

The development of uncertainty in national and subnational population projections: A New Zealand perspective

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Abstract Projections or forecasts of future population, and the age and sex structure of the population, are key inputs to decision-making. However, these projections have an associated uncertainty that is often underappreciated by decision-makers. Moreover, a decision-maker faced with multiple population projections has no clear basis on which to decide between alternative projections. In this chapter, we outline the sources of uncertainty in population projections. We then describe the development of population projection methods in New Zealand, where the representation of uncertainty has become central to official projections produced by Statistics New Zealand, as well as alternative projections produced by the National Institute of Demographic and Economic Analysis. Finally, we outline a model-averaging approach that can be used by decision-makers to combine the information from multiple independent population projections. This may provide a more accurate approach to the use of population projections in decision-making, without requiring substantial modelling capability. This approach is illustrated with the example of subnational areas for the Waikato Region of New Zealand.

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1. Introduction

Projections (or forecasts)¹ of the future population are a key input for national and local government decision-makers, planners, private sector developers, businesses, community and advocacy groups, and many others. Having an indication of the future trajectory of population is valuable for guiding decisions on planning infrastructure and service provision, among other things (e.g. Auerbach and Lee 2001). Population projections can be used to anticipate and plan for future demands for public services and infrastructure, with lead times that reflect the nature of the resource base that will be required.

In most cases, population projections not only seek to establish the future level of the population, but also the age and sex structure of the population, and periods of differential rates of growth. Health, education and income support are key areas of public policy that involve long payback periods for investment, and where the investment varies across age groups. For example in New Zealand, the “From Birth to Death” reports of the Social Monitoring Group of the Planning Council in the 1980s spanned a comprehensive range of areas, providing analysis of future trends with relevance to social policy. This required not only projections of the population level, but also the age structure of the population. These reports were used extensively in the development of social policy through this period.

However, as American baseball player Yogi Berra is reputed to have once quipped, “It’s tough to make predictions, especially about the future”. Projections of future population are no different in this respect, and come with a high degree of *ex ante* uncertainty, particularly for small populations and over longer projection horizons (Cameron and Poot 2011). *Ex post*, when compared with actual populations, all projections are imperfect and result in error, where the actual population differs from what was projected.

The tolerance of projection error will differ with the use of the projection. There are different costs arising from underestimating demand compared with overestimating demand. These vary between service and infrastructure types depending on the economic, social and environmental

¹ The distinction between a projection and a forecast may appear to be semantic, but the distinction is fundamental to how end-users should interpret and use these information tools. A projection is a realisation of a model based on a particular (and known) combination of assumptions in terms of the model parameters. Thus, the projection is fully determined by the choice of parameters (which might be chosen empirically or stochastically, as we discuss later in the chapter). Moreover, a projection is based on current policy settings and does not typically try to anticipate future policy changes, some of which may be in response to projected population changes themselves. In contrast, a forecast is a prediction of a likely future. It is the preference for a particular set of parameter values that turns a projection into a forecast. For further discussion of this point, see Dion and Galbraith (2015). We prefer to use the term ‘projection’, which we maintain throughout the chapter, although we recognise that many end-users will interpret projections as forecasts.

costs of building over-capacity relative to the costs of building insufficient capacity. For example, in the mid-1970s when population projections had yet to take into account the impact of the then rapid decline in fertility, electricity planners were examining the need for nuclear generation. A Royal Commission on Nuclear Power Generation in New Zealand was set up in 1976, and by the time it had reported back to the Government in 1978, population projections had lowered, and the continued use of hydro-electric energy was judged as likely to meet foreseeable future need. As this example suggests, the many and varied uses of population projections require a consideration of both error and uncertainty, which has not always been well conveyed by conventional deterministic² projections.

In this chapter, we draw on a long literature considering the sources of error and uncertainty in population projections (Alho and Spencer 1985; 1997; Alho 1990; Alho et al. 2008). We use New Zealand as a case study, describing the historical development of population projections, from early efforts to more recent methodological advances. New Zealand is an appropriate case study, as Statistics New Zealand³ (Stats NZ) has long been at the forefront of developments in population projections methodology, most recently in the adoption of stochastic population projections at the national level.⁴ Importantly, the stochastic approach to projections helps to convey uncertainty in the projections in a much more meaningful way than the traditional deterministic approach. Alongside the official projections produced by Stats NZ, staff at the National Institute of Demographic and Economic Analysis (NIDEA; formerly the Population Studies Centre) at the University of Waikato, led by Emeritus Professor Jacques Poot (now at the Vrije Universiteit Amsterdam), pioneered the development of stochastic subnational population projections. Most recently, NIDEA has incorporated a gravity model of internal migration (Poot et al. 2016) into the population projections framework (Cameron and Poot 2014a; 2014b). The approaches adopted by Stats NZ and NIDEA are different but complementary, and we argue that taking a model averaging approach across these two different methodological approaches may actually lead to less dependence on a single projection source and better decision-making.

² A deterministic projection typically combines a given set of input parameters, compared with a stochastic projection which varies randomly according to the probability distributions of those input parameters.

³ For simplicity, we refer to Stats NZ, although before 1995 the organisation was known as the Department of Statistics.

⁴ New Zealand is currently one of only a few countries, along with the Netherlands, where the national statistics agency produces official population projections using a stochastic approach.

The remainder of the chapter proceeds as follows. First, we briefly discuss general methodological approaches to population projections, distinguishing time series econometric approaches from the cohort component model. We also describe the key differences between top-down and bottom-up approaches to subnational population projections. In Section 3, we identify the main sources of uncertainty in population projections, particularly model uncertainty and parameter uncertainty. Section 4 outlines the historical development of population projection methods in New Zealand, at both Stats NZ and NIDEA, and Section 5 briefly describes the *ex post* accuracy of projections. Finally, Section 6 describes a model averaging approach as a way forward in dealing with model uncertainty, and illustrates this with particular examples of subnational projections from the Waikato region of New Zealand. A novel contribution of this approach is the averaging of stochastic and deterministic projections by weighting them based on measures of their relative uncertainty. Section 7 concludes.

2. Population Projections

A variety of methods are available to demographers and population modelers for use in preparing population projections, whether at the national level or sub-nationally.⁵ These methods range from simple naïve models (e.g. extrapolation of past time series trends), through the ‘traditional’ demographic cohort component model, to sophisticated approaches such as agent-based modelling or microsimulation. There has been renewed interest in recent times in exploring the relative merits of different projection methodologies (e.g. see Raymer et al. 2012).

Naïve models are most appropriate where there exists a lack of data available for parameterizing more complex models. The simplest of these models involve an assumption of linear population growth based on past growth trends, or exponential growth (i.e. a constant population growth rate) based on past growth trends. Somewhat surprisingly, at the subnational level these naïve models tend to perform reasonably well, in terms of forecast accuracy, when compared with more sophisticated models (van der Gaag et al. 2003). Moreover, age- and/or

⁵ For simplicity, we set aside the case of small area projections, such as for suburbs or sub-county areas, and household projections, where in both cases an even wider variety of projections methods are arising (e.g. see Cameron and Cochrane 2017).

sex-specific projections models can be estimated, either separately or jointly, to derive a projected age-sex distribution for the population.

However, such naïve models fail to incorporate age structural effects, cohort effects, or major structural changes such as population ageing. Moreover, they are difficult to defend in discussions with planners or other end-users, as they lack a sound theoretical basis, and they fail to incorporate known or suspected drivers, including demographic and policy drivers, of population change. For a planner or policy maker who hopes that their plans or policies will influence future population growth or decline, these models offer little guidance.⁶ This can reduce the ‘buy-in’ from important end-users of projections, hampering their eventual use.⁷

In contrast, the traditional workhorse model for demographic projections – the cohort component model (CCM) – explicitly accounts for the *demographic* drivers of population change. The CCM is based on a demographic identity – the population at time $t+1$ is equal to the population at time t , plus any increases due to births or in-migration between times t and $t+1$, and minus any decreases in population due to deaths or out-migration between times t and $t+1$. Thus, the CCM requires the modeler to first project the three components of population change: (1) births, typically projected by means of age-specific fertility rates applied to women of childbearing ages; (2) deaths, typically projected by means of age-sex-specific mortality (or its complement, survivorship) rates applied to the population of each age and sex; and (3) migration, which may be projected in a number of different ways (van der Gaag et al. 2003). Moreover, other drivers of population change may be included in the model by first modelling their influences on fertility, mortality, and/or migration. Because these drivers can be included directly in the model, the CCM is more readily accepted as an appropriate model by end-users.⁸

A further appeal of the CCM is that it can be extended to other demographic projections, such as ethnic population or labor force projections, by including additional components. For ethnic population projections, this is typically the addition of a paternity component, to allow for ethnic births generated by males partnering with females of other ethnic groups; and an inter-ethnic mobility component, to allow for the net effect of people changing ethnic identification

⁶ On the other hand, in the hands of a savvy operator, a naïve model and a simple assumption about a change in the growth rate, might be used to justify almost any policy or plan.

⁷ For example, Cameron et al. (2007) used additional economic development as a driver of migration, in their subnational population projections for the Hamilton sub-region of New Zealand.

⁸ Although, end-users may disagree with the assumptions or the effect of drivers used in these models.

over time (Statistics NZ 2017a). For labor force projections this is typically the addition of labor force participation rate and hours worked components (Statistics NZ 2017b).

More sophisticated models incorporating agent-based modelling or microsimulation are also available (e.g. Benenson 1998; Fontaine and Rounsevell 2009). These approaches involve using large unit-record datasets to estimate transition probabilities between ‘states’. These states might include spatial location, partnership or marriage, birth of children, mortality, and so on. Thus, these transition probabilities can be used to project forward the population in terms of the components of demographic change in the CCM. However, these models are extraordinarily data-intensive, since they require projecting individual-level or household-level data (in the case of agent-based modelling) or representative individuals or households (in the case of microsimulation). Often, the transition probabilities are not directly measured or measurable, and have to be assumed, resulting in complex and non-transparent models.

In addition to the model, a relevant consideration for a suite of national and subnational population projections is whether they will be developed using a top-down or bottom-up approach (Willekens 1983). A top-down approach projects the population at the national level first, using a national-level model, then projects each sub-national area either separately or as part of a multi-regional model. The sub-national projections are constrained to sum to the previously determined national projection. The top-down approach has the advantage of ensuring a sufficiently bounded national population projection, driven by known drivers of population change at the national level. In contrast, a bottom-up approach projects each sub-national area (again, either separately or as part of a multi-regional model) in an unconstrained manner. A national population projection can then be derived by summing the sub-national projections. The bottom-up approach has the advantage of adhering to sub-national drivers of population change, without requiring *ex post* adjustments to the projections to ensure consistency with a national total.

However, the bottom-up approach also has several limitations. First, sub-national drivers of population change are likely to be subject to greater relative uncertainty than national drivers. A bottom-up approach may therefore lead to national projections with implausibly wide uncertainty, depending on how uncertainty is aggregated to larger geographies. Second, where the components of population change are defined by ethnicity, assumptions of commonality in basic structure across sub-national areas are unlikely to apply. Third, national and regional projections also become less timely using the bottom-up approach, as they are dependent on

the derivation of projections at smaller geographic levels. This is a disadvantage for producers wanting an agile national-level projection process. Fourth, small area projections need to be derived with full granularity (e.g. single-year of age, ethnicity) if this is desired for higher geographies.

Regardless of the overall approach adopted, even deterministic processes need constant vigilance in assessing the relevance of the parameters that are included. For example, Stats NZ had to decide during the early 1970s how much of the decline in fertility that rapidly arose in New Zealand reflected a significant permanent reduction in births, rather than a deferment of births. Stats NZ decided not to participate in the World Fertility Survey by the United Nations Fund for Population Activities in 1973. The first official projections to recognize the massive fall in fertility were not prepared until 1978.

3. Uncertainty

Understanding the uncertainty in population projections is important for their accurate interpretation. When measures of uncertainty are provided with projections, these can be used to (i) distinguish within projections what trends are more certain than others; (ii) make informed judgments about how uncertainty changes with different projection horizons; and (iii) to convey the important notion that the future does not follow smooth and certain paths, but is likely to fluctuate within a range of possibilities depending on interventions by policy agencies.

There are several sources of uncertainty in population projections. Starting with Alho's (1990) sources of error in population projections as a base, we distinguish three types of uncertainty in population projections. The types of uncertainty are not necessarily mutually exclusive, and some examples of uncertainty could be classified as belonging to more than one type. First, *model uncertainty* arises from an incomplete understanding of all the interacting and overlapping processes, their drivers and determinants, which contribute to future population outcomes. Any model is necessarily a simplification of the underlying reality, and as a result, there is uncertainty in terms of which model might be 'best'. The degree of covariance between model parameters, and the persistence or otherwise of shocks to time series, are further sources of model uncertainty.

Moreover, model choice is an important source of uncertainty, and while it may be attractive to argue otherwise, all population projection models require some degree of expert judgment.

This judgment may be exercised at a fine operational level, such as in determining the valid range of values that parameters may adopt, or at a broader methodological level, such as determining which drivers of population change are included, and which are excluded, from the model.

Second, *parameter uncertainty* arises due to sampling and measurement error in the model parameters. Sampling errors arise where parameters are estimated from sample data, including estimation from short or incomplete time series. For example, the introduction of the methodology for integrating administrative data records with the results of the 2018 Census enumeration in New Zealand has changed the share of each ethnic and age group counted in census statistics. In providing more accurate measures of how the population is distributed among these groups in 2018 compared to earlier census, the rate of change between 2013 and 2018 has a different error structure than any earlier measures of inter-censal change. Measurement error arises due to random errors or systematic bias in the estimation of the parameters. Both sources of error will be present in any model, even if the correct underlying model of the population processes was known.

Third, *random variation* is a source of uncertainty in any model. Population processes (migration, fertility, mortality) can be thought of as stochastic processes that, in addition to being measured with error, are subject to random fluctuations between different population groups, regions, and over time. As such, there will always be uncertainty in any projection of future population, arising from random variation that cannot be deterministically accounted for. In contrast, stochastic models (see below) account for the irregularity experienced in the past, so that it can be applied to later observations, usually of one or more of the key parameters in a deterministic model.

Uncertainty in population projections can be modelled or expressed in different ways. To simplify, population projection models can be generally characterized in terms of whether they are deterministic or stochastic (probabilistic). A deterministic projection tracks a single path forward in time, based on a single parameterization of the model. The uncertainty in a deterministic projection may be determined analytically from the uncertainty in the underlying parameter distributions or in the components of population change (e.g. see Alho and Spencer 1985; 1997). In contrast, a stochastic projection attempts to directly model the uncertainty in the projected population (e.g. see Tuljapurkar 1992; Tuljapurkar et al. 2004; Cameron and Poot 2011). Often this is achieved by simulation, i.e. running the model many times with different

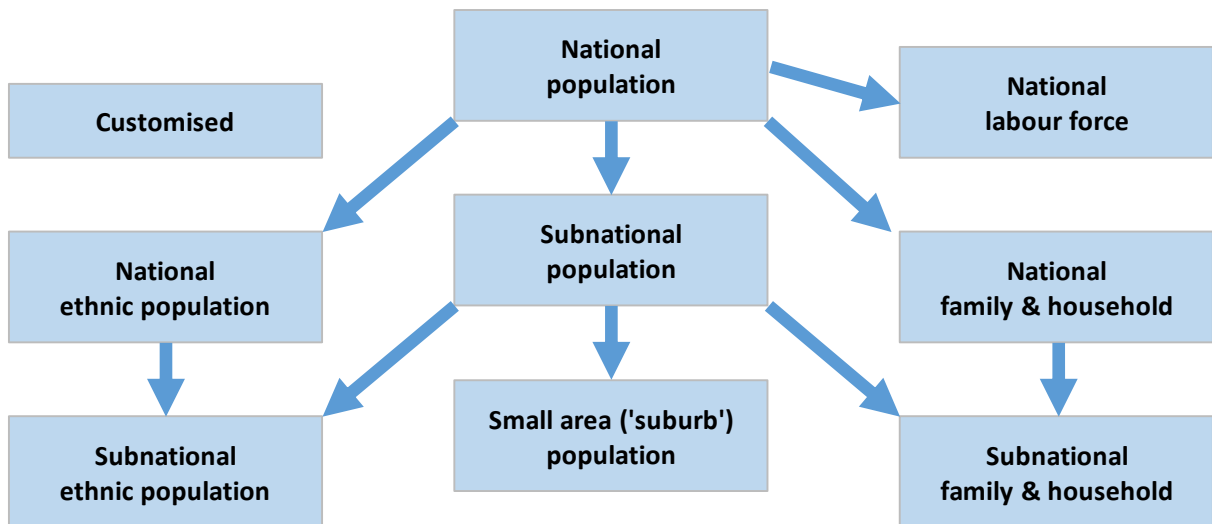
parameter values (drawn from distinct distributions for each parameter), and summarizing the projection interval based on the number of simulated projections that fall within given bounds (Pflaumer 1988).

However, in both cases – analytically or simulation-based – measures of uncertainty are somewhat limited to measures of the parameter uncertainty and random variation in a given model. In practice, most researchers do not adopt and run many different model and covariance structures in order to obtain an assessment of model uncertainty (for a notable exception, albeit based on relatively naïve time series models of total population, see Abel et al. 2013). Instead, a single modelling approach is adopted, and small variations on the model are used to derive measures of uncertainty. Most often, some variant of the CCM is used.

4. Historical Development of Population Projections in New Zealand

Stats NZ has a long history of producing demographic projections. National population projections were regularly produced from the 1950s, while labor force and Māori population projections were produced less regularly. For instance, the Hunn report in 1961 included projections, which were probably the first special purpose population projections to specifically capture the distinct demographic characteristics of New Zealand Māori for some fifty years ahead. In the mid-1970s, Stats NZ began projecting the population of subnational areas, a role that was previously undertaken by the Ministry of Works. An internally consistent suite of projections developed through the 1990s, which reflected New Zealand’s changing population composition and demand for demographic projections at different geographies (Figure 1).

Fig. 1 Demographic projections produced by Statistics NZ



The advantages of an independent national statistical organization (NSO) producing projections for all local government areas go beyond the consistency with national-level projections. The motivations of local councils – typically in terms of economic growth – often encourage aspirational population projections. These serve a role but are invariably based on aspirational demographic trends rather than prevailing trends. The role of the NSO is to provide objective statistical information according to international best practice methodology, not to customize its statistics according to national or local marketing or political aspirations. A NSO should also have a rich understanding of the range of quality issues associated with its sources. While there is a role for local experts to inform local population assumptions, NSOs provide a holistic perspective on all local areas across the country.⁹ The elicitation of expert opinion on projection assumptions is a science in itself, and is discussed in Garthwaite et al (2005), O’Hagan et al (2006) and Dias et al (2018); see also Statistics Canada (2019) for a practical example of elicitation used in official projections.

Pacific population projections were first published in the mid-1990s, to complement Māori ethnic population projections. By 2017, Stats NZ were publishing national population projections for five overlapping ‘level 1’ ethnic groupings – Māori; Pacific; Asian; European or Other (including New Zealander); and Middle Eastern/Latin American/African – as well as

⁹ Local council input has been a feature of Stats NZ’s small area (‘suburb’) population projections since they began in the 1990s. Local planners are well placed to inform assumptions about the timing and pace of local developments such as new subdivisions and major building projects (e.g. retirement homes, prisons), and about zoning changes that can impact housing and population density. Their input makes the resulting projections more attuned to local plans, if not attuned to local aspirations.

for the three largest ‘level 2’ ethnic groups (Chinese; Indian; and Samoan) (Statistics NZ 2017a).

Methodologically, official projections have continued to evolve. The cohort component method and a top-down approach – which enables the progressive release of higher level (e.g. national) projections without dependence on the lowest level projections – have persisted as the backbone of official projections.

One of the more significant methodological developments by Stats NZ has been the application of a stochastic approach to national-level projections. The adoption of a stochastic approach has been progressive, beginning with national population projection in 2012, national labor force projections in 2015, and national ethnic population projections in 2015. The advantages of a stochastic approach are discussed in Alho (1997; 2005), Booth (2006), Bryant (2003; 2005), Dunstan and Ball (2016), and Keilman (1991), among others.

The driver for Stats NZ to adopt stochastic projections has been less about improving the accuracy of projections, and more about improving their interpretability. This reflects that conventional deterministic projections are poor at conveying uncertainty, especially for key demographic characteristics (e.g. dependency ratios, death numbers). For most characteristics, the uncertainty indicated by deterministic scenarios is neither consistent between characteristics, nor consistent across the projection period. Given the inherent uncertainty of the future, encouraging users of projections to think about uncertainty is important. However, it is only feasible for users to think about uncertainty if that uncertainty is conveyed to them appropriately.

Rather than using scenario variants that convey uncertainty only qualitatively (e.g. high, medium, low), stochastic projections will typically be summarized by means of percentiles that convey the probability of certain outcomes occurring (within the constraints of existing policy settings). For example, stochastic projections can be used to indicate the probability that a certain population characteristic will reach a given level by a given year; or will be within a given range by a given year; or will reach a given level within a given timeframe. They can also provide a projection interval that represents, for a given probability, the range within which the population is projected to be at certain dates. Different percentiles need to be provided for users with different risk appetites. As percentiles are non-additive, flexibility is also needed to derive percentiles for customized purposes (e.g. for ad hoc age groupings).

There are many different ways to model uncertainty and to produce stochastic projections. Both expectation (e.g. Billari et al. 2014; Lutz et al. 2014; Lutz 2009; Lutz and Scherbov 1998) and empirical approaches (e.g. Lee and Carter 1992; Woods and Dunstan 2014) are used internationally to model uncertainty, and both have their advantages and disadvantages. Empirical models have been most intensively applied where demographic trends have been largely monotonic and sustained, as in the case of death rates and life expectancy. Stats NZ has taken a pragmatic approach by using a mix of expectation and empirical methods to produce projections (Table 1).

Table 1 Method of assumption formulation for New Zealand population projections (published 2016)

Assumption	Median (50th percentile)	Variance/distribution of values
Base population	Empirical model: official population estimates based on census and post-enumeration survey	Expectation (judgement): variance varies by age-sex
Fertility	Expectation (judgement): long-term total fertility rate of 1.85 births per woman	Empirical model: ARIMA (0,1,0) model fitted to total fertility rate for 1977–2016 June years ^a
Mortality	Empirical model: coherent functional demographic model fitted to age-specific death rates for 1977–2015 June years	
Migration	Expectation (judgement): long-term annual net migration of 15,000	Empirical model: ARIMA (1,0,1) model fitted to net migration for 1988–2016 June years ^a
Sex ratio at birth	Empirical model: median and variance from sex ratio at birth for 1900–2015 December years	

^a It is worth noting that ARIMA models assume that future projection errors will have the same pattern as past errors.

Stats NZ provides an independent and comprehensive set of public-good demographic projections, but of course it is not the sole producer of projections in New Zealand. We focus our attention on one such producer, which has also been an innovator in population projections methods. In the early 2000s, the Population Studies Centre (PSC) at the University of Waikato began to provide bespoke subnational population projections for local councils and central government agencies. This began with work for the Bay of Plenty region (Bedford and Dharmalingam 2002; Bedford 2005), in response to a view by local government, particularly in the fast-growing Tauranga City and Western Bay of Plenty District, that official projections did not reflect what they perceived as expansive population growth prospects. This was a view shared by many local council planners, and soon after, Jacques Poot at PSC was contracted to

produce projections for the Hamilton sub-region (Cameron et al. 2007; 2008a) and for Thames-Coromandel District (Cameron et al. 2008b). One of the key innovations in that early PSC work was that the projections were ‘end-user informed’. This meant that end-users had input into the assumptions underlying the population projections model, i.e. that the models would “incorporate local information by experts and end-users with respect to the assumptions that drive the projections” (Cameron et al. 2007). This allowed these bespoke projections to satisfy end-users (i.e. local council planners) that the projections would accurately reflect their beliefs about the growth trajectory of their local area.¹⁰

One additional innovation in these early projections was that migration was modelled on the basis of age-sex-specific net migration *rates*, as opposed to modelling a total number for net migration, which would then be distributed by age and sex (Cameron et al. 2007). The advantage of net migration rates is that they adjust dynamically to reflect the underlying size of the population, without further intervention on the part of the modeler. Subsequently, the PSC population model was included within the Waikato Regional Council’s integrated WISE (Waikato Integrated Scenario Explorer) model (Rutledge et al. 2008; 2010).

Given end-users’ concerns about accuracy of the projections, understanding and representing uncertainty in the projections was an important consideration in the PSC work. This led to the projections model being extended to produce stochastic population projections (Cameron and Poot 2011). This model was then used in subsequent subnational projections for the Waikato and Bay of Plenty regions (Cameron and Cochrane 2014; Cameron et al. 2014).

The methodology for population projections at the re-branded National Institute of Demographic and Economic Analysis (NIDEA; formerly the Population Studies Centre) has continued to evolve. Recent developments include revising the projections model so that internal migration within New Zealand is estimated using a gravity model (Poot et al. 2016; Cameron and Poot 2014a; 2014b). Along with other origin-destination models, the gravity model has several advantages over time-series-based approaches for projecting net migration, or net migration rates. First, it explicitly models all of the underlying inward and outward flows of migrants to and from each subnational area. This allows the projections model to demonstrate not only the total migration flows, but also their origins and destinations, which has been a key request from end-users. Second, it more explicitly accounts for the underlying drivers of population movement, at least for internal migration (international migration, which

¹⁰ Although, as it turns out, those beliefs were often aspirational rather than realistic. See later in this section.

is much more volatile, still requires more traditional methods for projection). Third, the uncertainty in the gravity model parameters are estimated, and importantly the covariance between the parameters is estimated, and these can be used in deriving stochastic population projections. Fourth, the role of end-users in pre-determining the outcome of the projections is limited, although recent projections by NIDEA had adopted a Bayesian approach to avoid overly optimistic assumptions driving the projections (Cameron and Cochrane 2014; Cameron et al. 2014).

5. The Accuracy of Historical Population Projections

In a world increasingly bombarded by ‘forecasts’, it is also worth reflecting on the relative predictability of demographic events. Fundamentally, the population can only change through births, deaths and migration – although ethnic populations are susceptible to changing patterns of identification over time (O’Donnell and Raymer 2015), and local populations are susceptible to volatility in migration flows that are often the most significant component of population change. But this fundamental simplicity does make population size and age-sex composition more predictable than other phenomena.

Somewhat trivially, even if population projections are not used by a decision-maker, or are used but not understood, then an assumption about the future population structure and level will be implicit in the action taken. The Population Monitoring Group of the New Zealand Planning Council played a pivotal role in bringing to the attention of the public the policy implications of population change, and since this was disestablished in 1991, no other similar ongoing forum of population experts has existed in New Zealand. In policy areas such as housing, justice, health, urban infrastructure and immigration, policy is less likely to respond in good time to the certainties we already are aware of that will drive the future population, and its structure, in each place. A Governmental Committee to Review Power Requirements met each year up to the late 1970s to advise Parliament of the implications of population change on future electricity needs and inform planning of major energy projects by the State. The most recent expert review of population change was prepared by the Royal Society of New Zealand in 2014, but it has not had the attention it merited (see Royal Society of New Zealand 2014).

As Patrice Dion, chief demographer at Statistics Canada observed, “A good population projection is not defined by whether or not it matches reality. It is defined by whether it was

plausible and useful to policy-makers at the time the projection was created.”¹¹ This is not to say that projection accuracy is unimportant, but that policy-makers and other users should question assumptions at the time the projections are produced, not some time later with the benefit of hindsight. Nonetheless, an assessment of projection accuracy can be useful for both producers and users, especially as an indicator of potential uncertainty in current projections.

Surprisingly, routine published assessments of accuracy are rare among national statistical agencies. This is in spite of errors in past projections providing an indication of likely uncertainty in more recent projections (Keilman 1990; 1997). Stats NZ has published two reports, the first in 2008, and an update (which also assessed the accuracy of population estimates) in 2016 (Statistics NZ 2016). The general findings about accuracy are not surprising:

- Projection accuracy tends to diminish the further out from the starting point; and
- Projection accuracy tends to diminish as populations becomes smaller, such as for smaller geographic areas or age groups.

Each of the key assumptions – fertility, mortality, migration – have contributed to projection inaccuracy at different times. Despite New Zealand fertility levels around replacement level for the last four decades, small changes in the timing of childbearing has caused ups and downs in birth numbers, periodically placing strains on local maternity services (Sceats 1999). Mortality assumptions have a relatively small impact on total population projections, but a significant impact on projections of the population in older ages. An important development in the accuracy of mortality forecasting are models that draw on the latest age-and-sex-specific shifts in mortality patterns, such as those that use functional demographic models (Woods and Dunstan 2014). These are preferable to mortality models that focus on extrapolating life expectancy at birth, which usually assume age-specific mortality patterns are changing at the same proportionate rate.

Migration assumptions have usually been the most inaccurate, for several reasons. First, it reflects the volatility of New Zealand’s international migration, with large swings in arrivals, departures and net migration experienced over short periods of time. Second, it reflects that internal migration becomes an increasingly significant driver of population change as geography becomes smaller. Third, it reflects that migration is a much more complex process

¹¹ <https://www.statcan.gc.ca/eng/blog-blogue/cs-sc/ptp>.

to model than fertility or mortality, given the multitude of factors that drive migration both into, and out of, an area, as well as its temporal and spatial variability.

Quantifying the accuracy of past projections is useful as it reinforces that accuracy and uncertainty are intrinsically linked. Sub-populations that have high projection uncertainty are generally those where projections are less accurate. This does not mean that projections are worthless. It does, however, underscore the importance of regularly updating projections to reflect new demographic data and policy settings. It also reinforces the value of including quantified measures of uncertainty in projections to enable end-users to differentiate between more certain and less certain demographic outcomes.

Indeed, this differential uncertainty is conveyed by stochastic projections. The 2016-base New Zealand population projections (Statistics NZ 2016) indicate a 65+ population of 1.62-2.06 million in 2068 (90% projection interval), or relative uncertainty of ± 12 percent around the median. In contrast, the 15-39 population has a projection of 1.32-2.31 million in 2068, or relative uncertainty of ± 28 percent around the median. Both future births and migration, and uncertainty thereof, play a role in the greater relative uncertainty for the younger population.

6. A Way Forward

Having recognized that population projections are subject to uncertainty, that there are several alternative population projections available from different providers, and that the degree of uncertainty (as conveyed by the accuracy of historical population projections) can be quite high, what is a decision-maker to do? How should a 'preferred' population projection for decision-making purposes be determined? In our experience, there is no one correct answer to this question. However, in this section we propose one way forward for the decision-maker, which does not involve deciding on a single preferred population projection, but instead makes use of multiple projections and their implicit or explicit uncertainty.

Returning to the concepts of uncertainty we outlined in Section 3, there are essentially three sources of uncertainty: (1) model uncertainty; (2) parameter uncertainty; and (3) random variation. The latter two sources of uncertainty are explicitly accounted for in a stochastic population projection, and as Alho et al. (2008) note, in the case of deterministic scenario projections (high/medium/low) this uncertainty may be inferred from the range between the

high and low projections. However, stochastic population projections do not typically account for model uncertainty.¹²

When faced with model uncertainty, and a multitude of possible (deterministic and/or stochastic) population projections, one fruitful solution may be to employ ‘model averaging’ (Claeskens and Hjort 2008; Bijak 2010). The overall approach is very simple – each projection is weighted using some schema of weights, and an overall projection, with or without an associated measure of uncertainty, is derived from the weighted average of the projections. This approach is increasingly common in forecasting applications – for instance, it is used in combining climate projections from different climate models to derive an overall ensemble projection for future climate (Tebaldi and Knutti 2007; Knutti et al. 2010).

There are many model averaging approaches that could be applied. One challenge in model averaging is how two stochastic projections can be averaged, or how stochastic projections may be averaged with deterministic projections. One solution is to use the median projection to represent a stochastic projection, and then to weight the median projection based on a measure of its uncertainty. Deterministic projections can similarly be weighted on the basis of their uncertainty, with their uncertainty determined by the accuracy of past projections, or based on expert judgement.

We illustrate the model-averaging approach with a simple case study based on Stats NZ and NIDEA subnational population projections for ten territorial authorities¹³ in the Waikato Region of New Zealand. Specifically, we use 2013-base population projections (February 2015 release) from the all three variant deterministic projections (high, medium, and low) from Stats NZ, and the median stochastic projection and associated measures of uncertainty from NIDEA. We take a simple weighted average of the medium Stats NZ projection and the median NIDEA projection, where the weights are the inverse of the projection interval. This approach ensures that the projection with the least uncertainty (in terms of parameter uncertainty and random

¹² More accurately, while stochastic population projections could be structured to incorporate some forms of model uncertainty, to date we are unaware of any systematic attempt to incorporate model uncertainty into stochastic population projections. For a notable exception that employs Bayesian model averaging, albeit based on relatively naïve time series models of total population, see Abel et al. (2012).

¹³ Territorial authorities are the smallest administrative unit in New Zealand. They do not necessarily share contiguous boundaries with regions. The Waikato region is comprised of all, or part, of eleven territorial authorities. However, the proportion of Rotorua District that lies within the Waikato Region is small, so it is excluded from consideration here. Taupo and Waitomo Districts also contain parts that lie outside the Waikato Region, but the populations of those areas are small. However, it should be noted that the NIDEA projections only project the parts of each territorial authority that lie within the Waikato Region, and so will be underestimates relative to the total population for Taupo and Waitomo Districts.

variation, as discussed above) receives a higher weighting. Following Stoto (1983), we assume the range from high to low projection is a 67 percent projection interval for the Stats NZ projections (see also Alho et al. 2008). To ensure comparability, we use the 67 percent projection interval from the NIDEA stochastic projections.

We then compare the two projections (Stats NZ and NIDEA) and the model-average projection with the realized population, based on the 2018 subnational population estimates, for each area. The currently available subnational population estimates for 2018, despite being only partly based on the 2018 Census, are still the best available estimate of subnational populations at the time of writing.

The results of the model-average projection, along with the contributing NIDEA and Stats NZ projections, are summarized in Table 2. The Stats NZ medium projections are somewhat higher than the NIDEA median projections for all territorial authorities. However, the range from high to low projection for the Stats NZ projections is similar in percentage terms to the 67 percent projection interval for the NIDEA projections. This provides some confidence that interpreting the range from high to low variant as a 67 percent projection interval is not untenable. For most territorial authorities, the NIDEA projection interval is slightly smaller than the range for the Stats NZ projections, so the model-average projection is slightly closer to the NIDEA median projections than the Stats NZ medium projections. The last four columns of Table 2 present a median and 67% projection interval for the combined projections. This is derived by taking 100 draws from the NIDEA stochastic projections, and 100 draws from a normal distribution for the Stats NZ projections,¹⁴ and weighting each draw equally. The median projection by this method is similar to the model-average projection, but the range is instructive. When the two models (NIDEA and Stats NZ) agree, the range of the averaged projection is similar to those of each of the contributing projections, whereas when the two models disagree the range of the averaged projection is much wider. This demonstrates that a failure to account for model uncertainty likely leads to measures of projection uncertainty that are overly conservative.

Table 3 compares the three projections (NIDEA median; Stats NZ medium; model average) with the subnational population estimate for 2018. The error in the last three columns of Table 3 is expressed as the percentage error (with the population estimate as the denominator). The final row shows the Weighted Mean Absolute Percentage Error (WMAPE) for each of the three

¹⁴ This normal distribution assumes a mean equal to the medium projection, and a standard deviation equal to half the difference between the low and high projections.

projections. WMAPE is a suitable summary measure of the relative size of errors in population projections (Siegel 2002). It is clear from Table 3 that both the NIDEA and Stats NZ projections systematically under-projected the 2018 population. This is due to the historically high and unexpectedly sustained net international migration gain for New Zealand over this period. The projection error ranges from -10.4% to -1.3% for the Stats NZ medium projection and from -13.1% to -5.0% for the NIDEA median projection. Being a weighted combination of the two other projections, the error for the model-average projection is in between, ranging from -3.4% to -10.7%. The WMAPE is lowest for the Stats NZ medium projection (4.8%), and highest for the NIDEA median projection (8.5%), with the model-average projection in-between (7.0%).

The results in Table 3 have implications for decision-makers. A decision-maker is faced with a choice of population projection on which to base their decision. However, they will not know in advance the error in any of the projections they could choose. Choosing one of the available projections could result in a projection with a higher, or lower, relative error. The decision-maker faces a risk that they choose the projection that turns out to have the highest relative error. A risk averse decision-maker could avoid the risk of choosing the *ex post* worst-performing projection by using a weighted average of the available projections. In our illustrative case, that would reduce the worst outcome (in terms of relative error) from a WMAPE of 8.5% to a WMAPE of 7.0%. Obviously, this is worse than the best case WMAPE of 4.8%; however, the decision-maker would not know *ex ante* which of the available projections will result in the smallest relative error.

Table 2 Projections and model-averaging projections for territorial authorities in the Waikato region

Territorial authority	NIDEA 2013-base projection for 2018				Stats NZ 2013-base projection for 2018				Model-average projection				
	P16.5	Median	P83.5	Range	Low	Medium	High	Range	Average	P16.5	Median	P83.5	Range
Thames-Coromandel	26,833	27,410	27,504	4.2%	27,000	27,600	28,200	4.3%	27,504	26,898	27,459	28,088	4.3%
Hauraki	18,412	18,789	18,795	4.0%	18,450	18,800	19,150	3.7%	18,795	18,440	18,771	19,173	3.9%
Waikato	68,162	69,121	70,275	2.8%	70,500	71,900	73,300	3.9%	70,275	68,470	70,235	72,452	5.7%
Matamata-Piako	32,205	32,580	33,101	2.3%	33,300	34,100	34,800	4.4%	33,101	32,327	33,082	34,395	6.3%
Hamilton	155,487	157,550	160,178	2.6%	160,800	163,800	166,700	3.6%	160,178	156,262	160,149	164,962	5.4%
Waipa	47,635	48,364	49,916	3.0%	50,800	51,800	52,700	3.7%	49,916	47,935	49,620	52,174	8.5%
Otorohanga	8,993	9,176	9,462	4.0%	9,570	9,810	10,050	4.9%	9,462	9,069	9,435	9,905	8.9%
South Waikato	21,224	21,561	22,251	3.1%	22,700	23,200	23,700	4.3%	22,251	21,367	22,139	23,397	9.2%
Waitomo	8,825	8,987	9,083	3.6%	8,980	9,220	9,460	5.2%	9,083	8,865	9,090	9,315	4.9%
Taupo	34,676	35,108	35,336	2.5%	35,000	35,700	36,400	3.9%	35,336	34,774	35,374	35,976	3.4%

Table 3 Projection errors for 2018 for territorial authorities in the Waikato region

Territorial authority	Projection			SNPE	Error		
	NIDEA Median	Stats NZ Medium	Model Average		NIDEA Median	Stats NZ Medium	Model Average
Thames-Coromandel District	27,410	27,600	27,504	30,800	-11.0%	-10.4%	-10.7%
Hauraki District	18,789	18,800	18,795	20,600	-8.8%	-8.7%	-8.8%
Waikato District	69,121	71,900	70,275	77,800	-11.2%	-7.6%	-9.7%
Matamata-Piako District	32,580	34,100	33,101	35,500	-8.2%	-3.9%	-6.8%
Hamilton City	157,550	163,800	160,178	165,900	-5.0%	-1.3%	-3.4%
Waipa District	48,364	51,800	49,916	54,800	-11.7%	-5.5%	-8.9%
Otorohanga District	9,176	9,810	9,462	10,400	-11.8%	-5.7%	-9.0%
South Waikato District	21,561	23,200	22,251	24,800	-13.1%	-6.5%	-10.3%
Waitomo District	8,987	9,220	9,083	9,570	-6.1%	-3.7%	-5.1%
Taupo District	35,108	35,700	35,336	38,300	-8.3%	-6.8%	-7.7%
WMAPE					8.5%	4.8%	7.0%

7. Conclusion

Projections or forecasts of future population, and the age and sex structure of the population, are key inputs to decision-making. However, projections are not perfect predictions of a certain future; they must contain uncertainty. This uncertainty arises from model uncertainty, parameter uncertainty, and random variation. Decision-makers must therefore be cognizant of this uncertainty and how it will affect their decisions.

Moreover, decision-makers are often faced with multiple alternative population projections for the same area and/or group of interest. How should a decision-maker decide between these alternative projections? One consideration is to attempt to choose the projection that has the

lowest degree of error. However, there is no way for the decision-maker to determine this *ex ante*, and therefore no clear way to decide on a single preferred projection.

We argue that alternative projections should be seen as complementary, rather than competing, inputs into decision-making. A decision-maker can make use of all of the available projections, rather than limiting themselves to one. This limits the risk of the decision-maker choosing as their preferred projection the projection that *ex post* results in the highest relative error.

We have illustrated a model-averaging approach that makes use of all of the available projections, and creates a single projection by weighting the available based on estimated uncertainty. This approach limits the risk of choosing the projection with the highest relative error. We acknowledge that our illustration is relatively simple, although the illustration does demonstrate how deterministic and stochastic projections may be combined. Nevertheless, we believe that illustrating the approach with a simple case study is useful in opening the conversation about model averaging as a useful tool for applied demography. We leave as an exercise for future researchers to determine an optimal model averaging approach. In the meantime, the simple approach should provide decision-makers with a useful heuristic to assist in using a range of population projections for improved decision-making.

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