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Building Conceptual Understanding: Exploring Teachers' Use of Virtual Manipulatives in Primary Mathematics Education

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Contents

Abstract

Despite the emphasis on representations and conceptual understanding in the 2024 refresh of the New Zealand Mathematics Curriculum, the integration of virtual manipulatives (VMs) remains limited in classroom practice. Using a mixed-methods action research (MMAR) design, this research investigated how a professional development intervention can shape teachers' perceptions and integration of VMs into mathematical instruction, following evidence of limited uptake in an initial survey.

Quantitative data were collected from 36 New Zealand intermediate school teachers using a modified Constructivist Learning Environment Survey (CLES -VM) to examine teachers' perceptions and reported use of VMs. This informed a subsequent action research phase with seven participants, incorporating pre- and post- focus groups and a series of professional development sessions following a design-based structure. Qualitative data were analysed using reflective thematic analysis, while survey data were analysed using descriptive statistics.

Findings indicated that while teachers initially faced challenges such as the pragmatic constraints of using VMs and the tension between constructivist ideals and classroom management, the professional development intervention helped to mitigate these barriers. The professional development intervention supported teachers to adopt more intentional and flexible approaches to VM use, deepening their understanding of VMs as tools for conceptual development rather than representational extras. An unanticipated finding was the participant-initiated use of generative AI to design custom-built VMs, which contributed to a perceived shift in teacher autonomy. By transitioning from consumers of generic digital tools to creators of tailored resources, teachers were able to address the specific needs of students more effectively. Additionally, participants identified the role of VMs in reducing cognitive load and supporting instructional equity and accessibility.

The research suggests that sustainable digital integration requires a hybrid professional development model that balances the immediate need for ready to use resources with the long-term, design-based development of teachers TPACK (Technological Pedagogical Content Knowledge).

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Chapter 1: Introduction

Digital technology has become a central part of our everyday lives. It is constantly evolving and developing, impacting the way we learn, work, and communicate. The rapid growth of artificial intelligence in recent years has only increased our reliance on digital technology. Consequently, teachers need to continually develop their knowledge of digital technology and how to implement it effectively. In mathematics the use of virtual manipulative tools provide opportunities for teachers and students to explore mathematical ideas in different ways (Moyer-Packenham & Westenskow, 2013). However, the pedagogical potential of these digital tools has yet to be fully realised in students' mathematical performance.

In the past 25 years mathematics achievement in New Zealand has fallen (Morrow et al., 2022; OECD, 2023). This decline is evident in large scale international study achievement indicators. Since 2000, the Program for International Student Assessment (PISA) has shown a continuous decrease in performance of New Zealand's 15-year olds (OECD, 2023). In 2019, The Trends in International Mathematics and Science Study (TIMSS) reported that Year 9 students in New Zealand recorded their worst ever results (Mullis et al., 2020). More recently in 2022, the National Monitoring Study of Student Achievement (NMSSA) revealed only 42% of year 8 students met the expected curriculum level for mathematics (Asil et al., 2023). In 2025, data released by the Curriculum Insights and Progress Study suggests that 62% of year 8 students are more than one year behind end-of-phase curriculum expectations (Ministry of Education [MoE], 2025). Such a significant widening of achievement suggests that a large proportion of students are entering secondary education without the foundational mathematical knowledge required to succeed.

Despite various government funded initiatives and curriculum changes in recent decades, addressing achievement trends remains a persistent challenge. In 2000, the Numeracy Development Project (NDP) was released aimed at addressing perceived inadequacies of mathematics teaching (Morrow et al., 2022). Part of a complete curriculum review, the NDP would sit alongside a new curriculum released in 2007. This outcome-based curriculum marked significant change of direction and encouraged an enquiry approach to teaching with student-centred learning (MoE, 2007). Although popular with schools, concerns regarding this curriculum often focused on its 'openness' and lack of guidance (Aitken & Wood, 2023). Morrow et al. (2022) suggests that four interconnected elements, central to the 2007 New Zealand curriculum, rather than improve teaching, increased the downward trend in mathematics performance. The four elements being: "(1) its highly generic non-prescriptive nature, (2) a commitment to teacher autonomy in curriculum knowledge selection, (3) competency-based outcomes approach, and (4) a commitment to localisation in curriculum selection" (p. 62). In 2021, the Ministry of Education began working on a refreshed curriculum that involved transitioning from a outcome based curriculum to a progression based curriculum (McPhail et al., 2023).

The introduction of a refreshed New Zealand Curriculum aims to address the underachievement of New Zealand students. The document was released in October of 2024 and sees a shift towards explicit instruction and structured progression. *Te Mātaiaho* (MoE 2024), as it was initially titled, indicates that the new curriculum is knowledge rich and backed by the science of learning. The science of learning refers to the use of evidence-based research from cognitive psychology and neuroscience to ensure teaching methods align with how the human brain processes and retains information (Weinstein et al., 2018). The move to a more structured approach aims to increase consistency of what is being taught and ensure teachers have clear and comprehensive progression outcomes (MoE, 2024). Another key component that has been identified in the new curriculum is the greater use of explicit teaching. Explicit teaching refers to a teacher-directed approach, where concepts are clearly explained and demonstrated (Hughes et al., 2022). This combined with advice from the Ministry of Education that, 'teachers must plan so that all students experience all the statements in the sequence for their year level' (2024, p. 24) has led to a whole-class approach to teaching mathematics gaining momentum.

Under the updated curriculum, teachers are encouraged to use physical and visual tools during explicit instruction to scaffold student learning (MoE, 2024). This ensures a strong conceptual foundation is built before students move toward more abstract forms, such as equations. Manipulatives, in a mathematical sense, are concrete objects that can be used to help represent, visualise, and interact with abstract mathematical concepts. Teachers need to use tools and representations that make sense to the student and provide support for students' thinking (Anthony & Walshaw, 2009). Examples of common manipulatives in primary classrooms are items such as, pretend money, counters, beans, base 10 blocks, and fraction tiles to name a few. Unfortunately, as students get older, many of these resources are not evident in classrooms. Whether due to limited access, presumptions that using manipulatives are for younger students, or teachers lack of knowledge in using them, the absence of these tools restricts opportunity to develop conceptual understanding (Johnson et al., 2021).

The use of manipulatives in a virtual setting can provide opportunities that are not possible with physical manipulatives and allow students to communicate thinking that otherwise would be difficult to share (Anthony & Walshaw, 2009). For example, when working with fraction tiles, tiles can be split or renamed instantly. When working with 3D shapes, the object can be dissected or deconstructed to show internal angles or the net of the object. Another affordance that virtual manipulatives provide is instant feedback. When students are using virtual manipulatives, errors are often identified by the app or by the user and can be acted on immediately rather than waiting for the teacher (Calder & Murphy, 2018).

Like with the use of manipulatives, one inhibiting factor in the use of virtual manipulatives is teacher capability (Fletcher et al., 2020). Virtual tools often need the teacher to spend time experimenting and learning how to use them effectively. As technology increases, greater computer literacy is required. Without targeted instruction and professional development, teachers are generally reluctant to take on something new unless they have an interest or relevant prior knowledge (Johnson et al., 2021).

My particular interest in this area of research stems from my own experiences in mathematics leadership and the implementation of manipulatives and virtual manipulatives to support the conceptual understanding of students and teachers. Over the years, I have seen first-hand a large number of teachers who struggle to implement digital resources and tools into their teaching. The combination of anxiety when relying on digital resources and a lack of knowledge of how to utilise them effectively leads to them being avoided. I have also been involved with a range of professional development experiences that have varied considerably in their effectiveness. With that in mind, finding a suitable and successful professional development approach that can support teachers' implementation of virtual manipulatives has been a focus of mine.

This research seeks to explore the perceptions, assumptions, and post-teaching reflections of intermediate teachers as they use virtual manipulatives to support the new curriculum. A mixed-methods action research (MMAR) approach will be adopted, combining a constructivist learning environment survey with action research. Focus will be directed towards virtual manipulatives main affordances for their adoption, and how virtual manipulatives can be used to support whole-class explicit instruction. The project hopes to offer new insights and a unique perspective of virtual manipulative implementation.

Chapter 2: Literature Review

2.1 Introduction

This research project's review of literature will discuss important theoretical, conceptual, and empirical studies that relate to the use of virtual manipulatives in mathematical education. The review will begin by briefly discussing foundational learning theories establishing constructivism as the primary framework for understanding mathematical development in this context. Next, the literature review will then summarise the history of education and mathematics in New Zealand. Building on this historical foundation, the review will discuss the different mathematical pedagogies that have shaped current mathematics teaching in recent years.

Next, the review will focus on the importance of the use of mathematical manipulatives. This discussion will lead logically to an examination of virtual manipulatives in which they will be defined, their affordances identified, and studies on their effectiveness in supporting mathematical understanding reviewed. Subsequently, the vital role of teacher professional development in supporting effective teaching will be discussed. To address the rapidly evolving digital landscape, the review will then consider the relevance and emergence of AI in mathematics teaching.

Finally, the review of literature will synthesise these areas to identify gaps in existing studies that relate to whole-class teaching and virtual manipulatives, thus explaining how this study will contribute to the field of research.

2.2 Foundational Theories of Learning

Understanding how individuals acquire new knowledge and skills is central to educational research and practice. Since the late 19th Century, many theories and understandings of how we learn have been proposed, each taking different angles and epistemological approaches. Early models were dominated by behaviourism, which as defined by Kimble (1961), views learning as "a relatively permanent change in behavioural potentiality that occurs as a result of reinforced practice" (p. 6). In education, this is often displayed through repetitive practice and immediate feedback (Westover et al., 2021). However, the limitations of observing only external stimuli led to the rise of Cognitivism and Humanism. Cognitivists like Piaget (1952) and Gagné (1965) shifted focus towards mental processes, such as the development of schemata and the encoding of information. At a similar time, Humanist theorist such as Maslow (1962) and Rogers (1969) advocated for a more holistic, student-centred approach, whereby each person autonomously shapes their own behaviour and decisions (Muhajirah, 2020). Constructivism synthesises these perspectives by viewing learning as an active process of meaning-making rather that a passive collection of facts. Grounded in the work of Piaget (1952) and his work on cognitive development, modern constructivism has emerged as a dominant and popular paradigm in education (Ebrahimi, 2015; Zajda, 2021). The Constructivist Paradigm anchors this study, by providing a lens to view how teachers approach the delivery of mathematics in their classrooms.

2.2.1 Constructivism in Mathematics Learning

Within this study, a constructivist learning environment was anticipated, as such environments support learners to build mathematical ideas both actively and socially. In a mathematics classroom, this paradigm shifts the focus from rote memorisation of procedures to the active construction of conceptual understanding. Piaget (1952) theorised that children construct their own knowledge through their experiences. This may occur either by assimilating new information with existing schema or through a process of 'accommodation' when the new information doesn't fit to existing schema and the learner is motivated to make changes to make it fit (Dingman et al., 2019). Following a constructivist approach, the teacher is responsible for creating an environment that provides opportunities for students to construct mathematical knowledge. This involves the teacher using scaffolding to support students' construction of knowledge, providing opportunities for investigation and autonomy, and enabling students to collaborate and reflect on their thinking. The social collaborative component is supported by Vygotsky (1978) and his theory that learning is a collaborative process, with understanding co-constructed through social interaction.

To understand why these constructivist practices are now the expected standard in the local context, it is necessary to examine the historical shifts in New Zealand education that moved the focus from colonial rote-learning to student-centred inquiry.

2.3 Education in Aotearoa, New Zealand

Formal education in New Zealand has a history that can be traced back over 200 years to the arrival of colonial settlers and missionaries who formed the first schools. As settler numbers increased, so did the number of missionary schools but it was not until the Education Act of 1877 that free compulsory schooling was established nationwide. This deeply sociological shift aimed to educate New Zealanders as the country grew and help support the foundations of a new society (Gordon, 2016). At the same time, the Natives School Act was introduced to integrate Māori into European culture with the main goal of teaching English. Beyond the core subjects of reading writing and mathematics, the curriculum was mainly driven by the instruction of skills to support manual and domestic jobs (Calman, 2016). Attitudes towards the use of Te Reo Māori led to the language being actively discouraged and Māori children being punished for speaking it. The adoption of a colonised education system displaced Māori systems of knowledge transfer that had been in place prior (Berryman et al., 2025). The marginalisation of mātauranga Māori reflected a greater effort of colonisation to supress Māori. This historical reality stands in contradiction to how education was viewed by academics at the time, who saw it as fair and universally beneficial, providing opportunities for all as part of a liberal society (Openshaw et al., as cited in Gordon, 2016).

Approaches to teaching in the early 20th century were dominated by traditionalism, with subjects and content being taken from, and heavily influenced by, England (McGee, 2017). During this time a didactic, whole-class approach was the preferred form of transmission. It was common for classes to be streamed, meaning students were in classes with students of a similar ability. Teachers were the authoritarian figure who held the knowledge, and it was their job to transfer this knowledge to the student. Around the world, progressive approaches to teaching were emerging and began to have an impact in New Zealand. Influenced by John Dewey's (1938) progressive philosophy of child-centred schooling and experimental learning, alternative pedagogical approaches were gaining interest.

An influential figure in the development of New Zealand education at this time was Clarence Beeby (Alcorn, 1999). Building on international trends, Beeby challenged current education structure that privileged the wealthy and academically able, with an egalitarian philosophy that aimed to provide a fairer education system where everyone, whatever their background, had access to a free, high quality education (McDonald, 2002). The core of his influence is captured in the famous 1939 statement (developed with then Minister of Education Peter Fraser)

> "The Government's objective, broadly expressed, is that every person, whatever his level of academic ability, whether he be rich or poor, whether he live in town or country, has a right as a citizen, to a free education of the kind for which he is best fitted and to the fullest extent of his powers" (Beeby 1939 cited in Alcorn, 1999, p. 202)

Beeby would continue to influence education in New Zealand in the following years, proposing child-centred pedagogies and progressive education principles. Revisions of syllabuses began to show a shift away from heavily prescribed documents towards more open-ended and suggestive frameworks (McGee, 2017). Progressive educators opposed what they perceived as an excessive focus on fact memorisation, repetitive drills, and curriculum material that had no bearing on students daily lives (McGee, 2017). The work of Piaget (1952), Bruner (1966), Vygotsky (1978) and others also began to influence teacher's understanding of how students construct knowledge from experience.

Syllabus and subject revisions continued throughout the twentieth century, but it was not until 1993 when a comprehensive and integrated curriculum was introduced. The New Zealand Curriculum Framework (NZCF) contained seven key learning areas and included essential skills, attitudes and values (MoE, 1993). At the turn of the new millennium, feedback from schools saw the Ministry of Education begin a phase of revision to this curriculum (Cowie et al., 2009). In 2007 a new curriculum (MoE, 2007) was released, the redesign was more flexible, principle based, with a focus on key competencies over content. The document was seen as a blend of progressive ideals and economic priorities by promoting inclusive, child-centred learning alongside values that emphasise individual achievement and lifelong learning (Hughson, 2022). The 2007 curriculum was generally well received both in New Zealand and overseas with its high-autonomy and outcome based principles being heralded as 'world-class' (Hughson, 2022). The focus on key competencies encourages a more holistic view of learning and success but critics of the 2007 curriculum such as Morrow et al. (2022) believe it was too open ended, had limited focus on knowledge, and lacked meaningful integration of Te Ao Māori. The reduction of knowledge from the maths curriculum in particular has prompted Morrow (2022) to suggest that the "emptying out of mathematical knowledge" led to a "curriculum without content" (p. 62). A change of government in 2008 led to the implementation of national standards in an attempt to address these concerns and make the curriculum more content focused. The idea was that National Standards would provide parents with benchmarks against which their child's progress and achievement could be judged and compared (Mutch, 2013). Driven by falling performance in international studies such as PISA, the government pushed for more accountability and mandated further education policies that would challenge New Zealand's Liberal progressive education (Mutch, 2013).

A return to government by the more liberal aligned Labour Party in 2017 saw an immediate abolishment of National Standards and discussion began around a 'Refresh' of the New Zealand Curriculum. The refresh began in 2019 with consultation comprising of teachers, academics, industry interest groups, and Māori and Pasifika representation (McPhail et al., 2023). After a number of draft versions were published, and another change of government, the finalised Maths and English documents were made available in October 2024, with implementation expected in the following school year. Initially titled *Te Mātaiaho*, the new curriculum aims to provide a clear and structured learning framework, a strong foundation of maths and literacy, and an evidence-based approach to teaching and learning (MoE, 2024). The Ministry of Education announced that the new curriculum would focus on the basics of mathematics and be delivered through explicit teaching (Luxon & Stanford, 2024). The Ministry stated that direction for the new knowledge rich curriculum was influenced by countries across the OECD (primarily Singapore, Australia) and adapted for New Zealand.

The continual review and reform of the NZ primary curriculum has led to a range of different pedagogical approaches being implemented. In mathematics, this has influenced a range of different teaching strategies that are currently being used in primary classrooms across New Zealand. The following section will discuss these approaches and strategies in more detail.

2.4 Shifting Trends in Primary Mathematics (2000–Present)

The evolution of primary mathematics in New Zealand has been defined by a constant tension between teacher led instructional approaches and student centred, inquiry based methods (Hunter, 2008; Hunter et al., 2018; Morrow et al., 2022; Mutch, 2013). This pedagogical divide centres on whether learners should first master procedural fluency through structured content or develop relational understanding through problem solving. The

21st Century has seen contrasting, and often debated approaches influence mathematics education in New Zealand, largely driven by curriculum changes and national initiatives (Morrow et al., 2022). These changes and initiatives have fundamentally shaped primary classroom pedagogy over the last 25 years, resulting in the pendulum swing between relational understanding and structured content. This section will trace this progression chronologically, beginning with the strategy and stage-based frameworks of the early 2000s, moving into the collaborative inquiry models of the 2010s, and concluding with the current shift toward systematic mastery and aligned national resources.

2.4.1 The Strategy and Stage-Based Framework (Early 2000s)

2.4.1.1 Numeracy Development Project

Since 2001, a major influence on the direction of mathematics teaching in New Zealand primary schools has been the Numeracy Development Project (NDP) (MoE, 2008). The project changed the way mathematics is taught, promoting the teaching of multiple mental strategies to solve problems. Widespread and intensive professional development was provided across New Zealand at a cost of $70 million in order to introduce teachers to the new approach (Patterson, 2015). The implementation of the NDP coincided with falling mathematics performance in international comparisons such as PISA and TIMSS. In 2009 funding for the Project finished, but its philosophy continues and many of the ideas underpin the 2007 National Curriculum (Patterson, 2015). Early reviews of the NDP reported improvements in student knowledge and identified its valuable structure for teachers but also acknowledged there were still a noticeable achievement gap between Pakeha and Māori student performance (Bobis et al., 2005; Thomas & Tagg, 2006; Young-Loveridge, 2006). After analysing data from over 73,000 students, Bobis et al. (2005) reported that the NDP had been beneficial to all students involved but also stated that, "the project did not narrow the 'achievement gap' as hoped but instead widened the gap slightly" (p. 48). Thomas and Tagg (2006) also observed that students did not have strategies when working with large, more complex calculations, possibly due to the focus on mental strategies and limited instruction of formal written methods.

2.4.1.2 Ability Grouping for Organisation

In classrooms, the NDP influenced how teachers organised lessons and grouped their students. The project required teachers to determine a students' ability level in relation to number strategy development and group students accordingly. This was advised in NDP teacher guides that suggested "grouping students by their strategy stages makes it easier for you [the teacher] to pose problems that are broadly in the students' zone of proximal development" (MoE 2008, p.11) The result was extensive ability grouping of students at all primary age levels. In the following years, ability grouping was continued by many schools. During a lesson, the teacher works with a group, while the rest of the class work independently or with a partner on other tasks. Teachers perception of this approach are positive due to belief that it is an effective way to support diverse student needs and drive achievement (Anthony & Hunter, 2017). By using this differentiated approach, the goal is that students receive the right amount of challenge for their level. This approach has been challenged (Boaler, 2013, 2015; Boaler & Dweck, 2015; Francis et al., 2017) with suggestions that it has negative impacts on student self-efficacy and mathematic identity. Students placed into the lower ability groups may not get the opportunity to engage in more challenging mathematical activities. This restriction can prevent progress and is supported by Slavin and Karweit's (1985) seminal research into grouping and streaming who found that students placed into lower ability groups made less progress than students in more able groups.

Classroom studies have also shown a deficit theory mentality with students in low ability groups, where teachers assume students lack the capacity to succeed and are therefore provided with material that is low level, using less complex language, and generally closed questions (Anthony, 1996; Anthony & Hunter, 2017; Jorgensen et al., 2014). Additionally, analysis by Hattie (2009) of educational interventions found a positive effect size of 0.12 for ability grouping on children's achievement, well below the average of 0.4 for all interventions. Further research by Boaler (2013) suggests that student mindset is negatively impacted by grouping and streaming of students. Liljedahl (2021) observes that students placed in low set ability groups often leads to non-thinking behaviours while also reinforcing a belief that they are not good at mathematics. In addition, a negative fixed mindset can occur in high ability groups when students who have always been deemed as 'smart' are challenged. Having never failed before, these students can struggle accepting the work is challenging and lack the skills to respond effectively (Dweck, 2006). In light of these findings, there has been a push in recent years towards a more heterogenous approach with mixed ability grouping such as those found in inquiry-based pedagogies discussed in the following section.

2.4.3 The Shift Toward Collaborative Inquiry (2010–2020)

2.4.3.1 Inquiry-Based Learning

Inquiry based learning shifts students away from following direct instruction and procedures towards problem solving and investigating. According to Lee et al. (2004) inquiry based learning involves "an array of classroom practices that promote student learning through guided and, increasingly, independent investigation of complex questions and problems." (p.9). Authentic and genuine problems are often utilised which don't have an obvious or readily apparent solution (Gibbs et al., 2022). Students are required to work together to find a possible solution by looking for patterns, making conjectures, and justifying their solutions (Hunter et al., 2018). Inquiry based learning encourages students to engage in productive struggle and reason mathematically, shifting them from passive to active learners (Fry et al., 2025). In New Zealand, an inquiry based approach called DMIC (Developing Mathematical Inquiry Communities) which prioritised collaborative learning and culturally responsive pedagogy was devised (Hunter et al., 2018).

2.4.3.2 DMIC (Developing Mathematical Inquiry Communities)

DMIC (Hunter, 2008) initially began as an ambitious project looking to address the inequalities and underachievement occurring with students in low socioeconomic areas of predominantly Māori and Pacific Island heritage. This approach to teaching mathematics was broadly adopted in New Zealand with over 140 schools across the country involved (Education Gazette, 2019). DMIC is designed to provide students with opportunities to engage in rich mathematical talk while the teacher's roll shifts from being the mathematical authority to one of a facilitator of learning (Hunter et al., 2018). Based on the five practices for orchestrating mathematics discussions identified by Smith and Stein (2011), teachers work through anticipating, monitoring, selecting, sequencing and connecting. Lessons follow a structure of an initial launch where students are introduced to a problem, followed by a chance to work collaboratively in heterogenous groups to find a solution. Subsequently, groups are selected to share their mathematical thinking with the class before the teacher finishes the lesson by connecting the learning that has occurred. The DMIC approach requires teachers to be trained in its facilitation and students need to be guided through the use of 'talk moves' that help to establish the social norms related to communication and participation (Hunter et al., 2018). Critics of inquiry-based approaches argue that insufficient guidance from the teacher risks a lack of focus on foundational skills, increases cognitive load, and leaves students susceptible to acquiring misconceptions (Kirschner et al., 2006). The DMIC approach advocates for mixed ability grouping which will be discussed in the following section.

2.4.3.2 Mixed Ability Grouping

Mixed ability grouping (also referred to as heterogeneous grouping) is used in inquiry-based approaches and seen as an alternative to some of the inequitable outcomes often associated with ability grouping that were mentioned prior. This approach fosters collaborative learning while promoting inclusion and empathy among peers (Sullivan, 2014). By removing set ability groups, teachers can group students socially, allowing for compatible peers to work together effectively (MoE, 2008). Through the use of questioning, explaining, and justifying, students of differing abilities can learn from each other (Slavin & Karweit, 1985). Furthermore, this environment ensures students are not restricted by pre-determined levels, thereby increasing their exposure to challenging content and enhancing learning potential. The challenge of mixed ability grouping lies with the teachers' ability to set clear expectations around collaborative discourse and discussion. As Sullivan (2014) notes, the efficacy lies less in the grouping method itself and more in the teacher's proactive approach to addressing classroom diversity.

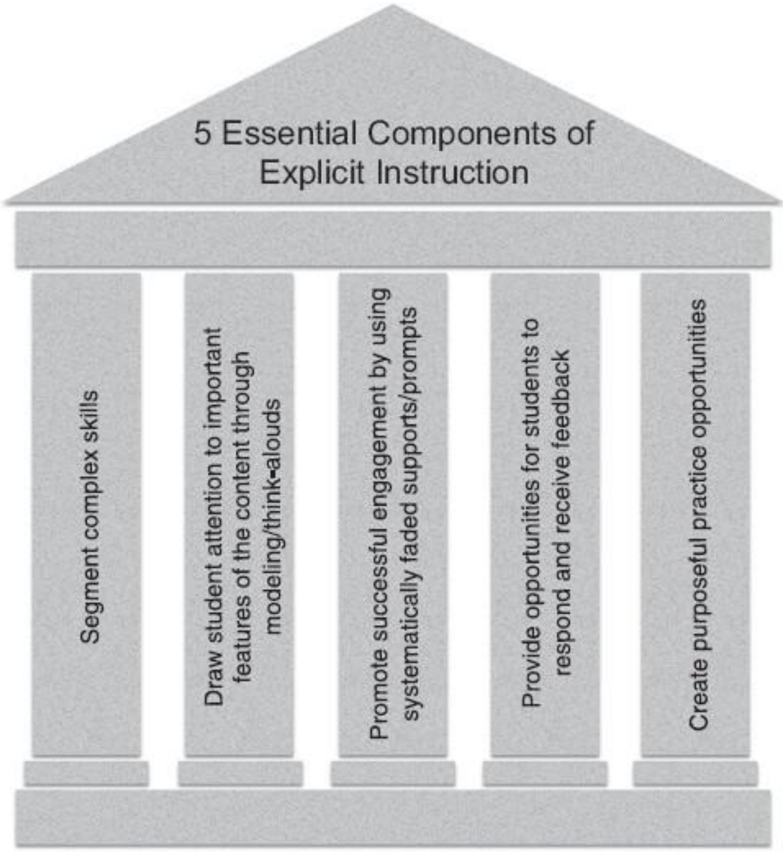
2.4.4 Current Directives: Structured Mathematics and Curriculum Alignment (2020–Present)

2.4.4.1 Explicit Teaching

To address falling achievement, the Ministry of Education announced that the new curriculum would address the basics of mathematics with a focus on explicit teaching (Luxon & Stanford, 2024). Within the refreshed curriculum document, the term is used 15 times and defined as "a structured, carefully sequenced approach to teaching." (MoE, 2024, p25). Explicit teaching is not a singular entity but a combination of teaching behaviours that support successful instruction. Hughes et al. (2022) explain that at its core "explicit instruction is a group of effective teaching 'elements' that, when implemented appropriately, can provide appropriate amounts of clarity, guidance, and support to students who would not be able to learn in their absence" (p.236). Explicit teaching requires a high level of teacher involvement compared to more student-centred pedagogies. Teachers are required to be active, modelling, scaffolding and questioning during the learning process. After an extensive review of literature, Hughes et al. (2017) identified five essential components of explicit instruction. Figure 1 shows these five components as 'pillars' supporting the successful implementation of explicit instruction.

Figure 1

Five Pillars of Explicit Instruction



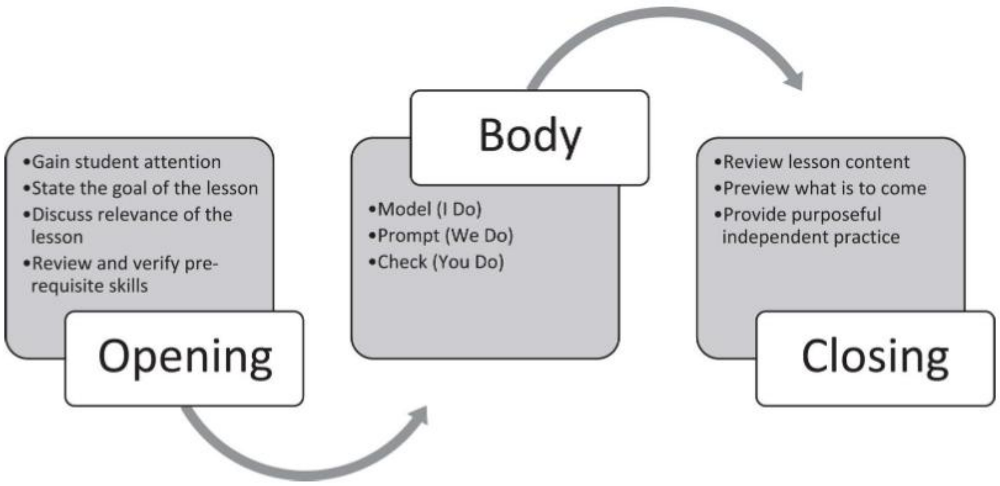
Note. From "Explicit instruction: Historical and contemporary contexts." By Hughes et al., 2017, Journal Learning Disabilities Research & Practice, 32(3), 140–148

Teachers segment complex skills into manageable chunks, model important content features using clear language, and then, building on initial success, gradually and systematically withdraw scaffolding. Throughout the process, teachers elicit frequent student response while creating purposeful practice opportunities to consolidate learning (Hughes et al., 2017).

The phrase 'I do, we do, you do' (Figure 2), has been adopted to describe a gradual release of responsibility that aligns with an explicit approach to teaching mathematics (Mariage et al., 2018). To implement this approach, the teacher will work with the whole-class, initially modelling and instructing, before students engage in worked examples with sufficient scaffolding. Later in the lesson, when students are ready, they work independently. This approach is a move away from ability groups and heterogenous grouping that has been discussed earlier and traditionally used in many primary classrooms.

Figure 2

Structure and Components of a Typical Explicit Instruction Lesson



Note. From "Explicit instruction," by Hughes et al., 2022, in High Leverage Practices for Inclusive Classrooms, Routledge.

2.4.4.2 Maths Mastery and a Whole-class Approach

Since 2015 the term 'maths mastery' has been a central policy in mathematics education in the United Kingdom (Boylan et al., 2018). Influenced by East Asian educations systems such as those found in Shanghai and Singapore, this approach aims to provide deep, secure understanding of mathematical concepts for all students (NCETM, 2022). This is achieved through the use of a process of broken-down steps incorporating concrete, pictorial and abstract representations to scaffold and challenge learners. A central concept to 'mastery' is that all students are working on the same concept at the same time (Boyd & Ash, 2018). To enable this to occur, a whole-class approach to teaching mathematics has been adopted as the primary form of instruction. The National Centre of Excellence for the Teaching of Mathematics (NCETM, 2022) state that in a mastery approach, "Pupils are taught through whole-class interactive teaching, enabling all to master the concepts necessary for the next part of the curriculum sequence." (Section 'In the classroom' para. 1). In New Zealand, the recently implemented mathematics curriculum uses similar language and the document advises teachers to, "Plan for all students to experience all the statements in the sequence for their year level." (MoE, 2024, p. 24). This advice, combined with the focus on explicit teaching, has seen a shift towards a whole-class approach to teaching mathematics.

Whole-class teaching is a classroom approach where all students are taught together as a single group. Slater and Chambers (2024) define the concept of whole-class teaching as a "particular social and organisational context for the activities, interactions and pedagogical approaches employed in the classroom" (p. 537). Traditionally, this approach has been linked with direct, teacher-led instruction, but more recent views recognise that it can also include interactive, student-centred methods (Willemsen et al., 2020). There is no clear consensus on when best to utilise a whole-class approach, with some suggesting its effective for introducing new concepts and summarising lessons, while others believe its beneficial to use frequently to address misconceptions and reinforce learning (Slater & Chambers, 2024). Rosenshine (2009) identified that the most effective teachers began instruction with full teacher control, with diminishing control occurring throughout the lesson until students are working independently.

Perceptions of a whole-class pedagogy recognise its merits but also acknowledge there are limitations. A whole-class approach is seen as a practical and effective use of teachers' time that allows for teachers to easily manage focus and student engagement (Muijs & Reynolds, 2018). Students' collective progress and collaborative peer learning are also seen as a major benefit of a whole-class approach (Slater & Chambers, 2024). In addition, whole-class discussions provide valuable opportunities for students to learn from their peers and learn collaboratively (Willemsen et al., 2020).

From a negative viewpoint, a whole-class approach provides a challenge to effectively accommodate the differing needs of students. Pitching the lesson at the right level and appropriate pace while accommodating the needs of all students, can be challenging to achieve (Blausten et al., 2020; Burns & Myhill, 2004). In heterogeneous classes with a range of abilities, lower-ability students may find the instruction too difficult, while more able students may find it too easy. In whole-class maths lessons, some students lack prerequisites while others progress quickly, meaning only a few benefit from the lesson (Slavin & Karweit, 1985). In a whole-class approach, students' individual prior knowledge can be effectively overlooked, leading to boredom for more able students or frustration for less able students. Applying uniform teaching sequences and methods across all students also contradicts Vygotsky's (1978) concept of the Zone of Proximal Development. His concept emphasises the need for instruction that challenges students just beyond their current abilities with appropriate levels of support. Tomlinson (2001) also suggests that flexible grouping and differentiated instruction are crucial to address the individual needs of the students.

Hunter et al. (2018) also indicates that a didactic approach based on Eurocentric ideas of learning such as whole-class teaching, is not suited to the relational and communal approaches of indigenous education. A shift to a whole-class approach risks marginalising the students who require the most support. This is important as In a New Zealand context as Māori students' underachievement has been an ongoing issue for many years (Asil et al., 2023; Berryman et al., 2025; Morrow et al., 2022).

2.4.4.3 Government Funded Resources

To support teacher's integration of the maths component of *Te Mātaiaho* (MoE 2024), the government funded a number of resources from overseas companies. Minster of Education, Erica Stanford, announced that the resources will be used "alongside the new knowledge-rich curriculum and will support teachers, reduce workload, and lift student achievement" (Luxon & Stanford, 2024, p. 1). Two of the four options, *PR1ME* and *Maths - No Problem!,* are influenced by the Singapore education system. Mainly textbook focused, the resources encourage the use of manipulatives and a maths mastery approach. The promotion of these textbooks follows moves in the United Kingdom where the use of high quality textbooks to support a mastery approach are encouraged (Marks et al., 2019). Response to their introduction in the UK has been mixed. Boyd and Ash (2018) identified that a dependence on textbooks and teacher guidance is concerning as it could potentially reduce teachers to a role of simply delivering the curriculum while also placing considerable pressure on schools to trust that the materials are effective and evidence-based. Wrathall (2024) acknowledges that funding of resources by the Ministry of Education has been in decline and although these new resources are welcomed, there is a risk of a "one-size-fits-all" approach that doesn't meet the needs of all students. Significant time and training are also needed to ensure that the use of textbooks provides support through structure and progression without limiting teacher autonomy (Marks et al., 2019).

2.4.5 Summary

This section has described a range of different pedagogical approaches to teaching mathematics that are currently being implemented in New Zealand primary schools. It has tracked the shift towards a more explicit and directed approach, guided by the Ministry of Education's curriculum refresh. The introduction of funded textbook resources to support the new curriculum is also influencing current teaching pedagogy in New Zealand. In any approach, the use of materials and resources will continue to be utilised by teachers to support understanding. The following section will discuss the use of mathematical manipulatives and their importance in developing conceptual understanding.

2.5 Mathematical Manipulatives

Manipulatives, both physical and virtual, are fundamental tools in mathematics education, bridging the gap between concrete experience and abstract concepts. This section will focus on the use of manipulatives in primary classrooms, exploring their theoretical foundation and practical application. Firstly, the use of physical manipulatives will be explained before moving to a more detailed discussion of virtual manipulatives.

2.5.1 Physical Manipulatives in the Mathematics Classroom

Since the beginning of the 19th Century, teachers have utilised concrete objects to teach mathematical concepts by engaging students with moveable physical items (O'Meara et al., 2020). Johnson et al., (2021) describe these physical manipulatives as "objects that allow users to model, and engage with, both real world and abstract structures in mathematics education in a more tactile manner." (p. 1008). These physical manipulatives can either represent everyday resources such as money or be made specifically for a mathematics concept such as fraction pieces or deci-pipes.

The use of manipulatives aligns with the first stage of Jerome Bruner's (1966) concrete-pictorial-abstract (CPA) approach. The CPA approach evolved from his hypothesis that the path of intellectual development moves through three stages "enactive-iconic-symbolic". The theory proposes that students must first engage with concrete material and experiences (enactive), then move to a representation stage using diagrams (iconic), before lastly moving into an abstract or symbolic understanding (Johnson et al., 2021). In a similar constructivist approach, Piaget (1978) advocated for the use of manipulatives as they provide opportunities for students to actively construct understanding rather than passively receive it. The ability to link different representations supports a robust conceptual understanding and aids the construction of more comprehensive schemas (Moyer-Packenham et al., 2008).

The use of manipulatives in developing conceptual understanding in mathematics is significant. Conceptual understanding is more than just learning certain facts and procedures, students who have conceptual understanding can see relationships between ideas and apply their knowledge to new situations (Mills, 2019). In contrast, procedural knowledge relates to the rules, algorithms, and procedures used when solving a mathematical problem (Goldman & Hasselbring, 1997). Mathematical literacy requires a synthesis of both conceptual and procedural knowledge in order to be successful. Students who are predominantly taught with a procedural approach often lack conceptual understanding leading to frustration and a negative attitude to mathematics (Boaler & Dweck, 2015). Furthermore, students who rely on procedural understanding struggle to identify when they have made a mistake, overgeneralise rules, and fail to transfer understanding to unfamiliar contexts. This is supported by Goldman and Hasselbring (1997) who indicate that "If students do not understand why they are using specific procedures in particular contexts, then there can be little possibility of transfer to other contexts" (p. 201). This highlights the importance of using manipulatives to provide visual representations of the mathematic concept allowing for greater connection and conceptual understanding.

The focus on the use of manipulatives in the new curriculum and Ministry-funded textbooks directly promotes the use of physical tools to support understanding. Within the explicit teaching section of *Te Mātaiaho* (MoE, 2024), it states "Using materials and visual representations throughout explicit teaching supports students to develop conceptual understandings as they move towards more abstract forms of representation" (p. 23). In the funded textbooks, such as *Maths - No Problem!*, a list of essential manipulatives is provided. The promotion of manipulatives is important, especially as their use at the upper primary and intermediate level reduces as students get older (Johnson et al., 2021). In a review of 503 primary teachers, Uribe-Flórez and Wilkins (2010) found teachers believed that the use of manipulatives was more appropriate for younger learners. Reasons for this range from mistakenly held perceptions that manipulatives only benefit low-ability students to concerns about classroom management and disruption (Johnson et al., 2021; Moyer & Jones, 2004).

2.5.2 Virtual Manipulatives in Mathematics Education

Digital technology is becoming more and more prevalent in classrooms in the twenty-first century, promoting future-focused learning and preparing students for success in a digital future (Cullen et al., 2025, Attard, 2018; Calder and Murphy, 2018). In many primary schools, digital tablets and chrome books are used daily by students to access the internet, interact with educational games and software, and as a source of output for writing and presentations. As students get older, many New Zealand schools encourage students to bring their own device (BYOD) or provide a class set of Chrome books, leading to 1:1 access to digital technology. This easy access to digital technology has many benefits but must be managed effectively by teachers to ensure it is used productively (Reiten, 2021). In mathematics, digital technology can be used to engage students, to manipulate data, and provide a tool to represent mathematical concepts using virtual manipulatives.

The definition of what is a virtual manipulative has evolved over time as technological advances have increased. Moyer et al (2002) defined a virtual manipulative as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373). Many early virtual manipulatives were merely a digital pictorial representation of a physical manipulative. Moyer-Packenham and Bolyard (2016) revised the definition and identified that virtual manipulatives were more than representation and “It is the interactive and dynamic capabilities of the manipulative representation that makes it a virtual manipulative” (p. 9). For learning to be effective, students must be able to make meaningful connections between pictorial representations, the actions applied to those representations, and the related symbolic forms, as these links help illuminate the mathematical concepts under instruction (Moyer-Packenham & Westenskow, 2013). Virtual manipulatives allow students to interact in a digital sense, providing opportunities that may not be possible with ordinary manipulatives (Rich, 2024). An example is the splitting and renaming of fractions. With concrete materials often made of plastic, students can not break or cut them into smaller pieces. In a virtual setting, students can split fractions multiple times, re-join fractions, and rename them at the touch of a button. The term 'affordances' is used to refer to the features or learning interactions that a virtual manipulative can provide to explore a mathematical concept. Affordances will now be discussed.

The concept of affordance was first proposed by Gibson, (1977) who defined affordances as the properties of the environment taken relative to an observer. In other words, affordances are the potential allowable actions between the environment or object and the person (Calder, 2011a). When applied to digital objects, affordance refers to the qualities of an interface or digital object that allow the user to interact with it. Calder and Campbell (2016) discuss affordances by highlighting the "symbiotic relationship between the digital medium and the user" (p. 52). Common positive affordances of virtual manipulatives are the interactivity they provide, the ability to receive immediate feedback, and the opportunity to view multiple representations at the same time (Calder, 2011b; Moyer et al., 2002; Moyer-Packenham, 2016; Moyer-Packenham & Westenskow, 2013). The ability to manipulate mathematical objects supports students reorganisation of mathematical ideas and promotes more flexible thinking (Calder, 2011b). The affordance of being able to undo and redo actions or reset and modify previous actions also promotes flexible thinking and experimentation (Moyer et al., 2002). Multiple representations allow students to link and explore ideas simultaneously providing opportunity to make connections and identify differences. Virtual manipulative also offer affordances that allow students to personalise their learning through individual exploration and in turn strengthen student agency and engagement. (Bray & Tangney, 2015; Calder & Campbell, 2016; Dueker & Desai, 2022; Willacy et al., 2017). The emergence of touchscreen devices has also led to the identification of haptic affordances, which relate to the direct interaction through touch (Hegedus, 2013). Research by Lee & Tan (2014) focused on the role of manipulatives in a CPA approach and found that the affordance of smooth movement between the concrete to pictorial to abstract stages supported students' conceptual understanding.

Research into the effectiveness of virtual manipulatives has highlighted the benefit to student engagement and understanding but their success is often influenced by the level of teacher expertise (Fletcher et al., 2020; Moyer-Packenham & Westenskow, 2013; Reiten, 2021; Willacy et al., 2017). Teachers need to familiarise themselves with how the tool functions and plan carefully how they will utilise the tool in lessons. In addition, using digital devices adds another layer of classroom management for teachers to navigate (Drijvers et al., 2010;

Johler et al., 2022). Reiten (2021) identified that when professional development was provided, teachers preparedness and confidence in implementing virtual manipulatives was enhanced. Dueker & Desai (2022) concluded a similar view, highlighting that teacher guidance and consistent integration into lessons led to greater effectiveness of the virtual manipulatives. In comparison, Keldgord and Ching (2022) identified that technical difficulties and limited access provided a challenge to teachers fully integrating virtual manipulatives into teaching practices.

2.6 Role of Teacher Professional Development

This section will discuss the different approaches to facilitating professional development and identify common traits of successful programs. In particular, it will focus on nuances of digital technology professional development and explore strategies that can lead to effective professional development in this area.

Darling-Hammond et al. (2017) characterise professional development as "structured professional learning that results in changes in teacher practices and improvements in student learning outcomes" (p. v). Professional development can be delivered through a range of methods, from formal, structured approaches and collaborative initiatives to individual, self-directed practices. In New Zealand, the expectation for teacher professional development is guided by the Teaching Council of New Zealand. Their Professional Growth Cycle framework replaced teacher appraisals in 2020 and outlines expectations that teachers are to engage in continuous professional learning and development throughout their careers (Teaching Council of Aotearoa New Zealand, 2020). Ultimately, the expectation is that professional development leads to improved outcomes for all learners. This approach has been informed by an initial synthesis of effective professional development programmes in New Zealand by Timperley et al. (2007) which in turn led to the identification of 10 principles for effective professional learning and development (see Timperley, 2008). The principles function as a responsive framework that reflects the complexity of the classroom, acknowledging that for professional learning to be effective, it must respect teachers' prior knowledge and the specific socio-cultural contexts of their schools. Within these principles, Timperley (2008) highlights the need for an inquiry cycle that centres on using learner data to identify the knowledge, 'and skills teachers need to improve their practice. A crucial element is the role of school leaders in fostering a culture of inquiry and leading by example by engaging in their own inquiry (Timperley, 2008).

Internationally, a more recent review of 35 studies by Darling-Hammond et al. (2017), identified 7 key aspects of effective professional development that are listed below:

1. Is content focused
2. Incorporates active learning utilising adult learning theory
3. Supports collaboration, typically in job-embedded contexts
4. Uses models and modelling of effective practice
5. Provides coaching and expert support
6. Offers opportunities for feedback and reflection
7. Is of sustained duration

(Darling-Hammond et al., 2017, p. 4)

Successful professional development generally needs to feature multiple components simultaneously rather than focusing on a single element (Desimone, 2009). When these components are fragmented, initiatives often fail due to systemic barriers such as inadequate resources, a lack of shared vision, a lack of foundational knowledge, and conflicting requirements (Desimone, 2009; Timperley, 2008).

Professional development aimed at developing teachers' use of digital tools in the classroom has seen greater attention in recent years (Haynes-Brown, 2025). The COVID-19 pandemic's acceleration of online learning, coupled with the evolution of robotics and coding and the swift emergence of artificial intelligence, has reinforced a significant need for teacher upskilling in digital technologies (Haynes-Brown, 2025; Moorhouse & Wong, 2022). A common issue with digital technology directed professional development is insufficient unpacking of resources and integration into the curriculum. When teachers are exposed to apps, websites or tools, the key aspects identified by Darling-Hammond et al. (2017) need to be considered. It is particularly important that professional development is provided in a way that develops the different fields of knowledge collectively rather than in isolation. Several conceptual frameworks, which are introduced below, have been designed to support teachers' use of digital technologies.

2.6.1 Technological Pedagogical Content Knowledge (TPACK)

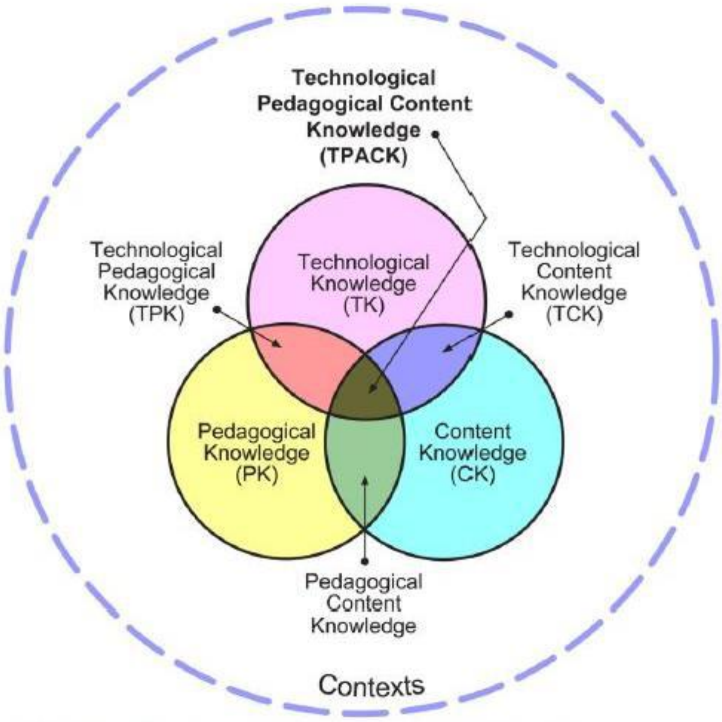
Almost 40 years ago, Shulman (1987) introduced the concept of pedagogical content knowledge (PCK) as a core component of teachers' professional knowledge. His seminal work highlighted that teaching quality is more than just a grasp of content knowledge (CK) and pedagogical knowledge (PK) but also requires knowledge of how to teach specific content (Koehler & Mishra, 2009). Shulman explains that PCK represents "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8). Specifically, in the realm of mathematics, teachers' PCK involves knowledge of teaching strategies that utilise appropriate conceptual representations in order to address misunderstandings and foster meaningful learning (Koehler & Mishra, 2009).

Since the first identification of PCK, developments in technology and its use have increased rapidly. On top of knowing what to teach and how to teach, teachers must also have a knowledge of how to use technology. This technology knowledge (TK) refers to all technology, but in our modern world, it is heavily comprised of digital mediums and the use of web-based tools and applications. Over time, technology becomes ubiquitous and in turn transparent due to its seamless integration into teaching. For example, whiteboards or calculators, once revolutionary, have now become commonplace in classrooms. Weiser's (1991) formative quote summarises this, "The most profound technologies are those that disappear. They weave themselves into the fabric of everyday life until they are indistinguishable from it" (p. 94). Due to its constant evolution and adaptation, teachers are required to be constantly upskilling in their own TK.

Koehler and Mishra (2009) built on Shulman's initial formulation of PCK and formed the TPACK framework. This framework identifies that the three knowledges of content knowledge (CK), pedagogical knowledge (PK), and technology knowledge (TK) are all connected and overlap. Figure 3 shows the three overlapping elements of knowledge.

Figure 3

TPACK Framework and its Knowledge Components



Note. From "What is technological pedagogical content knowledge (TPACK)?," by Koehler and Mishra, 2009, Contemporary Issues in Technology and Teacher Education, 9(1) p. 63.

Within the diagram, PCK can be identified where PK and CK overlap. At the centre of the diagram, is technological pedagogical content knowledge (TPACK) which integrates all three areas of knowledge in a dynamic and transactional relationship. Koehler and Mishra (2009) indicate that mastering TPACK requires teachers to have a comprehensive understanding of several key areas. They are as follows.

Representing concepts: How to effectively represent subject matter concepts using various technologies.

Pedagogical techniques: Constructive pedagogical approaches that utilise technology to teach content.

Addressing learning challenges: What makes certain concepts difficult or easy to learn, and how technology can be used to overcome common student difficulties.

Technology for Knowledge Construction: How technologies can be used to build upon existing knowledge and to develop new ways of knowing, or strengthen established ones.

(Koehler & Mishra, 2009)

When delivering professional development that involves TPACK, Mishra and Koehler (2006) recommend that the different areas of knowledge are not taught independently. From experience, they found that a learning by design approach was most effective. Mishra and Koehler (2006) suggest that "design-based activities provide a rich context for learning and lend themselves to sustained inquiry and revision," which makes them appropriate to support "the deep understanding needed to apply knowledge in the complex domains of real-world practice" (p. 1029).

The TPACK model was expanded upon by (Niess et al., 2009) who proposed that teachers are at varying levels of knowledge with regards to integrating technology. It is not a simple case that a teacher has TPACK or not. This new model took a mathematics viewpoint towards technology integration. After reviewing the original model, they designed a five-stage developmental process when learning to integrate a particular technology in mathematics. The five stages are described below and visually represented in Figure 4.

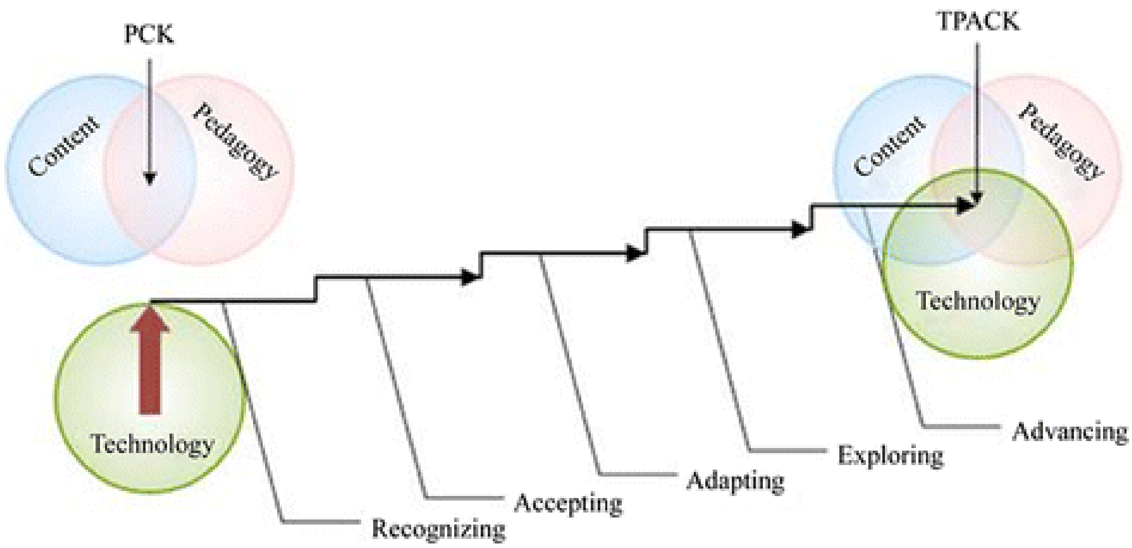
1. **Recognising (knowledge),** where teachers are able to use the technology and recognise the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.
2. **Accepting (persuasion),** where teachers form a favourable or unfavourable attitude toward teaching and learning mathematics with an appropriate technology.
3. **Adapting (decision),** where teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with an appropriate technology.
4. **Exploring (implementation),** where teachers actively integrate teaching and learning of mathematics with an appropriate technology.
5. **Advancing (confirmation),** where teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology.

(Niess et al., 2009)

It is important to note that while the stages appear linear, the model is proposed to display a more iterative process with regards to the development of TPACK. As technology develops, the emergence of new tools and resources requires rethinking and a return to a prior stage in the process (Niess et al., 2009).

Figure 4

A proposed five-stage developmental process to integrating technology



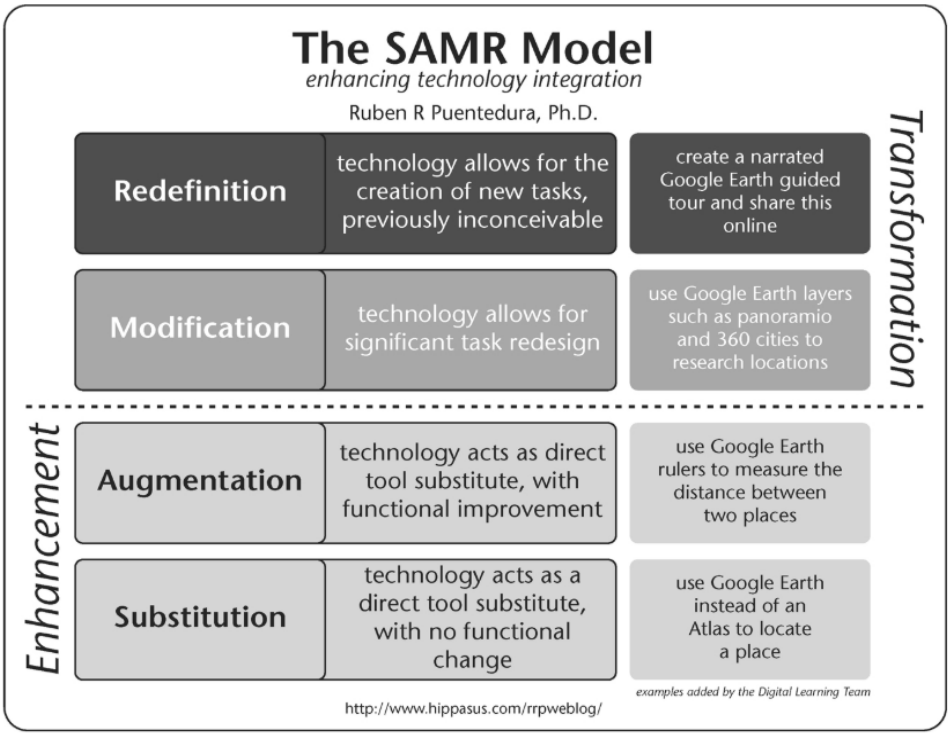
Note. From "Mathematics teacher TPACK standards and development model," by Niess et al., 2009, Contemporary Issues in Technology and Teacher Education, 9(1), p.10.

2.6.2 The Substitution, Augmentation, Modification, Redefinition Model (SAMR)

The SAMR model of delivery, initially proposed by Puentedura (2006), is another framework used to help teachers improve technology integration with their teaching (Adulyasas et al., 2021). SAMR is a taxonomy-based approach, consisting of four levels, that focuses on how teachers can move from digital enhancement to transformation (Hunter, 2015). The diagram in Figure 5 shows the levels of this model and provides examples of how this might look in the classroom.

Figure 5

The SAMR model of enhancing technology integration



Note. From "Transformation, technology, and education," by R. Puentedura, 2006, *Hippasus* (https://hippasus.com/blog/). Copyright 2006 by R. Puentedura.

The first level begins with substitution, where the technology is replacing what was currently being used. Linking this to mathematics, this might involve replacing a plastic fraction piece with a digital version. The second stage, called augmentation, is where the substitute technology provides some functional improvement. In maths, a fraction piece could be split, renamed or converted into a decimal. The third and fourth stages see a shift away from enhancement of learning towards the transformation of learning. Stage three, the modification stage, allows for the technology to redesign the task. Students using the fraction virtual manipulatives to create their own fraction problems and adding on-screen annotations to show their reasoning is an example of this within a mathematic context. The final stage is redefinition which provides opportunities for the technology to create new experiences that were previously unachievable. A mathematic example of this could be students collaborating on an online digital whiteboard to design a fraction farm where different animals and crops are represented by different fractions. Their work could then be presented or shared online.

The plain language and the easy to follow diagram (see Figure 5) has made the SAMR model accessible and extensively used around the world (Blundell et al., 2022). Although widely discussed and referred to, this model is not without is critics. The model's lack of context, its overly rigid structure, and emphasis on transformation over enhancement have all been identified as limiting factors of the model (Blundell et al., 2022).

The landscape of digital technology integration continues to evolve, most notably with the increasing availability of generative artificial intelligence models. As these tools become more established, it is important to consider their specific implications for classroom practice and student engagement.

2.7 Generative Artificial Intelligence (GenAI)

While artificial intelligence has been a field of academic study and technical development since the 1950s (McCarthy et al., 1955), its integration into education has recently been accelerated by the development of generative models. Since 2022, the rapid expansion of generative artificial intelligence (GenAI) has provided a range of new opportunities for educators (Song et al., 2025). Mishra et al. (2023) define GenAI as "artificial intelligence (AI) applications which are designed to use a variety of machine learning algorithms to create new content" (p. 236). Its accessibility has led an increasing number of teachers to utilise GenAI to support planning efficiency and assist in the design and creation of educational resources.

Studies in New Zealand and Australia report a sense of cautious optimism about the use of AI, while also noting that teachers remain wary of its limitations and potential negative impacts (Ashby, 2024; Coblentz et al., 2025). Concerns such as an over reliance on AI tools, difficulty differentiating between student work and AI outputs, and a decrease in critical thinking have been identified (Lan & Chen, 2024). Integrating AI into classroom teaching presents great challenges. Coblentz et al. (2025) suggests that it is important that schools are having conversations around how to adopt these tools ethically and incorporate them effectively to maximise the learning benefits. The shift from traditional teaching methods to an environment powered by AI is not an incremental process, but a fundamental leap that cannot be achieved without deliberate action (Pratschke, 2024). Merely providing teachers with new tools is insufficient. For successful, ethical, and constructive integration of AI, teachers require robust digital and AI literacy. This involves going beyond the basic operational skills to include an understanding of the tools' capabilities, an awareness of their related risks and issues (e.g. bias, privacy) and the pedagogical ability to utilise them constructively to enhance learning (Pratschke, 2024).

To address the educational impact of GenAI, Mishra et al. (2023) returned to the TPACK model to explore the types of knowledge teachers need to effectively use GenAI. They proposed an expanded model to include contextual understanding, referring to the new type of knowledge GenAI requires. Mishra et al. (2023) state "It is not only crucial for teachers to understand how GenAI works, and how to repurpose it for their specific pedagogical needs; they need to understand that their actions are grounded within and circumscribed by external factors" (p. 244). This crucial concept of contextual awareness indicates that the use of GenAI is not an isolated decision but is influenced and limited by a range of external factors such as social and cultural norms, legal and ethical frameworks, and institutional policies.

In mathematics, GenAI is thought to have the potential to enhance conceptual understanding and student efficacy while also providing opportunities for personalised instruction, adaptive learning, and discovery (Mishra et al., 2025). For teachers, rather than spending considerable time searching for relevant resources that meet their needs, AI can be used to create resources that are tailored to the needs of the student (Almuhanna, 2024; Berry, 2024). This creative approach is supported by Gabriel et al. (2025) who envisions that, "AI is not merely used as a tool for product-oriented tasks but as a transformative force, where human-computer interactions lead to an expansion of practice"(p.6). Achieving this vision, however, relies on educators developing the necessary AI literacy and contextual understanding required to navigate its ethical intricacies and maximise its pedagogical potential.

2.8 Gaps in Literature

While foundational research has established the value of using virtual manipulatives, the existing literature is largely characterised by a focus on student-centred outcomes and is increasingly limited by its age. For example, the meta-analysis conducted by Moyer-Packenham and Westenskow (2013) into the use of virtual manipulatives in mathematics instruction provides evidence that virtual manipulatives generally have a positive effect on student achievement. Similarly, Lee and Tan's (2014) study into the role of virtual manipulatives in supporting the concrete-pictorial-abstract (CPA) found that these tools supported student success. However, since these studies were conducted the growth and availability of digital devices means that they offer limited insight into how VMs function within modern day classrooms.

Within a local context, studies in New Zealand by Calder and Campbell (2016), and in Australia by Attard (2018) focused on how mathematical apps and mobile technologies enhance student engagement. Both studies used a qualitative approach, including a mixture of interviews, observations and case studies. The studies revealed similar findings that indicated mobile technology has the potential to boost engagement but is dependent on the type of task and teachers' pedagogical practices. Despite this acknowledgment of the teacher's role in integrating digital devices, these studies stop short of investigating the teacher as the primary subject of inquiry.

Addressing the role of the teacher in integrating virtual manipulatives into classroom practice, Fletcher et al. (2020) examined how digital technologies are integrated into New Zealand schools. Their findings indicated that digital tools could enhance collaboration and student agency, but success relied heavily on teacher confidence and ongoing professional development. These finding were echoed by Dueker and Desai (2022) in their study on the use of virtual manipulatives and explicit instruction in a virtual learning environment. Their findings suggested that while virtual manipulatives can enhance conceptual understanding and engagement, their effectiveness greatly depends on the guidance of the teacher and consistent integration into lessons. Following the Covid 19 pandemic, there was increased interest in online learning and instruction. Keldgord & Ching (2022) explored teachers' experiences and perceptions using virtual manipulatives during hybrid lessons and supporting online learning environments. Their findings indicated that teachers saw the benefits and potential of VMs to support engagement and conceptual understanding, but full integration was hindered by technical difficulties, lack of training and limited access to resources.

Relatively few studies exist that have predominantly focused on the role of professional development in the use of virtual manipulatives. A design-based research study by Suh (2016) focused on developing effective strategies for pre-service maths teachers to integrate technology into mathematics classrooms. Through a series of practice-based assignments, these trainee teachers were able to gain a better 'picture of practice' on integrating technology. The use of experienced teachers providing exemplary models of teaching integrating technology was identified as a key element of success. A"…how study by Reiten (2021) explored how targeted professional development can support middle and high school mathematics teachers in integrating virtual manipulatives. The study involved a professional development program over eight months that provided teachers with a repository of VM resources. The study found that the intervention led to more effective and reflective use of digital technology in mathematics teaching. Participants of the study were specialist mathematics teachers, rather than general classroom teachers found in primary and intermediate schools. Therefore, the limited number of studies into how a professional development intervention can impact non-specialist teachers' perceptions and use of VMs, provides a gap worthy of further investigation.

2.9 Summary

In summary, this literature review has traced the evolution of mathematics education in New Zealand, from its historical roots to the current shift towards a more structured, explicit teaching approach outlined by the *Te Mātaiaho* curriculum. This pedagogical shift, influenced by approaches like 'maths mastery' and the availability of Singapore influenced resources has seen a renewed interest in whole-class teaching. While this approach offers practical benefits like efficiency and collective learning, it also presents challenges in catering to diverse student needs. The use of virtual manipulatives as a tool for scaffolding and promoting conceptual understanding, has been identified as a potential solution to this challenge. The literature consistently shows that virtual manipulatives can enhance engagement and achievement, but their effectiveness is heavily dependent on the teacher's expertise and integration. Relatively few studies have examined the role of professional development in supporting teachers with virtual manipulatives and have generally focused on specialist mathematics teachers. Therefore, an apparent gap remains in understanding how a targeted professional development intervention can influence the perceptions and instructional use of virtual manipulatives among primary and intermediate school teachers. This research seeks to address this gap by examining the affordances that lead to the adoption of virtual manipulatives and investigating whether a professional development intervention can facilitate their effective implementation in a whole-class setting. By doing so, this study will offer unique insights into supporting teachers as they navigate a new curriculum and integrate digital tools to promote equitable learning outcomes for all students.

Chapter 3: Methodology

3.1 Introduction to Methodology

Building upon the literature review in Chapter 2, which established the broader research context for this study, this methodology chapter outlines the design and key factors that influenced its direction. It begins by discussing my paradigmatic positioning, exploring the nature of the pragmatic paradigm. Following this, the chapter provides the research rationale, detailing the decision to adopt an action research approach within a mixed methods framework (MMAR). The research design is then unpacked in greater detail, providing both a theoretical and practical justification for the selection of the quantitative (Constructivist Learning Environment Survey, CLES) and qualitative (action research) methods used in the study. Next, the process of participant recruitment is explained, along with a description of the instruments utilised for data collection, specifically the CLES and qualitative focus groups. A detailed account of the data analysis techniques follows, covering the use of descriptive statistics for the quantitative CLES data and a thematic analysis of the qualitative data. Finally, the chapter concludes by considering the validity and authenticity of the study, addressing ethical considerations, and reflecting on the researcher's own positionality.

To inform this research and to obtain a clearer understanding of the current landscape of virtual manipulatives use, an initial constructivist learning survey (CLES) was conducted with 36 classroom teachers. The CLES is a quantitative instrument used to measure teachers' perceptions of the classroom learning environment. The CLES used was adapted to include the use of virtual manipulatives and perceptions on a whole-class approach to teaching mathematics. Primary analysis of the CLES results revealed a significant proportion of the survey group had limited prior experience with the use of virtual manipulatives as a teaching tool. This identified ‘gap’ in practical experience along with the acknowledged need to support student conceptual understanding in a whole-class environment, directly informed the design of this study. Thus, the following research question was formed:

How does a professional development intervention shape teachers’ perception and instructional use of virtual manipulatives?

To answer this, the following two supporting questions will be considered, What barriers prevent teachers from using virtual manipulatives?

How can virtual manipulative be best utilised to support student understanding in a whole-class approach?

3.2 Research Method

3.2.1 Research Paradigm

This research project is based on the assumption that a practical and pluralist approach provides the best way to understand human behaviour. Kivunja and Kuyini (2017) explain that paradigms in educational research relate to the researchers' world view, their way of thinking, their shared thoughts, that guide their interpretation of data. The pragmatic paradigm was developed by philosophers who argued that accessing the truth about the real world could not be achieved solely through a positivist or interpretivist approach. The pragmatic paradigm, rather than adhering to a singular ontological or epistemological position allows a combination of methods that when combined provide insights into the behaviours, the beliefs behind those behaviours, and the consequences that may follow from different behaviours (Kivunja & Kuyini, 2017). In the context of this study, this allows for the use of both quantitative and qualitative methods to provide a comprehensive understanding of teacher perceptions and the beliefs behind their practices.

Epistemologically, pragmatism accepts that knowledge is not solely constructed nor exclusively based on the objective reality we experience. Its philosophy is guided by the pragmatic rule or maxim that states that the meaning of an idea or truth comes down to its experiences or practical consequences (Heras-Escribano, 2021). Johnson and Turner (2003) reinforce pragmatism's focus on the consequences of the research by stating, "it offers a practical and outcome-oriented method of inquiry that is based on action and leads, iteratively, to further action and the elimination of doubt." (p. 17). This directly aligns with the iterative action research approach used in this study, where each cycle of reflection and refinement aims to develop teachers' use of virtual manipulatives in their mathematics teaching. This philosophy also embraces fallibility, meaning it is accepting that our beliefs or theories are rarely perfect or final and always open for revision. Most relevant to this study, a pragmatic philosophy has a clear, value orientated approach. It acknowledges research is driven by value, and in the case of this research, the value of education of children and adults.

3.2.2 Rationale

In selecting a suitable methodology for this research, the pragmatic paradigm served as the guiding principle. The decision was further guided by several key factors. First and foremost, as a fellow teacher and a familiar presence to the research group, the chosen approach needed to be collaborative and empowering, ensuring that participants felt like co-creators of knowledge rather than subjects being studied. Furthermore, it was essential that the research was clearly relevant to the teachers' own practice and grounded in their everyday reality.

Given these considerations, an experimental design with a control group was deemed unsuitable due to the ethical concerns of providing professional development to some teachers while withholding it from others. This approach would also have positioned the participants as subjects of an experiment, directly conflicting with the study’s empowering goals. Similarly, a quasi-experimental design with pre-existing groups was not favoured, as it also risked participants feeling like subjects and the lack of group randomisation could impact the validity of attributing change to the intervention (Maciejewski, 2020).

While a case study approach was more favourable, it was ultimately rejected. Although case studies provide rich and detailed data, it was felt that this approach would lead to observation rather than active intervention. Additionally, suggesting broader application from a single case study can be challenging (Wellington, 2015). To align more directly with the research’s core goals of collaboration, empowerment, and practical relevance, the decision was made to adopt an action research approach.

An action research approach provides opportunities for teachers to be active agents in their own professional development, which in turn fosters greater engagement and knowledge growth (Jones, 2023). The cyclic nature of action research makes it iterative and responsive, promoting reflection and refinement. This design approach can also be comfortably applied within a school setting and can complement prior and ongoing professional development (Jones, 2023). As a researcher, this approach allows for greater exploration of how teachers' perceptions and practices evolve over time in response to the intervention, rather than just measuring a ‘before and after’. An in-depth review of action research and participatory action research are found later in section 3.2.5.

During the formative stages of planning this research, discussions with my supervisors drew attention to the potential benefits of integrating quantitative data to strengthen the study's overall design. This led to the decision to utilise a Constructivist Learning Environment

Survey (CLES). A modified CLES, incorporating questions that focused on virtual manipulative use and a whole-class approach to teaching maths, was specifically chosen to gain an initial quantitative baseline, and provide a general perspective on teachers' existing perceptions of learning environments. This information could then be further explored and investigated by the in-depth qualitative inquiry.

3.2.3 Mixed Methods Action Research (MMAR)

The collection of both quantitative and qualitative data within the action research phases places this research within a mixed methods action research (MMAR) framework. According to Ivankova (2015) mixed methods action research is being increasing utilised due to the many features they both share. Specifically, both approaches are rooted in a pragmatic philosophy that focus on solving problems by combining different types of evidence and iterative cycles to improve the validity of the findings (Ivankova 2015).

Lochmiller and Lester (2016) further define the pragmatic orientation as one in which "the practitioner-scholar draws upon the strengths of qualitative and quantitative research methods to wholly describe and study the research topic, phenomenon, or experience." (p. 213). The advantage of using a mixed methods approach is that it can provide a rich, more complete picture through maximising the strengths and minimising the weaknesses of a particular methodology (Lochmiller & Lester, 2017). Johnson and Turner (2003) suggest that the fundamental principle of mixed methods is to collect data through multiple strategies and methods so that it results in complementary strengths and nonoverlapping weaknesses. The use of quantitative data from the CLES and the collection of qualitative data from the action research phases is guided by this principle.

By gathering the quantitative CLES data first, this research aligns with an explanatory sequential design (Shields et al., 2024). In this design, research is completed in two phases, with the quantitative data collected first informing and guiding the collection of qualitative data. (Lochmiller & Lester, 2017). An explanatory sequential design's main advantage is that it provides researchers with both depth and breadth in a single study (Shields et al., 2024). This is achieved by using qualitative analysis to explore the underlying reasons and contextual factors behind the initial numerical trends identified in the quantitative data (Shields et al., 2024). A negative to this approach is the increase in time to conduct the two phases of data collection. Also, the researcher needs to be sensitive to potential bias with Shields et al. (2024) suggesting that researchers "may unconsciously seek to confirm quantitative findings in the qualitative data, potentially overlooking contradictory findings."(p. 73).

3.2.4 Quantitative Phase: Constructivist Learning Environment Survey (CLES)

Developed for researchers and teachers, the CLES helps evaluate how closely a classroom's environment matches constructivist ideas, and supports teachers in reflecting on their own epistemological views to improve their teaching (Fraser, 2002). The first Constructivist Learning Environment Survey was designed by Taylor and Fraser (1991) to enable teacher-researchers to monitor their development of constructivist approaches to teaching. The first versions of the CLES focused on three key themes: students' use and reflection on prior knowledge, their learning autonomy, and opportunities for peer negotiation of understanding (Taylor, 2022). Embracing ideas of critical constructivism, the instrument was modified in 1997 to include five key dimensions. This change was crucial for fostering an emancipatory ethos, enabling teachers and students to critically challenge the hidden myths of objectivism and control in schools, and thereby cultivate more equitable and enriching learning environments (Taylor et al., 1997). Initially designed for teachers to provide a means of learning about their students' perceptions of the classroom environment, these surveys have been adapted and used to gain teachers perceptions of their own classroom environment. Ebrahimi (2015) used an adapted CLES to analyse English language teachers perceptions of a constructivist approach to teaching English in Iran. Mavuru and Mbonanwe (2022) more recently used CLES to analyse how science teachers in Lebanon create constructivist learning environments in their classrooms. For this research, the instrument was modified to meet the needs of the research. A detailed description of the changes and how the CLES was administered can be found later in section 3.4.1.

3.2.5 Qualitative Phase: Action Research

Action Research (AR) is a qualitative approach to research, with a fundamental goal of bringing about change. AR differs from traditional definitions of research by focusing on conducting research *with* rather than *for* individuals and communities (Cohen et al, 2007). The origins of AR can be traced back to the 1940s and the work of German-American social psychologist Kurt Lewin (Adelman, 1993; Stark, 2014). Lewin's work was exemplified in active participation by those involved in a problem's identification, investigation, decision-making, and monitoring of consequences (Adelman, 1993). AR aligns with the ideas of

Dewey (1938), who in the early 20th century, proposed a more democratic approach to research and the solving of problems through social inquiry (Edwards & Burns, 2016; Stark, 2014). Wadsworth (2020) suggests that all research could in fact be described as action research but emphasises that the 'action' in AR that comes about from the consequences of observation and reflection, when explicitly recognised, defines AR. At the heart of all AR is a cycle that repeats, utilising an inquiry spiral of action, reflection, learning, and new informed action (Piggot-Irvine & Bartlett, 2008). It is iterative, with each cycle building on the previous one, allowing for refinement and adaptation of interventions. A key feature of AR is the active involvement of those who are experiencing or directly involved in the identified problem or issue. Rather than study the participants, the researcher works alongside them, enhancing ownership and commitment to change (Piggot-Irvine & Bartlett, 2008). Critical reflection is integral throughout the action research process, with participants and researcher constantly evaluating their actions and impact of interventions.

This form of research is well suited to an educational setting as teachers are encouraged to be reflective practitioners with a goal of improving their teaching practice. It also aligns with the view of educational theorist Lawrence Stenhouse (1975) who suggested that when a teacher's work is researched, the research should be conducted by the teachers themselves. Fordham (2016) indicates that there are two types of AR in education that differ due to the aims and audience of the research. Firstly, research that focuses on professional development and emphasises reflective practice and secondly, research that focuses on identifying good pedagogical practice that can be disseminated to a wider audience (Fordham, 2016). This study adopts the latter approach, seeking to generate knowledge that goes beyond just the development of the participants to inform broader educational practice.

Participatory Action Research (PAR) builds upon the core elements of action research but explicitly involves the participants in all stages of research. This research method adopts elements of PAR during the inquiry cycle. As with AR, it combines action and reflection, theory and practice, to seek practical solutions to the issue at hand (Bradbury-Huang, 2015). The collaborative nature of this approach allows teachers to actively engage in the research process and thus empowering them to take greater ownership of their learning. In this research, the objective of the research had already been defined, but participants were involved in decisions that affect the direction and outcomes of the research.

The PAR approach is underpinned by four fundamental principles that empower individuals and foster meaningful change. Firstly, PAR supports the value of expertise gained through experience (Cornish et al., 2023). For example, in education, it's crucial that teachers are not just consulted but are owners and participants in any project designed to modify their professional practice. Secondly, PAR embraces the concept of ‘knowledge in action,’ where active learning from experiences and the following implementation of changes directly lead to the creation of new knowledge (Bradbury-Huang, 2015). Thirdly, PAR recognises research itself as a transformative process. This means the journey of inquiry is as vital as the research's eventual outcome, as it serves to empower those who are deeply involved. The final connecting element is collaboration through discourse, a principle that draws upon the varied expertise and collective wisdom of all participants through open dialogue. During the focus group stage of research, how and what was to be delivered during the professional development sessions was identified through the collaborative inquiry of the participants.

3.3 Participants and Sampling

3.3.1 Target Population

The target population for this study comprised of one large provincial intermediate school in the Bay of Plenty region of New Zealand. The school has approximately 930 students on roll comprising of a diverse range of cultural groups. Each class at the school is a single grade class (either year 7 or year 8) and mathematics is taught by the classroom teacher. There is a stable school environment with a low staff turnover rate, and teacher experience varies widely, ranging from first-year teachers to those with over 30 years in the profession.

The school took on the *Maths - No Problem! (MNP)* resource at the start of the year with teachers frequently engaged in professional learning and development initiatives. The resource is grounded in a mastery-based approach that advocates for whole-class teaching of mathematics. The MNP approach is extremely structured with units of work broken down into a series of well-defined lessons containing materials and questions. Students have access to a textbook and workbook, and each lesson begins with an engagement task or question. As a whole-class, students are guided through the problem by the teacher who models possible solutions. This is followed by a second, teacher led, section where students work with a partner applying the strategies modelled by the teacher to a series of questions. The third and final stage sees students work independently in the workbook while the teacher roams the room checking for understanding.

3.3.2 Initial Survey Participants (CLES Survey Group)

The initial quantitative phase of this study involved a group of 36 classroom teachers from the school. Participants were recruited using a convenience sampling approach. This approach was chosen for its practicality and accessibility, allowing for an efficient collection of data. Golzar et al. (2022) identify that convenience sampling is frequently utilised in education due to researchers having available access to existing target populations. An invitation to complete the CLES was provided via an arranged meeting between the teaching staff and researcher at the school. While convivence sampling may limit the generalisability of the findings to the broader teaching population, it provides foundational insight into initial teacher perceptions regarding the use of virtual manipulatives and a whole-class mathematics approach.

3.2.3 Action Research Working Group Participants

Following the administration of the CLES survey, an invitation was extended to all participating teachers to express their interest in joining the smaller action research group. Teachers were made aware of the commitments involved, including participation in focus groups and professional development sessions. From the pool of interested volunteers, a group of 7 teachers were invited to the research, to explore more deeply their practice and perceptions. A brief description of each participant can be found in table 1 (Pseudonyms have been used to maintain anonymity).

While the selection process was primarily guided by voluntary participation, the school leadership, in consultation with the researcher, invited a pre-existing group of teachers who were already regularly meeting to discuss the implementation of the *Maths No Problem!* resource. Winlow et al. (2013), suggest that "In an educational context, a purposeful sample, or natural group, may be of greater value than a random sample." (p.295). The group consisted of three female teachers and four male teachers. Teaching experience was varied with one teacher currently in their second year of teaching and two of teachers having over 15 years of experience. For this study, the already established collaborative dynamic was considered advantageous for developing a stable and productive action research environment. All seven teachers within this group willingly consented to participate in this study.

Table 1

Participant descriptions

Name	Teaching Experience	Teacher description
Nicole	15+ years	Nicole has worked at the school for 13 years. She has taught both year 7 and 8. She currently leads a remedial maths group and has some prior experience of using virtual manipulatives.
Neil	10 years	Neil has worked at the school for 8 years. He currently teaches year 7 and is a team leader. Neil has an interest in integrating ICT into his program.
Steve	15+ years	Steve is an experienced teacher who has worked at the school for 6 years. Previously, Steve taught at a middle primary level. He has a keen interest in mathematics but admits that he has limited ICT knowledge.
Mike	9 years	Mike has worked at the school for 2 years. Mike has taught at a range of year levels both in New Zealand and overseas in China. Mike is always willing to try new programs and teaching approaches.
Pippa	11 years	Pippa has worked at the school for most of her teaching career. She currently holds a within school maths leadership role and has minimal experience using virtual manipulatives.
Karen	10 years	Karen has worked at the school for 7 years. She currently teaches year 8 in a job share arrangement with another teacher.
Brian	2 years	Brian is a recently qualified teacher, currently completing his third year of teaching. Brian is enthusiastic and motivated to develop his teaching.

Note: All names are pseudonyms to ensure participant confidentiality.

3.4 Data Collection Instruments and Procedures

3.4.1 Quantitative Data Collection: Constructivist Learning Environment Survey (CLES)

3.4.1.1 Instrument Description CLES-VM

To provide valid and reliable data for assessing mathematics classroom environments from a constructivist perspective, a modified CLES was used that addressed the focus of the research. The CLES-VM adapted two sections to address teachers' perceptions of their maths classroom and the use of virtual manipulatives. The survey in this research consisted of 25 items split into five subscales. Participants responded to each item on a five-point Likert scale (1 = Almost Never, 5 = Almost always). On the front page of each of the forms there were details of the purpose of the questionnaire and directions on how to answer each question. Table 2 provides a description of each sub scale (Learning about the maths classroom, learning about the use of manipulatives, learning to speak out, learning to learn, learning to communicate). A sample of one item from each of the scales is included. Language within each sub section focused on a whole-class approach to teaching mathematics when applicable.

3.4.1.2 Administration Procedures

A pre-arranged meeting at the school site between the researcher and classroom teaching staff was used to administer the CLES-VM. At this meeting, the researcher provided a brief overview of the research aim and answered any questions. It was made clear that participation was voluntary. Teachers who expressed interest received a consent form and a paper copy of the CLES-VM.

Table 2

Description for each scale of the CLES-VM and one sample item

Scale	Description	Sample Item
Learning about the maths classroom	The extent of teachers' perceptions of mathematics teaching with reference to a whole-class approach, the new curriculum, and ensuring students experience every statement for their year level.	I see the whole-class approach as a meaningful way to support students' mathematical understanding.
Learning about the use of manipulatives	The extent of manipulatives and virtual manipulatives use, with items that address teachers use of virtual manipulatives as well as items that address student access and engagement with virtual manipulatives.	I have experimented using virtual manipulatives for instruction.
Learning to speak out	The extent to which teachers feel it is appropriate and beneficial for students to question the teachers' pedagogical plans and methods, and to express concerns about any barriers to their learning.	It's OK for them to question the way they are being taught.
Learning to learn	The extent in which teachers allow students to share their ideas regarding the design and management of the learning activities, assessment criteria and social norms of the classroom.	I reflect on how well whole-class activities are meeting the needs of all students.
Learning to communicate	The extent to which teachers provide opportunities for students to explain and justify their ideas.	Students talk with other students about how to solve problems.

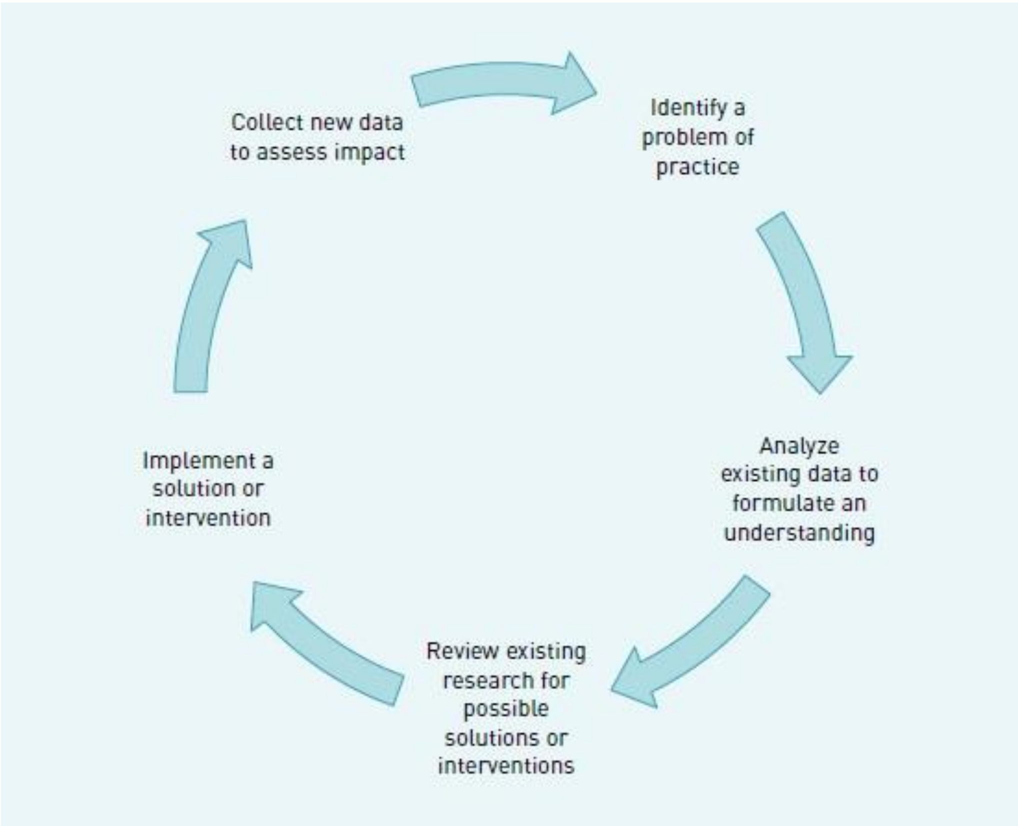
3.4.2 Qualitative Data Collection: Action Research Cycle

3.4.2.1 Action Research Framework

Action research utilises a multi stage, inquiry spiral that can be repeated to systematically investigate and effectively design solutions to an identified issue (Lochmiller & Lester, 2017). The initial framework for the action research component followed the cycle shown in Figure 6.

Figure 6

Action research cycle

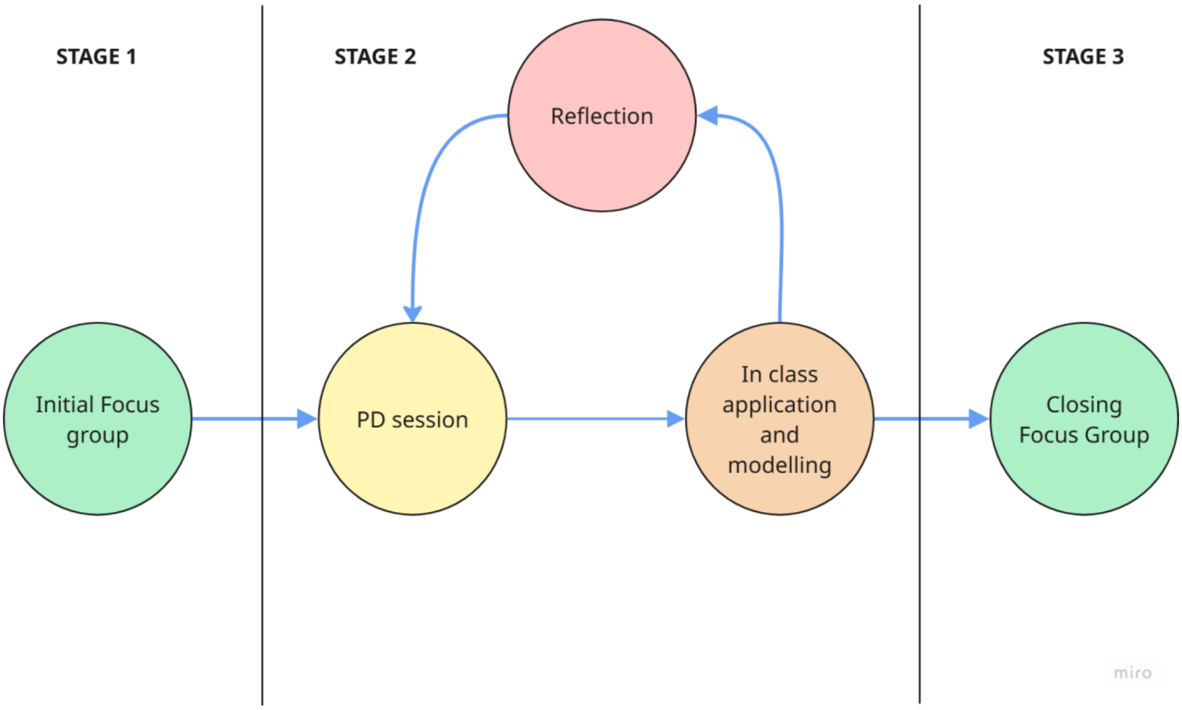


Note. From "Introducing Action Research," by Lochmiller & Lester, 2017, in An Introduction to Educational Research: Connecting Methods to Practice, SAGE Publications.

The CLES-VM helped to identify the problem of practice alongside the literature review on the topic. The first action of the cycle was to implement a focus group to gain initial thoughts and experience of virtual manipulative use. The next stage of the cycle was administering of professional development. The content and style of delivery was directly informed by the research group's insights. A combination of interactive instruction and active design, supported by modelling and team-teaching sessions was selected. After each session with the group, the participants used virtual manipulatives in their classrooms, either independently or alongside the researcher. At the start of the next group session, a mini reflective discussion was initiated. At the end of the professional development period, a second focus group was scheduled. The diagram in Figure 7 illustrates the simplified three stage model for this research. Stage one consisted of a 1-hour focus group with the participants. Stage 2 involved a cycle of professional development sessions, in class application and modelling and reflection. Over a five-week period three cycles of stage 2 were conducted. The final stage involved a 1 hour closing focus group.

Figure 7

Simplified 3 stage model of action research for this study



3.4.2.2 Focus Groups

Focus groups are defined by (Katz-Buonincontro, 2022) as "socially constructed speech acts that give researchers valuable insight into perspectives and interpersonal interactions in a group setting" (p.50). Focus groups allow the researcher to utilise three fundamental strengths of qualitative research: exploration and discovery, context and depth, and interpretation

(Morgan, 1997). Focus groups provide an environment that allows the researcher and participants to explore topics and as people share their thoughts, new perspectives come to light that might otherwise have been hidden in a one-on-one interview (Katz-Buonincontro, 2022). They also allow the research to get an understanding of the background behind participants thoughts and feelings. As a group, participants are encouraged to see similarities and differences in their thoughts and experiences while questioning and supporting their ideas. Morgan (1997) observes that focus group participants want to understand each other and their discussions support interpretation and understanding of the why and how things have occurred.

3.4.2.3 Design and Purpose

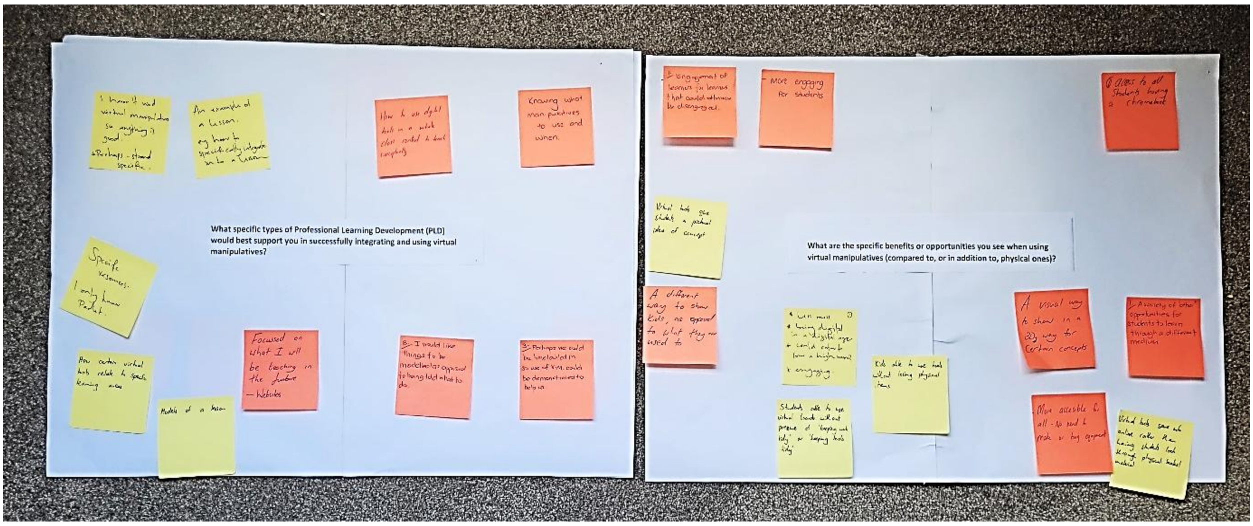
The first focus group was conducted towards the end of the second school term providing time for the data to be transcribed and analysed. To structure the focus group sessions and support the generation of ideas, a modified Nominal Group Technique (NGT) was employed. NGT is a commonly used consensus development method that uses pre-determined questions to elicit responses from every group member (Harvey & Holmes, 2012). Through the use of a pre-determined set of questions that are discussed in order, NGT can provide more reliable information and mitigate the influence of social dynamics (Bhandari & Hallowell, 2021). A key element of this technique, which was adopted for this study, is the silent generation of ideas. This approach ensures all participants have an equal opportunity to contribute, provides dedicated time for thoughtful reflection on each question, and encourages engagement (Bhandari & Hallowell, 2021).

For this study, participants responded to a series of initial questions and prompts by recording their thoughts on post-it notes. Participants were allocated 5 minutes to record their ideas for each question or prompt. These post-it notes were then displayed on a shared surface and referred to by both participants and the researcher during the subsequent discussion. As the researcher, this was valuable, as I was able to group similar ideas, identify key themes, and direct follow-up questions that promoted deeper discussion (see Figure 8). Additionally, the physical evidence of the post-it notes provided a valuable visual record that could be cross-referenced with the transcribed audio of the focus group sessions, enhancing the rigor of data analysis.

As part of an action research design, the planning and content of the professional development emerged from the focus group discussion. Prior to the start of term 3, the professional development was planned and created to align with the ideas and requests of the group. The second and closing focus group was conducted in week six of term three and followed a similar structure to the first focus group. This provided enough time for teachers to implement the professional development into their classrooms and reflect on their experiences.

Figure 8.

Photo showing participants responses using post-it notes



3.4.2.4 Professional Development Sessions

Following the initial focus group, professional development sessions were designed to directly address the identified needs of the participants. The group specifically highlighted fractions as the mathematical area they would be teaching during the first half of term three. Polypad (https://polypad.amplify.com) was selected as the virtual manipulative (VM) to support the teaching of fractions due to its versatility and the participants' prior experience with the tool.

To align with Koehler and Mishra's (2009) work on TPACK (Technological Pedagogical Content Knowledge), a learning by design model was adopted to promote deep understanding. This approach began with an initial sharing of potential uses for the Polypad tool. Participants then identified upcoming lessons where VMs could support their teaching. Subsequently, they collaboratively created a resource for their classrooms that utilised the VM and supported the fractions concept being taught. This approach to professional TPACK development directly addresses the needs identified by Koehler et al (2013) who state that it should "centre on teachers' experiences with defining, designing, and refining educational artefacts to solve particular learning challenge" (p. 18). It integrates their pedagogical knowledge, content knowledge, and technological knowledge simultaneously while developing a resource that has an active purpose. An interesting development during the research was the utilisation of AI. Two of the teachers identified the need for specific tools that were not available, and this led them to investigate the use of AI to code custom-built virtual manipulatives.

3.4.3 Role of Researcher

The researcher served a dual role as both the lead researcher and the provider of the professional development. During the focus group discussions, my role was to facilitate the guiding of conversations to explore the research questions. I created the discussion guide and posed the initial questions. Participants were more comfortable sharing candid feedback on their classroom practices and challenges because my established role as a colleague had already fostered a significant level of trust and rapport. This allowed for a more open and genuine exchange, as they viewed me as a peer engaged in a shared process rather than an outside researcher. Possessing an insider status can help to overcome potential barriers of formality and encouraged more authentic discussions (Cornish et al., 2023). However, I was also mindful of my insider role potentially leading to shared assumptions or a reluctance to challenge me as a facilitator.

Following the focus group, my role shifted from facilitating discussion to that of an interventionist. I designed and led a series of professional development sessions, based on the themes that emerged from the focus groups and integrating the learning by design model. This was followed by modelled and co-constructed lessons with the teachers who requested them. My role as a peer was crucial during this phase, as I was able to demonstrate the strategies within a familiar context, using the same resources and dealing with the same students as the teachers. This practical, in-class approach was viewed by the participants as more relevant and valuable than traditional professional development sessions. I continually sought feedback from the teachers on the effectiveness of the strategies and adapted the modelled lessons in response to their input and questions.

In summary, my role as a researcher and peer of the research group was an important component of this study, fostering trust and providing contextual understanding of the data.

This insider perspective was a key strength, providing a foundation for a collaborative and responsive process that aligns with action research.

3.5 Data Analysis

Due to the mixed methods approach quantitative and qualitative data analysis methods were required. After administration of the CLES -VM, the forms were collated and responses compiled into a spreadsheet. Following the focus group sessions, audio files were transcribed in preparation for thematic analysis. The following section will describe the data analysis procedure in more detail.

3.5.1 Quantitative Data Analysis (CLES-VM Data)

The process of quantitative data analysis began with the raw CLES -VM data being transcribed and organised into a spreadsheet using Microsoft Excel. The dataset was systematically checked for transcription errors and missing values. Following this, descriptive statistics, including averages (means) and standard deviations, were created to summarise the CLES-VM data. Average scores were calculated for each individual question, and then a mean score was computed for each subscale by averaging the scores of the questions within that subscale. Averages and standard deviations were selected as the key statistical measures to provide a clear and concise overview of the data and to provide a foundation for the more detailed findings from the thematic analysis without introducing complex inferential statistics. Standard deviation was calculated using the STDEV.S function in Excel, as the participants in this study represent a sample of a larger teacher population. The results of this analysis are presented in a series of tables and graphs, with all mean and standard deviation values rounded to one decimal place for clarity. Each set of subscale results were analysed for patterns that could be further investigated during the subsequent qualitative data analysis.

3.5.2 Qualitative Data Analysis (Focus Groups, Professional Development Sessions, Field Notes)

3.5.2.1 Analytical Approach

The qualitative data collected from the focus groups and professional development sessions' notes were analysed using reflexive thematic analysis (RTA), as outlined by Braun and

Clarke (2006, 2021). Thematic analysis (TA) involves the exploring of data, following a series of stages, to identify repeated patterns of meaning (Braun & Clarke, 2006). In reflective thematic analysis, the researcher's subjectivity is viewed as a key resource for analysis, with reflexivity applied throughout their engagement with theory, data, and meaning-making (Braun & Clarke, 2021). A six-stage process for data engagement, coding and theme development was initially identified by Braun and Clarke (2006) and can be seen in Table 2.

Table 3.

Six-stage process for data engagement

Phase	Name	Description
1	Familiarise with data	Transcribe; re-read the data, note down initial idea
2	Generate Initial code	Code Interesting feature of the data
3	Search for themes	Collate code into potential themes, gather all data relevant to each potential theme
4	Review themes	Check if the themes work, in relation to the coded extract and the entire data set, generate a thematic 'map' of the analysis
5	Define and name theme	On- going analysis to refine the specifics of each theme, and the overall story the analysis tells, generate clear definitions and names for each theme
6	Produce the report	Select vivid, compelling extract examples, analysis of selected extracts, relating back to the research question and literature, producing a report of the analysis

Note. Adapted from "Using Thematic Analysis in Psychology," by Braun and Clarke, 2006, In Qualitative Research in Psychology, 3(2), p. 87.

More recently, Braun and Clarke (2021) updated their articulation of their guidelines, and the following were used to guide the analysis of data in this research.

"1) Data familiarisation and writing familiarisation notes; 2) Systematic data coding; 3) Generating initial themes from coded and collated data; 4) Developing and reviewing themes; 5) Refining, defining and naming themes; and 6) Writing the report."

(Braun & Clarke, 2021, p.331)

It is important to note that although the six phases are organised into a logical sequential order, the analysis is not necessarily a process of moving through the phases. (Byrne, 2021). Analysis is often recursive and iterative with the researcher moving forwards and backwards through the stages as required. Braun and Clarke (2021) also emphasis that simply following procedures doesn't guarantee good thematic analysis. It is important to understand what these procedures enable, what insights they offer, and to see them as tools in the process rather than the end goal. During both data generation and analysis, I was conscious of my role as a researcher in the co-creation of meaning. To support my thematic journey, I kept a reflexive journal for tracking my thoughts, choices, and challenges throughout (Braun & Clarke, 2021). The journal was a useful tool to support my research during data analysis as writing down my thoughts and observations helped me to gain clarity and ensured passing ideas were not lost.

3.5.2.2 Process

Phase 1 – Data familiarisation and writing familiarisation notes

The first phase focused on becoming familiar with the data by listening to the focus group recordings once before transcribing. In order to actively listen during the first playback of each recording, no notes were taken at this point. Byrne (2021) suggests that taking an 'active-listen' approach aids the development of understanding of the primary areas addressed in each recording. After this initial active listen, each recording was transcribed initially using the Microsoft Word transcription tool. When transcription of a recording was complete, transcripts were reread and checked several times. During this time, casual notes of initial trends in the data and potentially interesting exchanges in the transcripts were recorded.

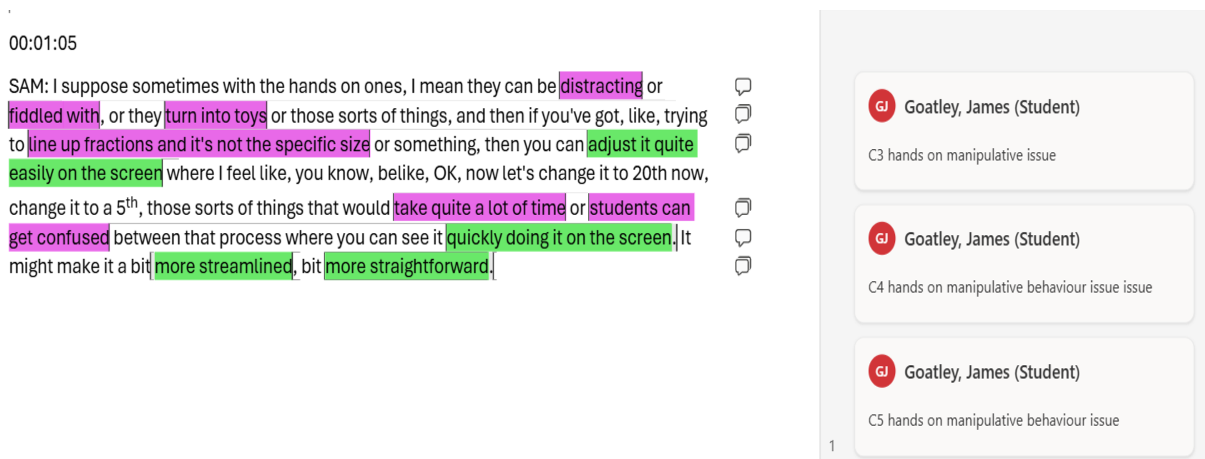
Phase 2 – Systematic data coding

Codes are the foundational bricks of the data analysis, and are integral to theme development (Braun & Clarke, 2021). Byrne (2021) further explains the coding process, stating, "coding is undertaken to produce succinct, shorthand descriptive or interpretive labels for pieces of information that may be of relevance to the research question(s)" (p. 1399). For this research, semantic and latent coding were both utilised. Semantic coding involves a purely descriptive analysis of the data, recording and presenting exactly what the participant has said. Latent coding, on the other hand, goes further, attempting to identify hidden meanings and assumptions that may help explain the semantic data (Byrne, 2021). This interpretive approach requires the researcher to 'read between the lines' and relies heavily on their judgment.

The coding process for this research used the comment feature in Microsoft Word documents. Relevant pieces of information were first highlighted, and then codes pertaining to that information were recorded in the comment section (see Figure 9). To support the subsequent stage of identifying initial themes, related or connected pieces of information were highlighted in different colours (Kushnir, 2025). Following Byrne's (2021) suggestion, the information and codes were later transferred to a spreadsheet so that any subsequent iterations of coding could be recorded.

Figure 9

Example of systematic coding during this study



Phase 3 - Generating initial themes from coded and collated data

The next stage involved interpretation and aggregation of meaning in relation to the data identified in the coding process. Links between codes were identified and, in some cases, combined into one single code. Highly repeated codes or codes with similar underlying concepts may represent a key element within the data and thus be upgraded or put forward as a possible sub theme (Braun & Clarke, 2012). It is important to note that themes do not emerge from the data but are identified and constructed by the researcher focusing on relevance to the research question (Braun & Clarke, 2021; Byrne, 2021). This stage saw multiple iterations as I organised the codes into different groups. It was at this point that I realised my codes were not detailed enough, and in some cases, I had recorded a theme or sub theme rather than a code that represented the data. My reflective journal described my process at this point.

> *Now that I have my codes completed, I am finding that what I have written for some of the codes is not helpful. For example, a number of my codes say 'virtual manipulative use'. I have realised that this is more of a possible theme rather than a code and doesn't tell me anything about the data it represents. I have begun the process of reviewing the codes and where needed adding more detail.*

I returned to the initial transcript and codes and added more precise information to ensure the code represented what had been spoken by the participant. Using this second iteration of coding I was able to compile a table and began the process of generating initial themes. Table 3 shows the process of how a group of codes relating to modelling with VMs were first redefined before being synthesised into possible themes. Alongside this process I took the coded data and formed an initial thematic map using Miro software (see Figure 10) so that connections could be easily viewed.

Table 4

Table showing the review and development of codes

Text extract	Initial code	Code with more detail	Synthesised code
It would definitely be easier to have that on the TV for them to just watch you move things around rather than, yeah, all crowding around something.	VM Modelling	VMs easier to model or explain to the class	Benefits of VMs for Whole-Class Instruction
It's not very easy to draw either. I struggle with drawing, trying to draw something up on the board.	Drawing models difficult	VMs don't require you to draw objects to model to class	VMs as a Superior Alternative to Traditional Modelling
That was great for those teachers because they were trying to pick up a place value chart with the discs on it.	VM Modelling	using concrete materials awkward to model to class	Benefits of VMs for Whole-Class Instruction
I've had it up on the TV and I feel to model it worked really well to help the understanding and also with engaging in the concept as well.	VM Modelling	benefit of conceptual modelling to the whole-class.	VMs as a Superior Alternative to Traditional Modelling
When modelling, it's not as good when you just colour it in or rub it out. Using the VMs, students could actually see it happen, and think, 'ohh that comes from there', I found it really useful.	VM Modelling	Benefits of being able to manipulate a diagram when modelling	VMs as a Superior Alternative to Traditional Modelling

Figure 10

Initial thematic map of coded data

Phase 4: Developing and reviewing themes

This stage required a recursive review by the researcher of the identified themes and their links to the data codes and larger data set (Byrne, 2021). (Braun & Clarke, 2012) proposed a series of questions that researchers need to address when revisiting potential themes. Key questions to ask are:

- Is this a theme (it could be just a code)? And if it is,
- What is the quality of this theme (does it tell me something useful about the dataset, and my research question)?
- What are the boundaries of this theme (what does it include and exclude)?
- Are there enough (meaningful) data to support this theme (is the theme 'thin' or 'thick')?
- Are the data too diverse and wide-ranging (does the theme lack coherence)?

(Braun & Clarke, 2012, p. 65)

By following these questions, it was possible to refine the identified themes and review the sub themes and supporting data. At this point I became aware that much of my analysis had focused on what had been discussed but not necessarily on how perceptions had changed. This led me to return to the coded data and focus on shifts in the participants perceptions and themes linked to them. My reflective journal captured the clarity I gained from this change of focus.

> *For some time, I have felt like I was making progress, but the more I look at the themes, sub themes and codes, the more lost I have become. I realise that I have been coming at this from the wrong direction and not focusing on how the participants perceptions have changed. I have been too caught up in looking for general themes and connections.*

This iterative process is common during analysis, Byrne (2021) suggests that, 'It may be necessary to recode some data items, collapse some codes into one, remove some codes, or promote some codes as sub-themes or themes.' (p.1407). Although it felt like I had taken a backward step, it was clear that the new direction aligned more closely with my research aim.

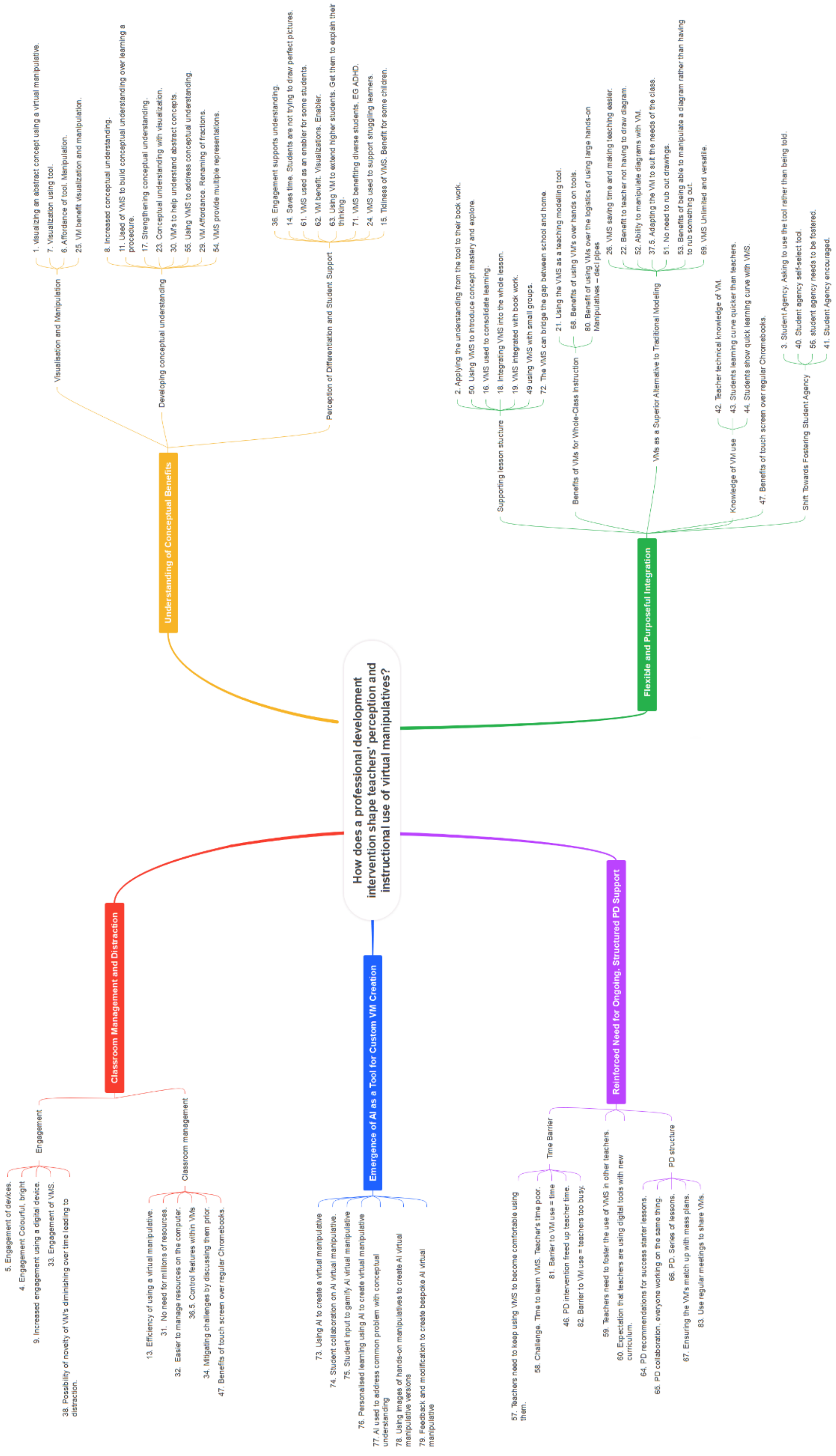
Phase 5: Refining, defining and naming themes

Phase 5 begins when the researcher has a satisfactory thematic map of their data (Braun & Clarke, 2021). I created a revised thematic map which I identified five major themes (see Figure 11). I was confident that I had identified the 'essence' of what each theme was about and how they captured the key aspects of the data (Braun & Clarke, 2021). It is at this point that I needed to identify which data items to use as extracts to support the defining of the themes. I spent some time to discuss and articulate my themes to critical friends and my supervisors. Discussing and explaining my themes aided my clarity and helped to form an overall picture of the data.

Byrne (2021) advises that data can be presented illustratively or analytically. For my write up, following advice from my supervisors, I presented the results and discussion separately. This requires an illustrative approach, providing a surface level description of what was said in the results, followed by deeper analysis and links to literature in the discussion section (Byrne, 2021).

Figure 11

Revised Thematic Map



Phase Six: Writing the report

Braun & Clarke (2006) explain the task required at this phase is 'to tell the complicated story of your data in a way which convinces the reader of the merit and validity of your analysis.' (p.93). As I drafted my results chapter, I found I was gaining greater clarity with each theme. The reflexive thematic process remained iterative, as I returned to data, codes, and mind maps. During the writing phase, Byrne (2021) suggests it is beneficial to determine the order in which themes are reported, highlighting 'Themes should connect in a logical and meaningful manner, building a cogent narrative of the data.' (p.1410.). As part of a mixed methods approach, linking between the quantitative and qualitative results would become an important element during the discussion section.

3.6 Validity and Authenticity

In order to achieve validity and authenticity in the data, a number of strategies were used. A primary strategy for enhancing validity was the triangulation of data, which was achieved through a mixed-methods explanatory sequential design. This approach, which aims to provide both depth and breadth to the research, facilitated a more comprehensive and holistic understanding of the phenomenon being explored (Shields et al., 2024). The initial quantitative phase provided a broad overview, while the subsequent qualitative phase offered an opportunity to uncover the underlying reasons for the numerical trends, thereby strengthening the overall conclusions.

To build credibility and dependability in the qualitative data, the thematic analysis was conducted with care. I utilised rich, detailed descriptions and direct participant quotes to illustrate the themes, providing an authentic and accurate representation of the teachers' experiences. To maintain rigor, a detailed audit trail of all actions and discussions was kept throughout the data generation and analysis process, allowing the research to be auditable and the findings to be dependable.

The authenticity of the research was strengthened further through a commitment to ethical practice. As a researcher, I remained flexible and responsive to the timelines of the participating teachers to ensure my presence was not a burden. Furthermore, as a novice researcher, I engaged in regular debriefing with my supervisors. Their invaluable guidance and feedback were critical to challenging my assumptions, while also fostering my own reflexivity and understanding of the research process.

3.7 Ethical Considerations

Ethical approval was obtained from the University of Waikato Human Ethics Committee (FEDU012/25) before the research commenced.

3.7.1 Informed Consent

Ethical considerations were paramount throughout this research, ensuring the well-being and rights of all participants. Prior to the research beginning, informed consent was gained from the school principal (Appendix A) to conduct the research on school grounds.

The administration of the Constructivist Learning Environment Survey (CLES) occurred on school grounds. All participants were provided with a comprehensive information sheet detailing the research and a consent form to complete (Appendix B). It was explicitly communicated that participation was voluntary, and individuals could withdraw at any time without penalty. It was important that senior management did not attend the CLES administration to ensure there was no perceived coercion to participate. After completing the CLES, participants were able to indicate their interest in being part of the main research group.

Participants for the main research group were recruited using a convenience sampling approach. These participants were met on the school site, where they received a detailed explanation of what their participation would entail. Care was taken to ensure participants fully understood their rights and the implications of their involvement. Participants included in this group were provided with a second, more detailed consent form, reflecting the additional commitment required for this phase of the study (Appendix C).

3.7.2 Anonymity and Confidentiality

To ensure confidentiality, all participant data was anonymised at the point of collection. Participants were assured that their identities would remain anonymous throughout the research and pseudonyms were used in research documents to maintain anonymity. All data was treated with the strictest confidentiality, only being accessible by the researcher and advisors. Whilst the identities of the participants remain confidential, the data shared by participants were collected with the intention of being reported on within the study. Furthermore, while participants were reminded about issues relating to confidentiality, it was not possible to determine what participants might share of the study with others. Therefore, even with every effort made, anonymity couldn't be guaranteed.

3.7.3 Right to Withdraw

Participants were able to contact the researcher and withdraw from the study at any time up until data analysis began. If any participant had withdrawn, all data that had been contributed would have been destroyed and not used in the study.

3.7.4 Data Storage and Security

All research data, including interview transcripts, survey responses, and researcher notes, were securely stored on university-provided cloud storage and an encrypted external hard drive. Any physical documents were kept in a locked cabinet at my home. Although participants' identities may be protected, the data shared will not remain confidential since it is reported in the project. Participants were made aware of their own obligation to protect the privacy of other participants.

3.7.5 Researcher Positionality

My role in this study extended beyond that of a neutral observer. My positionality is influenced by my professional identity as a teacher, a colleague, and an active participant in the same educational context as the research group. This insider status, while providing unique access and understanding, required careful reflection to manage potential biases (Holmes, 2020). The following section outlines how my professional background and my desire to improve teaching outcomes have informed the research process, and the reflexive practices I employed to ensure the integrity of the findings.

My motivation to conduct this study stems from my own experience implementing virtual manipulatives in my mathematics lessons. This pragmatic approach guided the research, leading to a focus on tangible outcomes and actionable strategies for improving teaching. Rather than simply exploring the problem, the study was designed to actively seek solutions.

My positionality as a peer of the research group was a significant factor in the study's design and execution. As a colleague, not a superior or an outsider, I was able to build an environment of trust where participants felt comfortable sharing their honest opinions and experiences. To ensure an unbiased approach and avoid focusing on a particular outcome, I maintained a professional role during data collection by adhering to a clear protocol for the focus groups.

Throughout the research, I engaged in ongoing self-reflection using a reflective journal. This practice was crucial, as it provided a continuous record of my thoughts and feelings as the study transitioned through the action research model, allowing me to critically examine my own biases and assumptions.

3.8 Chapter Summary

This chapter reviewed the research rationale and design, explaining why a mixed-methods action research (MMAR) approach was the most suitable way to achieve the project's aims. The research was framed within a pragmatist approach, which justified the combined use of a constructivist learning survey with a participatory action research framework. Following a review of action research and participatory action research, the participants, instruments, and research were identified and explained. The use of focus groups was chosen as the main source of data generation. For data analysis, means and standard deviations were used for the quantitative data, while thematic analysis was applied to the qualitative data. Finally, ethical considerations were discussed and the researcher positionality identified.

Chapter 4: Results

4.1 Introduction

This chapter explores the results of the study focusing on how a professional development intervention can impact teacher perceptions and instructional use of virtual manipulatives. First, this chapter will focus on the Constructivist Learning Environment Survey (CLES) data collected from a group of 36 classroom teachers at the school. This is followed by a detailed review of the qualitative data collected from the smaller research group consisting of seven teachers. This includes the identification of five main themes, supported by extracts from the transcripts. To conclude this chapter, a synthesis of the quantitative and qualitative data is included to demonstrate how the two streams of data are connected.

4.2 Constructivist Learning Environment Survey (CLES) Findings

Before presenting the findings, it is important to explain the adapted instrument used to collect the data. The CLES-VM was designed with two unique sections to directly assess variables relevant to this study: teachers' use of virtual manipulatives, and their perception of mathematics teaching with reference to a whole-class approach. These adapted sections, along with three standard CLES subscales, form the 25-item survey. Participants provided their responses on a 5-point Likert scale (1 = Almost Never to 5 = Almost Always).

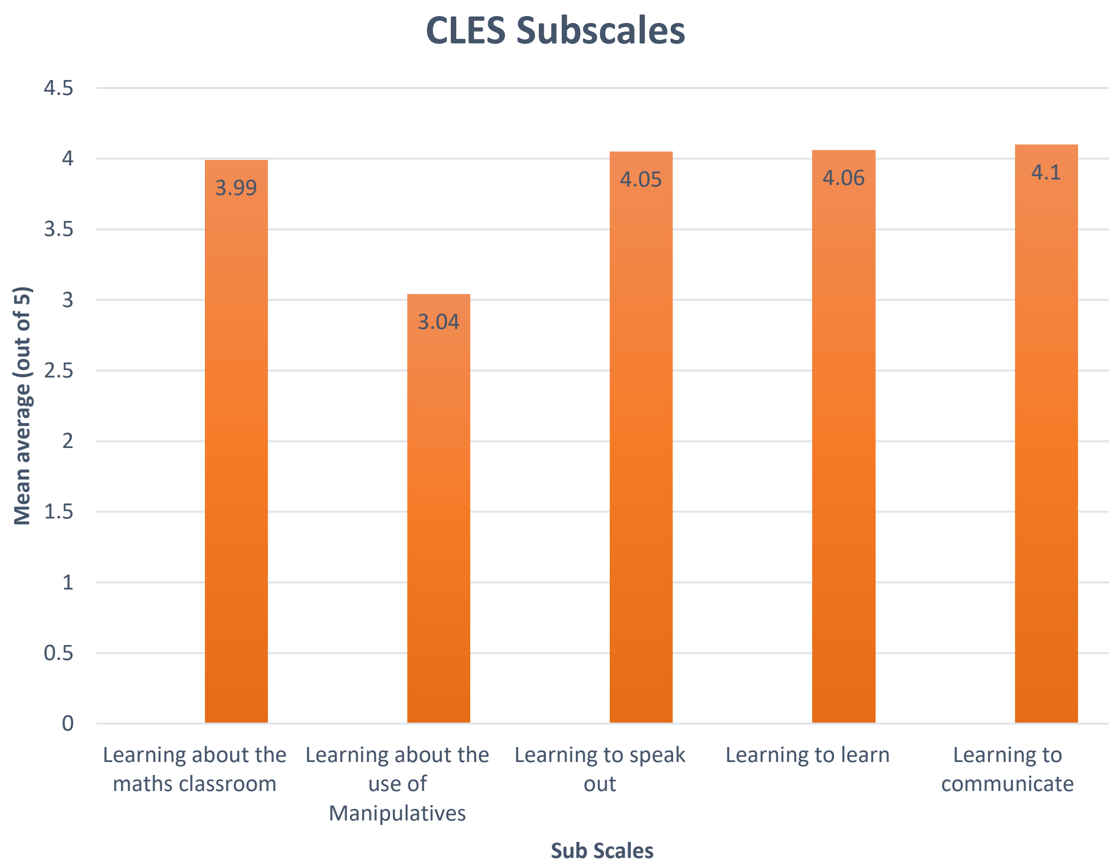
In this section, the CLES results are presented and analysed using statistical measures. The analysis focuses primarily on the mean scores and standard deviations for each question and subscale. It begins with a comparison of the five subscales, followed by a more detailed analysis of each subscale broken down with a table representation of the data. Consideration is then given to the school's recent adoption of the *Maths - No Problem* (MNP) program, with links made between this context and the results.

4.2.2 Overview of CLES Subscale Results

Overall, the findings indicate several key principles of social learning and agency are evident across the schools' mathematics classroom environments. Student collaboration, learning from mistakes, and increased student agency are all indicated as highly valued. Furthermore, the findings indicate strong teacher commitment to the whole-class pedagogical philosophies of the MNP program, while also highlighting specific challenges related to student-led control and the integration of new technologies. This is particularly relevant to the present study and is illustrated in Figure 1, where the mean score of 3.04 out of 5 for '*Learning about the use of manipulatives'* is noticeably lower than the other four subscales. Within this subscale, the questions relating to virtual manipulatives produced the lowest scores of the entire CLES. These results provide evidence of the need for further investigation into how professional development interventions can enhance teacher competence in the use of virtual manipulatives.

Figure 12

Graph showing comparison of the five CLES Subscales



4.2.3 CLES Subscales

4.2.3.1 Subscale 1: Learning About the Maths Classroom

The overall teacher response mean average of 3.99 on the 'learning about the maths classroom' subscale suggests that teachers have a generally strong perception of their classroom practices and beliefs around the new whole-class mathematics approach. A key finding is the high level of agreement among teachers regarding the creation of an environment where students can learn from mistakes (Q3 – 4.42). From a constructivist perspective, errors or mistakes are not viewed as failures but as opportunities to address understanding in the learning process. Specifically, a mistake is considered valuable feedback that reveals a learner's current level of conceptual understanding or highlights a misconception. This allows the student to refine and restructure their existing schema, leading to deeper knowledge construction. This result, therefore, indicates that teachers value fostering a supportive learning environment and actively leveraging the benefits of a growth mindset, through seeing errors as important steps towards mastery.

Another finding can be identified from the score of 3.94 for Question 4, 'Ensuring all students experience every statement at their year level is/will be challenging'. This result indicates some agreement that the directive from the Ministry of Education requiring all students to experience every curriculum statement for their year level, presents a practical challenge for teachers in the classroom.

The lowest individual score for this subscale was for the question, 'The new curriculum aligns with how I believe maths should be taught,' with a score of 3.83 (Q2). Whilst still a high score, this may suggest that the new curriculum does not align with the personal teaching philosophies of all teachers. The relatively low standard deviations for all questions (SD 0.5 – 0.7) are also relevant, suggesting a strong consensus among the teachers.

Table 5

CLES Subscale 1: Learning about the maths classroom

	CLES Question	Average score (1-5)	Standard Deviation (SD)
Q1	I see the whole-class approach as a meaningful way to support students' mathematical understanding.	3.89	0.6
Q2	The new curriculum aligns with how I believe maths should be taught	3.83	0.7
Q3	I create an environment where students can learn from mistakes and see them as part of the learning process.	4.42	0.6
Q4	Ensuring all students experience every statement at their year level is/will be challenging.	3.94	0.6
Q5	Students are engaged when using a whole-class approach	3.86	0.5
	Overall Subscale Average	3.99	

4.2.3.2 Subscale 2: Learning About the Use of Manipulatives

Teachers' responses on the 'Learning about the use of manipulatives' recorded the lowest mean subscale score of the entire survey. The score of 3.04 was almost a whole point lower than any of the other subscales indicating a lack of knowledge of manipulative use.

Teachers' responses within the subscale showed a marked difference between their views on traditional manipulatives and their engagement with virtual ones. The highest mean score of the subscale of 3.92 for Question 7, indicates teachers have a strong belief in the pedagogical value of manipulatives. This is an important element of a constructivist approach, which maintains that students actively build or 'construct' knowledge by interacting with their environment, rather than passively receiving information. The adoption of the MNP resource and its emphasis on manipulative use likely influenced this positive score. However, understanding of how to integrate them was slightly lower (Q6 – 3.50) suggesting that teachers were still learning how to use them effectively.

In noticeable contrast, all scores related to virtual manipulatives were markedly lower, suggesting a lack of experience and integration. The lowest mean score of 2.25 for student access to virtual manipulatives, paired with a score of 2.56 for teachers' own experimentation, highlights a clear gap in classroom practice. The high standard deviations for the virtual manipulative items (1.1 and 1.2) further suggest a wide variation in the limited experiences teachers have had, indicating that virtual manipulatives are not being consistently or widely used in the classroom.

Table 6

CLES Subscale 2: Learning about the use of manipulatives

	CLES Question	Average score (1-5)	Standard Deviation (SD)
Q6	I have a good understanding of integrating manipulatives	3.50	0.9
Q7	The use of manipulatives enhances my ability to teach mathematical concepts	3.92	0.7
Q8	Students have access to virtual manipulatives to support understanding	2.25	1.1
Q9	I have experimented using virtual manipulatives for instruction.	2.56	1.1
Q10	Students are engaged when using virtual manipulatives	2.97	1.2
	Overall Subscale Average	3.04	

4.2.3.3 Subscale 3: Learning to Speak out

Responses to the third subscale, "Learning to speak out," suggests a tendency among most teachers to agree that students need to have a voice in their learning. This belief is a key component of student agency, which is the capacity for students to have active participation in decisions regarding how and what they learn. The focus on agency is a valuable element of a student-centred pedagogical approach. This is reflected in the high scores for questions concerning a student's right to ask about or question the relevance of what they are learning (Q11: 4.08, Q12: 4.06). The highest score on this subscale, 4.42 for "It’s OK for them to express their opinions," shows that teachers highly value student input and actively welcome their ideas as a part of classroom dynamics.

The lowest score for this subscale was for Question 13, "It’s OK for them to complain about activities that are confusing," with a score of 3.67. This lower score likely reflects the negative connotation of the word "complain." This was highlighted by one teacher who wrote on the form, "Interesting word choice," suggesting that teachers may perceive a difference between students "complaining" about an activity versus "expressing concern" or "questioning the teacher" about it. In contrast, the more positive score of 4.00 for Question 14, ‘It’s OK for them to complain about anything that prevents them from learning,’ may be due to teachers perceiving these issues as often stemming from external factors, such as peer behaviour, rather than from their own direct actions.

Table 7

CLES Subscale 3: Learning to Speak Out

	CLES Question	Average score (1-5)	Standard Deviation (SD)
Q11	It's OK for students to ask me "why do I have to learn this?"	4.08	0.7
Q12	It's OK for them to question the way they are being taught.	4.06	0.8
Q13	It's OK for them to complain about activities that are confusing.	3.67	0.8
Q14	It's OK for them to complain about anything that prevents them from learning.	4.00	1.0
Q15	It's OK for them to express their opinions.	4.42	0.7
	Overall Subscale Average	4.05	

4.2.3.4 Subscale 4: Learning to Learn

The overall subscale average of 4.06 suggests a strong belief among the teachers in the importance of adapting their instruction to meet student needs. However, a closer look reveals a clear distinction between teacher-led and student-led aspects of the learning process.

The three questions related to teachers’ responses to whole-class teaching (Q16 – 4.61, Q17 – 4.53, Q18 – 4.42) scored particularly high, reflecting a strong commitment to the new mathematics program. These results indicate that, while following the highly prescriptive MNP framework, teachers still feel they have the agency and professional judgement to adjust lesson pace and delivery to meet students’ needs. These decisions are directly made by the teacher in response to formative assessment and the engagement of students during the lesson.

In contrast, the questions related to student voice and input (Q19 – 3.78, Q20 – 2.97) scored noticeably lower. The more directive nature of the MNP framework and the Ministry of Education's emphasis on explicit teaching and structured learning could explain these lower scores. The results also suggest that although supportive of students having agency (See previous subscale), there is a reluctance to fully embrace student-led learning. The higher standard deviations for the student decision questions (Q19 – 1.1, Q20 – 1.0) suggest that, although most teachers are adhering to the MNP program, there is some variation in practice, with certain teachers allowing more student choice in activities.

Table 8

CLES Subscale 4: Learning to learn

	CLES Question	Average score (1-5)	Standard Deviation (SD)
Q16	I am open to adjusting my approach based on student engagement with whole-class activities.	4.61	0.6
Q17	Students' learning progress influences how I pace and sequence whole-class lessons.	4.53	0.6
Q18	I reflect on how well whole-class activities are meeting the needs of all students.	4.42	0.5
Q19	Students help me to decide how much time they spend on activities.	3.78	1.1
Q20	Students help me to decide which activities they do.	2.97	1.0
	Overall Subscale Average	4.06	

4.2.3.5 Subscale 5: Learning to Communicate

Responses to the fifth and final subscale, learning to communicate, recorded a mean average of 4.10, indicating the high value teachers place on student collaboration. The questions relating to direct teacher actions regarding communication scored highest (Q21 – 4.64, Q22 – 4.44, Q23 – 4.28), likely reflecting the structured approach provided by the MNP framework. In this framework, students are encouraged to work with a partner during the explore and guided practice stages, to co-construct understanding through dialogue. These are intentional, planned opportunities for communication.

In noticeable contrast, the lower scores for 'students ask other students to explain their ideas' (Q24 – 3.56) and 'other students listen carefully to classmates' ideas' (Q25 – 3.58) reflect more spontaneous and peer-led communication. The lower scores may indicate that teachers perceive this level of independent, peer-led communication as occurring less frequently or being more difficult to cultivate. This observation aligns with the previous subscale results, where teachers reported a much stronger sense of control over their own teaching actions than over student-led decisions.

Table 9

CLES Subscale 5: Learning to Communicate

	CLES Subscale: Learning to communicate	Average score (1-5)	Standard Deviation (SD)
Q21	Students get the chance to talk to other students.	4.64	0.49
Q22	Students talk with other students about how to solve problems.	4.44	0.61
Q23	Students explain their ideas to other students.	4.28	0.66
Q24	Students ask other students to explain their ideas.	3.56	0.88
Q25	Other students listen carefully to classmates' ideas.	3.58	0.6
	Overall Subscale Average	4.10	

4.2.4 Summary

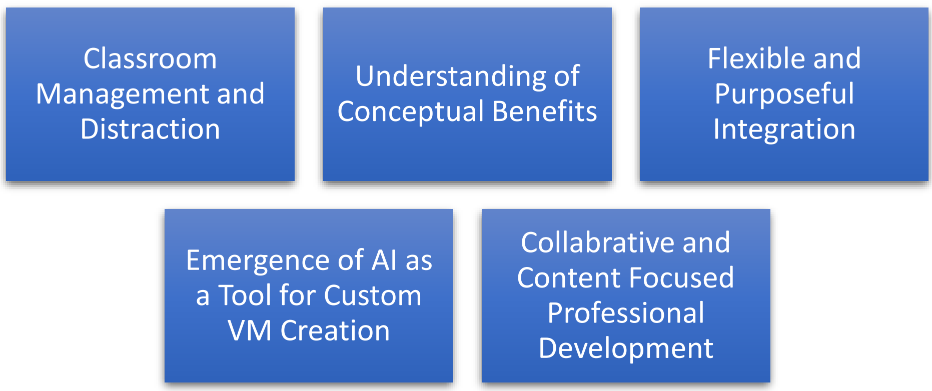
In summary, the analysed CLES data provides a snapshot of teachers' attitudes towards their mathematics learning environment from a constructivist perspective. The introduction of the MNP program has been widely accepted, with teachers demonstrating a strong commitment to the constructivist principles of knowledge being actively constructed, learning being an active process, collaborative learning, and student choice. This is evident in their positive views toward collaborative student talk, learning from mistakes, and student voice. The results also indicate a favourable perception of the shift toward a whole-class approach and the increased use of manipulatives. However, the data also highlights some tension regarding a purely constructivist approach with specific challenges, particularly in incorporating student-led decisions and use of virtual tools to support learning. The limited integration of virtual manipulatives provides the foundational evidence and rationale for the follow-up action research, focusing on a targeted intervention into virtual manipulative use and best practices within a whole-class approach.

4.3 Themes

This section presents five themes related to how the professional development intervention shaped teachers' perception and instructional use of virtual manipulatives (see Figure 12). Themes were generated through thematic analysis of focus groups and a professional development intervention. The five themes that were identified in this study represent the multifaceted and nuanced use of virtual manipulatives in mathematics. The first theme *classroom management and engagement* focus on teachers' perceptions of how VMs are implemented and organised in the classroom and the resulting influence on student participation and focus. The second theme, *benefits to conceptual understanding*, explores teachers' perceptions on the impact of VMs towards student understanding. The third theme, *flexibility and purposeful integration*, considers teachers perceptions of how and when to integrate VMs into mathematics lessons. The fourth theme, e*mergence of AI as a tool for custom VM creation*, traces teachers' perception development regarding the fast-evolving realm of AI and its potential in developing custom built tools. The fifth and final theme, c*ollaborative and content focused professional development*, addresses teachers' perceptions of effective components of professional development involving VMs.

Figure 13

The five identified themes



Each of these themes will be explored in more detail with supporting extracts from the transcripts. Throughout this section, participants' pseudonyms will be used, and participant voice is indicated with italics.

4.3.1 Classroom Management and Engagement

Prior to the professional development intervention, teachers primarily focused on the potential for increased distraction and management difficulties with virtual manipulatives. Teachers expressed concerns of students getting off task and clicking on links or other functions of the tool that were not relevant to the lesson. Neil, a teacher participant, mentioned students often ending up *'down rabbit holes'* and how they *'quickly find a way to find something else'*, leading to distraction and off task behaviour. Teachers were also concerned with the limited ability to monitor what students are doing on their screens as opposed to working in their books or with hands-on resources.

> *It's also harder to see what they're doing in that case. It's harder to see all their screens with it. When they've got (manipulatives) in front of them, you can see what they're doing or what they're manipulating, whereas when they're behind the screen you can't. (Brian – Teacher)*

The teachers acknowledged that this was not unique to the use of virtual manipulatives and needed to be managed in all lessons when using digital devices. The possibility of students getting off task was balanced by the view that virtual manipulatives would increase engagement. Another participant, Pippa, commented on the fact that her students *'just have a natural way of being into something that's digital'* and that anything new or different is often engaging.

Another possible cause of distraction that was discussed was the learning curve for students. On top of learning a new maths concept, students would be required to learn how to use the virtual manipulative effectively. Mike indicated that students might get lost if the instructions for using the tool were not immediately clear and suggested multiple lessons might be needed to teach how to use a virtual manipulative.

Understandably, comparisons were made to hands-on manipulatives that the teachers were currently using. The teachers acknowledged that these hands-on manipulatives could be equally distracting and were often fiddled with or turned into toys. The teachers also mentioned that using hands-on manipulatives can be quite messy with the number of resources being used around the classroom and the chance of pieces getting lost or misplaced. Pippa saw using virtual manipulatives mitigating these issues, allowing student to *'make a mess on the screen'* and not having to worry about containing all the resources.

Following the professional development intervention and implementation of VMs in classrooms, the teachers' perceptions appeared to shift, with many initial concerns proving to be less problematic than anticipated. For example, the teachers found that students getting distracted was considerably lower than they initially predicted. Karen observed that her students were really engaged and focused on the task, and she did not see any students distracted or mucking about. This corresponded with teachers seeing higher levels of engagement, something that they identified as a key factor in mitigating against distraction. Pippa noted that her students *'loved using it'* and that the novelty of using the virtual manipulative reinforced her idea that this supported student engagement. Karen commented on the colourful and bright nature of the virtual manipulatives was also a factor in student engagement. The teachers were also realistic and discussed whether student engagement might decrease once the uniqueness of using them waned. In addition, Brian pondered if distraction might increase with prolonged use of a term or two with students "getting distracted and mucking about" as they became more aware of the other features available on the Polypad app.

An advantage of VMs over their hands-on counterparts that the teachers observed during their time implementing them, was to do with general classroom management. The elimination of missing pieces and mess, expressed prior to the professional development intervention, was confirmed and identified as beneficial when using VMs. Karen commented on the "unlimited" nature of virtual resources and that it removed the constraint of having a finite number of physical manipulatives, which could lead to management issues if not enough materials were available for all students or groups. Steve echoed this, praising the versatility of virtual manipulatives and provided an observation that *'Sometimes you don't have enough disks for what you're doing depending on the task, while online there is no problem.'*

During the professional development sessions, teachers provided feedback regarding the Polypad tool. A number of teachers asked if it was possible to limit the number of tools and options for the students. After some combined research, an authorising mode was discovered which allowed for some options to be restricted. This was seen as a valuable feature for managing student focus and reinforced the value of the action research approach with the researcher working collaboratively with the participants.

Another interesting observation made during implementation of the VMs was the improved ease of use for touch screen users over standard Chromebook track pads. Neil noted that students could manipulate elements *'a lot faster'* when using a touchscreen potentially reducing frustration and, by extension, any possible off task behaviour.

4.3.2 Benefits to Conceptual Understanding

Teachers' perception and understanding of the conceptual benefits of VMs underwent a noticeable shift following the professional development intervention. Before the intervention, teachers foresaw VMs primarily offering practical advantages that would support conceptual understanding. Their actual experience post intervention revealed a much broader and deeper impact on student learning.

Prior to the intervention, teachers identified the pictorial idea of a concept being an important conceptual component and that VMs provide interactive images and models to support this. Mike highlighted the benefit of using a VM with fractions and the ability to *'easily adjust'* the image and *'show parts of a whole'*. Steve was aware that VMs provided the opportunity to show fractions in different ways at the same time. The ability to show multiple representations simultaneously and make comparisons in the same digital space was deemed advantageous. The teachers also identified the ability to use VMs to model to the whole-class and support conceptual understanding. Teachers mentioned the difficulties they have had attempting to model concepts to a class with physical manipulatives.

> *I've had the lid of the fractions board like this (holds up in front of her) and I'm trying to move things around to show the kids. It would definitely be easier to have that on the TV for them to watch you move things around rather than all crowding around something.* (Pippa – Teacher)

Following the professional development intervention, the conceptual benefits were tangibly experienced and more deeply understood. The topic focus at the time was fractions and teachers commented on how the VMs had supported understanding with multiplying fractions by fractions. The ability to represent a complex topic in a conceptual way rather than following a procedure was seen as beneficial.

> *Having that visual representation of the conceptual nature of multiplying fractions by fractions rather than just the old being taught to times across the*

> *top and then times across the bottom. It gives evidence to like why is that happening*? (Nicole – Teacher)

The teachers also commented on how easy the virtual manipulatives made converting between mixed numbers and improper fractions. Steve mentioned the ability to dynamically manipulate and rename fractions instantly (e.g., renaming 6/8 as 3/4) aided abstract understanding in his class. Nicole agreed and added that the virtual manipulatives helped her address common misconceptions with fractions, such as understanding that fractions from different wholes can still combine. She explained that students can easily colour in ¾ of a shape but when presented with ¾ from multiple objects they struggle to make the connection. Being able to reorganise the fraction pieces by manipulating them on the screen allowed Nicole to address the students' need. She added, *'you are addressing the conceptual knowledge and understanding that you just take for granted that the kids know, but they don't.'*

Another benefit that was identified which supported conceptual understanding was the unlimited nature of virtual manipulatives. The VM resources were praised for allowing more versatile and comprehensive demonstrations and modelling. Although more of a teaching benefit, the infinite number of resources was seen as beneficial to students conceptual understanding due to the multiple representations they could produce. Nicole and Mike identified the affordance of seeing multiple representations of a concept at the same time as being highly beneficial. Both referred to the fraction options the VM offered, having both circles and bars. Working with multiple representations using physical resources is challenging due to the number of resources and space required. Pippa added that it was also great for students *'to display their thinking in different ways'* and make comparisons with other students' representations.

Supporting students with diverse learning needs was also discussed post professional development intervention. Mike observed that using VMs really resonated with a certain group of his class that often found learning mathematics difficult.

> *There are certain types of students that when we went on to the virtual manipulatives it really clicked for them. … I started noticing there was a certain group, including the ADHD kids, that it just really helped. (Mike – Teacher)*

This observation was supported by Karen, who shared that she thought for some students *'their brain might work better seeing it on a screen'* emphasising that this flexibility was important given the *'variety of personalities and differences in our classrooms right now'*. Nicole added that using VMs was an *'enabler'* for these students, possibly providing the scaffolding and support that they require. She was also aware that for some students, the need to draw perfect representations and diagrams overrides the core learning objective. Mike agreed, referring to the *'creative'* students who *'spend ages trying to get the perfect picture or that sort of thing in their books'*. The use of VMs was seen to mitigate this difficulty, providing a quicker and more efficient path, which in turn allowed for a greater focus on the underlying mathematics concept.

4.3.3 Flexible and Purposeful Integration

Prior to the professional development Intervention, teachers were unsure about when and how to best integrate VMs into lessons. It was recognised that it was important to have a justifiable reason to utilise VMs in their maths lessons. Steve referred to this by suggesting it's about *'knowing when to use it rather than just using it because it's there'*. Teachers were also curious about where best to integrate VMs within the lesson sequence. The 'explore' and 'mastery' stage of the *Maths - No Problem!* lesson structure were identified as suitable for VM use. The teachers noted that this was where they were currently using physical manipulatives.

There was also discussion prior to the professional development intervention regarding how to use VMs to model effectively to the class. Teachers commented on the need to draw diagrams or visual representations on the board and the difficulty of producing accurate images. Brian identified drawing accurate diagrams was something that he often struggled with and saw VMs as a solution to this. The teachers noted that inaccurate drawings have the potential to interfere or restrict student learning. It was also identified that manipulating and showing changes to a static drawing was also challenging and this could be potentially avoided by using a VM.

Utilising VMs to support the shift to a whole-class approach was also discussed. The group felt that maintaining the partner work, a key element of the *Maths - No Problem!* approach, was important. Teachers envisaged a one-between-two student ratio of Chromebooks to maintain student collaboration and discussion. This was highlighted by Nicole in her comment,

> *'Its good to keep that communication going between the pairs, it's one of the big reasons why we have the pairs.' (Nicole – Teacher)*

Mike went further, adding that sharing a device would benefit students who might struggle with using a VM, and that having a buddy could help them work through issues. Brian also commented on the fact that less screens would also be more manageable with regards to ensuring everyone was on task.

A final notable perception that was identified by the teachers prior to the professional development intervention was around student agency. The group saw the idea of students having agency over choosing when to use VMs as a desired outcome. Nicole commented on students selecting either a VM or concrete materials depending on what suits their learning style.

Following the professional development intervention, teachers gained increased competence and clarity around how to integrate VMs into their lessons. A compelling development in teacher understanding was the ability to use VMs throughout the learning sequence. It was evident that the VMs helped support teachers model maths concepts during the 'explore' and 'mastery' sections of their lessons. Steve commented on how easy it was to manipulate items on the screen rather than the white board.

> *At the start of all my lessons I found that rather than a still picture, like in the guided practice or the mastery, being able to actually have pieces and move them around so that all the students could see them, and also use the renaming feature was really helpful. (Steve – Teacher)*

The use of VMs was not confined to these whole-class, teacher-led stages of the lesson. The teachers also integrated VMs to support consolidation during small group work, independent work, and follow up activities. Brian found that he used the VMs mainly when he was working with small groups so that they could *'show their understanding'* to other members of the group. Mike encouraged students to use the VMs during independent work and as a follow up to previous lessons. He found that it was useful supporting students who were struggling.

I used the VMs as a follow on for some of those students that were still battling with understanding on the worksheet. We used the Polypad instead, and that was better for them, just to help with their understanding. (Mike – Teacher)

One particular takeaway for Neil came during one of the modelled lessons. His initial perception of a VM was as a stand-alone tool and something that you would *'jump onto'* to show a maths concept. The ability to use Polypad to integrate the 'explore' and 'mastery questions alongside the VM tools all in one place, was seen as a huge advantage and streamlined the learning process.

They didn't have to go look at the book, look at the TV, look at the Chromebook, they had it all there. So, seeing it in Polypad in that way it was pretty interesting. (Neil – Teacher)

The affordances VMs provide with regards to modelling concepts and not needing to draw diagrams were identified prior to the professional development intervention. These perceived benefits were confirmed during the teacher's implementation and made it easier for teachers during whole-class instruction. Brian indicated that he didn't need to worry about drawing perfect pictures and that the VMs *'helped to really hammer that conceptual understanding'* more effectively. Steve commented on his experience of manipulating fractions that would have been far more difficult on a whiteboard.

…it's not as good when you colour it in or rub it out. When using the VM, the students could actually see it happen, and think, 'oh that comes from there', I found it really useful. (Steve – Teacher)

When it came to student agency, teachers saw direct evidence of students independently choosing to use VMs. Karen reported that in her class students asked to use Polypad following a lesson, using it to help them solve problems when they were working independently. She also noted that students were seen drawing diagrams in their books that replicated the VM models they had been using in prior lessons for multiplying fractions. The group agreed that this behaviour may not happen naturally and that it would need to be actively fostered through modelling and consistent reminders to become the 'norm'.

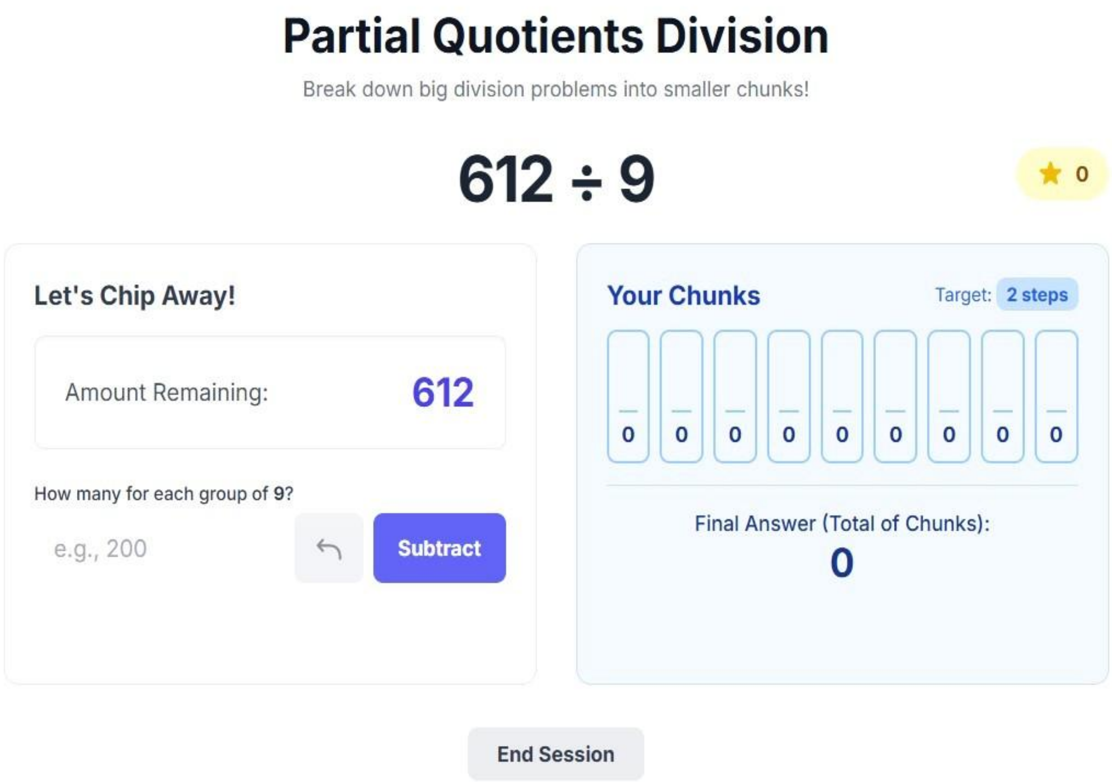
4.3.4 Emergence of AI as a Tool for Custom VM Creation

During the research, an unexpected new direction for virtual manipulatives developed. Several of the teachers had been using generative artificial intelligence (GenAI) to support their planning and resource creation and saw an opportunity to design a VM using *Google Gemini Canvas* to create the required computer code. Rather than spend time looking for the right VM for the concept they were teaching, they decided to build their own VM. Using a photo of a student's written informal model of division, Neil used AI to create a virtual model that all the class could use (see Figure 14). During an afternoon, he encouraged students to try out the new VM and provide feedback. Neil commented on the instant engagement saying *'they were hooked, for like 45 minutes'* and found that the students had lots of ideas for how to improve the VM. The majority of suggestions related to how the students interacted with the tool and led to a more gamified focus with different levels and confetti falling down the screen when they were successful. With each suggestion, Neil used the GenAI tool to modify the VM based on the student feedback, leading to high student engagement and ownership. In his explanation, Neil emphasised how transformative this process could be for a student efficacy.

> *It's quite a personal example but imagine if you had a problem that a student provides a method for, then the next day it's a game. Like how cool would that be for the student? (Neil – Teacher)*

Figure 134

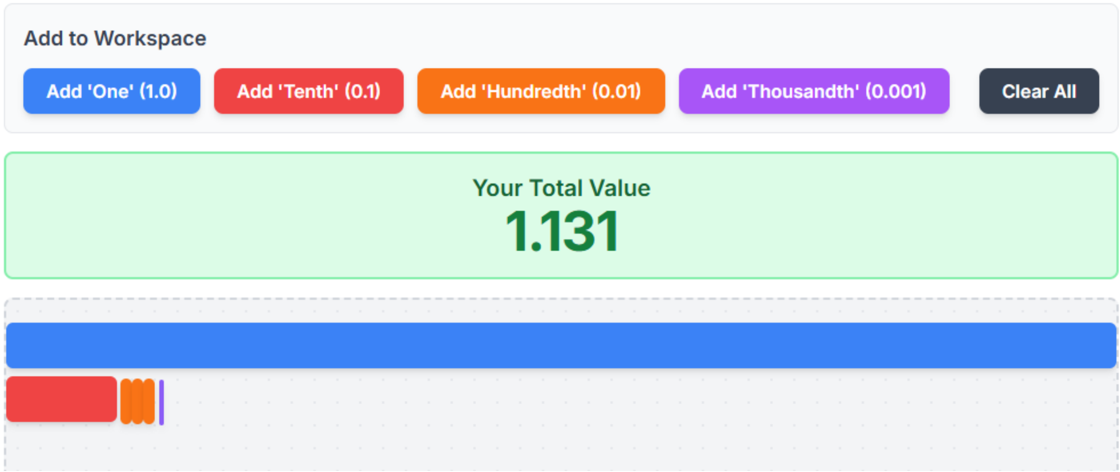
Screen shot of a division VM created by a teacher using GenAI



After the success of the first use of AI to create a VM, Neil and Nicole identified a need to create a VM to replicate Deci Pipes. Nicole commented on Deci Pipes being a great tool to show a conceptual model of decimals, but their size and design made them awkward to use. Their size also meant that it was impossible for all of the class to be using them at the same time. Using AI, they were able to create a virtual model that students could interact with. It had added features such as pieces snapping together and automatic conversion of ten-tenths to a whole, which physical manipulatives or other VMs lacked (see Figure 15). This new direction with VMs, positions AI as a personal coder that overcomes time and skill barriers in creating bespoke educational resources.

Figure 15

Screen shot of Deci-pipe VM created by teachers using GenAI



4.3.5 Collaborative and Content Focused Professional Development

Analysis of the CLES data identified limited use of virtual manipulatives across the school. This provided the evidence for a professional development intervention aimed to see if teachers' perceptions and use changed. Discussion during the initial focus group mirrored the CLES data and low use of virtual manipulatives. Comments such as "*I haven't really used them (Chrome Books) at all in maths*" (Pippa) and "*I don't understand it well enough at the moment*" (Karen) provides evidence that virtual manipulatives are not being regularly utilised.

The implementation and design of the professional development came from initial discussions with the research group as part of the action research design. The teachers' general consensus was that a combination of instructional sessions alongside modelled lessons would be most beneficial. Steph suggested having an example lesson would be useful to see how the VMs were used. Steve emphasised that *"seeing something modelled for you"* was more effective than merely being *"told to do something,"* highlighting his own need for *"serious scaffolding"*. The group also mentioned that modelling was already happening in other areas of the curriculum and had been successful.

The need for targeted use of VMs was identified ensuring that it was content focused. The group agreed that learning about VMs that were not relevant to what they would be teaching in the immediate future would not be beneficial. Nicole suggested that the VMs could help to address common misconceptions, as she was often seeing them especially with proportional thinking and rational numbers. The group were also aware that it can be overwhelming if too many VMs are introduced at once. Mike commented on the need to *‘narrow the focus’* and stick to one or two VMs that they could get to know really well. Neil echoed this claiming that *‘it's easy to forget or struggle with which tool fits what lesson’* if there are too many options. The use of Polypad was identified as a versatile tool that some of the group and the researcher had used already. With three weeks between the first focus group and the professional development intervention starting, there was enough time for the researcher to look at how Polypad could meet the group’s needs.

Following the intervention, teachers' perceptions evolved, demonstrating a greater appreciation for integrated and collaborative professional development, particularly in how it informs content delivery. The concrete examples and expert support were valued by the group. Nicole acknowledged the researcher's support as *‘definitely valuable’* particularly because it provided the *‘time and headspace to engage really solidly with one tool’* and shared *‘gems’*. Neil agreed, sharing that there were many features he had not previously seen within the Polypad tool and found it *‘handy’* to have them explained. The group worked collaboratively, identifying specific content that they planned to teach and identifying possible VMs to address student needs. Within the Polypad app, the teachers were able to combine content material from the *Maths - No Problem!* resource directly with VMs that could support the mathematics concepts involved.

The modelled lessons were also seen as valuable opportunities to observe the integration of VMs into a lesson structure. The teachers commented on seeing VMs used at various stages of the lesson reinforced the notion that VMs don’t need to be a standalone tool. Neil highlighted this when he reflected on his key takeaway from a modelled lesson, which involved observing the integration of ‘Explore’ tasks, ‘Guided Practice’ questions, and visual models within a single Polypad environment.

The group identified not having enough time as a major barrier to learning and using VMs. The demands of teaching multiple subjects and currently learning new curriculums leaves teachers less time to invest in learning how to integrate digital resources.

> *It's because we're time poor, right? That's the major problem with teachers. The time we don't have and the headspace to engage really solidly with one*

tool because there's so many things going on, you have got all the tabs open in your brain. (Nicole – Teacher)

Being able to spend time experimenting and using the VMs was seen as a way to consolidate understanding. Karen reinforced this suggesting that *'the more you use something, the more confident you become, the more you're likely to use it'*. The group agreed that moving forward, it was important that time was put aside possibly in team meetings, to practice and experiment with different VMs. The group discussed the creation of a set of starter lessons that could be used to support integration of VMs by other teachers at the school. The idea being that this could aid with learning how to use the VM while not adding to the workload of teachers.

I think if you want more buy in, it would be through creating templates or starter lessons. This is what they look like and here are a few instructions. Provide a series of them and then go, OK now try and make these yourself. (Karen – Teacher)

Continued collaborative planning and sharing of created resources were seen as crucial for broader adoption across the school, especially in alignment with new curriculum documents that explicitly mandate digital tool use in maths.

4.4 Synthesis of Findings

The preceding sections presented the quantitative data from the CLES, and the qualitative themes generated from the focus groups and professional development intervention. The purpose of this section is to integrate and synthesis these two streams of data, moving beyond the simple presentation of findings to establish clear connections between the two. A convergence approach will be utilised to map the CLES data scores against the five identified qualitative themes. The objective is to establish clear links between the teachers' quantitative observations of their mathematics classrooms and the qualitative explanations of their perceptions and instructional experiences of virtual manipulative use, thereby framing the subsequent discussion and connection to relevant literature.

The following convergence matrix (Table 9) provides a detailed mapping of key quantitative scores against corresponding qualitative themes and supporting findings.

Table 10

Convergence Matrix of Quantitative and Qualitative data

CLES Subscale/Question (Quantitative)	Mean Score (1-5)	Corresponding Qualitative Theme	Supporting Qualitative Finding (Quote/Observation)
Q1: I see the whole-class approach as a meaningful way to support students' mathematical understanding.	3.89	Flexible and purposeful integration	Steve: At the start of my all my lessons I found that rather than a still picture, like in the guided practice or the mastery, being able to actually have pieces and move them around so that all the students could see them, and also use the renaming feature was really helpful.
Q3: I create an environment where students can learn from mistakes...	4.42	Benefits to Conceptual Understanding	Nicole observed the VM helped students "recognise their errors" with fractions, supporting a growth mindset.
Q:7 The use of manipulatives enhances my ability to teach mathematical concepts	3.92	Benefits to Conceptual Understanding	Being able to reorganise the fraction pieces by manipulating them on the screen allowed Nicole to address the students' need. She added, 'you are addressing the conceptual knowledge and understanding that you just take for granted that the kids know, but they don't'.
Q9: I have experimented using virtual manipulatives for instruction.	2.56	Collaborative and content focused professional development	"...I've found maths is probably the one (subject) that I use it (digital technology) the least." "I'll be honest, I haven't really used them (virtual manipulatives) at all in maths"
Q10: Students are engaged when using virtual manipulatives.	2.97	Classroom Management and Engagement	Teachers reported "High levels of engagement... which was seen as a key factor in mitigating against distraction" after PD.
Q12: It's OK for them to question the way they are being taught.	4.06	Flexible and purposeful integration	Success probably would also be the kids using it. Having used maybe a few of these tools and wanting to go ahead and use those that enrich their learning, you know, for them to kind of have a bit of agency over them. (Pippa)
Q16: I am open to adjusting my approach based on student engagement with whole-class activities.	4.61	Flexible and purposeful integration	I used it quite often, especially for my lower ones who were struggling because I could put it on the TV and make it for them, especially with the mixed numbers, where they could actually see it in front of them and then use it to manipulate them. (Brian)
Q.18 I reflect on how well whole-class activities are meeting the needs of all students.	4.42	Collaborative and content focused professional development	...there's lots of conversations going on at the moment, which are great because we're no longer talking about content, We're talking about pedagogy and how we teach and how we teach conceptually. (Nicole)
Q22 Students talk with other students about how to solve problems.	4.44	Classroom Management and Engagement	Teachers envisaged a one-between-two student ratio of Chromebooks to maintain student collaboration and discussion. This was highlighted by Nicole in her comment, "It's good to keep that communication going between the pairs, it's one of the big reasons why we have the pairs."

This synthesis demonstrates clear links between the CLES results and quantitative themes. The CLES data established the baseline gap in teachers VM use which the qualitative themes subsequently explained through teachers' reflections of instructional uncertainty, conceptual understanding, and classroom management concerns. These integrated findings provide a unified and evidence-based foundation for critical discussion and interpretation against relevant literature in Chapter Five.

Chapter 5: Discussion

5.1 Introduction

In this study, five themes were generated through thematic analysis. These themes were synthesised with the collected CLES data to support a cohesive understanding and represent how a professional development (PD) intervention shapes teachers' perception and instructional use of virtual manipulatives.

The five themes identified were as follows:

- Classroom management and engagement
- Conceptual understanding
- Flexible and purposeful integration
- Emergence of AI as a tool for custom VM creation
- Collaborative and content focused professional development

This chapter will explore each theme in depth, highlighting its connections with the CLES results and existing literature. Throughout the discussion, I will also consider the barriers that prevent teachers from implementing VMs and how VMs can effectively support whole-class instruction.

5.2 Classroom Management and Engagement

Classroom management and engagement was a major theme in the results. Teachers' initial anxieties focused on the potential for distraction, the difficulties with monitoring what students were doing, and the cognitive load of learning both a new mathematics concept and a new tool. These perceptions were not unexpected as using technology often requires adaptions to teaching techniques that add to the complexity and challenge of integration (Drijvers et al., 2010). Following the professional development intervention many of the teachers' concerns were less problematic than anticipated. Good levels of student engagement were observed (Karen, Pippa) which was seen as a key factor in mitigating against distraction. Literature suggests agreement that use of digital technologies in mathematics classrooms helps to increases student engagement and can in turn lead to improved learning outcomes (Attard, 2018; Bray & Tangney, 2015; Calder & Campbell, 2016; Hilton, 2016).

The engagement element of virtual manipulatives has also been observed in other studies through the 'focus constraint' VMs afford (Moyer-Packenham & Westenskow, 2013). The VMs focus and channel student attention on the mathematical objects and processes relevant to the concept being taught.

In this research, the teachers experience using the VMs also reinforced their prior perceptions regarding VMs eliminating mess and logistical issues of distributing physical manipulatives. Students were able to *'make a mess on the screen'* (Pippa) and the VMs eliminated the risk of missing pieces (Mike) which saved time and reduced classroom disruption. A benefit observed was that of unlimited accurate representations (Karen, Steve). This affordance of VMs allows for efficient use and, according to Moyer-Packenham (2016), promotes more methodical and purposeful student actions. Teachers expressed concern that they do not have enough physical manipulatives for all students, a common observation in other studies (Moyer et al., 2002; Moyer-Packenham, 2016; Suh, 2016). VM use removes a material constraint that limits the complexity of tasks teachers can assign and improves equity in resource access.

During the professional development stage of the research, the teachers requested more control over what students could access within the Polypad tool. This aligns with a finding by Johler et al. (2022) in their case study, which observed that despite teachers self-reporting the importance of "trust, good relationships, and risk-taking," more than half of them still found that they "should have control over pupils' screens at all times." (p. 11). This need for control is not limited to virtual manipulatives, Moyer and Jones (2004) found that for certain teachers the "decisions to use or not to use manipulatives are based on the amount of control they believe they will be able to maintain in their classrooms" (p. 28). This need for control corelates with the CLES data and teachers' strong sense of control of their own teaching actions versus student-led decisions. Their desire to mitigate the perceived challenges of student distraction demonstrates, implicitly, a lack of confidence in student agency and self-discipline regarding technology.

As with any subject, conducting whole-class teaching while students work behind a screen might make it harder for the teacher to monitor student engagement and focus. The identification of an 'authoring mode' by the researcher provided control options that reduced the chance of students getting off task. The inbuilt design directly addressed teacher management concerns, effectively providing an internal control mechanism to replace the loss of physical management and supervision. This reflective and critical approach to the use of the VM reinforces the use of collaborative action research in discovering such solutions (Edwards & Burns, 2016).

Overall, the research group were realistic with regards to classroom management and the use of VMs. It was acknowledged (by Brian and others) that these tools are not universal solutions to the demands of modern classrooms and that the initial novelty of the technology may not translate into sustained engagement. Specifically, concern existed that once the uniqueness of the tool faded, student engagement might decrease. In addition, the increasing familiarity with the tool could be counterproductive, with the availability of more options and features distracting students from the primary task. Assumptions that the device alone will maintain engagement, coupled with poor pedagogical practices have been shown to result in student disengagement (Attard, 2018). Consequently, to sustain engagement beyond the initial novelty effect, the teachers' long term utilisation of VMs would need to be addressed through ongoing professional development.

An interesting observation that was not identified prior to the professional development sessions was how students interact with the VM interface when a touch screen was used. It was observed (Neil) that students who used touch-screen devices had an improved experience, leading to a perceived increase in engagement. This can be related to haptic affordances (from the Greek word haptikos, meaning 'ability to touch'), which are the properties of a digital tool's interface that invite direct interaction. Research into apps and devices with touch-screen properties has identified these haptic affordances (Calder & Campbell, 2016; Calder & Murphy, 2018; Hegedus, 2013) as improving the learning experience and strengthening the connection between physical and abstract representation. An affordance relevant to Neil's observation is the direct interaction with the virtual object, which is not mediated through a mouse or keyboard (Calder & Murphy, 2018). This closer, more intuitive connection allows for the simultaneous linking of the student's actions with a virtual object. As a result, it provides a greater "sensory environment to connect to the symbolic formalisms of the mathematical ideas" (Hegedus, 2013, p. 93). These observations suggest that, where possible, more touch-screen devices should be made available in class. Furthermore, school-wide advice to parents purchasing student devices may need to prioritise touch-screen options to enhance student engagement with VMs.

5.3 Benefits to Conceptual Understanding

The findings suggest an apparent shift in teachers' perceptions and understanding of the conceptual benefits of VMs following the professional development intervention. Prior to the intervention, the teachers recognised the positive affordances that VMs provide through their ability to create interactive, pictorial representations of concepts. For example, Mike's comment on the ability to easily modify fraction pieces and Steve's observation of multiple representations, shows awareness of VMs as a more dynamic alternative to physical manipulatives. Moyer-Packenham and Bolyard (2016) describe the user's ability to 'interact with, move, or manipulate the dynamic mathematical representation' as a defining feature of a VM and what distinguishes them from other digital tools. (p. 5)

When conducting whole-class teaching, the research group were also conscious of the fact that some physical manipulatives are not suitable when modelling to the class. When a teacher models a concept, the tools they use must be easily visible and simple to manipulate. Pippa noted that using physical fraction pieces was particularly challenging, as they were difficult to move effectively and weren't easily visible to all students. To address the issue, she saw VMs as a possible solution, making her instruction more effective. This reflects findings by Keldgord and Ching (2022) who reported teachers perceived VM use was beneficial in conditions when using physical manipulatives was impractical. Clear modelling using VMs supports students in making connections between visual and abstract concepts, which is crucial for conceptual understanding. The benefits of VMs to aide lesson efficiency while supporting conceptual understanding aligns with finding by Lee and Tan (2014) into the role of VMs when using a C-P-A approach.

The post-intervention data shows that the teachers' perceptions and understanding of VMs' conceptual benefits appeared to evolve. Nicole's experience with using VMs to teach multiplying fractions is an illustration of this. The VM did not only allow Nicole to *'show'* the concept, but it helped her to provide the evidence of *'why this is happening'*, taking her students from a procedural understanding to a deeper conceptual one. When multiplying fractions, a procedural approach will instruct students to multiply the numerators together and then the denominators. Success can be achieved, but students may not be able to explain why it works. Lee and Tan (2014) highlight the importance of using the C-P-A approach to achieve sound conceptualisation of abstract concepts. By utilising the fraction bars and rectangle tool, Students in Nicole's class were able to see an area model of the sum, strengthening their conceptual understanding. The model provides visible and tangible evidence that the product of the two fractions is smaller (when both fractions are less than one). This challenges students' ideas around multiplying always producing a larger product. Rich (2024) explains that many students' struggles and misunderstandings with fractions result from incomplete and incorrect attempts to merge their developing knowledge of fractions with their existing knowledge of the properties of whole numbers. The VM for this task provided easily manipulatable components and immediate feedback, aiding the students' understanding.

Another affordance of VMs that the teachers identified was the strength of multiple representations. Following the professional development intervention, their perceptions around how a VM can provide an environment for comparing representations seemed to strengthen. Nicole and Mike identified the use of different fraction representations and how they aided understanding. Rich (2024) supports this observation explaining that VMs with linked representations support students to transition between visual and symbolic representations. The ability to dynamically connect representations so that when one representation is changed, the other matches it, is a valuable affordance that VMs provide (Moyer-Packenham & Bolyard, 2016; Rich, 2024).

Post professional development intervention, the ability to manipulate and move elements within a VM was seen as a valuable affordance for dispelling misunderstandings. VMs provide an environment where students can explore and experiment with maths concepts, gaining instant feedback from their actions (Moyer-Packenham & Westenskow, 2013). This immediate, error-driven learning directly reflects evidence in the CLES data that demonstrates a strong commitment to fostering a supportive learning environment where "students can learn from mistakes and see them as part of the learning process" (CLES-VM, Q3). Nicole's observations regarding students' misconceptions of fractions and how the tool allowed the students to recognise their errors, reinforces this constructivist principle and supports a growth mindset. To effectively address misconceptions, Mackle (2016) advises that student need to manipulate fractions when exploring questions and observing patterns, opportunities the VMs effectively fulfilled.

The research group observed that the use of VMs was beneficial in supporting the diverse needs of learners including those with ADHD and autism. In their evidence-based practice synthesis of VMs for students with ASD (Autism Spectrum Disorder) and IDD (Intellectual

Developmental Disorder), Long et al. (2023) found that VMs are being increasingly used to support students with disabilities.

During the final focus group, several of the teachers reported observing that the VMs increased student engagement and focus while removing barriers that restricted learning. In particular, they focused on how VMs removed the need for students to draw perfect representations, an issue for many ASD and ADHD students. Their fixation on perfection, places a heavy demand on executive functions, which can trigger demand avoidance, and increase anxiety (Nugent, 2000). Another affordance that can particularly help students with learning difficulties identified by Moyer-Packenham et al. (2008) is the "built-in constraint systems in many virtual tools" that "provide support for sense-making and make mathematics ideas explicit as the user interacts with the tool" (p. 214). By making the mathematical concepts more explicit, the VMs become the pedagogical scaffold or 'enabler' Nicole mentioned, increasing the chance of student success.

5.4 Flexible and Purposeful Integration

The CLES data collected highlighted a noticeable gap in teacher competence and integration of VMs. Within the 'learning about manipulatives' subscale, the questions specifically relating to virtual manipulatives produced the lowest scores of the entire CLES. Teacher comments during the initial focus group supported the CLES data with teachers acknowledging their own lack of experience and need for guidance in their use. This result is consistent with other studies (Dueker & Desai, 2022; Fletcher et al., 2020; Reiten, 2021) that also identified a lack of knowledge of digital tools being a key barrier to their pedagogical use. This lack of knowledge leads to the teachers not making the most of what VMs can provide.

Reiten (2021) suggests that effective use of technology means teaching *with* not *near* technology. Teaching *near* technology often fails to leverage the opportunities VMs provide to connect mathematical representations and build conceptual understanding. Teaching *near* technology is a misuse of the technology and is often only used to merely grab student attention (Suh, 2016). Teachers' perceptions during this study clearly recognised the importance of the manipulatives being more than just an additional tool to increase student participation and involvement. While they saw increased student engagement through using the VMs as helpful and beneficial, their focus ultimately shifted to how the VMs supported students' understanding of the maths concepts. This is reflected in Steve's comments about not just using something because it is there but understanding the purpose for using it. Steve identified himself as someone who had limited digital knowledge. After the professional development intervention, Steve's experience using the VMs to teach shifted his understanding and he acknowledged the ability of VMs to clarify abstract concepts that are difficult to teach otherwise. This evolution from simple presence to thoughtful, content driven application highlights the need for teachers to be clear on the why, how, what, and when of technology integration. Consequently, these decisions need to be determined by the explicit needs of the students and their own current knowledge of the technology's potential (Fletcher et al., 2020).

This essential shift toward purposeful technological integration also provided a practical solution to initial concerns of the research group regarding tool selection. Discussion during the initial focus group highlighted the challenge of selecting the right tool from the huge range of VM tools available. Teachers commented on feeling overwhelmed by the wide variety of tools and expressed confusion over selecting the right tool. This observation mirrors Keldgord and Ching (2022) findings that a major barrier to VM use is the time needed to sort through the various VMs available and set them up. Following the group's request to narrow the focus of the professional development, the use of the multi-functional Polypad tool effectively removed the need to learn about a range of separate VMs. This targeted approach provided a space to develop their confidence and knowledge in how the tool could be integrated to support conceptual understanding.

Prior perceptions regarding the benefits of VMs for modelling were reinforced during professional development sessions and in-class implementation. The research group identified possible positive affordances of VM use, including improvements with display and visibility to the class, practicality with regards to manipulation, and removing the need to draw diagrams. These perceptions were strengthened following the professional development intervention, aligning with findings by Lee and Tan (2014) that the use of VMs aids teacher efficiency when teaching. For instance, Pippa and Steve commented on how much easier it was to display and manipulate objects on the screen so that all students could see. Additionally, Brian, conscious that he struggled to draw diagrams correctly, found that VMs provided clear, accurate representations. This echoes the meta-analysis findings of Moyer-Packenham and Westenskow (2013) who identified that VMs contain precise representations, allowing accurate and efficient use. Although Moyer-Packenham and Westenskow's research focused on student use of VMs, this study demonstrates that these benefits are also reflected in teacher practice.

A key development during the professional development intervention was the realisation that the use of VMs could be integrated throughout an entire lesson sequence. Initial discussions had only identified the 'explore' and 'guided practice' stages as opportunities for the teacher to model how students could use them. However, through the use of the Polypad tool, teachers' perceptions evolved to view the VM as a holistic workspace for the whole lesson. This shift was highlighted by Neil's observations of a lesson, noting how it became possible to integrate the 'explore' activities and mastery questions in one place alongside the relevant VMs. Neil, who had initially seen VMs as a stand-alone tool to be used only when needed, observed that by providing everything in a single location, students were more focused without the need to switch attention between the TV screen or textbook. This successful transition to a single, integrated platform illustrates how the adoption of digital technology is greatly increased when teachers can easily and feasibly access and integrate it into their lessons (Loong & Herbert, 2018).

When discussing ultimate goals of the professional development, the research group identified increasing student agency. The ability for students to eventually select a VM to support their learning without teacher intervention was seen as a key advantage. Developing this disposition, where students take ownership of and responsibility for their own learning, has been identified as helping to create confident and connected lifelong learners (Alison, 2019). Early evidence of this shift developed immediately following a co-taught lesson with the researcher, when Karen shared that students from her class had asked to use a VM to support their work. She added that she had also observed students replicating the diagrams produced by the VM in their maths books. This student-led action aligns with the constructivist approach, where learners are self-regulated and actively involved in their own learning (Zajda, 2021). Furthermore, this increased sense of student controllability and ownership of their learning has the potential to increase motivation and perceived self-efficacy (Willacy et al., 2017). Although these observations were highly encouraging, the research group acknowledged that student agency would not happen naturally and would need to be deliberately fostered.

5.5 Emergence of AI as a Tool for Custom VM Creation

The discovery and subsequent experimentation with AI to create a custom-built VM to meet specific student needs led to a shift in teachers' perception of VM use. Rather than being constrained by the capabilities of existing, generic tools, the teachers were able to utilise AI to rapidly customise a VM to address highly specific conceptual challenges. This application of AI advances previously documented use in lesson planning and generic resource creation (Ashby, 2024; Berry, 2024; Coblentz et al., 2025), unlocking opportunities to strengthen students' conceptual understanding.

Drawing on the work of Song et al. (2025) teachers' implementation of generative AI typically proceed from low level to higher level activities. When considering the SAMR model of technology integration (Puentedura, 2006), teachers are likely to first experiment at a level of *substitution,* such as organising existing material and generating questions and examples (Song et al., 2025). This initial use of AI automates and decreases the time taken on administrative tasks, freeing teachers to focus on higher-order cognitive activities (Almuhanna, 2024). In the case of this research however, two of the teachers rapidly progressed to a higher level of AI integration, *redefinition* (Puentedura, 2006), by creating new student experiences that were previously unachievable. The ability to write the amount of computer code accurately and rapidly would have been previously impossible for an untrained computer coder. The advancement of AI has opened up potential for teachers who know how to leverage the technology to meet specific instructional goals.

The initial application of AI during the research utilised an example of a student's written work. Following identification of a student's personal method to solve a division equation, the teacher photographed it and then used AI to create a VM version. Neil's observations of the transformative impact this had on student efficacy are powerful and highlight how AI can be utilised to create a more individualised learning environment (Almuhanna, 2024). Students often hold a fixed mindset regarding their maths ability, which can be hard to break (Dweck 2006). Activities such as this that elevate the student's conceptual thinking as an example for other students to follow, promotes a growth mindset, fundamental to efficacy (Boaler & Dweck, 2015). Through his iterative development of the AI generated VM, Neil was able to engage the students in its customisation. This process appears to challenge traditional mathematics approaches that can, as Boaler (2015) suggests, lead to students 'surrendering agency and thought'(p.27). It would appear from Neil's observation that this engaging approach, utilising the power of AI, led to increased engagement and motivation from the students.

At a later stage in the research, two of the teachers (Neil and Nicole) worked together to use AI to build a VM that replicated deci pipes. This informal collaboration and experimentation was a catalyst for innovative action and occurred in response to a pedagogical challenge (Moorhouse & Wong, 2022). Driven by the lack of a suitable virtual model, the teachers innovatively leveraged GenAI to meet this specific need. Berry (2024) identifies the ability to support visualisation as a powerful feature of AI for illustrating complex ideas. Early versions of their VM contained issues and bugs, requiring constant refining. AI still requires human oversight, and this can be time consuming. This requirement for human oversight and the time-intensive iterative process of evaluating and modifying AI-generated content reflects findings by Almuhanna, (2024) regarding 'considerable time-based limitations when integrating AI technologies'(p, 10365). The pair's development of a custom-built VM was shared with the rest of the group leading to its application in other classes. Moorhouse and Wong (2022) suggest that the mere novelty of using something new does not automatically lead to advancement and that pedagogical innovation needs to be principled and evidence based. However, Neil and Nicole's commitment to solving a school-wide instructional need ensured the principled development, acceptance, and eventual implementation of the new VM.

The innovative use of AI demonstrates that VMs are not just static, existing resources, but dynamic, customisable teaching aids that can be instantly generated to serve the specific pedagogical needs of the class, offering a powerful improvement over both generic virtual and physical manipulatives.

5.6 Collaborative and content focused professional development

The CLES data indicated a clear need for professional development to effectively support the integration of VMs, specifically highlighting the support of conceptual understanding and whole-class teaching. Responses to questions that focused on teachers' use of VMs were noticeably lower with scores of 2.24 (Q8) and 2.56 (Q9) out of 5. In line with the action research of this study, the teachers in the group were involved in how the professional development was delivered. The group's shared preference for a specific and conceptualised approach, combining directed instruction and modelling, aligned with the goals of the TPACK model, which emphasises the synergistic development of 'technological knowledge' alongside pedagogical and content knowledge. (Koehler et al., 2013). From their prior experience, the group identified in class modelling as a highly beneficial approach. A study into effective teacher professional development by Darling-Hammond et al. (2017) identified that 'modelling of instruction helps teachers to have a vision of practice on which to anchor their own learning and growth' (p. 11). This was directly echoed by Steve's comments, who noted that 'Seeing' something integrated rather than being 'told' about it, creates a sense of efficacy and a feeling of 'Yeah, I can do that'.

Alongside side the in-class modelling, the direct instruction and guidance provided to the group was valued. This aligns with observations by Darling-Hammond et al. (2017) and Haynes-Brown (2025) who indicate that a component of successful professional development is the sharing of professional expertise and practical classroom implementation. For this study, the researcher took on the role of the expert, to share ideas based on their own knowledge of VM use. Timperley (2008) identifies that the use of expertise is necessary as "substantive new learning requires teachers to understand new content, learn new skills, and think about their existing practice in new ways" (p. 20). This approach ensured that the research group received the explicit, content-specific guidance required to expand their current practice, establishing a strong foundational knowledge of VM use that supported subsequent implementation efforts.

A key component identified by the group that they wanted addressed in the professional development was a narrowing of focus due to the overwhelming number of VMs available. This aligns with observations by Reiten (2021) that teachers spend too much time searching for a suitable VM and advised that focusing attention on a limited number of tools was beneficial. To address the desire for a narrowing of focus, the professional development centred on the use of Polypad. As a multipurpose tool familiar to several of the group, it was selected for its versatility and its capacity to integrate multiple VM representations in one place.

An apparent barrier to the integration of VMs was the challenge of insufficient time, a finding that directly mirrors results in other studies (Hansen et al., 2016; Haynes-Brown, 2025; Reiten, 2021). Teachers attributed this difficulty to the *'lack of headspace'* (Karen, Brian) stemming from the numerous competing demands of teaching and implementing a new curriculum. For example, Nicole referred to having *'multiple tabs open in her brain'* and being time poor, making it hard to prioritise integrating VMs. Reiten (2021) identified time as a crucial support theme that enables teachers to learn, integrate, and reflect effectively on VM integration. The group also expressed concern that the lack of time would make it hard to sustain momentum with VM usage going forward. This is reflected in the seventh characteristic of effective professional development outlined by Darling-Hammond et al. (2017) that of having a sustained duration. They propose that, "offering multiple opportunities for teachers to engage in learning around a single set of concepts or practices, has a greater chance of transforming teaching practices and student learning" (p. 15).

In response to this issue and with consideration of how the professional development intervention could be applied to the rest of the school, the teachers proposed several actionable ideas. Firstly, they suggested the allocation of dedicated release time to allow them to collaborate, create personalised resources, deepen their understanding of the tool, or to experiment with emerging technologies like GenAI (Karen, Nicole). Collaboration is a key feature of effective professional development but according to Timperley (2008), it must be responsive to the needs of the students. In this case, the group have become aware of the value VMs can provide to support their students conceptual understanding and the need to increase VM use. Secondly, they recommended committing time at team meetings for sharing and disseminating new knowledge with other teachers. Lastly, they called for the creation of ready-made resources or a series of lessons that could be used immediately, reducing the burden of having to create everything from scratch. These suggestions collectively demonstrate the group's active interest in expanding the use of VMs through collaborative inquiry, directly aligning with key recommendations on successful professional development by Timperley (2008) and Darling-Hammond et al. (2017).

The discussion of how the professional development intervention could be rolled out across the school to all teachers raised a notable point of tension. To gain acceptance and encourage sustained use, the group recommended creating a series of starter lessons or templates. However, this directly contrasts with the learning by design model that Koehler and Mishra (2005) proposed for developing TPACK and adopted for this professional development intervention. By providing lessons and templates, teachers are less likely to develop deeper understandings of the nuances and complexities of technology integration (Koehler & Mishra 2005). Through a learning by design process, teachers are actively identifying issues and collaboratively finding solutions, addressing their pedagogical knowledge, content knowledge and technological knowledge at the same time. This mirrors an active learning approach identified by Darling-Hammond et al. (2017) who indicate effective professional development requires opportunities for “sense making” and chances to “analyse, try out, and reflect on new strategies”(p. 7). Ultimately, the groups’ request highlights a common challenge in professional development, that of the need to balance the practical demands for immediate, ready-made resources with the pedagogical requirement of fostering deep conceptual understanding through active, design-based inquiry.

Chapter 6: Conclusions and Implementations

The aim of this study was to explore the following question:

> *How does a professional development intervention shape teachers' perception and instructional use of virtual manipulatives?*

This question was investigated by considering the potential barriers to adoption and the practicalities of implementing virtual manipulatives (VMs) within a whole-class mathematics framework. While the initial data highlighted barriers such as the pragmatic constraints of using VMs and the tension between constructivist ideals and classroom management, the findings suggest that these difficulties can be overcome. In fact, some of issues encountered during this study served as a catalyst for a shift in how digital tools are used and positioned within the mathematics classroom.

This chapter synthesises the findings to suggest that the integration of VMs, particularly when supported by generative AI, can support the procedural and conceptual demands of mathematics teaching. It moves beyond a summary of results to offer four core conclusions.

1. *The emergence of teacher-led custom VM design*
2. *Balancing teacher control and student inquiry in whole-class settings*
3. *Teachers' developing perception on VM use to scaffold and support learners*
4. *The TPACK Sustainability Paradox: Depth Versus Efficiency*

Following these conclusions, the chapter outlines the practical implementations for teachers and school leadership, identifies the limitations of the current study, and suggests possible directions for future research into the design and use of VMs in mathematics pedagogy.

6.1 Conclusions

6.1.1 The Emergence of Teacher-led custom VM Design

The emergence of generative AI as a tool for creating custom-built VMs represents a growing evolution in teachers' digital autonomy. This development has the potential to redefine teacher perceptions regarding VMs' use at a fundamental level, moving them from consumers of generic tools to co-creators of tailored VMs that directly address identified student needs. This study concludes that this potential paradigm shift redefines the teacher's role while breaking traditional barriers of time and skill in resource creation.

This transition moves the application of VMs beyond the lower levels of the SAMR model (Puentedura, 2006) of 'Substitution' and 'Augmentation' and into the realm of 'Redefinition'. When a teacher can take an informal, student-generated strategy (as seen in the case of Neil) and digitise it into a functional tool, the VM becomes a responsive pedagogical resource rather than a pre-packaged supplement. This practice validates student voice and transforms the classroom into a place where digital mathematical tools are deliberately crafted to align with the conceptual demands of the lesson, rather than teachers depending on generic resources that may not fully address learner variation.

Furthermore, the use of Gen AI addresses the time constraints identified by the research group as a barrier. Issues such as the time taken to find the right VM for the concept being taught and knowledge of how it works, limit their use. The direction taken by Neil and Nicole to leverage Gen AI, to build a deci-pipe VM, provides a possible solution to these challenges. Rather than spending time searching websites to find a VM that meets the requirements, teachers can move beyond the exhausting search for pre-made resources. By providing prompts that directly relate to the immediate conceptual needs of the students, AI generated VMs shift the technology from a pre-planned addition to a timely targeted response.

This study concludes that the possible future of VM integration lies not in more extensive collections of ready-made tools, but in the development of teacher AI prompting competencies that allow for the immediate creation of customised learning environments.

6.1.2 Balancing Teacher Control and Student Inquiry in Whole-class Settings

A conclusion of this study is that VMs can serve as a mediating device that helps teachers resolve the control-agency tension that is in inherent in constructivist philosophy (Mishra et al., 2025). In this study, the school adoption of *Maths – No Problem!* which favours a whole-class approach to teaching mathematics provides an added layer. The initial CLES data revealed a pedagogical paradox of high theoretical endorsement for student led learning yet a practical hesitation to relinquish control. This suggests that the barrier to teacher constructivist practice is not a lack of belief in its effectiveness but rather a lack of tools that provide the security to implement in a classroom environment.

During this study the identification of built-in constraints within the Polypad tool provided a 'bounded agency' that many VMs and other digital resources often lack. By utilising the built-in constraints of the software, teachers were able to shift their concern and focus from constant monitoring (ensuring students are on task) to a more productive role of facilitating the mathematical learning (guiding and supporting the students). This transition is crucial as it suggests that for digital integration to be successful, the technology must suit the teachers need for control while simultaneously providing the opportunity and freedom for students to explore. In a whole-class approach, the teacher's ability to manage the class is enhanced not by restricting the mathematical exploration, but by narrowing the technical distractions and ensuring learning is central to the lesson.

In addition, the shift in student ownership that was evident in Karen's observations of students proactively requesting VMs, represents a maturing of the classroom's mathematical culture. This research concludes that pedagogically authentic integration of VMs has the potential to transform student agency from something granted by the teacher into something claimed by the learner. The study demonstrated that the divide between constructivist theory and classroom practice can be reduced when technology is positioned not as a distraction that needs to be managed but as a scaffold to support independent student inquiry.

6.1.3 Teachers Developing Perception on VM Use to Scaffold and Support Learners

Rather than being seen as just a tool to increase active participation, involvement, and promote student interest, teachers' observation and comments reflect a developing awareness of the benefits VMs provide. This was evident in the structured approach to mathematics being implemented due to the new curriculum. Instead of students repeatedly following procedural steps, VMs provided new avenues to deepen students' conceptual understanding. The research group specifically noted the value of students visually interacting with mathematical structures, transitioning from passive recipients of information (whether from a teacher or textbook) to active participants in their learning. The use of VMs as an instruction aide during whole-class delivery streamlines the modelling process, replacing the physical challenges of traditional manipulatives with a shared representation that is easily visible for all students.

Furthermore, this study concludes that VMs serve as a supportive tool for managing cognitive load, particularly for diverse learners. While the benefits of VMs for students with learning needs are well-documented (Long et al., 2023; Nugent, 2000), the teachers' observations regarding the drawing of diagrams were particularly noteworthy. The finding that VMs prevent frustration by removing the requirement for perfect (in the mind of the student) hand drawn diagrams is more than a matter of convenience, but a matter of instructional equity. When the fine motor skills of drawing representation are separated from the meaning of the mathematics, students can dedicate their full cognitive attention to the underlying mathematical concept. This suggests that the use of VMs is not just an alternative tool to support instruction, but a more efficient use of time that minimises cognitive overload.

6.1.4 The TPACK Sustainability Paradox: Depth Versus Efficiency

This study concludes that while a 'learning-by-design' structure is a key element in developing deep Technological Pedagogical Content Knowledge (TPACK), the long-term viability of this approach is challenged by the pragmatic realities of teacher workload. The intervention demonstrated that when teachers actively engage in collaborative design of VM incorporated lessons, they move beyond surface level tech integration to a thoughtful synthesis of content, pedagogy, and technology. However, this study also reveals a persistent tension. The very process that creates ownership and deepens understanding is the same process that teachers fear is unsustainable due to their lack of time and demands of a congested curriculum.

The suggestion from the research group to provide 'ready-made' resources for their colleagues highlights a challenge in professional development. This research concludes that the composition of a series of ready to go lessons, while effective for initial teacher acceptance and immediate relief in workload, risks sidestepping the essential work required to truly develop teacher's TPACK. Therefore, this study concludes that a hybrid model of dissemination is required where ready-made templates are provided alongside modelling of a "learning-by-design" approach during staff meetings.

6.2 Implications for Practice

As the findings indicate, the use of virtual manipulatives had a range of benefits to the teachers involved. Moving forward, the use of Gen AI to custom build VMs provides not just greater efficiency for teachers but allows them to be co-creators in the design of the VM ensuring that it meets the cognitive needs of the students. The use of refined and carefully constructed VMs that respond to the students' gaps in understanding has the potential to increase maths standards and lower teacher workload.

The research also identified how VMs can support diverse learners by helping to remove the frustration and cognitive load when drawing diagrams and representations. Encouraging teachers to explore ways in which VMs can reduce the cognitive load and allow students to focus on the core concepts of the lesson provides greater instructional equity.

At a school leadership level, the findings of this research suggest that professional development involving technology needs to find a balance between deep pedagogical understanding with the current demands of teacher workload. As mentioned, a hybrid model that provides resources that can support the initial implementation of new technology while developing teachers' TPACK through modelling and a learning by design approach provides the best chance of success.

6.3 Limitations

Firstly, this study comprised of a small purposeful sample of seven teachers from a single intermediate school. Due to this small sample size, the external validity and generalisability of the findings are limited. The results reflect the specific cultural and institutional context of one site rather than the broader teaching population. However, many of the findings align with existing literature, enhancing the transferability of the study, and allows other practitioners to determine the applicability of these insights to their own professional settings.

Secondly, my positionality as an insider researcher created a dual role conflict. As a fellow teacher, I benefited from an established rapport and a deep understanding of the site's context. However, I recognised participants can be influenced by loyalty or a social desire to be liked and say what they believe the researcher wants them to say. To mitigate researcher bias, member checking was employed. Participants were invited to review transcripts and confirm that the findings reflected their authentic perspectives. During the study, I also met regularly with my supervisors to debrief on my progress and discuss the best approach to mitigate any researcher bias.

Finally, the temporal scope of the study presents a limitation. The professional development intervention occurred over a 5–6-week period, which captured the initial adoption phase rather than any long-term pedagogical change. In addition, although a mixed method action research approach, the impact of the professional development relied primarily on the qualitative data. While this provided rich insights into teacher perception, the lack of longitudinal quantitative data means the long-term impact on teachers' use of virtual manipulatives could not be statistically measured.

6.4 Suggestions for future research

As identified, one of the limitations of this study was the small sample size of teachers from a single intermediate school. A logical progression for research into VM use would be to expand this study to include a range of different schools at different educational levels. The need for conceptual understanding at younger ages is important, especially with the expectations of the newly refreshed curriculum. A second limitation was the limited temporal scope of the study. A longitudinal study could be conducted to see if teachers continue to use AI for resource design once the initial professional development support is removed.

This study focused on teachers' use of virtual manipulatives. Further research that focuses on the student use of VMs in developing their conceptual mathematical understanding would provide data on the impact of student VM use. Additionally, future research could explore how students feel their agency is impacted when using tools designed specifically around their own thinking.

As the integration of Gen AI in education is an emerging field, further investigation into its capacity to facilitate the creation of custom-built VMs is warranted. Specifically, future studies could use comparative methodologies to compare the impact of these tailored VMs against that of generic resources. In addition, given the time demands on teachers identified in this study, research into how Gen AI affects teacher efficacy and reduces implementation demands of VM use would provide valuable insights into the sustainability of such innovations.

6.5 Concluding thoughts

Utilising the CLES-VM at the start of this study provided evidence of the perceived gap in teachers' virtual manipulative knowledge and use. Due to my mathematical background, I was initially more comfortable working with the statistical quantitative data. In comparison, the prospect of analysing the qualitative data collected during the action research phase was daunting. Qualitative data is messy and unstructured, and during my analysis I often had to pause to question and rethink my approach in search for clarity. The iterative nature of thematic analysis, although time consuming, led to me becoming deeply emersed in the data, ultimately strengthening my understanding. As I reflect on this journey, I have become more comfortable and gained a greater appreciation for the benefits of qualitative data and its ability to provide insight and meaning as well as its openness that provides opportunities for unexpected findings.

While this research began by questioning how a professional development intervention shapes teachers' perceptions and instructional use of virtual manipulatives, the findings ultimately point toward a broader transformation in the relationship between the teacher and the digital tool. The integration of generative AI showed that the pragmatic barriers of generic VMs can be reduced, repositioning the teacher not as a mere user of technology but as a creator of custom-built digital tools that meet the mathematic needs of the students. As identified earlier, the future of VM use may lie not in the continued collection of additional VMs, but in fostering teachers' capacity to create tailored digital tools in direct response to student thinking. Ultimately, as the digital landscape continues to shift, the lasting value of this work lies in the reminder that the most effective classroom innovations are those that empower the teacher to remain the primary facilitator of the learning experience.

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Appendices

Appendix A

Principal letter and consent

James GOATLEY
University of Waikato Masters student
[redacted]

20/05/25

Dear [redacted],

I am writing to seek your permission to conduct a short-term research project at Otumoetai Intermediate School as part of my Master's thesis. This project will involve working with a group of teachers, and I have provided a summary of my research intentions, an overview of the study, and a consent form for your review. Initially I would like to ask teachers to complete a short questionnaire, this should take no longer than five minutes in duration and is completely voluntary. I have attached this form for your consideration. There will be an opportunity for teachers who wish to be involved with the main research group to indicate their interest. A small group of teachers who have indicated interest will be invited to work with me in closer association to explore the research in a more a depth manner. This group of teachers will be the action research group in which I intend to work with on the mathematics project. Please feel free to contact me if you have any questions or would like to discuss the research further.

This project proposes to take place across term two, sits within the mathematics curriculum, and investigates the use of virtual manipulatives to support whole-class teaching. The project's title is 'Building Conceptual Understanding: The Role of Virtual Manipulatives in Whole-Class Mathematics Teaching.' It aims to understand better how the use of virtual manipulatives can be used by teachers to support their instruction to the whole-class.

The study follows a Participatory Action Research (PAR) approach, meaning that teachers will be actively involved in shaping the research and its outcomes. To achieve the study's aim, the teachers and researcher will be involved in professional development sessions that will model the use of virtual manipulatives in relation to the topics being covered in class. After the teachers have trialled using the virtual manipulatives, the teachers and researcher will meet to reflect and share experiences.

At present, there is a growing focus on ensuring all students develop strong mathematical foundations through engaging and effective teaching practices. With the recent implementation of the refreshed New Zealand mathematics curriculum, there is an emphasis on all students experiencing the full range of mathematical concepts outlined for their year level. This shift has seen a move towards a whole-class approach to teaching mathematics, ensuring that every student has equitable access to high-quality learning opportunities.

Despite the curriculum directive, teachers may face challenges in implementing whole-class mathematics instruction in a way that supports diverse learning needs. Virtual manipulatives provide a promising avenue for enhancing conceptual understanding and engagement, yet there is minimal research on how professional learning can support teachers in effectively integrating these tools into their practice. This research aims to explore how targeted professional development can help teachers build confidence and capability in using virtual manipulatives to support whole-class mathematics teaching. The study seeks to contribute to the broader understanding of how digital tools can enhance learning outcomes and align with the goals of the new mathematics curriculum.

All information about the participants will be treated confidentially, with pseudonyms used for participants, school, teacher and area names. While every effort will be made to ensure confidentiality, anonymity can't be guaranteed. Participants have the right to withdraw up until analysis has begun. At the end of the study, a copy of the research's findings will be sent to your school.

If you wish to contact me directly for further clarification, please call me at +64 [redacted] or email me at [redacted] If there is a need, you may contact my chief supervisor: Dr Simon Taylor [redacted]

Thank you.

Yours faithfully

James Goatley

This research has been approved by the University of Waikato Faculty of Education Ethics Committee on May 13th, 2025. Approval number: FEDU012/25.

Project title: Building Conceptual Understanding: The Role of Virtual Manipulatives in Whole-Class Mathematics Teaching.

Please tick in the boxes if you agree with the following:

☐ I have read and understood the information sheet providing information and explanation of the research project's nature and purpose. I consent for the school to participate in this study. I understand that I may withdraw the school from the research project up until analysis has begun. I understand that the school's participation in this study is private and that no material which could identify the school, or teachers will be used in any reports on this study.

Name of Principal: ....

Signature..Date..........................

Appendix B

Letter and consent for CLES participants

James Goatley
University of Waikato Masters student
[redacted]

30/05/25

Tēnā koe,

As part of my current post graduate Master's research for 2025 here at the school, I am conducting a Constructivist Learning Environment Survey (CLES). The aim of my research is to better understand how professional learning can support teachers to use virtual manipulatives effectively in mathematics teaching. This questionnaire is intended to gather a general picture of the current learning environment in mathematics classrooms.

Participation Information

- Participation in this questionnaire is completely voluntary. The questionnaire will take approximately 5–10 minutes to complete.
- Responses are anonymous and will not be linked to your name or school in any reporting.
- You are free to decline participation without any consequence.
- By submitting the completed questionnaire, you are giving consent for your data to be included in the research.

Consent Declaration

Please tick each box to indicate your agreement:

☐ I have read and understood the information above.
☐ I understand that my participation is voluntary and I can choose not to take part.
☐ I understand that my responses will be anonymous and confidential.
☐ I give consent for my responses to be used in this research.

Name of participant:..

Signature...Date..........................

This research has been approved by the University of Waikato Faculty of Education Ethics Committee on May 13th, 20025. Approval number: FEDU012/25

Appendix C

Information Letter and consent for staff participants

James GOATLEY
University of Waikato Masters student
[redacted]

10/05/25

Tēnā koe,

The following is an invitation to be part of my current post graduate Master's research for 2025 here at the school. Thank you for completing the initial questionnaire and indicating you were interested in being part of the research group. The following information explains what the research will involve and asks for your consent to be involved in the study.

This research proposes to take place across term two, sits within the mathematics curriculum, and investigates the use of virtual manipulatives to support whole-class instruction. It aims to understand better how professional learning sessions can impact teachers to incorporate virtual manipulative tools in mathematics instruction.

Summary of what Participation Involves

The study follows a Participatory Action Research (PAR) approach, meaning that you will be actively involved in shaping the research and its outcomes. If you choose to take part in this stage of the study, you will be involved in:

- One focus group discussion (approx. 60 minutes)
- 1-2 professional learning session (approx. 30-40 minutes)
- Trialling the use of virtual manipulatives in your classroom
- A follow-up focus group (approx. 60 minutes) to reflect on your experience
- Participation is expected to take around 3-4 hours over a 5-6 week period.

Significance of Research

As you are aware, there is a push to ensure that all students build solid mathematical foundations through engaging and effective teaching. With the recent introduction of the refreshed New Zealand mathematics curriculum, there is a clear expectation that all students will experience the full range of mathematical concepts outlined for their year level. This has led to a shift towards a whole-class approach to teaching mathematics, ensuring that every student has access to meaningful learning experiences. However, implementing whole-class mathematics instruction can be challenging, particularly when catering to students with diverse learning needs. Virtual manipulatives offer a promising way to enhance student understanding and engagement, yet many teachers have had limited opportunities to explore their full potential in a whole-class setting.

Confidentiality and right to withdraw

All information about the participants will be treated confidentially, with pseudonyms used for participants, school, teacher and area names. While every effort will be made to ensure confidentiality, anonymity can't be guaranteed. Participants have the right to withdraw up until data analysis begins. If you choose to withdraw before this, any data you have provided will be destroyed and not included in the study.

If you wish to contact me directly for further clarification, please call me at +64 0211909588 or email me at jgoatley@otuinter.school.nz. If there is a need, you may contact my chief supervisor: Dr Simon Taylor simont@waikato.ac.nz

Thank you.

Yours faithfully,

James Goatley

<u>Consent Form for Teacher Participants</u>

Project title: Building Conceptual Understanding: The Role of Virtual Manipulatives in Whole-Class Mathematics Teaching.

Please tick each box to indicate your agreement:

☐ I have read and understood the information above.

☐ I understand what participation involves and the time commitment.

☐ I understand that my participation is voluntary and I can withdraw up until data analysis begins.

☐ I understand that data will be kept confidential and used only for research purposes.

☐ I give consent to take part in the main research group as described

Name of participant:..

Signature...Date..........................

Appendix D

CLES-VM

What happens in my mathematics lessons?

Teacher form 2025

I have been teaching for:	Current year level:	My gender:
	7 / 8 / Both	

DIRECTIONS

1. **Purpose of the Questionnaire**
This questionnaire asks you to describe important aspects of your mathematics classroom. There are no right or wrong answers. <u>Your opinion is what is wanted</u>. Your answers will help to guide the direction of future research.

2. **How to Answer Each Question**
On the next few pages you will find 25 sentences. For each sentence, circle <u>only one</u> number corresponding to your answer. For example:

		Almost Always	Often	Some-times	Seldom	Almost Never
In this class . . .						
8	I ask students questions.	5	4	3	2	1

- If you think you *almost always* ask students questions, circle the 5.
- If you think you *almost never* ask students questions, circle the 1.
- Or you can choose the number 2, 3 or 4 if one of these seems like a more accurate answer.

3. **How to Change Your Answer**
If you want to change your answer, <u>cross it out</u> and circle a new number, For example:

8	I ask students questions.	~~5~~	4	3	2	1

4. **Completing the Questionnaire**
Now turn the page and please give an answer for <u>every</u> question.

			Almost Always	Often	Some-times	Seldom	Almost Never
Learning about the maths classroom							
In my lessons . . .							
1	I see the whole-class approach as a meaningful way to support students' mathematical understanding.		5	4	3	2	1
2	The new curriculum aligns with how I believe maths should be taught		5	4	3	2	1
3	I create an environment where students can learn from mistakes and see them as part of the learning process.		5	4	3	2	1
In this class . . .							
4	Ensuring all students experience every statement at their year level is/will be challenging.		5	4	3	2	1
5	Students are engaged when using a whole-class approach		5	4	3	2	1
Learning about the use of manipulatives			Almost Always	Often	Some-times	Seldom	Almost Never
In my lessons . . .							
6	I have a good understanding of integrating manipulatives		5	4	3	2	1
7	The use of manipulatives enhances my ability to teach mathematical concepts		5	4	3	2	1
In my lessons . . .							
8	Students have access to virtual manipulatives to support understanding		5	4	3	2	1
9	I have experimented using virtual manipulatives for instruction.		5	4	3	2	1
10	Students are engaged when using virtual manipulatives		5	4	3	2	1
Learning to speak out			Almost Always	Often	Some-times	Seldom	Almost Never
In this class . . .							
11	It's OK for students to ask me "why do I have to learn this?"		5	4	3	2	1
12	It's OK for them to question the way they are being taught.		5	4	3	2	1
13	It's OK for them to complain about activities that are confusing.		5	4	3	2	1
In this class . . .							
14	It's OK for them to complain about anything that prevents them from learning.		5	4	3	2	1

15	It's OK for them to express their opinions.		5	4	3	2	1

Learning to learn			Almost Always	Often	Some-times	Seldom	Almost Never
In this class . . .							
16	I am open to adjusting my approach based on student engagement with whole-class activities.		5	4	3	2	1
17	Students' learning progress influences how I pace and sequence whole-class lessons.		5	4	3	2	1
18	I reflect on how well whole-class activities are meeting the needs of all students.		5	4	3	2	1
In this class . . .							
19	Students help me to decide how much time they spend on activities.		5	4	3	2	1
20	Students help me to decide which activities they do.		5	4	3	2	1
Learning to communicate			Almost Always	Often	Some-times	Seldom	Almost Never
In this class . . .							
21	Students get the chance to talk to other students.		5	4	3	2	1
22	Students talk with other students about how to solve problems.		5	4	3	2	1
23	Students explain their ideas to other students.		5	4	3	2	1
In this class . . .							
24	Students ask other students to explain their ideas.		5	4	3	2	1
25	Other students listen carefully to class mates' ideas.		5	4	3	2	1
			Almost Always	Often	Some-times	Seldom	Almost Never

If you wish you can add any written comments about their learning here:

Appendix E

Reflective journal extract

17/8/25

Readings this week.

- Towards good practice in thematic analysis: Avoiding common problems and becomming a knowing researcher. BRAUN AND CLARKE (2023)
- A worked example of Braun and Clarke's approach to reflexive thematic analysis BYRNE (2021)

Reading Byrne's worked example and the challenges of identifying themes I think I need to have a rethink. For some time I have felt like I was making progress but the more I look at my themes and codes, the more lost I have become. I think I have realised that I am coming at this from the wrong direction and not focusing on how the participants perceptions have changed. I think I have been too caught up in looking for general themes and connections.

I am going to go back to my intial codes and look for different groupings and connections. Maybe look for some other software options other than excel.

- Idea from reading

  - Use Braun and Clarke's key questions
    - Is a theme truely a theme or just a code
    - If it is a theme, what is the quality of this
    - what are the boundaries of the theme
    - Are there enough meaningful data to support this theme
    - Are the data too diverse and wide ranging.

Things to do -
- Get in touch with Nigel - arrange meeting
-