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EFFECTIVE THERMAL CONDUCTIVITY PREDICTION OF FOODS USING COMPOSITION AND TEMPERATURE DATA

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ABSTRACT

Thermal conductivity data are important for food process modelling and design. Where reliable thermal conductivity data are not available, they need to be predicted. The most accurate 'first approximation' methodology for predicting the isotropic thermal conductivity of foods based only on data for composition, initial freezing temperature and temperature dependent thermal conductivity of the major food components was sought. A key feature of the methodology was that no experimental measurements were to be required. A multi-step prediction procedure employing the Parallel, Levy and Effective Medium Theory models sequentially for the components other than ice and air, ice and then air respectively is recommended. It was found to provide the most accurate predictions over the range of foods considered (both frozen and unfrozen, porous and non-porous). The Co-Continuous model applied in a single step also provided prediction accuracy within ±20% (on average), except for the porous frozen foods considered. For greater accuracy more rigorous modelling approaches based on knowledge of the foods structure would be required.

Keywords: thermal conductivity prediction, foods

1. Introduction

Thermal property data are needed for modelling and design of food processing operations. Datta (2007) argued that the implementation of advanced thermal processing models of food is now limited more by the accuracy and availability of input parameters (which includes thermal conductivity) than by computing power or modelling expertise.

A large number of thermal conductivity data may be found in the literature (Houska et al., 1994; Houska et al. 1997; Rahman 2009, ASHRAE, 2010) and online (e.g. *Nelfood.com*, Nesvadba et al., 2004) but most of these data are for minimally processed foods. In the event that thermal property data for the food of interest are not available, to predict them with similar precision to thermal conductivity measurement using relatively simple thermal conductivity models would be highly desirable.

The literature describes a large number of thermal conductivity models (Murakami and Okos, 1989; Maroulis et al., 2002; Carson et al. 2006; Carson 2006, Wang et al. 2006; Rahman, 2009; ASHRAE, 2010). Many of them are simply empirical data-reduction models, and hence have a limited range of applicability. A large number have theoretical bases, although many of them include one or more parameters the values of which must be determined empirically, and these often perform well in model validation exercises (e.g. Murakami and Okos, 1989; Hamdami et al., 2003). However, if the numerical value of the empirical parameter is an unknown, these models are of little use if the user intends to perform a prediction without performing any measurements), particularly if very little is known about the microstructure of the food. The aim of this paper was to determine the most accurate model/method for obtaining a first approximation (preferably to within ±20%) of the thermal conductivity of any isotropic food product, by referring just to its composition data, initial freezing temperature (if applicable) and the temperature of the food, without the need to perform any measurements.

2. Thermal Conductivity Prediction for Foods

The thermal conductivity of food products depends on three basic factors: composition, processing conditions, and structure (Rahman, 2009). Foods may be considered as mixtures of the following major components: water, protein, fat, carbohydrate and ash (i.e. non-combustible solids such as minerals etc). Some foods may contain a significant volume fraction of ice and/or air (porosity). Temperature is the most critical processing condition in solid and liquid phases although pressure can be significant too e.g. high pressure processing (HPP). The temperature-dependent thermal conductivities of the major food components were measured by Choi and Okos (1986) and have been reproduced in a number of other sources (e.g. Rahman, 2009; ASHRAE, 2010). In general terms, the thermal conductivities of protein, fat, carbohydrates and ash are similar; about three times lower than that of water, nine times lower than that of ice and ten times higher than that of air.

It is the dependence of the thermal conductivity of the food on structure that is accounted for by the thermal conductivity model. In this study only models which are functions of the composition of the food and thermal conductivities of the major food components only, and do not involve any parameters which must be measured experimentally.

For first approximations, one of two approaches may be employed:

1) Predict the thermal conductivity of the food of concern in a single step by using a single model equation
2) Use an algorithm consisting of a number of steps in which more than one model may be used to predict the thermal conductivity

2.1 Single-Step Approach

Such an approach is desirable because of its simplicity and relative ease of implementation. In addition to the requirement that they must only require the volumetric fractions and thermal conductivities of the components as inputs, suitable models for first approximations will need to be able to be applied to multi-component mixtures and they should treat each component equally, and hence require no knowledge of the food structure.

The simplest thermal conductivity models that meet the Single-step criteria are, respectively, the arithmetic, harmonic and geometric weighted means of the thermal conductivities of the components of the food, where the weighting coefficients being provided by the volumetric fractions of the food:

Parallel Model (Rahman, 2009): $$k_e = \sum_i k_i v_i \quad (1)$$

Series Model (Rahman, 2009): $$k_e = \frac{1}{\sum_i \frac{v_i}{k_i}} \quad (2)$$

Geometric Model (Rahman, 2009): $$k_e = \prod_i k_i^{v_i} \quad (3)$$

The Series and Parallel models physically match structures where the components are in layers perpendicular or parallel to the heat flow direction respectively. The geometric model represents no particular physical structure but it is mathematically simple. The Series and Parallel models respectively represent the theoretical lower and upper bounds of the thermal conductivity of mixtures, provided thermal conduction is the only transport mechanism involved (Carson, 2005). It is therefore unlikely that they will provide the most accurate predictions; however, since they provide limits it is useful to consider their predictions in any modelling exercise. The predicted values by the Geometric model always lie between those predicted by the Series and Parallel models.

Two other models which meet the single-step criteria are the well-known Effective Medium Theory model (EMT) (Landauer, 1952):

$$\sum_i v_i \frac{k_e - k_i}{2k_i + k_e} = 0 \quad (4)$$

and Wang's Co-continuous model (CC) (Wang et al., 2008):

$$k_e = \frac{\sum_i \frac{v_i}{k_i}}{2}\left(\sqrt{1 + \frac{8\sum_i k_i v_i}{\sum_i \frac{v_i}{k_i}}} - 1\right) \quad (5)$$

The EMT model represents the physical structure where all of the components are randomly dispersed with each other (co-dispersed) i.e. no component necessarily represents a continuous phase. The Co-Continuous (CC) model represents a physical structure where all of the components are continuous but intertwined and none is dispersed. Figure 1 shows plots of these five models (Eqs. 1 to 5) for a food with two components in which the ratio of thermal conductivities of the components (k_1/k_2) is 20.

The well-known Maxwell-Eucken model (described below, Eqs. 8 and 9) represents the physical structure where a component is dispersed in another one which is continuous. The above single-step criteria rule out the Maxwell-Eucken model for use in a single step, since it requires the designation of a continuous, and a dispersed phase, and is only capable of handling two components at a time. The Maxwell-Eucken model is, however, suitable for use in a multi-step approach since it does not contain any empirical parameters.

2.2 Multi-Step Approach

While the single-step, single model approach offers simplicity, there is the potential for greater accuracy from the same input data using a multi-step method, since more than one structural model may be employed. Also, components in foods seldom exist in a single well-defined micro-structure. Multi-step thermal conductivity prediction methods have been proposed and implemented previously (e.g. Maroulis et al., 2002; Carson, 2006; Cogné et al., 2003); however, only the method proposed by Wang et al. (2010) does not employ models containing parameters which typically must be determined by experimental measurement, so a similar procedure will be used in this study, as outlined below.

Figure 2 shows plots of the predictions of the Series and Parallel models for four different thermal conductivity ratios (k_1/k_2). The difference between the Series and Parallel models increases with increasing in k_1/k_2, and, since this region contains all the possible effective thermal conductivities (provided k_i and v_i are accurate, and only conduction is involved), it follows that the uncertainty involved in thermal conductivity prediction also increases as k_1/k_2 increases. Based on the ratio of the maximum and minimum thermal conductivity components, the problem of thermal conductivity prediction for foods can be divided into four classes (Carson et al., 2006):

I. Unfrozen, non-porous foods ($k_{water}/k_{solids} \approx 3$)
II. Frozen, non-porous foods ($k_{ice}/k_{solids} \approx 12$)
III. Unfrozen, porous foods ($k_{water}/k_{air} \approx 25$)
IV. Frozen, porous foods ($k_{ice}/k_{air} \approx 100$)

Class I foods with the lowest maximum thermal conductivity ratio are the simplest foods to predict thermal properties for since the uncertainty involved is relatively low, as indicated by the small region bounded by the Series and Parallel models (Fig. 2a). In this case, most thermal conductivity models commonly found in the food engineering and refrigeration literature will provide predictions of sufficient accuracy. However, food Classes II, III and IV provide greater challenges to thermal conductivity prediction since the thermal conductivity of ice is an order of magnitude higher than the thermal conductivities of the other components, and the thermal conductivity of air is an order of magnitude lower (Figs. 2b – 2d). Further, for frozen or porous foods the location (structure) of the ice or air component may be definitive. The approach recommended by Wang et al. (2010) is to break the problem down and deal with the different food components sequentially, i.e. Class II foods are considered as being a mixture of a Class I food and ice, Class III foods are considered as being a mixture of a Class I food and air, and Class IV foods are considered as being a mixture of Class II foods and air. This approach is schematically illustrated in Figure 3.

Wang et al. (2010) recommended that thermal conductivity model for Class I food components (as indicated in Figure 3) should be the Parallel model, since it is the simplest model yet provides sufficient accuracy and allows for any number of components. This approach was also adopted for this study.

Wang et al. (2009) and Wang et al. (2010) recommended that for Class II foods the presence of ice should be accounted for using Levy's model (Levy 1981):

$$k_{\mathrm{II}} = k_{Levy} = k_{ice}\,\frac{2k_{ice} + k_{\mathrm{I}} - 2(k_{ice} - k_{\mathrm{I}})F}{2k_{ice} + k_{\mathrm{I}} + (k_{ice} - k_{\mathrm{I}})F} \qquad (6)$$

$$F = \frac{2/G - 1 + 2(1 - v_{ice}) - \sqrt{[2/G - 1 + 2(1 - v_{ice})]^2 - 8(1 - v_{ice})/G}}{2} \qquad (7)$$

$$G = \frac{(k_{ice} - k_{\mathrm{I}})^2}{(k_{ice} + k_{\mathrm{I}})^2 + k_{ice}k_{\mathrm{I}}/2} \qquad (8)$$

The Levy model physically represents the structure where the two components are mixed in a combination of one dispersed in the other which is continuous and vice versa in proportions such that the composite conductivity is the same (i.e. a mixture of the two versions of the Maxwell-Eucken model as shown in Figure 4).

Class III and IV foods require consideration of porosity in addition to ice, water and other components. The effect of porosity on thermal conductivity is complicated by the widely differing structures that porous foods may have, e.g. air may be dispersed as bubbles in continuous matrix, or it may form a continuous phase, as is the case with particulate foods, or in some cases it may exist in both dispersed and continuous phases. For a given thermal conductivity of the so-called condensed phase (i.e. the phase containing the solid and liquid components), the thermal conductivity of food will be significantly higher if the air forms a dispersed phase rather than a continuous phase (Carson et al., 2005). This is best illustrated by the Maxwell-Eucken model, which assumes a structure of one phase sparsely dispersed within another. If air forms the dispersed phase then the Maxwell-Eucken model as the following form ("ME1"):

$$k_{ME1} = k_{\mathrm{I}} \frac{2k_{\mathrm{II}} + k_a - 2(k_{\mathrm{II}} - k_a)v_a}{2k_{\mathrm{II}} + k_a + (k_{\mathrm{II}} - k_a)v_a} \qquad (9)$$

If air forms the continuous phase then it has the following form ("ME2"):

$$k_{ME2} = k_a \frac{2k_a + k_{\mathrm{I}} - 2(k_a - k_{\mathrm{I}})(1 - v_a)}{2k_a + k_{\mathrm{I}} + (k_a - k_{\mathrm{I}})(1 - v_a)} \qquad (10)$$

Figure 4 shows plots of the two forms of the Maxwell-Eucken model, and it is clear that the form in which air is the dispersed phase (sponge/foam-like materials) predicts significantly higher thermal conductivities, than when air is the continuous phase (particulate materials), other than for very low or very high porosities. The EMT, CC and levy models are also shown in Figure 4. They each traverse the space between ME1 and ME2 in different ways due to the structures they represent.

The objective of this paper was to recommend a procedure in which nothing, including the nature of the air distribution, is assumed to be known about the structure of the food; however, some general inferences may be drawn about the air-distribution from the porosity of the food. Specifically, if the porosity is ‘low’ (e.g. < 0.3) then it is reasonable to assume (in general) that the air is dispersed as bubbles. Likewise, if the porosity is ‘high’ (e.g. > 0.7) then it is reasonable to assume (in general) that air forms a continuous phase, and the food is most probably in particulate form. In the mid-porosity range both particulates (air continuous) and sponge/foam structures (air dispersed) are possible. Ideally, a model accounting for porosity would provide similar predictions to the Maxwell-Eucken model with air as the dispersed phase for low porosities, similar predictions to the Maxwell-Eucken model with air as the continuous phase for high porosities, and predictions which are mid-range between the two forms of the Maxwell-Eucken model for mid-range porosities. Figure 4 shows that the EMT model fulfils these requirements adequately for first approximations (see also Carson et al., 2005), and therefore it is recommended for use in the multi-step procedure, to account for porosity, i.e. Class III and IV food. The EMT model for the multi-step process is:

$$k_{\mathrm{III}} = \frac{(3v_a - 1)k_a + [3(1 - v_a) - 1]k_{\mathrm{I}} + \sqrt{\{(3v_a - 1)k_a + [3(1 - v_a) - 1]k_{\mathrm{I}}\}^2 + 8k_{\mathrm{I}}k_a}}{4} \qquad (11)$$

$$k_{\mathrm{IV}} = \frac{(3v_a - 1)k_a + [3(1 - v_a) - 1]k_{\mathrm{II}} + \sqrt{\{(3v_a - 1)k_a + [3(1 - v_a) - 1]k_{\mathrm{II}}\}^2 + 8k_{\mathrm{II}}k_a}}{4} \qquad (12)$$

Eqs. 11 and 12 are simply the two-component forms of Eq. (4) rearranged to be explicit in terms of the effective thermal conductivity.

2.3 Composition data and ice fractions

When using a multi-step method for Class II, III, or IV foods, it is important to bear in mind that intermediate volume fractions must be used at the intermediate stages. For example, if a Class II (frozen, non-porous) food is being modelled, then the first stage is to determine the conductivity of the 'non-ice' phase, using the Parallel model. In this case the volume fractions employed must be the volume fractions for the non-ice phase, e.g. for protein:

$$\upsilon_p = \frac{v_p}{v_w + v_p + v_f + v_c + v_{ash}} = \frac{v_p}{1 - v_{ice}} \quad (13)$$

and similarly for the other components. This may be implemented most conveniently by using the following form of the Parallel model, rather than Eq. (1):

$$k_I = \frac{k_w v_w + k_p v_p + k_f v_f + k_c v_c + k_{ash} v_{ash}}{v_w + v_p + v_f + v_c + v_{ash}} \quad (14)$$

For a complete worked example of this method, refer to Wang et al. (2010).

It is also important to recognise that composition data for foods are usually available on a mass basis, and yet the thermal conductivity models employ volume fractions, since thermal conductivity is a volumetric property. The conversion between mass and volume fractions for liquid and solid components may be modelled by the following relationship (Choi and Okos, 1986; Rahman, 2009):

$$v_i = x_i \frac{\rho_{cond}}{\rho_i} \quad (15)$$

where:

$$\rho_{cond} = \frac{1}{\sum_i \frac{x_i}{\rho_i}} \quad (16)$$

If the food is porous and the porosity (i.e. the volume fraction of air, v_a) has not been measured it may be estimated from the apparent (bulk) density (ρ_e) (Choi and Okos, 1986; Rahman, 2009):

$$\frac{1-v_a}{\rho_e} = \sum_i \frac{x_i}{\rho_i} \tag{17}$$

Density data for the major food components as functions of temperature may be found in the same sources as the thermal conductivity data (i.e. Choi and Okos, 1986; Rahman, 2009; ASHRAE, 2010). Rahman (2009) discusses other models for predicting porosity of foods from composition and density data; however, Eqs. (15) to (17) were deemed to be sufficiently accurate for this exercise.

The thermal conductivity of frozen foods is strongly dependent on the ice fraction, which in turn is strongly dependent on temperature, and therefore an ice fraction model is required for thermal conductivity prediction. There are several ice fraction models in the literature (Pham, 1987; Fikiin, 1998; Boonsupthip and Heldman, 2007; Rahman 2009). Many of these require calculations of mole fractions, which in turn requires estimation of molar masses for the macromolecules (proteins and complex carbohydrates). Many contain empirical parameters, and most require knowledge of the amount of bound or un-freezable water. The empirical model proposed by Tchigeov based only on total water content, system temperature, and initial freezing temperature (T_F) has been found to work well for –45°C < T < T_F and –2 < T_F < –0.4°C (Fikiin, 1998, Pham, 2014):

$$x_{ice} = \frac{1.105 x_{w,total}}{\left[1 + \dfrac{0.7318}{\ln(T_F - T + 1)}\right]} \tag{18}$$

For initial freezing temperatures below –2°C a more general relationship may be used:

$$x_{ice} = (x_{w,total} - x_b)\left(1 - \frac{T_F}{T}\right) \quad (19)$$

Note that in both Eq. (18) and (19) temperature is in degrees Celsius rather than Kelvin. If the fraction of bound water (x_b) is unknown it may be related to the composition of the food. For example, Pham (1987) recommended that for meat products

$$x_b = 0.4 x_p \quad (20)$$

In this study, Tchigeov's method (Eq. 18) was used for foods with an initial freezing temperature above –2 °C, and Pham's method based on protein composition (Eq. 20) was used for foods with initial freezing temperatures below –2 °C. No simple but sufficiently accurate ice fraction prediction method without empirical parameters was found in the literature. However, unlike the thermal conductivity models that contain empirical parameters which are specific to the food in question and typically need to be determined from a thermal conductivity measurement, the empirical parameters in Eq. (18) and (Eq. 20) apply generally and do not need to be determined from an ice fraction measurement, provided, in the case of Eq. (18) that the initial freezing temperature is in the specified range, and, in the case of Eqs. (19) and (20), the food contains a non-zero protein content. Therefore their use is consistent with the aim of predicting thermal conductivity without experimental measurements being required.

Initial freezing temperature data are available in the literature for some foods, and some predictive models have also been proposed (Boonsupthip and Heldman, 2007; Rahman, 2009). In the absence of any measured data, Fikiin (2014) recommends the use of –1.0 °C for first approximations. In this study, measured initial freezing temperature data were used for all foods considered.

The amount of unfrozen water in frozen foods is simply the difference between the ice fraction and the total water content:

$$x_w = x_{w,total} - x_{ice} \quad (21)$$

3. Comparison of Single Step and Multi-step Predictions with Measured Data

The predictions from each of the five single-step models (Eqs. 1 – 5) along with the multistep procedures (as illustrated in Fig. 3) have been compared to measured thermal conductivity data from the literature for the four different Classes of foods. The difference is defined as:

$$\delta = \frac{|k_{\exp} - k_{\mathrm{mod}}|}{k_{\exp}} \times 100\% \quad (22)$$

Only measured thermal conductivity data where the measurement methodology was proven accurate and the composition of the food (including porosity) and, in the case of frozen foods, initial freezing temperature were available were considered for this assessment exercise.

3.1 Class I Foods

Figure 5 shows plots of thermal conductivity predictions from Eqs. (1) to (5) as a function of combined solids content for a range of Class 1 foods at 20 °C (data from Willix et al., 1998). Table 1 summarises the average differences (δ) between the model predictions and experimental data for each of the different foods. All the model predictions are within ±20%, which is quite acceptable for a first approximation. Hence the decision to employ the Parallel model in the multi-step procedure is justified, since it is the simplest model, and for the foods considered actually produced the lowest average difference.

3.2 Class II foods

Table 2 shows the differences between the model predictions and experimental data for the same selection of foods (data from Willix et al., 1998); however, this time the temperature is –20°C. The Parallel model no longer provides sufficient accuracy; however, the Co-continuous and Geometric models, as well as the multi-step procedure (i.e. using the Parallel model for the thermal conductivity of the non-ice components, followed by Levy’s model to account for the ice fraction) all provide predictions within, on average, ±20%.

3.3 Class III foods

Suitable data for testing the predictions for porous non-frozen foods were difficult to obtain, since very often only minimal composition data are provided. In particular, bulk or apparent density or porosity data is often not available in the literature. Many of the data for which composition and temperature data were supplied were highly questionable, since they lay outside the Series and Parallel model bounds. Table 3 shows the differences between the model predictions and experimental data for four different Class III foods where all of the following data were provided: porosity (or bulk density), moisture content, measurement temperature. Where these data were not supplied, the solids contents were assumed based on typical compositions for the particular food in question (Rahman, 2009). Table 3 shows that the multi-step procedure (i.e. using the Parallel model for the thermal conductivity of the non-ice components, followed by the EMT model to account for porosity) and the single-step

CC and EMT equations on average provide predictions of sufficient accuracy for first approximations, with the multi-step procedure providing the greatest accuracy. However, differences from individual measurements were sometimes greater than 20%, which highlights the greater uncertainty involved in thermal conductivity prediction once porosity is introduced.

3.4 Class IV foods

The number of examples of Class IV foods is relatively small, and the group is mainly comprised of frozen desserts. Of these, ice cream is probably the most widely studied in the food engineering literature, and the data from Cogné et al. (2003) were used since all the necessary data (composition, temperature, initial freezing temperature) were available. Table 4 shows the differences between the model predictions and experimental data at two different temperatures (–15 °C, and –30 °C). In this instance the multi-step procedure (i.e. using the Parallel model for the thermal conductivity of the non-ice components, followed by Levy's model to account for the ice fraction, and the EMT model to account for porosity) has the clear advantage over the single-step models, and none of the single-step procedures provided predictions which are accurate to within ±30 %.

Note that while the average difference between the thermal conductivities predicted by the standard Multi-step procedure is within ±20%, individual values are considerably higher (up to 50%). The difference between the predictions and the data increases as the porosity increases. This may be explained by the fact that the air remains dispersed as discrete bubbles even though the porosity of ice cream increases well beyond 0.3, and therefore the Maxwell-Eucken model with air as the dispersed phase is the more suitable model of the porous structure than the EMT model, which assumes that air begins to form a continuous phase as the porosity increases. The final column in Table 4 shows the difference between experimental data and model predictions when the porosity is accounted for by the Maxwell Eucken model with air is the dispersed phase, rather than the EMT model. As expected the predictions are more accurate with the average difference being almost half of the average difference when the standard multi-step model using the EMT model is employed. Figure 6 serves to further illustrate this point.

4. Discussion

The results of the prediction comparison exercises show that for foods containing porosity, both frozen and unfrozen, the multi-step thermal conductivity prediction procedure proved to be the most accurate. The multi-step procedure also has the advantage over the single-step procedure in that while it can be employed without any knowledge of the structure of the food, there is scope for knowledge of the structure of the food to be incorporated into the method (as was illustrated in Section 3.4). Other than for the Class IV foods, the single-step Co-Continuous model provided, on average, prediction accuracies within the ‘first approximation’ range of ±20% and is simpler to implement than the multi-step method.

On balance, the authors recommend that the multi-step procedure be used since it provided the greatest prediction accuracy over the range of foods considered, it has the scope for improving prediction accuracy by allowing for equations at each stage of the procedure to be changed if structural information about the food is known, and yet it can also be used with reasonable confidence in the form presented here without any knowledge of the structure of the food.

5. Conclusion

Using only composition and initial freezing temperature data and knowledge of the food's temperature, a multi-step thermal conductivity prediction procedure provided the most accurate thermal conductivity predictions for the range of foods considered. However, the single-step Co-Continuous model also provided predictions within ±20% other than for food containing both ice and air voids. On balance, however, the multi-step procedure is recommended for general use, since it provided the most accurate predictions over the widest range of foods, and also because there is scope for enhancements to be made within its framework, unlike the single-step method. It is emphasised that this methodology is intended for first approximations based on the minimum of input data, rather than as a rigorous modelling framework.

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NOMENCLATURE

F	intermediate variable (Eq. 7)
G	intermediate variable (Eq. 8)
k	thermal conductivity ($W\ m^{-1}\ K^{-1}$)
T	temperature (°C)
v	volume fraction
x	mass fraction

δ	difference between experimental value and model prediction
ρ	density ($kg\ m^{-3}$)
υ	intermediate volume fractions

Subscripts

1	property of component 1

2	property of component 2
a	property of air
ash	property of ash
b	property of bound water
c	property of carbohydrate
cond	property of condensed (i.e. solid/liquid) phase
e	effective property
exp	experimental property
f	property of fat
F	initial freezing property
i	*i*th component
I, II, III, IV	property relating the class of food as defined in Section 2.2
ice	property of ice
mod	property predicted by a model
p	property of protein
w	property of water

Figure and Table Captions

Table 1: Comparison of the differences between predicted and experimental thermal conductivity values for unfrozen, non-porous (Class I) foods at 20^{o}C (experimental data from Willix, et al., 1998).

Table 2: Comparison of the differences between predicted and experimental thermal conductivity values for frozen, non-porous (Class II) foods at -20^{o}C (experimental data from Willix, et al., 1998).

Table 3: Comparison of the differences between predicted and experimental thermal conductivity values for unfrozen, porous (Class III) foods [experimental data Sources: a - Baik et al. (1999), b - Rahman (2009), c - Houska et al. (1997), d - Houska et al. (1994), e - Carson et al. (2004), f - Muramatsu et al., (2008), g – Carson (2014), h - Carson and Kemp (2014)]

Table 4: Comparison of the differences between predicted and experimental thermal conductivity values for ice cream (Class IV food) at -15°C and -20°C (experimental data from Cogné, et al., 2003)

Figure 1: Plots of the Series, Parallel, Geometric, EMT and Co-continuous models for a binary mixture in which $k_1/k_2 = 20$

Figure 2 Plots of Series and Parallel models: 2a) $k_1/k_2 = 3$, 2b) $k_1/k_2 = 12$, 2c) $k_1/k_2 = 25$, 2d) $k_1/k_2 = 100$.

Figure 3: Schematic representation of the sequential approach for predicting the thermal conductivity of foods

Figure 4: Plots of the Maxwell-Eucken model with air as the dispersed phase (“ME1”), air as the continuous phase (“ME2”) plus the EMT, Co-Continuous (CC) and Levy models

Figure 5: Plots of the thermal conductivity predictions of Series, Parallel, Geometric, EMT and Co-Continuous models with experimental data for unfrozen, non-porous (Class I) foods

Figure 6: Plots of the thermal conductivity predictions of Series, Parallel, Geometric, EMT and Co-continuous models, standard Multi-step prediction method, and modified Multi-step prediction method with experimental data for ice cream (Class IV food)

	δ				
	Parallel	Series	Geometric	Co-continuous	EMT
Lean beef	0.2	11.6	1.3	0.3	2.0
Beef mince	13.1	9.0	6.1	9.0	12.6
Boneless chicken	5.4	6.8	4.1	5.1	8.2
Pork Sausage meat	11.2	11.6	3.3	6.2	8.9
Trim Pork Mince	13.8	4.2	9.6	11.4	15.2
Veal mince	10.1	6.8	7.1	9.1	10.9
Venison	5.2	6.5	4.3	5.3	7.9
Lemon Fish fillets	0.2	12.6	2.7	1.8	2.6
Snapper fillets	5.0	5.7	4.2	5.0	7.7
Tarakihi fillets	5.5	5.9	4.3	5.2	7.8
Cheddar cheese	0.0	21.3	9.9	4.8	6.9
Edam Cheese	0.9	21.5	9.6	4.6	5.1
Mozzarella cheese	0.2	20.8	8.2	3.7	4.6
Average	**5.5**	**11.1**	**5.7**	**5.5**	**7.7**

Table 1: Comparison of the differences between predicted and experimental thermal conductivity values for unfrozen, non-porous (Class I) foods at 20°C (experimental data from Willix, et al., 1998).

	δ					
	Parallel	Series	Geometric	Co-continuous	EMT	Multi-step
Lean beef	29.4	59.7	11.0	16.0	13.3	3.0
Beef mince	87.7	49.6	10.0	14.6	48.9	32.3
Boneless chicken	33.8	58.5	8.2	13.4	17.1	6.4
Pork Sausage meat	48.8	57.7	13.4	7.0	15.0	5.3
Trim Pork Mince	94.1	43.0	25.5	23.0	64.2	47.5
Veal mince	71.1	51.3	7.5	7.0	42.3	27.2
Venison	39.5	56.2	3.7	9.3	22.3	11.3
Lemon Fish fillets	42.2	55.4	0.9	7.5	25.5	14.4
Snapper fillets	36.7	54.3	2.1	8.8	21.9	11.8
Tarakihi fillets	37.6	54.4	2.1	8.5	22.3	12.0
Cheddar cheese	22.7	58.2	37.3	17.5	29.1	20.1
Edam Cheese	13.8	63.1	42.3	24.9	33.0	25.9
Mozzarella cheese	33.4	62.0	33.9	16.5	18.1	15.3
Average	**45.4**	**55.6**	**15.2**	**13.4**	**28.7**	**17.9**

Table 2: Comparison of the differences between predicted and experimental thermal conductivity values for frozen, non-porous (Class II) foods at -20^{o}C (experimental data from Willix, et al., 1998).

		δ					
	v_a range	Parallel	Series	Geometric	Co-continuous	EMT	Multi-step
Cup cake[a]	0.29 - 0.83	32.1	59.4	34.4	15.0	27.6	24.6
Defatted soy flour[b]	0.54 - 0.72	59.7	43.4	13.2	11.1	5.9	7.8
Dried beef[c]	0.67 - 0.73	25.4	46.8	26.6	8.1	18.5	17.5
Milk powder[d]	0.40 - 0.43	17.8	64.5	43.5	28.2	19.8	15.1
Model food[e]	0.04 - 0.65	10.8	75.4	41.0	39.3	19.2	19.2
Rice[f]	0.36 - 0.41	22.4	62.3	28.9	20.9	13.4	2.7
Sponge Cake[g]	0.45 - 0.61	40.0	64.7	35.0	16.7	18.9	10.5
Sucrose powder[h]	0.44 - 0.51	34.8	48.4	8.2	5.1	1.2	3.1
Average		**28.0**	**58.6**	**31.3**	**20.4**	**17.4**	**14.5**

Sources: a - Baik et al. (1999), b - Rahman (2009), c - Houska et al. (1997), d - Houska et al. (1994), e - Carson et al. (2004), f - Muramatsu et al., (2008), g - Carson (2014), h - Carson and Kemp (2014)

Table 3: Comparison of the differences between predicted and experimental thermal conductivity values for unfrozen, porous (Class III) foods [experimental data Sources: a - Baik et al. (1999), b - Rahman (2009), c - Houska et al. (1997), d - Houska et al. (1994), e - Carson et al. (2004), f - Muramatsu et al., (2008), g – Carson (2014), h - Carson and Kemp (2014)]

Porosity	δ						
	Parallel	Series	Geometric	Co-continuous	EMT	Multi-step	
						Standard	Max-Euck
-15 °C							
0.13	51.4	84.9	33.4	39.4	8.4	4.5	5.8
0.23	62.3	88.2	44.6	43.8	2.4	3.8	8.7
0.33	70.0	89.6	54.2	45.5	9.8	2.1	9.4
0.41	73.2	90.1	60.8	46.2	23.8	11.3	8.2
0.46	72.1	90.3	64.8	46.9	34.6	19.9	5.7
0.6	102.6	88.1	67.1	36.2	54.2	38.3	18.9
0.67	105.5	86.8	69.1	32.8	64.0	53.9	18.4
Average	**76.7**	**88.3**	**56.3**	**41.5**	**28.2**	**19.1**	**10.7**
-30 °C							
0.13	60.4	86.4	32.8	40.3	15.8	7.0	8.5
0.23	63.5	89.9	47.5	47.4	3.8	1.1	6.0
0.33	75.5	90.9	56.1	47.8	7.0	2.5	9.3
0.41	73.7	91.6	63.9	49.9	24.8	14.4	4.9
0.46	77.7	91.5	66.8	49.1	34.5	20.7	5.4
0.6	115.6	89.2	68.6	36.9	55.8	38.6	22.2
0.67	104.6	88.8	72.6	37.7	68.5	58.4	13.7
Average	**81.6**	**89.7**	**58.3**	**44.2**	**30.0**	**20.4**	**10.0**

Table 4: Comparison of the differences between predicted and experimental thermal conductivity values for ice cream (Class IV food) at -15^{o}C and -20^{o}C (experimental data from Cogné, et al., 2003)

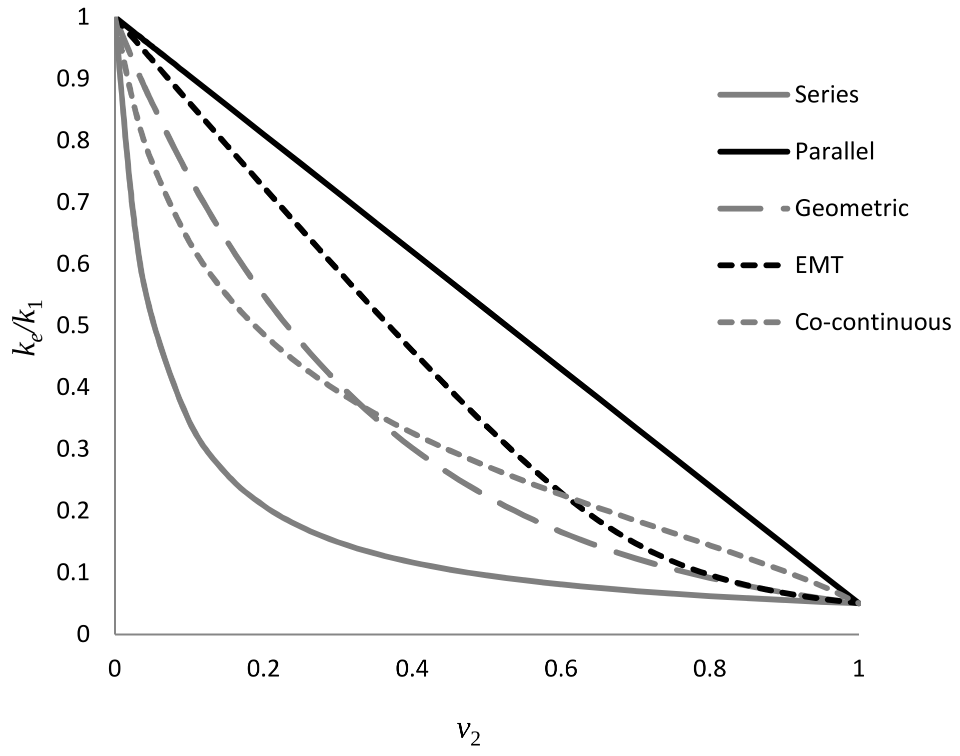

Figure 1: Plots of the Series, Parallel, Geometric, EMT and Co-continuous models for a binary mixture in which $k_1/k_2 = 20$

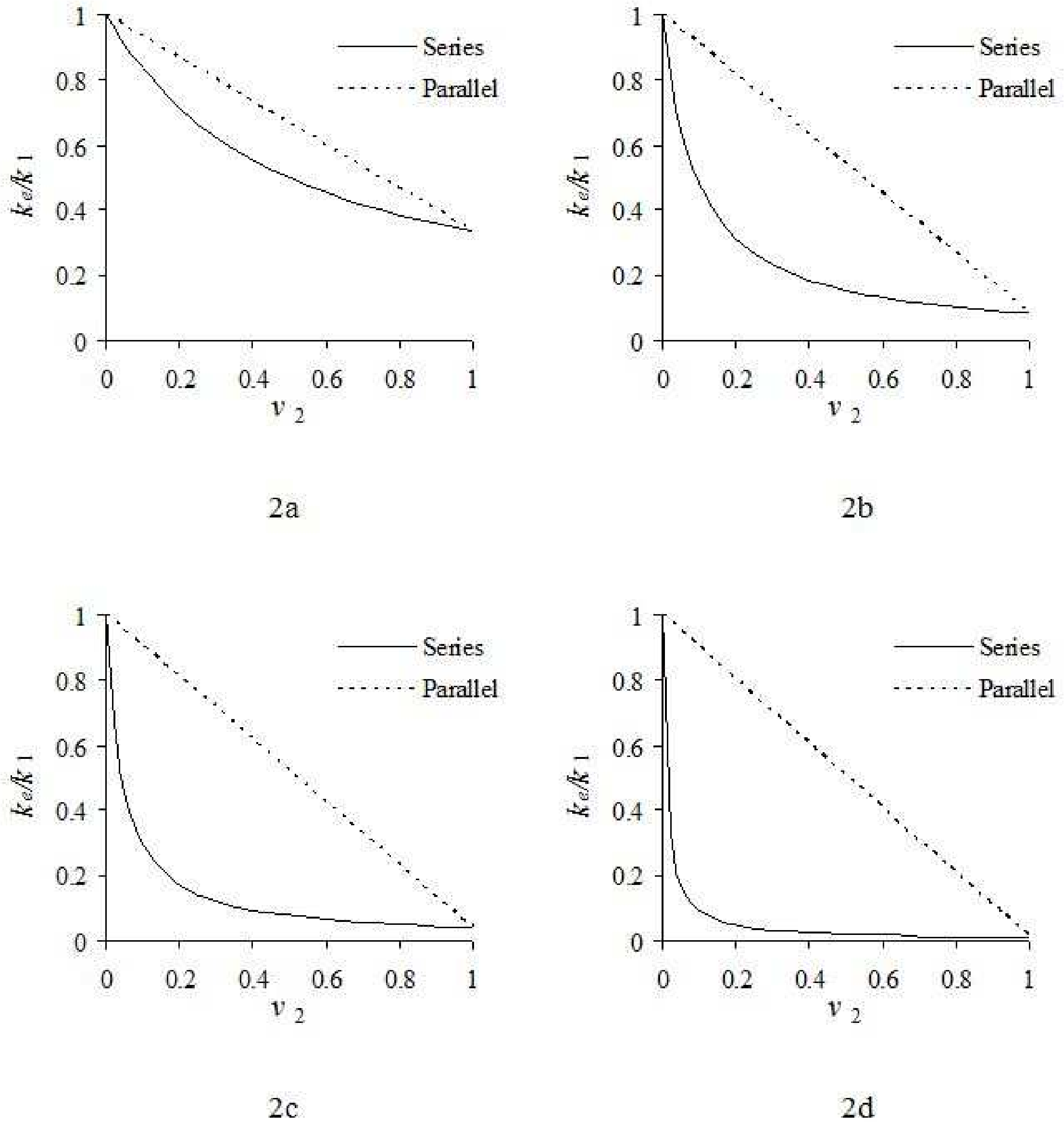

Figure 2 Plots of Series and Parallel models: 2a) $k_1/k_2 = 3$, 2b) $k_1/k_2 = 12$, 2c) $k_1/k_2 = 25$, 2d) $k_1/k_2 = 100$.

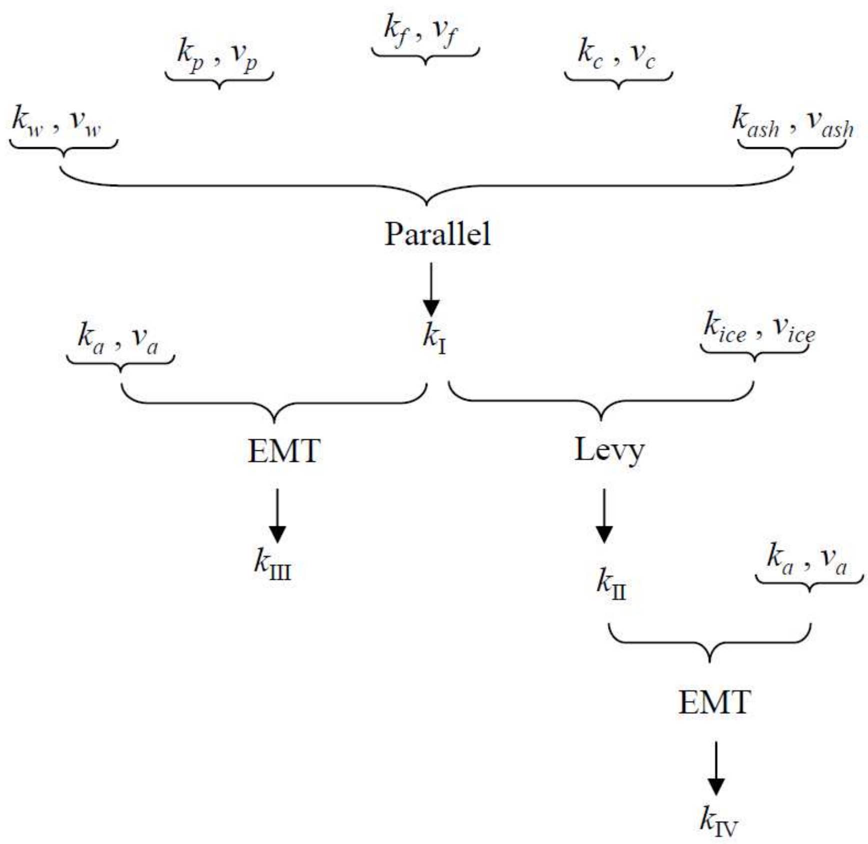

Figure 3: Schematic representation of the sequential approach for predicting the thermal conductivity of foods

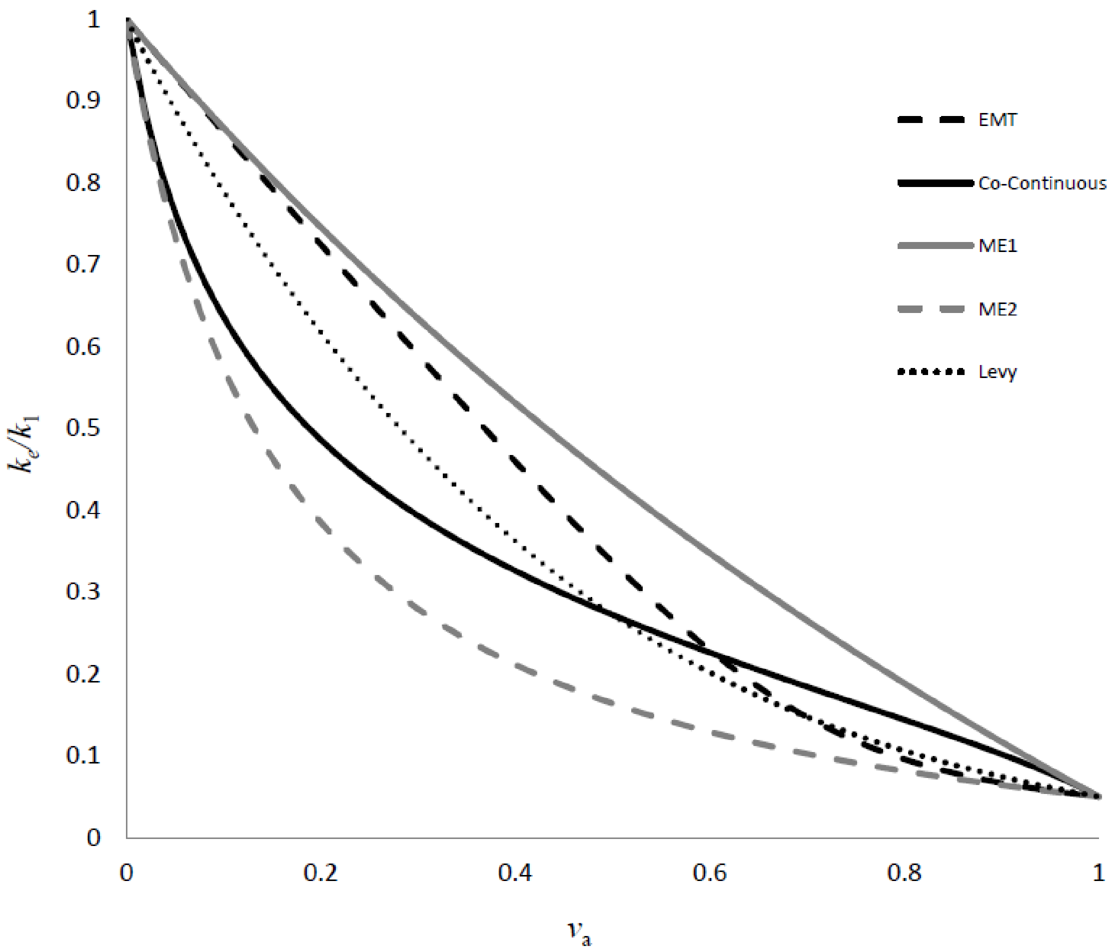

Figure 4: Plots of the Maxwell-Eucken model with air as the dispersed phase (“ME1”), air as the continuous phase (“ME2”) plus the EMT, Co-Continuous (CC) and Levy models

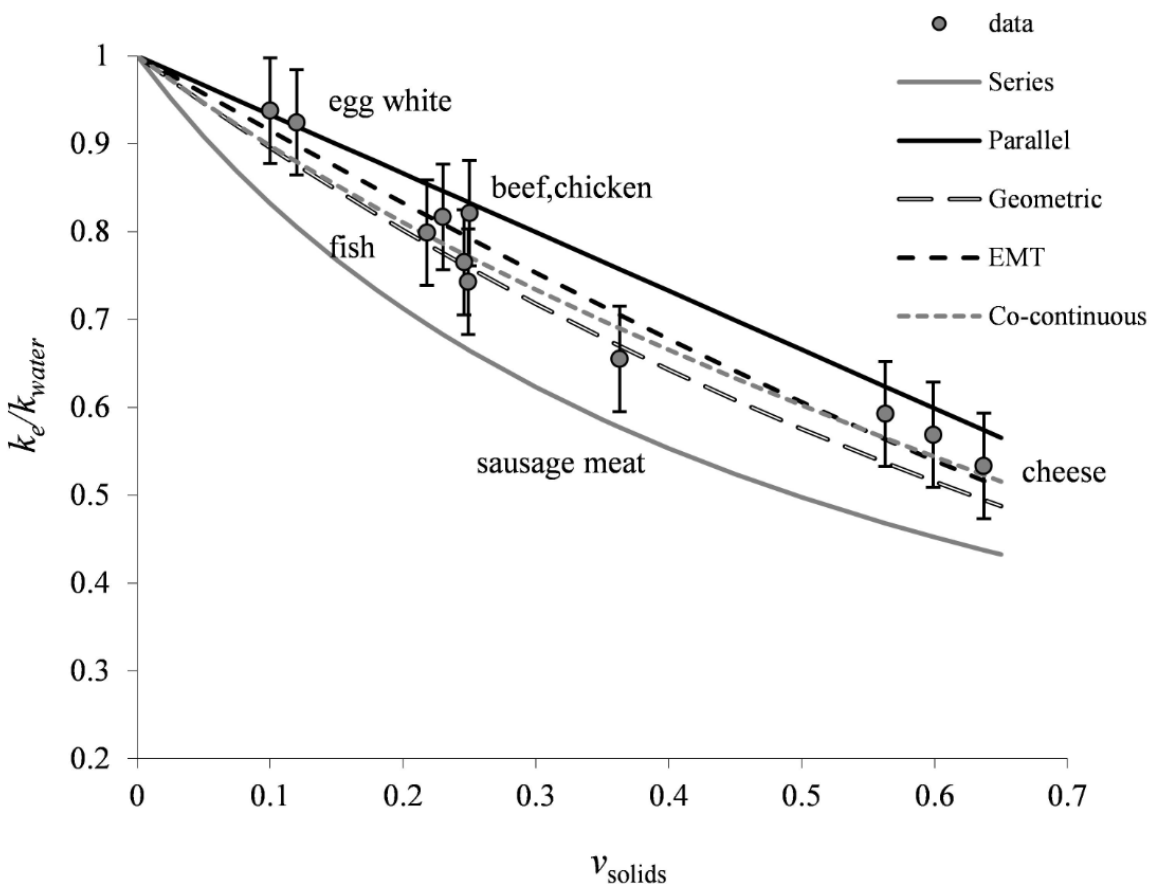

Figure 5: Plots of the thermal conductivity predictions of Series, Parallel, Geometric, EMT and Co-Continuous models with experimental data for unfrozen, non-porous (Class I) foods

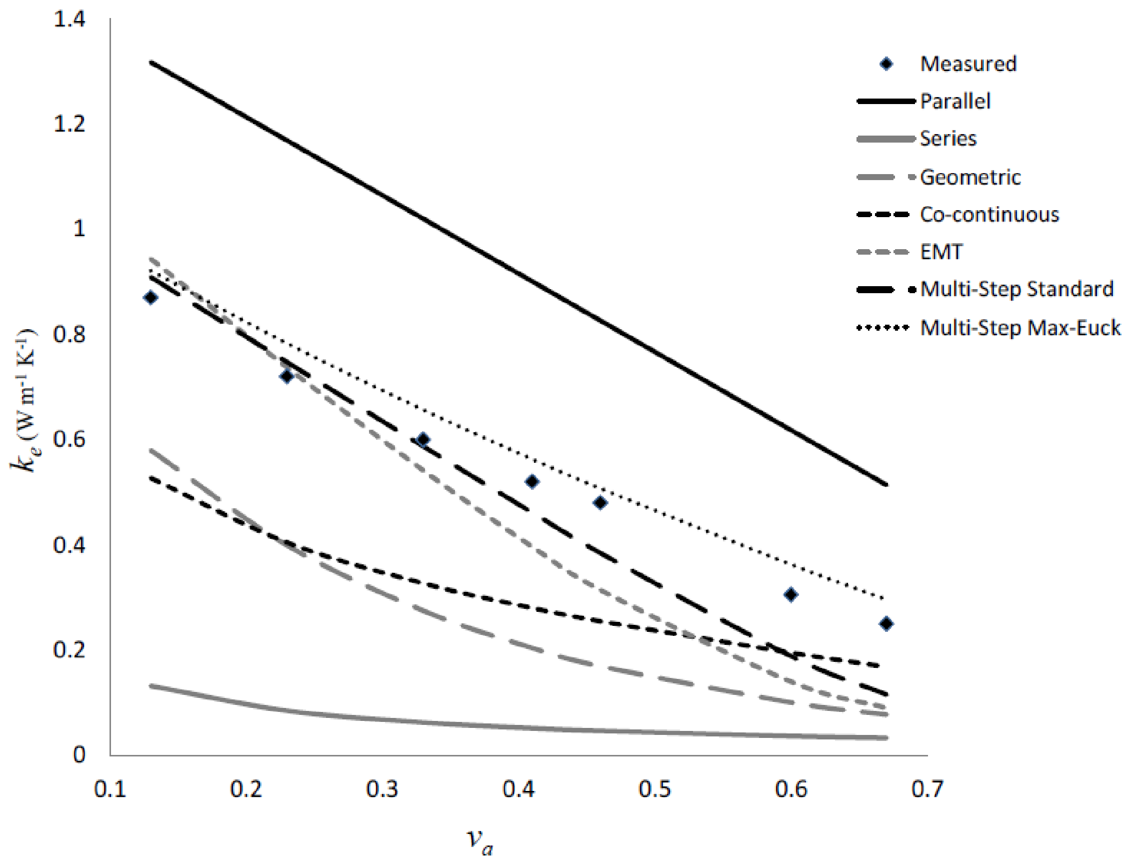

Figure 6: Plots of the thermal conductivity predictions of Series, Parallel, Geometric, EMT and Co-continuous models, standard Multi-step prediction method, and modified Multi-step prediction method with experimental data for ice cream (Class IV food)

- Different methods for predicting thermal conductivity of foods solely from composition and temperature data were compared against measured data
- Multi-step procedure involving sequential application of Parallel, Levy and Maxwell-Eucken model provided most accurate predictions on average
- Other than for frozen, porous foods, the Co-continuous models also provided predictions within ±20% on average