



Promoting statistical thinking in year 12 multilingual classrooms: a collaborative study

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Abstract

There are challenges making connections between language use and mathematics in mathematics education. A lack of connections between the two domains can have negative consequences on student learning and performance. The challenges in statistics classrooms, where language and contexts are important as a medium of instruction, have received little scrutiny. This article reports on collaborative research carried out in three largely Pasifika-dominated year 12 classes. Specifically, we explored the language resources and strategies that appear to enhance the statistical understanding of Pasifika students. Findings from the teacher reflection aspect of the study indicated that some strategies to incorporate student language and communicative resources in their learning worked better than others. Teachers may need to re-evaluate their teaching practices, especially if part of their population is learning English as a second language.

Keywords Statistical thinking · Linguistic challenges · Collaborative research · Design research · Connecting language and statistics · Statistical investigations · Translanguaging practices

Introduction

The issues of mathematics teaching and learning in multilingual and multicultural contexts are strongly located in the dynamics of a highly globalized society of the twenty-first century. Classrooms deemed to be multicultural are places where learners have different linguistic and cultural backgrounds, where they may speak one language at home and another language at school, where teachers and students may not share a common language or cultural background, and where some or all of the students learn the language of instruction as a second language (Dockrell et al.,

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2022; Lesser & Winsor, 2009; Robertson & Graven, 2020). These students may need additional support in social or academic situations.

According to several researchers (Barwell, 2005, 2020; Goldenberg, 2008; Latu, 2005; Morris, 2021; Moschkovich, 2018), mathematics is strongly connected with language and culture. For example, Barwell (2005) claimed that students from minority cultural or linguistic backgrounds tended to have greater difficulties with word problems. Latu (2005) noted that word problems involving mathematical implication and logical structures such as conditionals and negation were a particular issue for Pasifika students. The same point was made by Morris (2021) in his study with Tongan students. Concepts such as “probability” and “randomness” and comparative terms like “very likely,” “probable,” and “almost certain” have no equivalents in Tongan language.

To be able to perform well in mathematics, students must be proficient in the language of instruction and use language effectively in diverse contexts (Kempert et al., 2011; Nacarato & Grando, 2014; Xi & Yeping, 2008). It follows that students must be able to move between every day and academic ways of communicating ideas and relate those ideas to mathematical symbols and text (Goldenberg, 2008). This situation may present some unique challenges for students as they must simultaneously learn ordinary English and mathematical English and be able to differentiate between those types of English (Adler, 1998; Farsani, 2016; Kaplan et al., 2009; Moschkovich, 2018). Furthermore, to be able to perform competently, students must understand the highly technical language used specifically in mathematics (Brown et al., 2009; Goldenberg, 2008; Xi & Yeping, 2008). This language is not used in everyday English and therefore is less likely to be familiar or understood by English language learners (ELLs). In multicultural classrooms, it takes more effort for ELL to process ideas than native English speakers (Bose & Choudhury, 2010; Clarkson, 2007; Nacarato & Grando, 2014). These students can miss out on mathematical learning because they may spend too much time trying to understand what is required to complete mathematical tasks.

New trends in multilingual education have provided opportunities to change traditional approaches to teaching and to explore the potential advantages of translanguaging (García & Leiva, 2014). Translanguaging is a dynamic process whereby multilingual users mediate complex social and cognitive activities through strategic and flexible employment of the multiple communicative resources they possess (Canagarajah, 2018). These communicative resources include gestures, objects, everyday experiences, home language, and a mixing of languages and mathematical representations (Moschkovich, 2018). Other researchers, such as Creese and Blackledge (2010) and Farsani (2016), have employed the term translanguaging to depict language fluidity and movements during teaching and learning.

The practice of code-mixing is a language swap between two languages at the same time (Bose & Choudhury, 2010). Warren and Miller (2015) claimed that mathematics learning starts with exploring one concept using one representation, moving to using two representations in parallel, linking the parallel representations, and finally integrating representations. From their perspective, translanguaging relates to switching between representations, and code-mixing relates to changing the oral language used to assist in making connections between the representations.

Given that the nature of a language affects its speakers' worldview and cognition and thus people's perceptions are relative to their spoken language, researching in various culturally and linguistically sites can potentially depict how mathematics could be perceived differently in different languages (Barwell et al., 2019; Morris, 2021; Moschkovich, 2018; Prediger et al., 2019; Robertson & Graven, 2020). Although the role of language in the teaching and learning of mathematics has become well established in the literature (Clarkson, 2007; Hoffert, 2009; Moschkovich, 2018; Planas & Civil, 2013), there have been few research studies about language issues in statistics education (Kaplan et al., 2009; Lavy & Mashiach-Eizenberg, 2009). It is, however, important to gain insights into how ELL learn statistics (Kazima, 2006; Lesser & Winsor, 2009; Sharma, 2014). This paper provides a contribution and stimulus for more debate and research in this area. The overall purpose of the study reported in this paper was to explore the language resources and strategies that appear to enhance the statistical understanding of Pasifika students.

Literature review

This section describes language strategies that teachers can use to address some of the linguistic challenges faced when the language of instruction is different to the home language(s) of students in an educational setting.

Mathematical register

Within any language, there are many distinct registers, including everyday conversation, mathematics, and statistics (Lesser & Winsor, 2009; Marin, 2018; Schleppegrell, 2011). Halliday (1978) used the term *register* to refer to the specialised method of communication used in a particular social practice. The statistics register, for example, includes words unique to statistical communication, but also specialised uses of everyday words, which take on unique meaning in statistical contexts. To succeed in a statistics classroom, students need to be not only familiar with and competent in their ordinary English register, so they can communicate with their classmates, but also fluent in multiple mathematical registers (Kazima, 2006; Moschkovich, 2005).

One example of confusion between the everyday and statistical English (Lesser & Winsor, 2009) involves the term independent. Since the everyday meaning of independent can be associated with separateness (i.e. independent nations), the authors conjectured that this leads students (incorrectly) to equate independence with disjoint sets (i.e. mutually exclusive). Another technology-related example provided by Lesser and Winsor was the term mode. The mode button on a calculator, for example, has nothing to do with the most frequent observation in statistical context. The mode button is used on a calculator to change the functionality of the calculator.

It is not only the meaning of words used within a statistical context that can be confusing. While some terms have different meanings in everyday usage and in statistics, some are also used in mathematics in more than one way (Kaplan et al., 2009; Lesser & Winsor, 2009; Moschkovich, 2018). For example, ambiguous words such

as random hold more than one meaning, definition, or use. Everyday uses of the term “random” tend to relate to something being unusual or surprising, whereas in a statistics classroom randomness relates to predictability of outcomes. This causes a disconnect between our real-world experiences and what we are taught in statistics, creating confusion around randomness. The researchers advised that attending to the multiple meanings of the concept was necessary and recommended focusing on the statistical process of random selection rather than the outcomes. Furthermore, the researchers suggested that the lexical ambiguity of randomness can be used to foster a deeper understanding of the concept.

Role of context

Context refers to the setting in which information is communicated and may include content, people, or environment. In statistics, the context is at the heart of any investigation, and all aspects of a statistical investigation must directly relate to the context in which the investigation is situated (Neill, 2012; Watson et al., 2018). The need for context is demonstrated in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report (Carver et al., 2016, p. 17) where *Recommendation 3: Integrate real data with a context and a purpose* indicates that context plays a role in helping students to attain learning goals. When students find context meaningful, their motivation to learn and communicate increases (Lesser & Winsor, 2009; Watson et al., 2018). Goldenberg (2008) claimed that statistics is a more natural vehicle for context-embedded instruction than mathematics; hence, context-embedded instruction can have benefits for teaching ELLs statistics. Indeed, the most important clue needed to deduce the meaning of a word or sentence is generally its context (Lesser & Winsor, 2009; Watson, 2006).

Sharma (2014) reported that students, when carrying out statistical investigations, focussed on real-life contexts but can get side-tracked by irrelevant details, while ignoring relevant information. For example, some students interpreted sample as an example of consumer goods rather than a subset containing the characteristics of a larger population. Moreover, a few student’s phonetically translated sample into everyday Hindi language. Lesser and Winsor (2009) reported a student’s confusion from a context-rich exercise about correlation because the term “ski resort” was unfamiliar in her high-poverty urban city in a desert region. Situations like this prevent students from interpreting the results of an investigation in terms of the context from the data and in relation to the research question posed.

Code switching

ELLs are known to employ code-switching to clarify their understanding and to express their arguments and ideas generally (Clarkson, 2007; Moschkovich, 2005). Code-switching involves the movement between languages in a single speech act. It can involve switching a word, a phrase, a sentence, or several sentences (Adler, 1998). Bose and Choudhury (2010) state that in addition to switching between two languages, for example, English and Hindi, the teacher may also switch from a

formal version of Hindi to a very colloquial form of the same language. In this case, the code-switch takes place as a language-swap from English to Hindi, as well as from the formal form to an informal form within one language.

Warren and Miller (2015) argued that communicating mathematically involves aspects of both code-switching and codemixing, they claim that these aspects are more than simply translating from one language to as suggested by some researchers. Code-switching has been shown to promote ELL student–student and student–teacher interactions (Farsani, 2016; Setati et al., 2002). In addition, Makgato (2014) found that code-switching to home language is a common practice to sustain continuous communication between teachers and learners. Sometimes students switch their languages because they find problems difficult to solve in English. Clarkson (2007) explained how ELL may comprehend target language texts using their first learnt language (L1). He claimed that the first language scaffolds semantic processing, while if a learner were to process the input exclusively in second language/formal language of instruction, then s/he might run into trouble handling syntactically complex sentences.

It appears, however, that code-switching and school language instruction can cause tension. Clarkson (2007) explained how translation is not always beneficial or reliable as it might not reflect the exact meaning. Thus, switching between languages can add an extra layer of challenge to language learners, as they may find themselves working between a multitude of registers in both the medium of instruction and their home language (Mady & Garbati, 2014; Schleppegrell, 2011). Moreover, students' use and perceptions of the value of a particular language in different settings varies (Planas & Setati-Phakeng, 2014; Sharma, 2014). For example, Planas and Setati-Phakeng (2014) reported that while students used their home language in small group settings, they did not do this during whole-class discussions.

Integrating reading and writing

Traditionally, school mathematics has been a subject in which children have done little reading or writing although assessment work relies heavily on written work (Morgan et al., 2014). Calls for increasing reading and writing in mathematics curricula are based on the understanding that students learn mathematics more effectively and more deeply when reading and writing are directed at mathematics (Goldenberg, 2008; Jaafar, 2016; Meaney & Kirsten, 2009). Supporting ELL in reading and writing is vital in multilingual settings. It is important to have daily routines of writing, reading, and speaking about statistics content (Hoffert, 2009; Sharma et al., 2011; Winsor, 2007).

Statistics writing may be informal (i.e. journals, exit slips) or formal (i.e. writing report, Winsor, 2007). Groth et al. (2016) asked students to write a letter to a student who had been absent, explaining the meanings of probability terms from the lesson. All the students in their study included correct explanations for the benchmark terms, certain, impossible, and evenly likely. From the students' writing, the authors found that students were assigning incorrect numerical probabilities for in-between terms such as unlikely and almost impossible. The presence of this pattern

in the students' writing made the researchers aware of the need to emphasize the distinction between the benchmark and in-between terms in future lessons. In multilingual statistics classrooms, students need both language and statistical skills to relate their thinking to the real-life context and to communicate their ideas both verbally and in writing. However, ELLs may not have sound statistical literacy skills to communicate their thinking. Writing activities can be useful in helping students communicate their ideas to their peers and hence deepen their understanding of statistical concepts.

Using non-linguistic cues

Providing non-linguistic cues such as visual diagrams, demonstrations, physical items, and gestures can make more complex language accessible for all learners as these are less language-dependent modes. Farsani (2016) suggested that having such multiple entry points and scaffolds are helpful to not only communicate directly but also to create a low-anxiety environment. Visual diagrams such as graphic organisers can be especially beneficial when the graphic organisers are filled in with both English and the student's home language (Kaplan et al., 2014; Nguyen & Cortes, 2013; Winsor, 2007). Graphic organisers can enable English language learners see the relationships between key mathematical concepts and vocabulary. In the case of words with multiple meanings, it is important to emphasize the similarities and differences, so the student can assimilate them. This helps them become flexible, adaptable thinkers (Benjamin, 2011).

Furthermore, the use of physical resources or pictures can support students in their comprehension of statistical terms (Brown et al., 2009; Nguyen & Cortes, 2013). One fundamental word in statistics that has lexical ambiguity for students is "random" (Kaplan et al., 2014). Following the activation activity, the instructor in the Kaplan et al. study showed her students two pictures. The first was of three people dressed in rainbow-striped zebra costumes on a street in Shanghai to represent the colloquial definition of random: something that is weird, haphazard, or out of the ordinary. The other was an upside-down hat to represent the statistical definition of random: where choices or outcomes are based on probability. This introduction provided the instructor with the zebra-versus-hat mnemonic image for *random*, which she used during the rest of the semester to contrast the statistical and colloquial meanings of random.

Using home language

Several international agencies recognise the contribution that multilingual education can make to engaging diverse learners. In addition to supporting academic achievement, students using multiple languages can also assist in the development of positive identities associated with home culture (European Commission, 2015), which is a basic human right. Several researchers in mathematics education (Moschkovich, 2018; Planas & Civil, 2013; Setati et al., 2002; Winsor, 2007) have identified that students' home language(s) can serve as a resource for

thinking and communicating as students simultaneously learn and develop proficiency in the language of instruction and the learning of mathematics. Lesser et al., (2016, p. 146) stated: “Full participation of ELLs into the learning community is essential, not only with regard to issues of equity, but also to recognize the assets ELLs bring to a classroom environment”.

One of the key strategies to support ELL mathematical development is accepting and encouraging the use of students’ home languages in the mathematics classroom. To best support students, home languages should be recognised as resources, not as hindrances (Moschkovich, 2013; Robertson & Graven, 2020). When students can access and use their home language, their development of mathematical vocabulary is faster (Xi & Yeping, 2008). There are deliberate strategies that utilize a student’s home language to facilitate their mathematical learning and language development. One of these is to use a mathematical word bank, where students are given mathematical words with translations in their home language (Lee et al., 2011). These also facilitate relationships with parents, as the word banks enable them to understand the language used in their child’s mathematics homework and other work (Lee et al., 2011).

Some research shows that many teachers believe using home language is detrimental to learning (Mady & Garbati, 2014; Planas & Setati-Phakeng, 2014; Robertson & Graven, 2020; Winsor, 2007). The students themselves do not necessarily regard this as being particularly salient (Setati et al., 2002; Sharma, 2014). Robertson and Graven (2020) stated that many South African parents are sceptical of the suitability of their home languages as vehicles for their children’s social and economic advancement. This led many parents to choosing English as the language for instruction for their children.

Collaborative learning

To enhance the communication skills of students, it is recommended that teachers increase dialogue in English by means of small-group discussion, exploratory talk, and argumentation. Collaborative learning is a powerful tool for all students, but especially ELL (Takeuchi, 2016). When ELL can work alongside a partner, they are given the opportunity for interaction and support, enhancing their learning (Brown et al., 2009). Collaboration affords ELL the chance to ask questions and make mistakes in a safe setting, where they can receive direct and immediate feedback. Furthermore, when students are engaged in authentic conversation and interaction, their language development is fostered. This is especially true when ELL are partnered with a peer who has a higher degree of English language proficiency (Takeuchi, 2016). This strategy aligns with suggestions that grouping students with mixed ability improves learning outcomes for students who are not high achievers (Anthony & Hunter, 2017; Boaler, 2014).

Using game-based learning

Games have always played an important role in learning mathematics as they encourage mathematical thinking. Research has shown that game-based learning enriches the learning environment, improves the students' performance, increases the students' motivation, provides the opportunity to work with the group, and provides engaging learning environments (Burguillo, 2010; Lavy, & Mashiach-Eizenberg, 2009; Nisbet & Williams, 2009). Games also help to build strong relationships between school and home learning environments.

Chow et al. (2011) claimed that games are very effective alternative activities that provide students with a learning environment that is engaging and educational. Additionally, games help in creating opportunities for independent learning and overcome challenges for English language learners. It appears that children who are reluctant to participate in other mathematical activities because of language barriers will often join in a game, and so gain access to mathematical learning as well as engage in structured social interaction. Also, Naresh et al. (2014) argued in favour of using cultural games for promoting learning of probability for culturally diverse students. The authors suggested that probability lessons embedded in cultural context help students to build connections between content (probability) and cultural context, and as a result, students can broaden their perception of mathematics.

The research literature presented in this paper describes many language strategies that teachers can use to address some of the linguistic challenges faced when the language medium of instruction is different to the home language(s) of students in an educational setting. Effective teaching in linguistically diverse classroom requires sophisticated skills among educators who need to walk a fine line between teaching content and providing the linguistic support needed by students to learn that content. To date, few studies have explored the issues related to Pasifika students in relation to the learning and teaching of statistics. It will be interesting to explore language resources and strategies used by teachers to develop statistical thinking of Pasifika students.

Research design and data collection methods

Historically, there has been a tendency to view practitioners as consumers of research by the educational research community rather than active participants in the generation of new knowledge about the teaching and learning (Groth, 2007; Kieran et al., 2013; Sharma, 2017; Shaughnessy, 2014). This may have occurred because researchers are often interested in theoretical aspects and general questions, whereas teachers are usually interested in solving problems related to situations that arise in the classroom on daily basis. The base of this dichotomy is based on the notion that theoretical knowledge is delivered in the university, while practical knowledge is delivered in classrooms (Anderson & Freebody, 2012; Makar & O'Brien, 2013). Hence, teachers have not always been involved in the design of research, the generation of research questions, the analysis of data, or the dissemination of results. More recently, researchers in mathematics education have begun to involve teachers

as key stakeholders in research to forge closer links between research and practice (Arbaugh et al., 2010; Kieran et al., 2013; Shaughnessy, 2014). The methodology chosen for the study reported in this paper was selected to maximise the opportunity to link theory and practice.

Design-based research theory (Cobb & McClain, 2004; Sharma, 2017) was used to conceptualize the study. Design research is a cyclic process with action and critical reflection taking place in turn (Arbaugh et al., 2010; Cobb & McClain, 2004; Nilsson, 2013). There are benefits for both teachers and researchers when undertaking a design research partnership: the research plan can be flexible and adaptable to unforeseen effects or constraints (Nilsson, 2013). Furthermore, all participants are equal partners in the research process with no hierarchy existing between researchers and practitioners (Arbaugh et al., 2010; Shaughnessy, 2014). The approach also fosters theory development (Arbaugh et al., 2010; Kieran et al., 2013; Shaughnessy, 2014).

The following research question guided our study:

What language resources and strategies enhance the statistical understanding of Pasifika students?

To answer the research question, the study involved a cycle of three phases: a preparation and design phase; a teaching experiment phase; and a retrospective analysis phase.

Phase 1: preparation and design for the teaching phase

One of the purposes of the preparation phase was to ensure the teachers were positioned as members of the research team and given opportunities to be active participants in the research. The phase began with a discussion of research findings on language challenges and language-as-resource pedagogical strategies for ELLs. The team also hypothesised how dialogue and statistical activity might unfold because of planned statistical investigation. This helped develop a deeper shared understanding of the challenges teachers might face and the opportunities they can create, as well as potential strategies that teachers could use to reduce obstacles for student learning.

Phase 2: teaching experiment

The teaching took place as part of regular classroom statistics teaching in three largely Pasifika student-dominated year 12 classes (each comprising about 25 students). As part of the learning activities, students carried out investigations of existing Kiwi Kapers data sets using the statistical enquiry cycle method (Ministry of Education, 2007). Kiwi Kapers uses a simulated data set of Kiwis to explore some of the big ideas including sampling variation and making inferences.

The investigation was spread over three lessons in term two of the school year to suit the school schedule.

Phase 3: retrospective analysis

The three teachers and researcher performed a retrospective analysis together after each lesson to reflect on and refine the lesson plans, while the teaching experiment was in progress. The updated lesson plans were used for teaching future lessons. In addition, the team analysed the whole unit on completion of the teaching experiment. Teachers were given the opportunity to reflect on the implementation process and identify what they will do to enhance student learning and interest differently.

Ethical considerations

Research with human participants may give rise to ethical concerns that need to be considered before commencing the data collection process. As a researcher, there is an expectation to adhere to Waikato University's Ethical Conduct in Human Research and Related Activities Regulations, relating to activities involving interests and rights of others (University of Waikato, 2008). The Kura Toi Tangata Faculty of the University of Ethics Committee approved our ethics application. The project was developed in consultation with the teachers. We followed all ethical protocols, including informed consent, the right of withdrawal, the right to confidentiality, and the preservation of anonymity, using pseudonyms in referring to the relevant institutions and the individuals within them.

Context of study

The teaching took place as part of regular classroom statistics teaching in three largely Pasifika student-dominated year 12 classes. Pasifika students come from many island nations such as Samoa, Cook Islands, Tonga, Fiji, and Tokelau. Pasifika is a term of convenience to encompass this diversity. The students had different cultural backgrounds, values, and belief systems and were not a homogenous group. The school was designated as a Decile 1 (govt.nz/secondarieschool/secondary-schooling-in-nz/deciles/) catholic boys' school. This category was applied because many of its students lived in low-socio-economic or poorer communities.

The school had approximately 1000 students with 90% Pasifika, 5% Māori, and 5% Asian. About half of these boys are ELLs.

Statistics is one of the three strands of mathematics learning area (Ministry of Education, 2007). The statistics strand of the *New Zealand Curriculum* is made up of three threads: statistical investigations (statistical thinking), statistical literacy, and probability.

Statistical investigations involve working through a five-stage process (Wild & Pfannkuch, 1999), which includes *problem*, *plan*, *data*, *analysis*, and *conclusion* (PPDAC). In New Zealand classrooms, statistics is taught using the PPADC statistical enquiry cycle across all year levels of schooling.

PPDAC statistical enquiry cycle

Problem—identifying what is to be investigated and why it is important.

Plan—deciding what data needs to be collected, who to collect it from, how to gather it, and how to use the resulting data to answer the original question.

Data—collecting and organising the data.

Analysis—exploring the data and reasoning with it, usually by creating graphs to display the data and calculating statistics (e.g. the number in each category; the average; the range) to summarise it.

Conclusion—answering the original question (from the problem stage) and drawing on the data and analysis to justify this answer.

As part of the learning activities, students carried out investigations of existing data sets (<https://new.censusatschool.org.nz/resource/kiwi-kapers->) using the statistical enquiry cycle (Ministry of Education, 2007). This data file is comprised of a fictitious kiwi population based on actual counts and values from NZ kiwi populations. There are 700 kiwis in the population. The student investigation was conducted over three lessons, the duration of each was 50 min. Kiwis are birds native to NZ. The kiwi is a unique and curious bird, it cannot fly and has loose, hair-like feathers, strong legs, and no tail (see image below) (Fig. 1).

This investigation is related to the achievement objectives: carry out investigations of phenomena, using the statistical enquiry cycle—using existing data sets; evaluating the choice of sampling and data collection methods used; using relevant contextual knowledge, exploratory data analysis, and statistical inference; making inferences from surveys; using sample statistics to make point estimates of population parameters; and recognising the effect of sample size on the variability of an estimate in the statistics strand of the mathematics and statistics learning area (Ministry of Education, 2007).



Fig. 1 Kiwi bird (https://www.google.com/search?q=image+of+kiwi+bird&rlz=1C5GCEM_enNZ948NZ948&oq=&gs_lcrp=EgZjaHJvbWUqCQgBEEUYOxjCAzIJCAAQRrg7GMIDMgkIARBFgDsYwgMyCQgCEEUYOxjCAzIJCAMQRrg7GMIDMgkIBBBFGDsYwgMyCQgFEEUYOxjCAzIJCAYQRrg7GMIDMgkIBxBFgDsYwgPSAQw3Mjg3NzYzajBqMTWoAgiwAgE&sourceid=chrome&ie=UTF-8)

Findings and discussion

A thematic analysis was used to generate emerging themes (Braun & Clarke, 2006) from teacher reflections data. The researcher examined transcribed audio-recorded data to identify emerging themes. These themes were then coded in the summaries, and an online meeting was held with the teachers to discuss whether the themes were supported and if any themes needed to be divided or blended. The discussion is supported using the participants' voice through direct quotations.

Focus on reading and writing

One of the themes that emerged from data analysis involved a focus on reading and writing. All teachers mentioned students' difficulties with reading, speaking, and writing in English. This affected the students' ability to engage in class work. Teacher C's reflection suggests that sometimes mathematics teachers might not have the skills to teach the written component of statistics.

I agree. It comes across a range of disciplines that writing is a problem for most of our students. It comes out in assessments. Students leave sections blank. They need to develop writing skills. Mathematics teachers find it hard when you got to do scaffolding. We are not natural teachers of writing. It is okay in mathematics and then as maths teacher [that]we are not good at a particular way of writing and helping with statistics requires a different way of writing. (Teacher C)

Although concerns were expressed about integrating writing in statistics, the teachers supported their students in their writing by integrating language strategies in their lessons, which is an important component in statistics lessons, in particular statistical projects. All the teachers reported that for their statistics sessions, they had to write and draw a lot on the whiteboard and used class notes so the students could follow what was being discussed. Writing down the key terms helped students see them and connect them to the spoken word.

Teacher A and Teacher C also provided a writing frame and cloze activities to help students analyse data and draw conclusions. For example, the following writing frame was provided for writing conclusions about kiwi population data.

The evidence from mysuggests that of kiwis is betweenand Approximately of kiwi arebetween..... and.....

Students completed the statements from their sample analysis and then wrote a statement about what they thought might be happening in the population. Some statements were collected on the board so that the class could look at the overall results.

Teacher A further reflected:

I believe writing in statistics can take various forms, including writing the processes involved in a statistical investigation and writing the conclusion that answers the statistical question. Although students complain that writing is time

consuming, it improves communication skills and helps them grasp statistical language and concepts. They can apply these skills in other learning areas.

All the teachers reported that writing down the key terms helped students see the written and connect the spelling of the word to the spoken word. This use of writing on the board to aid language learning and comprehension of the students concurs with the findings of Sharma et al. (2011) in a different study and Winsor (2007). Sharma et al. (2011) found that writing words/vocab on the board re-enforced learning for Pasifika students. Teacher A's reflections concur with the findings of Jaafar (2016), who claimed that the integration of writing activities not only help students cement mathematical knowledge, deepen understanding, and develop appreciation for the rigor and concision of mathematical language, but also enable them to develop learning habits essential to their success in any field.

Adjusting teaching approaches

What may seem a normal speaking pace to a native speaker of any language may seem too fast for an ELL to comprehend the narrative. The addition of complex terms and concepts in statistics can make learning even more difficult. All the teachers adjusted their teaching and learning approaches based on evidence from research literature to improve students' achievement of learning outcomes. Teacher A drew diagrams, wrote questions, and added instructions and explanations to the whiteboard. He checked before proceeding that the students could read and understand what was written the board before he started talking. Teacher B gave explanations and instructions in clear and simple language to make sure students understood the instructions.

I try to give instructions step-by-step before asking students to do independent, pair, or group work. Then I ask one of the boys to repeat the instructions aloud for the rest of the class to make sure all have understood what is required. (Teacher B)

In his whole class sessions, Teacher A slowed down his speaking pace. At times, Teacher C reported modifying the linguistic complexity of her speech by using shorter sentences and rephrasing questions.

As well as modifying speech, Teacher C also wrote notes and questions on a mini white board she used during her small group interactions. Teacher C reflected: *the best technique was to focus on writing throughout out the statistical inquiry cycle; you can see it in the books. I will do this more often in my teaching.*

Providing non-linguistic cues such as visual diagrams, drawings, and gestures can make more complex language accessible for all learners and the teachers seemed to be intuitively aware of this. The teachers used strategies that supported students visually and were helpful in scaffolding students who may not have the language skills to match their statistical ability.

The findings are consistent with the studies done by Nguyen and Cortes (2013) and Lee et al. (2011). Nguyen and Cortes claimed that visual aids, such as diagrams and posters can enable students who may not have the ability to pose their questions in English, or who do not have the confidence to approach their teachers, to find answers.

Posing statistical questions

There are three different types of questions that students' progress through as they conduct statistical investigation.

Summary questions—a description of the data, usually involves single variable data.
 Comparison questions—comparing two or more sets of data across a common variable.
 Relationship—involves investigating a relationship between two variables (Arnold & Pfannkuch, 2018).

Year 12 students in this study had difficulty posing appropriate statistical questions from kiwi data. All the three teachers found students struggling to write appropriate comparison and relationship questions. Teacher A reflected:

I found students struggling to write good comparison and relationship questions. I put a summary, comparison and a relationship question on the board and asked them to critique them using “What makes a good question?” criteria. I spent one whole session on posing statistical questions.

Teacher A's modelling helped students realise statistical questions can be classified into three categories.

Reflection regarding the need to support the writing of different types of statistical questions based on data is consistent with claims made by Arnold and Pfannkuch (2018). These authors state that while students can pose comparison questions, they have trouble posing summary questions. The authors suggested that students might need clarity around the purpose of questions at each phase of the statistical inquiry cycle. This was addressed by Teachers B and C when they reminded the students to keep the question in mind at every phase of the statistical enquiry cycle.

Collaborative learning

The three teachers used collaborative learning strategies when implementing the statistical investigations in the classroom. For most statistics sessions, the students were asked to form groups to discuss the ideas and questions they might have relating to the statistical enquiry cycle. Group work allowed the students to ask questions and get feedback in a safe learning environment as pointed out by Teacher C.

Students are often not eager to share their ideas in front of the whole class. It is not productive to ask the boys to give answers to the entire class. They may not feel confident with their level of English and content [knowledge]and going public may make them more uncomfortable.

All the teachers mentioned that they are careful about how they group the students. They found that sometimes the boys did not engage in productive talk, so they had to use different grouping methods.

I group students of the same home language so they can process information together using their home language. The more proficient English speakers can support [others in the group] in making sense of the information. (Teacher A)
Sometimes I use ability grouping to deal with the range of ability levels, so I don't hold some students. I can help students who are having difficulty. and improve learning outcomes for all students. (Teacher B).

Collaborative work allowed the students to collaborate in their learning and ties in with the work of Brown et al. (2009) and Goldenberg (2008), who explained that when language learners can work alongside a partner, they are given the opportunity for interaction and support, thereby enhancing their learning. However, this study indicates that simply putting students in groups does not mean students will engage in productive discussion. It seems that Teacher B constructs ability grouping in the best interests of students. His belief seems to be based on the thought that constructing ability groups enables him to target students' learning needs. The construction of ability groups by teacher B may promote performance goals (results based on tests) and a fixed mindset (Boaler, 2014).

Home language

In this study, students were supported by teachers and peers to use their home language, English, and mathematical/statistical English to discuss and develop their understanding about statistics as reflected in the following quotations:

Students can use their home language in groups. On whole class discussions they are required to use English. Some of them don't want to use their home language in [the] mathematics class. (Teacher B)

The teachers identified that students' first language(s) can serve as a resource for thinking and communication as students simultaneously learn and develop proficiency in the language of instruction as they learn statistics.

They use home language outside the classroom. In mathematics classes they sometimes mix language. Even teachers sometimes code switch although they don't realise it to gain student attention or to build positive relationships with students. (Teacher C)

While research shows that many teachers believe using home language is detrimental to learning (Edwards, 2012; Mady & Garbati, 2014; Planas & Setati-Phakeng, 2014; Robertson & Graven, 2020; Winsor, 2007), this was not the case for the three participants who could see the educational value of learners being able to use their home language(s) in the classroom. However, the teachers' use and perceptions of the value of a particular language in different settings varied. For example, while students used home language in group settings, they were required to communicate in English in whole class discussions. One reason for this could be that none of the teachers in the current study was proficient in a Pacific language. The findings concur with Planas and Setati-Phakeng (2014) who reported that while students used their home language in small group settings, they did not do this during whole-class discussions.

The research also highlights the constraints imposed on ELL learning when the teacher is not of their ethnicity or familiar or fluent with their cultural backgrounds.

Using real-life contexts

When students explore statistics through a meaningful context, they can derive meaning of abstract concepts, such as variation, and see that statistics is an integral part of their lives (Lesser, & Winsor, 2009; Watson, 2006). The teachers in this study were aware of making connections to the experiences and cultures of Pasifika students as reflected in the following quotations.

If the learning context is familiar to students, it makes more sense to them. They will be more engaged and motivated in the lesson. (Teacher C)

When I look back should have done investigation outside the classroom. Students need to pose questions on something that is relevant to them or their community. Maybe, involve the parents as well. Benefit the students' experiences. They need to understand the whole process of the statistical enquiry cycle. (Teacher A)

Teacher C commented on the importance of building contextual knowledge in statistics to avoid confusion and misinterpretation of the data and the context.

It is important they understand what population and variables they can make links to. Some of them have never seen a kiwi, they don't understand the context. Halfway, I realised that some of them were interpreting kiwi as kiwi people. They related the data to people. Next time I will spend more time on context, may be show them a video clip about kiwi population. (Teacher C)

Students, when carrying out statistical investigations focused on real-life contexts, can often get side-tracked by irrelevant details while ignoring relevant information. For example, some students in Teacher C's class interpreted kiwi birds as kiwi people. The students were not familiar with the context that produced the data. The findings concur with the findings of Lesser and Winsor (2009) and Sharma (2014). Lesser and Winsor (2009) reported a student's confusion from a context-rich exercise about correlation because the term "ski resort" was unfamiliar in her high-poverty urban city in a desert region. In this study, a simulated data set of kiwis was used to conduct a statistical investigation. Students need to spend more time understanding the nature of the raw data and what it means for data analysis. Students in this study could have explored data sets that had cultural relevance from Censu-AtSchool or Statistics New Zealand.

Using games

Students love games and playing games in the classroom improved students' attitudes and motivation towards and encouraged students to participate actively. Teacher A and Teacher C mentioned using card games to help Pasifika students develop their statistical vocabulary. For example, Teacher B used a fun game called *Forbidden*

Words to start or end a lesson. The idea of the game was for one player to try and describe a statistical term or phrase without using certain forbidden words. The other players had to try and guess the word. For example, Fila picked out word card “standard deviation” He had to describe the phrase without using the words mean, variance, square root, and sigma. Teacher B reflected:

The game allowed the students to interact with each other while practicing statistical terminology. It encouraged the boys to listen carefully, and pay attention to what others were saying, concentrate and remember. It was a fun way to help the boys negotiate meanings of statistical words without even realizing it. However, if a classroom culture has not yet been strongly established, it can negatively impact on student engagement and learning.

Teacher C added that games can boost student confidence because they are learning in a safe learning environment. She stated:

I realise that hands-on activities like the forbidden words game have the potential to boost student confidence, engage students and contribute positively to their vocabulary learning. I will use games like this for other topics. To make the game easier teachers can allow the students to use one of the forbidden words or have a scoring system based on the number of forbidden words used.

Teacher reflections about using games resonate with the findings of Chow et al. (2011) and Nisbet and Williams (2009). Chow et al. claim that games are very effective alternative activities that provide students with a learning environment that is fun and educational. Additionally, games help in creating opportunities for independent learning and overcoming challenges for ELL students who are reluctant to participate in other mathematical activities because of language barriers will often join in a game, and so gain access to learning as well as engage in structured social interaction.

Limitations and implications for practice and research

Due to the internationalization and globalization of mathematics education, there has been a growing interest in language and cultural issues in multilingual settings. Hence, this research will be of interest to the international community because it involves looking at issues that are relevant for schools in English speaking nations worldwide. Teachers may need to re-evaluate their teaching practices, especially if part of their population is learning English as a second language.

Previous studies (Edwards, 2012; Robertson & Graven, 2020; Sharma et al., 2011) show that dealing with multiple languages in statistics classrooms can be challenging for teachers. The teachers in the present study demonstrated a range of specific strategies consistent with research-based effective language learning practice. Whether this was by virtue of prior learning in teacher education or professional development, or by experience in the collaborative setting cannot be determined here, but this could be an area for future investigation.

The three participants in the study used a variety of linguistic strategies that qualify as translanguaging during their statistics teaching. These strategies were used to make sense of the content and to elicit students' thinking but not necessarily to support students' first language. They also used translanguaging to establish rapport with students and to affirm knowledge of a common indent. More evidence on the benefits of translanguaging in statistics education needs to be gathered, ideally through interventions in which the effectiveness of translanguaging as a pedagogy is tested in linguistically diverse mathematics classrooms. Furthermore, research on translanguaging as a tool for assessing students' mathematical knowledge is necessary to clarify its benefits when used in formative and summative assessments.

This study did not intentionally look at ways in which features of Pasifika languages can help re-enforce concepts of statistics. For example, Lesser and Winsor (2009) noted that in Malay the expression for the mean is "mama rata", which translates roughly as "same level". Hence, the Malay language invokes the "levelling" conceptual interpretation of the mean. The authors added that similar examples might be identified through cognates in Spanish. This idea could be the focus of future research.

The number of participants in the study is small and non-random; thus, there are limitations on the generalizability of results. It was not possible to isolate whether language strategies used were because of age, gender, or prior experience. A study with more participants might well achieve these types of results, which would then have implications for constructing support to change teacher practices. In addition, it would be valuable to know what the students thought about strategies used by the teachers.

Conclusion

The focus of this article was to explore the language practices used by teachers in multilingual statistics classrooms. Findings from the teacher reflections show that the teachers developed a nuanced awareness of issues relating to language and communication in multilingual settings. They used a range of strategies such as visual strategies, collaborative learning, home language, meaningful contexts, teacher modelling, and writing strategies which are supported by the research literature to enhance comprehension and cognition amongst the multilingual students. However, some strategies such as visual strategies, collaborative learning, and writing strategies worked better than others.

Language diversity of students in schools is expanding worldwide, and educators need to improve how they teach these learners. The findings reported in this paper have the potential to generate more interest in language challenges and strategies in statistics education and collaborative research where teachers are regarded as key stakeholders in all aspects of the research process. There is a need to commission and or support research focused on multisite ethnographic studies of second-language mathematics classrooms. Such studies are necessary to develop the kind of cumulative findings called for by Barwell (2020) and de Araujo et al. (2018) because they can be used to compare mathematics teaching and learning processes and outcomes across a range of different settings. Consciously pursuing equity as an objective, mathematics teachers can create nurturing environments in which ELLs

can be successful at both learning mathematics with understanding and at becoming more proficient in the instruction of language without sacrificing their home language or culture. Teachers, curriculum developers, and researchers need to continue to work together to find ways to help all learners develop statistical thinking within the milieu of culture, language, teaching strategies, learning resources, and curriculum demands that characterise many mathematics classrooms.

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Declarations

Ethics approval and consent to participate Ethics approval was sought from the University of Waikato. Participants were provided with an information sheet with details of the research, and informed consent was sought.

Conflict of interest The author declare no competing interests.

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