Developing a new numerical surface/subsurface model for irrigation and drainage system design

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Abstract In recent decades many hydrologists have focused on surface/subsurface interactions and a number of numerical and physical models have been developed for simulating the interplay of flows between the saturated and unsaturated zones. This paper presents a numerical surface/subsurface flow model based on 2-D shallow water and 2-D Manning equations for surface flow, with 3-D Richards equations for subsurface flow in the unsaturated and saturated zones. The aim here is to provide a tool for designing more robust agricultural irrigation and drainage schemes. The model can be used in place of the classical design methods based on 1-D equations. The proposed model yields information about the water table location, depth of surface water, and water content in the unsaturated soil in any field location before and after spreading irrigation and drainage. The additional information has the possibility of decreasing expense and increasing the safety factor for both irrigation and drainage projects.

Key words surface/subsurface interactions; numerical model; irrigation; drainage; Richards equations; Saint Venant equations

INTRODUCTION

To approach an ideal design for irrigation and drainage systems, the water content in the surface and subsurface must be determined. Generally hydrologists and hydrogeologists study surface and subsurface water separately, and then estimate leakage from surface water in designing irrigation systems, or seepage from the subsurface top layer when designing drainage systems. However, by solving both surface and subsurface equations simultaneously this separate estimation step can be eliminated.

There has been recent focus on developing multi-use numerical and physical surface/sub-surface models. Many hydrologists have been concerned with surface/subsurface interactions and a number of numerical and physical models have been developed for simulating the interaction between surface flow and flows in the saturated/unsaturated zones (Weill *et al*., 2009).

Often 1-D analytical equations are used for designing drainage systems such as those developed by Hooghoudt (1940), Kirkham (1984), Hammad (1962), and Dagan (1964), or the 1-D Manning and Hazen-Williams equations used for designing irrigation systems. Drainage equations require two main parameters: maximum water table depth and drainage pipe diameter. Hooghout (1940) used the Dupuit-Forchheimer assumptions and differential subsurface water flow equations to develop his well-known 1-D expression for designing the distance between drains in homogenous flat soils:

$$H = \frac{RL}{K} F_H \tag{1}$$

$$F_H = \left[\frac{(L - d\sqrt{2})^2}{8dL} + \frac{1}{\pi} \ln \frac{1/2\sqrt{2}d}{r} \right] \tag{2}$$

where L is the distance between two drains [L], K is the hydraulic conductivity [LT^{-1}], R is the infiltration from the top layer [LT^{-1}], H is the maximum water table level [L], d is the drain elevation [L], and r is the drain radius [L].

Analytical methods of this type use simplifying assumptions for solving the main PDEs, possibly reducing design accuracy. In addition, the resulting equations cannot give a clear description of the surface/subsurface water flow before and after spreading irrigation or drainage systems. This paper introduces a new method for designing drainage systems by solving surface/subsurface equations with the use of numerical solutions. The method not only provides higher accuracy compared to the classic analytical methods because of reduced simplification assumptions, but also predicts the water table for every spatial point. This could help agricultural managers, for example, to cultivate a variety of different plants in a given field. In addition, the method could be used to evaluate an existing irrigation drainage system.

GOVERNING EQUATIONS

In this paper, surface water flow equations are used for water overland flow and for water movement in the irrigation and drainage canals or pipes. Subsurface flow equations are utilised for water moving in the saturated and unsaturated zones.

Subsurface flow equation

Water flow in the saturated/unsaturated zone is taken to follow Darcy's law, which was originally derived for saturated material. However, Richards (1931) and Childs & Collins-George (1950) have shown that by considering hydraulic conductivity to be dependent on water pressure, Darcy flow can be modified to unsaturated flow, with governing equation (Fredlund & Rahardjo, 1993):

$$\frac{\partial}{\partial x}(K(h)\frac{\partial(h+z)}{\partial x})+\frac{\partial}{\partial y}(K(h)\frac{\partial(h+z)}{\partial y})+\frac{\partial}{\partial z}(K(h)\frac{\partial(h+z)}{\partial z})=S(h)\frac{\partial(h+z)}{\partial t}-q \tag{3}$$

$$S(h)=\gamma_0 m_w$$

where h is the pressure head [L], z is the elevation [L], $K(h)$ is the hydraulic conductivity [LT^{-1}], q represents a sink and/or source of water [T^{-1}] , $S(h)$ is the specific volumetric storativity [L^{-1}], γ_0 is unit weight of water [$ML^{-2}T^{-2}$], and m_w is the slope of the storage curve.

Surface water flow equations

Two-dimensional Saint Venant equations comprise two momentum balance equations in the x and y directions, and one mass balance equation. The mass balance equation is:

$$\frac{\partial(h_s u_x)}{\partial x}+\frac{\partial(h_s u_y)}{\partial y}=\frac{\partial(h_s)}{\partial t}-q_s \tag{4}$$

where h_s is the water depth [L] and u_x and u_y are the depth-averaged flow velocity in the x and y directions [LT^{-1}], q_s is the total sink and/or source of water [LT^{-1}]. The Saint Venant equations are valid only for shallow water and gentle slope. Water depth h_s should be much smaller than wave length or the characteristic length of the water body L, that is, $h \ll L$. Generally, it is required that $h/L < 10^{-4}$ (Weiyan, 1992).

For determination of velocity in the present study, instead of using the complicated momentum balance equations, the Manning-Strickler law is used, which has been widely applied for surface water flow (Weillet *et al.*, 2009):

$$u_x=\frac{1}{n}h_s^{\frac{2}{3}}(\frac{\partial(h_s+z_l)}{\partial x})^{\frac{1}{2}} \tag{5}$$

$$u_y=\frac{1}{n}h_s^{\frac{2}{3}}(\frac{\partial(h_s+z_l)}{\partial y})^{\frac{1}{2}} \tag{6}$$

where n is the manning coefficient [$L^{1/3}T$], z_l is local ground surface elevation and $\partial(h_s+z_l)/\partial x$ and $\partial(h_s+z_l)/\partial y$ are the friction slope in the x and y directions, respectively. By assuming that the water depth gradient ($\partial(h_s)/\partial x$ or $\partial(h_s)/\partial y$) are much smaller than the surface elevation gradient ($\partial(z_l)/\partial x$ or $\partial(z_l)/\partial y$, respectively), the mass balance equation can be written as:

$$\frac{\partial}{\partial x}(\frac{h_s^{\frac{5}{3}}}{n\sqrt{S_x}}\frac{\partial(h_s+z_l)}{\partial x})+\frac{\partial}{\partial y}(\frac{h_s^{\frac{5}{3}}}{n\sqrt{S_y}}\frac{\partial(h_s+z_l)}{\partial y})=\frac{\partial(h_s)}{\partial t}-q_s \tag{7}$$

where S_x and S_y are the slopes of the local ground surface in the x and y directions. This equation has been widely used for describing water surface flow (Gottardi & Venutelli, 1993; Di Giammarco *et al*., 1996; Panday & Huyakorn 2004; Weillet *et al*., 2009).

METHOD OF DRAINAGE SYSTEM DESIGN

By increasing the distance between two drains, the water table level moves toward the initial state prior to drainage. By decreasing this distance the water table tends toward the elevation of the drains. For designing drainage and irrigation systems the model requires the minimum unsaturated soil depth for the root zone (*f*) which relates to the type of crop.

Designing the distance between consecutive drains requires initial specification of pipe diameter and length. The distance between two drains is first given an initial estimate and the model run for specified soil hydrodynamic parameters and fields boundary conditions which include irrigation systems. The result is then compared with root depth. Finally, the optimal design is found by changing irrigation conditions and estimating the parameters. With this method the water table level is determined after spreading drainage and irrigation systems through all parts of the field.

VERIFICATION

Numerical models should always be verified against standards before use. For verification of the model component using surface water equation for overland flow, a famous analytical V-catchment problem is used. Then the model is verified for coupled surface/subsurface conditions as well.

V-catchment problem

The V-catchment problem was first published by Di Giammarco (1996). Today it is known as a standard verification test for overland flow. This test has been used by a number of workers in the field including Vanderkwaak, (1999); Panday (2004); Kollet & Maxwell (2006); Weill *et al*. (2009); Mauro *et al*. (2010); Shen *et al*. (2010).

The V-catchment concept is a simplified catchment consisting of two $1000\ \text{m} \times 800\ \text{m}$ inclined planes that join together in the middle by a $1000\ \text{m} \times 20\ \text{m}$ channel (Fig. 1(a)). Surface slopes are 0.02 for parallel to channel direction and 0.05 for perpendicular to the channel. The Manning parameters are 0.015 $\text{sm}^{-1/3}$ for the planes and 0.15 $\text{sm}^{-1/3}$ for the channel. As the channel is symmetrical, just half of the domain needs be modelled. The plane channel is considered dry as the initial condition. A rainfall for 90 minutes with 10.8 mm h^{-1} intensity was used for this verification and the total simulation period was 180 minutes.

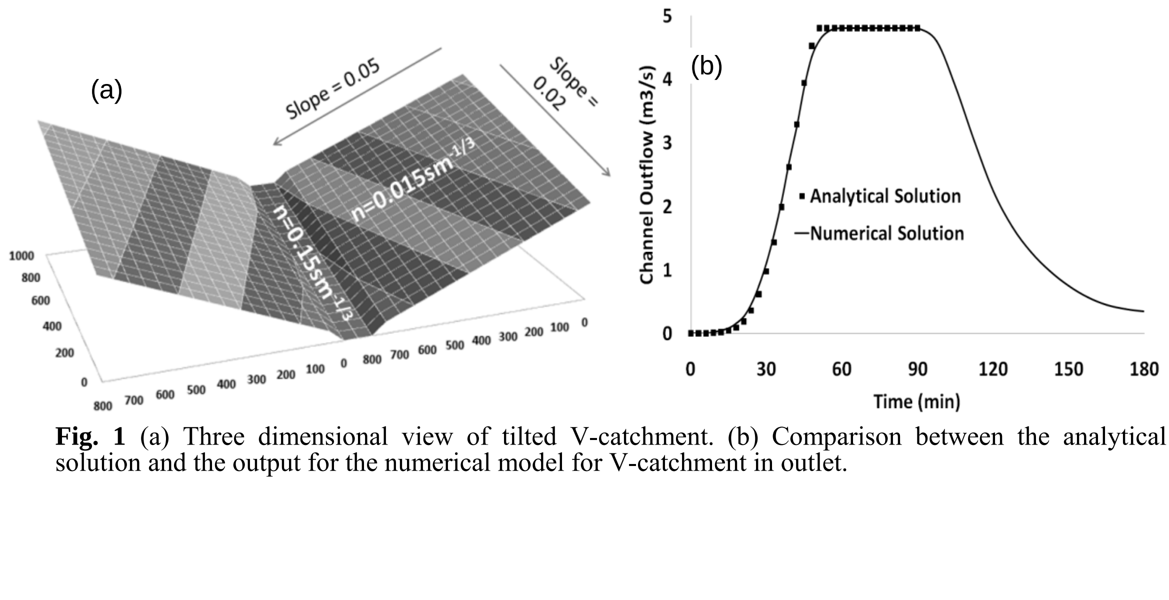

Fig. 1 (a) Three dimensional view of tilted V-catchment. (b) Comparison between the analytical solution and the output for the numerical model for V-catchment in outlet.

Figure 1(b) compares the channel outflow with analytical and numerical solution in the channel outlet. The result shows the analytical and numerical solutions match very well during the rising period. There is just a small oscillation around 30 min before the hydrograph rises to its maximum.

A sample of drainage / irrigation design

In this section, to check the surface/subsurface model component, drainage and irrigation systems for a 1000 m × 800 m area were designed (Fig. 2(a)). The land is taken as flat and surrounded by a drainage channel. For simplification in this stage, the surface/subsurface model water movement in the drainage channels was not modelled with the surface flow sub-model. Only water flow in the channel was considered as a constant head boundary condition for the subsurface equation.

The soil is defined as homogenous and the curve of the hydraulic conductivity *vs* pressure is specified as in Fig. 2(b). This simulation is designed on steady state conditions so volumetric water content conditions or volumetric storativity curve *vs* pressure is not required. Rainfall rate was defined to be 800 mm per year.

Due to technical limitations for drain installation, depth of drains and minimum unsaturated soil depths for root zones 2 m and 75 cm were considered, respectively. The method of irrigation is drop irrigation. The drippers are located at a distance of 2 m from each other and the discharge rate for each one is 7L/h.

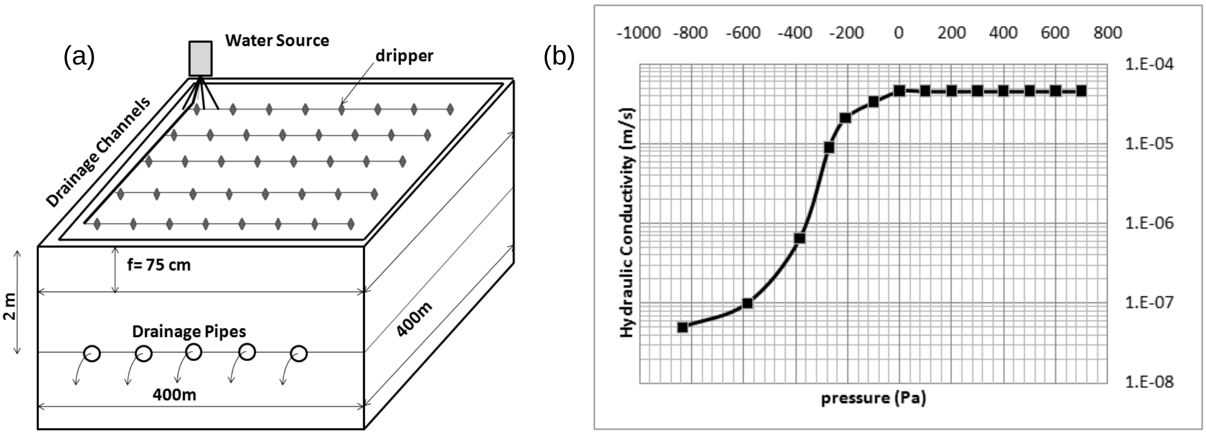

Fig. 2 (a) Three-dimensional schematic view of farm; (b) hydraulic conductivity via pressure.

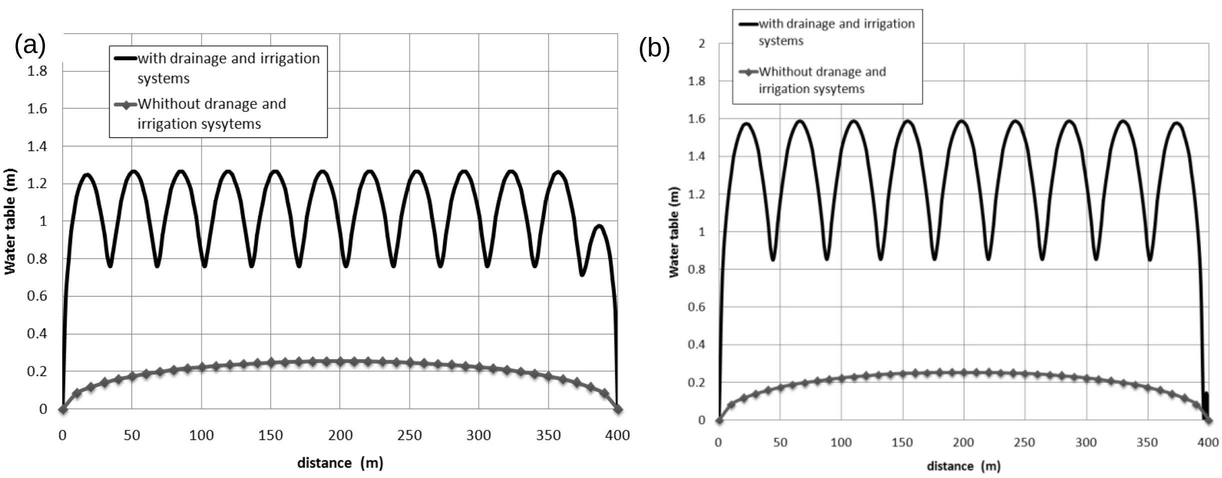

Fig 3 Simulated water tables without any irrigation and drainage systems and with irrigation and drainage systems: (a) design based on surface/subsurface model, (b) design based on Hooghoudt analytical equation.

The result gives 34 m as the minimum distance between drains which can satisfy f = 75 cm for all parts. Figure 3(a) shows the simulated water table in a section of the farm before and after spreading drainage and irrigation systems. With the same data, the analytical Hooghoudt equation gave 44 m for the distance between drains. The surface/subsurface model was run with the Hooghoudt result and this result shows, in many parts of the farm, the f factor could not be satisfied (Fig. 3(b)).

CONCLUSION

Due to their necessary simplifying assumptions, analytical models often cannot find the best design for drainage and irrigation systems. In addition, these methods do not account for diverse boundary conditions so they cannot accommodate the effects of different boundary conditions. On the other hand, the Surface/Subsurface model, outlined here, gives a full spatial view of the water table in farms and also shows the effect of possibly complicated boundary conditions.

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