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# Waikato Journal of Education

Te Hautaka Mātauranga o Waikato

Volume 19, Issue 2: 2014

## Special Edition:
## Emergent learning and threshold concepts in tertiary education

# Waikato Journal of Education
## Te Hautaka Mātauranga o Waikato



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# Waikato Journal of Education

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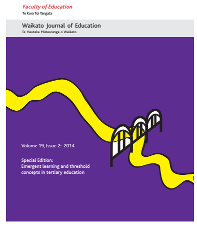

# Student and expert perceptions of the role of mathematics within physics

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## Abstract

*Students' perceptions of the role of mathematics within physics were examined. I propose that the identification of physics as a science based ultimately on experiment is a threshold concept: transformation from a naïve view that physics is based upon mathematics to an expert view that physics is based on experiment is difficult for students. Seven students taking first-year university physics were interviewed in two focus groups; nine practising physicists from academia and industry (considered as experts) were interviewed as six individuals plus one focus group of three participants. Of particular interest was the 'expert' view emphasizing the conceptual nature of physics. This was a threshold in understanding that had not been crossed by students. Rather, students viewed mathematics and physics as being more strongly connected than did practising physicists; specifically that "maths explains physics". Experts consider this view as holding back a student's understanding of the subject and preventing them from becoming effective physicists. It is troublesome to students because they are less able to identify the relevant concepts before trying to tackle a problem with mathematics, making their approach less likely to be effective, however, both groups (physicists and students) identified physics as belonging to 'the real world' and that mathematics shows how physical entities can be combined or related, indicating student responses are not completely naïve. Opinions on how best to teach mathematical concepts in physics varied considerably across participants.*

## Keywords

Physics, mathematics, science, tertiary education, threshold concepts

## Introduction

While it is possible to carry out a physics experiment without considering mathematics (for example, investigating the density of an object relative to that of water by observing whether it floats or sinks), it rapidly becomes clear to a student of physics that mathematics is an essential skill to have. Mathematics is somehow intertwined with physics. Physicist Paul Dirac summarized the relationship: "It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of a mathematical theory of great beauty and power" (Dirac, 1963). Physical laws can be embodied in mathematical form through equations, with remarkable conciseness and clarity.

Yet this relationship is not always useful for one's understanding of physics. A well-used example comes from Mazur (1997). A class of first year students was presented with a simple, conceptual problem regarding electrical circuits and current flow. While practising physicists would identify the

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problem as trivial, it caused great difficulty for the students, who demonstrated naïve assumptions about the nature of current flow. However, when the same problem was rephrased in terms of quantitative values of electrical resistances, and students were asked to calculate currents and voltages over components, it proved to be straightforward. In other words, the students' ability to perform mathematical calculations had masked their lack of understanding of the underlying concepts.

Students at tertiary level can often identify 'doing physics' as 'applying mathematics'. Tumarino and Redish (2007), based on observations of students solving physics problems, identified six 'epistemic games' that students use as problem-solving strategies. By 'epistemic game', they mean an approach that draws from particular processes and knowledge in a manner to construct an answer to a problem or to work out something new. Two of these games were identified by expert physicists as unhelpful to student learning—the 'recursive plug-and-chug' method and the 'transliteration to mathematics' method. Both relied on trying to phrase the problem in terms of mathematics while ignoring the underlying physical principles. The former involved placing quantities into equations and evaluating the results, without drawing on any conceptual awareness, in terms of physics, of what they were doing. The latter involved the copying of another problem into a new situation and ensuring the mathematical steps were the same; no thought was given to the purpose of these steps. Conversely, an example of a valuable approach, or game, would be 'Mapping meaning to mathematics'. In this game the students first start with an understanding of the physical situation, and then write this in terms of mathematics, using the mathematics as a language to describe what they understand.

Indeed, the much-used formative assessment tool The Force Concept Inventory (FCI, Halloun & Hestenes, 1985; Halloun, Hake, Mosca, & Hestenes, 1995; Hestenes, Wells, & Swackhammer, 1992), is deliberately focused on physics concepts (in this case Newton's Laws) as opposed to mathematical calculations. Mathematics is almost completely absent. It is designed to flush out students' deep thinking regarding Newton's Laws of Motion. While students can readily state what these laws are, and apply them in quantitative calculations, their application of them often betrays the fact that they have not understood them beyond a superficial level. Hake (1998) in a huge survey of 62 first-year physics courses in the United States found that traditional teaching resulted in little gain in students' understanding as measured by the FCI, suggesting that physical understanding was being obscured by other aspects of teaching.

Developing an expert-like view of physics is difficult for students, but can be achieved. Gire, Jones and Price (2009) found that tertiary students' views about physics in general do not become very expert-like until a fourth year of study. Elby (2001) showed that, while conceptual-based approaches often do not lead to changes in epistemology, it is possible for students' epistemological beliefs to be changed with appropriate instruction. The personal experience of the author in teaching physics across all undergraduate years is that students are held back from understanding the physical principles of a problem by a focus that is diverted towards the mathematics. Quale (2011) summarizes the issue as students holding the incorrect view that "valid mathematical reasoning generally leads to valid physics" (p. 368). He further suggests, without substantial evidence, that students acquire such a view from their teachers. If the researcher is correct, the implications for the teaching of physics are significant.

At a local level, the author's experience with teaching physics to physics and engineering students is that students often take two, equally unhelpful views of mathematics within physics. Students who are not strong in mathematics take the first view; they can assume that their lack of skill in mathematics will necessarily prevent them from understanding physics, and therefore they lack motivation in their studies. Some students who are strong in mathematics hold the second view; they can take the approach that their mathematics will carry them through the physics—that the physics doesn't add anything substantively new to what they know already. This article argues that both these views can hold students back from learning physics.

This article therefore asks the question: In what way do student views of the relationship between physics and mathematics align with those of physics experts? One could postulate that identifying physics as a science is a threshold concept (Meyer & Land, 2003). A threshold concept, as proposed by Meyer and Land (2003) has five distinctive features: It is *transformative*—once learned the student views the subject in a different way; it is likely to be *irreversible*—such a change of viewpoint is not lost; it is *integrative*—once grasped it shows how other things can be related; it may be *bounded*—there are inherent limits on its applicability; and it is *troublesome*—failing to grasp the concept causes difficulty for students but they are unlikely to see why. These concepts are hard; they are often built on hidden, underlying knowledge and deep-seated assumptions that are not evident to the student (Meyer & Land, 2006; Perkins, 2006). Educators need to think what is hidden and work at revealing it.

While most people would have little trouble in agreeing to the statement "physics is a science" the evidence above suggests at a deep level it is indeed troublesome for them—they would prefer to identify physics as applied mathematics. In other words, they hold to the fact that physics somehow arises from the application of mathematics, as opposed to being based on experiment. It is troublesome too to the educator because students are able to work through physics without understanding it. This article examines expert and student views in more detail and discusses them in terms of a threshold concept framework.

## Methodology

Ethical approval was obtained from the University of Waikato Faculty of Education Human Ethics Committee.

This qualitative study was concerned with how students and practising physicists view the connections between physics and mathematics. Since the study was primarily concerned with their experiences, thoughts and interpretations, an interpretive paradigm was adopted (Hennink, Hutter, & Bailey, 2011; Silverman, 2010). A descriptive methodology was employed, choosing interviews with open questions and focus groups in order to collect data from two different groups of participants.

The first group contained practising physicists. These are people who use physics as part of their professional work. They were recruited from the author's professional networks through personal approach. The group consisted of nine participants—three tenured university academics, one post-doctoral researcher and five participants taken from three different large employers in New Zealand (one of which was also undertaking a PhD while in industry). The ages of the participants in this group ranged from 20 to 60. All participants were male, reflecting the large gender bias in the physics community within New Zealand.

The second group consisted of first year students at the University of Waikato. Participants were recruited initially through an announcement on a web-based discussion page associated with a first year physics paper 'Physics for Scientists and Engineers 1' at the University of Waikato, and, when this was unsuccessful, a personal approach was made during laboratory classes. Seven students (five male, two female) agreed to take part out of a total of around 130 students within the first year physics paper. Five of the students indicated that they were studying for a Bachelor of Engineering degree; two that they were studying for a Bachelor of Science degree, majoring in physics. All participants in this group were under 25.

Questions were designed to draw out each participant's ontological and epistemological views of the way mathematics works within physics. The same set of questions was asked of each participant from both groups, but not always in the same order; also participants had an opportunity to respond freely with their thoughts on the role of mathematics within physics. The set of questions is shown in Appendix A. The participating physicists were interviewed mostly individually, but with one focus group of three participants from the same employer. The students were interviewed in two focus groups of three and four participants. The interviews were recorded and transcribed. Interviews of the practising physicists took place in early 2012; those of the students in the second half of 2012.

The transcripts were coded for themes on an inductive basis (Hennink et al., 2011; La Pelle, 2004). The key, recurrent themes were then extracted. Themes were compared and contrasted between the two sets.

## Results

### Practising physicists

Several strong themes were identified by the practising physicist group. Themes that were identified by four or more members of this group are summarized below.

#### *Physics applies to the real world*

Physicist participant one illustrated the link between physics and the real world by drawing on a short conversation:

> A physicist's view of the world is somewhat odd, somewhat different. A friend of mine [was] talking to me about what it means to be a physicist… 'I think you view the world differently—so when you see a car coming down the road you see a bunch of forces and torques and momenta, whereas I just see a car.' (Physicist participant 1).

This group strongly identified that the subject of physics was intrinsically linked to the world and universe in which we live. They viewed the subject as highly applied and useful. Physics is descriptive of the physical processes that occur in our world; physicists are, overall, realists (quantum mechanics notwithstanding).

#### *Physics is conceptual*

The strongest single theme was that physics is conceptual. When tackling a problem, the first and most important step is to identify the relevant physical concepts. "The ability to discern what is important from the less important is a paramount skill" (physicist participant 8); "[Physics is] a focusing on the important aspects and what are the parts that can be ignored" (physicist participant 1). In application of physics, any use of equations and mathematical calculations follow after the recognition of the underlying concepts. Without the identification of what is important in a particular situation, further analysis becomes pointless.

#### *Mathematics is useful*

Mathematics was viewed primarily as one of many tools for undertaking physics. Specifically, it shows how to relate different physical quantities together, through equations. The power of mathematics for physics comes from it being precise (unambiguous) and predictive. By analysing equations, it is possible to predict phenomena that have not been recognized—this provides a powerful tool for testing application of theories to practice. While it is clear to the practising physicists that certain aspects of physics could be tackled without use of mathematics, mathematics is an essential tool for someone who makes a career from physics. However, when mathematics is applied, it must always be linked back to the underlying physics: "Any mathematics you write down has to be securely anchored to the real world" (physicist participant 1).

#### *Practising physicists enjoy physics*

Finally, a strong theme was that practising physicists enjoyed working with physics. "Physics seemed unlimited" (physicist participant 7); "I've always liked to understand exactly what's happening" (physicist participant 3). While the result, in itself, is unsurprising, that it came across strongly through discussion of mathematics within physics was unexpected.

### Students

The following themes were strongly apparent in the student focus groups.

### *Physics applies to the real world*

Along with the practising physicists, students also strongly identified with the applicability of physics to the universe in which we live. Examples include: “A physics equation is a representation of real life” (student participant 4); “When a physicist writes an equation, it’s about finding something tangible” (student participant 3); “In the physics side they would … see what meaning it would have in nature” (student participant 6). One could also reasonably say that the student groups were, overall, realists in their ontology.

### *Physics demonstrates application of mathematics*

The student group also strongly viewed physics as an application of mathematics. In other words, mathematical reasoning can be applied in order to produce physics. Tackling problems in physics required the application of mathematical equations. One student emphasized the incompleteness and lack of purpose of mathematics without physics: “Math isn’t really useful until you do something with it that’s not on paper” (student participant 4). There was some student discussion on whether mathematics had a purpose without physics; for example, was it possible to have an application of mathematics which was unconnected to the physical world?

### *Maths explains physics*

Moreover, physics was seen as being *explained* (as opposed to *described*) by mathematics. That is, why certain physical relationships are the way they are is a result of the underlying mathematics: “Maths is used to explain physics in my eyes” (student participant 5); “Maths is simply physical things explained” (student participant 2). Students presented views in line with those described by Quale (2011): “Valid mathematical reasoning generally leads to valid physics” (p. 368). Students also saw mathematics as being an essential language of physics.

### *Mathematics and physics complement each other*

Students strongly identified with the complementarity of mathematics and physics. They considered the two subjects as having a large degree of overlap, and identified that each subject helped support the other. They emphasized that in the teaching of both subjects, the other should not be neglected: “When one thing is seriously linked to another thing, they need to be taught together” (student participant 4). Mathematics was considered as essential for the understanding of physics; it was not possible to understand physics without it.

While it is important to note that some student comments hinted at the view taken by practising physicists (Figure 1), the majority of comments supported an alternative viewpoint. Some students demonstrated multiple views through their comments. For example, student participant 2 commented “You do really have to understand the [physics] theory to read the math though, as you said before, because if you don’t really understand the theory then you will get wrong results” but also commented later in the focus group “Mathematics is simply physical things explained … if you can do something in math, you can add units to the numbers and do it in physics.” The first quotation shows an expert-like view; however, the second quotation suggests more naïve thinking.

One out of the seven students had views that were consistently expert-like; for example:

> I’ve got a little brother who’s just in high school, and I sometimes work with him with physics, and like he writes down formulas like F is equal to m a, but he doesn’t understand what it means, so they [teachers] just give formulas that they [students] actually use, but they don’t understand what they mean …” (Student participant 6)

## The pedagogy of physics teaching

The questions that were asked of the groups (see Appendix 1) were not designed to identify viewpoints on the teaching of mathematical concepts within physics, i.e. the pedagogy of physics teaching. However, all students and all-but-one of the practising physicists volunteered comments in this regard. This suggests that they viewed pedagogy as important. The ready expression of such views may have been a consequence of the university environment in which most of the interviews took place.

Students were consistent in their views, specifically that the two areas complement each other and need to be taught together, with reference to each other. Students in focus group 2 (participants 5–7) expressed frustration when, in their physics papers, mathematics is 'glossed over'. "Some of the formulas we've used for a long time … but we didn't actually get to prove them … I think it's good that you actually know where they are from" (student participant 6). The teaching of one subject (mathematics or physics) helps the learning of the other. One of the practising physicists presented similar thoughts, commenting, "I took in so much more maths during my physics classes than in my maths classes" (physicist participant 5). However, the pedagogies presented by individuals within the practising physicist group were disparate. There was no consistent view expressed. Pedagogies ranged from "let the maths do the work" (physicist participant 2) through to ignoring the mathematics until the physics has been established and presented. This range in pedagogies was surprising, especially given the consistency with which practising physicists identified with the conceptual nature of physics, and is worth further investigation in regard to how best to teach mathematically intensive material within physics. However, it is beyond the realm of this particular article and I do not discuss it further.

## Discussion

### The two groups see a differing role for mathematics within physics

Practising physicists presented viewpoints that showed physics as the central aspect with mathematics as a way of probing or describing the physics. Physicists talked about the enjoyment that comes from doing physics and the broad scope of its application. The connection to 'the real world', that is, to something that can be measured, sensed or experienced, was made strongly. A strong positivist nature was apparent in the group's ontology and epistemology. The nature of 'doing physics', to a physicist, was primarily about identifying the physical principles underlying a problem: Mathematics was seen as a powerful, descriptive tool for assisting the doing of physics. Specifically, mathematics was viewed as a way of relating and combining physical concepts, through equations. While essential to have, the mathematics was always seen as secondary to the physics, when tackling a problem in physics. The strong identification with the conceptual nature of science by practising scientists has already been well established (Duggan & Gott, 2002) and therefore this finding is unsurprising.

The broad relationship between physics and mathematics, as presented by practising physicists, can be summarized through Figure 1. Physics concepts, labelled P1, P2, P3, and so on, are used to describe phenomena (R1, R2, R3 and so on) in the world and universe in which we live. The mapping is not one-to-one; several concepts may need to be brought together in order to describe a particular phenomenon, and one concept could explain many phenomena. The role of the mathematics is to assist the description of the phenomena, not to provide an explanation of the phenomena. Several mathematics concepts (labelled M1, M2, M3 and so on) may be used to help with a particular description.

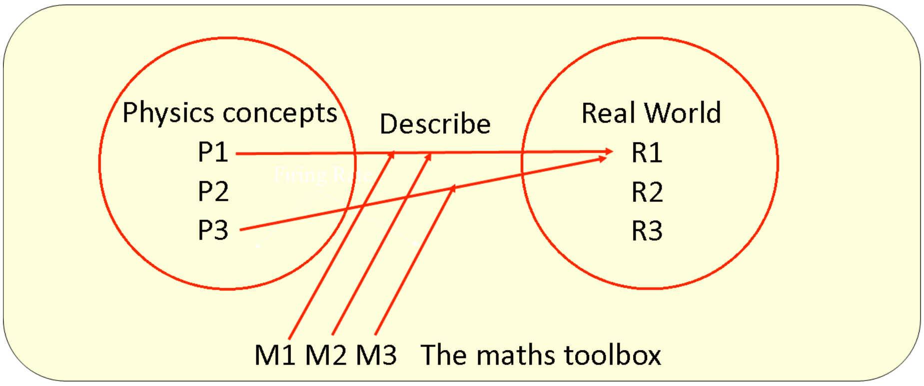


**Figure 1. The relationship between physics and mathematics as experienced by practising physicists**

Students also clearly identified with physics as belonging to the ‘real world’. A realist paradigm was clearly evident. However, in regard to the nature of mathematics within physics, the two groups differed. In contrast to the physicists, students saw mathematics as going beyond being a language for physics, to being essential to understanding physics. They viewed physics as being an application of mathematics, i.e., mathematics somehow implies physics (Quale, 2011), and mathematics being incomplete without physics: “Maths isn’t really useful until you do something with it that’s not on paper” (student participant 4). I note at this point all the students interviewed were studying for either a Bachelor of Engineering or Bachelor of Science degree, none were studying towards the Bachelor of Computing and Mathematical Sciences (BCMS) degree offered at the University of Waikato. It is plausible that views from students enrolled in the BCMS might be different.

The broad relationship between mathematics and physics, as presented by the student focus groups, can be summarized by Figure 2. Here, the mathematics concepts M1, M2, M3 and so on are brought together to *explain* physics concepts P1, P2, P3. The mapping is not one-to-one. Then the physics concepts can be used to *describe* the phenomena which are experienced in the world, labelled R1, R2, R3 and so on. Again, the mapping is not one-to-one.

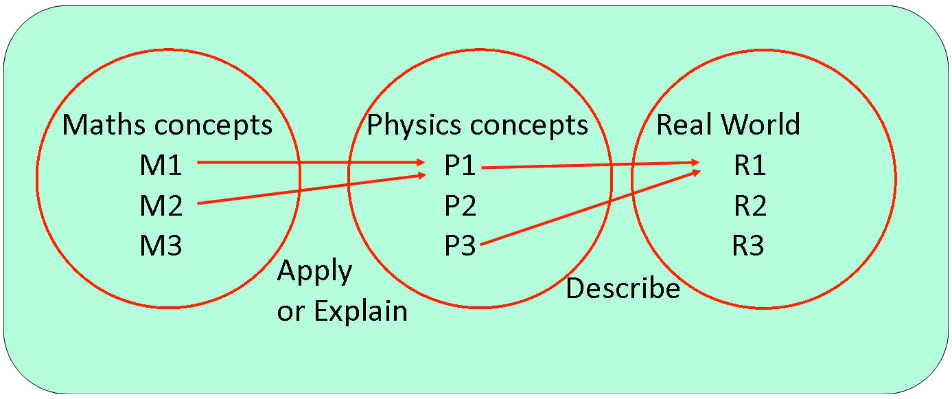


**Figure 2. Relationship between physics and mathematics as generally understood by first-year students**

## A threshold concept

At the beginning of this paper, I asked the question whether identifying physics as a science could be considered a threshold concept. What I mean by science is a body of knowledge that is fundamentally based upon empirical observation; one that is descriptive of the universe in which we live. A counter viewpoint, offered by the student group, would be that physics is the application of mathematics; i.e., the universe is explained by the mathematics.

Overall, the results show some support for the suggestion that first year physics students often fail to cross the threshold of identifying physics as a science. While the group identifies with physics as being a description of what is real, there is also a view that this can be somehow inferred or implied from mathematics: “Maths is used to explain physics in my eyes” (student participant 5). In other words, the viewpoint that physics is a branch of mathematics is held by this set of students, if not explicitly stated. There was little evidence of a similar view being held by any of the practising physicists.

The threshold of seeing that physics concepts can lie separate from mathematics has not been crossed, or has only partially been crossed, by some students. This is *troublesome* to students because they are less able to identify the relevant concepts before trying to tackle a problem with mathematics, making their approach less likely to be effective (Tumarino & Redish, 2007).

Students, particularly in the second focus group (student participants 5–7) expressed frustration that there was insufficient mathematics within the first year physics papers. They strongly felt the need to relate physics back to the mathematics in order for them to understand the physics, that is, they are trying to place new conceptual ideas into a pre-threshold framework. Ideas can therefore become *troublesome*.

It is unclear from where the naïve approach arises. While an obvious starting point is to look at the teaching (Quale, 2011), the range of different pedagogies presented by the practising physicists suggests that this may not be the case. It is plausible that, for the situations to which the students are exposed in physics teaching at secondary and early tertiary level, the view that “mathematics explains physics” serves them well. Only later in their university study and careers does this actually become troublesome.

Crossing the threshold would see a transformation in strategies. “When you learn a lot of concepts in physics, it makes it impossible to look at the world in such a naïve manner again” (physicist participant 9). In terms of Tumarino and Redish’s epistemic games (2007, p. 4), there would be a transformation from the naïve approaches (based on the idea that mathematics underlies the application of physics) of “transliteration to mathematics” and “recursive plug-and-chug” to the more effective games “mapping meaning to mathematics”, “mapping mathematics to meaning”, “physical mechanisms” and “pictorial analysis”. The first of these effective games, “mapping meaning to mathematics” is powerful—it involves the writing of physical reality in mathematical terms—i.e. describing (not explaining) the meaning in terms of mathematics. This is a key process carried out by practising physicists. The second, “mapping mathematics to meaning” is the reverse, inferring the physical meaning from a description written mathematically. Indeed, the “physical mechanisms” game abandons mathematics altogether and resorts to the identification of relevant physical process (what physicist participant 8 called a “paramount skill”) and the “pictorial analysis” game resorts to mathematics only in a graphical form.

Once the new games become apparent, they are likely not to be forgotten, and, as such, the change is *irreversible*. The power of being able to use these games in an appropriate way through the careful phrasing of a question was exemplified by a conversation in the practising physicist focus group:

> Interviewer: So a physicist is good at asking the right question?
>
> Participant 6: I delight in a good question as much as I delight in a good answer.
>
> Participant 7: A good question is half the question solved, really; if you pose the question in the correct way you are quite often halfway to the correct answer.

Moreover, it was clear to participant 7 that many ways of looking at things was something that developed with experience (i.e. an *irreversibility* in approach):

> Once you become a bit older, you realize that the laws of thermodynamics that govern the behaviour of gases and liquids will give you the behaviour of a crowd. That becomes very interesting, and the more doors you knock on the more they open and the more interesting stuff we find behind them. (Physicist participant 7)

I can now briefly comment on Meyer and Land’s (2003) five features of a threshold concept in turn. The evidence of this study suggests certainly that the role of maths within physics is *troublesome*—for example, students can correctly predict currents in electrical circuits using mathematics without any grasp of understanding current flow. Grasping that physics is underpinned by experiment, rather than mathematics, is likely to be *transformative* and *irreversible*, opening up a number of new strategies that are not unlearned later. One could argue that experiment-based physics exposes *integration* in terms of physical concepts, although such interactions are evident in purely mathematical approaches too; for example the analogy between the resonance of mass on a spring and that of an inductor-capacitor circuit is clear in equation-form. Any *boundaries* are less obvious.

### Educational significance

It is clear that practising physicists uphold the primary importance of identification of the relevant underlying physics concepts when approaching a physics problem as part of their work. The educational significance of this is that it is therefore reasonable to expect that the teaching of a Bachelor of Science degree (majoring in physics) would encourage students to develop this viewpoint. Indeed, this has long been the focus of many courses in physics at a tertiary level. However, I am mindful that transforming students' epistemological beliefs (crossing the threshold) is extremely difficult (Elby, 2001; Gire et al., 2009).

It is well established that merely presenting physics in a manner that emphasises concepts is insufficient for producing epistemological changes in students' understanding of physics (Elby 2001; Duggan & Gott, 2002; Etkina & Planinšič 2014). Physics teaching at tertiary level is now being driven towards developing other epistemic principles that are used in the professional application of physics, such as critical assessment of ideas, probing of assumptions and analysis of evidence (Etkina & Planinšič, 2014). Developing an appropriate understanding of the role of mathematics within this would be just one of the changes needed to produce an effective physics-teaching curriculum. Given the wide range of different pedagogies presented by practising physicists, helping teachers at tertiary level to identify their own epistemic beliefs may also be just as necessary for improved success of tertiary students. Indeed, Quale (2011) suggests (albeit with little evidence) that students acquire their beliefs about mathematics within physics from their teachers, implying that education of the teacher is just as valuable as education of the student.

It is interesting to note that, during the interviews with practising physicists, the epistemic principles needed in a university physics curriculum, as presented by Etkina and Planinšič (2014) were not strongly evident. It is possible that this was because the questions focused their thinking towards mathematics. Alternatively, they may not have identified critical assessment of ideas, analysis of evidence, etc., as physics per se. I therefore caution against taking the results of the study out of the context of the role of mathematics within physics.

Furthermore, I need to acknowledge that teaching of physics at tertiary level is complicated by the fact that most students who are undertaking first year physics study are not aiming to become professional physicists. They have a huge cross-section of careers in mind, notably engineering.

## Limitations

The student group sampled by this study was drawn from those studying a first-year physics paper at the University of Waikato, a regional university in New Zealand. This cohort consisted of around 130 students, the large majority male, and the majority undertaking various programmes in Engineering. These biases are typical of first-year physics papers at New Zealand universities. Recruiting students for the study proved particularly difficult. This may be because the students could not see the value in taking part in such a study.

It is likely that there would be some bias in the student group who took part, although this has not been established. The frustration felt by the second focus group about the perceived lack of mathematics within physics papers may be because those who were more confident with mathematics were more inclined to take part in this research. It is quite plausible that, by advertising the study as being about the way mathematics and physics work together, those who found mathematics difficult were implicitly discouraged from taking part.

Likewise, since the group of practising physicists were selected through the author's professional networks within New Zealand, there would again be the likelihood in bias in their opinions. Although it was considerably easier to recruit participants to this group, this group was again small (but consistent in size with the student group) and therefore I must therefore be careful with generalizing the findings.

## Conclusion

This study examined the ontological and epistemic beliefs regarding the role of mathematics within physics in two participant groups: practising physicists in New Zealand and first-year physics students at the University of Waikato. Data have been collected through interviews and focus groups and transcripts have been analysed for key themes.

While both groups identify strongly with physics as belonging to 'the real world', students viewed mathematics and physics as being more strongly connected than did practising physicists; specifically students took the broad viewpoint that "Maths is used to explain physics" (student participant 5) while practising physicists viewed mathematics as a descriptive tool to assist the application of physics. This may be troublesome to students because they are less able to identify the relevant concepts before trying to tackle a problem with mathematics, making their approach less likely to be effective.

However, it is clear that this is only part of the story. It is known that a conceptual approach to teaching physics 'on its own' is insufficient to transform students' epistemological beliefs. Grasping the expert view of the role of mathematics within physics can only be part of a larger threshold.

Opinions on how best to teach mathematical concepts in physics varied considerably across the practising physicists. However, the majority of students expressed the opinion that the mathematics and physics need to be taught simultaneously in a coherent manner.

At the start of this article it was suggested that the statement "physics is a science" may be a threshold concept. The study has provided some limited evidence towards this, focused on the role of mathematics within physics. The evidence within the teaching literature suggests there may be many other threshold concepts; critically analysing data, probing assumptions and so on. Perhaps one might better describe the phrase "physics is a science" as hiding a collection of threshold concepts, many of which must be crossed before a student acquires an expert view of physics commensurate with those who practise physics professionally. The understanding of the role of mathematics in physics is just one of these.

Moving forward requires looking at the whole way in which physics is presented, not just emphasizing the underlying concepts (Duggan & Gott, 2002; Etkina & Planinšič, 2014). The findings of this work will inform future development of the physics and engineering curricula at the University of Waikato.

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## Appendix 1. The set of questions asked of participants

The following questions were asked of participants:

1a. What degree(s) did you study? Why did you choose this degree? [To practising physicists].

1b. What degree are you studying? Why did you choose this degree? [To students].

2. What kind of skills are needed to be good at physics?

3. It is frequently stated that "mathematics is the language of physics". In what ways do you agree or disagree with this statement?

4. What is required in order to solve a physics problem?

5. What is meant when a physicist writes an equation? Is it the same as a maths equation?

6. Does maths help you understand physics? Or physics help you understand maths? Or both or neither? Explain.