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Dynamical age of solar wind turbulence in the outer heliosphere

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Abstract. In an evolving turbulent medium, a natural timescale can be defined in terms of the energy decay time. The time evolution may be complicated by other effects such as energy supply due to driving, and spatial inhomogeneity. In the solar wind the turbulence appears not to be simply engaging in free decay, but rather the energy level observed at a particular position in the heliosphere is affected by expansion, "mixing," and driving by stream shear. Here we discuss a new approach for estimating the "age" of solar wind turbulence as a function of heliocentric distance, using the local turbulent decay rate as the natural clock, but taking into account expansion and driving effects. The simplified formalism presented here is appropriate to low cross helicity (non-Alfvénic) turbulence in the outer heliosphere especially at low helio-latitudes. We employ Voyager data to illustrate our method, which improves upon the familiar estimates in terms of local eddy turnover times.

1. Introduction

During the roughly 30 years of observation of low-frequency fluctuations in the solar wind, it has become increasingly apparent that turbulence at magnetohydrodynamic (MHD) scales is an important feature of the heliosphere. MHD turbulence may provide the mechanisms for energy loss from large scale structures, cascade processes at intermediate scales, and excitation of kinetic processes at small scales. Consequently, turbulence may play an essential role in heating [*Coleman*, 1968] of the solar wind plasma, and possibly acceleration of both the slow [*Hollweg*, 1986] and fast [*McKenzie et al.*, 1995] wind. Similarly, turbulence properties have strong influence upon spatial transport of heat and wave energy [*Tu and Marsch*, 1993; *Matthaeus et al.*, 1994a]. A particularly important influence of turbulence is through scattering of energetic particles, including pickup ions and cosmic rays [e.g., *Bieber et al.*, 1994; *Zank et al.*, 1998], and this is closely associated with spatial transport and diffusion effects that can ultimately influence the entire structure of the heliosphere.

In each scenario in which MHD turbulence may be thought to be important, crucial questions arise concerning the relative strength of turbulence: How influential is turbulence relative to other effects? Is it so strong that it constrains other processes? An example would be the assumption that "scattering centers" are effective enough to enforce local isotropy of particle distributions. Or, is it weak enough that it is only a small correction to other effects? The latter would be consistent with adopting a wave dispersion relation or magnetostatic quasi-linear scattering theory. Sometimes this issue can become rather subtle, for example, when one questions the reasons for apparent accuracy of WKB theory in describing radial profiles of solar wind fluctuation levels from 1 to about 10 AU [e.g., *Verma and Roberts*, 1993; *Zank et al.*, 1996]. Often a simple way to address these questions is through comparison of characteristic timescales of turbulence and other relevant processes. The strength of turbulence can be meaningfully quantified by identification of a rate of turbulent evolution. This provides a natural definition for "aging" of the turbulence, in that arbitrary time intervals can be referred to units of the intrinsic dynamical timescale.

The purpose of the present paper is to explore the issue of dynamical aging of MHD turbulence in the solar wind between 1 AU and about 40 AU using a simple formalism that can be compared with Voyager observations. The goal is to place earlier qualitative estimates on firmer footing [e.g., *Matthaeus and Goldstein*, 1986] and to point the way toward more complete understanding of the role played by turbulence throughout the heliosphere.

We employ three related methods in estimating the age. There are important differences: The first method is the only one that does not utilize a theoretical model for energy decay. The second is entirely theoretical, having no input from observations except for boundary conditions. The third method uses no models for inhomogeneous effects, in contrast to the first two approaches. Comparison of these three approaches shows

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agreement to within factors of two from 1 AU to about 40 AU. Several ways in which the present methods might be improved are briefly described in the Discussion section.

2. Age in Decaying Turbulence

A convenient choice for the natural "clock" is the timescale associated with energy decay in homogeneous MHD turbulence. In the absence of any driving mechanism, the equation for decay of nonthermal fluid scale energy per unit mass E can always be written as

$$\frac{dE}{dt} = -\frac{E}{\tau}, \qquad (1)$$

which serves to define the timescale $\tau(t)$. The dimensionless age of the turbulence $\hat{t}$ is now defined by

$$\hat{t} = \int_0^t \frac{dt'}{\tau(t')}. \qquad (2)$$

Under the assumption of free decay, using (1) the age can be written as

$$\hat{t} = -\int_0^t \frac{1}{E}\frac{dE}{dt} dt' = \log\left[\frac{E(0)}{E(t)}\right]. \qquad (3)$$

Thus, to make a fair comparison between different samples of decaying homogeneous turbulence, one looks at the systems when there remains the same fraction of initial energy. Such turbulence is of equal "age." This convention is familiar in numerical simulations when comparing undriven dissipative runs having different parameters such as Reynolds numbers [e.g., *Matthaeus et al.*, 1996a].

One can go further if the decay law is related back to turbulence parameters. A well-known example adapted from hydrodynamics is the Taylor–Karman decay phenomenology in which the decay timescale becomes the "eddy turnover time,"

$$\tau = \frac{\lambda(t)}{u(t)}, \qquad (4)$$

where λ is the energy-containing scale (often taken to be a correlation or outer scale [see *Batchelor*, 1970]) and $u \sim (2E(t))^{1/2}$ is the characteristic speed of the turbulence, and E is the turbulent flow energy per unit mass. For simple decaying homogeneous turbulence this provides a complete phenomenology: $\dot{\lambda} \sim u$; $\dot{u^2} \sim -u^3/\lambda$. This type of model provides a closed form energy decay model [*von Karman and Howarth*, 1938] and has been found using simulations to be reasonably accurate (within say a factor of 2) for a wide range of both hydrodynamic and MHD parameters [*Hossain et al.*, 1995]. In MHD the magnetic energy per unit mass $b^2/2$ complements the flow energy, where b is the rms magnetic fluctuation in Alfvén speed units, i.e., $b = \delta B/(4\pi\rho)^{1/2}$ for mass density ρ and δB the variance of the magnetic fluctuations. The MHD Taylor–Karman phenomenology has been proposed as an approximate description of energy decay in locally homogeneous solar wind turbulence in the context of scale separated transport theory [*Matthaeus et al.*, 1994a].

The usefulness of the simplest MHD phenomenology is restricted to low or zero cross-helicity (correlation between magnetic and velocity fluctuations) turbulence, and for this case the single relevant Elsässer amplitude is $Z^2 = u^2 + b^2$ [*Marsch and Tu*, 1989]. The corresponding zero cross helicity MHD timescale is $\tau = \lambda/Z$ [*Kraichnan*, 1965; *Dobrowolny et al.*, 1980; *Hossain et al.*, 1995]. In the solar wind this condition is generally applicable to the outer heliosphere, especially at low latitudes where observations indicate that high cross-helicity Alfvénic fluctuations [*Belcher and Davis*, 1971] are relatively absent [*Roberts et al.*, 1987a,b]. The applicability of a low cross helicity phenomenology to high latitude solar wind is more in doubt, since Ulysses observations indicate that high cross helicity regions extend further outward in the polar wind [*Goldstein et al.*, 1995].

A straightforward estimate of the rate of aging of solar wind turbulence is obtained by calculating the eddy turnover time from well known near earth 1 AU solar wind parameters [e.g., *Matthaeus and Goldstein*, 1982]. This amounts to adopting the Taylor–Karman picture, i.e., a strong turbulence model, while ignoring, for example, cross helicity effects on decay [e.g., *Dobrowolny et al.*, 1980; *Grappin*, 1982, 1983; *Pouquet et al.*, 1986, 1988]. Using $\lambda \approx 1/50$ AU $= 3\times10^{11}$ cm, and $u = 10$ km/s, gives $\tau(1$ AU$) = 50$ to 100 hours. Noting that the transit time of the equatorial wind to 1 AU at 400 km/s is ≈ 100 hours, we would estimate that low helio-latitude solar wind turbulence ages at a rate of about 1–2 eddy turnover times per AU of radial convection. Such estimates, which are expected to vary with radial distance, are familiar in solar wind studies [e.g., *Velli et al.*, 1989; *Matthaeus and Goldstein*, 1986] but are of uncertain accuracy since they ignore the effects of cross helicity (especially in the inner heliosphere) and inhomogeneity, as well as shear, transport, and possibly pickup ion effects that may be important in the outer heliosphere [*Zank et al.*, 1996].

3. Age in Expanding, Driven Wind

Important generalizations to (1) are embodied by transport theories of the form

$$\frac{\partial E}{\partial t} + U\frac{\partial E}{\partial r} + \frac{AU}{r}E = -\frac{E}{\tau} + S, \qquad (5)$$

which describe radially symmetric transport of scale-separated inhomogeneous turbulence in a wind of constant speed U. Here r is the radial coordinate, A is a constant, and S represents driving (source) terms that inject energy into the turbulence field. A variety of transport formalisms can be expressed by (5), ranging from WKB transport of noninteracting waves [*Parker*, 1966; *Barnes*, 1979; *Hollweg*, 1974] to transport of strongly interacting MHD turbulence [*Zhou and

Matthaeus, 1990; *Marsch and Tu*, 1989; *Matthaeus et al.*, 1994b]. A simplified theory of this form was examined in detail by *Zank et al.* [1996] and *Matthaeus et al.* [1996b] using assumptions appropriate to solar wind fluctuations in the outer heliosphere. These assumptions include low cross helicity, fixed turbulence symmetry, and low large-scale Alfvén speed (compared to U). In general, the parameter A in (5) includes effects such as "mixing," expansion, compression, and shear and has thus been refered to as the "MECS" parameter by *Zank et al.* [1996]. In the simplified formalism, A is treated as a constant. Note that the value $A = 1$ corresponds to a WKB expansion if one also sets the decay and driving terms to zero. The simplified forms of the theory, including dissipation and driving, admit analytical solutions of interest in the present discussion.

For steady conditions we may write (5) as

$$U\frac{dE}{dr} \approx -\frac{E}{\tau} - \frac{E}{\tau_{\rm exp}} + \frac{E}{\tau_{\rm shear}} + \cdots \qquad (6)$$

The timescales τ (the energy decay time), $\tau_{\rm exp} = r/(A \times U)$ (the expansion time) and $\tau_{\rm shear}$ parameterize the corresponding terms of the transport equation (and in general depend upon position and time.) Here we consider only the case where driving is due to instability associated with solar wind streams [*Roberts et al.*, 1991, 1992; *Zank et al.*, 1996]. Other driving effects may include, for example, energy injection due to excitation of turbulence by interstellar pickup ions [*Williams et al.*, 1995]. The shear time is estimated by *Zank et al.* [1996] to be $\tau_{\rm shear} = r/(C_{\rm sh}U)$ and the constant $C_{\rm sh} \approx 10$ when estimated from 1 AU solar wind parameters.

Using a change of variables to the convection time at speed U relative to a reference position r_0,

$$t(r) = \int_{r_0}^{r} \frac{dr}{U}, \qquad (7)$$

we can formally integrate the steady transport equation to find that

$$\log\left[\frac{E_0}{E(t)}\right] = \hat{t} + \hat{t}_{\rm exp} - \hat{t}_{\rm shear}. \qquad (8)$$

The last two terms on the right-hand side represent the cumulative effects of expansion and shear, respectively, on the turbulence at convection time $t(r)$ (radial position r). The energy at reference position r_0 is E_0.

In spite of the complications that appear due to spatial inhomogeneity and driving effects, one can always compute the turbulence age directly if one adopts a theoretical model for the energy decay time. As mentioned above, for a Taylor–Karman MHD model $\tau(r) = \lambda(r)/Z(r)$ and the dimensionless age is

$$\hat{t} = \int_{r_0}^{r} \frac{dr}{U}\frac{Z(r)}{\lambda(r)}. \qquad (9)$$

Thus, if the spatial variation of the energy-containing scale and the turbulent energy density is known by any method whatsoever, the dynamical age of the turbulence can be directly calculated.

Recall that in (9) $Z(r) = \sqrt{u^2 + b^2} = \sqrt{2E}$ is the amplitude associated with the (incompressible) turbulent energy density per unit mass E, and is equivalent to an Elsässer amplitude for the assumption of zero cross helicity. For connection with observations [*Zhou and Matthaeus*, 1990] it is sometimes convenient to assume that the (Alfvén) ratio $r_A = u^2/b^2 = E_u/E_b$ is constant; for equipartition $r_A = 1$ and $Z = \sqrt{2}b = 2\sqrt{E_b}$, where $E_b = b^2/2$ is the energy per unit mass of the magnetic fluctuations. This is reasonably in accord with observations [*Roberts et al.*, 1990] in which averages of 1 day estimates of r_A decrease with radius from about 0.8 at 2 AU and remain constant near 0.5 beyond 5 AU or so.

Even without adopting a particular theoretical connection between turbulence parameters and the energy decay time, it is clear from (8) that the turbulence age $\hat{t}$ can be determined if the fraction of energy remaining is known along with the quantitative effects of shear and expansion. For example, suppose that there are no driving effects. Expansion effects are taken into account by $t_{\rm exp} = A\log(r/r_0)$. For this simplified case the age of the turbulence as a function of heliocentric distance is simply

$$\begin{aligned} \hat{t} &= \log\left(\frac{E_0}{E}\right) - \hat{t}_{\rm exp} \\ &= \log\left[\frac{E_0}{E}\left(\frac{r_0}{r}\right)^A\right]. \end{aligned} \qquad (10)$$

For simple WKB expansion the constant $A = 1$, and we see that a proper evaluation of age of turbulence in a uniformly expanding medium differs from the homogeneous case by correcting the fraction of remaining energy by the radial factor r_0/r prior to computing the logarithm. This "undoes" the effects of expansion, since the age calculation is intended to be a measure of decay due to turbulence not reduction of fluctuations by expansion.

Note that this example does not require that we make use of a particular model for the eddy turnover time. Corrections for shear can also be modeled by computing the correction factor due to shear $\hat{t}_{\rm shear} = \int_{r_0}^{r} dr\, C_{\rm sh}/r = C_{\rm sh}\log(r/r_0)$.

4. Methods of Evaluating the Age

The above considerations suggest three simple methods, each somewhat distinct, that can be used to compute the dynamical age of turbulence in the solar wind.

4.1. Method I

We may use theoretical models for $\hat{t}_{\rm shear}$ and $\hat{t}_{\rm exp}$ to represent the effects of shear and expansion. A simple calculation shows that for this case

$$\hat{t} = \log\left[\frac{E_0}{E}\left(\frac{r}{r_0}\right)^{C_{\rm sh}-A}\right] \qquad (11)$$

where typically we would use $A \approx 1$ and $C_{\rm sh} \approx 10$. The required values of E_0/E are then extracted from spacecraft data, allowing computation of the age.

4.2. Method II

If we employ a theoretical model for τ directly, and solve the transport equations that map the required parameters in the radial coordinate, we may then compute the age directly from the integral (2). Effects of shear and expansion are included in the transport model, so there is no need for observational data except for the boundary data required by the theoretical calculation. As an example we will use an MHD Taylor–Karman phenomenology for τ and make use of the analytically solvable models described by *Zank et al.* [1996] and *Matthaeus et al.* [1996b] including both shear and expansion effects. Using $\tau = \lambda/\sqrt{E_b}$ and the corresponding solution for $\lambda(r)$ and $E_b(r)$ given by Zank et al. (in their equations 25, 28 and 29), we find, from (9), and for the special case of $A = 1$, an explicit expression for the turbulence age,

$$\hat{t} = \log\left[1 + D\left\{\left(\frac{r}{r_0}\right)^{C_{sh}+1/2} - 1\right\}\right], \qquad (12)$$

where

$$D = \frac{\sqrt{E_{b0}}r_0}{(C_{sh} + 1/2)U\lambda_0}. \qquad (13)$$

This model requires specification of the turbulent magnetic energy density E_{b0} and the correlation scale λ_0 at the inner boundary at r_0. A closely related model with $\tau = \lambda/Z$ can also be solved exactly under the assumption of constant Alfvén ratio [see *Matthaeus et al.*, 1996b], and gives a result that would be identical to (12) except that the quantity E_{b0} in the constant D is replaced by Z_0, the Elsässer amplitude at reference position r_0.

4.3. Method III

The final method we consider is in a sense a hybrid of the first two. We use the Taylor–Karman form of the eddy turnover time $\tau = \lambda/Z$ to compute the age from (9) as in method II, but in this case we will extract the turbulence parameters $Z = \sqrt{2E(r)}$ and $\lambda(r)$ from observations. Note that method I also relied on observations for $E(r)$ but did not require $\lambda(r)$ since no phenomenology for τ was assumed. Method III requires no modeling of any kind for shear or expansion, in contrast to both method I and method II.

5. Turbulence Age Results

In each test case described here, we calculate the age of turbulence relative to its state at 1 AU. Where necessary, we evaluate the radially dependent energy density $E(r)$ by using the fluctuating magnetic energy per unit mass derived from spacecraft observations of the interplanetary magnetic field. We use only the N component of the magnetic field. The N direction is normal to the radial direction and the direction of solar rotation. At low latitudes N is normal to the solar equatorial plane. Use of the N component alone avoids complications due to stream structure and sector boundary effects [*Ness and Wilcox*, 1965]. (Sector rectification is a possible alternative approach, but might be unreliable in the outer heliosphere [*Burlaga and Ness*, 1993].) The magnetic field fluctuations are computed from spacecraft vector averages (with cadence for Voyager data of 1 hour) using 10 hour data intervals, and averaging the results of such samples over approximately a solar rotation period. Data from the Voyager 1 and 2 spacecraft are used because they span the range of radial distances from 1 AU to 40 AU.

To evaluate the age of turbulence using method I, we employ (11) with $C_{sh} = 10$ and $A = 1$. This corresponds to the case of no "MECS" or "mixing" effects as discussed by *Zank et al.* [1996], as well as their estimated value of the shear driving constant. We can estimate E/E_0 from δB_N^2, the energy density (per unit mass, in Alfvén speed units) in the normal component of magnetic field, by employing two assumptions, that the Alfvén ratio is independent of heliocentric distance, and that δB_N^2 is a constant fraction of the total magnetic variance δB^2. This is equivalent to assuming the turbulence has fixed symmetry with the mean field direction normal to N. (See *Klein et al.* [1991] for a discussion of variance ratio anisotropies in the outer heliosphere.) With these assumptions, (11) becomes

$$\hat{t} = \log\left[r_{AU}^9 \frac{\delta B_N^2(1\ AU)}{\delta B_N^2(r)}\right], \qquad (14)$$

where r_{AU} is the radial coordinate in units of AU (1.5×10^{13} cm). Note that the boundary value $\delta B_N^2(1\ AU)$ is computed from omnitape data [*King and Papitashvili*, 1994], making the evaluation by time-lagging back to 1 AU from the time of observation at a given r using a constant 450 km/s wind speed. The goal is to (nominally) look at the same plasma parcel at 1 AU and at r, thus providing a correction for solar cycle effects.

Figure 1 shows the results of this method I analysis using Voyager 1 data from 1 AU to about 40 AU and Voyager 2 data from 1 AU to about 30 AU. What is apparent is an increase in age by about 40 characteristic times (eddy turnover time) during convection from 1 to 40 AU. While there is on average an aging of about 1 turnover time per AU, it is evident that the rate of turbulent evolution seems to be clearly slowing with increasing heliocentric distance.

The formula (12) for method II evaluation of the turbulence age relative to its state at 1 AU is made concrete using the same shear and expansion constants ($C_{sh} = 10$, $A = 1$) as were used in method I. However, the constant D defined by (13) represents boundary data and is determined using estimates of 1 AU turbulence parameters ($\sqrt{2E_{b0}} \approx 20$ km/s, $\lambda_0 \approx 3 \times 10^{11}$ cm = 1/50 AU, $U \approx 400$ km/s, $r_0 = 1$ AU). Thus we estimate $D \approx 1/4$. The corresponding values of turbulence age for Voyager 1 observations from 1 AU to 40 AU are plotted as dashed lines in Figure 2. The two related phenomenologies, in terms of either $\tau = \lambda/\sqrt{E_b}$ ($D = 1/4$), or $\tau = \lambda/Z$ ($D = 1/2$) are both shown in the figure, using longer and shorter dashed lines, respectively. The cases differ only slightly from one another

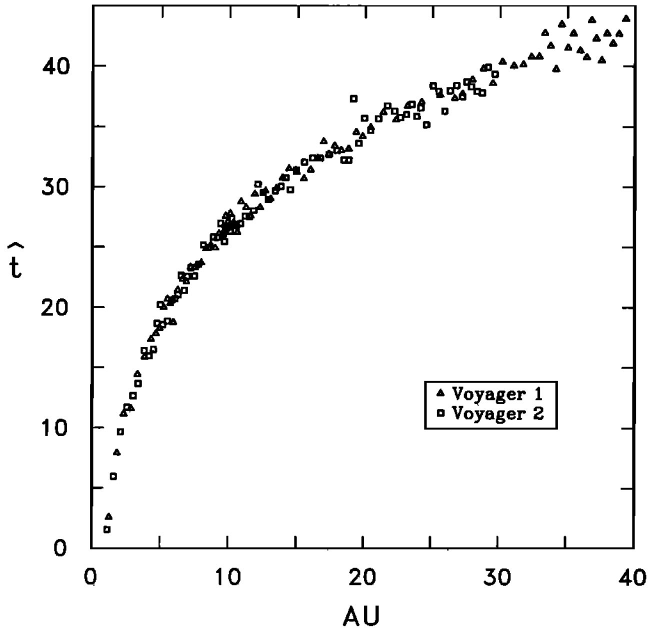

Figure 1. Dimensionless age of turbulence relative to state at 1 AU, for method I. Shear and expansion are accounted for using simple analytical models described in the text. Results shown are for constants $C_{sh} = 10$ and $A = 1$. Turbulent energy density as a function of heliocentric distance, normalized to 1 AU is obtained from Voyager 1 (triangles) and Voyager 2 (squares) data using the component of magnetic field fluctuation normal to the ecliptic. Data point values are obtained using (11).

and can be compared, in Figure 2, with the subset of method I results that were obtained from Voyager 1 data over the same interval. The models compare reasonably well over the entire range of radial distance, differing by about 15% at 40 AU. In Figure 3 the same two method II cases, represented by the same dashed lines, can be compared with the method I results from Voyager 2.

Method III is unique in that it requires computation of the energy-containing scale, taken here to be the correlation scale λ associated with the normal component of the magnetic fluctuations. As with extraction of any quantity that is sensitive to the long wavelength, low frequency part of the observed signal, there are considerable uncertainties in evaluation of the correlation scale from data intervals of finite duration. Here we make use of estimates of $\lambda(r)$ based upon the autocorrelation function of the observed δB_N, defined as $R_{NN}(\eta) = \langle \delta B_N \delta B'_N \rangle$, where η is the distance between the positions at which δB_N and $\delta B'_N$ are measured. We employed several approaches, most of which involved evaluation of λ as the normalized integral under the correlation function, $\lambda = \int_0^\infty R_{NN}(\eta) d\eta / R_{NN}(0)$. For finite data samples, this integral must be cut off, leading to errors that systematically depend upon unresolved or poorly resolved low frequency structures. A second approach is to approximate λ as the distance over which the correlation function falls to $1/e \approx 0.36787$ of its peak value at zero separation. This latter, so called "e-folding" definition of the correlation length, produced the most stable results, and the present results are entirely of this type. In this analysis we also make use of the approximation that the energy density can be computed from the magnetic normal component variances, and we approximate that $Z(r) = \sqrt{2}b$ (equipartition). In addition we assume that the fluctuations are transverse to, and rotationally symmetric about the mean magnetic field direction, which itself is assumed to be orthogonal to the N direction. (This is consistent at low latitudes with a fixed symmetry relative to a Parker mean magnetic field. See discussion prior to (14).) Thus $b \approx \sqrt{2} b_N$, where $b_N^2 = \delta B_N^2 / \sqrt{(4\pi\rho)}$ is the variance of the N component of the magnetic fluctuation in Alfvén speed units. In this case the method III approximation to (9) becomes

$$\hat{t} = 2 \int_{r_0}^{r} \frac{dr'}{U} \frac{b_N(r')}{\lambda(r')}. \quad (15)$$

Method III results for the turbulence age are shown in Figure 2, based upon Voyager 1 data, and in Figure 3, based upon Voyager 2 data, and in each case are represented by the solid curve.

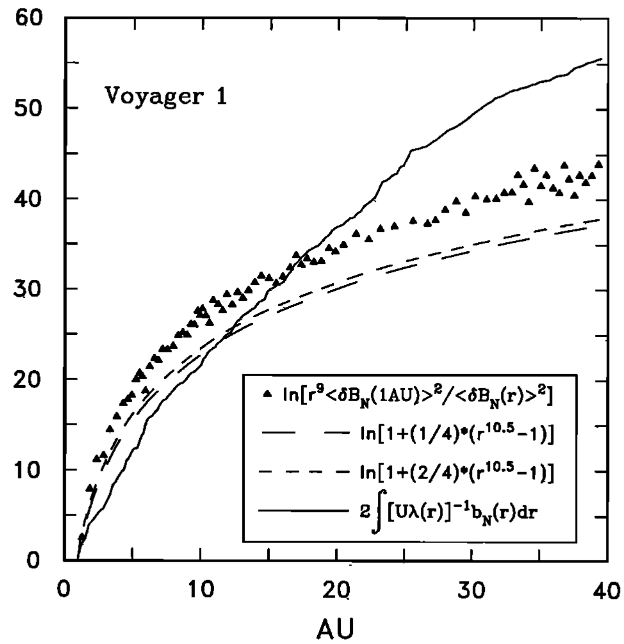

Figure 2. Turbulence age $\hat{t}$ calculated from Voyager 1 data using methods I, II, and III. Method I results (triangles) are a subset of those shown in Figure 1. Turbulence age obtained from method II is shown for the *Zank et al.* [1996] analytical decay model based upon $\tau = \lambda/\sqrt{E_b}$ (long dashed lines) as well as a similar model that employs $\tau = \lambda/Z$ (short dashed lines). The same values of C_{sh} and A are used for methods I and II. No observational data enter method II except for boundary data at 1 AU. The method III calculation of dynamical age, obtained from observed values of λ (via the e-folding technique) and energy (variance of normal magnetic component), is depicted by the solid line.

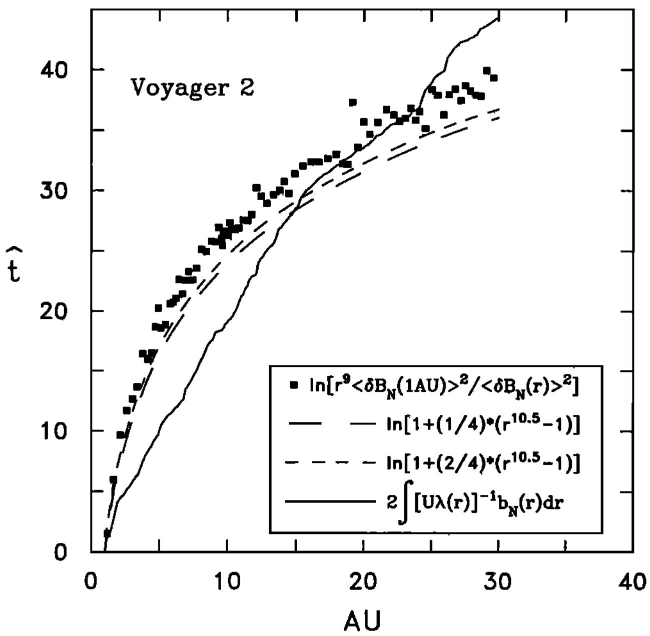

Figure 3. Similar to Figure 2, but using Voyager 2 data to calculate turbulence age $\hat{t}$ via Methods I, II, and III. Method I results (squares) are a subset of those shown in Figure 1. The turbulence age obtained from Method II is shown for the analytical decay models (dashed lines) as described in the text. The Method III calculation of dynamical age, obtained using observed values of the correlation lengths and energies, is depicted by a solid line.

6. Discussion

We have presented a determination of the "age" of turbulence, i.e., the passage of characteristic nonlinear or eddy turnover times, for solar wind turbulence from 1 to 40 AU near the ecliptic that have been explored by the Voyager magnetic field and plasma instruments. Three methods have been employed, and they agree moderately well out to 40 AU, i.e., within a factor of 2, although there is a suggestion that the discrepancies grow worse as the methods are extended for tens of AU. An optimistic interpretation of the results is that they confirm, at least roughly, the basic theoretical underpinnings of the methods. A more demanding and realistic interpretation is that the present results suggest avenues for future improvement of both the theory and method of analysis of the observations.

The data presented in Figures 2 and 3 summarize the results. Two models, methods I and II, make use of a simple model for shear driving, characterized by a "shear constant" $C_{sh} = 10$. Of these two, the method I result (triangles and squares) relies on observed values of turbulence energy, while the method II results (dashed lines) employ analytical models for radial variation of the turbulence level, making use of observed quantities only at the inner 1 AU boundary. The remaining trace in the figures (solid lines) are method III determinations, based upon the Taylor–Karman phenomenology and observed values of energy and correlation scale. It is important as well to recall what each model lacks. Method I assumes nothing about turbulence phenomenology. Method II ignores observations except at the inner boundary. Method III completely ignores inhomogeneous effects, such as shear and expansion.

The overall picture provides a fairly consistent view of the rate of aging of MHD turbulence in the solar wind from 1 to 40 AU. Given the intrinsic differences in the methods, and the lack of fine tuning in the examples presented, it is noteworthy that the aging estimates are as closely spaced as they are. On this basis, we cautiously claim that the results confirm some of the basic physics of the age determinations as we have defined them. There are a number of refinements that might render the various estimates in closer agreement. For example, an improved turbulence phenomenology (i.e., better theory for τ) would change the results for methods II and III, but leave method I results unaffected.

A very simple modification of method III would be a better observational definition of the energy-containing scale, or perhaps a better method for evaluating the correlation scale. Method I depends sensitively upon the assumed values of shear and expansion constants, and especially upon the form of the shear driving term in the transport equations. Sensitivity to both the phenomenology and the shear and expansion terms have influence upon method II results, which nonetheless agree rather well with those of method I. The reader should note that this correspondence would be expected on the basis of the comparison of transport theory and observation given by *Zank et al.* [1996] (see, for example, their Figure 4).

At any given distance between 1 and 40 AU, the estimates plotted in Figures 2 and 3 lie within about ±20% of their mean. In this interpretation the turbulence experiences about 40 eddy turnover times (or decay times) during the 170 days or so required for convection from 1 to 40 AU. On average this corresponds to about 4 days per eddy turnover, which is at the slower end of the range of expectations based upon 1 AU estimates. However, the models also indicate a gradual and monotonic slowing of the turbulence as it ages and evolves in radial distance, and Figures 2 and 3 support an estimate of about two turnover times per AU between 1 AU and about 10 AU. In the range 20–40 AU this slows to about 1/2 an eddy turnover time per AU.

Based upon the present simple models, a consistent picture is that solar wind turbulence experiences from 30 to 50 eddy turnover times between 1 and 40 AU, with the evolution from 1 to 10 AU being somewhat faster than the evolution between 10 and 40 AU. It has been our main purpose to examine the feasibility of such relatively simple descriptions of turbulence aging and evolution, and to this end we have simplified observational analysis, grouping together various types of solar wind fluctuation data, and have employed only very simple analytical models of shear, expansion, and decay.

Given the suggestive but not entirely satisfactory level of agreement seen in the three methods, it is tempting to envision improvements that might attain greater accuracy in general, and in particular better agreement. Indeed, a useful strategy would appear to be to use the differences in physical assumptions in the three methods to suggest and constrain improved models of several types. Perhaps most clearly indicated is a better theory for shear generation of turbulence, improving upon the simple approach based upon a single radially independent shear constant C_{sh}. The goal of such an improvement might be to make method I and II results move closer to the unchanged method III results.

We also must recall that the phenomenology that enters into methods II and III is specifically restricted [*Hossain et al.*, 1995; *Zank et al.*, 1996] to low cross helicity and thus would not be applicable directly to high latitude solar wind regions in which cross helicity remains high at least to 4 AU [*Goldstein et al.*, 1995]. In these Alfvénic regions, an age determination method would need to include an appropriate high cross helicity phenomenology [e.g., *Dobrowolny et al.*, 1980; *Grappin et al.*, 1982, 1983].

Most likely, several types of refinements will be needed to decrease discrepancies between the methods that grow at higher heliocentric distances. In fact all refinements might help in this regard, since the errors will accumulate as the methods march out from the inner boundary. However a future refinement that promises to specifically affect outer heliospheric results is modeling of the role of interstellar pickup ions in driving outer heliospheric turbulence [*Lee and Ip*, 1987; *Williams et al.*, 1995; *Zank et al.*, 1996]. In this regard we note that the results of the method II analysis (solid curves in Figures 2 and 3) may be quite sensitive to the presence of pickup ion-driven turbulence which might act to supply energy, increasing Z while decreasing λ, and thus pushing up the estimate of $\hat{t}$. This is a possible explanation of the more rapid increase of the method III results relative to both methods I and II.

In the future it will also be interesting to compare the rate of turbulent aging examined here to alternative measures based upon evolution of the low frequency spectral breakpoint [*Klein et al.*, 1992; *Horbury et al.*, 1996], and especially the possible variation of the rate of evolution with helio-latitude, e.g., a theoretically motivated comparison of Voyager and Ulysses results.

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