Perceiving Necessity

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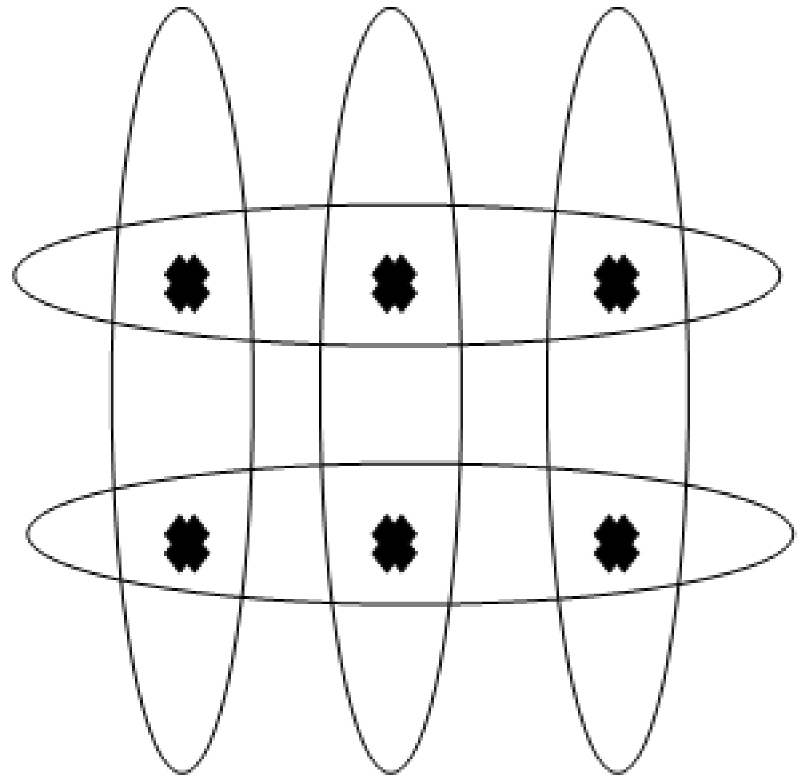


Fig 1. Why $2 \times 3 = 3 \times 2$

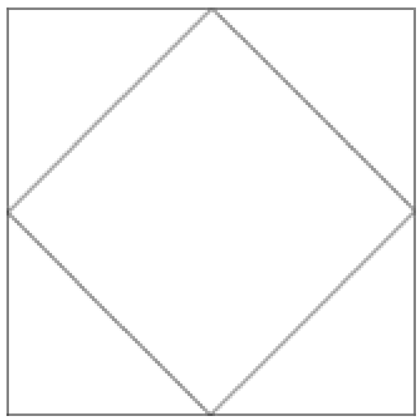
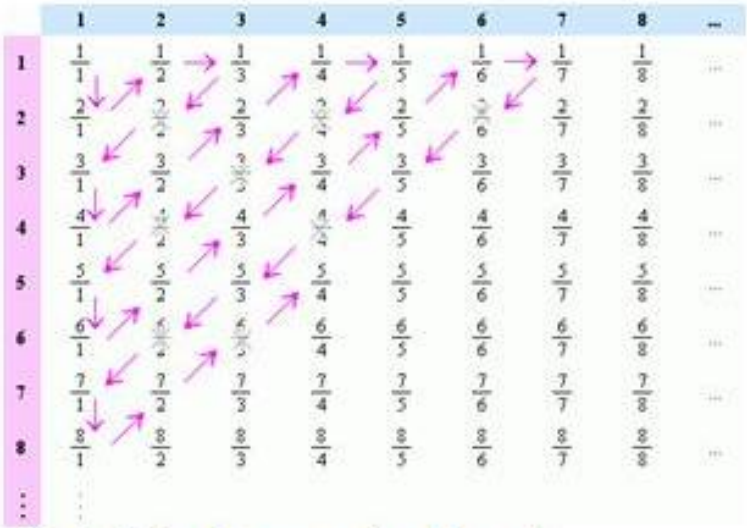


Fig 2. Why the inside square is half the area of the outside



	1	2	3	4	5	6	7	8	...
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	...
2	$\frac{2}{1}$	$\cancel{\frac{2}{2}}$	$\frac{2}{3}$	$\cancel{\frac{2}{4}}$	$\frac{2}{5}$	$\cancel{\frac{2}{6}}$	$\frac{2}{7}$	$\frac{2}{8}$	...
3	$\frac{3}{1}$	$\frac{3}{2}$	$\cancel{\frac{3}{3}}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	...
4	$\frac{4}{1}$	$\cancel{\frac{4}{2}}$	$\frac{4}{3}$	$\cancel{\frac{4}{4}}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	...
5	$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	...
6	$\frac{6}{1}$	$\cancel{\frac{6}{2}}$	$\cancel{\frac{6}{3}}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$	...
7	$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$	...
8	$\frac{8}{1}$	$\frac{8}{2}$	$\frac{8}{3}$	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$	...
⋮	⋮								

Fig 3. Why the set of rational numbers is countable.

- In each case it seems we can perceive that a mathematical truth *is* so, and also at the same time that it *must be* so. It is as though at the same time we see the truth *and the reason for it*.
- There has been a recent upsurge of interest in the role of diagrams in *mathematics* (e.g. Brown 1999, Giaquinto 2007, Mumma 2010: many inspired by Manders 2008) and *logic* (Shin 2002), a growing awareness that diagrams can serve as more than just a *post hoc* illustration of necessary reasoning in some more "serious" nondiagrammatic form.
- We believe this movement also holds profound implications for *general epistemology* currently blocked by ideas descending from Hume.

- What ideas?
- The claim that we perceive necessity is somehow *antinaturalistic*.:
  - **"'...we do not understand our own must-detecting faculty." Not only are we *aware* of no bodily mechanism attuned to modal aspects, it is unclear how such a mechanism could work even in principle....' (Wright citing Blackburn)**
- Relatedly, Benacerraf has made a career out of invoking scepticism about mathematical knowledge claiming that the usual "semantics for mathematics" does not "fit an acceptable epistemology", since it:
  - **"...will depict truth conditions in terms of... objects whose nature, as normally conceived, places them beyond the reach of the better understood means of human cognition (e.g. sense perception and the like)" (Benacerraf, 1973, p. 667).**

- Why are these view held?
- Arguably, the downstream legacy of Hume's empiricism
- This is encapsulated in the widely influential, supposedly common-sense Humean maxim:
  - **"There are no necessary connections between distinct existences"**.
- But what does this mean exactly? And is it true?

Hume writes:

- **"No connexions between distinct existences are ever discoverable among human understanding."** (*Treatise*, Appendix)

- **"There is no object, which implies the existence of any other if we consider these objects in themselves."** (*Treatise*, 1, III, vi)

- **"Any thing may produce any thing. Creation, annihilation, motion, reason, volition; all these may arise from one another, or from any other object we can imagine…no objects are contrary to each other, but existence and non-existence."** (*Treatise*, 1, III, xv)

***Quite remarkable! ↑**

- But what exactly is meant by 'existences', or 'objects'?
- Consider Hume's most famous application of his principle, to causality (famous passage about billiard balls):
  - **"I consider, in what objects necessity is commonly suppos'd to lie; and finding that it is always ascrib'd to causes and effects, I turn my eye to two objects suppos'd to be plac'd in that relation. . . . I immediately perceive, that they are contiguous in time and place, and that the object we call cause precedes the other we call effect. In no one instance can I go any farther…."** (*Treatise*, 1, III, xiv)
- Here Hume speaks as if the 'objects' are the balls themselves, but he is actually talking about their *motions*.
- Properties and events must therefore count as 'objects' or 'existences' for Hume.

- But then, what about the properties 'black' and 'white'? The events of 2 and 3 hours passing? Or, to return to mathematics, the 'objects' Ø and {Ø}?
- What Hume means by 'distinct' cannot be fully understood apart from his theory of perception and the epistemology he twines around it.
- Hume's theory of perception has two broad features we wish to argue against:
  - It is **passive**
  - It is **atomistic**

Passive:

i) Ideas are simple copies of impressions

ii) Impressions of reflexion consist solely in combinations of impressions of sensation:

- Sensory impressions are the building blocks of all thought.

iii) All mental activity is 'perception-like':

- Reflexion too is a form of perception – of ideas that are 'weaker' and 'less vivid':
  - **"To hate, to love, to think, to feel, to see; all this is nothing but to perceive"** (*Treatise* 1, II, vi)

iv) Denial of *abstract ideas*.

- Hume defines abstract ideas as ideas that are general in that at least some of their determinable properties lack determination. E.g. a 'general triangle': neither isosceles or scalene.
- He claims (following Berkeley) there are *no general ideas*, only *particular ideas used in a general way* (e.g. a proof about triangles to be valid might need to draw on the particular ideas of isosceles *and* scalene *and* equilateral triangles…)
- Allowing abstract ideas would render the mind active since it would need to choose *which* determinables to abstract from.
- Hume sees the denial of abstract ideas as a properly naturalistic position to take against scholastic obscurantism.

Atomistic:

i) Separate Imaginability Criterion of Distinctness

- When we distinguish shape from colour in an object such as a white globe, it is not that we examine the white globe and use reason to distinguish its whiteness and roundess *as abstract ideas* (medieval period: 'prescinding', 'prescission').
- Rather, what we do is imagine *black globes* and *white cubes*.
- Without such a literal, quasi-perceptual forcing apart of ideas we cannot distinguish them, though we might think we can, a cause of much confusion and wasted time in philosophy**:…"that distinction of reason, which is so much talked of, and is so little understood, in the schools.**" (*Treatise*, 1, I, vii)
- Thus Hume denies that we can *prescind without separating*: **"…all ideas, which are different, are separable…"** (*Treatise*, 1, I, vii). This is crucial.

- Recall Hume's remark:
  - **"…no objects are contrary to each other, but existence and non-existence"**
- This implies that objects as he understands them are all compossible – in other words, they have no natures which might produce impossibilities in combining them. Thus such necessities as do exist in the world may only consist in constant conjunction between discrete 'objects'.
- Hence: *Modal Combinatorialism*.
  - ***'top-down' combinatorialism***: any whole can be decomposed into some given set of atomic parts. (*HUME DOESN'T MEAN THIS)
  - ***'bottom-up' combinatorialism***: given some set of atomic parts, any permutation of them is possible. (*HE MEANS THIS)

- *Hume’s Fork* (Flew): strict division of knowledge into *relations between ideas*, determined a priori, and *matters of fact*, learned through experience.
- This actually follows from Modal Combinatorialism, in a weird way.
- If matters of fact are atomic and all compossible, and thus cannot generate any necessities from their own natures, then this means that Hume must confine knowledge of necessary connexion to ‘relations between ideas’ (often thought of as analytic / determinable merely by inspecting language).
    - *Simple physical example*: if the natures of *mass* and *motion* don’t rule out any combination of these ‘objects’, then a law of nature (e.g. **F = ma**) can be nothing more than an extremely widespread regularity. Thus causal necessity is not ‘real necessity’.

- The Fork has been problematic for *mathematics*, where the general agreement that mathematics does deal with relations of ideas, discoverable *a priori* forces the conclusion that it's entirely disconnected from real-world observation.
- Our mathematical examples can serve as particularly clear counterexamples to the Fork's overwhelming influence on contemporary analytic epistemology if rendered immune to Humean challenges.

- In using recent scientific results to bolster philosophical argument one risks falling hostage to empirical fortune.
- However there is a wealth of contemporary empirical research undermining both the passivity and atomism of Hume's view of perception.

i) Passivity:

- The brain's functioning is far from Hume's simple copying of impressions into ideas.
  - Andy Clark: **"Brains…are bundles of cells that support perception and action by constantly attempting to match incoming sensory inputs with top-down expectations or predictions"** (Clark, 2012)

ii) Atomism:

- Not only is the mind organised sufficiently holistically to support 'top-down predictions', it appears that it operates this way at a number of distinct functional levels which are richly inter-related, producing:
  - **"a cascade of cortical processing events in which higher-level systems attempt to predict the inputs to lower level ones on the basis of their own emerging models of the causal structure of the world"** (Clark, 2012)
- A particularly nice anti-atomistic example: the perception of *symmetry*. This is a high-level structural property.
- In fact the necessary truth perceivable in *fig 1* draws directly on symmetry-perception in recognising the sameness of the rows and columns, while the symmetry of the square and of the diamond plays a key role in recognising the necessary geometric truth in *fig 2*.

- E.g. Vertical bilateral symmetry is perceived much faster than other symmetries (e.g. about other axes, repetitions of shapes in friezes). It can be perceived when the stimulus array is presented for <160 milliseconds - less time than the brain takes to attend to anything.

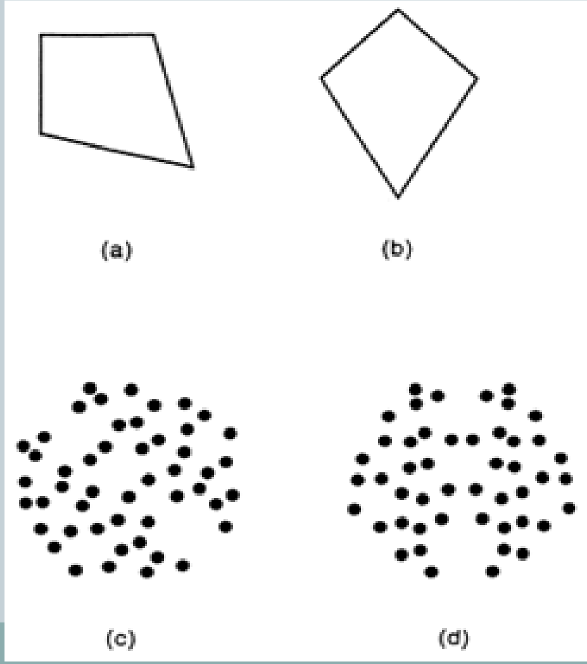


Fig 4: vertical bilateral symmetry has an immediate and salient "look" compared to symmetry about an oblique axis.

- Vertical Bilateral Symmetry is a global property of an array. Yet perception of it is as immediate as any perception can be – and unquestionably pre-reflective.
- The deeply automatic and pre-reflective nature of symmetry perception is confirmed by its appearance in animals, including very simple ones. E.g. bees - it is possible to train bees to prefer either symmetrical or asymmetrical patterns, but symmetry comes more naturally to them (Giurfa et al, 1996).
- Bees are not reflective animals, and it is hard to believe they deal in "relations of ideas", in Hume's sense. They just see.

Back to *Fig. 1*:

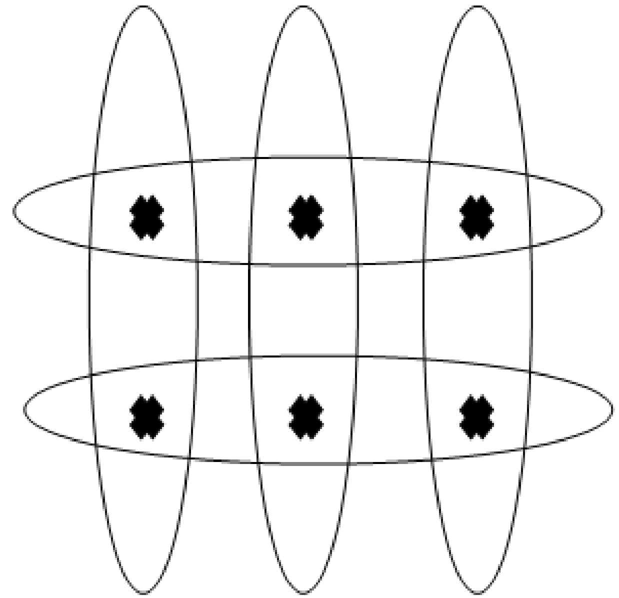
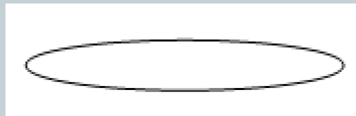
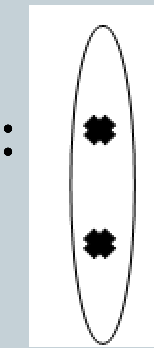


Fig 1. Why $2 \times 3 = 3 \times 2$

Does this constitute a necessary connection between distinct existences? Well, what are the 'objects' here? The Humean has some choices, all unsatisfactory:

i) *<u>Physical Mark View</u>*: The relevant objects are:

- 5 of these: 
- 6 of these: 
- which go together to make 5 oval + star combos, such as: 

This appears to be a natural choice in terms of the organisation of our visual field when regarding the page.

Recall the *Humean Separate Imaginability Criterion of Distinctness*: we can imagine each of these shapes existing on its own on the page.

- But then it is false that there are no necessary connections between these objects as positioned in *fig 1*.
- For instance, one cannot change the number of stars in the vertical ovals without changing the number of stars in the horizontal ovals.
- Interpreted thus, then, Hume's maxim is simply incorrect.

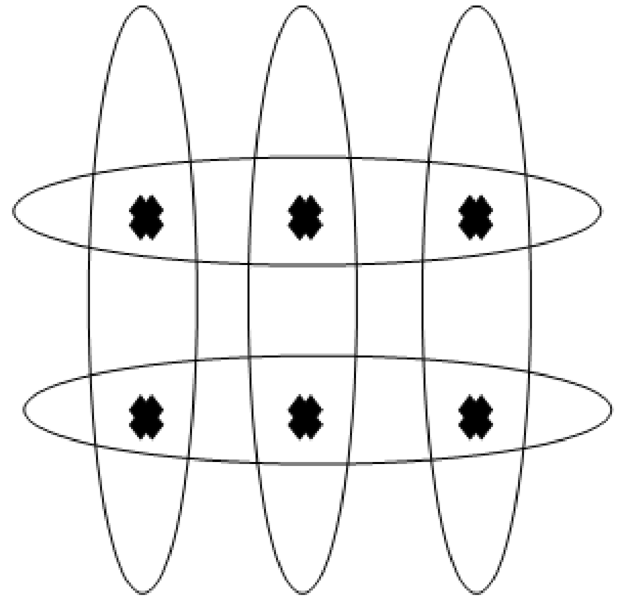


Fig 1. Why $2 \times 3 = 3 \times 2$

ii) *Abstract Object View*: On the other hand, we might claim that *fig.1* doesn't display a truth about physical marks but about something more purely mathematical or abstract – for instance the relevant objects/existences are ***three '2s'*** and ***two '3s'***.

- These objects are arguably not distinct. E.g. 2 is made up of 'two ones' and 3 is made up of 'three ones', so 2 is a proper part of 3.
- At this point, then, Hume might defend his maxim by stating that *fig 1* solely expresses relations between ideas, and he never meant to claim that *ideas* were distinct existences (c.f. 'The Fork').

- But there is something unsatisfying here. It seems puzzling to claim that we can gain mathematical knowledge by examining *fig 1*, and yet that mathematical objects are *entirely* separate from perceived experience.
- Furthermore, now Hume's claim that there are no necessary connections between distinct existences seems to beg the question, to arbitrarily rule out that we perceive the kinds of existences between which necessary connections hold, by labelling them as 'mere ideas'.
- He seems happy to apply a Separate Imaginability Criterion of Distinctness to the billiard balls, where it gives him the answer he wants about causation. But in *fig. 1* one may equally imagine the ovals with two and three stars existing on their own. Yet as assembled in *fig 1* these objects do seem to have necessary connections.

- If Hume then argues that the 5 star-oval combos cannot be distinct existences precisely because of these necessary connections, his maxim effectively becomes: **"there are no necessary connections between distinct existences, which are those existences between which there are no necessary connexions"**. This seems to rob it of all philosophical content.

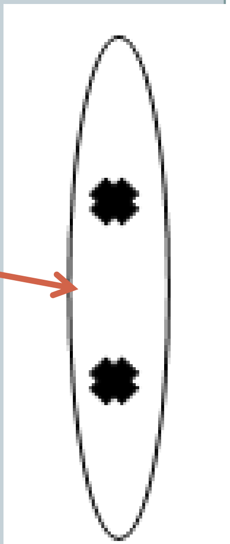
iii) *"Both" view*: One might think of compromising by combining the two views as follows: the objects represented by *fig 1* are **ovals and stars *and* '2s' and '3s'**.

- However this raises tricky questions of the relationship between physical marks and abstract objects. If they are all separate objects, why include the stars and ovals in the diagram at all? Why not lose the physical marks, keep the 2s and 3s and draw the mathematical moral straight from them? (Obviously impossible, thus the view risks incoherence…)

- This points the way to our preferred interpretation:

iv) *"Hybrid…but not both"*: Rather than understanding physical marks and numbers as separate objects, attribute to them *partial identities*.

- What does this mean? Just that 'twoness' is a property which may be *prescinded* from this, while precisely *not* being separable from it. This is of course exactly what Hume's epistemology rules out as impossible.



- Prescinding without separation is a form of thought which a great deal of mathematics rests on. It is essential for all structural reasoning. One might argue that *all* necessary reasoning is structural – that necessary reasoning in essence consists in recognising that a particular structure has the structure that it is fact has. (This idea is worked out in (Legg 2012)).

- We can perceive necessary truths.
- In such perception the mind is not *passive* and *atomistic* as Hume supposes, but *active* and *integrated*.
- If we work out the consequences of this for epistemology, where would it take us?
  - Suggest it could be helpful to look back to a rationalist view largely left behind by Anglo-American philosophy, which has so admired Hume. This view taught that we may attain certain knowledge from ideas that are sufficiently *clear and distinct*…. *Further work needed*.
  - Suggest we reconsider what we mean by 'naturalistic epistemology'…

- The last few decades have seen an outpouring of work in the metaphysics of modality remarkably mismatched by relatively slight investigation into the epistemology of modality.
- We suggest that the strictures imposed by Hume's relegation of necessary truths to mere 'relations between ideas', and his horror of empirically investigating the mind's capacity for abstract thought (as an 'anti-naturalistic', inevitably confused inquiry) have left the latter investigation on relatively barren ground.
- Recent attempts to address the mismatch again largely follow Hume in pursuing the idea that conceivability is our guide to possibility (Yablo 1993, Gendler and Hawthorne, 2002).
- Yet a clear, principled account of what exactly makes certain states of affairs 'inconceivable', and therefore impossible, still seems lacking.

Final critique of Modal Combinatorialism:

- Hume's notion that nothing is contrary but existence and non-existence has had enormous downstream philosophical influence.
- Mill (a law of logic) → Wittgenstein's *Tractatus* → Carnap's treatment of necessity as truth in all 'state-descriptions.
- From there, the development of possible worlds semantics in logic inspired an arguably 'un-Humean' leap to replacing state descriptions conceived of as linguistic entities with analogous metaphysical entities (of a variety of kinds) which 'truth-make' modal claims.
- Yet even in Lewis' most extreme form of modal realism (Lewis 1986), Modal Combinatorialism played a powerful role in his influential ('Humean') analysis of laws of nature as mere patterns of regularity across subsets of possible worlds.

Want to suggest this is a *syntactic approach to modality*.

Let us attempt to frame the issue in *schematic*, maximally general terms.

Consider a world consisting of 4 'idea / objects' (**a**, **b**, **c** and **d**) conceived of as particulars which may combine to make larger states of affairs. Imagine that these idea / objects are all *distinguishable*. Then according to Hume they must be *separable*. Thus, we have:

ab ac ad ba bc bd ca cb cd da db dc…

A toy universe in which Modal Combinatorialism is true. All combinations are possible

Let us now imagine a toy universe in which Modal Combinatorialism is false. This just means that not all combinations are realisable. Here is just one example:

ab ad ba bd ca cb cd da db…

- This toy universe is missing **ac**, **bc** and **dc** (for some reason, let us imagine it is to do with the nature of **c**).
- An intelligent mind inspecting the world above might summarise the combinations missing from it in a simple statement like, '**c** can never come last in combination'.
- This statement is obviously a rudimentary law, or universal.

- Our point is now merely that the second scenario is *not incoherent*.
- It is not analytically false to conceive constraints on the happy combination of any conceivable object with any other conceivable object (bearing in mind that of course these objects will have *natures*).
- Mathematics, as we have seen, rules out such combinations regularly: for example, the combination of 2×3 with $3 \times x$, for any x other than 2.
- Hume's Modal Combinatorialism is therefore an underhanded way of killing off a kind of realism about universals.
- Hume rules out such constraints by fiat, not by argument – but necessary mathematical truths should trump plausible philosophical overgeneralisations.