Children's Negotiation of Meanings About Geometric Shapes and Their Properties in a New Zealand Multilingual Primary Classroom

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Abstract

New Zealand is a nation of superdiversity in terms of ethnicities and languages spoken. This superdiversity is reflected in New Zealand multilingual classrooms. In the New Zealand primary school mathematics curriculum, the teaching and learning of early geometry focuses on recognising and understanding shapes, their properties, and symmetries, and on describing the position and movement of shapes. The Achievement Objectives suggest that the children at Curriculum Level 3, which roughly translates to Year 5/6 (9 to 11-year-old), are expected to identify, describe, and classify two-dimensional (2D) and three-dimensional (3D) shapes by spatial features. Acknowledging the multilingual context of a New Zealand classroom, this study investigated how children negotiate their meanings about 2D shapes, 3D shapes, and their properties as they engage in whole-class and/or group interactions in a New Zealand primary classroom. Accordingly, following research questions (RQ) guided this study:

1. *What discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
2. *How do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
3. *What characteristics of dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

A qualitative study informed by the Discursive Psychology perspective (Edwards & Potter, 1992) within the Critical Inquiry research paradigm was undertaken. Edwards and Potter (1992) argue that language-in-use is construed as an action in itself and, as a result, knowledge is taken as situated and constructed through language-in-use as people interact. Bakhtin's (1981) Dialogic Theory and Garfinkel's (1967) Ethnomethodology informed the theoretical framework of this study. Data were gathered from a Year 5/6 classroom in a New Zealand English-medium school. The participants were fifteen children (nine multilingual, six monolingual) and their mathematics teacher. Six geometry lessons on shapes and their properties were observed and audiovisually recorded. Additional data were gathered from a variety of sources, including semi-structured teacher interviews, four focus group interviews with children, a short questionnaire filled by the parents, children's work samples, and

teacher's unit plan. Data from different sources allowed me to establish the reliability and validity of the findings.

Data were analysed in three phases: thematic analysis, micro-level analysis, and macro-level analysis. Five themes were identified from thematic analysis of data to explore the discursive constructions that the children used to represent their understanding of shapes and their properties (RQ1). These themes are: (i) making sense of 2D shapes, (ii) making sense of 3D shapes, (iii) relating 2D shapes with 3D shapes, (iv) mathematical construct of dimension, and (v) naming shapes in Te Reo Māori (the Indigenous language of New Zealand). For the purpose of managing and presenting analysis, two Key Moments within each of the five themes were identified for further analysis at the micro-level and macro-level. For the micro-level analysis, I used selected Conversation Analysis (Schegloff & Sacks, 1973) techniques to explore what is said and how it is said (RQ2). Based on the micro-level analysis findings, the macro-level analysis was conducted using Bakhtinian concepts of speech genres, discourses, heteroglossia and unitary language, double-voicedness, and chronotopes to explore the characteristics of dialogic space that influence children's negotiations of meanings about shapes and their properties (RQ3).

The study reveals four novel findings. First, the analogy of "flat vs fat" may not be useful in developing children's geometric understanding of dimension. Second, the study indicates that multilingual children use prosodic repertoires from their multiple languages as they engage in whole-class or group interactions, and these prosodic repertoires may be interpreted differently by monolingual English-speaking children. Third, the study reveals the presence of several speech genres available to teachers and children within the dialogic space of a multilingual classroom. Fourth, the study shows that multiple meanings could be drawn out for each utterance, and the meaning of an utterance is dependent not only upon the interaction of unitary language and heteroglossia between the discourses but within the discourse as well.

The findings of this study suggest, first, that a comprehensive definition of dimension needs to be included in the school curriculum. Second, teachers may benefit from learning about prosodic features that multilingual children may use to show their confidence or doubt about their learning, along with several speech genres available within the dialogic space. Several ideas for further research in the mathematics education field with a focus on developing an understanding of geometry concepts such as dimension are also suggested. Overall, the study highlighted the need for teachers and teacher educators to recognise subtle yet powerful aspects of language use that influence children’s negotiation of meanings about geometric ideas as children engage in classroom interactions.

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Glossary

This glossary includes Māori terms and their interpretations. These interpretations/ translations were derived from Te Reo Pāngarau (Christensen, 2010).

Ahu - nga	Dimension as well as direction
Āhua ahu-toru	Three-dimensional solid
Ahu-rua	Two-dimensional
Ahu-tahi	One-dimensional
Ahu-toru	Three-dimensional
Iwa	Nine
Koeko tapatoru	Triangular pyramid
Ono	Six
Porowhita	Circle
Rima	Five
Rua	Two
Tahi	One
Taimana	Kite shape
Tapaiwa	Nonagon
Tapaono	Hexagon
Taparima	Pentagon
Tapa-tekau	Decagon
Tapatoru hāngai	Right-angled triangle
Tapatoru hikuwaru	Scalene triangle
Tapatoru koki hāpūpū	Obtuse-angled triangle
Tapatoru koki tāhapa	Acute-angled triangle
Tapatoru rite	Equilateral triangle

Tapatoru waerite	Isosceles triangle
Tapatoru	Triangle
Tapawaru	Octagon
Tapawhā hāngai	Rectangle
Tapawhā rite	Square
Tapawhā whakarara rite	Rhombus
Tapawhā whakarara	Parallelogram
Tapawhā	Quadrilateral
Tapawhitu	Heptagon
Tekau	Ten
Toru	Three
Waru	Eight
Whā	Four
Whitu	Seven

Chapter 1.
Introduction to the Study

This first chapter introduces the study and presents my interest in investigating classroom interactions[1] during geometry lessons on shapes and their properties in a New Zealand multilingual primary school. My interest in the topic began during my time working as a primary teacher in a government school in India. I give an account of my teaching experience, and how it led to this research in the first section (Section 1.1). The chapter then outlines the context of this study (Section 1.2), followed by current definitions and conceptions in geometry as per *The New Zealand Curriculum* (The NZC) (Ministry of Education, 2007) (Section 1.3). In the next section, I discuss the development of Te Reo Māori[2] terms for geometric shapes (Section 1.4). The chapter then briefly discusses insights from the current geometry education research about the role of language in developing children's understandings of geometric concepts (Section 1.5), followed by a brief review of geometry education research in New Zealand context (Section 1.6). Research questions are presented in the next section (Section 1.7). A brief account of the theoretical framework and research design is presented in the following section (Section 1.8). Finally, a synopsis of each of the chapters in the thesis is presented in the last section (Section 1.9).

1.1 Background and Research Interest

I worked as a primary teacher at a government school in Delhi, the capital of India, for almost three years. I taught the same group of students from Year 3 to 5 in the school. In Delhi government schools, either Hindi or English can be used as the medium of instruction. English is one of the official languages in India. To develop children's English language skills, the National Curriculum Framework (NCERT, 2005) introduces English as a language subject in Year 1. I noticed that the Year 3 students in my class had a minimal understanding of English, which was limited to knowledge of the alphabet, the names of body parts, flowers, vegetables, and a few opposite word pairs. Reciting a poem in English was a significant accomplishment for these students.

The students I taught came from low socio-economic backgrounds. They had migrated from the interior regions of Bihar, Uttar Pradesh, and Madhya Pradesh to the Northeast border of

[1] In this thesis, the term "classroom interactions" is used to denote interactions that occur in a classroom, including both whole-class and group interactions.
[2] Te Reo Māori is an indigenous language of New Zealand, and gained official status in 1987.

Delhi and Uttar Pradesh in India, where the school was located. As the students belonged to these different regions of a Hindi-speaking belt, they mainly spoke in Hindi with flavours of their regional languages, including Khadi Boli, Braj, Bundeli, and Bhojpuri. To appreciate students' sociolinguistic backgrounds in the school I taught, Hindi was selected as the language of instruction.

I had trouble using Hindi as the language of instruction for teaching mathematics even though I am a native speaker of Hindi. This was because I had had English as the language of instruction throughout my education and had learned many mathematical concepts through the medium of the English language. I brought English as an additional language to the languages already present in the classroom as I used English mathematical terms while teaching. It seemed that my students might have been experiencing difficulties in making sense of mathematical concepts in English, just as I was having difficulties understanding mathematical concepts in Hindi.

Through classroom conversations, I realised that language was not just a tool to transmit knowledge but also a way of making sense of mathematical concepts for my students and me. It was in the conversational space that we tried to make sense of what the other person was trying to say. Being educated in Piaget and Vygotsky's constructivist paradigm, I attempted to make maximum use of students' previous knowledge and provided them with concrete examples while teaching geometry. For instance, to introduce shapes in Year 3, I showed a duster to my students and asked them to draw the shapes they saw in it. They drew different shapes for the same object, the duster. They drew it as a square (*Varg/ Chaukor/ Samchaturbhuj* in Hindi), as a rectangle (*Aayat/ Samakon*, in Hindi), and some drew it by depicting its three faces together like a box (*Dibba jaisa*, in Hindi). While we discussed these drawings, there were two realizations. First, students could see different shapes in the duster, given that they drew different shapes for the same object. They could visualize and take note of both two-dimensional (2D) and three-dimensional (3D) shapes. Second, I used English terms for the shapes, and my students used Hindi words for the same shapes. For the students, it was either Varg/ Chaukor/ Samchaturbhuj for square, Aayat/ Samakon for rectangle, or Dibba or box for the rectangular prism shape of a duster. It was interesting that we had successfully negotiated our understandings of the shapes while navigating different languages for naming those shapes. However, it was not simply that we used various terms from other languages to talk about the same shape. My students had identified the shape I was talking about even though I had not explicitly mentioned the relationship between the terms from English and Hindi.

The process of negotiating the students' meanings of shapes involved, one, using their language to name the shape, and two, associating that shape's name with its use. The words had cultural meanings attached to them. For example, "Samakon" implies right angles, and "Chaukor" means four equal sides. As a result, the students may have connected the same object with different shapes. The students and I associated "Varg" with a square and "Aayat" with a rectangle. The students understood that "Dibba" is not a mathematical term, although they had no knowledge of the mathematical word for the 3D representation (the cuboid or rectangular prism) of the duster. This teaching and learning of geometry was happening in a milieu of different languages and different ways of understanding shapes. Although I made a conscious effort to use Hindi terms for mathematical concepts, the students preferred English terms. They also associated their learning of English terms in mathematics with achievement. These reflections inform the focus of this study: how children negotiate their meanings about 2D shapes, 3D shapes, and their properties while interacting with others in the multilingual context of a mathematics classroom.

In the next section, I outline the context of this study.

1.2 Context of the Study

This study was conducted with a Year 5/6 class (9 to 11 years) in an English-medium state school in New Zealand. The New Zealand primary education system recognises three kinds of schools: state school, state-integrated school, and private school. The schools are either English-medium or Te Reo Māori-medium. Te Reo Māori is an Indigenous language of New Zealand, which gained official status in 1987. The English-medium state and state-integrated schools follow The NZC (Ministry of Education, 2007), whereas Māori-medium schools follow *Te Marautanga o Aotearoa* (Ministry of Education, 2015). Though developed from different perspectives, both documents share the same vision and principles to guide schools in developing their design and curriculum to support their students' learning. Together, these two documents support the partnership between Māori (the indigenous people of New Zealand) and Non-Māori, a core principle of New Zealand's founding documents, Te Tiriti o Waitangi and The Treaty of Waitangi.

Te Tiriti o Waitangi is one of the fundamental documents that inform curriculum decision-making in English-medium and Māori-medium schools. The Treaty was first signed as an agreement between the Māori Rangatira (chiefs) and the British Crown on February 6, 1840 (NZHistory, 2021). It endorses the bicultural foundation of New Zealand history and aims to support partnership, participation, and protection for co-signatories. The principle of Te Tiriti o Waitangi in The NZC seeks to provide all students with the opportunity to acquire Te Reo Māori Me Ōna Tikanga (roughly translated as Māori language and culture) in Māori- as well as English-medium schools. In alignment with the Treaty, The NZC also upholds the principle of cultural diversity and values the diversity found in different cultures, languages, and heritages; therefore, in contemporary times, the Treaty is inclusive of early and new migrants (Teaching Council NZ, 2017; Whyte, 2015).

With the globalisation and movement of people, New Zealand has become a nation of superdiversity (Spoonley & Bedford, 2012; Vertovec, 2007) with multiple ethnicities and languages (Statistics New Zealand, 2020a). Recognising and appreciating learners' diverse language backgrounds when they come to school can promote positive identities and, thus, can help engage them in the learning process (Lo Bianco et al., 2016). Currently, the languages most commonly spoken at home in New Zealand other than English are Te Reo Māori, Samoan, Mandarin, and Hindi (Statistics New Zealand, 2020b). These languages belong to different language families[3] and this fact may augment the complexities of the multilingual context of English medium primary school classes in New Zealand.

Multilingualism has been defined differently in multilingual research. Three terms are related to the concept of multilingualism – (i) monolingual, (ii) bilingual, and (iii) multilingual. Kemp (2009) noted that researchers differentiate these terms based on the number of languages the terms refer to. Therefore, monolinguals are speakers who use one language. Monolinguals are also known as monoglots or unilinguals. Bilinguals are those individuals who use two languages. Multilinguals know three or more languages with different degrees of proficiency in these languages. However, developments in the field of multilingual research in the last decade show that the term multilingualism implies "various forms of social, institutional and individual usage as well as individual and group competence, plus various contexts of contact and involvement with more than one language" (Franceschini, 2009, p. 29). This understanding of multilingualism acknowledges language diversity, incorporates sensitivity towards socio-cultural diversity, and appreciates society's heterogeneity. It is now used as an umbrella term that covers research on bilingualism. For

[3] The language English belongs to the Indo-European languages, Māori and Samoan languages belong to the Polynesian languages, and Hindi belongs to the family of Indo-Aryan languages

this study, a contextual understanding of multilingualism is adopted. Accordingly, multilingualism implies the presence (covert or overt) of two or more languages in any classroom setting (Barwell, Clarkson, et al., 2016). Therefore, the multilingual context of the English-medium primary classroom provides the first reason for its selection as the appropriate context for this study.

The second reason for selecting a Year 5/6 class as the context for this study concerns the mathematics performance of New Zealand students at the primary school level. Mathematics is a subject that influences an individual's way of dealing with various spheres (private, social, or civil) of his/her life (Anthony & Walshaw, 2007). The teaching and learning of mathematics aims to equip a learner with the knowledge and skills required to deal with the mathematical needs of everyday life (Booker et al., 2014). The recent Programme for International Student Assessment (PISA) 2018 reported that the achievement levels of students in mathematics (with an average score of 494) were similar to the achievement levels reported in PISA 2015 (with 495 points as average score) (May et al., 2019). However, if scores are compared since PISA 2003, the average scores in mathematics have declined from 523 points in 2003 to 494 points in 2018 (Hipkins, 2019; May et al., 2019).

Geometry is one of the major threads of learning in mathematics (Atiyah, 2002). The focus of geometry education is to develop concepts and skills that enable learners to make sense of the world around them (Jones & Mooney, 2003). Developing a sound understanding of geometry concepts is vital to succeeding in mathematics (Education Review Office NZ, 2018). Specific to the achievement levels in geometry, the recent Trends in International Mathematics and Science Study (TIMSS) 2018/2019 results suggested that the average achievement of New Zealand Year 5 students was at its lowest since 2006 (Rendall et al., 2020). Thus, this study aims to contribute to geometry education research to support students' learning of geometry concepts.

In New Zealand, the primary school education years are from Year 1 to Year 8 and typically fall under Curriculum Level 1 to Level 4. The Curriculum Levels, rather than Years, define the Achievement Objectives for students. The Curriculum Levels are designed in such a way that the individual needs of all learners are met. Teachers have the freedom and flexibility to develop teaching programmes that best suit the needs of their students. A student usually enters at Curriculum Level 1 in Year 1 and follows a suggested progression of Achievement Objectives in the learning areas throughout their primary schooling (Year 1 to 8).

Mathematics and Statistics is one of the eight learning areas in The NZC. Maass et al. (2019) have recently argued that mathematics learning may support children in developing 21st-century skills and becoming active and responsible citizens. Thus, the Mathematics and Statistics learning area of The NZC focuses on developing logical and systematic thinking, flexibility, criticality, and creativity (Ministry of Education, 2007). Within Mathematics and Statistics, geometry is part of the sub-strand *Geometry and Measurement*.

The Geometry and Measurement sub-strand focuses on measurement, shape, position and orientation, and transformation. The teaching and learning of geometry involve "recognising and using the properties and symmetries of shapes and describing position and movement" (Ministry of Education, 2007, p. 26). To achieve these aims, The NZC also states the Achievement Objectives for each level. Table 1.1 shows the Achievement Objectives related to the learning of shapes and their properties.

Table 1.1

Curriculum Levels and Achievement Objectives Regarding Shapes

Curriculum levels	Achievement Objectives (Shapes)
Level 1	Sort objects by their appearance.
Level 2	Sort objects by their spatial features, with justification. Identify and describe the plane shapes found in objects.
Level 3	Classify plane shapes and prisms by their spatial features. Represent objects with drawings and models.
Level 4	Identify classes of two-and three-dimensional shapes by their geometric properties. Relate three-dimensional models to two-dimensional representations and vice versa.

Note. Adapted from *The New Zealand Curriculum* (The NZC), The Ministry of Education, 2007. Copyright 2007 by the Crown.

As per the Curriculum Levels and Achievement Objectives (see Table 1.1), the students at Level 3 will be able to identify and describe 2D shapes, and classify 2D and 3D shapes by spatial features. Hence, Curriculum Level 3, which translates to Year 5/6 (9 to 11 years), was selected as the appropriate age range for the participants in this study.

To support the learning of geometry at the primary school level, Anthony and Walshaw (2007) argued that learners' cultural knowledge provides plentiful opportunities. Nevertheless, these reservoirs of learners' cultural knowledge are often accessible only when the learners are provided with a supportive environment that appreciates their socio-cultural identities and includes space for their language(s). In line with this thought, Averill, Te Maro, et al. (2009) have suggested and encouraged the use of a culturally responsive mathematics teaching framework as an appropriate pedagogy. This teaching framework aims to ensure that mathematics learning is accessible to Māori students and students from all New Zealand's diverse cultures. The culturally responsive mathematics teaching framework provides classroom-teaching strategies that align well with and are inclusive of Māori conceptual understanding, values and traditions, on the one hand, and are also responsive to students from other cultures. Upholding the Te Tiriti o Waitangi (Treaty of Waitangi) and a culturally responsive teaching approach, one of the learning objectives is that the students should be able to "use both English and Te Reo Māori to describe different polygonal shapes" (NZMaths, 2021a).

This section has presented two reasons that inform the selection of the New Zealand English-medium Year 5/6 (9 to 11 years) as the context of this study: first, the context of English medium classroom is multilingual; second, the students in Year 5/6 class are expected to identify and describe 2D and 3D shapes according to The NZC.

The next section presents definitions and conceptions in geometry related to shapes and their properties in the New Zealand mathematics classroom.

1.3 Definitions and Conceptions in Geometry

This study explores children's interactions as they talk about shapes and their properties during classroom discussions. This section, therefore, presents definitions of the concepts related to shapes and their properties as presented in geometry teaching at the primary school level in New Zealand. Etymologically speaking, geometry is a combination of two Greek words, *geo* and *metron*. "Geo" means earth, and the meaning of the word metron is "measure". In 1986, Alan Bishop argued that geometry is the "mathematics of space" (p. 141). The teaching and learning of geometry should allow opportunities for "mathematising space" (p. 141) to explore mathematical interpretations of space. In other words, geometry is a web of concepts, their representation and ways of reasoning that help us explore and analyse the space around us, including shapes (Battista, 2007). The learning of geometry at the primary school level includes the learning of visual geometry as well as formal geometry (Booker et al., 2014).

Visual geometry focuses on spatial awareness and the ability to visualise spatial arrangements, that is, to develop a sense of space, shape, and form. When teaching geometry at lower primary school levels, the emphasis is often on developing this sense of shape and space. As students move to upper primary levels, formal geometry becomes an important part of teaching and learning geometry. Formal geometry involves greater accuracy in terms of the language used and the representations of space and shapes. As students move to upper primary levels, they are expected to use appropriate vocabulary for describing and explaining geometric concepts.

Two-dimensional (2D) shapes are plane shapes that have only two dimensions – length and breadth. These shapes do not have thickness or depth. 2D shapes are shapes that cannot be held (NZMaths, 2021a), for example, a print or a surface. Sides and corners are two terms used as part of geometry language to describe properties of 2D shapes at the primary school level. Sides represent the line segments of 2D shapes. A corner is used to indicate the point where two sides meet. Three-dimensional (3D) shapes are solid shapes that can be held, as these shapes have thickness or depth. Therefore, 3D shapes have three dimensions, which include length, breadth, and height. Faces, edges, and vertices form part of the geometry vocabulary for 3D shapes. Faces are the flat surfaces of the 3D shapes. Edges are formed as lines where two faces meet. Vertices is the plural of vertex, a point made by the intersection of two or more edges in a 3D shape.

These definitions of 2D and 3D shapes construe *dimension* as a parameter or measurement attribute that defines an object's characteristics. In the Geometry and Measurement sub-strand of Mathematics and Statistics learning area of The NZC, dimension is understood as a concept for measuring aspects of a shape. For example, when an object is measured in one direction, we are dealing with one dimension. In addition, if the object is measured in a direction perpendicular to the one measured earlier, it is the second dimension we are dealing with. Similarly, if the object can be measured in a direction perpendicular to both the previous directions, it demonstrates the third dimension (Schwartzman, 1994).

This study focuses on the ways children represent their understanding of these definitions and concepts of shapes and their properties during whole-class[4] and group interactions[5]. The next section discusses the evolution of the Te Reo Māori terms used for teaching and learning shapes in New Zealand primary schools.

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[4] In this thesis, the term "whole-class interactions" signals interactions where the teacher and students engage in discussion as a whole class.

[5] The term "group interactions" is used to signal interactions that happen when students engage in group work/activity and the groups are created by the teacher during the lesson.

1.4 Te Reo Māori Mathematical Register in the New Zealand Context

As noted previously, Te Reo Māori is the language of the Indigenous people of New Zealand. During the early and middle 20th century, the use of the Māori language was actively discouraged, resulting in the endangerment of the Māori language by the 1970s (Trinick et al., 2017). Against this backdrop, a movement for its revival was initiated, and Te Reo Māori became one of the three official languages of New Zealand (with English and New Zealand sign language) in 1987. Māori-medium schools were established to support the Māori language as well as the Māori cultural revitalisation. With the language revitalisation initiatives and re-introduction of Te Reo Māori in the school domain, a mathematical register in Te Reo Māori was developed for teaching and learning of learning area Pāngarau (mathematics) in Māori-medium schools (Barton et al., 1998).

The early development of Te Reo Māori vocabulary for teaching mathematics was largely "ad hoc coining of words by teachers and kaumātua (elders), using whatever word-creation strategies were available" (Trinick & May, 2013, p. 460) to each Māori-medium community and schools in different parts of New Zealand. The use of different wordlists resulted in educational as well as Māori language change issues on the national level (Barton et al., 1998). Some of these issues concerned the use of transliterations[6]. For example, the use of inconsistent words was deemed unsuitable by the Te Taura Whiri I Te Reo Māori (the Māori Language Commission). Moreover, Te Ohu Pāngarau (a group of school-based Māori-medium mathematics educators) questioned the dubious relationship between terms and their intended meanings (Barton et al., 1998; Trinick, 2015). These concerns led to a meeting between Te Taura Whiri I Te Reo Māori and Te Ohu Pāngarau. As a result, a list of overarching and consistent terms for teaching and learning of mathematics in Māori-medium schools was published in 1991.

From 1999 to 2009, the Numeracy Development Project in English-medium schools and the Poutama Tau Project in Māori-medium schools provided an impetus to develop mathematics terminology in Māori, such as terms required to describe various stages of The Number Framework (Trinick & Stevenson, 2005). Trinick and May (2013) stated that a number of strategies were used to develop Māori-medium terms for teaching and learning mathematics. These strategies included adding prefixes and suffixes, changing the meaning of existing

[6] Transliteration is to pronounce words from one language using the phonology of a different language (Mammadzada, 2021). For example, "motorcar" is transliterated as "motokaa", or "value" was transliterated as "wāriu" in Te Reo Māori.

words, compounding existing words, resurrection of old words with slightly modified meanings, and creating metaphors, among others.

Barton et al. (1998) suggested that the development of Māori terms in a short span of time led to translation of Western mathematical ideas into Māori, leaving out the Māori-mathematical constructs, and suggested that the development of Māori terms as translation of western mathematical ideas may not be as useful as intended in promoting the learning of Māori students. They suggested that translating English terms into Māori might still support the English mathematical discourse (Barton & Frank, 2001), thus displaying the epistemic dominance of Western mathematics over Indigenous mathematics knowledge (Parra & Trinick, 2018). Specific to the teaching and learning of shapes using Te Reo Māori in English-medium schools, a list of relevant terms is provided in the glossary at the start of this thesis from the recent dictionary *Te Reo Pāngarau* (Christensen, 2010). These terms can be used to teach about shapes in both Māori-medium as well as English-medium schools.

In addition to the list, a teaching unit was developed to support the learning of Te Reo Māori terms for shapes in an English medium primary school at Curriculum Level 3. The title of the unit is "Te Whānau Taparau – The polygon family" (NZMaths, 2021b) (see Appendix A). However, the unit describes the square as "tapawhā" as well as "tapawhā rite" (see Appendix A). Tapawhā indicates a shape with four sides and tapawhā rite indicates a shapes with four equal sides. The property of equal sides in the term tapawhā rite may support better understanding of the square shape.

Barton et al. (1998) suggested that in Māori language, mathematical ideas are expressed as either verb or adjective. For example, terms like "tapawhā rite" and "tapawhā hāngai" indicate square and rectangle respectively, by highlighting the properties of the shapes through the use of adjectives – rite and hāngai. As an adjective, "rite" draws our attention to equal lengths as the quality of four sides, and "hāngai" indicates perpendicular as the quality of four sides in shape. These qualities are not apparent when these shapes are referred to as nouns, as evident through English Mathematical terms, such as "square" and "rectangle". Barton et al. (1998) argued that thinking about the mathematical concept as an adjective or noun requires different ways of thinking mathematically. For example, thinking about a shape through an adjective emphasises the property of shapes (as explained earlier), whereas thinking about a shape as a noun may emphasise only the name of shapes without making any connection to its properties. Moreover, this approach of developing a mathematics register through translation may result in a set of fixed codes that are independent of context, people and their intentions, negating the active role of language in the co-construction of mathematical concepts.

Mathematics education research has explored the role of language by exploring classroom interactions. In the next section, I provide a brief account of how language has been explored in mathematics education research.

1.5 The Role of Language in Geometry Classrooms

Much of the research in geometry education has focused on the development of geometric concepts from diverse perspectives, including learning theories, visual-spatial abilities research, role of gestures from semiotics and embodied cognition perspectives.

For the teaching and learning of geometry concepts, van Hiele and Hiele-Geldof's theory (1959/1985) is widely used in research (see, Debrenti, 2016; Gunčaga et al., 2017; Hourigan & Leavy, 2015; Ismail & Rahman, 2017). Van Hiele's and van-Hiele-Geldof's theory provides a sequential order of five thought levels that a learner progresses through based on their knowledge and mastery of the previous thought level (see Section 2.1.1 for more details). They further argue that the child needs didactic instructions in order to move from one thought level to another.

Another theory that has informed the geometry education field is Duval's theory of figural apprehension (1995). Duval suggested four kinds of apprehensions allow understanding of geometric figures. These are perceptual, sequential, discursive, and operative. Perceptual apprehension enables one to link pictorial cues in order to comprehend the visual representation. Sequential apprehension is concerned with one's ability to dissect the figure into its smaller figural units. Discursive apprehension accounts for one's ability to understand the details of a figure given through written or spoken speech. Lastly, the operative apprehension accounts for one's ability to act/operate on the figure. Duval (1995) argued that for a figure to be recognised as a geometric figure it should allow perceptual apprehension along with one of three other kinds of apprehensions: sequential, discursive, and operative.

Most recently, Seah and Horne (2019) proposed their theory of learning progression for geometric reasoning (details are provided in Section 2.1.1). They suggested eight zones of learning progression from pre-cognition (zone 1) to logical inference-based reasoning (zone 8). Though these theories provide insights into the development of geometric thinking, they assign only a limited role to language in developing children's thinking of geometry concepts.

In addition to these learning theories, research based on visualisation and spatial abilities informs us about the visual-spatial skills that help children mentally imagine and act on objects that further facilitate their geometric reasoning (Lowrie et al., 2017; Sinclair & Bruce, 2014). Gestures also contribute to learning of geometric concepts (Arzarello et al., 2009; Maschietto & Bartolini Bussi, 2009). It has been argued that gestures, along with language, drawings, and other linguistic modes such as inscriptions, charts, and diagrams, help students develop an understanding of mathematical concepts, including geometric ideas (Chen & Herbst, 2013; Núñez, 2009; Sabena, 2017). However, these perspectives assume the role of language is limited to mathematical terms and their meaning, ignoring the communicational aspect of language use. In all these theories, the focus on language is limited to the learning of linguistic codes or, in other words, geometry vocabulary. This limited focus ignores the communicative function of language as people interact to co-construct their understanding of geometry concepts.

Broader mathematics education research has explored language as part of classroom interactions. These studies have used the Interactionist perspective (see, Krummheuer, 2007; Yackel & Cobb, 1996), the Conversation Analysis (CA) perspective (see, J. Ingram et al., 2019; Ingram et al., 2015), and the Discursive Psychology perspective (see, Barwell, 2003a, 2012b, 2016a). Studies from an Interactionist perspective draw our attention to the sociomathematical norms regarding how and when an explanation or argument is considered and accepted as a mathematical explanation (Yackel & Cobb, 1996) and argument (Krummheuer, 2007). These studies argue that children in a mathematics classroom learn not only the vocabulary pertinent to the mathematical concept; they also learn the meta-rules of how an argument should be presented in the mathematics classroom. However, they also explore mathematics classroom interactions as part of a broader mathematics discourse, ignoring the dynamic nature of local conversational moments that contribute to the meaning-making process in mathematics classrooms.

Research from a CA perspective has also informed us about the influence of language use in a local conversational moment that affects the participation of students and therefore contributes to the co-construction of mathematical concepts. These studies have highlighted subtle signals in the form of a teacher's preference for students' responses in mathematics language. Dispreference for students' out-of-turn utterances may also deter students from sharing their mathematical thinking with others (Ingram et al., 2015). Moreover, it has been found that prosodic features, for example, repetition of students' responses with different pitches of voice may indicate teachers' disapproval of students' responses (Hellermann, 2003). Studies from the CA perspective inform us about subtle yet powerful interactive practices that may contribute to students' participation in the mathematics classroom. However, these studies do not inform us about how broader socio-historical meanings become part of mathematical discussions.

The Discursive Perspective draws our attention to discursive practices as part of mathematics classroom research. This research has revealed how re-voicing or re-uttering others' responses may contribute to students' learning of mathematics (Boukafri et al., 2018; Eckert & Nilsson, 2017; Moschkovich, 1999, among others). These studies highlight that the interactional practice of re-voicing by a teacher may achieve various discursive functions, such as achieving collective argumentation by bringing students' contributions to the centre of classroom interactions (Moschkovich, 2015; Planas & Morera, 2011) or providing meta-messages to direct students' attention to details of how an argument needs to be presented (Forman & Larreamendy-Joerns, 1998). Barwell (2003b) investigated students' discursive practices to explore what students attend to while engaging in tasks in a mathematics classroom. He found that students attend to discursive practices of participating in a classroom while solving mathematics problems. Though the studies exploring discursive practices draw our attention to how students and teachers use their language to achieve certain action, it is not clear how students, in particular, negotiate their mathematical understanding as they engage in mathematics interactions.

In regard to a multilingual context, mathematics education research has supported the perspective of language as a resource (Adler, 2002) and promoted the practice of code-switching for teaching and learning of mathematical concepts in multilingual contexts (Gwee & Saravanan, 2018; Moschkovich, 2019; Setati, 1998). As a result, research has supported the learning of mathematical terminologies and grammatical patterns of mathematical registers for supporting the mathematical learning of culturally and linguistically diverse learners (Adler, 2002; Moschkovich, 2007; Planas & Setati-Phakeng, 2014). Yet, research exploring the development of geometric concepts such as shapes and their properties in the presence of multiple languages is rare. Moreover, these perspectives assume the role of language is limited to mathematical terms and their meaning, ignoring the meaning making aspect of language-in-use. Ward (2019) argued that language is more than vocabulary and syntax. He explained that patterns of stress and intonation in language influence the words, their meanings, and social significance. The complexity of these prosodic patterns may increase with the presence of a variety of languages in a mathematics classroom. It is the ordered nature of indexicality (i.e., the act of referring, pointing) of the language used in a specific conversational context that enables us to interpret what is said beyond the meanings of individual words (Barwell, 2003b). Therefore, in this study, I set out to explore how meanings about shapes and their properties are negotiated during classroom interactions in a New Zealand multilingual primary classroom.

The following section presents discussion of geometry education research in the New Zealand context.

1.6 Geometry Education Research in New Zealand

Much of the mathematics education research in New Zealand has been conducted with numeracy development (e.g., Averill et al., 2008; Hunter & Sawatzki, 2019; Jhagroo, 2015; Mills, 2018), and problem-solving (e.g., Bailey, 2017; Calder, 2010; Ingram et al., 2016; Jhagroo, 2015). Research investigating the teaching and learning of geometric concepts in the New Zealand context is limited to only three studies.

The first study on transformational geometry with Years 7-8 students in a Māori-medium school (see, Manuel et al., 2015). Manuel et al. (2015) presented students with three tasks. The first two tasks were designed to promote their understanding of symmetry and patterns. The third task involved students visiting a local marae[7] to allow them to explore and appreciate the cultural significance of transformation geometry in Māori culture. The researchers argued that these tasks not only helped in promoting students' mathematical learning and associated language development but also enabled teachers to practise the Māori fundamental concept of *Ako*, which means reciprocal teaching and learning with students.

The second study on spatial thinking (see Trinick et al., 2015) was based on interview data from Māori elders to explore Māori spatial orientation terms and spatial frames of references. Based on interview data, Trinick et al. (2015) developed a series of learning activities for teaching Māori spatial orientation concepts to Year 7-10 students at a Māori-medium school. They concluded that the activities displayed an increased understanding of Māori spatial orientation concepts.

The third NZ study was conducted with twelve Year 5-7 teachers and 281 students in nine English-medium New Zealand schools (see N. Ingram et al., 2019). N. Ingram et al. (2019)

[7] Marae is Māori word for a courtyard, where formal greetings and discussions take place in Māori cultural setting.

used nine challenging tasks related to angles and geometric reasoning to better understand this approach for teaching and learning mathematics. The authors argued that using challenging tasks enables teachers to assess students' geometric thinking. Although the study was centred on students' learning of the geometric concept of angles, it mentioned that the students' understanding of angles improved.

These three studies touched upon concepts linked with shapes and stated that the students developed a better understanding of symmetry, patterns, and orientation. None of the studies, however, examined how students conveyed their understanding of these geometric concepts. In light of the superdiverse character of New Zealand society and a scarcity of research in the field of geometry education in the New Zealand context, the significance of this study is three-fold. First, it adds to the knowledge base of geometry education research in the New Zealand context. Second, it opens up new avenues for further investigations for expanding research in geometry education. Third, this study offers insights into the complexities of the multilingual context that shapes the learners' sense-making processes in an English medium classroom. The findings may be of interest to educators, researchers, and policymakers across Aotearoa New Zealand.

In the next section, I present the research questions that guide this study.

1.7 Research Questions

The overarching research question that guides this thesis is:

How do children negotiate their meanings about 2D shapes, 3D shapes, and their properties in a Year 5/6 New Zealand multilingual primary classroom?

Specific research questions that inform this study are as following:

1. *What discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
2. *How do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
3. *What characteristics of the dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

The first research question concerns the discursive constructions that children use to represent the geometric ideas about shapes and their properties through interactions in a multilingual primary Year 5/6 classroom in New Zealand English medium school.

The second research question focuses on exploring the ways in which children interact to construct their understanding as they talk about geometric shapes and properties. For this question, the focus is on the "context dependency" of utterances (William, 1999) and *how* an utterance is made in a specific local context of interaction. The focus of this second research question is on the local conversational context of interaction within classroom interaction.

The third question focuses on illuminating the characteristics of dialogic space that influences children's negotiations of meanings about shapes and their properties. Dialogic space is a dynamic, conversational space made up of different perspectives based on disciplinary knowledge, ideologies, and interactional practices that open up the possibility of diverse meanings of utterances (Kazak et al., 2015). The dialogic space includes the teacher's and other children's utterances that influence a child's negotiation of meanings. Acknowledging others' presence in the dialogic space allows us to see how socio-historical meanings of shapes and their properties embedded in others' utterances become part of children's discursive constructions and hence contribute to the process of negotiation of meanings.

In the next section, I present the research design adopted to answer these research questions for this study.

1.8 Theoretical Framework and Research Design

With an interest in exploring classroom interactions to develop an understanding of students' learning of shapes and their properties, I situate my position as a researcher from the Discursive Psychology perspective (Edwards & Potter, 1992) in terms of theoretical foundations within the Critical Inquiry paradigm. Discursive Psychology offers an anti-cognitive and poststructuralist account of meaning-making and underscores language as the primary mode of social activity. It approaches language-in-use (talk and text) as a domain of action in its own right rather than construing it as an outcome of mental states and cognitive processes (Edwards & Potter, 2005). The focus of research is on what is said and how to explore how action and interaction unfold in real time.

Potter (2012) argued that studies from Discursive Psychology perspective explore how people construct, understand, and display their understanding of the world around as they interact in "everyday and institutional situations" (p. 113). This research, therefore, focuses on the children's and teacher's discursive constructions of their understandings of shapes and their properties along with their orientations. Rather than commenting on the underlying cognitive processes of thinking for one's participation (Barwell, 2003a, 2003b), the Discursive Psychology approach investigates how participants discursively construct their thinking while interacting in particular situations.

Within the Discursive Psychology perspective, the theoretical framework for this study is informed by two theories, which are Ethnomethodology (Garfinkel, 1967) and Bakhtin's (1981, 1986) Dialogic Theory. Ethnomethodology suggests that in day-to-day events, people usually make sense of social situations while they are participating. Garfinkel (1967) argued that participants display their interpretation of relevant aspects of social interactions through their actions and language. Thus, the ethnomethodological approach focuses on the social action performed through language instead of on the language itself (Ingram, 2018). According to Bakhtinian Dialogic Theory, language-in-use should be construed as a worldview laden with an ideology where the meanings are constructed on the basis of the relationships that participants hold with each other. Thus, language-in-use is not limited to the use of abstract codes of phonemes, morphemes, words, semantics, and syntax but provides a possibility of multiple meanings.

Qualitative research was adopted to seek answers to the research questions. The study took place in a Year 5/6 class in a New Zealand primary classroom. The school catered to a multilingual student population. Participants included fifteen students with their mathematics teacher. Nine of the fifteen students were multilingual. Students with two or more languages were considered as multilingual speakers for this study. The teacher had seven years of teaching experience. Six lessons on shapes and their properties were observed, and fieldnotes were taken. Each lesson lasted for 45 to 50 minutes. Data from other sources were also generated. All six observed lessons were audiovisually recorded. Three semi-structured interviews with the teacher were conducted. In addition, four focus group interviews with students were conducted once all the lessons had been observed. I also collected student work samples and the teacher's lesson plans.

For the first step of analysis, data from the teacher's interviews, focus group interviews with students, fieldnotes and audiovisually recorded lessons were thematically coded to identify the relevant Key Moments from six audiovisually recorded lessons. In total, ten Key Moments were selected for further analysis. As the study focuses on interactions, participants' utterances were considered as the unit of analysis. The selected Key Moments were analysed at two levels, which are micro-level analysis and macro-level analysis. It is crucial to note that the macro-level analysis is based on micro-level analysis even though the analysis at both levels is presented separately. The micro-level analysis used a number of Conversation Analysis (CA) techniques, and the macro-level analysis used Bakhtinian concepts from Dialogic Theory. The combination of CA techniques with Bakhtin's Dialogic Theory allowed me to explore how meanings are constructed situationally as well as institutionally as children participated in whole-class and/or group interactions.

I adopted a non-participant observer position. I tried to maintain independence from the participants and classroom conversations (Gray, 2014). My focus was to observe the natural setting with minimal disruption to the classroom practices. I strived not to influence participants' behaviours nor any classroom practices. However, as a teacher from a different context, I brought my own values, beliefs, and biases to the data gathering processes. In order to remain aware of my bias and critically reflect on it, I attempted to be reflexive (Creswell & Creswell, 2018) concerning my assumptions, experiences, and identities that might influence the research process in any way.

In the next section, I present synopsis of each of the chapters presented in this thesis.

1.9 Structure of the Thesis

This thesis is organised into seven chapters. Chapter 1 introduces the study and outlines the personal and theoretical factors that justify my interest in classroom interactions to explore students' understanding of shapes and their properties. This first chapter also presents three research questions that guided this study.

Chapter 2 is a review of the research literature relevant to the study. It reviews research that has examined the development of geometry concepts at the school level from different perspectives, including learning theories, visual-spatial abilities research, and the role of gestures in learning from semiotics and embodied cognition perspectives. The chapter provides a historical overview of multilingualism research in mathematics education. This chapter also reviews research from mathematics education on classroom interactions. The review of the literature highlights the gaps in research leading to the research questions.

Chapter 3 provides details of the methodology adopted for this study. It explains the research process and the associated procedures. It focuses on the rationale for a qualitative research design. The chapter provides details of the paradigm adopted with epistemological and ontological assumptions, theoretical framework, research design, data gathering procedures, and process of data analysis. The chapter also outlines the steps undertaken for establishing the study's reliability and validity. Details of the ethics procedures followed during the study are also provided.

Chapter 4 presents the first chapter of the analysis and the findings. I present findings from both thematic and micro-level analysis. Thematic analysis allowed me to identify relevant themes pertaining to major geometric ideas that were evident in the data. Micro-level analysis explores the selected Key Moments within each theme to draw out children's discursive constructions as well as the interactive ways that they used to construct their understanding of shapes. In this chapter, findings pertaining to *what* is said and *how* it is said are presented. The chapter primarily addresses the first two research questions.

Chapter 5, the second chapter of the analysis, offers macro-level analysis of the Key Moments analysed at micro-level analysis. The chapter explores the aspects of dialogic space that influence children's negotiation of meanings about shapes and their properties in the local conversational space of whole-class and/or group interactions. The Key Moments are analysed using Bakhtinian concepts of heteroglossia, unitary language, double-voicedness, addressee, and chronotopes. It is crucial to note that the analysis at the macro-level is founded on the insights gained from the micro-level analysis.

Chapter 6 is the discussion chapter. In this chapter, findings from the thematic analysis, micro-level analysis, and macro-level analysis are brought together to answer the three research questions. The findings are discussed in light of the broader mathematics education research.

The final chapter, Chapter 7, presents the study's overall conclusions along with a few identified limitations. Implications for primary school teachers, curriculum development, and teacher-educators are also discussed. It then discusses avenues for future research, presents a summary of contributions to knowledge in the field of geometry education and offer concluding thoughts.

Chapter 2.
Review of the Literature

The focus of this research is to investigate *how children negotiate their meanings about shapes and their properties in a New Zealand multilingual primary classroom.* This chapter presents a critical review of theoretical and research-based literature in relation to the teaching and learning of geometric shapes, including two-dimensional (2D) and three-dimensional (3D) shapes, and their properties. The review of literature in this chapter situates this study within the context of existing research and establishes the need for this study by highlighting the research gap. The first section reviews research from the field of geometry education pertaining to teaching and learning of shapes and their properties (see Section 2.1). In the second section, I present a review of studies that examined the multilingual context in the teaching and learning of mathematics (see Section 2.2). Following this, I review the literature investigating language-in-use as classroom interactions in the mathematics classroom (Section 2.3). In the next section, I present the gaps in literature and my positioning as the researcher (Section 2.4). The chapter concludes with a summary (see Section 2.5).

2.1 Geometry Education in Mathematics Education Research

The review of literature suggests that research on the conceptual development of shapes and their properties has been informed by different research fields. For the purpose of organisation, I present this section in four sub-sections. The first sub-section reviews the learning theories that contribute to our understanding of the conceptual development of geometric shapes (2D and 3D) and their properties (see Section 2.1.1). In the second sub-section, I review the research on developing an understanding of the mathematical construct of dimension at the primary school level (see Section 2.1.2). In the next section, the review focuses on the role of visual-spatial abilities in the development of geometric concepts (see Section 2.1.3). The final sub-section explores the role of gestures in geometry education (see Section 2.1.4).

2.1.1 Learning Theories in Teaching and Learning of Geometry

This section presents a critical discussion of van Hiele and van Hiele-Geldof's theory (1959/1985), Duval's (1995) theory of figural apprehension, and Seah and Horne's (2019) learning progression for geometric thinking. The literature reveals that most studies in geometry education have taken van Hiele and van Hiele-Geldof's theory (1959/1985) as their theoretical basis (e.g., Debrenti, 2016; Hourigan & Leavy, 2015) for developing an understanding of shapes and their properties. I briefly discuss van Hiele and van Hiele-Geldof's theory in the next section.

Van Hiele and Van Hiele-Geldof's Theory (1959/1985).

This theory is based on a constructivist approach, largely on the lines of the Piagetian developmental theory of cognitive development. Pierre van Hiele and his wife Dina van Hiele-Geldof developed a sequential theory for explaining how learners develop their geometry concepts, first described in Dutch. Fuys et al. (1988) translated the van Hiele theory and the levels into English, which were validated by van Hiele (van Hiele, 1999). According to this theory, learners progress through five sequential thought levels in their developmental trajectories, given appropriate instructional experience. The progress of the learners at each level is dependent upon their prior experiences, knowledge, and mastery gained at the previous level. These thought levels are described in Table 2.1.

Table 2.1

Van Hiele and Geldof-Hiele's Theory's Thought Levels

Levels	Descriptions
Level 0:	The student identifies names, compares and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance.
Level 1:	The student analyses figures in terms of their components and relationships among components and discovers properties/rules of a class of shapes empirically (e.g., by folding, measuring, and using a grid or diagram).
Level 2:	The student logically interrelates previously discovered properties/ rules by giving or following informal arguments.
Level 3:	The student proves theorems deductively and establishes interrelationships among networks of theorems.
Level 4:	The student establishes theorems in different postulational systems, and analyses/compares these systems.

Note. Adapted from "The van Hiele model of thinking in geometry among adolescents," by Fuys, D., Geddes, D., & Tischler, R., 1988, *Journal for Research in Mathematics Education,* Monograph, 3, p. 5. Copyright 1988 by the National Council of Teachers of Mathematics.

At Level 0, a child identifies a shape based on the visible geometric figure. For example, a child is able to identify the representation of shape, for example  as a square. However, at this level, the child is not able to describe any property of the shape. At Level 1, the child is able to analyse shape with regard to their properties. In this case, the child will be able to describe a square as a geometric shape with four equal sides and angles but will not be able to recognise that a square is a kind of rectangle. At Level 2, the child may be able to gauge that a square is a subset of the rectangle by logically interrelating their properties. At Level 3, a child may use properties of a square deductively to establish relationships between theorems, for example, congruency theorems, but is limited to Euclidean[8] geometry. Level 4 is the most advanced level, where a child is able to use properties of shapes in different kinds of geometry, for example, non-Euclidean[9] geometry. Van Hiele and van Hiele-Geldof suggested that the progression from one thought level to another is more reliant upon the instruction that the learner receives than the learner's maturity or age. Hence, providing appropriate instruction according to the sequenced phases of learning and thought level is of utmost importance. A substantial literature supports van Hiele and van Hiele-Geldof's theory for its instructional focus (e.g., Sinclair & Moss, 2012; Stumbles, 2018).

However, the theory is not free from drawbacks and criticisms. First, the theory has been criticised for emphasising that the development of geometric concepts takes place in a sequential manner. With specific concern for the development of shapes and their properties, Pyshkalo, a Russian psychologist and educator, drew heavily on the van Hiele theory to develop instructional plans for primary school learners. Pyshkalo (1968) found that "familiarising second graders with solids enabled them to reach the second level (van Hiele level 1), surpassing the progress of seventh graders in the traditional schools" (as cited in Hoffer, 1983, pp. 209-210). In support of this finding, research has shown that an individual student may possess simultaneously different van Hiele levels for different geometry concepts, casting doubt on the sequential order of thought levels (Battista, 2009; Burger & Shaughnessy, 1986; Gagnier et al., 2017). Therefore, it is difficult to identify the specific van Hiele level for an individual learner (Ness & Farenga, 2007). Secondly, the van Hiele theory focuses on the development of concepts of Euclidean geometry and seems to assume that students and teachers have a precise understanding of the mathematical concept of dimension. In Fuys et al. (1988)'s translation, van Hiele and van Hiele-Geldof's task modules and level descriptors clearly mention plane figures, their representations, their properties and axioms as important principles of school geometry. Battista (2009) suggests that this overemphasis on planar (2D) shapes and related concepts has resulted in confusion regarding diagrams and geometric figures of shapes. Irrelevant characteristics of diagrams are also often attributed to the geometry concept (Clements & Battista, 1992). For example, the narrowness of a triangle is often given as a reason for not considering it as a triangle (Devichi

[8] Euclidean geometry is primarily concerned with planar shapes.
[9] Non-Euclidean geometries include hyperbolic, spherical and elliptic geometries

& Munier, 2013). The theory also does not account for any developmental trajectory for non-Euclidean geometries that concern the non-flat world (Guven & Baki, 2010).

Finally, the van Hiele theory undertakes a limited approach to the role of language in the development of geometry concepts. The role of language is restricted in terms of linguistic symbols and a system of relations that are particular to a thought level. According to van Hiele (1999), the function of language is to define the geometry concepts of sides and angles. The theory situates the role of language within the issue of disharmony in communicating the features and properties of geometric structures. Disharmony arises because of misconceptions about the mathematical terms and their meanings. This limited understanding of language is concerned with the use of geometry vocabulary, neglecting the communicational function of language that fosters meaning constructions of geometry concepts.

The second theory that has informed teaching and learning of geometric shapes is Duval's theory of figural apprehension, which is presented in the next section.

Duval's Theory of Figural Apprehension (1995).

The second theory that has informed how geometric figures are learnt is the Duval (1995) theory of figural apprehension. According to Duval (1995), a given figure can be recognised in several distinct ways depending on the set of rules applied for visual representations. Visual representations refer to tools such as drawings, diagrams, and figures supporting the recognition of what is mathematically relevant (Thom & McGarvey, 2015). Thus, Duval suggested that to view figures geometrically, a set of rules is always present that must be followed if the given figure is to be viewed in the geometric sense (as a geometric shape). As a result, a considerable amount of cognitive input is required to view the figures geometrically as representations against their automatic perceptual recognition. Duval (2017) further adds that a learner also needs to perceive figural units of a figure in different dimensions. That is, to recognise a geometric shape as a cube, figural units of the cube (3D), its faces (2D), sides (lines, 1D), and vertices (0D) must be grasped. This breaking up of a figure according to different figural units is the process of dimensional deconstruction of shapes. Duval argued that to learn geometry one needs to deconstruct dimensionally all 2D shapes and use the figures as heuristics to understand the representations.

In addition to the idea of dimensional deconstruction, Duval (1995) suggested that for a figure to function as a geometric heuristic to solve the problems, it must evoke "cognitive apprehensions" (p. 143) to integrate different ways of looking at the shapes. He proposed four different kinds of cognitive apprehensions: perceptual, sequential, discursive, and operative.

Perceptual apprehension is concerned with the unconscious integration of the figural organisation laws and the pictorial cues that result in a particular visual representation. Sequential apprehension is related to the way the representations are deconstructed in terms of their figural units. Discursive apprehension informs us about the details of the figure that cannot be determined without additional information through speech (written and/or oral). Discursive apprehension works in a situation when the details of the representation are not clear from the figure. For example, a figure may look like a rectangle, but the details about its angles, length of sides, and the feature of parallelism will determine if it is one or not. The last apprehension is operative apprehension. Operative apprehensions involve operating with the figure in various ways – dividing it into parts to locate shapes, changing the orientation of the figure, spatially putting it in other places or in other ways, and/or getting an insight into the solution of a problem. Duval argued that for heuristic purposes, a figure must evoke perceptual and one of the other three cognitive apprehensions. Along with this, he argued that to recognise any shape a learner must also be able to distinguish the physical object (for example, a cardboard template) from its semiotic representation (geometric figure – rectangle). He emphasised the use of sign systems in developing the concept of “figures” and the underlying operations that work at different levels. It is this triadic structure among the object (3D), the mathematical object (what the figure represents, e.g., the shape as a rectangle), and the figure itself (the drawing) that develops the understanding of a shape.

Working with Duval’s concepts of perceptual and operative apprehensions, Hallowell et al. (2015) studied how 36 Grade 1 students matched shapes across 2D and 3D representations. They conducted an individual interview with each student while they completed a shape-matching task developed to elucidate their spatial-mathematical thinking. In the task, they included ten items, where a student was required to match the 2D representation of 2D/3D shapes with one of four manipulatives showing 2D or 3D shapes. They found that Grade 1 students find it challenging to relate a rectangle and triangle figure with a solid cylinder and a solid cone. Gal and Linchevski (2010) also used Duval’s concept of perceptual apprehension to explore difficulties faced by Israeli junior high school students (Grade 9, 13 to 14 years) in learning geometry. They found that the students failed to dimensionally deconstruct the figures provided in the tasks in order to infer mathematically relevant properties of the figure. They argued that visual perception of the shape might have resulted in this inability of students to dimensionally deconstruct the shape from the perceptual apprehension.

Although Duval's approach incorporates the aspect of discursive apprehension to understand and operate on the geometric figures and problems, these studies have not provided any account of the role of language in mediating the explanation or explication behind the reasoning.

Recently, Seah and Horne (2019) proposed a learning progression for geometric reasoning, which is presented in the following section.

Seah and Horne's Learning Progression for Geometric Reasoning (2019).

Recently, Seah and Horne proposed a learning progression for geometric reasonings. They developed eight zones to delineate learning progression based on data collected from 755 Year 7 to 10 students (aged 11-16). The process of identifying these zones of learning progression included: (i) construction of a hypothetical geometric learning progression, (ii) development of assessment items for formalising the learning progression zones, and (iii) standardising these zones of learning progression by establishing the model's reliability and validity based on the assessment data from students. Thirty-six assessment items were grouped in three domains: properties and hierarchies of shapes, transformation (including perspective drawing, mental rotation, among others) of relationships, and geometric measurement. The items correspond to two of the proposed zones of learning progression. Students' responses were analysed along with their scores on the assessment items. The eight zones of learning progression for geometric thinking are Pre-cognition, Recognition, Emerging informal reasoning, Informal and insufficient reasoning, Emerging analytical reasoning, Property-based analytical reasoning, Emerging deductive reasoning, and Logical inference-based reasoning (see Appendix B for behavioural descriptions of these zones).

Specific to the learning of shapes at the primary school level, Seah and Horne (2019) argued that a child in the pre-cognition zone (Zone 1) is able to recognise simple shapes on the basis of appearance and common orientation. Children also display their emerging recognition of 3D objects from perspectives, and they name 3D shapes using names of the common 2D shapes that they see in it. As they move to the second zone (Recognition), children are able to identify simple 2D shapes on 3D solid shapes and show an emergent understanding of representations of 3D solid shapes. In the third zone of learning progress (Emerging informal reasoning), children are able to use one or two properties of shapes to justify their reasoning about shapes. However, they are still not able to use much of the geometry language. Geometry vocabulary and language develop as the children move to the fourth zone (Informal and insufficient reasoning). Children develop analytical reasoning as they move to zone five, Emerging analytical reasoning. In this zone, a child is able to visualise and use nets to represent 3D shapes. They also develop relevant geometric language and learn to use it to provide reasoning about geometric shapes. As the child moves through these zones, they develop more complex thinking about shapes and their properties.

Although Seah and Horne (2019) analysed students' responses on geometry assessment items, the understanding of language use is limited to the knowledge of shape names and related geometric vocabulary. It is not clear what the authors imply with regard to "sound reasoning" (Zone 7, Emerging deductive reasoning). In addition, in the process of standardising this learning progression, data were collected from Year 7-10 students; it is possible that this learning progression may not provide useful insights for the geometric understanding of students from Year 1-6.

Using their model of learning progression, Seah et al. (2016) studied the understanding of a square in Australian Year 7-8 students (12 to 13 years). They analysed data from 214 students in terms of keywords, construction of narratives and representations. They found that all children drew a prototypical image of a square as . They noted that children might confuse a square, a 2D shape concept, with a cube, which is a 3D shape. Berenger (2018) also supported this finding. In a study with a Year 7 class in a suburban secondary school in Melbourne, Australia, he provided students with two tasks. For Task A, students were asked to respond to the question: *What is a square?* Then a teaching episode on the square was observed, and Task B was conducted to assess students' retention of key ideas from the teaching episode. Task B asked students to draw a square and list its properties. He found that students struggled with defining a square as a 2D shape with necessary aspects of the concept and suggested that this difficulty in identifying the key attributes may lead to several misconceptions, such as that a 3D square is a cube. However, the question that arises is whether children were trying to display their understanding of the two-dimensionality and three-dimensionality of shapes or were instead trying to provide a geometric definition of the shape.

The learning progression approach, like other theories (van Hiele and van Hiele-Geldof's, or Duval's theory), ignores the crucial role language plays as children interact with others and use language mathematically to represent their understanding of shapes. In this section, I have discussed three theoretical perspectives on how children's understanding of geometric concepts develops. These approaches included van Hiele and Hiele-Geldof's (1985) theory, Duval's theory (1995), and Seah and Horne's (2019) learning progression. These approaches take a limited approach to the role of language in developing geometric understanding. They focus on the learning of geometric-specific vocabulary, ignoring the communicational role of language in the co-construction of knowledge. Moreover, the studies taking these approaches fail to acknowledge the presence in contemporary classrooms of multiple languages, which might contribute to the meaning construction process in geometry classrooms.

With a focus on geometry concepts, the theories mentioned in this section and the studies using these theories have contributed to our understanding of how children learn about 2D and/or 3D shapes, with limited focus on dimensionality as a property of shapes required for developing a sound understanding of shapes. In the next section, I present a review of studies that have focused on teaching and learning of dimension at the school level.

2.1.2 Dimensionality as a Property of Shapes

The mathematical construct of dimension plays a crucial role in developing foundational skills in mathematics (Manin, 2006), particularly for construing two-dimensional (2D) and three-dimensional (3D) shapes and their properties. In The NZC (Ministry of Education, 2007), 2D shapes are defined as plane shapes that have only two dimensions – length and breadth. 3D shapes are defined as solid shapes with length, breadth, and thickness/depth. These definitions may highlight different ideas about the dimension as a mathematical construct of shapes. Taking Euclid's geometry perspective, dimension is understood as the characteristics of length, breadth, and height held by an object. That is, if an object has only length, it is considered to have only one dimension, whereas if an object has length and breadth, it has two dimensions. Consequently, an object with length, breadth and height will have three dimensions.

Skordoulis et al. (2009) presented a contrasting view and stated that topologically speaking, linear shapes like line, rectangle, curve, among others, are one-dimensional, whereas surfaces are two-dimensional, and will include sphere region, circular region, plane, polygonal shapes etc. Three-dimensional shapes will include solid objects like spherical region, cylindrical region and others (Manin, 2006; Ural, 2014). Following this perspective, the hollow sphere and the solid sphere will have different dimensions, two and three dimensions, respectively. Both of these perspectives can be interpreted from the definitions of 2D and 3D shapes provided earlier and may contribute to students' understanding of dimension. Yet, this mathematical construct is seldom studied in the field of mathematics education, with only a few exceptional studies (e.g., Lehrer et al., 1998; Morgan, 2005; Panorkou, 2011; Panorkou & Pratt, 2016; Tossavainen et al., 2017; Ural, 2014).

Lehrer et al. (1998) conducted a 3-year longitudinal investigation to explore how children's conceptions of 2D and 3D shapes develop. The participants included 30 children in total, with 10 (who moved from Grade 1- 3), 9 (who moved from Grade 2-4), and 11 (who moved from Grade 3 to 5). They used van Hiele's and Geldof-Hiele's theory to develop instruments for eliciting students' conceptions regarding 2D shapes and 3D shapes. They printed shapes for 2D shapes and made use of wooden models for 3D shapes. Two triads of wooden models were used in the second and last year of study. The first triad of wooden models comprised a cube, a cone, and a pyramid. The second triad consisted of a cube, a triangular prism, and a rectangular prism. For the 2D shapes, they reported that children reasoned about shapes based on the "fat or skinny" dimension (p.142). For the 3D shapes, they related 3D figures with known 2D figures, and students argued that the shapes could be morphed by "pulling" or "pushing" (p. 142). For example, students claimed that by "sitting" on a rectangular prism, it could be transformed into a cube. This way of describing 3D shapes may indicate that students view dimension as a malleable quality of shapes or objects.

Similar to Lehrer et al. (1998), Seah et al. (2016) found that children may refer to a cube as a 3D version of a square. They conducted a study with 214 Year 7-9 (11 to 14-year-old) students. Students' responses were coded for keywords that children used to describe the shape. They reported that students might consider the cube as a 3D square, as evident in this student's response: "a 2D square has 4 sides and a 3D square has 6 sides" (p. 590).

Morgan (2005) specifically focused on the definitions that students and teachers use to talk about dimensions. She analysed how Year 5 students (10 to 11 years) and their teacher defined their understanding of dimension during a classroom discussion (see extract presented in Appendix in Barwell, 2005b). She argued that the students and the teacher identified dimension as a multi-faceted notion that includes dimension in regard to either "thickness" or describing 2D as flat and 3D as fat, or something extra in 3D as compared to 2D. The understanding of dimension expressing the material attribute of thickness aligns with the finding reported by Panorkou (2011).

Panorkou (2011), in her phenomenographic study of students' experiences of dimension, studied twelve 10-year-old students' experiences of dimension using three tools: Elica applications, *Flatland* the film, and Google SketchUp. She found that students constructed their understanding of dimension as (i) a material attribute of an object, as thickness; (ii) as vector, expressing ideas of position, direction and orientation; and (iii) as capacity, where objects with higher dimensions can contain objects with lower dimensions (e.g., a cube contains a square). Panorkou's study shows how children may represent their understanding of dimensions in different ways (see Panorkou & Pratt, 2016).

Tossavainen et al. (2017) and Ural (2014) worked with mathematics teachers to gauge their understanding of the idea of dimension. Tossavainen et al. (2017) studied primary and secondary pre-service teachers' definitions of area and explored their understanding of its dimensional aspect. The study was conducted with 82 Finnish pre-service teachers (typically 20 year-olds), who were asked to complete a questionnaire, and the questions were provided in Finnish. The researchers found that teachers' understanding of area was incomplete, and the teachers found it difficult to comprehend the two-dimensional aspect of area. They argued that although the concept of area is central to elementary mathematics, the aspect of the two-dimensionality of an area is barely considered in the teaching and learning of shapes and their areas. Tossavainen et al. (2017) also stated that the use of the same word for the boundary of the shape as well as the space within the shape might add to the difficulty in construing dimension as an important attribute for understanding shapes. For example, in English, a *circle* may imply both the boundary as well as the area (as a disc) enclosed by the circle.

Ural (2014) also pointed to this usage of terminology in teaching and learning that may hinder the learning of geometry. Ural investigated how teachers make decisions about dimensions of geometric figures. He conducted his study with fifteen primary and secondary teachers from schools in Burdur Centre, Turkey. The teachers were provided with a test based on eighteen geometric figures. For each geometric figure, teachers were asked to specify the number of dimensions of that geometric figure. For example, the teacher was asked if a point is 0-, or 1-, or 2-, or 3-dimensional. Other geometric figures included the line, angle, parabola, circle, triangle, spherical region. Ural (2014) found that naming both the rectangular region (showing enclosed space) and rectangular boundary as a rectangle may lead to an inadequate geometric understanding of shapes and their properties. He argued that it is important to emphasise the difference between expressing shape as a region and as a boundary because it may influence one's understanding of dimensions.

Taking insights from the language being used when describing geometric shapes may help us to understand the difference between these two expressions. Bezgovšek Vodušek and Lipovec (2014) showed that in the Slovenian language the boundary of a circle is not considered a 2D shape and is called *Krožnica*, whereas a disk is considered a 2D shape of a circle, and is called *Krog*. Having different terms for denoting circumference and circle may highlight the idea that 2D shapes have filled spaces within their boundaries, thus underscoring the crucial dimensional aspect of 2D shapes.

It is to be noted that very few studies have explored students' understanding of dimension in a primary school setting. Even though these studies have noted how language might influence teachers' and children's understanding of dimension, the research exploring the impact of language-in-use on children's understanding of dimension in a multilingual context is negligible. This study aims to explore how children represent their understanding of dimensions as they participate in whole-class and/or group interactions.

In the next section, I review studies on children's learning of geometric shapes from the perspective of visual-spatial abilities.

2.1.3 Visual-Spatial Abilities in the Development of Geometric Concepts

The review of the literature reveals that the ability to imagine objects (static or dynamic) and to act on these objects mentally are crucial for the teaching and learning of geometry. These abilities include spatial and visualisation abilities. Spatial abilities have been found to facilitate the development of geometry concepts and reasoning during primary school education years (Cheng & Mix, 2014; Clements & Battista, 1992; Danisman & Erginer, 2017; Guay & McDaniel, 1977; Shumway, 2013; Sinclair & Bruce, 2014). In the case of 2D and 3D shapes, research has shown that visual-spatial abilities positively contribute to the learning of geometric shapes and their properties.

Cohrssen et al. (2017) designed a project-based programme and provided early childhood children with six activities within a play-based curriculum for the development of spatial thinking skills. They conducted their study with 19 children (4 to 6 years), five of whom attended all sessions. During these six activities, children were required to "i) draw the school, (ii) draw school signs, (iii) draw 2D maps of 3D environments, (iv) build 3D constructions from 2D images, (v) map the route from home to school, attending to landmarks along the way, and (vi) compare and discuss maps, identifying shapes on people's maps" (p. 97). Children were provided with typical early childhood materials such as blocks. The focus was on modelling consistent geometry vocabulary, including 2D and 3D shape names (cube, squares, cone, etc.) and spatial and directional language (e.g., in, on, up, down, etc.). They found that embedding young learners in the real-world spatial environment promotes their understanding of 2D and 3D shapes.

Different aspects of spatial abilities have also been found to support geometric concepts of shapes and their representations. For example, Bruce and Hawes (2015) conducted a lesson study with a team of seven teachers and 42 primary students (4 to 8 years) in a primary school in Ontario, Canada. Each student participated in pre- and post-test assessments, which included a 2D mental rotation task and a 3D mental rotation block task. They found that the students' spatial abilities of mental rotations are flexible and can be improved with practice. They further argued that the manipulatives to work with 2D and 3D shapes and specifically designed teacher delivered lessons could improve learners' understanding of shapes.

This finding was backed by Hawes et al. (2017). They implemented a 32-week teacher-led spatial reasoning intervention in a K-2 classroom with 12 female teachers and 39 students (4 to 7 years). They investigated the extent to which in-class spatial activities may lead to improved children's spatial and geometry performance. Children were asked to perform on three tasks: (i) a task assessing their spatial language focused on shape recognition and positional language, (ii) a 2D mental rotation task, and (iii) a task requiring students to use their understanding to reason about 2D shapes, transformations, symmetry, and composition/decomposition of 2D shapes. In addition to the earlier finding, Hawes et al. (2017) argued that spatial training might also improve students' numerical skills.

The spatial ability to take various perspectives from different positions is also considered to foster 2D and 3D geometry concepts (van den Heuvel-Panhuizen et al., 2015). Van den Heuvel-Panhuizen et al. (2015) conducted a study with 4 to 5-year-old students from the Netherlands (n= 334) and Cyprus (n= 304) to explore the link between imaginary-perspective-taking (based on visibility and appearance) and mathematics ability. The visibility competence tasks assessed students' abilities to deduce which objects are visible or not from other perspectives, whereas the appearance competence task concerns their ability to indicate how an object would appear or look like when seen from a different perspective. They found students' mathematical abilities were significantly related to their imaginary perspective-taking, signalling that spatial ability to see and imagine objects from a different perspective may foster the learning of 2D and 3D shapes and their properties.

In addition to acknowledging the role of mental rotation and spatial orientation, nets are also claimed to develop 3D geometric concepts by fostering spatial abilities (Wright & Smith, 2017). Wright and Smith (2017) conducted task-based interviews with 34 Year 6 students and asked them about their anticipation if a given net would fold to form the target solid (cube or square-based pyramid). They found that children used different strategies to decide if a net would fold into the target solid. These strategies included strategic choice of base and mental rotation of faces to create a folding sequence. They argued that directing students' attention to particular target shapes with nets would support their learning of diverse properties of 3D solids.

Sack and Vazquez (2016) argued that spatial orientations (concerning the position and direction of objects) play a crucial role in determining the representations that learners build for the 3D models. They conducted a longitudinal study for a period of seven years in an elementary school in the United States of America. There were 11 Grade 3 and 14 Grade 4 students (9 to 10 years). The purpose of the study was to explore the development of 2D and 3D geometry concepts using the Geocadabara Construction Box dynamic computer interface integrated with the Spatial Operational Capacity (SOC) model (Sack & van Niekerk, 2009). The SOC model is an instructional design with three sets of sub-models (full-scale models, conventional graphic models, and semiotic models) to address the complex nature of teaching and learning in geometry. The full-scale models (or scaled-down models) are real objects that can be manipulated by the student. The conventional-graphic models represent two-dimensional graphic (2D) representations of the real, three-dimensional (3D) objects. The semiotic models are abstract, symbolic representations that bear no resemblance to the actual objects, for example, floor plan diagrams (For more details, see Sack & Vazquez, 2016). They argued that to develop an understanding of geometry concepts a learner needs to develop competencies in these three different representational modes. The instructional plan moves from the 3D object to 2D representations of the 3D object and then to 2D representations of 2D objects (an abstract concept). They found that the learners were not able to visualise the hidden cubes in 3D structures (made up of unit cubes). They further stated that the learners experienced difficulties in using the terms "horizontal" and "vertical" while describing and making sense of what others are saying. For example, four cubes in line, whether standing or lying flat vertically, were stated as vertical. Sack and Vazquez (2016) suggest that the SOC model highlights the ambiguity in verbally describing the figures.

Recently, Fujita et al. (2020) conducted a study with 1357 students from Grade 4 to 9 (9 to 14 years) in Japan. They investigated how children use their spatial reasoning skills to solve geometric problems that require students to make sense of diagrams showing 2D representations of 3D geometric shapes. They found that for solving a geometric problem, students need to harmonise their spatial visualisation skills (identifying that the diagonal divides the square face of the cube in triangles) with analytical reasoning (identifying diagonals of a cube will make an equilateral triangle as all diagonals are equal), and domain-specific knowledge (that equilateral angles are all equal and measure 60°). They argued that students need to be given more opportunities to develop their spatial reasoning skills, enabling them to consolidate their spatial reasoning skills with domain-specific skills.

These studies draw our attention to the role of spatial abilities in developing an understanding of shapes and their properties. However, none of these studies had explored how students used their language in explaining and constructing their thinking, even when the data were conducted through interviews. Sack and Vazquez (2016) provided a few insights into the role of language in explaining students' thinking. Yet, the processes through which learners may have sailed through the ocean of different cultural meanings during conversations associated with shapes have not been taken into account, despite many studies having highly diverse populations.

An exploration of the processes through which learners communicate their understanding of geometry concepts while interacting in a diverse linguistic setting does not appear to have been conducted. Such an exploration of processes in a diverse linguistic setting may provide valuable insights into how learners navigate through multiple languages to develop their understanding of geometry concepts. The dearth of exploration of the multilingual context in the process of development of geometry concepts in visual-spatial abilities is evident.

In the next section, I present a critical review of studies that focused on the role of gestures in geometry education.

2.1.4 The Role of Gestures in Geometry Education

Research has emphasised the role of gestures in developing geometry concepts (Arzarello et al., 2009; Maschietto & Bartolini Bussi, 2009). Gestures are defined as hand and arm movements that people use as they talk (McNeill, 1992). I first discuss research where gestures have been studied as a part of a semiotic process (Arzarello, 2006; Bartolini Bussi & Baccaglini-Frank, 2015; Calero et al., 2019; Elia, 2018; Elia et al., 2014), followed by a review of the literature regarding embodied cognition perspective on gestures (Alibali & Nathan, 2012; Alibali et al., 2019; Flood, 2021; Flood et al., 2020; Kim et al., 2011). I also present a review of studies on the role of gestures in a multilingual context (Church et al., 2004; Wermelinger et al., 2020).

Gestures as Part of the Semiotic Process.

The concept of a “semiotic bundle” (Arzarello, 2006) has been used widely to study the role of gestures in the teaching and learning of geometry (Arzarello et al., 2014; Arzarello & Paola, 2007; Chen & Herbst, 2013). Arzarello (2006) proposed this concept of the semiotic bundle to investigate the role of various semiotic systems, for example, gestures, language, drawings, and extra-linguistic modes like charts in developing an understanding of mathematical concepts. A semiotic bundle contains different semiotic sets. Each semiotic set comprises (i) signs, (ii) modes, and (iii) a set of relationships among different signs and meanings embodied in signs (Arzarello, 2006). The signs are produced with actions and have an intentional character. The modes define the rules for using signs and transforming them in various manners to produce a variety of meanings embodied within the sign-mode structure. Different semiotic sets combine to make a semiotic bundle. A semiotic bundle is analysed in terms of a collection of semiotic sets and the relationships among these sets within the semiotic bundle. The structure of the semiotic bundle depends upon the semiotic activities of the subject at a particular time and space. The semiotic bundles are analysed in two different yet complementary ways. The first is the analysis of the relationships among the different semiotic resources activated by the learner, i.e. *synchronic analysis.* The second analysis, *diachronic analysis,* focuses on the roles played by the different semiotic resources in supporting the cognitive processing of the learner during mathematical activity.

Using the semiotic approach, researchers have argued that gestures act as semiotic tools and help students to display their understanding of geometry concepts. Maschietto and Bartolini Bussi (2009) conducted a teaching experiment with 25 students (10 to 11 years). The teaching experiment started at the end of Grade 4 and continued as part of the Grade 5 mathematics curriculum at the beginning of the academic year. The researchers explored how Grade 4/5 students constructed their mathematical meanings about perspective drawings using semiotic systems of gestures, oral and written speech, and drawings. They found that students used two kinds of gestures. One set of gestures allowed students to work on the artefact (in this case, Durer’s glass, an instrument made of wood, plaxiglas[10] and metal with three holes. Only one hole shows the drawing superimposed on the skeleton of the cube inside it. For more details, refer to Maschietto and Bartolini Bussi, 2009). The second kind of gestures aided students in representing geometrical properties to display their understanding of geometric concepts.

[10] A light thermoplastic

Bartolini Bussi and Baccaglini-Frank (2015) also supported the role of gestures in the development of geometry concepts. They conducted a teaching experiment where first-grade learners (6 to 7 years) were required to use a sequence of commands as a programme for a bee-bot (robot) to move along a defined path. The aim was to develop students' dynamic perceptions of paths as static boundaries of the geometric figure of squares and rectangles. They found that children often used gestures for missing words to show rotations (left or right). In addition, they found that children invented the term "*square O*" (p. 398) to represent their understanding of the right angle in a square. They argued that gestures along with language act as semiotic resources that promote the development of geometry concepts of rotations, angles and shapes.

In addition, Daher (2014) used a semiotic approach to investigate how Grade 5 (8 to 9 years) students construct their understanding of geometric relations among the three sides of a triangle using manipulatives as they worked in a group. Four children participated in this study, and they worked with numbered sticks of varying length to form a triangle. Their group discussion was video-recorded and analysed for how their interaction was reflected in their actions for creating triangles using different sticks. Daher (2014) argued that working with sticks enabled students to see the relationships between a triangle's edges (or sides) in a real-world sense. He also considered that gestures complemented speech and allowed students to make connections between real-life situations and geometric manipulatives, highlighting the semiotic function of gestures in developing sound geometric understanding.

Calero et al. (2019) also conducted an experimental study with 132 children (3 to 8 years) to explore how children use gestures to represent their implicit and explicit geometry concepts as they worked on a geometry judgement task. The task required students to identify an odd card from a set of six cards. Students were presented with 20 such sets of cards. The task required students to use their understanding of direction, topology, distance, angles, and parallelism. They argued that gestures might reflect children's implicit knowledge about geometry concepts, and that children may not use language alone to express their understanding of geometry concepts. They further argued that gestures are strongly associated with children's grasp of geometric concepts and may provide a useful assessment tool.

With a specific focus on shapes, Elia et al. (2014) explored the role of gestures[11] while 5-year-old kindergarten learners in Cyprus engaged in geometrical activity. They found that learners made use of iconic and deictic gestures (see Figure 2.1) throughout their activities to develop an understanding of shapes. Iconic gestures refer to the gestures that represent the characteristics of entities. In Figure 2.1A, the child seems to use her hands to represent a characteristic of an object that she is representing. In contrast, the child seems to point to the object that she referred through her gesture (see Figure 2.1B). These kinds of gestures are deictic in nature. Deictic gestures refer to the gestures that are used to point to objects and actions in space.

Figure 2.1

Iconic and Deictic Gestures

A. Iconic gesture A. Deictic gesture

Note. Adapted from "The role of gestures in making connections between space and shape aspect and their verbal representations in the early years: findings from a case study," by I. Elia, A. Gagatis, M. van den Heuvel- Panhuizen, 2014, *Mathematics Education Research Journal, 26,* p.747. Copyright 2014 by the Mathematics Education Research Group of Australasia.

Examining kindergarteners' learning of geometry concepts related to two-dimensional shapes, and transformations of two-dimensional shapes, Elia (2018) explored the way a 5-year-old girl approached a shape configuration task. In this task, the child was required to give instructions to the researcher so that the researcher could create the shape based on the child's instructions. She found that the child used deictic gestures to index the recognised shape or to indicate the place where the shape needs to be placed; thus, they were related to the child's spatial communication and thinking. However, iconic gestures were used as representational tools to indicate the transformation of shapes; thus, iconic gestures enabled the child to operate on the shape. The use of iconic gestures highlighted the child's implicit geometry knowledge of transformation even when the child was not able to verbalise her thinking.

[11] Elia et al. (2014) referred to McNeill's (1992) categorisation of gestures.

Going further, Elia et al. (2018) argued that iconic gestures help students to visually represent their understanding of the orientation of shapes, transformations, attributes of shapes such as parallel, vertical side, and other geometric concepts that are too complex for a child to state verbally. They suggested that iconic gestures served multiple purposes in developing a child's geometric understanding in comparison to deictic gestures.

Chen and Herbst (2013) too supported the finding that students may use both deictic and iconic gestures while engaging with geometry problems. They studied how high school (13 to 15 years) students interact with diagrams and publicly justify their understanding through gestural and verbal modalities. They focused on the interplay among gestures, language, and diagrams as a semiotic bundle to examine how these interactions reveal high school students' understandings of shapes. They collected video-recorded data, and the analysis focused on students' use of gestures and language in providing their geometric reasoning. They found that gestures, along with verbal descriptions, enable learners to engage with geometry problems and act as semiotic resources to compensate for limited information provided through drawing in geometry problems. Though this study focused on the language used in reasoning, much of the focus on language was on using geometry-specific language, therefore ignoring the communicational aspect of language-in-use.

Gestures have also been studied from an embodied cognition perspective, as is described in the next section.

Embodied Cognition Perspective and Gestures.

Research using an embodied cognition perspective argues that gestures constitute forms of reasoning and problem solving, and are not simply communicational tools to express geometric understanding (Alibali & Nathan, 2012; Flood, 2021; Flood et al., 2020; Kim et al., 2011). Alibali and Nathan (2012) explored gestures using an embodied cognition perspective and argued that the teachers' and students' use of gestures during teaching and learning of mathematics shows the embodied nature of mathematics cognition. They provided data from their studies exploring the role of gestures in the learning and teaching of mathematics. They argued that gestures manifest embodied aspects of mathematics learning through both iconic and deictic gestures. First, deictic gestures are used in pointing to objects, locations, and inscriptions with either the index finger or the hand. They argued that deictic gestures help learners or teachers to ground mathematical ideas expressed through speech in the physical material environment, making the link between abstract mathematical ideas and material work explicit. Second, iconic gestures allow teachers and students to represent the real-world objects and properties of mathematical ideas. For example, teachers may use gestures to simulate the 90° angle for the right angle.

Kim et al. (2011) analysed 15 one-hour video-recorded geometry lessons in a Grade 2 classroom with 23 children (age 7 years) in Canada. The geometry lessons focused on learning about three-dimensional objects, patterns, and movements of objects. They analysed four sets of gestures: “(a) gestures with no talking and no apparent communicative purpose, (b) gestures without talk but with apparent communicative purpose, (c) gestures accompanying talk oriented toward others, and (d) gestures accompanying talk not directed toward others” (p. 214). They found that children used their bodies (primarily through gestures) to think through new problems. That is, they used their bodies and gestures as resources to cognitively engage with new geometry problems. They used gestures to think about geometry problems, to co-construct and share their understanding of the problem with others, and to cope with the learning of abstract geometry concepts. One example of an abstract concept is thinking about the properties of the hemisphere by asking: will it roll or slide? On the basis of their findings, Kim et al. (2011) argued that there is a need to recognise and call attention to the central role that is played by gestures or bodily movement in a child’s learning. Therefore, teachers and curriculum designers need to develop new teaching and assessing methods that could explore gestures for developing an understanding of a child’s learning.

Walkington et al. (2019) also explored how 14 to 16-year-old high school students (Grades 9 and 10) collaboratively use gestures to prove geometric conjectures in a socio-technological context. The school was situated in the Southern United States and catered to students from economically disadvantaged groups. They worked with 51 high school students in 18 groups of two and five groups of three. They found that students’ collaborative gestures enhance their reasoning as well as their problem-solving abilities.

In another study, Ng et al. (2020) studied differences in Grade 6 (11 to 12 years) students’ learning of 3D shapes and their properties in two technologically enhanced environments. Seven teachers and 174 students participated in this study. In one environment, students worked with Dynamic Geometry Environment (DGE) (n= 65), a software that offers students dynamic modes of thinking and interacting with mathematical concepts. The other environment required students to work with 3D pens (n= 101). They found that students working with 3D pens showed better retention of the 3D shapes. They argued that this better retention of understanding might have resulted from the use of the concrete gestures of moving the hands to create 3D solid, therefore supporting the embodied nature of mathematical learning.

Recently, Flood et al. (2020) explored 23 Grade 4-6 (9 to 12 years old) students' gestures during a task-based interview while students worked with a technology-enabled embodied learning device for mathematics, called the Mathematics Imagery Trainer for Proportions. They analysed 70 minutes of video-recorded task-based interviews. During each interview, students were required to work on a proportion problem, and as the student explained and worked through the problem, the tutor focused on the mismatch between the student's speech and gestures. They found that students use their gestures as a medium to show words that are often missing in their speech, and therefore students call for inputs from listeners to provide those terms. They also found that students relied on gestures to display their understanding of kinaesthetic and tactile experiences, which are difficult to state in words, for example, showing the same pace through hands. They also argued that students might demonstrate their confidence about the claim that they make through their gestures, displaying their epistemic stance.

Elsewhere, Flood (2021) explored Grades 5 to 11 instructors' representational gestures (also known as iconic gestures) during classroom interactions. She analysed ten 80-minute video-recorded sessions on programming courses in Los Angeles, USA. She found that instructors use representational gestures to provide students with a clue about the problem, method or the format of the answers. Alternatively, instructors may also use these kinds of gestures to respond to students' contributions by repeating students' gestures, by extending students' gestures, or adding new gestures. She argued that instructors/teachers' use of gestures allows gestures to become a resource for promoting co-construction of classroom knowledge.

These studies inform us that gestures contribute not only by communicating mathematical understanding; they also play a crucial role in developing reasoning and problem solving. However, none of these studies examined the multilingual context of geometry classrooms where gestures were used. In the next section, I present a review of a few studies that have explored the role of gestures in a multilingual context.

Gestures in Multilingual Contexts.

Few studies have explored the role of gestures in a multilingual context (see, Church et al., 2004; Wermelinger et al., 2020). While working with high school students, Ng (2016) had shown that bilingual students relied on their use of iconic gestures to communicate about calculus in a calculus class supported by a dynamic geometry environment (DGEs). The participants included eight bilingual Grade 12 students (17 to18 years) from a culturally diverse school in Western Canada. The participants worked in pairs. Each pair took 35-40 minutes to complete the task. The group work was video-recorded. Ng (2016) found that bilingual students use gestures as visual mediators in the absence of words to engage in mathematical discourse.

Church et al. (2004) conducted a pre- and post-test experimental study of 51 Grade 1 (26 from a mainstream English-speaking classroom and 25 from a bilingual programme) students (6 to 7 years) in a Chicago Public Elementary School. Both classrooms were shown the same video. The students from the bilingual programme mainly spoke Spanish. Students were shown a testing video on the conservation task. The groups of students from each of the classrooms were then split and recombined to form control and experimental groups with both English-speaking and Spanish-speaking students in both groups. The control group was shown an instructional video in English with no gestures, whereas the experimental group was shown an instructional video in English with gestures. The researchers found that all students performed better in the experimental group in which the instructions in English were supported with gestures. They argued that gestures play a crucial role in getting the message across in case the speech in another language (in this case for Spanish students) is inaccessible. They further argued, “using speech and gesture is part of the natural communication process” (p. 314). However, the study focused on instructors’ gestures rather than students’ gestures, therefore providing no substantial insights into what kinds of gestures might be used by bilingual students.

Farsani (2016) also investigated the communicative repertoire used by the bilingual Farsi-English instructors in a mathematics classroom in the United Kingdom. For the communicative repertoire, Farsani focused on verbal as well as nonverbal features, including gestures. The study was conducted in a bilingual English-Iranian mathematics classroom with a focus on the teacher’s use of deictic gestures. He found the teacher used deictic gestures to convey instructional messages and to emphasise the verbal language by pointing to a visual object that sounded similar to the mathematical object. For example, in his research, he found that the teacher pointed to their eyes using index and middle finger to represent the mathematical idea of the “isosceles” triangle by emphasising “eyesosceles”. Thus, in this way, Farsani (2016) argued that deictic gestures helped in the learning of English mathematics registers.

Recently, Wermelinger et al. (2020) conducted an experimental study where they compared the ability of 4 to 5-year-old monolingual and bilingual pre-schoolers (n=80, monolingual= 40, and bilingual= 40) to comprehend and produce iconic gestures. Monolingual students spoke Swiss German, and bilingual students spoke two different languages, including English, Italian, Spanish, Dutch, Serbian, Croatian, Portuguese, Russian, Czech, Hungarian and Arabic. They found bilingual speakers used more iconic gestures than monolingual students. They argued that this might have been because of a smaller vocabulary in each of their languages, and the difficulty that the bilingual students may have faced while communicating in speech only. The researchers argued that the iconic gestures allowed the bilingual students to communicate clearly what they wanted their partners to understand.

Although these studies may suggest that bilingual students use iconic gestures more than deictic to complement their communication in their second language, there are few studies that state that children from different cultures may have a repertoire of culturally informed gestures. For example, Iverson et al. (2008) conducted a longitudinal study where they analysed speech-gesture production of six monolingual children (3 Italian and 3 American). The speech-gesture development of these children was followed between the ages of 10 to 24 months. They found that Italian students used representational/iconic gestures much more than American students. Thus, greater use of iconic gestures by bilingual/multilingual students can be attributed to their culture rather than lack of language proficiency.

The review of studies exploring the role of gestures informs us that gestures act as a semiotic tool, and provides evidence in support of an embodied cognition perspective. However, multilingual contexts of these studies are seldom mentioned or explored. Moreover, the review raises several questions, such as whether bilingual children use iconic gestures because of limited knowledge of the language of instruction or because they come from gesture-rich cultures. Thus, a study exploring gestures in a multilingual context may provide valuable inputs for furthering the research in the area of mathematics education.

In this section, I critically reviewed studies from geometry education research. The geometry education research is informed by studies from diverse perspectives including learning theories, visual-spatial abilities, gestures, and limited studies on mathematical construct of dimension.

In the next section, I present an historical development of mathematics education research regarding the conception of language(s) from the multilingual perspective and its role in the development of mathematical concepts.

2.2 Multilingualism in Mathematics Education

Having knowledge of more than one language has recently being recognised as being a "resource" for learning mathematics (Adler & Ronda, 2015; Moschkovich, 2015; Setati & Moschkovich, 2013). However, this has not been the case since the beginning of multilingual studies in the field of mathematics education research. This section of the literature review reveals the historical development that has taken place in terms of the role of language(s), from being seen as a "deficit" to being recognised as a "resource" in developing mathematical concepts (see Sections 2.2.1, 2.2.2, 2.2.3, and 2.2.4). It is noticeable that multilingualism research has greatly influenced research in the field of mathematics education; however, studies exploring the impact of multilingualism in developing geometric concepts are limited, with just a few exceptions.

2.2.1 Language(s) as a Deficit in Mathematics Education

Until the 1970s, the deficit model was a dominant point of view regarding the influence of multiple (or dual) languages on a child's development. This deficit model claimed that the knowledge of two or more languages negatively influenced an individual's cognitive, linguistic and educational development (Austin & Howson, 1979). Various studies justified this position by analysing the performance of monolingual and bilingual learners on verbal and non-verbal tasks. It was argued that knowledge of more than one language leads to language mixing and language confusion, lower IQ, and impairment of the intelligence of whole ethnic groups (Saunders, 1988).

Subsequently, in mathematics education research, knowledge of two languages was attributed as one of the major factors for underachievement in mathematics (Hargreaves, 1997; Phillips & Birrell, 1994). A critical analysis of these studies highlights two gaps in analysing the role of multiple languages in mathematics education. First of all, all these studies implicitly assumed Eurocentric mathematics as the only form of mathematics, thus negating the influence of cultural forms and practices of mathematics (Barwell, 2009). Secondly, the tasks were administered in only one language, which was often the second language for the learner (Saunders, 1988). Thus, it was not possible to clearly state the reasons for the poor performance of multilingual students on mathematics assessment.

The reasons for poor performance of multilingual students on mathematics assessment could be attributed to the limited proficiency of the learners in the language in which the tasks were administered, rather than a limited understanding of the mathematical concepts that were assessed. The cultural inappropriateness of the administered tasks could also be attributed as a reason for the poor performance of bilingual learners (Garratt & Kelly, 2007; Saxe, 1988). For example, Trumbull and Solano-Flores (2011) showed how a simple mathematics task on building an aquarium (as shown in Figure 2.2) could be culturally inappropriate.

Figure 2.2

The Aquarium Task

You are going to make a salt-water aquarium with your class. You have $100 to spend. The chart below shows the prices for different kinds of fish and equipment. Choose the items you want, being sure to keep your total expenses to no more than $100. Show your work.

Item	Price
12-gallon tank	$29
Water pump and filter	$12
Damselfish	$4 each
Clown goby fish	$5 each
Angel fish	$3.50 each
Yellow tang fish	$5.25 each
Blue tang fish	$4.50 each
Box of tropical fish food	$3.50

Note. Adapted from Kane and Mitchel, 1996, as mentioned in “The role of language in assessment,” by E. Trumbull and G. Solano-Flores, in M. R. Basterra, E. Trumbull, and G. Solano-Flores (Eds.), *Cultural validity in assessment: Addressing linguistic and cultural diversity* (pp. 28), 2011, Routledge (https://doi.org/10.4324/9780203850954). Copyright 2011 by Taylor & Francis.

They argued that the task might be interesting and apt for children belonging to the white middle class. However, the task can be alienating for American Indian communities, where the concept of keeping fish as a pet is alien and may make the task unappealing to American Indian students, thus lowering their performance on the task.

In 1960s, research started suggesting that the knowledge of two languages may act as an asset in promoting the learning of mathematics, which is presented in the next section.

2.2.2 Language(s) as an Asset in Mathematics Education

The language as an asset perspective is informed by the research mostly conducted in bilingual education. A seminal study by Peal and Lambert in 1962 (De Klerk, 1995) challenged the deficit model and argued in favour of an asset-based approach to bilingualism. Swain and Cummins (1979) supported this asset-based argument by testifying that bilingualism positively influences a learner’s intellectual development. The positive relationship between cognitive processing and bilingualism was also recognised (Bialystok et al., 2004). The role of dual languages was acknowledged in a positive frame, yet the nature of their role was not clearly mapped out.

The earliest argument in this direction was given by Cummins (1979) in the form of the Threshold Hypothesis (Cummins, 1979, p. 227). The hypothesis states that if multilingual students have low proficiency in all their languages it will result in cognitive disadvantage, while having high proficiency in all languages will result in cognitive advantages. However, being proficient in only one language offers neither advantage nor disadvantage.

Cummins's Threshold Hypothesis provided evidence in support of multilingualism in mathematics learning (Essien, 2010), yet his differentiation between "academic language proficiency" and "conversational language proficiency" (Morgan et al., 2014, p. 844) draws our attention towards a deficit model of language knowledge. Morgan et al. (2014) argued that "pedagogic approaches that draw on Cummins tend to focus on deficits. In this way, not only do children's existing language skills go largely unacknowledged, but a kind of linguistic 'target register' is propagated" (pp. 848-849). In other words, Cummins's approach upholds the idea that to participate in mathematical activity one requires a specific academic mathematics language. Inadvertently, the threshold hypothesis also implied that low mathematical attainment could result from a poor understanding of the mathematical language rather than low proficiency in the language in which mathematics is taught.

Indirectly Cummins's hypothesis drew attention to another kind of deficit, that is, the lack of mathematical language. Therefore, it can be argued that this deficit is not only relevant to multilingual learners; it is equally critical for monolingual students. Having low achievement in mathematics learning can be attributed to low proficiency at three levels – first, in one's language; second, in the language of instruction for mathematics teaching and learning; and third, in understanding the mathematical language.

The next section discusses an approach where language is considered from the perspective of registers specific to a particular social context, in this case, the mathematics classroom.

2.2.3 Language(s) as a Register in Mathematics Education

For an understanding of mathematical language, the work of Michael Halliday (1974) is noteworthy. He construed the human meaning-making process as a social practice. He argued that social practices must be studied in social and cultural contexts by taking account of the relationship between the linguistic systems and the social structures in which these practices are located (Halliday, 1974; Morgan, 2006). Realising the relationship between linguistic and social structures, he developed the concept of "register" (Halliday, 1975, p. 26). A register is that variety of language that is determined by what the person is doing. A person uses different registers of language in different socio-cultural contexts for different purposes.

Schleppegrell (2007) stated that there are two distinctive features of the classroom mathematics register: (i) multiple semiotic systems, and (ii) grammatical patterns. Multiple semiotic systems include symbolic notations, oral language, written language, graphs and visual displays such as gestures and actions. The interplay of these semiotic systems helps in developing a conceptual understanding of the mathematical concept along with the ability to solve mathematical problems. Grammatical patterns are also part of mathematical communication. For constructing concepts in mathematics, one needs knowledge of mathematical words along with the language patterns that are used to convey meanings mathematically (Herbel-Eisenmann et al., 2015; Kotsopoulos, 2007; Zolkower & Shreyar, 2007). The language patterns include defining the technical meanings of the words (e.g., sum, twice) in mathematics context, explains dense noun words (e.g., rectangle, volume, side), and delineates the logical relationships (for example, a square is an example of the quadrilateral).

Based on Halliday's account, it has been argued that the development of mathematical registers in a pupil's proficient language could promote mathematical thinking (Farrugia, 2009; Roberts, 1998). Undertaking Halliday's concept of registers, the Te Reo Māori register for teaching and learning of mathematics was developed during the 1980s and 1990s to promote mathematics learning of Māori students (Barton et al., 1998). Meaney et al. (2009) and Barton (2008) argued that Te Reo Māori terms for a number of mathematics concepts might promote better mathematical understanding. For example, they found that the teacher emphasised the words to highlight the equal sides of a square in the term *tapawhā rite,* which literally means a shape with four equal sides. However, to use Te Reo Māori mathematics register effectively in New Zealand classrooms for promoting students' learning, it is imperative to develop teachers' competencies to use Te Reo Māori fluently in a classroom (Averill, Anderson, et al., 2009; Averill, Te Maro, et al., 2009; Hāwera & Taylor, 2017).

Additionally, developing mathematical registers in Indigenous languages has been assessed as a difficult venture for various reasons (Farrugia, 2009; Roberts, 1998). In the context of the Te Reo Māori register, several techniques were used in the creation of a mathematics register in the early days. Early techniques included both ad hoc coining of terms by teachers and elders and transliterations. These word lists were developed on a local basis for teaching mathematics in Māori classrooms on a daily basis. However, as variation increased in these lists of Te Reo Māori mathematical terms, concern grew about the unsuitability of a few terms and the need for standardised lists became apparent (Trinick & May, 2013). The task of creating an Indigenous mathematical vocabulary is not straightforward, as it involves deciding which translation of several is appropriate and captures the concept most closely.

Moreover, underlying grammatical structures that operate in languages also need to be taken into account in developing mathematical registers in Indigenous languages; as different grammatical structures may produce different meanings. Parra and Trinick (2018) recently argued that the development of a mathematics register in an Indigenous language using transliterations might also indicate and reinstate the epistemic dominance of western mathematics over Indigenous mathematics thinking, which again may contribute to a deficit perspective signalling a lack of mathematical ideas in Indigenous communities.

In the next section, I discuss the language as a resource perspective that since the 2000s has been taken in mathematics education to support the learning of multilingual learners.

2.2.4 Language(s) as a Resource in Mathematics Education

Adler (2000, 2002) claimed that the languages must be construed as "re-source" (Adler, 2000, p. 207) which can be utilised repeatedly. Several researchers, thus, have claimed that code-switching among the language of instruction and Indigenous languages can foster mathematical understanding (Adler, 2002; Kaphesi, 2003; Moschkovich, 2007; Pourdavood et al., 2005; Setati & Adler, 2000; Sharma et al., 2011; Webb & Webb, 2008; Zazkis, 2000). Code-switching occurs when speakers switch between two or more languages or mix words from another language in an utterance in one language. Research has also suggested that code-switching allows the use of a learner's proficient language as a linguistic resource, thus bringing the learner's world and culture, and appreciation for them, into the classroom (Setati, 1998; Setati & Barwell, 2006).

In line with Adler, Moschkovich (2007) advocated that the practice of code-switching is inherently related to the student's choice of language when engaging in mathematical activity. This preference for one language over the other is informed by "the place, the purpose, the topic, the participants and the social relations among them" (p. 132). Thus, code-switching highlights the way multilingual learners blend their multilingual competencies to work out their understanding of mathematical concepts as they successfully participate in mathematical activity (Moschkovich, 2013, 2015; Setati & Moschkovich, 2013). This understanding of code-switching is comprehensive of the abilities that learners employ using their knowledge of diverse languages rather than a mark of their inability to express a concept in one language and, therefore, using the other language.

While supporting the practices of code-switching, Adler (1995) warned of three kinds of teaching dilemmas from a teacher's perspective. The first dilemma is concerned with code-switching itself, which Adler (1995) called the dilemma of code-switching. She argued that while teaching mathematics in a multilingual context, teachers often encounter the dilemma of either developing English (the language of instruction) or developing mathematical meaning (with the help of an additional language). A tension exists between developing mathematical understanding and English simultaneously without jeopardising either. This dilemma gets deeper where there are more than two languages. In addition to the already mentioned tension between English (the language of instruction, which is also a language of aspiration) and the local language, the issue of favouring one over the other Indigenous languages doubles the tension. The second dilemma is the dilemma of mediation, which is tension between developing mathematical communicative competence, and negotiating and developing mathematical meaning. This dilemma concerns the root of the difficulties experienced by the students in understanding and explaining mathematics. The tension arises as to what should be focused on in a multilingual class: mathematical language or the mathematical concept. The dilemma of mediation is about "how and when to act" (Adler, 2002, p. 69) to promote both mathematical understanding and mathematical language. The third dilemma is the dilemma of transparency. This dilemma concerns the tensions arising between the language used for thinking and the language used to display knowledge. That is, making the mathematical language more explicit shifts the learner's attention to the language rather than the mathematical concept under consideration.

These tensions have also been noted by Barwell, Chapsam, et al. (2016). The authors compared four case studies from mathematics classrooms in Canada, Malaysia and South Africa to explore how the tensions regarding code-switching are similar or different in these three different contexts. They argued that the teacher plays a crucial role in mediating these tensions. In addition, the mediation process may take several forms, including mixing languages to provide additional support to students for meaning-making, policing language-use in the classroom, using multiple discourses, and making meaning of students' utterances.

Additionally, the critical analysis of code-switching practices also elucidates the issues of power, dominance, and access to meanings of mathematical constructs along with their influence on a learner's self-concept and identity (Setati et al., 2002; Setati et al., 2009). Setati (2003) reported that teachers in South Africa used "Setswana as the language of solidarity and English as the language of authority" (p. 299). She explains that in South

Africa, the need for access to English tends to dominate in a milieu of two or more languages (Setati, 2008). This dominance of English in teaching and learning of mathematics promotes the elite status of the language and its hegemony in mathematics education.

In addition to this, the use of English as a powerful medium of instruction marginalises students belonging to non-dominant backgrounds and creates social inequality in terms of equitable access to mathematical knowledge (Parra & Trinick, 2018; Planas, 2014; Planas & Civil, 2013). This social inequality to mathematical knowledge privileges a particular set of mathematics knowledge – western mathematics. This draws our attention to the political dynamics within which languages operate in a mathematics classroom. It can be argued that languages are neither neutral nor supportive even when the multiple languages act as "resources" for promoting teaching and learning of mathematics in a multilingual setting.

Barwell (2018) challenged the notion of the language-as-resource perspective. He argued that the language-as-resource perspective is limited in two ways. The first argument against this is the static, material conception of language as a tool that can be re-sourced again and again (see Adler, 2002, 2010; Adler & Ronda, 2015). The material conception of language is evident in code-switching practices, which construct a fixed notion of language as a discrete, bounded and decontextualised entity with its own grammatical structure, and assumes that multilingual speakers move back and forth between languages with distinctive grammatical structures.

Moreover, with a focus on mathematical practices of understanding, reasoning, and communicating mathematically, Moschkovich (2019) proposed a framework of "academic literacy in mathematics" (p. 89) to investigate how hybrid language practices allow students to engage in mathematics practices. She aimed to explore how different connotations of the same word may impact students' mathematical thinking. Within the academic literacy in mathematics framework, she argued that solving mathematical problems involve "not only *mathematical proficiency* but also competencies in *mathematical discourse* as well as *mathematical practices*" (p. 97, emphasis added in original). She analysed an excerpt from a lesson on 2D geometric shapes in a Grade 4 bilingual classroom with 9 to10-year-old children. The teacher provided instruction in Spanish as well as in English. Moschkovich argued that bilingual students' pronunciations might indicate hybrid language practice rather than simply code-switching. Moreover, she argued that the teacher re-voiced students' contributions with a focus on promoting students' participation in the classroom instead of using specific mathematical language.

The perspective on language within the code-switching practices ignores the dynamic nature of language use evident in the diversity within and across the languages in terms of "vocalizations, expressions, intonations, styles, and bodily gestures that encode the process of meaning making" (Planas & Chronaki, 2021, p. 151). Secondly, Barwell (2018) argued that the studies focussing on the socio-political dimension of language as a resource for gaining access and opportunity to mathematical knowledge do not provide any insights into the process of how mathematical concepts are developed (see Planas & Setati-Phakeng, 2014; Setati & Barwell, 2006; Setati & Moschkovich, 2013). The language aspect explored in these studies clearly overshadows the learners' constructions of mathematical concepts, thus, shifting this aim of mathematics education research, in general, to the periphery.

In this section, I have discussed four perspectives that have been taken on the role of language(s) in the mathematics classroom. However, all these perspectives take language as a set of codes with specific meanings, ignoring the communicational aspect of language-in-use where children learn not only the mathematical vocabulary but the mathematical practices as a product of their participation in the mathematics classroom.

In the next section, I present a review of research from the field of mathematics education that has explored languages as part of classroom interactions.

2.3 Mathematics Education Research on Classroom Interactions

A review of the literature concerning language in mathematics education shows us that classroom interactions while discussing, explaining, and talking play a crucial role in developing mathematical understanding (see, Edwards & Mercer, 2013; Krummheuer, 2007; Mercer & Sams, 2006). This section presents in three sub-sections a brief discussion of different approaches that contribute to our understanding of how interactions help in developing mathematical thinking. These diverse approaches include (i) the Interactionist Perspective (see Section 2.3.1), (ii) the Conversation Analysis Perspective (see Section 2.3.2), and (iii) the Discursive Perspective (see Section 2.3.3).

The contributions of the Interactionist perspective to understanding the role of interaction in mathematics education are presented first.

2.3.1 Interactionist Perspective and Mathematics Education

The Interactionist perspective on mathematics learning assumes that our mathematical reasoning and sense-making processes develop from our participation in the social interactions whereby individuals co-construct taken-as-shared mathematical meanings

(Bauersfeld, 1994). In 1995, Mercer identified three kinds of talk that are evident in children's conversations during group work. These are: (i) disputational talk, (ii) cumulative talk, and (iii) exploratory talk. Talk that includes disagreements among peers and decisions that are made individually is called disputational talk. Students provide assertions and counter-assertions without reaching a consensus. Cumulative talk, on the other hand, is the talk where students build on each other's suggestions, and a collective decision is taken. This kind of talk is represented through repetitions and elaborations. The third kind of talk is exploratory talk. Mercer (1995) argued that exploratory talk allows students to engage constructively yet critically with each other's ideas; thus, exploratory talk provides opportunities for learning.

Using exploratory talk, Mercer and Sams (2006) developed an interventional teaching programme called "Thinking Together" (p. 507). The programme enabled children to talk and reason together as a group. The research included 406 children (9 to 10 years) and 14 teachers from Year 5 classrooms in the United Kingdom. Examining talk in the classroom, Mercer and Sams (2006) explored a mathematics teacher's role in expanding children's language skills in order to develop their reasoning. They argued that the intervention programme enabled children to work effectively as a group and to use language to reason collectively.

In line with Mercer and Sams (2006), Monaghan (2005) conducted an intervention study with Year 6 teachers and students (9 to 10 years) in four schools (two in London and two in Milton Keynes) for developing students' exploratory talk. Teachers were provided with professional development training which enabled them to use exploratory talk in their classrooms. Three lessons were planned in collaboration with teachers to introduce students to the exploratory talk to be used for peer or group discussions. In these lessons, students learned what exploratory talk was, were given ground rules for using it and practised using those rules before working on the mathematics content. In total, 12 lessons were observed in this class, including the first three lessons on learning about the exploratory task. Monaghan found that the teacher plays a crucial role not only in modelling the approach but also in providing feedback on the mathematics concepts. It was noted too that students became conscious and reflective about their own conversational practices in group discussions.

Edwards (2005) also explored the patterns of exploratory talk in high school students (11 to 15 years) group discussions. Group talks about the problem-solving activity were audio-recorded from three to seven consecutive lessons in four secondary mathematics classrooms

(Years 7, 8, 9, and 10). She found that students questioned, provided support, and accepted each other's explanations. Moreover, she argued that students displayed an understanding of the ways of working in the classroom, showed polite turn-taking, holding back, and talking aloud to develop a sense of mutual trust and acceptance of each other's contributions. However, her investigation of talk did not explore what makes an explanation mathematical in nature.

Yackel and Cobb (1996) explored normative aspects of mathematical discussions and how these normative aspects inform students' mathematical explanations, justifications and arguments. They proposed the idea of sociomathematical norms of classroom discussion to describe the discussion norms that interactively constituted and are specific to mathematical aspects of students' learning. That is, "what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms" (p. 461). They provided evidence from Grade 2 (7 to 8 years) classroom discussions of how these sociomathematical norms are constituted interactively by the teacher and the students. They argued that sociomathematical norms are interactionally constructed by the teacher and the students. One example of a sociomathematical norm is the way the students and the teacher co-constructed the meaning of mathematically different explanations of solutions to addition problems. Mathematically different explanations are one kind of sociomathematical norm where students provide their solutions using a different approach. For example, the addition of 16+14+8 can be achieved in two mathematically different ways, as shown below.

- 10 (from 16) + 10 (from 14) = 20,
  6 (left from 16) + 4 (left from 14) = 10
  20+10= 30
  30+8= 38
  OR
- 6 (from 16) + 14 = 20
  20 + 10 (from 16-6) = 30
  30+8 = 38

The above-mentioned ways of adding 16+14+8 show two mathematically different ways of finding the sum. Yackel and Cobb (1996) also found that explanations are constituted as mathematical explanations if they are backed by mathematical logic instead of on the basis of who provided those explanations. Thus, they argued that although sociomathematical norms are co-constructed by the teacher and students, teachers have more influence on these constructions as they act as a representative of the mathematics community in the classroom.

Acknowledging sociomathematical norms of classroom interactions, Krummheuer (2007) explored the nature of arguments provided in the mathematics classroom. He studied how the structure of a mathematical argument is achieved in the course of interactions in the mathematics classroom. He argued that six-year-old children (Grade 1) learn mathematics as they participate in collective argumentation. He explored the structure of argumentation that is accomplished in the mathematics classroom through interactions. In other words, what kinds of statements are provided by the participants in order to establish their argument. He found that students construct and display their arguments based either on the data and/or evidence, or with warrants which are statements that contribute to legitimising the argument, or by providing a counter statement.

In line with Krummheuer (2007), Gellert (2014) studied different interpretations of the mathematical objects that arise as mathematical contentions during mathematics lessons. Gellert (2014) suggested that a "mathematical contention" (p. 856) is demonstrated when a participant's statement is contradicted, challenged, or questioned by another participant's statement while interpreting a mathematical object. She studied moments of contention in small group interactions in upper primary Grade 3 and 4 classes (8 to 10 years) using the triad Initiation – Maintenance – Closing. Initiation is putting forward one or more perspectives on a specific mathematical problem; Maintenance involves an examination of emerging perspectives on that problem by different participants; and Closing examines the way the conversation closes with or without a consensus. She argued that the nature of the interaction during the Maintenance phase determines the interaction during the Closure. That is, the interaction can take a funnel pattern (Bauersfeld, 1988) or a focusing pattern (Wood, 1994). A funnel pattern is the pattern of interaction where the teacher asks multiple questions in small steps to lead the student to only one correct answer. A focusing pattern involves an interaction pattern where the teacher asks questions with the intention of allowing students to provide justification, reasons and explanations with their interpretation of the problem. Gellert (2014) argued that the focusing process is more productive than the funnel pattern as the focusing pattern is open for multiple interpretations of the problems instead of following only the teacher's interpretation of the problem, as in the case of the funnelling pattern.

Using data collected over a period of two months, Sfard and Kieran (2001) analysed the interaction of two 13-year-old boys learning algebra. They analysed the interactional data for signs of failure of communication between the two, which meant that their interaction did not lead to mathematical learning. The analysis focused on providing a detailed description of the micro-context along with the engagement of the students in the communication process. They found that talking in pairs or a group does not necessarily lead to learning. They proposed that if mathematical communication is to be effective and conducive for learning, “the art of communicating has to be taught” (p. 71). Building on this work, Sfard presented her “commognitive approach” (Sfard, 2008, p. 127) and argued that cognitive and interpersonal communicational processes are different manifestations of the same phenomenon (Sfard, 2008; Sfard & Kieran, 2001). Therefore, to make sense of a student’s cognitive activity, the overall communicative activity with all its patterns and the contexts of their demands need to be understood (Sfard & Kieran, 2001). Sfard (2012) argued that for communication to be effective, there must be two conditions: (i) it fulfils its communicative purpose by fulfilling expectations based on intentions, and (ii) the act of communication should have no evidence of a breach. Thus, the effectiveness of communication depends on the harmony of the intentions of the speaker with the expected outcome from the other. She argued that an act of communication is effective as long as there is no evidence of breach or incongruence between intentions and expectations (Sfard, 2012).

Using Sfard’s approach, Kaur (2015) explored how Grade 2-3 (7 to 8 years) children developed an understanding of the properties of the triangle while working in a technologically-enabled classroom using the Sketchpad program. Kaur’s study is part of a larger teaching experiment that investigated children’s geometric thinking in primary grades in an urban middle district in Western Canada. Twenty-four children from diverse ethnic backgrounds participated in the teaching experiment. Four lessons on teaching and learning about triangles were observed and video-recorded. Each lesson lasted approximately 2 hours and 30 minutes. Kaur (2015) noted that children began by describing informal aspects of triangles, for example, “everything moves with it except one point” (p. 418), to show their reasoning. Their reasoning moved to formal geometric properties in later lessons, as evident in statements like “the lines are the same” (p. 418). She argued that the discourse in students’ language moved along the proposed order of discourse, (i) the discourse of visual object recognition, (ii) the discourse of informal properties, and (iii) the discourse of definitions. Kaur emphasised that naming objects with the same word may help students to see commonalities between shapes and objects, which may promote understanding of the hierarchical relationship between triangles and different types of triangles. Her research underscored the importance of the communicational features of language that mediate the development of geometry concepts; however, her research focuses on 2D geometry and not 3D. Sfard's approach emphasises the collective nature of communication while keeping the interactional experience of individuals in the meaning-making process at the periphery.

In the studies outlined in this section, the focus of the interactional analysis is on the mathematical discourses rather than on the in-the-moment interactions among participants. These studies take account of the broader socio-cultural context that influences the routines and narratives of the mathematics classroom but ignore the specific circumstantial aspect of interactions that may affect the routines and discourses produced. In the next section, I review research that has shed light on the in-the-moment interactions that occur in a mathematics classroom from a Conversation Analysis perspective.

2.3.2 Conversation Analysis Perspective and Mathematics Education

Research on Conversation Analysis (Schegloff & Sacks, 1973) has also informed the field of mathematics education (e.g., Mushin et al., 2013; Radford et al., 2011; Roth & Gardener, 2012; Roth & Thom, 2009). Conversation Analysis (CA) explores the overall interactional structure of the classroom talk, including the sequential organisation of conversation (Schegloff, 2007), as well as the construction of participants' utterances, also known as turns (Drew, 2013), by exploring the intonation, stress, volume and silence in utterances.

Exploration of sequences of talk in mathematics classrooms highlighted various aspects of classroom talk. A three-step interactional pattern of Initiation-Response-Evaluation/ Feedback, abbreviated as IRE/F (McHoul, 1978; McHoul, 1985; Mehan, 1979; Sinclair & Coulthard, 1975), is often observed as part of the formal talk in the classroom (see Roth & Gardener, 2012; Mushin et al. 2013). It is an interactional pattern, representative of most of the classroom talk, whereby the teacher initiates a sequence of talk, students respond, and then the teacher evaluates the students' responses.

Roth and Gardener (2012) explored the IRE sequence of classroom talk with a theoretical framework based on ethnomethodological principles with the Vygotskian cultural-historical perspective. On the basis of Vygotskian perspective, they argued that what children learn today in the presence of others within a social and material context relation with others will be a psychological function in their future. They considered how second-grade students display their understanding of 3D shapes, analysing one episode where Grade 2 (6 to 7 years) students explained what makes a cube a cube. They found that children articulated multiple explanations for what makes a cube. They also found that the teacher's third evaluative turn in the IRE sequence can act as an intentional pointing on the part of the teacher (moreknowledgeable other) to direct students' attention to provide specific details about their construction of a cube to support their participation in the culture of the mathematics classroom.

Research has also shown that the rewording or reformulation of the statements by the teacher can be useful in providing students with appropriate feedback. For example, Mushin et al. (2013) investigated the role of language in Year 1 (5 to 6 years) oral assessments in an Australian Indigenous community school using CA. In their study, the teacher administered an assessment with one child at a time. During each assessment, 12 cut-outs (in three colours – red, yellow and blue; and in two sizes – big and small) were presented to the child. The teacher provided instructions orally. Students could respond orally or could show their understanding through their actions. Mushin et al. (2013) observed the sequential nature of the oral assessment exercises where students were asked to sort 2D shapes (in the form of paper cut-outs) based on sameness, shape or size. They argued that during assessments teachers play a crucial role as they reformulate the precise wording of the assessment tasks and provide students with further opportunity to demonstrate their understanding. They further reported that subtle factors such as repetition or reformulation of phrases, word choice, the silences between and within utterances, and falling and rising intonation patterns could have a significant impact on the student's interpretations of the geometry task, which may, in return, interfere with the student's capacity to demonstrate their understanding of mathematical tasks. The study acknowledged the students' cultural and linguistic backgrounds yet did not offer any insights into the different meanings that they might bring from these to the classroom.

Heller (2015) also investigated the IRE sequence in a Grade 5 mathematics classroom to explore how the teacher and students (9 to 10 years) displayed their orientation to discursive norms for providing explanations and arguments during classroom interactions in Germany. She video-recorded language and mathematics lessons and used genre-oriented approaches with CA to investigate subtle displays of orientation to discursive norms. Heller (2015) found that in mathematics classes teachers often initiate a turn using the phrase "give reasons" (p. 194) when asking for students' answers for a particular question. This act of *explicitly* specifying what counts as an appropriate argument or explanations (the teacher's use of the phrase "give reasons") lays conditions for what arguments or explanations are accepted or preferred. The preference in a classroom is dependent upon the content of the student's response and is independent of the teacher's liking or disliking. Similarly, students' explanations or arguments without reasons may be seen as dispreferred/negative responses. In CA, preference is a key idea. Preference accounts for the sequential organisation of two subsequent turns/utterances, where conditionally relevant actions are expected. For example, a question in the first turn is expected to be fulfilled by an answer in the following turn. Another example is when a request in the first utterance (by a speaker) is expected or preferred to be followed by acceptance in the second turn (by the second speaker). The notion of preference in CA studies is not about conversation participants indicating whether a certain response is liked or disliked. Instead, the idea of preference in social interactions refers to the use of responses to allow moving forward in the interactions without any conflict/tension. These responses allow the maintenance of the solidarity between the participants to achieve co-construction of reasoning in the interaction. Heller (2015) argued that in order to master the academic discourse of mathematics and provide preferred responses students need to be able to interpret subtle displays of orientation to discursive norms and to identify the different stances concealed within others' turns/utterances.

Like Heller (2015), J. Ingram et al. (2019) also investigated the classroom interactions where students gave explanations. However, they focused on the classroom interactions where explanations are given *without being explicitly* called for. They studied 42 video-recorded mathematics lessons from eight different schools (with 6 secondary schools taking students aged 11 to 18 years, one middle school taking students aged 7 to 13 years, and one high school taking students aged 13 to 18 years). They reported two major findings pertaining to how preference was organised in classroom interactions. First, they found that a student's response was considered a dispreferred response if it was given without the student having been asked. That is, if a teacher had assigned the next speaking turn to one student and another student responded then the second student's response was considered a dispreferred response. Second, they stated that in mathematics classrooms teachers and students considered overtly disagreeing with a student's response a dispreferred response when the response was incorrect. In these cases, when disagreement has occurred, students provided explanations for why the responses were incorrect the first time.

A similar finding was reported in another study by Ingram et al. (2015). They analysed 22 video-recorded mathematics lessons from seven different schools in the United Kingdom using a CA approach with a focus on how errors are handled interactionally in a classroom. Participants included secondary school students 11 to 14 years old and their teachers. Each lesson lasted for 45 minutes or 90 minutes, depending on the school. They found that teachers seldom provided a negative evaluation of students' responses. They argued that since negatively evaluating students' mistakes are considered as dispreferred responses, mistakes might be treated as embarrassing and problematic in an implicit manner. In other words, though teachers attempt to show mistakes as part of learning mathematics, interactional tools such as delaying a response by pausing for 0.6 seconds may indicate that mistakes are dispreferred responses.

In addition to these findings, both these studies (J. Ingram et al., 2019; Ingram et al., 2015) also highlighted the interactional role of prosody in classroom interactions. For example, they showed how a teacher might display acceptance of a student's response by mimicking the prosody and structure embedded in the student's response. Research from the field of sociolinguistics informs us that different intonation patterns perform a variety of interactional roles. For example, Hellermann (2003) found that repeating others' utterances with the same tonal pitch may indicate approval or acceptance; conversely, the repetition with different pitch may indicate disapproval. Similarly, the use of high rising intonation at the end of an utterance is often interpreted as a sign for questioning (Ward, 2019). However, in the New Zealand context, a high rising terminal (HRT) intonation pattern may indicate the speaker's intention to check if the listener is following the speaker or as a way to develop communicational solidarity (Metge & Kinloch, 1978; Warren, 2016).

In addition to the interactional role of prosody, Tainio and Laine (2015) argued that in a classroom the emotions and affective stances of teachers and students often differ, even though the main purpose is to promote students' learning. They analysed the emotions and affective stances of students and teachers as evidenced by their utterances in the Finnish mathematics classroom. They videotaped ten mathematics lessons in Grade 6 (11 to 12 years) classrooms in Finnish schools in Helsinki, Finland. Using conversation analytic techniques, they studied the verbal as well as non-verbal aspects of classroom conversations during moments when students gave incorrect answers to teachers' questions, with a focus on the emotional states of students. They maintained that when responding to students' incorrect responses "teachers, consciously or accidentally, display their stance towards student mistakes" (p. 84). If these stances are charged with embarrassment, as in the case of student mistakes, they may negatively affect the student's emotion.

Recently, and outside mathematics education research, Kamiloğlu et al. (2020) reviewed 108 published studies on prosody and its connection with positive emotions in English language, and argued that loud voice, pitch, and speech rate might indicate a variety of positive emotions. They argued, for example, that a higher pitch may indicate emotions of amusement, interest or relief, and a low pitch may indicate admiration. However, in mathematics education research, with few exceptions (e.g., Tainio & Laine, 2015), the link between prosody and emotions in a multilingual context appears to be unexplored.

This section has highlighted how verbal and non-verbal aspects of communication in the mathematics classroom might indicate teachers' and students' emotions and stances about their mathematics learning. The studies also inform us about the rules of participation that become evident as part of classroom interaction practices that students learn, and how subtle aspects such as prosody contribute to the process of meaning-making in the classroom process. However, the studies do not account for the role of prosody in the presence of multiple languages, as in a multilingual classroom context. This study aims to explore how prosody acts as an interactional tool and how children use prosody to interact as they construct their understanding of shapes and their properties.

Mathematics education researchers have also explored mathematics classrooms from a Discursive Perspective. This perspective draws our attention to what the participant aims to achieve within the broader discourse of mathematics education as they make use of certain discursive practices.

2.3.3 Discursive Perspective and Mathematics Education

Discursive Psychology allows us to explore interactions to see what actions participants orient to through language, explicitly or implicitly while engaged in interactions (Potter, 2013). The Discursive Psychology perspective views interactions as discursive practices, where the focus is on the social actions that are performed by the utterances, rather than on their content. Three kinds of discursive practices are identified in this section. These discursive practices are re-voicing, genres, and discursive demands. I first present a review of studies on revoicing as a discursive practice in mathematics classrooms.

Re-voicing as Discursive Practice.

The instructional practice of *re-voicing* in teacher-talk is considered a discursive practice and was first noted by O'Conner and Michaels (1993). They argued that the teacher engages in re-voicing by re-uttering student's responses with the aim of coordinating the elements of the academic task as stated by students while maximising their learning of social participation structures during classroom interactions. They argued that by re-voicing students' contributions in the classroom, the teacher shares power in knowledge building by offering conversational space to students to display their participant stance in relation to knowledge.

Moschkovich (1999) analysed the classroom discourse of a third grade (8 to 9 years) lesson on geometric shapes using a tangram puzzle to explore the classroom practices used by the teacher to support the participation of English language learners in an urban California school. Moschkovich noted that the students brought different ways of talking, including narrative and predictive and argumentative ways, to mathematically talk about mathematical objects, in this case, shapes. She found that the teacher often re-voiced English language learners' contributions by building on what they said and enabling them to clarify what they meant. This practice of re-voicing thus focused on developing mathematical content and argumentation rather than simply focusing on vocabulary development, thereby supporting English language learners' participation in mathematical discussions. Recently, Moschkovich (2015) further argued that the teacher's re-voicing provides not only micro-scaffolding to individual students but also "meso level scaffolding support" (p. 1076) for developing the argument skills of the whole class as a group, thus leading to collective argumentation (Planas & Morera, 2011).

Forman and Larreamendy-Joerns (1998) also supported the claim that the teacher's re-voicing of students' utterances helps students to understand how to develop a mathematical argument. They studied one teacher's re-voicing strategies in a Grade 2 (7 to 8 years) lesson on area measurement in United States. They considered that the teacher's re-voicing practices work as meta-messages to draw students' attention to explicit explanations while giving mathematical arguments.

Eckert and Nilsson (2017) further explored the interactional strategy of re-voicing in two classrooms in Swedish primary schools. Data from video-recorded series of probability lessons from the Grade 5 and 6 (11 to 13 years) mathematics classrooms were investigated with a focus on the interactional strategies of the two teachers. They found that there are two kinds of re-voicing: active and inactive. In active re-voicing, the teachers indicate their intention and interpretation of students' utterances as they re-voice students' contributions, whereas inactive re-voicing shows word-by-word repetition of students' utterance without indicating the teacher's intention or interpretation. Eckert and Nilsson (2017) argued that active re-voicing plays an important role in continuing the mathematical discussions. In contrast, inactive re-voicing may implicitly reject students' ideas, thus limiting opportunities for alternative explanations.

Boukafri et al. (2018) also studied teachers' practice of re-voicing for its linguistic form, discursive form and mathematical form. They analysed four geometry lessons where Grade 7

(12 years) students worked on finding the shortest path and the Pythagorean Theorem. They argued that though the teacher also engages in reporting, expanding and rephrasing students' responses, the practice of re-voicing may discursively indicate students' inadequate understanding. They suggested that future research should explore how students' re-voicing influences learning during mathematical discussions in the classroom.

Research outside the field of mathematics education (e.g., Copp Mökkönen, 2012; Tholander & Aronsson, 2003) on students' re-voicing of the teacher's talk may provide fruitful insights. Tholander and Aronsson (2003) explored group-work sessions in five Swedish junior high schools (13 to 15 years) to investigate the discursive practices that students use to engage in during group work. They used Goffman's concept of participation framework (1981) and Bakhtin's Dialogic Theory of language use (1981) to analyse the students' positionings as they engage in group work. Goffman's participation framework represents the ways in which participants position themselves and others as partners in conversations. These ways of positioning themselves in regard to others are inherently linked with who the other is, who the talk is addressed to, using a key Bakhtinian notion, that of the addressee. Tholander and Aronsson (2003) found that pedagogical routines typical of traditional classrooms can be identified in small-group talk, where a student may act as a sub-teacher, taking up the role of a teacher and embedding their utterances with teacher-talk. In Bakhtinian terms, students as sub-teachers may appropriate the teacher-talk in their utterances.

In line with how students discursively position each other in group settings, Evans et al. (2006) investigated the role of emotions in displaying discursive positionings of three students during small-group work on geometry problems on the Pythagorean theorem in a Grade 8 (12 to 13 years) mathematics classroom in Lisbon, Portugal. They used Bernstein's sociological approach to explore what positions are available to students in a group setting and how those positions influence the participants' emotions. They found that students may position other students as evaluator or evaluated, helper or seeker of help, leader or follower, among others. They also observed that students displayed emotions such as anxiety or excitement during group discussions as part of the discursive practices embedded in their talk to demonstrate the positions they attempted to adopt, modify, or claim.

Expanding research on sub-teaching to a multilingual setting, Copp Mökkönen (2012) investigated the practice of sub-teaching in whole-class discussion in the presence of the teacher in an English-medium Finnish school. She conducted her study with Grade 1 and 2 students (7 to 8 years) to explore how students may use adults' language in the form of genres (as suggested by Goodwin & Kyratzis, 2007) to accomplish certain social actions in the classroom and produce new meanings. She found that children sometimes used teacher-talk to maintain the social order of the classroom, which may provide evidence of Bakhtin's notion of double-voicedness (Bakhtin, 1981, 1986) in their utterances. Double-voicedness implies that a person can appropriate someone else's words as their own by embedding their own values and meanings. Copp Mökkönen (2012) argued that children used teacher-talk to indicate an authority taken from the authoritative discourse of teacher-talk to maintain the social order of the classroom. She further argued that directives (statements to get someone to do something) using teacher-talk were also evident in some of the children's talk, which may indicate that by using this way of talking, children discursively construct asymmetrical power relations, and position themselves and others differently in a classroom setting. These studies highlight the need to explore the ways in which students may use teacher-talk in the mathematics classroom. In addition to highlighting the need for further research exploring students' re-voicing, Herbel-Eisenmann et al. (2009)'s research with Grade 6-10 mathematics teacher-researchers and students (11 to 15 years) found that the students' interpretation of teachers' re-voicing moves may differ from a teacher's interpretation, and this also needs to be explored. They further argued that the teachers' re-voicing might lead to multiple meanings, which are often unexplored.

I next present a review of studies that have explored genres in mathematics classrooms as discursive practice.

Genres as Discursive Practice.

In mathematics education research, a few studies have investigated genres as a discursive practice. One such study is that of Gerofsky (1996), which analysed mathematical word problems as a genre and described their features. She argued that the genre of mathematical word problems is similar to the genre of a parable, as both word problems and parables use language that refers to some *other world* and is related to real-world life in a tangential manner. That is, the language used in both parables and word problems refers to abstract entities or ideas that are difficult to express. She also pointed out that in both parables and mathematical word problems, there is an illocutionary force which states what needs to be done (for example, using the recently learnt method or algorithm to solve the word problem).

Gerofsky (1999) later analysed the Initial Calculus lectures of four mathematics professors at Simon Fraser University to explore the generic features of the lectures. She found that the properties of the lecture genre were similar to the language of persuasion. She noted:

> Some prominent features of the “initial calculus lecture” genre included unusual uses of the first person plural pronoun (‘we’, ‘us’, ‘our’- see also Rowland, 1999), extensive use of rhetorical questions and tag questions, the attribution of questions or opinions to the audience and lecturers ‘answering’ these unasked questions or objections, and the structuring of the lecture as an inexorable chain of logic that could lead to no conclusions but the ones given. I was struck by the similarity of some of these features to the language of persuasion (p. 41).

In her analysis, Gerofsky found that the lecturers used ‘we’ in an unusual manner – who was included in the ‘we’ was not evident. This use of ‘we’ may indicate a relationship of power and dependency between the lecturer and their students and seeks conformity from the listener. Similarly, tag questions, such as (“Ok?” or “Right?”) were also used to elicit consent from the audience (in this case, students). She argued that there is a clear link between the language of persuasion and that of mathematics lectures. Her analysis of lectures may indicate some intention on the lecturer’s part to persuade their students.

Martínez (2018) too used the Bakhtinian concept of genres to identify genres in 85 teaching episodes in 12 lessons observed in a Grade 3 (6 to 7 years) mathematics and language integrated Spanish immersion classroom. The participants spoke English at home. For the genre analysis, the author based his study on Bakhtin’s idea of genres and used Gerofsky’s approach to analyse linguistic features and social conventions in each of the episodes. The linguistic features included modes of communication, mathematical content, mathematical practices, language used, attention given to the accuracy of the language used, and the social conventions concerned with who addressed whom, the direction of the conversation, and who asked questions. The author identified three genre categories: general genres (such as whole-class discussion, small-group tasks, task launching, teacher explanation, and student presentation), language teaching genres (role-playing, info gaps, and artefact description), and mathematics teaching genres (including textbook work, number talk, and test). Based on the genre analysis, Martínez (2018) argued that the teacher dynamically engaged in switching among these genres to successfully navigate the competing demands of mathematics and language teaching. He further suggested that the teaching of language and mathematics cannot be treated separately. Martínez’s study noted the presence of different genres used in the classroom by the teacher; however, his study did not account for the genres that are used in moment-to-moment interactions and how speakers embed their utterances with their intentions to influence negotiation of meanings in a conversational space.

Rezat and Rezat (2017) investigated specific generic features of a subject-specific genre of geometric construction text in a German Grade 7 geometry classroom. They based their understanding of genres on Sandig's (1997) definition, a similar concept to Bakhtin's speech genre. Sandig argued that genres essentially represent the type of act and text. The type of act constitutes communicative properties of a genre in a context in which it is used with others. The text type aspect of the genre concerns the language structure and "corresponding linguistic means to realize the genres" (p. 4196), for example, sequence patterns. Rezat and Rezat (2017) analysed both the act and the text properties of the geometric construction text genre in the context of mathematics textbooks, the relevant research literature on teacher education for teaching geometric constructions, and audio-recorded geometry lessons. They found that genre features differ in these different contexts. For example, they found that the type of act is justification in the context of mathematics textbooks, and reporting in the context of teacher education research. These different types of acts (that is, justification and reporting) reflected different expectations from the reader. For example, the mathematics textbooks reflected *what ought to be done* for geometric constructions, whereas research on teaching geometric constructions focused on *what was done.* For the classroom discourse, they argued that the teacher made use of different properties of the genre from both of the other contexts according to the moment-to-moment need. Rezat and Rezat (2017) suggested that there is a need for teachers to be aware of the genres that they use in their classrooms for better and more effective teaching and learning of mathematics. Although this study focused on the moment-to-moment change of genre in the classroom discourse, the implicit understanding of language adheres with an already given set of symbols with specific meanings, without acknowledging the dynamic nature of language-in-use in classrooms.

Above-mentioned studies focused on the analysis of genres that looked specifically at classroom discourse, either in the form of lectures or in the mathematical content of learning activities. These studies do not take account of the speaker's intentions and the values that they bring to the communicational field of the classroom. Both these aspects were taken up by Rockwell (2000), who used the Bakhtinian concept of speech genres to analyse the teaching of speech genres in a teacher's utterances during a lesson observed in a Mexican rural school. She argued that for Bakhtin (1986), speech genres are created historically as the language is used again and again and diversified and transformed by the unlimited possibilities of interaction within any human activity. She analysed a 60-minute lesson to illustrate the diversity of teaching genres in a Grade 6 science classroom in a rural Mexican school. She described a number of important findings. First, she noted that the teacher used a variety of speech genres, including informal talk, explanation, folklore and anecdote, although her analysis does not provide an exhaustive list of those speech genres. She found that the teacher often used two kinds of genre: "*plática* (informal talk or chat) and *explicación* (explanation)" (Rockwell, 2000, p. 267). She categorised the informal talk as speech genres because they reflected generic overtones, established a particular way of expressing knowledge, and included relatively open turns for student utterances. She argued that explanation was interactionally used as another kind of speech genre, displaying assumptions that the students knew about the topic in question and were expected to provide details about the elements of that topic. Second, she argued that these genres both underwent and caused transformations in the structure of the students' participation each time they were used. She suggested, too, that the teacher's use of a certain speech genre might create discursive conditions for the use of a particular kind of speech genre by the student. Finally, she claimed that teaching itself is like a complex speech genre, comprising several other speech genres which are embedded with "thoughts, values, and sentiments that are re-voiced and reinterpreted in each new situation" (p. 273).

In the next section, I present a review of research that explored how discursive demands are made through discursive practices in mathematics classrooms.

Discursive Demands in Mathematics Classroom.

Barwell (2012a) presented the notion of discursive demands that multilingual children may face while participating in whole-class and group interactions. He argued that discursive demands are the demands associated with situated language-in-use. He suggested that discursive demands are different from linguistic demands, as linguistic demands concern only the learning of certain language codes for representing learning, whereas discursive demands deal with when and how those linguistic codes can be used. Discursive demands, therefore, concern how one uses language in a particular situated context and how it impacts one's participation in interaction and relationship with others. He explored the participation of one 5-year-old refugee student during a mathematics lesson on halving and doubling in a class of 26 students in a British elementary school. He identified a number of discursive demands that the child learned to work with as they participated in classroom interactions. Barwell (2012a) argued that discursive demands create ambiguity around the suitability of one's contributions to in-class interactions. For example, a child is required to learn the rules of participating in mathematics classroom discussion, which is often implicit, and when there are multiple speakers, it might be difficult for a child to discern whose response has been evaluated by the teacher. He argued that the role of language in the mathematics classroom needs to be explored in terms of its linguistic as well as discursive demands.

Earlier, Barwell (2003b) had also explored students' attention as a discursive demand in terms of what Year 5 primary school students (9-10 years old) attended to as they solved an arithmetic word problem in class. He found that participants attend not only to different aspects of mathematics problems, such as typical features and structure of the mathematical problems, but also to the structure of participation required in doing mathematics as a collaborative task.

Recently, Barwell (2016a) examined the interaction in an elementary mathematics classroom in Quebec, Canada. The study was conducted with a Grades 5-6 (10 to 11-year-olds) class for new immigrant learners of French. Eighteen students with diverse language backgrounds participated in this study. Students' group and whole-class interactions were audio-recorded during mathematics lessons over a period of three weeks. The teacher focused on teaching mathematical vocabulary in French. Barwell focused on how polygons and non-polygons were introduced to the students during one of the lessons. He found that the understanding of what counts as informal and formal mathematics discourse about polygons and non-polygons is interactionally constructed during classroom discussion, and that the teacher played a crucial role in directing students' attention to the formulations of formal mathematics vocabulary pertaining to polygons. Moreover, the meaning of the utterance is also dependent upon the possible alternatives that it might lead to in a conversation.

Earlier, Barwell (2005a) argued that the ambiguity which arises in describing the property of a shape according to different perspectives has the potential to act as a learning opportunity. He analysed an extract from the Year 5 (9-10 years old) students' geometry lesson on the concept of dimension. During this lesson, the teacher showed a plastic circle to demonstrate the two-dimensional shape of a circle. The teacher during this instance displayed her dispreference for a plastic circle being used for learning about two-dimensional shapes because the same object (plastic circle) could be described as a 2D shape (circle) or 3D shape (cylinder). Barwell argued that the teacher's act of showing her dispreference added a degree of ambiguity to her explanation. However, that very ambiguity allowed students and the teacher to acknowledge that alternative perspectives could be used to describe the shape.

Most recently, Barwell (2020) has argued that learning mathematics is a process of socialisation whereby the discursive practices of doing mathematics that students learn involve not only what language, gestures, and other signs to use but also how to explain, justify, and argue. He analysed four classrooms (three Grade 5/6 classes and one Grade 3 class) in three schools with English-language learners and focused on socialisation events and socialisation practices. Socialisation events include "moments of significance in students' socialization into mathematics and language", and socialisation practices refer to the "language and discourse practice" of these socialisation events (p. 154). He contended that the classes that he observed could be classified in two categories: language-neutral classroom and language-positive classrooms. In language-neutral classrooms, the socialisation of second-language learners in mathematical explanations and genres is often implicit, whereas in language-positive classrooms English learners are explicitly supported in the learning of socialisation practices such as developing explanations and accounts of mathematical thinking as well as mathematics genres.

In this section, I explored the research literature from Interactionist, Conversation Analysis, and Discursive Psychology perspectives. The studies either explored the discourses of the mathematics classroom, as in the case of interactionist and discursive practices approach; or explored the in-the-moment development of talk that contributes to students' mathematical understanding as in the studies that used the CA approach. However, none of these approaches explored how in-the-moment development of mathematical understanding is influenced by the broader mathematics discourse. Moreover, only a few studies provided insights into the underlying processes that take place in the multilingual interactional space that may arise as a result of multilingual context (see Barwell, 2005a), and none of these studies related to the New Zealand context. The studies taking account of the multilingual character of interactional activities concerning geometry concepts are even fewer.

In the next section, I present the research gaps and my positioning as a researcher in this study.

2.4 Gaps in the Research Literature, and the Researcher's Positioning

This critical review of the literature on geometry education research has revealed that there is a dearth of studies exploring the development of 2D shapes, 3D shapes, and their properties within the context of multilingual geometry classes. Sinclair et al. (2016) published a review article focusing on the research contribution since 2008 in the field of geometry education. Interestingly, it fails to mention the complexities of teaching and learning of geometry in a multilingual context. The research presented in this thesis aims to fill this research gap by elucidating the processes through which children make sense of geometry concepts of 2D shapes, 3D shapes, and their properties in a multilingual context.

For this research, the researcher aligns with Barwell's (2018) "language as sources of meaning" (Barwell, 2018, p. 155) perspective, which is located in the Discursive Psychology approach. This perspective is largely based on Bakhtin's foundational ideas that assume that the process of meaning-making is relational, and language is agentive, diverse, stratified and stratifying. The "language as sources of meaning" perspective (Barwell, 2018) underscores the capacity of language to enunciate the diverse range of multiple meanings while acknowledging the within-language transformations taking place whenever the language is used (Barwell, 2005a, 2013, 2019).

Barwell's (2018) "sources of meaning" perspective on language closely aligns with the idea of translanguaging. Translanguaging is "a process by which students and teachers engage in complex discursive practices that include *all* the language practices of students in a class in order to develop new language practices and sustain old ones, communicate and appropriate knowledge, and give voice to new socio-political realities by interrogating linguistic inequalities" (García & Kano, 2014, p. 261). Using a dialogic-translanguaging perspective, Planas and Chronaki (2021) conducted a study with eighteen Grade 8 (12 to 13 years) multilingual classrooms in a low-income area of Barcelona. Fourteen of the eighteen students had Spanish and four had Catalan as their home language. The teacher could speak both Catalan and Spanish. In their study, Planas and Chronaki (2021) focused on the translanguaging that produced meaning that moved the mathematical task (involving arithmetic patterns of Fibonacci numbers) forward. They found that students used the words *baixar* and *bajar* (both implying "going down") from their everyday language to display their mathematical thinking. They also observed that students noted the subtle difference between the meanings of these words; however, the focus of their interaction was on the mathematical process required to represent their understanding of arithmetic patterns. They found that "an everyday word appears with diverse meanings in relation to thinking about the mathematical problem" (p. 161). For example, they reported that children adopted the everyday meaning of the word *sobras* which could mean "leftovers" or "very small pieces of a given unit" (p. 161) in the context of mathematical meaning-making. It can be argued that multilingual learners engage in languaging (a verb) rather than using language as a resource (a noun). This understanding emphasises the transformations that take place within the meanings of words and utterances as they are being used. This consideration seems to be overlooked in mathematics education research endorsing language as a resource approach.

The study presented in this thesis aims to unravel the complexities of language-in-use in a multilingual classroom by exploring the interactional ways through which participants represent their meanings about shapes and their properties as the meanings are developed from moment to moment as children engage in classroom interactions, on the one hand. On the other, the study aims to develop a critical understanding of the dialogic space of a multilingual classroom context that influences children's understanding of shapes and their properties.

Thus, the following research questions guide this study:

1. *What discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
2. *How do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
3. *What characteristics of the dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

2.5 Chapter Summary

This chapter situates the study presented in this thesis within the broader research field on teaching and learning of shapes and their properties in primary school. The first major section of the review focused on studies on geometry concepts in the broader mathematics education field. Within this first section, literature for the major theories that inform the teaching and learning of shapes and their properties at the primary school level, studies on dimension, inputs from research on visual-spatial abilities, and the role of gestures in contributing to the geometric understanding were reviewed. In the second section, I discussed four perspectives that inform our understanding of the impact of research in multilingualism in mathematics education research and the role of language(s) in the mathematics classroom. In the third section, mathematics education research on classroom interactions was reviewed. The review of literature in the first three sections has highlighted the research gap. The research gap and my theoretical positioning for this research culminated in a justification for the need to research the influence of language on children's negotiation of meanings about shapes and their properties in a New Zealand multilingual primary classroom. The next chapter presents the methodology adopted for this research.

Chapter 3.
Methodology

The purpose of this chapter is to outline the theoretical framework and the research design selected to answer the research questions (see Section 1.7). The first section (Section 3.1) discusses the Critical Inquiry paradigm chosen for the study, along with the epistemological and ontological assumptions. The second section (Section 3.2) of the chapter draws attention to the Discursive Psychology perspective as the theoretical framework of this study. The paradigm and the theoretical framework of the study lay the foundation for the methodological framework that has been adopted for undertaking the process of data gathering. The methodological framework forms the third section of the chapter (Section 3.3). This section presents the research design of the study, procedures of data gathering, and the researcher's position as a non-participant observer. Following the methodological framework, the data analysis framework is presented (Section 3.4). The section offers a detailed description of how the data were analysed to answer the research questions. Section 3.5 addresses the validity and reliability constructs of the study. Following a discussion on validity and reliability, the procedures for ethical approval and ethical conduct during the study are discussed in Section 3.6. The chapter concludes with a chapter summary (Section 3.7).

3.1 Critical Inquiry Paradigm

A paradigm is often called a school of thought, and it is concerned with a set of values and beliefs held by the research community within an intellectual tradition. It provides a realm within which concepts and practices operate. A paradigm defines the ontological and epistemological assumptions regarding the nature of being and knowledge, respectively. Ontology deals with the philosophical position that defines the notion of being or reality (Cohen et al., 2018). It answers questions on the nature of reality in its existence. Epistemology, on the other hand, is concerned with the nature and forms of knowledge. It is concerned with the questions of "what is knowledge? How is it acquired, and how should we know what we know?" (Ma, 2016, p. 23).

As noted in Chapter 2, the present study aims to explore children's negotiations of meanings about shapes and their properties in a New Zealand multilingual primary classroom. This research situates itself within the domains of the Critical Inquiry paradigm (Crotty, 1998). In the Critical Inquiry paradigm, "research is understood as a political enterprise with the ability to empower and emancipate" (Leavy, 2017, p. 13). Table 3.1 presents the ontological and epistemological assumptions within the Critical Inquiry paradigmatic position adopted for this study.

Table 3.1

Critical Inquiry Paradigm Adopted for the Study

Research Paradigm	Ontological assumptions	Epistemological assumptions
Critical Inquiry paradigm	• Realities are multiple, subjective and constructed within the socio-political power dynamics. • Truth is constructed through social interactions. • Interaction/language-in-use is dialogical, creates reality while embedded in social-political space and constructed through rhetorical and political purpose. • Communication works as transaction and decision-making.	• The researcher and the object of inquiry are interactive and, therefore, influence one another. • Knowledge is socially conditioned, co-constructed, situated and negotiated as arising in the process of interaction. • Idiographic statements can be generated about the known because of time and context boundedness.

Note. Adapted from (i) *The foundations of social research: Meaning and perspective in the research*, by M. Crotty, 1998, Sage Publications. Copyright 1998 by Michael Crotty; (ii) *Research design: Qualitative, quantitative, and mixed methods approaches,* by J. Creswell and J.D. Creswell (5th Ed.), Sage Publications. Copyright 2018 by Sage Publications; (iii) *Discursive Psychology*, by D. Edwards and J. Potter, 1992, Safe Publications. Copyright 1992 by Sage Publications.

Ontologically speaking, for the present study, language-in-use is assumed as social action. Therefore, participants generate their realities and notions of truth as they participate in social action of interactions (Potter et al., 1993). Based on this ontological position, the nature of knowledge in this study is taken to be socially conditioned and co-constructed within the socio-political dialogical space which is negotiated by the participants as they partake in everyday discursive practices (Edwards & Potter, 1992). On epistemological grounds, it is assumed that knowledge is co-constructed and negotiated during the process of interaction and in a specific conversational context bounded by time and broader context.

This study describes and presents a critical analysis of how children discursively construct and negotiate their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual English-medium primary school classroom. The Critical Inquiry paradigm allows for uncovering the processes through which participants construct meanings. In order to allow deeper investigation of the description and the critical understanding of meaning-making practices in a geometry classroom from the Critical Inquiry paradigm, Bakhtin's Dialogic Theory and Ethnomethodology within a Discursive Psychology perspective as a theoretical framework was chosen to inform this study. The theoretical framework is discussed in the next section.

3.2 Discursive Psychology as a Theoretical Framework

A theoretical framework presents the theoretical underpinnings that guide the principles to understand any phenomenon. My research is situated within the theoretical foundations of Discursive Psychology. In Discursive Psychology, three key characteristics of language-in-use are defined. These are: language-in-use as (i) action-oriented, (ii) situated, and (iii) constructed and constructive (Potter, 2012, 2013). The first characteristic deals the orientation to action in language use, and draws our attention to how actions are performed through language, explicitly, or implicitly. The second characteristic, that language is situated, concerns how language-in-use is situated, both *situationally* as well as *institutionally*. The *situational* situatedness of the language concerns specific details of the organisation of talk in terms of what just happened and what is going to happen next in the micro-moment of interaction. The *institutional* situatedness of language-in-use defines how one is expected to use language in a certain institutional context. The third characteristic of language-in-use concerns its nature as being constructed and constructive. The idea that *language-in-use is constructed* draws our attention to how participants use language to state their orientations and their actions by using relevant prosody, timing and other resources. The idea that *language-in-use is constructive* draws our attention to the tendency of language to create psychological objects (in this case, mathematics ideas and concepts), actions, social organisations and histories as neutral, definite, stabilised versions independent of people.

Thus, the focus of inquiry from a Discursive Psychology perspective is on how participants display, construct, and negotiate their understanding as they participate in interactions as made evident through their utterances. Undertaking the Discursive Psychology perspective, the study uses Bakhtin's Dialogic Theory and Garfinkel's Ethnomethodology to inform the theoretical framework and provide conceptual tools to seek answers to the study's research questions. The following sections discuss these two theoretical positionings and lay the foundation for the study's methodological framework.

3.2.1 Bakhtin's Dialogic Theory

Bakhtin (1981, 1986) based his Dialogic Theory on his distinctive account of a dialogic relationship and the concept of dialogue (Barwell, 2016a; Kazak et al., 2015; Sullivan, 2010; Wegerif, 2008). Tannen (1997) argues that for Bakhtin, language is "fundamentally interactive and grounded in context" (p. 141). Language is not merely a cultural tool whose only function is to pass on cultural knowledge from one generation to the next. Instead, it provides a dialogic space that is shared by all participants to generate meanings through engaging in dialogue. According to Bakhtin, language should not be construed as a system of abstract categories of phonemes, morphemes, words, semantics, and syntax (focused on the structural view of language) but as a worldview laden with ideology; he thus emphasises the relational understanding of language. The dialogic space implies dynamic space, where all possible meanings are taken into account in a continuum of meaning. The specification of meaning is dependent upon the preceding and succeeding dialogues (Wegerif, 2011). The dialogic space allows for a dialogic gap where meaning is realised in the process of active, responsive understanding. Responsive understanding accounts for the ways one looks for meanings in everyday life. Bakhtin asserts that "anything that does not respond to something seems meaningless to us; it is removed from dialogue" (Bakhtin, 1986, p. 145). Thus, the dialogic space allows the possibility of multiple meanings by reducing but not resolving differences in diverse speakers' perspectives. The aim of dialogic understanding is not to reach one true meaning but to appreciate the diversity of multiple meanings (Wegerif, 2013). The possibility of multiple meanings can be explored through the Bakhtinian concepts of (a) unitary language and heteroglossia, (b) double-voicedness, addressee, and speech genres, and (c) chronotope.

The concepts of unitary language and heteroglossia are presented in the following sub-section.

Unitary Language and Heteroglossia.

Negotiation of meaning takes place because of the constant struggle between unifying and diversifying language forces operating simultaneously at different levels of interaction. On the one hand, the unifying forces account for something that Bakhtin labels "unitary language" (Bakhtin, 1981, p. 269). Unitary language presupposes a system of norms that are used to decide whether the utterance falls under the category of accurate use of language. It is a theoretical construct that attempts to specify limits to divergent meanings by guaranteeing mutual understanding and crystallised/sustained meaning of the utterance (Barwell, 2012b).

Diversifying language forces, on the other hand, strive to decentralise the meaning of the utterance, thus providing ground for "heteroglossia" (Bakhtin, 1981, p. 270). Decentralisation accounts for the process through which one embeds language with one's own meanings. These forces, unitary language and heteroglossia, can be realised in the individual concrete utterance (oral or written) of participants as they become part of various human activities (Bakhtin, 1986), making utterances the unit of speech communion. Importantly, according to Bakhtin, it is the specific circumstantial context as well as the socio-cultural milieu that informs how meanings are negotiated in a particular sphere of communication (Barwell, 2016b; Blackledge & Creese, 2014).

Analysing tensions between unitary language and heteroglossia enables the development of a macro-level understanding of the social, cultural, and historical dimensions that infiltrate the dialogic space. For a geometry class, the conceptual tools of unitary language and heteroglossia afford insights into how children structure and use their utterances to state their understanding of geometry concepts. Within this dialogic space, the concept of unitary language provides avenues to explore (i) the dominant discourses existing in the geometry classroom and (ii) the processes through which these dominant discourses permeate the children's utterances. The concept of heteroglossia allows the examination of the ways in which children negotiate their understanding of shapes and their properties within the milieu of unitary language forces. Recognising heteroglossia in the classroom, therefore, enables the identification of the role of colloquial language when children articulate their understanding through utterances while appropriating it within the discursive practices of the mathematics classroom. Additionally, Bakhtin's Dialogic Theory provides conceptual tools to analyse the dialogic quality of interactions within the utterance. The unifying and disunifying forces also work at the level of words in the process of making someone else's words one's own; this is discussed in the next sub-section.

Double-Voicedness, Addressee, and Speech Genres.

When one gives someone else's words one's own values and meaning, the utterance produced becomes "double-voiced" (Bakhtin, 1994, p. 102) in nature. Importantly, the manner in which values and meanings are attributed is dependent upon the characteristics of the "addressee" (Bakhtin, 1986, p. 95).

The idea of addressee can be linked with Bell's (1984) concept of Audience Design. Bell (1984) proposed, "speakers take most account of hearers in designing their talk" (p. 159). Thus, variation in the way language is used is essentially dependent upon the addressee. That is, speakers direct their attention to who their addressee is and use a specific linguistic style to communicate effectively with that intended addressee. Bell argued that speakers shift their styles to sound like others, often because of the variation among speakers' styles on different social dimensions. For example, students speak differently from teachers. He further stated that the shifts in speakers' utterances are open to evaluation by speakers from the same social group.

Bakhtin argued, moreover, that the relationship between the speaker and the addressee highlights the active role of the *other* in the process of speech. That is, a word is always oriented toward an addressee, whether physically present or not. Thus, the author or speaker takes into account all possibilities of anticipated response to the utterance and formulates the utterance accordingly. Therefore, language is not a neutral or transparent medium through which the meaning is conveyed unchanged; rather, language is populated with the intentions and values of the other. Hence, the word is always "half someone else's" (Bakhtin, 1981, p. 293).

To formulate the notions of addressee and double-voicedness in utterances, Bakhtin (1986) talked about "speech genres", which are "*relatively stable types* of utterances" (emphasis added in original, p. 60) and are specific to a particular sphere of life in which those utterances are used. Genres are often perceived as functional varieties or styles of language-in-use that are associated with certain kinds of speakers and situational contexts (Bauman, 2006; Maybin, 2008). This understanding of genre assumes a monologic perspective to the type of utterance. However, Bakhtin (1986) provided a dialogic approach to speech genres by underscoring the role of the addressee in the construction of an utterance addressed to somebody in the conversational context which defines the speech genre of that utterance. Speech genres reflect a speaker's ideology in terms of the participation role that they assign to their addressee through their use of common expressions (Joyner, 2018).

Moreover, for Bakhtin (1986), speech genres reflect specific conversational conditions at a particular moment of time as well as reveal the specific intention or action orientation of the speaker in that moment within the conversational space (Bakhtin, 1986; Sullivan, 2012). In other words, speakers use a specific kind of speech genre depending upon the social action they engage in. For example, in criminal court proceedings, the defendant makes use of justifications as one kind of speech genre when defending their actions. Sullivan (2012) argued that speech genres are important for understanding how the speaker brings "intonation or emotional attitudes" into speech (p. 109). Thus, speech genres account for the overall composition of the utterance with content, style, and intention embedded into it (Bakhtin, 1986; Sullivan, 2012). Bakhtin (1986) also emphasised the boundless possibility of speech genres:

> The wealth and diversity of speech genres are boundless because the various possibilities of human activity are inexhaustible, and because each sphere of activity contains an entire repertoire of speech genres that differentiate and grow as the particular sphere develops and becomes more complex. Special emphasis should be placed on the extreme heterogeneity of speech genres (oral and written) (Emphasis added in original, p. 60).

Therefore, speech genres provide an organising framework for participants that outlines the nature of their participation in a particular communicative interaction. In other words, speech genres display the participant's action through their utterance in anticipation of a response from an addressee. In a classroom context, these actions may include, but are not limited to, declaring knowledge, providing justification, and giving explanations and arguments. These actions solidify over time as the communicative environment of a classroom.

Bakhtin (1986) further argued that there are primarily two significantly different kinds of speech genres: primary and secondary. Primary speech genres are the simple genres used in everyday conversations. However, when these primary speech genres are adopted and shaped according to a context, and become an integral part of a specific context, these primary genres combine with others to create complex secondary speech genres. For example, the act of formulating an argument in everyday life (a primary genre) may become part of the mathematics classroom, creating argumentation as a secondary speech genre where students are expected to provide either logic, or evidence to support their claim.

The concepts of double-voicedness, addressee, and speech genres discussed in this section provide avenues to explore the dialogical processes of everyday interactions in geometry classrooms. These concepts will enable the investigation of (i) the processes through which the addressee influences the discursive constructions of children, and (ii) how children appropriate others' words as their own to develop their understanding of geometry concepts.

These concepts, outlined earlier, provide scope for critical analysis of language-in-use with focus on both the *said* and *unsaid*. To integrate the discourse and interaction level processes that influence the construction of meaning and its realisation in utterances, Bakhtin proposes the concept of the chronotope (Bakhtin, 1994, p. 18).

Chronotope.

Bakhtin (1981) defined a "chronotope" as an "intrinsic connectedness of temporal and spatial relationships" (p. 84). However, this definition of a chronotope is quite ambiguous and open to several interpretations based on what is understood by "forms of the most immediate reality" (Bakhtin, 1981, p. 85). For example, Brown and Renshaw (2006) conducted a chronotopic analysis of classroom interactions in one Year 7 (12 to 13 years) classroom in Brisbane, Australia, to elicit students' positioning as actors and authors in their learning. Brown and Renshaw (2006) suggested that a chronotope acts as a "spatiotemporal matrix" (p. 248) that defines the temporal and spatial situatedness of a participant's action. They found that children created a chronotope of participatory pedagogy by negotiating meanings of different words and defining their role as authors, in resistance to the already established traditional pedagogy chronotope of the classroom where the student's role is limited to that of passive recipient of official knowledge that is "immutable, unchanging and given by an authority" (p. 252).

In 2014, Renshaw proposed another interpretation of chronotope in terms of classroom chronotopes, to visualise the classroom context during different historical periods. He argued that different classroom contexts have been privileged at different times as a result of changing educational policies and the changing patterns of classroom activities. These conceptions include: (i) classrooms as factory time/spaces, (ii) classrooms as individualistic, inventive time/space, (iii) classrooms as self-regulated time/spaces, (iv) classrooms as relational time/spaces, and (v) classrooms as trading time/spaces. These different classroom chronotopes or times favour different participation roles for teachers and students.

Similar to Brown and Renshaw's (2006) and Renshaw's (2014) interpretations, Blommaert (2015) also interpreted chronotopes as "timespace frames" (p. 109) that can be invoked (or reminded) through the discourse and can define different aspects of the context, in terms of plot (what can happen and how), and actors (who can act and how). These timespace frames are invokable chunks of history that can define a set of assumptions about how people can act within a specific context in which they participate. Blommaert (2015) argued that this understanding takes account of a person's biographical as well as collective time.

However, Rosborough (2016) argued that these interpretations of chronotope have adopted a macro perspective of chronotope by either presupposing a shared understanding of time (e.g., Brown & Renshaw, 2006), or viewing time in terms of a thick segment of chronological time and space (see, Renshaw, 2014). She argued that these interpretations ignore the *quality* aspects of moments from the autobiographical time of participants that may influence the meaning-making process in conversational contexts by focusing on the classroom discourse.

Focusing on the quality aspect of experience from an individual’s perspective, Bemong and Borghart (2010) interpreted chronotope as epistemological in nature, and argued that chronotopes allow individuals to perceive or understand everyday reality by focusing on *time* as a constitutive dimension of their most immediate reality. On the same lines, Holquist (2010) suggested that one can explore chronotopes by investigating language for versions of time-space embedded in one’s utterances, with the question “for *whom* are time-space distinctions relevant?” (Holquist, 2010, p. 29). Thus, in this thesis, aligned with this study’s Discursive Psychology perspective, moments during which participants explicitly made reference to time by using words (such as “when”, “at home”, etc. from past or future) that denoted a particular time and space and thus displayed the emergence of new meanings with quality characteristics, were explored as Chronotopic Moments.

Dialogic Theory has been chosen for this study because it allows the exploration of the dialogical tensions at various levels of utterances that arose as the children developed and transformed their understanding of 2D shapes, 3D shapes, and their properties in a multilingual geometry classroom using the concepts outlined earlier. Additionally, Petrilli (2008) states that Bakhtin’s approach provides a critical lens to observe the dialogical confrontations of different languages operating in a multilingual context.

Dialogic Theory, therefore, aids in the examination of the processes through which socio-political-historical dimensions of language permeate day-to-day interaction and influence the children’s meaning constructions in multilingual mathematics classrooms (Morrell, 2004). The concepts of unitary language, heteroglossia, double-voicedness, addressee, speech genres and chronotope will help to elucidate the forces that shape the utterances that influenced the children’s sense-making processes about shapes and their properties.

However, the investigation of dialogical tensions using the concepts of unitary language and heteroglossia, double-voicedness, addressee and speech genres, and chronotope requires a detailed description of the classroom conversations that took place in the whole-class and group interactions. Thus, Ethnomethodology forms the second pillar of the theoretical framework adopted for this study, which is presented in the following section.

3.2.2 Ethnomethodology

Harold Garfinkel (1967) developed Ethnomethodology as an approach to investigate the properties of the indexical aspect of language-in-use and practical actions achieved through language as ongoing accomplishments of everyday life. Ethnomethodology is the study of how reasoning and activities are organised, within the limits and resources of a culture, as rational, identifiable events and occurrences (Heap, 1984). The focus of the inquiry in an ethnomethodological study is the everyday way of social life with an emphasis on the orderliness of the activities. Ethnomethodological research aims to provide a detailed description of how members make sense of any event as it unfolds in its mundane manner (Livingston, 1987). The entire intent of the ethnomethodological approach is the exploration and description of social events from the participants' perspectives in a non-judgemental manner (Livingston, 1987; Psathas, 1977). Thus, researchers position themselves within the zone of "ethnomethodological indifference" (Garfinkel & Sacks, 1986, p. 166) and keep notes about their beliefs, values, and ideological positions.

Garfinkel and Sacks (1986) emphasised the circumstantial unfolding of an event which identifies the common norms underlying any practical action that is "reflexively constitutive of the activities and unfolding circumstances to which they are applied" (Heritage, 1984, p. 109). The reflexive accountability of the action marks the practical reasoning that allows the construction of a world where the actors' actions are taken as accountable, intelligible, and sustainable in the course of development. This reflexive accountability of everyday actions also takes into account the deviations that occur from those normative actions.

In addition to accountability, Garfinkel and Sacks (1986) argue that the reflexivity of participants' language use can be investigated using indexical properties of natural language. Indexical properties draw our attention not only to what is said but also to how it is being said. The characteristic of how something is said calls for an understanding of language as practical action. Garfinkel (1996) argues that the understanding of an utterance is contingent upon the context in which it is being said. The context involves the background information about who made the utterance, where and when, in addition to what has been accomplished by making that utterance in light of other alternative utterances that could have been made (Heritage, 1984). Greiffenhagen and Sharrock (2006) state that a clear and detailed ethnomethodological account of activities provides concrete evidence through which the cultural practices are displayed in the conversations as the participants engage in interactions. Ethnomethodological research identifies the utterance as the starting point of analysis to investigate any natural phenomenon, in alignment with Bakhtin's Dialogic Theory. Therefore, the ethnomethodological approach is concerned with how people achieve interpersonal understanding through language (Francis & Hester, 2004). Within the ethnomethodological approach, speaking is considered a practical action capable of transformation.

For the present study, the moment-by-moment elicitation of practices, methods, and ways of sense-making used in Ethnomethodology provided opportunities for a micro-level analysis of the everyday working of teaching and learning of geometry. It was anticipated that the detailed ethnomethodological description would be help reveal the relevant instances of classroom interaction relevant to geometry concepts within the ordinary procedures and processes of the geometry lessons through which children were building their understanding of 2D shapes, 3D shapes, and their properties in a geometry classroom. A detailed description of the teaching and learning of geometry would allow the analysis of the different ways in which the children organised and displayed their understanding of shapes and their properties. Ethnomethodology, therefore, will be useful in developing a micro-level analysis of the everyday activities as it takes place during the teaching and learning of 2D shapes, 3D shapes, and their properties.

As the focus of the study is to develop a thorough understanding of how children negotiate their understanding of 2D shapes, 3D shapes, and their properties in a multilingual New Zealand classroom, it was crucial to identify an appropriate methodological framework aligned with the theoretical framework. The following section presents the details of the methodological framework developed for this study.

3.3 Methodological Framework

To seek answers to the research questions within the critical inquiry paradigm (see Section 3.1) and the theoretical framework (section 3.2), this section presents the study's methodological framework. The first sub-section presents the research design (see Section 3.3.1), followed by a description of data-gathering procedures and tools in section 3.3.2. I then offer insights into the researcher's positioning as a non-participant observer during the data-gathering procedure (see Section 3.3.3).

3.3.1 Research Design

This study focuses on exploring children's negotiation of meanings about geometric shapes and their properties in a New Zealand multilingual classroom. A qualitative research design

(Creswell & Creswell, 2018) was employed. The focus of qualitative research lies in the study of the natural environment with an emphasis on processes through which people make sense of their surroundings and experience the world (Denzin & Lincoln, 2018). Qualitative research thus highlights people's own accounts of meaning construction, the way people talk about their surroundings, how and what they say, and how they choose and formulate their words (Merriam & Tisdell, 2016). In addition to this, qualitative research enables the researcher to problematise the understandings that participants convey to each other and the processes through which they build and negotiate those understandings (Edwards & Mercer, 2013). Thus, following a qualitative research approach allows the exploration of a phenomenon as it unfolds by providing a detailed description rather than predicting a cause and effect relationship among various dependent and independent variables (Lincoln & Guba, 1985; Maykut & Morehouse, 2003). Lemesianou and Grinberg (2006) state that qualitative research must critically evaluate the data and allows us to unveil ideas and meanings that are subjugated or oppressed by dominant ideologies. The purpose of this study is to unravel the dominant and non-dominant meanings attributed to shapes and their properties. The study is an attempt to bring non-dominant meanings about shapes and their properties from the periphery of broader mathematical discussion to the centre, making the research transformative in nature (Creswell & Creswell, 2018).

The next section discusses procedures and tools that were used to gather relevant data for the study.

3.3.2 Data-Gathering Procedures and Tools

The section provides details of the (i) phases of data gathering, (ii) participants, and (iii) tools employed for gathering data. The data were gathered in two phases: (i) the Pilot Study, and (ii) the Main Study. Table 3.2 presents the summary of data-gathering procedures undertaken for the Pilot Study and the Main Study.

Table 3.2

Data-Gathering Procedures

Phases	School/ Lessons observed	Participants	Data-gathering tools	Participants' time requirement
Pilot Study	School A/ 3 lessons observed	Total children: 18 Multilingual children: 10 Monolingual children: 8 Teacher participant: 1	1. Classroom observation protocol and fieldnotes. 2. Semi-structured interview with the respective teacher. 3. Focus group interview with two groups of children, with five children per group. 4. Short questionnaire filled by the teacher for collecting demographic data about children (see Appendix J).	1. Three lessons observed. Each lesson lasted for 50 minutes. 2. One semi-structured interview (20 minutes long) with the teacher: 20 minutes. 3. Two focus group interviews with children: each focus group interview took 20 minutes.
Main Study	School B/ 6 lessons observed	Total children: 15; Multilingual children: 9; Monolingual children: 6; Teacher participant:1	1. Fieldnotes (n=6) 2. Audiovisual recordings of geometry lessons (n=6) 3. Three semi-structured interviews (see Appendix C) with the teacher. 4. Four focus group interviews with the children (n= 4) (see Appendix D) 5. Documents: The New Zealand Curriculum; teacher's unit plans, and children's worksheets. 6. Short questionnaire (See Appendix J) filled by parents for collecting demographic data about children.	1. Six lessons observed and audiovisually recorded. 50 minutes per lesson. 2. Three semi-structured interviews with the teacher: 10-15 minutes for each interview. 3. Four focus group interviews with children. 20 minutes for each interview.

The Pilot Study was conducted to trial data-gathering procedures, tools, and the data analysis framework. The following section discusses the rationale for conducting a Pilot Study and my reflections on the data-gathering and data analysis procedures.

Pilot Study.

The primary purposes for conducting the Pilot Study were two-fold. Firstly, the Pilot Study enabled the trialling of the data-gathering and analytical tools (as explained in Section 3.5.1) before undertaking the Main Study. Secondly, being new to New Zealand, I was able to use the Pilot Study to gain an understanding of New Zealand primary school classrooms and the associated educational philosophy. Observing lessons during the Pilot Study enabled me to develop an understanding of the teaching-learning process with respect to The New Zealand Curriculum (NZC) learning area of Mathematics and Statistics. The Pilot Study also alerted me to (a) any complexities that might arise during the consent-taking process, and (b) the ethical considerations that needed to be taken into account during the Main Study. In the following sections, I describe the participants, the data-gathering tools, and my reflections and learnings from the Pilot Study that informed the data-gathering procedure adopted for the Main Study.

Participants. One Year 5/6 teacher and 18 children (out of 25 children in the class) consented to participate in the research. There were three kinds of participants: (i) multilingual children (n=10), (ii) monolingual children (n=8), and (iii) the Year 5-6 teacher. Table 3.3 shows the home languages of the multilingual children who participated in the Pilot Study (as gathered through a small questionnaire filled by the teacher for each child, see Appendix J).

Table 3.3

Home Languages of the Multilingual Children in the Pilot Study

Year/School	No. of children	Pseudonyms	Home languages
Year 5/6, School A	5	Yu, Bo, Chen, Jiang, Lim	Chinese
	1	Amir	Arabic
	1	Adhi	Tamil
	1	Eiko	Japanese
	1	Ajayi	Afrikaans
	1	Atul	Bengali

Data-Gathering Tools. In addition to the questionnaire, data were gathered through: (i) classroom observation, using Gleason, Livers, and Zelkowski's (2015) *Mathematics Classroom Observation Protocol*; (ii) fieldnotes; (iii) one semi-structured interview with the teacher; and (iv) two semi-structured focus group interviews with two groups of children.

The Year 5/6 class in the Pilot Study was observed over three consecutive days. Three lessons on geometric shapes and their properties were observed. After the lessons were observed, two focus group interviews were conducted with the children, who provided consent for their participation in the focus group interviews. In the first group, there were five children (English: 1, Arabic: 1, Chinese: 3). In the second group, there were six children (English: 3, Tamil: 1, Chinese: 2). The interviews focused on exploring children's understanding of 2D and 3D shapes. They talked about their discursive construction of 2D and 3D shapes, and how these shapes are similar to and different from others. A semi-structured interview was conducted to gather the teacher's perspective on the children's learning of shapes and their representations. The focus of the interview was to elicit the teacher's ideas about assigning children to different groups and the tasks chosen for teaching the concept of shapes and their properties. The semi-structured interview with the teacher also addressed the teacher's interpretation of the discussion that had taken place during the observed lessons.

Reflections on the Data Gathering Procedure. The trialling of my proposed data collection during the Pilot Study enabled me to make two decisions for the Main Study. Firstly, I decided that using fieldnotes was more efficient than using the observation protocol. Having a fieldnotes journal helped me to jot down details of any in-moment conversations that I found relevant. Moreover, I could jot down details in the form of quickly sketched diagrams. I realised that the classroom observation protocol that I had adopted for the Pilot Study was too lengthy, and that in order to fill in the details that the observation protocol required I had missed some of the relevant classroom in-moment details. Secondly, I realised the importance of using video to capture participants' body language during their meaning-making. I noticed that a lot of children used their gestures to represent their understanding of the shapes that they were referring to.

The semi-structured focus group interviews with two groups of children primarily enabled me to practise my interviewing skills with children. For example, instead of asking, "How can you describe this shape [an object like a Jenga piece]?" I reworded it as "What can you tell me about this shape [object]?". The trialling of the semi-structured interview schedule for the focus group interview helped me be mindful of the words that I chose to elicit the children's understanding of shapes. The purpose of the focus group interviews was to identify the nature of any unverbalised content underlying the conversation that takes place when multilingual children interact with other children in a discussion. During the focus group interviews, I also noticed that children used not only different phrases to represent their understanding of shapes and their properties during these group interviews; there were other, more subtle kinds of utterance that drew my attention. For example, during focus group interviews, children sometimes paused and used a whispery voice to state their understanding. The subtle use of prosody became one of the focuses of my analysis for the Main Study data.

The semi-structured interview with the teacher was conducted for approximately 20 minutes. In terms of procedures for conducting the semi-structured interview, the Pilot Study enabled me to refine my interviewing skills with the teacher. During the interview process, I realised that I was continually switching between my researcher and teacher identities. For example, during the teacher interview, I provided my judgement about a child's understanding and expected the teacher to agree with my statement. The transcription of that interview allowed me to see the shifts between my researcher and teacher identities. This realisation helped me polish my interviewing skills and be constantly aware of the tension between my two identities. Having completed the Pilot Study, I made a conscious effort during the Main Study data collection to step back and listen to the teacher as she talked about her experiences of teaching geometry concepts to a diverse student population.

Reflections on the Data Analysis. One of the focus group interviews was used to trial the data analysis framework developed for this study and make any needed amendments to the data analysis framework before conducting the Main Study. The interview data were analysed at two levels, micro-level and macro-level. At micro-level analysis, a segment of audio-recorded focus group interview data was analysed using the Conversation Analysis (CA) techniques. For this purpose, I used V-Note software (details are provided in Section 3.4). The audio-recorded data were transcribed using Jefferson's CA transcription conventions (Jefferson, 2004). Repeated reading of the transcript enabled me to note different discursive constructions that children used to state their understanding of shapes. For example, a child stated that "square and cube are same; it just 3D" (Focus Group Interview 1, School A). It was noted that the children used three different terms (square, 3D square, and cube) to signify the same shape (cube). Interestingly, the children agreed that all these terms signified the same geometry shape. There was a collaborative understanding of the geometry shape as they proceeded through the conversation. Thus, the analysis of the Pilot Study data provided me with concrete evidence about what and how the meanings of 2D shapes, 3D shapes, and their properties were constructed through talk. These tangible pieces of evidence were then analysed to explore the heteroglossia and unitary language forces (Bakhtin, 1981) that resulted in the use of three different terms to denote the same shape.

The CA of the focus group interview segment also revealed that children displayed their confidence and doubt through subtle, nuanced ways of speaking, including stretching of words, emphasising, overlaps, and intonations. The prosodic aspect of the language could not be accessed without these CA transcription markers. Importantly, the transcribing process revealed that not all notations were relevant to my research questions; therefore, a simplified version of Jefferson (2004)'s CA guide was developed to transcribe the chosen instances for the Main Study (details are provided in the Main Study).

Main Study.

With insights from the Pilot Study, a few changes were employed in the procedure for gathering data for the Main Study. Firstly, audiovisually recorded data were prioritised for analysis, and segments for in depth analysis were selected from audiovisual data. Second, revised semi-structured interview schedules for teacher interviews and student focus group interviews were used. The Main Study data were gathered at a different Year 5/6 class in another New Zealand School (See Table 3.2). The following section provides details on (i) the participants and (ii) the tools used to gather data for the Main Study.

Participants. One Year 5/6 teacher and 15 children consented to participate in the research. There were three kinds of participants: (i) multilingual children (n=9), (ii) monolingual children (n=6), and (iii) the Year 5/6 teacher (n=1). Table 3.4 shows the home languages of the multilingual children who participated in the study.

Table 3.4

Home Languages of the Multilingual Children in the Main Study

Year/School	Number of children	Pseudonyms	Home languages
Year 5/6, School B	1	Ozan	Somali
	2	Tahi, Kimi	Tongan
	4	Matua, Zara, Tane, Nikau	Māori
	1	Yue	Chinese
	1	Garry	Filipino

Data-gathering tools. The purpose of the data-gathering process was to provide a detailed description of the teaching and learning of 2D shapes, 3D shapes, and their properties in the natural setting of the geometry class. In total, six lessons were observed in the Year 5/6 class (see Table 3.2). The study involved six different data-gathering tools. These were: (i) fieldnotes, (ii) audiovisual recordings of classroom and group interactions, (iii) semi-structured interviews with the teacher (audio-recorded), (iv) semi-structured focus group interviews with children (audio-recorded), (v) documents in the form of children's worksheets and the teacher's unit plans, and (vi) a short questionnaire. The combination of these different data tools enabled me to gather rich data.

Fieldnotes. Observation provides first-hand information about people and research sites. Fieldnotes are detailed descriptions of observations and interactions in the field that are kept as a chronological log (Emerson et al., 2011). Fieldnotes allowed me to observe the phenomenon as it occurred in the natural setting by taking down the notes in real time. For the Main Study, six geometry lessons on shapes and their properties were observed in School B. I assumed the position of the non-participant observer (Gray, 2014) and attempted not to influence the classroom events while acknowledging that my mere presence might have influenced some aspects of the classroom teaching and learning space. Importantly, taking fieldnotes requires good listening skills and careful attention to visual details (Creswell & Creswell, 2018) in order to note down "what" and "how" phenomena are unfolding (Punch & Oancea, 2014). Detailed description is achieved in the fieldnotes by noting down as many details as possible while carefully listening and writing down keywords. These jotted fieldnotes are later developed in expanded descriptive notes after each of the six lesson observations. Fieldnotes include a full description of settings and events along with the researcher's analytic ideas, inferences, memos, personal feelings, and reflections (as suggested in Bailey, 2018).

Audiovisual Recording of Classroom and Group Interactions. As discussed in Chapter 2 and reflected in the Pilot Study, non-verbal cues such as gestures, body movements, pointing, and facial expressions are of particular relevance in revealing mathematical thinking and meaning construction (see, e.g., Díez-Palomar & Olivé, 2015; Hwang et al., 2010; Nemirovsky et al., 2012). I used camera and audio recorders to capture video and audio data. Video recording makes it easier to revisit the "close to reality" (Otrel-Cass, 2018, p. 100) episodes of research sites to access the delicate, minute yet crucial details for examination and thus makes it a valuable data-gathering tool. As I attempted to discern the process of sense-making of geometric concepts by the children in a multilingual context, analysis of verbal and non-verbal communication used among children in whole-class and group settings as they worked on assigned tasks became pivotal. Video recordings provided me with many opportunities to explore the complex dynamism of the ongoing activity from diverse perspectives in order to produce careful, precise, and consistent analysis (Klette, 2009). A “polyphonic approach” (White, 2016, p. 4) to video recording was helpful to record two streams of data – audio and visual in real time from different perspectives. Two directional cameras were used to video-record the whole-class and group interactions in the Year 5/6 classroom at School B. I also used an eye-gear with an inbuilt camera and voice recorder to record any event in the classroom that caught my attention at the moment. Having this device made it easier to attend to an interesting event as it happened, which might not have been possible otherwise. In addition to these, five audio/voice recorders were kept on tabletops to record children’s talk-in-interaction in group settings. These four to five recorders helped to record the interactions of all groups.

Semi-Structured Interview with the Teacher. A semi-structured interview was conducted with the Year 5/6 teacher (School B) after every second lesson was observed. Three interviews were conducted in total. The interviews were short (15-20 minutes each) and were audio-recorded. A semi-structured interview schedule was used to conduct these interviews (see Appendix C). The purpose was to seek clarification regarding the grouping of children, the structuring of the unit/lesson, and tasks and materials/manipulatives used. The semi-structured interview also helped me to explore the teacher’s interpretations of settings, tasks, and purposes of the tasks for the observed lesson (Punch & Oancea, 2014). The interview transcripts were later sent to the teacher for member checking (Denzin & Lincoln, 2018).

Focus Group Interviews with the Children. The focus group interviews for exploring children’s understanding of shapes and their properties and their meaning-making processes recorded further children’s interactions in a group setting (Ho, 2006). Focus group interviews were conducted with four groups of children, using a semi-structured interview schedule (see Appendix D). Four focus groups interviews were conducted in total. The interviews were audio-recorded and transcribed for analysis. The focus group interview setting allows participants to voice their understanding in a comfortable environment (Cohen et al., 2018), and also provides a researcher with a quick and cost-effective way to gather data (Nuttavuthisit, 2019).

Questionnaire. A questionnaire consists of a structured set of questions that generate responses from the participants (Newby, 2014). In this study, a short parent questionnaire (see Appendix J) was used to generate information about the languages that the participants used at home or school, and find out how long they had been in New Zealand.

Documents. Different sets of documents were collected and thematically analysed for this study. These included the teacher's unit plan, resources used by the teacher to develop the unit plan, children's worksheets, pictures, and drawings and other classroom artefacts. Additionally, The NZC (Ministry of Education, 2007) was used to provide background information relevant to the teaching of mathematics, specifically geometry, at Levels 2 and 3 (Year 5/6). The NZC and the resources used by the teacher for teaching and learning of shapes provided evidence in terms of assumptions and beliefs held by different stakeholders, and expectations in teaching and learning of geometry in New Zealand classes at the targeted level.

3.3.3 The Researcher as Non-Participant Observer

As the research study is about the processes taking place in the natural setting of a classroom, I, as a researcher, assumed the position of the non-participant observer. Non-participant observation implies that I was not part of the teaching and learning process (Gray, 2014). As far as possible, I maintained a position of detachment and independence from the participants. The aim was to observe classroom practices in their natural setting with as little disturbance as possible. I strived not to influence participants' behaviours or any classroom practice. In addition, I tried to situate my non-participant observations with "ethnomethodological indifference" (Garfinkel & Sacks, 1986, p. 166). An ethnomethodologically indifferent position implies that the researcher's effort is put into describing participants' accounts of practices and methods employed rather than judging the "adequacy, value, importance, necessity, practicality, success, or consequentiality" (Garfinkel & Sacks, 1986, p. 166) of participants' accounts.

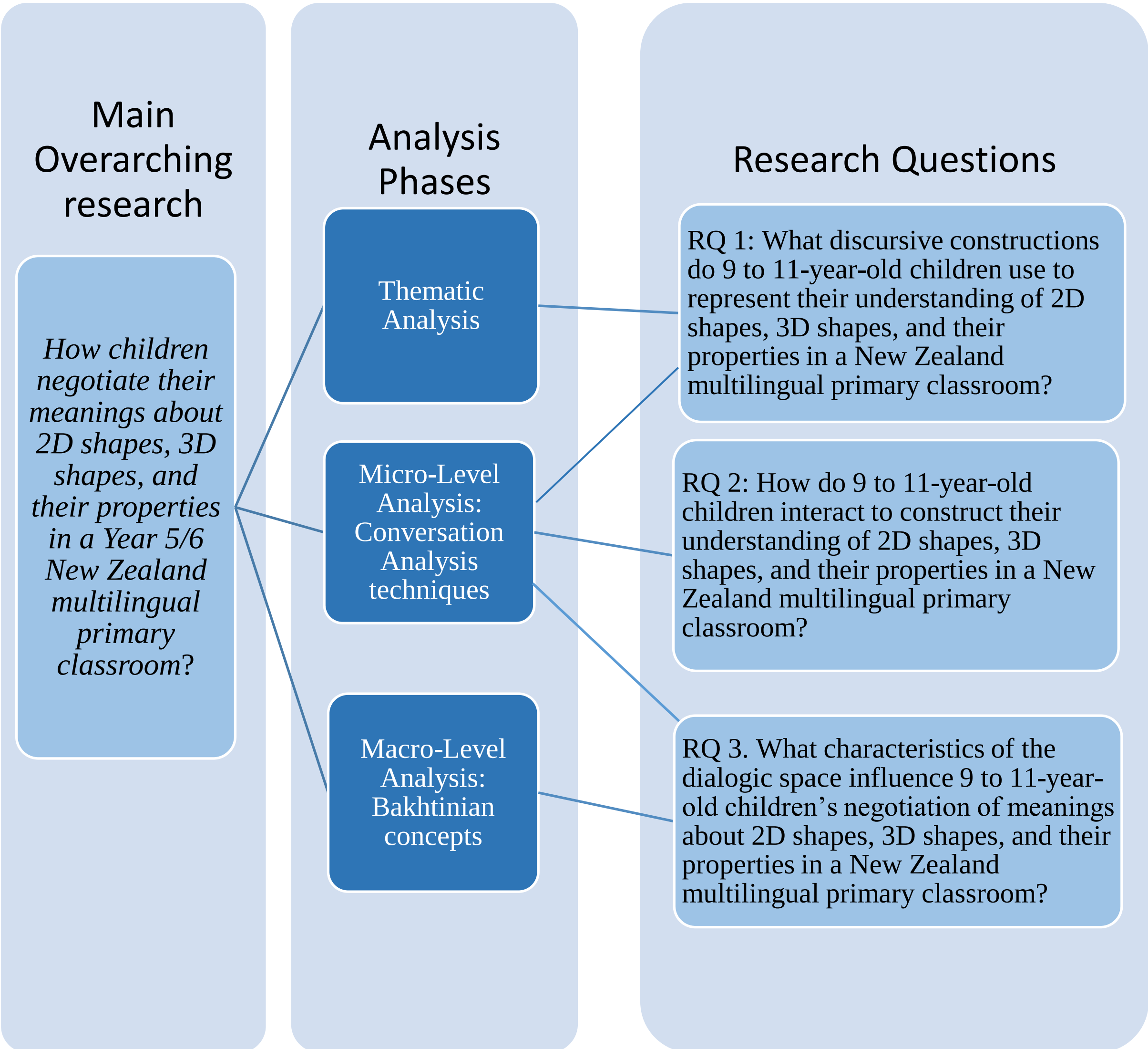
In addition to being a non-participant observer, I strived to be reflexive during the data collection and data analysis process in order to maintain the objective stance which is crucial in qualitative research (Creswell & Creswell, 2018). Being reflexive requires the researcher to engage in the process of critically reflecting on any of their own assumptions, biases, experiences and identities that might influence the process of research in any way. These values, beliefs, biases, and prejudices inform the choices a researcher makes about reporting and analysis of data (Creswell & Creswell, 2018). Thus, along with an "ethnomethodological indifference" stance on my part as a researcher, I maintained a reflective journal to document my assumptions, biases, prejudices, and value judgments that might influence data gathering and analysis. The next section presents details on how the gathered data were analysed to answer the research questions.

3.4 Data Analysis Framework

The study presents a critical exploration of the processes by which Year 5/6 children negotiate their understanding of geometric shapes and their properties in a New Zealand multilingual primary classroom. The theoretical framework (see Section 3.2) guided the data analysis framework developed for this study to seek answers to the research questions. Following the theoretical framework, participants' utterances were taken as the unit of analysis, and the analysis framework was followed to analyse the data. Figure 3.1 shows how the data analysis framework helped me to seek answers to the three research questions.

Figure 3.1

Data Analysis Framework



Bakhtinian concepts of heteroglossia and unitary language, addressee, double-voicedness, speech genres, and chronotopes (as explained in Section 3.2.1) were used to critically examine characteristics of dialogic space that influence children's negotiation of meanings concerning shapes and their properties. The analysis of data using these concepts required a detailed description of the whole-class and group conversations. Conversation Analysis (CA) was used as a tool to develop a detailed description of the data to enable thorough exploration of the participants' (both the teacher and children) conversations within whole-class and group interactions.

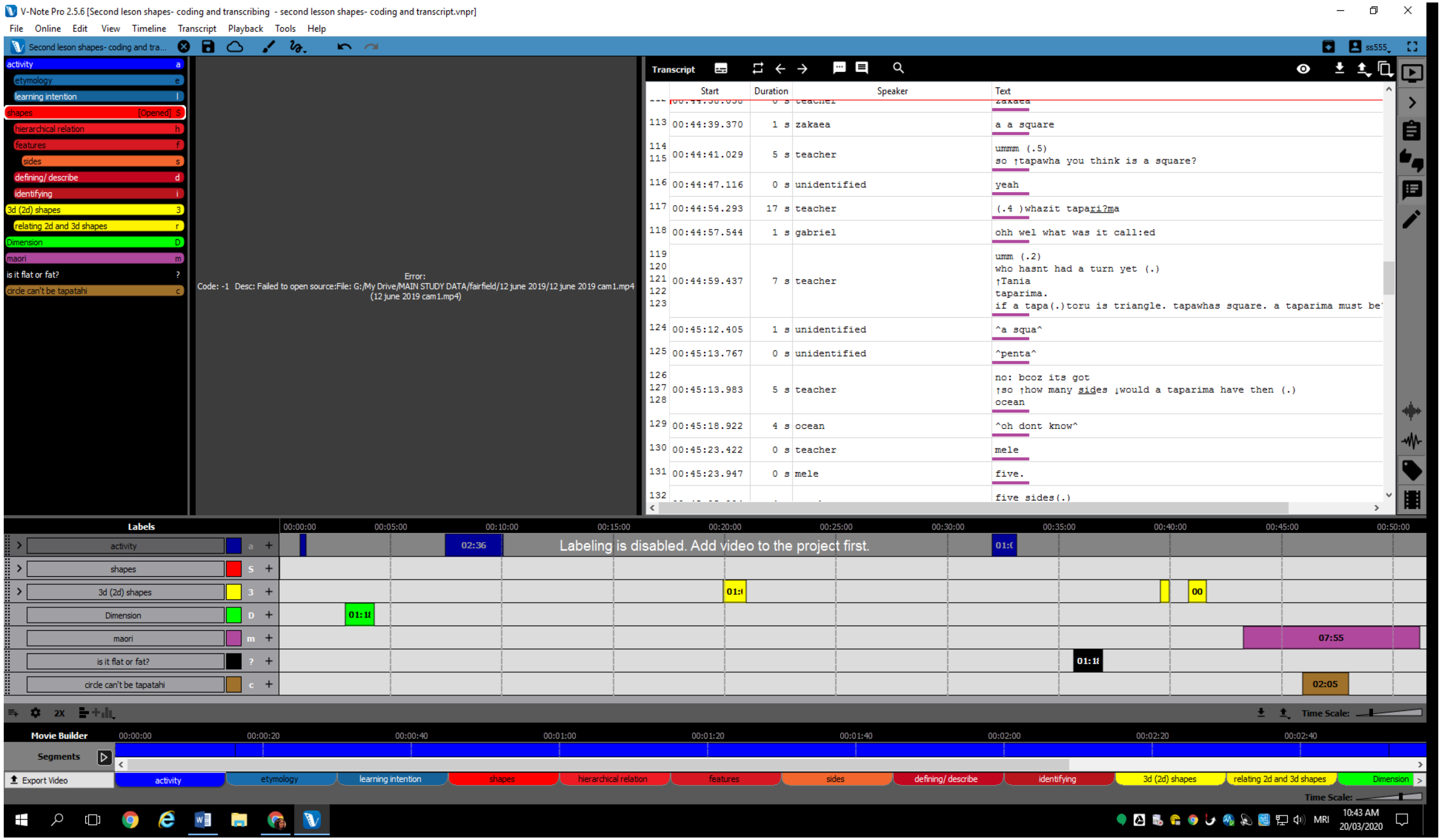
In order to manage the process of data analysis in a given time for this study, the analysis was completed in three phases. First, thematic analysis of the data from different data sources was conducted to identify relevant Key Moments (see Section 3.4.1). Key Moments are those specific moments in the audiovisually recorded lessons during which participants displayed their understanding of shapes and their properties as they participated in whole-class and group interactions. The identified Key Moments were, then, subjected to micro-level and macro-level analysis. The micro-level analysis formed the second phase of analysis (see Section 3.4.2) and used CA (Sacks, 1974) to develop a careful analysis of the selected Key Moments. Thematic analysis and micro-level analysis helped me to respond to the first and second research questions and provided the foundation for the macro-level analysis. The third phase of analysis, macro-level analysis, enabled me to explore the characteristics of the dialogic space of a New Zealand multilingual classroom that might influence children's negotiation of meanings about shapes and their properties (i.e. RQ3) in the micro-conversational space of whole-class and group interactions (see Section 3.4.3). The macro-level analysis, with a few insights from the micro-level analysis, enabled me to seek answers to the third research question. In the following section, I present a description of the thematic analysis that was followed to identify the relevant Key Moments.

3.4.1 Thematic Analysis

Data from audiovisually recordings of six observed lessons, fieldnotes, three semi-structured interviews from the teachers, four focus group interviews with children, along with documents (children's work samples, teacher's unit plan, and The New Zealand Curriculum) were analysed using thematic analysis. Thematic analysis allowed me to capture interesting features of data (Clarke & Braun, 2017) as relevant codes for further exploring ideas pertaining to 2D shapes, 3D shapes and their properties. Data were viewed multiple times for coding with a focus on the content relevant to geometric ideas about shapes and their properties. For audiovisually recorded data, the V-note software was used. Figure 3.2 shows an example of the coding process in the V-note software.

Figure 3.2

Coding of Audio and Video Recorded Lessons



The iterative process of coding enabled me to identify five topics of interest, discussed as themes for coding classroom interactions in which shapes and their properties were being identified. These themes are: (i) making sense of 2D shapes, (ii) making sense of 3D shapes, (iii) relating 2D shapes with 3D shapes, (iv) mathematical construct of dimension, and (v) naming shapes in Te Reo Māori.

For the purpose of managing the analysis of the data, two Key Moments for each theme were selected for intensive analysis at micro- and macro-level. Thus, ten Key Moments from the six audiovisually recorded geometry lessons were selected. The following sections present a detailed description of the analysis undertaken at each level.

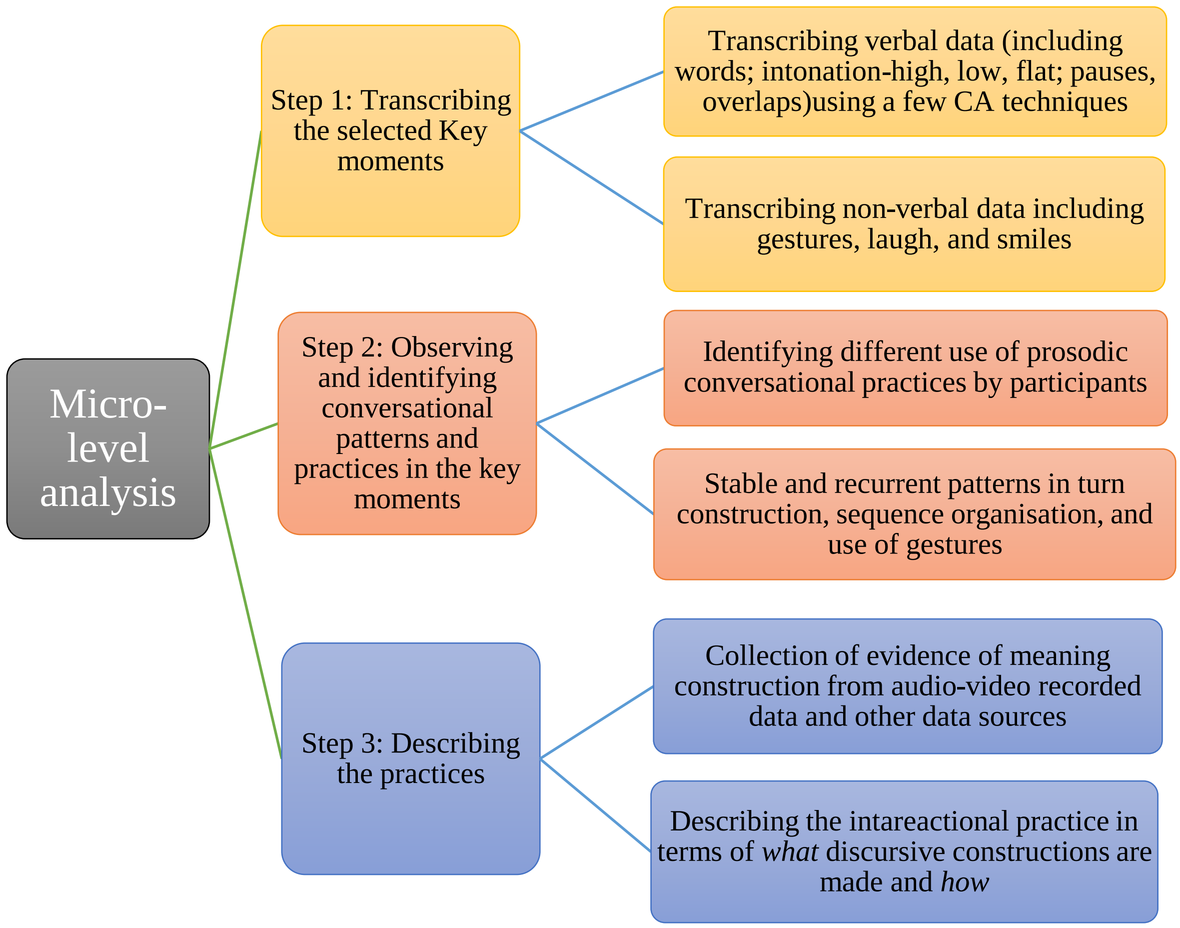
3.4.2 Micro-Level Analysis

The micro-level analysis made use of CA techniques to seek answers to the first research question (see Figure 3.3). Harvey Sacks and his colleagues at the University of California developed CA in the early 1960s (Sidnell & Stivers, 2013). Language-in-use in the everyday organisation of the social world became the focus of analysis in CA, as language-in-use in an everyday organisation helps us to gain interactional competence through the process of socialisation in a specific culture (Heritage, 1984). Goffman (1983) argued that CA can help us to illuminate the conversational patterns that are found in the interactional order of institutionalised talk, including classroom interactions. In the present study, three steps were followed for accomplishing micro-level analysis.

The first step involved transcribing identified Key Moments using selected CA techniques. In the second step, Key Moments and transcripts were examined to identify conversational patterns and practices. The final step in the micro-analysis involved describing the identified practices (see Figure 3.3).

Figure 3.3

Steps Undertaken in the Micro-Level Analysis



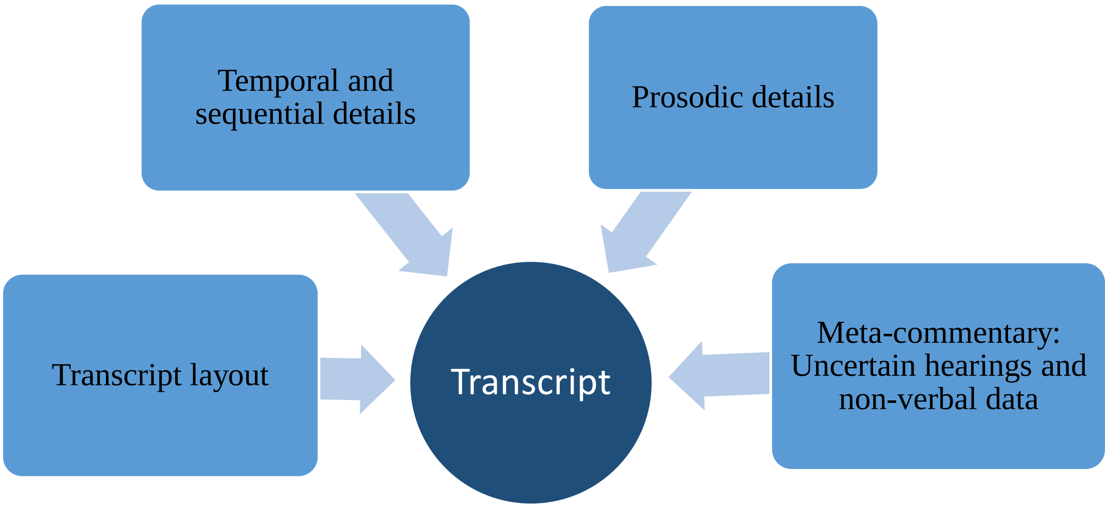
Note. Adapted from *The Handbook of Conversation Analysis*, by J. Sidnell and T. Stivers, 2013, Blackwell Publishing Ltd. (https://doi.org/10.1002/9781118325001). Copyright 2013 by Blackwell Publishing.

Step1. Transcribing Key Moments.

The ten Key Moments selected using the thematic analysis were transcribed using V-Note software. The software allows transcribing of the audio- and video-recorded data using CA transcription conventions; thus, it enabled me to transcribe data methodically, keeping track of the selected features of the interactions using selected CA techniques. The trialling of data from the Pilot Study enabled me to identify the relevant conversational markers that I needed to transcribe the ways through which participants showed their understanding of shapes and their properties, along with making sense of others' utterances. The transcripts were developed using Hepburn and Bolden's (2013) approach to indicate four different aspects of verbal data (see Figure 3.4). These aspects are concerned with *what* will be transcribed and *how*.

Figure 3.4

Process of Transcribing Selected Key Moments



Note. Adapted from "The conversation analytic approach to transcription," by A. Hepburn and G.B. Bolden, in J. Sidnell and T. Stivers (Eds.), *The handbook of conversation analysis* (pp. 57-76), 2013, Blackwell Publishing Ltd (https://doi.org/10.1002/9781118325001). Copyright 2013 by Blackwell Publishing Ltd.

The first aspect concerns the transcript layout. Transcript layout consisted of four key features. First, line numbers were used in the transcripts to signal the reader about the specific moments in interaction that are explored in the analysis. Second, speakers were identified by marking the transitions of the lines wherever relevant. Third, words were transcribed as they had been spoken by modifying spellings rather than modifying the word based on the researcher's view of what might have been intended or should have been produced (Liddicoat, 2011). For example, participants often said "kinda" instead of "kind of". The words were transcribed as they were spoken by the participant. Fourth, Courier New was used as a font for the transcripts as each of the font letters and characters occupy the same horizontal space, which made it easier to align overlapping talk and uncertain hearings as precisely as possible. Table 3.5 presents four different aspects of transcription with transcription symbols or markers and their related purposes.

Table 3.5

Simplified Transcription Guide for the Main Study

	Aspects of Transcript	Transcription symbols /Markers	To indicate	Purpose
1.	Transcript layout (Hepburn &	Line numbers	Reference points of analysis	To indicate the reader about the specific moments of interactions under analysis

	Bolden, 2013)	Speaker	Change of speaker	
		Text	Utterance of the speaker	Indicate the words as produced by the speaker
2.	Temporal and Sequential organisation of transcript	[ ]	Overlapping talk	May highlight the conversant's understanding of possible transition relevance points (Clayman, 2013)
		=	Latching	May mark the beat of silence smaller than one tenth of 1 second (Hepburn & Bolden, 2013)
		(.)	Pauses	May indicate silence for turn allocation or resolving a problem (Hayashi, 2013)
3.	Prosodic features	?	Rising intonation	May indicate suggestion, offer, invite, question, request (Ward, 2019), or to check if all conversants are following the speaker (Warren, 2016)
		.	Unit-final intonations	May indicate the completion of utterance or certainty of knowledge claim (Jeong, 2016)
		<u>underline</u>	Emphasis	May indicate the emphasis shown by the speaker (Couper-Kuhlen, 2004)
		↑	High pitch	May indicate new information (Pickering, 2009), submissiveness, or engagement, or insistence (Turnbull, 2017)
		↓	Low pitch	May indicate emotion or dominance of knowledge claim (Ward, 2019)
		°	Whispering	May indicate continuity of listening to a speaker (Hay et al., 2008b), or doubt (Reed, 2010)
		CAPITAL	Loud Voice	May indicate a speaker's confidence (Ward, 2004), the importance of knowledge claim (Ward, 2019)
		#	Creaky Voice	May indicate certainty, authority over knowledge claim, or distancing or lack of emotion (Ward, 2019)

		:	Stretching	May indicate a tendency to hold the speaking floor (Ward, 2019)
4.	Meta-commentary	( )	Uncertain hearing	To state the utterance which is not clear in the audio recording
		(( ))	Researcher's comments	To describe the contextual details and non-verbal details.

The second aspect of transcribing the data involved the transcription of temporal and sequential relationships by using conventions. First, to denote overlap onset and overlap offset of overlapping talk (by using square bracket '[ ]'). Second, to mark the beat of silence smaller than one tenth of a second through latching (=). Finally, gaps and pauses were calculated and presented in terms of lengths of silence, such as (0.2) indicating two tenths of one second.

The third aspect of transcription was concerned with the transcription of prosody in terms of intonation, volume, pitch, emphasis, speed of speech, and voice quality. The Pilot Study informed me about the ways in which participants use prosodic cues to display their intended action as well as their knowledge claims through their utterance. Therefore, participants' use of prosodic features in their utterances were transcribed using several CA techniques (see Table 3.5) in detail to account for the different invoked actions (e.g. request, offer, question, telling, among others). Table 3.5 shows how participants used different prosodic features in their discursive constructions of shapes and their properties. It should be noted that the symbols like a question mark '?' and period '.' are not used as punctuation marks in the transcripts; rather, they signal the intonation at the end of the utterance.

The fourth aspect of transcription was concerned with the meta-commentary and uncertain hearings. These were transcribed using '( )' and '(( ))'. Uncertain yet possible hearings were transcribed using '( )', and the researcher's comments were marked with double parentheses '(( ))'. The non-verbal details that include participants' gestures, smiles, laughter, body movements were also presented as comments within '(( ))' to transcribe non-verbal visual data.

Step 2. Observing Conversational Patterns and Practices.

The second step (see Figure 3.3) involved the repeated readings of transcribed data to document interactional patterns among participants' utterances/turns across Key Moments. In CA, a participant's utterance is called a turn, which is assumed to be designed to achieve a particular interactive action. Repeated observations of audiovisual recordings allowed me to develop a sense of recurrent and stable details of talk-in-interaction (Hutchby & Wooffitt, 2008). This step enabled me to explore two aspects of conversations. These are (i) turn design (which explores the action ascribed in a turn and the way a turn is constructed), and (ii) sequence organisation (which throws light on the achieved meanings and relationships among participants).

The exploration of turn-design (Drew, 2013) allowed me to explore the speaker's intended action along with the subtle cues they used to inform the listeners about their action. The turn design refers to "how a speaker constructs a turn-at-talk – what is selected or what goes into 'building' a turn to do the action it is designed to do, in such a way as to be understood as doing that action" (Drew, 2013, p. 132). A turn is an utterance by the speaker in which they employ a variety of linguistic (includes words, syntax, morphological and other grammatical forms) and paralinguistic (includes prosodic features of the pitch, the volume of voice, silence along with gestures, laughter, aspirations) features to convey the intended action. For example, a statement such as "Are you using that pen?" is not simply a question, it can also be counted as a request by the speaker. Moreover, prosodic cues are often present in the utterance to convey the intended meaning. For example, Sicoli et al. (2015) found that the initial pitch of the utterance marks the kind of action associated with the kind of question asked. That is, not all questions are in what/why form. Some questions could mark different actions, like a request. Thus, in a conversation, participants use prosodic cues to indicate "low-level coding of meaning" (Levinson, 2013, p. 113). Ward (2019) has shown that prosodic patterns may be used and interpreted differently by speakers of different languages. Hence, for the micro-level analysis, I referred to research from the sociolinguistic field to interpret the prosodic features used by multilingual children in the data presented in this thesis.

Scrutiny of the sequence organisation (Stivers, 2013) helped me to investigate how participants develop an understanding of taken-as-shared meanings in interactions. CA assumes that the utterances act as social action; thus, their position in the ongoing conversation is central to its meaning and its significance as an action (Schegloff, 2007). Analysis of conversational sequences has documented how various actions (Levinson, 2013), are performed, for example, sequences through which requests are made, granted or rejected. In CA, pairs of utterances that are strongly related and have a reflexive relationship with prior and following utterances are called adjacency pairs (Stivers, 2013). This is to say that "given the first [utterance], the second is expectable; upon its occurrence, it can be seen to be a second item to the first; upon its non-occurrence, it can be seen to be officially absent – all this provided by the occurrence of the first item" (Schegloff, 1968, p. 1083). The most common adjacency-pairs include: summons-answers, greetings-greetings, invitation-acceptance/declination, offer-acceptance/declination, request for action-grant/denial, among others (Stivers, 2013).

To explore sequence organisation in the classroom interactions, preference organisation and repair were analysed in the present study, along with the adjacency pairs. The principles of preference organisation allow the progression of interactional goals (Pomerantz & Heritage, 2013, p. 210). For example, in a classroom, a response to a teacher's question in terms of explanation or justification may be considered as a preferred response. Additionally, the teacher and other children may consider a mistake or a student speaking out of his/her turn as a dispreferred response (Ingram et al. 2016). Thus, preference organisation accounts for the recognition of the first-pair part and fulfilment of the expectation of the first-pair part in the second-pair part (Pomerantz & Heritage, 2013). For example, a question or request as a first pair-part expects an answer or a grant of request, respectively, as the second-pair part. The absence of the expected second-pair part is the dispreferred response. It has been shown that participants use different conversational devices such as hedging or withholding response before stating a dispreferred response (Pomerantz, 1984). Moreover, the analysis of preference organisation enables the elicitation of the cultural expectations about ways of participation in conversations (Pomerantz & Heritage, 2013, p. 224). For example, disagreement is often treated as a dispreferred response and is to be avoided (Pomerantz, 1984). However, Katriel (1986) argued that in Israeli Sabra culture disagreements are treated as the free expression of a speaker's thought, respect, and mutual trust.

When a dispreferred response is received in an utterance or trouble in understanding is identified, a conversational mechanism of repair is introduced by either of the participants. The conversational repair accounts for the interaction practices whereby one of the co-interactants interrupts the flow of the ongoing conversation to attend to the mis-articulations, mishearing, use of wrong words, or trouble in understanding, to ensure the continued flow of conversation (Schegloff, 2007). Therefore, the principles of repair ensure that "the interaction does not freeze in its place when trouble arises, that intersubjectivity is maintained or restored, and that the turn and sequence and activity can progress to possible completion" (Schegloff, 2007, p. xiv). The action of repair can be initiated by the speaker (self-initiated repair) or by the listener (other-initiated repair). Interestingly, in a lot of cultures, the preference principle that follows in the repair domain is to allow speakers to initiate self-repair in their talk rather than explicitly correct other's talk, that is, other-initiated repair (Sacks, 1987). However, there are some cultural differences in terms of how self-repair is initiated in different languages. One such example is reported by Fox et al. (2009). They reported that speakers of English, Japanese, and Finnish tend to initiate repair prior to recognisable completion of an incorrect word, whereas speakers of Sochiapam Chinantec and Mandarin initiated repair after the recognisable completion of the incorrect word.

The investigation of turn design and sequence organisation using CA for the data collected for this study enabled me to identify the actions that were generated through interaction in the observed geometry lessons on shapes and their properties, along with how these actions displayed participants' understandings. These features of sequence organisation enabled the exploration of the "procedural consequentiality" (Sidnell, 2010, p. 246) of the classroom interactions. Procedural consequentiality draws attention to the subtle aspects of the setting that determine the shape, content, and structure of the talk, along with the character of the interaction (Schegloff, 1991) that can take place in that particular setting.

Step 3. Describing the Practices.

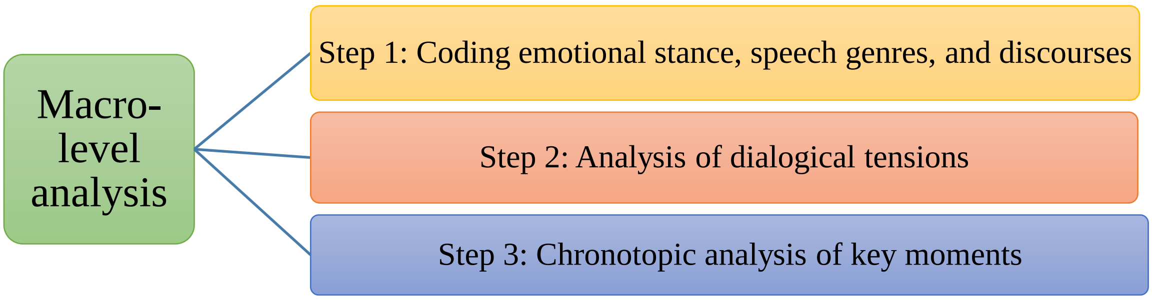
The last step is to describe the practices identified in the second step to answer questions of *what* is happening along with *how* it is happening (see Figure 3.3). The CA techniques used allowed me to document the conversational practices (Macbeth, 2010; Martin, 2009; Melander & Sahlström, 2009) that children used to represent their understanding of shapes and their properties. Thus, the micro-level analysis presented in Chapter 4 provides a detailed description of *what* discursive constructions were used by the Year 5/6 children during the whole-class and group interactions, along with *how* those discursive constructions were displayed during classroom and group interactions. However, it could not provide an account of what and how socio-cultural and historical dimensions permeated participants' utterances as they engaged in talk-in-interaction. Thus, a macro-level analysis was needed to explore the socio-cultural and historical aspects of the dialogic space of the classroom that influenced the children's meanings of 2D shapes, 3D shapes, and their properties. The next section presents a detailed description of the procedure followed for the macro-level analysis.

3.4.3 Macro-Level Analysis

At this level, Bakhtinian analytical tools were used on participants' discursive constructions to trace the influences of the social, cultural, and historical milieu in the dialogic space that influenced the children's conceptual understanding of shapes and their representations. These tools included unitary language, heteroglossia, double-voicedness, addressee, speech genres, and Chronotopic Moment (an interpretation of chronotope adopted in this thesis). This second level of analysis of data enabled me to delve deeper into the children's constructions to explore what understandings were shown, how, and why. The macro-level analysis explored the dynamic interplay of heteroglossia and unitary language forces working at different levels of talk-in-interaction within the dialogic space. A three-step analysis was undertaken (see Figure 3.5).

Figure 3.5

Steps for Macro-Level Analysis



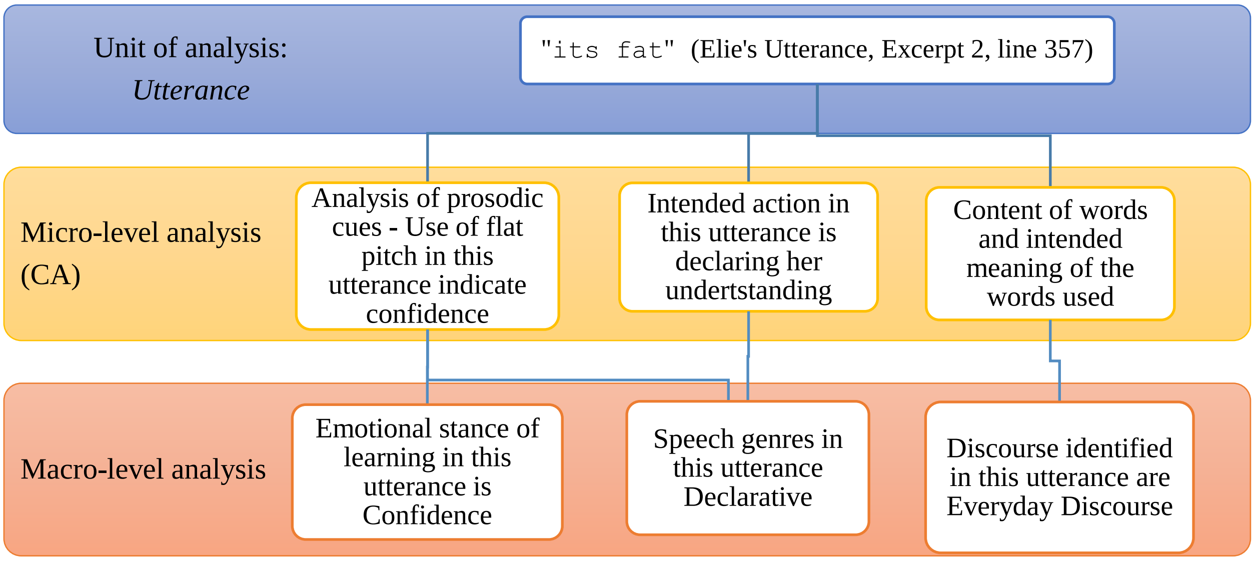
The steps of macro-level analysis are discussed in detail in the following sections.

Step 1. Coding of Key Moments.

Each Key Moment was coded to identify the (i) emotional stance of learning, (ii) speech genres, and (iii) discourses (as shown in Figure 3.6).

Figure 3.6

Coding of Utterances for the First Step of Macro-Level Analysis



Emotional Stances. The first aspect of coding each utterance involved identifying emotional stance. Research outside mathematics education has reported that prosodic patterns display different emotional stances of participants (Kamiloğlu et al., 2020; Ward, 2019). Thus, analysis of intonation patterns in participants' utterances explored at the micro-level analysis (see Table 3.5) enabled me to identify the emotional stances of the participants' learning (Cresswell & Sullivan, 2020).

Speech Genres. The second aspect of coding participants' utterances involved the identification of speech genres. The "relatively stable types" (Bakhtin, 1986, p. 60) of utterances account for the speech genres. Bakhtin (1986) argued that the speech genres are those utterances that reflect not only the specific conversational conditions of an interactional setting (in this case, the geometry classroom). Speech genres also reveal the specific purpose or action of those utterances (e.g. presenting an argument, asking a question, among others) in a particular conversational setting. In other words, speech genres help to identify the generic forms of utterances that are used to achieve a specific social action, such as providing an answer to a question or justification for a claim during classroom interactions (see Figure 3.6). For example, research has shown that teachers may use a persuasive way of talking while teaching (Gerofsky, 1999) which has been identified as Persuasive speech genre in this thesis. Similarly, children may provide mathematical arguments to justify their claim (Yackel & Cobb, 1996), which is identified as the Argumentative speech genre. Rockwell (2012) argued that intonation patterns (cues in the way utterances are made) act as an indication for the listener to identify the action and intention embedded in the speaker's utterance. Therefore, it is the content, style and intention embedded in the overall composition structure of utterance that define the speech genre (Bakhtin, 1986; Rockwell, 2012; Sullivan, 2012). Hence, in this study, the emotional stances (coded through the use of prosodic features) along with the content of the utterances in the sequential organisation of the interaction within the Key Moments are used to identify other speech genres in addition to Persuasive and Argumentative speech genres.

Discourses. In addition to emotional stances and speech genres, utterances were also coded to identify the visible discourses in the Key Moments. Bakhtin's understanding of "social languages" (Bakhtin, 1981, p. 460) guided the process of determining the different kinds of discourses in each Key Moment. According to Bakhtin (1981), "social language is a discourse peculiar to a specific stratum of society (professional, age group, etc.) within a given social system at a given time" (p. 460). Thus, discourses are ways of language use that are specific to a particular group at a particular time and place (Barwell, 2016a). Two major kinds of discourses were identified in this study – Everyday Discourse and Eurocentric-Academic Discourse. Everyday Discourse included the use of informal language by children to represent their understanding of shapes and their properties in the geometry classroom. Eurocentric-Academic Discourse included the use of geometry-specific vocabulary as suggested in The NZC for representing understanding of geometric ideas about shapes and their properties. The term “Eurocentric” highlights the presence of Western mathematical ideas in academic mathematics in New Zealand, as suggested by Barton et al. (1998), Barton and Frank (2001), and Parra and Trinick (2018) (see Section 1.4).

The coding of Key Moments for emotional stances, speech genres, and discourses at the step 1enabled me to investigate the interplay of heteroglossia and unitary languages that influences the negotiation of meanings at step 2, and which is described in the next section.

Step 2. Analysis of Dialogical Tensions.

The second step of macro-level analysis involves explicating the dialogical tensions between heteroglossia and unitary language forces at different levels of interaction. These include the voices, discourses, and languages used (Busch, 2014) that influence the meaning-making process in the geometry classroom. At this step, the analysis (i) investigated the interplay of language forces at the level of speech genres to explore the double-voicedness of utterances, (ii) scrutinised shifts in discourses, and (iii) studied the interaction of different languages present in the classroom during these lessons. Analysis at this level informed me about the changing relationship that the participant had with the addressee, the hierarchical nature of the relationship between the addressee and the speaker, and the tensions of heteroglossia and unitary language that influenced the utterances at various levels- voices, discourses, and languages. Analysing the speech genres with emotional stances of the utterances illuminated the double-voiced character (as explained in Section 3.2.1) of participants’ utterances as well as the role of the addressee in the construction of the utterances. Sullivan (2012) argued that speech genres express an emotionally invested point of view or position of participants embedded in their utterances. Therefore, the scrutiny of speech genres allowed me to get insights into how participants re-purpose someone else’s words as their own in the presence of different addressees. Analysis of tensions at the level of discourses allowed me to explore the influence of different discourses on participants’ utterances. At the level of language, the unitary language and heteroglossia allowed me to explore how multiple meanings can be intended through the same utterance. The third step of macro-level analysis investigated Chronotopic Moments within participants' utterances.

Step 3. Chronotopic Analysis of the Key Moments.

The chronotopic analysis of the Key Moments involved the identification of the Chronotopic Moment (as explained in Section 3.2.1). These Chronotopic Moments highlight the opportune moments of learning in interactions, where one or more participants make explicit reference to time (in past or future) that aids in the negotiation of meanings within the utterances at present. Chronotopic Moments were identified when participants made explicit reference to their moments of learning from the past or anticipated learning from the future by using time-denoting words in their utterances, such as "when I follow the pattern" to denote a future moment, or "my brother used to watch" to suggest the use of the past moment of learning.

Chronotopic analysis allows insights into *time* dimensions of the moments of learning from different time zones embedded in the utterances during these Key Moments in the observed classroom context. This exploration of Chronotopic Moments allowed me to identify the social, cultural, and historical traces of learning from different time zones (e.g., home-time, classroom-time, playtime) in participants' utterances. In addition, the scrutiny of explicit time elaborations by the participants allowed me to track the evolution of meanings from moment to moment within the milieu of dialogic tensions at different levels of interactions (Rosborough, 2016).

This section has discussed the data analysis framework and the procedures adopted to analyse the data in order to seek answers to the three research questions. In the next section, I briefly discuss the steps that were taken at different stages of this thesis to maintain the validity and reliability of findings.

3.5 Establishing Validity and Reliability

In research, it is crucial to ensure that the findings and interpretations are credible. In order to determine the credibility of findings, several steps were undertaken to maintain validity and reliability in this study. In the following sections, I present a discussion of procedures followed to ensure validity (see Section 3.5.1) and reliability (see Section 3.5.2) of the data-gathering and data-analysis processes.

3.5.1 Validity

Validity in qualitative research consists in the approach that a researcher takes to ensure the credibility of the findings using specific procedures (Creswell & Creswell, 2018; Rose &

Johnson, 2020). In other words, validity concerns the extent to which findings from a study can be warranted as trustworthy using the theories and evidence provided by the data (Ary et al., 2019). Trustworthiness refers to the truth-value of qualitative research. That is, trustworthiness is the rigour with which confidence in data, interpretation, and methods is achieved to ensure the quality of qualitative research (Connelly, 2016; Lincoln & Guba, 1986). In the following sections, I discuss the procedures that were taken to ensure the findings of this study are trustworthy and valid.

Conduct of Pilot Study.

To maximise the quality of research inquiry, Yin (2014) argued that the researcher should develop the data-collection protocols and test them before going into the field to gather data for a more extensive study. Therefore, I conducted the Pilot Study to trial the tools that were developed or adapted for gathering data as well as for trialling the data analytical framework. The Pilot Study helped me to become aware of teacher bias, and to practise my interviewing skills with the teacher and student participants for the Main Study. I was also able to trial and adjust my approach to data analysis.

Member Checking.

Member checking is a recommended procedure to ensure rigour in qualitative research (Denzin & Lincoln, 2018). It allows qualitative researchers to assess the trustworthiness of the qualitative results (Candela, 2019; Doyle, 2007). The interview transcripts (one for the Pilot Study and three for the Main Study) were sent back to the teacher participants for checking, and amending if necessary. This enabled the data to receive teacher participants' validation of the interview transcript which contributed to the accuracy of data along with the validity and reliability constructs of the study.

Transparency of the Data and Analysis.

CA provides a detailed description of the analytical procedures that makes the process of analysis transparent for the readers (Seedhouse, 2004). A detailed description of the transcripts and analysis procedure is provided to ensure the transparency and trustworthiness of the data and subsequent analysis (Lester, 2019). For this study, the micro-level and macro-level analysis of Key Moments provides a detailed description of the transcripts of Key Moments along with careful analysis. Additionally, this chapter offers a detailed description of the data analysis process to help readers clearly follow and understand the procedure.

Data Triangulation.

Triangulation is primarily done to increase the validity of a study (Scott & Morrison, 2006; Wilson, 2014). For this study, findings from different sources of data: audiovisually recorded lessons, fieldnotes, semi-structured interviews with the teachers, and focus group interview with children, together substantiated the validity of the study. Additionally, data gathered through different tools enabled me to provide a holistic and rich account of the data for the study.

3.5.2 Reliability

The matter of reliability is contested in the field of qualitative research. However, in qualitative studies, reliability can be understood in terms of the comprehensiveness of the coverage of what happens in the natural setting and its fit with what the researcher observes and records (Grabowski & Oh, 2018). For the research presented in this thesis, a number of steps were undertaken to ensure that reliability and validity concerns were dealt with in relation to the processes of data gathering and analysis. These steps are discussed briefly in the following sections.

Developing Thick Description.

A qualitative study should provide a “rich and thick description” (Onwuegbuzie & Leech, 2007, p. 244) of the data to allow readers to have a clear picture of the context of the study as well as to judge the transferability of findings. A detailed description of data enables the researcher to provide evidence for the claims to support and corroborate the findings (Cohen et al., 2018; Denham & Onwuegbuzie, 2013; Geertz, 1973). For this study, a rich and thick description along with a detailed transcription of the Key Moments is provided in the micro-level analysis (in Chapter 4) for readers in order to support their visualising of the context of the study.

Intra-Rater Reliability for the Transcripts.

The transcription process is an interpretative and active process (Lester, 2019); hence the final transcripts are the product of an iterative process. Nonetheless, different measures were taken to ensure that the transcripts were reliable to a greater degree. Intra-rater reliability accounts for the stability measure of the data representation over time. This was explicitly ensured regarding the transcripts of the audiovisual data. The same segments of the audiovisually recorded data were transcribed and re-transcribed using the same set of CA conventions, with a time gap of three weeks. When the transcripts were matched, 91% similarity was found between them, which is substantial (Roberts & Robinson, 2004).

Peer Consultation.

Peer consultation should be sought from other CA practitioners for comment and feedback on the analysis to ensure the reliability of the transcripts (Seedhouse, 2004). Thus, transcripts of three Key Moments (transcript of one focus group interview from the Pilot Study data and transcripts of two Key Moments from the Main Study data) were presented to peers and a supervisor to seek their inputs. This process of seeking input from CA practitioners helped me transcribe intonation patterns involving high and low pitch and rising intonation consistently, and to avoid my bias as a researcher while transcribing and analysing the data. Additionally, cultural advice was also sought from Māori colleagues for interpreting prosodic aspects of utterances as well as geometry concepts.

This section has outlined the steps that were followed to maintain the validity and reliability of this qualitative study. The next section discusses the procedures that I followed to ensure ethical conduct during the study and to mitigate any ethical issues that might arise due to the nature of my research.

3.6 Ethical Procedures

Qualitative research often seeks an exhaustive exploration of a phenomenon in natural settings where human participants are involved. Research with human participants may give rise to ethical concerns that need to be taken into account before commencing the data collection process. These concerns may include informing participants about the research, sharing information with participants, maintaining the anonymity of participants and confidentiality of data, among others. To cater to these ethical concerns, I gained ethics approval in accordance with the University of Waikato's Ethical Conduct in Human Research and Related Activities Regulation 2008. I received full ethics approval (Reference No. FEDU064/18) for my research before commencing my study (see Appendix E). In the following sub-sections, I discuss the ethical procedures that were maintained to ensure the ethical conduct of the study.

3.6.1 Access to Participants

I had no contact with the potential participants prior to the research. The same procedures for initial contact and access to participants were employed for both the Pilot Study and the Main Study. I emailed a number of primary school principals (six for the Pilot Study, 14 for the Main Study) to book an appointment to talk about my research; three principals (for the Pilot and the Main Study) responded to the email. Three teachers (one for the Pilot Study, two for the Main Study) showed their interest in being a part of the research and agreed to a meeting to talk about the research work, their role, and the roles of the student participants. I provided the principal and the teacher with the Information Sheets and the Informed Consent forms (Appendices F, F(a), G, and G(a)) to read and provide their consent to participate in the research. Teacher participants and the principals were assured that participation in the research was voluntary and that there would be no negative outcomes if they chose not to participate in the study. After gaining consent from the teacher and the principal, the Information Sheets and the Informed Consent forms for children (Appendices I and I(a)) and their parents/caregivers (Appendices H and H(a)) were given to the teacher. It was explained in the Information Sheets that the participation of children was voluntary, and that there would be no negative outcome if the children chose not to participate in the study.

3.6.2 Informed Consent

All potential participants were provided with the Information Sheets explaining the details of the research and how the data would be used. Those who decided to participate in the study were asked to complete and sign the Informed Consent forms (see attached Appendices F to I(a)). The Information Sheet and the Informed Consent form for children (Appendix I and I(a)) were written separately in an appropriate language. Consent was sought from each student every time before commencing data gathering concerning aspects such as if and when audio-recorders and/or the digital camera were to be switched on.

3.6.3 Anonymity and Confidentiality

The anonymity of the participants is preserved using pseudonyms. This was explained to participants in the Information Sheet and the Informed Consent forms (Appendices F to I(a)), and they were given the opportunity to suggest pseudonyms for themselves. As the data were collected from two different schools, pseudonyms ensure the anonymity of the participants. No one other than my supervisors, participants and I were allowed access to the raw data to maintain the confidentiality of the data.

3.6.4 Potential Harm to Participants

The researcher was in no position of power, in any way, over the potential participants. The researcher ensured that the participants knew that their participation in the research was voluntary, and that they could withdraw their participation from the research at any time of the data gathering without any repercussions.

3.6.5 Participants' Right to Decline to Participate and Withdraw Data

Through all information (written and verbal), I ensured that participants knew that their participation in the research was voluntary. Much effort was made to ensure that the participants were comfortable in declining to participate. They were advised that they could withdraw their participation at any time by contacting the researcher. No participant withdrew their data from the study.

3.6.6 Arrangements for Participants to Receive Information

Teacher participants, student participants and their parents (Appendices G(a), I(a), and H(a)) were asked if they would like to be sent a summary of the findings of the study in the Informed Consent form. If they ticked 'yes' and provided an address, a summary will be sent through the stated channel (Email or postal address).

3.6.7 Use of the Information

Data collected was to be used in writing a doctoral thesis, for publishing articles in academic journals, book chapters, and for presentations at academic conferences. This was explained to the participants in the Information Sheet and the Informed Consent form (see Appendices F to I(a)).

3.6.8 Conflict of Interest

There was no apparent conflict of interest. I was not in a position to assess children nor did I have any kind of authority over the participants. I had no professional relationship with the school, teachers, children, or their parents/caregivers.

3.6.9 Procedure for Resolution of Disputes

Participants were advised to contact me to address any concerns. If they were not then satisfied, they were advised to contact my chief supervisor. I ensured participants had postal and phone contact details for both of us (provided in the Information Sheets) to ensure that various lines of communication were open for them.

This section has presented a description of the ethical procedures that were followed to ensure that ethical conduct was maintained during the course of the study. In the next section, I present a chapter summary outlining the key ideas from this chapter on methodology.

3.7 Chapter Summary

The chapter began by situating this study in the Critical Inquiry paradigm. It then presented the theoretical framework within the Discursive Psychology approach used for the study. Bakhtin's Dialogic Theory and Garfinkel's (1967) Ethnomethodology were discussed. Following the theoretical framework, the data-gathering process was described in detail, including the different tools that were used to gather data. The chapter then described how the data were analysed in three phases: thematic analysis, micro-level analysis, and macro-level analysis. Following the description of the study's data analysis procedures, issues of validity, reliability and ethical conduct were discussed. The next two chapters (Chapters 4 and 5) present the findings. Chapter 4 presents the findings from the thematic analysis and micro-level analysis, which aims to present answers to the first and second research questions. Chapter 5 presents findings from the macro-level analysis, which analysed the Key Moments using the concepts of Bakhtin's Dialogic Theory to answer the third research question.

Chapter 4.
Findings I: Discursive Constructions of Shapes and Their Properties

The study aims to explore and analyse children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom. This chapter responds to the first two research questions.

The first research question is what discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom? This research question specifically focuses on the geometric ideas that children referred to through their discursive constructions as evident in their utterances to represent their understanding of shapes and their properties as they interacted during whole-class and/or group discussions.

The second research question, how do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?, focuses on the interactional tools that children used to construct their understanding of shapes and their properties. The purpose of this chapter is to dig deeper into the discursive constructions of children from one class in a primary school to reveal how discursive constructions are made as children participate in whole-class and/or group interactions. The findings presented in this chapter are drawn from six data sources at a New Zealand primary classroom. Table 4.1 presents the details of the data analysed in this chapter for thematic and micro-level analysis.

Table 4.1

Details of Data Analysed and Presented in This Chapter

School	Data Analysed
School B (Main Study)	a. Fieldnotes of six lessons (FN1, FN2, FN3, FN4, FN5, FN6) b. Audiovisual recordings of six lessons c. Four focus group interviews with four groups of consenting children (FG1, FG2, FG3, FG4) d. Three semi-structured interviews with the teacher Jenny (Interview 1, Interview 2, Interview 3) e. Documents including children's work samples and teacher's unit plan on shapes, and The New Zealand Curriculum (NZC) f. Short Questionnaire (completed by the parents)

In this chapter, abbreviations are used to signal the data source. For example, FN1 indicates fieldnotes data from lesson 1. Similarly, FN2 indicates fieldnotes data from lesson 2. I observed six lessons in total. Hence, I denoted these fieldnotes from these six lessons as FN1, FN2, FN3, FN4, FN5, and FN6. Similarly, I marked focus group interview data as FG1, where FG indicates Focus Group, and the number 1 shows the first group. I conducted four focus group interviews, which are denoted as FG1, FG2, FG3, and FG4.

Thematic analysis of three semi-structured teacher interviews, four focus group interviews with the children, and the fieldnotes were corroborated by repeated viewing and coding of audiovisual recordings of six geometry lessons to identify five themes pertaining to geometric shapes and their properties. These five themes are: (i) making sense of 2D shapes, (ii) making sense of 3D shapes, (iii) relating 2D shapes with 3D shapes, (iv) mathematical construct of dimension, and (v) naming shapes in Te Reo Māori.

Each theme explores the teaching and learning of one aspect of geometric shapes and their properties. As the study focuses on interactions, Key Moments were selected for micro-level analysis for each theme. Key Moments are those moments during which participants represented and co-constructed their understanding of shapes and their properties as they participated in whole-class and/or group interactions. To manage the micro-level analysis for this study, only two relevant Key Moments for each of these themes were selected from the audiovisually recorded data of six observed geometry lessons. Ten Key Moments were selected in total for micro-level analysis.

For the purpose of presentation, analysis of data pertaining to each theme is presented in the following sections (see Sections 4.1 to 4.5). Within each section, thematic analysis of data from fieldnotes, focus group interviews, and semi-structured teacher interviews is presented, along with a micro-level analysis of the two Key Moments selected for each of the themes. The presentation of findings from the ten Key Moments served two main purposes. Firstly, it enabled the management of the substantial amount of data gathered for the study. Secondly, it allowed me to examine classroom interactions in close detail in order to answer the first and second research questions.

Table 4.2 summarises the description of Key Moments identified for each theme, the classroom activities performed during those Key Moments, and the lessons these Key Moments were taken from.

Table 4.2

Description of Key Moments, Activities, and the Lessons

Themes pertaining to shapes and their properties	Key Moment	Activity	Whole-class or group interaction	Lesson
Making sense of 2D shapes	Key Moment 4.1a	Task Sheet A: Shapes in Everyday Life	Group interaction: Group *Whetū*[12] (Children: Ozan, Tahi, and Garry)	Lesson 1
	Key Moment 4.1b	Making shapes with play-dough and sticks with adhesive	Whole-class interaction	Lesson 2
Making sense of 3D shapes	Key Moment 4.2a	Task sheet B: Shapes in Everyday Life	Whole-class interaction discussing work of Group *Korere*[13]	Lesson 1
	Key Moment 4.2b	Making shapes with play-dough and sticks with adhesive	Group Interaction: Group *Marama*[14] (Children: Matiu, Garry, Tahi, Ethan, Ozan)	Lesson 2
Relating 2D shapes with 3D shapes	Key Moment 4.3a	Task sheet C: Shapes in Everyday Life	Whole-class interaction with Group *Taimana*[15]	Lesson 1
	Key Moment 4.3b	A poem on 3D shapes	Whole-class interaction	Lesson 4
Mathematical construct of dimension	Key Moment 4.4a	Making shapes with play-dough and sticks with adhesive	Whole-class interaction	Lesson 2
	Key Moment 4.4b	Completing a worksheet after watching a video on dimensions	Whole-class interaction	Lesson 3
Naming shapes in Te Reo Māori	Key Moment 4.5a	Making shapes with play-dough and sticks with adhesive	Whole-class interaction	Lesson 2
	Key Moment 4.5b	Completing a worksheet on shapes	Whole-class interaction	Lesson 3

[12] Māori word for star-shape
[13] Māori word for cone-shape
[14] Māori word for moon-shape
[15] Māori word for Kite shape

The first section explores the theme of children's discursive construction of 2D shapes (Section 4.1). The theme of the next section is how children talk about 3D shapes (Section 4.2). The third section (see Section 4.3) scrutinises the theme of how children relate 2D shapes with 3D shapes. The following section (Section 4.4) delves into the theme of children's discursive constructions of dimension as a mathematical construct. Finally, the fifth section (see Section 4.5) studies the fifth theme, how children discursively construct their understandings of naming shapes in Te Reo Māori. Findings drawn from different data sources for each of the themes helped in triangulating findings. Following the analysis of Key Moments pertaining to all five themes regarding shapes and their properties, the chapter concludes in Section 4.6 with overall findings that are drawn from thematic analysis and micro-level analysis of data pertaining to the five themes.

4.1 Theme: Making Sense of 2D Shapes

This section explores how multilingual children display their understanding of 2D shapes as they engage in peer and classroom conversation. The examination of data from fieldnotes and repeated viewing of audiovisual data of six lessons revealed two noteworthy patterns of talking about 2D shapes. Firstly, children used the terms "sides" and "corners" to talk about 2D shapes. The terms were used to imply line segments and angles of the shapes, respectively, as displayed in the following verbal descriptions:

> I forgot what this shape's called…it's got…one, two, three, four, five, six corners, and it's got six sides. (Elie, FN2)
>
> House shape, five sides, five corners. (Kayla, FN2)
>
> It's a square. It has four sides and four corners. (Alyssa, FN3)

Secondly, examination of the fieldnotes suggests that the name of the shape was identified by counting the number of sides it had. The following verbatim passage explicitly shows the pattern used, although some of the children did not seem to remember the names of some of the shapes they identified.

> I know but I just my brother used to watch a movie about of this kind of shapes that I know…their names are like twelve.…I thought it was and there's they were saying like a like a lot of shapes like one two three until they have passed eight, and then ten and twelve or something...I don't remember by how much it was but I do remember by…how many. (Ozan, FN1)
>
> Teacher: This 2D shape has eight sides and eight corners. What is it?
> Matiu: Octagon. (FN3)

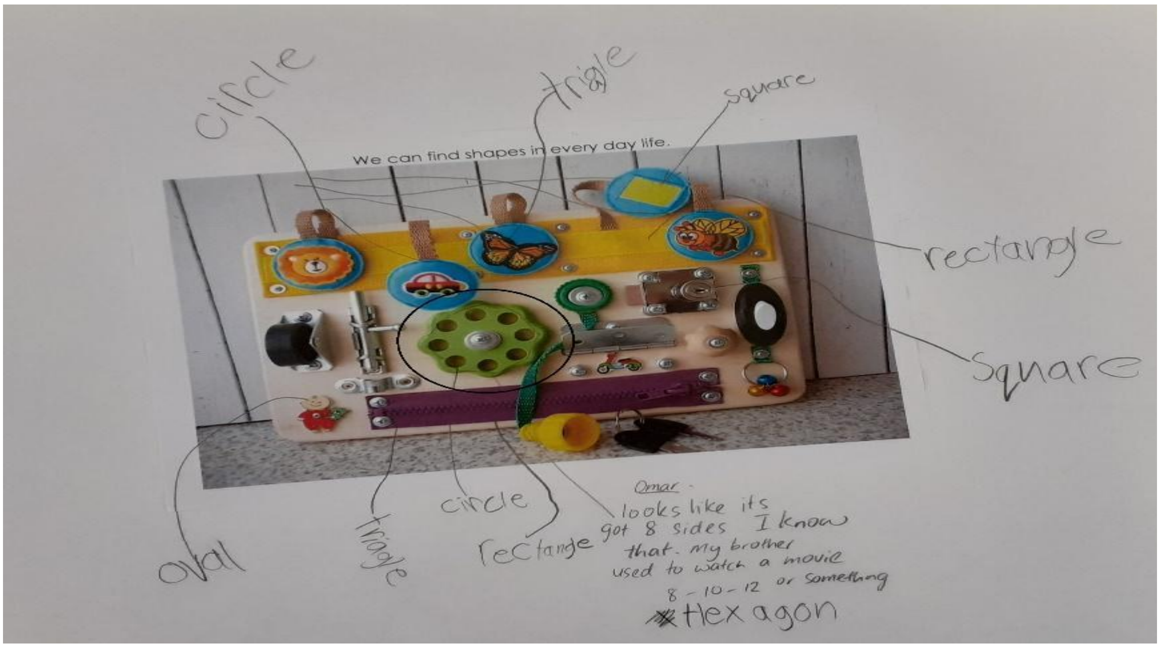
To explore these discursive constructions about 2D shapes and their properties further, two Key Moments pertaining to this theme were selected and analysed in detail. In the first Key Moment, a group of children had been provided with a task sheet to identify the shapes already known to them. In the second Key Moment, a child expressed the idea of "perfect square". The understanding of what "perfect square" might imply is scrutinised more closely.

4.1.1 Key Moment 4.1a: "I saw this as some kind of shape that I know."

This Key Moment is selected from the audiovisually recorded first lesson. During this lesson, the teacher had grouped children into groups of three to four children and provided each group with a task sheet to work collaboratively. The Key Moment presents a group discussion of Group Whetū (the name of the classroom group) during classroom teaching and learning about shapes. The group consisted of three 9 to 11-year-old multilingual children: Ozan, Tahi, and Garry. Ozan is a male 9-year-old bilingual Somali child with beginner's proficiency in English and Somali as his home language. Tahi is a male 11-year-old Tongan child with English as his second language and Tongan as his home language; and Garry is a male 11-year-old, Philippine child with advanced proficiency in English and Filipino as his second language. During this Key Moment, Ozan identified a shape in Task Sheet A provided to the group by the teacher (see the circled shape in Figure 4.1) for the activity "Shapes in Everyday Life" but could not remember the name of the shape.

Figure 4.1

Work Sample: Task Sheet A of Group Whetū



The transcript for this Key Moment is divided into three parts. The first excerpt, 4.1a (part i), shows the group interaction of three children, Ozan, Tahi and Garry. The second excerpt, 4.1a (part ii), shows the interaction of two children, Tahi and Ozan, with the teacher. Garry was not present in the group discussion in part ii. As Garry returned to the group, the teacher left the group. Thus, the interaction after the teacher left is presented in the last excerpt, 4.1a (part iii). The first excerpt, 4.1a (part i), shows the group interaction as the children started the task of identifying and naming shapes in Task Sheet A (see Figure 4.1).

Excerpt 4.1a (part i)

Speaker	Text
Garry	what sha:pes can you see right now
Tahi	circ::les(1.5)squa:res
	((Garry takes the picture sheet and turn it over to put glue to paste it on large white sheet as Tahi was still looking at it))
Ozan	I see a lot of circles over there (3.0)
	((Ozan looks at the sheet while Garry and Tahi make faces towards the camera))
Ozan	okay(.) what is this shape called ((pointing to shape))
Garry	↑so ↑whats tha:t whats [that Tahi?
Tahi	[squa::re
Garry	thats a ↓rectangle
Tahi	#square#
Garry	then Ill say squa::re
Tahi	↑Squ°are::°(.8) °thats° a square
	((Garry writes square as Tahi speaks))
Ozan	oh ↑I ↑SEE [One
	((Ozan looks at Garry who was given with the responsibility to write))
Tahi	[he::re ((Tahi points to different shape and laughs))
Ozan	THIS ONE ((points again to the shape))
Garry	wha:ts that
Ozan	I dont know what[it is called
Tahi	[°circle thats a circle°
Garry	cir(.)cle
Ozan	not °this° (2.0) ((put his hand to his head to show that it is not the shape that he was talking about))
	I am talking about whole thing(.) like like (2.0) ((drag his finger at the shape to show his imagination of sides)) (in jacks)(.5)what was it (2.0) [it=
Tahi	=ohh (.) °I know there is this thingy like this° ((points to another shape))
	[theres like ((makes the shape with his finger on the sheet to show the shape he implies))
Garry	[there is: no thingy (you images)
Ozan	((aspires)) Oh↑ I see
Tahi	no:: theres a thing(.) that they had tha:t goes ↑like (then) ((Tahi moves his finger in a curved motion))
Garry	[((draws a line and Tahi sees him))
Ozan	[oh ↑I see one (.5)Agai::n
Tahi	RECtangle:s (and like) (2.0)

Garry	↑Oval:((marks an arrow for a shape))
Ozan	[I SEE ↑one I see one °I see one°
Tahi	[okhay
Garry	wha::t
Ozan	this one like(.) not the:se circle.(.5) °like this° ((he drags his finger on the shape making the straight lines of the sides of the shape))
Tahi	OH ↑the shape(.)tha:ts not a shape
Garry	this not a shape
Tahi	what are another shape(.4) (I drop out)
Ozan	its some kind of [cir
Garry	[its an oval

Garry selected himself as the first speaker (line 205) and constructed his turn to put a question directed towards the other two group members. He could have started the discussion by identifying a few shapes in the task sheet. However, he posed the question, “what shapes can you see right now” (line 205). During this lesson (FN1), it was noted that the teacher had explicitly stated that Garry would help Tahi and Ozan to write the names of the shape. During the first semi-structured teacher interview (conducted after the first lesson was observed), the teacher was asked why she had asked Garry to help Tahi and Ozan in writing the names of the shapes. The teacher replied that Garry was a better English speaker than Ozan and Tahi, as well as one of the high achievers in mathematics (Teacher, Interview 1). This might explain why Garry did not start by naming the shapes but instead asked Ozan and Tahi to identify them.

Though Garry had not selected the next speaker, Tahi selected himself (line 206) and stated “circles” and “squares”. He stretched the words “circles” and “square” along with a long pause of 1.5 seconds (line 206). It seems that Tahi used stretching as a way to hold the speaking floor while Garry was writing these shape names on the task sheet. Tahi did not select the next speaker in his utterance; however, Ozan self-selected and stated that he saw circles in the picture (line 209). Through his utterance, Ozan stated that there were many circles in the task sheet. The utterance was constructed as a suggestion of different circles being present in the task sheet. Ozan paused for three seconds after finishing his utterance. It is probable that he was waiting for Garry and Tahi to respond to his utterance. Schegloff (1982) has shown that speakers may fall silent in a lack of supportive feedback in the form of backchannel. Backchannel is an interactional practice that occurs when the listener provides verbal or non-verbal responses to the speaker as the speaker continues to speak. Garry and Tahi did not respond to Ozan’s last utterance; Ozan again self-selected and pointed to the shape (using a deictic gesture) that he had identified (line 212). Ozan stressed “what” and “shape” by using a slightly higher volume than the rest of the sentence to mark the focus of his utterance. In the Somali language, the focus of the utterance is often marked by the use of stress on a specific syllable or word in a particular utterance (Biber, 1984). It is possible that Ozan stressed the words to structure his utterance as a question, without using the rising intonation, which is often used by English-speakers to mark a statement as a question.

Ignoring Ozan's utterance, Garry self-selected and posed another question (line 213). This time, he selected Tahi as the next speaker by saying his name at the end of the utterance. Tahi claimed that the shape that Garry referred to was a square (line 214); a slight increase in volume at the "qua::" (in square) may be interpreted as a signal to the specificity of the shape. In Tongan intonation patterns, when a noun is made definite, the main stress falls on the second-last vowel (Anderson & Otsuka, 2006; Condax, 1989). Tongan speakers directly feed the grammatical structures into the prosody they use instead of using definitive articles such as "the", as in the case of English (Ahn, 2011).

Garry claimed that the shape was a rectangle using low pitch (line 215), where he used the article "a" with "a rectangle". Research informs us that in English speakers often use low pitch (Ward, 2019) or creaky voice (Ward, 2004) to show their authority over knowledge, and thus Garry was perhaps using his low pitch in this way here. However, his claim was rejected by Tahi in the next utterance (line 216). It is noteworthy that this time Tahi used his creaky voice to claim his knowing.

Acknowledging Tahi's authority over his knowledge, Garry (line 217) accepted Tahi's claim and wrote that the shape was a square. Tahi, in his following utterance, continued his claim (line 218). While the activity required children to discuss the shapes, it seems that Tahi and Garry were not considering Ozan's point of view to decide if the shape was to be called a square or a rectangle. This act of neglecting Ozan's idea could be because Ozan was new to the class, and had limited proficiency in English. However, Ozan did not seem to be bothered by this; instead, he again attempted to draw his fellow peer's attention to the shape (see Figure 4.1, green coloured object) that he identified (line 220).

Tahi again self-selected and overlapped his talk with Ozan (line 222). Tahi pointed to a different shape that he had identified and noted on the sheet. Noticing that he was losing Garry's attention to his shape, Ozan used a loud voice and again pointed to his shape (line 223). Couper-Kuhlen (2004) argues that speakers use loudness as a prosodic marker to mark the current turn as a new course of action. Thus, in this case, Ozan probably used a loud voice to begin a conversation about a new shape instead of continuing the ongoing conversation about the different shape. However, when Garry (line 224) asked him about the name of the shape, Ozan claimed that he did not know the name of the shape. Tahi again self-selected in the next turn and claimed that the shape was a circle (line 226). Garry provided his agreement with Tahi's statement that the shape is a circle (line 227) by repeating, "circle". Research has shown that the next speaker often repeats the previous speaker's utterance to show mutual understanding and agreement (Rossi, 2020; Tannen, 2007).

The video-recorded data showed that Ozan (line 228) bowed his shoulders, lowered his head and kept one hand on his head at this moment in this interaction. This body language may be interpreted as his way of displaying his disappointment that Tahi and Garry were unable to identify the shape that he was referring to (in Figure 4.1). Ozan (line 230) again attempted to direct their attention to the shape when he dragged his finger to show them the sides of the shape (using iconic gesture). In line 233, it seems that Tahi realised that there is such a shape. However, when Garry dismissed the possibility of such a shape (line 237), both Ozan and Tahi went along with Garry. Again, both Tahi and Ozan acknowledged Garry's authority and knowledge about shapes.

It should also be noted that when Ozan again tried to show Garry the same shape (line 245), Garry seemed to be uninterested as he stretched his word "wha::t" (line 247). Lerner (1996) showed that a sound stretch could act as an indicator of a possible trouble, along with showing possible completion of utterance. In this case, it would imply both. Garry seems to end the conversation, as well as implying a possible problem with Ozan's identification of the shape, without further discussion. This time, Ozan again pointed to the same shape (line 248) and indicated that he was not talking about the circles within the shape. Instead, he gestured to make the lines of the shape to which he was referring to with his finger (using iconic gesture). Tahi and Garry both disagreed with his claim (lines 251 and 252).

The second excerpt 4.1a (part ii) of this Key Moment draws our attention to the different positions that children hold in the presence of the teacher and without the presence of the teacher in the group discussion. In the following excerpt, Garry was not present as he was doing other work. The interaction took place between the teacher, Ozan, and Tahi. The teacher had asked them about the shapes that they could see in the picture. Ozan, again attempted to show the shape he had identified (line 316). He used rising pitch with "I" and stressed "saw". Through this construction, Ozan attempted to show his conviction about the shape. It is interesting to note that Ozan, with his beginner's proficiency in New Zealand English, used a rising intonation at the end of his utterance to check if the teacher agreed with his telling of the shape in the same utterance (line 316). This use of High Rising Terminal

(HRT) intonation is a feature of New Zealand English and many other Englishes around the world (Warren, 2016).

Excerpt 4.1a (part ii)

#	Speaker	Text
314	Teacher	↓what else can you see so cir:cle: kinda: yeah
315		circle
316	Ozan	↑I saw this as some kind of sha:pe that I know? ((points to the shape))
317	Teacher	↑do you? ((teacher smiles))
318	Ozan	((nodding))
319	Teacher	↑I kno::w (.5)what what kinda shape can we call
320		that then
321	Ozan	its like(.)((drags his finger over the shape))
322		one of those shapes thats like its goes like this
323		((gestures with both hands to show sides of the shapes))
324		like this [like that
325	Tahi	[a square [°probably° ((use gestures to show lines))
326	Teacher	[ah:: (.5)so:(.) you↑you↑thinking like(.8)this
327		one(.5)↑lets see tho:se(.)if you would to(.2) give
328		it si::des ay?
329	Ozan	yeah yeah [third one
330	Teacher	[°one two°three four five six seven eight [ni:ne
331	Tahi	[theres ↑only eight
332		
333	Teacher	one so ↑theres EIGHT (.2)one two three four fi:ve
334		six seven eight(.) do you think eight (.)so: do you
335		know the ei:ght one? ((looked at Ozan))
336	Tahi	↑ITS a rect↑(.) oh no ((tries to think))
337	Teacher	so do you do you know what the eight one is ((the
338		question is explicitly directed to Ozan))
339	Ozan	I ↑know but(.2)I just my brother used to watch a
340		movie about(.2)of this kind o shapes↑(.5)that I
341		know(.5)their na:mes are are like like twelch(.4)I
342		I thought it was and theres they were saying like
343		a like a lot of shapes like one two three
344		until(.5) they have passed eight(.) and then ten an
345		twelve or something(.5)I dont remember by how much
346		it was (.2)but I do remember by [how many
347	Teacher	[NA:: I think some I think somehow you kno:w but
348		you are not you cant remember so ((coughed and cleared throat))
349	Tahi	I ↑know but I dont know the name
350	Teacher	yes okay so(.5)eight sided figure (.2) is::
351		((cleared throat)) is a octagon. remember octagon ((looks at
352		the camera)) okay so um
353	Tahi	octa:gon: theres a six one I am pretty su:re

To this, the teacher responded with an appreciative acknowledgement as she constructed her turn as “do you?”, while smiling (line 317). It has been argued that teachers often use a smile as a conversational marker to build rapport with students (Nguyen, 2007). Thus, it seems that the teacher smiled to build rapport with Ozan and to make him comfortable in sharing his thoughts about the shape. Noticing that Ozan was confident in his claim about the shape, the teacher provided him with a space to allow him to name the shape that he was thinking of (line 319). The video shows that Ozan dragged his finger over the shape (using an iconic gesture again) to show the sides of the shape. It is to be noted here that Tahi, who earlier dismissed the shape, now acknowledged that there could be a shape if it had sides (line 325).

Taking this cue from Tahi, the teacher drew sides to the shape (line 326). In the next turn, Ozan expressed his agreement that this is what he meant. As the teacher drew straight lines to the sides of the shape, Tahi counted the sides and claimed there were eight sides. He attempted to recall the name of the shape (line 336). The teacher explicitly directed her question to Ozan using her gaze (from the video-recorded data) and asking if he knew the name of the shape (lines 334 and 335). To this question, Ozan responded that there were shapes with a different number of sides. He claimed that these shapes could have from one to twelve sides (lines 339 to 346). Ozan stressed the words "this kind o shapes", "their names", "like twelch" (lines 340 and 341). It seems that Ozan was confident that the names of the shapes were linked to the number of sides. He did not use the word "side" during the whole interaction, which may suggest that his focus was on the number of sides that provide a clue for naming shapes. Moreover, he again displayed his knowledge (line 346) that he knew these shapes but could not remember the names of the shapes.

As the teacher expressed her agreement with the presence of these shapes, Tahi (line 349) claimed that he knew the shape but did not know the name of the shape. It should be noted here that Tahi, in the earlier excerpt, had denied knowing this shape (line 252). In his turn here, Tahi tried to agree with the teacher who held more authority over knowing. As the conversation proceeded, the teacher stated that the shape was an octagon (lines 350 and 351). Tahi, in his utterances, repeated the name of the shape (octagon) as suggested by the teacher, and stated that he was confident about a six-sided shape. Research has shown that students may attempt to upgrade their epistemic stance of knowing by repeating the teacher's claim (Skarbø Solem, 2016).

The third excerpt (excerpt 4.1a, part iii) shows the conversation that followed once the teacher left and Garry came back to the group. As Garry came back, Ozan selected himself to state that the shape that he was referring to is "a shape that has eight" (line 376). The teacher had identified the straight lines that Ozan referred to as "sides" but Ozan did not use this word in his construction.

Excerpt 4.1a (part iii)

#	Speaker	Text
375	Ozan	°okay this is(.)did you say° this is some kind of
376		tri:angle? (.)and this is like (.5)a shape that has
377		eight
378	Tahi	↑thats (1.0)octagon oh ((aspiration & smiles)) °so thats
379		something like ()°
380	Ozan	its like a ↑cir:cle some kind of circle (.5) ((makes
381		circle with his hands)) but is (1.0)↑is the ↑SHape that have
382		the most number (2.5) I remember the movie
383		[which is called ()
384	Garry	[((coughs)) HEXAGON
385	Tahi	its hexa:gon
386	Ozan	↑oh yeah hexa:go::n

Ozan mentioned that the shape has "the most number" (lines 381 and 382). Tahi claimed to know about the shape by stating the name of the shape (line 378). However, as he smiled and whispered, it seems that he was not confident about the name. He structured his utterance as if to seek agreement or acceptance of his statement. Despite the teacher mentioning the name of the shape, Ozan had not been able to recall the name and reiterated that the shape was "some kind of circle". Garry's response (line 384) is worth noting. He coughed the way the teacher had coughed to show her presence earlier (line 348, Except 4.1a, part ii), and the loud voice of his utterance can be interpreted as his intent to display his authority over the knowledge. He claimed the shape to be a hexagon (line 384) while ignoring the name offered by Tahi as an octagon (line 378). Both Tahi and Ozan agreed (lines 385-386) to the claim presented by Garry in his authoritative voice. In this whole interaction, Ozan did not use the term "side" to denote what he meant to count for naming the shapes. His utterance was focused on the "number" of sides.

In this Key Moment 4.1a, the analysis explored the talk-in-interaction through which children discursively constructed and represented their understanding of 2D shape. In terms of geometric understanding of 2D shapes, the data suggest that children used the "number of sides" as a rule for recalling the names of the 2D shapes, even when they had not developed "side" as part of geometry vocabulary. In terms of how utterances were constructed, the analysis suggests that multilingual children may use prosodic patterns from their home languages. In addition to prosodic cues, the analysis seems to suggest that some children (Ozan, in this Key Moment) may use iconic gestures more than deictic if they are unable to express their understanding in English. Ozan used his iconic gesture of displaying the shape by dragging his finger (lines 231, 249, 250, 321) or using his hands (line 323) more than deictic gestures (lines 212, 223, 316) to talk about his intended shape to his listeners, including the teacher and his group members Garry and Tahi.

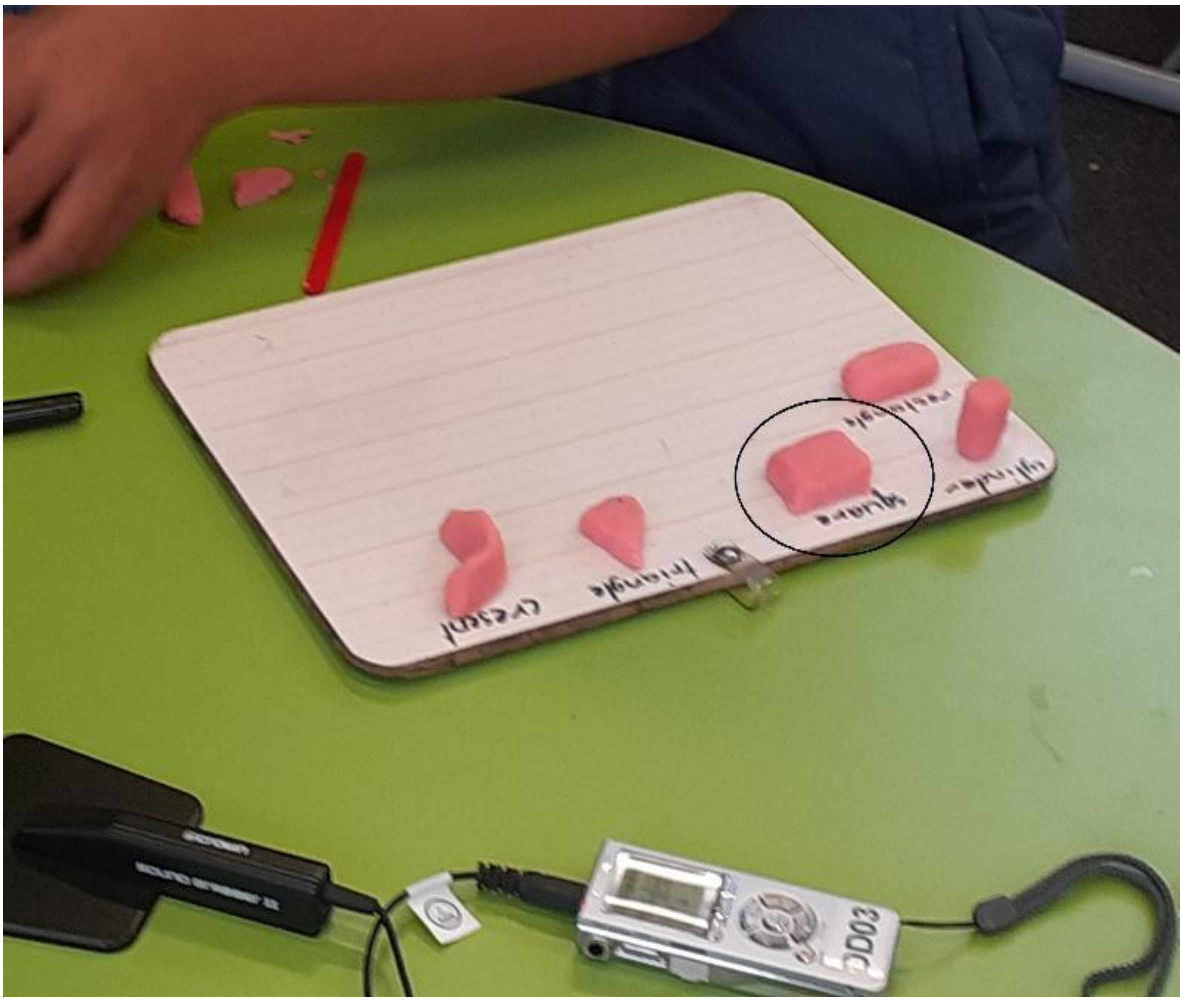
In the following Key Moment, I explore a child's idea of "perfect square" in terms of what she implied and how the transformation of the meaning of perfect square occurred as the conversation proceeded.

4.1.2 Key Moment 4.1b: "Whaea, look, a perfect square."

This episode is extracted from the audiovisual data of the second lesson. In this lesson, the teacher had provided children with playdough or sticks with adhesive to glue sticks together. In this Key Moment, Zara (female 9-year-old Māori-English bilingual Maori child) claimed that the shape that she made was a "perfect square" (see the circled shape in Figure 4.2).

Figure 4.2

Work Sample: Zara's Playdough Shapes



The transcribed data (in Excerpt 4.1b) show the classroom conversation that followed.

Excerpt 4.1b

#	Speaker	Text
189	Zara	>look whaea[16] Jenny:< (1.0) whaea Jenny (.) a
190		perfect square ((shows the shape by holding it in her hands))
191	Teacher	is it perf (.) why is it a perfect square? zara
192	zara	I dun↓no
193	Teacher	what makes it a perfect square(2.0)>come on zara ↑I
194		need< to ↑kno:w(0.5)because you said its perfect so
195		what makes a perfect square a perfect square
196		(1.0)=
197	Matiu	a ↑s[quare
198	Teacher	=[↑anyone ↑know why a perfect squa:re a perfect
199		square
200	Matiu	becoz its a square?
201	Garry	(h)(h)
202	Teacher	yeah because its a square doesnt tell me
203		much(1.0)ELIE what do you think
204	Elie	becau::se um: [if you have to= (2.0)
205	Zara	[you put(on)((rolled her eyes))
206	Elie	=um: because um::(1.0) if you have the right type
207		of shape. or if you (1.0)if or if you have(.2)
208		havin:g a right(0.5)type of equipment (.) °you can
209		have°
210	Teacher	°okay°
211	Elie	so:: if you are trying to make square of that
212		one(1.0)you can roll into a ball then you
213		start pressing it down the other side >the
214		other side and you can [get square<
215	Teacher	[oh thank thank you Elie
216		(0.5)↑can any↑one ↑tell me why a perfect square
217		might be (0.2)might be perfect square using geometry
218		language
219	Zara	[um ((looks at the roof trying to figure out how to say what she wants to
220		say))
221	Matiu	[um: °its got°
222	Teacher	Matiu
223	Matiu	because the face °no:(0.2)the si::des°(2.5) nah
224		°I dun know°
225	Teacher	yeah you re on the right track. the si:des what
226		(.)what would the sides be here
227	Matiu	perfectly:: aligned? with each other?=
228	Teacher	=aligned with each other?
229	Matiu	ah(1.0) perfectly the same?
230	Teacher	perfectly the sa:me the sides ↑are perfectly
231		the same (1.0)UM::: (1.0) zara(.2) did you hear
232		that(2.0)<a perfect squa:re is when the si:des
233		are per:fectly the same>
234	Zara	oh↑ ((exclamation)) [yeah
235	Teacher	[the si:des are the same. okay
236		(.)thats why you get a perfect square

[16] Whaea means teacher in Te Reo Māori

The class was making shapes using the material (playdough or sticks with adhesive) provided to them to make shapes that they knew. Zara (female 9-year-old Māori-English bilingual Māori child with greater proficiency in English) self-selected and tagged the teacher as the next speaker by naming her (lines 189 and 190). In her utterance, Zara claimed that the shape that she had made of playdough and was holding up (deictic gestures) was a perfect square. To this claim, the teacher responded with a question to Zara, thus initiating the Initiation-Response-Evaluation/Feedback conversational pattern in the classroom. The teacher initially structured her question to ask if the shape was a perfect square (line 191); however, in the same turn, she rearticulated her question as "why is it a perfect square?", thereby leaving more space for the children's explanations.

Zara, in the following turn (line 192), stated she did not know, by saying "I dunno". She initially used a flat pitch and then lowered her pitch. Ward (2019) has shown that this type of construction, using a flat and low pitch, is often made to signal the listener that the speaker has given in and is not able to provide any further explanation. Thus, it can be interpreted that Zara initially intended to show the shape (line 189) that she made using playdough was perfect in terms of the physical appearance of the shape as smooth and flat, and she was not expecting a question from the teacher about the shape. Recognising that Zara had not thought of the reason, the teacher, in her next utterance, rephrased the question and emphasised "what" (line 193) to encourage Zara to think about the shape's properties. The teacher used longer pauses of two seconds (line 193) and one second (line 196) in the same utterance to allow Zara to bring some explanation of her claim.

In the next utterance, Matiu (male 11-year-old Māori-English bilingual child) self-selected and stated that being a square makes it a perfect square (line 197). In the following utterance (line 198), the teacher appeared to ignore Matiu's utterance. This may be because the teacher required children to raise their hands before speaking (FN1 to FN6). It seems that the teacher ignored Matiu's response, probably because he had not followed this classroom norm. The teacher used high pitch at the beginning of "anyone" and "know" to open the floor for all children to respond (line 198). Ward (2019) showed that high onset is often used for initiating a new topic. This time, the teacher looked at Matiu and provided him with her consent to speak. Matiu responded that a perfect square is perfect because it is a square, as he stressed the word "square" (line 200). The use of the HRT in English spoken in New Zealand often implies the speaker's intention to check if the listener follows what the speaker is trying to say (Warren, 2016). Thus, Matiu's use of HRT at the end of his utterance may be interpreted as his way to check with the teacher whether she agrees with his response.

In the next utterance, Garry (male 11-year-old Filipino-English bilingual Philippines child) might not have understood the use of this specific intonation pattern in New Zealand English as he laughed at Matiu's response (line 201). Jefferson (1984) showed that laughter within talk-in-interaction sometimes signals trouble, as the recipient engages in laughter to embarrass the speaker. In this case, it seems that Garry might have evaluated Matiu's response as wrong and redundant. In the following utterance, it appears that the teacher did not accept Matiu's response as she said that "Being a square doesn't tell me much" (line 202). Matiu (line 200) had used HRT to seek approval from the teacher. She selected Elie as the next speaker to answer "why a perfect square is a perfect square". In lines 204, 206-209, Elie used "um", stretches and pause of one second to construct her utterance. These features are often a mark of a non-response (Sacks, 1987). Thus, it may be that she was not sure of what the teacher wanted her to comment on about the square.

It appears that because Elie had not used geometry-specific language, the teacher did not accept her response but thanked her for her attempt (line 215). The teacher's utterance may be interpreted as an implicit rejection of Elie's response. The teacher did not overtly evaluate her response as incorrect. It seems that the explicit negative evaluation by the teacher was considered dispreferred. Moreover, the teacher rephrased her question (line 216) and stressed the words "geometry language" to direct the children's attention to the geometry-specific features of the shape that made it a perfect square. Following this cue, Zara and Matiu self-selected. However, Zara used "um" as a filler and started looking at the ceiling of the classroom in an attempt to recall the shape (line 219). Matiu (line 221) used "um" to hold the floor, and then he used his low tone (whispering) to state his utterance.

The teacher selected Matiu as the next speaker (line 222). He attempted to answer (line 223) by emphasising the word "face", but then he changed the term "face" to "sides". He used his whispering tone for his utterance. Gobl and Chasaide (2003) reported that speakers often use whispering at the end of their utterances to signal diffidence. Thus, Matiu's use of whispering tone and pauses of 2.5 seconds (line 223) may be interpreted as doubt and uncertainty. And as he was uncertain, Matiu, in this utterance, self-initiated a repair (a conversational mechanism to correct the use of the wrong word, as explained in Section 3.5.2) (Kitzinger, 2013). He realised that he might be wrong, and therefore he stated that he did not know. In the following turn (line 225), the teacher provided positive feedback and again stressed the word “side” as she stretched it and used slightly high volume to signify to children that the answer she was looking for was related to the properties of a square in terms of equal sides. In doing so, she clearly showed her intent for children to use geometry-specific language by explicitly asking about the property of sides in the square.

After receiving positive feedback from the teacher, Matiu responded that sides needed to be perfectly aligned with each other (line 227). However, this time as compared to his previous utterance (line 223), Matiu used HRT to check if the teacher agreed with him. It seems that the teacher acknowledged that Matiu might have been looking for agreement as he used HRT; thus, in the following utterance (line 228), she responded with a question to Matiu to let him reconsider his response. She used HRT at the end of her utterance, probably to signal the partial correctness of Matiu’s response. Matiu (line 229) realised that his answer was partially correct, but that he needed to restructure his response to meet the teacher’s expectation. Thus, he used a filler and paused for one second to hold the floor while looking for the right word (line 229). He again used HRT with his utterance “perfectly the same” (line 229) to check with the teacher. This time, the teacher stretched the word “same” to emphasise its use (line 230). She used a slightly high pitch along with stretching the first syllable. Moreover, she reiterated the phrase “sides are perfectly the same” three times in her following utterance (lines 230-233). During the focus group interview, I asked Matiu to explain what he meant by his discursive construction of “perfectly the same” in this Key Moment. He responded that “the square has equal sides” (Matiu, FG2). The focus group interview data revealed similar discursive constructions. For example, during the same focus group interview, Ethan stated, “a square needs same sides” (FG2). Moreover, the fieldnotes revealed that the teacher often asked children to use geometry-specific language to talk about shapes and their properties (FN1 to FN6).

The micro-analysis of this Key Moment (4.1b) revealed several findings. Firstly, pertaining to the geometric understanding of shapes, the data revealed that children may use everyday language to imply the property of equal sides by using words such as “aligned” or “perfect”. However, the teacher considered the responses given in everyday language as dispreferred responses and focused on developing children’s use of geometry-specific language. Secondly, regarding the conversation patterns, the analysis revealed that classroom interaction followed the Initiation-Response-Evaluation/Feedback pattern. The teacher initiated the conversation with a question. Children provided a response, and the teacher evaluated the response. If the child provided an incorrect response, the teacher provided feedback to assist the child in providing the correct answer. Moreover, if a child responded out-of-assigned turn, his/her response was treated as dispreferred by the children and the teacher. Children were required to follow classroom norms of participation. The teacher's explicit evaluation of a child's incorrect response also appeared to be dispreferred. Finally, the analysis suggests that native and non-native speakers of English may perceive the HRT intonation differently. For example, in this Key Moment, Matiu used HRT to check if the teacher agreed with him, rather than by asking a question (Britain, 1992). This intonation was apparently interpreted by Garry as a question or marker of uncertainty (Ward, 2019) in Matiu's response, which is probably why he laughed in the following utterance.

4.1.3 Summary: Making Sense of 2D Shapes

This section firstly presented the thematic analysis of data from fieldnotes, teacher interviews and focus group interviews, then a detailed micro-level analysis of two Key Moments (4.1a and 4.1b) identified to explore the discursive constructions of 2D shapes. With particular regard to what discursive constructions about geometry ideas were made (RQ1), the findings suggest that children may discursively use words from their everyday language mathematically to display their understanding of geometric properties of shapes. For example, the words "perfect" or "aligned" may signal children's understanding of "equal" sides as part of the geometric property of square; or the use of "aligned" may signal the understanding of sides as "parallel". Interestingly, "aligned" was not considered as geometry-specific language, whereas "side" was considered. Concerning the question of how these discursive constructions were made (RQ2), the findings reveal, first, that multilingual children may use prosodic cues available to them from their multiple languages, and that native and non-native English speakers may interpret the same prosodic cues of the English language differently. Second, the analysis suggests that some children (see Ozan, in Key Moment 4.1a) may use iconic gestures more than deictic gestures to represent their understanding of shapes. Third, the analysis also suggests that two kinds of children's responses may be considered dispreferred by the teacher and the children in a geometry classroom. These responses include (i) children's responses given in everyday language, and (ii) children's responses that are given out-of-assigned turn during classroom interactions. The teacher's overt negative evaluation of children's incorrect responses is also considered dispreferred.

The following section explores children's discursive construction of 3D shapes in two Key Moments during whole-class and group interactions.

4.2 Theme: Making Sense of 3D Shapes

In this section, the discursive constructions in which children displayed their understanding of 3D shapes are explored. During the first lesson, children were asked to identify the shapes in a given picture on a task sheet entitled “Shapes in Everyday Life”. One of the groups identified the sphere as a “3D circle” (FN1). Similar ways of describing 3D shapes were observed during other lessons. Some of the discursive constructions are presented below:

> [3D circle is] sphere. (Alyssa, FN1)
>
> [cube] a square box. (Ozan, FN4)

The children made similar discursive constructions even after the lessons were taught during focus group interviews. Some of these are:

> A dice is 3D square, jenga is 3D rectangle. (Zara, FG4)
>
> It’s like a sphere circle shape…like as a circle..and because it’s a 3D shape. (Elie, FG4)

To further explore these discursive constructions pertaining to 3D shapes, two Key Moments were selected from two different lessons (Lessons 1 and 2). These two Key Moments were selected for two main reasons. Firstly, in each of these Key Moments, “3D triangle” or “triangle 3D” were discursively constructed to refer to a triangular prism and/or pyramid. Secondly, two children, Matiu and Garry, participated actively in the discussion in both Key Moments. The first Key Moment (Key Moment 4.2a) is taken from the first lesson and presents a classroom interaction concerning a shape that the children identified as “triangle 3D”. The Key Moment explores how children developed and negotiated their understanding of what the phrase “triangle 3D” implies. The second Key Moment (4.2b) is presented from the second lesson, where children again conversed about the same shape.

4.2.1 Key Moment 4.2a: “What’s a triangle 3D? A triangular prism.”

As explained earlier, during the first lesson the teacher provided children with a group task. She divided the class into groups of three to four children for the activity called “Shapes in Everyday Life” (see Table 4.2), and provided each group with a different task sheet to identify the shapes they already knew. Once the children had completed the task in their groups she asked the whole class to sit on the mat. Each group was called to come in front of the class and talk about the shapes that they had identified. The group work (see Figure 4.3) was provided by the Group Korere (name of the classroom group) with children Alyssa, Tane, and Olivia, who worked on Task Sheet B together (see Figure 4.3).

Figure 4.3

Work Sample: Task Sheet B by Group Korere

As they reported the names of the identified shapes, the teacher wrote the names on the whiteboard. The group had labelled one of the shapes as "triangle 3D" (see the circled shape in Figure 4.3). The excerpt 4.2a shows the transcribed whole-class discussion that followed.

Excerpt 4.2a

#	Speaker	Text
547	Teacher	so they ve got(0.2)square(0.5)two d:(1.0)triangle.
548		three d:(0.5) ↑what is: a tri::angle three d
549	Ethan	((raised his hand)) it. is. [a:
550	Teacher	[<can anyone remember> what (1.0)a tri (1.0) Yue?
551	Yue	cube
552	Teacher	CU::BE(0.5)um kori cu:↑be is (1.0)a cube is a bit
553		Different (.)um::: Matiu ((teacher smiled and pointed to Matiu))
554	Matiu	tri:angular (0.5)a[::
555	Tane	[prism
556	Matiu	prism
557	Teacher	triangular prism gre:at.
558	Garry	I WAS ABOUT TO SAY Cone (1.0)
559		[anyone
560	Teacher	[triangular ((trailed off as she wrote on the board))
561	Tane	[(h)(h)(h)
562	Zara	°its triangular prism?°
563	Teacher	um ↑whats a ↑really really good example of a
564		triangular prism (.5)that. is. quite famous (1.5)=
565	Matiu	uaaa
566	Teacher	=that we see overseas and in lots of pictures

567		((points to Matiu for answering))
568	Matiu	um: the e::gy↑pt (1.0)[um:: ↑mountain thingy
569	Garry	[((puts his hands up))
570	Teacher	↑yeah the egypt mountain [thingy
571	Garry	[pyramid
572	Un	[pyramid
573	Teacher	yeah[the pyramid
574	Zara	[PYRAMID
575	Matiu	I said tha:t

Ethan (male, 11 years) is a monolingual English speaker. Yue (female, 10 years) is a Chinese-English bilingual Chinese child, and Tane (male, 11 years), Zara (female, 9 years), and Matiu (male, 11 years) are Māori-English bilingual Māori children. The teacher read the names of the shapes (line 547) from the task sheet (see Figure 4.3). She used a high pitch on "what" to draw children's attention to the following question (line 548). Walker (2017) shows that a high pitch at the beginning of the utterance may be used in English to draw attention, and this appears to be what the teacher is doing here. She also stretched the word "triangle" in this utterance while emphasising "three d" as she stretched "d" (line 548). It seems that she acknowledged the possibility of a three-dimensional shape that resembles a triangle, and she asked children to recall the geometric name for that shape. As previously noted, the teacher repeatedly asked children to use geometry-specific language to describe the shape that they had found (FN1 to FN6).

Ethan (male 11-year-old English-speaking monolingual) raised his hand to answer and began to speak without permission from the teacher (line 549). Although he had raised his hand to take the turn to speak, he was not selected by the teacher as the next speaker. She instead ignored his utterance (line 550) and selected Yue to take the next turn. Yue answered that the shape was a cube, using a flat pitch in her intonation (line 551). It has been argued that bilingual Chinese speakers often use flat pitch while informing (Pickering, 2001; Wu, 2004); whereas English speakers often use pitch peaks such as high onsets or HRT while responding to a question to seek confirmation or backchannel feedback from their listeners or addressee (Ward, 2019; Warren, 2016). Wu (2004) has also found that unlike English speakers, Chinese speakers tend to focus on information rather than the addressee. The teacher emphasised the word "cube" by using both increased volume and stretching (line 552). She may have used these prosodic features to get Yue's attention at the start of her utterance. Moreover, the teacher's use of "um" as a hedging device (Schegloff, 2007) at the start of her utterance may imply a polite rejection of Yue's response (line 552). The teacher's response may indicate that she regarded blatant negative evaluation of children's incorrect answers as dispreferred.

The teacher selected Matiu as the next speaker (line 553). Matiu (line 554) used stretching and a pause to hold the floor so that he could recall and state the full name of the shape. When Matiu was unable to do so, Tane self-selected and constructed his utterance (line 556) in alignment with Matiu's utterance. It should be noted that Tane's utterance (line 555) provided delayed feedback in relation to Matiu's utterance (line 554). Stubbe and Holmes (2000) have shown that explicit verbal feedback is a norm of Pākeha[17] (European-descent New Zealanders) conversation, which suggests that silence may be considered awkward by English speakers. However, Māori speakers do not consider silence during a conversation problematic and may, therefore, refrain from giving immediate feedback in order to facilitate communicational solidarity (Metge & Kinloch, 1978). Māori participants have also been shown to engage in co-operating by overlapping, where they expand and elaborate on each other's suggestions (Stubbe, 1998). It seems that Tane used his Māori interactional conventions (Stubbe & Holmes, 2000) to provide feedback to Matiu so that Matiu could complete his utterance. The teacher accepted Tane's response and started writing on the whiteboard as Matiu repeated (line 556) Tane's words to emphasise the shape. According to Tainio (2012), English speakers often use repetition as a way to emphasise. It should also be noted that the teacher positively evaluated Matiu's and Tane's cooperative response (line 557). Reed (2004) has shown that falling tone acts as one of the markers that signify completion in English language utterances. Here, the teacher seemed to use it with "great" to signify the completion of the task of naming the "triangle 3D" in geometry language.

Following the conversation, Garry (male 11-year-old Filipino-English bilingual Filipino) used high volume (line 558) in his utterance. Ward (2019) has shown that English speakers may use loudness to signal important information. In this case, it seems that Garry intends to draw the teacher's attention to his suggestion of "cone" as the name for the shape in question. Garry's utterance may also be interpreted as his attempt to continue the discussion on the possible geometry term for the shape by engaging in parallel talk. In his following utterance, Garry asked for agreement from his classmates (line 559). As the teacher went to write "triangular prism" on the whiteboard (line 560), Tane laughed at Garry's suggestion of "cone" (line 61), perhaps to convey that he had received his suggestion while rejecting it at the same time (Jefferson et al., 1987). Zara repeated "triangular prism" (line 562). Zara's repetition of the phrase shows her emphasis on the shape; however, Zara's repetition of the phrase shows her emphasis on the shape; however, her rising intonation at the end of the

[17] Pākeha is Māori term for New Zealander of European descent.

utterance and whispery voice probably signal her engagement in learning and intent for continued listening to the teacher (Ward & Tsukahara, 2000). The teacher (line 563) then constructed a question for the children when she asked about the example for the triangular prism. In the following turns, it is noteworthy that children discursively used both “prism” and “pyramid” for the same shape (lines 568-572).

The micro-level analysis of this Key Moment (4.2a) reveals several findings that respond to the first and second research questions. Regarding the discursive constructions that children used (RQ1), the analysis suggests that children discursively used “triangle 3D” to represent both a pyramid and a triangular prism during whole-class interactions. With specific relevance to the second research question, concerning how children interact to construct their understanding of shapes and their properties, the analysis revealed that multilingual children may use prosodic features from their multiple languages. Moreover, it was observed that multilingual children may interpret silence differently. In this case, the Māori child Tane delayed feedback in alignment with Matiu’s utterance after a silence of 0.5 seconds. The use of delayed feedback displays the use of a cultural conversational practice intended to show solidarity with the speaker. Contrastingly, English speakers may interpret silence as a sign of trouble (Hay et al., 2008a). The analysis also suggests that overt negative evaluations of children’s incorrect responses by the teacher are considered dispreferred responses.

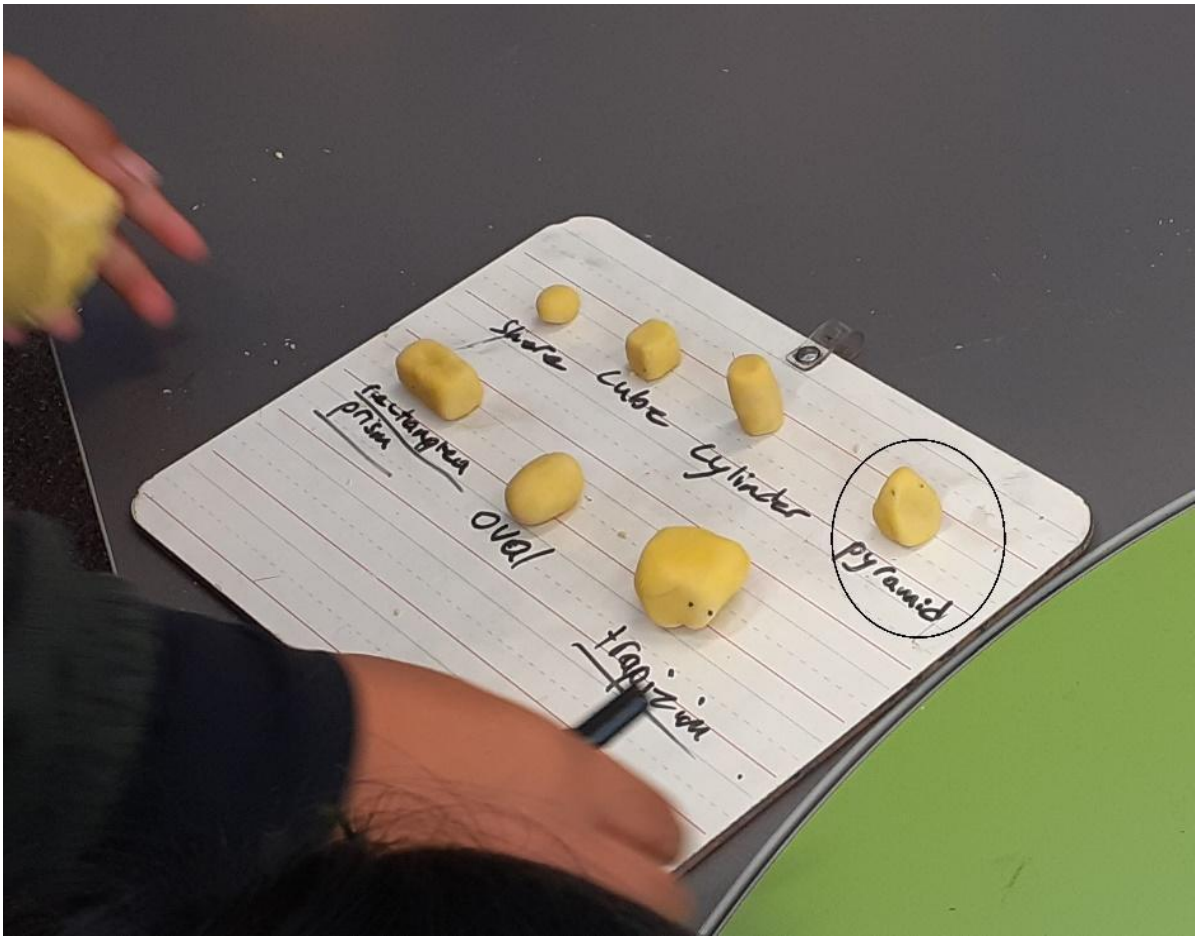
The analysis of the following Key Moment (4.2b) delves deeper into the children’s discursive constructions as they talked about the same shape in groups during the next lesson.

4.2.2 Key Moment 4.2b: “What’s a triangular prism?”

This Key Moment is taken from the second lesson. During this lesson, children were given playdough or sticks and adhesive to glue the sticks together. As explained earlier, the teacher divided the class into groups of four to five children. The interaction explored in this Key Moment is the group interaction of Group Marama, one of the groups observed during Lesson 2. This group had five children: Matiu, Ethan, Garry, Tahi, and Ozan. They were making solid 3D shapes using playdough. During this Key Moment, Matiu used the playdough to make a shape (see the circled item in Figure 4.4).

Figure 4.4

Work Sample: Matiu's Playdough Shape of Pyramid



The excerpt 4.2b presents the transcribed data of the group conversation about the shape made by Matiu.

Excerpt 4.2b

#	Speaker	Text
12	Matiu	I am gonna trying to (1.0) its like they are ol just
13		three: d:: you cant like (.) make (.) not make a (0.8)
14		fat
15	Garry	yes you ca:n↑

((Group started making different shapes))

#	Speaker	Text
42	Matiu	my next one is a (.)probably triangular prism
43	Garry	tri:angul[ar prism]=
44	Tahi	[°how you make°] tha:t
45	Garry	=I am ju↑st gonna make a tri:↓angle
46	Matiu	°a triangle?°
47	Garry	what↑s A triangular prism
48	Matiu	a three d one?
49	Garry	↑YES ITS A PYRA(H)mid (2.0)
50	Matiu	OR that

51	Garry/Tahi	((laughing))(3.0)
52	Matiu	↑then ↑whats a tri:angular prism (2.0) yayy
53		°simply a pyramid as well°
54	Ethan	°i dont know°

As the children started making playdough shapes, Matiu (male 11-year-old Māori-English bilingual Māori child) stressed "three d" by stretching (Couper-Kuhlen, 2009) and using pauses (lines 12-14). It seems that Matiu intended to say that it is not possible to make a flat shape because all the shapes that he will make using playdough would be 3D. In the next utterance (line 15), Garry (male 11-year-old Filipino-English bilingual child with more proficiency in English than Filipino) used his late pitch peak to suggest that it is in fact possible to make flat shapes. The use of the high pitch at the end of utterance with stretching of "can" (line 15) may be interpreted as indicating his confidence in his knowledge claim while correcting Matiu's misperception (Ward, 2019). The discussion ended abruptly here as children started making shapes using the playdough.

Matiu announced that the next shape that he was going to make is "probably triangular prism" (line 42). In the following utterance (line 43), Garry engaged in parallel talk (Speicher, 1993) to take part in the discussion. His use of stretching may indicate that he emphasised "triangular" to understand what shape Matiu was referring to. He did not specifically respond to Matiu's statement. It is possible that his utterance was directed to all the group members. Following Matiu's utterance, Tahi (male 11-year-old Tongan-English bilingual Tongan child) constructed his utterance by emphasising the last word of utterance, that is, "that" (line 44). It appears that Tahi employed this intonation pattern to stress triangular prism by stretching "that" (line 44). Tahi constructed his utterance as a question, asking how to make the shape. Matiu (line 46) ignored Tahi's question and responded to Garry's utterance (line 45). Matiu used a whispery voice along with HRT for this utterance. It seems that he was surprised to hear that Garry was going to make a triangle. The use of HRT may also indicate the possibility that Matiu had structured his utterance as a Yes/No question directed to Garry. In the following utterance (line 47), Garry responded to Matiu's earlier announcement about making a triangular prism. He used high pitch to initiate a question directed to Matiu regarding a triangular prism.

Following the question, Matiu responded with an answer "a three-d one" (line 48). It should be noted that Matiu, in this utterance (line 48), made use of HRT, probably to check (Warren, 2016) with Garry if he understood what Matiu meant. Acknowledging the cue, Garry, in his next utterance (line 49), used his high pitch and loud volume to respond to Matiu's utterance.

Wells and Corrin (2004) have shown that English speakers may use high pitch and loud volume to construct a competitive turn within the conversational space in order for a speaker to get hold of the conversational floor, that is, to hold his talking turn. Thus, it is probable that Garry constructed his utterance to argue for a different understanding of the shape. In this case, it seems that, according to Garry the shape in question was a pyramid and not a triangular prism. Potter and Hepburn (2010) have shown that the presence of a laugh in the current speaker's utterance (as displayed by (H), line 49) can sometimes mark a problematic use of a word or lexical item in a previous speaker's utterance. Thus, Garry's laugh may indicate that Garry believed that Matiu's understanding of shape as a triangular prism was incorrect.

Matiu, in his utterance (line 50), agreed to Garry's pronouncement of the shape as a pyramid, not a triangular prism. However, Matiu appears to use his high pitch (line 52) to emphasize his confusion about these two shapes. Moreover, he also used a pause and a whispery voice in his utterance. His use of a whispery voice may suggest that Matiu was doubtful if both pyramid and triangular prism are the same shapes. Gobl and Chasaide (2003) have shown that English speakers may use a whispery voice to show doubt. It also seems that Matiu did not direct his utterance to any one of the group members. Ethan (male 11-year-old monolingual English child) self-selected and stated that he was not sure of the two shapes. He used the low voice (line 53) probably to indicate uncertainty. Ozan (male 9-year-old Somali-English bilingual child with beginner English proficiency) did not participate in the conversation.

Keeping in mind the first research question about what discursive constructions children use to represent their understanding of shapes and their properties, the analysis reveals that one of the children (Matiu) suggested that it was not possible to construct 2D shapes using playdough. Second, the meanings of the discursive construction of "triangle 3D" are contingent upon the discussion that follows at a particular moment. Third, children constructed triangular prism and pyramid as the same 3D shape. In relation to the second research question that focuses on how children interact as they construct their understanding of shapes, the analysis suggested that prosodic (including high pitch, whispery voice, loudness of voice) features of children's interactional patterns may inform us about participants' emotional stances while they are learning geometric ideas.

4.2.3 Summary: Making Sense of 3D Shapes

This section has presented findings in relation to children's understandings represented through discursive constructions about 3D shapes. The findings are drawn from the thematic analysis of the data from fieldnotes, teacher interviews, focus group interviews, and micro-level analysis of the two audiovisually recorded Key Moments. For the first research question, which explores discursive constructions that children use to represent their understanding of shapes and their properties, two major discursive constructions were identified. The first discursive construction draws attention to the way children discursively represented 3D shapes as "triangle 3D" to refer to triangular prism and pyramid, "3D circle" was used to refer to a sphere, and the cube was referred to as "a square box" or "3D square". The second discursive construction was that anything made using playdough was perhaps perceived by some children as being "fat", implying that it is not possible to make 2D shapes with playdough. It is to be noted that only one child used this discursive construction to show his understanding of 3D shapes. The third discursive construction that children used concerns triangular prisms and pyramids. It is evident that the children used the discursive construction of "triangle 3D" to represent their understanding of triangular prism and/or pyramid. Moreover, the meaning of a term such as "triangle 3D" seems to emerge as the conversation proceeds. Pertaining to the second research question, that focuses on how these discursive constructions are made, the analysis suggests that prosodic cues play a crucial role in eliciting the emotional states that children hold while they are learning geometric ideas.

The next section presents a detailed analysis of two Key Moments where children discursively constructed the relationship between 2D and 3D shapes.

4.3 Theme: Relating 2D Shapes with 3D Shapes

This section explores children's discursive constructions about the relationships between 2D shapes and 3D shapes. The fieldnotes of six lessons observed for the study reported that children often used "sides" and "corners" to describe properties of both 2D and 3D shapes.

> house shape, five sides, five corners. (Kayla, FN2)
>
> [cube] it has six sides… no, six faces…twelve corners. (Alyssa, FN2)
>
> it's a square, it has four sides and four corners. (Alyssa, FN3)
>
> [triangular pyramid] this has three sides at the bottom. And [square based pyramid] has four sides at the bottom. (Matiu, FN5)

However, in the case of 3D shapes, the use of word "sides" may imply either edges or faces. For example, in Alyssa's description of a cube, the use of "sides" refers to the faces of the cube, whereas in Matiu's utterance "sides" refers to the edges of the bottom shape of the pyramid shape. It was noted that the teacher was responsive to this inconsistent use during the classroom interactions (FN2, FN3, FN4). During the semi-structured interviews with the teacher, she was asked about her thoughts on this inconsistent use of "sides". She said that she tried to reinforce the use of "faces" or "edges" while describing the properties of 3D shapes. She stated: "I tend to emphasise the use of edges, faces, and vertices while talking about 3D shapes as part of the geometry-specific language, especially while talking about 3D shapes" (Teacher, Interview 2).

In order to examine the discursive constructions that children use to represent their understanding of the relationship between 2D and 3D shapes (RQ1), two Key Moments were selected. The first Key Moment (extracted from the audiovisually recorded Lesson 1) explores the classroom interaction to elicit children's understanding of shapes and their properties as they describe and differentiate between 2D and 3D shapes during whole-class interactions.

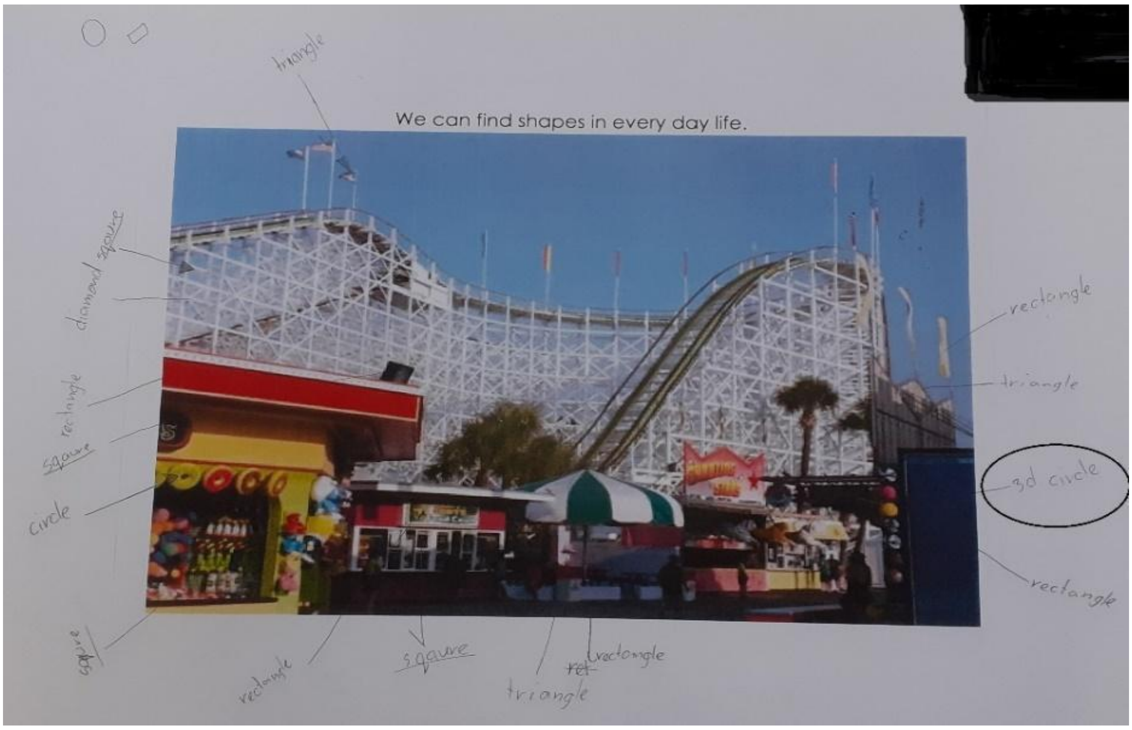
The second Key Moment was selected from Lesson 4. During the second Key Moment (4.4b), the teacher and children engaged in classroom discussion in which they identified 2D shapes within 3D shapes. This Key Moment presents a discussion about the 2D shapes that can be identified in a prism and how a distinction between a prism and pyramid is made. This distinction was not evident in the earlier discussion about prisms and pyramids (see Key Moment 4.3a).

4.3.1 Key Moment 4.3a: "sphere is a fat circle; a circle is a flat circle."

This Key Moment is from Lesson 1. As mentioned earlier, the teacher had divided the class into groups with four to five children in each group. During this Key Moment, Group Taimana (different from the earlier groups) had come to the front of the class to discuss the shapes that they had identified in the task sheet. There were three children in the group: Zara, Liliko, and Nikau. The group was given a different task sheet from the other groups (presented in Key Moment 4.1a and 4.2a) for the activity named "Shapes in Everyday Life" (see Figure 4.5).

Figure 4.5

Work Sample: Task Sheet C by Group Taimana



Zara is a female 9-year-old Māori-English bilingual Māori child. Liliko is a female 10-year-old Tongan-English bilingual child; and Nikau is a male 9-year-old monolingual English-speaking Māori child. This Key Moment presents the classroom interaction that followed when the group presented the shapes that they had identified. Excerpt 4.3a shows the transcribed data of the whole-class interaction.

Excerpt 4.3a

#	Speaker	Text
487	Teacher	is there anything in there tha:ts different(1.0)
488		fro:m um: some of them other shapes that we ve got
489		here (0.5) anything you can add
490	Zara	°no°
491	Teacher	so: you basically got(1.0)↑diamonds squa:res
492		<have I got> diamond on there?
493	Children	yes ((in chorus))
494	Teacher	Yup diamonds squares circle:s rectan::gles (1.0)
495	Teacher	um::↑three ↑d circle. oh (0.4)I have just got
496		((interruption))
497	Teacher	I have just got(.) um: a three: d: cir:cle=
498	Kayla	we had a (0.5) a three d? an a two d?
499	Teacher	=three d circle(0.5)↑ can ↑anyone (0.5)um: think of
500		the geometry term for three: d: circle
		((children raised their hands up to answer))
501	Teacher	um Alyssa
502	Alyssa	sphere?
503	Teacher	sph↑ere good gir:l(0.4) so ↑sphe:re (1.0) is: the

504		geometry ter::m (.)↑for a three d. so um: a sphere
505		is the(0.6)fat (1.0) cir:cle (1.5) a circle is the
506		flat (0.5) circle ((Kayla raised her hand)) (2.0) yes
508	Kayla	in our we had a two d(1.0) and a (.) three d
509	Teacher	you had a two d and a three d? in in yours too

The teacher (line 487) asked about the shapes that had not been mentioned by the earlier groups of children. Zara (line 490) self-selected and used her whispery voice to say "No". Stubbe (1998) has shown that Māori speakers often use silence and non-verbal signals as communicative devices for providing verbal feedback to the speakers. However, in this case, Zara's use of a whispery voice seems to imply a way to provide backchannel feedback to the question without interrupting the teacher's talk. Ward (2019) has shown that English speakers may provide backchannel feedback to show their intent of continuing listening. This use of backchannel utterance on Zara's part may signal a bilingual speaker's ability to use prosodic cues from each of her two languages. Acknowledging Zara's subtle response, the teacher continued her utterance (line 491) and read the names of the shapes from the group's task sheet. The teacher asked the children if she had written "diamond" as the shape on the whiteboard (line 492) to which the children responded: "yes" (line 493). In the following utterance (line 494), the teacher stretched the words "circle:s" and "rectan::gles" as she looked for a new shape in the task sheet (see Figure 4.5). At this time, the teacher identified a shape labelled as "3d circle" (see Figure 4.5) in the task sheet and used high pitch at the onset of the words "three" and "d" (line 495). Couper-Kuhlen (2004) has shown that the use of early high pitch by the speaker alerts the listener about upcoming new information or events. Thus, the teacher's use of high pitch may be interpreted as signalling a new topic for consideration.

The teacher then (line 497) emphasised the words "three d circle" by stretching and using a slightly higher volume to draw children's attention to the newfound shape. Kayla (female 11-year-old monolingual English-speaker) self-selected (line 498) and claimed that her group had also identified these "three-d" and "two-d" shapes. The use of HRT in her utterance displays her intention to check with the teacher if the teacher noticed these shapes in her task sheet (Warren, 2016). Since Kayla had not followed the classroom norm of raising a hand before speaking, the teacher ignored her utterance (line 499) and again emphasised the phrase "three-d" (line 500) by stretching. This emphasis may be interpreted as the teacher's acknowledgement of the phrase "three-d circle" to denote the shape. The teacher's use of high pitch (line 499) drew children's attention to a forthcoming question (Couper-Kuhlen, 2004).

Moreover, the teacher stated explicitly that she was looking for a geometry term for “three-d circle” (line 500). It is probable that through her construction (see lines 499-500), the teacher aimed to acknowledge the children’s way of describing the shape in question while, at the same time, initiating a conversation about the geometric term for the shape in question, which was a sphere. The teacher thus initiated a conversational mechanism of repair (Kitzinger, 2013) without explicitly rejecting the use of the phrase “three-d circle”. In this way, she displayed to her children that the preferable way of stating the name of the shape was to use geometry-specific language. The teacher had asked the question in lines 499-500. Children raised their hands to respond to the question and waited for the teacher to select the next speaker. The teacher selected Alyssa as the next speaker (line 501). Alyssa, (line 502) used HRT while answering the question. Alyssa is a New Zealand Pākeha child. The use of the HRT may indicate her intention to check if the teacher agrees with her (Warren, 2016). Following this cue, the teacher, in her next utterance (line 503), responded positively to Alyssa’s suggestion with “good girl”, and then explained that the geometry term for the shape in question is a sphere.

Analysis of fieldnotes data from the six observed lessons and the focus group interviews also revealed that the analogy of “flat vs fat” was used to distinguish between 2D and 3D shapes. Some of the transcripts from the fieldnotes are presented below:

> 2D is flat. 3D is fat. 2D, straight onto the ground, 3D, you can hold it, it’s fat, it’s solid. (Teacher, FN1)
>
> a rectangle has four sides…and four corners. If it is a 3D rectangle, it is very fat. If it is a 2D rectangle, it is very flat. (Elie, FN2)
>
> [Deck] card is flat rectangle… but Jenga is a fat rectangle…. A lot of stuff. Like a 3D has some stuff in it….2D is like flat, and it has nothing. It's like his, his body was like… it just.., it's like squished over from the car. (Ozan, FG1)

The micro-level analysis of this Key Moment highlighted a few findings about the discursive constructions of shapes when 2D and 3D shapes are studied simultaneously. Pertaining to the discursive constructions that children use to represent their understanding of shapes, the analysis revealed that children constructed the names of the 3D shapes in reference to 2D shapes (e.g., 3D circle, triangle 3D). The finding is also evident in the micro-analysis of the previous Key Moments (4.2a and 4.2b). Secondly, it was found that 2D shapes were differentiated from the 3D shapes by using the analogy of “flat vs fat”. In relation to the second research question, that seeks the ways in which children interact to construct their understanding of shapes, the analysis revealed that when a child gave responses out-of-their turns, their responses were considered dispreferred by the teacher. The finding was also evident in the Key Moment 4.1b. The use of prosodic features of multiple languages is also evident in the micro-level analysis of this Key Moment (4.3a).

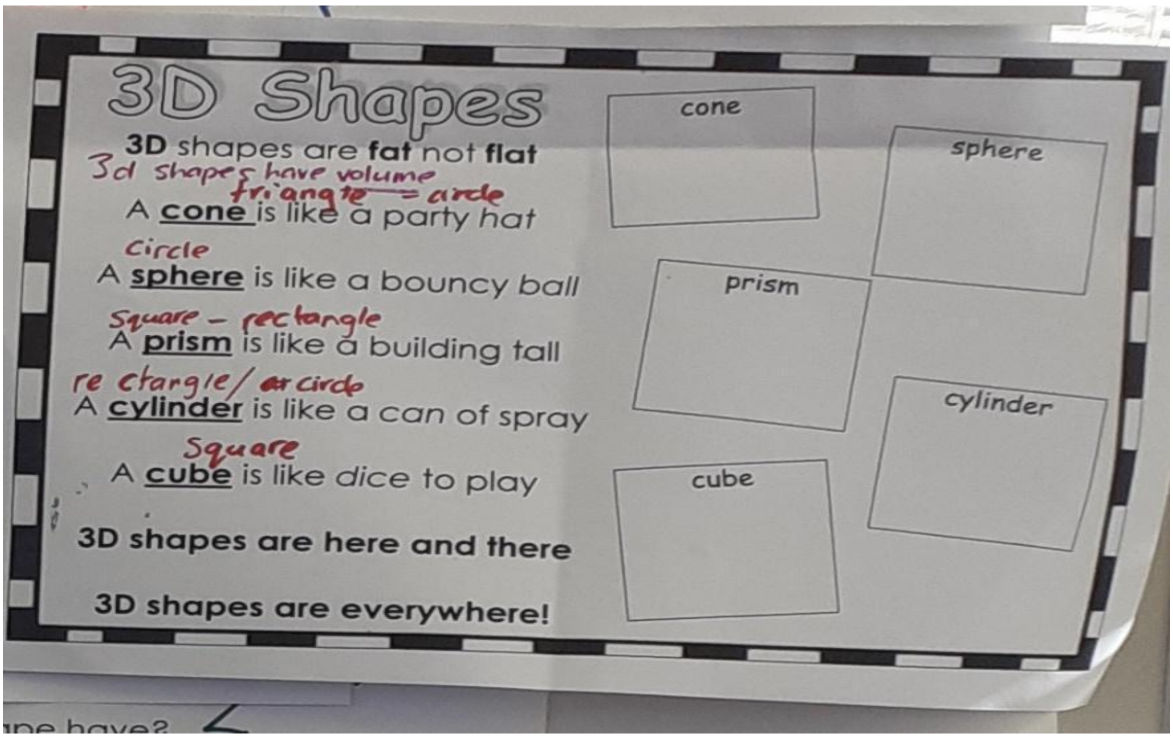
The next Key Moment explores another whole-class interaction to explore how children identify 2D shapes in 3D shapes.

4.3.2 Key Moment 4.3b: "the flat shapes within those ones with the volume."

This Key Moment is extracted from the fourth lesson. In this lesson, the teacher recited a poem for helping children recognise 3D shapes in real life. Using the poem, children were encouraged to imagine 3D shapes with 2D shapes within those 3D shapes. For example, a dice was given as an example for a cube (see Figure 4.6).

Figure 4.6

Work Sample: Whole-Class Interaction, Lesson 4



Fieldnotes inform us that during this lesson the teacher repeatedly mentioned the Learning Intention for the lesson. The Learning Intention was "We are learning to identify the properties of shapes using the language of geometry" (Teacher, FN4). The Learning Intention implied children were to learn to talk about shapes and their properties using the language of geometry. She also mentioned that she intended to replace the words "fat shapes" with "shapes with volume". Excerpt 4.3b presents the transcribed data of the classroom conversation that followed after the teacher recited the poem with the children.

Excerpt 4.3b

#	Speaker	Text
247	Teacher	lets start with a spe:re(.) ((children raised their hands
248		to answer)) whats(.)the flat shape(.) thats 0.5)
249		that(1.5)thats the other si:de of the.
250		two d:s(.)so two d is your flat.(0.5)
251		three d is your volume (0.8) um:: Olivia
252	Olivia	°circle°
252	Teacher	↑yeah cir:cle(0.8)so the fla:t or the two: d::
253		oops you alright? (0.5)the two d: to a:(1.0)
254		sp: sp: (1.0) spe::↑re(.5)is a circle?(.) >a
255		round circle on a piece of a paper<(1.5)but a
256		SPEre[18](0.4)thats got VOL:ume(.)thats
257		round(1.0) like this (4.0)is it ↑ still a
258		↑cir:cle: ((the teacher brings a ball))
259	Tahi	yeah
260	Children	yeah
261	Teacher	yea(0.5)its a↑circu:lar sha:pe but(0.4) its
262		not flat any↑more I cant (1.0) #squish it#
263		like tha:t (0.5)in↑side all of there is
264		volume(1.0)isnt it
265	Alyssa	°its a sphere°
266	Teacher	↑yeah (.) its: become: (0.5)three d: (.)its
267		go:ne ou:t from the wall (0.5)↑can you see it
268		(1.0)>so three d kinda goes out from the
269		wall< its goes outwards. an:d backwards.
270		(0.6)um: any questions to that so far
271		what would be the sha:pe(1.5)in ↑the cyli:nder?
272		(0.2)if you think of how a cylinde:r is ma:de
273		(3.5) ((gestures to show the shape of cylinder))
274		think about the cylinder (0.5) Zara
275	Zara	is it a rectan:gle?
276	Teacher	a rectan:gle (.) good gir:l (.6) Its a
277		[rec(1.0)
278	Matiu	[°rec°
279	Teacher	rec(0.5) recta:ngle: an:d(1.5)what is at
280		the ba:se? of a cylinder (1.0)or at the
281		bottom of the cylinder [so]
282	Zara	[oh]
283	Teacher	its the rectangle that makes it (1.0) ((children
284		raised their hands)) Ethan
285	Ethan	a circle?
286	Teacher	Yeah (1.0)so can ↑you see how three um: (1.0)
287		°o what it°(2.0)
288	Alyssa	circle (3.0)
289	Teacher	so our three d sha:pes have essentially got
290		(0.5)
291	Alyssa	two d
292	Teacher	two d insi:de them(0.5)a pri:sm (1.5)((the teacher gestured
293		to show the shape of a prism to children)) what shapes are inside
294		a pri:sm(0.5)>its like< a buil:di:ng(1.0) big(0.5)

[18] Here, the teacher is referring to "sphere", it is transcribed as it was said instead of how it should be as explained in Chpater 3 Methodology, Section 3.4.2.

295		ta:ll (.)buildi:ng. what(0.5)um: two d shapes are in
296		there (2.5)=
297	Zara	a bui[lding?
298	Teacher	=[if you think about a pri:sm (2.0) Garry
299	Garry	↓square (1.2)
300	Teacher	Squa:re(1.0)at(2.0)bo:ttom(1.0) what about
301		arou[nd the side(0.5)=
302	Zara	[outsi:de
303	Teacher	=a prism (1.5) think of a a big tall prism(1.0)
304		[pri:sn] (.) but a prism(0.5) ((children raised their hands))=
305	Zara	[OH]
306	Teacher	=Ethan
307	Ethan	a triangle?
308	Teacher	triangle? (0.5) its like that ((shows with her hands))
309		(2.5)=
309	Elie	°oh [no outside°
310	Teacher	[prism(.2) not a PYRA:MID (2.0)=
311	Matiu	ah ha (h)(h)(h)
312	Teacher	= Pri[sm =
313	Elie	[°he doesnt understa:nd this°
314	Teacher	=(3.5)um Matiu
315	Matiu	a recta:ngle
316	Teacher	↓yes. (2.0)

The teacher asked the children to tell her the 2D shapes that they could identify in the sphere (line 247). Children raised their hands to answer the question. The teacher used pauses within her utterance to hold the floor and to choose the next speaker. Moreover, she extended her utterance to give her time to decide the next speaker (lines 248-250). The teacher selected Olivia (line 251), who had raised her hand to answer the question. Olivia is a female 10-year-old monolingual English-speaker. She displayed her understanding with a whispering voice. It seems that Olivia used a whispery voice as a backchannel to the teacher's talk. Ward and Tsukahara (2000) have shown that feedback using a whispery voice often implies the speaker's tendency to end the turn quickly and resume listening. Olivia's response is assessed positively by the teacher in the next turn, when she used (line 252) her high pitch at the beginning of the turn to respond positively to Olivia's answer. The use of high pitch confirms the positive evaluation of Olivia's response (Lee, 2013).

The teacher stretched words "two-d" (line 252) and "sphere" (line 254), used high volume for "sphere" and "volume" (line 256) and repeated words "two-d" (lines 252-253) and "sphere" (lines 254, 256). Tainio (2012) has shown that repetition often acts as a powerful way of emphasising crucial information. Thus, the act of repeating words – "sphere", "two-d" – can be interpreted as the teacher's way of drawing children's attention to these terms so that children can grasp the terms' meanings. The teacher again emphasised the understanding of a sphere as a circular shape which is not flat (lines 261-264). Alyssa used a whispery voice to repeat that the shape is a sphere. The use of a whispery voice seems to indicate that Alyssa did not want to interrupt the teacher's talk (Ward & Tsukahara, 2000). In lines 266-267, the teacher used a high onset. Ward (2019) has shown that English speakers often use high onset as a marker to signal new information. It seems that the teacher used high onset to draw children's attention to a new explanation about 3D shapes, as she said "three-d kinda goes out from the wall" (lines 266-270).

Following the discussion about a circle as a 2D shape identified in the sphere, the teacher asked children about the 2D shape that they could see in the "cylinder" (line 271). In this utterance, she made use of several pauses within her turn. Goodwin (1980) has shown that speakers often use within-turn pauses to ensure the attention of all the participants. The teacher seems to use pauses here for this purpose, to make sure that all children are attentive to her incoming question. The teacher took 3.5 seconds (line 273) to allow time for children to think, as only two children had raised their hands to answer the teacher's question. During this time, she made use of gestures to help children recognise the two-dimensional shapes that she was referring to in the cylinder (line 273). In the following turn (line 274), the teacher selected Zara (female 9-year-old Māori-English bilingual child) as the next speaker because she had raised her hand. Zara (line 275) framed her answer to the question posed by the teacher in the form of a question. It seems that she stretched "rectangle" and used rising intonation to imply that "rectangle" would be the answer. The teacher evaluated Zara's response positively as she repeated Zara's response and used a positive marker, "good girl", in her utterance (line 276). She then wrote "rectangle" beside the "cylinder" on the task sheet (see Figure 4.6).

Although the teacher had gestured to draw the children's attention to consider the bottom of the cylinder (line 273), Zara did not name the shape at the bottom part of the cylinder. The teacher rephrased her question in her following utterances (lines 279 and 280) and explicitly drew the children's attention to the base shape of the cylinder. Zara's utterance (line 282) shows that Zara realised that she had missed the shape that the teacher was referring to when she gestured the shape. Following the cue from the teacher, eight children raised their hands to answer the question. The teacher selected Ethan (male 11-year-old monolingual English-speaking child) as the next speaker (line 284). Ethan took the next turn (line 285) and used a high rising intonation at the end of the utterance. The HRT may indicate his intention to check if the teacher agrees with him (Warren, 2016). Following Ethan's response, the teacher confirmed Ethan's claim (line 286) as she used a high onset in her utterance with "yeah". She took three pauses within her utterance. Her use of pauses may convey that the teacher got lost about what she was writing and where. Gauging this, Alyssa (female 11-year-old monolingual English-speaking child) self-selected and stated “circle” to remind the teacher (line 288). Ward and Tsukahara (2000) have shown that speakers often take conversational space to support the previous speaker by providing backchannelling feedback. Thus, in this case, Alyssa’s utterance may be interpreted as backchannelling feedback to show active listening and support for the teacher.

In the following turn (line 289), the teacher reiterated that “three-d shapes have two-d shapes” to direct children’s attention to imagine the 2D shapes in other 3D shapes. The teacher was completing her turn, with a gap of 0.9 seconds, Alyssa again self-selected and stated: “two-d” (line 291). As Alyssa took the turn after a long gap by the teacher, it seems that she formed her utterance as a supportive turn to provide feedback to complete the teacher’s utterance and signal her continuous listening (Couper-Kuhlen, 2009).

The teacher again took a gap of 0.5 seconds to signal the next shape (line 292), which was a prism. She paused for 1.5 seconds to see how many children raised their hands to answer the question. However, not many children raised their hands. The teacher therefore constructed her following utterance by stretching her words and gaps at several places to direct her children’s attention to the essential features of the shape (lines 293-296). Zara self-selected and used an HRT at the end of her utterance (line 297). Gussenhoven (2004) stated that rising intonation can indicate uncertainty or doubt. In this utterance, Zara probably used rising intonation to show her doubt rather than checking with the teacher. Since the teacher had not selected Zara as the next speaker, the teacher ignored Zara’s comment and instead selected Garry as the next speaker (line 298). In the following turn (line 299), Garry (male 11-year-old Filipino-English bilingual child with English as his proficient language) responded “square” with a low pitch. Ward (2019) has shown that English speakers use low pitch to signal authority over a knowledge claim. Thus, Garry’s low pitch may be interpreted as his confidence in his answer.

The teacher (line 300) used high pitch at the beginning of the utterance along with stretching the word “square” to approve of his answer (Ward, 2019) and used pauses within an utterance to take time to write. She asked another question in the same utterance. This time, Zara self-selected (line 302) and stated “outside” in the middle of the teacher’s utterance. Her utterance overlapped with the teacher’s utterance. It seems that she constructed her utterance as backchannel feedback to the teacher to assure the teacher of her continued attention.

Taking this cue, the teacher again expanded on her previous utterance (line 303) and selected Ethan (male 11-year-old monolingual English-speaker) as the next speaker (line 306). Ethan used HRT (Warren, 2016) at the end of the utterance to check if the teacher agreed with him (line 307). The teacher used HRT with the second half of "triangle" (line 308) to let Ethan reconsider his answer. The teacher again provided children with gestures to think about the shape that she was referring to. The teacher understood that the shape that the children were thinking of was a pyramid, and she wanted children to think of a rectangular prism. It should be noted that a triangular prism could be made using squares and triangles, a possibility that was not explored.

As the teacher explicitly mentioned that the shape was not a pyramid (line 310), Matiu (male 11-year-old Māori-English bilingual child) self-selected and tried to answer the question (line 311). However, he used only fillers in his response and raised his hand to respond to the question. He had realised that to get his response considered for the question, he needed to follow the classroom norm of raising a hand before answering a question. The teacher selected Matiu as the next speaker (line 314). Matiu gave his answer in a flat pitch (line 315). The use of flat pitch was probably to display his confidence in his knowledge claim (Ward, 2019). The teacher used her low onset and pitch fall with "yes" (line 316) to respond to Matiu's answer. The pitch fall signalled the completion of the task (Ward, 2019).

In this Key Moment, children's discursive constructions about the relationship between 2D shapes and 3D shapes were explored. The micro-level analysis of this Key Moment (4.3b) provides valuable insights towards answering the first two research questions. The first question concerns the children's discursive constructions about shapes and their properties. The analysis suggests that the children identified different 2D shapes in a 3D shape based on the *face* of the 3D shape that was referred during the interaction. Secondly, the analysis revealed that some children constructed prism and pyramid as the same 3D shape. This finding was also evident in the Key Moment 4.2b. With regard to the second research question, that focuses on the way children interacted to construct their understanding of shapes, the analysis revealed that a child's response was considered dispreferred if it was given an out-of-assigned turn during interactions; this finding is also highlighted in Section 4.1.

4.3.3 Summary: Relating 2D Shapes with 3D Shapes

In this section, data pertaining to children's understandings of relationships among 2D shapes and 3D shapes were explored. Data from fieldnotes, semi-structured teacher interviews, focus group interviews with children, and two selected Key Moments from two audiovisually recorded lessons were presented. With reference to the first research question, four discursive constructions were noted. First, it was found that children discursively used the word “sides” to denote both line segments of 2D shapes and faces or edges of 3D shapes. Second, children used the analogy of flat vs fat to differentiate 2D shapes from 3D shapes. Third, children discursively constructed the names of 3D shapes by referring to the prominent 2D shape and adding “3D” as a prefix. For example, “3D circle” (in Key Moment 4.3a). Fourth, children discursively constructed triangular prism and pyramid as the same 3D shapes. The second research question concerns how these discursive constructions were made. The analysis revealed two key findings. First, children’s responses provided out-of-assigned turn were considered dispreferred responses. Second, the analysis revealed that multilingual children used prosodic features of their multiple languages as they interacted in whole-class interactions.

In the next section, I present the analysis of the data pertaining to the next theme, that is, the mathematical construct of dimension.

4.4 Theme: Mathematical Construct of Dimension

This section explores children’s understanding of dimension as a mathematical construct. This section investigated data from different sources to elicit how children discursively constructed their understandings of dimension/s during whole-class and group interactions. During the first two lessons observed, it was noted that whenever children were probed to talk about what they understand by “D” in 2D and 3D shapes, they often stated:

> D is dimensions. (Kayla, FN1)
>
> like another world. (Matiu, FN2)
>
> 2D is flat...and 3D is fat. (Zara, FN2)

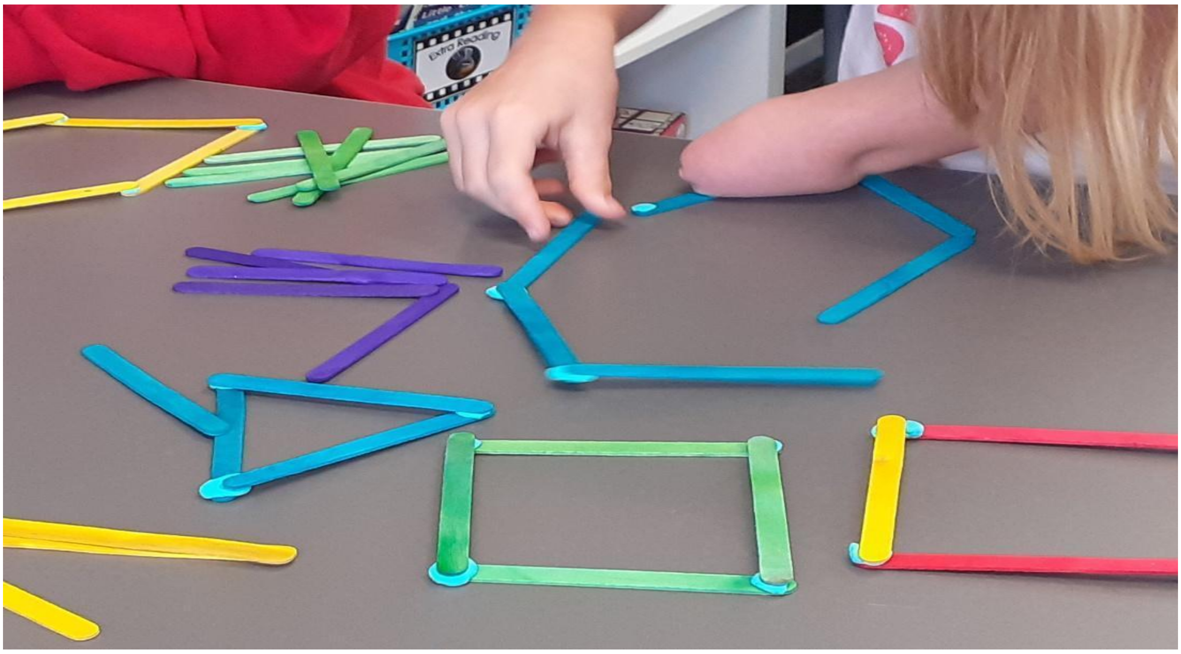
During the second semi-structured interview, the teacher reported, “Dimension is a difficult concept for children to understand… that’s why I used that video to show how dimensions work” (Teacher, Interview 2). The two Key Moments used for further analysis were selected on the basis of this conversation. The first Key Moment (4.4a) is from the second lesson and the second Key Moment (4.4b) is from the third lesson observed. Both the Key Moments explore the classroom interaction to investigate the discursive constructions that children and the teacher made to understand the mathematical construct of dimension. It is to be noted that although The NZC emphasises building children’s understanding of the mathematical construct of dimension; analysis of The NZC and key resources (NZMaths, 2021a) for teaching and learning of shapes reveals that no definition is provided for dimension as a mathematical construct (Ministry of Education, 2007; NZMaths, 2021a).

4.4.1 Key Moment 4.4a: "I think it's 3D because it's not 2D."

This Key Moment was extracted from the second lesson observed. During this lesson, the teacher provided children with playdough or sticks and adhesive to make shapes they already knew. Children were organised in either pairs or groups. Later, the teacher asked the children to come and sit on the floor with the shapes that they had made. The teacher initiated the classroom discussion and invited one child at a time to describe the shape that they had made. At the beginning of the activity, the teacher clearly asked children to describe their shapes using the language of geometry (noted from fieldnotes and audiovisually recorded data). In the excerpt that follows, the teacher invites Elie to describe the shape that she had made using sticks and adhesive (see Figure 4.7). Elie was a female 10-year-old English-Māori bilingual child with more proficiency in English than Māori. She had made a hexagonal skeleton or hexagon using sticks and adhesive (see Figure 4.7). The teacher asked her if the shape she made was 2D or 3D.

Figure 4.7

Work Sample: Elie's Shapes with Sticks and Adhesive



The following excerpt 4.4a shows the interaction presenting how the teacher and children displayed their understanding of dimension.

Excerpt 4.4a

#	Speaker	Text
341	Teacher	>↑anyone else got some right< (.) um: Elie
342		with your sticks
343	Elie	um::: (0.4) I forgot what this shapes called
344	Teacher	very good? ↑so ↑how many (0.6) so (1.6) ↑so (.)
345		um: [describe it]
346	Elie	°[its got] one two (2.0)° its: got one: two:
347		three four °five six° ↑its got six (0.2)
348		corne:rs
349	Teacher	got (.) six (1.5)
350	Elie	an::d its (.) go:t (1.5)
351	Teacher	Elie just hang on a minute (.) is it three d:
352		or two d: (1.0)
353	Elie	um:: I think its three d because °its not (.) a
354		two d° ((she was holding the shape and rolling it around her finger))
355	Teacher	put it down on a on the grou:nd (1.0) is it (.)
356		flat (.) or fat (0.5)
357	Elie	its fat (1.5)
358	Teacher	its fa:t (.) is it ↑coming ou:t towards you (1.0)
359	Elie	((looks at the shape holding it near the eye level))
360	Teacher	=okay lay it on the grou[nd (1.5)
361	Kimi	[°no its flat°
362	Teacher	its its okay. so: its not actually coming out
363		of the ground or going through the grou:nd (.)
364		so we call so we call (.) ↑we call that a two
365		d? (0.5) okay [so:(.2)
366	Elie	[↓uhm::
367	Teacher	↑its ↑got six co:rners (.) yeah
368	Elie	and its got (2.8) ((counted the number of sides)) and it
369		got six si:des
370	Teacher	six si:des good girl. I like you brought that
371		language (0.5)Okhay (0.3) ↑can ↑anyone help um
372		Elie (0.4) on what has ↑six si:des and ↑six
373		corners and is a and it is a two d shape
374		(1.0)um::: (1.3) Yue
375	Yue	a hexa:gon?
376	Teacher	ka pai so um you have actually made a hexa:gon
377		um: (0.6)
378	Elie	I know that thats called a hexagon ((hold and shows
379		the shape to the T))
380	Teacher	yeah a hexago:n has got six sides yeah (0.4)so°sort
381		of sort of a flat° (0.5) flat (2.0)um:

The teacher selected Elie as the next speaker (line 341) and asked her to describe the shape that she had made using sticks and adhesive. Elie used “um” as a hedging device and paused to get more time (line 343) in her response. Pomerantz and Heritage (2013) have shown that speakers often use “um” as a hedging device before providing a dispreferred response. In this case, not knowing the name of the shape was a dispreferred response.

The teacher (line 344) provided a positive evaluation of the shape that Elie had made and ignored her dispreferred response. She used high pitch at several places in her utterance, along with the use of pauses in her turn (line 345) to rephrase her question. Reed (2010) has shown that the use of high pitch occurs with the "interactional events that are designed as sequentially contrastive, or new" (p. 865). It seems that through the use of high pitch the teacher intended to design her turn to ask for new information about the shape. The teacher constructed her utterance with "how many?" probably signalling the number of sides, paused for 0.6 seconds and rephrased the question as a command (line 345), "describe it" (Hayano, 2013). The teacher's utterance seems to provide Elie with cues to direct her response in alignment with the teacher's expectation of stating the number of sides. Elie noticed the teacher's cue "how many" (line 344), and she started by counting the number of sticks that she had used to make the shape (line 346). At the end of her utterance, she stated that the shape had six corners. The teacher approved of Elie's response (line 349) as she repeated "six" and started to write on the whiteboard. Hellermann (2003) has shown that teachers use repetition of a child's response with level pitch as a conversational marker to show their approval of children's responses. The teacher (line 344) provided a positive evaluation of the shape that Elie had made but did not attend to her dispreferred response and asked Elie if the shape was 2D or 3D (line 351). To this question, Elie responded that the shape that she had made is 3D (line 353-354). Elie used a flat pitch for the first half of her utterance and a whispery voice for the second half. Research has shown that English speakers may use a flat pitch to display their authority or confidence (Couper-Kuhlen, 2004; Ward, 2019). However, a whispery voice at the end of utterance may indicate diffidence (Gobl & Chasaide, 2003). Thus, the use of a flat and whispery voice may indicate that Elie was partially confident of her claim. Moreover, the video-recorded data showed that Elie was holding the shape and spinning it around her fingers. It is to be noted that during the first lesson, the teacher had explained the difference between 2D and 3D shapes, as "2D is flat. 3D is fat. 2D, straight onto the ground, 3D, you can hold it, it's fat, it's solid" (Teacher, FN1). It is possible that Elie understood the shape that she made as three-dimensional as she could hold it.

In the following utterance, the teacher asked Elie to put the shape on the ground (line 355). As the teacher did not repeat Elie's previous utterance or use markers like "good girl", it is probable that the teacher evaluated Elie's response as incorrect (line 355). Moreover, she stretched "ground" to emphasise it, probably to provide Elie with a cue. The fieldnotes indicate that during this activity, the teacher often stated that if the shape is coming out of the ground, it is 3D, otherwise, 2D. It seems the teacher intended to use the same principle to help Elie to identify that the shape was 2D. The teacher rephrased her question and asked Elie if the shape was flat or fat (lines 355-356). The teacher did not emphasise "flat" or "fat" in her utterance. This lack of emphasis may imply that the teacher was expecting Elie to recall the “fat vs flat” distinction of shapes. Fieldnotes and audiovisually recorded data show that the “flat vs fat” analogy was often used in this class to describe 2D and 3D shapes. To this question, Elie (line 357) responded that the shape is fat. Elie’s flat pitch shows that Elie was sure of her answer (Ward, 2019).

The teacher (line 358) waited for 1.5 seconds before constructing her turn and then repeated Elie’s response (358); however, she stretched “fat” for emphasis. Hellermann (2003) has shown that silence in between turns can be interpreted as the current speaker’s (in this case, the teacher) orientation to the previous speaker’s (in this case, Elie) utterance as a dispreferred response. Moreover, the teacher used different intonation patterns (line 358) with the same words used by Elie (line 357). The use of different intonational patterns with the same words often implies contrast rather than agreement (Hellermann, 2003). Thus, it seems that the teacher again evaluated Elie’s response as incorrect. Therefore, she again provided Elie with feedback to reconsider her response (line 358).

The video-recorded data reveals that Elie held the shape at her eye level instead of verbally responding (line 359) to the teacher’s feedback in the previous turn. This may be interpreted as Elie’s way of restating that the shape is 3D as she could hold the shape in her hand. Noticing this, the teacher (line 360) asked Elie to put it on the ground. While the teacher was talking to Elie, Kimi self-selected and offered a repair on Elie’s turn. Kimi structured her response in low pitch (line 361) so that she did not interrupt the flow of conversation (Hay et al., 2008b).

The teacher attempted to build an understanding of the shape as 2D with Elie (lines 362-365). She used the HRT (denoted by question mark “?”) as a way to overcome a barrier to comprehension and build solidarity (Warren, 2016). Therefore, through her utterances, the teacher attempted to develop a mutual understanding with Elie, as she explained that the shape was not “coming out of or going through the ground” (lines 362-365). Moreover, the teacher used the “so we call” phrase (line 364) twice in her utterance; this use of “we” could be interpreted as displaying the teacher’s intention to persuade Elie to agree with her. Gerofsky (1999) has shown that teachers often use “we” in an unusual manner where who constitutes “we” is unclear to persuade children to agree with the teacher.

After Elie’s response in line 357, the teacher had provided multiple cues for the response that she was expecting from Elie. In her cues, the teacher had paused for 1 second (line 358), 1.5 seconds (line 360), and 0.5 seconds (line 365). In the her utterance (lines 362-365), the teacher provided an extended explanation of why the shape in question was 2D and had used HRT to check if Elie agreed with her. The teacher waited for 0.5 seconds for Elie's response. At this point, it seemed that Elie did not agree with the teacher's explanation.

Also, in the following turn (line 366), Elie used "um" as a hedging device, probably to convey that she was not convinced (Drew, 2013). In addition to this, Ward (2019) has shown that low and/or falling pitch may also be interpreted as a way to show declining interest in continuing a discussion. Thus, Elie's use of low pitch in this context may be interpreted as her way of indicating that she was not interested in carrying on with the conversation. With Elie's response (line 366), the teacher reiterated the first information that Elie had provided about the shape (lines 346-348). Ward (2019) has noted that high onset is often used in conversations to mark a change in the topic of conversation. Thus, the teacher's use of high onset (line 367) may be interpreted as an intended action to change the topic of discussion. Moreover, it might be that the teacher had realised that Elie was not convinced by her explanation; thus, the teacher attempted to change the topic of discussion. Elie picked up the cue and responded with the next property of the shape, stating: "it's got six sides" (lines 368-369). The teacher accepted Elie's response and showed her appreciation with the phrase "good girl" (line 370). The teacher explicitly made a comment about Elie's use of language. Here, the teacher referred to the use of geometry-specific language. As mentioned earlier, the teacher explicitly asked children to talk about shapes and their properties using the language of geometry.

The teacher then constructed her turn (lines 371-374) as a question asking for the name of the shape. She tagged Yue as the next speaker (line 374). Here, Yue (female 10-year-old Chinese-English bilingual child) used HRT (line 375). It seems that Yue aimed to check if the teacher agreed with her answer. The teacher approved Yue's response (line 376) and responded with "Ka pai", a Māori phrase, which is used to imply positive assessment and means 'well done'. The teacher (lines 380-381) acknowledged Elie's response and described the shape as "sort of a flat" instead of claiming it as 2D.

This Key Moment highlights several findings pertaining to the mathematical construct of dimension. The first research question concerns the discursive constructions about shapes and their properties, and the analysis of this Key Moment revealed that children used discursive constructions of "flat" shapes as 2D shapes and "fat" shapes as 3D shapes. However, the analysis of this Key Moment suggests that these discursive constructions may not be helpful in developing an understanding of dimension as a mathematical construct for some children.

In line with this observation, Matiu's (male 11-year-old Māori-English bilingual child) comment during Key Moment 4.2b, "I am trying it's like all three-d you can't like make.. not make a fat" can be interpreted as displaying the same confusion as experienced by Elie in this Key Moment. With specific concern to the second research question that focuses on the conversational patterns, the analysis revealed that Yue (female 10-year-old Chinese-English bilingual child) used HRT in her utterance. We have seen in Key Moment 4.3a that Yue also used her Chinese prosodic cues, which suggest that multilingual children may have multiple repertoires of intonational patterns at their disposal.

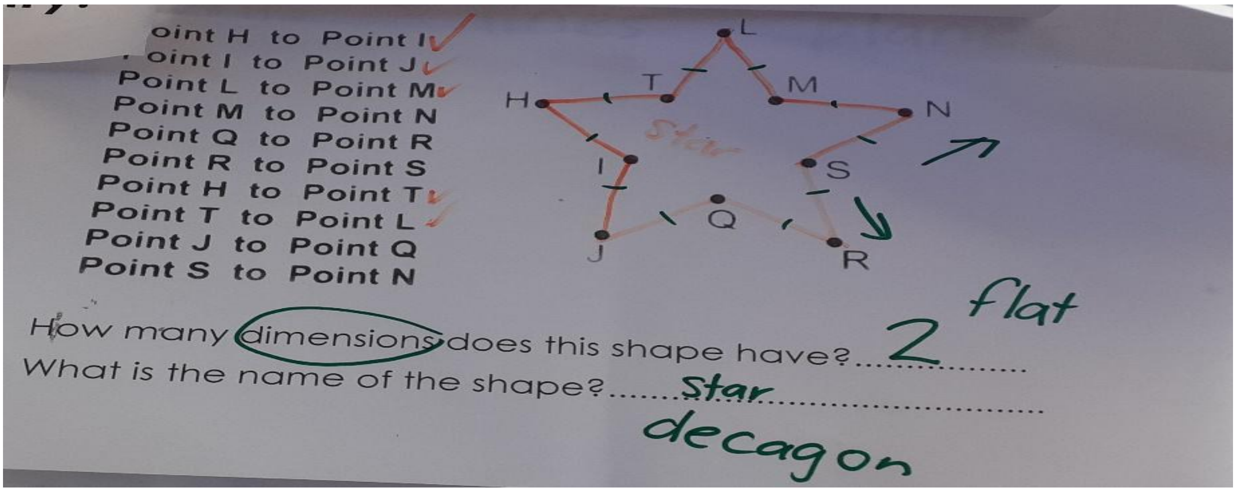
In the next section, I present another Key Moment for this theme. In Key Moment 4.4b, the teacher asked children how many dimensions a shape had after showing an instructional video designed to develop an understanding of dimensions.

4.4.2 Key Moment 4.4b: "How many dimension does this shape has?- umm ten."

This Key Moment is extracted from the third lesson. During the semi-structured interview with the teacher she stated that children tend to stick with the "flat vs fat" analogy for describing the shapes and are not really aware of what "D" implies in 2D and 3D shapes (Teacher, Interview 2). Thus, with the intention of developing children's understanding of dimensions, the teacher showed the class a video about dimensions. After showing the video, she gave the children a task sheet (see Figure 4.8) in which they were required to join the dots and then answer the questions asked in the task sheet. Children were given ten minutes to finish the task. Once they had finished the sheet, the teacher asked all the children to come on the floor to discuss the shape and answer the questions.

Figure 4.8

Work Sample: Whole-Class Discussion, Lesson 3



Excerpt 4.4b presents the transcribed data of the classroom conversation that took place when the children gathered on the floor with the teacher.

Excerpt 4.4b

#	Speaker	Text
585	Teacher	how many di:mensions does this sha:pe have(2.6)=
586	Zara	((puts her hand up for answering the question))
587	Teacher	=Zara
588	Zara	ten?
589	Teacher	(1.5) no↑(1.0)Ethan
590	Ethan	nine
591	Matiu	((laughing))
592	Teacher	↓no(1.0)think about what I am asking. I am
593		saying how many di:mensio:ns (.)does this
594		sta:r have(0.5)nikau
595	Nikau	°i dont know like°
596	Elie	°o my god°
597	Ethan	OH
598	Teacher	((circling the word ‘dimension on task sheet))
599	Ethan	TWO D::
600	Teacher	=how many dimensions in the space does this
601		star ha:ve (4.0)=
602	Zara	°Matiu whats dimension°
603	Teacher	=Elie ((as Elie raised her hand to answer))
604	Elie	five
605	Olivia	a[y?
606	Teacher	[no(1.0)Alyssa
607	Alyssa	ten
608	Teacher	no:(0.5)think about what the word we talking.
609		with di:mension (1.5)=
610	Zara	di:[mension
611	Teacher	=[do we go upto ten dimensions(1.0)
613		WE WOULD be on the movies if we went into ten
614		di:mensio:ns(1.0)would be on the sci fi
615		movie(0.5) Matiu
617	Matiu	two
618	Teacher	↑thank ↑you:[very much ((exhalation of breath, closed her eyes and slightly tilted her head back))
619	Ethan	[I SAID THAT

The teacher asked her children a question emphasising the word “dimensions” as she stretched the first part of the word (line 585). She paused for 2.6 seconds to see how many children had raised their hands to answer the question. As Zara (female 9-year-old Māori bilingual speaker) had raised her hand, the teacher looked towards her and selected her as the next speaker (line 587). Zara used HRT at the end of the word “ten” to check with the teacher if she agreed with her answer (Warren, 2016). However, in the following turn (line 589) the teacher responded with “no” with high pitch, indicating rejection of Zara’s response. The teacher did not provide any feedback at this point; instead, she selected another child, Ethan, by calling his name (line 589) as the next speaker. Ethan answered confidently (line 590) that the shape had nine dimensions. He used a flat pitch to construct his utterance with decisiveness (Ward, 2004). As Ethan said that the shape had nine dimensions, Matiu laughed (line 591). His laugh probably conveys his assessment that the answer was not correct (Jefferson et al., 1987).

The teacher took the following turn (line 592) and used a low onset while rejecting Ethan's response. Gussenhoven (2016) has shown that the use of the low pitch at the beginning of an utterance may imply a continuation of the topic. Thus, the use of low pitch here may indicate the teacher's intention to keep the flow of conversation. Moreover, in the same utterance (line 592), the teacher took a pause of one second. Rowe (1974) found that teachers often use pauses to wait for children to respond. It seems that the teacher here employed pauses to let children understand the question. Hence, she again paraphrased the question for the children. She emphasised the word "di:mensio:ns" by stretching (line 593) to draw children's attention to what they had just watched in the video before this task. She selected Nikau (male 9-year-old monolingual English-speaking child) as the next speaker. Nikau responded in his whispering voice (Gobl & Chasaide, 2003), showing that he was not sure of the answer (line 595).

Elie engaged in a parallel talk (line 596). She uttered, "oh my god", with her whispering voice. The use of a whispering voice was probably not to disturb the teacher's talk. However, her bodily gestures, such as rolling her eyes, probably indicated her disappointment with her classmates, who could not answer the question. It is interesting to note that by this time, Ethan had probably realised the correct answer. He expressed his thinking by saying "OH" (line 597). As the teacher circled the word "dimension" on the task sheet, Ethan used his loud voice (Ward, 2004) to state his response with confidence (line 599). However, Ethan's response was not considered in the classroom interaction, as he was not selected by the teacher as the next speaker. The teacher repeated the question (line 600). Zara (line 602) engaged in a parallel talk with Matiu to ask about the correct answer. The teacher selected Elie as the next speaker (line 604), and Elie confidently responded "five", which was again declared incorrect by the teacher. Zara repeated the word "dimension" said by the teacher (line 610), indicating her active listening (Hay et al., 2008b).

Following this discussion, the teacher explained why the answer could not be ten and selected Matiu as the next speaker (lines 611-615). Matiu responded with the correct answer (line 617), using a flat pitch. Ward (2019) has shown that flat pitch is often associated with being confident with knowledge claims. Thus, Matiu's use of flat pitch can probably be interpreted as showing his confidence in stating his response. The teacher, in her following utterance (line 618), thanked Matiu for the correct answer. Moreover, the teacher's gestures informed us that she was relieved to get the correct response, as she slightly tilted her head back. As the teacher approved Matiu's response, Ethan used a loud voice (line 619) to claim his authority over the response, as he had proposed it in his previous utterance (line 599).

The analysis of focus group interview data may suggest that even after watching the instructional video on dimensions, most children continued to have difficulty in verbally expressing their understanding of the mathematical construct of dimensions. Some of their explanations from focus group interviews are presented below:

> d is dimension. 2D is flat and 3D is fat. 3D has a lot of stuff. Like a 3D has some stuff in it. 2D is like flat and it has nothing. It's like his, his body was like he just, it's like squished over from the car. (Ozan, FG1)
>
> 3D is three ways to go and two 2D is two ways to go. (Matiu, FG2)
>
> it's like a different world. (Alyssa, FG3)
>
> D is dimension. Like they are at different place. (Zara, FG4)

The children's discursive construction during focus group interviews may be interpreted as evidence that a few of the children developed some understanding of the mathematical construct of dimension. In reference to the first research question, the micro-level analysis and the data from focus group interviews show that the children continued to discursively construct the understanding of dimension in reference to "flat vs fat". However, the use of this analogy may not help to develop a conceptual understanding of dimension. Second, children also discursively constructed dimension in terms of "ways to move" in different directions, or as "another world". Concerning the second research question, the micro-level analysis revealed two important findings. First, the children were required to raise their hands and be selected by the teacher as the next speaker before stating their response, or else their responses might be considered dispreferred. Second, the micro-level analysis of this Key Moment supports the finding (mentioned in the Key Moment 4.2b) that children may convey through prosodic cues their emotional stances of confidence or doubt in knowing geometric ideas.

4.4.3 Summary: Mathematical Construct of Dimension

In this section, children's discursive constructions about their understanding of dimension as mathematical construct were explored. Several findings can be drawn from the analysis. For the first research question, the analysis revealed that children discursively constructed dimension either through using the analogy of “flat vs fat” shapes for 2D and 3D shapes, or in terms of freedom to move in different directions as “different ways to move”, or as “another world”. The micro-level analysis also seems to suggest that the use of the flat and fat distinction of shapes may not help children to construe dimension as a mathematical construct. The discursive constructions also suggest that children find it difficult to express their understanding of dimension verbally. Pertaining to the question of how children interact to construct their understanding of shapes (i.e., RQ2), the analysis indicates that children are required to follow classroom norms of participation for their responses to be considered preferred. Second, it was found that the use of prosodic cues in children’s utterances may inform their emotional stances.

In the next section, I present an analysis pertaining to the last theme, which is “Naming shapes in Te Reo Māori”.

4.5 Theme: Naming Shapes in Te Reo Māori

The next set of Key Moments explores whole-class and group interaction pertaining to the use of Te Reo Māori for teaching and learning of shapes and their properties in a Year 5/6 New Zealand classroom. All the observed lessons were scheduled during the morning. Fieldnotes show that during the first ten minutes of each mathematics lesson the whole class engaged in naming numbers in Te Reo Māori and in telling the date for that particular day. The teacher, during the last semi-structured interview, said that children were “really aware of Te Reo number names” (Teacher, Interview 3).

The Key Moments presented in the following sub-sections discuss the names of the 2D shapes in Te Reo Māori. Interestingly, Te Reo Māori names for 3D shapes were not discussed in the Year 5/6 class. During the focus group interviews, multilingual children were asked if they knew what 3D shapes were called in their heritage language. Zara stated that the sphere is *poi* in Te Reo Māori.

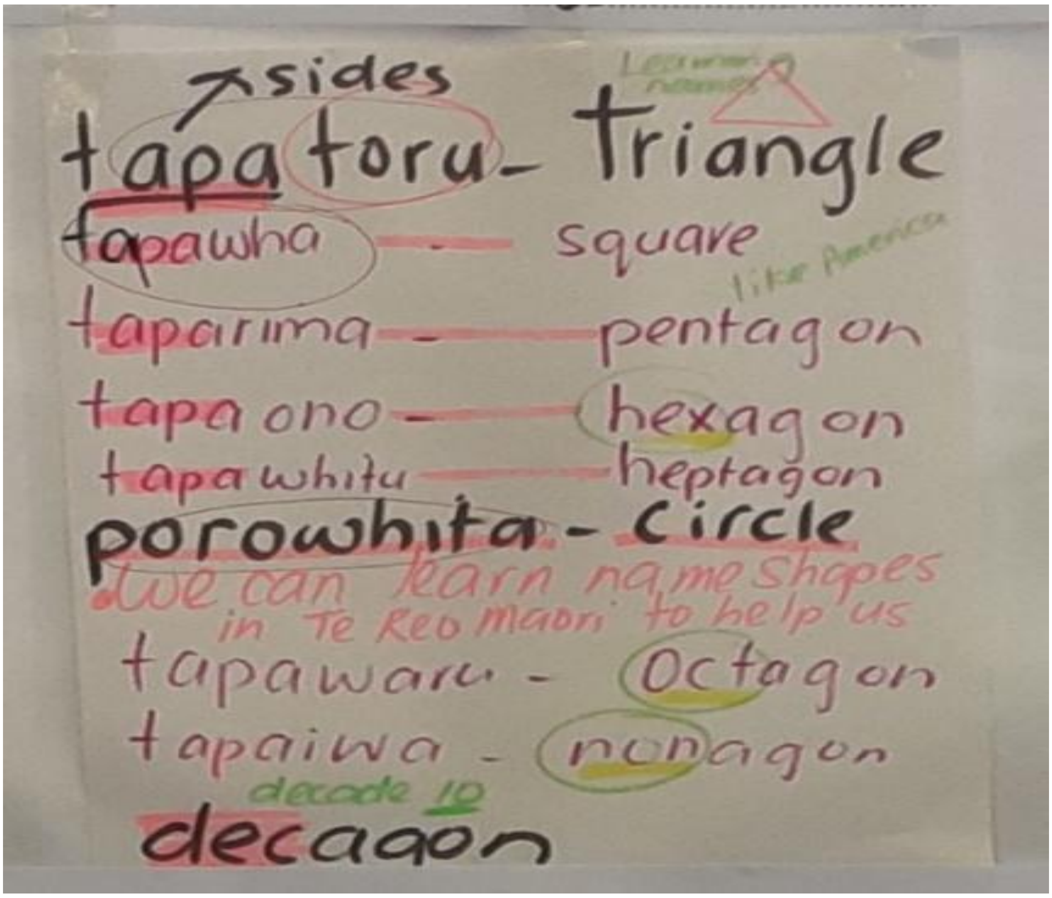
The first Key Moment (4.5a) presents a classroom interaction about the names of geometry shapes in Te Reo Māori. During this Key Moment, the teacher uses Te Reo Māori number names- *Tahi* for 1, *Rua* for 2, *Toru* for 3, *Whā* for 4, *Rima* for 5, *Ono* for 6, among others. The second Key Moment (4.5b) presents a group interaction about the name of the square in Te Reo Māori.

4.5.1 Key Moment 4.5a: "A circle has no sides."

This Key Moment is selected from the second lesson. During this Key Moment the classroom discussion was focused on the Te Reo Māori names for the shapes. The teacher engaged children in exploring the meanings of Te Reo Māori shape names by linking their knowledge of Te Reo Māori numbers to identify the shape (see Figure 4.9). During the classroom interaction, the teacher repeatedly encouraged children to state what they thought without wondering whether it was correct or not. Soon, children started following the pattern. During the discussion, they displayed their understanding that since toru in Te Reo Māori means three, and *tapa* means edge, *tapatoru* would imply a triangle. Similarly, *tapawhā* is a square as whā in Te Reo Māori means four, and *taparima* is a pentagon because rima in Te Reo Māori is five. As the children followed the pattern and identified the shape denoted by the Te Reo Māori shape name, the teacher wrote these on a white sheet, as shown in Figure 4.9.

Figure 4.9

Work Sample: Te Reo Māori Names for Shapes, Lesson 2



The excerpt (excerpt 4.5a) shows the transcribed classroom interaction that followed as the teacher asked them about *tapaono*.

Excerpt 4.5a

#	Speaker	Text
557	Teacher	↑what abo:ut um:::(1.0) whats a
558		tapa:ono(6.0)=
559	Zara	hhhhhhh (aspiration)
560	Teacher	=umm ↑Kayla(.4) what wud a tapaono be (8.0)
561		↑tapari:ma pentagon(.)tapaono: Yue↑
562	Elie	((holding the hexgonal skelton that she made using sticks to show its a
563		hexagon))
564	Yue	hexa↑go:n
565	Teacher	good girl. hexagon(4.5)
566	Elie	°I told you°
567	Teacher	um:↑(h)(1.2) heres my question.(1.0)
568		could ↑I ha:ve(2) a tapatahi:(1.5)
569		((Elie, Yue, Matiu, Ethan said no in chorus))
570	Zara	YES a cir ((teacher smiled as Zara responded))
571	Matiu	a circle has no ↓side
572	Elie	[no >↑you cant ↑you cant< °you cant°
573	Ethan	[CIR↑CLE::
574	Matiu	[°no a circle has no side°
575	Elie	[coz ((put her hand up for answering))
576	Teacher	can ↑I have a tapatahi.(.4)can I.(1.0)↑what
577		↑what woud tap ↑if I followed that pattern what
578		would tapatahi be Elie
579	Elie	a circle. but ↑you ca:nt have it bcoz circles
580		have no si:des [and] no corners
581	Teacher	[hmm] ↑you are ↑brilliant
582	Teacher	accepted.
583	Teacher	so ↑what do we ca:ll a circle
584	Tane	a ha::o cylinder. (h)
585	Teacher	<what do we call a cir:cle> when ↑I say (.)
586		can you get into a umdumdumdumd..
587	Ethan	↑PORA°whita°
588	Teacher	[umdumdumdumd(h)(h)(h)
589	Teacher	>what is it<(h)(h)
590	Ethan	porowhita
591	Teacher	ya↑ya↑ porowhita you get all the time (.4)
592		so a poro:whi:ta is a cir:cle (2.2)

The teacher used high pitch with "what" (line 557) to draw children's attention to the new question; moreover, she used "um" as a filler and a pause of one second to allow time for thinking about the next shape (Ward, 2004). The teacher waited for six seconds (line 558) to select the next speaker as the children raised their hands. The teacher selected Kayla (female 11-year-old monolingual student), who had not raised her hand to answer, to take the next turn (line 560). Kayla did not answer in the next turn. The teacher stressed "rima" by stretching in taparima probably to draw children's attention to the meaning of rima and its relation to "pentagon" (line 561). In the same utterance, she constructed the question again for tapaono, where she stressed the "ono" to encourage the children to think of it in terms of

Te Reo Māori numbers. In addition to this, she selected Yue as the next speaker as she called her name at the end of her utterance.

It should be noted that though Yue (female 10-year-old Chinese-English bilingual speaker) took the turn allocated to her by the teacher (line 564), Elie stated her answers with her deictic gesture as she was holding the hexagonal shape she had made with sticks and adhesive. Yue responded with the use of high pitch and stretching the last syllable of "hexagon" (line 564). Research indicates that English speakers usually use high pitch and stretching for stressing and insistence (Ward, 2019). However, Chinese speakers usually use flat pitch for stressing, where they focus on information, not the addressee (Wu, 2004). For showing authority, Badan and Cheng (2015) have shown that Mandarin Chinese speakers often use high pitch and stress at the end of the utterance to show their orientation to the addressee and their stance on the statement. Whereas English speakers use low pitch for showing dominance or authority over their knowledge claim (Ward, 2019) and rising intonation or pitch for showing their orientation to the addressee or low pitch for showing dominance or authority over their knowledge claim (Ward, 2019). Following these interpretations of prosodic cues, Yue's use of these intonational patterns may be interpreted as (i) her attempt to highlight her authority over knowledge claim and (ii) to emphasize her answer to the addressee, in this case, the teacher.

In the following turn, the teacher responded with "good girl" (line 565) to show a positive assessment of Yue's response. At this moment, Elie too claimed that she knew the answer (line 566). Her use of whispering probably conveys that she did not intend to break the flow of conversation yet was interested in claiming the knowledge. It seems that Elie engaged in parallel talk.

The teacher used a high pitched "um" and a pause of 1.2 seconds (line 567) to think about how she wanted to construct her next question (Ward, 2019). She composed her question as "could I have a tapatahi?" with two seconds pause within the turn. It is interesting to note that earlier, the teacher structured her utterances as "what would tapatoru be?", or "what would tapawhā be?" or "what's tapaono?". However, when the teacher asked children what they thought *tapatahi* might be, the pattern of her utterances changed. In place of "what would tapatahi be?", the teacher framed her utterance to ask her children if she "could have a tapatahi". It was revealed in the semi-structured interview that the teacher asked about tapatahi to encourage discussion about the possibility of there being a one-sided figure (Teacher, Interview 3). Thus, it is probable that she used "could I have" to engage children to explore the possibility of having a one-sided shape as well as to provide them with an implicit cue for contrastive thinking. Thus, the utterance “could I ha:ve (.2) a tapatahi” (line 568) implied to children that tapatahi might not be possible.

Following the teacher’s utterance, four children (Yue, Elie, Matiu, and Ethan) replied “no” (line 569). However, it seems that Zara (female 9-year-old Māori bilingual child) constructed tapatahi as a circle. In the following utterance, she self-selected and responded loudly “YES” (line 570), probably to signal her confidence (Ward, 2004) that tapatahi would be a circle. However, her falling pitch with “a cir” seems to indicate that she realised she was wrong the moment she saw her teacher smiling. Research has shown that a smile has several interactional purposes (Haakana, 2010). Sert and Jacknick (2015) have shown that a smile may indicate a trouble with the previous turn. Thus, in this case, the teacher’s smile may be interpreted as displaying the inacceptibility of Zara’s response (line 570). It seems that the teacher used her smile to let her children think in the opposite direction instead of signalling yes to the possibility of tapatahi.

Noticing the teacher’s smile, Matiu (male 11-year-old Māori-English bilingual child) self-selected as the next speaker and stated that circles have no side (line 571), using flat pitch that signalled his confidence in his claim (Ward, 2004). Although the teacher had not asked children for an explanation, Matiu and Elie understood that this time the teacher was looking for justification along with their response. Elie (female 10-year-old English-Māori bilingual Pākeha child with beginner fluency in Te Reo Māori) self-selected (line 572) and negated the possibility of having a tapatahi. Elie’s utterance overlapped with Ethan’s utterance (line 573).

Ethan’s use of a loud voice in this utterance shows that he was confident (Couper-Kuhlen, 2009) about the possibility of having a tapatahi. Ethan also constructed tapatahi as a circle. However, Matiu, who was sitting next to Ethan, again repeated with a whispering voice his earlier claim that a circle has no sides. He may have used his low pitched voice to make his claim without disturbing the flow of classroom interaction (Hay et al., 2008). It seems that Elie realised that she needed to raise her hand to take the next turn in the classroom interaction (line 575), after not being picked by the teacher as the next speaker in the earlier turns.

At this point the teacher changed her utterance to convey her expectation that the children should be able to work out the answer by considering the pattern, on the one hand, and thinking of a reason for its exception, on the other hand (lines 576-578). Through this utterance, the teacher hinted to the children to frame their answers in a particular manner.

That is, the answer to the question of "what would tapatahi be?" should make use of geometry vocabulary. Elie had raised her hand to get the next turn to speak, and the teacher selected her as the next speaker (line 578). Elie, in her following turn (line 579), started her utterance with "a circle" to imply that if the pattern was followed, tapatahi should imply a circle. However, she stressed the words "you can't" by a slightly high pitch and stretching to emphasise that the shape cannot be a circle as circles have no sides and no corners. The same understanding was iterated by another child, Ozan, during a focused group interview, as presented below:

> shape like a circles uhm circles zero side.. is like is like no sides. So people say it has one side. It doesn't. Because this is a circle (counter) and the circle doesn't have sides. like a quarter like these quarters like these quarters doesn't have corner or side. (Ozan, FG1)

The teacher provided a positive evaluation of Elie's statement (lines 579-580) with "hmm" (line 581), which overlapped with Elie's utterance. Realising that Elie had not finished, the teacher waited for her to finish. As Elie finished her utterance (line 580), the teacher presented with a positive marker, "you are brilliant" (line 581). Following the conversation, the teacher, in her next turn, used high pitch with "what" (line 583) to emphasise the Te Reo Māori name for circle, *porowhita*. Tane (male 11-year-old Tongan child) attempted to make a guess (line 584). Realising that children were not able to recall the shape's name, the teacher provided them with a hint (line 585) to think of the word that she often used when she asked them to sit in a circular shape. It is to be noted that the teacher had not selected the next speaker at this time. However, Ethan (male 11-year-old monolingual English-speaking child) shouted the first part of "porowhita" in his utterance (line 587) that indicated his excitement. Wilkinson and Kitzinger (2006) have shown that English speakers often use a very loud and high pitched voice to mark their sense of surprise and excitement. Since the teacher had not selected Ethan as the next speaker, noticing that Ethan had come up with the right name for the circle, she then selected him as the next speaker (line 589) through her gesture. This time Ethan responded in his flat pitch (line 590). The teacher acknowledged his response (line 591) and for emphasis repeated that a porowhita is a circle (Hellermann, 2003). The fieldnotes indicate that the teacher often used "porowhita" to ask children to sit on the mat in a circle.

The teacher reported in the last semi-structured interview (Interview 3) that she referred to the unit "Te Whānau Taparau – the polygon family" (See Appendix A) for teaching shape names in Te Reo Māori. The unit focuses on investigating the spatial features of shapes, primarily regular polygons, and uses both English and Te Reo Māori to describe different polygonal shapes. In this document, regular polygons are defined as the enclosed shapes formed by line segments of equal lengths, such as equilateral triangle, square, regular pentagon, and so on. This document introduces the circle as a shape that is not a polygonal shape, but it does not give the reason why a circle is not a polygon. The text indicates that all polygons have sides, which are straight. This assumption may explain Elie's explanation of why a circle cannot be tapatahi. Moreover, the document analysis of this unit also reveals some of the ambiguous terms used for naming shapes, for example, both tapawhā and tapawhā rite have been used for naming a square, which may lead to confusion, as evident in the next Key Moment (Key Moment 4.5b).

The analysis of this Key Moment explored children's discursive construction of naming shapes in Te Reo Māori (RQ1) and examined the ways in which children interacted to construct their understanding of shapes and their properties in Te Reo Māori (RQ2). For the first research question, the analysis of Key Moment (4.5a) suggested that children constructed the names of the 2D shapes by using the prefix "tapa" with the number of sides of the shape. Using this rule, a few children constructed "tapatahi" as circle. The shapes that were discussed were polygons but without specifying this category of 2D shapes, which was probably not clear to every child. Second, as the shapes discussed were only 2D polygonal shapes, it seems that children used the term "side" to signal the straight sides of the shape. The curved side of the circle was not considered as a "side". In other words, the analysis of this Key Moment draws our attention to the taken-for-granted understanding of "side" which assumes straightness is its innate characteristic. Moreover, Te Reo Māori names for 3D shapes were not discussed in the class. With respect to the second research question, the analysis suggests that children's emotional stances are visible through subtle yet significant prosodic features.

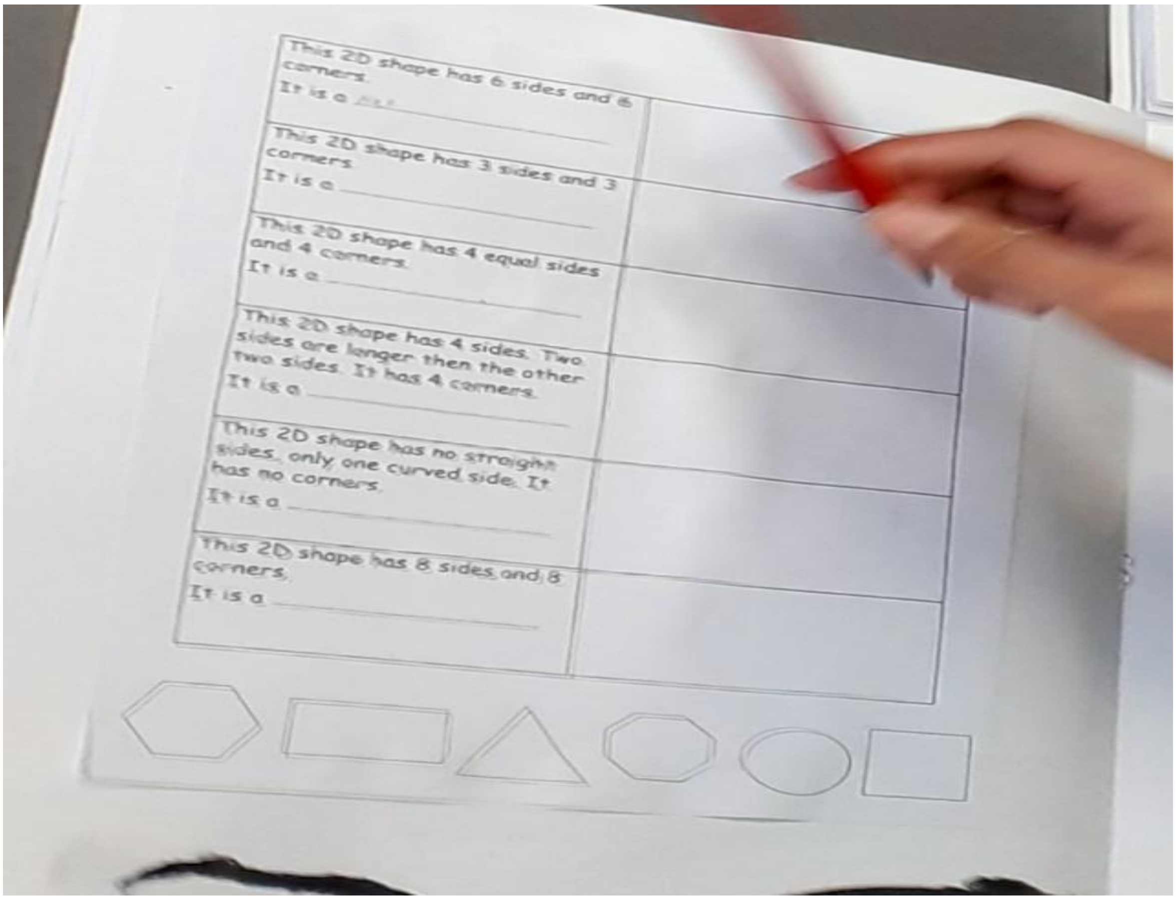
The next Key Moment is taken from Lesson 3 to show how ideas from this lesson were carried to the following lesson. Key Moment 4.5b demonstrates how different shapes can be represented through "tapawhā", following the western world rule of naming shapes.

4.5.2 Key Moment 4.5b: "If square is tapawhā, what's rectangle then?"

This Key Moment is taken from the third lesson. In the previous lesson, the class had discussed the shape names in Te Reo Māori. In this lesson, the teacher gave the children a worksheet (see Figure 4.10) to complete. They were then asked to identify shapes based on their properties. As the teacher discussed the task with the children, she asked them to write the Te Reo Māori names for each shape as well in the task sheet provided to them (see Figure 4.10). She grouped children and asked them to discuss the shape with their group members if they had any difficulties. Zara and Olivia were paired to work together. Zara is a female 9-year-old Māori-English bilingual child. Olivia is a female 10-year-old monolingual English-speaker. During this interaction, Zara asked Olivia about the name of the rectangle in Te Reo Māori.

Figure 4.10

Work Sample: Worksheet on Naming Shapes Based on Properties



Excerpt 4.5b shows the transcribed data of a group interaction among Zara, Olivia and the teacher.

Excerpt 4.5b

#	Speaker	Text
54	Zara	whats rectan:gle in ma:ori (0.5) tapawha: I
55		think thats four (2.5) whaea Jenny:?(0.5)if
56		square is tapawha: wha:ts rectan:gle?
57	Teacher	rectan:gle::s um:::(2.0)I think
58		[honestly] I am not su:re(1.0)

Zara	[is it]
Teacher	so just just leave leave it (2.0)↑good question
Olivia	tapari:ma is (.) actually (2.5) is actually a
	↓a pentagon °i thought so°
Teacher	No. i dont think it is (3.0)
Zara	I am gonna find the actual name for it
Teacher	°can you find it out for me° yeah I dont think
	it is I know definitely tapa:wha: is an:d I
	know definitely porowhita is >but I only threw
	the ↑other ones in< (1.0) yeah I put the other
	ones in because it help us remember what they
	are (.) okay

Zara started filling the task sheet. However, as she reached the description of the fourth shape in the task sheet, she wondered what would be the name for a rectangle in Te Reo Māori. Zara verbalised the question as she tried to think of the Te Reo Māori name for the rectangle in Māori (line 54). This was a form of self-talk. It seems that Zara supposed that the shape might be tapawhā, as she took a pause of 0.5 seconds. She also provided an explanation for her response. However, her use of "I think" indicates her doubt about the term. This is because, in the earlier lesson, tapawhā was discussed as the name of the square. Her explanation of "that's four" indicates that she was following the pattern of a number of sides with the shape name that they had noticed in the previous lesson. She took 2.5 seconds to think about the shape name (line 55).

Wondering about the correct Te Reo Māori name for the rectangle, Zara used high rising intonation (lines 55-56) to construct her utterance as a question directed to the teacher (Couper-Kuhlen, 2009). It should be noted that she tagged the teacher as the next speaker at the beginning of her utterance. She did this to draw the teacher's attention, as the teacher was not near her group. To respond to Zara's question, the teacher stretched "rectangle", used "um" and a pause of two seconds (line 57). Asp and Villers (2010) have shown that fillers like "um" articulated with a level tone and long pauses are used by English speakers to show hesitation. Thus, in this case, the teacher's turn may be interpreted as displaying her hesitation. Moreover, in the same utterance, she admitted that she was not sure what a rectangle would be in Te Reo Māori (line 58). The teacher appreciated the question and responded positively to the question with "good question" (line 60).

In the following utterance, Olivia claimed that taparima is a pentagon (line 61). To this, the teacher displayed her doubt, as she stated, "I don't think it is" (line 63). The teacher (lines 65-70) accepted her lack of knowledge about Te Reo Māori shape names and shared her vulnerability with her children. Moreover, as the teacher had said that she was not sure (line

58), Zara showed an interest in finding the actual names for the pentagon in Te Reo Māori (line 64).

During the semi-structured interview with the teacher, the teacher had stated that she was a Te Reo Māori learner with beginner's level fluency (Teacher, Interview 2). The teacher also mentioned that she used the unit "Te Whānau Taparau – the polygon family" (NZMaths, 2021b). This unit (see Appendix A) initially introduces the shape "square" as "tapawhā". However, later in the same list, a square is named "tapawhā rite". Similarly, a hexagon is named "tapaono rite". The lack of clarity in the unit for the Te Reo Māori terms for shapes may not have supported the teacher in responding to Zara's question, "If square is tapawhā, what's rectangle?" (line 56).

This Key Moment highlighted one major finding, which is relevant to the first research question. The analysis revealed that children may question the discursive constructions used in previous lessons to display their developing understanding of shapes in Te Reo Māori. In the last Key Moment (4.5a), it was noted that tapawhā was discursively used to mean a square. Zara's question "If square is tapawhā, what's rectangle" (in this Key Moment 4.5b) draws our attention to the different shapes that can be denoted by the term "tapawhā", which means four-sided shape.

4.5.3 Summary: Naming Shapes in Te Reo Māori

In this section, children's discursive constructions of shapes in Te Reo Māori were explored. With regard to the understanding of geometric shapes in Te Reo Māori, the analysis of the audiovisually recorded lessons, fieldnotes, and the semi-structured interview with the teacher revealed that children had a sound knowledge of number names in Te Reo Māori. Secondly, the analysis of Key Moments displayed that the children followed the pattern of counting the number of sides for finding the name of the 2D shape in Te Reo Māori. Thirdly, the second Key Moment highlighted that for the children tapawhā may imply both square and rectangle. The lack of clarity in the unit for the Te Reo Māori terms for square may suggest a need for clarification of the Māori terms used for teaching shape names in an English-medium primary school. Fourthly, the teacher was able to share her lack of knowledge in naming shapes in Te Reo Māori. The next section (Section 4.6) pulls together the findings from the analysis of ten Key Moments pertaining to the geometric ideas explored in this chapter.

The following section presents the overall findings and lays the groundwork for the next level of analysis, the macro-level analysis.

4.6 Overall Findings: Thematic Analysis and Micro-Level Analysis

The overall aim of this study is to explore how children negotiate their meanings about shapes and their properties in a New Zealand multilingual primary classroom. This chapter presented findings from the thematic analysis and micro-level analysis. The findings were drawn from several data sources, including six audiovisually lessons, fieldnotes, three semi-structured teacher-interviews, four focus group interviews with the Year 5/6 children, and documents (children's work samples, teacher's unit-plan, and The NZC). The analysis reported in this chapter responds to the first two research questions, and to selected findings that respond to the third research question.

The first research question focuses on the discursive constructions that children used to represent their understanding of shapes and their properties in a multilingual Year 5/6 New Zealand primary classroom. The analysis revealed five discursive constructions about shapes and their properties.

- First, children used the word "sides" to mean line segments of 2D shapes and faces or edges of 3D shapes (see Section 4.1 and 4.3). Moreover, children discursively associated the meaning of "side" with the straight side (see Key Moment 4.5a).
- Second, children used words from everyday language to imply mathematical ideas (e.g., the use of "aligned" and "perfect", Key Moment 4.1b).
- Third, children constructed the names of 3D shapes in reference to the 2D shapes. For example, a rectangular prism was named as a "3D rectangle", a sphere as a "3D circle", and a triangular prism and pyramid as a "triangle 3D" (see Sections 4.2 and 4.3).
- Fourth, children used three different discursive constructions to represent their understanding of dimensions. These discursive constructions were: (a) dimension expressed as "flat vs fat", (b) dimensions as "ways to move", and (c) dimension as "another world" (see Section 4.4).
- Fifth, children discursively used the rule of "number of sides" with the prefix "tapa" to name the shapes in Te Reo Māori (see Section 4.5). However, the lack of clarity in naming shapes names in the NZ maths unit "Te Whānau Taparau – the polygon family" (see Appendix A) may indicate the need for further development of Māori terms in the unit designed for teaching shape names in New Zealand English-medium schools.

The second research questions concerned the way children interacted to construct their understanding of shapes and their properties in a New Zealand multilingual primary classroom. The analysis revealed three interactional practices that children used while representing their discursive constructions during whole-class and group interactions.

- First, the analysis indicated that multilingual children used prosodic features of their multiple languages to convey their meanings (see Sections 4.1 to 4.5). Moreover, non-native New Zealand English speakers may perceive the same prosodic cues, for example, HRT intonation, differently from native New Zealand English speakers (for example, see Section 4.1.2).
- Second, children used gestures while constructing their understanding of 2D shapes, 3D shapes and their properties (Key Moment 4.1a, 4.1b, 4.4a, and 4.5a). The analysis also seems to suggest that some children with less proficiency in the language in instruction may use iconic gestures more than deictic gestures (see Section 4.1.1).

The analysis also revealed a finding which responds to the third research question, and concerns the charateristics of the dialogic space that influence children's negotiation of meanings. The micro-level analysis draws our attention to children's and the teacher's understanding of preferred and dispreferred responses as one of the characteristics of the dialogic space of a New Zealand multilingual primary classroom that influence children's negotiation of meanings (See Sections 4.1, 4.3, 4.4). The analysis suggests that the children and the teacher considered children's responses as a preferred response if the responses were given in geometry-specific language. Moreover, whenever children spoke out-of-assigned turn, their utterances were treated as dispreferred. The teacher's overt negative evaluation of children's incorrect responses were also considered as dispreferred responses.

To find other processes that may influence children's negotiation of meanings about shapes and their properties in a New Zealand multilingual classroom, I analysed the data at the macro-level using Bakhtinian concepts of heteroglossia, unitary language, double-voicedness and Chronotopic Moments. The analysis helped me to explore how the meanings of words like "side" and "perfect" are negotiated in the presence of dialogic space, in order to answer the third research question.

Importantly, as discussed in the methodology section (see Section 3.4), the micro-level analysis forms the basis of the macro-level analysis. The micro-level analysis of the ten Key Moments revealed that the prosodic features reveal the emotional stances embedded in children's utterances. For example, prosodic features of high pitch, low pitch, whispery voice, and the loudness of voice were often used to convey children's emotional stances of confidence, authority, doubt, diffidence, and excitement when learning about geometry shapes and their properties. These emotional stances embedded in participants' utterances form the basis of the first step of analysis at the macro-level. Prosodic cues explored in this chapter reflect individual participants' emotional stance with regard to learning. And, the content of the utterances in the local conversational context enabled the analysis of speech genres required for exploring double-voicedness of utterances (more details are presented in the next chapter). Additionally, the content of utterances and what is perceived as a correct response during whole-class and group interactions illuminates the discourses available in classroom interactions. Following the coding of the ten selected Key Moments for emotional stances, speech genres and discourses, I used Bakhtin's concepts to explore the tensions that inform the negotiation of meanings.

The next chapter presents the study's macro-level analysis in order to explore how dialogic space influences children's negotiation of meanings about shapes and their properties.

Chapter 5.

Findings II: Dialogic Space and Negotiation of Children's Meanings

The purpose of this chapter is to report analysis of the data in order to find a response to the third research question, which asks what characteristics of dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom. To investigate the processes that result in the negotiation of meanings in the dialogic space of a Year 5/6 New Zealand geometry multilingual primary classroom, I used the Bakhtinian analytical concepts of unitary language, heteroglossia, double-voicedness, and Chronotopic Moments (interpretation of the chronotope as used in this thesis and explained in Section 3.2.1). Dialogic space can be defined as the dynamic interactional space that contains the active process of meaning-making and within which all possible meanings of utterances are taken into account. The specific meaning of an utterance depends upon the preceding and succeeding utterances. Therefore, for this study the dialogic space takes account of various meanings derived from different perspectives embedded in the dominant discourses and the interactional practices that influence children's discursive constructions about shapes and their properties in a New Zealand multilingual primary classroom.

The analysis presented in this chapter is based on the micro-level analysis presented in the previous chapter (see Chapter 4, Findings 1). As explained in Section 3.4.3, the first step of macro-level analysis involves identifying three aspects of utterances in the ten selected Key Moments. These three aspects are: (a) emotional stances as evidenced through the prosodic features embedded in children's utterances, (b) speech genres, and (c) discourses in children's and teacher's utterances. This micro-level analysis of the ten Key Moments (see the preceding chapter) informed us that the children's use of prosodic features in their utterances may reflect their emotional stances towards their learning. Those emotional stances form the basis of the macro-level analysis presented in this chapter. The analysis of emotional stances along with the social action embedded in the way of talking allowed identification of speech genres. The exploration of the meanings ascribed to the content of the utterance enabled me to identify the dominant discourses embedded in the utterances (See Appendix K for detailed analysis of one of the Key Moments, showing all three aspects at the first step of macro-level analysis). The same iterative process was followed for each of the ten selected Key Moments.

The first section presents findings relevant to the identification of emotional stances, speech genres, and discourses (see Section 5.1). The second section (see Section 5.2) presents the second step of the macro-level analysis and uses the Bakhtinian concepts of unitary language, heteroglossia, addressee and double-voicedness by investigating the dialogical tensions embedded in children's and teacher's utterances at the level of voices, discourses, and languages using identified emotional stances, speech genres, and discourses at step 1. The third section presents the chronotopic analysis of Key Moments (see Section 5.3). Chronotopic Moments are identified in two of the ten selected Key Moments, drawing our attention to how children can use moments of learning from their past or future to make sense of learning at a specific in-the-moment during interactions. The chapter concludes with overall findings.

5.1 Identified Emotional Stances, Speech Genres, and Discourses in Key Moments

This section presents findings pertaining to the (i) emotional stances of learning, (ii) speech genres, and (iii) discourses. Each of the ten selected Key Moments was analysed (using the same iterative process presented in Appendix K) to identify its emotional stances, speech genres and discourses. I present findings from the identified emotional stances in the first sub-section (see Section 5.1.1). In the next sub-section, I examine the identified speech genres and the relevant findings (see Section 5.1.2), followed by findings pertaining to discourses in the last sub-section (see Section 5.1.3).

5.1.1 Emotional Stances of the Learning

Emotional stance can be defined as the emotional aspect of children's learning as embedded in their utterances; that is, the emotional stances revealed through their use of prosody suggest the children's emotions. The emotional stances of the children towards their learning were identified through interpretation of the prosodic cues at micro-level analysis (see Chapter 4).

Two main findings can be drawn related to emotional stances of learning. The first finding suggests that different emotional stances may be identified for a child within the same Key Moment. For example, during Key Moment 4.4a, Elie displayed three different emotional stances within the same Key Moment, as evident in the following utterances:

(a) `353    Elie       um:: I think its three d because °its not (.) a two d°` ((she was holding the shape and rolling it around her finger)).

(b) 357 Elie its fat (1.5)

(c) 366 Elie [↓uhm::

The use of whispery voice in the start of Elie's utterance (a) may indicate doubt (Gobl & Chasaide, 2003), and the use of flat pitch in the second half of same utterance may indicate confidence (Ward, 2019). Thus, the use of these two prosodic features in utterance (a) may indicate Elie was partially confident. In the second utterance, (b), the use of flat pitch may indicate confidence. In the last utterance, (c), the use of low pitch with hedging and stretching may indicate falling interest in conversation (Drew, 2013). These three utterances within the same Key Moment indicate three different emotional stances of learning (See Appendix K for detailed analysis of the emotional stances of Elie in Key Moment 4.4a).

Similarly, Matiu (male 11-year-old Māori-English bilingual child) expressed confidence and then confusion through his baffled response at different micro-moments in the same Key Moment 4.2b, as displayed in the following utterances:

(d) 42 Matiu my next one is a (.)probably triangular prism

(e) 52 Matiu ↑then ↑whats a tri:angular prism (2.0) yayy °simply a pyramid as well°

The use of flat pitch in Matiu's first utterance, (d), indicate confidence. However, in the second utterance, (e), Matiu used his high pitch in the beginning to probably indicate his confusion about triangular prism and pyramid. He also used a pause and a whispery voice in his utterance, which again indicate doubt (Gobl & Chasaide, 2003).

The second finding suggests that children may display various emotional stances at micro-moments during interactions (see Appendix L for the identified emotional stances in the selected ten Key Moments). For example, Matiu, who was often seen as a confident mathematics learner, displayed various emotional stances during the ten selected Key Moments. He displayed confidence during Key Moment 4.4b and doubt during Key Moment 4.1b, as shown in the following utterances:

(f) 617 Matiu two (Key Moment 4.4b)

(g) 223 Matiu because the face °no:(0.2)the si::des°(2.5) nah °I dun know° (Key Moment 4.1b)

Matiu showed confidence through the use of flat pitch (Ward, 2019) while saying "two" during Key Moment 4.4b, see utterance (f). In utterance (g), Matiu displayed doubt as he used whispering tone in the end of his utterance (Gobl & Chasaide, 2003) in Key Moment 4.1b.

Similarly, Ethan (male English-speaking monolingual child) displayed various emotional stances during these ten Key Moments. The following utterances show this range of emotional stances.

(h) 54 Ethan °i dont know° (Key Moment 4.2b)

(i) 599 Ethan TWO D::" (Key Moment 4.4b)

(j) 587 Ethan ↑PORA°whita° (Key Moment 4.5a)

These three utterances shows different emotional stances expressed by Ethan during different Key Moments. In utterance (h), Ethan's whispery voice shows doubt (Gobl & Chasaide, 2003). Ethan's confidence can be interpreted through his use of loud voice in utterance (i) (Ward, 2004). In the third utterance, (j), Ethan used loud voice for the first part of porowhita indicating surprise and excitement (Wilkinson & Kitzinger, 2006).

Two main findings can be drawn pertaining to the children's emotional stances of learning. First, children may use different emotional stances within the same Key Moment. Second, these children displayed a variety of emotional stances towards their learning across the selected ten Key Moments which reflected their changing levels of confidence and engagement in displaying their knowledge of shapes and their properties while interacting with their fellow classmates and the teacher. Further examination of these identified emotional stances with the words that the children used to achieve a social action while representing their understanding of geometry shapes helped me to identify a variety of speech genres. The following section presents findings pertaining to speech genres identified in each of the Key Moments.

5.1.2 Speech Genres

The investigation of emotional stances (in the form of prosody) in Key Moments (as presented in Appendix L) along with the intended social action embedded in the utterances led to the identification of various styles of talking (see Appendix M), which I labelled speech genres (this step of analysis is outlined in Section 3.4.3). In this section, I present the speech genres identified in the teacher's utterances, in children's utterances, and in both the teacher's and children's utterances, as one of the aspects of dialogic space within which children negotiated their understanding of shapes in their classroom interactions (RQ3). Identified speech genres with their labels and definitions are presented in Table 5.1.

Table 5.1

Speech Genres and Their Definitions

Label	Definitions
Appreciative speech genre	Utterances through which the teacher showed appreciation for the children's responses are considered Appreciative speech genres. These utterances include use of both verbal response and non-verbal aspects such as a smile or body language.
Pedagogical speech genre	These utterances share features with the Initiation-Response-Feedback sequence often evident in classroom interactions and enable the teacher to support children's thinking and to manage classroom participation. They include the teacher's initiating turn of asking a question to begin the discussion and providing feedback in the third turn to support the learning of the child whose response is being evaluated.
Assessment speech genre	These utterances include those through which the teacher shows evaluation of children's incorrect or incomplete responses without overtly showing her negative evaluation; no feedback is provided to the child whose response is being evaluated, and often another child is selected to answer the question. The focus during the interaction is on getting the correct response from the child rather than providing feedback to support a child's learning.
Giving-up speech genre	These utterances signal the presence of a participant's tendency to withdraw from an argument, either because they have lost interest in carrying on the discussion or because they consider that they lack the knowledge required to continue the discussion.
Persuasive speech genre	This speech genre reflects those utterances whereby the speaker seeks consent or agreement by using the language of persuasion. The use of "we" (Gerofsky, 1999) or repetition of the same statement without justification in utterances often signals this genre.
Argumentative speech genre	The Argumentative speech genre signifies children's presentation of an acceptable mathematical explanation or argument to support their knowledge claim and/or to establish with confidence their stance or position in a whole-class or group interaction.
Declarative speech genre	This genre includes children's utterances where children declared their knowledge claim without any supporting justification or explanation. The use of prosody such as low pitch or loud voice displayed their confidence over their knowledge claim.

It is to be noted that the speech genres presented in Table 5.1 do not constitute an exhaustive list of the speech genres that can be observed during classroom interactions, as noted by Rockwell (2000).

The first speech genre identified in the teacher's utterances is the Appreciative speech genre. These utterances included the use of smiling as a positive rapport-building marker, as evident in the following utterance:

317 Teacher ↑do you? ((teacher smiles)). (Key Moment 4.1a)

The teacher also repeated the child's response to show her acknowledgement and approval of the child's response. An example of this kind of utterance is provided below:

509 Teacher you had a two d and a three d? in in yours too. (Key Moment 4.3a)

The teacher used overt positive evaluations of children's responses while appreciating the child's overall effort, as evidenced in the following utterances:

591 Teacher ya↑ya↑ <u>poro</u>whita you get all the time (.4). (Key Moment 4.5a)

618 Teacher ↑thank ↑you:[very much ((exhalation of breath, closed her eyes and slightly tilted her head back)). (Teacher response to Matiu's answer, Key Moment 4.4b)

By structuring her utterance in these manners, the teacher not only positively evaluated the child's response to the question, but also displayed her appreciation for the correct answer at that moment. The Appreciative speech genre includes both verbal response and non-verbal aspects like a smile or body language to show appreciation for the children's responses. These positive utterances seem to build teacher's rapport and relationships with children.

The second kind of speech genre identified in the data which I labelled the Pedagogical speech genre included most of the teacher's talk. This speech genre shares features with the Initiation-Response-Feedback (McHoul, 1978). In the classroom interaction, the sequence of teacher's question in the first turn, child's response in the second turn, and teacher's feedback in the third turn was observed in the data at the micro-level analysis. This speech genre includes the teacher's initiating turn of asking a question to begin the discussion and providing feedback in the third turn to support the learning of the child whose response is being evaluated. The purpose of these utterances was both to support children's thinking and to manage classroom participation. Some of the examples for this speech genre are:

(a) 191 Teacher is it perf (.) why is it a perfect square? zara. (Key Moment 4.1b)

(b) 547 Teacher so they ve got(0.2)square(0.5)two d:(1.0)triangle. three d:(0.5) ↑what is: a tri::angle three d [<can anyone remember> what (1.0)a tri (1.0) Yue? (Key Moment 4.2a)

(c) 499 Teacher =three d circle(0.5)↑ can ↑anyone (0.5)um: think of the geometry term for three: d: circle ((students raised their hands up to answer)). (Key Moment 4.3a)

(d) 326 Teacher [ah:: (.5)so:(.) you ↑you ↑thinking like(.8)this one(.5)↑lets see tho:se(.)if you would to(.2) give it si::des ay? (Key Moment 4.1a)

(e) 333 Teacher one so ↑theres EIGHT (.2)one two three four fi:ve six seven eight(.) do you think eight (.)so: do you know the ei:ght one? ((looked at Ozan)). (Key Moment 4.1a)

(f) 225 Teacher yeah you re on the right track. the si:des what (.)what would the sides be here. (Key Moment 4.1b)

(g) 576 Teacher can ↑I have a tapatahi.(.4)can I.(1.0)↑what ↑what woud tap ↑if I followed that pattern what would tapatahi be Elie. (Key Moment 4.5a)

In the first three utterances (a-c), the teacher initiated the interaction and selected the next speaker, which shows that the teacher held the power to select the next speaker. The purpose of selecting the next speaker seems to maintain the classroom interaction and provide opportunities for children to respond to the question. And in the utterances (d) and (e), the teacher provided a positive evaluation of Ozan's thinking and gave further feedback by saying, "if you would to give it sides, ay?" to further support his thinking in naming the shape in question. Similarly, in the utterance (f), the teacher provided a positive evaluation in order to support Matiu's explanation for why a square is a perfect square. She initiated the sentence for Matiu to complete as she stated, "the sides…what would the sides be here". The teacher used this kind of utterance with the whole class as well (see the last utterance, (g)). The focus of these kinds of utterances within the Pedagogical speech genre was to elicit additional information from the children and support their further learning, as well as to maintain classroom interaction.

The third speech genre identified in the teacher's utterances during whole-class and group interactions in these selected ten Key Moments is the Assessment speech genre. By using this speech genre, the teacher showed evaluation of children's incorrect or incomplete responses without any overt negative evaluation, as demonstrated in the following utterances:

202 Teacher yeah because its a square doesnt tell me much (1.0) ELIE what do you think (Teacher, Key Moment 4.1b)

Teacher provided negative evaluation of Matiu's response and asked Elie to respond to the question. In the following utterance, the teacher assessed Yue's response as incorrect and asked Matiu to answer the question:

552 Teacher CU::BE(0.5)um kori cu:↑be is (1.0)a cube is a bit Different (.)um::: Matiu (teacher smiled and pointed to Matiu)). (Teacher, Key Moment 4.2a)

It is important to note that when using this genre, the teacher did not provide feedback to the child whose response is being evaluated but selected another child to answer the question. The focus during the interaction was on getting the correct response from the child rather than on providing feedback to support a child's learning.

It seemed that one child, Garry, made use of both Pedagogical and Assessment speech genres during Key Moment 4.1a. These speech genres were often identified in teacher's utterances during whole-class interactions. During Key Moment 4.1a, it was observed that Garry also initiated the group interaction with a question instead of identifying shapes in the task sheet provided to the group, as evident in the following excerpt.

205 Garry what sha:pes can you see right now
206 Tahi circ::les(1.5)squa:res
207 ((Garry takes the picture sheet and turn it over to put glue to paste it on large
208 white sheet as Tahi was still looking at it))
209 Ozan I see a lot of circles over there (3.0)
210 ((Ozan looks at the sheet while Garry and Tahi make faces towards the
211 camera))
212 Ozan okay(.) what is this shape called ((pointing to shape))
213 Garry ↑so ↑whats tha:t whats [that Tahi?
214 Tahi [squa::re
215 Garry thats a ↓rectangle
216 Tahi #square#
217 Garry then Ill say squa::re
218 Tahi ↑Squ°are::°(.8) °thats° a square
219 ((Garry writes square as Tahi speaks))
220 Ozan oh ↑I ↑SEE [One
221 ((Ozan looks at Garry who was given with the responsibility to write))
222 Tahi [he::re ((Tahi points to different shape and laughs))
223 Ozan THIS ONE ((points again to the shape))
224 Garry wha:ts that
225 Ozan I dont know what[it is called
226 Tahi [°circle thats a circle°
227 Garry cir(.)cle
228 Ozan not °this° (2.0) ((put his hand to his head to show that
229 it is not the shape that he was talking about))

230 I am talking about whole thing, like like (2.0)
231 ((drag his finger at the shape to show his imagination of sides))
232 (in jacks)(.5)what was it (2.0) [it=
233 Tahi =ohh (.) °I know there is this thingy like
234 this° ((points to another shape))
235 [theres like ((makes the shape with his finger on the sheet to
236 show the shape he implies))
237 Garry [there is: no thingy (you images)
238 Ozan ((aspires)) Oh↑ I see
239 Tahi no:: theres a thing(.) that they had tha:t goes
240 ↑like (then) ((Tahi moves his finger in a curved motion))
241 Garry [((draws a line and Tahi sees him))
242 Ozan [oh ↑I see one (.5)Agai::n
243 Tahi RECtangle:s (and like) (2.0)
244 Garry ↑Oval:((marks an arrow for a shape))
245 Ozan [I SEE ↑one I see one °I see one°
246 Tahi [okhay
247 Garry wha::t
248 Ozan this one like(.) not the:se circle.(.5) °like
249 this° ((he drags his finger on the shape making the straight lines of the
250 sides of the shape))
251 Tahi OH ↑the shape(.)tha:ts not a shape
252 Garry this not a shape
253 Tahi what are another shape(.4) (I drop out)
254 Ozan its some kind of [cir
255 Garry [its an oval

During this Key Moment, Garry started the conversation with the Pedagogical speech genre (line 213) to elicit Ozan's and Tahi's knowledge of shapes. He selected the next speaker at the beginning of the conversation (line 213), which seems to suggest that he may have assumed the role of the teacher in this group discussion. Moreover, he provided his negative evaluations at various moments (see lines 237, 252) without giving any feedback on why the response was incorrect, which may indicate the use of the Assessment speech genre. The micro-analysis of the third part of this Key Moment also indicates that Garry seemed to use the teacher's way of talking during this Key Moment, when he coughed and used a loud voice to declare that the shape in question is a hexagon (see Key Moment 4.1a).

In addition to the use of the Pedagogic and Assessment speech genres, the Giving-up speech genre and the Persuasive speech genre were also identified in both children's and teacher's utterances. The Giving-up speech genre signalled the presence of a participant's disinterest to continue the discussion (see Table 5.1). Interestingly, this genre was seen in the child's as well as teacher's utterances in two of the Key Moments, as evident in their utterances given below:

366 Elie [↓uhm:: (Elie, Key Moment 4.4a while explaining if the shape she made is 2D or 3D)

> 65 Teacher °can you find it out for me° yeah I dont think it is I know definitely tapa:wha: is an:d I know definitely porowhita is >but I only threw the ↑other ones in< (1.0) yeah I put the other ones in because it help us remember what they are (.) okay (Teacher, Key Moment 4.5b).

In Elie's utterance (line 366), she made use of the hedging device "uhm" to display her declining interest in continuing the discussion about the hexagonal shape that she had made. The teacher's utterance displays her acknowledgement of a gap in her own knowledge, after which she attempts to give up the discussion of Te Reo Māori names of geometric shapes during Key Moment 4.5b.

The Persuasive speech genre includes those utterances where the speaker intends to seek consent or agreement by using the language of persuasion. Speakers working with this genre did not provide any mathematical logic or justification. Instead, they used repeated statements to draw the listener's attention, or tag questions such as "Right?" and "Ok?" to seek consent or agreement (Gerofsky, 1999). Some of the utterances from the data are provided below:

> 228 Ozan not °this° (2.0) ((put his hand to his head to show that it is not the shape that he was talking about)) I am talking about whole thing(.) like like (2.0) (Ozan, Key Moment 4.1a)
>
> 362 Teacher its its okay. so: its not actually coming out of the ground or going through the grou:nd (.) so we call so we call (.) ↑we call that a two d? (0.5) okay [so:(.2) (Teacher, lines 362-365, Key Moment 4.4a)

Repetition in Ozan's utterance and the use of "we" in the teacher's utterance may indicate that they both tried to persuade the other to agree with them.

Apart from these speech genres, two more speech genres were identified in children's utterances: the Argumentative speech genre and the Declarative speech genre. Children's utterances within the Argumentative speech genre were those in which children used mathematical explanations or justifications to support their knowledge claim and to establish their stance or position with confidence in a whole-class or group interaction. This use can be seen in the following utterances:

> 339 Ozan I ↑know but(.2)I just my brother used to watch a movie about(.2)of this kind o shapes ↑(.5)that I know(.5) their na:mes are are like like twelch(.4)I I thought it was and theres they were saying like a like a lot of shapes like one two three until(.5) they have passed eight, and then ten an twelve or something(.5)I

dont remember by how much it was (.2)but I do remember by [how many" (Ozan, Key Moment 4.1a)

579 Elie a circle. but ↑you ca:nt have it bcoz circles have no si:des [and] no corners" (Elie, Key Moment 4.5a)

In Ozan's utterance, he used the mathematical rule of counting the number of sides for naming polygonal shapes, which is an acceptable mathematical explanation for justifying that there is a shape with eight sides. In Elie's response, she argued that the circle could not be named as tapatahi, as there was no straight side, an argument which is required for naming shapes with straight sides, such as triangle, rectangle, hexagon, and so forth.

The Declarative speech genre encompasses children's utterances where children declared their knowledge claim without providing any supporting justification or explanation. The micro-level analysis showed that in these kinds of utterances, speakers often use low pitch or loud voice to display their authority or confidence over their knowledge claim. Some of examples from the data are presented below:

384 Garry ((coughs)) HEXAGON (Garry, Key Moment 4.1a)

216 Tahi #square# (Tahi, Key Moment 4.1a)

49 Garry ↑YES ITS A PYRA(H)mid (2.0)(Garry, Key Moment 4.2b)

551 Yue cube (Yue, Key Moment 4.2a)

498 Kayla in our we had a two d(1.0) and a (.) three d (Kayla, Key Moment 4.3a)

315 Matiu a recta:ngle (Matiu, Key Moment 4.3b)

357 Elie its fat (1.5) (Elie, Key Moment 4.4a)

590 Ethan Nine (Ethan, Key Moment 4.4b)

564 Yue hexa↑go:n (Yue, Key Moment 4.5a)

It is evident that prosody is used to display authority over the knowledge claim without any supporting mathematical explanation. For example, Garry, in the first Key Moment (4.1a), used a loud voice to claim authority over knowledge, but did not provide any justification for why the shape was a hexagon. It is to be noted that he was referring to an eight-sided polygonal shape in this utterance (see Key Moment 4.1a, part iii).

In this section, I have examined children's and teacher's utterances to identify the speech genres embedded in their utterances. Speech genres were identified as one characteristic of the dialogic space that influences children's meanings of shapes and their properties in a New Zealand primary classroom (RQ3). Seven speech genres were identified in the selected ten Key Moments observed in six audiovisually recorded lessons in a geometry classroom. Analysis of the speech genres in the ten selected Key Moments reveals two main findings. First, it suggests that the participants may make use of different speech genres within the same Key Moment to influence other participants' understanding of the idea proposed by the speaker. Second, the analysis of the ten selected Key Moments also reveals that the use of speech genres is not static; rather, all participants (both children and the teacher) in a dialogic space have access to these different speech genres and may use them according to the conversational context. It is evident that the teacher and the children made use of the Pedagogical, Persuasive, and Giving-up speech genres.

The next section explores the discourses that were present in the observed geometry classroom. The same iterative process (as explained in Appendix K) was employed to identify different discourses present during geometry lessons in the Year 5/6 class.

5.1.3 Discourses

The identification of discourses focused on the content as well as the form of the utterances (as explained in Section 3.4.2). I identified two major discourses for each of the Key Moments. These discourses are (i) Everyday Discourse, and (ii) Eurocentric-Academic Discourse. Data from six audiovisually recorded lessons and fieldnotes of the six observed lessons) were used to triangulate these discourses.

Everyday Discourse included the use of informal everyday language during classroom interactions. Everyday Discourse involves the forms of language that are primarily used outside the school in informal settings to describe the properties of shapes as evident in the following utterances:

> (a) [Dimension] is like another world. (Matiu, FN2)
>
> (b) `189 Zara >look whaea Jenny:< (1.0) whaea Jenny (.) a perfect square` (Zara, Key Moment 4.1b)
>
> (c) `227 Matiu perfectly:: aligned? with each other?`" (Matiu stating that the sides of square are perfectly aligned, Key Moment 4.1b)
>
> (d) `568 Matiu um: the e::gy↑pt (1.0)[um:: ↑mountain thingy` (Matiu talking about triangle 3D, Key Moment 4.2a)

(e) 300 Teacher Squa:re(1.0)at(2.0)bo:ttom(1.0) what about arou[nd the side(0.5) (Teacher asking about the shape of the faces of prism, Key Moment 4.3b)

(f) 357 Elie its fat (Elie explaining the shape she made is fat, Key Moment 4.4a)

For example, words such as "another world" in utterance (a), "side" in utterance (b), "perfectly aligned" in utterance (c), "side" in utterance (e), and "fat" in utterance (f) demonstrate the use of Everyday Discourse during the classroom interactions. The micro-level analysis revealed that the word "side" was also used as part of geometry-specific vocabulary to describe the shapes and their properties, as evident in the following utterances:

(g) [Cube] it has 6 sides..no 6 faces…12 corners. (Alyssa, FN2)

(h) [triangular pyramid] this has three sides at the bottom. And [square based pyramid] has four sides at the bottom. (Matiu, FN5)

Similarly, "corners" was also often used as part of geometry-specific vocabulary to describe the properties of 2D and 3D shapes in utterances given below:

(i) 346 Elie °[its got] one two (2.0)° its: got one: two: three four °five six° ↑its got six (0.2) corne:rs (Elie, Key Moment 4.4a)

(j) 229 Matiu ah(1.0) perfectly the same? (Matiu talking about sides of square, Key moment 4.2b)

I argue that this use of "side/s" in utterances (g) and (h) and "corners" in utterances (i) and (j) and in Key Moment 4.4a (see Appendix K) can be seen as part of a geometry-specific vocabulary that exemplifies the Eurocentric-Academic Discourse. This discourse takes account of the use of geometry-specific vocabulary and language as given in The New Zealand Curriculum (NZC) to communicate geometric ideas about shapes and their properties during classroom and group interactions. In The NZC, the specific geometry vocabulary for describing 2D shapes includes terms like "sides" and "corners" and "faces", "edges", and "vertices" for 3D shapes. The fieldnotes (six observed lessons) and semi-structured interviews with the teachers (Interview 2 with Teacher) also inform us that the teacher attempted to develop children's geometry-specific academic language. Analysis of discourses revealed that the same terms, such as "sides" and "corners" could be used as part of both Eurocentric-Academic Discourse and Everyday Discourse during classroom interactions. In response to the third research question, the analysis reveals these discourses as another characteristic, which is present as part of dialogic space and influences children's meanings of terms used during classroom interactions.

5.1.4 Summary: Emotional Stances, Speech Genres, and Discourses

The purpose of this section was to provide a link between micro-level analysis and macro-level analysis by building the foundation for further analysis to answer the third research question. The third research question aims to explore the characteristics of dialogic space in terms of emotional stances, speech genres, and discourses that influence the negotiation of meanings of shapes and their properties as children engage in whole-class and group interactions. Three main findings are drawn from this first step of the macro-level analysis. First, examination of children's emotional stances emphasised that the same child participant may exhibit different emotional stances of learning within and across Key Moments. Moreover, these emotional stances may be available to other participants in the dialogic space of a geometry classroom. In addition, some children displayed both confidence and doubt in their knowledge claims regarding geometric ideas. Second, speech genres were explored as another characteristic of dialogic space. Seven different speech genres were identified (see Table 5.1). The analysis of these genres provided evidence that participants (both the teacher and children) had access to different speech genres within the dialogic space of the geometry classroom, and they might use different speech genres at different moments of interaction to influence others' understanding. Finally, the analysis revealed that terms such as "sides" and "corners" may be used in both Everyday Discourse and Eurocentric-Academic Discourse. While the analysis identified several speech genres and two major discourses as two characteristics of dialogic space that influence children's negotiation of meanings, what is of particular interest is how and when these speech genres and discourses interact and inform the meanings of geometric terms, when those terms are used in classroom interactions.

The next section explores the interaction of these speech genres and discourses within the dialogic space embedded with the contesting language forces of unitary language and heteroglossia.

5.2 Interplay Between Unitary Language and Heteroglossia

According to Bakhtin, the negotiation of meaning is dependent upon the preceding and succeeding dialogues within the dialogic space. Moreover, two opposing language forces operate simultaneously at different levels of interaction. The force of "unifying language" (Bakhtin, 1981, p. 269) tends to guarantee mutual understanding of the meanings of utterances by shaping their meanings within the domains of prevalent dominant discourses. Concomitantly, the "heteroglossia" (Bakhtin, 1981, p. 270) or diversifying force attempts to decentralise the already established meanings of the utterances by embedding individualised meanings into the language. It is the ongoing interplay of unifying and diversifying language forces in a specific circumstantial context (i.e., whole-class and/or group interaction at a particular time) as well as the socio-cultural milieu that informs the negotiation of meanings in any classroom setting. The tension between the unitary language forces and heteroglossia is explored at three different levels in the data. These levels are the voices, discourses, and languages used in the classroom and group interactions (Busch, 2014). The following subsections explore the interplay of these language forces at the level of voices (Section 5.2.1), discourses (see Section 5.2.2), and languages (see Section 5.2.3).

5.2.1 Interplay of the Competing Voices

This section explores the interplay of competing voices embedded in participants' utterances during whole-class and/or group interactions in the selected ten Key Moments. Bakhtin (1981) provided the analytical concept of double-voicedness to elicit the different voices embedded in any utterance. The concept of double-voicedness allows unravelling of the process through which an utterance responds to the previous and the upcoming utterances in the flow of interactions. It also explores the process through which speakers adopt someone else's words as their own. That is, each utterance carries the historical meanings rooted in words used in the utterance while expanding and innovating the meanings as speakers imbue their utterances with their individual responses and perspectives. Bakhtin argued that a speaker infuses the already encapsulated words (bearing someone else's meanings and values) with their own meanings as they participate in conversations. Moreover, the double-voicedness of the utterance is also influenced by the addressee for whom the utterance is structured.

In the following analysis, I explore the competing voices in each utterance using the categories of speech genres identified in participants' utterances in Section 5.1.2. Two (Key Moment 4.1a and 4.1b) of the ten selected Key Moments are discussed in detail in this section to unpack the double-voiced nature of the utterances and the influence of the addressee on the utterance. These two Key Moments were selected to underscore two aspects of double-voicedness: (i) the process through which a participant uses someone else's words as their own, and (ii) the process through which participants imbue the same word with their own meanings.

During Key Moment 4.1a – "I saw this as some kind of shape that I know" – Garry, Ozan, and Tahi engaged in identifying the shapes in the task sheet (See Figure 4.1) as requested by the teacher. Excerpt 4.1a presents the transcribed data of the Key Moment to display the voices embedded in participants' utterances.

Excerpt 4.1a (part i)

#	Speaker	Text
205	Garry	wh:at sha:pes can you see ri:ght now
206	Tahi	circ::les(1.5)squa:res
207		((Garry takes the picture sheet and turn it over to put glue to paste it on large
208		white sheet as Tahi was still looking at it))
209	Ozan	I see a lot of circles over there (3.0)
210		((Ozan looks at the sheet while Garry and Tahi make faces towards the
211		camera))
212	Ozan	okay(.) what is this shape called
213	Garry	↑so ↑whats tha:t whats [that Tahi?
214	Tahi	[thats a squa::re

During the interaction, Garry initiated the conversation with a question (line 205). The fieldnotes (FN1) inform us that Garry's utterance is similar to that with which the teacher had initiated the conversation with the whole class while introducing and modelling the activity. This way of initiating the conversation was also noted during other Key Moments (e.g., see Excerpt 4.2a (part ii), Excerpt 4.3b Excerpt 4.4b, in Chapter 4). The use of utterances in this manner for eliciting response from others was identified as the use of the Pedagogical speech genre in the analysis (see Section 5.1.2). The emotional stance identified during this Key Moment also suggests that Garry was confident and acted as an authority, by asking questions instead of providing answers to the question that the teacher had asked. The second utterance by Garry (line 213) is again a question. However, this time Garry directed this question to Tahi by naming him as the next speaker. This act also supports the interpretation that Garry assumed authority to select the next speaker, like the teacher, ignoring Ozan's effort (line 212) to draw attention to his intended shape (See Figure 4.1).

The use of the Pedagogical speech genre with prosodic features (displaying confidence and authority) indicates the teacher-voice embedded in Garry's utterance. During this interaction, it seems that Garry assumed the role of the teacher and used a speech genre often used by the teacher to initiate classroom interactions. He constructed his utterances in reference to his group members as his addressees. Through his utterances, Garry brought his understanding of how to use language when he is endowed with some pedagogical responsibility by the teacher (in Key Moment 4.1a, the teacher asked Garry to write the names of the shapes that Ozan and Tahi identified; see Section 4.1) to facilitate discussion and understanding of his group members, in this case, Ozan and Tahi. It is probable that Garry attempted to use the teacher's way of talking and showing her intentions to her children and brought it to the conversation.

In other Key Moments (e.g., Key Moment 4.3a) Garry did not use this Pedagogical speech genre when addressing the teacher, showing his ability to differentiate speech genres according to the audience. The use of different speech genres according to the audience may signal Garry's understanding of the conversational power that the teacher holds in the classroom. Therefore, it seems that children may use speech genres used by the teacher to take on the authority of the teacher in the presence of other children.

The second Key Moment 4.1b, "Whaea look, a perfect square", shows how a phrase "perfect square" gets laden with several layers of meanings as the teacher and children use it in their utterances. During this Key Moment, Zara made a shape using playdough and called it "a perfect square" (see the circled shape in Figure 4.2). Excerpt 4.1b shows the classroom conversation that followed.

Excerpt 4.1b

#	Speaker	Text
189	Zara	>look Whaea Jenny:< (1.0) whaea Jenny (.) a
190		perfect square
191	Teacher	is it perf (.) why is it a perfect square? Zara
192	Zara	I dun↓no
193	Teacher	what makes it a perfect square(2.0)>come on zara ↑I
194		need< to ↑kno:w(0.5)because you said its perfect so
195		what makes a perfect square a perfect square
196		(1.0)=
197	Matiu	a ↑s[quare
198	Teacher	=[↑anyone ↑know why a perfect squa:re a perfect
199		square
200	Matiu	becoz its a square?
201	Garry	(h)(h)
202	Teacher	yeah because its a square doesnt tell me much
		((Classroom interaction involving Elie and Teacher as main participants))
221	Matiu	[um: °its got°
222	Teacher	Matiu
223	Matiu	because the face °no:(0.2)the si::des°(2.5) nah
224		°I dun know°
225	Teacher	yeah youre on the right track. the si:des what
226		(.)what would the sides be here
227	Matiu	perfectly:: aligned? with each other?=
228	Teacher	=aligned with each other?
229	Matiu	ah(1.0) perfectly the same?
230	Teacher	perfectly the sa:me the sides ↑are perfectly
231		the same (1.0)UM::: (1.0) zara(.2) did you hear
232		that(2.0)<a perfect squa:re is when the si:des
233		are per:fectly the same>

Zara seems to have called the shape "a perfect square" (line 189) on the basis of its physical appearance, which is that the shape was smooth and flat. Fieldnotes (FN1 to FN6) show that during classroom instruction in each of the lessons, the teacher reminded children to provide their explanation using geometry language. At this moment (line 191), although the teacher did not explicitly say that she intended children to provide an explanation in geometry-specific language, her use of the question "why is it a perfect square" indicates the use of the Pedagogical speech genre, indicating to her children that she is requiring the use of geometry language. This is also evident in her utterance when she stated that "because it's a square doesn't tell me much" (line 202). Moreover, she provided positive feedback on Matiu's utterance (line 223) using geometric language, saying that he was on the right track (line 225).

The teacher used the phrase "perfect square" (line 191), implanting her geometrical meaning onto Zara's meaning of "perfect square". The phrase "perfect square" now accounts for not only its physical attributes of being smooth and flat but also the geometrical properties of equal sides for the shape. As the conversation proceeded, the teacher overtly expressed her intention for children to provide an explanation by describing the properties of the shape as she reframed the question (lines 193-195). Matiu (lines 197 and 200) stated that being a square makes it perfect.

However, in the first utterance (line 197), Matiu did not address the teacher overtly, and he may have been addressing himself or his group members instead. In his second utterance (line 200), he stated, "becoz it's a square" with HRT and his emphasis on the word "square". The stress on the word "square" in both of these utterances (lines 197 and 200) seems to suggest that he attempted to embed the word "square" with its geometrical properties (i.e., all four sides are equal). The teacher's utterance (line 202), however, suggests that the teacher may have used "square" as a name for a particular shape rather than implying the geometric concept of "square" signalling its properties. Matiu tried to explicitly display his understanding that a square has equal sides in his utterances (lines 221 and 223). In line 225, the teacher made explicit her expectation that the explanation should use the geometric vocabulary to explain why the shape is a perfect square. Using this feedback from the teacher, Matiu attempted to state the properties of the shape "square" (lines 227-229) to his addressee, the teacher.

In this Key Moment, the phrase "perfect square" was embedded with different meanings at different moments in the interaction. The phrase "perfect square" was initially used to imply the physical appearance of the shape (made of playdough), as displayed in Zara's utterance. The second meaning of the phrase "perfect square" seems to suggest the geometric concept of the square, a shape with four equal sides. The third meaning ascribed to the phrase was in terms of using the word “square” as a name of the shape. These different meanings display the language force of heteroglossia, which may provide different meanings to the same phrase. However, the emphasis on providing a mathematically appropriate justification for why a shape is “perfect square” seems to suggest the role of unitary language force. This language force streamlined the meaning of the phrase “perfect square” from a physically flat and smooth shape to a geometric concept of a shape with four equal sides.

This section explored the interaction of different voices in participants’ utterances. The interplay of heteroglossia and unitary language forces within the participants’ utterances at the level of speech genres in the two Key Moments described in this section elicited the double-voicedness of the utterances at two levels. First, the analysis of the first Key Moment (Key Moment 4.1a) revealed that the children might make use of the teacher’s voice to display authority. Second, the analysis of the second Key Moment (Key Moment 4.1b) revealed that different meanings of the same words, in this case, “perfect” (as flat and smooth shape, and shape with perfectly equal sides) and “square” (as the name of the shape and geometric concept) may emerge as the children and teacher engaged in conversational space. The next section presents the interaction of the competing discourses within the field of unitary language and heteroglossia.

5.2.2 Interplay of the Competing Discourses

This section explores how the presence of two discourses in a Key Moment influenced the negotiation of meanings constructed about the shapes and their properties. On the dimension of discourse, two major discourses (see Section 5.1.3) are at play during the classroom and group discussions. These discourses are Everyday Discourse and Eurocentric-Academic Discourse. The macro-level analysis highlights the ongoing tussle between heteroglossia and unitary language forces operating on the use of two discourses within the dialogic space. In this section, I explore three Key Moments (Key Moment 4.3b, Key Moment 4.4a, and Key Moment 4.5a). During these three Key Moments, different understandings of the word “side” seem to emerge because of the interaction of two discourses present in the dialogic space. In the following discussion, I explore the interaction of discourses that resulted in the negotiation of meanings.

During Key Moment 4.3b, the teacher initiated the classroom discussion about what 2D shapes can be found in 3D shapes. Excerpt 4.3b shows the interaction during which the dominant meaning of the word “side” is embedded in the Everyday Discourse.

Excerpt 4.3b

#	Speaker	Text
298	Teacher	=[if you think about a pri:sm (2.0) Garry
299	Garry	↓square (1.2)
300	Teacher	Squa:re(1.0)at(2.0)bo:ttom(1.0) what about
301		arou[nd the side(0.5)=
302	Zara	[outsi:de

This Key Moment is from the fourth lesson. The teacher asked children about the 2D shapes that they could identify in a prism (line 298). Garry stated "square" (line 299), and it seems the teacher displayed her assumption that the child had identified the shape of the bottom face of the prism. Therefore, the teacher used the phrase "around the side" to let children imagine the shape from a different position (line 301).

The use of the "around the side" phrase may suggest a side-view orientation to look at the shape, a perspective embedded in Eurocentric-Academic Discourse, to view shapes from different viewpoints like side-view, top-view. This is because the teacher used her gestures to draw children's attention to the side-view orientation of the shape. However, the teacher did not use the term "side-view", which would have distinguished the phrase "around the side" from other interpretations, such as "sideways", "along the boundary". In line 302, it seems that Zara understood the phrase "around the side" as "outside", highlighting the everyday way of understanding the term "side". This utterance indicates the use of Everyday Discourse.

Thus, the use of "side" in this teacher's utterance (line 301) highlights both the Eurocentric-Academic Discourse (with "side-view" as one possible meaning) on one hand, and also underscores the presence of Everyday Discourse (with "outside" as another possible meaning) on the other. The presence of different meanings that can be elicited from the use of the word "side" in this utterance indicates the presence of heteroglossia in the interaction, as the meaning of "side" could be understood as "side-view" (embedded in Eurocentric-Academic Discourse) or "sideways" or "outside" (as embedded in Everyday Discourse). It was Zara's interpretation of the phrase "around the side" as "the outside" that smoothly facilitated ongoing interaction. Therefore, during this Key Moment, the meaning of the word "side" was established or shaped as implying "outside", a meaning embedded in the Everyday Discourse. Thus, it can be argued that the discourse supported by the unitary language force during this Key Moment is Everyday Discourse.

During the second Key Moment, which is 4.4a, the teacher asked children to describe the shape that they had made using playdough or sticks and adhesive. Excerpt 4.4a presents the transcribed data.

Excerpt 4.4a

#	Speaker	Text
341	Teacher	>↑anyone else got some right< (.) um: Elie
342		with your sticks
342	Elie	um::: (0.4) I forgot what this shape's called
343	Teacher	very good? ↑so ↑how many (0.6) so (1.6) ↑so (.)
344		um: [describe it]
345	Elie	°[its got] one two (2.0)° its: got one: two:
346		three four °five six° ↑its got six (0.2)
347		corne:rs
348	Teacher	got (.) six (1.5)
349	Elie	an::d its (.) go:t (1.5)
350	Teacher	Elie just hang on a minute (.) is it three d:
351		or two d: (1.0)
352	Elie	um:: I think its three d because °its not (.) a
353		two d° ((she was holding the shape and rolling it around her finger))
350	Teacher	put it down on a on the grou:nd (1.0) is it (.)
351		flat (.) or fat (0.5)
352	Elie	its fat (1.5)
353	Teacher	its fat (.) is it ↑coming ou:t towards you (1.0)
354	Elie	((looks at the shape holding it near the eye level))
355	Teacher	=okay lay it on the grou[nd (1.5)
356	Kimi	[^no its flat^
357	Teacher	its its okay. so: its not actually coming out
358		of the ground or going through the grou:nd (.)
359		so we call so we call (.) ↑we call that a two
360		d? (0.5) okay [so:(.2)
361	Elie	[↓uhm::
362	Teacher	its ↑got six co:rners (.) yeah
363	Elie	and its got (2.8) ((counted the number of sides)) and it
364		got six si:des
365	Teacher	six si:des good girl. I like you brought that
366		language (0.5)Okhay (0.3) ↑can ↑anyone help um
367		Elie (0.4) on what has ↑six si:des and ↑six
368		corners and is a and it is a two d shape
369		(1.0)um::: (1.3) Yue
370	Yue	a hexa:gon?
371	Teacher	ka pai so um you have actually made a hexa:gon
372		um: (0.6)
373	Elie	I know that thats called a hexagon ((hold and shows
374		the shape to the T))
375	Teacher	yeah a hexago:n has got six sides yeah (0.4)so°sort
378		of sort of a flat° (0.5) flat (2.0)um:

The use of phrases like "describe it" (line 344) highlights the use of unitary language force that constrains the language used in this geometry lesson. The phrase "describe it" is an everyday request, yet at this moment, the purpose of the teacher's utterance is to draw the child's (in this case Elie's) attention to the use of geometry-specific academic language. Elie responded by counting the number of corners in shape (line 345). In everyday language, the word "corner" may imply a position or an area. She later responded by talking about the number of "sides" for the shape that she made (line 363). The word "side" in everyday language may imply a surface, or an edge, or a position (left, right), or a place close to someone's position, or ideological position, and so on. However, Elie's use of Everyday Discourse for describing a shape with "corners" (line 347) and "sides" (line 364) is acknowledged by the teacher as an accepted way of describing geometry shapes. These terms are loaded with the everyday meanings of "corners" and "sides" yet are also an acceptable part of geometry discourse. When speaking in a geometry classroom, the word "corner" may imply interior angles/vertices, thus, becoming part of Eurocentric-Academic Discourse. Similarly, the use of "sides" may imply line segments/edges of two-dimensional shapes, again underscoring its use in terms of Eurocentric-Academic Discourse.

Thus, the analysis of Excerpt 4.4a shows that heteroglossia (of several meanings attached to the words "side" and "corner") permeated the Eurocentric-Academic Discourse when children and teachers used the terms "corners" and "sides" of 2D shapes to refer to "interior angles/vertices" and "line segments/edges" respectively. It is evident that the teacher focuses on using the geometry-specific language, acknowledging the use of "sides" and "corners" as part of the geometry-specific language, and reinforcing the use of "side" and "corners" in her utterance (line 365). This reflects the teacher's tendency to direct her children's attention towards using the Eurocentric-Academic Discourse as the language force that determines the meanings of the words used (in this case, "sides" and "corners") in a mathematics classroom interaction. When the Eurocentric-Academic Discourse determines the meanings of the words used, this discourse becomes part of the unitary language force instead of Everyday Discourse. That is, the focus on geometry-specific language highlights the unitary language forces embedded in the teacher's utterances and its operation in the classroom discussion.

Moreover, data from fieldnotes and audio-visual recordings of the six lessons suggest that children used the word "sides" to describe the properties of 3D shapes. Fieldnotes show that children tended to imply either "edge" or "face" when they used "side" when describing 3D shapes. In the focus group interviews, children displayed the use of "sides" to indicate faces or edges of the solid shapes, such as:

> This [deck card] is a rectangle this one's got four sides. This [Jenga piece] rectangular prism one's got eight sides. (Tahi, FG2)

The second semi-structured interview with the teacher indicated that she believed that the use of “side” as a way to describe properties of 3D shapes often leads to confusion, as the children tend to use this word colloquially as well as in geometry-specific context to describe shapes. This could be because when children are required to describe a 2D shape, they may use “side” as an appropriate geometry term to signal the edges of the shape, and there may not be any confusion about the face of the shape. However, when describing a 3D shape, children continue to use “sides” to describe the edges of the 3D shape. The confusion arises when children are required to talk about the faces and edges of 3D shapes simultaneously.

The third Key Moment draws attention to the third understanding of the word “sides”, which emerged during this Key Moment 4.5a. During this Key Moment, children and teacher engaged in classroom discussion around the Te Reo Māori shape names. The teacher structured her utterances as “what would tapatoru be?”, or “what would tapawhā be?” or “what would a tapaono be?” (line 560). These utterances display the dominance of the unitary language force of Eurocentric-Academic Discourse, which favours the tendency to follow the pattern of naming shapes in Te Reo Māori using the number of sides based in the western mathematical framework and evident in English names for shapes, such as hexagon and octagon.

Excerpt 4.5.a

#	Speaker	Text
557	teacher	↑what abo:ut um:::(1.0)whats a
558		tapa:ono(6.0)=
559	Zara	hhhhhhh (aspiration)
560	teacher	=umm ↑Kayla(.4) what wud a tapaono be (8.0)
561		↑taparima pentagon(.)tapaono Yue↑
562	Elie	((holding the hexgonal skelton that she made using sticks to show its a
563		hexagon))
564	Yue	hexa↑go:n
565	teacher	good girl. hexagon(4.5)
566	Elie	^I told you^
567	teacher	um:↑(h)(1.2)heres my question.(1.0)
568		could ↑I ha:ve(2)a tapatahi:(1.5)
569		((Elie, Yue, Matiu, Ethan said no in chorus))
570	Zara	YES a cir
571	Matiu	a circle has no ↓side
572	Elie	[no >↑you cant ↑you cant< ^you cant^
573	Ethan	[CIR↑CLE::
574	Matiu	[^no a circle has no side^
575	Elie	[coz ((put her hand up for answering))
576	teacher	can ↑I have a tapatahi.(.4)can I.(1.0)↑what
577		↑what woud tap ↑if I followed that pattern what
578		would tapatahi be Elie
579	Elie	a circle. but you cant have it bcoz circles
580		have no si:des [and] no corners

teacher	[hmm] ↑you are ↑brilliant
teacher	accepted.
teacher	so ↑what do we ca:ll a circle
Tane	a ha::o cylinder. (h)
teacher	<what do we call a cir:cle> when ↑I say (.)
	can you get into a umdumdumdumd..
Ethan	PORA^whita^
teacher	[umdumdumdumd(h)(h)(h)
teacher	>what is it<(h)(h)
Ethan	porowhita
teacher	ya↑ya↑ porowhita you get all the time (.4)
	so a poro:whi:ta is a cir:cle (2.2)

However, when the teacher asked children about what they think of "tapatahi", the pattern of her utterances changed. She did not structure her utterance as "what would tapatahi be" as this might have indicated the possibility of there being a one-sided figure. Instead, the teacher framed her utterance as "could I have a tapatahi" (line 568). The structure of the teacher's utterance is a result of unifying as well as heteroglossic language forces. The use of "tapatahi" may indicate the unitary language force. The unitary language force is present in the pattern of "tapa + number of sides (n)", evident in terms like "taparima" and "tapaono", which reflect the pattern used in "pentagon" and "hexagon" respectively, a pattern based in the Eurocentric-Academic Discourse embedded in western mathematical framework. The use of this pattern would allow the flow of conversation by indicating the possibility of having a one-sided shape. However, the use of the phrase "could I have?" was probably intended to elicit a different interpretation or meaning of the question "what would tapatahi be?", which could be interpreted as it being the exception to the rule or the pattern followed. Therefore, this utterance "could I have?" highlights the heteroglossia embedded in the teacher's utterance. In other words, the use of "what would tapatahi be?" would have implied the possibility of having a one-sided shape to students with certainty; however, the use of "could I have" called for questioning this possibility and thus engaging students in a dialogic space.

The teacher then changed the structure of her following utterance (line 576) and again asked children, "what would tapatahi be?". This utterance can be interpreted as her second attempt at hinting that children should frame their answers in a manner that was acceptable to her. That is, the answer to the question of "what would tapatahi be" would probably have resulted in an answer following the pattern evident in earlier discussion (lines 576-578) about Māori names for shapes, again underscoring the unitary language force embedded in westernised way of naming shapes. This requirement is acknowledged in Elie's response (line 579) when she suggested it would be "a circle" if she followed the pattern, therefore highlighting the use of Eurocentric-Academic Discourse embedded with westernised mathematical idea of naming shapes by counting the number of sides in her justification. Since the focus is on counting the number of sides, the interpretation of tapatahi as circle assumes the possibility of having a curved side, one interpretation of “side”. However, in the second part of her utterance (lines 579-580), Elie argued that it is not possible to have tapatahi as “circles have no sides and no corners”. The understanding of “side” here assumes inherent straightness. The two meanings of “side” (meaning curved side and straight side) in the same utterance display the embedded heteroglossia.

In addition to the unifying language force evident in the first part of her utterance, “a circle” (line 579) makes use of the Eurocentric-Academic Discourse to support the possibility of tapatahi. The second part of her utterance, which is “but you can’t have it because circles have no sides and no corners” (lines 579-580), presents a divergent possibility, thus highlighting the heteroglossia of the utterance. In the second part of her utterance, Elie draws on the same Eurocentric-Academic Discourse to provide a divergent understanding of why tapatahi cannot be a circle. Elie argued that a circle has no sides and corners (lines 579 and 580). In her utterance, she adopted her words “sides” and “corners” as geometry vocabulary to demonstrate her understanding of a circle and support her argument that a circle cannot be a tapatahi. She used “corners” to represent the angles of the two-dimensional shape. The understanding of “sides” seems to suggest edges of 2D shapes that assume straightness as its innate characteristic, which is part of the dominant westernised understanding of sides, as evident in the unit “Te Whānau Taparau – the polygon family” (explained in section 4.5a). Document analysis of The NZC revealed that the assumption of the side being straight may arise from the terminology used in achievement objectives stated in The NZC pertaining to learning about shapes at Year 5/6. The analysis of this Key Moment 4.5a showed that the word “sides” is used to imply line segments of the polygonal shapes without any explanation of the curved side of the circle.

It seems that Elie brought her westernised understanding of straight sides to justify her argument, which is “a circle has no sides and corners”. Elie’s use of “sides” and “corners” is laden with westernised mathematical ideas pertaining to geometry meanings as well as everyday meanings, therefore reflecting the heteroglossia of her utterance. It is noticeable that the teacher accepted the utterance made by Elie with “hmm” (line 581) even before Elie completed her utterance (line 580). This acknowledgement (line 581) shows that the teacher found Elie’s response acceptable. This classroom discussion draws our attention to the ongoing struggle of unitary language force and heteroglossia within the use of Eurocentric-Academic Discourse.

It was noted that the teacher often used porowhita (circle) when asking children to sit in a circular formation (FN1, FN2, FN4, and FN6). Thus, it was evident that the children had some knowledge of Te Reo Māori terminology related to “circle”. It seems that children developed an understanding of porowhita as a verb to indicate to be circular instead of a noun indicating a circle shape. Therefore, the analysis of unitary language and heteroglossia in this Key Moment also draws attention towards the possibility that Te Reo Māori vocabulary for geometric shapes may have permeated in the meaning-making process in this New Zealand mathematics classroom. It can be argued that the unitary language force may support the Eurocentric-Academic Discourse that defines an accepted and widespread way of teaching and learning of geometry, whereas the use of Māori vocabulary permits heteroglossia to provide alternative meanings within the dialogic space dominated by Eurocentric-Academic Discourse during the teaching and learning of geometry shapes.

Two discourses were identified in all the ten Key Moments at step 1 of the macro-level analysis (See Section 5.1.3): Eurocentric-Academic Discourse, and Everyday Discourse. In this section, unitary language and heteroglossia were used to investigate the interaction of these two discourses and to explicate how different meanings of the same term, “side”, are negotiated during three of the ten Key Moments. Three different meanings of “side” emerged during these three Key Moments. This analysis contributes to the third research question, which concerns the process of interaction of discourses as characteristic of the dialogic space that influences children’s construction and negotiation of meaning related to shapes and their properties. The analysis presented in this section made two main findings. First, the analysis revealed that the unitary language force and heteroglossia may support either of the two discourses present in the classroom to ensure the flow of meaning during interactions (see Key Moments 4.3b and the 4.4a). Second, the analysis also highlighted that unitary language force and heteroglossia might emerge from the same discourse; for example, in the last Key Moment (Key Moment 4.5a) Eurocentric-Academic Discourse became the basis for both heteroglossia and unitary language force. Thus, the interplay of discourses within the dialogic space is dynamic in nature.

The next section highlights the interplay of unifying (unitary language) and diversifying (heteroglossia) language forces at the level of languages, often reflected through participants’ utterances.

5.2.3 Interplay of the Competing Languages

This section explores the interaction of heteroglossia and unitary language forces at the level of languages embedded in classroom interactions. Demographic data about the children revealed that children in the Year 5/6 class included nine multilingual children with varying degrees of proficiency in the different languages in their linguistic repertoire.

The micro-level analysis of the Key Moments (See section 4.2 to 4.5 in Chapter 4) suggested that multilingual children may tend to use prosodic features of their first language when speaking English. This micro-level analysis underscored the presence of multiple languages that operated at different times as the multilingual children engaged in group or whole-class interactions. The interplay of these languages influenced the meaning-making process as participants (teacher and children) engaged in group and/or whole-class interactions.

For the purpose of analysis at the macro-level, Key Moments (see Table 5.2) are explored to account for the heteroglossia and unitary language forces operating at the level of languages during classroom and group interactions. All the languages present in the classroom interactions are clearly present in these Key Moments.

Table 5.2

Competing Languages in Selected Key Moments

Key Moment	Participants	Competing languages identified through prosodic patterns
Key Moment 4.2a: “I saw this as some kind of shape”	Ozan (bilingual in Somali and English)	Somali (slightly higher volume to mark the stress, see Section 4.2, Chapter 4)
	Tahi (bilingual in English and Tongan)	Tongan (use of main stress on second last vowel - a Tongan intonation pattern for stress, see Section 4.2, Chapter 4)
Key Moment 4.2b: “Whaea look, a perfect square”	Matiu (bilingual in Māori and English)	New Zealand English (use of HRT to state his answer, see Section 4.2, Chapter 4)
Key Moment 4.3a: “what’s a triangle 3D?”	Yue (bilingual in Chinese and English)	Chinese (use of flat pitch for informing her answer, see Section 4.3, Section 4)
Key Moment 4.5a: “I think it’s 3D because it’s not 2D.”	Kimi (bilingual in English and Tongan)	Tongan Conversational pattern (use of whispery voice to state her answer but not to interrupt teacher’s talk, see Section 4.5, Chapter 4)

In these Key Moments, the use of English as the medium of instruction demonstrates its status as the unitary language force. However, the prosodic features from participants’ first/heritage language in their utterances is evidence of heteroglossic forces that inform the intended meaning of the participants (as shown in micro-analysis of Key Moments, see

Sections 4.2-4.6, chapter 4). The use of Somali (Ozan), Tongan (Tahi and Kimi), and Chinese (Yue) prosodic features in these Key Moments highlights the heteroglossic features of participants' utterances and their influence on meaning-making. Particular to New Zealand, the use of HRT highlights the influence of Te Reo Māori on English (Stubbe & Holmes, 2000). In addition to this, the teacher often used "ka pai" as a way to show her appreciation for children's responses (fieldnotes - Lessons 1-6). "Ka pai" is a Māori phrase that is used to imply a positive evaluation and means "well done". Therefore, the use of Māori phrases brought Te Reo Māori into the classroom discussion with English as the dominant language.

5.2.4 Summary: Interplay Between Unitary Language and Heteroglossia

The third research question explores the ways in which different characteristics of dialogic space influence children's negotiation of meanings of shapes and their properties as they interact in a geometry classroom. Seven speech genres and two major discourses were identified in section 5.1 as two main sets of characteristics of dialogic space. This section focused on exploring tensions and interactions among speech genres and discourses using the concepts of unitary language and heteroglossia at three levels, voices, discourses, and languages, to respond to the third research question. The interaction of heteroglossia and unitary language drew our attention to the social and historical meanings of the words used that are preferred within the interactional moment. The macro-level analysis in this section highlighted several main findings. At the level of voices, the analysis revealed that the children and the teacher may imbue the same term, such as "perfect" and "square", with their different meanings; and the children may use teacher's voice to display authority. At the level of discourses, the analysis suggests two main findings. First, heteroglossic and unitary language provide the term "side" with different interpretations as it is used in classroom and group interactions. Importantly, the analysis also revealed that unitary language force and heteroglossia may support both Everyday Discourse and Eurocentric-Academic Discourse depending upon the circumstantial context, that is, the minute moment of conversation. Second, the interplay of unitary language and heteroglossia may be evident within the same discourse. Finally, the interaction of competing languages in this classroom highlighted the presence of multiple languages, even though the dominant language was English.

The following section presents chronotopic analyses of different Key Moments to explicate a variety of chronotopes that influence the process of meaning-making.

5.3 Chronotopic Analysis of Classroom Interactions

Chronotopic analysis of the Key Moments highlighted the moments of learning in interactions, where one or more participants made explicit reference to time (in the past or future) that influenced the negotiation of meanings within the utterances in the present. These moments are labelled as Chronotopic Moments. Chronotopic Moments are identified as another characteristic of dialogic space that is made available to other participants through utterances that influence children's negotiation of meanings; thus, the analysis of Chronotopic Moments in this study responds to the third research question. Chronotopic analysis allows insights into the time aspect of the specific moments of learning from the different times (past or future) embedded in the utterances during Key Moments. It should be noted that not all the Key Moments had Chronotopic Moments within them. Chronotopic analysis of two of the Key Moments (Key Moments 41a and 4.5a) is presented in the following sub-sections (see Sections 5.3.1 and 5.3.2) to illuminate the role of Chronotopic Moments in the process of meaning-making during classroom interactions.

5.3.1 Chronotopic Analysis of Key Moment 4.1a

The first Key Moment selected for chronotopic analysis is Key Moment 4.1a (see Figure 5.1). Chronotopic analysis of this Key Moment focuses on the participation of one of the children, Ozan who participated in the group discussion as the group identified shapes in the task sheet. In Key Moment 4.1a, Ozan attempted multiple times to draw his peers' attention to the shape he was referring to in the task sheet (see Figure 4.1, Section 4.1, Chapter 4). His reasoning for why he identified a shape as an eight-sided shape is evident only in part of the conversation where his teacher conversed with the group (see Key Moment 4.1a (part ii) in Section 4.1, Chapter 4). The moment in Ozan's utterance (lines 339-345) where he refers to another experience in which he learned about the identified shape is labelled a Chronotopic Moment (see Figure 5.1).

Figure 5.1

Chronotopic Analysis of the Key Moment 4.1a

Extract 4.1a (part ii)

#	Speaker	Text
314	teacher	↓what else can you see so cir:cle: kinda: yeah
315		circle
316	Ozan	↑I saw this as some kind of sha:pe that i know?
317	teacher	↑do you? ((teacher smiles))
318	Ozan	((nodding))
319	teacher	↑I kno::w (.5)what what kinda shape can we call
320		that then
321	Ozan	its like(.)((drags his finger over the shape))
322		one of those shapes thats like its goes like this
323		((gestures with both hands to show sides of the shapes))
324		like this [like that
325	Tahi	[a square [°probably° ((use gestures to show lines))
326	teacher	[ah:: (.5)so:(.) you ↑you ↑thinking like(.8)this
327		one(.5)↑lets see tho:se(.)if you would to(.2) give
328		it si::des ay?
329	Ozan	yeah yeah [third one
330	teacher	[°one two°three four five six seven eight [ni:ne
331	Tahi	[theres ↑only eight
332		
333	teacher	one so ↑theres EIGHT (.2)one two three four fi:ve
334		six seven eight, do you think eight,so: do you know
335		the ei:ght one? ((looked at Ozan))
336	Tahi	↑ITS a rect↑(.) oh no ((tried to think))
337	teacher	so do you do you know what the eight one is ((the
338		question is explicitly directed to Ozan))
339	Ozan	I ↑know but(.2)I just my brother used to watch a
340		movie about(.2)of this kind o shapes ↑(.5)that I
341		know(.5)their na:mes are are like like twelch(.4)I
342		I thought it was and there's they were saying like
343		a like a lot of shapes like one two three
344		until(.5) they have passed eight, and then ten an
345		twelve or something(.5)I dont remember by how much
346		it was (.2)but I do remember by [how many
347	teacher	[NA:: I think some I think somehow you kno:w but
348		you are not you cant remember so ((coughed and cleared throat))
349	Tahi	I ↑know but I dont know the name
350	teacher	yes okay so(.5)eight sided figure (.2) is::
351		((cleared throat)) is a octagon. remember octagon ((looks at the
352		camera)) okay so um
353	Tahi	octa:gon: there's a six one I am pretty su:re

Chronotopic Moment: Indicative of past moment

The use of high onset (line 316) may indicate that Ozan intended to draw the teacher's attention to his identified shape. Additionally, the use of a Persuasive speech genre in this utterance may indicate Ozan's active participation during this time, even though he was not confident of his geometry claim. In the following conversation, the teacher acted as the facilitator (see lines 317, 319, 326-328, 333-335) and helped Ozan to verbalise his thinking.

As the teacher provided feedback and assistance, Ozan explained the criteria for naming the shape without being able to recall the name of the shape (lines 339-346). In the beginning of his utterance (line 339), Ozan stated, “my brother used to watch”, which signals a different time in which he learnt about this shape. The utterance can be interpreted as his attempt to bring a learning moment from his past, probably at home, to make sense of the name of the shape. This part of the Key Moment displays how Ozan brought one of his moments of learning about shapes from a different time-space nexus (i.e., home environment) to his present moment of learning to display his knowledge during the present task (see Figure 5.1). As time is referred to explicitly in this utterance, this moment is identified as a Chronotopic Moment in this Key Moment. It seems that Ozan learnt about the relationship between the number of sides and the name of the shape while his brother watched television. What is of interest here is that the statement that “my brother used to watch” may indicate that he was not actively watching television with his brother. His participation at that moment in time that he is referring to was passive in nature.

Another Key Moment in which a Chronotopic Moment was identified is Key Moment 4.5a. Chronotopic analysis of this Key Moment is presented in the following section.

5.3.2 Chronotopic Analysis of Key Moment 4.5a

The second Key Moment in which a Chronotopic Moment was identified is the Key Moment 4.5a, where children and the teacher discuss the Te Reo Māori names for 2D shapes. The teacher (Interview 3) stated that she referred to The NZC, along with the “Te Whānau Taparau – the polygon family” unit, regarding teaching about shapes and their properties in Te Reo Māori. The “Te Whānau Taparau” unit (NZMaths, 2021) forms the macro-context for this Key Moment (see Appendix A for the unit). In the unit, names for the regular polygons are provided, for example, tapatoru as a triangle, tapawhā as a square. The classroom discussion followed the rule of naming the shapes by identifying the number of sides. This rule is introduced as a way to classify 2D shapes at The NZC Level 1, which is based on the Eurocentric-Academic Discourse embedded with western mathematical ideas. During this Key Moment 4.5a, the sequential nature of classroom interaction revealed that the teacher and children were engaged in a discussion as a whole class for identifying the pattern of naming shapes in Te Reo Māori (see Figure 5.2). Moreover, the children used their prior knowledge of number names in Te Reo Māori with the rule of naming shapes by counting the number of sides and suffixing the number name to the word “tapa” (meaning edge or side) to identify the Māori shape name.

In this Key Moment, two Chronotopic Moments were identified (see Figure 5.2). Both Chronotopic Moments were identified in the teacher's utterances (see line 577 and line 585). The first Chronotopic Moment (line 577) signals a moment in the future whereas the second Chronotopic Moment (line 585) elaborates a moment in the past.

Figure 5.2

Chronotopic Analysis of the Key Moment 4.5a

Excerpt 4.5a

#	Speaker	Text
557	teacher	↑what abo:ut um:::(1.0) whats a
558		tapa:ono(6.0)=
559	Zara	hhhhhhh (aspiration)
560	teacher	=umm ↑Kayla(.4) what wud a tapaono be (8.0)
561		↑tapari:ma pentagon(.)tapaono: Yue↑
562	Elie	((holding the hexgonal skelton that she made using sticks to show its a
563		hexagon))
564	Yue	hexa↑go:n
565	teacher	good girl. hexagon(4.5)
566	Elie	°I told you°
567	teacher	um:↑(h)(1.2) heres my question.(1.0)
568		could ↑I ha:ve(2) a tapatahi:(1.5)
569		((Elie, Yue, Matiu, Ethan said no in chorus))
570	Zara	YES a cir ((teacher smiled as Zara responded))
571	Matiu	a circle has no ↓side
572	Elie	[no >↑you cant ↑you cant< °you cant°
573	Ethan	[CIR↑CLE::
574	Matiu	[°no a circle has no side°
575	Elie	[coz ((put her hand up for answering))
576	teacher	can ↑I have a tapatahi.(.4)can I.(1.0)↑what
577		↑what would tap ↑if I followed that pattern what
578		would tapatahi be Elie
579	Elie	a circle. but ↑you ca:nt have it bcoz circles
580		have no si:des [and] no corners
581	teacher	[hmm] ↑you are ↑brilliant
582	teacher	accepted.
583	teacher	so ↑what do we ca:ll a circle
584	Tane	a ha::o cylinder. (h)
585	teacher	<what do we call a cir:cle> when ↑I say (.)
586		can you get into a umdumdumdumd..
587	Ethan	↑PORA°whita°
588	teacher	[umdumdumdumd(h)(h)(h)
589	teacher	>what is it<(h)(h)
590	Ethan	porowhita
591	teacher	ya↑ya↑ porowhita you get all the time (.4)

Chronotopic Moment 1: Indicative of future moment

Chronotopic Moment 2: Indicative of past moment

The first Chronotopic Moment is when the teacher said: "Can I have tapatahi If I followed that pattern, what would tapatahi be" (lines 576-578), and Elie responded with her argument. The teacher's question, "what would tap if I follow the pattern" (line 577), makes an explicit reference to a moment in future. However, restructuring the utterance as "could I have tapatahi" signalled the possibility of an exception to the rule. Therefore, it is this moment when Elie built and displayed her understanding of why tapatahi cannot be a circle.

As the conversation proceeded, the teacher structured her utterance (line 583) so that it would activate children's earlier moments of learning by asking, "what do we call a circle". This statement shows that the teacher attempted to help her children to recall the Te Reo Māori term that they use for a circle. In her following utterance (line 585), by saying "when I say can you get", the teacher referred to a previous moment of learning in classroom history during this lesson and previous lessons, when she had asked children to sit in a circular formation by saying "can you get into porowhita" (Teacher Interview 2, FN1, FN2, FN4, and FN6). This moment in interaction is identified as the second Chronotopic Moment. The teacher did not refer to the Māori-mathematical framework for naming a circle; rather, she allowed her children to bring their prior knowledge. The explicit reference to earlier moments of learning allowed Ethan to recall the Māori name of the circle (line 587) and allowed a Māori-mathematical framework of naming shapes to come to the centre from the periphery in order to contribute to the meaning-making process in the current utterance.

It is to be noted that the macro-context remains the same for all these micro-moments within the same Key Moment. The children and teacher were aware that they were talking about the Māori names for shapes. Children used their knowledge of Māori number names to identify the pattern in tapatoru, tapawhā, taparima, and tapaono. The chronotopic analysis of this Key Moment suggests that the Chronotopic Moment may allow explicit use of Māori-mathematical knowledge, that is, the name of a circle in Te Reo Māori in the presence of Eurocentric-Academic Discourse with western mathematical ideas.

5.3.3 Summary: Chronotopic Analysis of the Key Moments

The chronotopic analysis of two Key Moments may suggest that the explicit reference to time may aid in the negotiation of meanings about shapes and their properties in a primary classroom. In this section, chronotopic analyses of two of the ten Key Moments were presented. These Key Moments revealed that Chronotopic Moments as one of the characteristics of dialogic space can act as pivotal moments of learning when a moment from another time zone is used to develop an understanding of shapes and their properties. The next section pulls out the main findings from the macro-level analysis.

5.4 Overall Findings II: Macro-Level Analysis

The macro-level analysis of the ten Key Moments sought answers to the third research question with a focus on exploring the characteristics of dialogic space and how these characteristics influence the process of negotiation of meanings pertaining to shapes and their properties. The macro-level analysis of the ten Key Moments presented in this chapter explored the underlying emotional stances, speech genres, and discourses during whole-class and group interactions in a Year 5/6 classroom. Following the identification of emotional stances, speech genres and discourses, the analysis explored the interaction of unitary language and heteroglossia at three levels – voices, discourses, and languages. Double-voicedness and the interplay of unitary language with heteroglossia allowed the exploration of socio-cultural dimensions that influence the negotiation of shapes and their properties. In the last step of the analysis, the Bakhtinian concept of chronotope was used to identify the Chronotopic Moments within two of the ten selected Key Moments. In terms of the characteristics of dialogic space, the analysis made the following findings:

- First, the analysis revealed that children display a variety of emotional stances within a Key Moment and across Key Moments (see Section 5.1.1).
- Second, the analysis showed that children and teachers have access to a variety of speech genres within the dialogic space that they use during whole-class and group interactions (see Section 5.1.2), and children may use any of these speech genres to negotiate meanings about shapes and their properties.
- Third, two discourses were identified in the data: Eurocentric-Academic Discourse, and Everyday Discourse.
- Fourth, the analysis uncovered the time aspect of the learning and highlighted the notion of the Chronotopic Moment within two of the ten selected Key Moments (see Section 5.3) as one of the characteristics of dialogic space that may act as a tangible and fruitful link between the previous, present, and future moments of learning.

In terms of how these characteristics influence the negotiation of meanings, the analysis revealed:

- The interaction of voices as speech genres indicated that the meanings of words like “sides” or “perfect” are in a state of constant flux as the participants (including the teacher and the children) embed the same word with their own meanings as they proceed in the conversation (See Section 5.2.1).

- The unitary language forces may support either of the discourses, Eurocentric-Academic Discourse and Everyday Discourse, for ascribing meaning to the utterance. Moreover, the interaction of unitary language forces and heteroglossia forces may arise within the same discourse present in a classroom (see section 5.2.2).

As the macro-level analysis is developed using the insights from micro-level analysis, the next chapter combines findings from the thematic analysis, micro-level analysis, and macro-level analysis to present a coherent discussion of the findings outlined in both Chapter 4 and Chapter 5.

Chapter 6.

Discussion: Answers to Research Questions

This chapter brings together the key findings from the analyses to answer the three research questions central to this thesis and discusses the findings within the field of mathematics and geometry education research. The following research questions guided this study:

1. *What discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
2. *How do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*
3. *What characteristics of the dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

To answer these research questions, Ethnomethodology and Bakhtin's Dialogic Theory informed the theoretical underpinnings and the methodological framework of the study. Six geometry lessons on shapes and their properties in a New Zealand Year 5/6 multilingual primary classroom were observed. Data were collected using six data-gathering tools, which included the audiovisual recording of six lessons, fieldnotes, semi-structured teacher interviews, focus group interviews with four groups of children, a questionnaire, and documents including The New Zealand Curriculum (NZC), teacher's unit plan and children's work samples. Data were analysed in three phases: thematic analysis (Chapter 4), micro-level analysis (Chapter 4), and macro-level analysis (Chapter 5).

Thematic coding of data enabled me to identify five themes: (i) making sense of 2D shapes, (ii) making sense of 3D shapes, (iii) relating 2D shapes with 3D shapes, (iv) mathematical construct of dimension, and (v) naming shapes in Te Reo Māori. For each theme, two Key Moments which were identified from the audiovisually recorded data were analysed at the micro-level and macro-level of analysis. At the micro-level analysis, CA techniques were used. In Chapter 4, I presented findings from thematic analysis and micro-level analysis to respond to the first and second research questions. Based on the findings from the micro-level analysis, the macro-level analysis explored answers to the third research question regarding the characteristics of the dialogic space that influenced children's negotiation of meanings about shapes and their properties.

As the analysis at the macro-level was built upon the findings from the micro-level analysis, I present related findings from these analyses in the form of a table to help me to answer the three research questions that guided this study (see Appendix N for findings from each phase of analysis). Table 6.1 (below) presents the research questions, relevant findings, and the sub-sections in which these findings are discussed.

Table 6.1

Research Questions and Discussion of Findings

Research Questions	Findings	Sub-Sections
RQ1. What discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?	**Finding.** Children used different discursive constructions to represent their understanding of the mathematical construct of dimension, including "flat vs fat", "different ways to go", or "another world".	**6.1** Discursive Constructions of "dimension"
	Finding. Children used discursive constructions of "3D rectangle" to talk about cuboid or rectangular prism. Similar discursive constructions, for example, "3D Triangle" and "3D circle" were used to represent pyramid/prism and sphere, respectively.	**6.2** Discursive Constructions of "3D Rectangle", "3D Triangle", and "3D Circle"
	Finding. Children discursively used Te Reo Māori number names with prefix "tapa" to represent 2D shapes in Te Reo Māori.	**6.3** Te Reo Māori in an English-medium New Zealand classroom
RQ 2. How do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a Year 5/6 New Zealand multilingual primary classroom?	**Finding.** Children's use of prosody plays an interactive role in the meaning-making process. **Finding.** Multilingual children used prosodic features of their multiple languages while interacting with others to construct their understanding of shapes and their properties. **Finding.** Children displayed different emotional stances through their prosody as embedded in their utterances within the same Key Moment.	**6.4** Role of Prosody in a Multilingual Classroom

	Finding. Children used two kinds of gestures during classroom interactions to represent their understanding of shapes and their properties.	**6.5** Gestures as Features of Turn-Design
RQ 3. What characteristics of the dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?	**Finding.** Children and the teacher considered responses given in geometry-specific language as preferred responses. **Finding**. The teacher's overt negative evaluation of children's incorrect responses was considered a dispreferred response. **Finding.** Children and the teacher considered children's responses as dispreferred if they provided a response without being given the turn to speak.	**6.6** Preferred and Dispreferred Response in a Geometry Classroom
	Finding. A variety of speech genres is available in a classroom. **Finding.** Children may use any of the speech genres (including teacher's speech genres) available to them in the classroom dialogic space to negotiate meanings about shapes and their properties.	**6.7** Speech Genres and the Negotiation of Meanings
	Finding. The negotiation of meanings about shapes and their properties is influenced by the interplay of unitary language and heteroglossia *between* two discourses. **Finding.** The negotiation of meanings about shapes and their properties is influenced by the interplay of unitary language and heteroglossia *within* the same discourse.	**6.8** Dominant Discourses, their Interaction and Negotiation of Meanings
	Finding. Chronotopic Moments embedded in children's utterances may act as the link between different time zones of learning and, thus play an important role in the negotiation of meanings during classroom interactions.	**6.9** Chronotopic Moments and Negotiation of Meanings

In the following sections, I first discuss findings pertaining to the geometric ideas that are represented through children's discursive constructions in their utterances in Sections 6.1, 6.2, and 6.3. These three sections respond to the first research question. The first section, 6.1, discusses the discursive constructions about dimension in an English-medium primary school in New Zealand.

6.1 Discursive Constructions of "Dimension"

In mathematics education research, the mathematical construct of dimension has rarely been explored (Panorkou, 2011; Panorkou & Pratt, 2016), even when the studies focused on geometric shapes and their properties (e.g., Fujita et al., 2020; Roth & Gardener, 2012; Seah & Horne, 2019). The lack of explicit explanation of what "dimension" means in these studies may suggest that the mathematical construct of dimension is assumed to be understood without any need for explicit teaching or learning. This study adds to the research literature on children's understandings of the mathematical construct of dimension at the primary school level. The micro-level analysis (see Section 4.4, Chapter 4) revealed that children discursively construct dimension in a variety of ways. Three major discursive constructions were identified: (i) dimension is expressed as "flat vs fat" for the "D" in 2D and 3D shapes, (iii) dimension is seen as "ways to move", and (iii) dimension is "another world". In the following sub-sections, I discuss each of these discursive constructions about dimension in light of current geometry education research.

6.1.1 Dimension as "Flat vs Fat"

The micro-level analysis revealed that children may resort to the "fat vs flat" analogy for explaining what the "D" stands for in "2D" and "3D". The finding is consistent with the research of Morgan (2005) and Panorkou (2011). These studies reported that children may describe 2D shapes as "flat" and 3D as "fat". However, the micro-level analysis in this study indicated that the use of the "flat vs fat" analogy may not be useful for describing the dimensional property of shapes. In Key Moment 4.4a, we saw that one of the children (Elie) was confident that the shape that she made using sticks and adhesive was 3D as she could see the thickness of the sticks. Even when the teacher reworded the "flat vs fat" analogy as "coming out of the ground", Elie maintained her argument. This Key Moment draws our attention to her understanding that even though the objects (sticks, in this Key Moment) that she used were thin, the shape could be held in her hands or "was coming out of the ground", therefore making it a 3D shape instead of 2D. A similar understanding was observed in Matiu's utterance in Key Moment 4.2b (see Section 4.2.2), when he was trying to make a triangular prism using playdough. He said that all the shapes they were making were going to be 3D because it was not possible to make a shape which was "not fat" using playdough (lines 12-14, Key Moment 4.2b). Again, this excerpt emphasises that the "flat vs fat" analogy was not helpful in explaining the difference between 2D and 3D shapes.

The next discursive construction talks about mathematical construct of dimension as “ways to move”.

6.1.2 Dimension as “Ways to Move”

The second kind of discursive construction was evident in another child utterance: “3D is three ways to go, and 2D is two ways to go” (Matiu, FG2), which may indicate some understanding of dimension as an ability to move in different ways. There are very few studies with a specific focus on children’s understandings of dimension as a mathematical construct. One study by Panorkou and Pratt (2016) focused on exploring twelve 10-year-old students’ experiences of dimensions using Google SketchUp software. The software provides students with the experience of a simulated 3D environment. Students were required to work on the 2D floor and 3D space (in the Google SketchUp software) to experience different dimensions. Panorkou and Pratt (2016) found that students were able to express an intuitive understanding of dimension. They reported that students could describe the difference between 2D and 3D in terms of the directions, positions and orientations in which the students could move; one of the students stated, “because, that’s two D, you go only left and right and up and down, while if you are in three D you go everywhere. For example, let’s say a house if it was two D, you wouldn’t be able to go into the house” (Panorkou & Pratt, 2016, p. 219). This way of expressing dimension is evident in Matiu’s utterance, as presented earlier.

The third discursive construction described dimension as “another world”.

6.1.3 Dimension as “Another World”

The discursive construction of dimension as “another world” is evident in several children’s utterances: “another world” (Matiu, FN2), “different world” (Alyssa, FG3), “different place” (Zara, FG4). Discursively constructing dimension as “another world or place” may be interpreted as signalling a lack of sound understanding of dimension as a mathematical construct. These discursive constructions about dimension do not construe dimension as a property of shape or object. Instead, they seem to display a conception of dimension as an experience of some other world, not representative of our surrounding world, which may lead to a misconception that our world is 2D rather than 3D. Alternatively, these utterances may be interpreted as signalling the difficulty that children had in expressing their understanding of dimension using language. Interestingly, this discursive construction of dimension has not previously been reported in research.

None of the above three ways of expressing an understanding of dimension accords with the view taken in The NZC, which takes dimension as a measurement attribute of the object. The mathematical construct of dimension is not clearly defined in any of the major curriculum documents, including The NZC, on teaching and learning of shapes. Nevertheless, other curriculum documents (e.g. units on teaching and learning of shapes at The NZC levels 1, 2 and 3) define 2D shapes as planar shapes having length and width, and 3D shapes as solid shapes having length, breadth, and height (NZMaths, 2021a). These definitions highlight two different understandings of dimension.

First, defining 2D shapes as planar shapes and 3D shapes as solid shapes underscores the understanding of dimension from Euclid's boundary notation perspective (Manin, 2006). According to Euclid's boundary notation, the points are the boundaries of lines, lines are the boundaries of the surface, and the surface accounts for the boundary of the solid object (Skordoulis et al., 2009). As a result, the dimension of, let us say, a sphere (hollow sphere) and a sphere region (solid sphere) would be different (Skordoulis et al., 2009; Ural, 2014). That is, to mark the position of a point on the curved surface of the sphere, we require only two dimensions, latitudes and longitudes, whereas to mark the position of a point in the spherical region that is inside the solid sphere we would require longitudes, latitudes and an additional dimension (probably, depth). This way of understanding dimension may account for the reasons why in Slovenian language the *boundary* of the circle, which is a circumference (*krožnica*) is not considered a 2D shape, whereas a *disk* is the 2D shape of a circle (*krog*) (Bezgovšek Vodušek & Lipovec, 2014).

Second, defining 2D shapes as having length and breadth, and 3D shapes as having length, breadth, and height underscores the understanding of dimension from a measurement perspective. This construction emphasises dimension as a measurement attribute that helps in measuring different sides of shapes and does not require an understanding of "planes" as a necessary criterion. This understanding of dimension does not align with any of the descriptions demonstrated in children's utterances in this study, as discussed earlier.

The discussion of the findings in regard to an understanding of dimension highlights the need to develop a clear and comprehensive understanding of what dimension implies when presented as a mathematical construct in curriculum documents. The mathematical construct of dimension needs to be understood and defined from both Euclid's boundary notation perspective and a measurement perspective to provide a comprehensive understanding of dimension as a mathematical construct or idea. A comprehensive understanding may enable teachers and children to be acknowledge the contexts within which the construct of dimension is used and for what purposes. In addition, more opportunities need to be provided for children to appreciate how context may inform the dimensions of the object in question. Hence, the present study adds to the knowledge field of geometry education on understanding dimension and suggests future opportunities for further research in this area.

In the following section, I discuss children's discursive constructions about 3D shapes as "3D rectangle", "3D triangle" and "3D circle".

6.2 Discursive Constructions of "3D Rectangle", "3D Triangle" and "3D Circle"

The second kind of discursive constructions that were evident in this study concerned the naming of 3D shapes. Micro-level analysis of several Key Moments revealed the constructions that children used to identify and name 3D shapes (see Key Moment 4.2a, 4.3a). These discursive constructions included naming the cuboid/rectangular prism a "3D rectangle", the cube as a "3D square", the pyramid and the triangular prism as a "triangle 3D" or "3D triangle", and the sphere as a "3D circle". The children were demonstrably confident about their knowledge claims when they used such discursive constructions. From a Te Reo Māori mathematical framework perspective, discursive constructions of the 3D shapes of cube, sphere, and rectangular prism or cuboid as 3D square, 3D circle, and 3D rectangle (see Key Moment 4.3a, Chapter 4) may represent children's emerging understandings of the difference between the 2D and 3D shapes. In Te Reo Māori, a rectangular prism is *poro-tapawhā hāngai*. As stated earlier, tapawhā hāngai implies a rectangle; and *poro* in Māori means a block or a prism. Thus, naming a rectangular prism a "3D rectangle" may signal children's emerging understandings of shapes based on their cultural mathematical knowledge (Barton, 2008; Meaney et al., 2009).

Lehrer et al. (1998) reported a similar finding. They conducted a 3-year longitudinal study with 30 primary school students in the United States and reported that primary school students may relate 3D shapes to already known 2D shapes. They also noted that children may show an increased tendency to talk about similarity between 2D and 3D shapes and may talk about "squishing" or "pulling" a shape to show the difference between shapes.

Recently, Seah et al. (2016) conducted a study in Australia with 214 Year 7-9 students and reported the use of "3D square" in one student's response: "a 2D square has four sides and a 3D square has six sides" (p. 590). They argued that these kinds of responses may indicate students' confusion of a 2D shapes, in this case, a square, with 3D shapes. Berenger (2018) too reported this discursive construction as confusion and argued that Year 7 students in Australia may refer to a cube as a 3D square.

However, the present study supports Lehrer et al.'s (1998) argument and suggests that naming 3D shapes in relation to already known 2D shapes is a discursive attempt by children to represent their understanding of 2D and 3D shapes rather than an indication of confusion, as noted by Seah et al. (2016) and Berenger (2018).

In the next section, I discuss discursive constructions pertaining to the use of Te Reo Māori in New Zealand Year 5/6 classroom.

6.3 Te Reo Māori in an English-Medium New Zealand Classroom

Micro-level analysis found that children discursively constructed the names of shapes in Te Reo Māori by using Māori number names with the prefix "tapa" (see Section 4.5). One such example is tapatoru for a triangle. The macro-level analysis of the Key Moments analysed under this theme revealed that the use of Te Reo Māori for naming shapes supported the dominance of Eurocentric-Academic Discourse of geometry learning (see Section 5.2.2).

The learning of shapes and their properties at level 3 in The NZC (Ministry of Education, 2007) focuses on enabling children to "use both English and Te Reo Māori to describe different polygonal shapes" (NZMaths, 2021a). That is, Te Reo Māori terms for naming 2D shapes are introduced to the children at level 3. Micro-level analysis revealed that the children constructed the names of shapes in Te Reo Māori by suffixing "tapa" (which means edge or side) with the number of sides (see Section 4.5). The macro-level analysis (Section 5.2.2) revealed that this pattern of naming shapes by counting the number of sides is embedded in the Eurocentric-Academic Discourse on geometry teaching and learning. During Key Moment 4.5a, children identified the pattern in the terms tapatoru (triangle), tapawhā (square), taparima (pentagon), tapaono (hexagon), tapawhitu (heptagon), tapawaru (octagon), tapatikau (decagon). However, using the pattern of combining the suffix "tapa" with the number of sides, tapawhā should be used to refer to a quadrilateral (a shape with four sides) (Māori Dictionary, 2021c) instead of a square. This is because the square indicates a shape with four equal sides perpendicular to each other as opposed to simply a shape with four sides.

The unit "Te Whānau Taparau – The polygon family" (NZMaths, 2021b) provides the basis of teaching and learning of shapes and their properties using Te Reo Māori at Curriculum

Level 3, or Year 5/6. The unit identifies tapawhā as a square. The use of tapawhā for square may result in confusion between square and rectangle, as was evident in Key Moment 4.5b, when Zara asked the teacher, "if square is tapawhā, what's rectangle?". In the same unit, tapawhā rite is also used to name the square shape. Tapawhā rite is the Māori word for square, and literally implies four equal sides. Rite in Te Reo Māori means "to be like, equal, in proportion" (Māori Dictionary, 2021a). Thus, stating that "square" in English is equivalent to "tapawhā rite" in Te Reo Māori may support the idea that the sides are equal; however, the same term may also signal the shape of the rhombus (a closed flat shape with four equal sides).

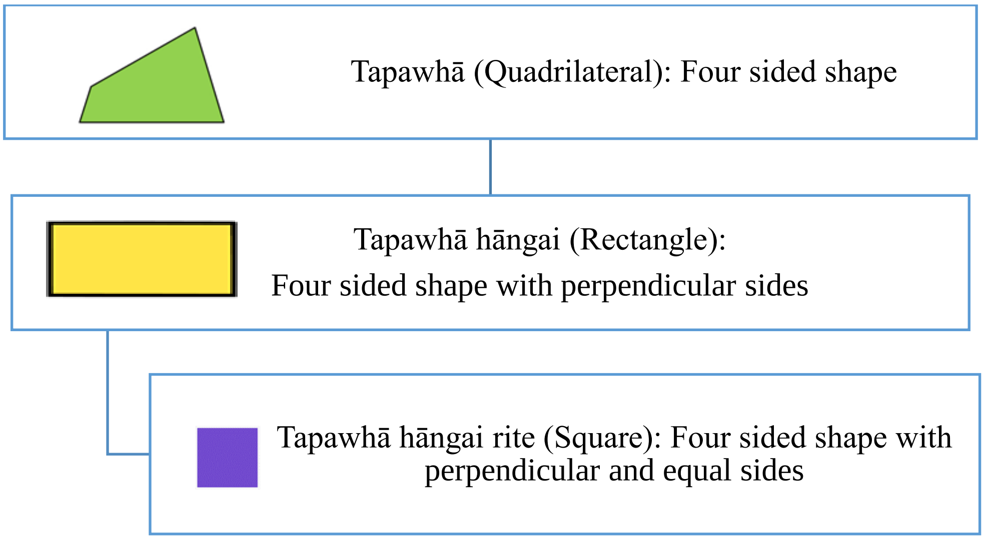
The unit "Te Whānau Taparau – The polygon family" informs the teaching and learning of shape names using Te Reo Māori in English-medium state schools in New Zealand (Teacher, Interview 3). Hence the attribution of shape names in this unit may have a direct influence on the meanings constructions (a focus of RQ3) about geometric shapes using Te Reo Māori in an English-medium primary classroom. The use of the Westernised rule for naming shapes and the lack of clarity and uniformity of the Te Reo Māori shape names in the unit highlight two major issues.

Firstly, the geometric idea of naming shapes according to the number of sides as used in the unit is Eurocentric in nature, even though the language being used is Te Reo Māori (Trinick et al., 2015). Therefore, the use of Te Reo Māori in this instance supports the epistemic power of mathematical knowledge embedded in Eurocentric-Academic Discourse (Parra & Trinick, 2018). Trinick (2015) has also argued that Indigenous languages are often used to support the teaching and learning of western school mathematics, thereby maintaining the power differential between the knowledge systems of the two languages.

Secondly, this use of Te Reo Māori may also underutilise the mathematical understanding (of western school mathematics) that could be accessed through the active use of Te Reo Māori in geometry classrooms. For example, the "Te Whānau Taparau" unit uses tapawhā hāngai to refer to a non-square rectangle or simply a rectangle or oblong (NZMaths, 2021b). Specifying a rectangle as tapawhā hāngai does reiterate the idea of a quadrilateral shape with perpendicular sides, thus having right angles. The meanings of "rite" and "hāngai" are equal and perpendicular, respectively. Therefore, stating the names of rectangle and square as tapawhā hāngai and tapawhā hāngai rite, respectively, may underline the hierarchical relationships among quadrilateral, rectangle, and square, as shown in the Figure 6.1 below.

Figure 6.1

Hierarchical Relationship among Quadrilateral, Rectangle, and Square



The adjective hāngai in Māori names for both rectangle (as tapawhā hāngai), and square (as tapawhā hāngai rite) indicates that the sides are perpendicular in both. The additional adjective of rite for square indicates the additional property of equal sides. Therefore, using Māori terms may provide an easy way to comprehend geometrical ideas about shapes, their properties, and hierarchical relationships (Barton, 2008) that may be difficult to follow using English.

In line with this, Bartolini Bussi and Baccaglini-Frank (2015) found that children may create their own terms for a category encompassing all rectangles and squares. In their research with Italian Grade 1 (6-7 years) students using bee-bots, they asked students to represent a sequence of commands for bee-bots in order to display their understanding of mathematical definitions of rectangles, including squares. They found that Year 1 children invented the term "squarized O" for shapes (in this case, square) with four right angles. They argued that inventing this term helped students to identify hierarchical relationships between rectangles and squares. They further argued that in European languages, including Italian and English, the terms for square (*quadrato*) and rectangle (*rettangolo*) may signify total separation of these kinds of geometric shapes. Furthering their argument, they stated that some languages, for example, Chinese, have the potential to linguistically mark the similarity between squares and rectangles as two kinds of the same shape, unlike English and Italian.

Accordingly, the present study suggests there is a need for further research to explore how Te Reo Māori can be used to promote better understanding of geometric shapes, their properties and the hierarchical relationships among different geometric shapes in New Zealand English-medium schools. Future research exploring the use of Te Reo Māori in English-medium schools may provide ways to integrate Māori mathematical knowledge in English medium state schools in New Zealand while furthering the use of culturally responsive pedagogies (Averill, Te Maro, et al., 2009).

In the previous sections (Section 6.1 to 6.3), I have discussed three discursive constructions that were found in the data. These discursive constructions respond to the first research question posed in this study, which concerns the discursive constructions that 9 to 11-year-old children used to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom.

In the following sections (Sections 6.4-6.6), I present a discussion of findings relevant to the second research question, which focuses on how 9 to 11-year-old children interact to construct their understanding of shapes and their properties. The second question concentrated on the interactional tools that children used during classroom interactions. In the next section (Section 6.4), I present a discussion about how prosody acts as an interactional tool and contributes to the meaning-making process during interactions.

6.4 Role of Prosody in a Multilingual Classroom

The second research question aimed to examine how the children in this study interacted to construct their understandings of shapes and their properties. The use of CA techniques made visible the role of prosody in the children's meaning-making. In this section, I will discuss findings pertaining to the role of prosody in classroom interactions. The section is divided into three sub-sections. The first sub-section draws attention to the interactive role of prosody in the classroom by reviewing research in the field of sociolinguistics (Section 6.4.1). Following this, I discuss how English speakers and speakers of other languages (Section 6.4.2) use and can interpret the same intonation pattern differently. In the last sub-section (Section 6.4.3), I discuss how prosodic features embedded in utterances may provide an insight into the emotional stances taken by the participants.

6.4.1 Interactive Work of Prosody

Research has shown that prosodic features such as stress and the intonation patterns of low, high, and flat pitch are some of the interactional devices that participants use to draw listeners' attention to the focus of their utterances (Ward, 2019). The micro-level analysis of participants' utterances (see Chapter 4) provided evidence of the role of prosody in the meaning-making process during classroom interactions in the geometry classroom.

The finding showed that how language-in-use can provide different meanings through the subtle yet powerful effects of prosody embedded in utterances. For example, in Key Moment 4.4a, the child (Elie) and the teacher both used the same words “it’s fat” (see Section 4.4.1); however, the use of flat pitch in Elie’s utterance (line 357, Excerpt 4.4a) and a pause of 1.5 seconds and the stretching of “fat” in the teacher’s utterance signalled different meanings of the same sentence “it’s fat”. Elie stated that the shape was fat, whereas the teacher’s utterance implied that she did not agree with Elie’s response.

Hellermann (2003) studied the interactive work of prosody in classroom interactions. He focused on the intonation patterns in teacher feedback on student responses, and found that teachers often use different prosodic patterns with the same words used in the students’ responses to achieve different interactive results. That is, the teacher may use slightly high pitch (or mid-level pitch) with the same words to indicate incompleteness of students’ responses and to provide supportive feedback. This was evident in the teacher’s response (lines 225 and 228) to Matiu’s response during Key Moment 4.1b. Conversely, low or flat pitch may indicate a correct response with no need for further feedback, as evident in the teacher’s response to Matiu in another Key Moment (line 557, Key Moment 4.2a).

Ward (2019) found that high onset in pitch may be used to signal a new topic; and this was noted in this study (e.g., Garry’s utterance in line 213 in Key Moment 4.1a, and the teacher’s utterance in line 216 in Key Moment 4.1b). Other intonation patterns were also observed in the data. For example, children used high rising terminal intonation (HRT) to signal the speaker’s intention to check with the listener (Hay et al., 2008b). Similarly, a whispery voice (in Key Moment 4.3a) may act as a feedback-providing mechanism (Ward & Tsukahara, 2000). The current study is one of the first studies situated in New Zealand to use research on prosody from other fields of research to explore its use by children as an interactive device in a mathematics classroom and its role in the meaning-making process.

In addition to the finding that prosody plays a crucial role as an interactive device, the study also noted that multilingual speakers may use prosody from their repertoire of multiple languages (see Sections 4.1.1, 4.2.1, 4.3.1, 4.4.2). Moreover, the same prosody may be used and interpreted differently by different multilingual speakers (see Section 4.1.2), and this is discussed in the next section.

6.4.2 Prosodic Repertoire of Multilingual Children

The micro-level analysis revealed that multilingual children appeared to make use of a variety of prosodic features from across their linguistic repertoire. Moreover, multilingual children may perceive these prosodic cues differently from English-speaking children. For example, Yue (a bilingual child with Chinese and English as her languages) used flat pitch to state her response in Key Moment 4.2a. Research has shown that Chinese speakers often use flat pitch to emphasise their response (Pickering, 2001; Wu, 2004), as opposed to English speakers, who use low pitch or increased volume to draw attention to their responses (Ward, 2019). Chinese speakers focus on the information rather than the addressee (Wu, 2004); while in the case of English, HRT (as used by Kayla, an English-speaker, in Key Moment 4.3a) is used to establish whether the listener (in this case, the teacher) is following the speaker (i.e., Kayla) (Warren, 2016). This finding suggests that children may use all available resources within their repertoire of multiple languages, including prosodic patterns of those languages, to meet their communicational needs in a particular situation or context.

The study suggests, then, that the language practices of multilingual speakers involve the use of the overall repertoire of their languages, including prosodic features. Thus, it can be argued that the language practices of children in a multilingual context are diverse within the language, providing evidence of translanguaging rather than code-switching. Planas and Chronaki (2021) argued that, unlike code-switching, the translanguaging perspective is not limited to lexicon and grammar but incorporates intonations and vocalisations in how language is used as well. Children's use of prosodic features from multiple languages shows that their language repertoire includes both linguistic and paralinguistic (i.e., prosodic) features to produce meaning through one utterance. Moreover, in the study, children did not engage in switching codes between different languages; rather they actively used prosody as part of their whole language repertoire without considering those prosodic features as parts of different languages. Therefore, the study provides evidence that may signal that children engage in translanguaging rather than using their multiple languages as distinct entities.

The research literature which has focused on the language-as-resource perspective often ignores the role played by the prosodic repertoire in contributing to the meanings displayed in the utterance (e.g., Adler, 2002; Planas & Setati-Phakeng, 2009; Setati et al., 2002, among others). Therefore, classrooms like the one observed in this study with English as the language of instruction and the majority of children as speakers of English may be considered a monolingual classroom. This finding supports the argument presented at the beginning of this thesis that the New Zealand classrooms are indeed multilingual spaces even when the words being used are English, an argument supported by Barwell, Clarkson, et al. (2016). This study, therefore, adds to the knowledge base exploring multilingualism in mathematics education. It also suggests a need for further research into the use of prosodic features as part of languaging in classrooms where the utterances are apparently monolingual at the lexical and syntactic level. The analysis also revealed that prosodic features may provide insights into the emotional stance of a child's learning, and this is discussed in the following section.

6.4.3 Prosody and Emotions

The macro-level analysis used the prosodic features identified at the micro-level analysis to identify the children's emotional stances (see Section 5.1.1). The study found that the children displayed doubt, confidence, authority, and an inclination to give up a discussion when they lost interest in continuing the interaction. From a Discursive Psychology perspective, these emotional stances are observable indicators and practices that participants use to show their emotions. The present study suggests that the children's emotional stances were always in flux and were constructed within any Key Moment through whole-class and group interactions. Thus, negative emotional stances can also be constructed through interaction as evident in one of the Key Moments. In Key Moment 4.4a, Elie displayed her lack of interest, a negative emotional stance, in continuing her discussion with the teacher about the shape (a hexagon) that she had made using sticks and adhesive (see Section 4.4.1). Tainio and Laine (2015) have claimed that a teacher's display through prosody of dispreference for a student's incorrect response may lead to a negative emotional stance in students. It is important, therefore, for teachers to be aware of the subtleties that have an impact on the meaning-making process and on learning.

In this section on the role of prosody in a multilingual classroom, three findings pertaining to the role of prosody (see Table 6.1) in construction of meanings were discussed. These findings respond to the second research question. The second research question focuses on the interactional tools that children in a New Zealand multilingual primary classroom used as they interacted to construct their understanding of 2D shapes, 3D shapes, and their properties. The first finding highlighted the part played by prosody in interactions. The second finding revealed that multilingual children may draw from the prosodic repertoire of their different languages. Thirdly, the study displayed the role of prosody in illuminating the emotional stance held by participants at different moments, which is fluid and constantly changing. This study thus contributes to the field of research that focuses on issues of multilingualism in the classroom.

In the following section, I discuss how gestures contribute to the meaning-making process during classroom interactions.

6.5 Gestures as Features of Turn-Design

Gestures include all the hand and other body movements that participants make as they engage in their daily activities (McNeill, 1992). The present study takes a Discursive Psychology perspective and interprets gestures as part of the communication act that children engaged in to display their understanding of shapes and their properties. This study focused on hand movements that participants (teacher and children) used to convey their understanding of geometry to others. The micro-level analysis of the data collected from six audiovisually recorded lessons on geometry (see Chapter 4) suggest that children (both multilingual and English only/proficient speakers) made use of gestures to convey their mathematical understanding of geometry concepts (in this case, geometric shapes) to their listeners. This finding aligns with the work of Calero et al. (2019), Chen and Herbst (2013), Elia et al. (2014), Kim et al. (2011), and Maschietto and Bartolini Bussi (2009). Two kinds of gestures were noted in the data from this study – deictic and iconic gestures. Findings pertaining to these two kinds of gestures are presented in the following two sub-sections: deictic gestures (Section 6.5.1), and iconic gestures (Section 6.5.2). The last section discuss findings pertaining to the use of deictic and iconic gestures by multilingual children (Section 6.5.3).

6.5.1 Deictic Gestures

Deictic gestures include the hand movements that participants made either to point to an object or when they showed an object which they considered relevant to the shape they were talking about. The use of deictic gestures by multilingual children and English proficient children was evident in four of the Key Moments. For example, in Key Moment 4.1a, Ozan (a male Somali-English bilingual child with Somali as his first language and with beginner's proficiency in English) used deictic gesture by pointing to the shape. Similarly, Zara (Maori bilingual child with English and Māori as her languages) used deictic gesture to show the shape that she had made (see Key Moment 4.1b). Proficient English language users, like Elie, also used deictic gestures during geometry lessons (see Key Moments 4.4a and 4.5a). During Key Moment 4.4a, Elie held the shape in her hand and rolled it around her finger to show the shape to the teacher even when she was not able to convince the teacher about her thinking. In the another Key Moment 4.5a, Elie again showed the shape that she had made to indicate the answer when she was not selected as the next speaker by the teacher. It seems that the children used their deictic gestures to ground their thinking process about the abstract mathematical concept in a physical environment, as suggested by Alibali and Nathan (2012).

In addition to deictic gestures, children also used iconic gestures. Discussion of the finding pertaining to iconic gestures is presented in the following section.

6.5.2 Iconic Gestures

Iconic gestures refer to those gestures that involve hand and arm movements (e.g., in this study, displaying the properties of shapes) showing the semantic aspects of the concept that children attempt to verbalise in their description. The use of iconic gestures was also reported in Key Moments by children (see Key Moment 4.1a). For example, Ozan used iconic gesture when he dragged his finger over the task sheet (see Figure 4.1, Picture A) to show the sides of the shape that he was talking about (line 231, Excerpt 4.1a), when he realised that his group members (Garry and Tahi) were not able to recognise the shape from his decription. Ozan used another iconic gesture by moving his hands in the air (line 323, Key Moment 4.1a) to emphasise the property of a shape where the name of the shape can be identified by counting the number of sides of the object shown in the picture, even though he was unable to state the name of the shape. Like Ozan, Tahi (another bilingual child with Tongan and English as his languages) made use of a similar iconic gesture when he showed agreement with the shape identified by Ozan (lines 235-236). This demonstration of conceptual understanding of geometric shapes using iconic gestures resonates with findings reported by Calero et al. (2019) and Elia (2018), who suggested that iconic gestures can convey and reveal students' implicit knowledge. In a multilingual context, Ng (2016) reported that high school bilingual learners use gestures to complement their linguistic communication because gestures may reduce their need of language to communicate mathematical ideas. This argument can be used to interpret the use of iconic gestures by both Ozan and Tahi. Their use of iconic gestures may be interpreted as their attempt to use their non-linguistic ways of communication when they were unable to recall or were not able to verbalise the shape name or properties.

In the following section, I discuss multilingual children's use of deictic and iconic gestures.

6.5.3 Multilingual Children and Gestures

The analysis of Key Moment 4.1a seems to suggest that multilingual children may use iconic gestures more than deictic gestures. For example, during Key Moment 4.1a, Ozan (Somali-English bilingual child) used iconic gesture (five times) more than his deictic gesture (three times) to signal the number of sides when recalling the shape name. The analysis supports the finding by Elia et al. (2018), who claimed that students use iconic gestures more than deictic gestures when expressing geometric concepts. This may be because the iconic gestures support students by complementing their verbal descriptions of shapes if they are unclear or incomplete, whereas deictic gestures only point or signal to the object or shape in question. The same finding was reported in a multilingual context by Wermelinger et al. (2020) and Church et al. (2004). Both of these studies investigated the use of gestures by bilingual students within the age range of 3.5 to 5 years. They reported that bilingual students used iconic gestures more than their monolingual or English-proficient speakers to get their meanings across in conversations. Alternatively, Wermelinger et al. (2020) suggested that the use of iconic gestures by bilingual or multilingual speakers could also result because of the different cultures that students belong to; in other words, their gestures may have resulted from a culturally-informed gesture repertoire, as noted by Iverson et al. (2008).

In this section on gestures, findings pertaining to the use of deictic and iconic gestures have been discussed. Only a limited number of studies have explored the use of gestures by bilingual or multilingual speakers. The studies mentioned earlier explored the use of gestures in mathematics learning and teaching in multilingual contexts at either the high school level or pre-primary school level. This study adds to the knowledge base in geometry education research and multilingualism research with regard to the use of gestures at primary school level. Awareness and knowledge of gesture used during interactions can help researchers and teachers to develop insights not only into children's conceptual understandings of geometry concepts (Alibali & Nathan, 2012; Elia et al., 2018) but also their epistemic stance on their knowledge claims (Flood et al., 2020). Knowledge about how gestures support the learning of geometric concepts can support educators and teachers to provide children with more opportunities to contribute, even when the children are not able to verbalise their thinking. Moreover, an understanding of gestures can help teachers to appropriately assess children's performance on geometric as well as mathematical tasks (Alibali & Nathan, 2012; Elia et al., 2014; Elia et al., 2018).

In the last two sections (6.4 and 6.5), I discussed findings pertaining to the interactional role of prosody and gestures which contribute to the second research question. The following sections answer the third research question, which aims to explore the characteristics of the dialogic space which influence children's negotiation of meanings in the geometry classroom. In the next section, I discuss findings pertaining to how children construct an understanding of what is considered preferred or dispreferred in a geometry classroom.

6.6 Preferred and Dispreferred Response in a Geometry Classroom

The teacher and the children considered some responses preferred and some as dispreferred. According to Pomerantz and Heritage (2013), preference principles account for the way the first pair-part of the adjacency pairs is recognised and how the next participant fulfils their expectation in the following turn by providing the second pair-part. The exploration of preferred and dispreferred responses relates to the third research question, investigating the preference characteristics of dialogic space that influence children's negotiation of meanings.

The investigation of sequence organisation in the selected Key Moments explored at the micro-level analysis (Chapter 4) revealed that the classroom interactions followed the often documented Initiation-Response-Evaluation/Feedback (IRE/F) sequence (McHoul, 1978; Mehan, 1979). The IRF sequence implies that classroom interactions typically follow the teacher's initiation move, followed by the student's response move, which is further followed by the teacher's feedback move. Once a response is received from a student who has been given a turn to speak, the teacher evaluates their response. If the response is taken as correct, the teacher then asks another question and the pattern mentioned earlier is followed (for example, see Key Moment 4.2a and 4.3a, Chapter 4). This IRE/F sequence acts as an interactional practice within the dialogic space that contributes to the negotiation of meanings in the geometry classroom.

On exploring children's utterances within the IRF sequence (e.g., Key Moments 4.1b, 4.2a, 4.3a, 4.3b, and others), the analysis revealed three major findings about what is considered to be preferred or dispreferred by the teacher and the children. The first finding suggests that the children and the teacher considered responses given in geometry-specific language to be preferred responses (see Section 6.6.1). The second finding was that the teacher's overt negative evaluation of children's incorrect responses was considered as dispreferred response (see Section 6.6.2). Third, children and the teacher considered children's responses as dispreferred if children provided responses without being given the turn to speak (see Section 6.6.3). These findings are discussed in the following sub-sections.

6.6.1 Preferred Response: Children's Responses in Geometry-Specific Language

The first finding showed that children's responses which used geometry-specific language were considered to be preferred responses; however, a response which did not do so was considered dispreferred by both the children and the teacher. For preferred responses, the analysis showed that the children might interactionally signal their response as preferred by providing it using geometry-specific language in an unmarked manner (i.e. without "um", "uhm", silence or withholding of response), even when the content of the utterance was mathematically incorrect. For example, in Key Moment 4.2a, the teacher asked children "what is: a tri::angle three d" (line 548, excerpt 4.2a). Yue responded that the shape would be "cube" (line 551). She had used a geometry term to talk about the shape in question without using any hedging device, such as "um" or taking a pause before answering, which may indicate her understanding of her response as preferred.

Children may also indicate if they consider their response likely to be dispreferred. For example, during the Key Moment 4.1b, when the teacher asked Elie to explain why the shape that Zara made was "a perfect square" (excerpt 4.1b, see chapter 4). Elie also stretched the filler "um::" (line 206) and withheld her reasoning for the shape being perfect while making her utterance (line 204, Key Moment 4.1b). This use of a filler and stretching in her utterance suggests that she was aware that her response might be considered dispreferred by the teacher as she had not explained her thinking using geometry-specific language but had instead used everyday language to show how a square could be made using playdough. Pomerantz and Heritage (2013) have shown that participants use fillers such as "um" as a hedging device before providing a dispreferred response. Elie's use of "um" shows her own understanding that her response would be considered dispreferred.

Moreover, the study found that the teacher might also implicitly show her dispreference for a response not given in the geometry-specific language by means of subtle interactional tools. For example, the teacher might select another child as the next speaker or repeat a child's response with a different intonation pattern, as noted in the Key Moment mentioned in the earlier section (Key Moment 4.1b). During this Key Moment, the teacher treated Elie's response as dispreferred due to the absence of geometry-specific language. Her dispreference was evident as she thanked Elie for her response, explicitly advised her children to use geometry-specific language in their responses, and selected another child to take the next turn (lines 216-218, Key Moment 4.1b). Heller (2015) too has reported that the teacher may signal a problem with a student's response if the explanation is not provided in a particular linguistic formatting, which is a "discursive norm" (p. 190). Discursive norms account for the way the explanation or an explanation is provided in interaction using appropriate academic language. It is to be noted that the focus of the teacher's feedback is not on the mathematical content. Rather, it is on the use of the specified language, as evident in the Key Moment 4.1b, when Matiu stated that sides of a square are "perfectly aligned". Yackel and Cobb (1996) provided a similar account of "what counts" as an acceptable mathematical explanation. They argued that sociomathematical norms define the normative aspects of mathematical discussions that students must be able to adhere to in order to provide acceptable mathematical explanations.

This sub-section provides evidence of how children and teacher organise their preference for children's responses in geometry-specific language as part of the dialogic space that contributes to the negotiation of meanings during interactions. In the next section, I discuss a second finding regarding how the teacher's negative evaluation of children's mistakes is considered a dispreferred response by the teacher herself.

6.6.2 Dispreferred Response: Explicit Negative Evaluations of Children's Responses

This study suggests that the teacher may treat negative evaluations of children's incorrect responses as dispreferred responses. For example, during the Key Moment 4.2a (mentioned in the previous section), Yue's response was implicitly treated as incorrect by the teacher. The teacher used two pauses (0.5 seconds and 1 second) and "um" within her next utterance (lines 552-553). The use of pauses and "ums" may indicate her dispreference for an overt negative evaluation of Yue's response. Ingram et al. (2015) reported a similar finding when they analysed how teachers handled errors in 22 mathematics lessons. They found that explicit negative evaluations of students' incorrect responses by the teacher are interactionally constructed as a dispreferred response.

In addition to pauses and "ums", the teacher may also repeat a child's utterance to implicitly signal a child's incorrect response. For example, during Key Moment 4.4a, the teacher repeated Elie's utterance "it's fat" with pauses and stretching the word "fat" to signal Elie's incorrect response (see line 358, excerpt 4.4a, Section 4.4.1). Recently, in another study, J. Ingram et al. (2019) again reported that secondary, high, and middle school teachers interactionally constructed negative evaluations of students' mistakes or incorrect answers to questions as dispreferred responses. Teachers' dispreference was shown either through their repetition of a child's incorrect response with different intonations, or by selecting another student to respond. Dispreference for Elie's response is evident in the teacher's repetition of that response with different intonation, which might have led to Elie's waning interest in the conversation, demonstrated by her saying "um" with falling pitch. Ingram et al. (2015) argued that avoiding negative evaluations of students' incorrect responses may indicate that "errors are to be avoided as they are face-threatening and embarrassing" (p. 192). This interactional message that mistakes are embarrassing may conflict with an understanding that errors are part of learning. Thus, dispreference for children's incorrect responses as a part of preference organisation in dialogic space may inhibit children's reasoning and argumentation.

In addition to children's mistakes being apparently categorised as dispreferred responses, the analysis reported that if a child provided a response when not asked, that response too was considered dispreferred. This finding is discussed in the next section.

6.6.3 Dispreferred Response: Speaking Out-Of-Turn

The analysis revealed that at several moments during classroom interactions, the teacher ignored a child's response if the child spoke without being given the turn to speak or selected as the next speaker (e.g., Matiu's utterance, line 197, Key Moments 4.1b; Garry's utterance, line 558, Key Moment 4.2a; Kayla's utterance, line 498, Key Moment 4.3a; among others). The act of ignoring a child's response when given out of turn displayed the teacher's dispreference for the child's response. This finding is also reported by J. Ingram et al. (2019) in their study with teachers from secondary and middle schools. They argued that students and teachers might consider a turn as dispreferred if it deviates from the usual rule of turn-taking (often seen in the IRF sequence) in classroom interaction.

The present study extends this finding concerning out-of-turn children's responses as dispreferred responses to a multilingual context. Moreover, this present study argues that the tendency to consider an-out-of-turn response as a dispreferred response may result in the loss of opportunities for constructive debate leading to mathematical ideas, as also suggested by J. Ingram et al. (2019). The study suggests that we need to develop an alternative understanding of these out-of-turn utterances as positive discursive practices in order to support child initiation and participation in a mathematics classroom.

In this section on preferred and dispreferred responses in a geometry classroom, I have discussed how children and teacher display their understanding of preferred and dispreferred responses during classroom interactions as part of the dialogic space that influence children's negotiation of meanings. This section emphasises that the practices of preference and dispreference are established *in situ* to ensure children's responses are taken as preferred responses during mathematical interactions. It is important to note that these practices form the part of the dialogic space within which children and a teacher interact and make use of these practices to influence the process of negotiation of meanings about shapes and their properties in a geometry classroom. Research has shown that the display of a teacher's dispreference for students' responses can have an impact on students' levels of participation in the classroom (J. Ingram et al., 2019; Ingram et al., 2015; Tainio & Laine, 2015). The present study argues that it may be fruitful to educate teachers about the subtle ways in which their display of dispreferred responses through interactional tools may influence the dialogic space of learning. The present study adds to the research literature by paying attention to the discursive practices that children use to interact as they construct their understanding of shapes.

In the next section, I present a discussion of different speech genres that are available to the participants in the dialogic space of classroom interactions that influence the negotiation of meanings about shapes and their properties.

6.7 Speech Genres and the Negotiation of Meanings

Speech genres are those preferred utterances that speakers use to accomplish a certain social action, and are always embedded with the speaker's intentions, values and sentiments (Bakhtin, 1986; Rockwell, 2012; Sullivan, 2012). In this study, I focused on the speech genres embedded in participants' (children and teacher) utterances to investigate how the presence of these speech genres influenced the negotiation of meanings within the dialogic space; this responds to the third research question. Two main findings were reported. First, a variety of speech genres are available to all participants in a classroom (discussed in Section 6.7.1). The second finding states that children may use any of the identified speech genres, including teacher's speech genres. Teacher's and children's speech genres are discussed in sub-sections 6.7.2 and 6.7.3.

6.7.1 Variety of Speech Genres in Dialogic Space

The findings showed that a variety of speech genres is present in the dialogic space. The findings also suggested that the children and the teacher's use of speech genres depended upon the situational context within the micro-moments during the Key Moments. In total, seven speech genres were identified (see Section 5.1.2): Appreciative speech genre, Pedagogical speech genre, Persuasive speech genre, Assessment speech genre, Argumentative speech genre, Declarative speech genre, and Giving-up speech genre. The children and the teacher might use any of these speech genres to display their intention and social action, along with acknowledging and establishing the role of the other the addressee. This finding supports the argument put forward by Rockwell (2000), who used the Bakhtinian perspective to analyse primary speech genres in teaching episodes in a Mexican rural school. Rockwell (2000) argued that teaching is one of the spheres of activity that uses a pool of speech genres from different areas of life which are adopted, developed, and blended together over time to form a composite teaching genre. She found that the teacher used speech genres from different aspects of life, including informal talk, explanation, folklore, and anecdotes. She also argued that these generic ways of talking as speech genres often displayed assumptions of what children knew about a topic along with establishing the nature of participation expected from the children during classroom interactions.

In the next sub-section, I discuss the teacher's speech genres and their influence on children's negotiation of meanings in classroom.

6.7.2 Teacher's Speech Genres and Negotiation of Meanings

In the present study, the macro-level analysis identified the use of primarily three kinds of speech genres in the teacher's utterances: Pedagogic speech genre, Assessment speech genre, and Appreciative speech genre. These three speech genres established the teacher's position as the authority, even though the purpose of each kind of speech genre was distinctive. Rockwell (2000) suggested that the use of speech genres in a teacher's utterances may create specific discursive conditions that support the use of specific speech genres by the student. This was also evident in this study. For example, the use of the Pedagogic speech genre by the teacher often called for the use of the Argumentative or Persuasive speech genre by the child, where the child is supposed to respond to the question asked by the teacher. However, the use of the Assessment speech genre may only highlight a child's incorrect response and provide no further feedback to the child, which may hinder further participation of the child whose response is assessed. Thus, it can be argued that the teacher's practices in use of these two speech genres (Pedagogic and Assessment speech genres) may work as meta-messages to indicate what kind of participation is expected from the children. Research has shown that the teacher's discursive practices while re-voicing may work as implicit clues to students about whether or not an explicit explanation is expected from them to support their claim (Forman & Larreamendy-Joerns, 1998; Moschkovich, 1999, 2007).

Pedagogic and Assessment speech genres may also be interpreted as similar to Eckert and Nilsson's (2017) active and inactive re-voicing. They investigated two teachers' re-voicing strategies in a Grade 5-6 lesson on probability and found that the teachers used two forms of re-voicing: active and inactive. They argued that active re-voicing contributed to the continuation of mathematical discussions, as evident in the use of the Pedagogical speech genre, whereas inactive re-voicing may implicitly reject students' contributions without asking for further explanations, as seen with the Assessment speech genre. Thus, it can be argued that awareness of these speech genres on the part of the teacher may enable them to use Pedagogic and Appreciative speech genres consciously instead of using the Assessment genre, which may deter children from participating in classroom interactions. In addition, Boukafri et al. (2018) studied teachers' practice of re-voicing in their research in Grade 7 geometry lessons with 12-year-old students. They argued that, discursively speaking, the teacher's re-voicing of students' responses might point out students' inadequate understandings.

In addition to the Pedagogic and Assessment speech genres, the Persuasive speech genre was also evident in this teacher's utterances. For example, during Key Moment 4.4a, the teacher tried to persuade Elie to agree with her by repeating her statement and using "we", as evident in this utterance:

> 362 Teacher its its okay. so: its not actually coming out of the ground or going through the grou:nd(.)so we call so we call (.) ↑we call that a two d? (0.5) okay [so:(.2) (lines 362-365, Key Moment 4.4a)

Gerofsky (1999) argued that lecturers often use the language of persuasion to seek agreement from the students. She conducted genre analysis of mathematics lectures' speech during initial calculus lectures in a Canadian university. She found that in using this language, the lecturers use "we" in an unusual manner or may ask tag questions, such as "right?, Okay?", as was evident in the teacher's utterance presented earlier.

In the next sub-section I discuss speech genres identified in children's utterances that contribute to negotiation of meanings as those genres become part of dialogic space in a multilingual classroom.

6.7.3 Children's Speech Genres and Negotiation of Meanings

The findings from the macro-level analysis identified several kinds of speech genres in children's utterances: Argumentative speech genre, Declarative speech genre, Giving-up speech genre, and Persuasive speech genre. Moschkovich (1999), in her study with Grade 3 students on learning of geometric shapes using tangram, found that students may bring different ways of talking, including narrative and argumentative forms. The Argumentative speech genre identified in children's utterances in this study may correspond to the argumentative way of talking that Moschkovich identified. Argumentative speech genres may also be interpreted as indicating the presence of sociomathematical norms that inform the use of mathematical logic and reasoning to develop a mathematical argument (Krummheuer, 2007; Yackel & Cobb, 1996).

The findings of the present study revealed that children may use speech genres that are often used by the teacher in a group interaction setting. For example, during Key Moment 4.1a, Garry used Pedagogic and Assessment speech genres. During this Key Moment, Garry shaped the group conversation in a manner the teacher often used to structure classroom discussion, by starting a conversation with a question. At the time, he assessed his peer's response as incorrect without providing further feedback. Garry's use of the Pedagogic speech genre and similar prosodic patterns may indicate that Garry embedded his utterance with the teacher's voice while interacting with his peers. The use of this specific speech genre perhaps indicated Garry's intention to define the kind of participation that he expected from his peers in a group setting. In contrast, he did not embed his utterances with the teacher's voice while talking to the teacher. Research outside mathematics education has shown that students draw on teacher-like talk to negotiate peer relations and maintain their social positionings, therefore displaying power (Copp Mökkönen, 2012; Goodwin & Kyratzis, 2007; Maybin, 2008; Tholander & Aronsson, 2003).

Using Goffman's (1981) participation framework and Bakhtin's (1981) Dialogic Theory, Tholander and Aronsson (2003) investigated interactions during group work sessions in five Swedish junior high schools (with 13-15 years old students). They found that students may take up the teacher role and use pedagogical routines quite emblematic of teacher-talk. They called this way of using language as "sub-teaching". Goodwin and Kyratzis (2007) too argued that children often appropriate adults' language (spoken, identified as speech genres in this study) in the form of genres. Copp Mökkönen (2012) further extended the investigation of sub-teaching in whole-class discussions (Grades 1 and 2) in the presence of the teacher in an English-medium Finnish school. She found that children may use genres present in teacher-talk (the authority discourse) and act as an authority to maintain classroom order even in the presence of a teacher. She further argued that this practice may indicate the presence of double-voicedness in children's utterances, as their utterances are embedded in the teacher-talk with their own intentions (Bakhtin, 1981, 1986). As was evident in Garry's utterances during the Key Moment 4.1a, Copp Mökkönen (2012) found that the children may also use directive statements, often to get somebody to do something, as they engage in classroom discussions. She argued that children use directive statements and teacher-talk to discursively construct asymmetrical power relations to display their own and others' positions in the classroom.

In this section, I have discussed findings pertaining to the speech genres and their role in negotiation of meanings in geometry classroom. Studies within the mathematics education field have commonly focused on genres of mathematics content (see, Gerofsky, 1996; Gerofsky, 1999; Rezat & Rezat, 2017; see Section 3.2.1). Interestingly, none of the studies explored speakers' intentions in the communicational field of the classroom except Gerofsky (1999). This study adds to the knowledge base in mathematics education regarding the speech genres that are available in the dialogic space as part of participants' utterances, which contribute significantly to the process of meaning-making of mathematical concepts in a primary mathematics classroom. The focus on oral utterances allowed me to identify the specific kind of expressions that participants use to elicit a certain participation expectation from their addressee. In this study, I have explored speech genres embedded in children's and teacher's utterances to investigate the speech genres that reflect a speaker's ideology in terms of the participation role they assign to their addressees through their use of common expressions (Joyner, 2018).

The present study adds to the research literature on speech genres and calls for future research to examine the speech genres that are used by the children in negotiating meaning in the mathematics classroom. The study also contributes to our knowledge regarding the re-voicing of teacher-talk by children, and argues that there is a need to explore how children use the teacher's voice during group and whole-class discussions in mathematics classroom and how it can support peer learning. Boukafri et al. (2018) too have argued in favour of more research for exploring the pedagogical potential of students re-voicing teacher-talk.

Moreover, the analysis of speech genres provides a window into the discursive practices that children and the teacher may use in a dialogic space. The analysis revealed that these speech genres contribute to the negotiations of meanings about shapes and their properties, which is a focus of the third research question. Speech genres contribute to our understanding of how a teacher and children use language mathematically to represent their understanding of shapes and their properties and how these speech genres are used to elicit a certain participation-expectation from their addressee.

In the next section, I discuss the role of the interplay of discourses identified in the participants' utterances in the negotiation of meanings about shapes and their properties.

6.8 Dominant Discourses, Their Interaction and Negotiation of Meanings

Bakhtin (1981) defined discourses as social languages that are specific to a particular group of people within society at a given time and place. Two discourses were evident in the children's and the teacher's utterances during classroom interactions. These discourses are Everyday Discourse and Eurocentric-Academic Discourse. The interaction of these two discourses was explored using the concepts of unitary language and heteroglossia at the macro-level analysis (see Section 5.2.2). On the level of discourses, two main findings were reported in this study. First, the study suggests that the meanings of words (e.g., side) may be associated with either of two discourses (Eurocentric-Academic Discourse or Everyday Discourse) available in the dialogic space of the classroom (See Table 6.1). That is, the unitary language may support either of these two discourses to shape the meaning of the word used in the specific interactional moment. The first sub-section discusses the first finding (see Section 6.8.1) followed by the discussion of second finding. The second finding suggests that the meaning of the words may also be influenced by the interplay of unitary language and heteroglossia within the same discourse (see Section 6.8.2).

6.8.1 Unitary Language and Heteroglossia: Between the Discourses

The macro-level analysis revealed that the meanings of the terms in geometry classroom depend on the discourse supported by the unitary language, and the unitary language may support either the Eurocentric-Academic Discourse or Everyday Discourse. For example, the analysis revealed that children used words, such as "side", to imply both everyday meanings and geometry meanings. "Side" is a part of the everyday vocabulary that can be used in different scenarios. It may imply position (e.g., Delhi is on the right side of Haryana state), stance (e.g., are you on the opposition side?), a person's character (e.g., he is on the disagreeable side), or an object of less importance (e.g., a side-dish of potato salad). There were instances where the everyday meaning of the word "side" was also evident, highlighting the construction of its use as part of Everyday Discourse (see Key Moment 4.3b, where the use of "side" implied a position from which to look at an object, i.e., from outside).

However, in the data, "side" was often interactionally treated as part of geometry-specific language to describe 2D shapes (see Key Moment 4.1a). For example, "side" was given geometry meanings, such as line segments of 2D shapes (Key Moment 4.1a), faces and edges of 3D shapes (Tahi, FG2 and FN2 and FN5). The underlying assumption of straight sides in these meanings (line segments of 2D shapes, faces and edges for 3D shapes) is drawn from the academic geometry embedded in The NZC that highlights the presence of Western mathematical ideas as suggested by Barton et al. (1998) and Parra and Trinick (2018). This underlying assumption of straight sides is not problematised in geometry education research. The term “side/sides” often implies the straight sides of polygons without explicitly mentioning the supposition of straightness (e.g., Bruce & Hawes, 2015; Herbel-Eisenmann et al., 2015; Kaur, 2015). This assumption marginalises the possibilities of different meanings, such as curved sides as in case of a circle.

The use of “side” as part of both Eurocentric-Academic Discourse and Everyday Discourse (informal mathematical expressions) highlights the tension between children’s use of informal mathematical expressions of language use and the need to use more formal mathematical discourse, which is often reported in Mathematics Education research (Adler, 2002; Barwell, 2016a; Barwell, Chapsam, et al., 2016). The findings in the present study provide evidence that the meanings of the terms/words in the utterances were interactionally treated and defined as part of either Eurocentric-Academic or Everyday Discourse. From the Bakhtinian perspective of unitary language and heteroglossia, it can be argued that the meaning of the utterances is dependent upon the discourse supported by the unitary language force within the milieu of discourses available in any particular interactional moment. Unitary language is a theoretical language force that tends to homogenise the meaning of the utterance to facilitate the flow of interaction (Barwell, 2018). It was evident in the data that the unitary language force may support either of the discourses depending on the interactional context; thereby providing different meanings to the same word as and when embedded in different discourses. The finding supports the argument put forth by Barwell (2016a) that participants may treat an everyday term mathematically during a particular interaction. In the present study, children used their everyday language in the form of mathematical language, embedding everyday words with mathematical ideas. That is, the meaning of the word “sides” was derived not only from its everyday use but was also informed through its mathematical significance. In other words, the formality or informality of the mathematical expressions were created *in situ* (Barwell, 2016a).

The finding presented in this sub-section has a number of implications. Firstly, the use of the word “side” as part of both Eurocentric-Academic and Everyday Discourse displays the problematic strict distinction between formal and informal mathematics discourse. Barwell (2016a) and Moschkovich (2019) have also suggested that it is not possible to demarcate clear boundaries between these two kinds of discourses. Secondly, the finding indicates that the spontaneous use of everyday terminology like “sides” to talk about shapes may provide an indication of a well-developed understanding of the shapes rather than suggesting a lack of understanding or low understanding of geometrical concepts (Barwell, 2013).

However, the teacher, during a semi-structured interview (see Section 4.3), suggested that the inconsistent use of the term “side” might also result in possible confusion in developing a robust understanding of shapes, more so while transitioning from 2D shapes to 3D shapes, a possibility that cannot be denied. Therefore, it may be argued that although children are able to determine the meaning of the word “side” in a given context, the use of the everyday word “side” in teaching and learning of shapes may result in potential difficulties not only for multilingual children but also for proficient English speakers (Bartolini Bussi & Baccaglini-Frank, 2015). An alternative term is needed to talk about the line segments of a 2D shape. One possibility could be “edge”, an English translation of “tapa”. In Te Reo Māori, the names of the polygonal shapes use “tapa”, as in tapawhā, taparima and so on. Tapa implies an edge, margin, or rim of the shape (Māori Dictionary, 2021b). The use of “edge” or “tapa” rather than “side” might help children to develop a better understanding of shape as an enclosed space.

In the next section, I discuss the second finding pertaining to interplay of unitary language and heteroglossia within the same dominant discourse.

6.8.2 Unitary Language and Heteroglossia: Within a Discourse

This section discusses the finding that the tensions between unitary language and heteroglossia can be observed within the use of the same discourse (see Section 5.2.2). For example, in Key Moment 4.5a, the class was involved in a discussion where they identified the respective English names for Te Reo Māori terms for shapes like taparima (pentagon) or tapaono (hexagon). During this Key Moment, the teacher asked if she could have a tapatahi. In response to this, Elie argued that a tapatahi would be a circle if the pattern for naming shapes was to be followed; however, in the same utterance (lines 579-580), she went on to explain that it could not be a circle as a circle has no sides and no corners. The interaction of unitary language and heteroglossia is evident within the use of Eurocentric-Academic Discourse as Elie used only this Eurocentric-Academic Discourse to account for the exception in this case. A unitary language force is evident in using the westernised rule for naming shapes in Te Reo Māori, according to which rule the name should have been “tapatahi”. The meaning of the word “side” in tapatahi, seems to suggest an understanding of the curved side. The other meaning of “side” is also evident in the same utterance, which assumes “sides” are straight, a meaning embedded in Eurocentric-Academic Discourse (as explained in Section 6.8.1). The presence of two meanings in the same utterance reveals heteroglossia. Heteroglossia can also be located in the possibilities of meaning at the level of the whole utterance: one possibility that supports the possibility of tapatahi (using the Eurocentric discourse of “number of sides” to name the shape), and another that rejects the possibility of tapatahi (using the Eurocentric discourse for assuming sides as straight). The interplay of unitary language and heteroglossia in this Key Moment presents concrete evidence of this interaction within the use of the same discourse.

This study adds to the research literature exploring the tensions between unitary language and heteroglossia in mathematics classes. In this section on dominant discourses and their interactions, the study provides concrete evidence of the interplay of unitary language and heteroglossia not only among different discourses but also within a particular discourse, as part of the dialogic space. However, there is a need for future research to investigate how this interplay of these language forces can be used to support meaning construction in a mathematics classroom. The study adds evidence to support the consideration of language as a “source of meaning” (Barwell, 2018, 2020) by displaying how language-in-use can lead to heteroglossia of meanings at various levels.

In the next section, I discuss findings pertaining to Chronotopic Moments as one of the characteristics of dialogic space that contribute to the negotiation of meanings (RQ3).

6.9 Chronotropic Moments and Negotiation of Meanings

In this section, I discuss the finding pertaining to the Chronotopic Moments in the data. In the perspective adopted in this study, I identified Chronotopic Moments as those moments during which children explicitly used time-denoting words relating to a moment in the past or future in order to make sense of learning in present moment. Therefore, children’s utterances were investigated for explicit reference to moments in time (past or future) that helped them to develop a new understanding as they engaged in classroom interactions. Three Chronotopic Moments were identified in two of the ten Key Moments, one in Key Moment 4.1a and two in Key Moment 4.5a. The Chronotopic Moments embedded in children’s utterances displayed how children brought moments of learning from different zones in time (a past moment, as presented in Key Moment 4.1a, or a moment of learning in the future, as presented in Key Moment 4.5a) to make sense of their geometry learning in the present moment. The study suggests that these Chronotopic Moments might provide children with an explicit link to their prior as well as future learning.

In classroom research, Bakhtin's concept of chronotope is often used to explore chronotopes as part of "invokable histories" (Blommaert, 2015) or as different classroom chronotopes (see, Renshaw, 2014)**.** However, the chronotopic analysis in these studies focused on classroom interaction as a single discourse. As a result, chronotopic analysis of classrooms is often divorced from the individual participant's disclosure of what time and place are most relevant to their learning (Holquist, 2010). The exploration of the time aspect of learning from the participant's point of view is a crucial manifestation of a chronotope, and was investigated by identifying Chronotopic Moments in this study.

Moreover, considering classroom context and time as a thick chunk of chronological time ignores the moment-to-moment flux in terms of its quality of in-the-moment interactions within conversational contexts (Rosborough, 2016). The analysis of speech genres and emotional stances in this study revealed that the nature of the child's participation can change qualitatively quite quickly from moment-to-moment within the same Key Moment. The study explored moments that children brought from different zones in time not only to make sense of learning for themselves but also to negotiate their learning in the dialogic space. The analysis in this study used a Discursive Psychology approach, which focuses on how participants construct and display their pivotal moments of learning. It also provided a fresh perspective by analysing chronotopes as Chronotopic Moments evidenced in children's utterances when they used words which referred to time. The study is one of the first pieces of research to explore the explicit linking by children of present time with time past or future in their utterances about their learning.

The last four sections (6.6, 6.7, 6.8, 6.9) discussed findings relevant to the third research question, which concerns the characteristics of dialogic space that influence children's negotiation of meanings about shapes and their properties. In the next section, I present a chapter summary.

6.10 Chapter Summary

This study contributes insights into the ways children discursively constructed and negotiated geometric understandings of shapes and their properties as they interacted during whole-class and group discussions in a Year 5/6 classroom. Findings from both micro-level and macro-level analyses were pulled together and discussed in this chapter.

Regarding the discursive constructions that children used to represent their understanding, the study revealed three main findings. First, children discursively constructed dimension in a variety of ways. The discussion of this finding highlighted the need for more research into the mathematical construct of dimension. Second, children used discursive constructions of "3D rectangle", "3D triangle", and "3D circle" to represent cuboid, triangular prism/pyramid and sphere, respectively. The discussion highlighted that these terms may represent children's emerging understanding of 3D shapes rather than being an evidence of confusion. Third, children used the suffix "tapa" with Te Reo Māori number names to represent 2D shapes. The discussion of this finding highlighted the epistemic power of mathematical knowledge embedded in Western mathematical ideas.

The second research question explored how children interact to construct their understanding of shapes and their properties. The study revealed that the children used prosody and gestures to construct and convey their understanding as they engaged in classroom interactions. The study revealed that multilingual children may use prosodic features from their repertoire of multiple languages. In addition, the study suggests that children may use iconic gestures more than deictic gestures. The discussion of these findings highlighted the need for further research exploring both prosody and gestures in mathematics classrooms.

The third research question examined the characteristics of dialogic space that influence children's negotiation of meanings. The study suggests that an understanding of preference organisation, speech genres, discourses, unitary language, heteroglossia, and Chronotopic Moments as characteristics of dialogic space that contribute to negotiation of meanings. The study revealed that children and teacher treated responses in geometry-specific language as preferred, and out-of-turn responses as dispreferred responses. The discussion suggests that the teacher's dispreference for children's incorrect response or responses in other than geometry-specific language may negatively influence children's participation in class. Various speech genres were also identified in children's and teacher's utterances as being available for use in dialogic space. It was found that different speech genres result in different expectations from other participants. Moreover, the study revealed that children may use the teacher's voice to display their authority and to influence the process of meaning-making. Dominant discourses of Eurocentric-Academic Discourse and Everyday Discourse were noted as characteristic of dialogic space. Finally, Chronotopic Moments were identified as moments from different zones in time that children used to make sense of their learning in the present.

Overall, the study suggests that meanings of utterances are constructed *in situ* by the participants within the micro-moment of interaction; however, they may be influenced by the macro-level context of the interaction. The next chapter draws key conclusions, identifies limitations, and identifies implications for future research in mathematics education.

Chapter 7.

Conclusions, Limitations, and Implications

The overarching research question for this study was how do 9 to 11-year-old children negotiate their meanings about 2D shapes, 3D shapes and their properties in a New Zealand multilingual primary classroom. In the first section of this chapter, I present the main conclusions that can be drawn from the study in response to this overarching research question (see Section 7.1) by answering the three research questions that guided this study. In the following section, I discuss the limitations of this study (Section 7.2). I, then, consider the study's implications for primary school teachers, curriculum development and teacher-educators (Section 7.3). In the next section, avenues for future research are recommended (Section 7.4), followed by an outline of the study's contributions to the field (Section 7.5). Finally, I present my concluding thoughts (Section 7.6).

7.1 Conclusions From the Study

In this section, I present major conclusions that can be drawn in response to each of the three research questions.

7.1.1 Conclusions to Research Question 1

> *What discursive constructions do 9 to 11-year-old children use to represent their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

The study reported three major discursive constructions that children during classroom interactions to represent their understanding of shapes and their properties. First, it noted that children used different discursive constructions to represent their understanding of dimension as a mathematical construct. They constructed dimension in terms of "flat vs fat", as "ways to move", or as "another world". Further research exploring the understanding of dimension at primary school level is suggested.

The second discursive construction evident in this study was the children's use of terms such as "3D rectangle", "3D square", "3D triangle", and "3D circle" to talk about 3D shapes of cuboid, cube, triangular prism or pyramid, and sphere. These discursive constructions may indicate children's emerging understanding of 3D shapes rather than signalling their misconceptions.

Third, children discursively constructed the names of shapes using Te Reo Māori number names with the prefix "tapa" to represent 2D shapes in Te Reo Māori. A few children discursively constructed a circle as tapatahi though the possibility of having a tapatahi was refuted later on the basis of the taken-as-shared understanding of "side" as a straight side. The underlying assumption of straight sides indicates the epistemic dominance of western mathematical ideas over Māori mathematical knowledge.

7.1.2 Conclusions to Research Question 2

> *How do 9 to 11-year-old children interact to construct their understanding of 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

In response to the second research question, the study presented two major findings. Firstly, it indicated that prosody in children's utterances plays a crucial role in meaning constructions. The study also found that multilingual children may use prosodic features of their multiple languages while representing their understanding of shapes, which may contribute to the construction as well as the negotiation of their understanding of shapes.

Secondly, the study suggested that children may use iconic as well as deictic gestures as part of their interactional tools to represent and construct their understanding of shapes. Iconic gestures help children to demonstrate the semantic aspects of the shape they are referring to, should they lack the relevant geometry vocabulary. Deictic gestures help children to ground their learning in the concrete physical environment. The study provides some evidence to suggest that multilingual children may use iconic gestures more than deictic gestures, which calls for more research in this area.

7.1.3 Conclusions to Research Question 3

> *What characteristics of the dialogic space influence 9 to 11-year-old children's negotiation of meanings about 2D shapes, 3D shapes, and their properties in a New Zealand multilingual primary classroom?*

For the third research question, the study identified several characteristics of the dialogic space that contributed to children's negotiation of meanings about shapes and their properties. First, the study indicated that the children and the teacher displayed their understanding of what is preferred and what is dispreferred as a response in a geometry classroom. This understanding is present and available to each participant in the dialogic space; thus, it may influence children's understanding as well as their participation during whole-class and group interactions. The study suggests that the teacher considered children's responses with geometry-specific language as preferred responses, which may deter students from providing their understanding using everyday language. Moreover, a teacher's implicit negative evaluation may also influence children's participation in the classroom discussion.

Second, the study identified the use of a range of speech genres, identified as Persuasive, Pedagogic, Assessment, and Argumentative. These speech genres are available to the children as well as the teacher within the dialogic space and are used to state, influence, and negotiate their knowledge claims during whole-class and/or group interactions. In addition, the study's exploration of double-voicedness of participants' utterances concluded that children might use the teacher's voice by structuring their utterances using the Pedagogical speech genre.

Third, the study also suggested that the meanings of the utterances are interactionally constructed and constantly influenced by the interaction of unitary language and heteroglossia at two levels of discourse, that is, between two discourses as well as within the same discourse. For example, children discursively used "side" to imply line segments of 2D shapes and faces or edges of 3D shapes. It was found that the meaning of "side" was constructed within the minute moments of interaction and was dependent upon the preceding and following turns. Nevertheless, the study proposes that "side" may not be an appropriate geometry term to develop an understanding of shapes and their properties. Instead, "edges" could be used in place of "sides" to highlight the dimensional aspect of shapes as well as the idea of "plane" in 2D shapes.

Finally, the study found that children may bring their moments of learning, identified in the study as Chronotopic Moments, from other zones in time in order to make sense of their learning at the present moment. These Chronotopic Moments are made available to all participants in the dialogic space to help others to see connections that a child is making with his/her previous or future learning. Thus, the study highlights how Chronotopic Moments influence children's meaning construction in a dialogic space.

Though the study has reported several crucial findings, the study is not without limitations.

7.2 Limitations of the Study

It is important to consider some of the limitations of the study alongside the conclusions that have been drawn from it. The aim of the study was to provide a detailed account of the practices that influenced children's meaning-making processes during geometry lessons, focusing on six lessons at one English-medium school in a New Zealand setting. Data from a variety of sources were collected to triangulate and ensure the validity of the findings. The focus on audiovisually recorded data allowed me to present details of the practices-in-use rather than abstract categories of practices. The findings were developed through rich and thick descriptions that may help readers to imagine the context of the study and decide about the transferability of findings (Onwuegbuzie & Leech, 2007). However, the findings account for only a small set of practices, and these could usefully be investigated further. Moreover, as these findings are specific to a particular mathematics class, they provide no guarantee that these practices would be evidenced in other schools in New Zealand or other parts of the world.

Moreover, the study focused on a school setting in a country where English is the language of the majority and is spoken widely at home as well. The findings from this study may differ to contexts such as India and South Africa, where English may or may not be spoken at home. A comparative study of the use and role of prosody in those and other contexts may provide further insights.

In addition, although I assumed the position of a non-participant observer in this study, my mere presence in the classroom might have influenced the children's and teacher's participation in some ways during the six observed lessons, and hence the data. Moreover, it cannot be denied that my own values, beliefs, biases, and prejudices as an Indian primary school teacher and researcher in a New Zealand English-medium classroom may have influenced the data collection and analysis procedures (Creswell & Creswell, 2018). Several procedures were undertaken to establish the validity and reliability of findings while minimising the impact of these influences on the study. For example, the Pilot Study allowed me to trial the data-gathering process before the Main Study data collection, and helped make me aware of my beliefs and biases while gathering data for the Main Study (Yin, 2014). To minimise the influence of my biases, beliefs and values during data analysis, I used several procedures, including member checking (Denzin & Lincoln, 2018), intra-rater reliability of transcripts, and peer consultation for Conversation Analysis (CA) (Lester, 2019; Sidnell & Stivers, 2013).

Finally, the study used focus group interviews with the children to gather information about their thinking pertaining to shapes and their properties. It was noted during the analysis that stimulated recall interviews with children could have prompted them to provide further useful insights into some of the ideas that they had presented during classroom interactions (Lyle, 2003). For example, during interviews children were asked about their understanding of dimension. A segment of video data of one of the Key Moments could have been shown to probe their thinking about what they meant about dimension during the Key Moment.

The conclusions presented in the previous section, along with the limitations identified in the study, have implications for New Zealand primary school teachers, curriculum development, and teacher-educators in the New Zealand context. These implications are presented in the next section.

7.3 Implications

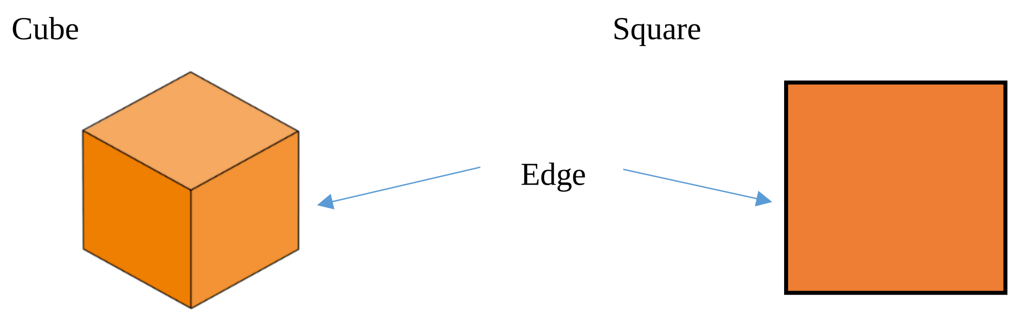
The teaching and learning of shapes at New Zealand Curriculum Level 3 focuses on developing children's understanding of shapes and their properties. The study made a number of findings with implications for different stakeholders involved in the educational process. The following sub-sections present implications for New Zealand primary school teachers (see sub-section 7.3.1), curriculum development (see sub-section 7.3.2), and teacher-educators (see Sub-section 7.3.3).

7.3.1 Implications for New Zealand Primary School Teachers

A number of implications can be drawn for New Zealand primary school teachers from this study. Firstly, the study draws our attention to the need for an alternative vocabulary that could be used in place of "side" for teaching and learning of 2D shapes in primary school. The use of "edges" rather than "sides" is proposed to help children make easier transitions from 2D shapes to 3D shapes. For example, in Figure 7.1, the term "edge" denotes the line segment that we get where two surfaces of the cube (a 3D shape) meet.

Figure 7.1

Edge for a 3D Shape (Cube) and a 2D Shape (Square)



If we use the same term "edge" to denote the end of the 2D shape, square in this example, the term "edge" will emphasize the aspect of the surface, which is crucial for emphasizing the importance of “plane” for 2D shapes, and why 2D shapes are called planar shapes. In addition, the use of “edge” for 2D shapes may help in smooth transition from description of 2D shapes to 3D shapes, which in turn may promote better child outcomes in regard to learning of 3D shapes. This is because the term “edge” can denote the same idea signifying the end of surfaces, often symbolized by a line segment in representations of 2D as well as 3D shapes.

Moreover, in everyday life, “edge” often implies the outside limit of an area or surface of an object, which signals the idea of enclosed space, an important property of both 2D and 3D shapes. Therefore, using the everyday meaning of “edge” as part of geometry vocabulary may support children’s transition from everyday meanings to mathematical meanings.

The second implication that can be drawn from the study is the need for teachers to be aware of speech genres that they may use during whole-class interactions. The study revealed that the teacher’s use of the Pedagogic speech genre is helpful in eliciting children’s knowledge. However, use of the Assessment speech genre may deter children from participating in classroom interactions, even though negative evaluations are not provided explicitly. Being aware of speech genres may enable teachers to consciously use Pedagogic speech genres in preference to Assessment speech genre, in order to promote student participation in classrooms.

The study revealed that the teacher’s preference for geometry-specific language may deter children from participating in whole-class interactions. It also suggests that children may use everyday words mathematically to represent their understanding. Therefore, it is suggested that the primary teachers focus initially on eliciting children’s understandings rather than on their acquiring of geometry-specific language.

Finally, the study suggests that children use both deictic and iconic gestures as useful resources for representing their understanding of shapes and their properties. It appeared, moreover, that children may use iconic gestures more than deictic gestures because iconic gestures allow children to represent properties of shapes when they are not yet able to verbalise those properties. Therefore, it is suggested that teachers can make use of children’s gestures to assess their understanding and provide feedback.

7.3.2 Implications for Curriculum Development

Two implications are drawn from the study for the development of curriculum. The study revealed that the children used different discursive constructions for representing their understanding of dimension. The study suggests that there is a need for a clear definition of "dimension" in a mathematical context to support teachers in providing a sound understanding of dimension at the primary school level. The definition might usefully be presented in curriculum documents and relevant supporting resources for teaching and learning of geometry. It is crucial for children to understand that the mathematical construct dimension includes *both* the boundary notation perspective *and* the measurement perspective rather than either one of these.

Second, the study indicated that there is a lack of clarity in the unit "Te Whānau Taparau" developed for the teaching and learning of shapes in Te Reo Māori. In addition, Te Reo Māori has the potential to support children's learning of shapes, their properties and the hierarchical relationships among shapes. The study suggests that more teaching and learning resources in Te Reo Māori need to be developed to promote better outcomes in mathematics for children from culturally and linguistically diverse backgrounds, especially Māori and Pasifika children.

7.3.3 Implications for Teacher-Educators

The study has two implications for teacher-educators. First, this study reported that multilingual children use prosodic features from their repertoire of multiple languages during classroom interactions. The study recommends professional development is required for teachers to develop an understanding of these prosodic features in order to understand how they are used by children while engaging in their negotiation of meaning during classroom activities.

Second, the study highlighted that children make use of iconic gestures and deictic gestures to represent and situate their understanding of shapes and their properties. However, instructional strategies currently taught in teacher-education courses overwhelmingly concentrate on the verbal language that children use to express their understanding. The study suggests that teacher education courses may benefit from incorporating a focus on the understanding of gestures for both instructional and assessment practices, as suggested by Alibali and Nathan (2012).

In this section, I discussed implications from this study. In the next section, I discuss avenues for future research.

7.4 Future Research

This study has suggested that a number of avenues could usefully be pursued in future research. First, only a few studies have explored teachers' and children's understanding of dimension as a geometry concept (e.g., Morgan, 2005; Panorkou & Pratt, 2016). This study adds to the mathematics education research field and suggests that children often find it difficult to express their understanding of dimension at the primary school level. The study also highlights the research gap in the field of geometry education research regarding the mathematical construct of dimension. Future studies could focus on teachers' and children's understanding of dimension in the New Zealand context.

Second, research has shown that participants' smiles and laughter may contribute to the meaning-making process in several ways (Haakana, 2010), such as building rapport (Nguyen, 2007) or displaying incorrectness of response (Sert & Jacknick, 2015). This was noted in the micro-level analysis (see Chapter 4). However, due to time constraints, this aspect of classroom interaction was not fully explored in this study, and therefore could be investigated in future research.

Third, research has shown that children use deictic gestures to situate their understanding of mathematics ideas in the concrete environment (Alibali & Nathan, 2012), or iconic gestures to represent properties of a mathematical idea (Elia, 2018). In support of that research this study, too, found that gestures play an important role in conveying meanings during classroom interactions. For future research, this study's findings suggest that a comparative study exploring the use of gestures by multilingual and monolingual children may provide useful insights into how this aspect of communication can facilitate conceptual understanding of geometry as well as other mathematical ideas. Additionally, Wermelinger et al. (2020) have demonstrated that gestures are culturally informed; therefore, an investigation into how teachers' and children's gestures contribute to the meaning-making process in multilingual settings may also provide fruitful insights.

Fourth, the study explored speech genres embedded in both teacher's and children's utterances. It demonstrated that speech genres may indicate expectations in terms of what speakers aim to achieve with their speech genres, in line with the studies of Rockwell (2000) and Gerofsky (1999). Speech genres may also suggest what a speaker expects from listeners while signalling their epistemic stances on how certain they feel about their knowledge claim, as observed by Flood et al., 2020. This study adds to research into speech genres in a multilingual context. However, only a very small amount of audiovisually recorded data from one New Zealand classroom was analysed for this study. More research exploring these speech genres may provide useful insights into how the teacher's use of specific speech genres may improve learning outcomes for children in mathematics classrooms.

Next, this study explored moments of learning from different zones in time that children used to throw light on geometric concepts they were considering in the present moment. I labelled these moments as "Chronotopic Moments". The study suggested that Chronotopic Moments may contribute to children's conceptual development. Further exploration of the use of explicit time references in teaching and learning may advance our understanding of the learning process.

Finally, the study revealed that children sometimes use teacher-talk during group interactions. Such use in children's utterances may signal children's attempts to act as a teacher in the group setting. The phenomenon is studied as sub-teaching in the research. This study suggests that further investigation of this phenomenon in a multilingual setting in mathematics classrooms may provide us with additional pedagogical insights.

In the next section, I discuss some of the contributions of this study to the broader field of mathematics education.

7.5 Contributions to the Field

The study aimed to explore how children negotiate their meanings about 2D shapes, 3D shapes, and their properties as they interact during classroom discussions in a New Zealand multilingual primary school classroom. The study has made a contribution to the current body of knowledge in mathematics education, specifically geometry education, in a range of ways.

First, the study is one of the first studies in New Zealand to explore children's understanding of geometric concepts in the multilingual context of a New Zealand English-medium school. The study revealed that multilingual children may use prosodic features of their multiple languages to influence and contribute to the negotiation of meanings about shapes and their properties. Thus, the study challenges the monolingual assumption of New Zealand classrooms and adds to the discussion and understanding of multilingualism in New Zealand classrooms.

Second, in the broader mathematics education field, this research adds to the few studies that have explored the mathematical construct of dimension in a mathematics classroom, and is the first one to present research that focuses on the construct of dimension in a multilingual context.

Next, methodologically this research is one of the first studies to use insights from CA research from sociolinguistics and research on the pragmatics of language-in-use to interpret how multilingual children's use of prosody contributes to the meaning-making process in a multilingual mathematics classroom.

The study is also novel in combining CA techniques and Bakhtinian concepts to analyse audio-visual data. This analytical framework allowed me to provide concrete evidence of how socio-cultural and historical meanings are materialised in multilingual mathematics education interaction. Thus, the study revealed that the mathematical meanings of utterances are constructed through interactions which demonstrate the mutual influence of both micro- (prosody, sequence organisation, preference as part of CA) and macro- (discourses, heteroglossia, addressee as part of dialogic space) features of interaction that contribute to the meaning-making process. For example, the study revealed how the meanings of utterances**,** for example, "side", is interactionally constructed either in mathematical terms signalling line segments of 2D shapes or in everyday terms as an aspect of object or direction (as evident in Section 5.2.2). The analytic approach allowed moment-by-moment explication of language-in-use, therefore making available the "seen-but-unnoticed" (Garfinkel, 2002) aspects of the mathematics classroom. This analytical framework may serve as a resource to educational research seeking to explore the finer details of the meaning-making process that aid in developing an understanding of mathematical ideas.

Finally, on the theoretical front, the study provides concrete evidence of tensions emerging from the interplay of unitary language and heteroglossia within the use of the same discourse that influences the meaning-making process. Thus, the study suggests that the tensions between the unitary language and heteroglossia are not necessarily evident only between the discourses but may also be seen within the same discourse that influences the meaning of the utterance.

7.6 Concluding Thoughts/ Envoi

> There are no "neutral" words and forms – words and forms that can belong to "no one"; *language has been completely taken over, shot through intentions and accents*. For any individual consciousness living in it, *language is not an abstract system of normative forms but rather a concrete heteroglot conception of the world …. Each word tastes of the context and contexts in which it has lived its socially charged life; all words and forms are populated by intentions. Contextual overtones (generic, tendentious, individualistic) are inevitable in the word*. (Bakhtin, 1981, p. 293; emphasis added).

This thesis shows how we, humans, appropriate and adapt our words and our utterances not only to the given meanings but to our intentions, as well as to re-invent our meanings. It shows how the meanings are populated not just by the *context* of the moment but the social *contexts* in which the word has been used.

I began my journey with an overarching question: *how do 9 to 11 years old children negotiate their understanding of shapes and their properties in a New Zealand multilingual primary classroom?* My main focus was to investigate the interactional aspects of children's utterances and the aspects of dialogic space of the New Zealand classroom that influence children's interaction during whole-class and/or group interactions. The findings showed that children embed their everyday language with mathematical meanings. Additionally, the prosodic features of multilingual children and children's gestures contribute to the negotiation of meanings as they talk about their understanding during classroom interactions. Moreover, there are several aspects of the dialogic space that also influence the negotiation of meanings, including children's understanding of what are considered preferred and dispreferred responses along with the variety of speech genres that are available to children to influence each other's understanding. Overall, the study highlighted the need for teachers and teacher educators to recognise subtle yet powerful aspects of classroom interactions that influence children's learning in multilingual New Zealand classrooms.

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Appendices

Appendix A. Te Whānau Taparau – The Polygon Family

Te whānau taparau - the polygon family

Purpose

This unit examines the properties of polygons and how these are related. It also gives the names in both Māori and English.

Achievement Objectives

GM3-3: Classify plane shapes and prisms by their spatial features.
AO elaboration and other teaching resources

Specific Learning Outcomes

- Investigate properties of symmetry in shapes.
- Investigate spatial features of shapes.
- Use both English and Te Reo Māori to describe different polygonal shapes.

Description of Mathematics

This unit allows students to develop an understanding of the geometrical features of polygons and how classes of polygons are defined. It also aims to develop aspects of symmetry (reflective and rotational) through a problem solving approach.

A polygon is a planar (flat) shape that is bounded by straight sides. The relationships between sides and angles are used to create classes (groups) of shapes. In this unit students form quadrilaterals and triangles. Each class of shapes contains sub-classes. For example, a quadrilateral might have pairs of opposite parallel sides. If a quadrilateral has only one pair of parallel sides, it is called a trapezium. If it has two pairs parallel sides, it is called parallelogram. Some parallelograms have internal angles that are right angles. That class of shapes is called rectangles. Definition, and reasoning with those definitions, is an important feature of geometric thinking.

In this unit, mathematical language is also explored particularly in terms of te reo Māori. It is envisaged that such an exploration will give rise to descriptions that can incorporate both languages, to support students to make sense of defining properties of 2-dimensional and 3-dimensional shapes.

The unit begins with string geometry to set the scene for investigating shapes and their properties using folding and possible turning techniques. Activity progresses to an examination of regular polygons where Māori terms are introduced. The concept of whānau or family is introduced to reinforce the fact that polygons are linked in a range of ways.

For this unit you will need to know, and the students will need to find out, the following:

porowhita = circle
whānau = family
taparau = polygons
tapatoru = triangle
tapawhā = square
tapawhā whakarara = parallelogram
whitianga = diameter
pūtoro = radius
paenga = circumference
tapatoru rite = equilateral triangle
puku = tummy
e toru, nga tapatoru rite = made from 3 smaller equilateral triangles
tapawhā whakarara rite = a rhombus
taparara = trapezium
tapawhā rite = square
tapawhā hāngai = an oblong
koeko tapatoru = a triangular pyramid
ahu-3 = 3 dimensions
tapaono rite = regular hexagon
kai = food
e ono, tapatoru rite = 6 equilateral triangles

Opportunities for Adaptation and Differentiation

The learning opportunities in this unit can be differentiated by providing or removing support to students, or by varying the task requirements. Ways to support students include:

- ensuring that students have access to the physical manipulatives suggested, so they can experiment with forming and folding shapes, before being required to visualise transformations
- explicit modelling of forming shapes folding, reflections and rotations, where students need support. Expect students to copy your actions before attempting problems independently
- helping students to represent shape forming, reflections and rotations diagrammatically to ease memory load and support thinking, e.g. drawing lines of symmetry on a paper copy of a shape
- providing a list of mathematical terms, and definitions for students to refer to
- using the art of story telling to provide a motivational setting for mathematical inquiry.

Tasks can be varied in many ways including:

Appendix B. Behavioural Descriptions for Zones of Learning Progression for Geometric Thinking (Seah & Horne, 2019)

Zones	Behavioural descriptions
Zone 1: Pre-cognition –	• Recognise simple shapes by appearance and common orientation; • show emerging recognition of objects from different perspectives; • name and describe 3D objects based on common 2D shape names; • identify some standard nets; identify location using a simple referencing system. • In measurement situations, recognise comparisons in one dimension without using units.
Zone 2: Recognition	• Identify simple shapes in situ and on simple solids; • recognise some reflective symmetry, some nets of simple solids and some simple shapes. • Show emerging representation of 3D objects; use some simple geometric language; • show emerging perception of measurement concepts such as length, area, and angle but do not coordinate information or justify thinking. • Beginning to represent and move between representations but focuses mostly on one property (isolated features).
Zone 3: Emerging informal reasoning	• Use one or two properties or attributes (insufficient) to explain their reasoning about shapes and measurement but often do not recognise properties in non-standard representations. • Demonstrate awareness of measurement attributes. • Tend to visualise objects from their own perspective. Use simple coordinates. • Tend to see objects and groups of objects as a whole but unable to analyse components independently.
Zone 4: Informal and insufficient reasoning	• Use some geometric language in context, name some 3D objects and are able to visualise some objects from a different perspective; • show incomplete reasoning in geometric and measurement situations, attending to necessary properties but not recognising redundancy. • Use some properties to identify shapes/objects. • Perform measurement calculations but attend to only one attribute. • Give directions from a map from personal rather than other viewer's perspective when situations are more complex.
Zone 5: Emerging analytical reasoning	• Able to visualise and represent 3D objects using 2D platforms (such as Nets); • recognise properties in non-standard orientations and are starting to use properties to identify classes; • begin to use but not recognise sufficient conditions; • use either properties or orientations to reason in geometric situations; • access relevant geometric language; • demonstrate knowledge of dilation and coordinate systems and recognise some rotational symmetry; • use landmarks but retain personal orientation when providing directions; • provide partial solutions and explanations when calculating measurement situations.

	• Begin to coordinate multiple components
Zone 6: Property based analytical reasoning	• Use properties accurately when reasoning about spatial situations but lack knowledge of geometry hierarchy. • Understand properties of 2D shapes but not special cases (e.g. regular). • Geometric and measurement arguments rely on examples/counter examples. • Provide accurate directions from a map using appropriate language and describe directions from a walker's perspective. • Understand impact of doubling dimensions on volume, able to visualise volume and calculate when numbers are small. • Omit one step when calculating multi-step measurement problems. • Able to make deductions about angle situations with limited explanations. • Beginning to reason deductively but not able to coordinate all aspects.
Zone 7: Emerging deductive reasoning	• Work analytically with properties of rectangles. • Beginning to recognise necessary and sufficient conditions. • Use sound reasoning in argument/explanations, though explanations often are procedurally based or based on an example. • Able to recognise the relationship between length, area and volume. • Using multiple properties to reason but in measurement situations may rely on procedural explanations.
Zone 8: Logical inference-based reasoning	• Construct arguments based on multiple properties of 2D shapes and 3D objects, • use the necessary and sufficient conditions to reason about geometric and measurement situations, conjectures and propositions (theorems), • demonstrate analysis of both reflectional and rotational symmetry

Note: Adapted from Seah and Horne (2019), p. 173-174.

Appendix C. Semi-Structured Interview Protocols

Note: This is the range of questions, I choose few questions for 15-20 minutes of the semi-structured interview for each lesson.

Prompts: About planning of the unit.

1. How did you plan for this class?
2. Did it go as you intended?
3. What went well?
4. What understanding of shape do you want your students to have?

Prompts: About the task.

5. Why did you structure the task as a group/pair task?
6. Why did you select this particular activity?
7. How much time did you expect it to take?
8. Did it go as planned?
9. Why did you make use of this particular material/ manipulative for the task?
10. Are there any other activities you could use?

Prompts: About the child.

11. How did you make these groups?
12. Are there any particular criteria to make these groups?
13. Do these groups change?
14. Why did you put (a multilingual child) in that particular group?

Appendix D. Semi-Structured Focus Group Interview Protocol

Prompts for student

1. What have you learned about shapes during the geometry unit?
2. What new language did you use?
3. How did you find working with others in the group on the geometry tasks?
4. Can you describe a shape?
5. What shape do you think this (object) is?
6. What can you tell me about this (object)? How would you describe it?
7. What is a 2D shape? Is it different? How is it different from 3D shape?
8. Can you give me an example?
9. How were your lessons on shapes?
10. What did you learn about shapes?
11. What do you mean by edges? Faces? Vertices?
12. Are these shapes different? (Show counter and ball, paper cut-out rectangle and Jenga Piece), how?
13. What is d in 2D and 3D?
14. Can you say a bit more about dimension? What do you understand by dimension?

Appendix E. Ethics Approval

Shweta Sharma <ss555@students.waikato.ac.nz>

FEDU Ethics Application Approved

1 message

Ethics Application <fedu.ethics@waikato.ac.nz> Sun, Aug 5, 2018 at 1:21 PM
To: Shweta Sharma <ss555@students.waikato.ac.nz>

The following is an automated email sent from the Ethics Review Application.

Congratulations **Shweta Sharma** your ethics application "**Making sense of 2D, 3D shapes, and their representations in New Zealand multilingual primary classes**" has been approved.

Dean's Office
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THE UNIVERSITY OF WAIKATO
Te Whare Wānanga o Waikato

MEMORANDUM

To: **Shweta Sharma**

cc: Dr Sashi Sharma
APROf Brenda Bicknell
Dr Nicola Daly

From: **Dr Sonja Arndt**
Co-chair Faculty of Education Research Ethics Committee

Date: 16/10/18

Request for Extension to Research Ethics Approval – Student (FEDU064/18)

Thank you for your request for an extension to the ethics approval for the project:

Making sense of 2D, 3D shapes, and their representations in New Zealand multilingual primary classes

It is noted that you wish to extend the Ethics application (FEDU064/18) as follows:

1. The working title is changed from "Making sense of 2D, 3D shapes, and their representations in New Zealand multilingual primary classes" to "Making sense of two-dimensional (2D) shapes, three-dimensional (3D) shapes, and their representations in New Zealand primary classes".

2. The classes (Class teacher and the students) will include a range of Year levels rather than Year 5-6 as specified for the present research. The participants will be recruited depending on the availability of the teacher participants and the Year levels they teach.

3. A small 15-20 minutes semi-structured group interview (see Appendix L) will also be conducted with student participants to seek clarifications about their understanding of geometry concepts. The group interview will take place after the end of the geometry unit at a time and place convenient to both teachers and students as per their schedule.

4. A questionnaire (as Appendix M) has been added as another data collection tool to gather background information of the students. The questionnaire will be completed by the Parents/Caregivers.

I am pleased to advise that this extension has received approval.

Please note that researchers are asked to consult with the Faculty's Research Ethics Committee in the first instance if any further changes to the approved research design are proposed.

The Committee wishes you all the best with your research.

Dr Sonja Arndt
Co-chair Faculty of Education Research Ethics Committee

Appendix F. Information Sheet- Principal

Making sense of 2D, 3D shapes, and their representations in New Zealand multilingual primary mathematics classes

My name is Shweta Sharma. I am a PhD scholar at Te Kura Toi Tangata Faculty of Education, University of Waikato. For my PhD, I am conducting this research to understand the processes through which multilingual students make sense of 2D shapes, 3D shapes, and their representations in primary classes.

I would like to invite your school to be a part of this study. I would like to observe classes across several Year levels in your school for a period of 2-3weeks (3-5 lessons in each class) in the second term (May to June) 2019 for geometry lessons on shapes and their representations.

I would like to take observation notes of the geometry lessons, and video-audio recording of whole/group teaching and learning tasks. I would also like to take copies of school documents (lesson plans, unit plans, and students' written work) and to conduct a short interview (approximately 20 minutes) with each class teacher after each observed lesson(s) at a time and place convenient to the teacher. The interview will be recorded, then transcribed, and given back to the teacher for checking and amendment before any analysis takes place. At the end of the observed lessons, I would also like to invite students to participate in a short (15-20 minutes) interview to seek clarification regarding their understanding of geometry concepts. The group interview will be audio-recorded. The group interview will take place at a time and place convenient to the students and the teachers.

The focus of the research is to describe how multilingual students understand 2D shapes, and 3D shapes, and their representations.

Points to be noted:

1. Participation in the research is voluntary. Any teacher/student can withdraw their participation from the research without being answerable to anyone. If the teacher/student chooses to withdraw from the research at any point in time, I would not audiotape them or use any video-image of them in the research. However, note that the data collected until that time cannot be withdrawn due to the nature of whole class data.
2. The identity of the teacher and the students will be kept anonymous and data will be confidential. No reports will identify school, teacher or students. Teacher and students may provide a pseudonym to use in my writing. While every effort will be made to protect the anonymity of all participants, this cannot be guaranteed.
3. The information gained through the research will be used mainly for producing a PhD thesis. When the thesis is finished, it will be published on the University of Waikato's Research Commons.
4. Parts of the research may also be used in writing articles, book chapters, and for presentations at conferences.
5. Throughout the study, all communication channels will be kept open so that you may contact my supervisor or me regarding queries about the research study.

If you are happy for your school to be involved in the study, please provide me with details of your teachers who may like to participate in the study. Also, please sign the attached Informed Consent form.

If you have any queries, please contact me in the first instance or my supervisor. Contact details are as follows:

- Shweta Sharma,

Faculty of Education, University of Waikato.
Ph. no. 02102587570,
Email: ss555@students.waikato.ac.nz

- Dr Sashi Sharma, Senior Lecturer,

Te Hononga Curriculum and Pedagogy
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Appendix F(a). Informed Consent Form – Principal

Making sense of 2D shapes, 3D shapes, and their representations in New Zealand primary classes

I, ___________________________, principal of _______________________ primary school, have discussed the research project with Shweta Sharma, principal investigator of the project, and I fully understand the purpose and extent of the project.

I give my approval to her to conduct this research in the school and understand that:

- the study and what it requires of the staff, students in my school;
- the study will involve classes across several Year levels (class teacher and students) for a period of 2-3 weeks (3-5 lessons) in second term (May to June) 2019 for geometry lessons on shapes and their representations;
- participants can withdraw at any time by contacting Shweta, however, data collected until that time cannot be withdrawn due to the nature of whole class data;
- the data collected from the school will be used to write a doctoral thesis, may be published in research papers, book chapters, and for presentations in conferences;
- while every effort will be made to protect the anonymity of all participants, this cannot be guaranteed; and
- I may ask Shweta or her supervisor any questions about the study.

Note: If you would like a summary of the findings, please choose Yes/No, and provide an email or postal address for it to be sent to at the end of the research.

Name: ____________________________________ Date ________________________

Signature of the Principal: _____________________

Email/ Postal Address ____

Appendix G. Information Sheet- Teacher

Making sense of 2D, 3D shapes and their representations in New Zealand multilingual primary classrooms

My name is Shweta Sharma. I am a PhD scholar at Te Kura Toi Tangata Faculty of Education, University of Waikato. For my PhD, I am conducting this research to understand the processes through which multilingual students make sense of 2D shapes, 3D shapes, and their representations in New Zealand primary classes.

I would like to invite you to be a part of this study. I would like to observe your class for a period of 2-3 weeks (3-5 lessons) in second school term (May to June) 2019 for geometry lessons on shapes and their representations. I would take observation notes of the whole lessons and take video-audio recordings of whole/group teaching and learning tasks. I would also like to take copies of your unit and/or lesson plans and to conduct a short interview with you for approximately 20 minutes after each lesson(s) at a time and place convenient to you. The interview will be recorded, then transcribed, and given back to you for checking and amending your response before any analysis takes place.

The focus of research is to describe how multilingual students understand 2D and 3D shapes, and their representations.

If you are happy to be involved in this study, please sign the attached consent form.

Points to be noted:

1. Your participation in this research is voluntary. You can withdraw your participation from the research at any time by contacting me.
2. However, it must be noted that the data collected until you withdraw your participation cannot be withdrawn due to the nature of whole class data.
3. Your identity will be kept anonymous. Data will be kept confidential. No reports will identify you in any form. You may nominate a pseudonym to be used in the thesis and any other publication. While every effort will be made to protect the anonymity of all participants, this cannot be guaranteed.
4. The information gained through the research will be used mainly for producing a PhD thesis. When the thesis is finished, it will be published on the University of Waikato's Research Commons.
5. Parts of the research may also be used in writing articles, book chapters, and for presentations at conferences.
6. Throughout the study, all communication channels will be kept open so that you may contact my supervisor or me regarding queries about the research study.

If you have any queries, please contact me in the first instance or my supervisor. Contact details are as follows:

- Shweta Sharma,

Faculty of Education, University of Waikato.
Ph. no. 02102587570,
Email: ss555@students.waikato.ac.nz

- Dr Sashi Sharma, Senior Lecturer,

Te Hononga Curriculum and Pedagogy

Faculty of Education, University of Waikato
Ph.no. +64 7 838 4466
Email: sashi@waikato.ac.nz

Appendix G(a). Informed Consent Form- Teacher

Making sense of 2D shapes, 3D shapes, and their representations in New Zealand primary classes

I have discussed the research project with Shweta Sharma, principal investigator of the project, and I fully understand the purpose and extent of the project.

This entails:

- Allowing Shweta to observe my geometry lessons in Year ______ on shapes and their representations for 2-3 weeks (3-5 lessons) in _____ (term/month) 2019;
- Allowing her to video and audio record the whole/group teaching and learning tasks in my class;
- Providing her with copies of my unit/lesson plans for the observed lessons; and
- Allowing her to conduct semi-structured interviews (for approximately 20 minutes at a convenient time and place) with me to seek clarification about the lessons.

I give my approval to be involved in the research study on the conditions,

- That the Principal has given written consent for the project to be carried out;
- I may ask Shweta or her supervisor any questions about the study;
- I may ask at any time for the video and/or audio recorder to be turned off;
- The recording (audio-video) will be treated confidentially and stored securely;
- The identity of the students, principal and me will be anonymous in any written record or report, and while every effort will be made to protect the anonymity of all participants, this cannot be guaranteed;
- I suggest ________ as pseudonym for me.

Note: If you would like a summary of the findings, please give an email address (or postal address) for this to be sent to you at the end of the research.

Name: ____________________________________ Date __________________________

Signature of the Teacher: _______________________

Email/ Postal Address ____

Appendix H. Information Sheet- Parents/Caregivers

Making sense of 2D, 3D shapes, and their representations in New Zealand multilingual primary classes

My name is Shweta Sharma. I am a PhD scholar at Te Kura Toi Tangata Faculty of Education, University of Waikato. For my PhD, I am conducting this research to understand the processes through which multilingual students make sense of two-dimensional (2D) shapes (e.g. circles and squares), three-dimensional (3D) shapes (e.g. spheres and cubes), and their representations in New Zealand primary classes.

Being part of the study will involve me coming to your child's class to observe geometry lessons on shapes and their representations for 3-5 lessons during the second term (May to June) in 2019.

I would like to invite your child to be involved in the study. I would make observation notes of your child's interactions during their regular geometry lessons. I would also like to audio- and video record your child's interactions with other students in whole/group teaching and learning tasks. In addition, I would require making copies of his/her drawings and written work. Your child may also be invited to participate in a short (15-20 minutes) group interview. The interview will be audio-recorded. The group interview will take place at a time convenient for the students and the teacher.

The focus of the research is to describe how multilingual students understand 2D and 3D shapes and their representations.

If you are happy for your child to participate in this research, kindly fill in the Questionnaire and sign the Informed Consent form attached with the Information Sheet. I will make a copy of your signed Informed Consent form and will provide that copy to your child.

Points to be noted:

1. Participation of your child in the research is voluntary. Your child can withdraw his/her participation from the research without being answerable to the teacher or me.
2. If your child chooses to withdraw from the research at any point in time, I would not audiotape him/her or use any video-image of him/her in the research. However, it must be noted that the data collected until you withdraw your participation cannot be withdrawn due to the nature of whole class data. Note that prior to any recording, I will check with your child for his approval.
3. The identity of your child will be kept anonymous and confidential. No reports will identify any student. Students may nominate a pseudonym to use in written reports. While every effort will be made to protect the anonymity of all participants, this cannot be guaranteed.
4. The information gained through the research will be used mainly for producing a PhD thesis. When the thesis is finished, it will be published on the University of Waikato's Research Commons.
5. Parts of the research may also be used in writing articles, book chapters, and for presentations at conferences.
6. Throughout the study, all communication channels will be kept open so that you may contact my supervisor or me regarding queries about the research study.

If you have any queries, please contact me in the first instance or my supervisor. Contact details are as follows:

- Shweta Sharma,

Faculty of Education, University of Waikato.
Ph. no. 02102587570,
Email: ss555@students.waikato.ac.nz

- Dr Sashi Sharma, Senior Lecturer,

Te Hononga Curriculum and Pedagogy
Faculty of Education, University of Waikato
Ph.no. +64 7 838 4466
Email: sashi@waikato.ac.nz

Appendix H(a). Informed Consent Form- Parents/Caregivers

Making sense of 2D shapes, 3D shapes, and their representations in New Zealand primary classes

I have read the information given about the research project with Shweta Sharma, principal investigator of the project, and I fully understand the purpose and extent of the project. This entails:

- Allowing Shweta to observe my child during his/her geometry classes on shapes and their representations for 3-5 lessons during second term 2019;
- Allowing her to video and audio record my child's interactions in whole/group teaching and learning tasks during the same geometry lessons, and allowing her to take copies of my child's work related to the observed geometry lessons.
- Allowing her to conduct a short (15-20 minutes) group interview with my child.

I give my approval for my child to be involved in the research study on the conditions,

- That the Principal/ Board of Trustees has given written consent for the project to be carried out;
- My child can withdraw from being involved in the study at any stage, without having to give any reasons, if so, he/she will not be audio and/ or videotaped, and no reference to him/her will be made in the reporting of the study. However, data collected until that time cannot be withdrawn;
- I may ask Shweta or her supervisor any questions about the study;
- My child and I may ask at any time for the video and/or audio recorder to be turned off;
- The recording (audio-video) will be treated confidentially and stored securely;
- The identity of my child will be anonymous in any written record or report. My child may provide a pseudonym for him/herself, and while every effort will be made to protect the anonymity of all participants, this cannot be guaranteed.

Note: If you would like a summary of the findings, please give an email address (or postal address) for this to be sent to you at the end of the research.

Name: ____________________________
Signature of the Parent/ Caregiver: ______________Date___________
Email/Postal Address ____
Name of the child ______________ Year _________

Appendix I. Information Sheet- Student

Making sense of 2D, 3D shapes, and their representations in New Zealand multilingual primary classes

My name is Shweta Sharma. I am a student at Te Kura Toi Tangata Faculty of Education, University of Waikato. For this research, I will be looking at the ways through which you make sense of two-dimensional (2D) shapes (e.g. circles and squares), three-dimensional (3D) shapes (e.g. spheres and cubes), and their representations.

I would like to spend some time in your class during geometry lessons for 3-5 lessons during second term 2019. I would be audio and video recording you during those geometry lessons. I would like to put an audio-recorder on your table while you do your whole class/group work. I would also like to have copies of your of drawings and written work related to those geometry lessons. I would also like to invite you to participate in a short group interview or discussion to talk about geometry.

Please note:

- You can say ‘no’ to participation at any time, and you do not have to explain why to want to stop.
- You can ask me to turn off the video and/ or audio recorder at any time.
- I will keep the video and audio-recordings safely and will not show them to anyone else except other than my supervisors.
- I will not tell your name to anyone. You may tell me a name to use for you in my research.
- You can ask me any questions about the study at any time or you can ask my supervisor.

If you have any questions at any time, you can ask me when you see me in your class or you can ring me on 02102587570. Or if you want, you can talk to my supervisor, her details are

Dr Sashi Sharma, Senior Lecturer,
Te Hononga Curriculum and Pedagogy
Faculty of Education, University of Waikato
Ph.no. +64 7 838 4466
Email: sashi@waikato.ac.nz

If you are happy to be a part of this study, please sign the attached form. Your parents/ caregivers must also sign it.

Appendix I(a). Informed Consent Form- Student

Making sense of 2D shapes, 3D shapes, and their representations in New Zealand primary classes

I have discussed the project with Shweta Sharma., and I fully understand the information sheet.

This involves:

- I choose to be part of this project.
- I can say ‘no’ to be part of the project at any stage, without having to give any reasons to my teachers, parents or anyone.

- I agree to the audio-video recording of my work during whole class/group work during geometry classes.
- I can ask to have audio and/or video recorders turned off at any time.
- Shweta will store audio-video recordings safely and no one other than Shweta or her supervisors can watch that.
- I will let Shweta have copies of my drawings and written work for the geometry lessons.
- My name will not appear in any written work by Shweta and I would like to use this name _________ for me in her research.
- I can ask Shweta or her supervisor any questions.

Name: ____________________________________

Date ________________________

Signature of the Student: _____________________

Appendix J: Questionnaire

1. Name ___________________

2. What are the languages that are spoken at home?

   ____

3. Age of the child ______________________

4. For how long have you been in New Zealand? _____________________

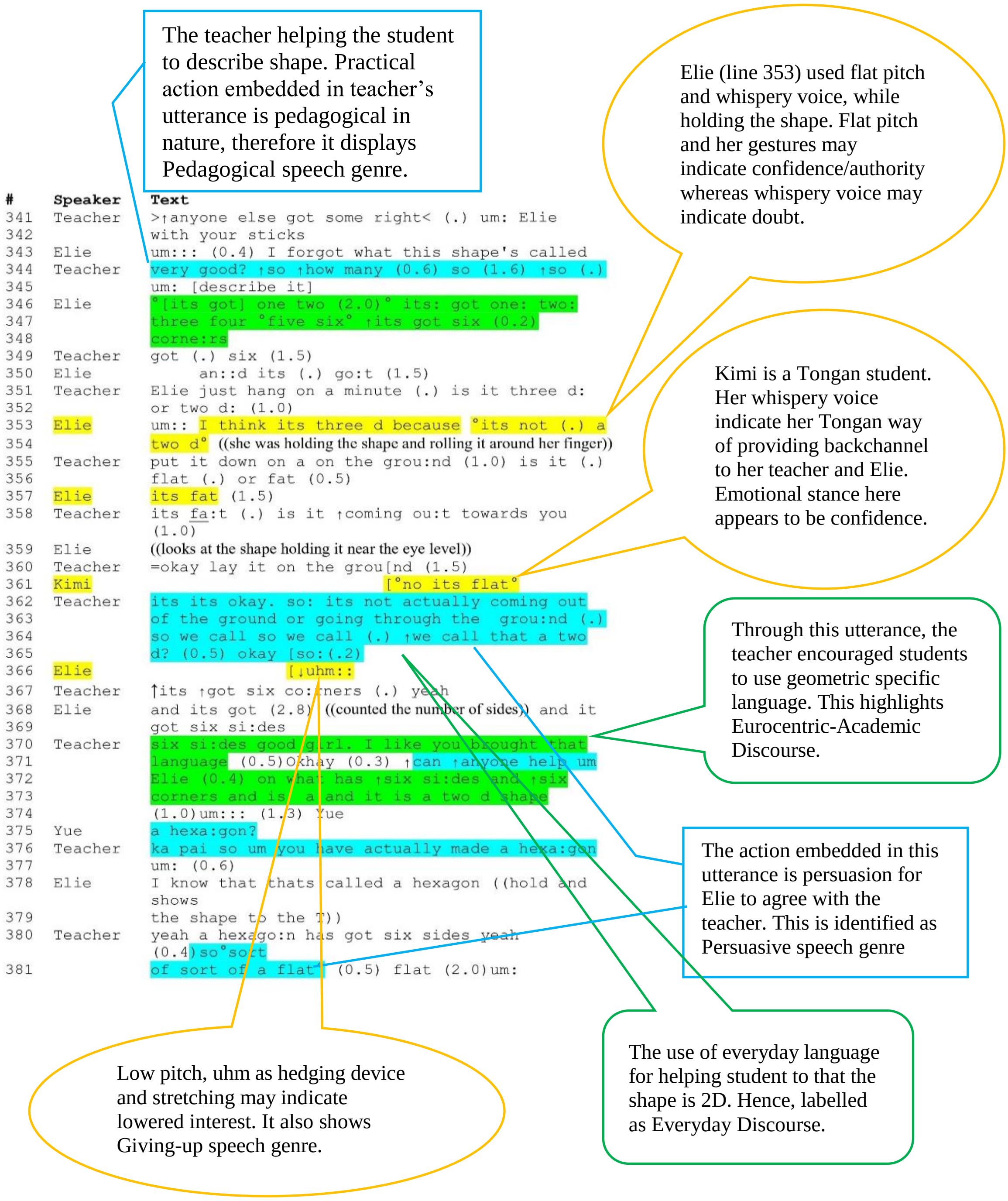
5. Where was the child born? _______________________________

6. How long have your child been in New Zealand School? ____________________

Appendix K. Coding of Emotional Stances, Speech Genres, and Discourses in Key Moment 4.4a.

For the purpose of the presentation and managing data analysis in the thesis, I provide details of the first step of macro-level analysis for only one Key Moment (see Figure below). Analysis of one Key Moment (Key Moment 4.4a) is presented to enable the reader to understand the analytical procedure followed for identifying emotional stance, speech genres, and discourses . The same iterative procedure was used to identify these aspects of emotional stance, speech genres, and discourses in the other nine Key Moments. For the first step, each utterance in each Key Moment was re-examined to identify (i) emotional stances based on the prosodic patterns used in utterances, (ii) speech genres by exploring the social action intended and prosody embedded in each utterance, and (iii) discourses by examining the content and form of utterance for discourses. It is to be noted that although the study focuses on children's negotiation of meanings about shapes and their properties, emotional stances, speech genres and discourses embedded in teacher's utterances are also examined here as part of dialogic space.

In the presented Key Moment 4.4a (see Figure below), each utterance was examined to identify the emotional stances (in yellow highlight), speech genres (in blue highlight) and the discourses (in green highlight) used in the conversations. Based on the prosodic cues used by Elie during this Key Moment, it seems that she displayed three different emotional stances about her learning at different micro-moments within this Key Moment.



Macro-level coding of a Key Moment 4.4a.
Yellow highlight shows emotional stance, blue highlight shows speech genres, and green highlights show dominant discourses.

Elie initially used the whispery voice (line 346) to count the number of sides of the shape that she had made and later used high onset and flat pitch (line 347-348). Research has indicated that English language users may use high onset/high pitch (↑) to display new information and may use flat pitch to display their confidence in their knowledge claim (Ward, 2019). It may be argued that the first emotional stance Elie displayed was of *confidence*. That is, she was confident about her learning and knowledge claim. However, there seems to be a change in her emotional stance as the teacher asked her if the shape she made is 3D or 2D (line 351-352). Elie used a flat pitch and whispery voice in her response (line 353-354) and held her shape in her hand (from the video-recorded data). The use of a whispery voice within her utterance may indicate her doubt (Gobl & Chasaide, 2003); however, the use of flat pitch may indicate confidence. It may be argued that at this moment, the emotional stance about her knowledge claim at this moment displayed through her utterance was *partial confidence*. This partial confidence may indicate the second emotional stance embedded in Elie's utterance about her knowledge claim. Elie again used flat pitch in her following utterance (line 357). However, as the conversation proceeded, Elie's utterance may indicate her falling interest in the conversation as she used low pitch with hedging and stretching (Drew, 2013; Ward, 2019). This *falling interest* indicates her third emotional stance of learning. The chronological change in Elie's emotional stance may suggest that children may show different emotional stances at different moments during interactions.

In terms of the speech genres used, Elie seems to display her social action intertwined with her emotional stance. For example, in her utterances in line 346 and 357, Elie appears to be confident (emotional stance as evident through her use of flat pitch). Also, it seems that she intended to declare (social action) her understanding of the shape that she made. Based on these interpretations, these kinds of utterances are labelled as *Declarative speech genre.* In her following utterance (line 353), it seems that Elie supported her understanding of her shape as 3D shapes by explicitly providing her assumption that the shape is not 2D. As stated earlier, it seems that at this moment, Elie was partially confident. These kinds of utterances where participants seemed to engage in the social act of providing argument were labelled as *Argumentative speech genres.* Thus, at this time, it looks as if Elie used an Argumentative speech genre. Following the conversation, Elie displayed her intention of not continuing the conversation (line 366). At this moment, Elie constructed an utterance "↓uhm::". These kinds of utterances where participants displayed an act of withdrawal with lack of interest were called the *Giving-Up speech genre*.

Two different discourses were also identified by examining the content and form of the utterances. Initially, Elie stated that she did not remember the name of the shape. The teacher asked "how many" in her utterance, probably to prompt Elie to state the number of sides. Elie stated that the shape had six corners. Counting the number of sides or corners for formulating the name shape displays the Eurocentric framework of geometry knowledge. Moreover, "side" is an everyday word that is used as part of academic geometry vocabulary in The New

Zealand Curriculum. Thus, though the *content* of the utterance used everyday words, the socio-historical meaning embedded in the use of the word “side” at this moment signals the use of “side” as representing the line segments of 2D shapes. Hence, the use of a Eurocentric framework of geometry knowledge including the accepted academic geometric language in children’s utterances is labelled as Eurocentric-Academic Discourse. As the conversation proceeded, the teacher asked if the shape “is coming out of the ground” (line 362-363) to help Elie to see the shape as 2D. The teacher used the phrase “coming out of ground” to signal the difference between 2D and 3D shapes. However, this phrase’s content and form seem to suggest everyday language for making sense of 3D geometry concepts. The use of phrases and words in this manner, where the content and form signal the meanings from everyday language, are labelled as using the Everyday Discourse in this study.

Appendix L. Identification of Emotional Stances

Key Moment	Participants	Emotional stance of the learning
Key Moment 4.1a: "*I saw this as some sort of shape*"	Ozan, Garry, Tahi, Teacher	Disappointment, Confidence, Conviction (Ozan); Confidence, Authority (Gary) Confidence (Tahi); Acknowledgement and Appreciation (Teacher)
Key Moment 4.1b. *"Whaea look, a perfect square"*	Zara, Matiu, Garry, Elie Teacher	Confidence, Uncertainty (Zara); Confident, Doubtful, Relieved (Matiu); Confident, Funny (Garry); Uncertain (Elie); Authority (teacher)
Key Moment 4.2a: *"what's a triangle 3D? A triangular prism"*	Teacher, Matiu, Tane, Garry, Zara, Ethan	Excited, Surprised (teacher); Confidence (Matiu) Confidence, Co-operation (Tane); Confidence, Attention-seeking (Garry); Engaged in learning, Doubtful (Zara); Unsure (Ethan)
Key Moment 4.2b. *"what's a triangular prism"*	Matiu, Ethan Garry, Tahi, Ozan	Confident, Confused (Matiu); Unsure (Ethan); Confident (Garry); Confident, Eager (Tahi) Ozan was present in the group but did not speak
Key Moment 4.3a *"sphere is a fat circle, a circle is a flat circle"*	Teacher, Alyssa, Zara, Kayla,	Excited (Teacher); Confident (Alyssa); Unsure (Zara); Authority over knowledge (Kayla)
Key Moment 4.3b: *"the flat shapes within those with the volume"*	Teacher, Olivia, Tahi, Alyssa, Zara, Matiu, Ethan, Garry, Elie	Motivating (Teacher Doubtful (Olivia); Supportive (Tahi); Confident (Alyssa) Disappointed (Zara); Confident (Matiu Doubtful (Ethan) Doubtful (Garry) Confident, disappointment (Elie)
Key Moment 4.4a: *"I think it's 3D because it's not 2D"*	Teacher, Elie, Kimi, Yue,	Supportive (teacher); Confident, doubtful, not interested (Elie); Reassuring, confident (Kimi); Confident (Yue)
Key Moment 4.4b:	Teacher, Zara, Ethan,	Authority, Relieved (Teacher) Confident, Unsure (Zara) Confident, (Ethan)

"How many dimensions does this shape has?"	Matiu, Nikau, Elie, Olivia, Alyssa	Confident (Matiu) Unsure (Nikau) Annoyed (Elie) Confused (Olivia) Confident (Alyssa)
Key Moment 4.5a: *"a circle has no side"*	Teacher, Zara, Elie, Yue, Matiu, Ethan, Tane	Supportive, Doubtful (Zara, Ethan, Tane) Confident, (Elie), Confident (Yue) Confident (Matiu) Confident, Surprised (Ethan) Unsure (Tane)
Key Moment 4.5b: *"if square is tapawha, what's rectangle then"*	Teacher, Zara, Olivia,	Unsure (teacher) Curious (Zara) Unsure, Curious (Olivia)

Appendix M. Coding of Key Moments for Speech Genres and Discourses

Key Moment 4.1

#	Speaker	Text
205	Garry	What sha:pes can you see right now
206	Tahi	circ::les(1.5)squa:res
207		((Garry takes the picture sheet and turn it over to put glue to paste it on large
208		white sheet as Tahi was still looking at it))
209	Ozan	I see a lot of circles over there (3.0)
210		((Ozan looks at the sheet while Garry and Tahi make faces towards the
211		camera))
212	Ozan	okay(.) what is this shape called ((pointing to shape))
213	Garry	↑so ↑whats tha:t whats [that Tahi?
214	Tahi	[squa::re
215	Garry	thats a ↓rectangle
216	Tahi	#square#
217	Garry	then Ill say squa:!re
218	Tahi	↑Squ°are::°(.8) °thats° a square
219		((Garry writes square as Tahi speaks))
220	Ozan	oh ↑I ↑SEE [One
221		((Ozan looks at Garry who was given with the responsibility to write))
222	Tahi	[he::re ((Tahi points to different shape and laughs))
223	Ozan	THIS ONE ((points again to the shape))
224	Garry	wha:ts that
225	Ozan	I dont know what[it is called
226	Tahi	[°circle thats a circle°
227	Garry	cir(.)cle
228	Ozan	not °this° (2.0) ((put his hand to his head to show that
229		it is not the shape that he was talking about))
230		I am talking about whole thing, like like (2.0)
231		((drag his finger at the shape to show his imagination of sides))
232		(in jacks)(.5)what was it (2.0) [it=
233	Tahi	=ohh (.) °I know there is this thingy like
234		this° ((points to another shape))
235		[theres like ((makes the shape with his finger on the sheet to
236		show the shape he implies))
237	Garry	[there is: no thingy (you images/
238	Ozan	((aspires)) Oh↑ I see
239	Tahi	no:: theres a thing(.) that they had tha:t goes
240		↑like (then) ((Tahi moves his finger in curved motion))
241	Garry	[((draw a line and Tahi see him))
242	Ozan	[oh ↑I see one (.5)Agai::n
243	Tahi	RECtangle:s (and like) (2.0)
244	Garry	↑Oval:((marks an arrow for a shape))
245	Ozan	[I SEE ↑one I see one °I see one°
246	Tahi	[okhay
247	Garry	wha::t
248	Ozan	this one like, not the:se circle.(.5) °like
249		this° ((he drag his finger on the shape making straight lines of the
250		sides of the shape))
251	Tahi	OH ↑the shape(.)tha:ts not a shape
252	Garry	this not a shape
253	Tahi	what are another shape(.4) (I drop out)
254	Ozan	its some kind of [cir
255	Garry	[its an oval
314	teacher	↓what else can you see so cir:cle: kinda: (yeah
315		circle
316	Ozan	↑I saw this as some kind of sha:pe that I know?
317	teacher	↑do you? ((teacher smiles))

- 205: → asking Question to elicit response → Pedagogical in nature
- 216: — creaky voice - showing confidence - emotional stance.
- 223–225: showing confidence because of High volume but display lack of knowledge.
- 230–231: → using gestures and utterance to convince Garry & Tahi Persuasive genre.
- 233: confidence
- 237: → stresses, confident about claim – Authoritative genre. with no supporting statement.
- 248–250: — again persuasive in nature
- 251: — negating the idea & with confidence. Declarative / or Authoritative
- 314: Asking Question to generate response → Pedagogical in nature
- 316: Ozan trying to convince teacher.
- 317: Plus appreciative as well. supporting → ∴ utterance is Pedagogical in nature

318	Ozan	((nodding))
319	teacher	↑I kno::w (.5)what what kinda shape can we call
320		that then
321	Ozan	its like(.) ((drags his finger over the shape))
322		one of those shapes thats like its goes like this
323		((gestures with both hands to show sides of the shapes))
324		like this [like that
325	Tahi	[a square [°probably° ((use gestures to show lines))
326	teacher	[ah:: (.5)so:(.) you ↑you ↑thinking like(.8)this
327		one(.5)↑lets see tho:se(.)if you would to(.2) give
328		it si::des ay?
329	Ozan	yeah yeah [third one
330	teacher	[°one two°three four five six seven eight [ni:ne
331	Tahi	[theres ↑only eight
332		
333	teacher	one so ↑theres EIGHT (.2)one two three four fi:ve
334		six seven eight, do you think eight,so: do you know
335		the ei:ght one? ((looked at Ozan))
336	Tahi	↑ITS a rect↑(.) oh no ((tried to think))
337	teacher	so do you do you know what the eight one is ((the
338		question is explicitly directed to Ozan))
339	Ozan	I ↑know but(.2)I just my brother used to watch a
340		movie about(.2)of this kind o shapes↑(.5)that I
341		know(.5)their na:mes are are like like twelch(.4)I
342		I thought it was and theres they were saying like
343		a like a lot of shapes like one two three
344		until(.5) they have passed eight, and then ten an
345		twelve or something(.5)I dont remember by how much
346		it was (.2)but I do remember by [how many
347	teacher	[NA:: I think some I think somehow you kno:w but
348		you are not you cant remember so ((coughed and cleared throat))
349	Tahi	I ↑know but I dont know the name
350	teacher	yes okay so(.5)eight sided figure (.2) is::
351		((cleared throat)) is a octagon. remember octagon ((looks at
352		the camera)) okay so um
353	Tahi	octa:gon: there's a six one I am pretty su:re
375	Ozan	°okay this is(.)did you say° this is some kind of
376		tri:angle? (.)and this is like (.5)a shape that has
377		eight
378	Tahi	↑that's (1.0)octogon oh ((aspiration & smiles)) °so thats
379		something like ()°
380	Ozan	its like a ↑cir:cle, some kind of circle (.5) ((makes
381		circle with his hands)) but is (1.0)↑is the ↑SHape that have
382		the most number (2.5) I remember the movie
383		[which is called ()
384	Garry	[((coughs)) HEXAGON
385	Tahi	its hexa:gon
386	Ozan	↑oh yeah hexa:go::n

Pedagogic genre.

Also, showing sides as part of Eurocentric academic discourse.

Eurocentric academic discourse

Time elaboration Chronotopic moment.

using n-gon for naming shapes

showing confidence with knowledge claim

Appreciative genre.

showing confidence but

again Authoritative genre.

Key Moment 4.1b

Speaker	Text
Zara	>look whaea Kerry:< (1.0) whaea Kerry (.) a perfect square ((shows the shape by holding it in her hands))
teacher	is it perf (.) why is it a perfect square? zara
zara	I dun↓no
teacher	what makes it a perfect square(2.0)>come on zara ↑I need< to ↑kno:w(0.5)because you said its perfect so what makes a perfect square a perfect square (1.0)=
Matiu	a ↑s[quare
teacher	=[↑anyone ↑know why a perfect squa:re a perfect square
Matiu	becoz its a square?
Garry	(h)(h)
teacher	yeah because its a square doesnt tell me much(1.0)ELIE what do you think
Elie	becau::se um: [if you have to= (2.0)
Zara	[you put (on) ((rolled her eyes))
Elie	=um: because um::(1.0) if you have the right type of shape. or if you (1.0)if or if you have(.2) havin:g a right(0.5)type of equipment (.) °you can have°
teacher	°okay°
Elie	so:: if you are trying to make square of that one(1.0)you can roll into a ball then you start pressing it down the other side >the other side and you can [get square<
teacher	[oh thank thank you Elie (0.5)↑can any↑one ↑tell me why a perfect square might be (0.2)might be perfect square using geometry language
Zara	[um ((looks at the roof trying to figure out how to say what she wants to say))
Matiu	[um: °its got°
teacher	Matiu
Matiu	because the face °no:(0.2)the si::des°(2.5) hah °I dun know°
teacher	yeah you re on the right track. the si:des what (.)what would the sides be here
Matiu	perfectly:: aligned? with each other?=
teacher	=aligned with each other?
Matiu	ah(1.0) perfectly the same?
teacher	perfectly the sa:me the sides ↑are perfectly the same (1.0)UM::: (1.0) zara(.2) did you hear that(2.0)<a perfect squa:re is when the si:des are per:fectly the same>
Zara	oh↑ ((exclamation)) [yeah
teacher	[the si:des are the same. okay (.)thats why you get a perfect square

- teacher trying to provide scaffolding to student – pedagogic genre
- confirmation-seeking genre
- use of HRT, looks for confirmation from teacher. Confirmation-seeking genre
- teacher evaluated Matiu's utterance and asked another student → ∴ Assessment genre.
- Explaining how to get square using everyday language and no use of geometry language. ↓ Everyday discourse.
- Teacher explicitly mentioning use of geometry specific language
- showing Euro-centric-academic discourse
- whispering shows doubt
- showing Pedagogic genre and support to Euro-centric-academic discourse.

Key Moment 4.2a

#	Speaker	Text
547	Teacher	so they ve got(0.2)square(0.5)two d:(1.0)triangle.
548		three d:(0.5) ↑what is: a tri::angle three d'
549	Ethan	((raised his hand)) it. is. [a:
550	Teacher	[<can anyone remember> what (1.0)a tri (1.0) Yue?
551	Yue	cube
552	Teacher	CU::BE(0.5)um kori cu:↑be is (1.0)a cube is a bit
553		Different (.)um::: Matiu ((teacher smiled and pointed to Matiu))
554	Matiu	tri:angular (0.5)a[::
555	Tane	[prism
556	Matiu	prism
557	Teacher	triangular prism gre:at.
558	Garry	°I WAS ABOUT TO SAY Cone° (1.0)
559		[anyone]
560	Teacher	[triangular ((trailed off as she wrote on the board))
561	Tane	[(h)(h)(h)
562	Zara	°its triangular prism?°
563	Teacher	um ↑whats a ↑really really good example of a
564		triangular prism (.5)that. is. quite famous (1.5)=
565	Matiu	uaaa
566	Teacher	=that we see overseas and in lots of pictures
567		((points to Matiu for answering))
568	Matiu	um: the e::gy↑pt (1.0)[um:: ↑mountain thingy
569	Garry	[((puts his hands up))
570	Teacher	↑yeah the egypt mountain [thingy
571	Garry	[pyramid
572	Un	[pyramid
573	Teacher	yeah[the pyramid
574	Zara	[PYRAMID
575	Matiu	I said tha:t

Assessment genre. / Supportive speech genre. / pedagogical genre to support students in giving justification / Flat pitch, confidence, Authoritative/Declarative / assessment of Yue's response / Tane supporting Matiu / to signal incorrect response. / Loud voice - flat pitch - Confidence. Authoritative genre. / Maori conversation pattern / whisper may indicate doubt/confidence [illegible] interrupting flow. / again pedagogical.

Key Moment 4.2b

#	Speaker	Text
12	Matiu	I am gonna trying to (1.0) its like they are ol just
13		three: d:: you cant like (.) make (.) not make a (0.8) fat-
	~~(0.8)~~	
14		~~fat~~
15	Garry	yes you ca:n↑ — confident, Authoritative genre.
	((Group started making different shapes))	
42	Matiu	my next one is a (.)probably triangular prism
43	Garry	tri:angul[ar prism]=
44	Tahi	[°how you make°] tha:t
45	Garry	=I am ju↑st gonna make a tri:↓angle
46	Matiu	°a triangle?°
47	Garry	what↑s A triangular prism
48	Matiu	a three d one? — showing confidence, the use of HRT is more like checking (Garry agrees.)
49	Garry	↑YES ITS A PYRA(H)mid (2.0)
50	Matiu	OR that
51	Garry/Tahi	((laughing))(3.0)
52	Matiu	↑then ↑whats a tri:angular prism (2.0) yayy
53		°simply a pyramid as well°
54	Ethan	°i dont know°

Authoritative genre. / Curious - ~~[illegible]~~ - doubt / Confirmation seeking genre.

Key Moment 4.3a

#	Speaker	Text
487	teacher	is there anything in there tha:ts different(1.0)
488		fro:m um: some of them other shapes that we've got
489		here (0.5) anything you can add
490	Zara	°no°
491	teacher	so: you basically got(1.0)↑diamonds squa:res
492		<have I got> diamond on there?
493	students	yes ((in chorus))
494	teacher	Yup diamonds squares circle:s rectan::gles (1.0)
495	teacher	um::↑three ↑d circle. oh (0.4)I hve just got
496		((interruption))
497	teacher	I have just got(.) um: a three: d: cir:cle=
498	Kayla	we had a (0.5) a three d? an a two d?
499	teacher	=three d circle(0.5)↑ can ↑anyone (0.5)um: think of
500		the geometry term for three: d: circle
		((students raised their hands up to answer))
501	teacher	um Alyssa
502	Alyssa	sphere?
503	teacher	sph↑ere good gir:l(0.4) so ↑sphe:re (1.0) is: the
504		geometry ter::m (.)↑for a three d. so um: a sphere
505		is the(0.6)fat (1.0) cir:cle (1.5) a circle is the
506		flat (0.5) circle ((Kayla raised her hand)) (2.0) yes
508	Kayla	in our we had a two d(1.0) and a (.) three d
509	teacher	you had a two d and a three d? in in yours too

pedagogical.

— trying to bring students' attention to this.

— showing

→ focussing on curricular-academic discourse.

use of HRT, Yet just to check if teacher is with us.

assessment of response,

assessment genre.

Key Moment 4.3b

#	Speaker	Text	
247	teacher	lets start with a spe:re(.) ((students raised their hands	Eurocentric-academic discourse.
248		to answer)) whats(.)the flat shape, thats 0.5)	
249		that(1.5)thats the other si:de of the.	
250		two d:s(.)so two d is your flat.(0.5)	
251		three d is your volume (0.8) um:: Olivia	
252	Olivia	°circle°	
252	teacher	↑yeah cir:cle(0.8)so the fla:t or the two: d::	
253		oops you alright? (0.5)the two d: to a:(1.0)	
254		sp: sp: (1.0) spe::↑re(.5)is a circle?(.) >a	
255		round circle on a piece of a paper<(1.5)but a	
256		SPEre(0.4)thats got VOL:ume(.)that's	
257		round(1.0) like this (4.0)is it ↑ still a	
258		↑cir:cle: ((the teacher brings a ball))	
259	Tahi	yeah	
260	students	yeah	
261	teacher	yea(0.5)its a ↑circu:lar sha:pe but(0.4) its	
262		not flat any↑more I cant (1.0) #squish it#	
263		like tha:t (0.5)in↑side all of there is	
264		volume(1.0)isnt it	
265	Alyssa	°its a sphere°	
266	teacher	↑yeah (.) its: become: (0.5)three d: (.)its	
267		go:ne ou:t from the wall (0.5)↑can you see it	
268	Kimi	(1.0)>so three d kinda goes out from the	
269		wall< its goes outwards. an:d backwards.	
270		(0.6)um: any questions to that so far	
271	teacher	what would be the sha:pe(1.5)in ↑the cyli:nder?	
272		(0.2)if you think of how a cylinde:r is ma:de	
273		(3.5) ((gestures to show the shape of cylinder))	
274		think about the cylinder (0.5) Zara	
275	Zara	is it a rectan:gle?	Confirmation-seeking genre
276	teacher	a rectan:gle (.) good gir:l (.6) Its a	Assessment genre.
277		[rec(1.0)	
278	Matiu	[°rec°	
279	teacher	rec(0.5) recta:ngle: an:d(1.5)what is at	asking question to support students to think - Pedagogical genre.
280		the ba:se? of a cylinder (1.0)or at the	
281		bottom of the cylinder [so]	
282	Zara	[oh]	
283	Teacher	its the rectangle that makes it (1.0) ((students	
284		raised their hands)) Ethan	
285	Ethan	a circle?	use of ~~[illegible]~~ HRT – Confirmation-seeking
286	teacher	Yeah (1.0)so can ↑you see how three um: (1.0)	
287		°o what it°(2.0)	
288	Alyssa	circle (3.0)	flat pitch - Confidence. ~~Authoritative/ genre~~ Declarative genre.
289	teacher	so our three d sha:pes have essentially got	
290		(0.5)	
291	Alyssa	two d	
292	Teacher	two d insi:de them(0.5)a pri:sm (1.5)((the teacher gestured	
293		to show the shape of a prism to students)) what shapes are inside	
294		a pri:sm(0.5)>its like< a buil:di:ng(1.0) big(0.5)	
295		ta:ll (.)buildi:ng. what(0.5)um: two d shapes are in	
296		there (2.5)=	
297	Zara	a bui[lding?	
298	teacher	=[if you think about a pri:sm (2.0) Garry	
299	Garry	↓square (1.2)	
300	teacher	Squa:re(1.0)at(2.0)bo:ttom(1.0) what about	pedagogic in nature.
301		arou[nd the side(0.5)=	
302	Zara	[outsi:de	

303	teacher	=a prism (1.5) think of a a big tall prism(1.0)
304		[pri:sn] (.) but a prism(0.5) ((students raised their hands))=
305	Zara	[OH]
306	teacher	=Ethan
307	Ethan	a triangle? *— HRT, however, it seems more like confirming- confirmation - seeking genre.*
308	teacher	triangle? (0.5) its like that? ((shows with her hands)) *— Assessment genre.*
309		(2.5)=
309	Elie	°oh [no outside°
310	teacher	[prism(.2) not a PYRA:MID (2.0)=
311	Matiu	ah ha (h)(h)(h)
312	teacher	= Pri[sm =
313	Elie	[°he doesnt understa:nd this°
314	teacher	=(3.5)um Matiu
315	Matiu	a recta:ngle
316	teacher	↓yes. (2.0)

Key Moment 4.4a

#	Speaker	Text
341	Teacher	>↑anyone else got some right< (.) um: Elie
342		with your sticks
343	Elie	um::: (0.4) I forgot what this shapes called
344	Teacher	very good? ↑so ↑how many (0.6) so (1.6) ↑so (.
345		um: [describe it]
346	Elie	°[its got] one two (2.0)° its: got one: two:
347		three four °five six° ↑its got six (0.2)
348		corne:rs
349	Teacher	got (.) six (1.5)
350	Elie	an::d its (.) go:t (1.5)
351	Teacher	Elie just hang on a minute (.) is it three d:
352		or two d: (1.0)
353	Elie	um:: I think its three d because °its not (.) a
354		two d° ((she was holding the shape and rolling it around her finger))
355	Teacher	put it down on a on the grou:nd (1.0) is it (.)
356		flat (.) or fat (0.5)
357	Elie	its fat (1.5)
358	Teacher	its fa:t (.) is it ↑coming ou:t towards you (1.0)
359	Elie	((looks at the shape holding it near the eye level))
360	Teacher	=okay lay it on the grou[nd (1.5)
361	Kimi	[°no its flat°
362	Teacher	its its okay. so: its not actually coming out
363		of the ground or going through the grou:nd (.)
364		so we call so we call (.) ↑we call that a two
365		d? (0.5) okay [so:(.2)
366	Elie	[↓uhm::
367	Teacher	↑its ↑got six co:rners (.) yeah
368	Elie	and its got (2.8) ((counted the number of sides)) and it
369		got six si:des
370	Teacher	six si:des good girl. I like you brought that
371		language (0.5)Okhay (0.3) ↑can ↑anyone help um
372		Elie (0.4) on what has ↑six si:des and ↑six
373		corners and is a and it is a two d shape
374		(1.0)um::: (1.3) Yue
375	Yue	a hexa:gon?
376	Teacher	ka pai so um you have actually made a hexa:gon
377		um: (0.6)
378	Elie	I know that thats called a hexagon ((hold and shows
379		the shape to the T))
380	Teacher	yeah a hexago:n has got six sides yeah (0.4)so°sort
381		of sort of a flat° (0.5) flat (2.0)um:

pedagogical genre.

Eurocentric-academic discourse.

flat pitch here — whispering here, probably partially confident

seeking confirmation.

flat pitch, confident, Authoritative genre.

persuasive genre

— Starting low pitch, use of stretching - loosing interest, probably showing giving up.

Euro-centric-academic discourse.

confident, diff prosody than English.

assessment genre

persuasion, persuasive genre.

Key Moment 4.4b

#	Speaker	Text	
			pedagogical.
585	teacher	how many di:mensions does this sha:pe have(2.6)=	
586	Zara	((puts her hand up for answering the question))	
587	teacher	=Zara	
588	Zara	ten?	– confirmation-seeking genre / or showing confidence
589	teacher	(1.5) no↑(1.0)Ethan	
590	Ethan	nine	flat pitch, confidence, declarative genre
591	Matiu	((laughing))	
592	teacher	[no(1.0)think about what I am asking. I am	assessment genre
593		saying how many di:mensio:ns (.)does this	→ trying to scaffold – pedagogical.
594		sta:r have(0.5)nikau,	
595	Nikau	°i dont know like°	
596	Elie	°o my god°	
597	Ethan	OH	
598	teacher	((circling the word 'dimension on task sheet))	
599	Ethan	TWO D::	High volume – Confidence, declarative genre.
600	teacher	=how many dimensions in the space does this	
601		star ha:ve (4.0)=	
602	Zara	°Matiu whats dimension°	– asking another student w/o disturbing class-interaction flow
603	teacher	=Elie ((as Elie raised her hand to answer))	
604	Elie	five	flat pitch confidence.
605	Olivia	a[y?	doubt-
606	teacher	[no(1.0)Alyssa	
607	Alyssa	ten	
608	teacher	no:(0.5)think about what the word we talking.	
609		with di:mension (1.5)=	
610	Zara	di:[mension	
611	teacher	=[do we go upto ten dimensions(1.0)	
613		WE WOULD be on the movies if we went into ten	
614		di:mensio:ns(1.0)would be on the sci fi	
615		movie(0.5) Matiu	
617	Matiu	two	declarative
618	teacher	↑thank ↑you:[very much ((exhalation of breath, closed her eyes and slightly tilted her head back))	→ assessment genre.
619	Ethan	[I SAID THAT	

Key Moment 4.5a

#	Speaker	Text
557	teacher	↑what abo:ut um:::(1.0) whats a
558		tapa:ono(6.0)=
559	Zara	hhhhhhh (aspiration)
560	teacher	=umm ↑Kayla(.4) what wud a tapaono be (8.0)
561		↑tapari:ma pentagon(.)tapaono: Yue↑
562	Elie	((holding the hexgonal skelton that she made using sticks to show its a
563		hexagon))
564	Yue	hexa↑go:n
565	teacher	good girl. hexagon(4.5)
566	Elie	°I told you°
567	teacher	um:↑(h)(1.2) heres my question.(1.0)
568		could ↑I ha:ve(2) a tapatahi:(1.5)
569		((Elie, Yue, Matiu, Ethan said no in chorus))
570	Zara	YES a cir ((teacher smiled as Zara responded))
571	Matiu	a circle has no ↓side
572	Elie	[no >↑you cant ↑you cant< °you cant°
573	Ethan	[CIR↑CLE::
574	Matiu	[°no a circle has no side°
575	Elie	[coz ((put her hand up for answering))
576	teacher	can ↑I have a tapatahi.(.4)can I.(1.0)↑what
577		↑what woud tap ↑if I followed that pattern what
578		would tapatahi be Elie
579	Elie	a circle. but ↑you ca:nt have it bcoz circles
580		have no si:des [and] no corners
581	teacher	[hmm] ↑you are ↑brilliant
582	teacher	accepted.
583	teacher	so ↑what do we ca:ll a circle
584	Tane	a ha::o cylinder. (h)
585	teacher	<what do we call a cir:cle> when ↑I say (.)
586		can you get into a umdumdumdumd.↑
587	Ethan	↑PORA°whita↑
588	teacher	[umdumdumdumd(h)(h)(h)
589	teacher	>what is it<(h)(h)
590	Ethan	porowhita
591	teacher	ya↑ya↑ porowhita you get all the time (.4)
592		so a poro:whi:ta is a cir:cle (2.2)

Eurocentric-academic discourse. / declaration / authoritative / with confidence / assessment genre / confident, declarative, Eurocentric-academic discourse. / High volume, confidence / future orientation / Eurocentric-academic discourse. / with supporting statement ∴ Argumentative genre. / past / -assessment / Te Reo Māori discourse / Māori-mathematical discourse.

Key Moment 4.5b

#	Speaker	Text
54	Zara	whats rectan:gle in ma:ori (0.5) tapawha: I
55		think thats four (2.5) whaea Kerry:?(0.5)if
56		square is tapawha: wha:ts rectan:gle?
57	teacher	rectan:gle::s um:::(2.0)I think
58		[honestly] I am not su:re(1.0)
59	Zara	[is it]
60	teacher	so just just leave leave it (2.0)↑good question
61	Olivia	tapari:ma is (.) actually (2.5) is actually a
62		↓a pentagon °i thought so°
63	Teacher	No. i dont think it is (3.0)
64	Zara	I am gonna find the actual name for it
65	Teacher	°can you find it out for me° yeah I don't think
66		it is I know definitely tapa:wha: is an:d I
67		know definitely porowhita is >but I only threw
68		the ↑other ones in< (1.0) yeah I put the other
69		ones in because it help us remember what they
70		are (.) okay

Eurocentric-academic discourse. / Indicating of future moment / — future reference.

Appendix N. Phases of Analysis and Key Findings of the Study

Phases of Analysis	Key findings
Thematic analysis and Micro-level analysis	**Finding 1.** Children discursively used "sides" to represent line segments of 2D shapes and faces and edges of 3D shapes. **Finding 2.** Children used different discursive constructions to represent their understanding of the mathematical construct of dimension as "flat vs fat, or "ways to go" or "another world". **Finding 3.** Children used discursive constructions of "3D rectangle" to talk about cuboid or rectangular prism. Similar discursive constructions, for example, "3D circle" was used to represent a sphere. **Finding 4.** Children discursively used Te Reo Māori number names with the prefix "tapa" to represent 2D shapes in Te Reo Māori. **Finding 5.** Children's use of prosody plays an interactive role in the meaning-making process. **Finding 6.** Multilingual children used prosodic features of their multiple languages while interacting with others to construct their understanding of shapes and their properties. **Finding 7.** Children used two kinds of gestures during classroom interactions to represent their understanding of shapes and their properties. **Finding 8.** Children and the teacher considered responses given in geometry-specific language as preferred responses. **Finding 9.** The teacher's overt negative evaluation of children' incorrect responses are considered dispreferred responses. **Finding 10.** Children and the teacher considered children's response as dispreferred if they provided a response without being given the turn to speak.
Macro-level analysis	**Finding 11.** Children displayed different emotional stances through their prosody embedded in their utterances within the same Key Moment. **Finding 12.** A variety of speech genres is available in a classroom. **Finding 13.** Children may use any of the speech genres available to them (including teacher's speech genres) in the classroom dialogic space to negotiate the meanings of shapes and their properties. **Finding 14.** The negotiation of meanings about shapes and their properties is influenced by the interplay of unitary language and heteroglossia *between* two discourses **Finding 15.** The negotiation of meanings about shapes and their properties is influenced by the interplay of unitary language and heteroglossia *within* the same discourse. **Finding 16.** The Chronotopic Moment embedded in children's utterances may act as the link between different time zones of learning and thus, plays an important role in the negotiation of meanings during classroom interactions.