation—making adjustments. *Arithmetic Teacher*, 34(7), 24–25.

Cockcroft, W.H. (1982). *Mathematics Counts.* Report of the Committee of Inquiry into the Teaching of Mathematics in School under the Chairmanship of Dr W.H. Cockcroft. London: Her Majesty's Stationery Office.

Cohen, L. and Manion,. (1994) (4th ed.). *Research Methods in Education.* London and New York: Routledge.

Collocott, E.E.V. (1925a). Supplementary Tongan vocabulary. *Journal of the Polynesian Society*, 34(2), 146–169.

Collocott, E.E.V. (1925b). Supplementary Tongan vocabulary. Notes on measuring and counting, proverbial expressions and phases of the moon. *Journal of the Polynesian Society*, 34(3), 193–213.

Confrey, J. (1990). What constructivism implies for teaching. In R.B. Davis, C.A. Maher and N. Noddings (eds.). *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education Monograph Number 4.* Reston, VA: National Council of Teachers of Mathematics, 107–122.

Connors, J. (1990). When mathematics meets anthropology: the need for interdisciplinary dialogue. *Educational Studies in Mathematics*, 21, 461–469.

Cooney, T.J. (1988). Teachers' decision making. *Mathematics, Teachers and Children.* London, Sydney, Auckland, Toronto: Hodder and Stoughton in association with Open University, 273-286.

Corle, C.G. (1960). A study of the quantitative values of fifth and sixth grade pupils. *Arithmetic Teacher*, VII(7), 333–340.

Corle, C.G. (1963). Estimates of quantity by elementary teachers and college students. *Arithmetic Teacher*, 10, 347–353.

Crites, T.W. (1993). Strategies for estimating discrete quantities. *Arithmetic Teacher,* 41(2), 106–108.

Cummins, H.G. (1977). *School and Society in Tonga 1826 to 1854.* Unpublished M.A. Thesis. Canberra: Australian National University.

Davis, R.B. and Maher, C.A., (1993). (eds.). *Schools, Mathematics and the World of Reality.* Boston: Allyn and Bacon .

Davis, R.B., Maher, C.A. and Noddings, N. (1990). Suggestions for the improvement of mathematics. In R.B. Davis, C.A. Maher and N. Noddings (eds.). *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education Monograph Number 4.* Reston, VA: National Council of Teachers of Mathematics, 187–191.

De Corte, E. and Somers, R. (1982). Estimating the outcome of a task as a heuristic strategy in arithmetic problem solving: a teaching experiment with sixth–graders. *Human Learning*, 1, 105–121.

De Corte, E. and Verschaffel, L. (1981). Children's solution processes in elementary arithmetic problems: Analysis and improvement. *Journal of Educational Psychology*, 73(6), 765–779.

Department of Education. (1982). *The Second IEA Mathematics Study: Preliminary Item Analyses New Zealand Population.* Wellington: Research and Statistics Division, Department of Education.

Department of Education. (1987). *Mathematics Achievement in New Zealand Secondary Schools. A Report on the Conduct in New Zealand of the Second International Mathematics Study for the Evaluation of Educational Achievement.* Wellington: Department of Education.

Department of Education and Science and the Welsh Office. (1991). *Mathematics in the National Curriculum.* London: Her Majesty's Stationery Office.

Department for Education and the Welsh Office. (1995). *Mathematics in the National Curriculum.* London: Her Majesty's Stationery Office.

Devender, E.M. (1992). Helping students acquire thinking skills through mathematics instruction. *School Science and Mathematics*, 92(6), 331–334.

Dorward, J. and Archibald, S. (1994). Linking teacher interests or background to real–world experiences for students. *Arithmetic Teacher,* 41(6), 300–303.

Dowker, A.D. (1992). Computational estimation strategies of professional mathematicians. *Journal for Research in Mathematics Education,* 23(1), 45–55.

Dubinsky, E. and Lewin, D.C. (1986). Reflective abstraction and mathematics education: the genetic decomposition of induction and compactness. *Journal of Mathematical Behavior*, 5(1), 55–92.

Easley, J.A.Jr. (1982). Naturalistic case studies exploring social-cognitive mechanisms, and some methodological issues in research on problems of teachers. *Journal of Research in Science Education,* 19(3), 191–203.

Easley, J. and Easley, E. (1992). *Changing Mathematics Teaching in the Elementary School.* Geelong: Deakin University.

Edwards, A. (1984). Computational estimation for numeracy. *Educational Studies in Mathematics*, 15(1), 59–73.

Ellerton, N.F. and Clements, M.A. (1992). Some pluses and minuses of radical constructivism in mathematics education. *Mathematics Education Research Journal*, 4(2), 1–22.

Erlwanger, S.H. (1973). Benny's conception of rules and answers in IPI mathematics. *The Journal of Children's Mathematical Behaviour*, 1(2), 7–26.

Erlwanger, S.H. (1975). Case studies of children's conceptions of mathematics–part 1. *The Journal of Children's Mathematical Behaviour*, 1(3), 157–283.

Fennema, E., Carpenter, T.P., Franke, M.L. and Carey, D.A. (1993). Learning to use children's mathematics thinking: A case study. *In.* R.B. Davis and C.A. Maher (eds.). *Schools, Mathematics and the World of Reality.* Boston: Allyn and Bacon

Foliaki, L. (1994). Commonalities and diversities. In H. McQueen (ed.). *Education is Change: twenty viewpoints.* Wellington: Bridget Williams Books.

Fuller, S. (1994). The reflexive politics of constructivism. *History of Human Sciences*, 71(1), 87–93.

Gadanidis, G. (1994). Deconstructing Constructivisim. *Mathematics Teacher,* 87(2), 91–97.

Gagne, R.M. (1968). Contribution of learning to human development. *Psychological Review,* 75, 177–191.

Garofalo, J. and Bryant, J. (1992). Assessing reasonableness: some observations and suggestions. *Arithmetic Teacher,* 40(4), 210–213.

Gay, J. and Cole, M. (1967). *The New Mathematics and an Old Culture A Study of Learning Among the Kpelle Liberia.* New York: Rinehart and Winston.

Gliner, G.S. (1991). Factors contributing to success in mathematical estimation in preservice teachers: types of problems and previous mathematical experience. *Educational Studies in Mathematics*, 22, 595–606.

Graeber, A.O. (1993). In. R.B. Davis and C.A. Maher (eds.). *Schools, Mathematics and the World of Reality.* Boston: Allyn and Bacon.

Greeno, J.G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education,* 22(3), 170–218.

Greenwood, J.J. (1993). On the nature of teaching and assessing "mathematical power" and "mathematical thinking". *Arithmetic Teacher,* 41(3), 144–153.

Haigh, W. (1993). Using the computer to solve problems by the guess–and–test method. *School Mathematics and Science,* 93(2), 92–95.

Hall, L.T.Jr. (1984). Estimation and approximation–not synonyms. *The Mathematics Teacher,* 77(7), 516–517.

Harte, W. and Glover, M.J. (1993). Estimation is mathematical thinking. *Arithmetic Teacher,* 41(2), 75–77.

Hawkins, D. (1994). Constructivism: some history. In P.T. Fensham, R.F. Gunstone and R.T. White (eds). *The Content of Science: a constructivist approach to its teaching and learning.* London: Falmer Press.

Herman, J.F., Norton, L.M. and Roth, S.F. (1983). Children and adults' distance estimation in a large–scale environment: effects of time and clutter. *Journal of Experimental Child Psychology,* 36, 453–470.

Hiebert, J. (1984). Children mathematics learning: the struggle to link form and understanding. *The Elementary School Journal,* 84(5), 497–513.

Hildreth, D.T. (1983). The use of strategies in estimating measurements. *Arithmetic Teacher,* 30, 50–56.

Hilton, P. and Pedersen, J. (1986). Approximation as an arithmetic process. In H.L. Schoen and M.J. Zweng (eds.). *Estimation and Mental Computation* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 55–71.

Hope, J.A. (1986). Mental calculation: anachronism or basic skill? In H.L. Sochen and M.J. Zweng (eds). *Estimation and Mental Computation,* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 45–54.

Hope, J.A. (1987). A case study of a highly skilled mental calculator. *Journal for Research in Mathematics Education,* 18(15), 331–342.

Jacobs, H.H. (1993). One point: mathematics integration: a common–sense approach to curriculum development. *Arithmetic Teacher,* 301–302.

Jaworski, B. (1988). 'Is' versus 'seeing as': constructivism and the mathematics classroom. In D. Pimm (ed.). *Mathematics, Teachers and Children.* London: Hodder and Stoughton, 287–296.

Kamii, C., Lewis, B.A., and Livingston, S.J. (1993). Primary arithmetic: children inventing their own procedures. *Arithmetic Teacher,* 41(4), 200–203.

Kavaliku, L. (1966). *Educational Re–organisation for National Development in Tonga.* Unpublished D. Phil Thesis. Victoria University, Wellington.

Knight, G., Arnold, G., Carter, M., Kelly, P. and Thornley, G. (1992). *The Mathematical Needs of New Zealand School Leavers.* A *Research Report.* Palmerston North: Massey University, Department of Mathematics.

Koloto, 'A.H. (1989). *Estimation skills in mathematics. Strategies used by 13 and 14 Year olds Students*. Unpublished M.A. Thesis. Hamilton: University of Waikato.

Koloto, 'A.H. (1993). Tongan system of numeration. *SAMEpapers 1993,* 194–209.

Krulik, S. and Rudnick, J.A. (1994). Reflect...for better problem solving and reasoning. *Arithmetic Teacher,* 41(6), 334–338.

Langford, P.E. (1986). Arithmetical word problems: thinking in the head versus thinking on the table. *Educational Studies in Mathematics*, 17, 193–199.

Lawry, W. (1851). *A Second Missionary Visit to the Friendly and Feejee Islands in the Year MDCCCL.* London: J.Mason and C.Gilpin.

Lee, K.S. (1991). Left–to–right computations and estimation. *School Science and Mathematics*, 91(5), 199–201.

Lerman, S. (ed.) (1994). *Cultural Perspectives on the Mathematics Classroom.* Dordrecht: Kluwer Academic Publishers.

Leutzinger, L.P., Rathmell, E.C. and Urbatsch, T.D. (1986). Developing estimation skills in the primary schools. In H.L. Schoen and M.J. Zweng (eds.). *Estimation and Mental Computation.* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 82–92.

Levin, J.A. (1981). Estimation techniques for arithmetic: everyday math and mathematics instruction. *Educational Studies in Mathematics, 12(4), 421–434.*

Levine, D.R. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education*, 13(5), 350–359.

Leyard, P. (1982). *The Tongan Past.* Nuku'alofa: Government Printing Press.

Lindquist, M.M. (1987). Estimation and mental computation. Measurement. *Arithmetic Teacher,* 34(5), 16–17.

Lobato, J.E. (1993). Making connections with estimations. *Arithmetic Teacher,* 40(6), 347–351.

Lochhead, J. (1991). Making math mean. In E. von Glasersfeld (ed.). *Radical Constructivism in Mathematics Education.* Dordrecht: Kluwer Academic Publishers, 75–87.

Lochhead, J. (1992). Knocking down the building blocks of learning: constructivism and the ventures program. *Educational Studies in Mathematics,* 23(5), 543–553.

Macmillan, A. (1990). Constructivism in a kindergarten mathematics class. *Mathematics Education Research Journal,* 2(2), 12–27.

Maher, C.A. (1988). The teacher as designer, implementer and evaluator of children's mathematical environments. *Journal of Mathematical Behaviour*, 6(3), 295–303.

Mara, D., Foliaki, L. and Coxon, E. (1994). In E. Coxon, K. Jenkins, J. Marshall, and L. Massey (eds.). *The Politics of Learning and Teaching in Aotearoa–New Zealand.* Palmerston North: Dunmore Press.

McFee, G. (1992). Knocking down the building blocks of learning: Constructivism and the ventures program. *Educational Studies in Mathematics* 23(5), 543–553.

Mcintosh, A., Reys, B.J. and Reys, R.E. (1992). A proposed framework for examining basic number sense. *For the learning of Mathematics*, 12(3), 2–8,44.

Mercer, J. (1992). What is left to teach if students can use calculators. *Mathematics Teacher*, 85(6), 415–417.

Ministry of Education (1992). *Mathematics in the New Zealand Curriculum.* Wellington: Learning Media, Ministry of Education.

Mitchell, C.E. (1991). Elementary school mathematics: keeping in step with our society. *School Science and Mathematics*, 91(8), 353–356.

Moore, T.F. (1986). More on mental computation. *Mathematics Teacher*, 79(3), 168–169.

Moulton, J.E. (1900). *Ko e 'Uluaki Buka a Eukaliti.* Siteni: S.E. Lees.

Mulhern, G. (1989). Between the ears: making inferences about internal processes. In B. Greer and G. Mulhern (eds.). *New Directions in Mathematics Education.* New York: Routledge, 29–62.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Newman, R.S. (1984). Children's numerical skill and judgement of confidence in estimation. *Journal of Experimental Child Psychology*, 37, 107–123.

Newman, R.S. and Berger, C.F. (1984). Children's numerical flexibility in the use of counting. *Journal of Educational Psychology*, 76(1), 55–64.

Noddings, N. (1990). Constructivism in mathematics education. In R.B. Davis, C.A. Maher and N. Noddings (eds.). *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education Monograph Number 4.* Reston, VA: National Council of Teachers of Mathematics, 7–18.

Noddings, N. (1993). Constructivism and caring. In. R.B. Davis and C.A. Maher (eds.). *Schools, Mathematics and the World of Reality.* Boston: Allyn and Bacon.

Nunes, T. (1992). Cognitive invariants and cultural variation in mathematical concepts. *International Journal of Behavioural Development*, 15(4), 433–453.

O'Daffer, P. (1979). A case and techniques for estimation: estimation experiences in elementary school mathematics—essential, not extra! *Arithmetic Teacher*, 26(6), 47–51.

Onslow, B. (1991). Linking reality and symbolism: a primary function of mathematics education. *For the Learning of Mathematics,* 11(1), 33–37.

Pirie, S and Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics,* 23(5), 505–528.

Resnick, L.B. (1989). Developing mathematical knowledge. *American Psychologist,* 44(2), 162–169.

Resnick, L.B. (1991). Shared cognition: thinking as social practice In L.B. Resnick, J.M. Levine and S.P. Teasley (eds.). *Respectives on socially shared cognition.* Washington: American Psychological Association, 1–20.

Resnick, L.B. and Ford, W.W. (1981). *The Psychology of Mathematics for Instruction.* Hillsdale, N.J : Earlbaum.

Resnick, L.B., Wang, M.C. and Kaplan, J. (1973). Task analysis in curriculum design: a hierarchically sequenced introductory mathematics curriculm. Journal of Applied Behavior, 6, 179–?.

Reys, B.J. (1986). Teaching computational estimation: concepts and strategies. In H.L. Sochen and M.J. Zweng (eds). *Estimation and Mental Computation,* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 31–44.

Reys, B.J. and Reys, R.E. (1990). Estimation directions from the standards. *Arithmetic Teacher,* 37(7), 22–25.

Reys, B.J, Reys, R.E. and Penafiel, A.F. (1991). Estimation performance and strategy use of Mexican 5th and 8th grade student sample. *Educational Studies in Mathematics,* 22,353–357.

Reys, R.E. (1984). Mental computation and estimation: past, present and future. *The Elementary School Journal,* 84(5), 547–557.

Reys, R.E. (1985). Testing computation skills. *Arithmetic Teacher,* 33(3), 14–16.

Reys, R.E. (1993). Research on computational estimation: what it tells us and some questions that need to be addressed. *Hiroshima Journal of Mathematics Education, 1, 105–*112.

Reys, R.E. and Bestgen, B.J. (1981). Teaching and assessing computational estimation skills. *The Elementary School Journal,* 82(2), 117–127.

Reys, R.E., Bestgen, B.J., Rybolt, J.F. and Wyatt, J.W. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education,* 13(3), 183–201.

Reys, R.E., and Reys, B.J. (1987). Estimation and mental computation. *Arithmetic Teacher,* 16–17.

Reys, R.E., Reys, B.J., Nohda, N., Ishida, J., Yoshikawa, S. and Shimizu, K. (1991). Computational estimation performance and strategies used by fifth and eighth grade Japanese students. *Journal of Research in Mathematics Education,* 22(1), 39–58.

Robitaille, D.F. and Garden, R.A. (eds). (1989). *The IEA Study of Mathematics II: Contexts and Outcomes of School Mathematics*. New York: Pergamon Press.

Rubenstein, R.N. (1985). Computational estimation and related mathematical skills. *Journal for Research in Mathematics Education*, 16(2), 106–119.

Rubenstein, R.N. (1986). Varieties of estimation. In H.L. Schoen and M.J. Zwend (ed). *Estimation and Mental Computation* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 157–161.

Sachar, T. (1978). An instrument for evaluating mental arithmetic skills. *Journal for Research in Mathematics Education*, 9, 233–237.

Saunders, H. (1980). When are we ever gonna have to use this? *The Mathematics Teacher*, 73(1), 7–16.

Saunders, W.L. (1992). The constructivist perspective: implications and teaching strategies for science. *School Science and Mathematics,* 92(3), 136–144.

Schoen, H.L. (1987). Estimation and mental computation. *Arithmetic Teacher*, 34(6), 28–29.

Schoen, H.L., Blume, G. and Hoover, H.D. (1990). Outcomes and processes on estimation test items in different formats. *Journal for Research in Mathematics Education*, 21(1), 61–73.

Schoen, H.L., Friesen, C.D., Jarrett., J.A. and Urbatsch, T.D. (1981). Instruction in estimating solutions of whole number computations. *Journal for Research in Mathematics Education*, 12(3), 165–78.

Schoen, H.L. and Zweng, M.J. (eds.). (1986). *Estimation and Mental Computation.* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Sharma, S.V. (1993). *Capabilities With Decimal Numbers: An Investigation of how Calculators Affect Children's Understanding in Decimals.* Unpublished M.Ed Thesis. Hamilton: University of Waikato.

Siegel, A.W. and McBurney, D.H. (1970). Estimation of line length and number: a developmental study. *Journal of Experimental Child Psychology*, 10(2), 170–180.

Siegel, A.W., Goldsmith, L.T. and Madson, C.R. (1982). Skills in estimation problems of extent and numerosity. *Journal for Research in Mathematics Education*, 13(3), 211–232.

Sowder, J. T. (1992). Estimation and number sense. In D.A. Grouws (ed.). *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan.

Sowder, J. T. and Wheeler, M. M. (1989). The development of concepts and skills related to computational estimation. *Journal for Research in Mathematics Education*, 20(2), 130–146.

Steen, L.A. (1990). Making math matter. *Instructor*, 99(8). 21–23.

Steffe, L.P. and Wiegel, H.G. (1992). On reforming practice in mathematics education. *Educational Studies in Mathematics*, 23(5), 445–465.

Stigler, J.W. (1989). Mathematics meets culture. *Journal for Research in Mathematics Education*, 20(4), 367–370.

Thompson, G.H. (1979). Estimating and approximation. *School Science and Mathematics*, 79(8), 575-580.

Thorndyke, P.W. (1981). Distance estimation from cognitive maps. *Cognitive Psychology*, 13(4), 526–550.

Threadgill–Sowder, J. (1984). Computational estimation procedures of school children. *Journal of Educational Research*, 77(6), 332–336.

Tonga Government (1934). *Tonga Government Gazette for the Year 1934.* Nuku'alofa: Government Printer.

Tonga Government (1946). *Tonga Government Gazette for the Year 1946.* Nuku'alofa: Government Printer.

Tonga Government (1959). *Tonga Government Gazette for the Year 1959.* Nuku'alofa: Government Printer.

Tonga Ministry of Education. (1974). *Report of the Ministry of Education for the Year 1974.* Nuku'alofa: Ministry of Education.

Tonga Ministry of Education. (1987). *Secondary School Entrance Examination Mathematics.* Nuku'alofa: Ministry of Education.

Tonga Ministry of Education. (1989). *Report of the Ministry of Education for the Year 1989.* Nuku'alofa: Ministry of Education.

Tonga Ministry of Education. (1990). *Secondary School Entrance Examination Mathematics.* Nuku'alofa: Ministry of Education.

Tonga Ministry of Education (1992a). *Mathematics Education in Tonga Draft Mathematics Syllabus for Secondary Schools Forms 1 to 5.* Nuku'alofa: Curriculum Development Unit, Ministry of Education.

Tonga Ministry of Education (1992b). *Report of the Ministry of Education for the Year 1992.* Nuku'alofa: Ministry of Education.

Tonga National Centre. (1988). *Pathways to the Tongan Past.* Nuku'alofa: Tonga National Centre.

Trafton, P.R. (1978). Estimation and mental arithmetic: important components of computation. In M.N. Suydam and R.E. Reys (eds.). *Developing Computational Skills* 1978 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Trafton, R. (1986). Teaching computational estimation : Establishing an estimation mind–set. *In* H.L. Schoen and M.J. Zweng (eds). *Estimation and Mental Computation* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 16–30.

Tupouniua, P. (1977). *A Polynesian village the process of change.* Suva: South Pacific Social Sciences Association.

Underhill, B. (1983). Estimation and reasonableness of results. In G. Shufelt and J.R. Smart (eds.). *The Agenda in Action* 1983 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 97–104.

Usiskin, Z. (1986). Reasons for estimating. In H.L. Schoen and M.J. Zweng (eds.). *Estimation and Mental Computation* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 1–15.

Usiskin, Z. (1993). If everybody counts, why do so few survive? In G. Cuevas and M. Driscoll (eds.). *Reaching All Students With Mathematics*, Reston, VA: National Council of Teachers of Mathematics, 7–22.

Van Devender, E.M. (1992). Helping students acquire thinking skills through mathematics instruction. *School Science and Mathematics*, 92(6), 331–333.

Vance, J.H. (1986). Estimating decimal products : an instructional sequence. In H.L. Schoen and M.J. Zweng (eds.). *Estimation and Mental Computation.* 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 127–134.

von Glasersfeld, E. (1989a). Constructivism in education. *In The International Encyclopaedia of Education : Research Studies.* Supplementary Volume. Oxford (England), New York: Pergamon Press, 162–163.

von Glasersfeld, E. (1989b). Cognition, construction of knowledge, and teaching. *Synthese*, 80, 121–140.

von Glasersfeld, E. (1990). An exposition of constructivism: why some like it radical. In R.B. Davis, C.A. Maher and N. Noddings (eds.). *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education Monograph Number 4.* Reston, VA: National Council of Teachers of Mathematics, 19–29.

von Glasersfeld, E. (ed.) (1991). *Radical Constructivism in Mathematics Education.* Dordrecht: Kluwer Academic Publishers.

von Glasersfeld, E. (1992). Guest editorial. *Educational Studies in Mathematics,* 23(5), 443–444.

Wearne, D. and Hiebert, J. (1994). Place value and addition and subtraction. *Arithmetic Teacher,* 41(5), 272–274.

Wheatley (1980). Calculators in the classroom : a proposal for curricular change. *Arithmetic Teacher*, 28(40), 37–39.

Wheatley, G.H. (1991). Constructive perspectives on science and mathematics learning. *Science Education*, 75(1), 9–21.

Wheatley, G.H. (1992). The role of reflection in mathematics learning. *Educational Studies in Mathematics* 23(5), 529–54.

Whitin, D.J. (1994). Exploring estimation through children's literature. *Arithmetic Teacher*. 41(8), 436–441.

Yackel, E. Cobb, P., Wood, T., Wheatley, G.H. and Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In T.J. Cooney and C.R. Hirch (eds.). *Teaching and Learning Mathematics in the 1990s*. Reston, VA: National Council of Teachers of Mathematics, 12–21.

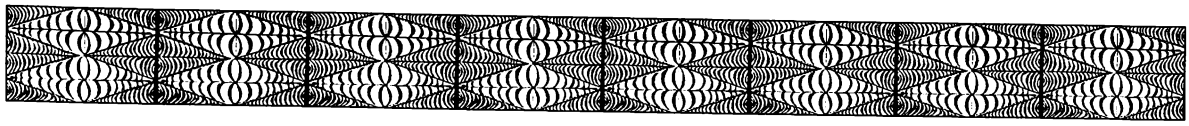
Zevenbergen, R. (1991). Constructivism in the classroom. In J. O'Reilly and S. Wettenhall (eds.). *Mathematics Ideas,* Parkville: Mathematical Association of Victoria, 15–18.

GLOSSARY

TONGAN	ENGLISH
fakafuofua	- estimate - estimation - conjecture - guess - closer to - nearly - prediction - mentally figure out - to think
fakafuofua lelei	- reasonable estimate
fakafuofua ‘oku fu‘u lahi	- overestimation
fakafuofua ‘oku fu‘u si‘isi‘i	- underestimation
fakafuofua ngaue‘aki ‘a e mata‘ifika taupotu ki to‘ohema	- front-end strategy or estimation using leftmost digits
fakaofiofi	- estimation - approximation - rounding
fakataha‘i	- add - addition
fika	- mathematics - computation - arithmetic - exercise
fika‘i	- compute - figure out
kole	- subtract - subtraction - ask - borrow
liunga	- multiply - multiplication
mahu‘inga faka-tu‘unga	- place value
mata‘ifika	- number - digit
mo‘oni	- answer - result of a computation - true - truth - right

ngaahi fakafuofua lelei	- reasonable range
ngaahi mata'ifika 'oku faingofua ange hono fakataha'i	- compatible numbers
ngaue'i	- compute - work out an answer
tali	- answer - response - respond
tanaki	- add - addition
totonu	- exact - precise - absolute
vahevahe	- divide - division - share

AFTER THOUGHTS



INTRODUCTION

In hindsight, there are two aspects of the results of the present study that need to be reexamined. These are:

i. the importance of culturally relevant mathematical activities; and

ii. relationship between computational and measurement estimation.

The following sections contain some after thoughts regarding the above issues.

CULTURALLY RELEVANT MATHEMATICAL ACTIVITIES

One of the important outcomes of the analysis of the mathematics curriculum in Tonga reported in chapter two, was the lack of emphasis on traditional mathematical activities such as ways of measuring and counting.

Everyday Life Experiences

Interview data from both primary and secondary students (see chapter six) showed that out-of-school experiences provided a rich source of ideas estimating measures of lengths, weights and capacities. The results of the present study support the claim by mathematics educators (Ascher, 1991; Atweh, Cooper, and Kanes, 1992; Barton, 1993; Bishop, 1988; Carraher, Carraher, and Schliemann; D'Ambrosio, 1985; Gerdes and Bulafo, 1994; Gillespie, 1995) who have argued for the need to give more consideration to the cultural relevance of content and activities in the development of mathematics curriculum, teaching and learning.

Conducting research in a cultural context would involve studying mathematics in the living contexts which are meaningful and relevant to the students. Such contexts would include the languages, cultures, everyday lives as well as their school-based experiences (Bishop, 1995; Howard, 1995). As Bishop (1995) suggested ethnomathematics is situated in the linguistic and social context of the communities concerned and can only be abstracted from those contexts after careful analysis.

Tongan Language and Mathematics

The data also showed that the Tongan language provides a rich source of alternative terms for fakafuofua (estimation). As reported in chapter six, the definition for fakafuofua (estimation) include: fakaofiofi (closer to); fakamahamahalo (conjecture); mate'i (guess); fakafuofua'i pe 'i loto (mentally figure out something); fua pe 'i loto (mentally work out the measure);

fakakaukau (to think); and sio pe ke ma'u 'a e anga 'o e me'a ko ia (look at an object and work out its charactersistics). It must be noted that all these term except for 'fua pe 'i loto' (mentally work out the measure), which may be limitied to measurement estimation, could be applied to other types of estimation in mathematics. Language as the tool for developing mathematical thinking ought to be carefully considered. As Bishop (1995) suggested, ethnomathematics research needs to be carried out in the communities concerned and with due reference to the language(s) of the those communities .

Implications for Ethnomathematics

Although the theme of ethnomathematics was explored in the thesis, this was not fully discussed due to a number of reasons. First, this aspect of the study emerged later in the analysis of the data. The main aim of the research was to study estimation in the Tongan schools, therefore, the emphasis of the data analysis focused on this. Second, as a Tongan researcher, there was a tendency to take it for granted that these are the way things are and it was not always possible to see the significant of the data in the context of ethnomathematics. Outsiders (ie. people not entrenched in the culture, including my supervisors and examiners) were in a much better position to identify the significant contribution that some of the research findings have in the development of the research data in the field of ethnomathematics.

The results of the present study point to the need to study estimation and mathematics in the everyday life experiences of the Tongan students, and Tongan language and culture. These are important future research areas for the researcher. In the Tongan contexts, some research in ethnomathematics is beginning to emerge. For instance, a study of the *Geometrical Skill Behind the Tongan Tapa Designs* was reported by Finau and Stillman (1995). In addition, a preliminary study of the Tongan number systems was conducted by Koloto (1993). In order to incorporate ethnomathematics into the content mathematics curriculum, learning and teaching activities, the following points need to be addressed:

i. Research and careful analysis of the mathematical skills inherent in the everyday life activities of the Tongan people, particularly in measurement, counting, weaving, tapa design, navigation, fishing, planting, cooking and selling.

ii. A major change in teachers' attitudes to acceptance of the cultural mathematical activities as important and relevant topics to be used in the mathematics classroom. This ought be accompanied with the adoption of a variety of teaching approaches and strategies.

iii. Students coming into a classroom where the mathematical skills and concepts used in their everyday life experiences are valued and utilised.

iv. Parents also ought to aware of the relevance of the mathematics in the Tongan culture and language in the formal classroom mathematics.

The above issues will be addressed when the Tonga Ministry of Education Officers and Overseas Mathematics Curriculum Consultants see these as important areas for development.

RELATIONSHIP BETWEEN COMPUTATIONAL AND MEASUREMENT ESTIMATION

The other important issue regarding the present study is the relationship between computational estimation and measurement estimation. The results of this study together with the review of the literature point to similarities as well as qualitatively different network of skills and concepts involved in measurement and computational estimation. Furthermore, the results not only showed differences between measurement and computational estimation strategies but also differences in estimating measures of length and other measures.

Similarities Between Measurement and Computational Estimation

The review of the research literature on estimation reported in chapter three discussed some of the similarities of the measurement and computational estimation process.

i. Nature of the estimation process and result

Both measurement and computational estimation involves the mental skill obtaining and approximate answer. Thus, proficiency in one type of estimation (eg. measurement estimation) most familiar to students could be used to further develop skills in another types of estimation (eg. computational estimation).

Although the result of the measurement and computational estimation process is an inexact answer and that both involve a mental process, there are also differences between these two types of estimation.

ii. Variety of strategies

The interview data as well as the review of research on measurement estimation provide amble evidence of the variety of strategies that could be formulated in making an estimate to measurement estimation tasks. For instance, in the present study five different appropriate strategies were used for estimating measures of lengths. The review of the literature also provided research data on the variety of strategies used by proficient computational estimators

(Dowker, 1992; Koloto, 1989; Levine, 1982; Reys, Bestgen, Rybolt and Wyatt, 1982; Siegel, Goldsmith, and Madson, 1982).

Differences Between Measurement and Computational Estimation

The differences between the nature and variety of measurement and computational estimation could be due to the following factors:

i. Differences in emphasis in teaching these topics

The results showed differences in emphasis given to these topics in mathematics teaching. Computational estimation had not been part of the students mathematical experiences in the classroom. Whereas, measurement estimation had been part of students' experiences in the classroom as well as everyday life activities. Measurement estimation allows for a variety of answers and strategies. On the other hand, computation has been taught as a process that gives exact answers therefore students mathematical thinking in the context of computational estimation were restricted to the limited techniques taught in the schools. Thus, the differences in the variety of strategies could be a direct result of teaching measurement estimation activities or lack of teaching of computational estimation.

ii. Relevance to everyday life experiences

Whereas, measurement estimation strategies were based on everyday life experiences, computational estimation strategies were based mainly on school taught rules and techniques. It is also possible that in comparison to computational estimation, measurement estimation activities occur more frequently in the everyday life experiences of students and more accessible to students and therefore has more relevance.

iii. Different cognitive skills required

Another important factor that need to be examined is the cognitive skills involved in the different types of estimation. For instance, measurement estimation involves making a judgement about the size of measures, computational estimation tasks would involve mentally operating with numbers. Whereas, measurement estimation tasks might involve a combination of judging the relative sizes of measures as well as mental computational estimation skills (eg. working out the width of a classroom window using a multiple of a 30 centimetre ruler), computational estimation might involve mentally computing with approximate numbers.

Differences Between Estimating Different Measures

Taking measurement estimation, one might argue that estimation in each of the quantities involved (ie. length, weight and capacity) would involve different set skills. Whereas, length and capacity would involve the subject making a perceptual judgement about the size of the given length or capacity, and/or making comparison of relative sizes of objects, measures of weight would involve much more than a visual judgement. A 'feel for' the size of the object is necessary.

CONCLUDING THOUGHTS

The above after thoughts considered two of the most important issues identified by the results of the present study. It is important that future research on estimation in a cultural context consider not only the social context and the languages for estimation in that culture (both in and out of school) but that it examines very carefully the nature of the relationship between the types of estimation being studied – measurment and computational estimation in this case.

This study has examined estimation in the context of the school setting, the next step is to conduct research involving a careful analysis of estimation in the traditional as well as contemporary everyday life activties in the Tongan society. Resutls of such an analysis should provided a better picture of the nature of estimation in the Tongan cultural context and and consequently the mathematical activities that ought to be emphasised in the schools. Estimation has the potential to provide much needed variety in the Tongan mathematics classrooms.

REFERENCES

Ascher, M. (1991). *Ethnomathematics A Multicultural View of Mathematical Ideas*. Pacific Grove, California: Brooks/Cole Publishing

Atweh, B., Cooper, T. & Kanes, C. (1992). The Social And Cultural Context of Mathematics Education. In B. Atweh & J. Watson (eds.). *Research in Mathematics Education*. Queensland University of Technology, Brisbane: Mathematics Education Research Group of Australasia.

Barton, B. (1993). Contemplating cultural constructs. In B. Atweh, C. Kanes, M. Carss, & G. Booker (eds.). *Contexts in Mathematics Education*, Proceedings of the Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia. Brisbane: MERGA.

Bishop, A.J. (1988). *Mathematical Enculturation - A Cultural Perspective on Mathematics Education*. Dordrecht: Kluwer Academic Publishers.

Carraher, T.N., Carraher, D.W., and Schliemann, A.D. (1985). Mathematics in the streets and in Schools. *British Journal of Developmental Psychology*, 3, 21–29.

D'Ambrosio U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, 5(1), 46–48.

Dowker, A. D. (1992). Computational estimation strategies of professional mathematicians. *Journal for Research in Mathematics Education,* 23(1), 45–55.

Finau, K. and Stillman, G. (1995). *Geometrical Skill Behind the Tongan Tapa Design.* A paper presented in the History and Pedagogy of Mathematics Conference, Cairns, Australia.

Gerdes, P. and Bulafo, G. (1994). *SIPATSI Technology, Art and Geometry in Inhambane.* Instituto Superior Pedagofico Mocambique.

Gillespie, K. (1995). Mathematics Acquisition for Aboriginal Students – The Importance of Contextual Teaching. In A. Richards (ed.) *FLAIR - Forging Links and Integrating Resources.* Proceedings of the 15th Biennial Conference of Australian Association of Mathematics Teachers. Darwin, Australia.

Howard, P. (1995). *Mathematics in Multicultural Australia: A focus on Indigenous Students.* A Paper presented in the History and Pedagogy of Mathematics Conference, Cairns, Australia, July 1995.

Koloto, 'A.H. (1989). *Estimation Skills in Mathematics. Strategies used by 13 and 14 Year olds Students.* Unpublished M.A. Thesis. Hamilton: University of Waikato.

Koloto, 'A. H. (1993). Tongan system of numeration. *SAMEpapers 1993*, 194–209.

Levine, D.R. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education*, 13(5), 350–359.

Reys, R.E., Bestgen, B.J., Rybolt, J.F. and Wyatt, J.W. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education*, 13(3), 183–201.

Siegel, A.W., Goldsmith, L.T. and Madson, C.R. (1982). Skills in estimation problems of extent and numerosity. *Journal for Research in Mathematics Education*, 13(3), 211–232.