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DYNAMICAL FINITE-AMPLITUDE MAGNETIC RECONNECTION AT AN X-TYPE NEUTRAL POINT

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ABSTRACT

The linear theory of magnetic reconnection demonstrates that the rate of energy release at an X-type neutral point is "fast"—only logarithmically dependent on the plasma resistivity—if the field is strictly two-dimensional, gas pressure is absent, and perturbations are small. The present paper explores the response of the X-point to finite amplitude disturbances under the more realistic conditions of limited compressibility and a finite nonplanar magnetic field component.

We show that fast reconnection is not inhibited by large amplitudes of the perturbation—in fact, both the reconnection rate and the ohmic dissipation rate increase with decreasing plasma resistivity. This "super fast" scaling can be understood by a simple, one-dimensional dynamic collapse model.

However, the presence of finite gas pressure or an axial magnetic field component stalls the collapse by providing backpressure which retards the imploding magnetic wave: the current sheet is prevented from thinning and reconnection drops toward the static diffusion rate. Thus we overrule the *initial* implosion as a means of rapidly liberating magnetic energy when gas pressure or an axial magnetic field are present, as they are in the solar corona.

But in this case the initial collapse does not provide the complete picture. Gas is subsequently squeezed out of the current sheet, allowing a higher current density to be attained. Thus the possibility of fast reconnection remains open. The dynamics of the evolution are complicated; it is not yet clear under exactly what conditions fast reconnection may be attainable.

Subject headings: MHD — Sun: activity — Sun: flares — Sun: magnetic fields

1. INTRODUCTION

The central problem of solar flare theory is that the normal rate of resistive diffusion in the solar atmosphere is extremely small: for macroscopic structures the diffusion time, on which magnetic energy can be liberated, is of order 10^{15} Alfvén times, or 10^8 yr. On the other hand, magnetic energy is released during flares and other transient events on a timescale of 10–100 Alfvén times, in agreement with the behavior of laboratory plasmas (e.g., Tsuneta 1993). To dissipate magnetic energy so rapidly, either an extremely short length scale or a greatly enhanced resistivity is required. Plasma microinstabilities which raise the effective collision frequency can enhance the resistivity by at most a factor of perhaps 10^6, so very short length scales are still essential to the process. Near-singular current sheets arise naturally in the solar atmosphere when topologically distinct magnetic structures collide. Since flares may result from such dynamic rearrangement of magnetic fields following a large-scale magnetohydrodynamic instability, in this paper we examine the rates of magnetic reconnection and of magnetic energy dissipation attained during the initial collapse of these current sheets. Energy release might be expected to occur on the Alfvén time for large resistivities, and to slow down for smaller amounts of resistivity. Thus the energy release rate must have a very weak dependence on resistivity if the rate decreases by a factor of only 10–100 for a factor of at least 10^6 decrease in resistivity. This suggests a logarithmic dependence.

Recently, several authors have developed a linear theory for the dissipation of small disturbances imposed on the potential magnetic field of an X-type neutral point in a plasma of negligible gas pressure. Craig & McClymont (1991) and Hassam (1992) showed that reconnective perturbations decay resistively on a "fast" timescale $\sim |\ln \eta|^2$, where η is the nondimensional resistivity. Craig & McClymont (1993) extended this treatment to include nonreconnective perturbations which displace field lines without changing their topology, and demonstrated that these also decay "fast" ($\sim |\ln \eta|^3$). Craig & Watson (1992) showed that these logarithmic scalings represent only a lower limit to the effectiveness of dissipation, and that perturbations generated at the boundaries on the Alfvén timescale decay in just a few Alfvén times. The above work demonstrates that the required small length scale occurs naturally in a two-dimensional magnetic field containing an X-type neutral point, provided that gas pressure is neglected, there is no axial magnetic field, and perturbations are small. Further studies of linear reconnection were carried out by Ofman, Morrison, & Steinolfson (1993) and Roumeliotis & Moore (1993).

The linear theory is made tractable by the fact that the acceleration $\boldsymbol{J} \times \boldsymbol{B}$ is perpendicular to the field lines: the introduction of gas pressure or a nonplanar component of the magnetic field upsets this orthogonality. To examine the effect of these parameters we must resort to numerical methods. The study of finite amplitude disturbances also requires a numerical treatment. For small enough perturbations (amplitude $\lesssim \eta$) the current which builds up at the neutral point is a cylindrical line current whose radius scales as $\eta^{1/2}$ and whose amplitude scales as η^{-1}. For larger disturbances, the exponentially growing inward-propagating wave overwhelms the background field before reaching the diffusion region around the neutral point. The collapse then becomes highly nonlinear, cylindrical symmetry is lost, and the line current flattens into a two-dimensional current sheet (Craig & Watson 1992). The nature of the nonlinear collapse and formation and dissipation of the current sheet is the subject of the present investigation.

We find that for large perturbation amplitudes the reconnection rate actually *increases* as η is reduced, provided gas pressure and axial magnetic field are absent. These results show excellent agreement with the one-dimensional dynamic collapse analyzed by Forbes (1982), which predicts the "super fast" rates of flux transfer, $d\psi/dt \sim \eta^{-0.045}$, and of ohmic heating, $d\epsilon/dt \sim \eta^{-0.198}$. Craig & McClymont (1993) suggested, on the basis of their linear analysis, that the presence of any significant gas pressure or axial magnetic field would inhibit fast reconnection. We verify that the reconnection rate associated with the initial implosion indeed stalls, in common with the linear problem. However, in the nonlinear problem, the current sheet reforms after expelling some of the gas, and reconnection takes place at a faster rate than in the initial implosion. The dynamics are complicated and not yet fully explored. Here we show that the gas is expelled from the current sheet at the local sound speed, and argue that for a sufficiently large magnetic perturbation, and a sufficiently small gas pressure (comparable to the coronal gas pressure), fast reconnection and energy release may be feasible.

The assumptions and equations of the model are described in § 2; in § 3 we present results of the simulations, and in § 4 we discuss our results in relation to other work.

2. THE DYNAMIC X-POINT MODEL

We first introduce the X-point equations before going on to outline the numerical methods we use to compute solutions. Sections 2.2 and 2.3 provide a theoretical rationale for the collapsing arbitrarily compressible X-point. In particular, we present scaling laws to be expected during the collapse.

2.1. *The Dynamical Evolution Equations and Their Numerical Simulation*

We examine the dynamic evolution and decay of finite amplitude perturbations imposed on an X-point potential magnetic field. The system is $2\frac{1}{2}$ dimensional: we allow for a velocity component and a magnetic field component in the z-direction, but neglect gradients in that direction. In order to examine the behavior of simple modes, we impose symmetry, so that the development is the same in all four quadrants of the square domain. (For computational simplicity we adopt a Cartesian system rather than the cylindrical coordinates employed for analytic purposes.) The simulation describes compressible adiabatic magnetohydrodynamics, neglecting the heat produced in the dissipation region. The variables comprise the density, ρ, the three components of velocity, v_x, v_y, and v_z, the flux function ψ, and the axial component of the magnetic field, B_z. The planar (x and y) components of the magnetic field are given by $B_\perp = \nabla \times \psi\hat{z}$. Since z is an ignorable coordinate, our vector operators will denote only differentiation with respect to x and y. We denote the planar components of velocity by $\boldsymbol{v} = (v_x, v_y)$. The evolution equations then consist of the continuity equation,

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v}) , \qquad (2.1)$$

the momentum equations,

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} - \nabla\left(P + \frac{1}{2} B_z^2\right) - \nabla^2\psi\nabla\psi + \rho \boldsymbol{V} \qquad (2.2)$$

and

$$\rho \frac{\partial v_z}{\partial t} = -\rho(\boldsymbol{v} \cdot \nabla)v_z + (\boldsymbol{B} \cdot \nabla)B_z + \rho \boldsymbol{W} , \qquad (2.3)$$

the induction equations

$$\frac{\partial \psi}{\partial t} = -(\boldsymbol{v} \cdot \nabla)\psi + \eta\nabla^2\psi \qquad (2.4)$$

and

$$\frac{\partial B_z}{\partial t} = -(\boldsymbol{v} \cdot \nabla)B_z - B_z \nabla \cdot v + (\boldsymbol{B} \cdot \nabla)v_z + \eta\nabla^2 B_z , \qquad (2.5)$$

together with the equation of state

$$P = P_0(\rho/\rho_0)^\gamma . \qquad (2.6)$$

The viscous terms, $\boldsymbol{V}$ and $\boldsymbol{W}$, include both physical viscosity (with a constant coefficient), and artificial viscosity, whose tensor form is modeled on the physical viscosity (e.g., Potter 1973) but whose coefficient is given by the Tyler form (Roache 1982), proportional to the sum of the advection speed and wave propagation speed. In most simulations, the physical viscosity is set to zero and artificial viscosity employed to control overshoot in steep wavefronts.

Finite difference analogs of these equations are solved as follows. First, the continuity equation and momentum equations are written in conservative form, and advanced from time t^j to t^{j+1} using the Lax-Wendroff method, then the induction equation, in nonconservative form, is advanced from t^j to t^{j+1} using an implicit Crank-Nicholson scheme, solved by the alternating-direction-implicit method (see, e. g., Potter 1973). The resulting system is time-centered and second-order accurate except for the magnetic force in the momentum equation, which is evaluated at the old time level. This scheme was first used by Craig & Watson (1992); more recently it has been applied by Rickard & Craig (1993) in their study of the coalescence instability, and by Craig, Henton, & Rickard (1993) in their periodic X-point analysis.

An independent code based on an adaptation of the semi-implicit method (Mikic, Barnes, & Schnack 1988), was used to check the results. In this case viscosity was represented by a simple scalar diffusion, $a\nabla^2\boldsymbol{v}$. Despite their differences, the two codes agree very well.

The initial magnetic field consists of the equilibrium X-point field,

$$\psi_0 = \tfrac{1}{2}(x^2 - y^2); \; B_z = \text{constant} \qquad (2.7)$$

with the perturbation

$$\delta\psi = \tfrac{1}{2} \cos \alpha \, (1 - x^2)(1 - y^2) \qquad (2.8)$$

imposed, where α is the angle between the perturbed separatrices near the neutral point (90° for the equilibrium field). This perturbation is the cylindrically symmetric perturbation $\delta\psi \propto 1 - r^2 = 1 - (x^2 + y^2)$ giving a uniform initial current density, modified to leave the flux distribution at the square boundaries unperturbed. The density and axial magnetic field are taken to be uniform initially, and the velocity is set to zero. The boundaries at x, y, $= \pm 1$ specify line-tying and are impermeable. In these units, the Alfvén speed at the outer boundary is unity, and time is measured in Alfvén times (the distance from the outer boundary to the neutral point divided by the Alfvén speed at the outer boundary).

The above perturbation moves magnetic flux into two opposing lobes of the X-point field at the expense of the other two, so that when the system is released at $t = 0$, the lobes expand in an attempt to return to equilibrium. However, the flux cannot return to the other lobes without passing through the neutral point, i.e., reconnecting. As the opposing lobes of the field drive together, a current sheet forms along the central part of the y-axis.

2.2. *A Scaling Argument for the Collapsing X-point*

For large perturbations, the wave of current which converges on the neutral point behaves differently from the cylindrically symmetric current of the linear regime. In the linear case, we equate the wave speed ($\propto r$) to the diffusion speed, η/r, to obtain the radius of the diffusion region, $r_c = \eta^{1/2}$. In the nonlinear case, the magnetic field of the wave, which is basically in the azimuthal direction, i.e., B_ϕ, overpowers the background field, whose azimuthal component is $B_{0\phi} \propto r \cos 2\phi$, at a macroscopic radius. As the perturbation sweeps in, it first cancels the background field on the y-axis: the radius at which this happens defines the length of the nascent current sheet. Craig & Watson (1992) show that this length scales as $\epsilon^{1/4}$, where ϵ is the energy of the perturbation. From that point on, the collapse takes on a one-dimensional character, with the magnetic wave sweeping in from the sides until the speed at which energy is swept in by the wave matches the diffusion speed, or $\eta/w \simeq v_A$, where w is the width of the current sheet and v_A is the Alfvén speed near the sides of the sheet.

Because the evolution is nonlinear, the Alfvén speed cannot be identified with that of the background X-point field. In fact both the plasma density and the magnetic field are affected by the quasi one-dimensional collapse. Consider first the distortions of the magnetic field: we expect these to provide the greatest influence on the local Alfvén speed. We know from considerations of the magnetofrictional collapse of an arbitrarily compressible X-point that the magnetic field at the edges of the sheet, B_s, saturates at some level independent of η. In fact, the singular Y-point configuration—the end result of the collapse—implies that $B_s \propto L$, where L is the length of the Y-point singularity (e.g., Craig 1994). Therefore, we expect that, to first order, the field strength B_s is independent of η.

Consider now the density inhomogeneity. As the density builds up at the head of the wave, a hole is carved out at the rear, at least within a radius L of the nonlinearity. This exacerbates any tendency for the wave to localize as it propagates inward. The extent of the density inhomogeneity depends on the severity of the nonlinearity; a precise description requires a detailed dynamic analysis. If we neglect the density inhomogeneities we obtain the approximate scalings,

$$w \propto \eta\ , \quad B \propto \epsilon^{1/4}\eta^0\ , \quad J \propto \epsilon^{1/4}\eta^{-1}\ , \tag{2.9}$$

with $v_A \propto B/\rho \propto \eta^0$ invariant. These scalings suggest that the flux transfer rate $d\psi/dt = \eta J$ and the ohmic heating rate $d\epsilon/dt = \eta J^2 wL$ are independent of η, consistent with fast reconnection. We now examine how these approximate results hold up when compared to an exact model of one-dimensional collapse.

2.3. *One-Dimensional Collapse Model*

The one-dimensional collapse has been studied by Forbes (1982), based on an analysis by Forbes & Speiser (1979). Forbes considers the collapse of a "diffuse current sheet" of uniform magnetic field gradient, surrounded by uniform field

$$B_y = \begin{cases} -1 & x < -1 \\ x & -1 < x < +1 \\ +1 & x > +1 \end{cases} \tag{2.10}$$

with initially uniform density, $\rho = 1$. Gas pressure is neglected. In the region between the characteristics propagating in from the discontinuities at $x = \pm 1$, the time-dependent solution for the density ρ, velocity v_x, and field strength B_y, are given by

$$\rho^{-1}\sqrt{\rho - 1} + \tan^{-1}\sqrt{\rho - 1} = \sqrt{2}t\ , \tag{2.11}$$

$$v_x = -\sqrt{2}\sqrt{\rho - 1}\rho x\ , \tag{2.12}$$

$$B_y = \rho^2 x\ , \tag{2.13}$$

while the half-thickness of the current sheet is given by

$$w = \rho^{-1}(\sqrt{\rho} + \sqrt{\rho - 1})^{-\sqrt{2}}\ . \tag{2.14}$$

Equation (2.11) indicates that, in the absence of resistivity, a singularity is reached in finite time $t_c = \pi/[2(2)^{1/2}] \simeq 1.1$ Alfvén times. We assume that the collapse is halted when the inflow speed is balanced by resistive diffusion across the thickness w, and that the resistivity is small enough to allow the density to build up to $\rho \gg 1$. Then we find the following scalings (after evaluating a weird assortment of $(2)^{1/2}$'s)

$$\rho \sim \eta^{-0.522}\ , \quad w \sim \eta^{0.892}\ , \quad B_s \sim \eta^{-0.153}\ , \quad J \sim \eta^{-1.045}\ . \tag{2.15}$$

Solution (2.15) provides scalings that are consistent with the approximate description of § 2.2. This suggests that, as anticipated, density inhomogeneities have only a secondary influence on the gross properties of the collapse. In the two-dimensional X-point field, we can expect these scalings to hold once the sheet has collapsed to the point where the perturbation field overwhelms the background field.

3. RESULTS

3.1. *Time of Peak Dissipation Rate*

To quantify the reconnection rate in a dynamic simulation, we use the peak rates of flux transfer and of ohmic heating attained as the opposing flux first sweeps in toward the neutral point. This peak is attained after one or two Alfvén times.

For small perturbations (Craig & McClymont 1993), the wave speed is determined by the background equilibrium field ($v_A \propto r$), so the time at which the perturbation reaches the diffusion region is about $\frac{1}{2}|\ln \eta|$, which for small η is many Alfvén times. In contrast, in the nonlinear case, the wave speed exceeds the Alfvén speed of the background field, so a much weaker dependence of the time to peak dissipation on the resistivity is to be expected. In the one-dimensional nonlinear collapse with $\eta = \beta = 0$ (Forbes 1982), the current sheet becomes singular at time $t_{\rm crit} = \pi/[2(2)^{1/2}] \approx 1.11$. The changeover from linear to nonlinear behavior should occur for perturbation amplitudes $\delta\psi/\psi_0 \simeq \eta$ (Craig & McClymont 1993).

Figure 1 shows the time of peak reconnection rate as a function of η for zero gas pressure and a fixed perturbation amplitude, $\delta\psi/\psi_0 \simeq 2.5 \times 10^{-2}$. For $\eta \gtrsim 10^{-2}$, the perturbation lies in the linear regime, and the timescale varies in agreement with the linear theory, $\tau = \frac{1}{2}|\ln \eta|$ (for the $m = 0$ mode), as shown by the straight line. For $\eta \lesssim 10^{-2}$, the perturbation becomes more nonlinear as η decreases, and the time to peak dissipation begins to decrease. The change from the linear to the nonlinear regime begins at $\delta\psi/\psi_0 \simeq \eta$, as anticipated.

3.2. *Dependence of Reconnection Rate on Resistivity and Perturbation Amplitude*

Here we examine the scaling of the reconnection rate with resistivity and perturbation amplitude *for a perfectly compressible plasma* ($\beta = B_z = 0$). Do our numerical simulations support the linearized analytic results of Craig & McClymont (1991, 1993) for small perturbations in the absence of gas pressure and axial magnetic field? Does the reconnection remain fast for large-amplitude perturbations? How do the flux transfer rate and ohmic heating rate depend on resistivity in the nonlinear regime? We find that the behavior of the system in the nonlinear regime is quite different from the small amplitude behavior described by the linearized analysis. In fact, the reconnection rate and ohmic dissipation rate become *faster* as the resistivity is decreased, in general agreement with the one-dimensional current sheet collapse discussed by Forbes (1982).

As a measure of the rate of dynamic reconnection, we use the peak flux transfer rate $d\psi/dt = \eta J$, and the peak ohmic dissipation rate $d\epsilon/dt = \int \eta J^2\, dx\, dy \simeq \eta J^2 A$, where A is the current sheet cross-sectional area, for a range of resistivities $10^{-4} \le \eta \le 10^{-1}$, and for three different amplitudes of perturbation. The first perturbation, marked with a cross ($\times$) in the following figures, lies in the linear regime, with amplitude $\delta\psi/\psi_0 = 0.3\eta$. The second, marked with a plus (+), has a fixed amplitude of $\delta\psi/\psi_0 = 0.025$, which we expect to transition from linear (or perhaps marginally nonlinear) behavior for the larger resistivities to nonlinear behavior for the smaller resistivities, Lastly, the third perturbation, marked with an asterisk (*), has amplitude $\delta\psi/\psi_0 = 0.1$, and so should behave nonlinearly across the whole range of resistivities.

In Figure 2 the current sheet thickness is shown as a function of η. For small amplitude perturbations ($\delta\psi/\psi_0 < \eta$) the "thickness"—actually the diameter of the line current—scales as $\eta^{1/2}$, as predicted by the linear theory. For large amplitude perturbations the current sheet thick-

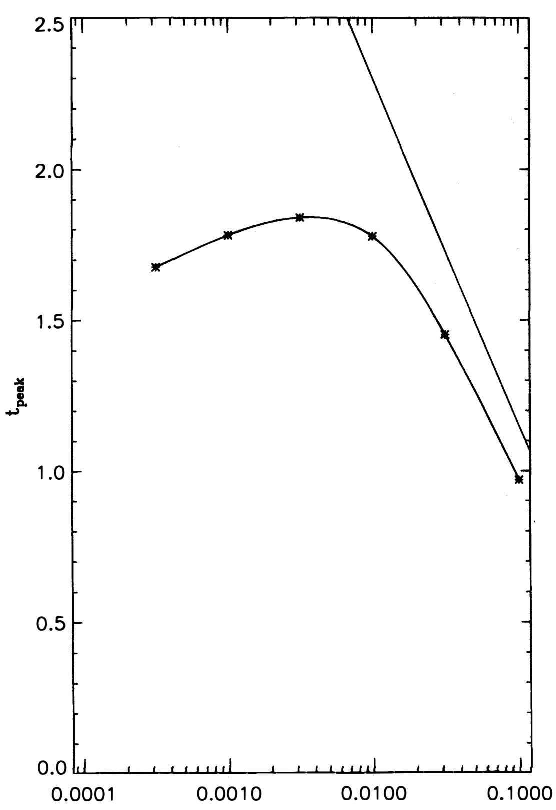

FIG. 1.—Time of peak reconnection rate as a function of η, for zero gas pressure and fixed perturbation amplitude. For $\eta \gtrsim 7 \times 10^{-3}$, the perturbation amplitude ($\delta\psi/\psi_0 \simeq 2.5 \times 10^{-2}$) is small enough to lie in the linear regime, and the timescale varies as $|\ln \eta|$, as shown by the straight line. As η decreases, the perturbation moves into the nonlinear regime, and the time of peak dissipation decreases slowly.

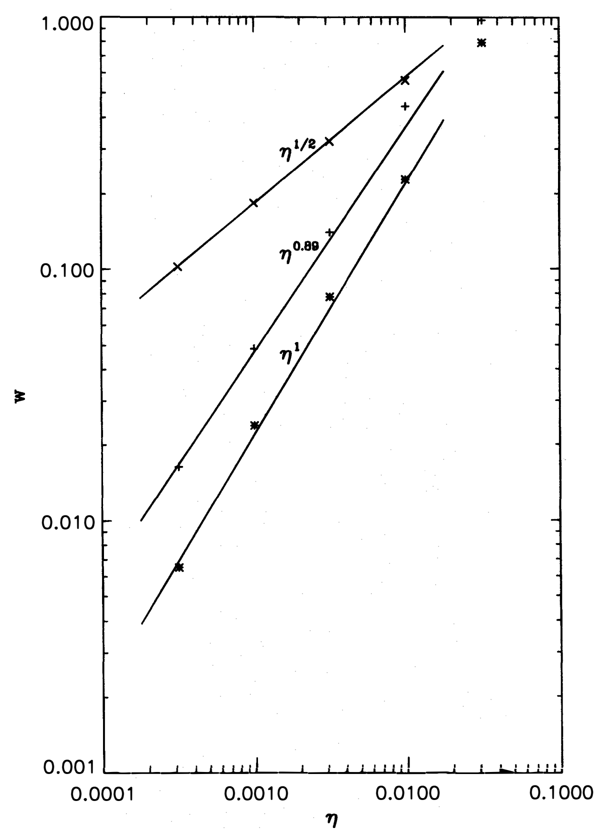

FIG. 2.—Current sheet thickness, w, as a function of resistivity, at the time of peak ohmic dissipation. For small perturbations in the linear regime ($\times$), the "thickness" (diameter of the cylindrical line current at the neutral point) scales as $\eta^{1/2}$, in agreement with the linear theory. Larger perturbations in the nonlinear regime show the current sheet thickness varying approximately as η^1. The scaling $\eta^{0.89}$ given by Forbes (1982) analysis is also shown.

ness scales approximately as η^1. The scaling found by Forbes (1982), $w \propto \eta^{0.892}$, is also shown; our results hint at a scaling faster than η^1, rather than slower.

In Figure 3, we compare the computed reconnection rate with the two-dimensional linear theory (Craig & McClymont 1991) and the one-dimensional nonlinear theory (Forbes 1982). In this plot of peak ohmic dissipation rate versus resistivity, we might expect small perturbations ($\times$) which lie in the linear regime to exhibit the $|\ln \eta|^{-2}$ scaling (shown by the dashed line) found analytically for the azimuthally symmetric $m = 0$ eigenmodes. Clearly the numerical results do not behave in this way in the η range studied: the small-perturbation rates ($\times$) decrease only slightly with decreasing η. This is consistent with the fact that the linear analysis yields an asymptotic result which holds only for $\eta \ll 10^{-4}$, and with the conclusion of Craig & Watson (1992) that disturbances initially localized in the outer field dissipate in a few Alfvén times, rather than on the slower asymptotic timescale. For perturbations in the nonlinear regime (+) and (*) the variation of the dissipation rate shows excellent agreement with the scaling found by Forbes (1982) for the highly nonlinear collapse of a one-dimensional current sheet: $d\epsilon/dt = \eta J^2 A \propto \eta^{-0.198}$ (shown by the solid line). Our approximate treatment (2.9) gives a dissipation rate independent of η: clearly Forbes's detailed description is more accurate.

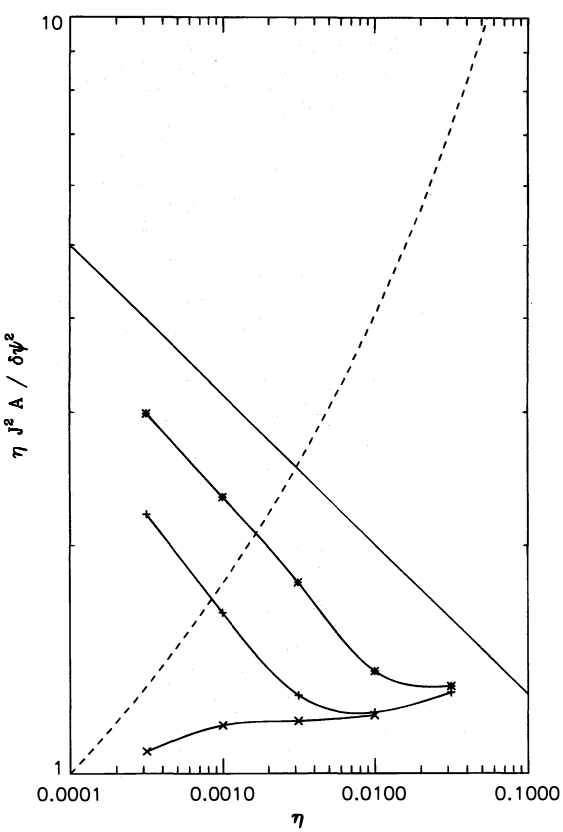

FIG. 3.—Peak ohmic dissipation rate as a function of resistivity, for small linear-regime perturbations (*cross*) medium perturbations (*plus*), and larger nonlinear perturbations (*asterisk*). The results are compared with the two-dimensional linear theory of Craig & McClymont (1993) (*dashed line*), and the one-dimensional nonlinear theory of Forbes (1982) (*solid line*). Small perturbations show signs of behaving as predicted by the linear theory for the smaller resistivities, while the larger perturbations show excellent agreement with Forbes's prediction of an *increasing* dissipation rate with decreasing resistivity.

3.3. *Dependence of Reconnection Rate on Gas Pressure and Axial Field Strength*

The linear theory is valid for negligible gas pressure, meaning that $\beta \ll \eta$, where $\beta = P/(\frac{1}{2}B_0^2)$, and B_0 is the background field strength at the wall. We expect backpressure in the current sheet to slow the reconnection if the gas pressure exceeds this threshold. The scaling with η of the peak reconnection rate attained on the initial collapse of the current sheet, in the presence of gas pressure, is investigated in Figure 4. The reconnection rate begins to fall away from the "fast" rate as η drops below β. For $\eta \ll \beta$, $d\psi/dt \propto \eta^{0.6}$. If the current sheet were in a state of quasi-steady flow, the reconnection should be roughly described by the Sweet-Parker mechanism, which predicts a slope of $\frac{1}{2}$. However, if the gas pressure dominates over magnetic stresses, so that the current sheet remains much too thick to dissipate quickly, reconnection must drop to the static diffusion rate ($d\psi/dt \propto \eta$).

The broken lines in Figure 4 show estimates of the reconnection rate derived from Forbes's (1982) study. Forbes obtains the criterion for fast reconnection $\beta \lesssim 0.2\eta^{0.56}$ (assuming $\rho \gg 1$, and corrected here for numerical errors). We extend his treatment by (a) using the full nonasymptotic form of equations (2.11)–(2.14), and (b) *estimating* the effects of β and η when these are of comparable magnitude by assuming that the effective thickness of the current sheet is given by the sum of the thicknesses due to gas pressure and resistive diffusion each acting alone. The numerical simulation results shown in Figure 4 appear to be consistent with the fast reconnection criterion $\beta \lesssim \eta$ obtained from the linear equations (Craig & McClymont 1993), since, in the "$\beta \gg \eta$" regime, the curves for values of β a decade apart lie a decade apart in η. Neither Forbes's asymptotic criterion nor the analytic curves in Figure 4 show this relationship. Nevertheless, the general features of the analytic and simulation results are in excellent agreement. Appreciable gas

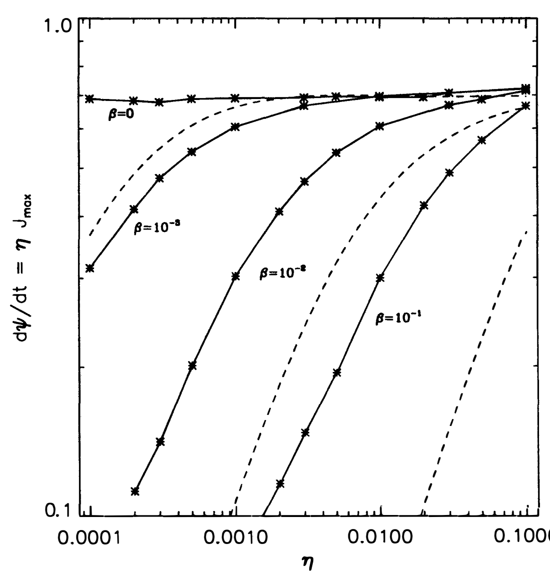

FIG. 4.—The peak flux reconnection rate, $d\psi/dt = \eta J$ at the neutral point, as a function of resistivity η, for different gas pressures, $\beta = 10^{-1}$, 10^{-2}, and 10^{-3}, and well as $\beta = 0$. These results confirm that reconnection (at the first peak) drops below the fast rate whenever $\beta \gtrsim \eta$. The curves corresponding to larger values of β have slopes of $\simeq 0.6$ as they drop away from the fast rate. The dashed lines show the approximate analytic results for $\beta > 0$ estimated from Forbes's (1982) analysis.

pressure (such as exists in the solar corona) undoubtedly inhibits fast reconnection, at least during the initial implosion.

We note that Ofman et al. (1993), in a similar study, conclude that even in the extreme case of gas pressure importance—incompressible plasma in an impermeable box—the reconnection rates are completely unchanged from the linear, zero gas pressure results. Contrary to our results, they also find that a moderate gas pressure ($\beta \simeq 0.1$) *enhances* the reconnection rate over the zero gas pressure case. They appear to find that the dissipation on the first "bounce" is practically unaffected by gas pressure. These results seem counterintuitive.

Craig & McClymont (1993) noted that the pressure of an axial component of the magnetic field, B_z, in the current sheet, acts just like gas pressure in preventing the collapse. The results illustrated in Figure 5 confirm that this is indeed the case (at least during the initial collapse). The curves labeled with values of $\frac{1}{2}B_z^2$ lie on top of the curves in Figure 4 labeled with the same values of β.

3.4. *Beyond the Initial Implosion*

Our main motivation in this paper has been to extend our linear analysis into the regime of finite amplitude perturbations, finite gas pressure, and finite axial magnetic field. To this end we have concentrated on understanding the initial dynamic collapse of the field and the formation of the current sheet. In the zero gas pressure (and axial field) cases, there are only two timescales in the problem; the Alfvén time and the resistive diffusion time. Balancing these timescales leads to analytic descriptions: two-dimensional in the small amplitude limit, and an approximate one-dimensional treatment for large amplitudes.

The introduction of gas pressure and a third timescale—the acoustic time—leads to a much more complicated system. Gas swept into the forming current sheet halts the collapse, preventing "fast" reconnection from being attained immediately. At least part of the trapped gas is subsequently squirted out of the current sheet—at roughly the sound speed—forming jets along the separatrices, and allowing a more intense current sheet to reform, so that a higher rate of reconnection is reached on a second bounce.

The dynamic evolution of the reconnection rate for a large gas pressure, $\beta = 0.1$, is illustrated in Figure 6 for three different resistivities. The build-up and decay of current density at the neutral point is shown, as the magnetic wave sweeps in and is dissipated or reflected. The reconnection rate at the first peak is reduced dramatically, falling by a factor of 6, as the resistivity is reduced from 10^{-2} to 10^{-3}. However, the reconnection rate at the second peak falls by only a factor of 2.

Although we have not yet determined whether the reconnection rate at the second (or a subsequent) peak is "fast," or examined the subtleties of the evolution, we offer the following "zeroth order" analysis, which supports the observation that, in the numerical simulation, gas is ejected from the current sheet at the local sound speed; we further suggest that the expulsion of gas is efficient, provided the magnetic perturbation is substantial and the gas pressure not too large.

Approximating the gas trapped in the current sheet as an incompressible fluid squeezed between two converging, rigid parallel plates, it is evident that the difference between the pressure applied to the plates to drive them together, and the pressure at the outflow, provides the force which accelerates the mass of gas. Assuming that the velocity of the trapped gas can be written in the form

$$\boldsymbol{v}(x, y, t) = u'(t)(\boldsymbol{x} - \boldsymbol{y}) , \qquad (3.1)$$

where $-L < x < +L$ measures distance parallel to the sheet, and $-w < y < +w$ is distance across the sheet, we find that the initial evolution of the current sheet thickness is given by

$$w = w_0 e^{-(1/2)(t/\tau)^2} , \quad t \lesssim \tau , \qquad (3.2)$$

where $\tau = L/(2\,\Delta P/\rho)^{1/2}$, ΔP is the pressure difference, and ρ is the density of gas in the sheet. For $t \gtrsim \tau$, the thickness decreases like $w \sim \exp(-t/\tau)$. If the pressure driving the thinning of the sheet, $\frac{1}{2}B_s^2$, is much larger than the pressure at the outflow, which will be of order the initial gas pressure, β, then $\tau \simeq L/v_A$, where v_A is a hybrid Alfvén speed defined by the magnetic field strength at the edges of the sheet together with the density of the trapped gas. However, since the thin sheet must be in a quasi-static state in which the magnetic pressure $\frac{1}{2}B_s^2$ is equal to the gas pressure in the sheet; this characteristic speed is also the sound speed in the current sheet [to within a factor of $(2/\gamma)^{1/2}$].

How large a magnetic perturbation is required to overpower the gas pressure in the sheet? First, we examine the one-dimensional collapse described by equations (2.11)–(2.14), in which the perturbation comprises the whole field. Equating the magnetic pressure $\frac{1}{2}B_s^2$ to the gas pressure in the sheet, $\beta\rho^\gamma$, we find (in the asymptotic limit $\rho \gg 1$) $\rho = (\beta/0.070)^{-0.925}$. Requiring the magnetic pressure to be much larger than the outflow pressure, assumed to be of order β, then implies $\beta \ll 0.070$. Moving from the strictly one-dimensional case to a magnetic perturbation imposed on the two-dimensional X-point field, we note that the magnetic energy of the collapsed current sheet is approximately $\frac{1}{2}B_s^2\,\pi L^2$, and that in dimensionless units $B_s \simeq L$, since the ends of the current sheet are defined when the imploding

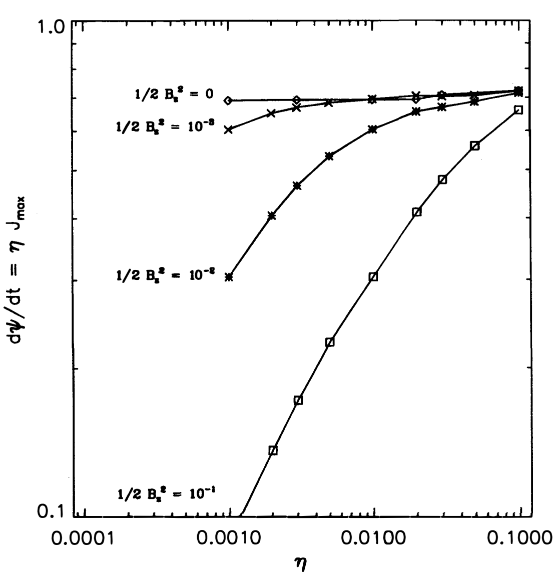

FIG. 5.—The peak reconnection rate, $d\psi/dt = \eta J$ at the neutral point, as a function of the resistivity η, for different axial field strengths, as measured by the parameter $\frac{1}{2}B_z^2$ which corresponds to the gas pressure parameter β. The results are almost identical to those of Fig. 4, confirming that the pressure of the axial magnetic field acts just like gas pressure and that the reconnection rate drops below the fast rate whenever $\frac{1}{2}B_z^2 \gtrsim \eta$.

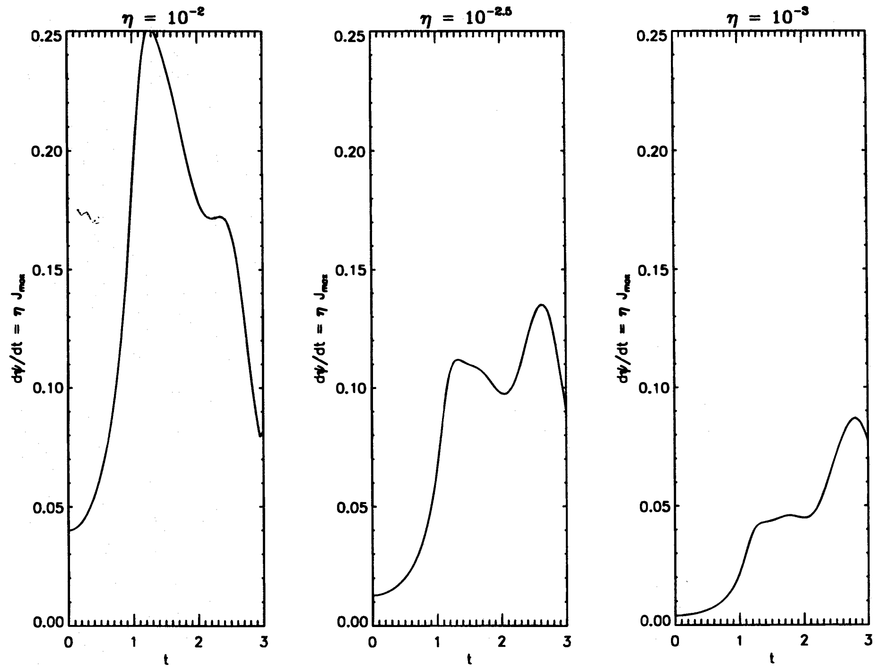

FIG. 6.—Evolution of the reconnection rate, $d\psi/dt = \eta J$, in the presence of gas pressure, $\beta = 0.1$, for three different resistivities, $\eta = 10^{-2}$, $\eta = 10^{-2.5}$, and $\eta = 10^{-3}$. The peak reconnection rate attained on the initial implosion drops rapidly as η is decreased, because the gas pressure prevents the current sheet from thinning enough to maintain dissipation. It is this drop that is analyzed in Fig. 4. However, a second peak in the reconnection rate occurs when the system bounces back after gas has been ejected from the sheet along the separatrices. The reconnection rate at the second peak drops off more slowly with decreasing resistivity, suggesting that fast reconnection may be possible following the expulsion of gas.

perturbation wave overwhelms the background potential field (Craig & Watson 1992; Craig 1994). We equate this energy to the energy $\langle \frac{1}{2} B_1^2 \rangle$ of the initial perturbation (assumed to cover a substantial volume of space). We then find that, if the system comes to equilibrium without resistive dissipation (as in the "magnetofrictional" collapse to a singular current sheet), the magnetic field at the edge of the current sheet is $B_s \simeq L \simeq B_1^{1/2}$. Since the equivalent equilibrium field strength in the one-dimensional case is just $B_s = \frac{1}{2}$, in the two-dimensional case the magnetic pressure is effectively reduced by the factor B_1. The net effect is then to replace β by β/B_1 throughout the analysis. We then find that the condition $\frac{1}{2} B_s^2 \gg \beta$ requires

$$\beta/B_1 \ll 0.070 . \quad (3.3)$$

Provided this condition is satisfied, the gas trapped in the current sheet should be ejected in a few Alfvén times. (The condition is not satisfied for the example shown in Fig. 6 for which $B_1 \simeq \beta = 0.1$.) Whether *all* the gas is expelled may depend (according to a steady state argument) on the pressure built up in the outflow lobes. However, we also note that the simulations indicate that the ejecta leave behind a partial vacuum in the current sheet. This highly dynamical event could be of major importance. In general, it seems that a major restructuring of the magnetic field has a better chance of producing fast reconnection than a small perturbation.

It is apparent in Figure 6 that a third peak seems to rise between the main peaks. The peaks are caused by the interactions of waves reflecting from the boundaries in both the inflow and outflow region, as noted by Sato et al. (1992), and by Sakai and collaborators (e.g., Sakai & Ohsawa 1987), who also obtain an approximate analytic description of such oscillations. A similar oscillating or steplike structure during the rise to peak current density is seen in the simulations of Strauss (1990), in which a strong axial magnetic field is present, but no gas pressure. All the above authors claim that the reconnection they observe is "fast," but only Strauss investigates the scaling of the reconnection rate with resistivity—unfortunately only over less than a decade in η. The others base their claims on the fact that the reconnection takes place in a few Alfvén times, which we do not believe to be an adequate criterion at the large resistivities typical of numerical simulations. For instance, while we have argued above that gas is ejected from the current sheet, it is not certain that all the gas is expelled. A remnant will have much greater consequences at $\eta = 10^{-8}$ than at $\eta = 10^{-4}$. Nevertheless, condition (3.3) suggests that fast reconnection in the low-β solar corona remains a possibility.

3.5. *Summary*

We have confirmed by analysis and by numerical simulation that the fast reconnection rate of the linear theory, which is valid in the absence of gas pressure and axial magnetic field, and for small perturbation amplitudes, is maintained for large-amplitude perturbations. The nonlinear results agree excellently with the analytic description of one-dimensional dynamic collapse developed by Forbes (1982).

If an appreciable backpressure is exerted by the plasma ($\beta \gtrsim \eta$) or by an axial component of the magnetic field ($\frac{1}{2} B_z^2 \gtrsim \eta$), the current sheet is prevented from thinning, and the efficiency of reconnection on the initial collapse falls to a slow rate. However, there are indications that gas trapped in the current sheet can be subsequently expelled along the separatrices, allowing a faster reconnection rate to be attained ultimately. We have not yet investigated thoroughly the complex dynamics subsequent to the initial implosion.

4. DISCUSSION

In the absence of gas pressure and axial magnetic field, magnetic reconnection is fast, even for large nonlinear perturbations. In this case there is no physical parameter other than resistivity to determine the current sheet thickness, so flux is reconnected as fast as it is carried into the diffusion region.

It is not clear yet whether reconnection remains fast in the presence of significant gas pressure or axial magnetic field. A quasi-steady balance between the pressure of gas (or axial field) swept into the current sheet and the external magnetic forces driving the collapse could be set up, in which the current sheet is not controlled by resistivity and therefore cannot thin enough for fast reconnection. This is what appears to happen on the "first bounce." But gas compressed by the imploding magnetic field then begins to stream from the ends of the current sheet. Provided the magnetic perturbation is large and the plasma β is small, it seems possible that fast reconnection can be ultimately attained. It is possible that the period of gas expulsion from the current sheet corresponds to the "preheating" phase of a flare, and that particle acceleration in the "vacuum hole" left behind by the streams of ejected gas produces the impulsive phase of a flare.

How do our results compare with previous work? The variety of contradictory answers that have emerged to date seems baffling. Previous studies fall into three classes: steady state incompressible, dynamical incompressible, and dynamical compressible. Steady state solutions have either no exterior boundary conditions imposed, "flow through" boundaries, or periodic boundary conditions. Dynamical simulations generally have "closed box" boundaries, with the advantage that the system is isolated and perfectly defined.

Proponents of analytic studies of steady state incompressible reconnection which rely in patching together solutions and which have no exterior boundary conditions imposed (see Priest & Forbes 1992) are confident of fast incompressible reconnection, yet the analyses have been shown to have some fundamental problems (Craig & Rickard 1994; Craig & Henton 1994). Specifically, in the usual sub-Alfvénic approximation neglecting $\boldsymbol{v} \cdot \nabla \boldsymbol{v}$, there is no flow across the separatrices, and so no reconnection. Relaxing this restriction, it is found that a plasma β of at least $\eta^{-1/2}$ is required in the advection region to drive fast reconnection. That is, if the plasma is incompressible throughout, reconnection is driven by thermal, rather than magnetic, forces, and the magnetic energy released is negligible compared to the thermal energy of the plasma. This implies that the incompressible solution can only describe a small volume of the reconnection region: it must ultimately merge into a low β plasma with magnetic pressure comparable to the gas pressure in the sheet.

Numerical studies of incompressible reconnection with periodic boundary conditions (Biskamp & Welter 1980; Biskamp 1986; Deluca & Craig 1992; Craig et al. 1993) have found either that reconnection is slow, or that reconnection which appeared to be fast for large resistivities drops to the Sweet-Parker rate for smaller resistivities ($\eta \lesssim 10^{-4}$). Important criticisms of the boundary conditions used in numerical simulations (Forbes & Priest 1987; Priest & Forbes 1992) do not seem to apply to these periodic cases.

Strauss (1990) shows evidence for fast reconnection in a system dominated by a strong axial magnetic field, with zero gas pressure. However, the minimum resistivity he studies is $\eta \simeq 4 \times 10^{-4}$, approximately the value at which Biskamp & Welter (1980) and Craig et al. (1993) find that incompressible reconnection begins dropping to a slower rate, so the result cannot be regarded as conclusive. Ugai (1993) finds that the introduction of a B_z component does not substantially affect the reconnection rate.

Ofman et al. (1993) study reconnection in a closed box with $\eta = 10^{-4}$, but they did not study the dependence of the reconnection rate on η. They claim that the same (fast) reconnection rate applies in an incompressible gas as in the $\beta = 0$ case, and that reconnection is even faster for $\beta \simeq 0.1$. On the other hand, Sato et al. (1992) claim that compressibility is essential to fast reconnection. Studies which introduce compressibility as a perturbation on incompressible reconnection (Jardine & Priest 1989; Parker 1963) find that compressibility has a minor effect.

"Anomalous" resistivity has been claimed as an essential ingredient for fast reconnection (Ugai 1986, 1992; Yokoyama & Shibata 1994). These authors find that uniform resistivity leads to slow Sweet-Parker reconnection, but that a resistivity which turns on locally when the current density surpasses a threshold yields unsteady Petschek-type fast reconnection.

Finally, we mention that Alfvén shear waves impinging on a neutral point avoid the problem of backpressure encountered in the fast mode waves considered here, but nevertheless do not lead to fast dissipation (Bulanov et al. 1990; Craig & Henton 1995; Craig & McClymont 1996).

Our experience has been that simulations with η larger than 10^{-4} do not provide sufficient separation between the advection and diffusion timescales to indicate behavior in more highly conducting plasmas. Craig & McClymont (1993) found their asymptotic analytic description of linear reconnection to be accurate only for $\eta \lesssim 10^{-6}$. We conclude that reconnection occurring in a few Alfvén times in the case of relatively large resistivity is not a conclusive indication that the reconnection is fast for smaller (realistic) resistivities, and that it is important to examine the dependence of the reconnection rate on η. Since direct numerical simulation of realistic resistivities does not seem to be feasible, it is important to understand the physical processes taking place, and to develop analytic models as a guide to extrapolating simulation results. We also note that the perturbation amplitude is a significant parameter.

We conclude that in a plasma of sensible gas pressure, such as the solar corona, the initial implosion onto the X-point of a magnetic disturbance does not yield fast reconnection. Plasma swept into the current sheet halts the collapse before significant dissipation occurs. If the disturbance is sufficiently large, however, the magnetic pressure of the imploding wave expels the trapped gas from the ends of the current sheet at the local sound speed, allowing the sheet to thin and faster reconnection to occur subsequently. The dynamics of wave interactions are interesting and complex, and not yet fully understood. Truly "fast" reconnection in this situation seems a distinct possibility, but we feel that none of the work to date has proved the case.

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